SREENIVASA INSTITUTE OF TECHNOLOGY AND MANAGEMENT STUDIES, CHITTOOR.
(AUTONOMOUS)

Department of Mechanical Engineering
(NBA & NAAC Accredited)

II B.Tech II Semester **Regulation-R18**

THEORY Of MACHINES-I

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Unit-1

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Basics of Mechanism

* **Theory of Machines:-**

It is a branch of Science which deals with the Study of the relative motion of Various parts of a machine and also forces which acts on them.

It is classified into two types, they are

1) Kinematics of Motion
2) Dynamics of Motion

1) **Kinematics of Motion:-**

It is a study of relative motion b/w the Various parts of the machine. Here the Various forces involved in the motion are not Considered.

Kinematics is the Study to known the displacement, Velocity & Acceleration of a part of a machine.

2) **Dynamics of Motion:-**

It is a Study of relative motion b/w the Various parts of the machine. It involves with Considering forces. There are two types,

a) Kinetics
b) Static

a) Kinetics:-

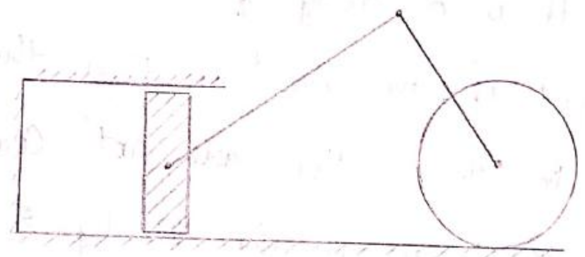
Various forces are Considered on a machine when a body is under motion

b) Statics:-

Various forces are Considered on a machine when a body is under rest (or) Stationary.

* Mechanism:-

It is a Combination of rigid and restraining links (or) bodies which are so shaped and Connected that they move upon each other with definite relative motion.



* Force:-

It is an External Agent which produces or tends to produce destroy or tends to destroy motion

* Resultant Force:-

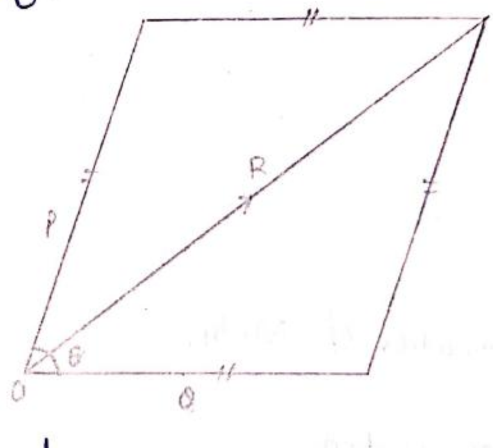
If a number of forces acting Simultaneously on a particle (or) Body then a single force which will produce a same effect as that of all the given forces.

* Composition of Forces:-

The process of getting resultant forces by Component forces [P, Q, R, etc.,] is Called as Composition of forces.

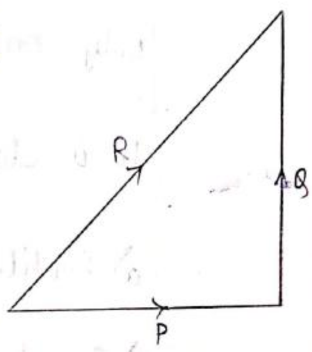
* Parallelogram law of forces:-

It states that if two forces acting Simultaneously on a particle be represented in magnitude and direction by the two adjacent Sides of a parallelogram their resultant may be represented in magnitude and direction by the diagonal of the parallelogram passing through the point 'O'.



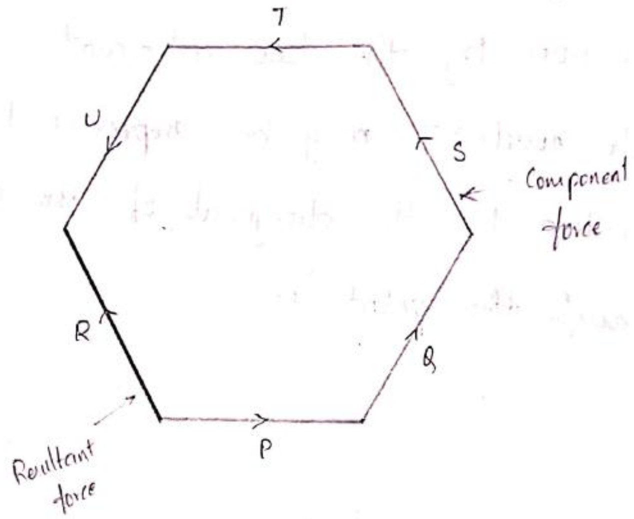
* Triangular law of forces:-

It states that if two forces acting Simultaneously on a particle be represented in magnitude and direction by the two Sides of a triangle their resultant may be represented in magnitude and direction by the third Side of a triangle taken in Opposite Order.



* **polygon law of forces:-**

It States that the number of forces acting Simultaneously on a particle be represented in magnitude and direction by the Side of polygon then the resultant may be represented in magnitude and direction by the closing Side of Polygon taken in Opposite Order.



* **Kinematics of Motion:-**

1) **Plane motion:-**

When the motion of a body is Confined to only one plane then it is known as plane motion.

It is classified into two types, they are

a) Rectilinear Motion

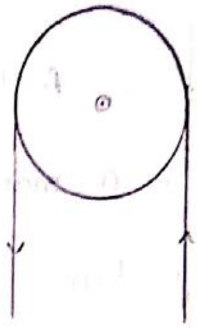
b) Curvilinear Motion

a) **Rectilinear Motion:-**

It is a Straight line path. It is also known as Translatory motion

b) Curvilinear Motion:-

Moving along a curved path



Pulley

* Kinematic Link:-

Each part of a machine which moves relative to the Some other part is known as kinematic link

Ex:- Reciprocating Steam Engine

* Machine:-

It is a device which receives energy and transforms it into Some useful work.

Machine Consists of number of parts.

* Types of Links:-

There are three types of links, They are

1) Rigid Link

2) Flexible Link

3) Fluid Link

1) Rigid Link:-

A Rigid link is one which does not undergo any formation while transmitting motion.

Ex:- Connecting Rod, Crank of a Reciprocating steam Engine etc.,

2) Flexible Link:-

A Flexible Link is one which is partly deformed in a manner not to affect the transmission of motion

Ex:- Belts System, chain System, etc.,

3) Fluid Link:-

A Fluid Link is one which is formed by having a fluid in a receptacle and the motion is transmitted through the fluid by pressure or Compression only

Ex:- Hydraulic presses, Hydraulic brakes, Jacks, etc.,

* Kinematic pair:-

★★★

"Pair" is Any two links (or) elements of a machine when it Contact with each other is known as Pair.

A Joint of two links having relative motion between them is known as kinematic pair.

(or)

If the Relative motion b/w two links Completely (or) Succesfully Constrained motion [In a definite direction] that pair is known as kinematic pair.

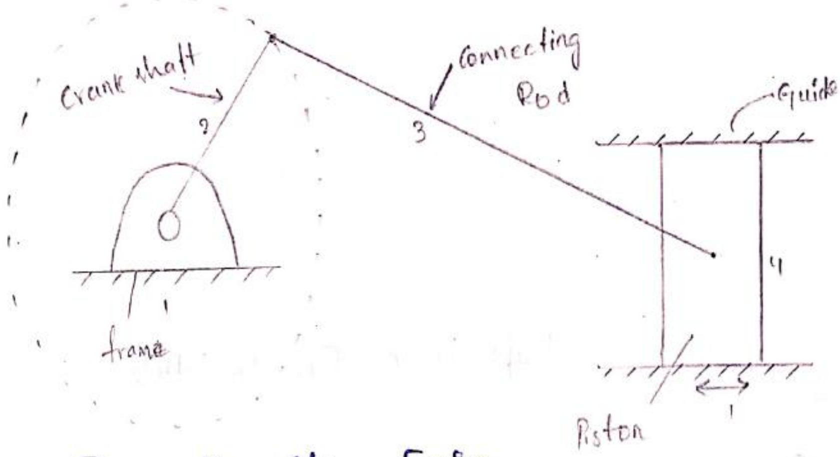
* Constrained Motions:-

Imp- There are classified into three types, they are

1) Complete Constrained motion [CCM]
2) Incomplete Constrained motion [ICM]
3) Succesfully Constrained motion [SCM]

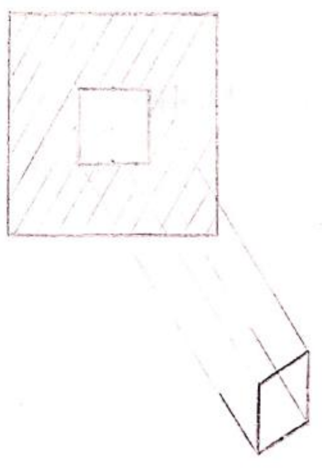
1) Complete Constrained Motion:-

1)



Reciprocating Steam Engine

2)



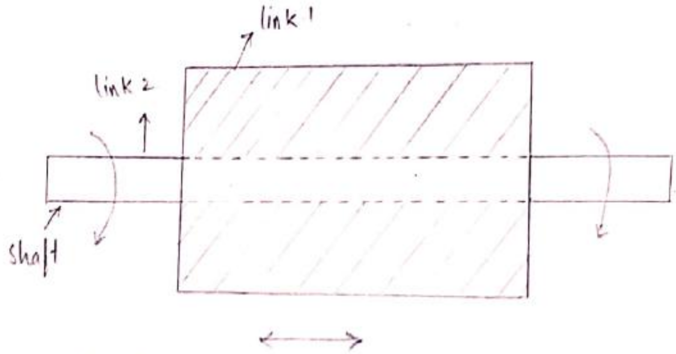
Square Bar in a Square Hole

When the Motion between a pair is limited to a definite direction irrespective of the direction of force applied. Then it is known as Completely Constrained motion

Ex:- 1) The piston & Cylinder in a Steam Engine
2) Motion of a Square bar in a Square plate (or) Hole

2) Incomplete Constrained Motion:-

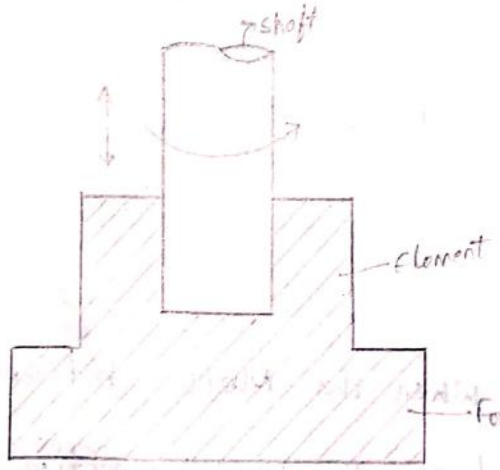
When the Motion between a pair Can takes Place more than one direction then it is known as Incomplete Constrained Motion.



Shaft in a Circular Hole

Ex:-
1) A Circular Shaft in a Circular Hole

3) Succesfully Constrained Motion:-



Shaft on a foot step Bearing

When the motion between the elements forming the pair is such that the constrained motion is not completed by itself but by some other means then the motion is said to be successfully constrained motion.

Ex:- Shaft on a foot step bearing, The shaft may rotate in a bearing or it may move upwards in that case that type of motion is Incomplete constrained motion. But if the load is placed on the shaft to prevent axial upward moment. Then the motion of the pair is said to be successfully constrained motion

* Classification of kinematics pair:-

★★★

1) According to the relative motion b/w two links
   - → Sliding pair
   - → Turning pair
   - → Rolling pair
   - → Screw pair
   - → Spherical pair

2) According to the type of contact
   - → Higher pair
   - → Lower pair

3) According to type of closure
   - → Self closed pair
   - → Forced closed (or) Open pair

* Sliding pair:- [Completely Constrained Motion]

When the two elements of a pair are Connected in such a way that one can only slide relative to the other.

Ex:- 1) The piston and Cylinder

2) Tail Stock on a lathe bed

3) A Square bar in a Square hole

Sliding pair has a Completely Constrained motion

* Turning pair:-

When the two elements of a pair are Connected in such a way that one can only turn or revolve about a fixed axis of another link.

Ex:- 1) A circular shaft with Collars at both ends fitted in a Circular hole

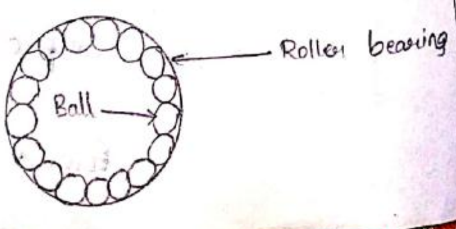
2) Spindle with lathe machine

Turning pair is also known as Completely Constrained motion

* Rolling pair:-

When the two elements of a pair are Connected in such a way that one rolls over the another fixed link

Ex:- Ball and Roller Bearings



* Screw pair:-

When the two elements of a pair are Connected in such a way that one element Can turn about the Other by Screw threads

Ex:- 1) Bolt & Nut

2) Water bottle & Cap

* Spherical pair:-

When the two elements of a pair are Connected in such a way that one element [Spherical shape] turns about the other fixed element

Ex:- 1) Attachment of a Car mirror

2) Pen stand

* Lower pair:-

When the two elements of a pair have a Surface Contact when relative motion takes place and the Surface of one element slides over the surface of the Other.

Ex:- A Square bar in a Square hole

Sliding pair is known as lower pair & Screw pair, Turning pair is also known as lower pair

* Higher pair:-

When the two elements of a pair have a line (or) Point Contact when the relative motion takes place and the motion between the two elements is partly turning and partly sliding.

Ex:- 1) Ball and roller bearings

2) Cam & followers

3) Toothed gearings

* Self closed pair:-

When the two elements of a pair are Connected together mechanically in such a way that only a required kind of relative motion occurs.

Ex:- 1) Lower pairs

* Open pair:-

When the two elements of a pair are not Connected mechanically but are kept in Contact by the action of External force.

Ex:- Cam & follower

* Types of joints:-

They are classified into 3 types, they are

1) Binary joint → 

2) Ternary joint → 

3) Quaternary joint → 

According to A.W. Klein,

$$J + \frac{h}{2} = \frac{3}{2}l - 2$$

where,

J = No. of joints

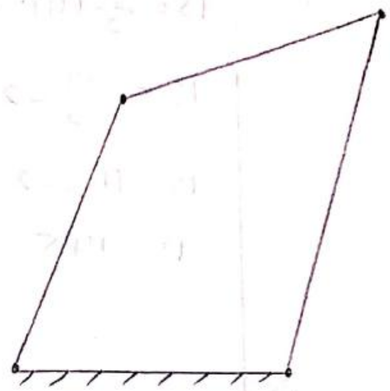
h = No. of higher pairs

l = No. of links

1)

links = 4

Joints = 4



$$J + \frac{h}{2} = \frac{3}{2}l - 2$$

$$4 + \frac{0}{2} = \frac{3}{2}(4) - 2$$

$$4 = 6 - 2$$

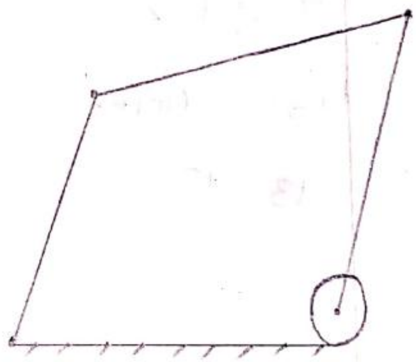
$$4 = 4$$

2)

links = 4

Joints = 3

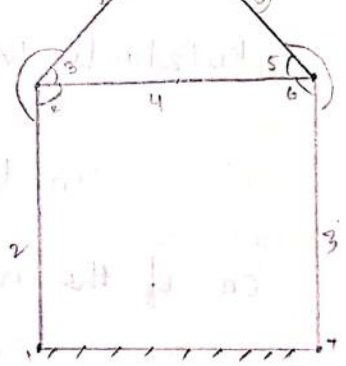
Higher pair = 1



$$J + \frac{h}{2} = \frac{3}{2}l - 2$$

$$3 + \frac{1}{2} = \frac{3}{2}(4) - 2$$

$$3.5 = 4$$



3) $l = 6$, $J = 7$,

$$J = \frac{3}{2}l - 2$$

$$7 = \frac{3}{2} \times 6 - 2$$

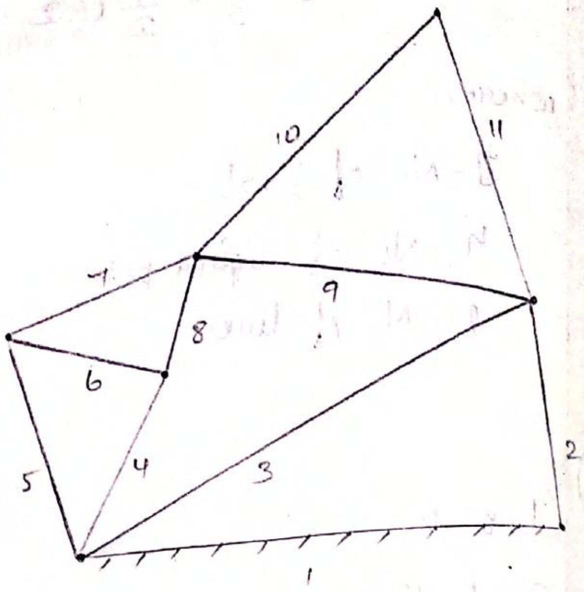
$$7 = 7$$

4) 2 Binary → 2

2 Terenary → 4

3 Quatenary → 9

$\overline{15}$



J = 15

L = 11

$J = \frac{3}{2}l - 2$

$15 = \frac{3}{2}(11) - 2$

$15 = \frac{33}{2} - 2$

$15 = 16.5 - 2$

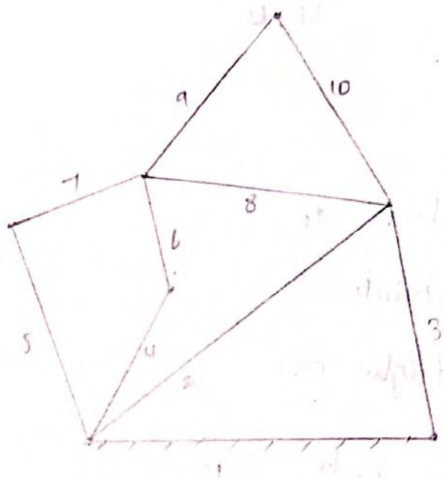
$15 = 14.5$

5)

4 binary → 4

3 quatenary → 9

$\overline{13}$



$J = \frac{3}{2}l - 2$

$13 = \frac{3}{2}(10) - 2$

$13 = 13$

* Degrees of freedom for a plane mechanism (or) Kutzbach Mechanism :-

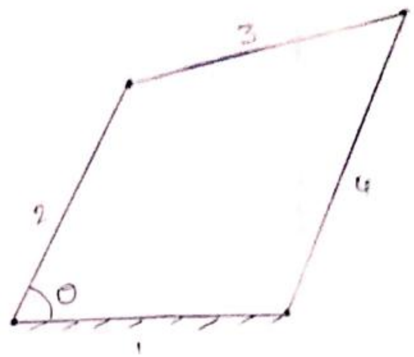
In the design (or) Analysis of a mechanism one of the most Concern is the no. of degrees of freedom

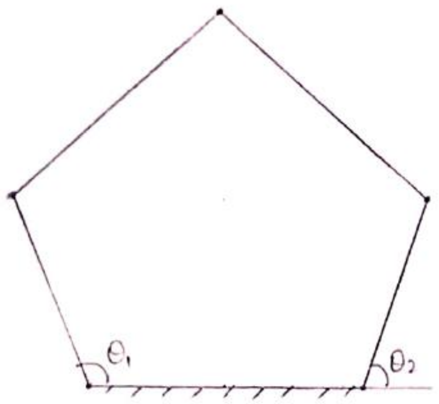
Degrees of freedom:-

The No. of input parameters [pair Variables] which must be independently Control in Order to bring the mechanism into useful Engineer purpose is called degrees of freedom

Ex:- 1) Four Bar Mechanism

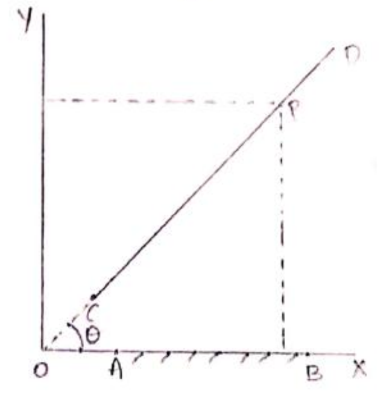
2) Five Bar Mechanism

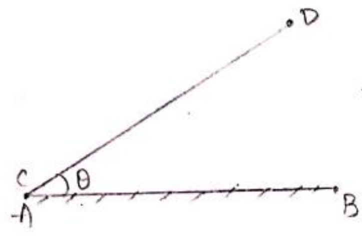




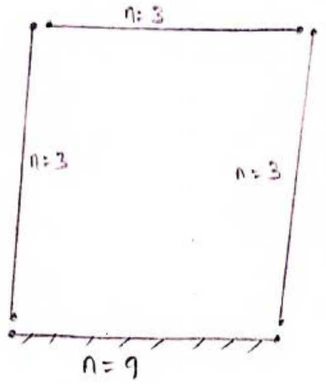
Proof:-

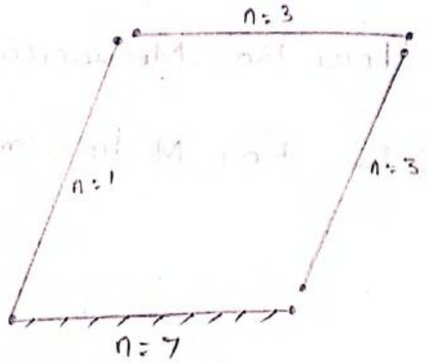
The link AB with Co-ordinate System Oxy as the reference link [Fixed link]. The position of point P on a movable link CD can be Completely Specified by 3 Variable i.e., the Co-ordinates of point 'P' is denoted by 'x' and 'y' and the inclination 'θ' with the x-axis

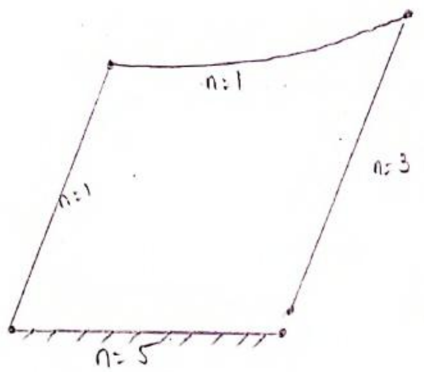


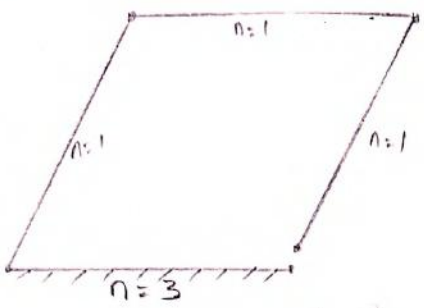


Each link of a mechanism has three degrees of freedom before it is Connected to any other link. But when the link is Connected to Other link by a turning pair by a Single Variable 'θ'.







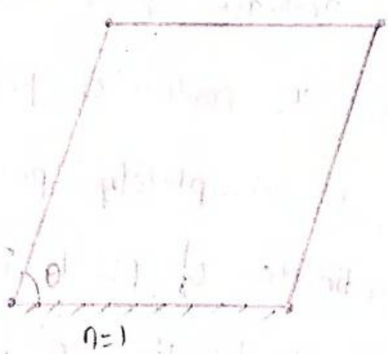


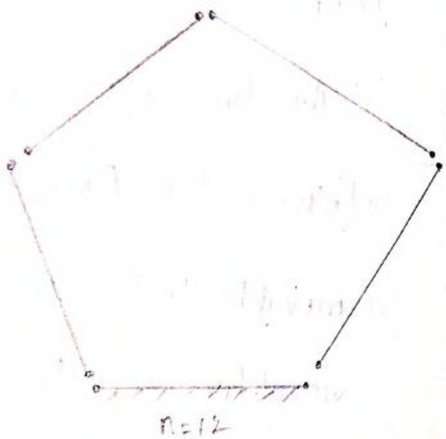
$n = 3(l-1) - 2j - h$

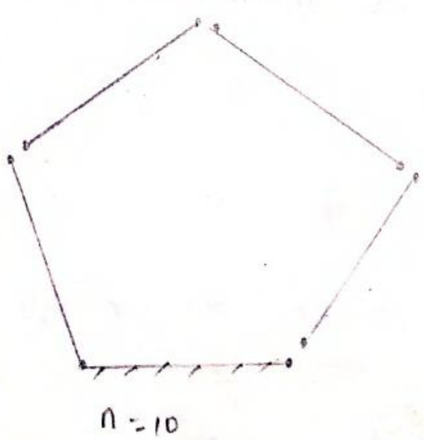
$= 3(4-1) - 2(4) - 0$

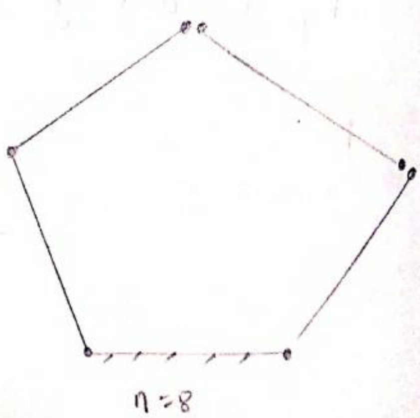
$= 9 - 8$

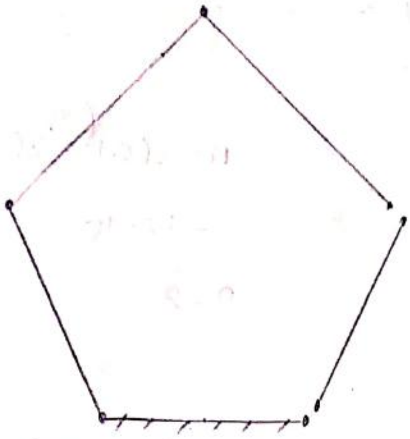
$n = 1$



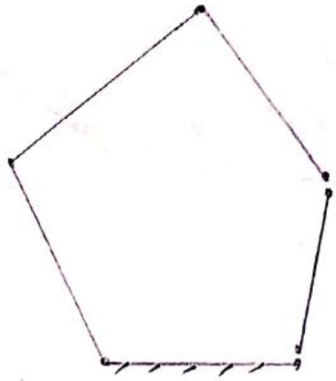




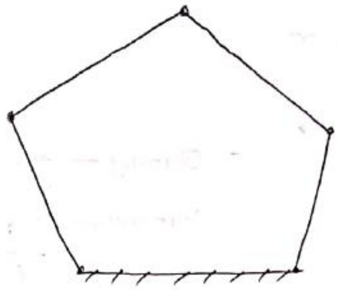




$n:6$



$n=4$



$n:2$

$$n = 3(l-1) - 2j - h$$
$$= 3(5-1) - 2(5)$$
$$= 12 - 10$$
$$n = 2$$

* Applications of Kutzbach Criterion:-

With no higher pair

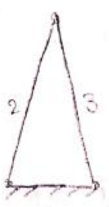
With higher pair

$$n = 3(l-1) - 2j - h$$

h → Higher pair

With No Higher Pair:-

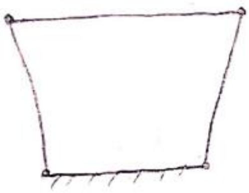
3 Bar Mechanism:-



$$n = 3(3-1) - 2(3) - 0$$
$$= 6 - 6$$
$$n = 0$$

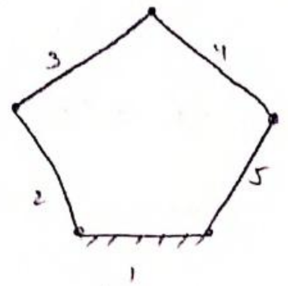
This mechanism is locked chain (or) Structure

4 Bar Mechanism:-



$$n = 3(4-1) - 2(4) - 0$$
$$= 9 - 8$$
$$n = 1$$

5 Bar Mechanism:-

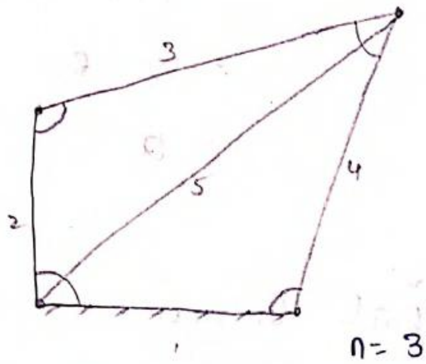


$n = 3(5-1) - 2(5) - 0$

$= 12 - 10$

$n = 2$

Two Seperate input motions to produce Constrained motion for the mechanism



2 Binary $\longrightarrow$ 2

2 Terenary $\longrightarrow$ 4

$J = \underline{6}$

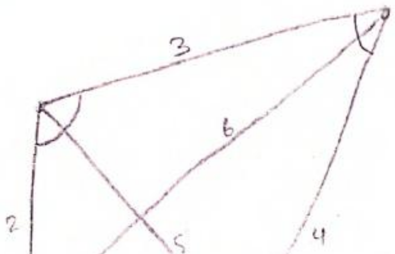
$L = 5$

$n = 3(l-1) - 2j - h$

$= 3(5-1) - 2(6) - 0$

$= 12 - 12$

$n = 0$ (Structure)



4 Terenary $\longrightarrow$ $\underline{8}$

$J = 8$

$L = 6$

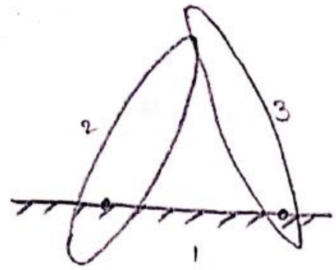
$n = 3(6-1) - 2(8) - 0$

$= 15 - 16$

$n = -1$ (Indeterminate Structure)

It is Reduntant Constrain (or) Indeterminate Structure

Now Considering a Higher pair Mechanism,



$l = 3$

$J = 2$

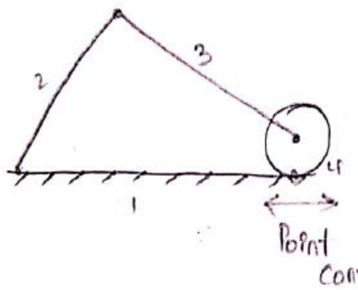
$H = 1$

$n = 3(l-1) - 2j - h$

$= 3(3-1) - 2(2) - 1$

$= 6 - 4 - 1$

$n = 1$



$L = 4$

$J = 3$

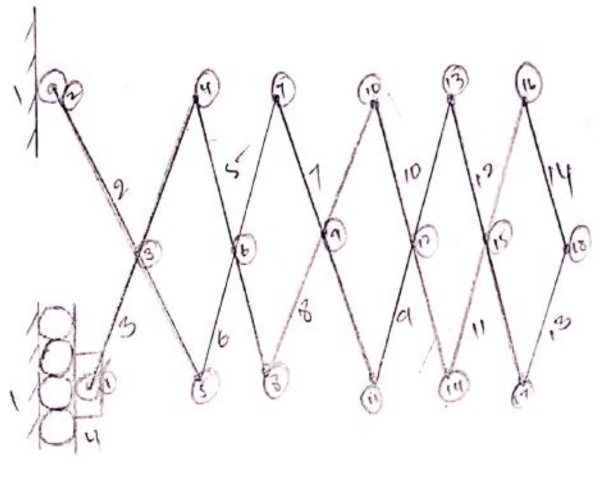
$H = 1$

$n = 3(4-1) - 2(3) - 1$

$= 9 - 6 - 1$

$= 9 - 7$

$n = 2$



$L = 14$

$J = 18$

$H = 1$

$n = 3(14-1) - 2(18) - 1$

$= 3(13) - 2(18) - 1$

$= 39 - 36 - 1$

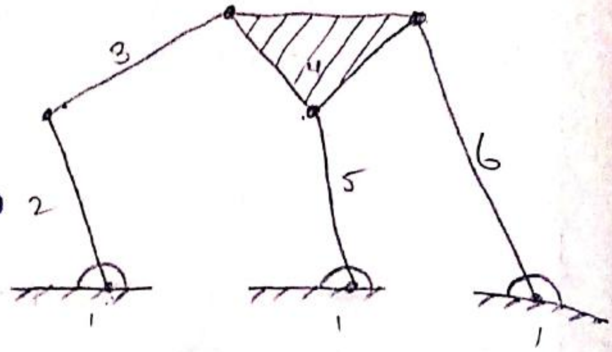
$= 39 - 37$

$n = 2$

$l=6$

$J=7$

$H=0$



$n=3(6-1)-2(7)-0$

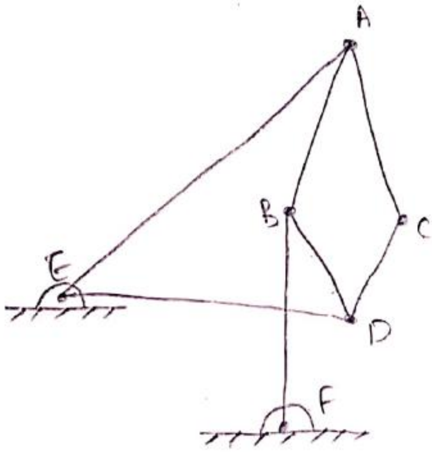
$=15-14$

$n=1$

$l=8$

$J=10$

$H=0$

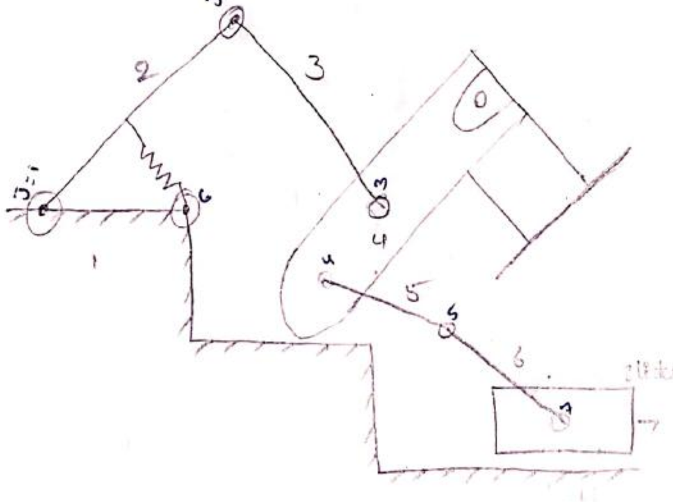


$n=3(l-1)-2j-h$

$=3(8-1)-2(10)-0$

$=21-20$

$n=1$



$l=7$

$J=$

$H=1$

$n=3(l-1)-2j-h$

$=3(7-1)-2(7))-1$

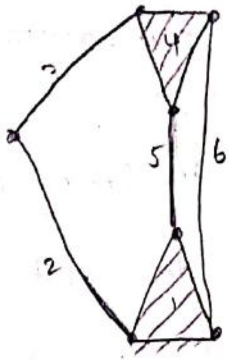
$n=18-15$

$n=3$

$L=6$

$J=7$

$H=0$



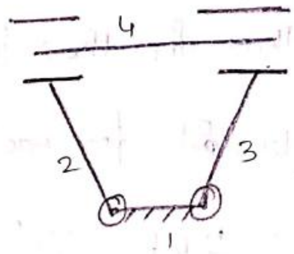
$n=3(l-1)-2j-h$

$=3(6-1)-2(7)-0$

$=15-14$

$n=1$

In this mechanism, there are 4 links all are lower pairs only. But, link '4' is Capable of



sliding without the help of remaining links. Therefore, This mechanism is a locked System. and also link '4' has redundant degree of freedom

* Grubler's Criterion:-

Grubler's Criterion applys to the mechanism with only Single degree of freedom joints. Therefore, the overall movability of mechanism is unity. Therefore, Substitute in kutzbach Criterion

$n=1,\ h=0$

$n=3(l-1)-2j-h$

$1=3(l-1)-2j-0$

$3l-3-2j-1=0$

$$\boxed{3l-2j-4=0}$$

* Space mechanism:-

For space mechanism each link has 6 Degree of freedom and in a mechanism one link is fixed. Therefore, No. of movable links are '$l-1$'. Then '$l-1$' movable link will have '$6(l-1)$' Degree of freedom. But, Some of the joints will have only one D.O.F. Hence, for these joints 5 degree of freedom [D.O.F] will be lost for each joint. Similarly, some joints will have 2 D.O.F which means 4 D.O.F will be lost for each joint

$$n = 6(l-1) - 5\times g - 4C - 3S$$

where,

g = Total No. of sliding pairs [$n=1$]

C = Total No. of cylindrical pairs [$n=2$]

S = Total No. of spherical pairs [$n=3$]

* Inversion of Mechanism:-

In any mechanism, there is a one fixed link in 'n' number of mechanisms there is 'n' number of fixed links then invertion of mechanism is nothing but in this mechanism for finding different links fixed in a kinematic change is known as Inversion of mechanism.

It may be noted that the relative motion b/w the links is not changed in any manner through the process of inversion but their absolute motion [measured w.r.to fixed link]. May be changed drastically.

Note:-

The part of the mechanism which initially moves w.r. to frame (or) fixed link is called as driver. and that part of the mechanism to which motion is transmitted. is called as follower. Most of the mechanism are reversible. So that the same link can play the role of driver as well as follower.

Ex:-1) In a reciprocating steam Engine → Piston – driver
Crankshaft - Follower

2) Reciprocating Air Compressor → Crankshaft - driver
Piston - Follower

→ classification [Inversion of M/m]:-

1) Four Bar Chain (or) quadric Cycle chain
- Beam Engine
- Coupling Rod
- Watt's Indicator m/m

2) Single slider Crank chain
- Bull Engine
- Oscillating Engine
- Crank & Slotted lever quick return motion m/m
- Whitworth quick return motion m/m
- Rotary I.C. Engine (or) Gnome Engine

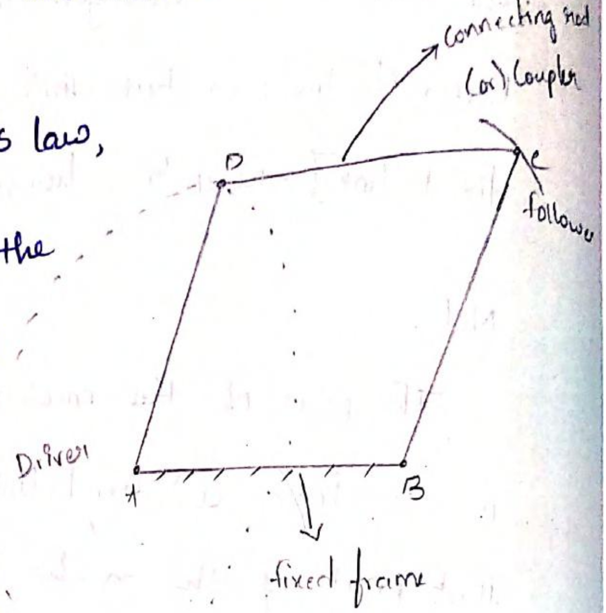
3) Double Slider Crank chain
- Oldham's Coupling
- Scotch yoke m/m
- Elliptical Trammels

* four Bar chain Mechanism :-

In a four bar chain mechanism, it is the simplest mechanism. It consists of only 4 links all are turning pairs. One of the link is fixed and the remaining links are movable

According to Grashof's law,

In any mechanism, the Sum of Smallest and longest link is not greater than the sum of the remaining links

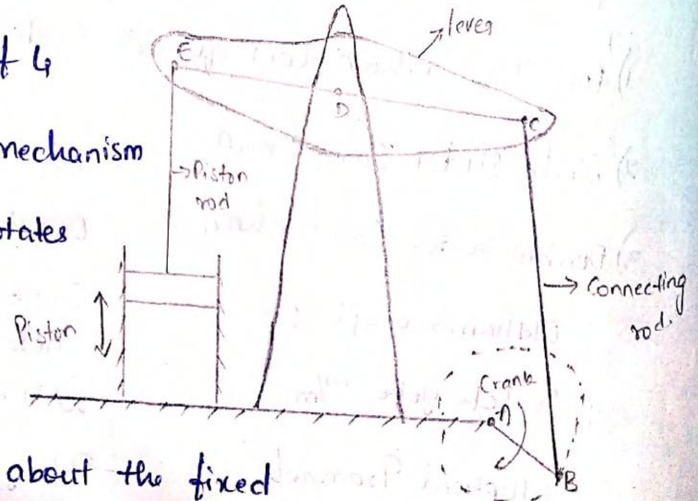


→ Inversion of four bar chain M/m :-

1) Beam Engine [Crank & lever M/m]
2) Coupling rod of locomotive [Double Crank M/m]
3) watt's M/m [straightened line motion M/m] (or) [Double lever M/m]

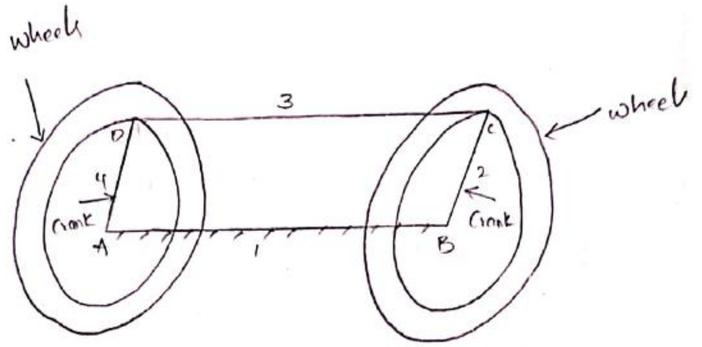
1) Beam Engine :-

It consists of 4 links. In this mechanism when the crank rotates about the fixed Centre 'A'. The lever oscillates about the fixed



Centre at 'D'. The end 'E' of the lever CDE is Connected to a piston rod which reciprocates due to the rotation of the Crank. This mechanism is to Convert rotary motion to reciprocating motion.

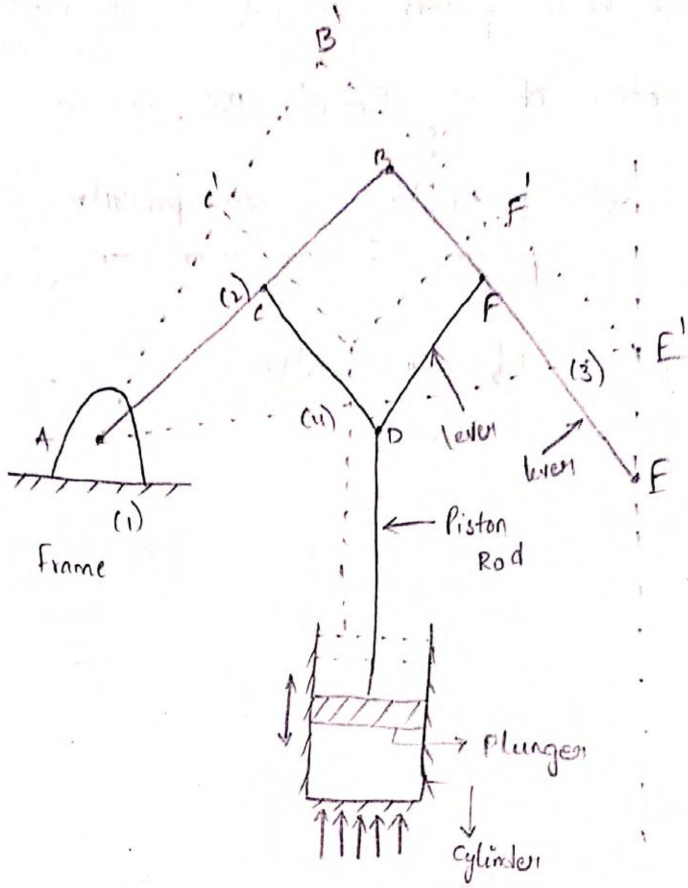
2) Coupling Rod of Locomotive:-



The mechanism of a Coupling rod of locomotive which Consists of 4 links. In this mechanism AD and BC are having equal lengths act as Cranks and are Connected to the respective wheels. The link CD acts as a Coupling rod. and the link AB is fixed in Order to maintain a Constant Centre to Centre distance between them. This mechanism is transmitting rotary motion from wheel to the other wheel.

(P.T.O)

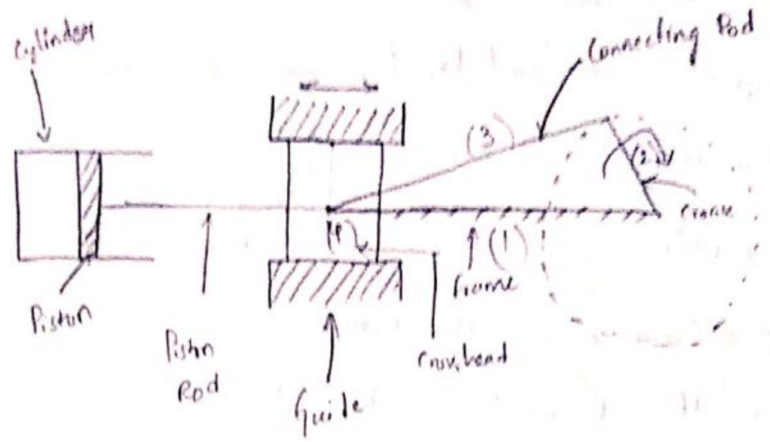
* Watt's indicator m/m :-



1) it Consists of 4 links, fixed link at 'A', link AB, BE, COF 2) It may be noted that CD and DF form one link because these two links have no relative motion between them. 3) The links BE and COF act as levers

4) The displacement of the link CDF is directly proportional to the pressure of gas or steam. which acts on the indicator plunger. on any small displacement of the m/m the tracing point 'E' at the end of the link 'BE' traces out approximately a straight line

* Single Slider Crank chain mechanism :-



1) It is modification of the basic 4 bar chain.

2) It Consists of 1 sliding (4-1) pair b/w Crosshead & guide and having 3 turning pair (1-2, 2-3, 3-4, ~~4-1~~)

3) It is mostly used in reciprocating steam engine

4) This mechanism Converts from rotary motion to reciprocating motion

* Inversion of Single Slider Crank chain m/m :-

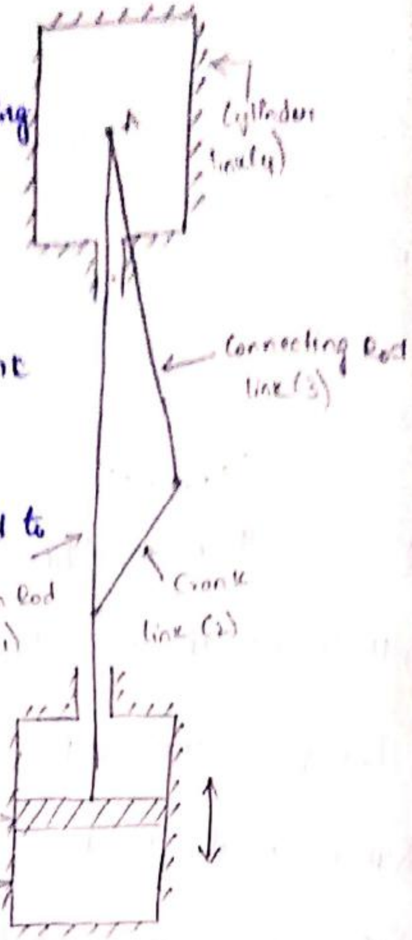
1) Bull Engine
2) Oscillating Engine
3) Crank & Slotted lever quick return motion m/m
4) whitworth quick return motion m/m
5) Rotary I.C Engine (or) Gnome Engine

In a 4 link mechanism.

(P.T.O)

1) Bull Engine:-

1) In this mechanism, the inversion is obtained by fixing the cylinder (or) link 4 i.e., Sliding pair.

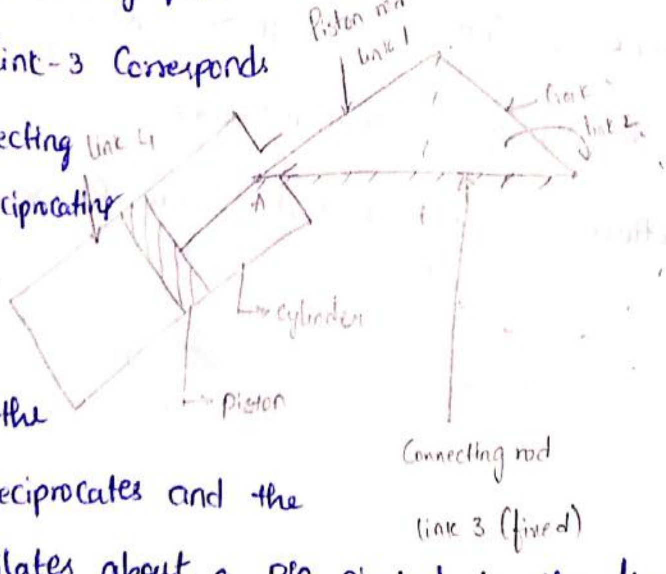


2) In this Case, when the Crank rotates the Connecting rod Oscillates about a pin pivoted to a fixed link at 'A'

3) The piston attached to the Piston rod reciprocates.

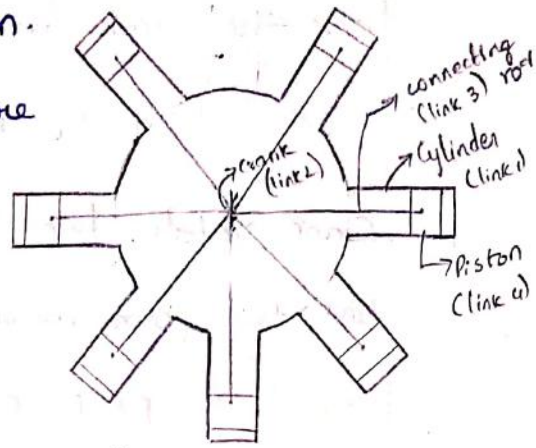
4) The duplex pump which is used to supply a feed water to boilers have two pistons attached to link-1.

2) Oscillating Engine:- It is used to Convert reciprocating motion to rotary motion. In this mechanism link 3 forming the turning pair fixed. The link-3 Corresponds to the Connecting rod of a reciprocating Steam Engine. When the Crank rotates the Piston rod reciprocates and the Cylinder oscilates about a pin pivoted to the fixed link at Point 'A'.



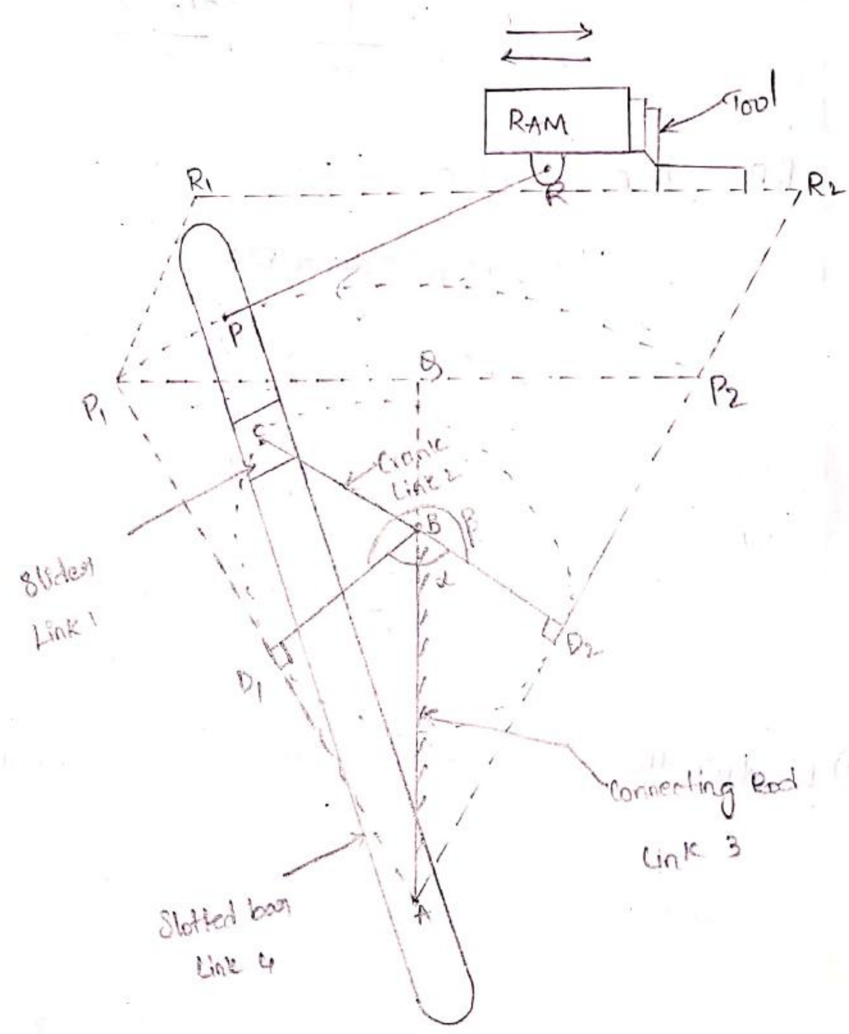
3) 7 Cylinders [Gnome Engine]

It is used in Navigation. Now a days, gas turbine are used in that place. It Consists of 7 Cylinders in one place and all revolves about the fixed Centre 'O'.



The Crank Link-2 is fixed in this mechanism. when the Connecting rod links rotates the piston link-4 reciprocates inside the Cylinder.

4) Crank and slotted lever Quick return motion Mechanism :-

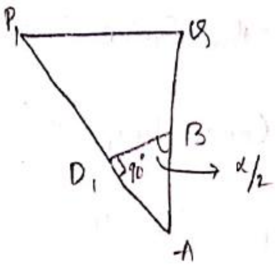


Crank rotates from BD_1 to BD_2 in clockwise direction and the ram moves

R_1 to $R_2 \Rightarrow$ Cutting Stroke

Crank rotates from BD_2 to BD_1 in Anti clockwise direction and the ram moves

R_2 to $R_1 \Rightarrow$ Return Stroke



$AQP_1 \parallel ABD_1$

$$\frac{\text{Time of Cutting stroke}}{\text{Time of return Stroke}} = \frac{\beta}{360 - \beta}$$

$$= \frac{\beta}{\alpha} = \frac{360 - \alpha}{\alpha}$$

$$P_1P_2 = R_1R_2 = 2P_1Q$$

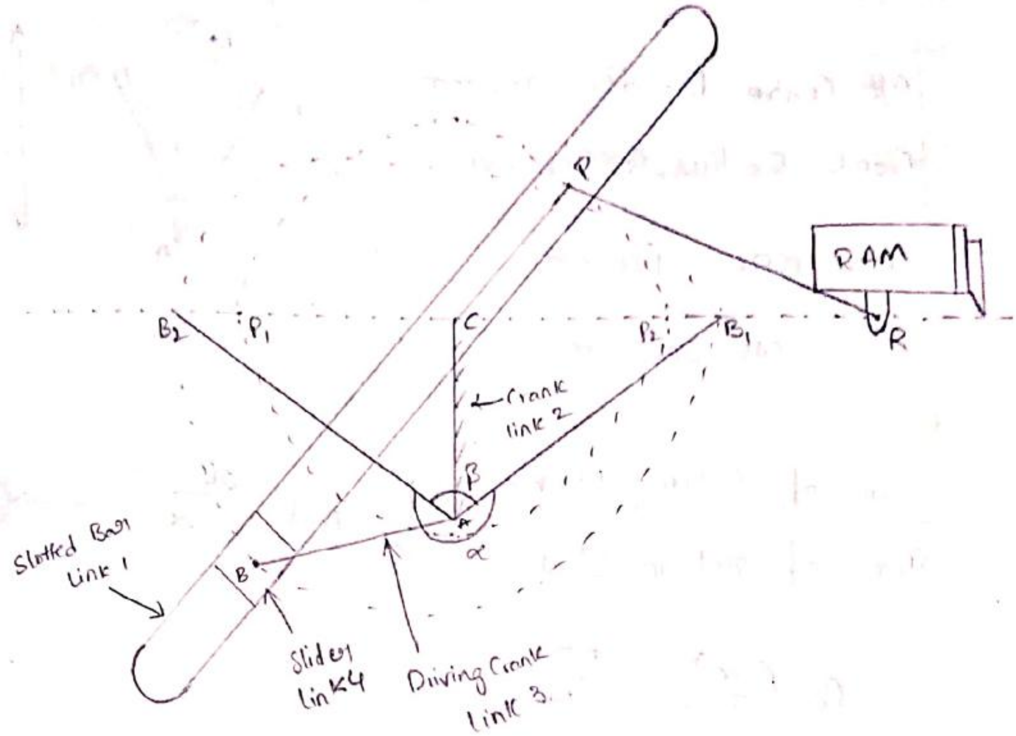
$$= 2 \times \sin \angle P_1AQ \times P_1A$$

$$= 2 \times \sin\left(90^\circ - \frac{\alpha}{2}\right) \times P_1A$$

$$= 2 \times \cos\frac{\alpha}{2} \times P_1A$$

$$= 2 \times \frac{BD_1}{AB} \times P_1A$$

5) Whitworth quick return motion mechanism:-



$$\frac{\text{Time of Cutting Stroke}}{\text{Time of return Stroke}} = \frac{\alpha}{\beta} = \frac{\alpha}{360-\alpha}$$

$$= \frac{360-\beta}{\beta}$$

AB_1 to AB_2 (α)

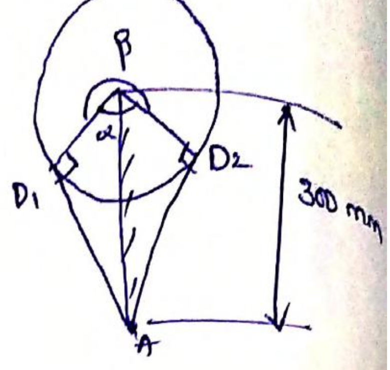
AB_2 to AB_1 (β)

Problems:-

1) A Crank & Slotted lever mechanism used in a shaper has a Centre distance of 300 mm between the Centre of Oscillation of the sloted lever and the Centre of rotation of the Crank. The radius of the Crank is

120 mm. Find the ratio of the time of Cutting to time of return stroke.

A. Given,



AB Centre distance: 300 mm

Crank Radius, R = 120 mm

$BD_1 = BD_2 = 120$ mm

AB = 300 mm

$$\frac{\text{Time of Cutting stroke}}{\text{Time of return stroke}} = \frac{\beta}{\alpha} \text{ (or) } \frac{360-\alpha}{\alpha} \longrightarrow ①$$

$$\cos\left(\frac{\alpha}{2}\right) = \frac{BD_1}{AB}$$

$$= \frac{120}{300}$$

$$\frac{\alpha}{2} = 66.421$$

$$\alpha = 132.84$$

From ①,

$$= \frac{360 - 132.84}{132.84}$$

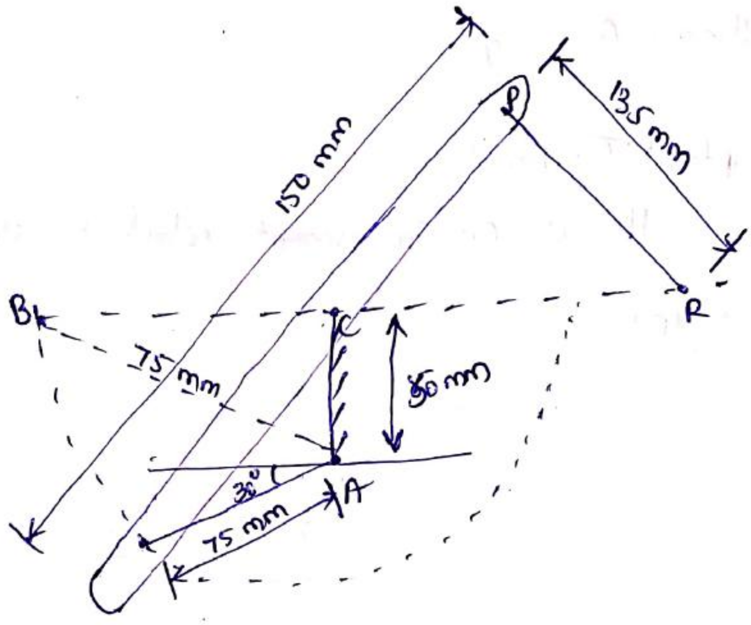
$$= 1.71$$

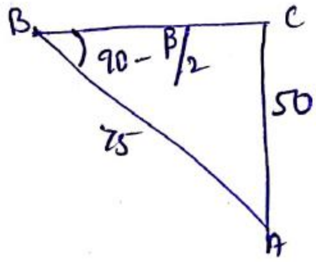
2) In a whitworth quick return motion mechanism The distance b/w the fixed Centres is 50 mm and the length of the driving Crank is 75 mm the length of the slotted lever is 150 mm and the

length of the Connecting rod is 135 mm. find the ratio of the time of cutting to the time of return stroke and also the length of the effective stroke.

Angle = 30°

A.





$$\sin\left[90 - \frac{\beta}{2}\right] = \frac{AC}{AB_1}$$

$$= \frac{50}{75}$$

$$\beta = 96.37$$

$$\alpha = 263.63$$

* Double Slider Crank chain Mechanism :-

It Consists of 2 turning pairs and 2 Sliding pairs

→ Types :-

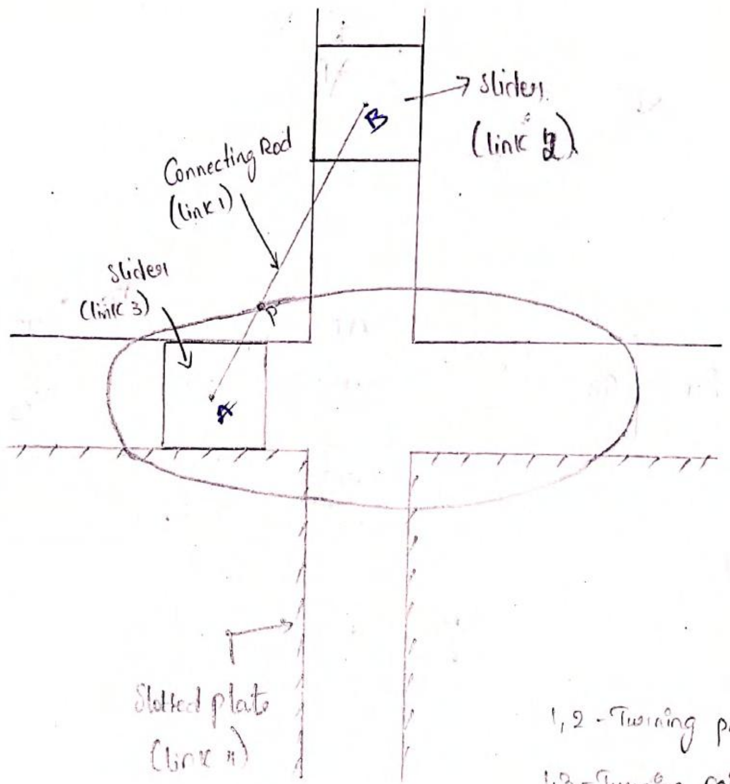
1) Elliptical Trammel

2) Scotch yoke m/m

3) Oldham's Coupling

1) Elliptical Trammel :-

It is an instrument which is used for drawing an ellipse



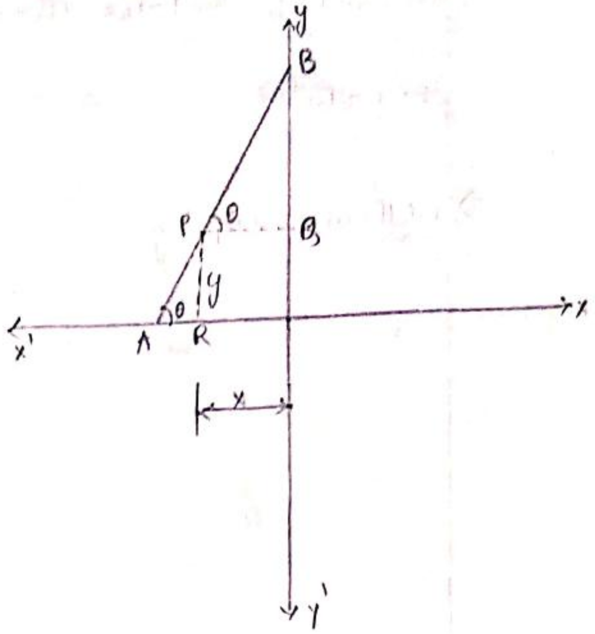
1, 2 - Turning pair

1, 3 - Turning pair

2, 4 - Sliding pair

3, 4 - Sliding pair

$\cos\theta = \frac{PQ}{BP}$



$\Rightarrow PQ = BP\cos\theta = (x)$

$\sin\theta = \frac{PR}{PA}$

$\Rightarrow PR = PA\sin\theta = (y)$

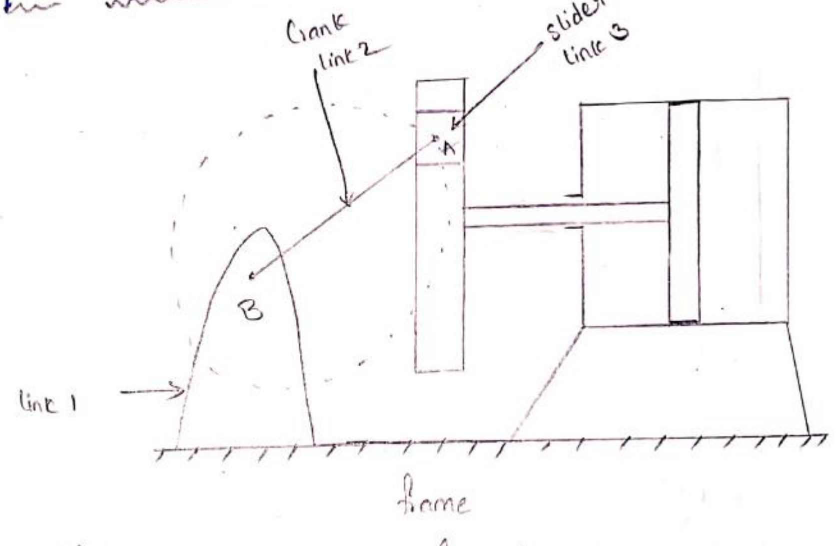
$\frac{x}{BP} = \cos\theta, \quad \frac{y}{PA} = \sin\theta$

Squaring and Adding,

$$\frac{x^2}{(BP)^2} + \frac{y^2}{(PA)^2} = \cos^2\theta + \sin^2\theta$$

$$\frac{x^2}{(BP)^2} + \frac{y^2}{(PA)^2} = 1 \implies \text{Ellipse}$$

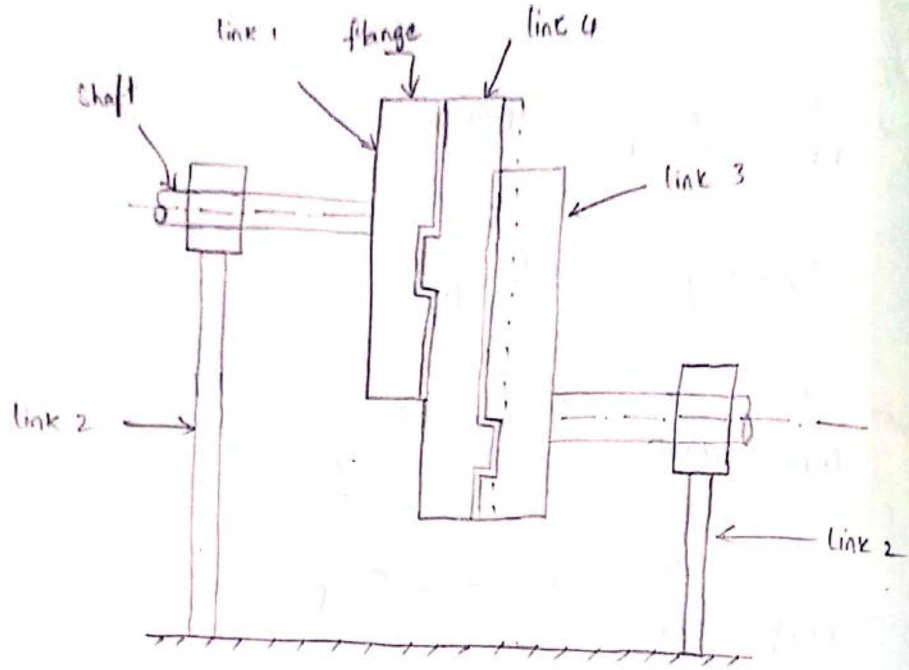
2) Scotch yoke mechanism:-

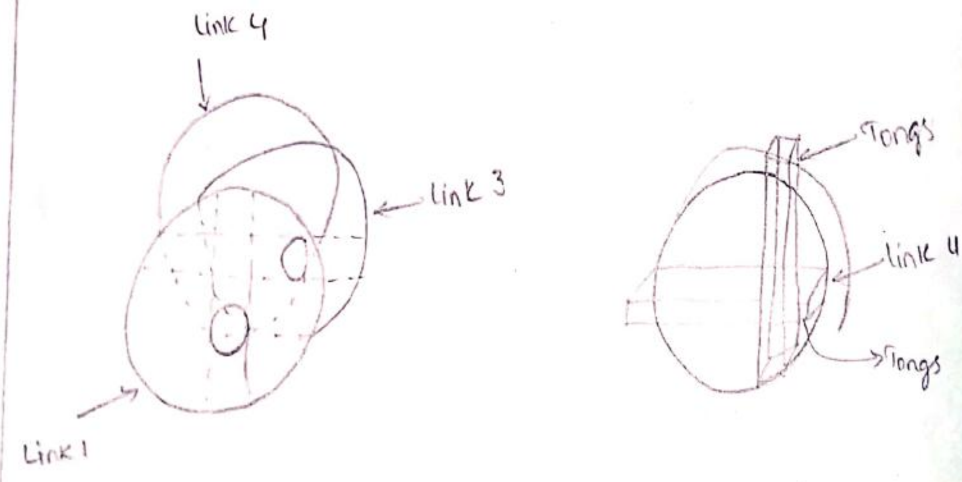


1,2 - Turning pair
2,3 - Turning pair
3,4 - Sliding pair
4,1 - Sliding pair

It is used for converting rotary motion to reciprocating motion. This inversion is obtained by fixing either the link 1 (or) link 3. Here, link 1 is fixed when the link 2 rotates about 'B' as a centre the link 4 reciprocates

3) Oldham's Coupling:-





It is used for connecting two parallel shafts whose axis are at a small distance apart. The shafts are coupled in such a way that if one shaft rotates the other shafts also rotates at the same speed. Therefore as shown in figure the shafts should be connected have two flanges rigidly fastened [joined] at their ends by forging operation. The link '1' and '3' forms turning pair with link '2'. These flanges have diametrical slots on their inner face. The intermediate piece link '4' which is a circular disc have two tongs on each face at right angles to each other. The tongs of link '4' closely fit into the slots in the two flanges. Therefore, link '4' can slide and reciprocates in b/w link '1' & '3'.

* Straight line motion mechanism:-

1) Exact straight line motion m/m < Peaucellier m/m, Harts m/m

2) Approximate straight line motion m/m

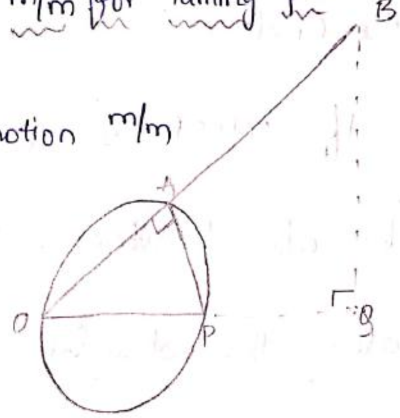
The straightened mechanism are of two types

1) Only turning pairs are used and

2) One sliding pair is used.

→ Exact straight line motion m/m for Turning pair:-

In Exact straight line motion m/m



$OA \times OB =$ Constant

Δ^{les} OAP & OBQ,

$$\frac{OA}{OP} = \frac{OQ}{OB}$$

$$\Rightarrow OA \times OB = OP \times OQ = \text{Const.} \times \text{Const.}$$

let, OB be a ~~chord~~ point on the Circumference of a Circle of a diameter OB.

let, OA be a any chord and 'B' is a Point on a parallel such that

$$OA \times OB = \text{Constant}$$

Then the locus of point 'B' will be a Straight line $\perp^r$ to the diameter OB.

This may be proved as follows,

1, Draw PQ $\perp^r$ to OB

2 Join AP.

Now $\triangle OAP$ & $\triangle OBQ$ are Similar

$$\therefore \frac{OA}{OP} = \frac{OQ}{OB}$$

$$OA \times OB = OP \times OQ$$

'OP' is Constant because it is a diameter of the Circle

$\therefore$ If $OA \times OB$ is Constant then OQ is (also) will be also Constant. Then the point 'P' meves along the straight path 'BQ' which is $\perp^r$ to OP

*** * Peaucellier Mechanism :-

It Consists of 8 links all are turning pairs only. It Consists of a fixed link O, O_1 and

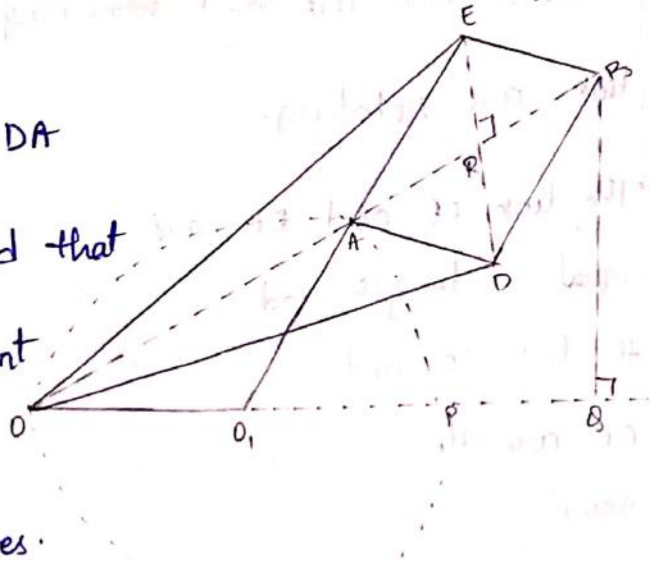
Other straight links O_1A, OE, OD, AD, DB, BE, EA.

The pin at 'A' is constrained to move along the circumference of a circle with the fixed diameter OP. By means of the link O_1A

$OO_1 = O_1A$

$OE = OD$

$AD = DB = BE = EA$



$AE = EB = BD = DA$

Now, we proved that

$OA \times OB =$ Constant

when the link O_1A rotates.

Join ED to bisect AB at 'R'. Now from right angled triangles ORE & BRE we have,

$$OE^2 = OR^2 + RE^2 \longrightarrow (1)$$

$$BE^2 = BR^2 + RE^2 \longrightarrow (2)$$

Substracting Eq. (2) from Eq. (1)

$$OE^2 - BE^2 = (OR^2 + RE^2) - (BR^2 + RE^2)$$

$$OE^2 - BE^2 = OR^2 - BR^2$$

$$= (OR + BR)(OR - BR)$$

$$= OB \times OA$$

$$\boxed{OA \times OB = \text{Constant}}$$

*** * Hart's Mechanism:-

It Consists of 6 Links and all are turning Pairs only

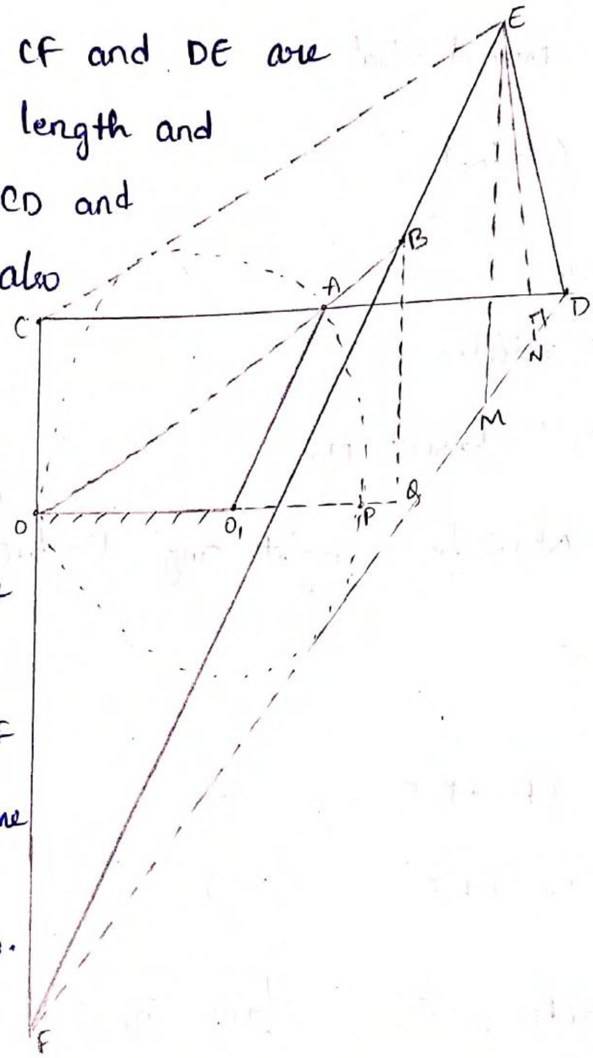
$OO_1 = O_1A$

$CF = ED$

$CD = EF$ are of Equal lengths

OO_1 are fixed link and remaining links are rotating.

The link CF and DE are equal in length and the link CD and EF are also equal.



The points 'O', 'A' & 'B' divides the links CF, CD and EF in the same ratio (or) equal ratio.

A little consideration will show that BOCE is a trapezium.

Similarly, OA and OB are parallel to CE and FD

Now, OAB is a straight line. It is proved that the Product $OA \times OB$ is Constant.

In triangle FCE, 'O' and 'B' divides 'FC' and 'EF' in the Same ratio. therefore,

$$\frac{CO}{CF} = \frac{EB}{EF}$$

Therefore, 'OB' is parallel to 'CE'

Similarly, In Δ^{le} FCD, 'O' and 'A' divides 'CF' and 'CD' in the same ratio then

$$\frac{CO}{CF} = \frac{CA}{CD}$$

Therefore, 'OA' is parallel to 'FD'.

from, similar Δ^{les} CFE and OFB,

$$\frac{CE}{CF} = \frac{OB}{OF}$$

$$OB = \frac{CE \times OF}{CF} \longrightarrow (1)$$

from, Similar Δ^{les} FCD and OCA,

$$\frac{FD}{FC} = \frac{OA}{OC}$$

$$OA = \frac{FD \times OC}{FC} \longrightarrow (2)$$

(1) × (2),

$$OA \times OB = \frac{FD \times OC}{FC} \times \frac{CE \times OF}{FC}$$

$$OA \times OB = \frac{FD \times OC \times CE \times OF}{FC^2}$$

$$= FD \times CE \times \left(\frac{OC \times OF}{FC^2}\right)$$

Since the lengths OC, OF and FC are fixed. Therefore,

OA × OB = Constant.

Now, from Point 'E' Draw EM $\parallel^{el}$ to CF and EN $\perp^r$ FD

Therefore, $FD \times CE = FD \times FM$

Here,

$FD = FN + ND$

$FM = FN - NM$ | $NM = ND$

$FM = FN - ND$

$FD \times CE = (FN + ND)(FN - ND)$

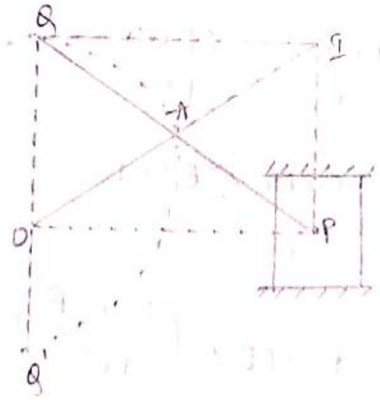
$= FN^2 - ND^2$

$= (FE^2 - EN^2) - (ED^2 - EN^2)$

$FD \times CE = FE^2 - ED^2 = \text{Constant}$

$FD \times CE = \text{Constant}$

* Exact Straight line motion mechanism with one Sliding pair by Scott russells mechanism :-



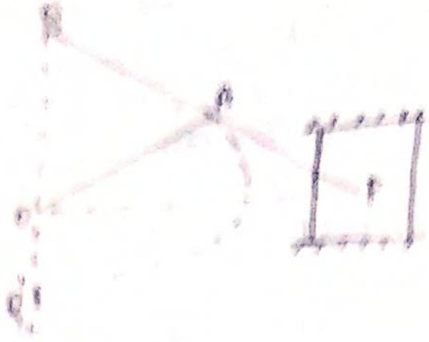
It Consists of a fixed link and a movable link 'P' of a sliding pair as shown in figure. The straight link PAQ is Connected by turning pairs to the link `OA' and the link `P'. The link `OA' rotates about `O'.

A little Consideration will show that the mechanism OAP is same as that of reciprocating steam engine. in which OA is Crank & AP is Connecting rod. In this mechanism the straight line motion is not generated but it is nearly Copied. In this mechanism, 'A' is the midpoint of `QP' and OA = AP = AQ. The instantaneous Centre for the link PAQ lies at `I' in OA produced and is such that `IP' is $\perp^r$ to `OP' and `IQ' is $\perp^r$ to `OQ'. Therefore, OQIP is a rectangle. `Q' moves along the Vertical line `OQ' for all positions of `QP'. Hence, `Q' traces a straight line `OQ'.

* Approximate Straight Line motion mechanism:-

1) Watts
2) Modified Scott Russell
3) Grasshoper
4) Tchebichef's
5) Robert's

* Modified Scott Russell :-



It is similar to Scott Russell mechanism. But In this Case, AB is not equal to AQ. And the Points 'P' & 'Q' are Constrained to move in the horizontal & Vertical direction. It is similar to elliptical tramnel. So, that any point 'A' on 'PQ' traces an Ellipse with Semi major axis 'AQ' & Semi minor axis 'AP'

* Watt's Mechanism:-

This mechanism Consists of 4 bar chain. In this figure $OABO'$ is a Crossed 4 bar chain. in which OO' are fixed in the mean position of the m/m. links OA and BO' are parallel and the Coupling rod AB is $\perp^r$ to OA and BO'. The tracing point 'p' traces out an approximate straight line over certain positions of its moment.

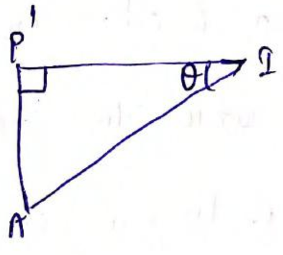
$$\text{If} \quad \frac{PA}{PB} = \frac{O'B}{OA}.$$

A little consideration will show that the initial position of the mechanism. The instantaneous centre of the link AB lies at infinity distance

Let $OA'B'O'$ be the new position of the m/m after the links OA & $O'B$ are displaced through an angle 'θ' & 'ϕ'. The instantaneous centre lies at 'I'

Since the angles 'θ' & 'ϕ' are very small

Arc AA' = Arc BB'



$$\therefore \quad OA \times \theta = O'B \times \phi$$

$$\frac{OA}{O'B} = \frac{\phi}{\theta}$$

Then,

$$A'P' = IP' \times \tan\theta$$

Similarly,

$$B'P' = IP' \times \tan\phi$$

$$\frac{B'P'}{A'P'} = \frac{\phi}{\theta} \longrightarrow ①$$

Hence,

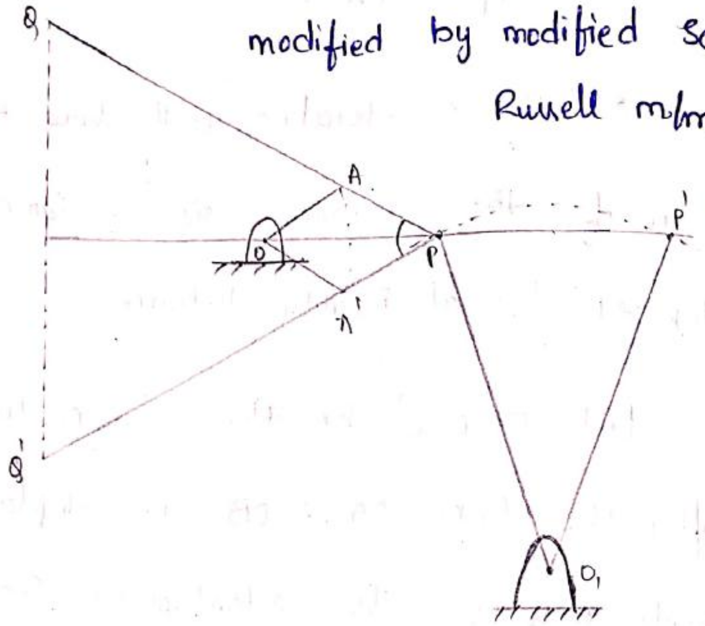
$$\frac{OA}{O'B} = \frac{\phi}{\theta} \longrightarrow ②$$

from ① & ②,

$$\boxed{\frac{B'P'}{A'P'} = \frac{OA}{O'B} = \frac{BP}{AP}}$$

* Grasshoper Mechanism :-

It is also 4 Bar chain m/m but it is slightly modified by modified Scott Russell m/m.

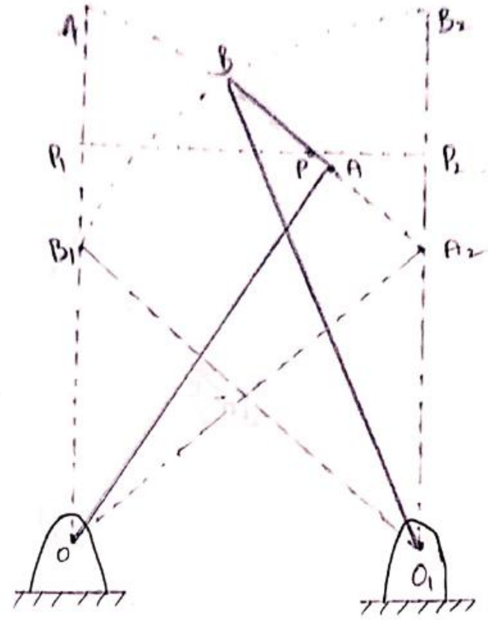


The link OA Osicilates about 'O' through an angle which Causes the pin 'p' to move along a Circular one with O_1 as Centre O_1P as radius. The small angular displacement at OP on each side of the Horizontal the point Q on the Extension of the Link PA traces out an approximately a straight path Q, Q' with length

$$OA = \frac{(AP)^2}{AQ}$$

* Tchebichef's m/m :-

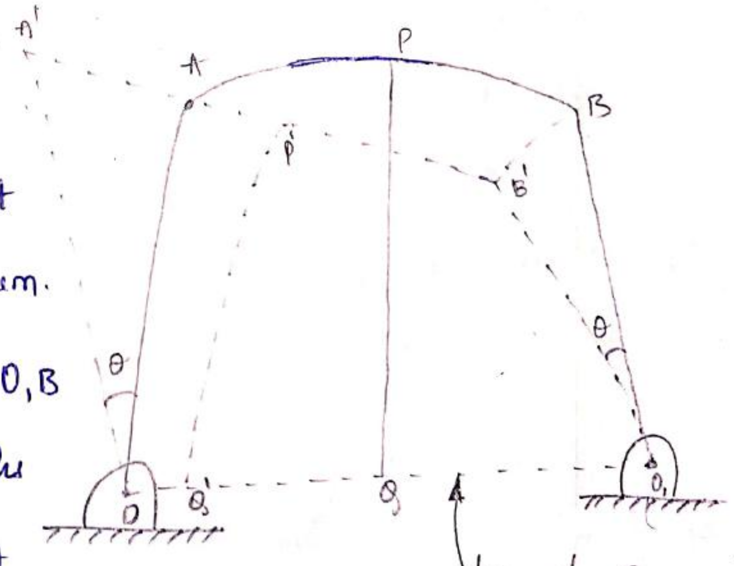
It Consists of 4 links. All are turning pairs only.



It is a Crossed links OA and O_1B are of equal length and the point 'P' is midpoint of AB traces out the approximate straight line parallel to OO_1. If the lengths of the links are in proportions AB : OO_1 : OA

* Robert's m/m :-

In this mechanism, totally 4 links in its mean position it look likes a trapezium. The links OA and O_1B are of equal lengths and OO_1 is fixed. A



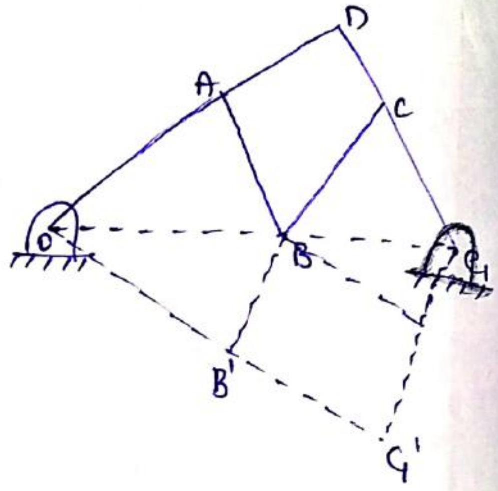
bar PQ is rigidly attached to the link AB at its middle

Point 'P'. A little bit displacement as shown by the dotted lines the point 'Q' traces out an approximate straight line.

* Pantograph:-

OAB, ODC

$$\frac{OA}{OB} = \frac{AB}{C_1D}$$



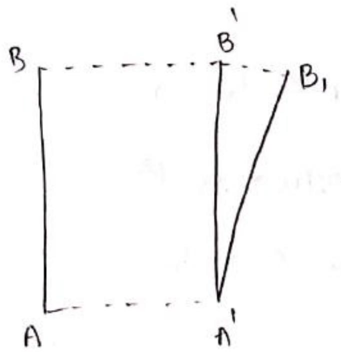
23/01/20

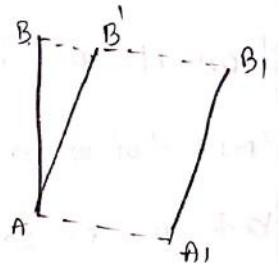
Unit – II

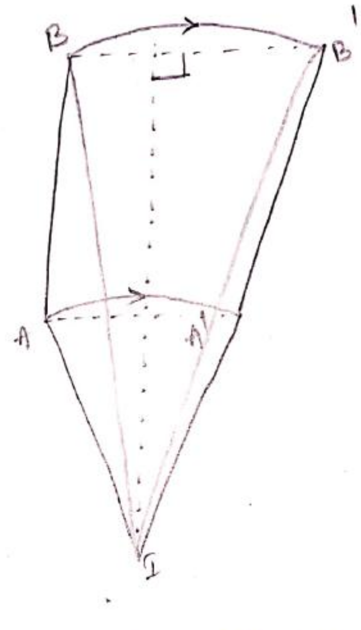
Velocity in Mechanisms

Velocity in mechanisms

- Instantaneous Centre
- Relative velocity





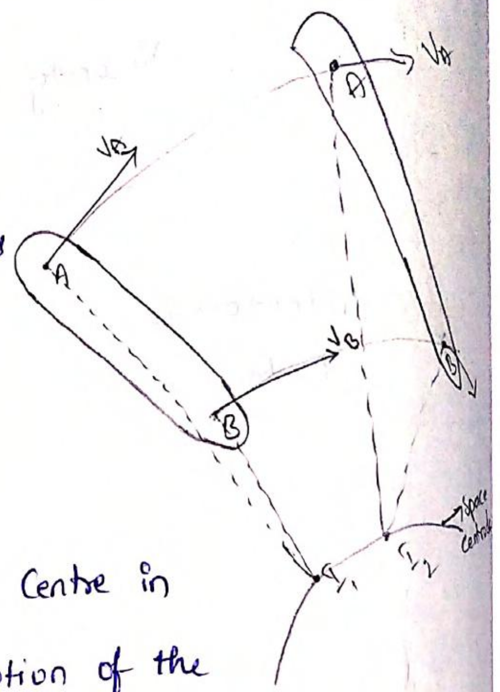


In actual practice, the motion of line AB is so gradual that it is difficult to see the two seperate motions. But, we see the two seperate motions the point 'B' moves faster than 'A'. This combination of rotation & Translation of the link AB may be assumed to be a motion of pure rotation about some Centre 'I' is known as instantaneous Centre of rotation.

The locus of all instantaneous Centres is known as Centrode. A line drawn through an instantaneous

Centre ⊥r to the plane of motion is called instantaneous axis

* Space & Body Centrodes:-

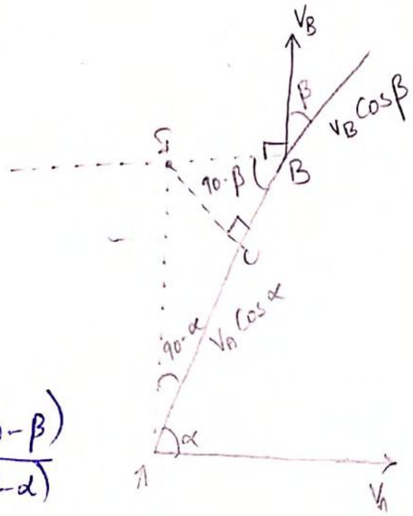


Locus of the instantaneous ~~Point~~ centre relative to the body itself is known as Body Centrode.

Locus of the instantaneous Centre in space during a definite motion of the body is known as Space Centrode

* Velocity of a body:-

Resolving the forces of the link AB,



$$V_A \cos\alpha = V_B \cos\beta$$

$$\frac{V_A}{V_B} = \frac{\cos\beta}{\cos\alpha} = \frac{\sin(90-\beta)}{\sin(90-\alpha)}$$

Now, Applying Lami's theorem,

$$\frac{AI}{\sin(90-\beta)} = \frac{BI}{\sin(90-\alpha)}$$

$$\frac{AI}{\cos\beta} = \frac{BI}{\sin\alpha}$$

$$\frac{AI}{BI} = \frac{\cos\beta}{\sin\alpha}$$

$$\frac{V_A}{V_B} = \frac{AI}{BI}$$

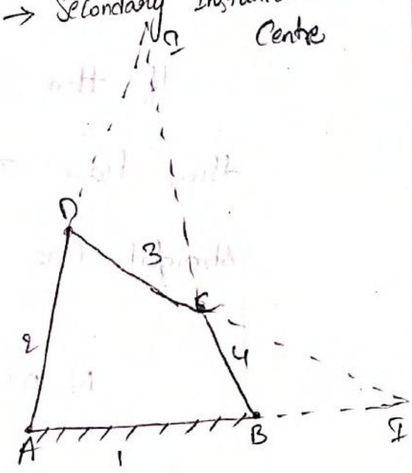
$$\frac{V_A}{AI} = \frac{V_B}{BI} \quad \left[\omega = \frac{V}{Radius}\right]$$

No. of instantaneous Centre in m/m, $N = \frac{n(n-1)}{2}$

* Types of Instantaneous Centre:-

AAA (2m)
1) fixed Instantaneous Centre } Primary Instantaneous Centre
2) Permanent " "
3) Neither fixed nor permanent I.C → Secondary Instantaneous Centre

for a four bar mechanism



AB - fixed I.C

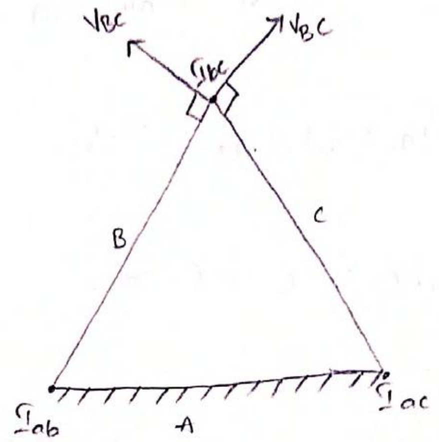
BC, CD, DA } Permanent I.C

DI, CI, BI, CI } Neither fixed nor permanent I.C

links	1	2	3	4
Instantaneous Centre	12 13 14	23 24	34	

* Aronhold's Kennedy's Theorem (or)

Three Centres inline Theorem :-



If three bodies moves relatively to each other they have 3 instantaneous Centre and lie on a straight line

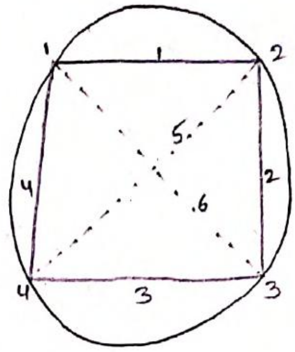
$$N = \frac{n(n-1)}{2} = \frac{3(3-1)}{2}$$

$$= \frac{6}{2} = 3 \text{ (Instantaneous Centre)}$$

The two instantaneous Centres at the pin joints of 'B' with 'A' and 'C' with 'A' and the two I.C is called as fixed I.C

According to Kennedy's Theorem, the 3rd Instantaneous Centre I_{BC} must lie on the line joining the I_{ab} and I_{AC}

* Circle diagram [To find I.C]:-

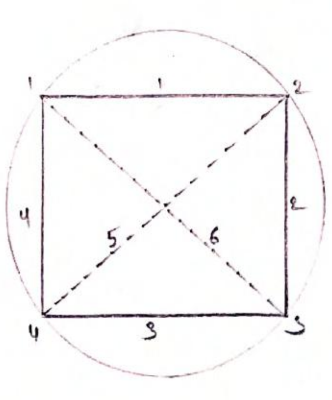


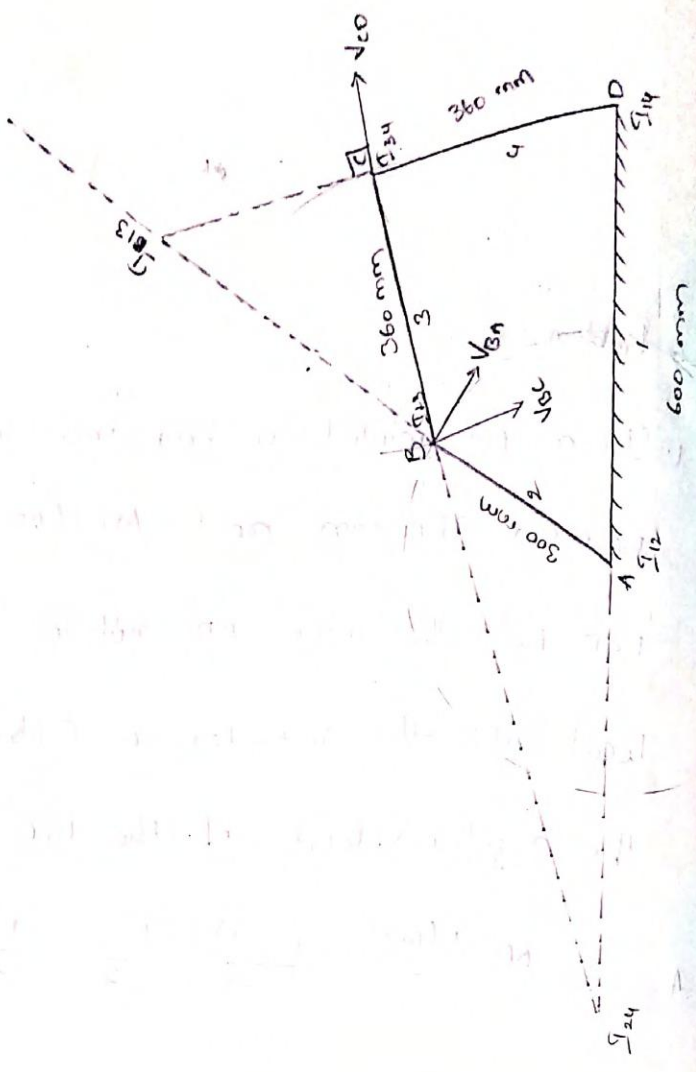
→ Problem:-

1. In a pin jointed 4 bar mechanism AB = 300 mm BC & CD = 360 mm and AD = 600 mm. The angle BAD = 60°. The crank AB rotates uniformly at 100 RPM locate all the instantaneous Centres and also find the angular velocity of the link BC?

A. $N = \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = \frac{4(3)}{2} = \frac{12}{2} = 6$

Links	1	2	3	4
Instantaneous Centre	12 ✓ 13 ✓ 14 ✓	23 ✓ 24 ✓	34 ✓	





$\omega_{BC} = ?$

$$\omega_{AB} = \frac{2\pi N}{60} = \frac{2\pi(100)}{60} = 10.47 \text{ rad/s}$$

$$\omega = \frac{V}{R}$$

$$V = \omega \times R$$

$$V_B = \omega_{AB} \times AB$$

$$= 10.47 \times 300$$

$$= \text{m/s} / 1000$$

$$V_B = 3.141 \text{ m/s}$$

$$V_B = \omega_{BC} \times I_{13} \times B$$

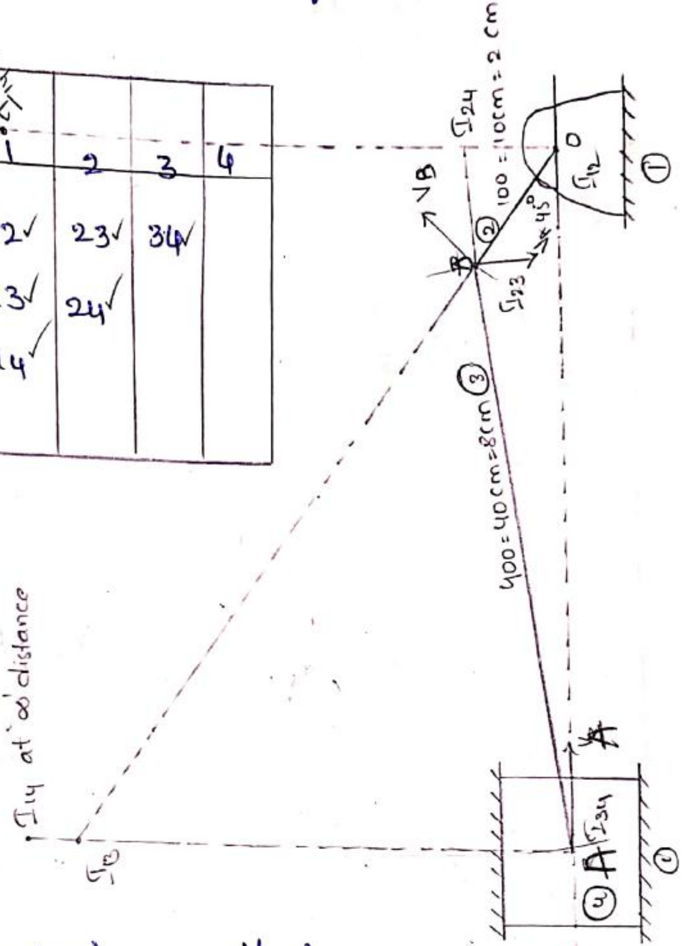
$$\omega_{BC} = \frac{V_B}{I_{13} \times B} = \frac{3.141}{0.54} = 5.81 \text{ rad/s}$$

2) ... the I.C of the Slider Crank m/m as shown in figure. The lengths of Crank OB and Connecting rod AB are 100 mm & 400 mm. Will the Crank rotates clockwise with an Angular velocity of 10 rad/s. find

i) Velocity of the slider A.

ii) Angular velocity of the Connecting rod AB

Links	1	2	3	4
Instantaneous Centre	12 ✓ 13 ✓ 14 ✓	23 ✓ 24 ✓	34 ✓	



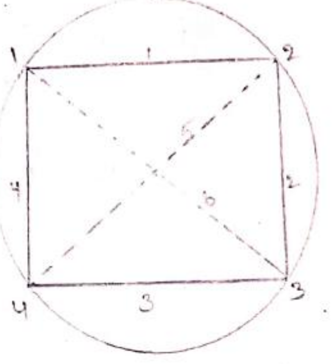
$N = \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$

$V_A = ?$

$\frac{V_A}{I_{13}A} = \frac{V_B}{I_{13}B}$

$V_B = \omega_{OB} \times OB = 10 \times 0.1 = 1$

$V_A = \frac{V_B \times I_{13}A}{I_{13}B} = 1 \times \frac{0.46}{0.56} = 0.82 \text{ m/s}.$

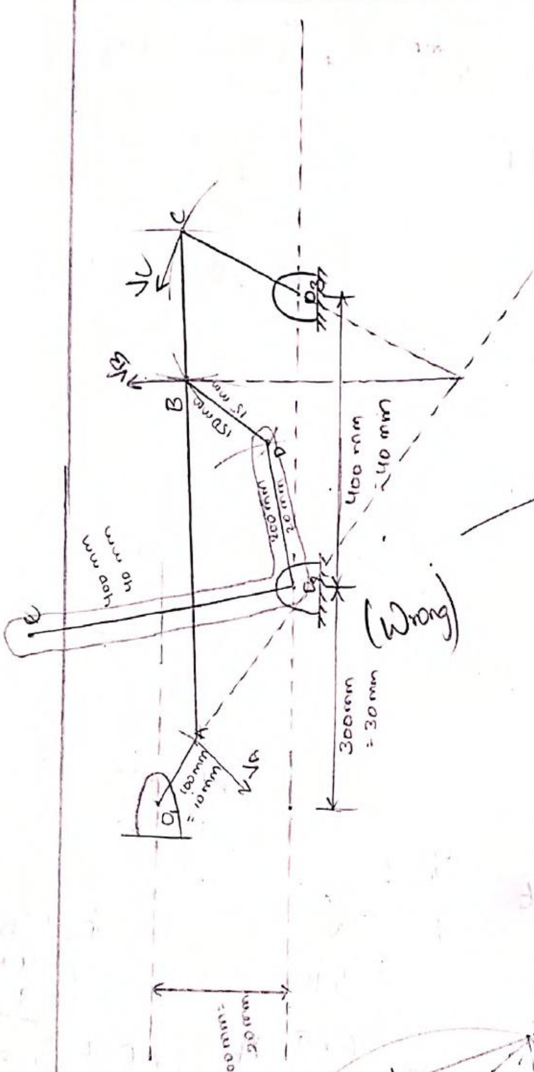


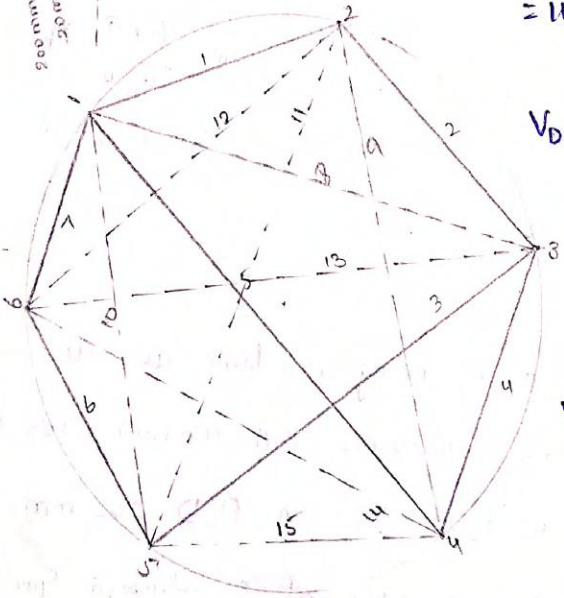
3) A m/m of a wrapping machine as shown in figure has the following dimensions. $O_1A = 100$ mm, $AC = 700$ mm, $BC = 200$ mm, $O_3C = 200$ mm, $O_2E = 400$ mm, $O_2D = 200$ mm and $BD = 150$ mm. The Crank O_1A rotates at a Uniform Speed of 100 rad/s

Find the velocity of the point 'E' of the bell crank lever by Instantaneous Centre method.

Links	1	2	3	4	5	6
I.C	12	23	34	45	56	
	13	24	35	46		
	14	25	36			
	15	26				
	16					

$N = \frac{n(n-1)}{2} = \frac{6(6-1)}{2} = \frac{30}{2} = 15$





$V_B = V_A \times \frac{I_{13}B}{I_{13}A}$

$= 10 \times \frac{0.86}{0.95}$

$= 9.05 \text{ m/s}$

$V_A = \omega_{OA} \times OA$

$= 100 \times 0.1$

$= 10$

$V_C = V_A \times \frac{I_{13}C}{I_{13}A}$

$= 10 \times \frac{0.94}{0.95} = 9.89 \text{ m/s}$

$V_D = V_A \times \frac{I_{15}D}{I_{15}B}$

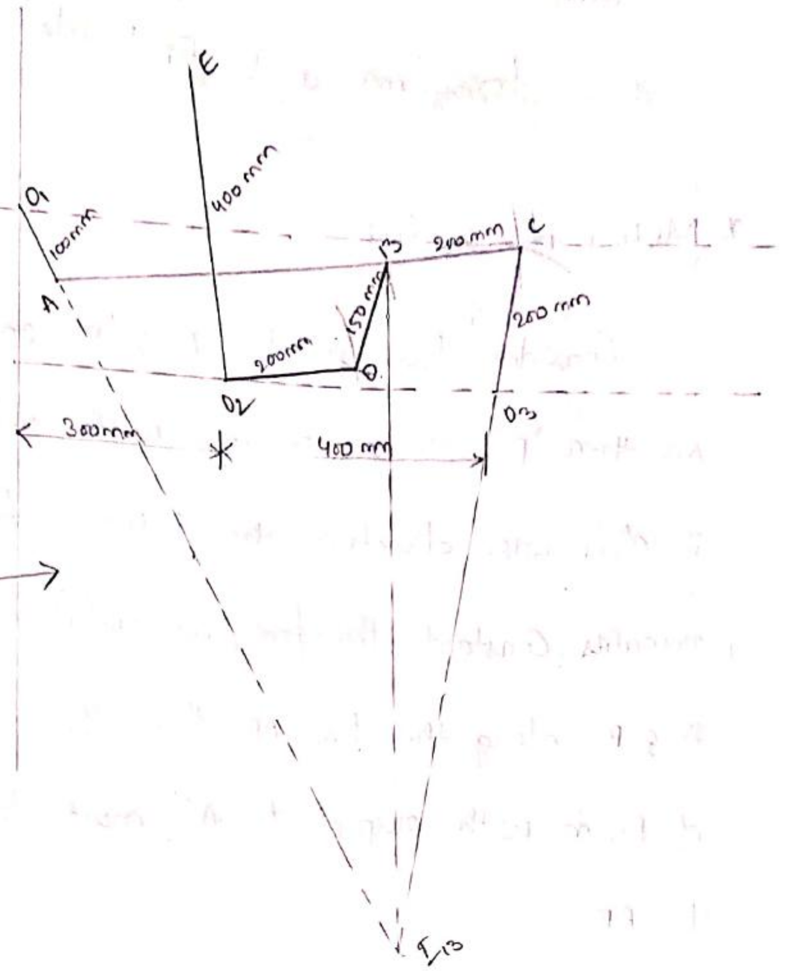
$= 10 \times \frac{0.05}{0.13}$

$= 3.84 \text{ m/s}$

$V_E = V_D \times \frac{I_{16}E}{I_{16}D}$

$= 3.84 \times \frac{0.004}{0.002}$

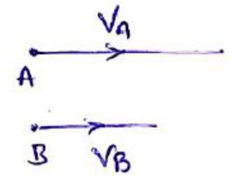
$= 7.68$

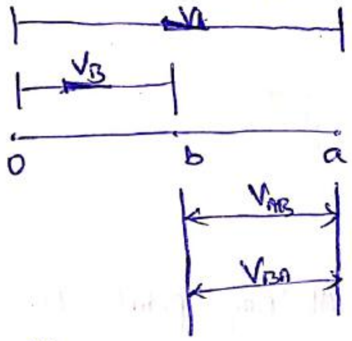


* Relative velocity of motion:-

Relative velocity of 2 bodies moving in Straight line,

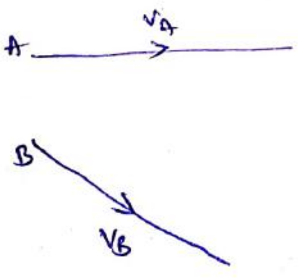
1)

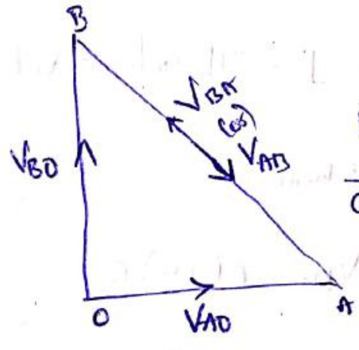




$\overline{V_{AB}} = \overline{V_A} - \overline{V_B}$

2)





$\overline{ba} = \overline{Oa} - \overline{Ob}$

$\overline{ab} = \overline{Ob} - \overline{Oa}$

$V_{AB} = -V_{BA}$

(Statement is writtened Back side)

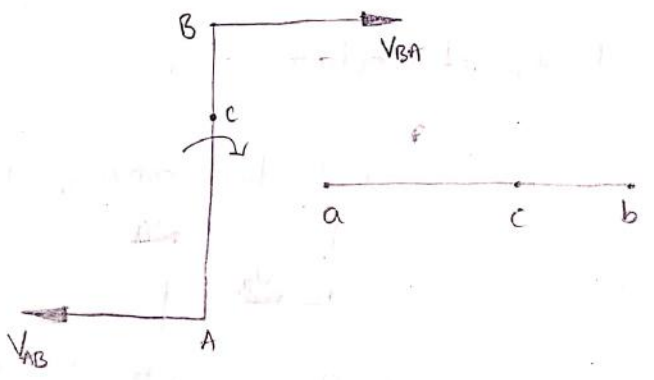
When the two bodies moving in inclined Condition,

(Diagram is in front side)

* Motion of a link:-

Consider two points 'A' & 'B' on a rigid link AB then 'P' will be moving with relative to 'A' in Clock wise direction the distance from 'A' to 'B' remains Constant. Therefore, no relative motion between A & B along the line AB then the relative motion of beam with respect to 'A' must be perpendicular to AB.

$V_{BA} = -V_{AB}$



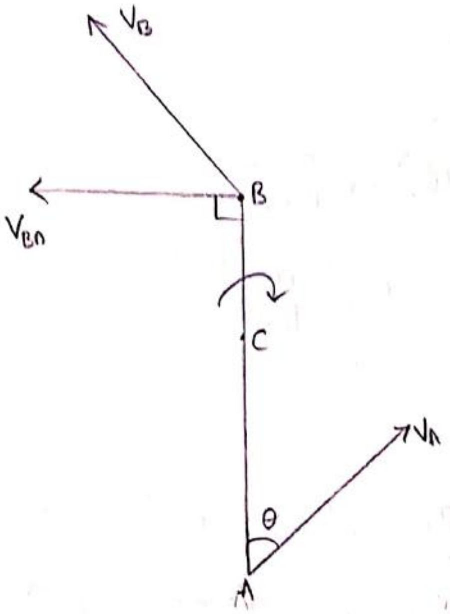
At any point on a link the velocity will be always $\perp^r$. Therefore, Velocity of 'B' w.r. to 'A' (V_{BA}) is $AB \times \omega_{AB}$

$$V_{BA} = AB \times \omega_{AB}$$

$$V_{CA} = AC \times \omega_{AC}$$

$\Rightarrow \dfrac{V_{CA}}{V_{BA}} = \dfrac{AC \times \omega_{AC}}{AB \times \omega_{AB}}$

* Velocity of a point on a link by relative velocity method:-

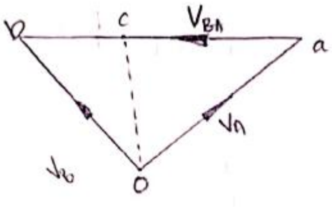


$\dfrac{bc}{ba} = \dfrac{BC}{BA}$

$bc = ba \times \dfrac{BC}{BA}$

$bc = 4.1 \times \dfrac{1.35}{5}$

$bc = 1.517$



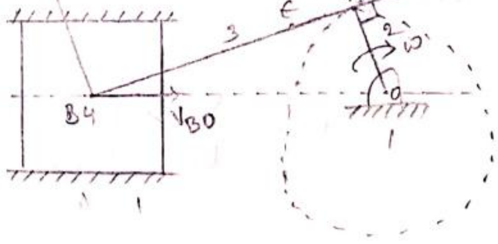
* Velocities in Slider Crank m/m.-

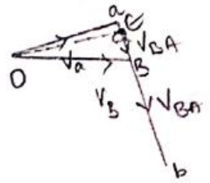
$\dfrac{ae}{ab} = \dfrac{AE}{AB}$

$ae = ab \times \dfrac{AE}{AB}$

$= 0.55 \times \dfrac{1}{4}$

$ae = 0.1375$



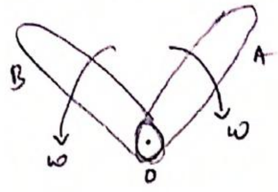


* **Rubbing velocity at a pin joint :-**

It is the algebraic sum of the Angular velocity of the two links joined by pin and multiplied by the pin radius.

$V = (\omega_1 - \omega_2) \times r$ [same direction]

$V = (\omega_1 + \omega_2) \times r$ [opposite direction]



1) In a four Bar chain mechanism ABCD. AD is a fixed link and it is 150 mm long. The Crank AB is 40 mm long and rotates at 120 R.P.M Clockwise. while the link CD is 80 mm Oscillates about D. 'BC' and 'AD' are of equal length. find the Angular velocity of the link CD. when an angle BAD is 60°

A.

$$\omega_{AB} = \frac{V_{BA}}{AB}$$

$$\omega_{AB} = \frac{2\pi N}{60}$$

$$= \frac{2\pi(120)}{60}$$

$$= 4\pi$$

$$\omega_{AB} = 12.566 \text{ rad/s}$$

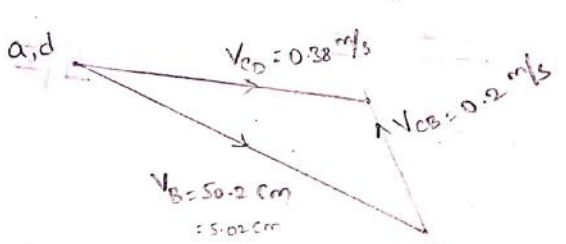
$$12.566 = \frac{V_{BA}}{40\,mm = 0.04\,m}$$

$$V_B = V_{BA} = 12.566 \times 0.04$$

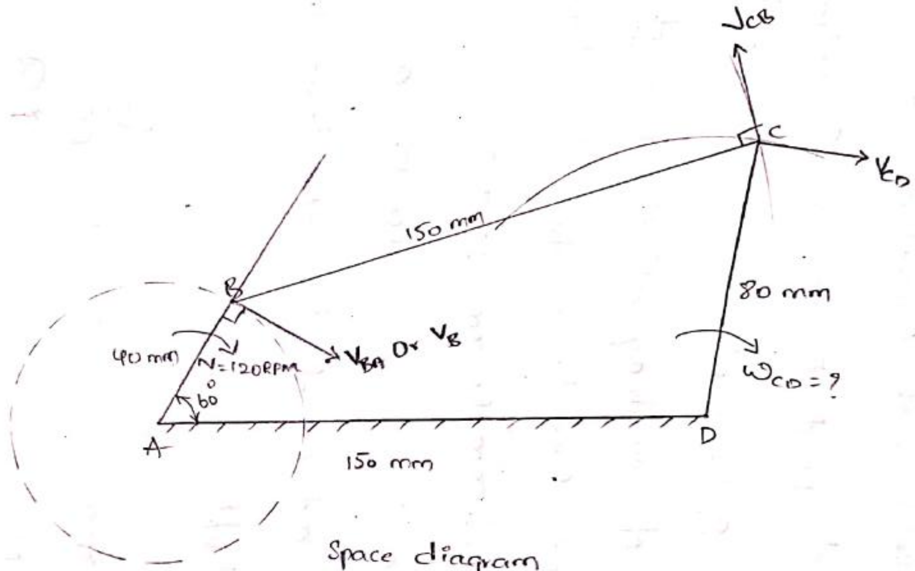
$$V_B = 0.502 \text{ m/s}$$

$$\omega_{CD} = \frac{V_{CD}}{DC} = \frac{0.38}{0.08} = 4.75 \text{ rad/s}$$

$$\omega_{CB} = \frac{V_{CB}}{CB} = \frac{0.2}{0.15} = 1.33 \text{ rad/s}$$



Scale - 1:10



Space diagram

Scale - 1:2

2) The Crank and Connecting rod of a steam Engine are 0.5 m and 2m long. The Crank rotates 180 RPM in the clockwise when it has turned 45° from Inner Dead Centre. Determine

1) Velocity of piston
2) Angular velocity of Connecting rod
3) Velocity of point 'E' on the Connecting rod 1.5 m from the gudgeon pin.
4) Velocities of rubbing at the pins of the Crank shaft, Crank and Cross head. when the diameters of the Pins are 50 mm, 60 mm, 30 mm
5) Position and linear velocity of any point G, on the Connecting rod which has the least velocity relative to Crankshaft

A.

$N_{OB} = 180$ rpm

$\omega_{OB} = \frac{2\pi N}{60}$

$= \frac{2\pi(180)}{60}$

$= 6\pi$

$\omega_{OB} = 18.84$ rad/sec

$V_{BO} = \omega_{OB} \times BO$

$= 18.84 \times 0.5$

$V_{BO} = 9.42$ m/s

ii) $\omega_{BP} = \frac{V_{PB}}{BP}$

$= \frac{7.2}{2}$

$= 3.6$ rad/s

iii) $\frac{be}{bp} = \frac{BE}{BP}$

$be = bp \times \frac{BE}{BP}$

$= 7.2 \times \frac{0.5}{2}$

$= 1.8$m

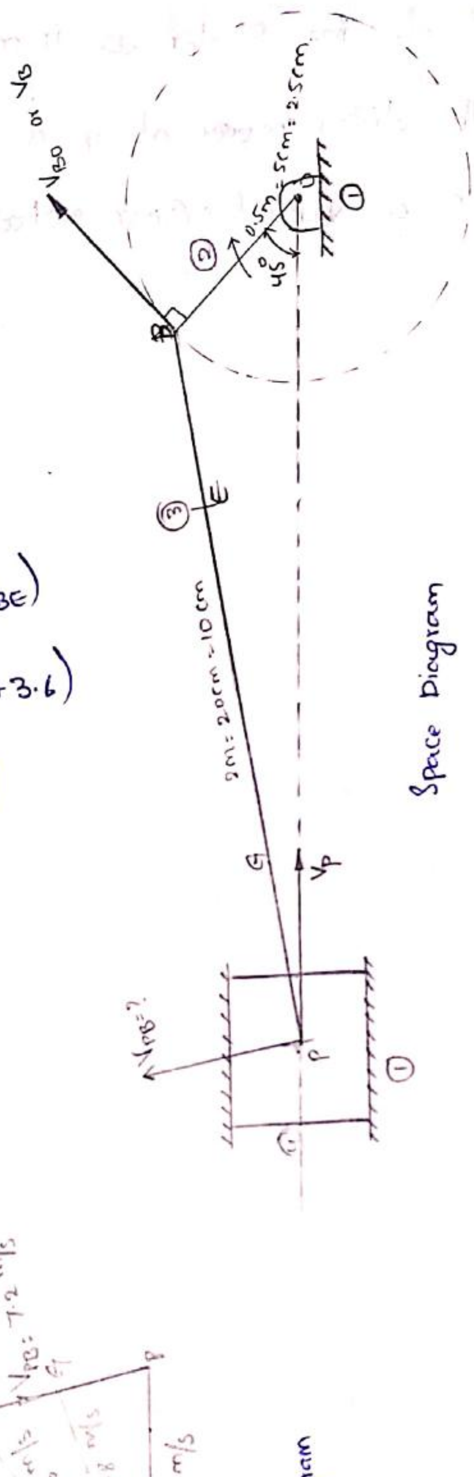
$\frac{bg}{bp} = \frac{BG}{BP}$

$bg = bp \times \frac{BG}{BP} = \frac{7.2 \times 1.5}{2} = 5.4$ m/s

Diameter of Crankshaft = 50 mm = 0.05 m

" " Crank = 60 mm = 0.06 m

" " Crosshead = 30 mm = 0.03 m



Velocity of rubbing at the pin of Crankshaft,

$V_O = r_O \times \omega_{OB}$

$= \frac{0.05}{2} \times 18.84$

$V_O = 0.471$ m/s

$V_B = r_B \times (\omega_{OB} + \omega_{BE})$

$= \frac{0.06}{2} \times (18.84 + 3.6)$

$= 0.03 (22.44)$

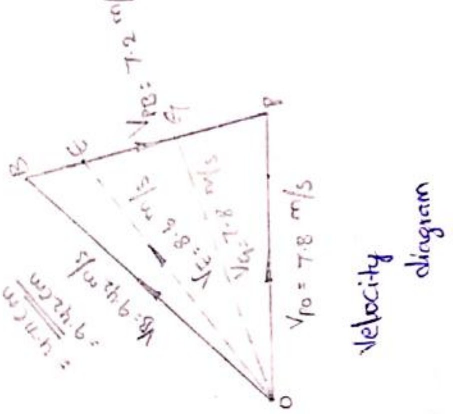
$= 0.6732$ m/s

$V_P = r_P \times \omega_{PB}$

$= \frac{0.03}{2} \times 3.6$

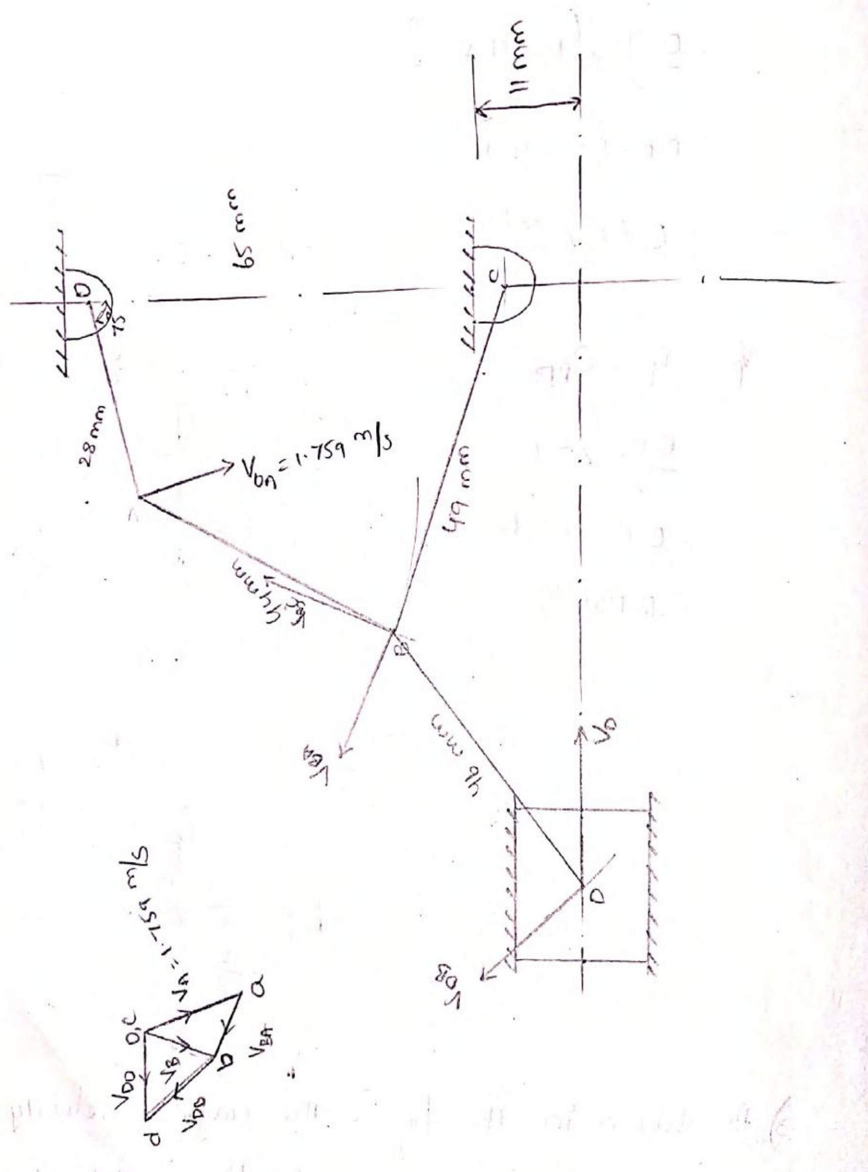
$= 0.015 \times 3.6$

$= 0.054$ m/s



3) As shown in the figure the angular velocity of the Crank OA is 600 rpm. Determine the linear velocity of the slider 'D' and the angular velocity of the link BD when the Crank is inclined at an angle of 75° to the Vertical

The dimensions of various links ... , AB = 44,
BC = 49 mm, BD = 46 mm. The centre distance b/w the centres of rotation 'O' and 'C' is 65 mm. The path of travel of the slider is 11 mm below the fixed point 'C'. The slider moves along a horizontal path and 'O' and 'C' is vertical. Crank rotates in Anticlockwise direction



$$\omega_{OA} = \frac{2\pi N}{60}$$

$$= \frac{2\pi(600)}{60}$$

$$= 20\pi$$

$$= 62.83 \text{ rad/s}$$

$V_{DB} = 1.7$ m/s

$V_{DO} = 1.6$ m/s

$V_{BA} = 1.1$ m/s

$V_{BD} = 1.5$ m/s

$$V_{AO} = OA \times \omega_{OA}$$

$$= 0.028 \times 62.83$$

$$= 1.759 \text{ m/s}$$

$V_D = 1.6$ m/s

$\omega_{BD} = ?$

$$V_{BD} = DB \times \omega_{BD}$$

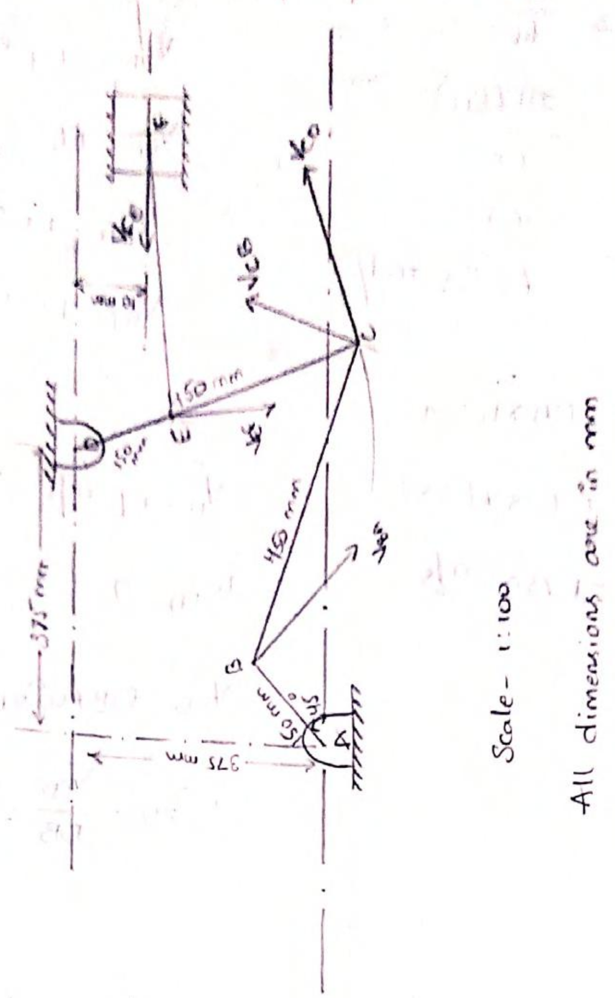
$$\omega_{BD} = \frac{V_{BD}}{DB} = \frac{1.7}{0.046} = 36.95 \text{ rad/sec}$$

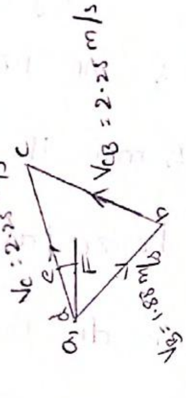
4) The mechanism as shown in figure has a dimensions of various links as follows AB = DE = 150 mm and BC & CD = 450 mm & EF = 375 mm. The crank AB makes an angle of 45° with the horizontal and rotates about 'A' in the ~~cront~~ clock wise direction at a uniform speed of 120 rpm. The lever DC ossicilates about the fixed point 'D' which is connected to AB by the coupler BC. The Block-F or Slider-F moves in the horizontal direction. Determine.

i) Velocity of slider F

ii) 'ω' of DC

iii) Rubbing speed at the pin 'C' which is 50 mm in diameter.





$V_c = 2.25$ m/s

$V_{CB} = 2.25$ m/s

$\frac{Ce}{Cd} = \frac{CE}{CD}$

$Ce = Cd \times \frac{CE}{CD}$

$= 2.25 \times \frac{0.3}{0.45}$

$Ce = 1.5$ m

$EF = 0.3$ m/s

$DF = 0.7$ m/s

ii) $\omega_{DC} = \frac{V_{CD}}{DC}$

$= \frac{2.25}{0.45}$

$\omega_{DC} = 5$ rad/s

$\omega_{CB} = \frac{V_{CB}}{BC}$

$= \frac{2.25}{0.45}$

$= 5$ rad/s

$N = 120\ rpm$

$\omega_{AB} = \frac{2\pi N}{60}$

$= 4\pi$

$\omega_{AB} = 12.56\ rad/sec$

V_{BA} or $V_B = \omega_{AB} \times BA$

$= 12.56 \times 0.15$

$= 1.884\ m/s$

iii) $V = r \times \omega$

$V = r \times (\omega_1 - \omega_2)$

$= r \times (\omega_{CB} - \omega_{CD})$

$= 0.025 \times (5 - 5)$

$V = 0$

5) In a mechanism as a shown in figure. The Various dimensions are AB = 125 mm, BP = 500 mm, PC = 125 mm CD = 250 mm, DE = 125 mm. The slider 'p' translates along an axis which is 25 mm vertically. Below the point 'A'. The Crank AB rotates at 120 rpm in the Anticlockwise direction. The Belt Crank lever CDE about fixed Centre 'D' draw the velocity diagram and Calculate the Velocity of point 'E' of the lever.

A. $\omega_{BA} = \frac{2\pi N_{AB}}{60}$

$= \frac{2\pi(120)}{60}$

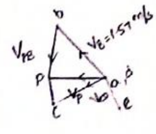
$= 12.56\ rad/s$

$V_{BA} = r \times \omega$

$= AB \times \omega_{BA}$

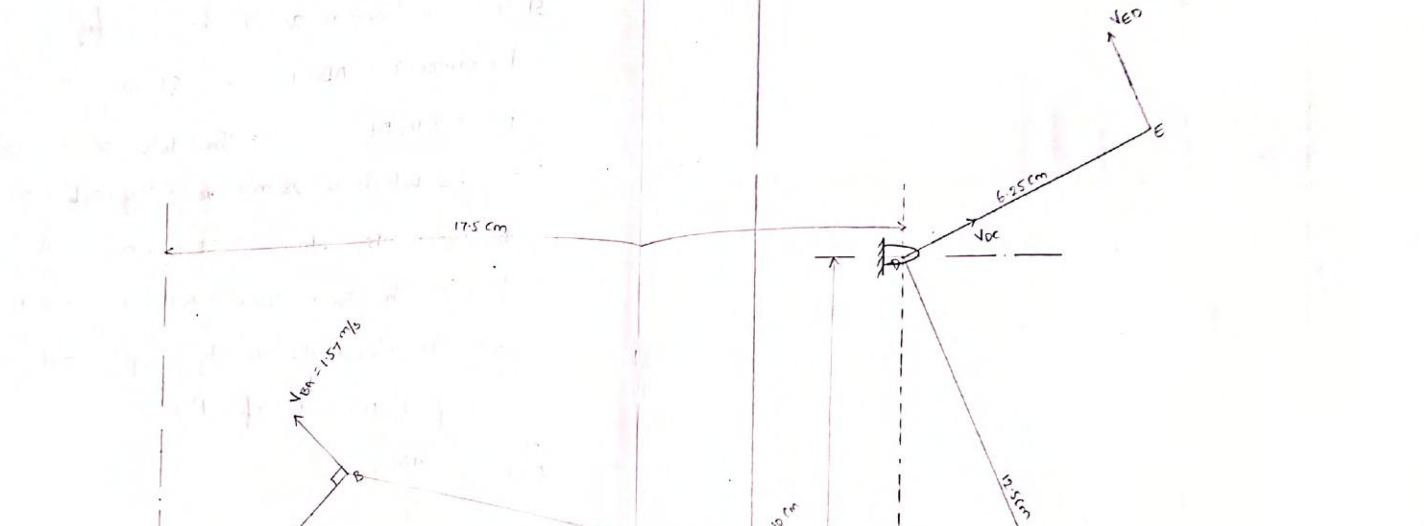
$= 0.125 \times 12.56$

$V_{BA} = V_B = 1.57\ m/s$



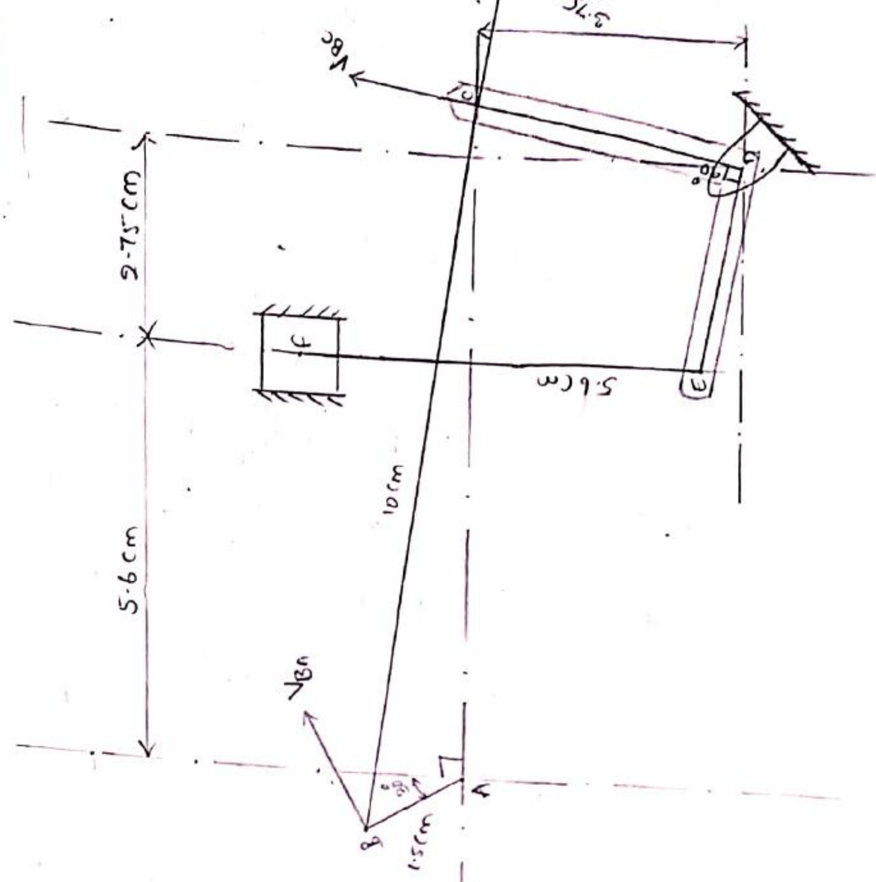
D
V_{PE}
$V_E = 1.57$ m/s
P
C
V_P
O, D
e

$V_{CD} = 1.34$



V_{ED}
E
6.25 cm
17.5 cm
V_{DC}
$V_{BA} = 1.57$ m/s
B
10 cm
12.5 cm
25 cm
6.25 cm
45°
A
V_{PB}
V_{PA}
P
6.25 cm
C
V_{CP}

b) The dimensions of a various links in a m/m as shown in figure, AB = 60 mm, BC = 400 mm, CD = 150 mm, DE = 115 mm and EF = 225 mm. find the velocity of the slider 'F', when the crank AB rotates uniformly in clockwise direction at a speed of 60RPM.

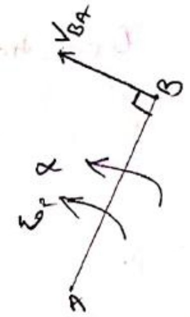


$$V_B = \omega_{AB} \times AB$$

$$= \frac{2\pi(60)}{60} \times \frac{60}{1000}$$

$$V_B = 3.6 \text{ cm}$$

* Acceleration in Mechanism:-



Consider a link 'AB'. let the Point 'B' moves w.r. to 'A' with an Angular velocity 'ω' $\left(\frac{rad}{s}\right)$ and α $\left(\frac{rad}{s^2}\right)$

for Acceleration, there is a two Component for which acceleration

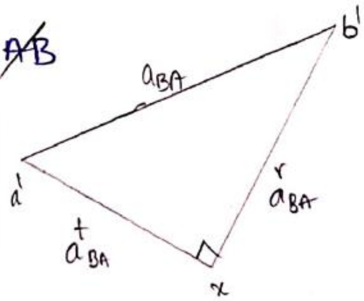
1) Radial Component → which is $\perp^r$ to the Velocity of particle
2) Tangential Component → which is $||^{el}$ to the " " "

Therefore, Radial Acceleration,

$$a^r_{BA} = \omega^2 \times \text{length of the link } AB$$

$$= \frac{(V_{BA})^2}{(AB)^{\cancel{2}}} \times \cancel{AB}$$

$$\boxed{a^r_{BA} = \frac{V_{BA}^2}{AB}}$$



Tangential Acceleration,

$$a^t_{BA} = \alpha \times \text{length of the Link } AB$$

$$\boxed{a^t_{BA} = \alpha \times AB}$$

1) The Crank of a Slider Crank m/m rotates clockwise at a speed of 300 RPM. The Crank is 150 mm and the Connecting rod is 600 mm long. Determine

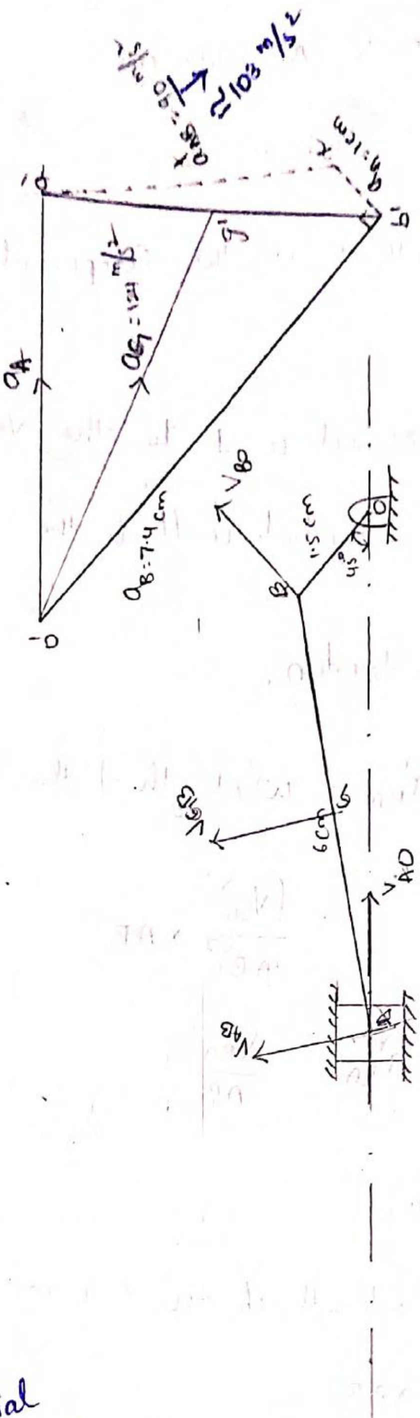
i) Linear velocity and Acceleration of the mid point of the Connecting rod

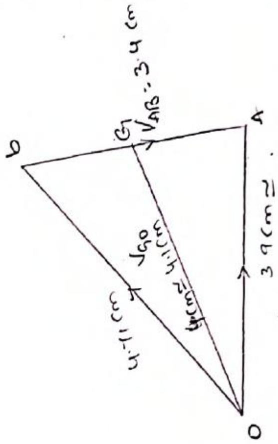
ii) Angular Velocity and Angular acceleration of the Connecting rod at a Crank angle of 45° from the IDC

$$\omega_{BO} = \frac{2\pi N}{60} = \frac{2\pi(300)}{60} = 31.41 \text{ rad/s}$$

$$V_{BO} = OB \times \omega_{BO}$$

$$= 0.15 \times 31.41 = 4.713 \text{ m/s}$$





Radial

Acceleration of B w.r.t to O,

$$a^r_{BO} = a_B = \frac{V_{BO}^2}{OB} = \frac{(4.713)^2}{0.15} = 148.1 \text{ m/s}^2$$

$$a^r_{AB} = a_A = \frac{V_{AB}^2}{BA} = \frac{(3.4)^2}{0.6} = 19.26 \text{ m/s}^2$$

(ii) Angular velocity of Connecting rod,

$$\omega_{AB} = \frac{V_{BA}}{AB} = \frac{3.4}{0.6} = 5.67 \text{ rad/sec}^2$$

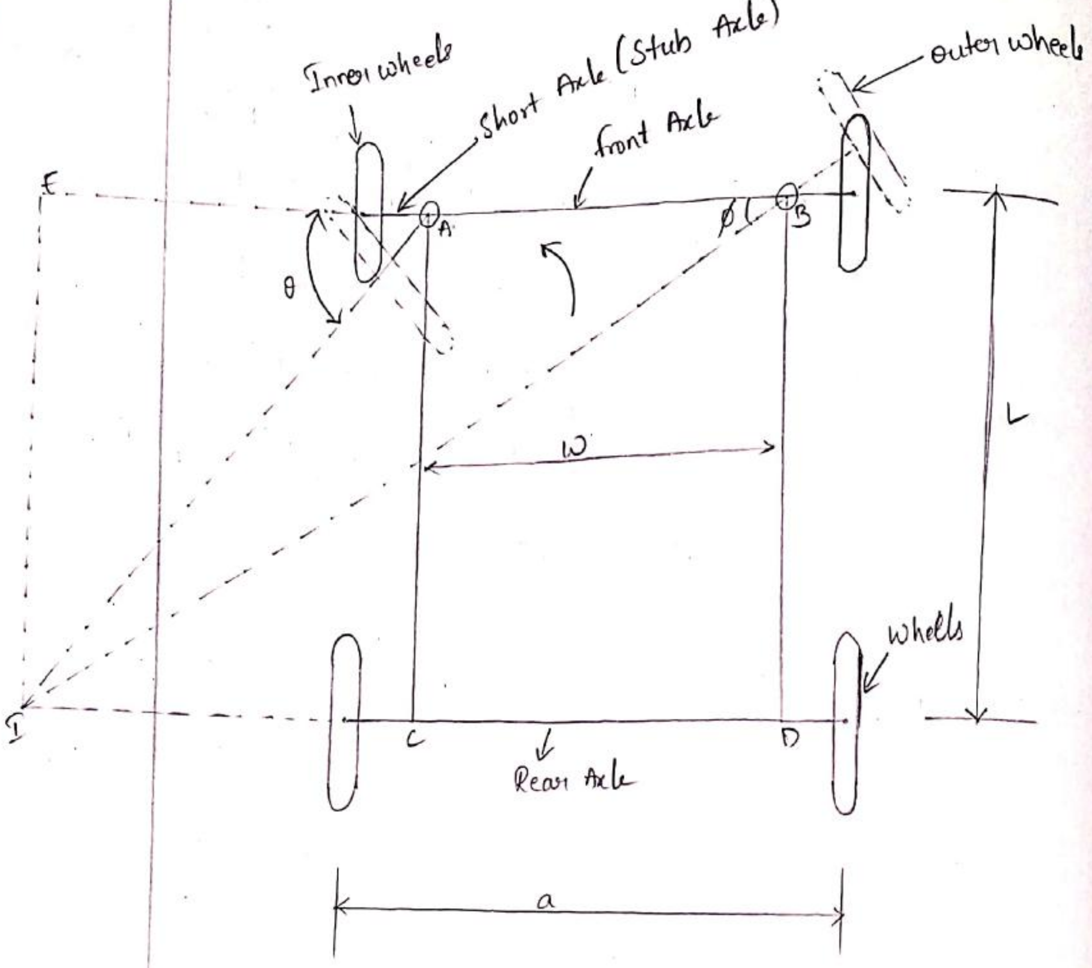
Angular acceleration of Connecting rod,

$$\alpha_{AB} = \frac{a^{t}_{AB}}{BA} = \frac{103}{0.6} = 171.66 \text{ rad/s}^2$$

10/03/2020

Unit - III

Steering Mechanism



* Derivation for Correct Steering m/m :-

Here,

W = Distance between the pivots of front Axle

L = wheel base

a = wheel track

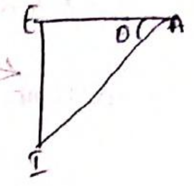
→ Definition for steering m/m :-

The m/m which is used for changing the direction of two (or) more of the wheel axles so as to move the automobile in any desired path is known as steering gear.

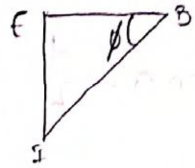
'θ' and 'φ' = Angle turned by the Stub Axle

Then,

$$\cot\theta = \frac{AE}{EI}$$



$$\cot\phi = \frac{EB}{EI} \qquad | EB = EA + AB \text{ (As per fig)}$$



$$\cot\phi = \frac{EA+AB}{EI}$$

$$\cot\phi - \cot\theta = \frac{EA+AB}{EI} - \frac{EA}{EI}$$

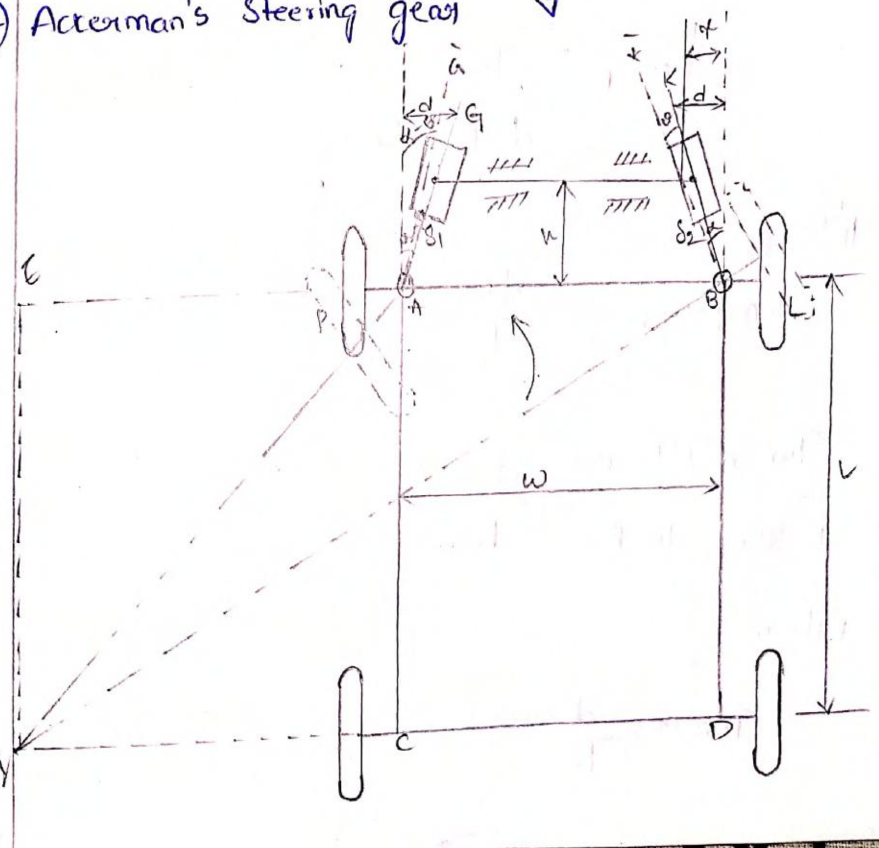
$$= \cancel{\frac{EA}{EI}} + \frac{AB}{EI} - \cancel{\frac{EA}{EI}}$$

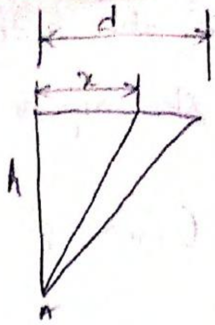
$$\boxed{\cot\phi - \cot\theta = \frac{AB}{EI} = \frac{W}{L}}$$

* Types of Steering gear :-

1) Davis Steering gear

2) Ackerman's Steering gear





$$\tan(\alpha - \theta) = \frac{d-x}{h}$$

$$\frac{\tan\alpha - \tan\theta}{1 + \tan\alpha \cdot \tan\theta} = \frac{d-x}{h}$$

where,

$$\tan\alpha = \frac{d}{h}$$

$$\frac{\frac{d}{h} - \tan\theta}{1 + \frac{d}{h}(\tan\theta)} = \frac{d-x}{h}$$

$$\frac{\frac{d - h\tan\theta}{\cancel{h}}}{\frac{h + d\tan\theta}{\cancel{h}}} = \frac{d-x}{h}$$

$$\frac{d - h\tan\theta}{h + d\tan\theta} = \frac{d-x}{h}$$

$$h(d - h\tan\theta) = (d-x)(h + d\tan\theta)$$

$$\boxed{\tan\theta = \frac{hx}{d^2 + h^2 - dx}}$$

$||^{ly}$

$$\tan(\alpha + \phi) = \frac{d+x}{h}$$

$$\frac{\tan\alpha + \tan\phi}{1 - \tan\alpha \cdot \tan\phi} = \frac{d+x}{h}$$

where,

$$\tan\alpha = \frac{d}{h}$$

$$\frac{\frac{d}{h} + \tan\phi}{1 - \frac{d}{h}\cdot\tan\phi} = \frac{d+x}{h}$$

$$\frac{\frac{d+h\tan\phi}{\cancel{h}}}{\frac{h-d\tan\phi}{\cancel{h}}} = \frac{d+x}{h}$$

$$h(d+h\tan\phi) = (d+x)(h-d\tan\phi)$$

$$hd + h^2\tan\phi = dh - d^2\tan\phi + xh - xd\tan\phi$$

$$\boxed{\tan\phi = \frac{hx}{d^2+h^2-dx}}$$

Correct Steering equation,

$$\cot\phi - \cot\theta = \frac{w}{L}$$

$$\frac{d^2+h^2+dx}{hx} - \frac{d^2+h^2-dx}{hx} = \frac{w}{L}$$

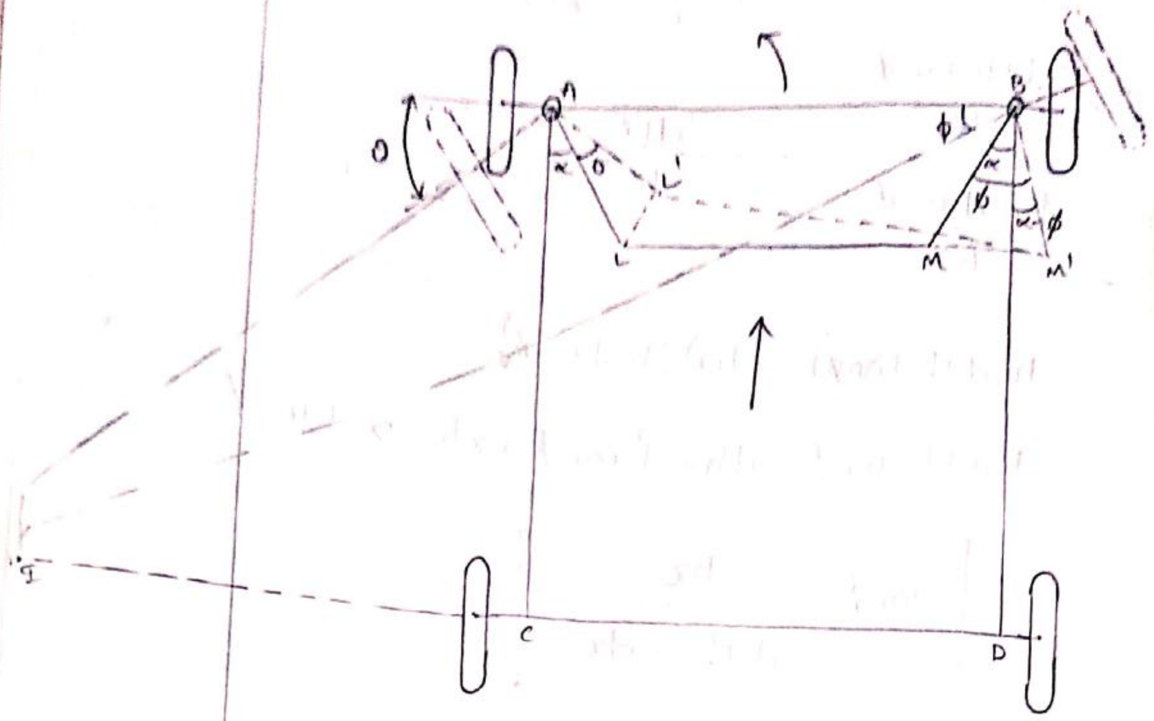
$$\frac{\cancel{d^2}+\cancel{h^2}+dx-\cancel{d^2}-\cancel{h^2}+dx}{hx} = \frac{w}{L}$$

$$\frac{2d\cancel{x}}{h\cancel{x}} = \frac{w}{L}$$

$$\frac{d}{h} = \frac{w}{2L}$$

$$\boxed{\tan\alpha = \frac{w}{2L}}$$

2) Ackerman's Steering gear:-



Projection of LL' on AB = Projection of MM' on AB

$$AL' \cos(90-(\alpha+\theta)) - AL\cos(90-\alpha) =$$

$$BM\cos(90-\alpha) + BM'\sin(\phi-\alpha)$$

$$AL'\sin(\alpha+\theta) - AL\sin\alpha = BM\sin\alpha + BM'\sin(\phi-\alpha)$$

$$[AL' = AL = BM = BM']$$

$$\sin(\alpha+\theta) - \sin\alpha = \sin\alpha + \sin(\phi-\alpha)$$

$$\sin\alpha\cos\theta + \cos\alpha\sin\theta - \sin\alpha = \sin\alpha + \sin\phi\cos\alpha - \cos\phi\sin\alpha$$

$$\sin\alpha\cos\theta - \sin\alpha - \sin\alpha = \sin\phi\cos\alpha - \cos\phi\sin\alpha - \cos\alpha\sin\theta$$

$$\sin\alpha\cos\theta - \sin\alpha - \sin\alpha + \cos\phi\sin\alpha = \sin\phi\cos\alpha - \cos\alpha\sin\theta$$

$$\sin\alpha(\cos\phi + \cos\theta - 2) = \cos\alpha(\sin\phi - \sin\theta)$$

$$\frac{\sin\alpha}{\cos\alpha} = \frac{\sin\phi - \sin\theta}{\cos\phi + \cos\theta - 2}$$

$$\boxed{\tan\alpha = \frac{\sin\phi - \sin\theta}{\cos\phi + \cos\theta - 2}}$$

* Universal Hook's Joint:-

→ What is Hook's Joint?

A) 1) It is used to 2 shafts which intersect at a small angle

2) It transmits power from the gear box of the Engine of the rear axle.

3) To transmit power to different spindles of multiple drilling machine.

4) The driving shaft rotates at a uniform angular speed where as the driven shaft rotates at a continuously varying angular speed

* Analysis of Hook's Joint:-

$$\tan\theta = \frac{EG_1}{EO}$$

$$\tan\phi = \frac{EG_2}{EO}$$

$$\frac{\tan\phi}{\tan\theta} = \frac{EG_2}{\cancel{EO}} \times \frac{\cancel{EO}}{EG_1}$$

$$\frac{\tan\theta}{\tan\phi} = \frac{EG_1}{EG_2}$$

$$= \frac{ON = ON_1 \cos\alpha}{OM = ON_1}$$

$$= \frac{ON_1 \cos\alpha}{ON_1}$$

$$\boxed{\tan\theta = \cos\alpha \times \tan\phi} \longrightarrow (1)$$

Ratio of Angular velocity of Shaft:-

$$\frac{\omega_1}{\omega_2} = ?$$

let,

ω_1 = Angular velocity of driving shaft

$$\omega_1 = \frac{d\theta}{dt}, \quad \omega_2 = \frac{d\phi}{dt}$$

$\therefore$ Differentiating Eq. (1) w.r.to time 't',

$$\frac{d}{dt}(\tan\theta) = \frac{d}{dt}(\cos\alpha \times \tan\phi)$$

$$\sec^2\theta \times \frac{d\theta}{dt} = \cos\alpha \times \sec^2\phi \frac{d\phi}{dt}$$

$$\sec^2\theta \times \omega_1 = \cos\alpha \times \sec^2\phi \times \omega_2$$

$$\frac{\omega_1}{\omega_2} = \frac{\cos\alpha \times \sec^2\phi}{\sec^2\theta}$$

$$\frac{\omega_2}{\omega_1} = \frac{Sec^2\theta}{Cos\alpha \times Sec^2\phi}$$

$$= \frac{1/cos^2\theta}{Cos\alpha \times Sec^2\phi}$$

$$\frac{\omega_2}{\omega_1} = \frac{1}{Cos^2\theta \times Cos\alpha \times Sec^2\phi} \longrightarrow ②$$

Now,

$$Sec^2\phi = 1 + Tan^2\phi$$

$$= 1 + \frac{Tan^2\theta}{Cos^2\alpha}$$

$$= \frac{Cos^2\alpha + Tan^2\theta}{Cos^2\alpha}$$

$$= \frac{Cos^2\alpha + \frac{Sin^2\theta}{Cos^2\theta}}{Cos^2\alpha}$$

$$= \frac{\cancel{Cos^2\alpha}\, \cancel{Cos^2\theta} + Sin^2\theta}{\cancel{Cos^2\theta} \times \cancel{Cos^2\alpha}\, Cos^2\theta \times Cos^2\alpha}$$

$$= 1 + \frac{Sin^2\theta}{Cos^2\theta \times Cos^2\alpha}$$

$$= \frac{Cos^2\theta(1 - Sin^2\alpha) + Sin^2\theta}{Cos^2\theta \times Cos^2\alpha}$$

$$= \frac{Cos^2\theta - Cos^2\theta Sin^2\alpha + Sin^2\theta}{Cos^2\theta \times Cos^2\alpha}$$

from ②

$$\frac{\omega_2}{\omega_1} = \frac{1}{\cancel{Cos^2\theta} \cdot \cancel{Cos\alpha} \times \left[\frac{Cos^2\theta - Cos^2\theta Sin^2\alpha + Sin^2\theta}{Cos^2\theta \times Cos^2\alpha}\right]}$$

$$= \frac{\cos\alpha}{\cos^2\theta - \cos^2\theta\sin^2\alpha + \sin^2\theta}$$

$$\frac{\omega_2}{\omega_1} = \frac{\cos\alpha}{1-\cos^2\theta\sin^2\alpha} \longrightarrow (3) = \frac{N_2}{N_1}$$

* Conditions for equal speeds of driven & driving shafts:-

If $\omega_2 = \omega_1$

from eq. (3)

$$\frac{\cancel{\omega_2}}{\cancel{\omega_2}} = \frac{\cos\alpha}{1-\cos^2\theta\sin^2\alpha}$$

$$\cos\alpha = 1-\cos^2\theta\sin^2\alpha$$

$$\cos^2\theta\sin^2\alpha = 1-\cos\alpha$$

$$\cos^2\theta = \frac{1-\cos\alpha}{\sin^2\alpha}$$

$$= \frac{1-\cos\alpha}{1-\cos^2\alpha}$$

$$\cos^2\theta = \frac{1-\cos\alpha}{(1-\cos\alpha)(1+\cos\alpha)}$$

$$\cos^2\theta = \frac{1}{1+\cos\alpha}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{1+\cos\alpha}$$

$$\cos^2\theta = \frac{\cos^2\theta\left[\frac{\sin^2\theta}{\cos^2\theta}+1\right]}{1+\cos\alpha}$$

$$\cancel{1} + \cos\alpha = \tan^2\theta + \cancel{1}$$

$$\tan^2\theta = \cos\alpha$$

$$\tan\theta = \pm\sqrt{\cos\alpha}$$

* Condition for maximum & minimum of driven Shaft :-

W.K.T, eq. (3)

driven

$$\frac{\omega_2}{\omega_1} = \frac{\cos\alpha}{1-\cos^2\theta\sin\alpha}$$

driving

for maximum speed of ω_2, the denominator's is maximum,

$$(1-\cos^2\theta\sin^2\alpha) = \text{minimum}$$

$$\theta = 0^\circ \text{ or } 180^\circ$$

$$\frac{\omega_2}{\omega_1} = \frac{\cos\alpha}{1-1(\sin^2\alpha)} = \frac{\cos\alpha}{\cos^2\alpha} = \frac{1}{\cos\alpha}$$

$$\boxed{\omega_2 = \frac{\omega_1}{\cos\alpha}}$$

for minimum speed of ω_2, the denominator is maximum.

$$1-\cos^2\theta\sin^2\alpha = \text{maximum}$$

$$\theta = 90^\circ \text{ or } 270^\circ$$

$$\frac{\omega_2}{\omega_1} = \frac{\cos\alpha}{1-\cos^2\theta\times\sin^2\alpha}$$

$$\frac{\omega_2}{\omega_1} = \frac{\cos\alpha}{1-\cos^2(90)\times\sin^2\alpha}$$

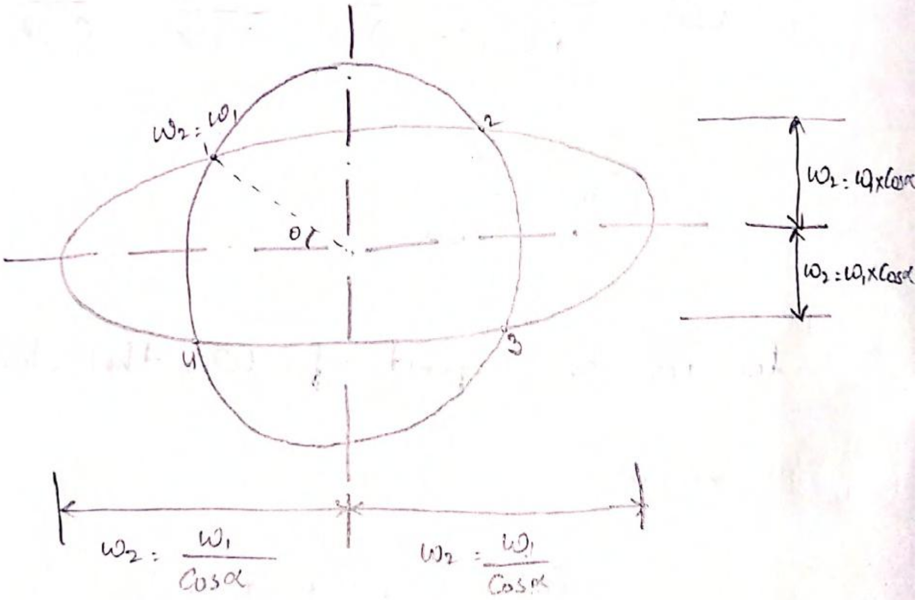
$$\frac{\omega_2}{\omega_1} = \frac{\cos\alpha}{1-0}$$

$$\frac{\omega_2}{\omega_1} = \cos\alpha$$

$$\boxed{\omega_2 = \omega_1 \times \cos\alpha}$$

5M. * <u>Polar velocity diagram</u>:-

The diagram which shows the Variation of Angular velocities of the driven shaft and driving shaft for one Complete revolution is known as Polar velocity diagram.



1) The Speed of the driven shaft is maximum when $\theta = 0^\circ$ or 180°, where as the speed is minimum then $\theta = 90^\circ$ or 270°

2) The speed of the driven shaft is equal to the speed of the driving shaft at 4 points.

3) The speed of the driving shaft is constant and hence it is represented by a circle of radius $= \omega_1$

4) The speed of the driven shaft is not constant. Therefore, the maximum value is $\frac{\omega_1}{\cos\alpha}$ and minimum value is $\omega_1 \times \cos\alpha$. It is represented by an ellipse of semi major axis $= \frac{\omega_1}{\cos\alpha}$ and semi minor axis $= \omega_1 \times \cos\alpha$

* Condition for maximum fluctuation speed of the driven shaft:-

Maximum fluctuation of the speed = max. speed − Min. Speed

$$= \frac{\omega_1}{\cos\alpha} - \omega_1 \times \cos\alpha$$

$$= \omega_1 \left[\frac{1}{\cos\alpha} - \cos\alpha\right]$$

$$= \omega_1 \left[\frac{1-\cos^2\alpha}{\cos\alpha}\right]$$

$$= \omega_1 \left[\frac{\sin^2\alpha}{\cos\alpha}\right]$$

$$= \omega_1 \left[\frac{\sin\alpha}{\cos\alpha} \times \sin\alpha\right]$$

$$= \omega_1 \left[\tan\alpha \times \sin\alpha\right]$$

where,

α is very small, so, $\sin\alpha = \tan\alpha = \alpha$

$$= \omega_1 \left[\alpha \times \alpha\right]$$

$= \omega_1 \times \alpha^2$

Fluctuation of the Speed = $\omega_1 \times \alpha^2$

* Condition for Angular Acceleration of the driven Shaft:

we know that the eq. (8) Can be written as

$$\frac{\omega_2}{\omega_1} = \frac{\cos\alpha}{1-\cos^2\theta\cdot\sin^2\alpha}$$

The Angular velocity of the driven shaft is given by,

$$\omega_2 = \frac{\omega_1\cos\alpha}{1-\cos^2\theta\cdot\sin^2\alpha}$$

The Angular acceleration of the driven shaft is,

$$\frac{d\omega_2}{dt} = \frac{\omega_1\cos\alpha}{1-\cos^2\theta\cdot\sin^2\alpha}$$

$$= \omega_1 \times \frac{d}{dt}\left[\frac{\cos\alpha}{1-\cos^2\theta\cdot\sin^2\alpha}\right]$$

$$= \omega_1 \times \cos\alpha \cdot \frac{d}{dt}\left[1-\cos^2\theta\cdot\sin^2\alpha\right]^{-1}$$

$$= \omega_1\cos\alpha\cdot(-1)\left[1-\cos^2\theta\sin^2\alpha\right]^{-2} \times$$

$$\left[0-\sin^2\alpha\times 2\cos\theta(-\sin\theta)\times\frac{d\theta}{dt}\right]$$

$$= \frac{-\omega_1\times\cos\alpha}{\left[1-\cos^2\theta\sin^2\alpha\right]^2}\times\left[-\sin^2\alpha\times-\sin 2\theta\times\frac{d\theta}{dt}\right]$$

$$= \frac{-\omega_1\times\cos\alpha}{\left[1-\cos^2\theta\sin^2\alpha\right]^2}\times\left[\sin^2\alpha\times\sin 2\theta\times\omega_1\right]$$

$$= \frac{-\omega_1^2\times\cos\alpha\times\sin^2\alpha\cdot\sin 2\theta}{\left[1-\cos^2\theta\sin^2\alpha\right]^2}$$

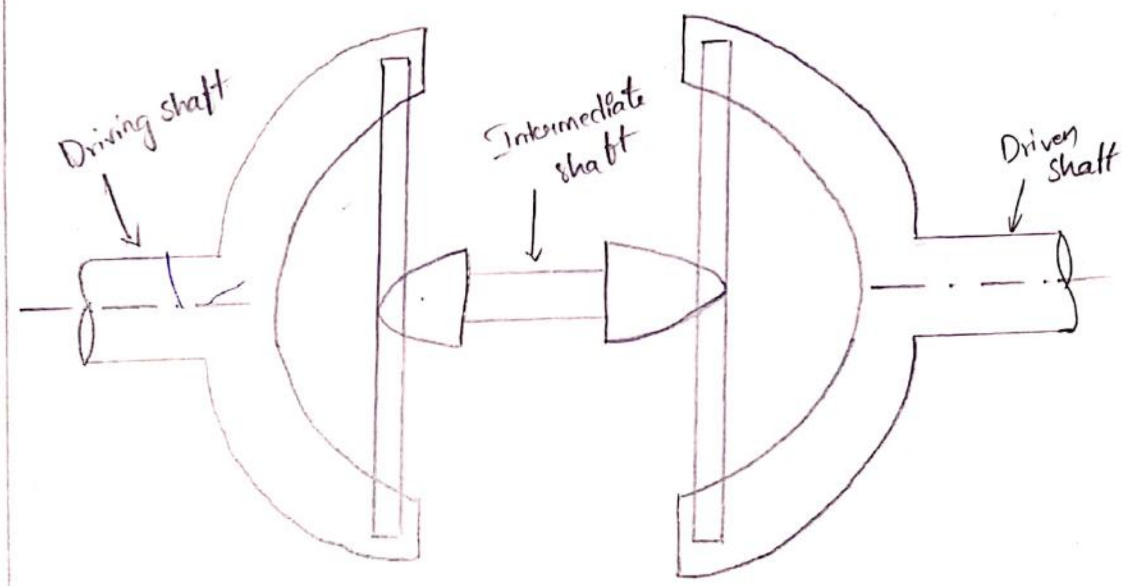
* Double Hooke's Joint:-

In a single Hooke's Joint, A driving shaft rotates at a constant speed whereas a driven shaft is varying speed. In order to have a constant velocity ratio of the driving and driven shaft a double hooke's Joint is used.

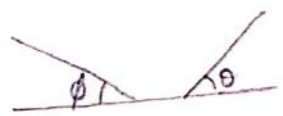
In a double Hook joint, two Hook Joints and intermediate shafts are used the speed of the driving & driven shafts will be equals, if.

1) The driving & driven shafts must be equally inclined to the intermediate shaft.

2) The two forces on the intermediate shaft lying in the same plane.



where,



θ = Angle turned by driving shaft

ϕ = Angle turned by driven shaft

γ = Angle turned by Intermediate shaft

α = Angle of inclination of the driving shaft with intermediate shaft

$$\tan\theta = \cos\alpha \cdot \tan\gamma$$

$$\tan\phi = \cos\alpha \cdot \tan\gamma$$

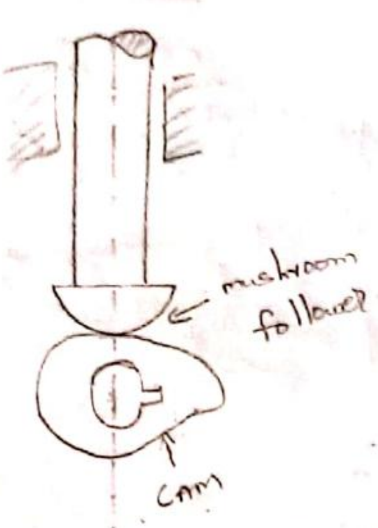
$$\therefore \tan\theta = \tan\phi$$

$$\boxed{\theta = \phi}$$

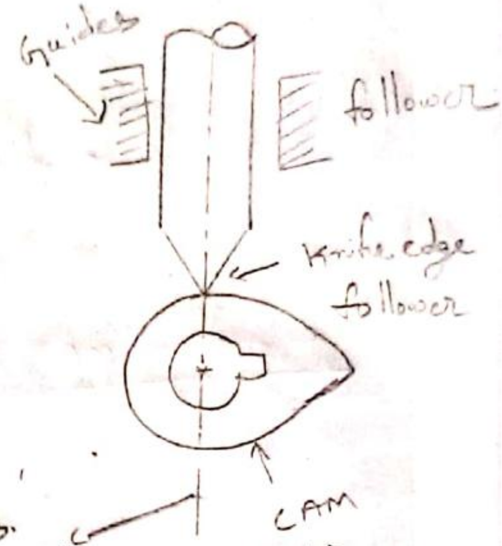
Unit - IV

CAMS

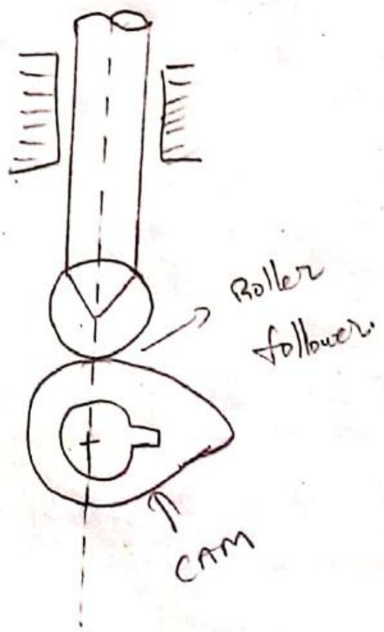
* mushroom follower:-



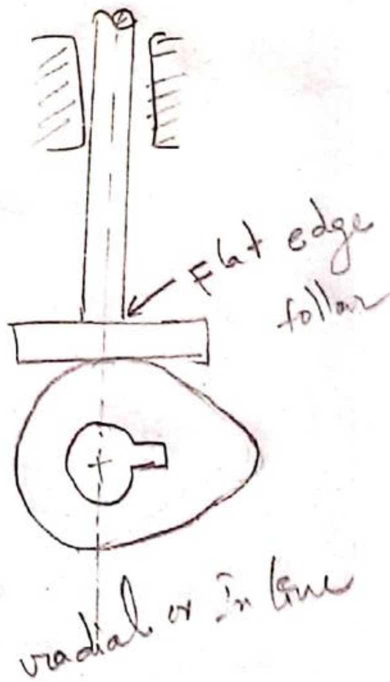
knife edge follower:-



Roller follower:-

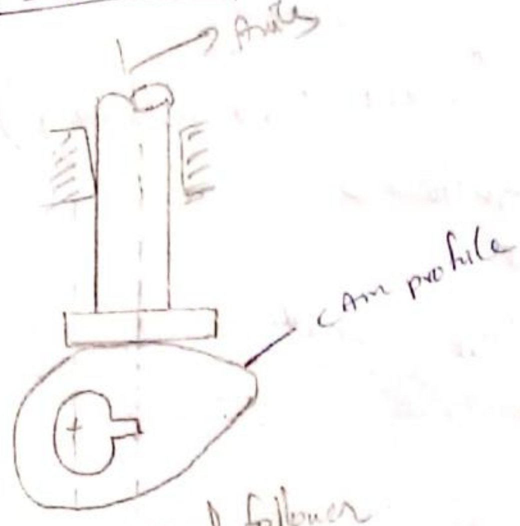


flat edge follower:-



radial or In line

OFF set follower;



* CAM's Follwer:- A rotating machine element which gives reciprocating or oscillating motion to a second element is known as 'CAM', The second element is known as Follower.

⇒ The CAM rotates at uniform(or)constant speed and drives the follower whose motion depends upon the shape of the CAM.

⇒ The CAM's are commonly used in IC Engines in printing machineries, Automatic machines in machine tools.

* Types of followers:- There are 3 types;

(1) According to the shape of the part which is in contact with the CAM

i) knife edge follower

ii) Roller follower

iii) flat edge foller

② According to the motion of the follower

i) Reciprocating follower (Translatory)

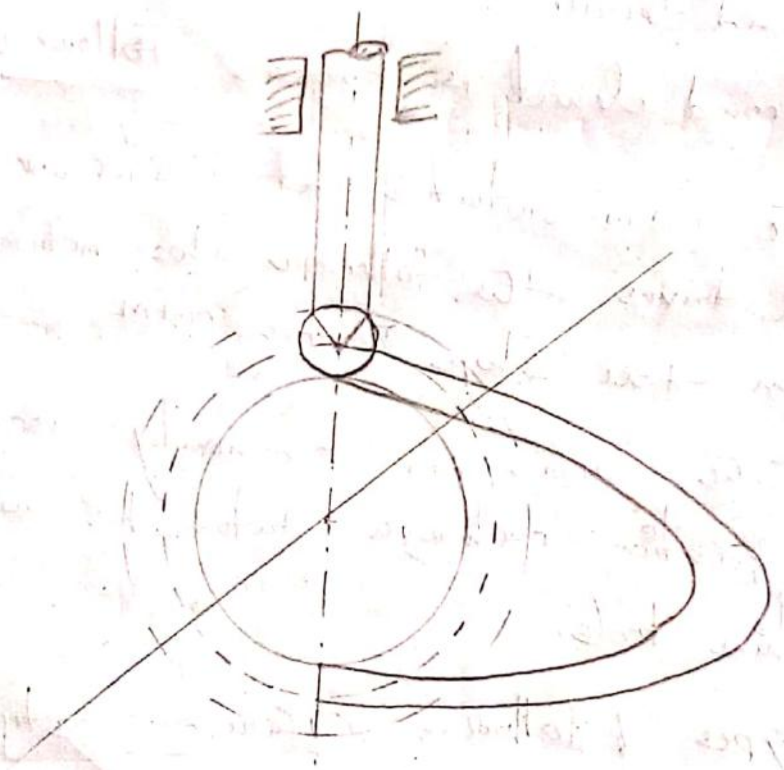
ii) oscillating follower

③ According to the location of the axis of the follower

i) ~~Infi~~ Inline or Radial follower

ii) OFF set follower.

2Ri* nomenclature of the CAM profile:-

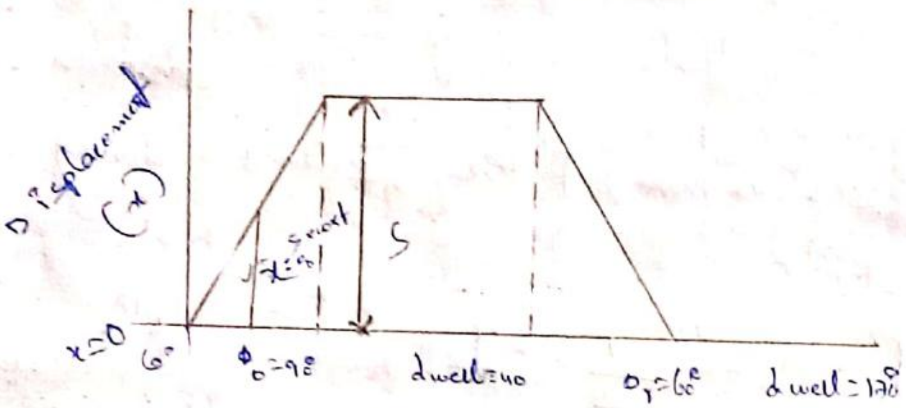


* motions of followers:-

1. uniform motion or uniform velocity
2. simple harmonic motion
3. uniform Acceleration & Retardation
4. cycloidal
5. Any other desired shape of motion.

① uniform motion or uniform velocity:-

Displacement Diagram:-



D. θ

x-axis - abscissa (Horizontal)
y-axis - ordinate (vertical)

where S = stroke of the follower

$x =$

w.k.T $\theta = \omega \times t \Rightarrow$ angular velocity × time.

$\theta_0 =$

$\therefore \frac{x}{\theta} = \frac{S}{\theta_0}$

$x = \frac{S}{\theta_0} \times \theta$

$\boxed{x = \frac{S}{\theta_0} \times \omega t}$ dis.

$$\text{velocity} = \frac{\text{displacement}}{\text{time}}$$

$$= \frac{dx}{dt}$$

$$= \frac{d}{dt}\left(\frac{s}{\theta_o} \times \omega \times t\right)$$

$$\boxed{V_o = \frac{s}{\theta_o} \times \omega \times 1}$$ V_o = velocity of outward stroke

$$\boxed{V_r = \frac{s}{\theta_r} \times \omega}$$ V_r = velocity of Return stroke

acceleration of the follower during outward stroke

$$f_o \text{ (or) } a_o = \frac{dv}{dt} = \text{const}$$ ~~velo~~ acceleration = $\frac{\text{velocity}}{\text{Time}}$

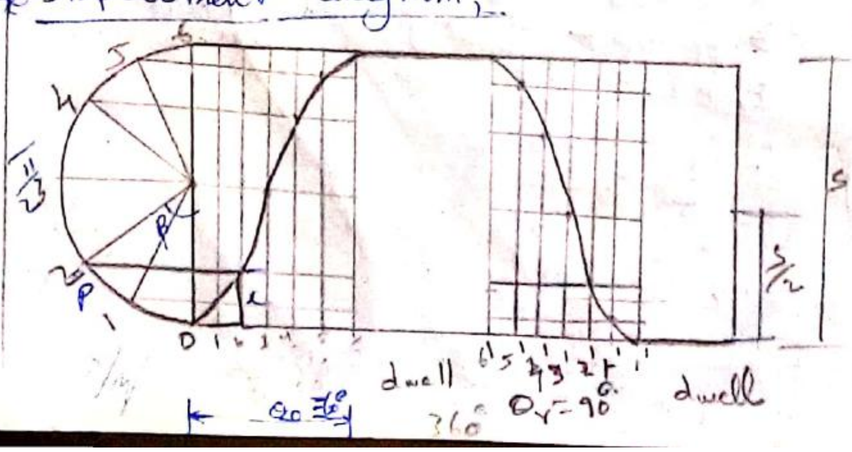
$$a_o = 0 \text{ or } \boxed{f_o = 0}$$

$$\boxed{f_r = 0}$$

10M

② Simple harmonic motion :-

Displacement diagram :-



w.k.T $\theta = \omega \times t$

$$\therefore \frac{x}{\theta} = \frac{S}{\theta_0}$$

$$x = \frac{S}{\theta_0} \times \theta$$

$$\boxed{\beta = \frac{S}{\theta_0} \times \theta}$$

$$\beta = \frac{S}{\theta_0} \times \theta = \frac{\pi}{\theta_0} \times \theta$$

now the displacement $x = \frac{S}{2} - \frac{S}{2}\cos\beta$

$$x = \frac{S}{2} - \frac{S}{2}\cos\left(\frac{\pi}{\theta_0} \times \theta\right)$$

$$V_0 = \frac{dx}{dt} = \frac{d}{dt}\left(\frac{S}{2} - \frac{S}{2}\cos\left(\frac{\pi}{\theta_0} \times \theta\right)\right)$$

$$\cancel{V_0 = \frac{S}{2} - \frac{S}{2}\sin\frac{\pi}{\theta_0} \times \theta}$$

$$V_0 = \frac{d}{dt}\left(\frac{S}{2}\left(1 - \cos\frac{\pi}{\theta_0} \times \omega \times T\right)\right)$$

$$\boxed{\therefore \theta = \omega \times t}$$

$$V_0 = \frac{S}{2}\left(0 - \left(-\sin\frac{\pi\omega t}{\theta_0}\right) \times \frac{\pi\omega}{\theta_0}\right)$$

$$= \frac{S}{2}\left(\sin\frac{\pi\omega t}{\theta_0} \times \frac{\pi\omega}{\theta_0}\right)$$

$$V_0 = \frac{S}{2}\left(\sin\frac{\pi\theta}{\theta_0}\right) \cdot \frac{\pi\omega}{\theta_0}$$

$$\boxed{V_0 = \frac{S}{2} \times \frac{\pi \times \omega}{\theta_0} \times \sin\left(\frac{\pi \times \theta}{\theta_0}\right)}$$

$V_{0(max)} = \sin\left(\frac{\pi\theta}{\theta_0}\right) = \sin\frac{\pi}{2}$

$$\frac{\pi\theta}{\theta_0} = \frac{\pi}{2}$$

$$\theta = \frac{\theta_0}{2}$$

$$V_{0(max)} = \frac{S}{2} \times \frac{\pi \times \omega}{\theta_0} \times \sin\left(\frac{\pi \times \theta_0}{2 \times \theta_0}\right)$$

$$\boxed{V_{0(max)} = \frac{S}{2} \times \frac{\pi \times \omega}{\theta_0}}$$ outward stroke

IIly $$\boxed{V_{r(max)} = \frac{S}{2} \times \frac{\pi \times \omega}{\theta_r}}$$ return stroke.

IIly acceleration (a_0) :- $\frac{dv_0}{dt}$

$$= \frac{d}{dt}\left(\frac{S}{2} \times \frac{\pi \times \omega}{\theta_0} \times \sin\frac{\pi \cdot \omega \times t}{\theta_0}\right)$$

$$= \left(\frac{S}{2} \times \frac{\pi \times \omega}{\theta_0} \times \cos\frac{\pi\omega t}{\theta_0} \times \frac{\pi\omega}{\theta_0}\right)$$

$$= \frac{S}{2}\left(\frac{\pi \times \omega}{\theta_0}\right)^2 \cos\frac{\pi\theta}{\theta_0}$$

max. acceleration $a_{0(max)} = \cos\frac{\pi\theta}{\theta_0} = \cos 0$

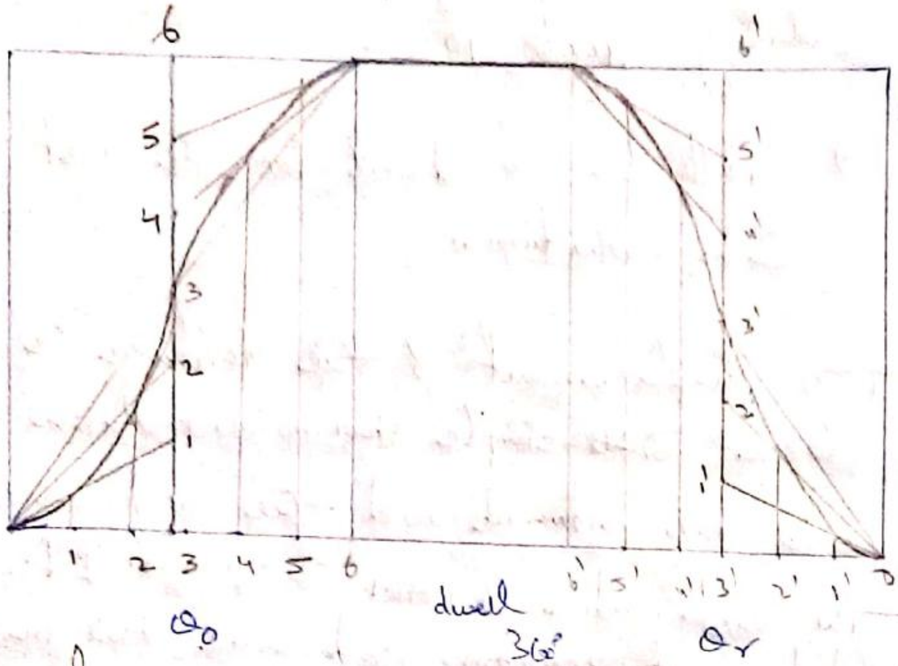
$$a_{0(max)} = \frac{S}{2}\left(\frac{\pi \times \omega}{\theta_0}\right)^2 \times \cos 0$$

$$\boxed{a_{0(max)} = \frac{S}{2}\left(\frac{\pi \times \omega}{\theta_0}\right)^2}$$ outward

$$\boxed{a_{r(max)} = \frac{S}{2}\left(\frac{\pi \times \omega}{\theta_r}\right)^2}$$ return

3. Uniform Acceleration & Retardation :-

Displacement diagram :



Formulas

$$\boxed{V_{0\,(max)} = \frac{2 \times S \times \omega}{\theta_0}}$$

$$\boxed{V_{r\,(max)} = \frac{2 \times S \times \omega}{\theta_r}}$$

$$\boxed{a_{0\,(max)} = \frac{4 \times S \times \omega^2}{\theta_0^2}}$$

$$\boxed{a_{r\,(max)} = \frac{4 \times S \times \omega^2}{\theta_r^2}}$$

Problems :- Draw the profile of a cam operating a knife edge follower when the axis of the follower passes through the axis of the cam shaft from the following data.

(i) follower to move outwards through 40mm during 60° of cam rotations

② Follower to dwell for the next 45°.

③ follower to return to the original position during next 90°.

④ Follower to dwell for the rest of the Cam rotation.

The displacement of the follower is to take place with simple Harmonic motion during both the outward and the returns stroke. The least radius of the CAM is 50mm. If the cam rotates at 300rpm. Determine the max velocity and acceleration of the followers during outward stroke & Return stroke.

Solution: Given data

~~S=~~ stroke of the follower (S) = 40mm

least radius of the CAm = 50mm

Cam rotates at N = 300rpm

velocity at outward :-

$\omega = \frac{2\pi N}{60} = 31.41$

① $V_0(max) = \frac{S}{2} \times \frac{\pi \times \omega}{\theta_0} = \frac{40}{2} \times \frac{\pi \times \frac{2\pi N}{60}}{\frac{60 \times \pi}{180}}$

~~1884.6~~ m/sec

$\omega = \frac{2\pi N}{60}$

$= \frac{2\pi\, 300}{60} = 31.41$

$= 1884.6mm$

$= 1.8846\, m/sec$

Note 2up
horizontal 10mm = 20°
vertical 10mm = 10mm

① velocity at Return stroke

$V_{r(max)} = \frac{S}{2} \times \frac{\pi \times \omega}{\theta_r}$

$= \frac{40}{2} \times \frac{\pi \times \frac{2\pi N}{60}}{90 \times \frac{\pi}{180}}$

= 1256.4 mm

= 1.256 m/se.

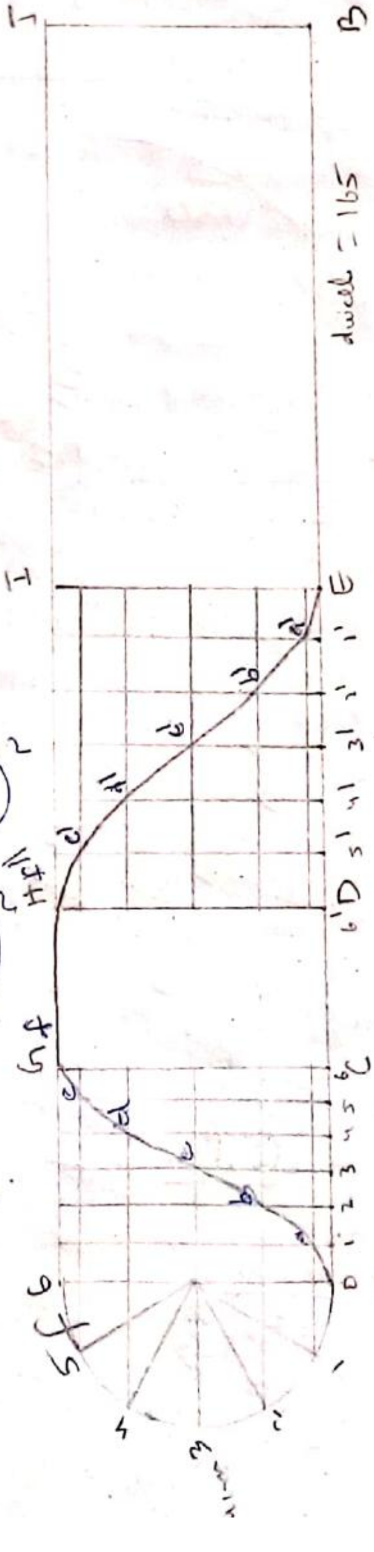
② ~~velocity~~ acceleration at outward:

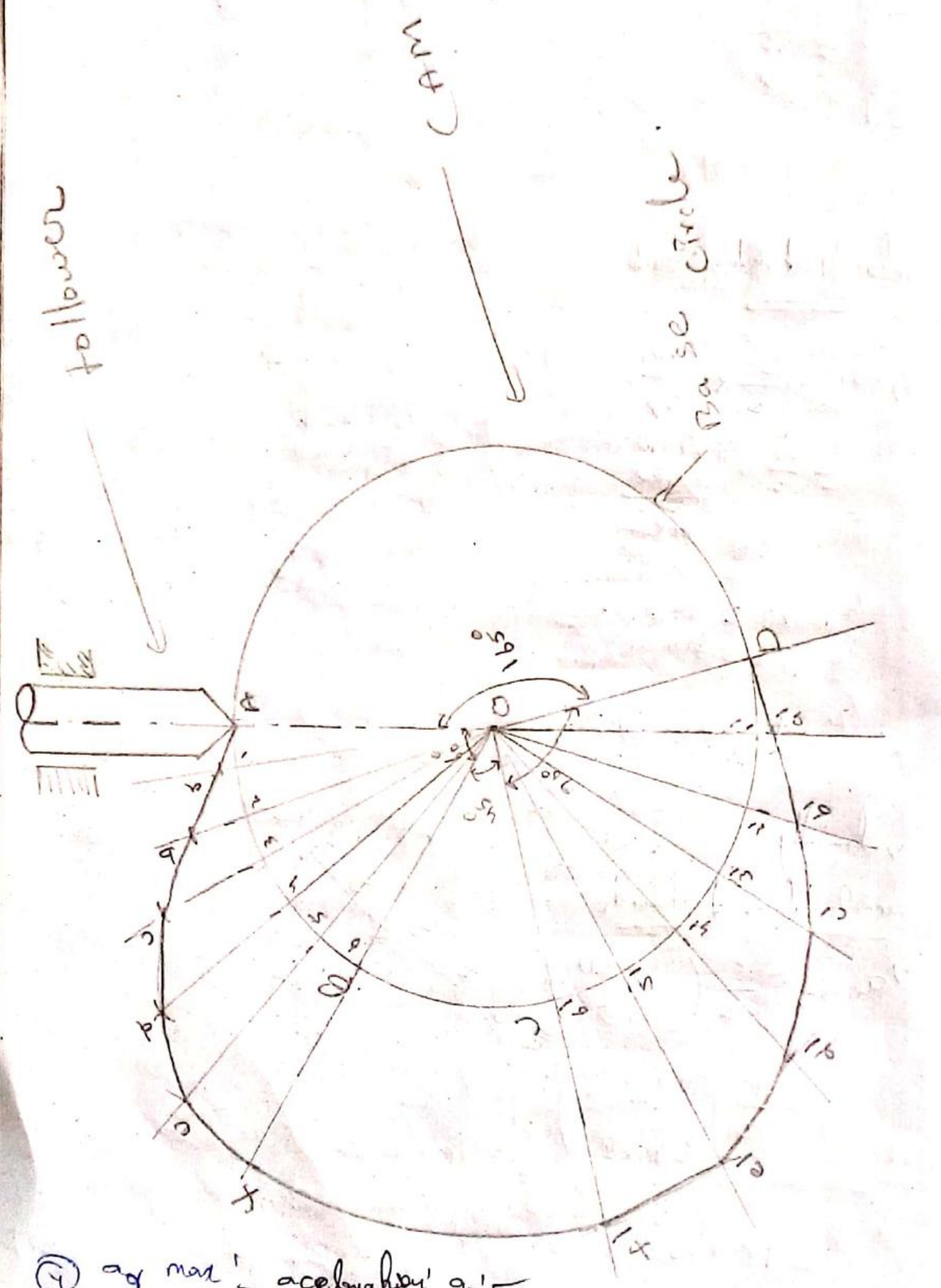
$a_{o(max)} = \frac{S}{2}\left(\frac{\pi \times \omega}{\theta_o}\right)^2$

$= \frac{40}{2}\left(\frac{\pi \times \frac{2\pi N}{60}}{60 \times \frac{\pi}{180}}\right)^2$

= 17758585

= 17.758585 m





(4) a_r max :- acceleration a_0 :-

$$a_r = \frac{s}{2}\left(\frac{\pi \times \omega}{\theta_r}\right)^2$$

$$= \frac{40}{2}\left(\frac{\pi \times \frac{271 N}{60}}{90 \times \frac{\pi}{180}}\right)^2$$

$= 78927.048$

$= 789.27$ //

(2) Draw the profile of a cam operating a knife edge follower when the axis of the follower is not passing through the axis of cam shaft, the it is hatoffset by 20mm from the axis of the cam shaft, Then draw the profile of the cam

(1) follower to move outwards through 40mm during 60° of cam rotation

(2) Follower to dwell the next 45°

(3) Follower to return to the original position during next 90°.

(4) Follower to dwell for the next angle of cam rotation

The displacement of the follower is simple harmonic motion. Both the out ward & return, The least radius is 50mm, cam rotates at 300rpm.

$$v_o = \frac{s}{2} \times \frac{\pi \times \omega}{\theta_o}, \quad a_o = \frac{s}{2}\left(\frac{\pi\omega}{\theta_o}\right)^2$$

$$v_r = \frac{s}{2} \times \frac{\pi \times \omega}{\theta_r} \quad a_r = \frac{s}{2}\left(\frac{\pi\omega}{\theta_r}\right)^2$$

Sol

Displacement Diagram

Same as 1st question.

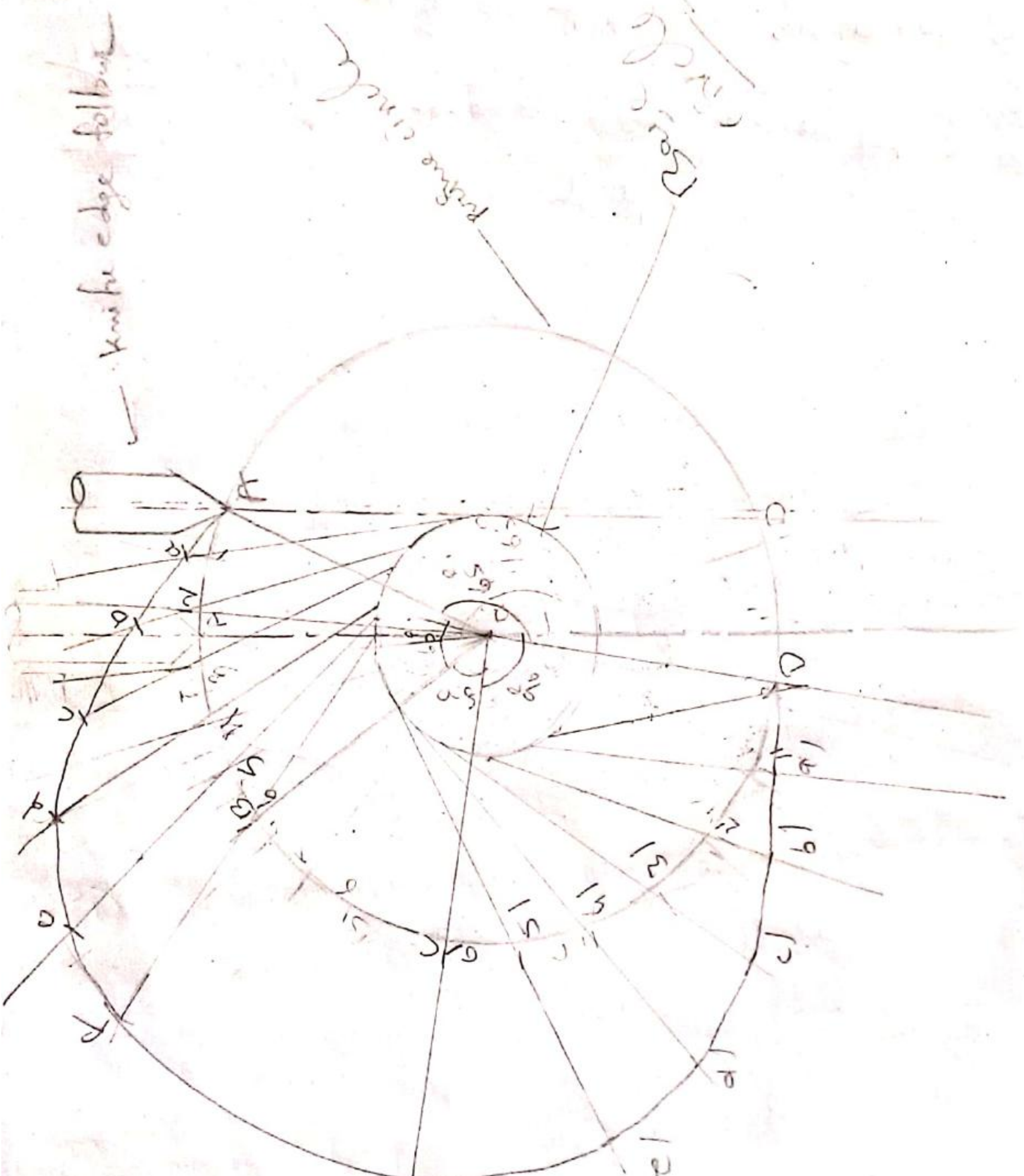
Calculations:—

velocity v_o = 1.886 m/sec

v_r = 1.256 m/sec

a_o = 17.75 m/s

a_r = 7.59.27 m/s



(3) Draw the profile of a cam operating ~~a~~ by ~~a~~ when the axis of the follower passes through the axis of cam

① Follower to move outward through 30mm with simple harmonic motion during 120° of cam rotation.

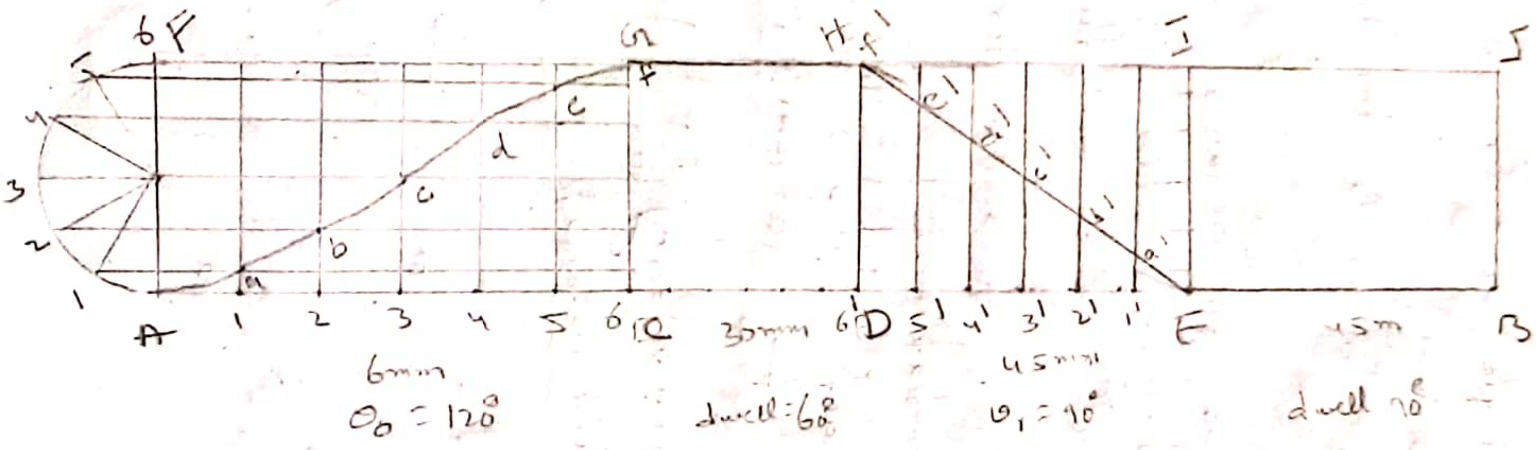
② follower to dwell for the next 60°.

③ Follower to move return through the original ~~condition~~ position with uniform velocity during 90° of cam rotation.

④ Follower to dwell for the rest of the cam rotation.

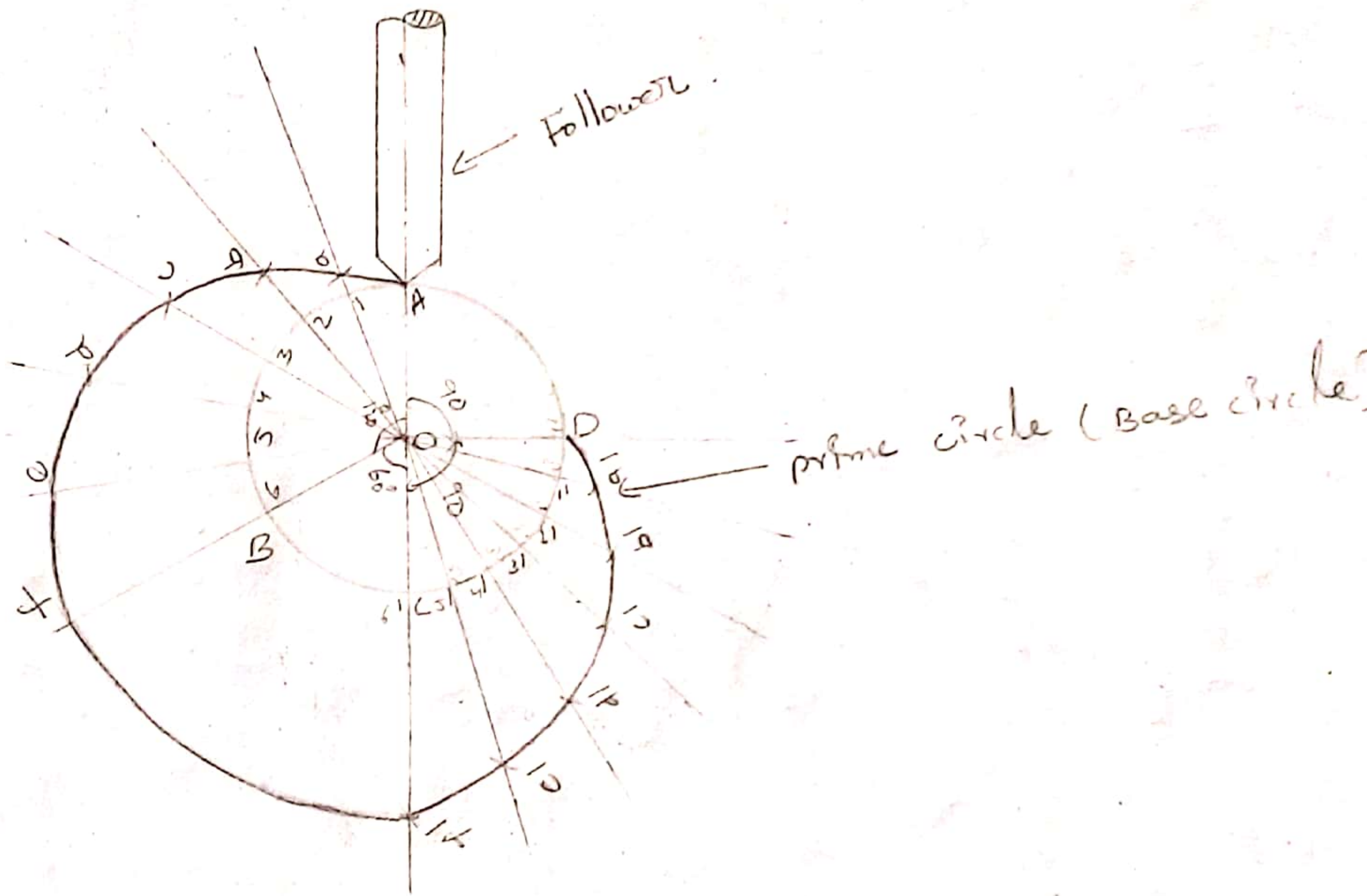
The least radius of the cam is 20mm and cam rotates at 240 r.p.m, Determine :-

① maximum velocity for outstroke, return stroke and maximum acceleration to ~~outst~~, return stroke
outward & return stroke



16 mm = 20°

60mm = 120°


Follower
prime circle (Base circle)
A
B
D
O
90
60

(4) Draw the profile of a cam, operating a knife edge follower from the following data.

(1) follower to move outward stroke through a distance of 20mm during 120° of cam rotation

(2) follower to dwell for the next 60° of cam rotation

(3) follower to return stroke to the original position during 90° of cam rotation.

(4) Follower to dwell for the remaining 90° of cam rotation

The cam is rotating clockwise at a uniform speed of 500 r.p.m the minimum radius of the cam is 40mm and the line of stroke of the follower is offset 15 mm from the axis of the cam & the displacement of the follower with uniform acceleration and retardation for both outward and return strokes (1) Determine max. velocity of the follower during outward and return stroke.

(2) The max. acceleration during outward and return stroke.

Calculation

$$V_{o(max)} = \frac{2\times S\times \omega}{\theta_o}$$

$$= \frac{2\times 20\times 52.35}{120\times\frac{\pi}{180}} = 999.81 \text{ mm} \Rightarrow 0.999 \text{ m/sec}$$

$$\therefore \omega = \frac{2\pi N}{60} = \frac{2\times\pi\times 500}{60} = 52.35$$

$$V_{r(max)} = \frac{2\times S\times \omega}{\theta_r}$$

$$= \frac{2\times 20\times 52.35}{90\times\frac{\pi}{180}}$$

$= 1333.08$ mm

$= \cancel{13.33}$ m/sec.

1.333 m/sec.

$$a_o = \frac{4\times S\times \omega^2}{\theta_o^2}$$

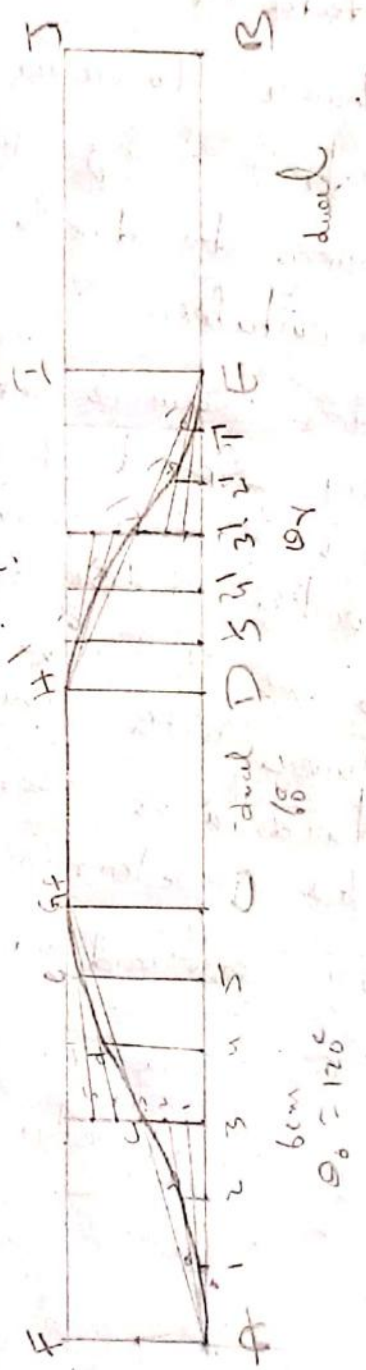
$= 49981.1$ mm

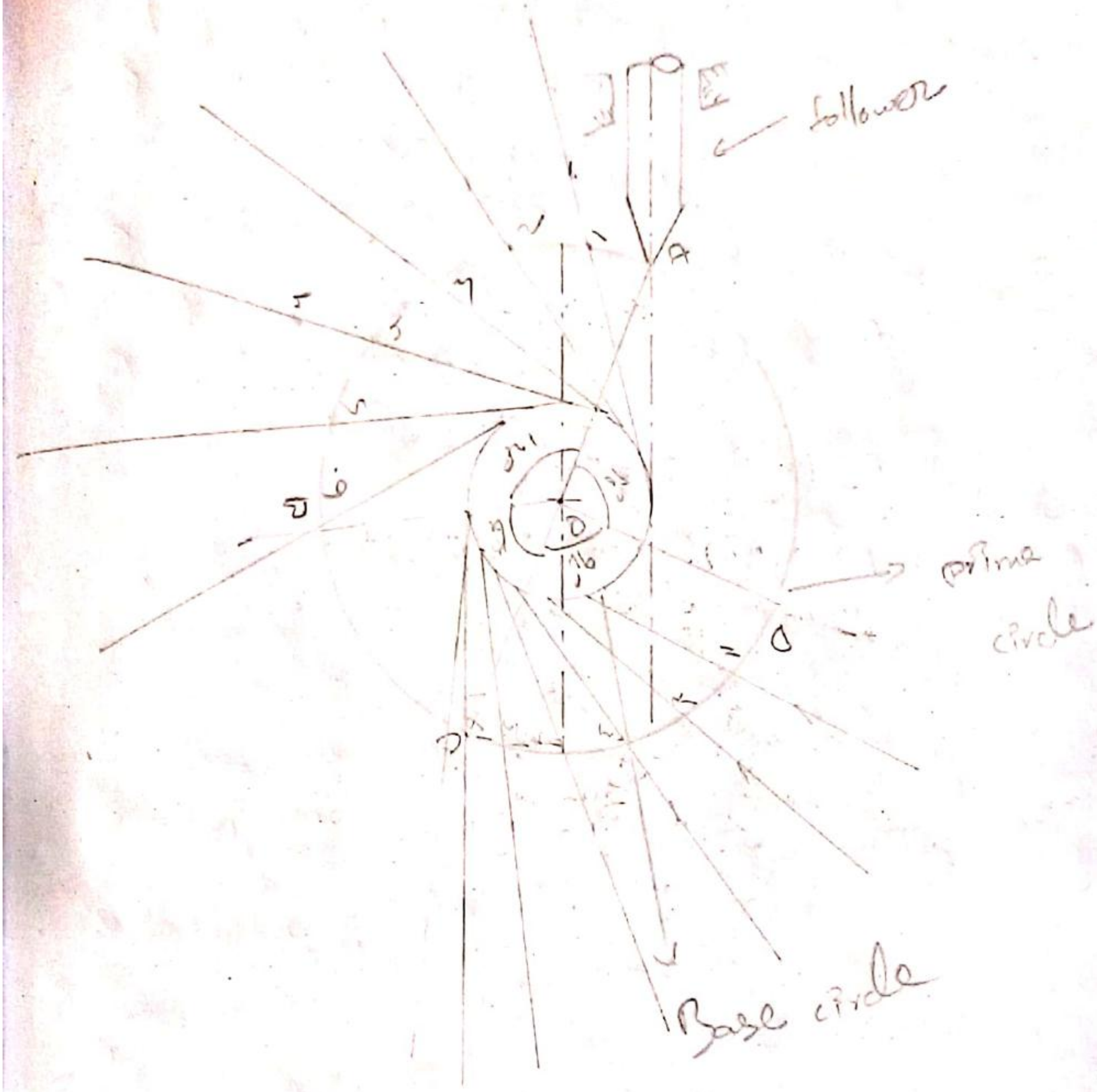
$= 49.981$ m/sec

$$a_r = \frac{4\times S\times \omega^2}{\theta_r^2}$$

$= 88855.25$ mm

$= 88.85$ m/sec





Scanned by CamScanner

5 A cam ~~with~~ is 30mm diameter is rotating clock wise at a uniform speed ~~at~~ of 1200 RPM & motion of a roller follower is 10mm diameter

① follower to outward stroke of 35 mm during 120° of cam rotation with uniform acceleration and Retardation.

② follower to dwell 60° of cam rotation

③ follower to return stroke 90° with uniform acceleration and retardation

④ Follower to dwell for the remaining 90° of cam rotation

Draw the cam profile if the axis of roller follower passes through the axis of the cam. Determine $V_{o\,max}$, $V_{r\,max}$, $a_{o\,max}$, $a_{r\,max}$.

Calculations:

$$v_{0(max)} = \frac{2\times S\times 10}{\theta_0}$$

$$\omega = \frac{2\pi N}{60} = \frac{2\times\pi\times \cancel{1200}\ 125.66}{\frac{120\times\pi}{180}}$$

$$= \frac{2\times\pi\times 1200}{60} = \cancel{3600}\ mm$$

$$= 125.66 = \cancel{2.6}\ m/sec.$$

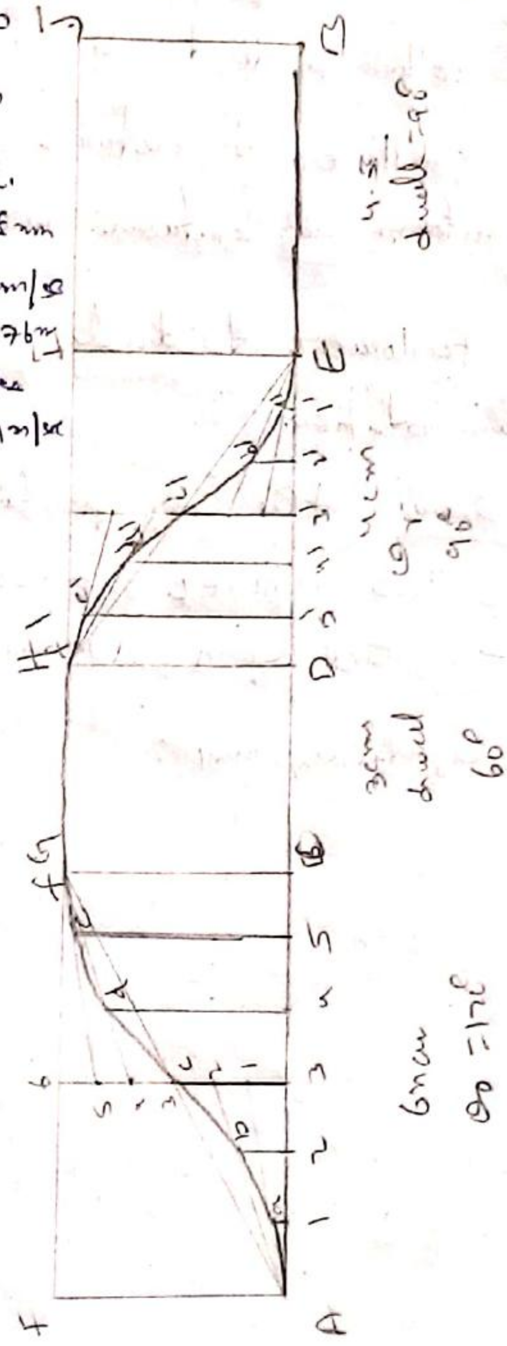
$$= \cancel{376.98}\ mm$$

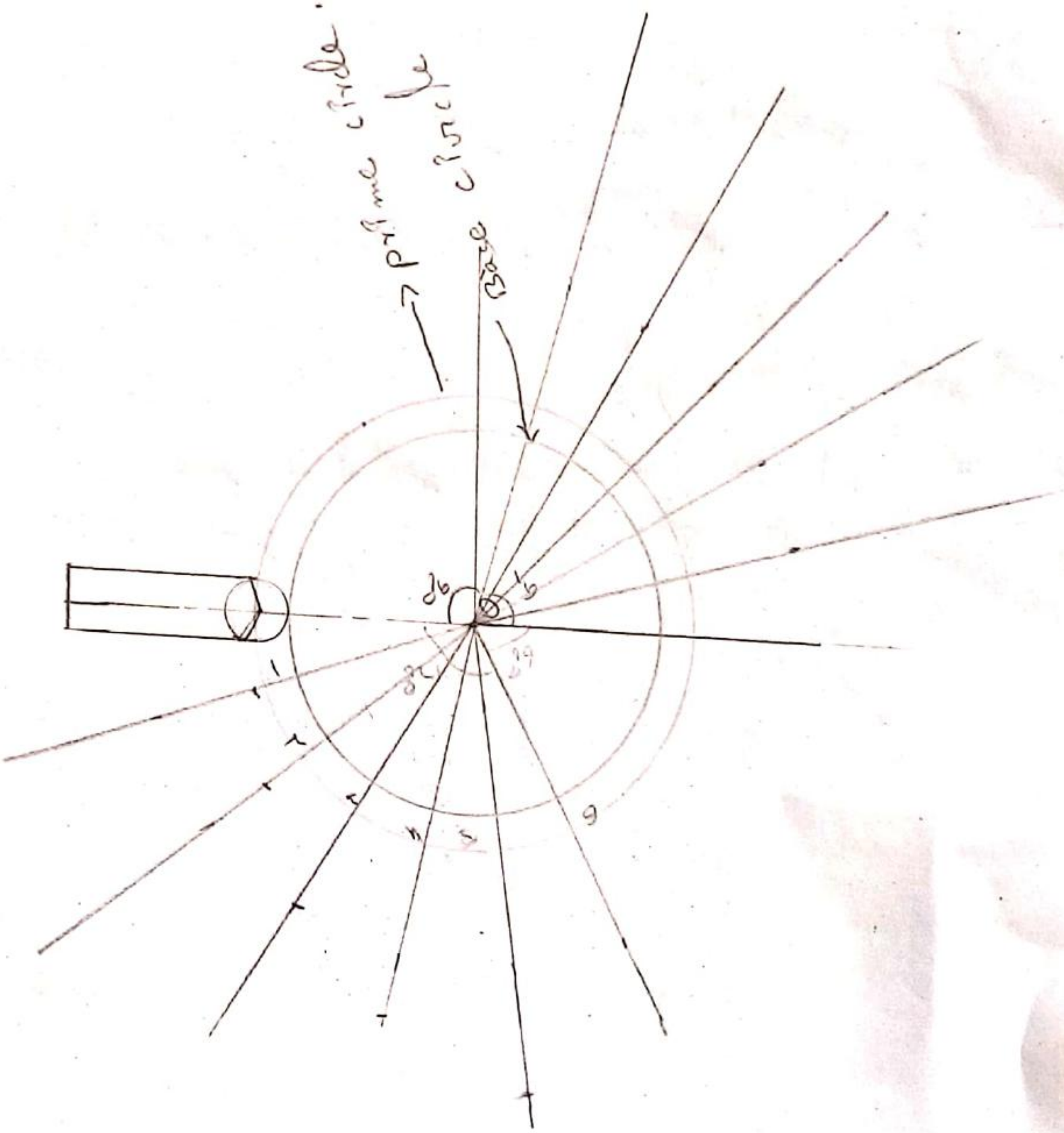
$$= 0.576\ m/s$$

$$= 4199.876\ m$$

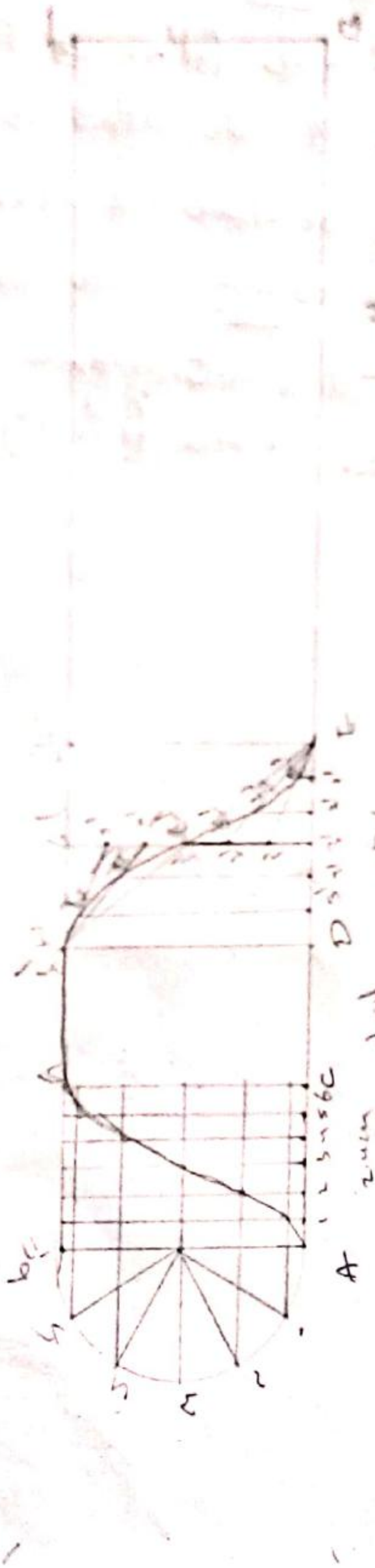
$$= \cancel{41.99}$$

$$V_{r(max)} = 2\cancel{x} = 4.199\ m/sec$$




prime circle
Base circle

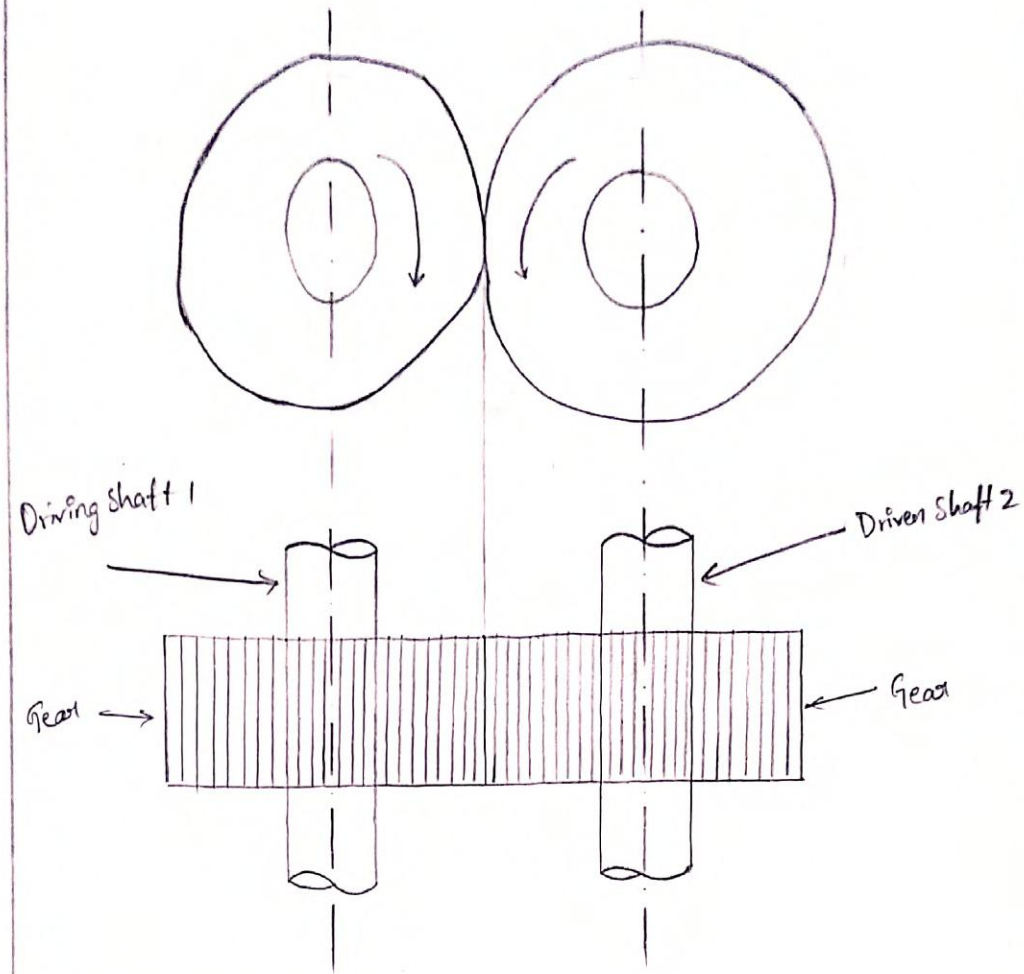
(6) Draw the profile of a cam, the follower moves with simple harmonic motion during ascent while it moves with uniform accelerated and decelerated during descent. least radius of the cam is 15mm, angle of ascent $\theta_o = 48^\circ$ angle of dwell 42°, angle of descent $\theta_d = 60$ and lift of the follower is 40mm and diameter of the roller is 30mm. Distance b/w line of action of the follower and axis of cam is 20mm, The cam rotates at 360 Rpm. Find the maximum velocity and acceleration of the follower during descent period



16/03/2020

Unit-V

Gear Trains



* **Gear Trains:-**

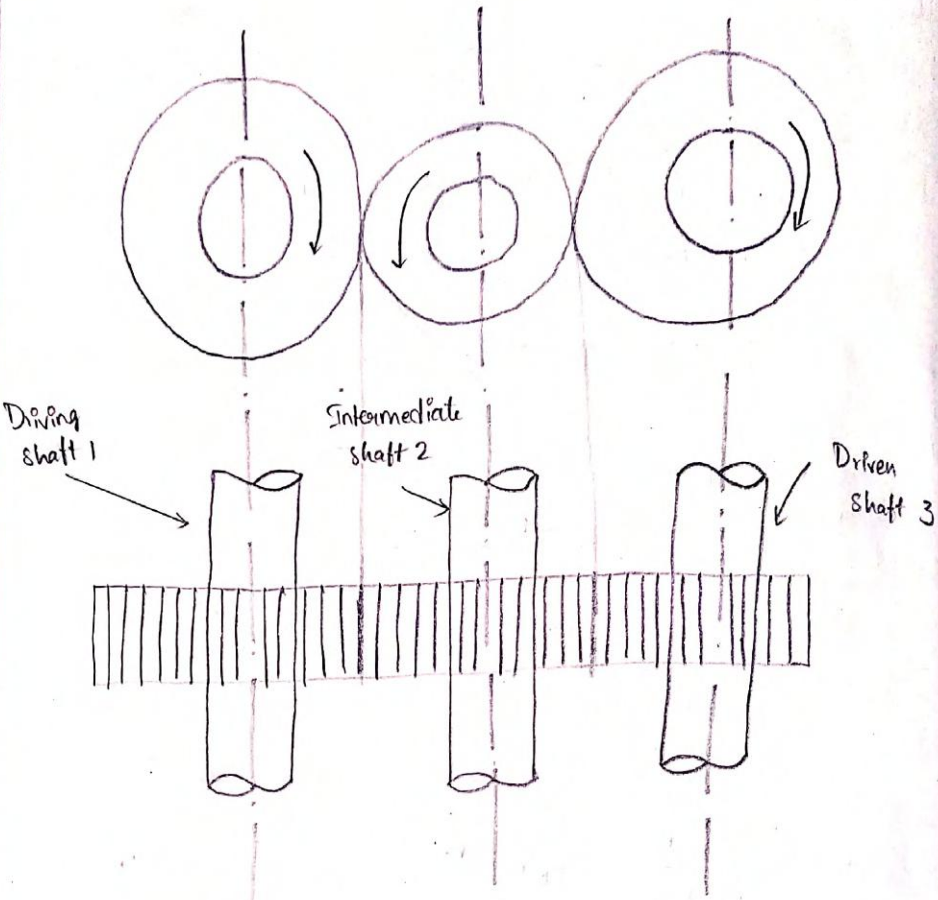
A Combination of two (or) more gears which are arranged in such a way that power is transmitted from a driving shaft to a driven shaft is known Gear Train. The Gear Train may Consist as spur gear, Bevel gear (or) Spiral Gear.

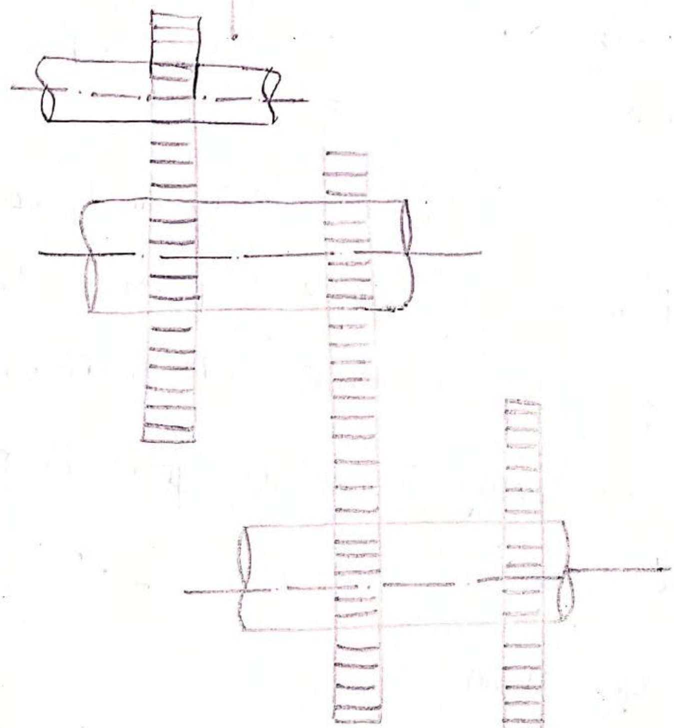
* **Types of Gear Train:-**

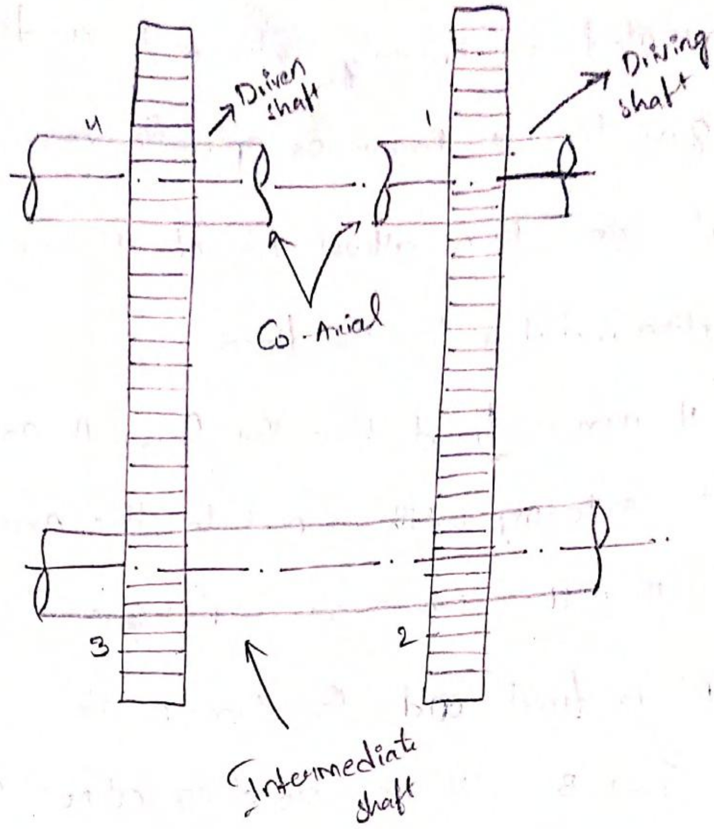
1) Simple Gear Train
2) Compound Gear Train

3) Reverted Gear Train

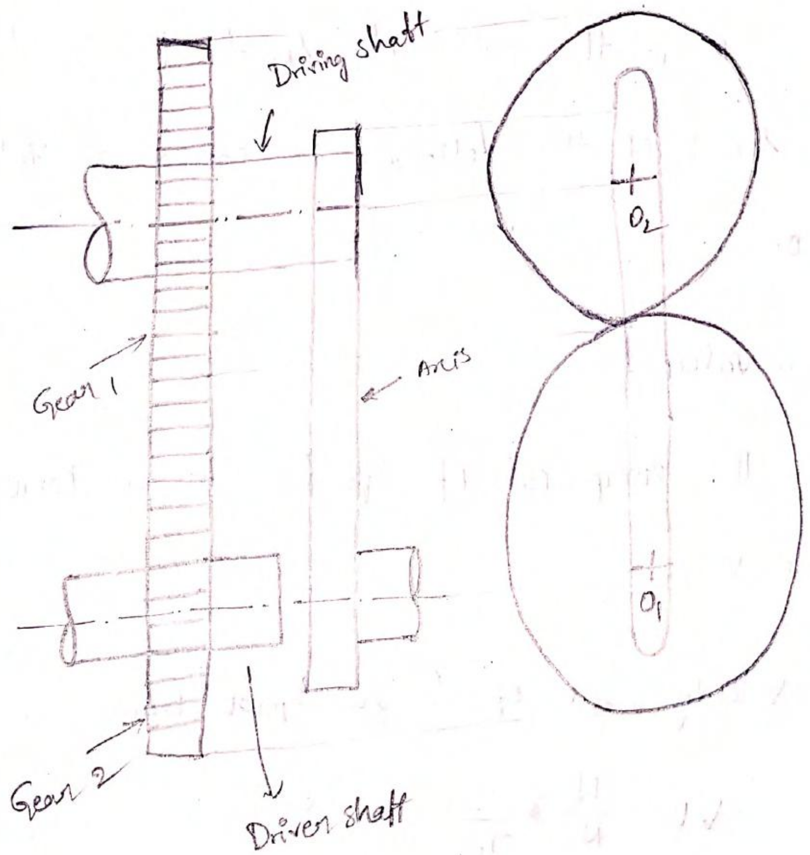
4) Epicyclic Gear Train







***Epicyclic Gear Train:-**



If the axis of the shafts over which the gears are mounted are moving relative to a fixed axis, The gear train is known as epicyclic gear train. In an epicyclic gear train atleast one of the gear axis is in motion relative to the frame.

If arm is fixed, then the Gear 'A' and Gear 'B' will be rotating with respect to the axis of the shaft. Then it is known as Simple gear train. But, if gear 'A' is fixed and the Arm is rotated about 'a' then Gear 'B' will be rotating about Gear 'A' and we get Epicyclic Gear Train.

* Velocity Ratio of Gear Trains:-

It is the ratio of speed of the driver to the speed of the follower is known as Velocity ratio.

* Train Value:-

The reciprocal of Speed ratio is known as Train Value.

* The Velocity ratio of Simple Gear Train:-

$$V.R = \frac{N_1}{N_2} = \frac{T_2}{T_1}$$

$$\text{Train value} = \frac{N_2}{N_1} = \frac{T_1}{T_2}$$

} for two gears

$$V.R = \frac{N_1}{\cancel{N_2}} \times \frac{\cancel{N_2}}{N_3} = \frac{\cancel{T_2}}{T_1} \times \frac{T_3}{\cancel{T_2}}$$ for three gears

$$\Rightarrow \frac{N_1}{N_3} = \frac{T_3}{T_1}$$

Problems

1) A Simple Gear Train Conisist of 2 geare only. Each gear mounted on Seperate shafts. The shafts are Parallel The Gear '1' is driving. the gear '2'. The Speed of the gear '1' is 1000 RPM. The no. of teeth on Gear '1' and '2' are 24 and 60. Determine

i) Speed Ratio of Gear train

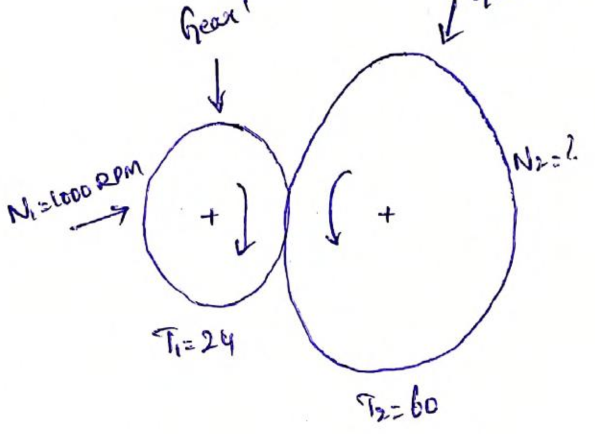
ii) Train value of " "

iii) Speed of the 2nd gear

iv) Direction of rotation of the 2nd gear if 1st gear is rotating clockwise

A.

i) Speed ratio $= \frac{N_1}{N_2} = \frac{T_2}{T_1}$



$= \frac{60}{24}$

$= 2.5$

$\frac{N_1}{N_2} = 2.5$

$\frac{1000}{2.5} = N_2$

$N_2 = 400$ RPM.