Define waves and particles:

Particels:

1. It is very easy to understand particles , Since physical quantity and can be seen physically
2. It has a well defined mass and occupies a particular position
3. If its position changes it leads to its velocity
4. The mass and velocity gives its momentum and hence energy
5. Therefore the characteristics of particles are
   - Mass
   - Velocity
   - Momentum
   - Energy

Waves:

1. It is not a physical quantity and cannot be seen physically
2. It is nothing but spreading of disturbances
3. The characteristics of waves are
   - Wavelength
   - Frequency
   - Time period
   - Amplitude
   - Phase
   - Intensity

Explain the concept of dual nature of light (or)
What are matter waves: Obtain the wavelength of matter waves. (or)
Show that wavelength "λ" associated with the electron of mass "m" and K.E "E" is given by $\lambda = \frac{h}{\sqrt{2mE}}$
(or) Discuss the de Broglies Hypothesis of duality of material particle.

Like radiations (ie., visible light, X-rays, UV rays) **material particles also exhibits dual nature** (ie., both particle and wave nature). This concept is given by the French Scientist who is called de Broglie, according to him when a material particle of mass "m", moving with velocity "v" will be associated with some sort of waves. Those waves are called as **de Broglie waves or particle waves or matter waves.**
The wavelength associated with the particle can be calculated in the following way.

Since this concept takes radiation as the basis, for the electromagnetic radiation that is light. According to Planck's Quantum theory, the light is nothing but emission of particles known as photons whose energy is given by

$E=h\nu$ ------ (1)

The Einstein's mass-energy relation is given by

$E=mc^2$ ------- (2)

From eq (1) & (2)
$h\nu = mc^2$

but $c= \nu\lambda$, $\nu = \frac{c}{\lambda}$

$h \times \frac{c}{\lambda} = mc^2$

$\lambda = \frac{h}{mc}$

m- mass of photon
c-velocity of photon

The above equation is showing the wavelength of light, but for a material particle (like electron, proton, or neutron, etc) the wavelength associated with it when it is moving with a velocity “v” is given by

$$\lambda = \frac{h}{mv}$$

m- mass of the particle
v- velocity of photon

Wavelength of de Broglie waves in terms of Kinetic energy:

$$E = \frac{1}{2} mv^2$$

$$v^2 = \frac{2E}{m}$$

$$v = \sqrt{\frac{2E}{m}}$$

$$\lambda = \frac{h}{m\sqrt{\frac{2E}{m}}} = \frac{h}{\sqrt{2mE}}$$

Wave length of Electron

If the particle is electron, then e^- will be accelerated by means of the potential “V”, then

$$eV = \frac{1}{2} mv^2$$

(Potential energy) (K.E of the electron)

$$v = \sqrt{\frac{2eV}{m}}$$

$$\lambda = \frac{h}{m\sqrt{\frac{2eV}{m}}} = \frac{h}{\sqrt{2eVm}}$$

If the particle is e^-

$$\lambda = \frac{6.624 \times 10^{-34}}{\sqrt{2 \times 1.602 \times 10^{-19} \times \mathbf{V} \times 9.11 \times 10^{-31}}} = \frac{12.27}{\sqrt{V}} \text{Å}$$

Properties of de Broglies waves:

We know that the wavelength of the de Broglies waves,

$$\lambda = \frac{h}{mv}$$

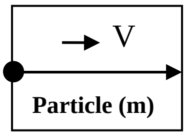
from the above we can tell that ,

1. If the mass(m) of the particle is less the wavelength(λ) is more
2. If the velocity (v) of the particle is less the wavelength(λ) is more
3. If the particle is at rest (v=0), the wavelength of the particle is “∞"
4. Both charged and uncharged particles can capable of acting as waves when they are in motion.
5. The velocity of de Broglie’s waves is more than the light waves.

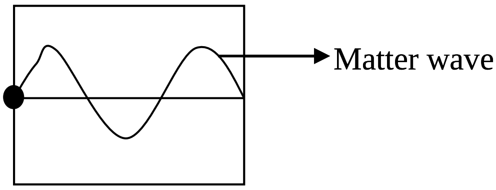
6. The de Broglies waves are not electromagnetic waves; they are the guiding waves or called pilot waves.
7. The wave nature of matter introduces the uncertainty in the predicting of the position of the particle

<u>Heisenberg's Uncertainty Principle:</u>

According to classical theory, each and every point has a fixed position and momentum in a given region, both position and momentum can be predicted simultaneously accurately at any time. The position of the particle is given by **x= vt,** the momentum is **p=mv**, therefore the position in terms of momentum is x=pt/m



According to de Broglie ,the moving particle is associated with matter waves, if it is the case , it is difficult to measure both position and momentum simultaneously accurately .If we try to calculated both of them simultaneously ,**error or uncertainty occurs**, therefore according to Heisenberg's Uncertainty principle, **<u>both position and momentum cannot be calculated simultaneously, accurately,</u>** if they are measured simultaneously, then error occurs ,if



The error in position is **Δx**
The error in momentum is **Δp**

Then $\boxed{\Delta x.\ \Delta p \geq \frac{\hbar}{2}\ or\ \frac{h}{4\pi}}$

As a result we can say that, it is impossible to conduct an experiment which is capable of demonstrating both particle and wave nature.

Similarly the two more parameter, energy and time cannot be measured accurately simultaneously

Then $\boxed{\Delta E.\ \Delta t\ \geq \frac{\hbar}{2}\ or\ \frac{h}{4\pi}}$

Applications of uncertainty Principle:

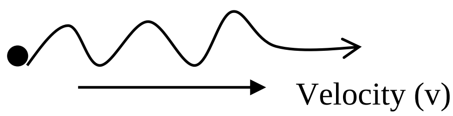
- This principle used to prove electrons are not persisting inside the nucleus
- Existence of protons, neutrons confirmed
- Binding energy of an electron in atoms can be calculated

Schrödinger's wave equation:

According to de Broglie hypothesis, the particle in motion always associated with waves. To describe the motion of such a particle which acts as waves , Schrödinger derived a equation known as Schrödinger's equation .
Let us consider a particle of mass "m" moving with velocity "v" along the x-direction. The displacement of the wave associated with the particle is given by means of the Classical wave equation is

$y = A \sin \frac{2\pi x}{\lambda}$

A-Amplitude



For the de Broglie's waves

$\Psi = \mathbf{A \sin} \frac{2\pi x}{\lambda}$

$\frac{\partial \psi}{\partial x} = A \left(\frac{2\pi}{\lambda}\right) \cos \frac{2\pi x}{\lambda}$

$$\frac{\partial^2\psi}{\partial x^2} = - A \left(\frac{2\pi}{\lambda}\right)\left(\frac{2\pi}{\lambda}\right) \text{Sin } \frac{2\pi x}{\lambda}$$

$$\frac{\partial^2\psi}{\partial x^2} = - \left(\frac{4\pi^2}{\lambda^2}\right) \Psi$$

$$\frac{1}{\lambda^2} = - \frac{\partial^2\psi}{\partial x^2}\frac{1}{4\pi^2 \psi} \quad \text{------------(1)}$$

This equation is meant for "de broglie's waves"

W.K.T

$$\lambda = \frac{h}{mv} = \frac{h}{p}$$

$$p = \frac{h}{\lambda}$$

but, $K.E = \frac{1}{2} mv^2 = \frac{1}{2} mv^2 \times \frac{m}{m} = \frac{p^2}{2m}$

$$K.E = \frac{h^2}{2m\lambda^2},$$

if we substitute the value for $\frac{1}{\lambda^2}$ in equation 1 gives,

$$K.E = - \frac{h^2}{2m}\frac{1}{4\pi^2 \psi}\left(\frac{\partial^2\psi}{\partial x^2}\right)\psi$$

$$K.E = - \frac{h^2}{8\pi^2 m\psi}\left(\frac{\partial^2\psi}{\partial x^2}\right)$$

But E + V = K.E

K.E = E-V

$$E - V = - \frac{h^2}{8\pi^2 m\psi}\left(\frac{\partial^2\psi}{\partial x^2}\right)$$

$$E - V + \frac{h^2}{8\pi^2 m\psi}\left(\frac{\partial^2\psi}{\partial x^2}\right) = 0$$

Divide throughout by $\frac{h^2}{8\pi^2 m\psi}$

$$\boxed{\left(\frac{\partial^2\psi}{\partial x^2}\right) + \frac{8\pi^2 m}{h^2} (E-V)\, \psi = 0}$$

Known as **Schrödinger's 1-Dim wave equation**

For 3-dim

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} + \frac{8\pi^2 m}{h^2} (E-V) = 0$$

Particle in a potential box:

Let us consider a particle of mass "m" is confined to move in a 1-dim potential box of length "a" & can capable of moving only along X-direction. It can capable of moving only inside the box since, the walls are given infinite potential, while reaching the walls it feels infinite potential and inside the box it possess purely kinetic energy that is its potential energy v=0

Therefore

V= 0 0 < x < a (Inside the potential box)

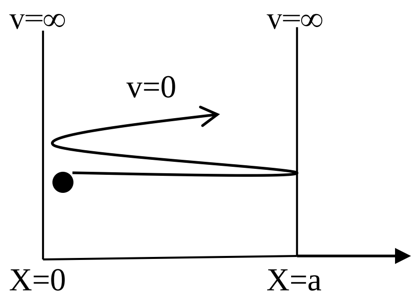
V= ∞ 0 ≥ x ≤ a (at the walls of the potential box)

In order to calculate the energy possessed by the electron and its wave function when it moves inside the potential box, let us consider Schrödinger's Time independent equation.

$$\left(\frac{\partial^2\psi}{\partial x^2}\right) + \frac{8\pi^2 m(E-V)}{h^2}\psi = 0$$

$$\left(\frac{\partial^2\psi}{\partial x^2}\right) + \frac{8\pi^2 mE}{h^2}\psi = 0 \quad \textbf{(inside the box V=0)}$$

$$\left(\frac{\partial^2\psi}{\partial x^2}\right) + k^2\psi = 0 \quad \text{------------------------ (1)}$$



$$k^2 = \frac{8\pi^2 mE}{h^2}$$

$$E = \frac{k^2h^2}{8\pi^2 m} \quad \textbf{("k" value is unknown)}$$

Calculation of "k":

In order to find the value of "k", consider the solution of the above equation (1) will be of the form,

ψ = A sin kx + B cos kx

In order to calculate the arbitrary constants A, B and k, let us apply the boundary conditions,

The first boundary condition, that is when x =0, ψ = 0

0= A sin k(0) + B cos k(0)

$\boxed{B = 0}$

While applying the second boundary condition that is When x =a , ψ =0

0= A sin k(a) + (0) cos k(a)

A sin ka = 0 (This equation will be satisfied when i) A=0 (But we cannot consider , since already B= 0 , if A is also equal to "0", no solution for the equation.)

Therefore let us consider **sin ka =0** , this is posible only if **ka =nπ**

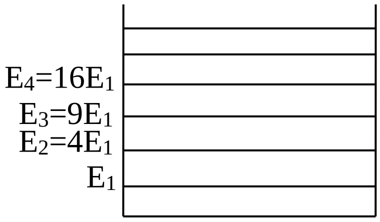
So $\boldsymbol{k = \frac{n\pi}{a}}$

$$E_n = \frac{n^2h^2}{8ma^2}$$

n = 1,2,3,……….

n =1

$E_1 = \frac{1^2 h^2}{8ma^2} = \frac{h^2}{8ma^2}$ called Zero point energy



$E_2 = \frac{2^2 h^2}{8ma^2} = 4\frac{h^2}{8ma^2} = 4E_1$ called 1st excited energy

$E_3 = \frac{3^2 h^2}{8ma^2} = 4\frac{h^2}{8ma^2} = 9E_1$ called 2nd excited energy

E_1, E_2, E_3…….. called Eigen values.