

# DC Generators

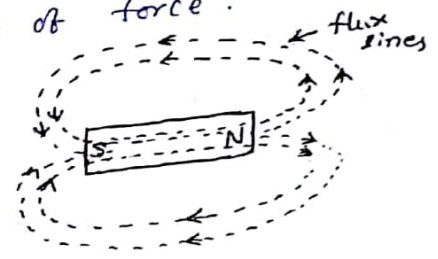
An electrical machine, deals with the energy transfer either from mechanical to electrical form (or) from electrical to mechanical form. This process is called electro-mechanical energy conversion.

An electrical machine which converts mechanical energy into an electrical energy is called an electric generator.

An electrical machine which converts an electrical energy into the mechanical energy is called an electric motor.

## Magnetic field and flux:-

The region around a magnet within which the influence of the magnetism can be experienced is called its magnetic field. It is represented by imaginary lines around a magnet called magnetic lines of force.



The total no. of lines of force existing in a particular magnetic field is called magnetic flux  $\phi$ .

1 weber =  $10^8$  lines of force.

## Revision of Electromagnetism:-

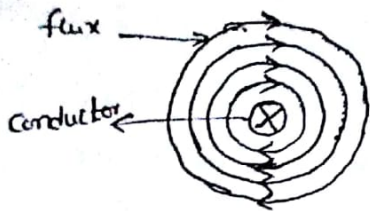
When a conductor carries a current, it creates a magnetic field around it. The direction of such magnetic field depends on the direction of the current passing through the conductor.

The direction of the flux produced by a current carrying conductor is given by:

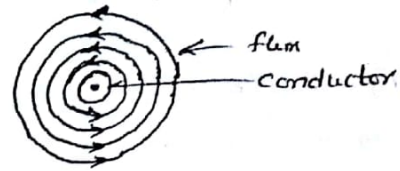
### Right hand thumb rule:-

It states that "Hold the current carrying conductor in the right hand such that the thumb is pointing in the direction of current and parallel to the conductor, then curled fingers point in the direction of the magnetic field (or) flux around it."





fig(a):- Current moving away from observer.



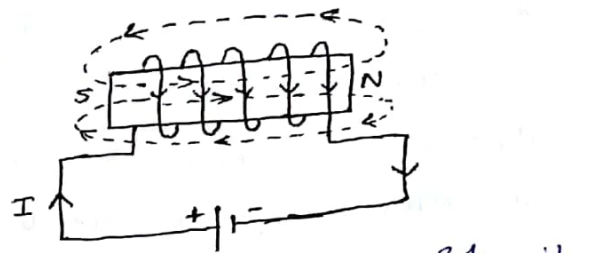
fig(b):- Current moving towards observer.

### Magnetic field due to circulating conductor:-

Consider an arrangement in which a long conductor is wound with no. of turns on the iron piece. Such an arrangement is called a Solenoid.

When a current is passed through the conductor, the flux is created and it will magnetize the iron piece. Poles will be created. The direction of flux around circular conductor is given by right hand thumb rule.

Hold the solenoid in the right hand such that curled fingers point in the direction of the current through the curled conductor, then the outstretched thumb along the axis of the solenoid points to the north pole of the solenoid.



### Principle of Operation of a DC Generator:-

All the generators work on a principle of dynamically induced emf. This principle is nothing but the Faraday's laws of electromagnetic induction.

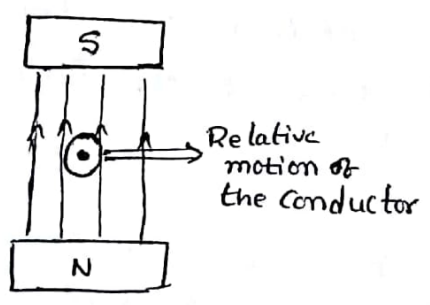
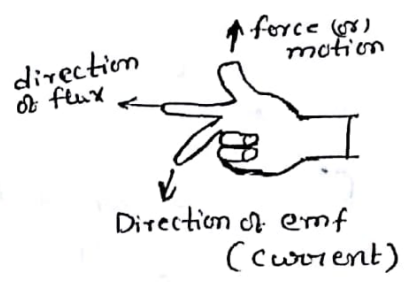
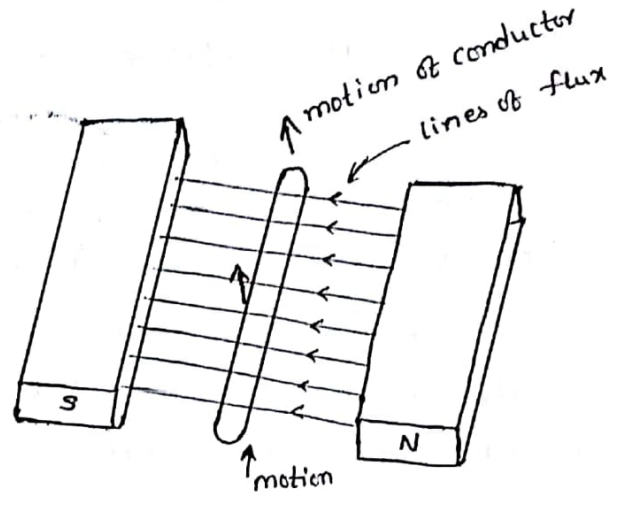
It states that, "whenever the no. of magnetic lines of force i.e., flux linking with a conductor or coil changes, an electromotive force is set up in that conductor or coil." The change in flux associated with the conductor can exist only when there exists a relative motion between a conductor and the flux. The relative motion can be achieved by rotating conductor w.r.t flux or by rotating

flux w.r.t to conductor.

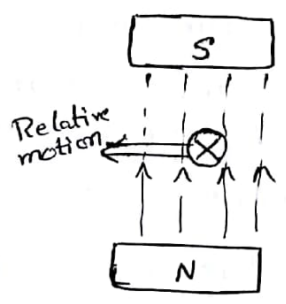
The direction of the induced emf can be obtained by using Flemings right hand rule.

Flemings right hand rule :-

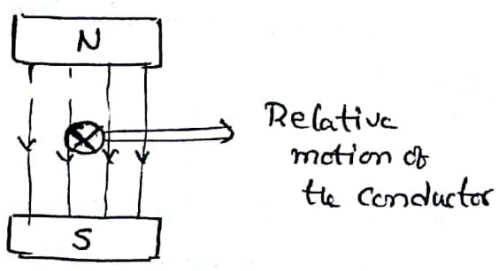
If three fingers of a right hand, namely Thumb, index finger and middle finger are outstretched so that everyone of them is at right angles with the remaining two, & if in this position index finger is made to point direction of flux  $\phi$ , thumb indicates the direction of the relative motion of the conductor w.r.t flux, then the outstretched middle finger gives the direction of the emf induced in the conductor.



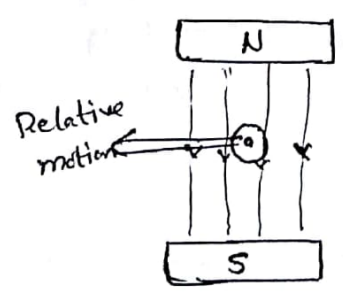
(a) Current flows outward, I coming out



(b) Current going away from the observer (or) Current going into



(c) Current going into



(d) I is coming out, (or) I coming towards observer.

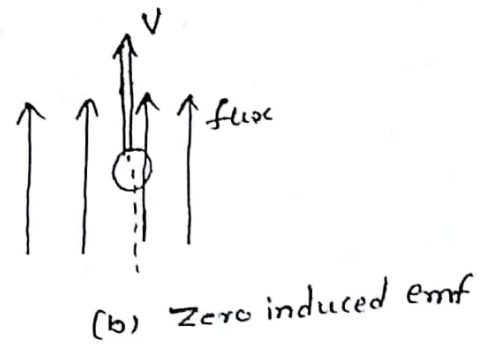
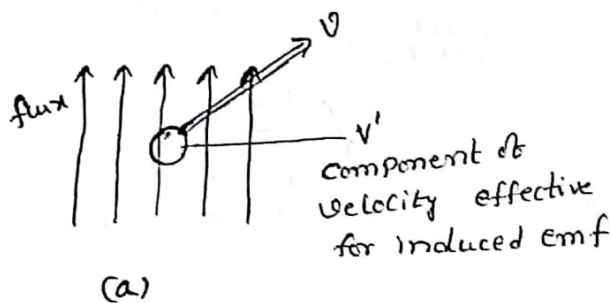
The magnitude of the induced emf is given by

$$E = B \times l \times v$$

Where  $l$  = Active length of conductor in 'm'  
 $v$  = Relative velocity component of conductor in m/sec in the direction perpendicular to the direction of the flux.

The active length means the length of conductor which is under the influence of magnetic field.

If the plane of the rotation of conductor is parallel to the plane of the flux, there will not be any cutting of flux & hence there cannot be any induced emf in the conductor.



If the angle b/w the plane of rotation and the plane of the flux is ' $\theta$ ' as measured from the axis of the plane of flux, then the induced emf is given by

$$E = Bl (v \sin \theta) \text{ volt}$$

Where  $v \sin \theta$  is the component of velocity which is perpendicular to the plane of flux and hence responsible for the induced emf.

To have d.c voltage, a device is used in a dc generator to convert the alternating emf to unidirectional emf. This device is called commutator.

## Constructional features of a D.C machine :-

(3)

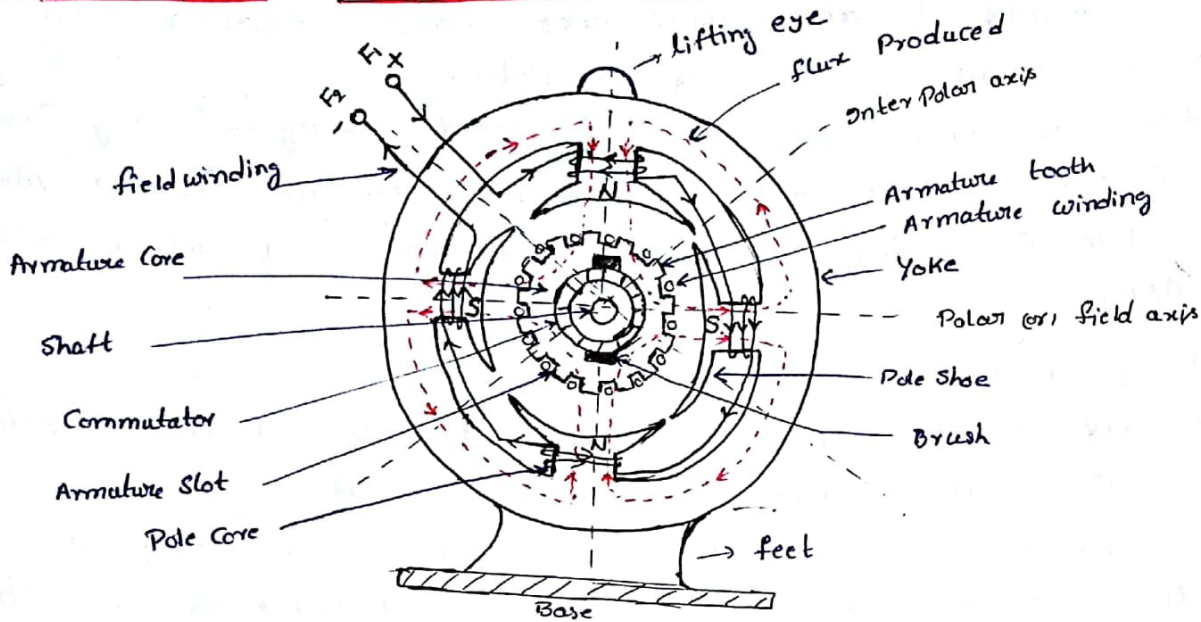


fig:- A cross-section of typical d.c machine. The stator of a DC machine consists of

(1) Yoke :- (a) frame - It is made of unlaminated ferromagnetic material: Cast iron (or) fabricated steel, Silicon steel - In order to provide low reluctance path to field & also gives good mechanical strength. It carries the magnetic flux produced by the poles. Since it is stationary part.

### Functions:-

- (1) It provides mechanical support to the poles
- (2) It serves as outermost cover, to protect the insulating material from harmful atmospheric elements like moisture, dust & various gases like  $SO_2$ , acidic fumes etc.
- (3) It forms a part of the magnetic circuit

### (2) Pole Core and Pole Shoe :-

These are made up of cast iron (or) cast steel. It is laminated of required size and shape & are stamped together to get a pole which is bolted to the yoke.

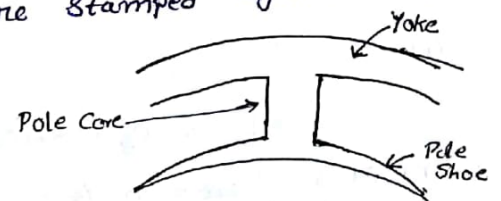


fig:- Pole structure.

### Functions of Pole Core & Pole Shoe :-

- Pole core basically carries a field winding which is necessary to produce the flux. (It is usually of circular section)
- It directs the flux produced through air gap to armature core, to the next pole.
- Pole shoe gives the mechanical support for field coil.
- Pole shoe reduces the reluctance due to its enlarged area, thereby more average value of flux under each pole, therefore emf increases in generator, & Torque increases in motor.

- Pole shoe spread out the flux over the armature periphery more uniformly & also made up of larger cross-section reduces the reluctance of magnetic path.
- Pole shoes are always laminated to avoid heating and any current losses caused by the any fluctuations in the flux distribution on the pole face due to the movement of armature slots and teeth.

### Field winding ( $F_1 - F_2$ ) :-

The ~~field~~ function of the field system is to produce uniform magnetic field within which the armature rotates.

The field coils are mounted on the poles and carry the dc exciting current. Due to which pole core behaves as an electromagnet, producing necessary flux.

As it helps in producing the magnetic field i.e., exciting the pole as an electromagnet it is called field winding (or) exciting winding.

The field coils are connected in such a way that the adjacent poles have opposite polarity, it is easily obtained with the help of right hand rule.

The magnetic flux produced by the field coils, passes through the pole pieces, the airgap, the armature and the frame.

Generally the air gaps ranging from 0.5mm to 1.5mm.

Field winding generally made up of conducting material such as copper or aluminium.

Armature :- It is further divided into two parts (i) Armature core & (ii) Armature winding.

#### (i) Armature Core :-

It is cylindrical in shape mounted on the shaft & it rotates between the field poles. It consists of slots on its periphery & are laminated (about 0.4mm to 0.6mm thick). The laminations are individually coated with a thin insulating film so that they do not come in electrical contact with each other.

The purpose of laminating the core is to reduce the eddy current loss.

And also air ducts are present to permit the air flow through

armature which serves coding purpose. (4)

### Functions :-

- (1) It provides house for armature winding i.e., armature conductor.
- (2) To provide a low reluctance path to the magnetic flux produced by the field winding.

So it is made up of magnetic material like cast iron (or) cast steel.

### (ii) Armature winding :-

It is the interconnection of the armature conductors, placed in the slots provided on the armature core periphery.

When the armature is rotated, in case of generator, magnetic flux gets cut by the conductors & emf gets induced in them.

The armature conductors are connected in series-parallel; the conductors being connected in series so as to increase the voltage and in parallel paths so as to increase the current.

The armature winding of a d.c. machine is a closed-circuit winding;

Functions :- (1) Generation of emf takes place in the armature winding in case of generator.

- (2) To carry the current supplied in case of d.c. motor.
- (3) To do the useful work in the external circuit.

It is made up of conducting material, which is copper.

### Commutator :-

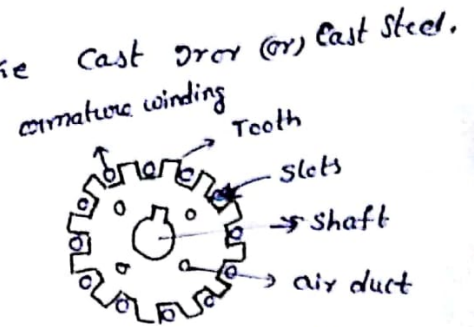


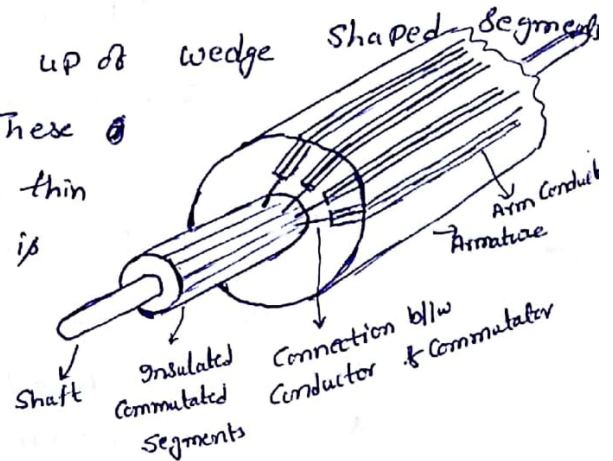
fig:- Armature core & Armature winding

Commutator :-

Functions :-

- (1) To facilitate the collection of current from the armature conductors.
- (2) To convert internally developed alternating emf to unidirectional emf.
- (3) To produce unidirectional torque in case of motors.

It is cylindrical in shape made up of wedge shaped segments of hard drawn, high conductivity copper. These segments are insulated from each other by thin layer of mica. Each commutator segment is connected to the armature conductor.



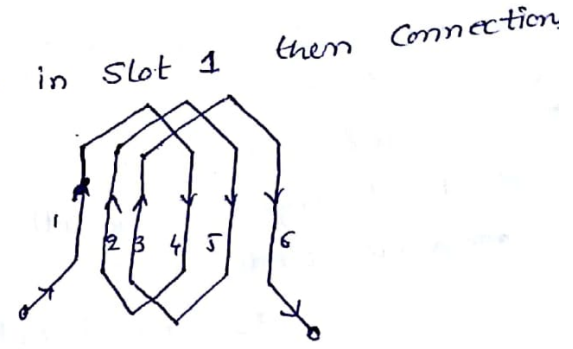
Types of Armature winding :-

Armature windings has basically two types namely,

- (a) Lap winding
- (b) wave winding

Lap winding :-

Its connection is started from conductor in slot 1 then connection overlap each other as winding proceeds, till starting point is reached again.

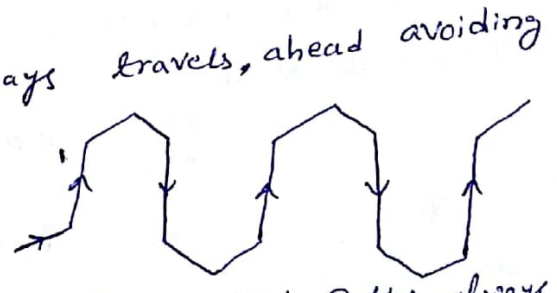


\* Total no. of conductors get divided into 'P' no. of parallel paths, where 'P' no. of poles

$$\therefore A = P$$

wave winding :-

On this type of connection, winding always travels, ahead avoiding overlapping. It travels like a progressive wave hence called wave winding.



Total no. of conductors get divided into '2' no. of parallel paths always irrespective of no. of poles of the machine.

$$A = 2$$



## Comparison of Lap and wave type winding :-

Lap winding	Wave winding
(1) No. of Parallel Paths (A) = Poles (P)	(1) No. of 11 <sup>th</sup> Paths A = 2 (Always)
(2) No. of brush sets = no. of Poles	(2) No. of brush sets = 2 "
(3) Preferable for High Current, Low Voltage Capacity generators	(3) Preferable for high Voltage, Low Current Capacity generators.
(4) Normally used for generators of Capacity more than 500A	(4) Preferred for generators of Capacity less than 500A.

## EMF Equation of DC generator :-

Let P = no. of Poles of generator

$\phi$  = Flux produced by each pole in webers (wb)

N = Speed of armature in r.p.m.

Z = Total no. of armature conductors

A = No. of Parallel Paths in which 'Z' no. of conductors are divided.

So A = P for lap type

A = 2 for wave type

Emf gets induced in the conductor according to Faraday's Law of Electromagnetic Induction. Hence avg value of emf induced in each armature conductor is,

$$e = \text{rate of cutting the flux} = \frac{d\phi}{dt}$$

Now consider one revolution of conductor, in one revolution, conductor will cut total flux produced by all the poles i.e.,  $\phi \times P$ .

while time required to complete one revolution is  $\frac{60}{N}$  seconds as

speed is N r.p.m.

$$\therefore e = \frac{\phi P}{60/N} = \phi P \frac{N}{60}$$

This is the emf induced in one conductor. Now the conductors in one parallel path are always in series. There are total Z conductors with 'A' parallel paths, hence  $\frac{Z}{A}$  no. of conductors are always in series and emf remains same across all the parallel paths.

$\therefore$  Total emf can be expressed as,

$$E = \phi P \frac{N}{60} \times \frac{Z}{A} \text{ Volts}$$

$$\therefore \begin{cases} E = \frac{\phi NZ}{6} & \text{for Lap wind} \\ & A = P \\ E = \frac{\phi NZ}{120} & \text{for wave wind} \\ & A = 2 \end{cases}$$

(Q) A 4P, lap wound, d.c generator has a useful flux of 0.07 wb per pole. Calculate the generated emf when it is rotated at a speed of 900 rpm with the help of Prème mover. Armature consists of 440 no. of conductors. Also calculate the generated emf if lap wound armature is replaced by wave wound armature.

Sol: -  $P=4$ ,  $Z=440$ ;  $\phi = 0.07 \text{ wb}$  &  $N=900 \text{ rpm}$ .

$$E = \frac{N\phi Z}{60} \cdot \frac{P}{A}$$

(i) Lap wound,  $A=P=4$ ,  $E = \frac{0.07 \times 900 \times 440}{60} \times \frac{4}{4} = 462 \text{ V}$

(ii) wave wound,  $A=2$ ,  $E = \frac{0.07 \times 900 \times 440}{60} \times \frac{4}{2} = 924 \text{ V}$

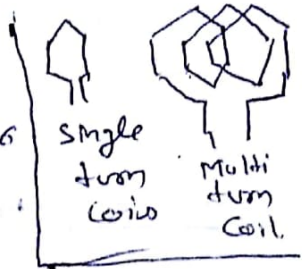
(Q) A 4P, lap wound, dc generator have 42 coils, with 8 turns per coils, it is driven at 1120 rpm. It useful flux per pole is 21 mwb, calculate the generated emf. Find the speed at which it is to be driven to generate the same emf as calculated above, with wave wound armature.

Sol: -  $P=4$ ,  $\phi = 21 \text{ mwb} = 21 \times 10^{-3} \text{ wb}$ ,  $N=1120 \text{ rpm}$ .

Coils = 42 & turns/coil = 8

Total turns = Coils  $\times$  turns/coil =  $42 \times 8 = 336$

$Z = 2 \times \text{total turns} = 2 \times 336 = 672$



(i) For lap wound,  $A=P$

$$\therefore E = \frac{N\phi Z}{60} \times \frac{P}{A} = \frac{21 \times 10^{-3} \times 1120 \times 672}{60} = 263.424 \text{ V}$$

(ii) For wave wound,  $A=2$

&  $E = 263.424 \text{ V}$ ;  $E = \frac{N\phi Z}{60} \cdot \frac{P}{A}$

$N = 560 \text{ rpm}$

(Q) An 8P, dc generator has per pole flux of 40 mwb, and winding is connected in lap with 960 conductors. calculate the generated emf on open circuit when it runs at 400 rpm. If the armature is wave wound at what speed must the machine be driven to generate the same voltage.

Sol: -  $P=8$ ,  $\phi = 40 \text{ mwb}$ ; Lap,  $A=P$ ,  $Z=960$ ,  $N=400 \text{ rpm}$ .

$$E_g = \frac{N\phi Z}{60} \times \frac{P}{A} = \frac{40 \times 10^{-3} \times 8 \times 400 \times 960}{60 \times 8} = 256 \text{ V}$$

Now it is wave connected hence  $A=2$ ,  $E_g = 256 \text{ V}$  (Same volt)

$$256 = \frac{40 \times 10^{-3} \times 8 \times N \times 960}{60 \times 2} = 100 \text{ rpm}$$

Q) A lap wound armature of a 4 pole generator armature has 51 slots. Each slot contains 20 conductors. what will be the emf generated in machine when driven at 1500 rpm, if useful flux per pole is 0.01 wb?

Sol:  $P=4$ , lap,  $A=P=4$ , 51 slots, 20 conductors/slot,  $\omega=1500$  rpm  
 $\phi=0.01$  wb.  
 Total conductors,  $Z = \text{slots} \times \text{conductors/slot} = 51 \times 20 = 1020$   
 $\therefore E_g = \frac{0.01 \times 4 \times 1500 \times 1020}{60 \times 4} = 255V$

Q) The armature of a 2 pole, 200V, generator has 4000 conductors and runs at 300 rpm. Calculate the useful flux per pole. If the no. of turns in each field coil is 1200, what is the average value of emf induced in each coil on breaking the field, if the flux dies away completely in 0.1 sec.

Sol:  $P=2$ ,  $E_g=200V$ ,  $Z=4000$ ,  $N=300$  rpm,  $A=P=2$   
 $E_g = \frac{N \phi Z P}{60 A} \Rightarrow$  i.e.,  $200 = \frac{\phi \times 2 \times 300 \times 4000}{60 \times 2} \Rightarrow \phi = 0.1$  wb

Now

$N = 1200$ ,  $\phi_1 = 0.1$  wb,  $\phi_2 = 0$ ;  $dt = 0.1$  sec

$$\therefore e = -N \frac{d\phi}{dt} = -1200 \left[ \frac{0 - 0.1}{0.1} \right] = 1200V$$

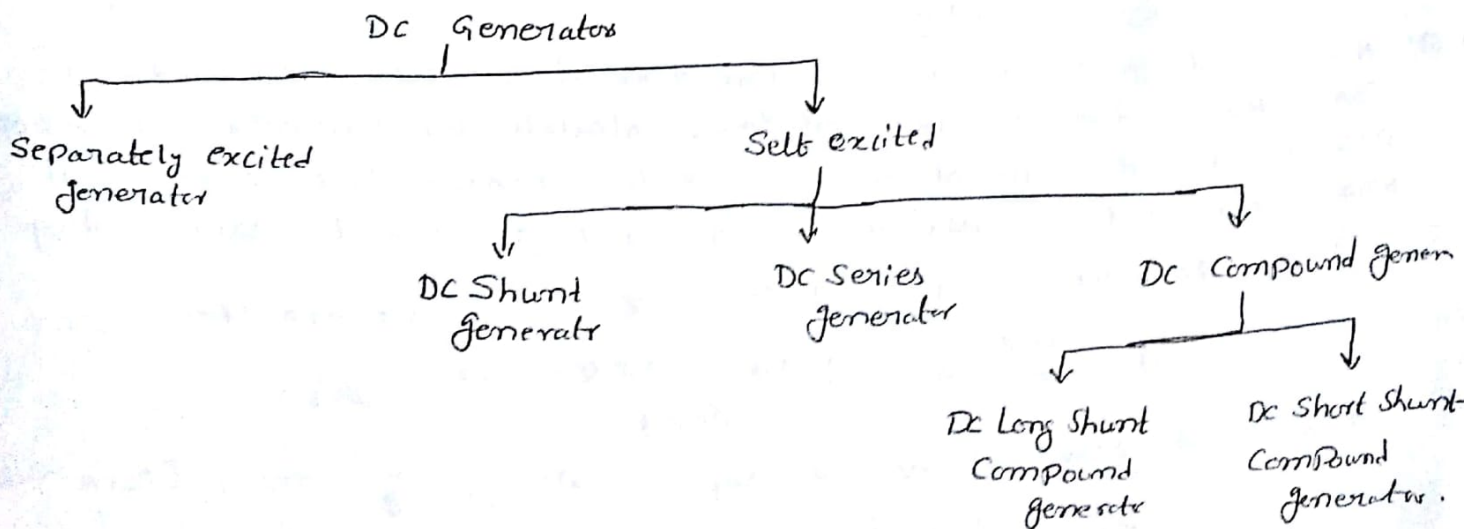
### Types of Generators :-

There are two methods of excitation used for dc generator,

- ① Separate excitation      ② self excitation

Depending on the method of excitation used, the d.c generator are classified as

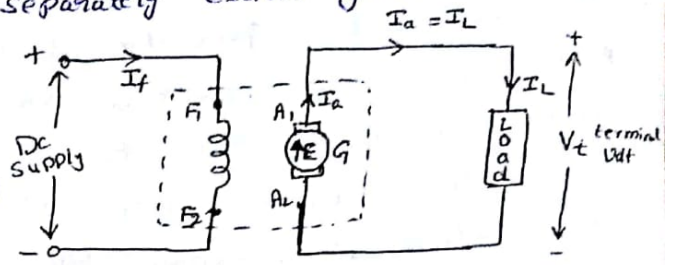
- ① Separately excited generator      ② self excited generator.



## Separately Excited generator

When separate dc supply i.e., excitation of field winding is separate then the generator is called separately excited generator.

The field winding of this has large no. of turns of thin wire. So length of such winding is more with less X-sectional area. So resistance of this field winding is high, in order to limit the  $I_f$ .



### Voltage and Current relations.

$I_f$  depends on supply voltage & resistance of field winding. For armature side, it is supplying a load, demanding a load current  $I_L$  at a voltage of  $V_t$  called terminal voltage.

$$\text{Now } \boxed{I_a = I_L}$$

The internally induced emf  $E$  is supplying the voltage of the load hence terminal volt  $V_t$  is a part of  $E$ .

But  $E$  is not equal to  $V_t$  while supplying a load. This is because when  $I_a$  flows through armature wind, due to armature resistance  $R_a \Omega$ , there is a voltage drop across armature wind equal to  $I_a R_a$  volts.

In addition to this drop, there is some volt drop at the contacts of the brush called brush contact drop. But this is negligible & hence neglected.

So, induced emf  $E$  has to supply three components namely,

- (i) terminal voltage  $V_t$
- (ii) Armature resistance drop  $I_a R_a$
- (iii) Brush Contact drop  $V_{brush}$

So voltage eqn for separately excited generator can be written, as

$$\boxed{E = V_t + I_a R_a + V_{brush}} \quad , \quad \text{where } E = \frac{N\Phi Z}{60} \times \frac{P}{A}$$

## Self Excited Generator:-

When the field winding is supplied from the armature of the generator itself then it is said to be self excited generator.

Here question is, without generated emf, field cannot be excited in such generators & without excitation there cannot be emf.

So, the answer is residual magnetism possessed by field poles, under normal condition.

When Generator is not working, without any current through field wind, the field poles possess some magnetic flux, called residual flux & the property is called residual magnetism.

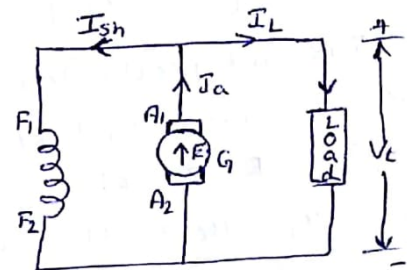
Thus, when generator is started, due to  $\phi_r$ , it develops a small emf which now drives a small current through the field winding. This tends to increase the  $\phi$ . This in turn increases the induced emf. This further increases the  $I_f$  & the  $\phi$ . This process is cumulative & continues till the generator develops rated volt across its armature. This is voltage building process.

It is further divided into 3 types:-

- (i) Shunt generator
- (ii) Series generator
- (iii) Compound generator.

### (i) Shunt generator:-

When field winding is connected in parallel with the armature & the combination across the load then the generator is called shunt generator.



The field winding has large no. of turns of thin wire, so it has high resistance.

Volt and Current Relations.

$$I_a = I_L + I_{sh}$$

$$I_{sh} = \frac{V_t}{R_{sh}}$$

Induced emf  $E$ , requires to supply volt drop  $I_a R_a$  and brush contact drop.

$$E = V_t + I_a R_a + V_{brush}, \quad \text{where } E = \frac{N\phi Z}{60} \times \frac{P}{A}$$

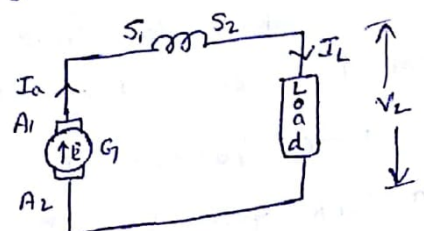
### Series Generator:-

When field winding is connected in series with the armature winding while supplying load, then the generator is called series generator.

The resistance of series field wind is very small, it has less no. of turns of thick X-section wire.

$$I_a = I_{se} = I_L$$

$$E = V_t + I_a R_a + I_a R_{se} + V_{brush}$$



$$\text{where } E = \frac{N\phi Z}{60} \cdot \frac{P}{A}$$

## Compound Generator:

(8)

It is further classified as (i) Long Shunt Compound generator  
(ii) Short " " " "

### (i) Long Shunt Compound generator:-

Shunt field wnd connected across the series combination of armature & series field winding.

Volt & Current relations are

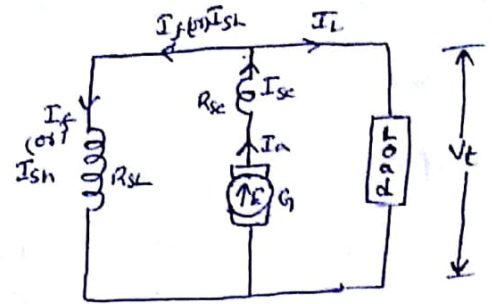
$$I_a = I_{se} \quad \& \quad I_a = I_{sh} + I_L$$

volt across shunt field winding is  $V_t$  . So

$$I_{sh} = \frac{V_t}{R_{sh}}$$

volt eqn is

$$E = V_t + I_a R_a + I_a R_{se} + V_{bruhl}$$



### (ii) Short Shunt Compound generator:-

on this shunt field winding is connected only across the armature excluding series field winding.

volt & Current relations are

$$I_a = I_{se} + I_{sh} \quad \& \quad I_{se} = I_L$$

$$I_a = I_L + I_{sh}$$

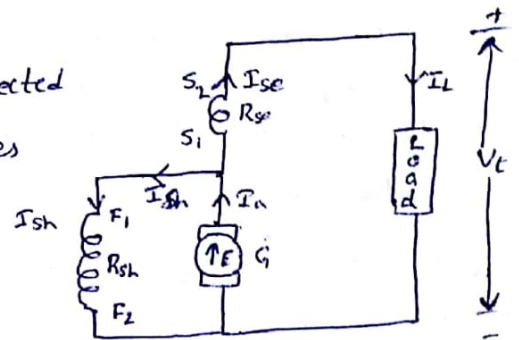
$$I_{sh} = \frac{E - I_a R_a}{R_{sh}}$$

volt eqn

$$E = V_t + I_a R_a + I_{se} R_{se} + V_{bruhl}$$

$$\therefore I_{se} = I_L$$

$$E = V_t + I_a R_a + I_L R_{se} + V_{bruhl}$$



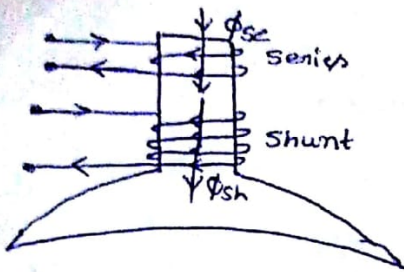
Neglecting  $V_{bruhl}$ , we can write

$$E = V_t + I_a R_a + I_L R_{se}$$

$$I_{sh} = \frac{V_t + I_L R_{se}}{R_{sh}}$$

Cumulative and Differential Compound generator.  
Two windings, shunt and series field are wound on the same poles. Depending on the direction of winding on the poles, two fluxes produced by shunt & series field may help or may oppose each other.

If two fluxes help each other, the generator is called Cumulative Compound generator.

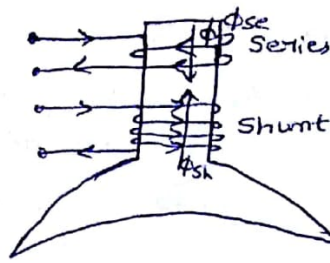


Cumulative Compound generator

$$\phi_T = \phi_{sh} + \phi_{sc}$$

where  $\phi_{sh}$  = Flux Produced by Shunt field winding.  
 $\phi_{sc}$  = flux Produced by Series field winding.

If two windings are wound in such a direction that the two fluxes produced by them oppose each other then the generator is called differential compound generator.



Differential Compound generator.

$$\phi_T = \phi_{sh} - \phi_{sc}$$

Q) A shunt generator has shunt field winding resistance of  $100 \Omega$ . It is supplying a load of  $5 \text{ kW}$  at a voltage of  $250 \text{ V}$ . Its armature resistance is  $0.22 \Omega$ . Calculate the induced emf of generator.

Sol:-

$$I_a = I_L + I_{sh} \quad ; \quad I_{sh} = \frac{V_t}{R_{sh}}$$

$$V_t = 250 \text{ V}, \quad R_{sh} = 100 \Omega \quad \therefore I_{sh} = \frac{250}{100} = 2.5 \text{ A}$$

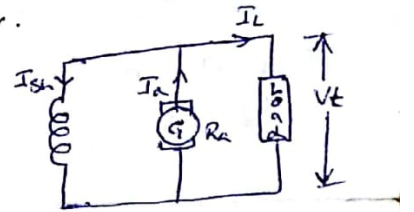
$$\text{Load Power} = 5 \text{ kW}$$

$$V_t \times I_L = 5 \text{ kW} \quad \Rightarrow \quad I_L = \frac{P}{V_t} = \frac{5 \times 10^3}{250} = 20 \text{ A}$$

$$I_a = I_L + I_{sh} = 20 + 2.5 = 22.5 \text{ A}$$

$$\text{Now } E = V_t + I_a R_a \quad (\text{neg } V_{\text{brush}})$$

$$E = 250 + 22.5 \times 0.22 = 254.95 \text{ V}$$



Q) A  $250 \text{ V}$ ,  $10 \text{ kW}$ , Separately excited generator has an induced emf of  $255 \text{ V}$  at full load. If the brush drop is  $2 \text{ V}$  per brush, Calculate the armature resistance of the generator.

Sol:- For Separately excited generator,

$$I_a = I_L \quad ; \quad V_t = 250 \text{ V}, \quad P = 10 \text{ kW}$$

$$P = V_t \times I_L \quad ; \quad I_L = \frac{10 \times 10^3}{250} = 40 \text{ A}$$

$$I_L = \frac{P}{V_t}$$

$$\therefore I_a = I_L = 40 \text{ A} \quad \text{Now } E = V_t + I_a R_a + V_{\text{brush}}$$

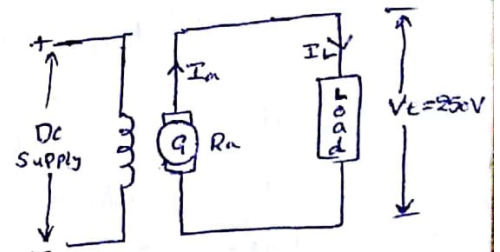
Now there are two brushes & brush drop is  $2 \text{ V/brush}$

$$\therefore V_{\text{brush}} = 2 \times 2 = 4 \text{ V}$$

$$\therefore E = 250 + 40 \times R_a + 4$$

$$; \quad \text{But } E = 255 \text{ on Full load}$$

$$\Rightarrow \boxed{R_a = 0.025 \Omega}$$



Q) A dc Series generator has armature resistance of  $0.5 \Omega$  and Series field resistance of  $0.03 \Omega$ . It drives a load of  $50 \text{ A}$ . It has 6 turns/coil and total 540 coils on the armature and is driven at  $1500 \text{ rpm}$ . Calculate the terminal voltage at the load. Assume 4 Poles, lap type winding, flux per pole as  $2 \text{ mwb}$  and total brush drop as  $2 \text{ V}$ .

Sol:-  $R_a = 0.5 \Omega$ ,  $R_{se} = 0.03 \Omega$ ,  $I_L = 50 \text{ A}$ ,  $P = 4$   
 Total coils = 540 ; 6 turns/coil

Total turns =  $540 \times 6 = 3240$

$\therefore$  Total Conductors =  $2 \times \text{turns} = 2 \times 3240 = 6480$

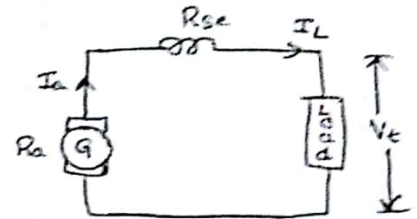
$E = \frac{N\phi Z}{60} \times \frac{P}{A}$  ; For lap wound  $A = P$  ;  $N = 1500 \text{ rpm}$   
 $A = 4$  ;  $\phi = 2 \times 10^{-3} \text{ wb}$

Total brush drop =  $2 \text{ V}$ .

$$E = \frac{1500 \times 2 \times 10^{-3} \times 6480}{60} \times \frac{4}{4} = 324 \text{ V}$$

$$E = V_t + I_a (R_a + R_{se}) + V_{\text{brush}} ; I_a = I_L = 50 \text{ A}$$

$$V_t = 324 - 50(0.5 + 0.03) + 2 = 295.5 \text{ V}$$



Q) A Short Shunt Compound dc generator supplies a current of  $75 \text{ A}$  at a voltage of  $225 \text{ V}$ . Calculate the generated voltage if the resistance of armature, shunt field & series field windings are  $0.04 \Omega$ ,  $90 \Omega$  and  $0.02 \Omega$  respectively.

Sol:-  $R_a = 0.04 \Omega$ ,  $R_{sh} = 90 \Omega$ ,  $R_{se} = 0.02 \Omega$

$V_t = 225 \text{ V}$ ,  $I_L = 75 \text{ A}$ ,  $E = ?$

$$I_a = I_L + I_{sh} ; E = V_t + I_a R_a + I_L R_{se}$$

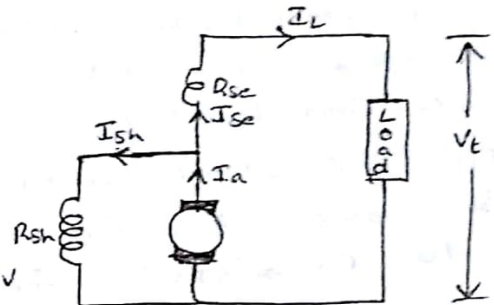
$$E - I_a R_a = V_t + I_L R_{se} = 225 + 75 \times 0.02 = 226.5 \text{ V}$$

$$I_{sh} = \frac{E - I_a R_a}{R_{sh}} = \frac{V_t + I_L R_{se}}{R_{sh}} = \frac{226.5}{90}$$

$$I_{sh} = 2.5167 \text{ A}$$

$$\therefore I_a = I_L + I_{sh} = 75 + 2.5167 = 77.5167 \text{ A}$$

$$E = V_t + I_a R_a + I_L R_{se} = 225 + 77.5167 \times 0.04 + 75 \times 0.02 = 229.6 \text{ V}$$





## Magnetization Characteristics of DC Generator:-

It is the graph of generated no load voltage  $E$  against the field current  $I_f$ , when speed of generator is maintained constant.

As it is plotted without load with open output terminals it is also called No-load characteristics or open circuit characteristics (O.C.C.).

$E_0$  vs  $I_f$  is magnetization characteristics.

But for generator  $E = \frac{N\phi Z}{60} \cdot \frac{P}{A}$

$E \propto \phi$  with  $\frac{PNZ}{60A}$  as constant  
 $E \propto I_f$  as  $\phi \propto I_f$

Thus induced emf increases directly as  $I_f$  increases.

But after certain  $I_f$  core gets saturated & flux  $\phi$  also remains constant though  $I_f$  increases.

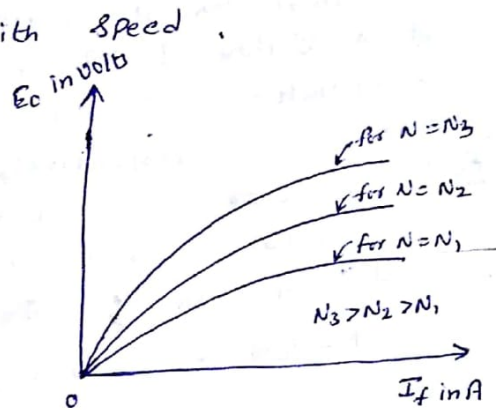
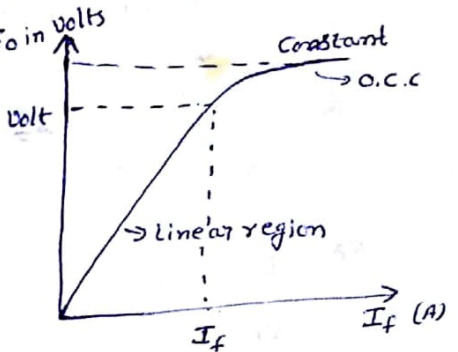
Hence after saturation volt also remains constant.

Now the induced emf also varies with speed.

Actually  $E \propto N\phi$

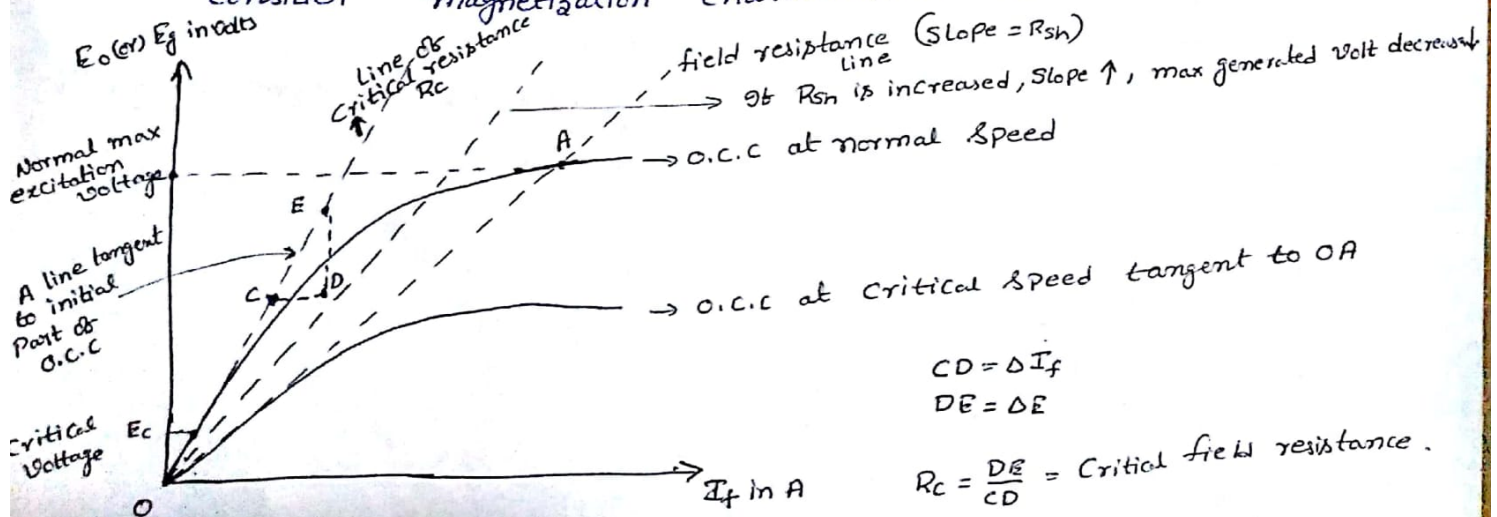
So its magnetization characteristics for various speeds are plotted we will get a family of parallel characteristics.

For lower speeds, generated voltages are less so characteristics for lower speeds are below the characteristics for higher speeds.



## Critical field resistance & Critical Speed in D.C Shunt generator:-

Consider magnetization characteristics of a d.c shunt generator



The generator voltage builds in step till Point A. This Point is intersection of field resistance line with O.C.C. Point A is the max volt it can generate.

If slope of  $R_{sh}$  line is reduced, max volt generator can build will be higher.  
 If it " " " " increased, " " " " " " " " less  
 corresponding to Point A, i.e., at B.

If slope of  $R_{sh}$  line is increased in such a way that it becomes tangent to the lower part of O.C.C., the volt corresponding to this point is  $E_c$ . This volt is just sufficient to drive the current through field resistance, so that cumulative process of building the volt starts.

This value of field resistance is called the critical resistance ( $R_c$ ).

$$R_c = \frac{\Delta E}{\Delta I_f} = \frac{DE}{CD} = \tan \theta$$

The critical resistance is the slope of the critical resistance line.

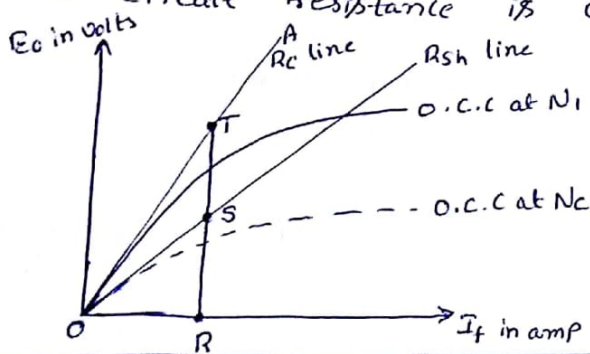
Note If field circuit resistance is more than  $R_c$  at start then the induced emf fail to drive current through field circuit and generator fails to excite at given speed.

Similarly  $E \propto N$ ,

As speed decreases the induced emf decreases & we get O.C.C below the O.C.C at normal speed

If we go on reducing the speed, at a particular speed we will O.C.C just tangential to normal field resistance line.

This speed at which the machine just excites for the given field circuit resistance is called the critical speed ( $N_c$ ).

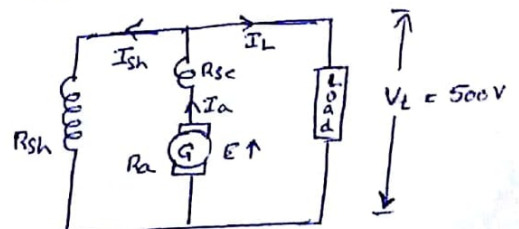


$$N_c = N_1 \times \frac{R_s}{R_T}$$

(Q) A 4P, lap wound lap wound long shunt compound generator has 1200 arm conductors. The armature, series field and shunt field resistance are  $0.1 \Omega$ ,  $0.15 \Omega$  and  $250 \Omega$ , respectively. If flux per pole is  $0.075 \text{ wb}$ . Calculate the speed at which the machine should be driven so that it can deliver the load of  $50 \text{ kW}$  at  $500 \text{ V}$ . Take overall volt drop due to brush contact as  $2 \text{ volts}$ .

Sol:-  $P = 4$ ,  $A = P = 4$  as lap winding,  
 $P = 50 \text{ kW}$ ,  $V_E = 500 \text{ V}$ .  $V_{\text{brush}} = 2 \text{ V}$   
 $R_a = 0.1 \Omega$ ,  $R_{sc} = 0.15 \Omega$ ,  $R_{sh} = 250 \Omega$ .  
 Consider a long shunt generator  
 $I_a = I_L + I_{sh}$

$Z = 1200$ ,  $\phi = 0.075 \text{ wb}$



$$I_{sh} = \frac{V_t}{R_{sh}} = \frac{500}{250} = 2 A.$$

$$P = V_t \times I_L \Rightarrow 50 \times 10^3 = 500 \times I_L$$

$$\therefore I_L = 100 A.$$

$$I_a = I_L + I_{sh} = 102 A$$

$$\text{Now, } E = V_t + I_a R_a + I_a R_{se} + V_{brist} = 500 + 102 \times 0.11 + 102 \times 0.15 + 2$$

$$E = 527.5 V$$

$$\& E = \frac{NPZ}{60} \frac{P}{A} \Rightarrow 527.5 = \frac{0.075 \times 4 \times N \times 1200}{60 \times 4}; \quad \boxed{N = 351.67 \text{ RPM}}$$

## DC motors

### Principle of operation of a DC motor:-

"When a current carrying conductor is placed in a magnetic field; it experiences a mechanical force", according to Lorentz force law.

The direction of force is given by magnitude

$$F = BIL \text{ Newtons (N)}$$

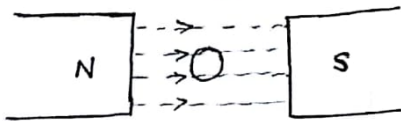
where  $F =$  Force in newton.

$B =$  magnetic flux density in  $Wb/m^2$ .

$I =$  current in ampere.

$L =$  length of the conductor in metre.

To understand the principle of operation of DC motor, let us consider a two pole motor



(a) Conductor in a magnetic field

Fig shows a uniform magnetic field in which a straight conductor carrying no current is placed.

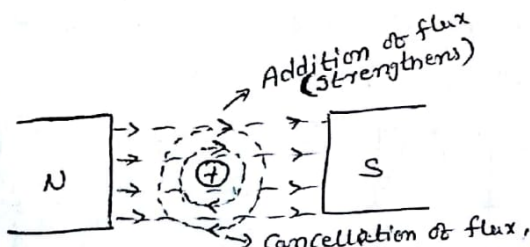
There is no movement of conductor.



(b) Flux Produced by current carrying conductor.

Fig (b) There is no exciting current flow through the field wind, & D.C current is sent through the conductor. It produces a magnetic flux lines around it in clock-wise direction.

There is no movement of conductor.



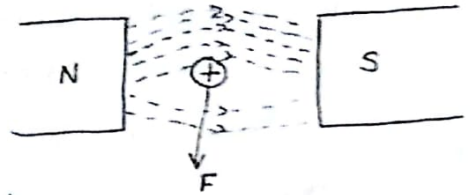
(c) Interaction of two fluxes

- Now there are two fluxes
- ① The flux produced by ~~Permanent~~ <sup>Electro</sup> magnet called main flux
  - ② The flux produced by the current carrying conductor.

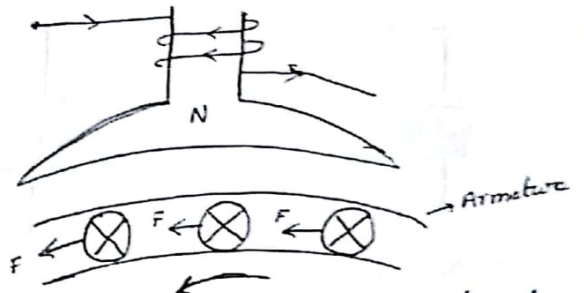
On top of the conductor, there is gathering of flux lines as two fluxes help each other.

On bottom of the conductor, two fluxes ~~help each~~ are in opposite direction & hence try to cancel each other. Due to this density of flux lines in this area gets weakened.

This exerts a mechanical force on the conductor which acts from high flux density area towards low flux density area.



||<sup>ly</sup> in DC motor, all the armature conductors, get subjected to the mechanical force.

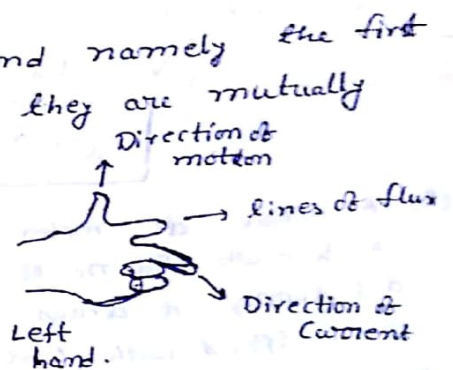


Due to this overall armature experiences a twisting force called torque & motor starts rotating.

Direction of Rotation of motor is given by Fleming's left hand rule.

Fleming's Left hand rule :-

Outstretch the 3 fingers of left hand namely the first finger, middle finger and thumb such that they are mutually perpendicular to each other.



- 1st finger → direction of magnetic field
- middle " → " " current
- thumb gives → " " force experienced by the Conductor.

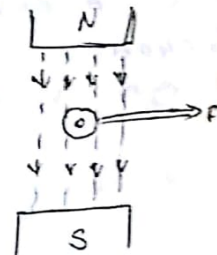
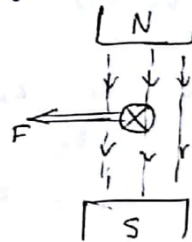
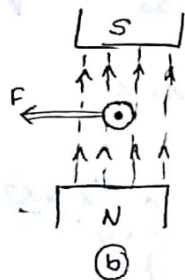
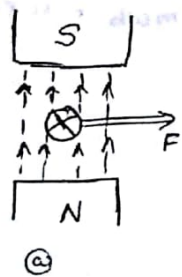


Fig:- Direction of force experienced by conductor.

Significance of Back Emf :-

After motoring action, armature starts rotating and armature conductors cut the main flux. So there is a generating action existing in a motor.

There is an induced emf in the rotating armature conductors according to "Faraday's law of electromagnetic induction".

This induced emf in the armature always acts in the opposite direction of the supply voltage. This is according to the Lenz's law.

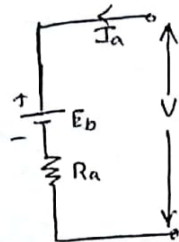
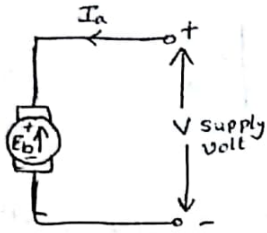
which states that the direction of the induced emf is always so as to oppose the cause producing it.

In d.c motor, electrical i/p i.e., supply volt is the cause & hence this induced emf opposes the supply voltage.

So this emf is called back emf ( $E_b$ ).

Its magnitude can be given by

$$E_b = \frac{N\phi Z}{60} \cdot \frac{P}{A} \text{ Volts}$$



(a) Back emf in a d.c motor.

(b) Equivalent circuit

The supply volt 'V' has to overcome back emf  $E_b$  which is opposing V and also various drops as armature resistance drop  $I_a R_a$  / brush drop etc.

$$V = E_b + I_a R_a + V_{\text{brush drop}}$$

$$I_a = \frac{V - E_b}{R_a}$$

(Q) A 4 Pole dc motor has lap connected arm wnd. The flux per pole is 30 mwb. The no. of arm conductors is 250. when connected to 230V d.c supply it draws an arm current of 40 A. Calculate the back emf & the speed with which motor is running. Assume arm resistance is 0.6  $\Omega$ .

Sol:-  $P = 4$ ,  $A = P = 4$  as lap,  $V = 230V$ ,  $Z = 250$ ,  $\phi = 30 \text{ mwb} = 30 \times 10^{-3} \text{ wb}$

$$I_a = 40 \text{ A.}$$

$$\text{volt eqn } V = E_b + I_a R_a \Rightarrow 230 = E_b + 40 \times 0.6$$

$$E_b = 206 \text{ V.}$$

$$\& E_b = \frac{N\phi Z}{60} \cdot \frac{P}{A} \Rightarrow 206 = \frac{30 \times 10^{-3} \times 4 \times N \times 250}{60 \times 4}$$

$$N = 1648 \text{ rpm}$$

Power eqn of a DC motor:-

$$V = E_b + I_a R_a$$

multiplying both sides of the above eqn by  $I_a$  we get,

$$V I_a = E_b I_a + I_a^2 R \Rightarrow \text{Power equation.}$$

$V I_a$  = Net electrical power i/p to the armature in watts.

$I_a^2 R_a$  = Power loss due to resistance of armature called arm. copper loss.

So  $E_b I_a = V I_a - I_a^2 R_a$

$E_b I_a = P_m =$  Electrical Equivalent of gross mechanical Power developed by the armature.

Condition for max Power, is obtained by  $\frac{dP_m}{dI_a} = 0$

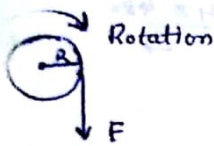
$0 = V - 2 I_a R_a$

$I_a = \frac{V}{2 R_a}$  i.e.,  $I_a R_a = \frac{V}{2}$

Substituting in Volt Eqn  $V = E_b + I_a R_a = E_b + \frac{V}{2}$

$E_b = \frac{V}{2} \Rightarrow$  Condition for max Power.

Torque Eqn of a DC motor:-



Turning or twisting force about an axis is called torque.

Consider a wheel of radius R metres acted upon by a Circumferential force F newtons as shown

The wheel is rotating at a speed of N r.p.m.

Then angular speed of the wheel is  $\omega = \frac{2\pi N}{60}$  rad/sec.

So work done in one revolution is

$W = F \times \text{Distance travelled in one revolution}$

$W = F \times 2\pi R$  Joules

$P = \text{Power developed} = \frac{\text{work done}}{\text{Time}}$

$P = \frac{F \times 2\pi R}{\text{Time for 1 rev}} = \frac{F \times 2\pi R}{(60/N)} = (F \times R) \cdot \left(\frac{2\pi N}{60}\right)$

$P = T \times \omega$  watts.

$\left[ \begin{aligned} \therefore T &= F \times R \\ \omega &= \frac{2\pi N}{60} \end{aligned} \right]$

where  $T =$  Torque in N-m

$\omega =$  Angular Speed in rad/sec

Let  $T_a$  be the gross torque developed by the armature of the motor. It is also called armature Torque.

The gross mechanical Power developed in the armature is  $E_b I_a$ .

So if speed of the motor is N r.p.m then

Power in armature = Arm torque  $\times \omega$

$E_b I_a = T_a \times \frac{2\pi N}{60}$

~~Civil - A - (111)  
12/15, 9/15  
19, 20, 27, 28, 29  
31, 32, 33, 34~~

~~17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50~~

But  $E_b$  in a motor is given by  $E_b = \frac{N\phi Z}{60} \cdot \frac{P}{A}$

∴  $E_b I_a = T_a \times \frac{2\pi N}{60}$

$$\frac{N\phi Z}{60} \cdot \frac{P}{A} \cdot I_a = T_a \cdot \frac{2\pi N}{60}$$

$$T_a = \frac{1}{2\pi} \phi I_a \cdot \frac{PZ}{A}$$

$$T_a = 0.159 \phi I_a \frac{PZ}{A} \quad \text{N-m}$$

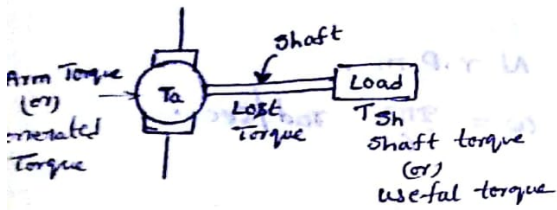
(Q) A 4 Pole d.c motor takes a 50A armature current. The armature has lap connected 480 conductors. The flux per pole is 20mwb. Calculate the gross torque developed by the armature of the motor.

Sol:-  $P=4, A=P=4, Z=480; \phi=20 \times 10^{-3} \text{ wb}; I_a=50\text{A}$

Now  $T_a = 0.159 \phi I_a \cdot \frac{PZ}{A} = 0.159 \times 20 \times 10^{-3} \times 50 \times \frac{4 \times 480}{4}$

$T_a = 76.394 \text{ N-m}$

Types of Torque in the motor:-



$T_a =$  gross torque developed in the armature

The mechanical power developed in the armature is transmitted to the load through the shaft of the motor. It is impossible to transmit the entire power developed by the armature to the load. This is because while transmitting the power through the shaft, there is a power loss due to the friction, windage & iron loss.

The torque required to overcome these losses is called lost torque denoted as  $T_f$ . These losses are also called stray losses.

The torque which is available at the shaft for doing the useful work is known as load torque (or) shaft torque  $T_{sh}$ .

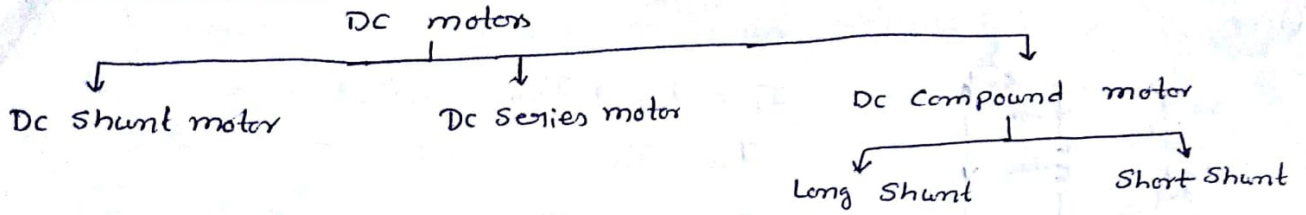
$$\therefore T_a = T_f + T_{sh}$$

The max power a motor can deliver to the load safely is called o/p rating of the motor.

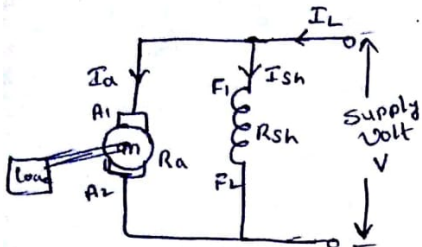
Generally it is expressed in H.P (Horse Power)

Net o/p of motor =  $P_{out} = T_{sh} \times \omega$

Types of DC motors :-



DC Shunt motor :-



The field wind is connected across the arm winding & the combination is connected across the supply.

Ra is very small & while Rsh is quite large.

Current & Volt relationship :-

$$I_L = I_a + I_{sh} \quad ; \quad I_{sh} = \frac{V}{R_{sh}}$$

$$\text{and } V = E_b + I_a R_a + V_{brush}$$

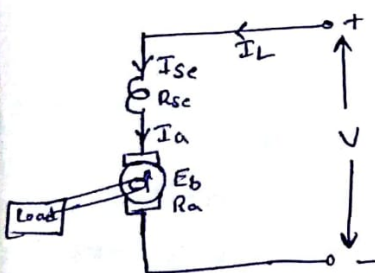
Vbrush is generally neglected.

Now flux produced by the field is proportional to the current passing through it, i.e., Ish

$$\phi \propto I_{sh}$$

As long as supply volt is constant, the  $\phi$  produced is constant. Hence d.c shunt motor is called constant flux motor.

DC Series motor :-



$$I_L = I_{se} = I_a$$

$$\text{and } V = E_b + I_a R_a + I_{se} R_{se} + V_{brush}$$

$$V = E_b + I_a (R_a + R_{se}) + V_{brush}$$

Supply volt has to overcome the drop across series field winding in addition to Eb and drop across arm winding.

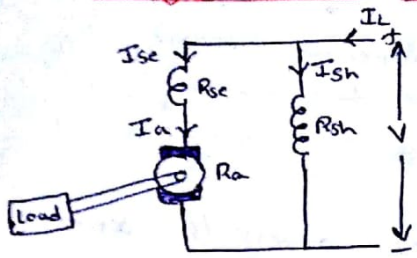
In series motor, entire arm current is passing through the series field winding. So flux produced is proportional to the armature current.

$$\boxed{\phi \propto I_{se} \propto I_a} \quad \text{for series motor}$$



## Dc Compound motor:-

### (i) Long Shunt Compound motor:-



$$I_L = I_{se} + I_{sh}$$

$$I_{se} = I_a$$

$$\therefore I_L = I_a + I_{sh}$$

$$\text{and } I_{sh} = \frac{V}{R_{sh}}$$

$$\text{and } V = E_b + I_a R_a + I_{se} R_{se} + V_{brusL}$$

$$\text{But as } I_{se} = I_a,$$

$$V = E_b + I_a (R_a + R_{se}) + V_{brusL}$$

### Short Shunt Compound motor:-

The shunt field is connected purely in parallel with armature and the series field is connected in series with combination.

$$I_L = I_{se}$$

The entire line current is passing through the series field winding.

$$\text{and } I_L = I_a + I_{sh}$$

Now the drop across the shunt field winding is to be calculated from the Volt Eqn.

$$\text{So } V = E_b + I_{se} R_{se} + I_a R_a + V_{brusL}$$

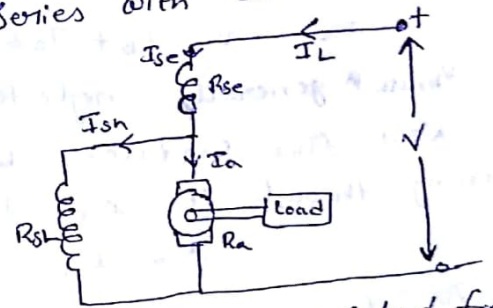
$$\text{But } I_{se} = I_L$$

$$\therefore V = E_b + I_L R_{se} + I_a R_a + V_{brusL}$$

$\therefore$  Drop across shunt field winding is,

$$= V - I_L R_{se} = E_b + I_a R_a + V_{brusL}$$

$$\therefore I_{sh} = \frac{V - I_L R_{se}}{R_{sh}} = \frac{E_b + I_a R_a + V_{brusL}}{R_{sh}}$$



(Q) A 4 Pole, 250V, dc series motor has a wave connected arm with 200 conductors. The flux per pole is 25 mwb. When motor is drawing 60A from the supply. Armature resistance is 0.15  $\Omega$  while series field winding resistance is 0.2  $\Omega$ . Calculate the speed under this condition.

Sol:-  $P = 4$ ,  $Z = 200$ ,  $A = 2$ ,  $\phi = 25 \times 10^{-3} \text{ Wb}$ .

$$I_a = I_L = 60 \text{ A}; \quad R_a = 0.15 \Omega; \quad R_{sc} = 0.2 \Omega$$

$$V = E_b + I_a R_a + I_a R_{sc}$$

$$250 = E_b + 60 (0.15 + 0.2)$$

$$E_b = 229 \text{ V}$$

$$E = \frac{NPZ}{60} \cdot \frac{P}{A} \Rightarrow 229 = \frac{25 \times 10^{-3} \times 4 \times N \times 200}{60 \times 2}$$

$$N = 1374 \text{ rpm}$$

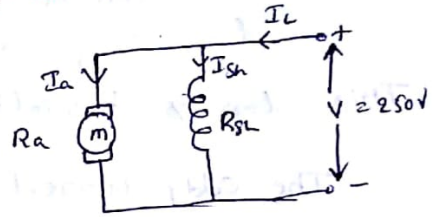
(Q) A 250V, dc shunt motor takes a line current of 20A. Resistance of shunt field wnd is  $200 \Omega$  and resistance of the arm is  $0.3 \Omega$ . Find the arm current and back emf?

Sol:-  $V = 250 \text{ V}$ ,  $I_L = 20 \text{ A}$ .  $R_a = 0.3 \Omega$ ,  $R_{sh} = 200 \Omega$ .

$$I_L = I_a + I_{sh}; \quad I_{sh} = \frac{V}{R_{sh}} = \frac{250}{200} = 1.25 \text{ A}$$

$$I_a = I_L - I_{sh} = 20 - 1.25 = 18.75 \text{ A}$$

$$\text{Now } V = E_b + I_a R_a \Rightarrow E_b = V - I_a R_a = 250 - 18.75 \times 0.3 = 244.375 \text{ V}$$



### Losses in a DC machine

The various losses in a dc machine are classified into 3 groups as:-

- (1) Copper losses.
- (2) Iron (or) Core losses.
- (3) mechanical losses.

#### Copper losses:-

The Cu losses are the losses that taking place due to the current flowing in a winding.

There are basically two windings in a d.c machine namely arm wnd and field wnd.

Cu losses are proportional to the square of the current flowing through these windings.

Thus the various copper losses can be given by

$$\text{Arm Cu loss} = I_a^2 R_a$$

$$\text{Shunt field copper loss} = I_{sh}^2 R_{sh}$$

$$\text{series field copper loss} = I_{sc}^2 R_{sc}$$

## ② Iron (or) Core losses:-

These losses are also called "magnetic losses". It includes:

- (a) Hysteresis loss                      (b) Eddy Current loss.

The hysteresis loss is Proportional to the freq and the maximum flux density  $B_m$  in the air gap and is given by

$$\text{Hysteresis loss} = \eta B_m^{1.6} V f \text{ Watts.}$$

where  $\eta$  = Steinmetz hysteresis coefficient

$V$  = Volume of Core in  $m^3$

$f$  = freq of magnetic reversal.

This loss is basically due to reversal of magnetisation of the arm core.

The eddy current loss exists due to eddy currents. When armature core rotates, it cuts the magnetic flux and emf gets induced in the core. This induced emf sets up eddy currents which cause the power loss. This loss is given by

$$\text{Eddy Current loss (} W_e \text{)} = K B_m^2 f^2 t^2 V \text{ Watts}$$

where  $k$  = constant ;  $t$  = thickness of each lamination

$V$  = Volume of core ;  $f$  = freq of magnetic reversals.

The  $W_h$  is minimized by selecting the core material having low hysteresis coefficient.

while  $W_e$  is minimized by selecting the laminated construction for the core.

These losses are almost constant for dc machines.

## Mechanical losses:-

Some power is required to overcome mechanical friction and wind resistance at the shaft. This loss is nothing but the friction & windage loss.

The mechanical losses are also constant for a d.c machine

The magnetic and mechanical losses together are called stray losses. For shunt & compound, where field current is constant, field  $C_u$  losses are also constant.

Thus stray losses along with constant field  $C_u$  losses are called constant losses.

while the arm current is dependent on load & thus armature Cu losses are called Variable losses. (15)

Thus for a d.c machine

$$\text{Total losses} = \text{Constant losses} + \text{Variable losses}$$

Efficiency of a Dc machine:-

$$\% \eta = \frac{\text{O/P Power}}{\text{I/P Power}} \times 100 = \frac{P_{out}}{P_{in}} \times 100$$

$$P_{in} = P_{out} + \text{Total losses} = P_{out} + P_{cu} + P_i$$

Where  $P_{cu}$  = Variable losses

$P_i$  = constant losses.

$$\% \eta = \frac{P_{out}}{P_{out} + \text{Losses}} \times 100$$

Condition for maximum efficiency:-

In case of a dc generator the output is given by

$$P_{out} = VI$$

$$P_{cu} = \text{Variable losses} = I_a^2 R_a = I^2 R_a$$

$I_a = I$  -- neglecting shunt field current.

$$\therefore \% \eta = \frac{VI}{VI + I^2 R_a + P_i} \times 100 = \frac{1}{1 + \left( \frac{I R_a}{V} + \frac{P_i}{VI} \right)} \times 100$$

The efficiency is max, when the denominator is minimum.  
According to maxima - minima theorem,

$$\frac{d}{dI} \left[ 1 + \left( \frac{I R_a}{V} + \frac{P_i}{VI} \right) \right] = 0$$

$$\therefore \frac{R_a}{V} - \frac{P_i}{VI^2} = 0 \quad \Rightarrow \quad I^2 R_a - P_i = 0$$

$$\therefore I^2 R_a = P_i = P_{cu}$$

Thus for maximum efficiency, the condition is

$$\boxed{\text{Variable losses} = \text{Constant losses}}$$

Current at maximum efficiency

For shunt machines; The  $I_{sh}$  is constant & loss  $VI_{sh}$  is treated to be part of constant losses. The variable losses are  $I_a^2 R_a$ .

At max  $\eta \Rightarrow I_a^2 R_a = P_i = (\text{Stray} + \text{shunt field losses})$

$$I_a = \sqrt{\frac{P_i}{R_a}} = \sqrt{\frac{\text{Constant losses}}{\text{Arm resistance}}}$$

For Series machines: The current through series field is same as arm current which is same as line current.

Here constant losses are only mechanical losses while the variable losses are the copper losses in arm as well as series field winding due to arm current.

At max  $\eta$ ,

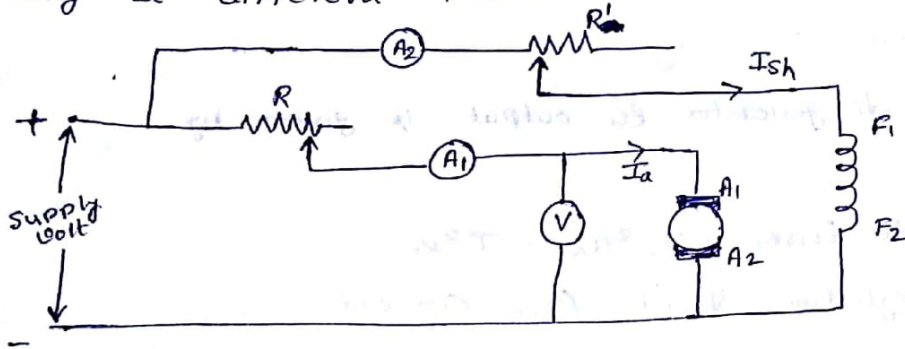
$$I_a^2 (R_a + R_{se}) = P_i = \text{mechanical losses}$$

$$I_a = \sqrt{\frac{P_i}{R_a + R_{se}}}$$

Swinburne's Test @ No-load Test :-

This is indirect method of testing d.c motors in which flux remains practically constant.

Without actually loading the motor, the losses and hence efficiency at different loads can be found out.



At starting some resistance is connected in series with the armature which is cut when motor attains sufficient speed. Speed of the motor is adjusted to the rated speed with the help of shunt field rheostat.

The no load armature current  $I_a$  is measured by  $A_1$ .

The shunt field current  $I_{sh}$  is measured by  $A_2$ .

If  $V$  = supply volt, then motor i/p at no-load will be

$$\text{Power i/p at no-load} = V(I_a + I_{sh}) \text{ watts.}$$

Cu losses in the field winding is

$$\text{Field Cu loss} = V \times I_{sh}$$

Let  $R_a$  be the armature resistance,

$$\text{Arm Cu loss} = I_a^2 R_a$$

Stray losses = i/p at no-load - Field Cu losses - No-load arm Cu losses

$$\text{Stray losses} = V(I_a + I_{sh}) - V \times I_{sh} - I_a^2 R_a = W_a$$

In the field and armature windings there will be copper loss due to flow of current which will increase the temperature of the field and armature winding when the motor is loaded. This increase in temperature will affect their resistances.

Thus the new value of field resistance  $R_{sh}'$  and that of armature  $R_a'$  can be found by considering that rise in temperature as about  $40^\circ C$ .

∴  $\alpha_1$  = Resistance temperature coefficient of copper at room temperature

$$R_a' = R_a (1 + \alpha_1 40)$$

At room temp the shunt field winding resistance will be,

$$R_{sh} = \frac{V}{I_{sh}}$$

$$\therefore R_{sh}' = R_{sh} (1 + \alpha_1 40)$$

Now shunt field winding current,  $I_{sh}' = \frac{V}{R_{sh}'}$

$$\text{New field Cu loss} = I_{sh}'^2 \times R_{sh}$$

Now if we want to find the efficiency of the motor at  $\frac{1}{4}$ th full load.

It can be calculated as follows,

Let  $I_{FL}$  = Full load current of motor.

$W_F$  = field Cu loss

$W$  = stray losses

$$\text{Load current at } \frac{1}{4} \text{th full load} = \frac{I_{FL}}{4}$$

$$\therefore \text{Motor input at } \frac{1}{4} \text{th full load} = V \cdot \frac{I_{FL}}{4} \text{ watts}$$

$$\text{Armature current at } \frac{1}{4} \text{th full load} \Rightarrow I_a' = \frac{I_{FL}}{4} - I_{sh}'$$

$$\text{Arm Cu loss at } \frac{1}{4} \text{th full load} = (I_a')^2 R_a = \left(\frac{I_{FL}}{4} - I_{sh}'\right)^2 \cdot R_a$$

$$\therefore \text{Motor output at } \frac{1}{4} \text{th full load} = \text{Motor i/p at } \frac{1}{4} \text{th load} - \text{Losses}$$

$$= \left(V \times \frac{I_{FL}}{4}\right) - \left(\frac{I_{FL}}{4} - I_{sh}'\right)^2 R_a - W_F - W$$

$$= \left(V \times \frac{I_{FL}}{4}\right) - (I_a')^2 R_a - W_F - W$$

$$\text{Efficiency at } \frac{1}{4} \text{th full load, } \eta = \frac{\text{Output}}{\text{Input}} = \frac{\text{Input} - \text{Losses}}{\text{Input}}$$

$$\eta\% = \frac{\left(V \times \frac{I_{FL}}{4}\right) - (I_a')^2 R_a - W_F - W}{V \times \frac{I_{FL}}{4}} \times 100$$

This is the efficiency of motor when the load on motor is  $\frac{1}{4}$ th of full load which can be found without loading the motor. The efficiencies at other loads can be calculated similarly.

### Advantages:-

- (1) Since constant losses are known, the  $\eta\%$  can be estimated at any load.
- (2) The method is convenient & economical as less power is required for testing even a large motor i.e., only no load power is to be supplied.
- (3) The motor is not required to be loaded i.e., only test to be carried out is the no-load test.

### Disadvantages:-

- (1) In this method, Iron losses are assumed to be constant which is not true, since they change from no-load to full load.
- (2) As it is a no load test it cannot be performed on a series motor.
- (3) We have assumed that there is rise in temperature of  $40^\circ\text{C}$  at full load which cannot be checked actually as we are not actually loading the motor.

### Efficiency as a motor :-

$$\text{Input} = VI, \quad \text{Armature Cu loss} = I_a^2 R_a$$

$$W = \text{Stray losses}, \quad W_F = \text{field Cu loss}$$

$$\therefore \eta\% = \frac{\text{Input} - \text{Losses}}{\text{Input}} \times 100 = \frac{VI - I_a^2 R_a - W_F - W}{VI} \times 100$$

### Efficiency as a generator :-

$$\text{Output} = V_t I_L, \quad \text{Armature Cu loss} = I_a^2 R_a$$

$$W = \text{Stray losses}, \quad W_F = \text{field copper loss}$$

$$\therefore \eta\% = \frac{\text{output}}{\text{output} + \text{losses}} \times 100 = \frac{V_t I_L}{V_t I_L + I_a^2 R_a + W_F + W} \times 100$$





$$T_{sh} = \text{Net Pull} \times R = 9.81(\omega_1 - \omega_2) R \quad \text{N-m}$$

Hence the output Power can be obtained as ,

$$P_{out} = T_{sh} \times \omega = 9.81 (\omega_1 - \omega_2) R \times \frac{2\pi N}{60} \quad \text{Watts .}$$

Now let ,  $V =$  Volt applied in Volts

$I =$  Total line current drawn in amps .

then  $P_{in} = VI$  watts .

Thus if the readings are taken on full load condition then the efficiency can be obtained as

$$\% \eta = \frac{P_{out}}{P_{in}} \times 100 .$$

Adjusting the load step by step till full load, no. of readings can be obtained. The Speed can be measured by tachometer.

Thus all the motor characteristics can be Plotted.

#### Advantages:-

- (1) Actual efficiency of the motor under working conditions can be found out.
- (2) The method is simple and easy to Perform.
- (3) Can be Performed on any type of d.c motor.

#### Disadvantages:-

- (1) Due to friction , heat is generated and hence there is large dissipation of energy .
- (2) Some type of cooling arrangement is necessary .
- (3) Convenient only for small machines due to limitations regarding heat dissipation arrangements .
- (4) The Power developed gets wasted hence method is expensive .

(Q) A 10kw, 250v d.c Shunt motor with an  $R_a = 0.8 \Omega$  &  $R_{sh} = 275 \Omega$  takes 3.91A, when running light at rated volt and rated speed. Calculate the machine efficiency as a generator when delivering an output of 10kw at rated voltage & speed and as a motor drawing an input of 10kw.

Sol:-  $V = 250v$ ,  $R_a = 0.8 \Omega$ ;  $R_{sh} = 275 \Omega$ ,  $I_o = 3.91A$

$$(a) I_{sh} = \frac{V}{R_{sh}} = \frac{250}{275} = 0.9090A$$

$$I_{ao} = I - I_{sh} = 3.91 - 0.9090 = 3A.$$

$$\text{No load armature Cu loss} = I_{ao}^2 R_a = (3)^2 \cdot (0.8) = 7.2W$$

$$\text{No-load input Power} = V \cdot I_o = 250 \times 3.91 = 977.5W$$

$$\begin{aligned} \text{Constant losses} &= \text{No-load i/p Power} - \text{No load Cu losses} \\ &= 977.5 - 7.2 = 970.3W. \end{aligned}$$

$$\text{Shunt field Cu losses} = I_{sh} \cdot R_{sh} = (0.9090)^2 (275) = 227.22W$$

$$\begin{aligned} \text{Stray losses} &= \text{Constant losses} - \text{shunt field Cu loss} \\ &= 970.3 - 227.22 = 743W \end{aligned}$$

(b) when machine is running as a generator, output = 10kw.

$$I_L = \frac{O/P}{V} = \frac{10 \times 10^3}{250} = 40A; \quad I_{sh} = 0.9090A.$$

$$I_a = I_L + I_{sh} = 40 + 0.9090 = 40.909A = 40.91A.$$

$$\text{Input} = O/P + \text{Losses}$$

$$= 10000 + (40.91)^2 \times 0.8 + 970.3$$

$$= [10000 + 1338.90 + 970.3] = 12309.2W$$

$$\text{Generator efficiency, } \eta_g = \frac{O/P}{i/p} \times 100$$

$$\eta_g = \frac{10,000}{12309.2} \times 100 = 81.24\%$$

(c) when machine is running as a motor, input = 10kw.

$$I_L = \frac{O/P}{V} = \frac{10 \times 10^3}{250} = 40A; \quad I_{sh} = 0.9090A.$$

$$I_a = I_L - I_{sh} = 40 - 0.9090 = 39.09A.$$

$$\text{Armature Cu loss} = I_a^2 \cdot R_a = (39.09)^2 (0.8) = 1222.48W.$$

$$\text{Total losses} = 1222.48 + 970.3 = 2192.78W.$$

$$\text{Output} = \text{I/P} - \text{losses} = 10000 - 2192.78 = 7807.22W.$$

$$\text{motor efficiency } \eta_m = \frac{O/P}{i/p} \times 100 = \frac{7807.22}{101000} \times 100 = 78.07\%$$

[Assumption:- stray losses at load has been neglected.]

Q) In a brake test conducted on a d.c shunt motor the full load readings are observed as, Tension on light side = 9.1 kg, Tension on black side = 0.8 kg, Total Current = 10 A. Supply Voltage = 110 V, Speed = 1320 r.p.m. The radius of the Pulley is 7.5 cm. Calculate its full load efficiency.

Sol:-  $w_1 = 9.1 \text{ kg}$  ;  $w_2 = 0.8 \text{ kg}$  ,  $I = 10 \text{ A}$  ,  $V = 110 \text{ V}$  ,  $R = 7.5 \text{ cm}$

$$T_{sh} = (w_1 - w_2) \times 9.81 \times R = (9.1 - 0.8) \times 9.81 \times 0.075 = 6.1067 \text{ N-m}$$

$$P_{out} = T_{sh} \times \omega = T_{sh} \times \frac{2\pi N}{60} = \frac{6.1067 \times 2\pi \times 1320}{60} = 844.133 \text{ W.}$$

$$P_{in} = VI = 110 \times 10 = 1100 \text{ W.}$$

$$\% \eta = \frac{P_{out}}{P_{in}} \times 100 = \frac{844.133}{1100} \times 100 = 76.74 \%$$

Q) The following readings are obtained when doing a load test on d.c shunt motor using a brake drum.  
 Spring balance reading = 10 kg and 35 kg; Diameter of drum = 40 cm  
 Speed of the motor = 950 rpm, Applied Voltage = 200 V

Line Current = 30 A.

Calculate the O/P Power and efficiency.

Sol:-  $w_1 = 10 \text{ kg}$  ;  $w_2 = 35 \text{ kg}$  ;  $D = 40 \text{ cm}$  ,  
 Radius =  $R = \frac{D}{2} = 20 \text{ cm} = 0.2 \text{ m}$ .

$I = 30 \text{ A}$  ,  $V = 200 \text{ V}$  ,  $N = 950 \text{ rpm}$ .

$T_{sh} = (w_2 - w_1) \times 9.81 \times R = (35 - 10) \times 9.81 \times 0.2 = 49.05 \text{ N-m}$

$P_{out} = T_{sh} \times \omega = \frac{2\pi N}{60} T_{sh} = \frac{2\pi \times 950 \times 49.05}{60} = 4879.67 \text{ W}$   
 $= 4.87 \text{ kW}$

$P_{in} = V \cdot I = (200)(30) = 6000 \text{ W} = 6 \text{ kW}$ .

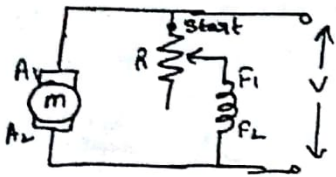
$\% \eta \text{ of DC motor} = \frac{P_{out}}{P_{in}} \times 100 = \frac{4.87}{6} \times 100 = 81.32 \%$ .

$\therefore$  Power output of DC shunt motor = 4.87 kW

## Speed Control of DC Motor:-

### (i) DC Shunt Motor:-

#### (a) Flux control:-



The Speed is inversely Proportional to the flux.

$$\text{Since } N \propto \frac{V}{\phi}$$

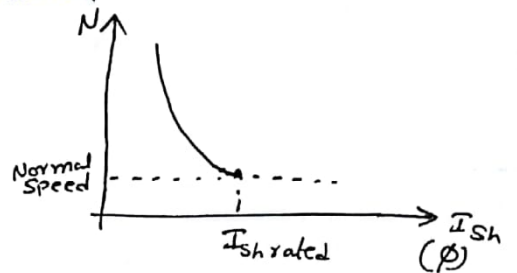
The flux is dependent on the Current through the Shunt field winding. Thus the flux can be controlled by adding a rheostat in series with the shunt field winding as shown in fig.

At the beginning the rheostat R is kept at minimum indicated as start. The supply voltage is at its rated value. So the Current in the field winding is also at its rated value. Hence the speed is also rated speed.

Then 'R' is increased due to which shunt field current  $I_{sh}$  decreases, decreasing the flux produced as  $N \propto \frac{1}{\phi}$ , the speed of the motor increases beyond its rated value.

Thus by this method, the speed control above rated value is possible.

#### Adv of Flux Control method:-



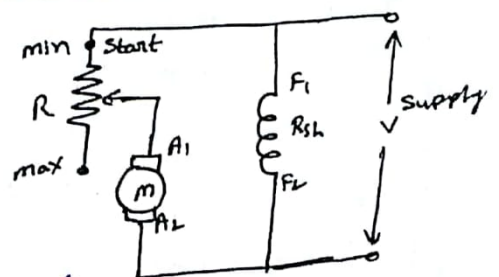
- (1) It provides relatively smooth and easy control
- (2) Speed control above rated speed is possible.
- (3) As the field winding resistance is high, the field current is small. Hence power loss ( $I_{sh}^2 R$ ) in the external resistance is very small.
- (4) As the field current is small, the size of the rheostat required is small.

#### Disadv of flux control method:-

- (1) The speed ~~above~~ <sup>below</sup> rated speed is not possible as flux can be increased only upto its rated value.
- (2) High speed affects the commutation, makes motor operation unstable.

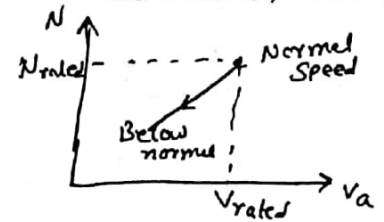
### (b) Armature voltage control (or) Rheostatic control method:-

The speed is directly proportional to the voltage applied across the armature. As supply voltage is normally constant, the volt across the armature can be controlled by adding a variable resistance in series with the armature as shown in fig.



Initially the rheostat position is minimum & rated voltage gets applied across the armature. So the speed is also rated. For a given load, armature current is fixed. So when extra resistance is added in the armature circuit,  $I_a$  remains same & there is volt drop across the resistance added. Hence the voltage across the armature decreases, decreasing the speed below normal value.

By this method, we obtain below rated speed.



### Adv of Rheostatic Control:-

- (1) Easy & smooth speed control below normal is possible.
- (2) In potential divider arrangement, rheostat can be used as a starter.

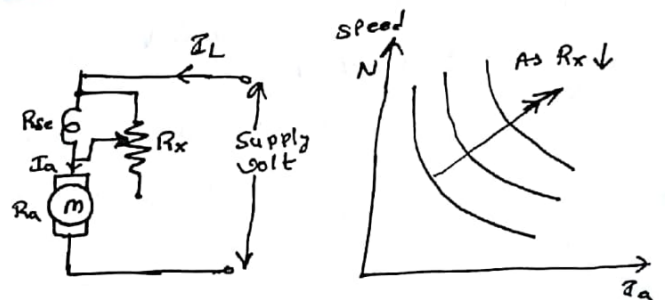
### Dis Adv :-

- (1) As the entire  $I_a$  passes through the external resistance, there are tremendous power losses.
- (2) As  $I_a > I_f$ , rheostat required is of large size & capacity.
- (3) Speed above rated is not possible by this method.
- (4) This method is expensive, wasteful & less efficient.

## ② Speed Control of Dc series motor:-

### (i) Flux Control:-

On this method, the series field winding is shunted by a variable resistance ( $R_x$ ) known as field diverter.

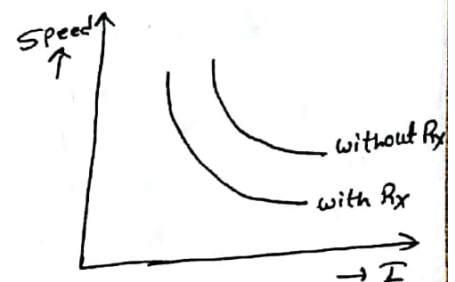
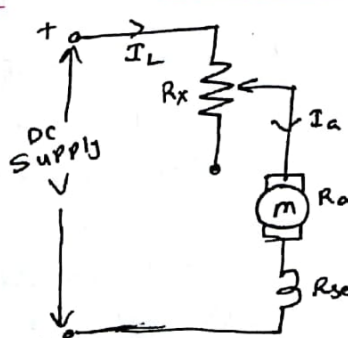


Due to parallel path of  $R_x$ , by adjusting the value of  $R_x$ , any amount of current can be diverted through the diverter. Hence current through the field winding can be adjusted as per the requirement. Due to this, the flux gets controlled & hence the speed of the motor gets controlled.

By this method the speed of the motor can be controlled above rated value.

### (ii) Armature Voltage Control (or) Rheostatic Control method:-

On this method, a variable resistance  $R_x$  is inserted in series with the motor circuit. As this ~~resistance~~ reduces the voltage across the armature.



As speed is directly Proportional to the voltage across the armature, the speed reduces. As the entire current passes through  $R_x$ , there is large Power loss. The Speed - armature current characteristics with change in  $R_x$  are shown in fig. (20) (11)

### Applications of DC motors

Types of motor	Characteristics	Applications
Dc shunt motor	Speed is constant & medium starting Torque	(1) Blowers & fans (2) Centrifugal & reciprocating PUMPS (3) Lathe machines (4) machine tools (5) milling machines (6) Drilling machines.
Dc series motor	High starting Torque. No-load speed is dangerous, variable speed.	(1) cranes (2) Hoists, Elevators (3) Trolleys (4) conveyors (5) Electric locomotives
Cummulative Compound motor	High starting Torque No-load condition is allowed	(1) Rolling mills (2) punches (3) Shears (4) Heavy planers (5) Elevators.
Differential Compound motor	Speed increases as load increases	Not suitable for any practical applications.

## Commutation :-

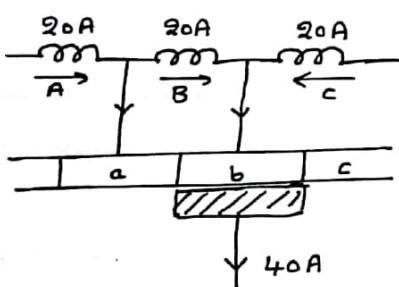
The currents induced in armature conductors of a d.c generator are alternating. To make their flow unidirectional in the external circuit, we need a commutator. These currents flow in one direction when armature conductors are under N-pole and in the opposite direction when they are under S-pole.

As conductors pass out of the influence of a N-pole & enter that of S-pole, the current in them is reversed. This reversal of current takes place along magnetic neutral axis or brush axis i.e., when the brush spans & hence short circuits that particular coil undergoing reversal of current through it.

This process of current reversals in a coil while it crosses the MNA is called Commutation.

The brief period during which coil remains short circuited is known as commutation period  $T_c$ .

### Commutation Process.



fig(a)

The brush width is equal to the width of one commutator segment & one mica insulation.

In fig(a), coil B is about to be short circuited because brush is about to come in touch with commutator segment 'a'.

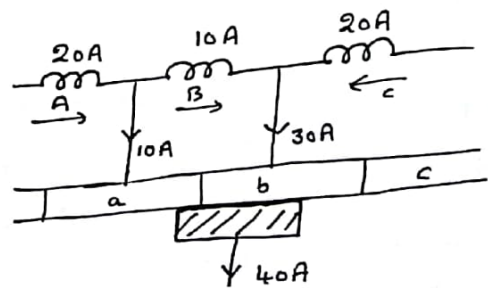
It is assumed that each coil carries 20A, so that brush current is 40A.

Prior to the beginning of short circuit, coil B belongs to the group of coils lying to the left of the brush and carries 20A from left to right.

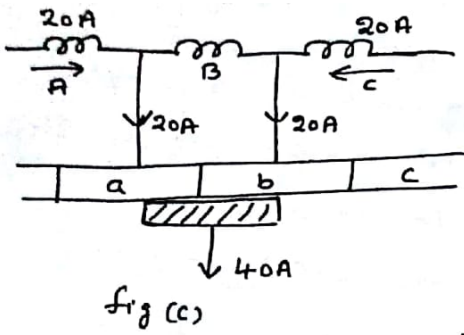
In fig(b), coil B has entered its period of short circuit and is approximately at  $1/3$ rd of this period.

The current through coil B has reduced down from 20A to 10A, because

the other 10A flows via segment 'a'. As area of contact of the brush is more with segment 'b' than with segment 'a', it receives 30A from the former, the total again being 40A.



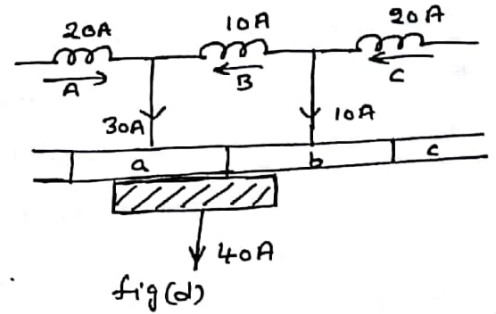
fig(b)



In fig(c), Coil B is in the middle of its short circuit period. The current through it has decreased to zero. The two currents of value 20A each, pass to the brush directly from Coil A & C as shown.

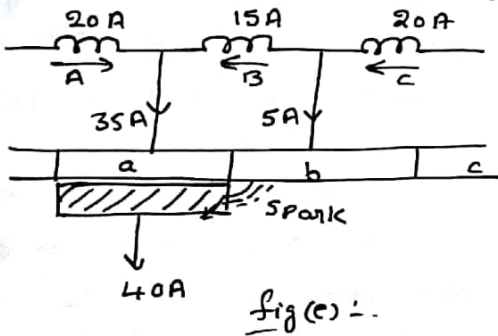
The brush contact area with two segments 'b' and 'a' are equal.

In fig(d), Coil B has become part of the group of coils lying to the right of the brush. It is seen that brush area with segment 'b' is decreasing rapidly whereas that with segment 'a' is increasing.



Coil B now carries 10A in reverse direction which combines with 20A supplied by Coil A to make up 30A that passes from segment 'a' to the brush.

The other 10A is supplied by Coil C and passes from segment 'b' to the brush, again giving a total of 40A at the brush.

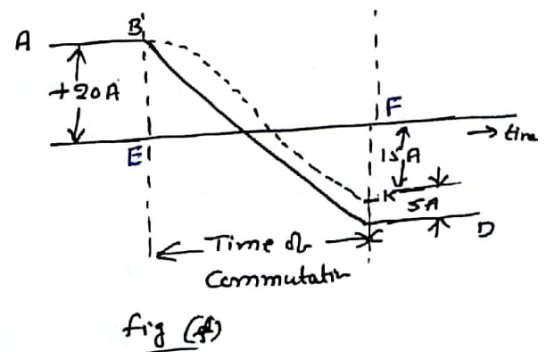


Fig(e), depicts the movement when Coil B is almost at the end of commutation or short circuit period.

For ideal commutation, current through it should have reversed by now but as shown it is carrying 15A only (instead of 20A).

The difference of current between coils 'C' & 'B' i.e.,  $20 - 15 = 5A$ , jumps directly from segment 'b' to the brush through air thus producing spark.

As the changes of current through Coil B are plotted on a time base in fig(f), it will be represented by a horizontal line AB i.e., 20A up to the time of beginning of commutation.



From the finish of commutation, the current will be represented by another horizontal line CD. Now again the current value is  $FC = 20A$ , although in the reverse direction.



If the Current varies at a uniform rate i.e., it BC is a straight line, then it is referred to as Linear Commutation.

However, due to the Production of self induced emf, in the coil the variations follow the dotted curve. On this current in coil B has reached only a value of  $KF = 15A$  in the reverse direction. Hence the difference of  $5A$  passes as a Spark. which results in Poor commutation.

The main reason which retards or delays this quick reversal is the Production of self-induced emf in the coil undergoing commutation. This self induced emf is known as reactance Voltage.

Value of Reactance Voltage :-

$$\text{Reactance Voltage} = \text{Co-efficient of self inductance} \times \text{rate of change of current.}$$

The time of Short Circuit on Commutation is the time required by the Commutator to move a distance equal to the Circumferential thickness of the brush minus the thickness of one insulating plate or Strip of mica

Let  $w_b =$  brush width in cm ;

$w_m =$  width of mica insulation in cm

$v =$  Peripheral Velocity of Commutator segments in cm/sec

$$T_c = \text{time of Commutation on Short Circuit} = \frac{w_b - w_m}{v} \text{ Sec.}$$

If  $I$  is the current through a conductor, then total charge during Commutation  $= I - (-I) = 2I$

$$\therefore \text{Self induced emf on reactance voltage} = L \times \frac{2I}{T_c} \rightarrow \text{if Commutation is linear}$$

$$= 1.11 L \frac{2I}{T_c} \rightarrow \text{if Commutation is Sinusoidal.}$$

This sparking will not only damage the brush & also the Commutator

Methods of improving Commutation:-

(i) Resistance Commutation

(ii) EMF Commutation

(iii) Interpoles or Compoles

3) The armature of a certain dynamo runs at 800 rpm. The commutator consists of 123 segments and the thickness of each brush is such that the brush spans three segments. Find the time during which the coil of an armature remains short-circuited.

Sol: As  $\omega_m$  is not given  $\Rightarrow$  So neglect

$$\omega_b = 3 \text{ segments} \quad \& \quad \omega = (800/60) \times 123 \text{ segments/sec}$$

$$\therefore \text{Commutation time} = \frac{3 \times 60}{800 \times 123} = 0.00183 \text{ sec} = 1.83 \text{ msec}$$

(Q) A 4 Pole, lap wound armature running at 1500 rpm delivers a current of 150 A & has 64 commutator segments. The brush spans 1.2 segments & inductance of each armature coil is 0.05 mH. Calculate the value of reactance voltage assuming (i) linear commutation (ii) sinusoidal commutation. Neglect mica thickness.

Sol: -  $E = L \frac{2I}{T_c}$  ; Now  $L = 0.05 \text{ mH}$  ;  $\omega_b = 1.2 \text{ segments}$

$$\omega = \left(\frac{1500}{60}\right) \times 64 = 1600 \text{ segments/sec.}$$

$$T_c = \frac{1.2}{1600} = 7.5 \times 10^{-4} \text{ sec} ; \quad I = \frac{150}{4} \text{ A} = 37.5 \text{ A}$$

$$\therefore \frac{2I}{T_c} = \frac{2 \times 37.5}{7.5 \times 10^{-4}} = 10^5 \text{ A/sec}$$

$$\therefore \text{For linear commutation, } E = 0.05 \times 10^{-3} \times 10^5 = 5 \text{ V}$$

$$\therefore \text{" Sinusoidal " } E = 1.11 \times 5 = \underline{5.55 \text{ V}}$$

(Q) Calculate the reactance voltage for a machine having the following particulars. Number of commutator segments = 55. Revolutions per minute = 900. Brush width in commutator segments = 1.74. Coefficient of self induction =  $153 \times 10^{-6} \text{ H}$ . Current per coil = 27 A.

Sol:  $I = 27 \text{ A}$  ;  $L = 153 \times 10^{-6}$  ; No. of segments = 55.

$$\text{RPM} = 900 ; \quad \omega = \left(\frac{900}{60}\right) \times 55 = 825 \text{ segments/sec.}$$

$$T_c = \omega_b / \omega = 1.74 / 825 = 2.11 \times 10^{-3} \text{ sec.}$$

Assuming linear commutation,  $E = L \times \frac{2I}{T_c}$

$$\therefore E = 153 \times 10^{-6} \times 2 \times 27 / 2.11 \times 10^{-3} = \underline{3.91 \text{ V}}$$

## Action of Commutator :-

If, Somehow, connection of the coil side to the external load is reversed at the same instant the current in the coil side reverses, the current through the load will be direct current. This is what commutator does.

Fig (1) shows a commutator having two segments  $C_1$  &  $C_2$ . It consists of a cylindrical metal ring cut into two halves (or) segments  $C_1$  &  $C_2$  respectively separated by a thin sheet of mica insulation.

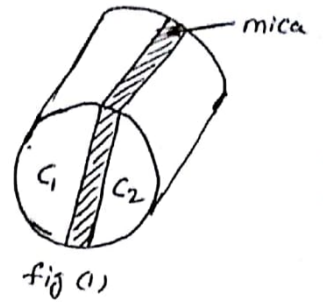


fig (1)

The commutator is mounted on but insulated from the rotor shaft.

The ends of coil sides AB & CD are connected to the segments  $C_1$  &  $C_2$  respectively as shown in fig (2).

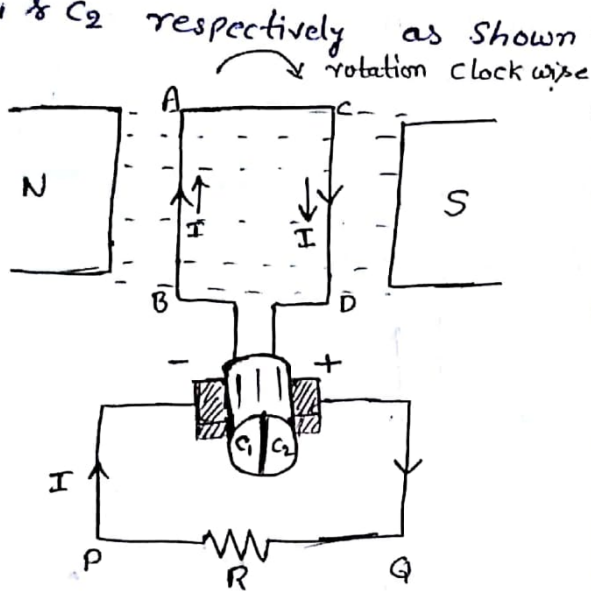


fig (2)

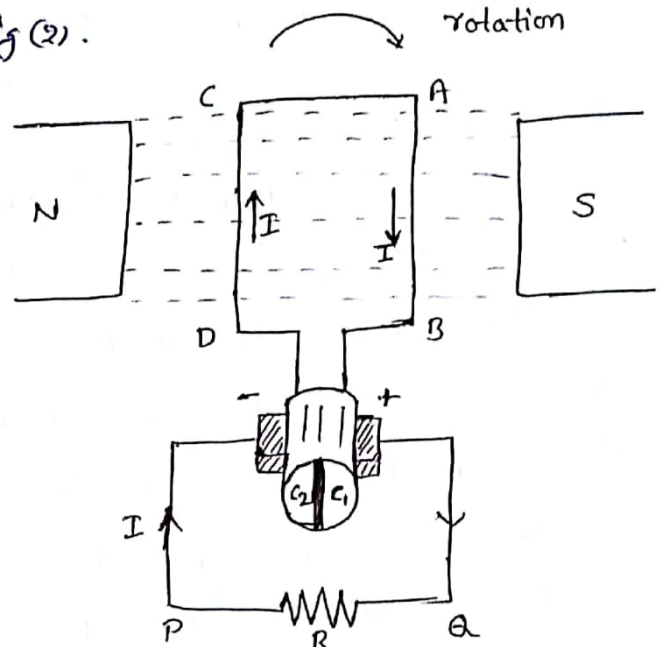


fig (3)

Two stationary carbon brushes rest on the commutator at all times connects the coil side under S-pole to the +ve brush and that under N-pole to the -ve brush.

(1) In fig (2), the coil sides AB & CD are under N-pole & S-pole respectively. Note that segment  $C_1$  connects the coil side AB to point 'P' of the load resistance R, and the segment  $C_2$  connects the coil side CD to point 'Q' of the load.

Also note the direction of current through load. It is from Q to P.

(ii) After half a revolution of the loop [i.e.,  $180^\circ$  rotation], the coil side AB is under S-Pole & the coil side CD under N-Pole as shown in fig (3). The currents in the coil sides now flow in the reverse direction but the segments  $C_1$  &  $C_2$  have also moved through  $180^\circ$  i.e., segment  $C_1$  is now in contact with +ve brush & segment  $C_2$  in contact with -ve brush.

Note that commutator has reversed the coil connection to the load, i.e., coil side AB is now connected to Point 'Q' of the load & coil side CD to connected to Point 'P' of load.

Also note the direction of current through the load. It is again from Q to P.

Thus the alternating voltage generated in the conductor (or) coil side will appear as direct voltage across the brushes. This is purpose of commutator.

The brushes purpose is to collect the current from the rotating loop (or) winding to the external stationary load.

The variation of voltage across the brushes with the angular displacement of the loop is shown in fig (4)

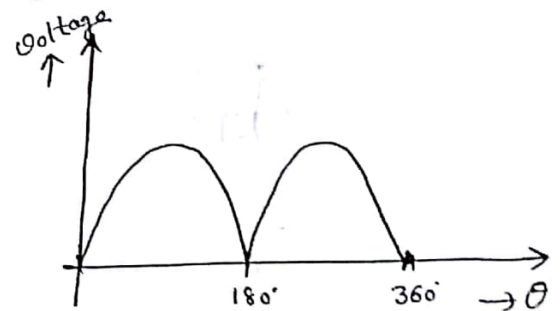


fig (4)

This is not a steady direct voltage but has a pulsating character. It is because the voltage appearing across the brushes varies from zero to maximum value and back to zero, twice for each revolution of the loop.

A steady dc voltage is achieved by using a large number of coils connected in series. The resulting arrangement is known as armature winding.

## Commutation :-

The currents induced in armature conductors of a d.c generator are alternating. To make their flow unidirectional in the external circuit, we need a commutator. These currents flow in one direction when armature conductors are under N-pole and in the opposite direction when they are under S-pole.

As conductors pass out of the influence of a N-pole & enter that of S-pole, the current in them is reversed. This reversal of current takes place along magnetic neutral axis or brush axis i.e., when the brush spans & hence short circuits that particular coil undergoing reversal of current through it.

This process of current reversal in a coil while it crosses the MNA is called Commutation.

The brief period during which coil remains short circuited is known as commutation period  $T_c$ .

### Commutation Process.

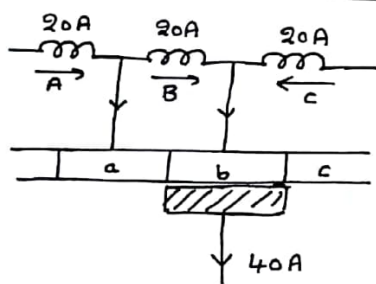


fig (a)

The brush width is equal to the width of one commutator segment & one mica insulation.

In fig (a), coil B is about to be short circuited because brush is about to come in touch with commutator segment 'a'.

It is assumed that each coil carries 20A, so that brush current is 40A.

Prior to the beginning of short circuit, coil B belongs to the group of coils lying to the left of the brush and carries 20A from left to right.

In fig (b), coil B has entered its period of short circuit and is approximately at  $1/3$ rd of this period.

The current through coil B has reduced down from 20A to 10A, because

the other 10A flows via segment 'a'. As area of contact of the brush is more with segment 'b' than with segment 'a', it receives 30A from the former, the total again being 40A.

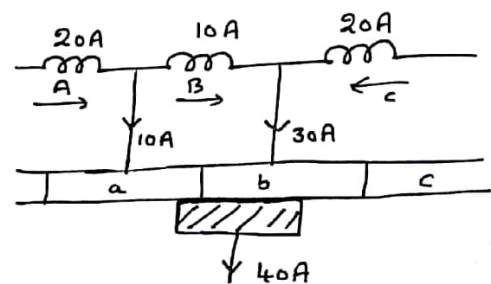
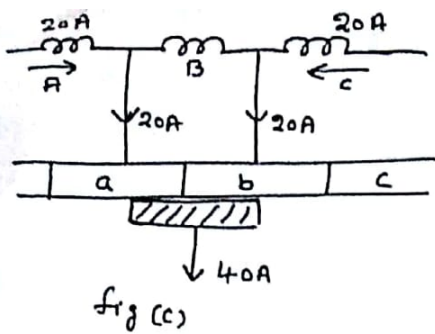


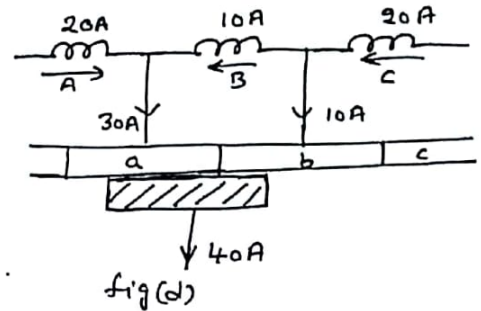
fig (b)



In fig(c), Coil B is in the middle of its Short Circuit period. The current through it has decreased to zero. The two currents of value 20A each, pass to the brush directly from Coil A & C as shown.

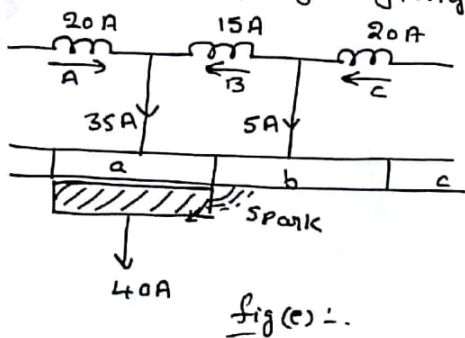
The brush contact area with two segments 'b' and 'a' are equal.

In fig(d), Coil B has become part of the group of coils lying to the right of the brush. It is seen that brush area with segment 'b' is decreasing rapidly whereas that with segment 'a' is increasing.



Coil B now carries 10A in reverse direction which combines with 20A supplied by Coil A to make up 30A that passes from segment 'a' to the brush.

The other 10A is supplied by Coil C and passes from segment 'b' to the brush, again giving a total of 40A at the brush.

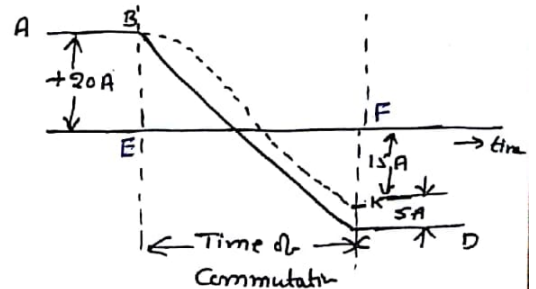


Fig(e), depicts the movement when Coil B is almost at the end of Commutation or Short Circuit Period.

For ideal commutation, current through it should have reversed by now but as shown it is carrying 15A only (instead of 20A).

The difference of current between coils 'C' & 'B' i.e.,  $20 - 15 = 5A$ , jumps directly from segment 'b' to the brush through air thus producing spark.

Of the changes of current through Coil B are plotted on a time base in fig(f), it will be represented by a horizontal line AB i.e., 20A up to the time of beginning of commutation.



From the finish of commutation, the current will be represented by another horizontal line CD. Now again the current value is  $FC = 20A$ , although in the reverse direction.

If the current varies at a uniform rate i.e., it BC is a straight line, then it is referred to as linear commutation.

However, due to the production of self induced emf, in the coil the variations follow the dotted curve. On this current in coil B has reached only a value of  $KF = 15A$  in the reverse direction. Hence the difference of  $5A$  passes as a spark. which results in poor commutation.

The main reason which retards or delays this quick reversal is the production of self-induced emf in the coil undergoing commutation. This self induced emf is known as reactance voltage.

Value of Reactance Voltage :-

$$\text{Reactance Voltage} = \text{Co-efficient of self inductance} \times \text{rate of change of current.}$$

The time of short circuit on commutation is the time required by the commutator to move a distance equal to the circumferential thickness of the brush minus the thickness of one insulating plate or strip of mica

Let  $w_b =$  brush width in cm ;

$w_m =$  width of mica insulation in cm

$v =$  Peripheral velocity of commutator segments in cm/sec

$$T_c = \text{time of commutation on short circuit} = \frac{w_b - w_m}{v} \text{ Sec.}$$

If  $I$  is the current through a conductor, then total charge during commutation  $= I - (-I) = 2I$

$$\therefore \text{Self induced emf on reactance voltage} = L \times \frac{2I}{T_c} \rightarrow \text{if commutation is linear}$$

$$= 1.11 L \frac{2I}{T_c} \rightarrow \text{if commutation is sinusoidal.}$$

This sparking will not only damage the brush & also the commutator.

Methods of improving commutation:-

- (i) Resistance commutation
- (ii) EMF commutation
- (iii) Interpoles or Compoles

(P) The armature of a certain dynamo runs at 800 rpm. The commutator consists of 123 segments and the thickness of each brush is such that the brush spans three segments. Find the time during which the coil of an armature remains short-circuited.

Sol: As  $\omega_m$  is not given so neglect

$$\omega_b = 3 \text{ segments} \quad \& \quad \omega = (800/60) \times 123 \text{ segments/sec}$$

$$\therefore \text{Commutation time} = \frac{3 \times 60}{800 \times 123} = 0.00183 \text{ sec} = 1.83 \text{ msec}$$

(Q) A 4 pole, lap wound armature running at 1500 rpm delivers a current of 150 A & has 64 commutator segments. The brush spans 1.2 segments & inductance of each armature coil is 0.05 mH. Calculate the value of reactance voltage assuming (i) linear commutation (ii) sinusoidal commutation. Neglect mica thickness.

Sol: -

$$E = L \frac{2I}{T_c} \quad ; \quad \text{Now } L = 0.05 \text{ mH} \quad ; \quad \omega_b = 1.2 \text{ segments}$$

$$\omega = \left(\frac{1500}{60}\right) \times 64 = 1600 \text{ segments/sec.}$$

$$T_c = \frac{1.2}{1600} = 7.5 \times 10^{-4} \text{ sec} \quad ; \quad I = \frac{150}{4} \text{ A} = 37.5 \text{ A.}$$

$$\therefore \frac{2I}{T_c} = \frac{2 \times 37.5}{7.5 \times 10^{-4}} = 10^5 \text{ A/sec}$$

$$\therefore \text{For linear commutation, } E = 0.05 \times 10^{-3} \times 10^5 = 5 \text{ V}$$

$$\therefore \text{" Sinusoidal " } \quad E = 1.11 \times 5 = \underline{\underline{5.55 \text{ V}}}$$

(Q) Calculate the reactance voltage for a machine having the following particulars. Number of commutator segments = 55. Revolutions per minute = 900. Brush width in commutator segments = 1.74. Coefficient of self induction =  $153 \times 10^{-6} \text{ H}$ . Current per coil = 27 A.

Sol:  $I = 27 \text{ A} \quad ; \quad L = 153 \times 10^{-6} \quad ; \quad \text{No. of segments} = 55.$

$$\text{RPM} = 900 \quad ; \quad \omega = \left(\frac{900}{60}\right) \times 55 = 825 \text{ segments/sec.}$$

$$T_c = \omega_b / \omega = 1.74 / 825 = 2.11 \times 10^{-3} \text{ sec.}$$

$$\text{Assuming linear commutation, } E = L \times \frac{2I}{T_c}$$

$$\therefore E = 153 \times 10^{-6} \times 2 \times 27 / 2.11 \times 10^{-3} = \underline{\underline{3.91 \text{ V}}}$$



## D.C. Generators

- The field Poles are made of a stack of steel plates or laminations 1 to 1.5 mm
- Both armature core & Yoke carry half of the flux per pole.
- armature core is made up of circular steel laminations of thickness 0.4 to 0.6 mm.
- $E_a \propto \frac{1}{A}$  ; Total current delivered by the armature is  $A$ .
- O.C.C are also called "No-load Saturation" Curve.
- Arm Char's :-  $I_f$  vs  $I_L$  at a constant terminal Volt & Speed.
- For Parallel operation of DC Generators :- (i) Correct Polarities  
(ii) No-load induced emf's are to be same in order to avoid circulating currents on no-load.
- Series Generator has rising nature of external Char's, it cannot operate in parallel successfully. In order to make them operate successfully in parallel, (i) cross connecting the fields.  
(ii) Providing an equalizing bar.
- 'PΦ' is called the magnetic loading &
- $I_a Z$  is " " arm ampere-conductor loading.
- On No-load, d.c series motor has tendency to run at dangerously high speed, this is referred to as the RACING of the series motor.
- Series motor should not be used in conjunction with belt-driven loads. However, if it is inevitable, a flywheel of sufficient moment of inertia is to be attached to the shaft.
- On Regenerative or Back-to-Back method, the diff b/w the i/p to M-G set and the o/p of G gives the sum total of the losses occurring in both the m/c's.
- Retardation or Running down test :- to determine the stray losses.
- The main drawback of 3-point starter is the possibility of the MAL-operation of the N.V.R, if the speed is controlled by the field flux control method.
- Field winds are concentrated wind
- DC m/c's invariably use double layer winds.
- DC arm winds are closed winds. The no. of parallel paths is  $\frac{2k}{p}$  where  $k$  is an integer  $\geq 1$ .
- In AC m/c's there may be one or more paths in parallel
- $120^\circ$  phase grouping is NOT possible in single-layer winds.

- wave winding poses a problem in end-connection, so it is not used in A.C. m/c's.

- Generally double layer lap wnds are used.

- Equalizer Rings =  $\frac{C}{(P/2)}$  = no. of arm coils / Pair of Poles

- The currents flowing in the equalizer rings will also be alternating.

- On wave and commutator pitch  $Y_c = \frac{2(\pm 1)}{P}$

$Y_b = \frac{2}{P} + 1$  ;  $Y_f = 2Y_c - Y_b$

Brush current  $I_{brush} = I_a / (P/2)$

\* - No. of dummy coils =  $C' - C$

where  $C' = \frac{1}{2} U S$  ;  $C = (\frac{P}{2}) Y_c \pm 1$

where  $U = \frac{2C}{S} = \text{no. of coil sides / slot} = (\text{even no.})$

Coil span  $Y_{cs} = \frac{S}{P}$

- A dc shunt motor is operating at no-load, If the arm is shunted by a resistance, its speed will decrease.

- The emf induced in a conductor rotating in a bipolar field is [ b ]  
 (a) dc (b) ac (c) dc & ac both (d) none

- If a current carrying coil is placed in a uniform magnetic field with its plane  $\perp$  to direction of magnetic induction, then ~~the~~ the net force and torque on the coil are both zero.

- Min no. of coils for 400V, 6-pole lap connected dc m/c for a max volt of 15V b/w adjacent commutator segments should be [ b ]  
 (a) 80 (b) 160 (c) 200 (d) 400

- A frog-leg wnd is used in a dc m/c to eliminate - ~~the~~  
 "the use of equalizers in lap wnd".

- The volt regulation of a dc generator at FL being zero implies that the generator is cumulatively compounded.

- Centrifugal pumps - dc series motor

- A fly wheel is normally fitted to cumulatively compounded motor driving torque load.

- In case of dc series motor it is possible to have a finite no-load speed if a resistance is connected across its armature.

## Alternators

- The highest rating of an alternator is 1700 MW.
- Reluctance torque =  $\frac{1}{\omega_s} \left\{ \left[ \frac{V_t^2}{2} \left( \frac{1}{x_q} - \frac{1}{x_d} \right) \right] \sin 2\delta \right\}$
- Modern alternators have SCR b/w 0.5 and 1.5.
- At leading P.f operation, an alternator is under excited.

## Principles of Electro-mechanical Energy Conversion

- The energy storage capacity of magnetic field is nearly 30,000 times that of an electric field.
- General eq:- 
$$W_{elec} = W_{mech} + W_{fld}$$
  - $W_{elec}$  = Net electrical energy i/e to the coupling field
  - $W_{mech}$  = Total energy converted to mechanical form  
(mech energy o/r + energy stored in the mech system + fric & winding losses)
  - $W_{fld}$  = Total energy absorbed by the coupling field  
(= energy stored in the field + coupling field energy losses).

- $e i dt = dW_{mech} + dW_{elec} \rightarrow$  Energy Balance eqn.  
(Law of Conservation of energy).
- $dW_{elec} = e i dt = i d\psi$   
( $\psi = N\phi$ ) =  $i N d\phi = (\text{m.m.f}) d\phi$
- When the movable part of any physical system is kept fixed, the entire electrical energy is stored in magnetic field.  
 $dW_{fld} = dW_{elec} = i d\psi = F \cdot d\phi$   
 $W_{fld} = \int_0^{\psi_1} F \cdot d\psi = \int_0^{\psi_1} i d\psi$
- Co-energy is useful in calculating the mechanical forces.  
magnetic stored energy density,  $W_{fld} = \frac{1}{2} \mu H^2$

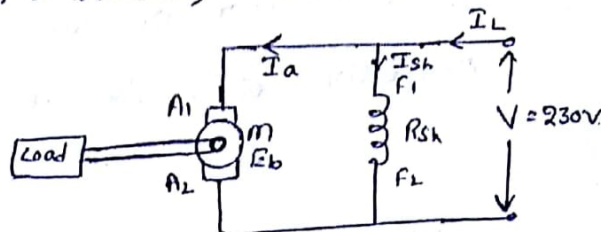
(a) A dc shunt motor, with armature circuit resistance of  $0.1 \Omega$ , runs at 1600 rpm. while taking an armature current of 100 A from 230 V dc source. The frictional & windage loss is 300 W, no-load core losses are 1200 W and the total  $I^2R$  loss is 2500 W, stray loss equals 1% of the output. Find the shaft torque of the motor and its efficiency.

Sol:-  $R_a = 0.1 \Omega$ ,  $N = 1600 \text{ rpm}$ ,  $I_a = 100 \text{ A}$ ,  $V = 230 \text{ V}$   
 Rotational losses = 300 W,  $I^2R$  loss = 2500 W, Core loss = 1200 W.

Dc shunt motor

$$V = E_b + I_a R_a$$

$$E_b = V - I_a R_a = 230 - 100 \times 0.1 = 220 \text{ V}$$



Gross mechanical Power developed in armature  $P_m = E_b I_a$

$$\therefore P_m = E_b I_a = 220 \times 100 = 22 \text{ kW}$$

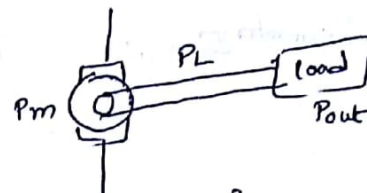
$$\therefore P_{out} = P_m - P_L (\text{rotational losses} + \text{Iron losses})$$

$$= 22 \times 10^3 - (300 + 1200)$$

$$P_{out} = 20.5 \text{ kW}$$

$$P_{out} = T_{sh} \cdot \omega \Rightarrow T_{sh} = \frac{P_{out}}{\omega} = \frac{P_{out}}{\frac{2\pi N}{60}} = \frac{20.5 \times 10^3}{\frac{2\pi \times 1600}{60}}$$

$$T_{sh} = 122.4 \text{ N-m}$$



$$\therefore \eta = \frac{P_{out}}{P_{in}} = \frac{P_{out}}{P_{out} + \text{total loss}}$$

$$= \frac{20.5 \times 10^3}{20.5 \times 10^3 + 2500 + 300 + 1200} \times 100 = 83.67\%$$

(b) A 4-Pole, 250V, wave-connected shunt motor gives 10kW when running at 1000 rpm and drawing armature and field currents of 60A and 1A respectively. It has 560 conductors. Its armature resistance is  $0.2 \Omega$ . Assuming a drop of 1V/brush.

Determine

(a) Total torque (b) useful torque (c) useful  $\phi$ /pole

(d) rotational losses (e) efficiency.

Sol:-  $P = 4$ ,  $V = 250 \text{ V}$ , wave  $A = 2$ ,  $P_{out} = 10 \text{ kW}$ ,  $V_{brush} = 1 \text{ V/brush}$   
 $N = 1000 \text{ rpm}$ ,  $I_{sh} = 1 \text{ A}$ ,  $I_a = 60 \text{ A}$ ,  $Z = 560$ ,  $R_a = 0.2 \Omega$

$$(a) \quad E_b = V - I_a R_a - V_{brush} = 250 - (60 \times 0.2) - (2 \times 1) = 236 \text{ V}$$

$$T_a = \frac{P_m}{\omega} = \frac{E_b I_a}{\left(\frac{2\pi N}{60}\right)} = 135.218 \text{ Nm}$$

$$(b) \quad T_{sh} = \frac{P_{out}}{\omega} = \frac{P_{out}}{\left(\frac{2\pi N}{60}\right)} = \frac{10 \times 10^3}{\frac{2\pi \times 10^3}{60}} = 95.49 \text{ Nm}$$

$$(c) \quad E_b = \frac{N\phi Z}{60} \cdot \frac{P}{A}$$

i.e.,  $\phi = \frac{E_b \times 60 \times A}{N \cdot Z \cdot P} = \frac{60 \times 2 \times 236}{4 \times 1000 \times 560} = 0.0126 \text{ wb.}$

$$(d) \quad \text{Rotational losses} = (T_a - T_{sh}) \times \omega$$

$$= (135.218 - 95.49) \times \frac{2\pi \times 1000}{60} = 4.16 \text{ kW}$$

$$(e) \quad I_L = I_a + I_{sh} = 61 \text{ A}$$

$$P_{in} = V \times I_L = 250 \times 61 = 15250 \text{ W}$$

$$\% \eta = \frac{P_{out}}{P_{in}} \times 100 = \frac{10 \times 10^3}{15250} \times 100 = \underline{\underline{65.57\%}}$$

(Q) A 400V DC shunt generator has full load current of 200A. Its armature resistance 0.06Ω, field resistance is 100Ω, & stray losses are 2000W. Find the efficiency at full load and constant losses.

Sol:- Load Volt  $V_L = 400\text{V}$ ; load current  $I_L = 200\text{A}$ .  
 stray losses = 2000W.

$$R_a = 0.06\Omega; \quad R_{sh} = 100\Omega;$$

$$\eta_{FL} \% = ? \quad \& \quad \text{Constant loss} = ?$$

$$\text{Generator Output Power} = V I_L = 400 \times 200 = 80000 \text{ W} = 80 \text{ kW}$$

$$\text{Total losses} = \text{Variable losses} + \text{Constant losses}$$

$$\text{Constant losses} = \text{Stray losses} + \text{Shunt field } Cu \text{ loss } (I_{sh})^2 R_{sh}$$

$$I_{sh} = \frac{V}{R_{sh}} = \frac{400}{100} = 4 \text{ A}$$

$$\text{Constant losses} = 2000 + (4)^2 \times 100 = 3600 \text{ W}$$

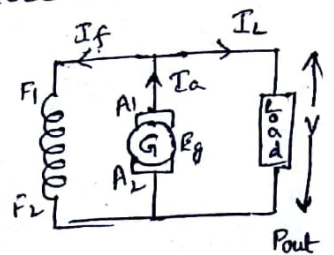
$$\text{Variable losses} = I_a^2 R_a$$

$$= (I_L + I_{sh})^2 R_a$$

$$= (200 + 4)^2 \times 0.06$$

$$= \underline{\underline{2496.96 \text{ W}}}$$

$$(\because I_a = I_L + I_{sh})$$



$$\begin{aligned} \text{Total losses} &= 3600 + 2496.96 \\ &= 6096.96 \text{ W.} \end{aligned}$$

$$\text{efficiency} = \frac{\text{O/P}}{\text{O/P} + \text{losses}} \times 100$$

$$= \frac{80,000}{80,000 + 6096.96} \times 100 = 92.91\%$$



## Swinburne's Test :-

This is indirect method of testing d.c motors in which flux remains practically constant. In this method, we are conducting without actually loading the motor, the losses and hence efficiency at different loads can be found out.

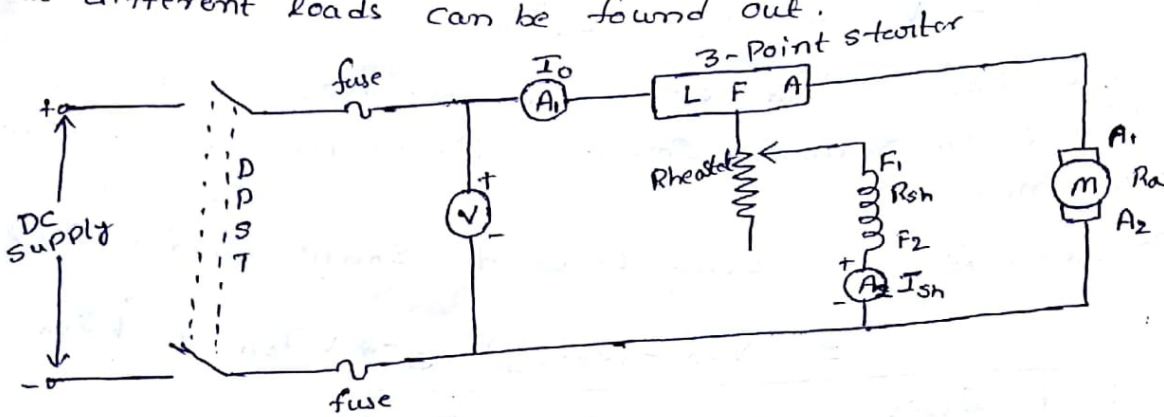


Figure :-

In this method,  $R_a$  &  $R_{sh}$  are measured separately so that losses can be calculated easily on various loads.

The circuit diagram for determining the no-load losses of a D.C shunt machine is as shown in fig.

Let the DC supply volt be 'V' applied which is measured by Voltmeter.

At starting some resistance is connected in series with the armature which is cut when motor attains sufficient speed. Speed of the motor is adjusted to rated speed with the help of shunt field rheostat.

$I_0$  is the no-load input motor current which is measured by ammeter  $A_1$ .

$I_{sh}$  is the shunt field current measured by ammeter  $A_2$ .

Since the machine is on no-load,

$$\text{Input} = \text{O/P} + \text{Losses}$$

$$\text{then Input power} = \text{Losses}$$

[  $\because$  O/P Power = 0 at no-load ]

$$\text{Power input at no-load} = V I_0$$

$$\text{Shunt field Copper losses} = V I_{sh}$$

$$\text{Armature Copper loss} = I_{a0}^2 R_a = (I_0 - I_{sh})^2 R_a$$

$$\text{where } I_{a0} = I_0 - I_{sh}$$

Voltage	No-load Current	Field Current	Speed
$V_{rated}$	$I_0$	$I_{sh}$	$N_{rated}$

∴ Power input = Total losses

$$VI_0 = VI_{sh} + I_{a0}^2 R_a + \text{Stray losses}$$

where Stray losses = (o/p at no load) - (Field Cu loss) - (No-load Armature Copper loss)

$$\therefore \text{Stray losses} = VI_0 - VI_{sh} - I_{a0}^2 R_a$$

∴ we add shunt field copper loss to stray loss we get constant loss which remains constant irrespective of the load on the machine.

∴ Constant losses = Stray losses + Shunt field copper loss.

$$= VI_0 - (I_0 - I_{sh})^2 R_a - \cancel{VI_{sh}} + \cancel{VI_{sh}}$$

$$\therefore \boxed{\text{Constant losses} = VI_0 - (I_0 - I_{sh})^2 R_a}$$

Efficiency when running as motor :- at say  $\frac{1}{4}$ th full load :-

~~motor input power = VI~~

Total losses = Armature copper loss + Constant losses

Let

$I_{FL}$  = Full load current of motor

$W_c$  = constant loss =  $VI_0 - (I_0 - I_{sh})^2 R_a$

Load current at  $\frac{1}{4}$ th full load =  $\frac{I_{FL}}{4}$

∴ motor input at  $\frac{1}{4}$ th full load =  $V \cdot \frac{I_{FL}}{4}$  watts.

Armature current at  $\frac{1}{4}$ th full load =  $I_a = \frac{I_{FL}}{4} - I_{sh}$

Armature copper loss at  $\frac{1}{4}$ th full load =  $I_a^2 R_a$   
 $= \left(\frac{I_{FL}}{4} - I_{sh}\right)^2 R_a$

∴ motor output at  $\frac{1}{4}$ th full load = motor i/p at  $\frac{1}{4}$ th full load - Total losses

$$\text{motor o/p at } \frac{1}{4} \text{th full load} = \left(V \cdot \frac{I_{FL}}{4}\right) - \left(\frac{I_{FL}}{4} - I_{sh}\right)^2 R_a - W_c$$

$$\therefore \text{Efficiency at } \frac{1}{4} \text{th full load, } \eta_{\frac{1}{4}} = \frac{\text{o/p}}{\text{i/p}} = \frac{\left(V \cdot \frac{I_{FL}}{4}\right) - \left(\frac{I_{FL}}{4} - I_{sh}\right)^2 R_a - W_c}{\left(V \cdot \frac{I_{FL}}{4}\right)}$$

The efficiency at other loads can be calculated, similarly.



### Efficiency when running as Generator :- at $\frac{1}{4}$ th full load.

Let  $I_{FL}$  = full load current of motor

$$W_c = \text{Constant loss} = VI_0 - (I_0 - I_{sh})^2 R_a$$

$$\text{Load current at } \frac{1}{4}\text{th full load} = \frac{I_{FL}}{4}$$

$$\therefore \text{Armature Current at } \frac{1}{4}\text{th full load} = I_a = \left(\frac{I_{FL}}{4} + I_{sh}\right)$$

$$\therefore \text{Generator output at } \frac{1}{4}\text{th full load} = V \cdot \frac{I_{FL}}{4}$$

$$\text{Armature Copper loss at } \frac{1}{4}\text{th full load} = I_a^2 R_a$$

$$= \left(\frac{I_{FL}}{4} + I_{sh}\right)^2 R_a$$

$$\text{Total losses} = \text{Arm Cu loss} + \text{Constant loss}$$

$$= \left(\frac{I_{FL}}{4} + I_{sh}\right)^2 R_a + W_c$$

$$\text{Generator Input} = \text{Generator Output} + \text{Total losses}$$

$$= V \left(\frac{I_{FL}}{4}\right) + \left(\frac{I_{FL}}{4} + I_{sh}\right)^2 R_a + W_c$$

$$\therefore \text{Efficiency at } \frac{1}{4}\text{th full load}$$

$$= \frac{\text{Generator O/P}}{\text{Generator i/P}}$$

$$\eta_{\frac{1}{4}FL}$$

$$= \frac{V \left(\frac{I_{FL}}{4}\right)}{V \left(\frac{I_{FL}}{4}\right) + \left(\frac{I_{FL}}{4} + I_{sh}\right)^2 R_a + W_c} \times 100$$

### Advantages:-

- (1) It is convenient and economical method of testing of DC machines. As the machine is run on No-load, hence the power required is less.
- (2) Efficiency of machine can be predetermined at any load since, constant losses are known.

### Disadvantages:-

- (1) This test is not applicable to series motor, since when it runs on No-load obtains dangerously high speed.
- (2) No account is taken for change in iron losses from no-load to full-load on account of distribution of flux due to armature reaction.



$$T_{sh} = \text{Net Pull} \times R = 9.81(\omega_1 - \omega_2) R \quad \text{N-m}$$

Hence the output Power can be obtained as ,

$$P_{out} = T_{sh} \times \omega = 9.81 (\omega_1 - \omega_2) R \times \frac{2\pi N}{60} \quad \text{Watts .}$$

Now let ,  $V =$  Volt applied in Volts

$I =$  Total line current drawn in amps .

then  $P_{in} = VI$  watts .

Thus if the readings are taken on full load condition then the efficiency can be obtained as

$$\% \eta = \frac{P_{out}}{P_{in}} \times 100 .$$

Adjusting the load step by step till full load, no. of readings can be obtained. The Speed can be measured by tachometer.

Thus all the motor characteristics can be Plotted.

#### Advantages:-

- (1) Actual efficiency of the motor under working conditions can be found out.
- (2) The method is simple and easy to perform.
- (3) Can be performed on any type of d.c motor.

#### Disadvantages:-

- (1) Due to friction, heat is generated and hence there is large dissipation of energy.
- (2) Some type of cooling arrangement is necessary.
- (3) Convenient only for small machines due to limitations regarding heat dissipation arrangements.
- (4) The Power developed gets wasted hence method is expensive.

# Transformers

①

The main advantage of alternating currents over direct currents is that, the alternating currents or voltages can be raised or lowered as per requirements at different stages of electrical network as generation, transmission, distribution and utilization. This is possible with a static device called transformer.

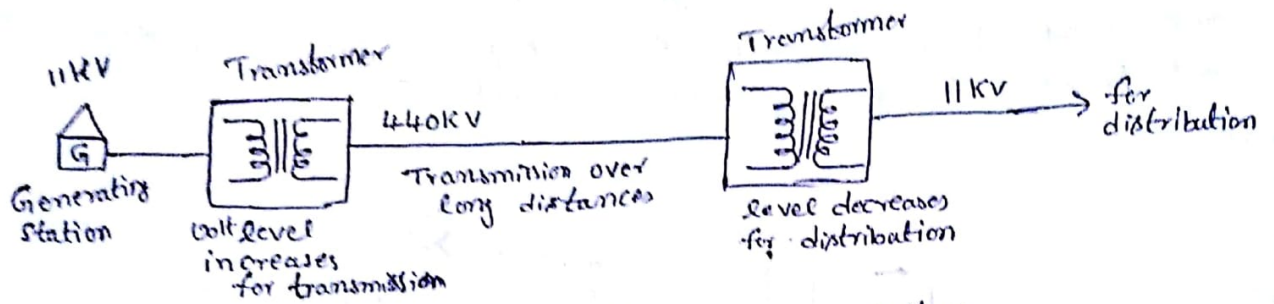
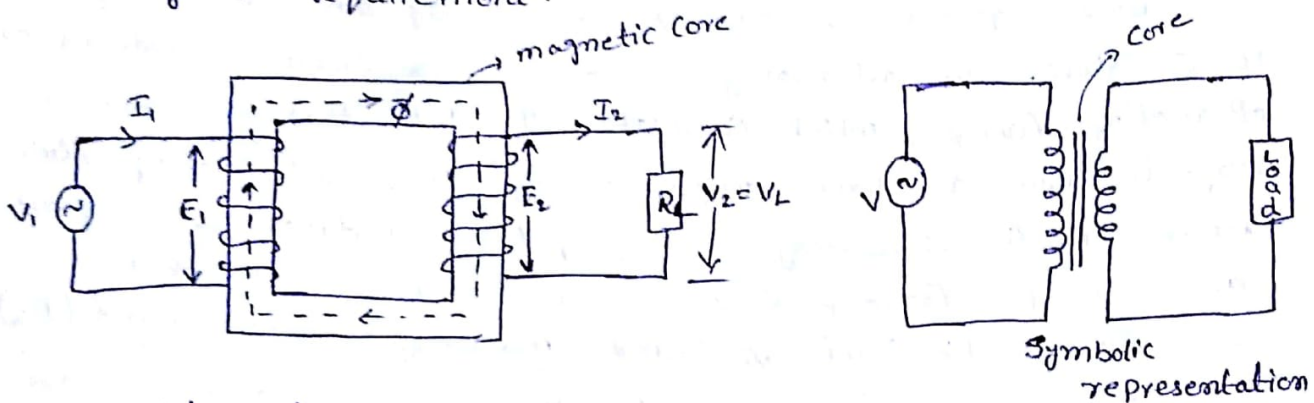


fig:- Use of transformers in transmission system.

~~Def:-~~ Def:- Transformer is a stationary device which transfers the electrical power from one circuit to another circuit at constant freq, but the voltage can be increased or decreased according to requirement.



When the AC voltage is applied to a winding, that winding is known as Primary winding. If the load is connected across the winding, that winding is called secondary winding.

If the secondary volt is greater than Primary voltage, that transformer will act as a step-up transformer. Similarly if the secondary volt is less than Primary volt, then that transformer is known as step down transformer.

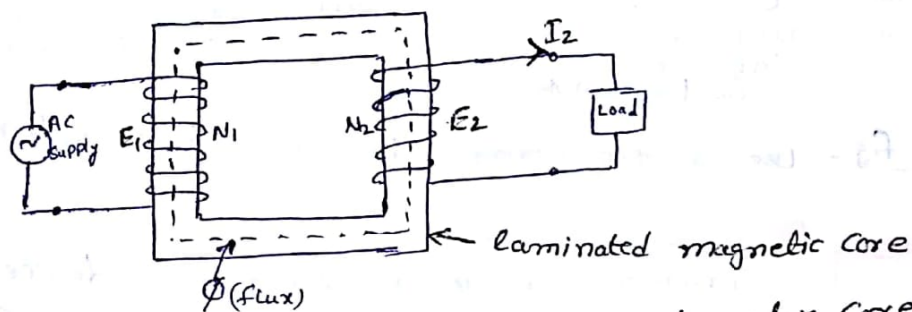
## Working Principle of transformer:-

The transformer will work on the principle of mutual induction.

"It states that when two coils are inductively coupled and if current in one coil is changed uniformly then an emf gets induced in the other coil. This emf can drive a current, when a closed path is provided to it."

The transformer works on the same principle.

Let us consider, a core type transformer as shown in fig. It consists of two inductive coils which are electrically separated but linked through a common magnetic core.



The transformer has a laminated rectangular core and two inductive coils are wound on each limb of the transformer.

When primary winding is excited by an alternating volt, it circulates an alternating current. This current produces an alternating flux  $\phi$ , which completes its path through common magnetic core as shown dotted in fig. This alternating flux links with the secondary winding, there induces some emf according to Faraday's laws of electromagnetic induction principle. This emf is called mutually induced emf ( $E_2$ ).

The same flux cut by the primary winding, an emf will generate called self induced emf ( $E_1$ ).

Since secondary winding is closed by the load, the secondary current  $I_2$  will pass through secondary winding and load.

Here we assume that the transformer is in ideal condition (i.e., losses of transformer are neglected).

$$\text{Therefore, the input Power} = \text{output Power}$$
$$V_1 I_1 = V_2 I_2 \quad (\text{or}) \quad \frac{I_1}{I_2} = \frac{V_2}{V_1}$$

### Can D.C supply be used for Transformers ?

The d.c supply cannot be used for the transformers.

The transformer works on the principle of mutual induction, for which current in one coil must change uniformly. If dc supply is given, the current will not change due to constant supply & transformer will not work.

Practically winding resistance is very small. For d.c, the inductive reactance  $X_L$  is zero as dc has no freq. So total impedance of winding is very low for d.c. Thus winding will draw very high current if dc supply is given to it. This may cause burning of windings due to extra heat generated & may cause permanent damage to the transformer.

Thus dc supply should not be connected to the transformer.

### EMF equation of transformer :-

When an AC volt  $V_1$  is applied to the Primary winding of the transformer, a sinusoidal flux is set up in the core which links both Primary and Secondary windings

Let us assume that

$N_1$  = no. of turns in Primary winding.

$N_2$  = no. of turns in secondary winding.

$E_1$  = Emf induced in Primary winding.

$E_2$  = " " " Secondary " "

$T$  = Time Period in sec

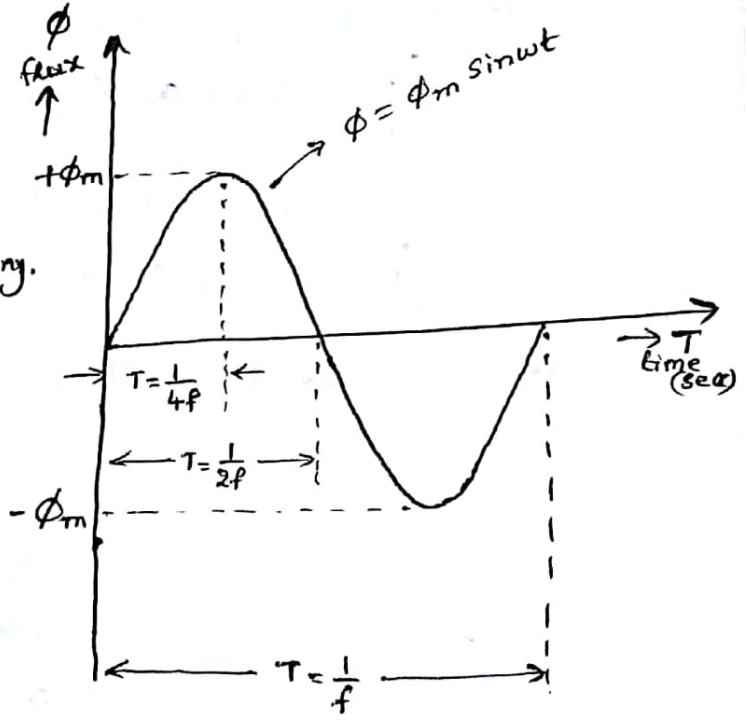
$f$  = frequency in Hz.

$\phi_m$  = maximum flux in Wb.

$$\phi_m = B_m \cdot A$$

$B_m$  = max. flux density.

$A$  = Area of x-section of the core (net Area).



From fig, it is clear that, an alternating flux attains its maximum value in a quarter cycle i.e.,  $\frac{1}{4f} = T$

Hence, change in flux =  $d\phi = \phi_m - 0 = \phi_m \omega b$

$\therefore$  Change in time =  $dt = \frac{1}{4f}$  sec.

According to Faradays laws of electromagnetic induction, the magnitude of induced emf equal to rate of change of flux linkages.

i.e., Avg induced emf / turn  $(e) = \frac{d\phi}{dt} = \frac{\phi_m}{\frac{1}{4f}}$

$e = 4f\phi_m$

Since flux is varying sinusoidally, form factor of sinusoidal flux is 1.11.

Form factor =  $\frac{\text{Rms value}}{\text{Avg value}}$

$\therefore$  Rms Value = 1.11 x avg value

= 1.11 x  $4f\phi_m$

$\therefore$  Rms Value  $E_{\text{turn}} = 4.44 \phi_m f$  Volt

If the primary winding has  $N_1$  turns, then the induced emf in the primary winding is

$E_1 = 4.44 \phi_m f \cdot N_1$  Volt  $\rightarrow$  ①

Similarly, emf induced in secondary winding is

$E_2 = 4.44 \phi_m f \cdot N_2$  Volt  $\rightarrow$  ②

If  $\phi_m = B_m \cdot A$ . then  $E_1 = 4.44 B_m A \cdot f \cdot N_1$  Volt.

$E_2 = 4.44 B_m A \cdot f \cdot N_2$  Volt.

Transformation ratio eqn:-

$\frac{e_{\text{①}}}{e_{\text{②}}} \Rightarrow \frac{E_2}{E_1} = \frac{4.44 \phi_m f \cdot N_2}{4.44 \phi_m f \cdot N_1} = \frac{N_2}{N_1}$

$\therefore$  Transformation ratio  $k = \frac{E_2}{E_1} = \frac{N_2}{N_1} = \frac{V_2}{V_1} = \frac{I_1}{I_2}$

(Q) A 25 kVA transformer has 500 turns on Primary winding and 40 turns on secondary winding. The Primary is connected to 3000V, 50 Hz supply. Calculate secondary induced emf, maximum flux in the core and Primary & Secondary currents in full load. (3)

Sol:- Given data  $N_1 = 500$  ;  $N_2 = 40$  turns ,  
 $f = 50 \text{ Hz}$  ,  $V_1 = 3000 \text{ V}$  ,  $V_1 = E_1$  ,  
 $V_2 = E_2 = ?$

We know  $k = \frac{E_2}{E_1} = \frac{N_2}{N_1} \Rightarrow E_2 = E_1 \times \frac{N_2}{N_1}$   
 $E_2 = 3000 \times \frac{40}{500}$   
 $E_2 = 240 \text{ V}$

If  $E_2 = 4.44 \phi_m f N_2$   
 From this  $\phi_m = \frac{E_2}{4.44 f N_2} = \frac{240}{4.44 \times 50 \times 40}$

$\phi_m = 27.027 \text{ mWb}$

From rating of transformer =  $V I$

$25 \times 10^3 \text{ VA} = V_1 I_1$

$I_1 = \frac{25 \times 10^3}{3000} = 8.33 \text{ A}$

||<sup>1</sup>g

$25 \times 10^3 \text{ VA} = V_2 I_2$

$I_2 = \frac{25 \times 10^3}{240} = 104.16 \text{ A}$

(Q) The maximum flux density in the core of a 250/3000 V, 50 Hz supply, single phase transformer is  $1.2 \text{ Wb/m}^2$ . If the emf/turn is 8V, determine (i) Primary & Secondary turns (ii) Area of the core

Sol:- Given that

1- $\phi$  Transformer 250/3000V , i.e.,  $V_1 = 250 \text{ V} = E_1$   
 $V_2 = 3000 \text{ V} = E_2$   
 $f = 50 \text{ Hz}$ .

$B_{\text{max}} = 1.2 \text{ Wb/m}^2$  ,  $\text{emf/turn} = 8 \text{ V}$

(i) We know that  $E_1 = 4.44 f \phi_m N_1$

$\text{emf/turn} = \frac{E_1}{N_1} = 4.44 f \phi_m$



$$N_1 = \frac{E_1}{\text{emf/turn}} = \frac{250}{8} = 31.25 \approx 32 \text{ turns}$$

$$N_2 = \frac{E_2}{\text{emf/turn}} = \frac{3000}{8} = 375 \text{ turns.}$$

(ii) Area of core = ?

$$B_m = \frac{\phi_m}{A} \Rightarrow A = \frac{\phi_m}{B_m}$$

$$E_1 = 4.44 \phi_m f N_1$$

$$E_1 = 4.44 B_m A f N_1$$

$$A = \frac{E_1}{4.44 \times B_m \times f \times N_1} = \frac{250}{4.44 \times 1.2 \times 50 \times 32} = 0.029 \text{ m}^2$$

(Q) A 1- $\phi$  T/F has 400 Primary and 1000 Secondary turns. The net cross sectional area of the core is  $60 \text{ cm}^2$ . If the Primary winding is connected to  $50 \text{ Hz}$  supply at  $520 \text{ V}$ . Find

- (i) Peak value of flux density.
- (ii) Volt induced in secondary winding.

Sol:-

$$N_1 = 400, N_2 = 1000; A = 60 \text{ cm}^2 = 60 \times 10^{-4} \text{ m}^2$$

$$f = 50 \text{ Hz}, E_1 = 520 \text{ V.}$$

(i)  $B_m = ?$

$$E_1 = 4.44 f B_m \cdot A \cdot N_1$$

$$\phi_m = \frac{520}{4.44 \times 50 \times 400 \times 60 \times 10^{-4}} = 0.975 \text{ wb/m}^2$$

(ii)  $K = \frac{E_2}{E_1} = \frac{N_2}{N_1}$

$$E_2 = E_1 \cdot \frac{N_2}{N_1} = 520 \times \frac{1000}{400} = 1300 \text{ V.}$$

(Q) A 1- $\phi$  T/F has ~~500~~ 500 turns in the Primary winding and 1200 turns in secondary winding. The cross sectional area of the core is  $80 \text{ cm}^2$ . If the Primary winding is connected to  $50 \text{ Hz}$  supply at  $500 \text{ V}$ . Find ~~Primary winding~~

- (i) Peak flux density
- (ii) Volt induced at Secondary.

Ans:-

- (i)  $\phi$
- (ii)  $E_2 = 1200 \text{ V}$

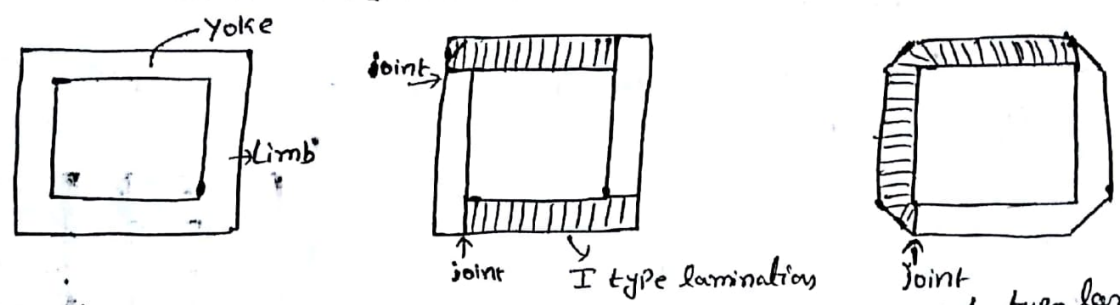
~~work is from~~ (1)

# Construction :-

There are ~~two~~ <sup>Six</sup> basic parts of a transformer

- (i) magnetic core
- (ii) winding or coils
- (iii) conservator tank
- (iv) Breather
- (v) Bushings
- (vi) Radiators.

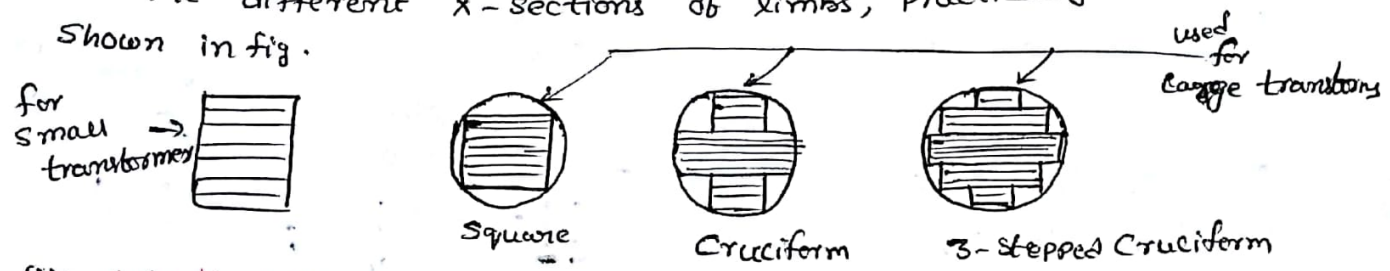
(i) Core: The core of the transformer is either square or rectangular in size. It is further divided into two parts. The vertical portion on which coils are wound is called limb, while the top and bottom horizontal portion is called Yoke of the core.



Core is made up of laminations, due to this eddy current losses get minimized. These laminations are insulated from each other by using insulation like red oxide, Varnish etc.

Generally high grade silicon steel laminations (0.3 to 0.5 mm) thick are used.

The different x-sections of limbs, practically used are



## (ii) Windings :-

The coils of transformer are of two types. They are

- (a) Concentric coils and
- (b) Sandwich coils.

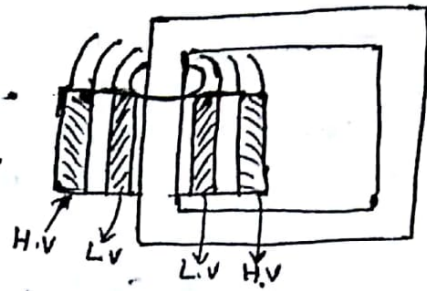
### (a) Concentric Coils :-

Generally, these coils are used in core type transformer. In this, two windings (Primary & Secondary) are split into no. of coils and are wound adjacent to each other on the same limb. Because of this arrangement, we can reduce the leakage flux b/w two windings.

The Low volt winding is placed close to the core of a transformer & H.V winding is placed on the top of L.V winding for the reason that the insulation involved is less.

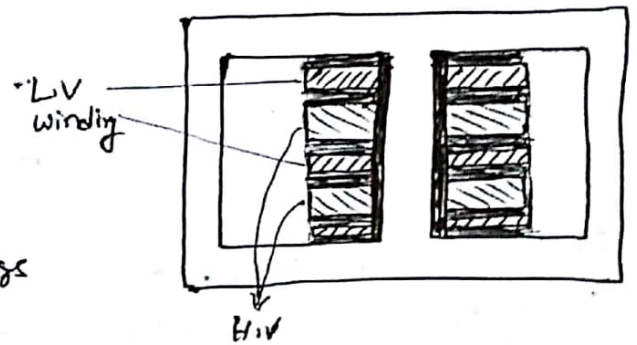
A Very Common arrangement is cylindrical concentric coils as shown in fig.

These coils are mechanically strong. These are wound in the helical layers. The different layers are insulated from each other by Paper, cloth or mica.



(b) Sandwiched Coils:-

Sandwiched coils are used in shell type transformers. Each high volt portion lies between the two low voltage portion i.e., sandwiching the H.V. portion.



Such subdivision of windings into small portions reduces the leakage flux.

The top and bottom coils are L.V. coils in order to reduce insulation as shown in fig. All the portions are insulated from each other by Paper.

(iii) Conservator Tank:-

The oil in the transformer main tank is subjected to expand and contract due to variations in load current.

The function of the conservator tank is to help the oil in the tank to settle down by expansion whenever heavy loads appears.

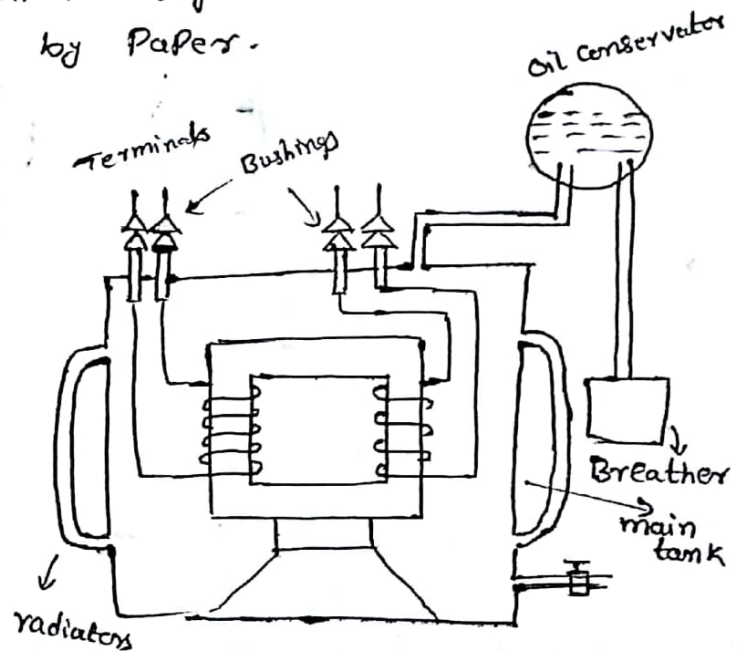


fig. Transformer Construction

Without a Conservator tank, the main tank may burst out, because of the high pressure developed inside the tank.

#### (iv) Breather :-

(5)

The main tank & some portion of the Conservator tank are filled with oil. This oil should not be exposed to the atmosphere directly, because it may absorb the moisture & dust and may lose its electrical properties within a very short time, & also harmful for transformer insulation.

A silica gel breather is most commonly used to filter air from moisture.

#### (v) Bushings :-

Connections from the transformer windings are brought out by means of bushings. The function of the bushings is to give proper insulation for the output leads. Bushings are fixed on the transformer tank. It is made up of Porcelain and available and can be used upto 33kV.

Capacitor & oil filled type of bushings are used for voltages above 33kV.

#### (vi) Radiators :-

Thin metal structures are connected round the T/F tank which acts as a heat sink. The function of the radiator is to cool the transformer tank gradually.

### Types of Transformers :-

There are two main types of transformers which are:

- (i) Core type                      (2) Shell type.

#### (i) Core type Transformer :-

It has a single magnetic circuit. The core is rectangular having two limbs. The winding encircles the core. The coils used are of cylindrical type, these coils are wound in helical layers with different layers insulated from each other by paper or mica. Both are placed on both the limbs. The low volt coil is placed inside near the core while high voltage coil surrounds the L.V coil. Core is made up of large no. of thin laminations.

As the windings are uniformly distributed over the two limbs the natural cooling is more effective. The coils can be easily removed by removing the laminations of the top yoke, for maintenance.

The fig (a) shows the schematic representation of the Core type transformer, while fig (b) shows the view of actual construction of the Core type transformer.

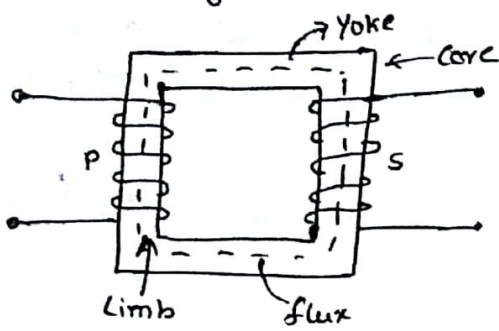


fig:- (a) Representation

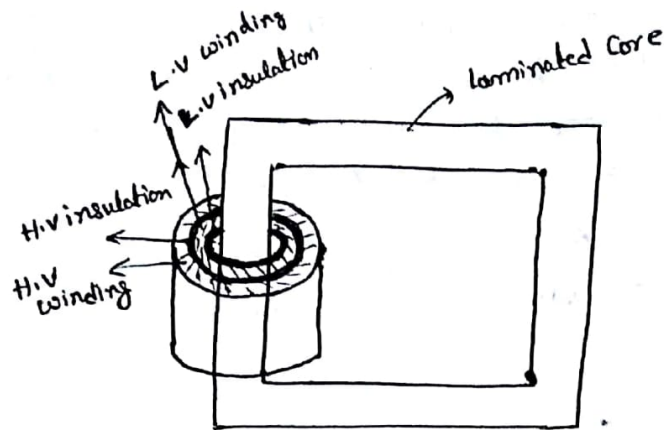


fig (b):- Construction.

Fig:- Core type transformer.

(ii) Shell type Transformer:-

It is double magnetic circuit. The Core has three limbs. Both the windings are placed on the central limb.

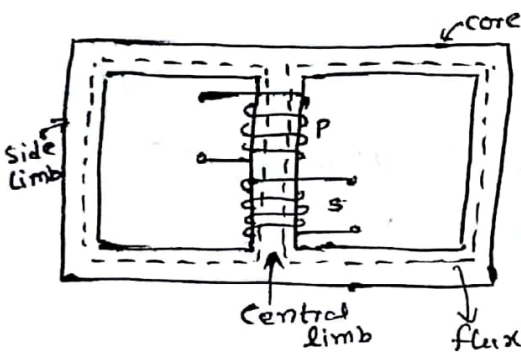


fig (a) Representation

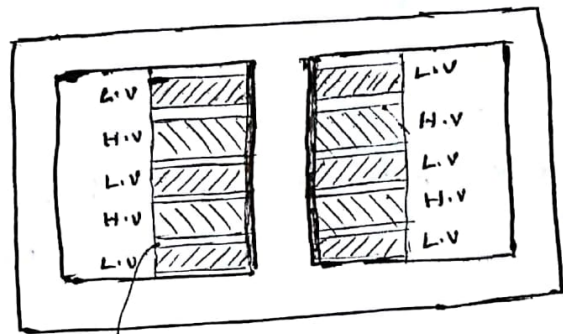


fig (b) Construction.

Fig:- Shell type transformer.

The Core encircles most part of the windings. The coils used are generally multilayer disc type or sandwich type coils. In which H.V coil is in between two L.V coils & L.V coils are nearest to top & bottom of the yokes.

The Core is laminated. Generally shell type transformer is preferred for very high volt transformers.

As the windings are surrounded by the core, the natural cooling does not exist. For removing any winding for maintenance, large no. of laminations are required to be removed.

Fig (a) shows the schematic representation while fig (b) shows the constructional view of shell type transformer.

## Comparison between Core and Shell type transformers:-

⑥

Core type Transformer	Shell type transformer
(1) The Core is encircled by the winding.	(1) most part of the winding is encircled by the core.
(2) It has single magnetic circuit	(2) It has a double magnetic circuit.
(3) The core has two limbs	(3) The core has three limbs.
(4) The coils used are concentric cylindrical coils.	(4) The coils used are multi-layer type or sandwich type coils.
(5) The windings are uniformly distributed on two limbs, hence natural cooling is effective.	(5) The natural cooling does not exist as the windings are surrounded by the core.
(6) The coils can be easily removed from maintenance point of view.	(6) The coils cannot be removed easily.
(7) Preferred for low voltage transformers	(7) Preferred for high voltage transformers.

## Voltage Ampere Rating:-

When electrical power is transferred from primary to secondary there are few power losses in between. These power losses appear in the form of heat which increase the temperature of the device. Now this temperature must be maintained below certain limiting value as it is always harmful from insulation point of view. As the current is the main cause in producing heat, the output maximum rating is generally specified as the product of output voltage and output current i.e.,  $V_2 I_2$ .

Actually output power available depends on  $\cos\phi_2$  which is P.f of the secondary. As  $\cos\phi_2$  can change depending on load, the rating is not specified in watts or kilowatts but indicated as the product of Volt & Current called VA rating.

This rating is generally expressed in kVA

$$\text{kVA rating of a transformer} = \frac{V_1 I_1}{1000} = \frac{V_2 I_2}{1000}$$

∵  $V_1$  &  $V_2$  are the terminal voltages of Primary & Secondary then from specified kVA rating we can decide full load currents of Primary & Secondary,  $I_1$  &  $I_2$ . This is the safe maximum current limit which may carry, keeping temperature rise below its limiting value.

$$I_{1 \text{ full load}} = \frac{\text{kVA rating} \times 1000}{V_1}$$

$$I_{2 \text{ full load}} = \frac{\text{kVA rating} \times 1000}{V_2}$$

(Q) For a single phase transformer having Primary & Secondary turns of 440 and 880 respectively, determine the transformer kVA rating if half load secondary current is 7.5A and maximum value of core flux is 2.25 mwb. Assume  $f = 50 \text{ Hz}$ .

Sol:-  $N_1 = 440$ ,  $N_2 = 880$ ,  $(I_2)_{HL} = 7.5 \text{ A}$ ;  $f = 50 \text{ Hz}$ .

$$\phi_m = 2.25 \text{ mwb}, \quad E_2 = 4.44 \phi_m f N_2$$

$$\therefore E_2 = 4.44 \times 2.25 \times 10^{-3} \times 50 \times 880 = 439.56 \text{ V}$$

$$(I_2)_{FL} = \frac{\text{kVA rating}}{E_2}$$

and  $(I_2)_{H.L} = \frac{1}{2} (I_2)_{FL}$

$$(I_2)_{H.L} = \frac{1}{2} \times \left( \frac{\text{kVA rating}}{E_2} \right)$$

$$7.5 = \frac{1}{2} \times \frac{\text{kVA rating}}{439.56} \Rightarrow \text{kVA rating} = 2 \times 7.5 \times 439.56 \times 10^{-3} = 6.5934 \text{ kVA}$$

## Voltage Regulation of Transformer:-

Because of presence of impedances on Primary and Secondary, it causes volt drop across impedances. It is observed that the secondary terminal volt drops from its no-load value ( $E_2$ ) to load value ( $V_2$ ) as load & load current increases.

This decrease in the secondary terminal volt expressed as a fraction of the no-load secondary terminal volt is called regulation of a transformer.

Let  $E_2$  = Secondary terminal volt on no-load.  
 $V_2$  = " " " " on given load.

then mathematically V.R at given load is given by

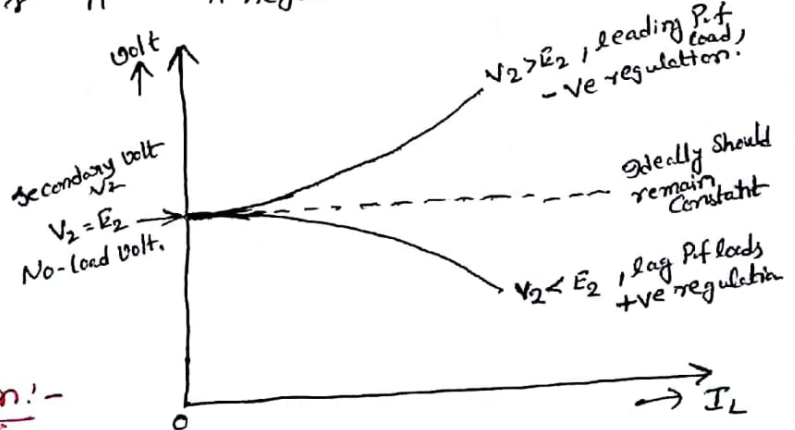
$$\% \text{ Volt regulation} = \frac{E_2 - V_2}{E_2} \times 100$$

The ratio  $\frac{E_2 - V_2}{E_2}$  is called Per unit regulation.

The secondary terminal volt  $\downarrow$  not only depends on the magnitude of the load current but also on the nature of the power factor of the load.

As load current  $I_L$  increases, volt drops tend to increase. In case of lagging P.f  $V_2 < E_2$  & we get positive volt regulation, while for leading P.f  $E_2 < V_2$  & " " negative " " "

Generally, The volt drop should be as small as possible, hence less the regulation better is the performance of a transformer.



### Expression for Volt Regulation:-

The volt regulation is defined as,

$$\% R = \frac{E_2 - V_2}{E_2} \times 100 = \frac{\text{Total Volt drop}}{E_2} \times 100$$



The regulation can be expressed as,

$$\%R = \frac{I_2 R_{2e} \cos\phi \pm I_2 X_{2e} \sin\phi}{E_2} \times 100$$

where  $I_2$  = Full load secondary current

$E_2$  = No-load secondary volt

$R_{2e}$  = Equivalent resistance referred to secondary.

$X_{2e}$  = " reactance " " " "

$\cos\phi$  = load power factor.

+ve sign for lagging power factors, while

-ve sign " leading " " " "

The regulation can be further expressed in terms of  $I_1$ ,  $E_1$ ,  $R_{1e}$  and  $X_{1e}$ .

we know  $k = \frac{E_2}{E_1} = \frac{I_1}{I_2}$

$\therefore E_2 = k E_1$ ,  $I_2 = \frac{I_1}{k}$

while  $R_{1e} = \frac{R_{2e}}{k^2}$ ,  $X_{1e} = \frac{X_{2e}}{k^2}$

Substituting in the regulation expression, we get

$$\%R = \frac{\frac{I_1}{k} (k^2 R_{1e}) \cos\phi \pm \frac{I_1}{k} (k^2 X_{1e}) \sin\phi}{k E_1} \times 100$$

$$\therefore \%R = \frac{I_1 R_{1e} \cos\phi \pm I_1 X_{1e} \sin\phi}{E_1} \times 100$$

(Q) A 250/125 V, 5KVA 1- $\phi$  transformer has Primary resistance of  $0.2\Omega$  and reactance of  $0.75\Omega$ . The secondary resistance is  $0.05\Omega$  and reactance of  $0.2\Omega$ .

- (i) Determine its regulation while supplying full load on 0.8 lead P.f  
 (ii) The secondary terminal Volt on Full load and 0.8 Lead P.f

Sol Given  $R_1 = 0.2\Omega$ ,  $X_1 = 0.75\Omega$ ,  $R_2 = 0.05\Omega$ ,  $X_2 = 0.2\Omega$

$\cos\phi = 0.8$  leading

$$K = \frac{E_2}{E_1} = \frac{125}{250} = \frac{1}{2} = 0.5.$$

$$I_{2FL} = \frac{\text{KVA} \times 1000}{V_2} = \frac{5 \times 10^3}{125} = 40A.$$

$$R_{2e} = R_2 + K^2 R_1 = 0.05 + (0.5)^2 \times 0.2 = 0.1\Omega$$

$$X_{2e} = X_2 + K^2 X_1 = 0.2 + (0.5)^2 \times 0.75 = 0.3875\Omega$$

- (i) Regulation on full load,  $\cos\phi = 0.8$  leading

$$\sin\phi = 0.6$$

$$\%R = \frac{I_2 R_{2e} \cos\phi - I_2 X_{2e} \sin\phi}{E_2} \times 100$$

$$= \frac{40 \times 0.1 \times 0.8 - 40 \times 0.3875 \times 0.6}{125} \times 100$$

$$\%R = -4.88\%$$

- (ii) For secondary terminal Volt, use basic expression for regulation.

$$\%R = \frac{E_2 - V_2}{E_2} \times 100$$

$$-4.88 = \frac{125 - V_2}{125} \times 100 \Rightarrow$$

$$-6.1 = 125 - V_2$$

$$V_2 = 131.1V$$

It can be seen that for leading P.f  $E_2 < V_2$ .

Equivalent resistance :- (Secondary resistance shifted to Primary) ③

The equivalent resistance of the transformer can be found by shifting either secondary resistance to primary resistance (or) by " primary " " to secondary " .

From fig(1), total Copper losses equal to

$$= I_1^2 R_1 + I_2^2 R_2$$

$$= I_1^2 \left[ R_1 + \frac{I_2^2}{I_1^2} R_2 \right]$$

$$= I_1^2 \left[ R_1 + \frac{R_2}{k^2} \right] \quad \left[ \because k = \frac{I_1}{I_2} \right]$$

Total Copper loss =  $I_1^2 R_{1e}$

$$R_{1e} = R_1 + \frac{R_2}{k^2} = R_1 + R_2'$$

where  $R_2' = \frac{R_2}{k^2}$  = is known as equivalent resistance of the secondary referred to primary.

$R_{1e}$  is known as Total equivalent resistance of the transformer referred to primary.

$$R_{1e} = R_1 + R_2'$$

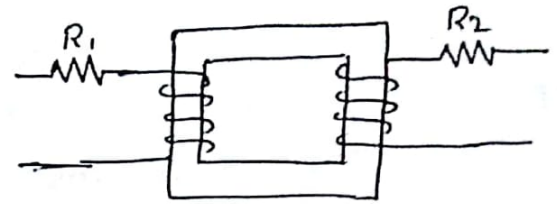
Similarly

$$\begin{aligned} \text{Total Cu loss} &= I_1^2 R_1 + I_2^2 R_2 \\ &= I_2^2 \left[ R_1 \left( \frac{I_1}{I_2} \right)^2 + R_2 \right] \\ &= I_2^2 \left[ R_1 \cdot k^2 + R_2 \right] \\ &= I_2^2 \left[ R_1' + R_2 \right] \end{aligned}$$

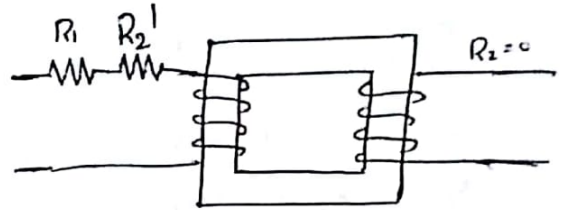
Total Cu loss =  $I_2^2 R_{2e}$

Here  $R_1' = k^2 R_1$  is primary resistance referred to secondary.

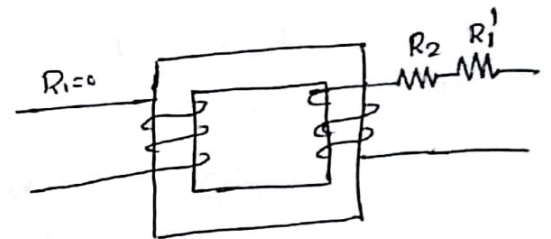
$R_{2e} = R_1' + R_2$  is the total equivalent resistance of the transformer referred to secondary.



fig(1)



fig(2)



### Losses in a Transformer :-

In a transformer, there exists two types of losses.

- (i) The core gets subjected to an alternating flux, causing Core losses.
- (ii) The windings carry currents when transformer is loaded, causing Copper losses.

#### (i) Core or Iron losses:-

Due to alternating flux set up in the magnetic core of the transformer, it undergoes a cycle of magnetisation and demagnetisation. Due to hysteresis effect there is loss of energy in this process which is called hysteresis loss ( $W_h$ ).

$$W_h = k_h B_m^{1.6} f V \text{ watts}$$

Where  $k_h$  = hysteresis constant depends on material.

$B_m$  = maximum flux density.

$f$  = frequency,

$V$  = Volume of the core.

These losses occurred due to the reversal of flux in the transformer core.

These losses can be minimised by using low hysteresis loss coefficient material. Ex:- CRGO Si steel"

The induced emf in the core tries to set up eddy currents in the core and hence responsible for the eddy current losses. The eddy current loss is given by

$$W_e = k_e B_m^2 f^2 t^2 V \text{ watts}$$

Where  $k_e$  = eddy current constant

$t$  = thickness of the core

We know, flux in the core is almost constant as supply volt 'V', at rated freq  $f$  is always constant. Hence the flux density  $B_m$  in the core is constant, and hence both hysteresis and eddy current losses are constant at all the loads. Hence the core (or) iron losses are known as constant losses (P<sub>I</sub>).

## (ii) Copper losses:-

The Copper losses are due to the Power wasted in the form of  $I^2R$  loss due to the resistances of the Primary and Secondary windings. The Copper loss depends on the magnitude of the currents flowing through the windings.

Let  $I_1$  = Primary current,  $R_1$  = Primary resistance

$I_2$  = Secondary " ,  $R_2$  = Secondary " ,

then the total Copper losses in the transformer  $P_{Cu}$

$$P_{Cu} = I_1^2 R_1 + I_2^2 R_2$$

From the above eqn, Copper losses varies with square of the current and are also known as Variable losses.

$$P_{Cu} \propto I^2 \propto (kVA)^2$$

Let the current through the windings is full load current, we get full load copper losses. Let the load on T/F is half then we get copper losses at half load which are less than full load copper losses. Thus Copper losses are called variable losses.

Thus for a transformer,

$$\begin{aligned} \text{Total losses} &= \text{Iron losses} + \text{Copper losses} \\ &= \text{Constant losses} + \text{Variable losses} \end{aligned}$$

$$\text{Total losses} = P_i + P_{Cu}$$

## Efficiency of a Transformer:-

Due to the losses in a transformer, the output Power of a transformer is less than the input Power supplied.

$$\therefore \text{Power output} = \text{Power input} - \text{Total losses}$$

$$\begin{aligned} \therefore \text{Power input} &= \text{Power output} + \text{Total losses} \\ &= P_{out} + P_i + P_{Cu} \end{aligned}$$

The efficiency of any device is defined as the ratio of the Power output to Power input. So for a transformer the efficiency can be expressed as

$$\eta = \frac{P_{out}}{P_{in}} = \frac{P_{out}}{P_{out} + P_i + P_{Cu}}$$

$$\text{Now } P_{\text{out}} = V_2 I_2 \cos\phi$$

Where  $\cos\phi =$  load Power factor.

The transformer supplies full load of current  $I_2$  and with terminal voltage  $V_2$ .

$$P_{\text{Cu}} = \text{Copper losses on full load} = I_2^2 R_{2e}$$

$$\therefore \eta = \frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + P_i + I_2^2 R_{2e}}$$

But  $V_2 I_2 =$  VA rating of a transformer

$$\therefore \eta = \frac{(\text{VA rating}) \times \cos\phi}{(\text{VA rating}) \times \cos\phi + P_i + I_2^2 R_{2e}} \times 100.$$

This is full load Percentage efficiency, with  $I_2$  is full load secondary current.

But if the transformer is subjected to fractional load, then efficiency can be calculated by using 'n'

$$n = \text{fraction by which load is less than full load} = \frac{\text{actual load}}{\text{full load}}$$

For Ex:- If T/F is subjected to half load, then

$$n = \frac{\text{half load}}{\text{full load}} = \frac{(1/2)}{1} = 1/2 = 0.5.$$

When load changes, the load current changes by same proportion.

$$\therefore \text{new } I_2 = n (I_2)_{\text{FL}}$$

Similarly, the output  $V_2 I_2 \cos\phi_2$  also reduces by the same fraction.

As Cu losses are proportional to square of current then,

$$\text{new } P_{\text{Cu}} = n^2 (P_{\text{CuFL}})$$

So copper losses get reduced by  $n^2$ .

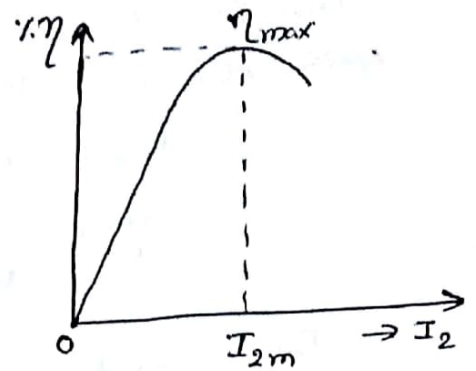
In general for fractional load, the efficiency is given by

$$\% \eta = \frac{n (\text{VA rating}) \cos\phi}{n (\text{VA rating}) \cos\phi + P_i + n^2 (P_{\text{FLCu}})} \times 100$$

For all types of load p.f.'s lagging, leading and unity the efficiency expression doesn't change.

Condition for maximum efficiency:-

When a T/F works on a constant  $V_1$  and  $f$  the  $\eta$  varies with the load.



As load increases, the  $\eta$  increases.

At a certain load current, it achieves a maximum value. At load further increased the  $\eta$  starts decreasing.

The load current at which the efficiency attains max value is denoted as  $I_{2m}$  & max efficiency is denoted as  $\eta_{max}$ .

So for maximum efficiency  $\frac{d\eta}{dI_2} = 0$

Now 
$$\eta = \frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + P_i + I_2^2 R_{2e}}$$

$$\therefore \frac{d\eta}{dI_2} = \frac{d}{dI_2} \left[ \frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + P_i + I_2^2 R_{2e}} \right] = 0$$

$$\therefore [V_2 I_2 \cos\phi_2 + P_i + I_2^2 R_{2e}] \frac{d}{dI_2} (V_2 I_2 \cos\phi_2) -$$

$$(V_2 I_2 \cos\phi_2) \cdot \frac{d}{dI_2} [V_2 I_2 \cos\phi_2 + P_i + I_2^2 R_{2e}] = 0$$

$$\therefore (V_2 I_2 \cos\phi_2 + P_i + I_2^2 R_{2e}) (V_2 \cos\phi_2) - (V_2 I_2 \cos\phi_2) (V_2 \cos\phi_2 + 2I_2 R_{2e}) = 0$$

Cancelling  $(V_2 \cos\phi_2)$  from both the terms we get,

$$V_2 I_2 \cos\phi_2 + P_i + I_2^2 R_{2e} - V_2 I_2 \cos\phi_2 - 2 I_2^2 R_{2e} = 0$$

$$P_i - I_2^2 R_{2e} = 0$$

$$\therefore \boxed{I_2^2 R_{2e} = P_{cu} = P_i} \rightarrow \text{Condition for maximum } \eta.$$

$$\boxed{\text{Copper losses} = \text{Iron loss}} \text{ i.e., } P_i = P_{cu}$$

### Load current $I_{2m}$ at maximum efficiency:-

(11)

$$\text{For } \eta_{\max} \Rightarrow I_2^2 R_{2e} = P_i \quad \text{but } I_2 = I_{2m}$$

$$I_{2m}^2 R_{2e} = P_i$$

$$\therefore \boxed{I_{2m} = \sqrt{\frac{P_i}{R_{2e}}}} \rightarrow \text{This is the load current at } \eta_{\max}$$

Let  $I_{2FL}$  = Full load current ,

$$\text{then } \frac{I_{2m}}{I_{2FL}} = \frac{1}{I_{2FL}} \sqrt{\frac{P_i}{R_{2e}}}$$

$$\frac{I_{2m}}{I_{2FL}} = \sqrt{\frac{P_i}{I_{2FL}^2 R_{2e}}}$$

$$\therefore \boxed{I_{2m} = I_{2FL} \sqrt{\frac{P_i}{(P_{Cu})_{FL}}}}$$

### KVA Supplied at maximum efficiency:-

For constant  $V_2$  the KVA Supplied is function of load current.

$$\therefore \text{KVA at } \eta_{\max} = I_{2m} V_2 = V_2 \cdot I_{2FL} \sqrt{\frac{P_i}{P_{FLCu}}}$$

$$\therefore \boxed{\text{KVA at } \eta_{\max} = (\text{KVA rating}) \times \sqrt{\frac{P_i}{P_{FLCu}}}}$$

Substituting condition for  $\eta_{\max}$  in the expression of efficiency, we can write expression for  $\eta_{\max}$  as,

$$\therefore \eta_{\max} = \frac{V_2 I_{2m} \cos \phi}{V_2 I_{2m} \cos \phi + 2 P_i} \times 100 \quad \text{as } P_{Cu} = P_i$$

(Q) A 3300/110V, 50Hz, 60KVA 1- $\phi$  T/F has iron losses of 600w. Primary & secondary winding resistances are 3.3 $\Omega$  and 0.011 $\Omega$  respectively. Determine the efficiency of the transformer on full load at 0.8 lag P.f load.

Sol:- Given  $V_1 = 3300V$ ,  $V_2 = 110V$ , KVA = 60,  $f = 50\text{Hz}$ ,  
 $R_1 = 3.3\Omega$ ,  $R_2 = 0.011\Omega$ ,  $P_i = 600w$ ,  $\cos \phi = 0.8 \text{ lag}$

$$(I_1)_{FL} = \frac{\text{KVA}}{V_1} = \frac{60 \times 10^3}{3300} = 18.1818 \text{ A.}$$

$$\text{Primary Cu loss} = I_1^2 R_1 = (18.1818)^2 \times 3.3 = 1090.90w.$$

$$(I_2)_{FL} = \frac{\text{KVA}}{V_2} = \frac{60 \times 10^3}{110} = 545.45 \text{ A.}$$



$$\text{Secondary Cu loss} = I_2^2 R_2 = (545.45)^2 \times 0.011 = 3272.72 \text{ W}$$

$$\therefore \text{Total Cu loss} = 1090.90 + 3272.72 = 4363.627 \text{ W on FL}$$

$$\therefore \% \eta = \frac{\text{KVA} \times \cos \phi}{\text{KVA} \cdot \cos \phi + P_i + P_{cu}} \times 100$$

$$\% \eta = \frac{60 \times 10^3 \times 0.8}{60 \times 10^3 \times 0.8 + 600 + 4363.627} \times 100 = 90.6282\%$$

(Q) A 4 kVA, 200/400V, 50Hz, 1- $\phi$  TTF has equivalent resistance referred to primary as 0.15  $\Omega$ . Calculate,

(i) The total copper losses on Full load.

(ii) The efficiency while supplying full load at 0.9 p.f lagging

(iii) " " " " half " " 0.8 " leading

Assume total iron losses equal to 60W.

Sol:- Given,  $V_1 = 200\text{V}$ ,  $V_2 = 400\text{V}$ ,  $S = 4\text{ kVA}$ ,  $R_{1e} = 0.15\Omega$

$$P_i = 60\text{W}; \quad k = \frac{V_2}{V_1} = \frac{400}{200} = 2$$

$$R_{2e} = k^2 R_{1e} = (2)^2 \times 0.15 = 0.6\Omega$$

$$I_{2,FL} = \frac{\text{KVA}}{V_2} = \frac{4 \times 10^3}{400} = 10\text{A}$$

(i) Total copper losses on Full load,

$$P_{cu,FL} = (I_{2,FL})^2 R_{2e} = (10)^2 \times 0.6 = 60\text{W}$$

(ii)  $\cos \phi = 0.9$ , lag and full load

$$\% \eta_{FL, 0.9 \text{ lag pf}} = \frac{\eta \cdot (\text{VA rating}) \cos \phi}{\eta (\text{VA rating}) \cos \phi + P_i + \eta^2 P_{FL, cu}} \times 100$$

$$\text{Here } \eta = \frac{\text{Actual load}}{\text{full load}} = \frac{FL}{FL} = 1$$

$$\therefore \% \eta_{FL, 0.9 \text{ pf}} = \frac{1 \times 4 \times 10^3 \times 0.9}{1 \times 4 \times 10^3 \times 0.9 + 60 + 60} \times 100 = 96.77\%$$

(iii)  $\cos \phi = 0.8$  leading p.f, half load

$$\eta = \frac{\text{half load}}{\text{Full load}} = \frac{1/2}{1} = 1/2$$

$$\% \eta_{HL, 0.8 \text{ lead pf}} = \frac{(1/2) \times 4 \times 10^3 \times 0.8}{1/2 \times 4 \times 10^3 \times 0.8 + 60 + (1/2)^2 \times 60} \times 100$$

$$= 95.52\%$$

- (Q) A 250 KVA 1-φ T/F has iron loss of 1.8 kW, The full load Copper loss is 2000 W. Calculate, (i) η at Full load, 0.8 lagging P.f.  
 (ii) KVA supplied at maximum efficiency,  
 (iii) maximum η at 0.8 lagging P.f.

Sol:- Given,  $P_i = 1800 \text{ W}$ ,  $P_{FLCu} = 2000 \text{ W}$ .

$$(i) \eta_{FL, 0.8 Pf} = \frac{\eta (\text{VA rating}) \cos\phi}{\eta (\text{VA rating}) \cos\phi + P_i + P_{FLCu} \cdot \eta^2} \times 100$$

$\eta = \frac{FL}{FL} = 1$

$$\therefore \eta_{FL, 0.8 Pf} = \frac{1 \times 250 \times 10^3 \times 0.8}{1 \times 250 \times 10^3 \times 0.8 + 1800 + (1)^2 \times 2000} \times 100$$

(ii) KVA at  $\eta_{max} = \frac{98.135\%}{(KVA \text{ rating})} \cdot \sqrt{\frac{P_i}{P_{FLCu}}} = 250 \times \sqrt{\frac{1800}{2000}}$

$= 237.17 \text{ KVA}$

(iii)  $\eta_{max, 0.8 Pf} = \frac{KVA \text{ at } \eta_{max} \times \cos\phi}{(KVA \text{ at } \eta_{max}) \cos\phi + 2 P_i} \times 100$  [∵  $P_{Cu} = P_i$ ]

$$= \frac{237.17 \times 10^3 \times 0.8}{237.17 \times 10^3 \times 0.8 + 2 \times 1800} \times 100 = 98.137\%$$

- (Q) A 20 KVA, 440/220V, 1-φ, 50 Hz T/F has iron loss of 324 W. The Cu loss is found to be 100 W when delivering  $\frac{1}{2}$  FL current. Determine (i) efficiency when delivering full-load current at 0.8 lag P.f & (ii) the Percent of FL when the efficiency will be maximum.

Sol:-

Half load Cu loss = 100 W

$P_{Cu} \propto I^2$

where  $I = \eta I_{FL}$  ;  $\eta = \frac{\text{fraction of load}}{\text{Full load}} = \frac{\text{actual load}}{\text{Full load}}$

$$\frac{P_{FLCu}}{P_{Cu \frac{1}{2} FL}} = \frac{(I_{FL})^2}{(\frac{1}{2} I_{FL})^2} \Rightarrow P_{FLCu} = (P_{Cu \frac{1}{2} FL}) \times 2^2$$

$$P_{FLCu} = 100 \times 4 = 400 \text{ W}$$

Iron loss = 324 W.

(i)  $\eta_{FL, 0.8 Pf} = \frac{(1) \times 20 \times 10^3 \times 0.8}{1 \times 20 \times 10^3 \times 0.8 + 324 + 400 (1)^2} \times 100 = 95.67\%$

(ii)  $I_{em} = I_{FL} \times \sqrt{\frac{P_i}{P_{FLCu}}} = I_{FL} \times \sqrt{\frac{324}{400}} = 0.9 I_{FL}$

∴ Efficiency would be maximum at 90% of FL

## Transformer Tests:-

The losses, volt regulation and efficiency of a transformer can be determined by performing the transformer tests. These tests are performed without actually loading the transformer. Due to this, these test is very economical and convenient.

There are two types of tests which are commonly employed for this purpose are,

- (1) Open Circuit (O.C) test.
- (2) Short Circuit (S.C) test.

### (1) Open Circuit test (or) No-load test :-

The purpose of this test is to determine no-load loss or core loss and no-load  $I_0$  which is helpful in finding  $X_0$  and  $R_0$ .

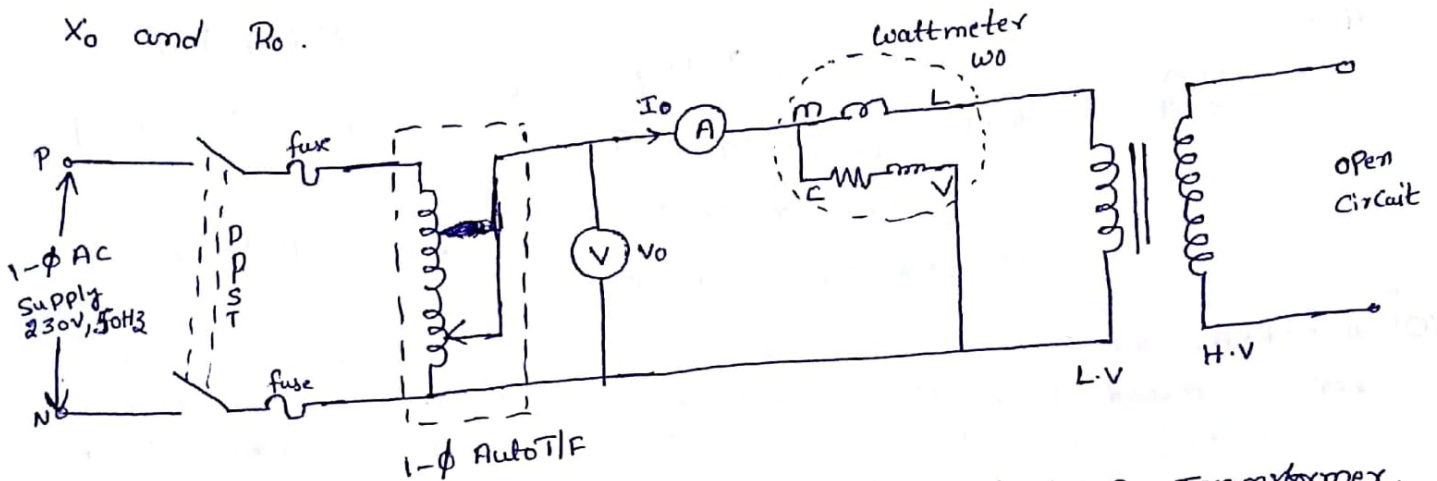


Fig:- Circuit diagram for open circuit test of transformer.

OC test is performed on the LV side of the transformer. On this test, rated voltage is applied to L.V side of the transformer with the help of 1- $\phi$  Auto transformer, and H.V side is open circuited.

Here L.V side is primary winding & H.V side is secondary winding.

The connections to the L.V side of transformer from the supply is through the wattmeter, voltmeter and ammeter, are made, as shown in fig.

As soon as the ~~test~~ rated voltage is applied on L.V side of transformer, note down the readings of

of all meters.

Tabular Column:-

Rated voltage	No-load Primary Current	Wattmeter reading
$V_0$	$I_0$	$W_0$

The wattmeter reading shows the Core losses of T/F, it also includes copper losses of primary winding but are negligible compared to core loss.

From these readings, various parameters of the circuit are calculated, as follows.

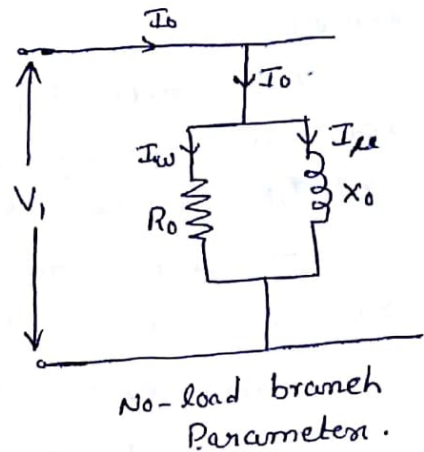
$$W_0 = V_0 I_0 \cos\phi_0 = \text{Core loss}$$

$$\therefore \cos\phi_0 = \frac{W_0}{V_0 I_0} = \text{No-load P.f}$$

$$I_{\mu} = I_0 \sin\phi_0 = \text{magnetising component of current}$$

$$I_w = I_0 \cos\phi_0 = \text{Active component of current}$$

$$\therefore X_0 = \frac{V_1}{I_{\mu}} ; R_0 = \frac{V_1}{I_w}$$



(ii) Short circuit test:-

This is an economical method for determining the following:-

- (i) Equivalent circuit parameters i.e.,  $R_{01}$  (or)  $R_{02}$ ,  $X_{01}$  (or)  $X_{02}$  &  $Z_{01}$  (or)  $Z_{02}$  of the T/F as referred to the winding in which the measuring meters are placed.
- (ii) Cu loss at full load. This loss is used in calculating  $\eta$  of transformer.
- (iii) Regulation of the transformer is determined.

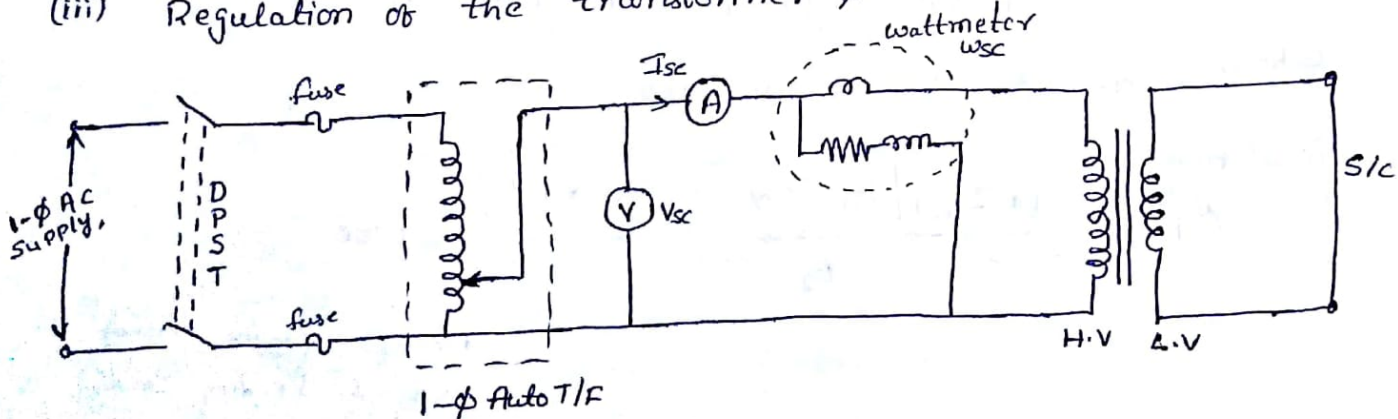


Fig:- Circuit diagram for short circuit test.

Tabular Column:-

S/C Volt ( $V_{sc}$ )	S/C Current (A)	Wattmeter reading
$V_{sc}$	$I_{sc}$	$W_{sc}$

SC test is performed on H.V side of Transformer. In this test, rated current is applied to the H.V side and the L.V side of the T/F is short circuited.

Here H.V side is Primary winding & L.V side is Secondary winding.

The rated current is applied to H.V side, which is obtained by slowly varying the Auto-transformer. This rated current is obtained for just 5 to 10% of normal primary voltage is applied.

The wattmeter reading shows <sup>Full load</sup> Copper losses of transformer both in Primary & Secondary windings. Thus, the <sup>losses</sup> of any T/F can be determined by conducting O.C & S.C test.

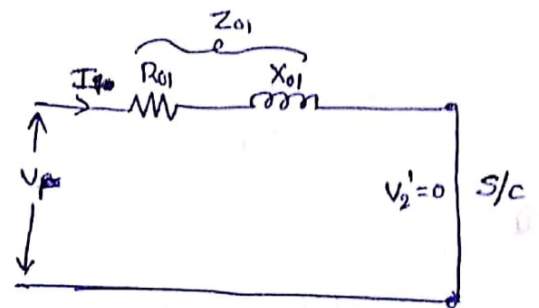
To determine the Equivalent Circuit Parameters of the SC test, we use the following formulae.

$$W_{sc} = V_{sc} I_{sc} \cos \phi_{sc} = \text{FL Cu loss}$$

$$W_{sc} = I_{sc}^2 R_{01}$$

$$\therefore R_{01} = \frac{W_{sc}}{I_{sc}^2} ; \quad Z_{01} = \frac{V_{sc}}{I_{sc}}$$

$$\therefore X_{01} = \sqrt{Z_{01}^2 - R_{01}^2}$$



From <sup>Knowing</sup> these values, we can find the efficiency & regulation of transformer at any different loads, and at any power factor of the load. By using the formulae;

$$\text{Efficiency } \eta = \frac{n (\text{VA rating}) \cos \phi}{n (\text{VA rating}) \cos \phi + P_i + n^2 P_{FLCu}} \times 100$$

where  $n =$  fraction of the load.

$$\text{Regulation } \%R = \frac{(n I_{2FL}) [R_{02} \cos \phi \pm X_{02} \sin \phi]}{E_2} \times 100$$

+ve  $\rightarrow$  lagging loads  
-ve  $\rightarrow$  leading loads.

(Q) A 5KVA, 500/250V, 50Hz 1-φ T/F gave the following readings:-

- O.C Test : 500V, 1A, 50W (L.V side open).
- S.C Test : 25V, 10A, 60W (L.V side Shorted).

- determine :-
- (1) The efficiency on FL, 0.8 lag P.f
  - (2) The Volt regulation on FL, 0.8 lead P.f
  - (3) The efficiency on 60% of FL, 0.8 leading P.f.

Sol:- Given  $V_1 = 500V$ ,  $V_2 = 250V$ ;  $f = 50Hz$ ;  $V_{Rating} = 5 \times 10^3 VA$ .

from O.C test :-  $I_0 = 1A$ ,  $P_i = 50W$   
 S.C Test :-  $I_{sc} = 10A$ ;  $P_{cu} = 60W = W_{sc}$   
 $\xrightarrow{\text{rated current}} P_{FLcu} = 60W$

$$(1) \% \eta_{FL, 0.8 \text{ lag}} = \frac{\eta (V_{Rating}) \cos \phi}{\eta (V_{Rating}) \cos \phi + P_i + \eta^2 P_{FLcu}} \times 100$$

where  $\eta = \frac{\text{actual load}}{FL} = \frac{FL}{FL} = 1$

$$\therefore \% \eta_{FL, 0.8 \text{ lag}} = \frac{1 \times 5 \times 10^3 \times 0.8}{1 \times 5 \times 10^3 \times 0.8 + 50 + (1)^2 60} \times 100$$

$$= 97.32\%$$

$$(2) \% R_{0.8 \text{ lead pf}} = \frac{(I_{FL}) [R_{ie} \cos \phi \pm X_{ie} \sin \phi]}{V_1} \times 100$$

S.C test is conducted on H.V side which is Primary.

$$\therefore I_{FL} = \frac{kVA \times 10^3}{V_1} = \frac{5 \times 10^3}{500} = 10A = I_{sc}$$

~~$$\% R = \frac{10}{500} \times 100 = 2\%$$~~

$$R_{ie} = \frac{W_{sc}}{I_{sc}^2} = \frac{60}{10^2} = 0.6 \Omega ; \quad X_{ie} = \frac{V_{sc}}{I_{sc}} = \frac{25}{10} = 2.5 \Omega$$

$$X_{ie} = \sqrt{Z_{ie}^2 - R_{ie}^2} = \sqrt{(2.5)^2 - (0.6)^2} = 2.426 \Omega$$

$$\therefore \% R = \frac{10 [0.6 \times 0.8 - 2.426 \times 0.6]}{500} \times 100 = -1.95\%$$

(3) For 60% FL,  $\eta = 0.6$ ;  $\cos \phi = 0.8$  lead.

$$\therefore \% \eta = \frac{(0.6) 5 \times 10^3 \times 0.8}{0.6 \times 5 \times 10^3 \times 0.8 + 50 + (0.6)^2 60} \times 100 = 97.103\%$$

(Q) A 10KVA, 440/3300 V, 1- $\phi$  T/F, when tested on O.C, gave the following figures on the Primary side:- 440V, 1.3 A, 115W. When tested on Short circuit with FL current flowing, the Power input was 140W. Calculate the efficiency of the transformer at (i) FL unity P.f (ii) one quarter FL 0.8 P.f.

Sol:- Given that, Rating = 10KVA ;  $E_1 = 440V$  ;  $E_2 = 3300V$ .

O.C Test :-  $V_1 = 440V$ ,  $I_0 = 1.3A$  ;  $W_0 = P_i = 115W$ .

S.C Test :-  $W_{sc} = 140W$ ,  $I_{sc} = I_{FL}$

(i)  $\eta_{FL, UPF} \% = ?$       (ii)  $\eta_{\frac{1}{4}FL, 0.8 Pf} \% = ?$

(i) FL, at UPF ;  $\eta = 1$  ;  $\cos\phi = 1$   
 $\overset{\text{Total Cu loss}}{P_{cu}} = \eta^2 P_{FLCU} = (1)^2 P_{FLCU} = (1)^2 \times 140 = 140W$

$$\% \eta_{FL, UPF} = \frac{(1) \times 10 \times 10^3 \times 1}{1 \times 10 \times 10^3 \times 1 + 115 + (1)^2 \times 140} \times 100$$

$$= \underline{\underline{97.51\%}}$$

(ii) one quarter FL at 0.8 Pf

$$\eta = \frac{1/4}{1} = 1/4 = 0.25 ; \cos\phi = 0.8$$

$$\therefore \% \eta_{\frac{1}{4}FL, 0.8 Pf} = \frac{(1/4) \times 10 \times 10^3 \times 0.8}{(1/4) \times 10 \times 10^3 \times 0.8 + 115 + (1/4)^2 \times 140} \times 100$$

$$= \underline{\underline{94.17\%}}$$

(Q) A 20KVA, 2000/200V, 1-φ T/F has the following Parameters,

- H.V winding  $R_1 = 3\Omega, X_1 = 5.3\Omega,$
- L.V winding  $R_2 = 0.05\Omega, X_2 = 0.1\Omega.$

Find the voltage regulation at,

- (i) at a 0.8 lagging P.f (ii) at UPF (iii) 0.707 P.f leading.

Sol:- Given that,  $R_1 = 3\Omega, X_1 = 5.3\Omega, R_2 = 0.05\Omega, X_2 = 0.1\Omega$

Find 20KVA  $\Rightarrow$  T/F,  $E_1/E_2 = 2000/200V.$

- (i) %R at 0.8 Lag P.f = ?
- (ii) %R<sub>UPF</sub> = ?
- (iii) %R<sub>0.707 lead P.f</sub> = ?

$$\%R = \frac{I_2 R_{2e} \cos\phi \pm I_2 X_{2e} \sin\phi}{E_2}$$

where  $K = \frac{E_2}{E_1} = \frac{2000}{2000} = 0.1$

Equivalent resistance referred to secondary  $R_{2e} = K^2 R_1 + R_2 = (0.1)^2 \cdot 3 + 0.05 = 0.08\Omega.$

Equivalent reactance referred to secondary  $X_{2e} = K^2 X_1 + X_2 = (0.1)^2 \cdot 5.3 + 0.1 = 0.153\Omega.$

Full load secondary current,

$$I_{2FL} = \frac{(KVA \text{ rating}) \times 10^3}{E_2} = \frac{20 \times 10^3}{200} = 100A.$$

(i) At P.f = 0.8 lag :-

$\cos\phi = 0.8, \sin\phi = 0.6$  +ve sign for lag

$$\%R_{FL, 0.8 \text{ lag}} = \frac{100 [0.08 \times 0.8 + 0.153 \times 0.6]}{200} \times 100 = 7.79\%$$

(ii) At P.f = Unity :-

$\cos\phi = 1, \sin\phi = 0$

$$\%R_{FL, UPF} = \frac{100 [0.08 \times 1 + 0.153 \times 0]}{200} \times 100 = 4\%$$

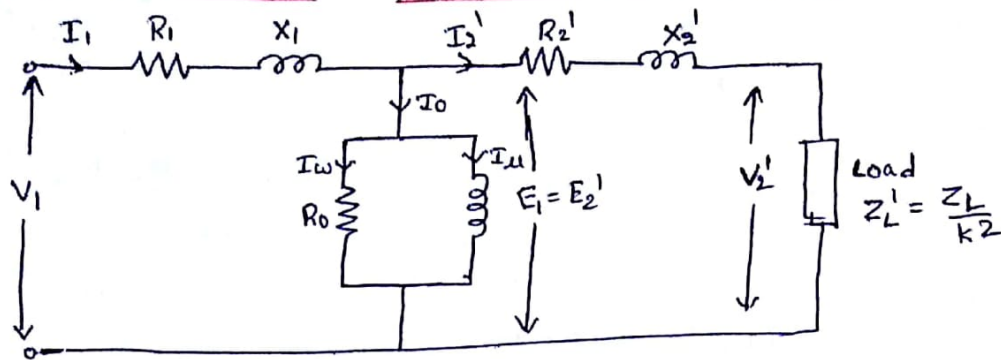
(iii) At P.f = 0.707 lead :-

$\cos\phi = 0.707, \sin\phi = 0.707$ , use -ve sign for leading P.f in regulat formula

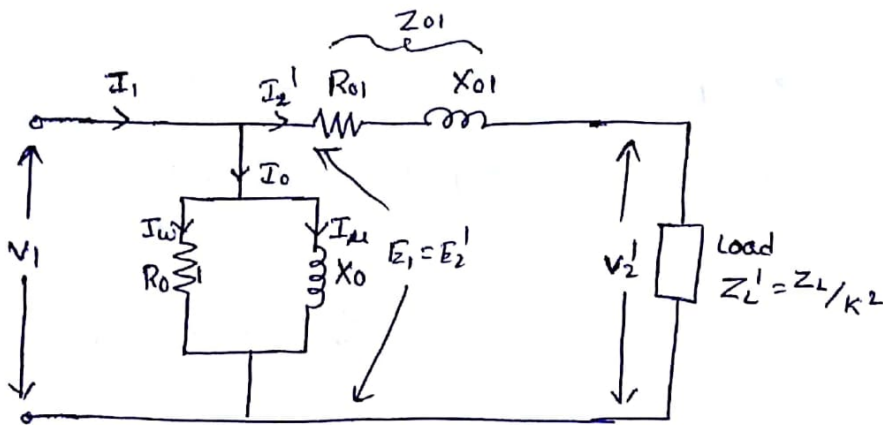
$$\%R_{FL, 0.707 \text{ lead}} = \frac{100 [0.08 \times 0.707 - 0.153 \times 0.707]}{200} \times 100 = -2.58\%$$



Equivalent circuit of a Transformer :-



fig(1) - Exact equivalent circuit of T/F referred to Primary side.



fig(2) - Approximate equivalent circuit of T/F referred to Primary side.

## Single - Phase Induction Motor

### Working Principle of 1- $\phi$ Induction motor:-

Constructionally, this motor is similar to a PolyPhase induction motor, except that

- (i) its stator is provided with a single phase winding &
- (ii) a centrifugal switch is used in some types of motors, in order to cut out a winding, used only for starting purposes.

It has distributed stator winding and a squirrel - cage rotor.

When fed from a 1- $\phi$  supply to its stator winding produces a flux which is only alternating i.e., one which alternates along one space axis only. It is not a rotating flux, as in case of 2 or 3- $\phi$  stator winding, fed from a 2 or 3- $\phi$  supply. Now, an alternating or pulsating flux acting on a stationary squirrel-cage rotor cannot produce rotation (only a revolving flux can). That is why a 1- $\phi$  IM is not self starting.

However, if the rotor of such a machine is given an initial start by hand (or small motor) or otherwise, in either direction, then immediately a torque arises and the motor accelerates to its final speed.

This peculiar behaviour of the motor has been explained in two ways:- (i) By two-field or Double field revolving theory.

(ii) By Cross-field Theory.

### Double - field Revolving Theory:-

This theory utilizes the Ferraris' Principle which states that "An alternating mmf can be resolved into two m.m.f's whose amplitude is half of the original m.m.f's and are rotating in opposite directions with equal angular velocity."

Accordingly, an alternating sinusoidal flux ( $\Phi_m$ ) can be resolved into two equal revolving fluxes, each equal to half the value of the alternating flux & each rotating synchronously ( $N_s = 120f/p$ ) in opposite direction.

From fig (a)

(A) Forward flux ( $\phi_m/2$ )  $\Rightarrow$  revolving in anticlockwise

(B) Backward flux ( $\phi_m/2$ )  $\Rightarrow$  " " " " clockwise direction

Let alternating flux have a max value of  $\phi_m$ . as shown in fig (a).

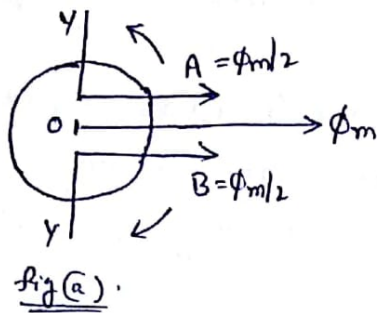


fig (a)

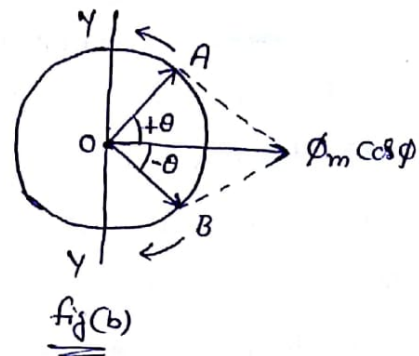


fig (b)

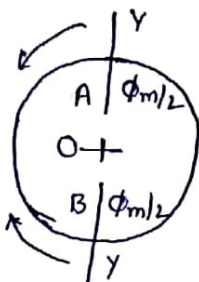


fig (c)

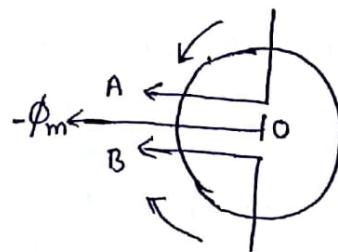


fig (d)

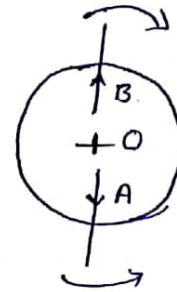


fig (e)

After some time, when A and B would have rotated through angle  $+\theta$ , and  $-\theta$ , as shown in fig (b), the resultant flux would be

$$= 2 \times \frac{\phi_m}{2} \cos \frac{2\theta}{2} = \phi_m \cos \theta.$$

After a quarter cycle of rotation, fluxes A and B will be oppositely-directed as shown in fig (c), so that the resultant flux would be zero.

After half a cycle, fluxes A and B will have a resultant of  $-2 \times \frac{\phi_m}{2} = -\phi_m$ .

After three-quarters of a cycle, again the resultant is zero as shown in fig (e) and so on.

If we plot the values of resultant flux against  $\theta$  b/w limits  $\theta = 0^\circ$  to  $360^\circ$ , then a curve is obtained as shown in fig (i).

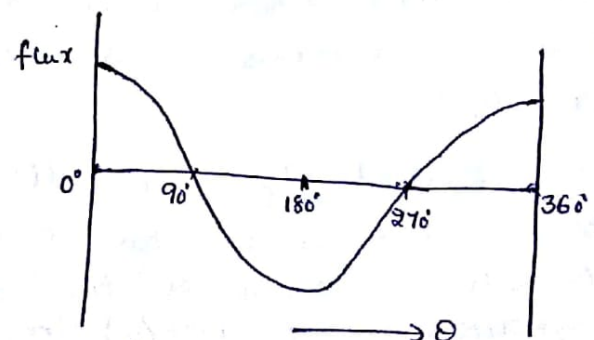


fig (i)

Note: If the slip of the rotor is 's' w.r.t the forward rotating flux, then its slip w.r.t the backward rotating flux is  $(2-s)$ .

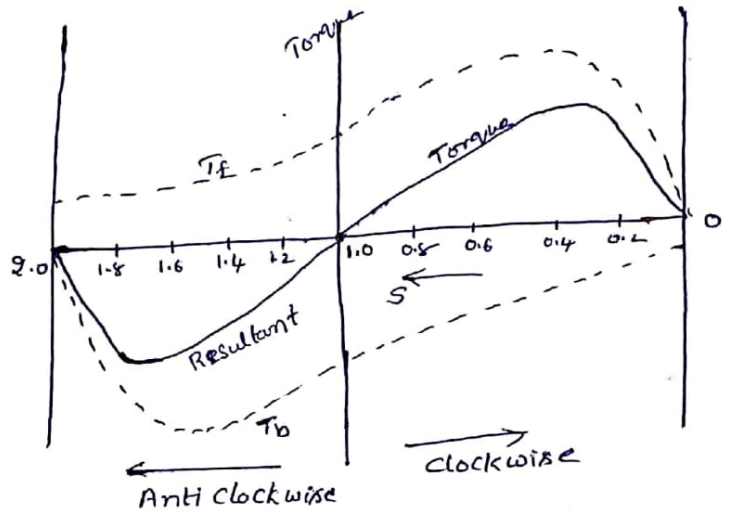
Each of the two components fluxes, while revolving round the stator, cuts the rotor, induces an emf & this produces its own torque.

$T_f$  = forward torque ;  $T_b$  = backward torque.

At stand still,  $T_f$  &  $T_b$  are numerically equal but, being oppositely directed, produce no resultant torque. That explains why there is no starting torque. (i.e., not self-starting).

However, if the rotor is started somehow, say in the clockwise direction, the clockwise torque starts increasing and, at the same time, the anticlockwise torque starts decreasing. Hence, there is a certain amount of net torque in the clockwise direction which accelerates the motor to full speed.

Fig: Shows both torques and the resultant torques for slips b/w zero and +2.



12) Permanent Capacitor IM:- (Capacitor run motor)

Centrifugal switch is not required

It is used in exhaust & intake fans, unit heaters, blowers etc.

## Applications of AC machines:-

- (1) Capacitor - Start  
Capacitor - run Induction motor :- Elevators, conveyors, large 1- $\phi$  Compressors, grinders, large 1- $\phi$  air conditioning, ceiling fans, air circulation, Compressor, Refrigerator, Vacuum Pumps, fans, drilling m/c's, Lathes etc.
- (2) Shaded Pole 1- $\phi$  IM :- Toys, hair dryers, photocopy m/c's, advertising displays, light duty fans such as small film projectors, window air conditioning, Paddle wheel - level sensors, Exhaust fans used in rest rooms etc.
- (3) Repulsion motors :- High Speed lifts, mining equipment, air compressors fans & pumps etc.
- (4) Universal motors :- It is used for domestic applications ex:- vacuum cleaners, food processors, mixers, hair driers, coffee grinders, electric shavers, portable tools like drilling machines etc.
- (5) Reluctance motor :- Analog electric meters, washing machine design, control rod drive machine of nuclear reactor, Hard disk drive meter etc.
- (6) Hysteresis motor :- Widely used in synchronous motor applications where smooth starting is req. such as in clocks, other timing devices, & Record players, turntables etc.
- (7) Split Phase IM :- (Poor T) Fans, Centrifugal Pumps etc. blowers, washing m/c's, grinders.
- (8) Synchronous motors :- used for constant speed load services, Reciprocating Compressor drives, Power factor corrections, drive crushers, rotary kilns, Volt regulation of Transmission lines etc.
- (9) SLIP ring Induction motor :- hoists, cranes, elevators, ~~etc~~ Compressors, Printing Presses, large ventilating fans etc.
- (10) Squirrel Cage I.M :- Lath machines, Compressors, Centrifugal Pump, in agriculture etc. and industrial pumps, industrial drives etc.
- (11) Capacitor Start IM :- (↑T) water pumps, grinders, lathe m/c's, Compressors, drilling m/c's etc.