Unit - 1

Electro static field

Introduction :-

Electromagnetics is a branch of physics or electrical Engineer-ing which is used to study electrical and magnetics phenomon the electric and magnetic fields are closely related to each other.

What is a field ?

Consider a magnet. It has its own effect in a region surrounding it. The effect can be experienced by placing another magnet near the first magnet, which is defined by a particular physical function.

So field can be defined as the region in which, at each point there exists a corresponding value of some physical function.

If at each point of a region or space there is a corresponding value of some physical function then the regio is called a field

If the field produced is due to magnetic effects, it is called magnetic field

An electric charge (positive or negative) produces a field around it which is called an electric field.

Moving charges produce a current & current carrying conductor produces a magnetic field

In such a case, electric and magnetic fields are related to each other. Such a field is called electromagnetic field.

To quantify the field, three dimensional representation plays an important role.

A complete pictorial representation & clear understanding of fields and laws governing such fields, is possible with the help of vector analysis.

Co-ordinate system

To describe a vector accurately and to express a vecto in terms of its components it is necessary to have some reference directions.

Such directions are represented in terms of various Co-ordinate systems. There are 3 major Co-ordinate Systems, they are:

1. Cartesian (or) rectangular Co-ordinate system.
2. Cylinderical Co-ordinate system.
3. Spherical Co-ordinate system

① Cartesian (or) rectangular Co-ordinate system

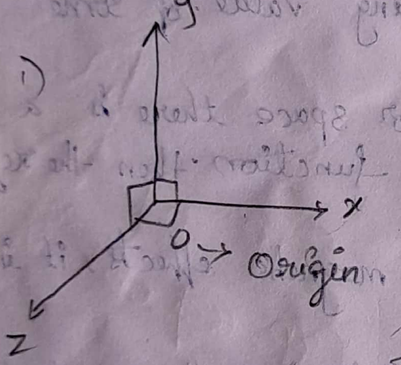
This is also rectangular Co-ordinate system.

This system has three Co-ordinate axes represented as x, y, z which are mutually at right angles to each other.

These three axis intersect at a Common point Called origin of the system.

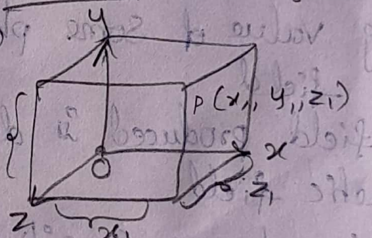
* Rectangular Co-ordinate system

1)



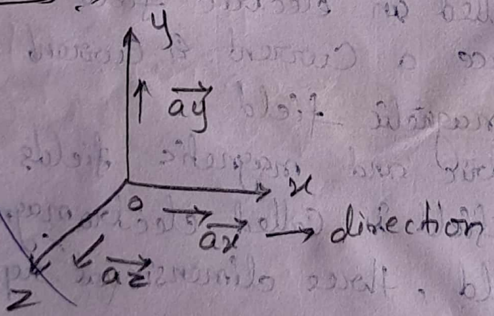
→ ISRO – Indian space Research organisation.

COMET – is a stone on a space



fig(a)

Unit Vector:



→ In Cartesian Co-ordinate system x=0 plane indicates two-dimensional y-z plane, y=0 plane indicates two-dimensional x-z plane and z=0 plane indicates two dimensional x-y plane. Representing a point in Rectangular Co-ordinate system.

 A point in a rectangular Co-ordinate system is located by three Co-ordinates namely x, y and z Co-ordinates

the point can be reached by moving from origin the distance x in x direction, then the distance y in y direction and the distance z in z direction

Consider a point 'p' having Co-ordinates x_1, y_1 and z_1. It is represented as $p(x_1, y_1, z_1)$ which is shown in fig(a)

The Co-ordinates x_1, y_1, and z_1 can be '+ve' or '-ve'

Unit Vectors :- (Base Vectors) :-

The unit Vector are the one which are strictly oriented. axis of the given Co-ordinate system. In Cartesian Co-ordinate system, $\vec{ax}, \vec{ay}$ and $\vec{az}$ are the base Vectors as shown in fig

$$\vec{P} = x_1\vec{ax} + y_1\vec{ay} + z_1\vec{az}$$

$$\vec{Q} = x_2\vec{ax} + y_2\vec{ay} + z_2\vec{az}$$

Then, the distance or displacement from P to Q is represented by a distance Vector $\vec{PQ}$ & is given by

$$\vec{PQ} = \vec{Q} - \vec{P}$$

$$= [x_2 - x_1]\vec{ax} + [y_2 - y_1]\vec{ay} + [z_2 - z_1]\vec{a_z}$$

The magnitude of this Vector is given by

$$|\vec{PQ}| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$$

The above eqn is called distance fromula which gives the distance b/w the two points.

② Cylindrical Co-ordinate System:-

The surfaces used to define the cylindrical Co-ordinate System are,

i) plane of Constant z which is parallel to xy plane

ii) A cylinder of radius r with z axis as the axis of the cylinder;

iii) A half plane perpendicular xy plane and at an angle ϕ with respect to xz plane

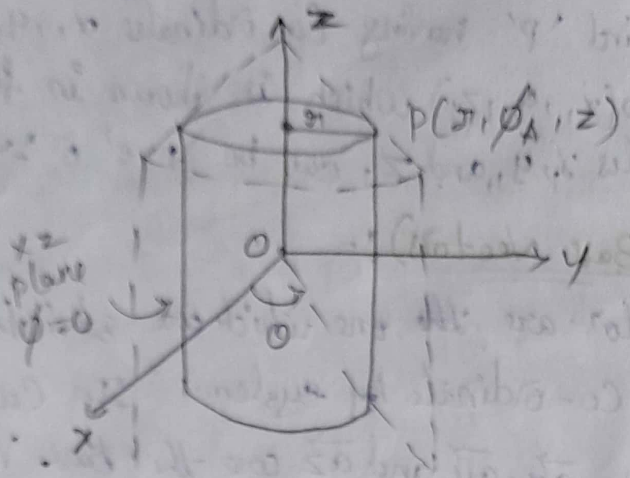
The angle ϕ is called azimuthal angle The ranges of the Variables are,

$$0 \leq r < \alpha \quad \text{—— ①}$$

$$0 \leq \phi \leq 2\pi \quad \text{—— ②}$$

$$0 \leq z \leq \alpha \quad \text{—— ③}$$

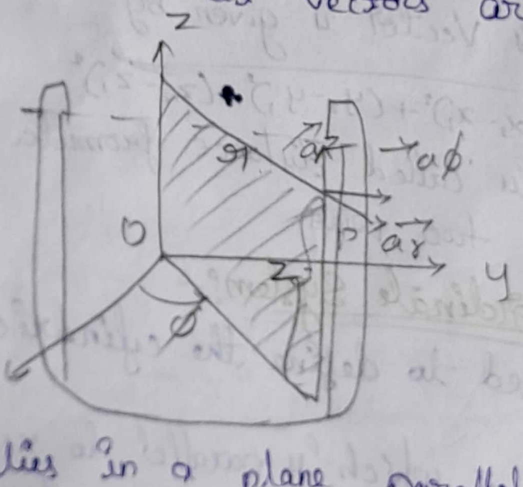
The point p in cylindrical Co-ordinate system has three Co-ordinates, r, ϕ and z whose values lie in the respective ranges given by the eqn ①, ② and ③



Note:- Angle ϕ is Expressed in radius, and for ϕ, anticlockwise measurement is treated positive while clockwis measurement is treated negative.

Unit Vector

Similar to Cartesian Co-ordinate system, there are three unit vectors in r, ϕ and z directions denoted as $\vec{a_r}$, $\vec{a_\phi}$, and $\vec{a_z}$ these unit vectors are shown in fig



The $\vec{a_r}$ lies in a plane parallel to the xy plane and is perpendicular to the surface of cylinder.

The unit vector $\vec{a_\phi}$ lies also in a plane parallel to xy plane but it is tangent to the cylinder

The unit vector $\vec{a_z}$ is parallel to z axis

Hence, vector of point p can be expressed as,

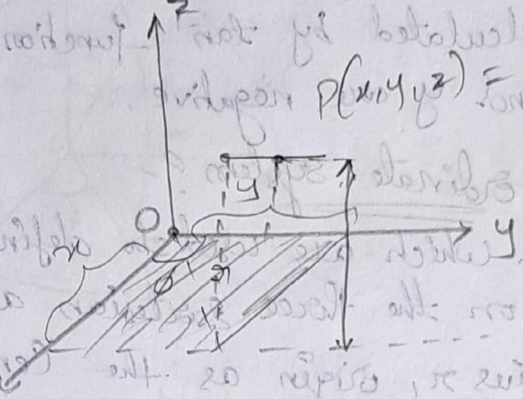
$$\vec{P} = P_r\vec{a_r} + P_\phi\,\vec{a_\phi} + P_z\,\vec{a_z}$$

where, P_r is radius r, P_ϕ is angle ϕ and P_z is z co-ordinate of point p.

Note :- In Cartesian Co-ordinate System, the unit vector are not dependent on the Co-ordinates. But in cylindrical Co-ordinate system $\vec{a_r}$ and $\vec{a_\phi}$ are functions of ϕ co-ordinates as their directions change as ϕ changes.

Relationship b/w Cartesian and cylindrical System

Consider a point p whose cartesian co-ordinates are x, y and z, while the cylindrical co-ordinates are r, ϕ and z as, shown in fig



Looking at the XY plane, we can write $x = r\cos\phi$, and $y = r\sin\phi$.

The z remains same in both the system.

Hence, transformation from cylinderical to Cartesian Can be obtained from the equations,

$x = r\cos\phi$, $y = r\sin\phi$, and $z = z$

It can be seen that, r can be expressed interms of x and y as

$$r = \sqrt{x^2+y^2}$$

while, $\tan\phi = y/x$

thus, the transformation from Cartesian to cylindrical Can be obtained as,

$$r = \sqrt{x^2+y^2}$$

$$\phi = \tan^{-1}\left(\frac{y}{x}\right) \text{ and } z = z$$

Note :-

→ r is positive or zero, hence positive sign of square root must be considered.

→ while Calculating ϕ make sure the sign of x and y

→ If both are positive, ϕ is positive in the first quadrant

→ If x is negative & y is positive then the point is in second quadrant hence ϕ must be within $+90°$ and $+180°$ (i.e $-180°$ to $-270°$).

→ when y is negative and x is positive then ϕ is in fourth quadrant i.e within $0°$ and $-90°$ (i.e $270°$ and $360°$).

→ Similarly, when x is negative and y is also negative, the point is in third quadrant and accordingly, ϕ must be b/w $-90°$ to $-180°$ (i.e $+180°$ and $+270°$). So $180°$ must be subtracted from the ϕ calculated by $\tan^{-1}$ function to get accurate ϕ when both x and y are negative.

3 Spherical Co-ordinate system :-

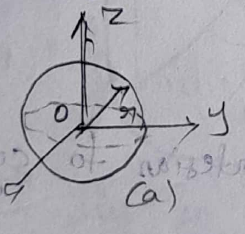
The surface which are used to define the spherical co-ordinate system on the three Cartesian axis are :-

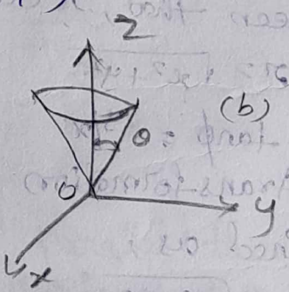
→ Sphere of radius r, origin as the centre of the sphere (fig a).

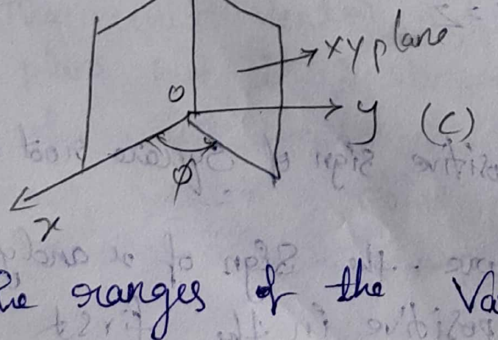
→ A right circular cone with its open at the origin and its axis as z axis. Its half angle is θ. It rotates about z axis and θ varies from $0°$ to $180°$ fig b.

→ A half plane perpendicular to xy plane containing z axis making an angle ϕ with the xy plane fig c.

Thus the three coordinates of point P in the spherical co-ordinate system are (r, θ, ϕ).







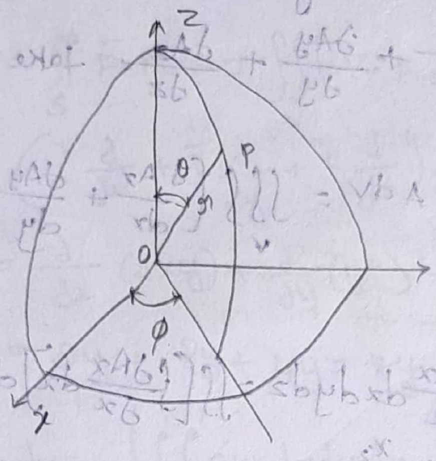
The ranges of the variables are

$0 \le r < \alpha$ — ①

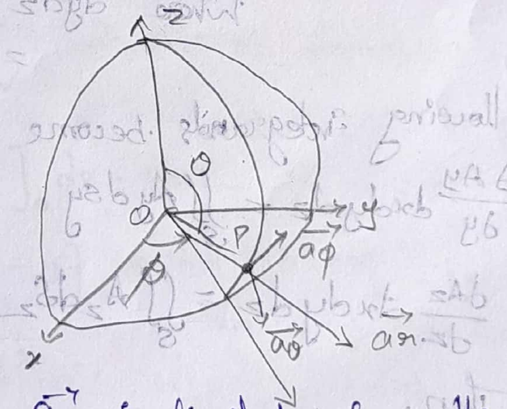
$0 \le \phi \le 2\pi$ — ②

$0 \le \theta \le \pi$ — ③ as half cycle

The point $P(r, \theta, \phi)$ can be represented in the sperical Co-ordinate system as shown in the figure. The angles θ & ϕ are measured in radians.



Similar to other two co-ordinate systems, there are three unit vectors in the r, θ and ϕ directions denoted $\vec{a_r}$, $\vec{a_\theta}$ and $\vec{a_\phi}$. These unit vectors are mutually perpendicular to each other and ~~and~~ are shown in fig. below



The unit vector $\vec{a_r}$ is directed from the Centre of the sphere i.e origin to the given point P.

It is directed radially outward, normal to the sphere the unit vector $\vec{a_\theta}$ is tangent to the sphere and oriented in the direction of increasing θ.

Divergence theorem :-

The volume integral of the divergence of a vector field over a volume is equal to the surface integral of the normal component of this vector over the surface bounding the volume $\iiint_V \nabla\cdot A\, dv = \oiint_S A\cdot ds$.

Proof :- The divergence ~~the~~ of any vector A is given by

$$\nabla\cdot A = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$ take the volume integral on b.s

$$\iiint_V \nabla\cdot A\, dv = \iiint_V \left[\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}\right] dx\,dy\,dz$$

$[\because dv = dx\,dy\,dz]$

$$\iiint \frac{\partial A_x}{\partial x} dx\,dy\,dz = \iint\left[\int \frac{\partial A_x}{\partial x} dx\right] dy\,dz$$

But $\int_{x_1}^{x_2} \frac{\partial A_x}{\partial x} dx = \left[A_x\right]_{x_1}^{x_2} = A_{x_2} - A_{x_1}$
$= A_x$

Then $\iiint_V \frac{\partial A_x}{\partial x} dx\,dy\,dz = \iint_S A_x\,dy\,dz = \iint_S A_x$

Where $dy\,dz = ds_x$
$= x$ component of surface area ds.

similarly the following integrands become

$$\iiint_V \frac{\partial A_y}{\partial y} dx\,dy\,dz = \iint_S A_y\, ds_y$$

$$\iiint_V \frac{\partial A_z}{\partial z} dx\,dy\,dz = \iint_S A_z\, ds_z$$

Then,

$$\iiint \nabla\cdot A\,dv = \iiint\left[\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}\right)$$

$$= \iint_S (A_x ds_x + A_y ds_y + A_z ds_z)$$

$$= \oiint_S A\cdot ds$$

The divergence theorem applies two time varying as well as static field.

This theorem is used to convert volume integral of a divergence Vector into a closed Surface ~~theorem~~. Integral.

Q) using divergence theorem evaluate $\iint_S \bar{F}\cdot n\, ds$, where $\bar{F} = 2xy\,\bar{i} + y^2\,\bar{j} + 4yz\,\bar{k}$ and S is the Surface of the Cube bounded by $x=0,\ x=1,\ y=0,\ y=1$ and $z=0,\ z=1$ using divergence theorem

Ans:-

$$\iint_S \bar{F}\cdot n\, ds = \iiint_V \nabla\cdot F\, dv$$

$$\nabla\cdot F = \left(\bar{i}\frac{\partial}{\partial x} + \bar{j}\frac{\partial}{\partial y} + \bar{k}\frac{\partial}{\partial z}\right)\cdot\left(2xy\,\bar{i} + y^2\,\bar{j} + 4yz\,\bar{k}\right)$$

$$= \frac{\partial}{\partial x}(2xy) + \frac{\partial}{\partial y}(y^2) + \frac{\partial}{\partial z}(4yz)$$

$$= 2y + 2y + 4y = 8y$$

$$\iiint \nabla\cdot F\, dv = \int_0^1\int_0^1\int_0^1 8y\, dx\,dy\,dz$$

$$= \int_0^1\int_0^1 [8yz]_0^1\, dx\,dy$$

$$= \int_0^1\int_0^1 8y\, dx\,dy$$

$$\iiint \nabla\cdot \bar{F}\, dv$$

$$= \int_0^1\left[\frac{8y^2}{2}\right]_0^1 dx$$

$$= \cancel{8}^{4}\left[\cancel{\tfrac{1}{2}} - 0\right]$$

$$= \int_0^1 4\, dx$$

$$= 4\int_0^1 dx$$

$$= 4[x]_0^1$$

$$= 4(1-0)$$

$$= 4$$

$$\iint_S \bar{F}\cdot n\, ds = 4$$

$$= \iiint \nabla\cdot F\, dv$$

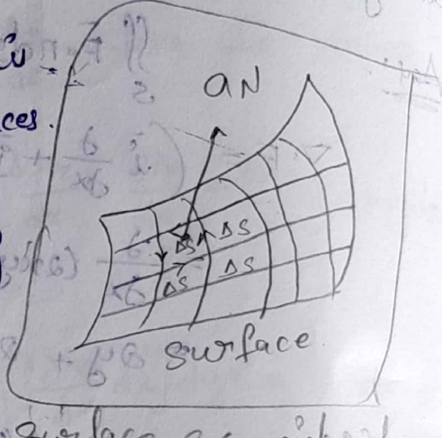
STOKe's Theorem

The line integral of a Vector around a closed path is equal to the Surface integral of the Normal Component of its Curl over any closed Surface. ~~The line integral of a Vector~~

$$\oint H \cdot dl = \iint_S \nabla \times H \cdot ds \quad -(1)$$

Proof:

Consider an arbitrary surface this is broken up into incremental surfaces of area Δs as shown in fig



If H is any field vector, then by defintion of the ~~two~~ to one of the Incremented Surfaces.

Surface S consists of small Surface. ΔS

$$\frac{\oint H \cdot dl \, \Delta s}{\Delta s} = (\nabla \times H) a_N \quad -(2).$$

where N indicates Normal to the surface and a $dl\Delta s$ Indicate that a closed path of an Incremented Area Δs.

~~and dl~~

→ The Curl of H Normal to the Surface Can be written as

$$\oint H \cdot dl \, \Delta s = (\nabla \times H) \cdot a_N \Delta s \quad -(3)$$
$$= (\nabla \times H) \cdot \Delta s \quad -(4)$$

where, a_N is a Unit Vector Normal to Δs ∴

→ The Closed Integral for each Surface, S is given by the Surface Integral of the Normal Component for H

→ This theorem is used to Convert the Surface integral of Curl of a Vector into ~~this theorem is used to~~ closed line integral.

Continuous charge distribution:-

If the charge are distributed instead of concentrated at one point it is better to define charge distribution in terms of charge density. The different types of charge density are given below 1. Linear charge density

1. Linear charge density :-

It is defined as the total charge distributed over a line (or) curve

$$\rho_l = \underset{\Delta l \to 0}{Lt} \left[\frac{\Delta Q}{\Delta l}\right]$$ this gives. the total charge per length i.e. is given by.

$$\rho_l = \frac{Q}{l} \text{ Coulomb/metre } (C/m)$$

2) Surface charge density :-

It is defined as the total charge distributed over a Surface

$$\rho_s = \underset{s \to 0}{Lt} \left[\frac{\Delta Q}{\Delta s}\right]$$ this gives. the total charge per area i.e. is given by

$$\rho_s = \frac{Q}{S} = \frac{Q}{A} \text{ coulomb/sq. metre } (C/m^2)$$

3) Volume charge density :-

It is defined as the total charge distributed over a Volume

$$\rho_v = \underset{\Delta V \to 0}{Lt} \left[\frac{\Delta Q}{\Delta V}\right]$$ this gives the total charge per volume i.e. is given by

$$\rho_v = \frac{Q}{V} \cdot \text{Coulomb / cubic metre } (C/m^3)$$

Electric field Intensity

Consider a charge 'Q' fixed in position and move another charge "q", call. it as test charge, around the fixed charge. the test charge experience a force around the fixed charge 'Q' i.e electric field is set up around the 'Q' and any charge brought into this

field will experience a force.
The electric field or electric field Intensity is defined as the electric force per unit charge i.e. is given by

$E = \frac{F}{q}$, according to ~~Colums~~ Coulomb's law

$F = \frac{Qq}{4\pi\varepsilon r^2}$, Electric field $\boxed{E = \frac{F}{q}}$

$$E = \frac{\frac{Qq}{4\pi\varepsilon r^2}}{q}$$

$$E = \frac{Q}{4\pi\varepsilon r^2} \text{ Newtons/coulomb}$$

The another unit of electric field is volts per meter

$$\boxed{E = \frac{Q}{4\pi\varepsilon r^2} \text{ volts/meter or V/m}}$$

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Electric field due to a charged circular disc

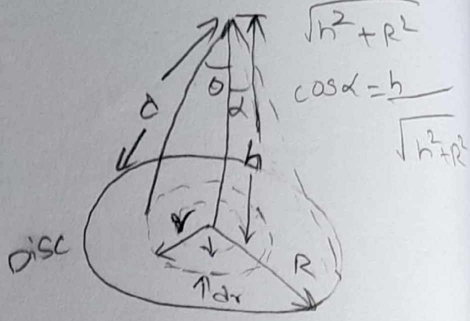
Charge density $\rightarrow \rho_s$ coulomb/m^2

Circular disc of Radius $\rightarrow R$

angular ring of Radius $\rightarrow r$

Radial thickness $\rightarrow dr$

Area of angular ring is $ds = 2\pi r dr$



charged circular disc

Field intensity

$$\boxed{dE = \frac{\rho_s ds}{4\pi\varepsilon d^2}}$$

$$\tan\theta = \frac{r}{h}$$

$$r = h\tan\theta$$

$$dr = h\sec^2\theta d\theta$$

$$\sin\theta = \frac{r}{d}$$

$$\boxed{d = r/\sin\theta}$$

$$dE_y = \frac{\rho_s \cdot ds}{4\pi\varepsilon d^2}\cos\theta$$

$$= \frac{\rho_s \cdot 2\pi r dr}{4\pi\varepsilon d^2}\cos\theta$$

$$= \frac{\rho_s\, 2\pi r\, h\sec^2\theta d\theta \cos\theta \sin^2\theta}{4\pi\varepsilon r^2}$$

$\sec\theta = \frac{1}{\cos\theta}$

$$= \frac{\rho_s h \sec\theta \sin^2\theta d\theta}{2\varepsilon r}$$

$\sec\theta \sin\theta = \tan\theta$

$$= \frac{\rho_s \sec\theta \sin^2\theta d\theta}{2\varepsilon \tan\theta}$$

$$\boxed{dE_y = \frac{\rho_s \tan\theta \sin\theta d\theta}{2\varepsilon \tan\theta}}$$

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The total electric field due to the charged disc

$$E = \int_{\theta=0}^{\alpha} dE = \frac{\rho_s}{2\varepsilon}\int_{0}^{\alpha} \sin\theta \, d\theta$$

$$= \frac{\rho_s}{2\varepsilon}\Big[-\cos\theta\Big]_{0}^{\alpha}$$

$$= \frac{\rho_s}{2\varepsilon}\Big\{-\left[\cos\alpha - \cos 0\right]\Big\}$$

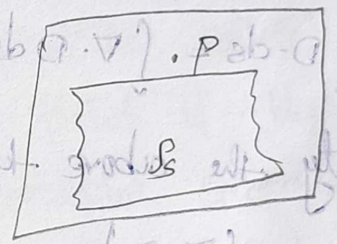
$$= \frac{\rho_s}{2\varepsilon}\left[\cos 0 - \cos\alpha\right]$$

$$E = \frac{\rho_s}{2\varepsilon}\left[1 - \cos\alpha\right]$$

$$E = \frac{\rho_s}{2\varepsilon}\left[1 - \frac{h}{\sqrt{h^2+R^2}}\right]$$

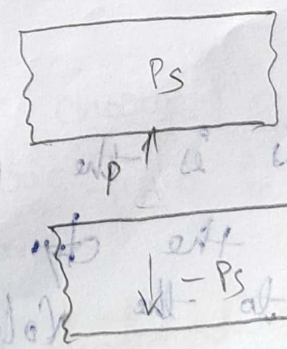
Electric field Due to a Infinite uniformly ~~long~~ changed sheet

$$E = \frac{\rho_s}{2\varepsilon}(1-\cos\alpha) \quad \alpha = 90^\circ$$



$$\boxed{E = \frac{\rho_s}{2\varepsilon}}$$

Electric field b/w two infinite uniformly charged sheets



$$E = \frac{\rho_s}{2\varepsilon} + \frac{\rho_s}{2\varepsilon}$$

$$= \frac{2\rho_s}{2\varepsilon}$$

$$\boxed{E = \frac{\rho_s}{\varepsilon}}$$

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Gauss's Law

Gauss's law in Integral and Differential forms

Gauss's law states that the total flux out of closed surface is equal to the Net charge with in the surface. It can be written in integral form has

$$\oint_S D \cdot ds = Q$$

If the Volume charge density is ρ_v, then the Net charge enclosed by the surface 'S' with in the volume v will be

$Q = \int_v \rho_v dv$. By Gauss' Law

$$\boxed{\oint D \cdot ds = \int_v \rho_v dv}$$

By applying Divergence theorem.

$$\oint_S D \cdot ds = \int_v \nabla \cdot D \, ds$$

By equality the above two equations.

$$\int_v \nabla \cdot D \, ds = \int_v \rho_v dv$$

$$\therefore \boxed{\nabla \cdot D = \rho_v}$$

This is the Gauss Law is differential form. it states that the divergence of electric displacement is equal to the Volume charge density.

Gauss's divergence theorem

Gauss law states that the closed surface integral of

$\oint D \cdot ds = Q$ (Charge Enclosed)

$$\oint_S D \cdot ds = Q$$

The volume charge density is ρ_v, then net charge enclosed by the surface 'S' with in the volume V is given by

$$Q = \int \rho_v dv$$

$$\oint D \cdot ds = \int_v \rho_v dv \Rightarrow \text{But } \boxed{\rho_v = \nabla \cdot D}$$

$$\boxed{\therefore \oint D \cdot ds = \int_v (\nabla \cdot D)\, dv}$$

This is Gauss divergence theorem or simple divergence theorem

Applications of Gauss law :- Gaussian Surface :-

Gauss Law is Applied to the Surface if the following conditions are Supplyed.

→ The Surface is closed

→ electric flux density D is either normal (or) tangential to the Surface at each point of the Surface.

→ Electric flux density is constant over the Part of the Surface where D is Normal

→ Surface over which Gauss Law is Applied is Called Gaussian Surface.

→ Gauss Law is used to determine the charge (Q) enclosed, or flux density (D) (or) electric field intensity (E) for Symmetrical charge distribution

Mention Some Applications of gauss law in electrostatics.

Ans Gauss Law is Applied to determine the electric field intensity (E) from a closed Surface

→ Electric field can be determined for charged shell to Concentrate shell or cylinders etc.

Q) why Gauss law cannot be applied to determine due to finite charge line

Ans Gauss law cannot be applied on non-gaussian surface. It can be applied if the surface encloses the volume completely.
∴ Gauss law cannot be applied to determine the electric field due to finite line charge

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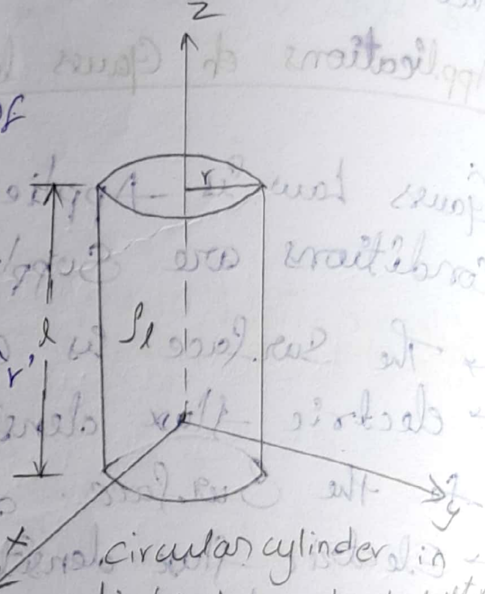
Applications of Gauss's law :-

Infinite line charge :-

Consider an infinite line charge of ρ_l of columb per meter (C/m) as the z-axis of cylindrical co-ordinate.

→ Consider a circular cylindral of radius 'r' with length 'l' as Gaussain surface as shown in figure.



circular cylinder in cylindrical co-ordinate system

→ The electric flux density ds is normal to the surface or the cylinder at everywhere.

→ By apply-ing Gauss's law to the closed surface (gaussian surface) of the cylinder.

$$Q = \oint D_s \, ds$$

$$= D_s \int_{sides} ds + D_s \int_{top} ds + D_s \int_{bottom} ds$$

→ Electric flux on top & bottom of cylinder (except surface of the cylinder) is '0' (zero).

→ Then, $Q = D_s \int_{sides} ds = D_s \int_{z=0}^{l} \int_{\phi=0}^{2\pi} r \, d\phi \, dz$

$$= D_S \int_{z=0}^{l} r\phi\Big]_0^{2\pi} dz = D_S \int_{z=0}^{l} 2\pi r\, dz$$

$$= D_S \cdot 2\pi r\, z\Big]_0^{l}$$

$$Q = D_S \cdot 2\pi r l$$

$$\therefore \boxed{D_S = \frac{Q}{2\pi r l}}$$

If $\rho_l = \frac{Q}{l}$ is the linear charge density.

$$\Rightarrow \boxed{D_S = \frac{\rho_l}{2\pi r}}$$

electric field intensity $E = \frac{D_S}{\varepsilon_0}$

$$E = \frac{\rho_l}{2\pi \varepsilon_0 r}$$

Single shell of charge

let a positive charge Q is uniformly distibuted over a spherical surface of radius as shown in figure

→ By applying Gauss law inside the ~~section~~ shell the integral of flux density over a spherical surface (gaussian surface is zero) as no charge is enclosed by the surface

$$\oint D \cdot ds = 0$$

$$E = \frac{D_S}{\varepsilon}$$

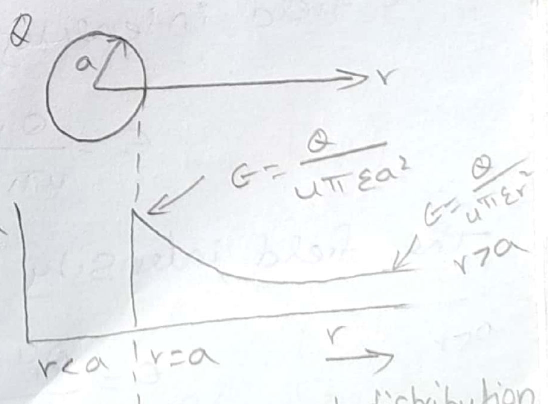
$r<a$

$$\varepsilon \oint E \cdot ds = 0$$

$$\boxed{E = 0} \; r<a$$

electric field is zero inside the shell.

By applying gauss law just a outside the shell, the integral of flux density over a spherical surface is the charge of the shell



single shell of charge and distribution electric field intensity.

$$\oint_S D \cdot ds = Q$$

$$\varepsilon \oint_S E \cdot ds = Q$$

$$\boxed{\oint E \cdot ds = \frac{Q}{\varepsilon}}$$

$$E \cdot (4\pi r^2) = Q/\varepsilon$$

$$\boxed{E = \frac{Q}{4\pi\varepsilon r^2}} \quad r > a$$

This is the electric field just outside the spherical shell.

5/5/22

Two concentric shells of charge

a) Electric field intensity b/w two shells.

Consider two spherical shells of radia 'a' and 'b'. Let Q_1 and Q_2 be the charges uniformly distibuted over the inner shell of radius 'a' and outer shell of radius 'b' Respectively.

→ By applying Gauss law the line integral of flux density 'D' over a closed surface is zero.

$$\oint D \cdot ds = 0 \qquad [\because D = \varepsilon E]$$

$$\therefore E = 0 \qquad r < a$$

The field intensity b/w the two shells $(a > r > b)$

$$E = \frac{Q_1}{4\pi\varepsilon r^2} \quad a > r > b$$

the field intensity just outside both the shells due to Q_1 and

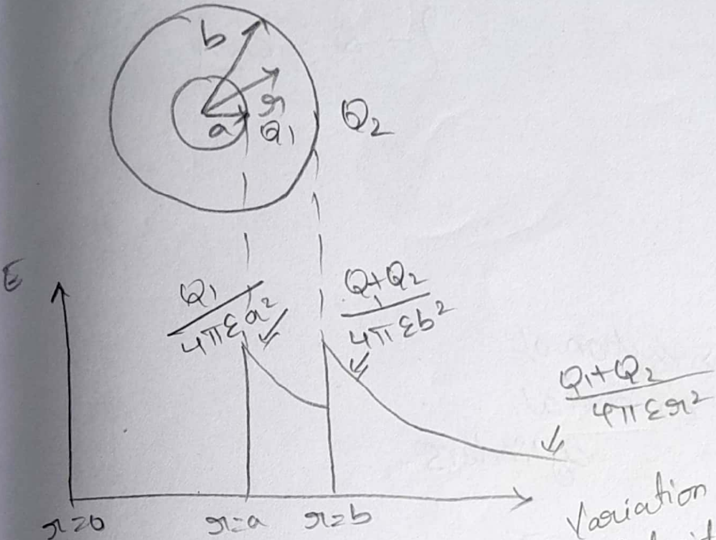
$$E = \frac{Q_1 + Q_2}{4\pi\varepsilon r^2} \quad a > r > b$$

At the inner shell $r=a$, the electric field intensity

$$\boxed{E = \frac{Q_1}{4\pi\varepsilon a^2}}$$

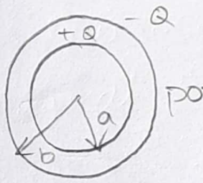
At the outer shell $r=b$, the electric field intensity

$$\boxed{E = \frac{Q_1+Q_2}{4\pi\varepsilon b^2}}$$



Variation of field intensity of concentric shell

potential between two concentric shells



potential b/w two shells

consider two concentric spherical shells of inner radius 'a' and outer radius 'b'.

let Q be the charge distibuted over the inner shell and -'Q' be the charge distibuted over the outer shell as shown in the figure.

→ The potential differences b/w the two shells is given by

$$V = -\int_b^a E\,dr$$

$$= -\int_b^a \frac{Q}{4\pi\varepsilon r^2}\,dr \qquad \left[\because E = Q/4\pi\varepsilon r^2\right]$$

$$= -\frac{Q}{4\pi\varepsilon}\int_b^a \frac{1}{r^2}\,dr$$

$$r^{-2} = \frac{r^{-2+1}}{-2+1} = \frac{r^{-1}}{-1} = \frac{-1}{r}$$

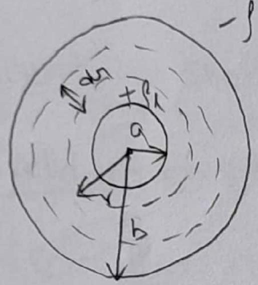
$$= \frac{-Q}{4\pi\varepsilon}\left[\frac{-1}{r}\right]_b^a$$

$$= \frac{Q}{4\pi\varepsilon}\left[\frac{1}{r}\right]_b^a = Q/4\pi\varepsilon\left[1/a - 1/b\right]$$

$$\left\{\begin{matrix} +Q = Q_1 \\ -Q = Q_2 \end{matrix}\right\}$$

$$V = \frac{1}{4\pi\varepsilon}\left[\frac{Q_1}{a} - \frac{Q_2}{b}\right]$$

Co-Axial cylinders :-



Cross section of co-axial cylinders.

By applying Gauss's law

$$\oint D\cdot ds = \rho_l l \quad [\because D = \varepsilon E]$$

$$\varepsilon\oint E\cdot ds = \rho_l l \qquad [\because \int ds = 2\pi r l]$$

$$E\, 2\pi r \cancel{l} = \frac{\rho_l \cdot \cancel{l}}{\varepsilon}$$

$$\boxed{E = \frac{\rho_l}{2\pi\varepsilon r}} \Rightarrow \boxed{E_r = \frac{\rho_l}{2\pi\varepsilon}}$$

The potential difference between two cylinders

$$V = -\int_b^a E\,dr$$

$$= -\int_b^a \frac{\rho_l}{2\pi\varepsilon r}\,dr = \frac{-\rho_l}{2\pi\varepsilon}\int_b^a \frac{1}{r}\,dr$$

$$= \frac{-\rho_l}{2\pi\varepsilon}\ln r\Big]_a^b$$

$$= \frac{-\rho_l}{2\pi\varepsilon}\left[\ln a - \ln b\right]$$

$$= \frac{-\rho_l}{2\pi\varepsilon} \ln(a/b)$$

$\frac{1}{r} = \ln r$

$$\boxed{V = \frac{\rho_l}{2\pi\varepsilon} \ln(b/a)}$$

The electric field can be written as interms of potential.

$$V = E r \ln(b/a)$$

$$\boxed{E = \frac{V}{r \ln(b/a)}}$$

$$a \leq r \leq b$$

6/5/22

Unit-2

Electro static applications

electric potential :-

Consider a uniform electric field 'E' and a unit positive, say test charge 'q' in the field 'E'. If the test charge moved from one **Point** to another point in the electric field 'E', A force acts on the test charge due to the electric field the force is given by

$$E = \frac{F}{q}$$

$$F = qE$$

Since there is a moment of charge in the electric field from one point 'r_1' to another point 'r_2'. against the direction E, there will be a work done again the force.

$$W = -\int_{r_1}^{r_2} qE \cdot dr$$

potential difference V is defined as the work done in moving a unit positive charge from one point to another ~~point~~ in an electric field. work done on unit positive charge. Per charge.

$$V = \frac{W}{q}$$

$$V = -\int_{r_1}^{r_2} E \cdot dr \text{ Joules/coulomb}$$

But,

$$\boxed{E = \frac{Q}{4\pi \varepsilon r^2}}$$

$$r^{-2} = \frac{r^{-2+1}}{-2+1}$$

$$= \frac{r^{-1}}{-1} = \frac{-1}{r}$$

$$V = -\frac{Q}{4\pi\varepsilon}\int_{r_1}^{r_2} \frac{1}{r^2}\, dr$$

$$V = -\frac{Q}{4\pi\varepsilon}\left[\frac{-1}{r}\right]_{r_1}^{r_2}$$

$$V = \frac{Q}{4\pi\varepsilon}\left[\frac{1}{r_2} - \frac{1}{r_1}\right] \text{volts}$$

This is the potential difference b/w two points r_1 & r_2

$$V = \frac{Q}{4\pi\varepsilon r_2} - \frac{Q}{4\pi\varepsilon r_1}$$

$$\boxed{V = V_1 - V_2}$$

If the test charge moved from infinity to a given point in the electric field

$r_1 = \alpha$

$$\frac{1}{r_1} = \frac{1}{\alpha} = 0$$

then $\boxed{V = V_1}$ $\quad V_2 = 0$

Absolute potential or potential at a point is defined as the work done will moving a unit positive charge from infinity to a given point in an electric field.

$$\boxed{V = \frac{Q}{4\pi\varepsilon r} \text{ volts}}$$

<u>Conservative</u> <u>Field</u> :-

The potential difference $V = -\int_{r_1}^{r_2} E \cdot dr$ is not independent of the path.

→ In a conservative field the work done moving from one point to another is independent of path. So $\oint E \cdot dr = 0$

This shows no work ~~done~~ is done in carrying unit charge around any closed path.

<u>Note</u> :- Any field where the closed line integral of the field is zero is ~~said~~ set to be a conservative field.

12/5/22 Imp v.v. ***

Electric potential due to electric dipole

An electric dipole (or) simple dipole is that two equal and opposite charges (+q & -q) separated by a very small distance.

⇒ The product of charge and spacing is called electric dipole moment.

⇒ Let Q & -Q be the two charges separated by a small distance 'd'.

$$\boxed{m = Qd} \quad \text{—— (A)}$$

⇒ Let p at any point at distance of r_1, r_2 & r from +Q, -Q & midpoint of dipole as respectively shown in fig.

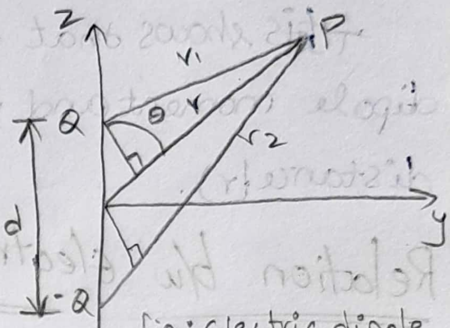


fig: Electric dipole

⇒ potential at p due to +Q is $v_1 = \frac{Q}{4\pi\varepsilon r_1}$ —— (1)

⇒ potential at p due to -Q is $v_2 = \frac{-Q}{4\pi\varepsilon r_2}$ —— (2)

The resultant potential at P, is $v = v_1 + v_2$

$$v = \frac{Q}{4\pi\varepsilon}\left[\frac{1}{r_1} - \frac{1}{r_2}\right] \text{ —— (3)}$$

If the point P is to faraway from the dipole the distances r_1 & r_2 are written as

$$r_1 = r - \frac{d}{2}\cos\theta \text{ —— (4)}$$

$$r_2 = r + d/2 \cos\theta \text{ —— (5)}$$

The potential at p due to the dipole

$$v = \frac{Q}{4\pi\varepsilon}\left[\frac{1}{r - d/2\cos\theta} - \frac{1}{r + d/2\cos\theta}\right] \text{ —— (6)}$$

$$= \frac{Q}{4\pi\varepsilon}\left(\frac{d\cos\theta}{(r - d/2\cos\theta)(r + d/2\cos\theta)}\right)$$

$$= \frac{Q}{4\pi\varepsilon}\left[\frac{d\cos\theta}{r^2 - (\frac{d}{2}\cos\theta)^2}\right] \quad \left[\because (\tfrac{d}{2}) << r^2\right]$$

$$= \frac{Q}{4\pi\varepsilon}\left[\frac{d\cos\theta}{r^2}\right]$$

$$\boxed{V = \frac{m\cos\theta}{4\pi\varepsilon r^2}} \quad [\because m = Qd]$$

This shows that the potential is directly proportional to the dipole moment and inversly proportional to the square of the distance (r).

Relation b/w electric potential & electric field intensity:

If two points are separated by an infinite distance 'dr' the work done by an external force in moving a unit positive charge from one point to another:

$$dw = dv = -\varepsilon \cdot dr \quad \text{——} \quad (1)$$

Since the scalar potential 'v' is a function of x, y, z the above equation written as

$$\frac{\partial v}{\partial x}dx + \frac{\partial v}{\partial y}dy + \frac{\partial v}{\partial z}dz = -\varepsilon\, dr$$

$$\left[\bar{a}_x \frac{\partial v}{\partial x} + \bar{a}_y \frac{\partial v}{\partial y} + \bar{a}_z \frac{\partial v}{\partial z}\right] \cdot \left[\bar{a}_x dx + \bar{a}_y dy + \bar{a}_z dz\right] = -\varepsilon \cdot dr$$

$$\nabla V \cdot dr = -\varepsilon \cdot dr$$

electric field $\boxed{\varepsilon = -\nabla V}$ electric potential

The -ve of the potential gradiant of that point

Thus, the electric field strength at any point p just the -ve sign shows that the direction of ε is opposite to the direction in which potential increases.

16/5/22

Current and current density:-

Conduction current

$I \propto V$

$I = V/R$

$R \propto l$

$R \propto \frac{1}{A}$

$R \propto l/A$

$R = \frac{\rho l}{A}$

$\rho \rightarrow$ Resistivity of the material of the conductor.

$R = \frac{l}{\sigma A}$

$\sigma = \frac{1}{\rho}$ = Conductivity of the material

If electric field ϵ exists in the conductor the potential V is

$$\boxed{V = \epsilon l}$$

sub the value of V,&R in current equation

$$I = \frac{V}{R} = \frac{\epsilon \cancel{l}}{\cancel{l}/\sigma A} = \epsilon \sigma A$$

$$\boxed{I = \epsilon \sigma A}$$ Amperes.

This is conduction current

conductor current density $\boxed{J = \frac{I}{A} = \sigma \epsilon}$ Amphere

$J \propto \epsilon$

$$\boxed{J = \sigma \epsilon}$$

This equation known as the point form of ohm's law.
It states that the electric field strength (ϵ) with in a conductor is proportional to current density 'J'. $\boxed{J \propto \epsilon}$

Displacement Current :-

The displacement current I_D is following through a capacitor 'C' when ac voltage V is applied across the capacitor

Electric current is a rate of change of electric charge

$$I_D = \frac{dQ}{dt}$$

But $Q = CV$

$$\boxed{\therefore I_D = \frac{C\,dV}{dt}}$$

For parallel plate capacitor, the capacitance is given by

$$\boxed{C = \frac{\varepsilon A}{d}}$$

where, ε is permittivity of the medium

A is the area of the plate

d is the distance between the two plates.

sub, the value of C in current equation

$$I_D = C\frac{dV}{dt}$$

$$= \frac{\varepsilon A}{d}\frac{dV}{dt}$$

But, $\boxed{V = Ed}$

$$\therefore I_D = \frac{\varepsilon A}{\cancel{d}}\frac{\cancel{d}\,dE}{dt} \quad [\because D = \varepsilon E]$$

$$\boxed{I_D = \frac{A\,dD}{dt}} \Rightarrow \frac{I_D}{A} = \frac{dD}{dt}$$

$$J_D = \frac{I_D}{A}$$

Displacement current density,

$$\boxed{J_D = \frac{dD}{dt}}$$

18/5/2

Continuity Equation of current

Current is, the rate of continuity equation of current.
Current is the rate of moment of electric charge passing a given reference point

$$I = \frac{dQ}{dt}$$

The small increment of current dI crossing an increment surface ds is given by

$$dI = J \cdot ds$$

$J \to$ current density (A/m^2)

The total current crossing the whole surface S is given by

$$I = \int_S J \cdot ds$$

Consider a region bounded by a closed surface the current through the closed surface is

$$I = \oint_S J \cdot ds$$

And, this outward flow of +ve charge must be balanced by a degrees of +ve charge within the closed surface.

→ If the charge inside the closed surface 'Q', then the rate of decrease is $-\frac{dQ}{dt}$ and the principle of conservation of charges requires

$$\therefore \oint J \cdot ds = \frac{-dQ}{dt}$$

This is the integral form of continuity equation of current

→ The differential or point form is obtained by changing the surface integral to a volume integral by using divergence theorem. (Now convert surface integral to volume integral.)

$$\oint_S J \cdot ds = \int (\nabla \cdot J) dv$$

$$\int_V (\nabla \cdot J)\, dv = \frac{-dQ}{dt}$$

$$[\because Q = \int \rho_v \, dv]$$

$$= \frac{-d}{dt} \int_V \rho_v \, dv$$

$$\int_V (\nabla \cdot J)\, dv = -\int_V \frac{d\rho_v}{dt} \cdot \cancel{dv}$$

$$\boxed{\nabla \cdot J = \frac{d\rho_v}{dt}}$$

this is the differential (or) point form continuity equation.

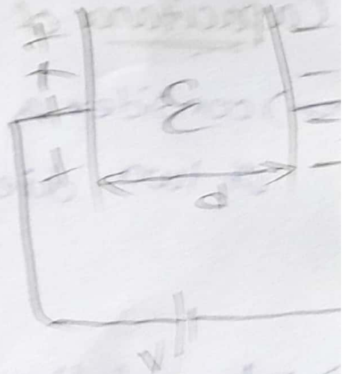
Capacitance

Concept of C:-

A capacitor is an electric device which consist of two conductors seperated by a dielectric medium.

=> consider a capacitor composed of two conducting plates of Area "A" separated by a dielectric medium whose permitivity of $(\varepsilon_0)\varepsilon$.

=> the space separation b/w



=> If the potential "V" is applied across the plates the +ve charge Q is deposited on one plate and the -ve charge -Q is deposite on other plate. the net charge is 0.

=> If the capacitance of two conducting plates is defined as the ratio of magnitude of charge on either of the conductor to the potential difference b/w conductors. It is given by

$$C = \frac{Q}{V}$$

-> The unit of capacitance is coulomb/volt (or) Farad

=> Assume that their is a uniform charge density "σ" over the plates and dielectric medium.

$$\boxed{D = \frac{Q}{A} \; C/m^2}$$

It is also written in terms of Electric field "E" as

$$\boxed{D = \varepsilon E}$$

$$Q/A = \varepsilon \epsilon$$

$$\therefore \boxed{Q = A \varepsilon \epsilon}$$

But electric field is given by $\boxed{\epsilon = \frac{v}{d}}$ v/m

Sub the value of ϵ in above equation

$$Q = A\varepsilon\, V/d$$

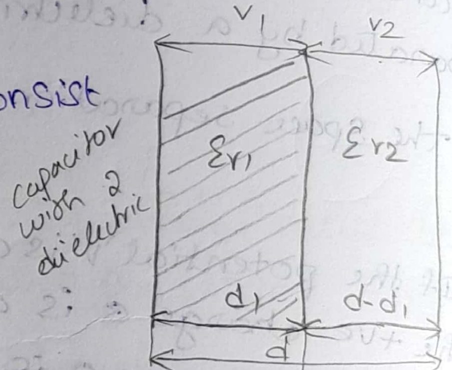
$$\boxed{C = Q/V = A\varepsilon/d}$$

where, the general permittivity is $\boxed{\varepsilon = \varepsilon_0 \varepsilon_r}$

$$\therefore \boxed{C = \frac{A\varepsilon_0\varepsilon_r}{d}} \text{ farad}$$

Capacitance of two dielectric media

⇒ consider a parallel plate capacitor consist of two dielectric as shown in fig.



⇒ The relative permitivity of dielectric medium 1 & 2 are $\varepsilon_{r1}, \varepsilon_{r2}$ respectively.

⇒ If the potential across the capacitance is V ($C \to V$)

⇒ The potential difference across medium 1 & 2 are V_1 & V_2 respectively

$$V = V_1 + V_2$$

Let $\varepsilon_1, \varepsilon_2$ be the field intensities in the medium 1 & 2 respectively

$$V_1 = \epsilon_1 d_1$$

$$V_2 = \epsilon_2 (d - d_1)$$

$$\therefore V_1 = V_1 + V_2 = \epsilon_1 d_1 + \epsilon_2 (d - d_1)$$

The electric flux density $\boxed{D = Q/A}$ will be same in both the medium.

The electric field intensities for both the medium are given by

$$E_1 = \frac{D}{\varepsilon_{r1}\varepsilon_0} \qquad D = \varepsilon E \qquad \boxed{D = \varepsilon_r \varepsilon_0 E}$$

$$= \frac{Q}{A\varepsilon_{r1}\varepsilon_0}$$

$$E_2 = \frac{D}{\varepsilon_{r2}\varepsilon_0} = \frac{Q}{A\varepsilon_{r2}\varepsilon_0}$$

The applied potential $V = E_1 d_1 + E_2 (d - d_1)$

$$V = \frac{Q}{A\varepsilon_{r1}\varepsilon_0} d_1 + \frac{Q}{A\varepsilon_{r2}\varepsilon_0}(d - d_1)$$

$$V = \frac{Q}{A\varepsilon_0}\left[\frac{d_1}{\varepsilon_{r1}} + \frac{d - d_1}{\varepsilon_{r2}}\right]$$

$$Q/V = \frac{A\varepsilon_0}{\frac{d_1}{\varepsilon_{r1}} + \frac{d-d_1}{\varepsilon_{r2}}}$$

$$C = Q/V = \frac{A\varepsilon_0 \varepsilon_{r1}\varepsilon_{r2}}{d_1\varepsilon_{r2} + (d - d_1)\varepsilon_{r1}}$$

If medium is air $\boxed{\varepsilon_{r1} = 1}$

medium 2 $\boxed{\varepsilon_{r2} = \varepsilon_r}$

$$\boxed{C = \frac{A\varepsilon_0\varepsilon_r}{d_1\varepsilon_r + (d - d_1)}}$$

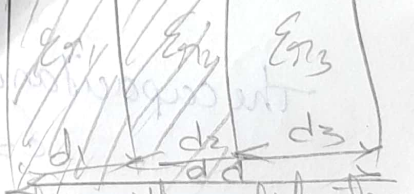
19/5/22 Capacitance of 3-dielectric media

Then,

$V = E_1 d_1 + E_2 d_2 + E_3 d_3$ flux density $D = \frac{Q}{A}$ $\boxed{D = \varepsilon E}$

Electric field intensities

$$E_1 = \frac{D}{\varepsilon} = \frac{D}{\varepsilon_0\varepsilon_{r1}} = \frac{Q}{A\varepsilon_0\varepsilon_{r1}}$$



capacitor with 3 dielectric

$$\epsilon_2 = \frac{Q}{A\epsilon_0\epsilon_{r2}}$$

$$\epsilon_3 = \frac{Q}{A\epsilon_0\epsilon_{r3}}$$

$$V = \frac{Q}{A\epsilon_0\epsilon_{r1}}d_1 + \frac{Q}{A\epsilon_0\epsilon_{r2}}d_2 + \frac{Q}{A\epsilon_0\epsilon_{r3}}d_3$$

$$V = \frac{Q}{A\epsilon_0}\left[\frac{d_1}{\epsilon_{r1}} + \frac{d_2}{\epsilon_{r2}} + \frac{d_3}{\epsilon_{r3}}\right]$$

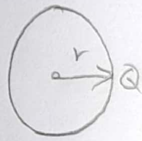
$$\boxed{C = \frac{Q}{V} = \frac{A\epsilon_0}{d_1/\epsilon_{r1} + d_2/\epsilon_{r2} + d_3/\epsilon_{r3}}}$$

Capacitance of an isolated sphere :-

consider a sphere of radius 'r' having a charge Q columbs as shown in fig.

→ the potential is the workdone per unit charge in carrying a positive test charge from infinity to the sphere.

→ The absolute potential is given by



$$V = -\int_{\infty}^{r} E \cdot dr = -\int_{\infty}^{r} \frac{Q}{4\pi\epsilon r^2} dr$$

But

$$E = \frac{Q}{4\pi\epsilon r^2}$$

$$= -\frac{Q}{4\pi\epsilon}\int_{\infty}^{r}\frac{dr}{r^2}$$

$$= \frac{Q}{4\pi\epsilon}\left[\frac{r^{-2+1}}{-2+1}\right]$$

$$= -\frac{Q}{4\pi\epsilon}\left[\frac{r}{-1}\right]$$

$$= \frac{Q}{4\pi\epsilon r}$$

$$\boxed{V = \frac{Q}{4\pi\epsilon r}}$$

$V = \frac{Q}{4\pi\epsilon r}$

$4\pi\epsilon r = \frac{Q}{V}$

The capacitance of the isolated sphere is

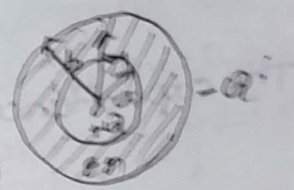
$$C = Q/V = 4\pi\epsilon r \text{ farad}$$

$$C = 4\pi\epsilon_0\epsilon_r \text{ farad}$$

$\boxed{C = 4\pi\epsilon_0 r}$ farad

Capacitance of concentric spheres:-

consider a two concentric spheres inner radius 'a' & outer sphere radius 'b'.



ε_r - permitivity of dielectric medium b/w inner & outer sphere

$$E = \frac{Q}{4\pi\varepsilon r^2} \quad (a \le r \le b)$$

$$V = -\int_b^a E \cdot dr$$

$$= -\int_b^a \frac{Q}{4\pi\varepsilon r^2} dr$$

$$V = -\frac{Q}{4\pi\varepsilon}\int_b^a \frac{dr}{r^2}$$

$$= Q/4\pi\varepsilon \left.\frac{1}{r}\right]_b^a$$

$$= \frac{Q}{4\pi\varepsilon}\left[1/a - 1/b\right]$$

$$V = \frac{Q}{4\pi\varepsilon}\left[\frac{b-a}{ab}\right]$$

The c of 2 concentric spheres

$$C = \frac{Q}{V} = 4\pi\varepsilon\left[\frac{ab}{b-a}\right]$$

$$\boxed{C = 4\pi\varepsilon\left[\frac{ab}{b-a}\right]}$$

Capacitance of co-axial cable cylinders

charged at the ρ_l c/m rate of

$$E = \frac{\rho_l}{2\pi\varepsilon r}$$

$$V = -\int_b^a E\,dr = \frac{-\rho_l}{2\pi\varepsilon}\int_b^a \frac{dr}{r} = \frac{-\rho_l}{2\pi\varepsilon}\left.\ln r\right]_b^a = \frac{-\rho_l}{2\pi\varepsilon}(\ln a - \ln b)$$

$$= \frac{-\rho_l}{2\pi\varepsilon} - (\ln b - \ln a) = \frac{\rho_l}{2\pi\varepsilon}\ln(b/a)$$

$$V = \frac{\rho_l}{2\pi\varepsilon}\ln(b/a)$$

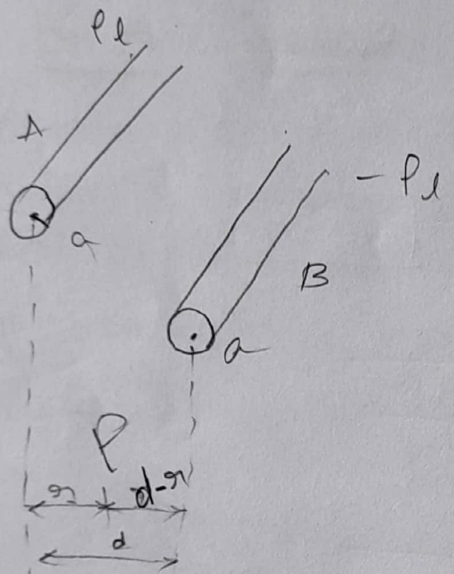
The capacitance of co-axial cable | unit length

$$C = \frac{\rho_l}{V}$$

$$C = \frac{2\pi\varepsilon}{\ln(b/a)} \text{ F/m} \qquad [\because \varepsilon = \varepsilon_0\varepsilon_r]$$

$$\boxed{C = \frac{2\pi\varepsilon_0\varepsilon_r}{\ln(b/a)} \text{ F/m}}$$

Capacitance of parallel conductors (Transmission lines)



$$E = \frac{\rho_l}{2\pi\varepsilon r} + \frac{\rho_l}{2\pi\varepsilon(d-r)}$$

$$E = \frac{\rho_l}{2\pi\varepsilon}\left(\frac{1}{r} + \frac{1}{d-r}\right)$$

$$V = -\int E \cdot dr$$

$$V = \frac{-\rho_l}{2\pi\varepsilon}\int_{d-a}^{a}\left(\frac{1}{r} + \frac{1}{d-r}\right)dr$$

$$= \frac{-\rho_l}{2\pi\varepsilon}\left[\ln\left(\frac{a}{d-a}\right) + \ln\frac{a}{d-a}\right]$$

$$= \frac{-\rho_l}{\cancel{2}\pi\varepsilon}\left(\cancel{2}\ln\frac{a}{d-a}\right)$$

$$V = \frac{\rho_l}{\pi\varepsilon}\ln\left(\frac{d-a}{a}\right)$$

$$C = \frac{\rho_l}{V} = \frac{\pi\varepsilon}{\ln\left(\frac{d-a}{a}\right)} \text{ F/m}$$

$$\varepsilon = \varepsilon_0\varepsilon_r$$

20/5/22 Electrostatic energy & energy Density

A capacitor stores a electro static energy equal to workdone to build up the charge. If a voltage Source is connected across the capacitor. The capacitor charges potential is defined as the workdone per unit charge.

$$\boxed{V = \frac{dW}{dQ}}$$

The workdone $\boxed{dW = V \cdot dQ}$

But, $\boxed{V = \frac{Q}{C}}$

A capacitor is charged to the value of Q.

The total workdone is $dW = \frac{Q}{C} \cdot dQ$

$$W = \int_0^Q \frac{Q}{C} dQ$$

$$= \frac{1}{C} \left[\frac{Q^2}{2}\right]_0^Q$$

$$= \frac{1}{C}\left[\frac{Q^2}{2} - 0\right]$$

$$\boxed{W = \frac{Q^2}{2C} \text{ Joules}}$$

But, $\because Q = CV$

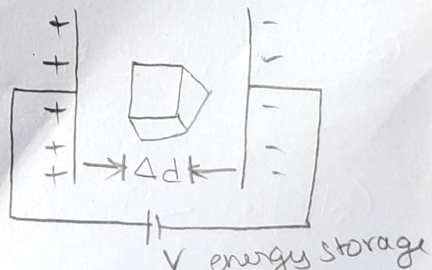
$$W = \frac{(CV)^2}{2C} = \frac{C^{\cancel{2}}V^2}{2\cancel{C}}$$

$$\boxed{W = \frac{1}{2} CV^2 \text{ Joules}}$$

$$\boxed{W = \frac{1}{2} CVV = \frac{1}{2} QV} \text{ Joules}$$

Energy Density :-

Consider a elementary cube of side Δd parallel to the plates of the capacitor as shown in figure.



V energy storage in capacitor

The capacitance of elemently capacitor is

$$\boxed{C = \frac{\varepsilon A}{d}}$$

$$\Delta C = \frac{\varepsilon A}{\Delta d}$$

$$= \frac{\varepsilon (\Delta d)^{\cancel{2}}}{\cancel{\Delta d}}$$

$$\boxed{\Delta C = \varepsilon \Delta d}$$

Energy stored in the elemental capacitor is

$$\Delta W = \frac{1}{2} \Delta C (\Delta V)^2$$

$$W = \frac{1}{2} C V^2$$

But potential drop across the elementary cube is

$$\boxed{\Delta V = \epsilon \cdot \Delta d}$$

ϵ is electric field exist in the cube.

Energy stored equation

$$\Delta W = \frac{1}{2} \Delta C (\Delta V)^2 = \frac{1}{2} (\varepsilon \cdot \Delta d)(\epsilon \cdot \Delta d)^2$$

$$= \frac{1}{2} \varepsilon \epsilon^2 \Delta d^3 = \frac{1}{2} \varepsilon \epsilon^2 \Delta V$$

$\boxed{\Delta V = (\Delta d)^3}$ is elementary volume

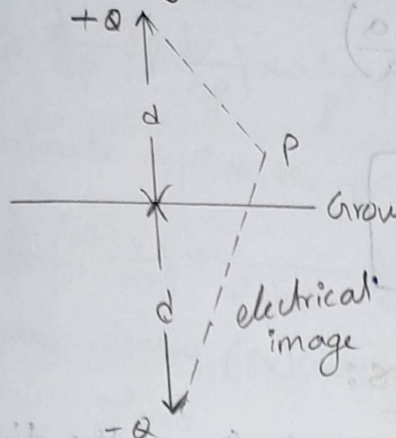
Energy density is given by

$$\boxed{\frac{\Delta W}{\Delta V} = \frac{1}{2} \varepsilon \epsilon^2}$$

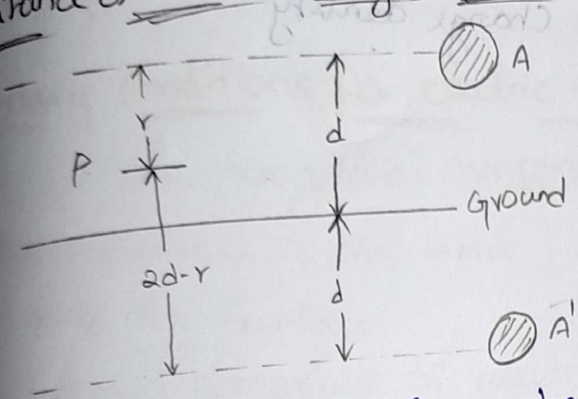
$$\frac{\Delta W}{\Delta V} = \frac{1}{2} \varepsilon \epsilon \epsilon \qquad [\because D = \varepsilon \epsilon]$$

$$= \frac{1}{2} D \epsilon \text{ Joules}/m^3$$

Method of images :-



Capacitance of infinite single wire transmission line :-



If the transmission line A has a charge of ρ_l C/m along its length this will induced charge of $-\rho_l$ c/m on the image line A'

The electric field intensity at point 'p' due to transmission line A and image line A'

$$E = \frac{\rho_l}{2\pi\varepsilon r} + \frac{\rho_l}{2\pi\varepsilon(2d-r)}$$

$$= \frac{\rho_l}{2\pi\varepsilon}\left(\frac{1}{r} + \frac{1}{2d-r}\right)$$

The potential difference between the transmission lines is given by

$$V = -\int E\,dr = -\frac{\rho_l}{2\pi\varepsilon}\int_{2d-a}^{a}\left(\frac{1}{r} + \frac{1}{2d-r}\right)dr$$

$$V = \frac{-\rho_l}{2\pi\varepsilon}\left[\ln\frac{a}{2d-a} + \ln\frac{a}{2d-a}\right]$$

$$= \frac{-\rho_l}{\cancel{2}\pi\varepsilon}\,\cancel{2}\left(\ln\frac{a}{2d-a}\right)$$

$$V = \frac{\rho_l}{\pi\varepsilon}\ln\left(\frac{2d-a}{a}\right)$$

$$C = \frac{\rho_l}{V/2} = \frac{\rho_l}{\frac{\rho_l}{2\pi\varepsilon}\ln\left(\frac{2d-a}{a}\right)}$$

$$\boxed{C = \frac{2\pi\varepsilon}{\ln\left(\frac{2d-a}{a}\right)} \ f/m}$$

***
v.v.imp

Poisson's and Laplace's Equations:-

According to gauss's law in point form, the divergens of electric flux density is equal to the volume charge density

Poisson's Eqn:- $\nabla \cdot D = \rho_v$

But,

$$D = \varepsilon E$$

$$\nabla \cdot (\varepsilon E) = \rho_v$$

$$\varepsilon \nabla \cdot E = \rho_v$$

$$\boxed{\nabla \cdot E = \frac{\rho_v}{\varepsilon}}$$

But, $E = -\nabla V$

$$\nabla \cdot \nabla V = \frac{\rho_v}{\varepsilon}$$

$$\nabla \cdot \nabla V = -\frac{\rho_v}{\varepsilon}$$

$$\boxed{\nabla^2 V = \frac{-\rho_v}{\varepsilon}}$$

This is the Poisson's Equation

Laplace Eqn :-

For cartesian co-ordinate system

$$\nabla \cdot \nabla V = \frac{\partial}{\partial x}\left(\frac{\partial V}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\partial V}{\partial y}\right) + \frac{\partial}{\partial z}\left(\frac{\partial V}{\partial z}\right)$$

$$\nabla \cdot \nabla V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

Poisson's equation for cartesian co-ordinate system is written as

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = \frac{-\rho_v}{\varepsilon}$$

For cylindrical co-ordinate system, the poisson's equation

$$\nabla^2 v = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{dv}{d\rho}\right) + \frac{1}{\rho^2}\left(\frac{\partial^2 v}{\partial \phi^2}\right) + \frac{\partial^2 v}{\partial z^2} = \frac{-\rho_v}{\varepsilon}$$

For spherical co-ordinate system, the poisson's equation is

$$\nabla^2 v = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial v}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\frac{\sin\theta\, v}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 v}{\partial \phi^2} = \frac{-\rho_v}{\varepsilon}$$

If the volume charge density (ρ_v) is zero, then

$$\nabla^2 v = 0$$

This is laplace's equation. The operator ∇^2 is called as Laplacian operator

/5/22
vv amp

Boundary Conditions for electric field :-

The tangential component of electric field 'E' is continuous at the surface. Is the same just outside the surface as it is just inside the surface.

⇒ The normal component of electric flux density is continuous if there is no surface charge density. Otherwise 'D' is discontinuous by an amount equal to the surface charge density.

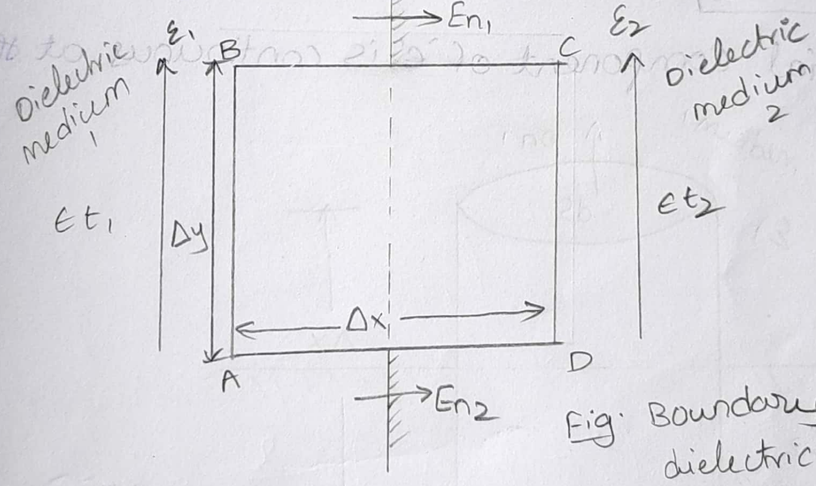


Fig: Boundary Surface b/w two dielectric media.

consider a interface or boundary b/w two dielectrics of dielectric constant $\varepsilon_1, \varepsilon_2$ in an electric field.

Consider a interface or boundary b/w two dielectric media

⇒ fig. Shows boundary surface b/w two dielectric media

⇒ consider a rectangle by length Δy & width Δx at the boundary of two dielectric media as shown in fig.

⇒ In an electrostatic field the voltage around any closed path must be zero.

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$V = \oint E \cdot dl = 0$

⇒ Apply this to the rectangular path ABCD in which AB is just inside medium 1 & CD just inside medium 2 as shown figure.

$$\oint E \cdot dl = E_{t_1} \Delta y + E_{n_1} \Delta x - E_{t_2} \Delta y - E_{n_2} \Delta x$$

where,

E_{t_1} & E_{t_2} are the average tangential components of $\vec{E}$ along box AB & CD

E_{n_1} & E_{n_2} are the average normal components of $\vec{E}$ along the box DC & AD

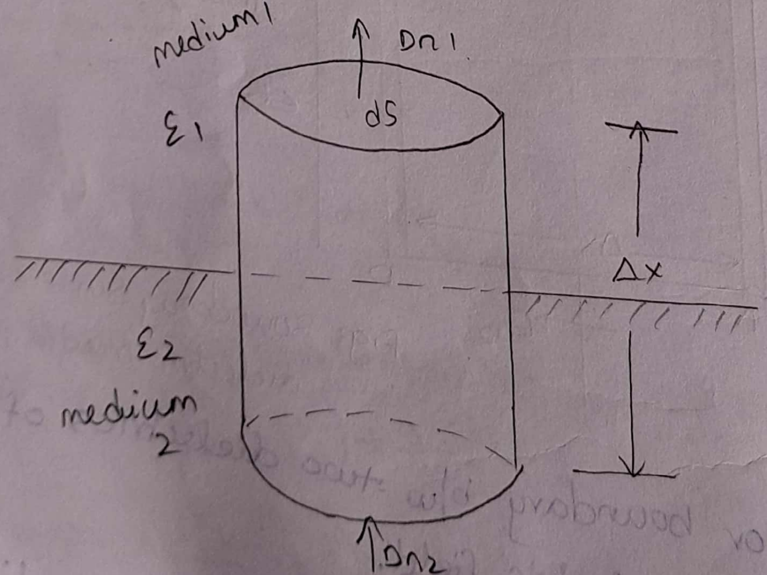
⇒ As the sides AB & CD are brought closer together the length BC & AD approach zero.

Then, $\Delta x \to 0$

$$E_{t_1} \Delta y - E_{t_2} \Delta y = \oint E \cdot dl = 0$$

$$\boxed{E_{t_1} = E_{t_2}}$$

the tangential component of $\vec{E}$ is continuous at the boundary



2/6/22 problems ① & ② chapters:-

1) find the electric-field intensity $\in$ at (111) if the potential is

$$V = [xyz^2 + x^2yz + xy^2z$$

Sol:- $V = xyz^2 + x^2yz + xy^2z$

$$\in = -\nabla V$$

$$= -\left[\frac{\partial}{\partial x}\vec{a_x} + \frac{\partial}{\partial y}\vec{a_y} + \frac{\partial}{\partial z}\vec{a_z}\right][xyz^2 + x^2yz + xy^2z]$$

$$= -\left[(yz^2 + 2xyz + y^2z)\vec{a_x} + (xz^2 + x^2z + 2yxz)\vec{a_y} + (2zxy + x^2y + xy^2)\vec{a_z}\right]$$

$\in$ at $(1,1,1) = -\left[4\vec{a_x} + 4\vec{a_y} + 4\vec{a_z}\right]$ V/m

2018 (10M)

1) Derive an expression for potential due to infinite line charge? 5M

2) find the electric field at a point $(1,-2,1)$m. if the given potential is

$$V = 3x^2y + 2yz^2 + 2xyz$$

Sol:- $V = 3x^2y + 2yz^2 + 2xyz$

$$\in = -\nabla V$$

$$= -\left[\frac{\partial}{\partial x}\vec{a_x} + \frac{\partial}{\partial y}\vec{a_y} + \frac{\partial}{\partial z}\vec{a_z}\right][3x^2y + 2yz^2 + 2xyz]$$

$$= \left[(3 2xy + 2yz^2 + 2yz)\vec{a_x} + (3x^2 + 2z^2 + 2xz)\vec{a_y} + (3x^2y + 2y2z + 2xy)\vec{a_z}\right]$$

$\in$ at $(1,-2,1) = \left[(-12-4-4)\vec{a_x} + (3+2+2)\vec{a_y} + (-6-8-4)\vec{a_z}\right]$

$\in$ at $(1,-2,1) = -\left[-20\vec{a_x} + 7\vec{a_y} - 18\vec{a_z}\right]$ V/m

3/6/22

2) find the energy stored in the 20pF 11^{el} plate capacitor with separation of 2cm the magnitude of electric field in the capacitor is $E = 1000$ V/m

Sol:- Given data

$C = 20$pF

$= 20 \times 10^{-12}$ PF

$d = 2$cm

$= 2 \times 10^{-2}$m

$\in = 1000$ V/m

Energy stored in a capacitor $W = \frac{1}{2} C V^2$

$$= \frac{1}{2} \times 20 \times 10^{-12} \times 20 \times 20$$

$$= 4000 \times 10^{-12}$$

$$V = Ed = 1000\,V/m \times 2 \times 10^{-2}\,m$$

$$= 4 \times 10^3 \times 10^{-12}$$

$$= \frac{2000}{100} = 20\,V$$

$$\boxed{W = 4 \times 10^{-9}\,J}$$

3) Determine the gradient of the scalar field $f = 5r^2 + r\sin\theta$

Sol:- Gradient of scalar field. ∇f

The given scalar field is in spherical co-ordinates

$$\nabla V = \frac{\partial V}{\partial r}\vec{a_r} + \frac{1}{r}\frac{\partial V}{\partial \theta}\vec{a_\theta} + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\vec{a_\phi}$$

$$\nabla f = \frac{\partial}{\partial r}(5r^2 + r\sin\theta)\vec{a_r} + \frac{1}{r}\frac{\partial}{\partial \theta}(5r^2 + r\sin\theta)\vec{a_\theta} + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}(5r^2 + r\sin\theta)\vec{a_\phi}$$

$$\nabla f = (10r + \sin\theta)\vec{a_r} + \frac{1}{r} r\cos\theta\,\vec{a_\theta} + 0$$

$$\boxed{\nabla f = (10r + \sin\theta)\vec{a_r} + \cos\theta\,\vec{a_\theta}}$$

4)

Sol:- Given data

$A = 25\,cm^2$

$= 25 \times 10^{-4}\,m^2$

Thickness $\cdot d = 0.5\,cm$

$= 0.5 \times 10^{-2}\,m$

$\varepsilon_r = 6$

$C = ?$ $\varepsilon_0 = 8.854 \times 10^{-12}$

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$\varepsilon = \varepsilon_0 \varepsilon_r$

$C = \frac{\varepsilon A}{d}$ $\Rightarrow$ $C = 2656 \cdot 2 \times 10^{-14}$

$= \frac{\varepsilon_0 \varepsilon_r A}{d}$ $= 26.562 \times 10^{-12}$ F

$\boxed{C = 26.562 \text{ pF}}$

$= \frac{8.854 \times 10^{-12} \times 6 \times 25 \times 10^{-4}}{0.5 \times 10^{-12}}$

unit-1

Q) If two charges of 50 μC & 10 μC are located at a (-1, +1, 3) and (3, 1, 0) respectively. Determine the nature of force and it's magnitude and direction, existing b/w the two point charges

Sol:- $\{1 \mu C = 10^{-6} C\}$

Q_1	Q_2
50 μC	10 μC
(-1, 1, 3)	(3, 1, 0)

$Q_1 = 50 \times 10^{-6} C$ at $r_1 = -\vec{a_x} + \vec{a_y} + 3\bar{a}_z$

$Q_2 = 10 \times 10^{-6} C$ at $r_2 = 3\vec{a_x} + \vec{a_y}$

$r = r_2 - r_1$

$= 3\bar{a}_x + \cancel{a_y} + \bar{a}_x - \cancel{a_y} - 3\bar{a}_z$

$r = 4\bar{a}_x - 3\bar{a}_z$

$\bar{a} = \frac{r}{|r|} = \frac{4a_x - 3\bar{a}_z}{\sqrt{(4^2) + (-3)^2}}$

$F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r^2} \bar{a}$

$\bar{F} = \frac{50 \times 10^{-6} \times 10 \times 10^{-6} \times (4\bar{a}_x - 3\bar{a}_z)}{4\pi \times 8.854 \times 10^{-12} (4^2 + 3^2)^{3/2}}$

$\bar{F} = (0.144\bar{a}_x - 0.108\bar{a}_z)$ Newton.

2) A point charge of 10 μC is located at in (1, 2, 3) and other point charge of −3 μC is located at (3, 0, 2) in vaccum, if find the force b/w the solution.

$Q_1 = 10\,\mu C = 10 \times 10^{-6} C \Rightarrow \vec{r_1} = \vec{a_x} + 2\vec{a_y} + 3\vec{a_z}$

$Q_2 = -3\,\mu C = -3 \times 10^{-6} C \Rightarrow r_2 = 3\vec{a_x} + 2\vec{a_z}$

$r = r_2 - r_1$

$= 3\bar{a}_x + 2\bar{a}_z - \bar{a}_x - 2\bar{a}_y + 3a_z$

$r = 2a_x - 2\bar{a}_y + 5\bar{a}_z$

$r = 3$

$F = \frac{1}{-}(2\bar{a}_x - 2\bar{a}_y - \bar{a}_z) \times 10^{-2}$ newton

3) compute the force b/w two charges of 1 columb, each separated in by a distance equal to 1m.
The diameter of the earth (1.27×10^7 m).

Sol:- (i) $Q_1 = Q_2 = 1\,c$

$r = 1$ metre

$$\text{force } F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r^2} = \frac{1}{4\pi \times 8.854 \times 10^{-12}} = 9 \times 10^9 \text{ Newtons}$$

(ii) $r = 1.27 \times 10^7$ m

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r^2} = \frac{1}{4\pi \times 8.854 \times 10^{-12} \times (1.27 \times 10^7)^2}$$

$F = 5.575 \times 10^{-5}$

~~$F = 7.087 \times 10^{-5}$~~ Newtons.

1) A two small identical conducting spheres have charge of −1 nano coloumbs (nc) and 2 nc respectively. If they are brought in contact and then seperated by 4cm. what is force b/w them?

Sol:- $Q_1 = -1 nc$

$= -1 \times 10^{-9} c$

$Q_2 = 2 nc$

$= 2 \times 10^{-9} c$

$r = 4cm = 4 \times 10^{-2} m$

when two charges Q_1 & Q_2 are brought in contact and separated the new charge is given by

$$Q = \frac{Q_1 + Q_2}{2}$$

$$= \frac{(-1+2) \times 10^{-9}}{2}$$

$$Q = \frac{1}{2} \times 10^{-9}$$

$$\boxed{Q = 0.5 \times 10^{-9} c}$$

$$F = \frac{Q_1 Q_2}{4\pi \varepsilon_0 r^2} \qquad [\because Q_1 = Q_2 = Q]$$

$$= \frac{(0.5 \times 10^{-9})^2}{4\pi \times 8.854 \times 10^{-12} \times (4 \times 10^{-2})^2}$$

F = 5.625 micronewtons)

6/6/22

Unit - 3

Steady electro-magnetic field

Magnetic flux density:-

$B = \phi / A$ weber/metre^2 ⇒ B - magnetic flux density in wb/m^2

$B = \mu H$ ϕ - flux lines in wb

$\mu = \mu_0 \mu_r$ A - Area in m^2

where,

μ is permeability of medium

$\mu = \mu_0 \mu_r$ (H/m)

$\mu_0 = 4\pi \times 10^{-7}$ (H/m) free space permeability

μ_r is relative permeability of the medium

H is magnetic field intensity (A/m)

magnetic flux density is defined as the magnetic flux passing per unit area its unit is web/m^2 (or) tesla.

⇒ If ϕ wb's is passing through an area of a.

Magnetic flux (ϕ):-

magnetic flux is defined as the flux ϕ passing through any area.

$$\boxed{\phi = \int_S B \cdot ds \text{ weber}}$$

⇒ According to the gauss's law for electric field the total flux passing through any closed surface is equal to the charge enclosed.

$$\boxed{\psi = \oint D \cdot ds = Q}$$

The charge 'Q' is the source of the lines of electric flux, and this lines begin on +ve charge and terminate on -ve charge.

⇒ No such source has found for the lines of magnetic ~~field~~ flux. The magnetic flux lines are closed and don't terminate on magnetic charge. hence the gauss's law for magnetic field is

$$\boxed{\oint_S B \cdot ds = 0}$$

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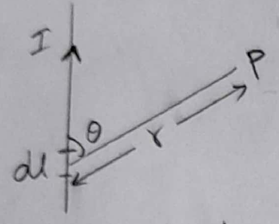
By appling divergence theorem

$$\boxed{\iint B\cdot ds = \nabla\cdot B}$$

Then, $\nabla\cdot B = 0$

The total magnetic flux passing through any closed surface is equal to zero.

BIOT - SAVART's LAW :-



current element

$$dB \propto \frac{I\,dl}{r^2}$$

$$dB = \frac{\mu I dl}{4\pi r^2}\bar{a}$$

where,
$\mu = \mu_0\mu_r$ is permeability of the medium
Idl is the current element
r is the distance the point P and current element
$\bar{a}$ is the unit vector

It's magnitude is

$$dB = \frac{\mu I dl \sin\theta}{4\pi r^2}$$

The magnetic field intensity is given by

$$B = \mu H$$
$$H = B/\mu$$

$$dH = \frac{I dl}{4\pi r^2}\bar{a}$$

Its magnitude is

$$\boxed{dH = \frac{Idl\sin\theta}{4\pi r^2}}$$

This is refered to as ampere's law for current element

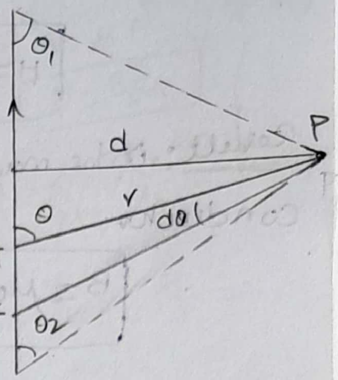
Statement :- magnetic flux density reduced by a current element at any point in a magnetic field is proportional to the current element and inversly proportional to square of the distance b/w them.

The magnetic flux density at any point 'p' due to current element 'Idl' is given by $\boxed{dB = \frac{Idl}{r^2}}$

Magnetic field intensity :-

Magnetic field intensity due to finite and infinite conductor

Consider a conductor of finite length carrying current 'I' and consider a small current element Idl in the conductor at a distance 'r' from the point p. where magnetic field is to be determined as shown in figure.



By biot-savart's law the flux density at p. due to the current element Idl is given by

$$dB = \frac{\mu_0 I dl \sin\theta}{4\pi r^2}$$

$$r d\theta = dl \sin\theta \text{ (arc)}$$

$$r = \frac{dl}{d\theta} \sin\theta$$

$$\cancel{dB = \mu_0 I dl \sin\theta} \quad dB = \frac{\mu_0 I r d\theta}{4\pi r^2} = \frac{\mu_0 I d\theta}{4\pi r}$$

$$\sin\theta = \frac{d}{r}$$

$$r = \frac{d}{\sin\theta}$$

$$dB = \frac{\mu_0 I d\theta}{4\pi d} \sin\theta$$

$$= \frac{\mu_0 I \sin\theta}{4\pi d} d\theta$$

The total flux density is

$$B = \frac{\mu_0 I}{4\pi d} \int_{\theta}^{\pi - \theta_2} \sin\theta \, d\theta$$

$$B = \frac{\mu_0 I}{4\pi d} \left[\cos\theta_1 + \cos\theta_2 \right] Wb/m^2$$

B due to finite length conductor

$$H = \frac{I}{4\pi d} \left(\cos\theta_1 + \cos\theta_2 \right) A/m$$

It is infinity $\boxed{\theta_1 = \theta_2 = 0}$

$$B = \frac{\mu_0 I}{\cancel{4}_2 \pi d} \times \cancel{2}$$

Infinite long conductor

$$\boxed{B = \frac{\mu_0 I}{2\pi d} \ \omega/m^2} \rightarrow \text{magnetic flux density}$$

$$\boxed{H = \frac{I}{2\pi d} \ A/m} \rightarrow \text{magnetic intensity.}$$

Result:- The magnetic flux density at any point due to infinitely long conductor.

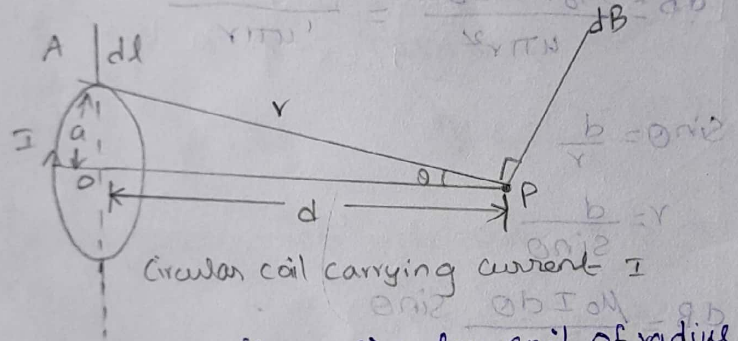
$$\boxed{B = \mu_0 I / 2\pi d \ \ wb/m^2}$$

2) The magnetic field intensity at any point due to infinitely long conductor

$$\boxed{H = \frac{I}{2\pi d} \ A/m}$$

3/6/22

Magnetic field intensity on the axis of circular coil



Circular coil carrying current I

consider a circular coil of radius 'a' carrying current 'I' and also consider a current element Idl. Let p be any point at a distance 'd' from the center of the coil as shown in fig.

The magnetic flux density at p due to the current element

$$dB = \frac{\mu_0 I dl}{4\pi r^2} \sin\theta$$

Δ AOP,

$$\sin\theta = \frac{a}{r} = \frac{a}{\sqrt{a^2+d^2}} \qquad \left\{ \begin{array}{l} \therefore r^2 = a^2+d^2 \\ r = \sqrt{a^2+d^2} \end{array} \right\}$$

$$dB = \frac{\mu_0 I dl}{4\pi (a^2+d^2)} \frac{a}{\sqrt{a^2+d^2}} \qquad \because (a^2+d^2)(a^2+d^2)^{1/2}$$

$$= \frac{\mu_0 a I dl}{4\pi (a^2+d^2)^{3/2}} \qquad = (a^2+d^2)^{1+1/2} = (a^2+d^2)^{3/2}$$

the magnetic flux density due to circular coil is given by

$$B = \frac{\mu_0 a I}{4\pi (a^2+d^2)^{3/2}} \int dl$$

$$= \frac{\mu_0 a I}{\cancel{4}_2\cancel{\pi} (a^2+d^2)^{3/2}} \cancel{2\pi} a$$

$$B = \frac{\mu_0 a^2 I}{2(a^2+d^2)^{3/2}} \text{ weber}/m^2$$

$\because B = \mu H$

$\boxed{H = B/\mu}$

$\mu = \mu_r \mu_0$

$\mu = \mu_0$

the magnetic field intensity

$$H = \frac{I a^2}{2(a^2+d^2)^{3/2}} \text{ A/m}$$

If $d=0$, the field at the centre

flux density $B = \frac{\mu_0 a^2 I}{2a^3}$

$$\boxed{B = \frac{\mu_0 I}{2a} \text{ webers}/m^2}$$

Field intensity $H = \frac{I a^2}{2(a^2+d^2)^{3/2}}$

$$= \frac{I a^2}{2a^3}$$

$$\boxed{H = I/2a \text{ A/m}}$$

Ampere's Circuital law and Applications :-

$$\oint H \cdot dl = I$$

Statement :- Ampere's circuital law states that the line integral of magnetic field intensity 'H' about any closed path is exactly equal to the direct current enclosed by that path.

* Ampere's law is special case of Biot-Savart's law.
* Ampere's law is similar to gauss's law and it is easily applied to determine the magnetic field intensity 'H' when the current distibution is symmetrical.

Applications :-

Applications of amphere's law to an infinite line current and infinite line current sheet and infinately long co-axial transmission line or discuss as follows.

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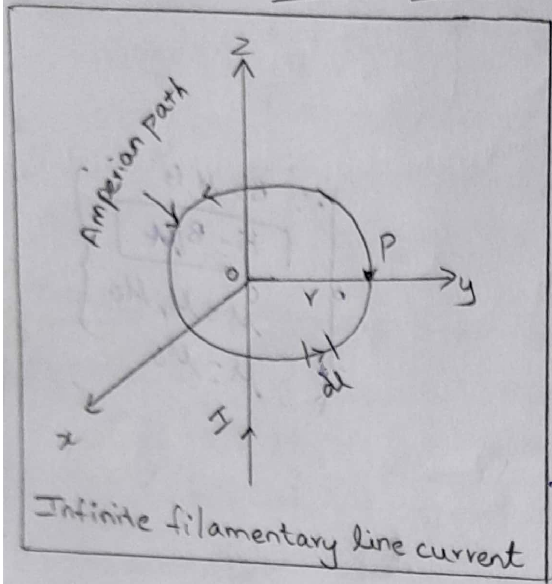
Infinite line Current :-



Infinite filamentary line current

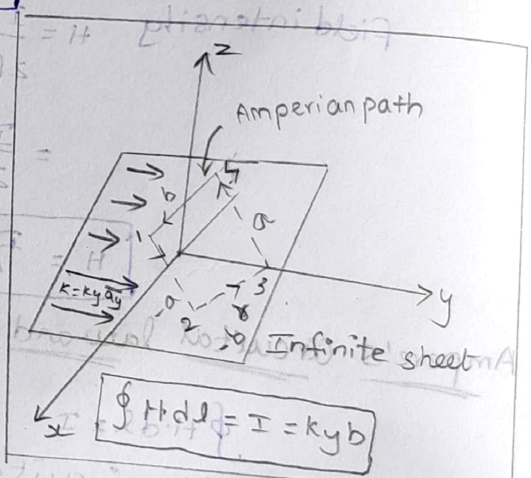
* Consider an infinitely long filimentary current I, along the z-axis as shown in figure.
* let p be any point at which H has to be determine
* consider a closed path passing through P which is known as ampere path
* since this path encloses whole current I, according to ampheres law.

$$I = \oint H \cdot dl$$

$$= H \cdot (2\pi r)$$

$$\boxed{H = \frac{I}{2\pi r} \bar{a}_r}$$

Infinite Sheet of Current



$$\boxed{\oint H \cdot dl = I = kyb}$$

* consider an infinite current sheet in the z=0 as shown in fig. let the current density of sheet is $K = ky\bar{a}_y$ A/m
* consider a rectangular (Amperian path) closed path by applying amphere's law

$$\boxed{\oint H \cdot dl = I = kyb}$$

* Due to the infinite sheet the sheet can be consider as consisting of filamentary pairs. so that the characterstics of H for a pair are the same for infinite sheet of the current.

$$H = \begin{cases} H_0 \bar{a}_x & z>0 \\ -H_0 \bar{a}_x & z<0 \end{cases}$$

* H on one side of the sheet is -ve of that on the other side. The line integral of rectangular path is given by

$$\oint H \cdot dl = \int_1^2 H \cdot dl + \int_2^3 H \cdot dl + \int_3^4 H \cdot dl + \int_4^1 H \cdot dl$$

$= 0(-a) + (-H_o)(-b) + 0\,(a) + H_o(b)$

$\oint H \cdot dl = H_o b + H_o b \Rightarrow 2H_o b$

Equating $\oint H \cdot dl$ equations

$2H_o \cancel{b} = ky \cancel{b}$

$$\boxed{H_o = \frac{ky}{2}}$$

Then,

$H = \frac{ky}{2}\bar{a}_x, z > 0$

$= -\frac{ky}{2}\bar{a}_x, z < 0$

In general for an infinite sheet of current density k A/m

$$\boxed{H = \frac{1}{2} k \times \bar{a}_n}$$

where,

a_n is the unit normal vector.//

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Magnetic potential :-

Scalar magnetic potential

Ampere's law stated that the line integral of field H around closed path is equal to the current enclosed.

$$\oint H \cdot dl = I$$

If no current is enclosed

$$\oint H \cdot dl = 0$$

magnetic field H can be expressed as -ve gradiant of a scalar function

$$H = -\nabla V_m$$

where,

V_m is called scalar magnetic potential

$$\boxed{V_m = -\oint H \cdot dl}$$

$\left\{ \begin{array}{l} B = \mu H \\ \mu = \mu_o \mu_r = \mu_o \end{array} \right\}$

this scalar potential also satisfies laplace's equation

In free space, $\nabla \cdot B = 0$

$\mu_o \nabla \cdot H = 0$

But, $H = -\nabla V_m$

$\mu_o \nabla \cdot (-\nabla V_m) = 0$

$-\mu_0 \nabla V_m = 0$

$$\boxed{\therefore \nabla^2 V_m = 0}$$

Vector Magnetic potential :-

scalar magnetic potential exists if their is no current enclosed
i.e., $\oint H \cdot dl = 0$

If current is enclosed the potential which depends upon current element (vector quantity) is ~~...~~ no more Scalar but it is vector quantity.

* since the divergence of a vector is a scalar, vector potential is expressed in curl.

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i.e., $\nabla \cdot B = 0$

$$B = \nabla \times A$$

where, A is magnetic vector potential.

Take curl on both sides, $\nabla \times B = \nabla \times \nabla \times A$

By the Identity,

$$\nabla \times \nabla \times A = \nabla(\nabla A) - \nabla^2 A$$

But, $\nabla \times B = \mu J$

$$\nabla(\nabla \cdot A) - \nabla^2 A = \mu J$$

for the steady dc,

$$\nabla \cdot A = 0$$

then $-\nabla^2 A = \mu J$

$$\bar{a}_x \nabla^2 A_x + \bar{a}_y \nabla^2 A_y + \bar{a}_z \nabla^2 A_z = -\mu(\bar{a}_x J_x + \bar{a}_y J_y + \bar{a}_z J_z)$$

Equating

$$\boxed{\begin{aligned} \nabla^2 A_x &= -\mu J_x \\ \nabla^2 A_y &= -\mu J_y \\ \nabla^2 A_z &= -\mu J_z \end{aligned}}$$

They are in the form of poisson's equation. from the above equations, the magnetic vector potential can be written as

$$A_x = \frac{\mu}{4\pi}\int_V \left(\frac{J_x}{r}\right) dv$$

$$A_y = \frac{\mu}{4\pi}\int_V \left(\frac{J_y}{r}\right) dv$$

$$A_z = \frac{\mu}{4\pi}\int_V \left(\frac{J_z}{r}\right) dv$$

the general magnetic vector potential can be expressed as

$$A = \frac{\mu}{4\pi}\iiint J/r \, dv$$

Magnetic Materials

(i) Diamagnetic
(ii) paramagnetic
(iii) ferromagnetic

Diamagnetic :- The metals & other elements having slight magnetic properties in which magnetization is opposite to the applied field.

paramagnetic :-

If the magnetization is in the same direction as the applied field

ferromagnetic :-

Very strong magnetic effects. The magnetization is in the same direction as the field

Ex:- Iron, Steel, Nickel, Cobalt.

If $\mu_r \leq 1$, the materials are diamagnetic

If $\mu_r \geq 1$, the materials are paramagnetic

If $\mu_r \gg 1$, the materials are ferromagnetic

Substance	Magnetic type ~~Diamagnetic~~	Relative permeability (μ_r)
Silver, copper	Diamagnetic	0.99998 0.99991
Vaccum	Non-magnetic	1
Alluminium, Palladium	Paramagnetic	1.0002 1.0008
Cobalt, Nickel	Ferromagnetic	250 600

gnetization :-

A bar magnet is compassed of large no. of magnetic dipoles as in fig.

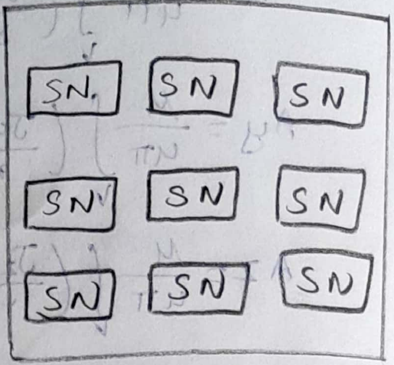

fig. Bar magnet composed of magnetic dipole.

As in the case of an electric dipole consist of equal and opposite charges with very small separation, a small bar magnet with pole ~~strength~~ Strength Q_m & length 'l' be treated as magnetic dipole. whose magnetic moment is $Q_m l$

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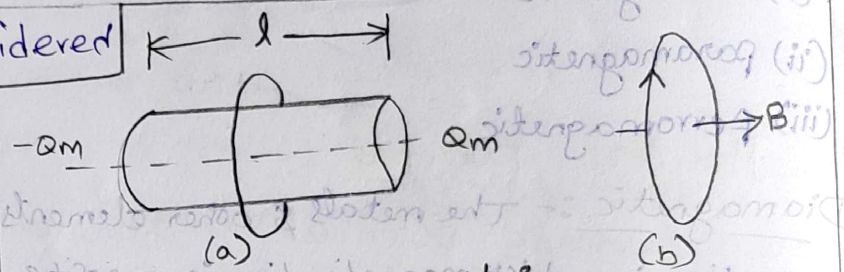
A magnetic dipole is considered as equivalent, to current loop with area A & carrying a current I as shown in fig.



Dipole moment & equivalent current loop

This loop has a magnetic moment

$$m = IA$$

$$\therefore \boxed{Q_m l = IA}$$

If A is the Area of cross section of the bar magnet l is axial length.

The volume of the magnet is $\boxed{V = Al}$

The dipole moment of the bar magnet is $Q_m l$

The net dipole moment per unit volume is called magnetization.

$$\text{Magnetization } (M) = \frac{\text{Dipole moment}}{\text{volume}}$$

It is represented ~~by (M)~~ as vector (M).

$$= \frac{Q_m \cancel{l}}{A\cancel{l}}$$

$$= \frac{Q_m}{A} \; \bar{a} \; A/m$$

M can be defined at any point in the bar magnet by

$$M = \lim_{\Delta v \to 0} \frac{m}{\Delta v} \; A/m$$

$[\because M = IA = Q_m l]$

Magnetic Susceptibility :-

$\chi_m = \frac{M}{H}$, it is dimensionless quantity

$= \frac{\text{Magnetization}}{\text{magnetic field intensity}}$

free space

$$\boxed{B = \mu_0 H}$$

Magnetic Susceptibility is defined as the ratio of magnetization to the magnetic field intensity.

* It is ~~sepr~~ denoted as χ_m

* for magnetic the flux intensity is given by $B = \mu_0 (H + M)$.

$$= \mu_0 H \left(1 + \frac{M}{H}\right)$$

$$B = \mu_0 H (1 + \chi_m) \quad \text{—} \; (1)$$

But $\boxed{B = \mu H}$ — (2)

Equating this two equation. (from (1) & (2))

$$\mu = \mu_0 (1 + \chi_m)$$

$$\mu = \mu_0 \mu_r$$

$$\boxed{\mu_r = 1 + \chi_m}$$

μ_r is relative permeability.

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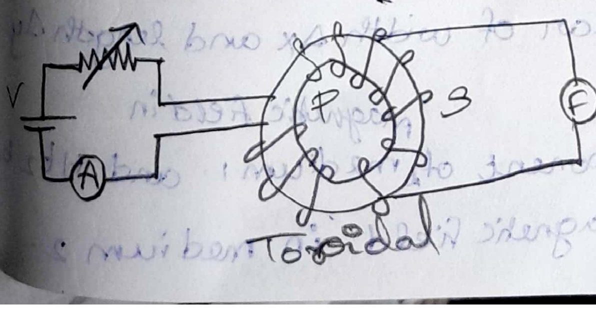
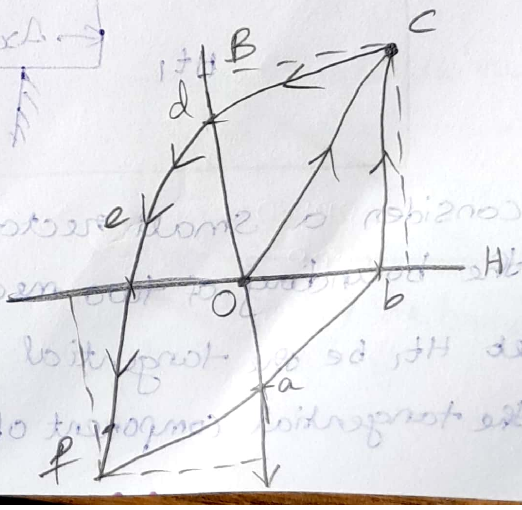
Magnetization (B-H) curve





* Magnetization curve

B with H.

→ Consider a toroid with ferromagnetic core.

→ primary & secondary coils
 ↳ DC supply
 ↳ flux meter

→ H ↑ ↓
 ↳ I ↓↑ toroid

→ H is ↑ oc in B-H curve is called magnetic saturation.

→ H ↓ to zero B will not decreased od → Residual or Remanent value

→ The field at E (H = -He) coercive force

→ f - Negative saturation

→ -ve saturation to +ve saturation cure fabc

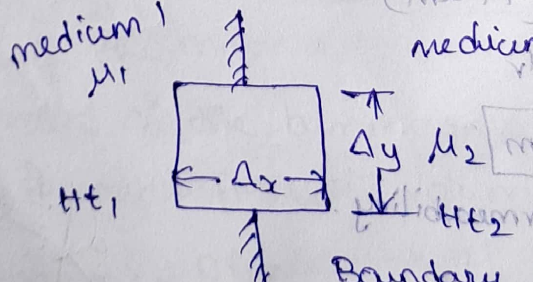
→ closed loop (defabc....) BH loop or hysteresis loop

Boundary conditions :-

→ The boundary conditions b/w the two different magnetic material are given below:-

1) The tangential component of magnetic field intensity is continuous across the boundary.

2) The normal component of magnetic flux density is continuous across the boundary. Consider a boundary b/w two isotropic homogenous media with ~~permeabilities~~ permiability μ_1 and μ_2 as shown in fig.



Boundary b/w two magnetic media

Consider a small rectangular of width Δx and length Δy at the boundary of two media

→ Let H_{t_1} be the tangential component of magnetic field in medium 1 and H_{t_2} be the tangential component of magnetic field in medium 2.

→ According to amper's law if there is no current enclosed by the path

$$\oint H \cdot dl = I$$

$$\oint H \cdot dl = 0$$

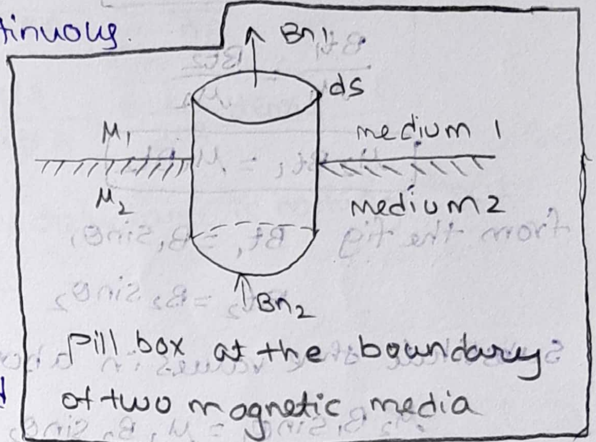
$$Ht_1 \Delta y - Ht_2 \Delta y = 0$$

$$\boxed{Ht_1 = Ht_2}$$

The tangential component of H in medium 1 is same as in medium 2.

∴ the tangential component of H is continuous.

→ consider a pill box surface radius ds b/w across the two isotropic homogeneous media as shown in fig.



Pill box at the boundary of two magnetic media

→ let Bn_1 be the normal component of magnetic flux density in medium 1 and Bn_2 be the normal component of magnetic flux density in medium 2.

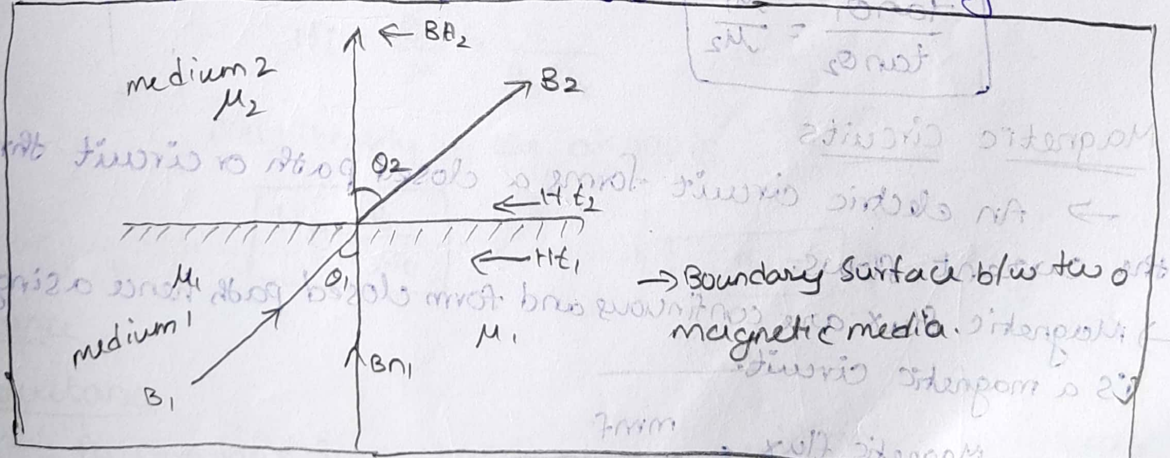
By Gauss's law for magnetic field,

$$\boxed{\iint_S B \cdot ds = 0}$$

$$Bn_1 ds - Bn_2 ds = 0$$

$$\boxed{Bn_1 = Bn_2}$$

The normal component of B is continuous across the boundary.



→ Boundary surface b/w two magnetic media.

→ Consider the magnetic lines away from the normal across the boundary as shown in fig.

$$Bn_1 = B_1 \cos\theta_1$$

$$Bn_2 = B_2 \cos\theta_2$$

But, $\boxed{B_{n_1} = B_{n_2}}$ (boundary condition)

$$B_1 \cos\theta_1 = B_2 \cos\theta_2 \quad \text{--- (1)}$$

$\boxed{B = \mu H}$

$$H_{t_1} = \frac{B_{t_1}}{\mu_1}$$

$$H_{t_2} = \frac{B_{t_2}}{\mu_2}$$

But, $\boxed{H_{t_1} = H_{t_2}}$

$$\frac{B_{t_1}}{\mu_1} = \frac{B_{t_2}}{\mu_2}$$

$$\boxed{\mu_2 B_{t_1} = \mu_1 B_{t_2}}$$

from the fig $B_{t_1} = B_1 \sin\theta_1$

$B_{t_2} = B_2 \sin\theta_2$

Substitute these values in above eqn

$$\mu_2 B_1 \sin\theta_1 = \mu_1 B_2 \sin\theta_2 \quad \text{--- (2)}$$

Dividing eqn (2) by (1)

$$\frac{\mu_2 \cancel{B_1} \sin\theta_1}{\cancel{B_1} \cos\theta_1} = \frac{\mu_1 \cancel{B_2} \sin\theta_2}{\cancel{B_2} \cos\theta_2}$$

$$\cancel{\frac{\mu_1 B_2 \sin\theta_2}{B_2 \cos\theta_2}}$$

$$\mu_2 \tan\theta_1 = \mu_1 \tan\theta_2$$

$$\boxed{\frac{\tan\theta_1}{\tan\theta_2} = \frac{\mu_1}{\mu_2}}$$

Magnetic Circuits

→ An electric circuit forms a closed path or circuit through which the current flows.

→ Magnetic flux are continuous and form closed path hence a single flux line is a magnetic circuit.

$$\text{Magnetic flux} = \frac{\text{mmf}}{\text{Reluctance}}$$

→ The magnetic flux through magnetic circuit is defined as the ratio of magnetic motive force to the Reluctance of the circuit.

magnetic motive force of a magnetic circuit to the line integral of magnetic field H around the closed circuit

$mmf = \oint H dl = NI$ Amp-turns

→ Reluctance $= \dfrac{mmf}{\text{magnetic flux}}$

→ The reluctance is defined as the ratio of total mmf of magnetic circuit to the flux through it

$$\because \; B = \phi/A \; wb/m^2 \qquad \boxed{\phi = BA} \; wb \qquad \boxed{B = \mu H}$$

$$R = \frac{\oint H \cdot dl}{BA} = \frac{H \cdot l}{BA}$$

$$R = \frac{\cancel{H} l}{\mu \cancel{H} A} = \boxed{\frac{l}{\mu A} \; \text{Henry}^{-1}}$$

Reciprocal of reluctance is nothing but permeance.

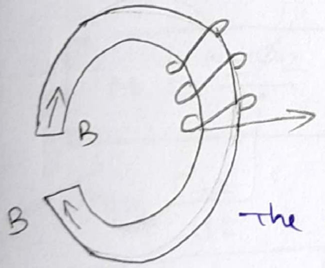
$$P = \frac{1}{R} = \frac{1}{l/\mu A} = \frac{\mu A}{l}$$

$$\boxed{P = \frac{\mu A}{l}} \; \text{Henry}$$

[illegible]/6/22

Magnetic circuit with Air-gap

I_2 *consider



* By the continuity of the normal component of 'd' the flux density in the gap is same as iron.

The field itensity in the iron is

$$H_i = \frac{B}{\mu} = \frac{B}{\mu_0 \mu_r}$$

field itensity in the air gap is

$$\boxed{H_g = \frac{B}{\mu_0}} \qquad \left\{ \boxed{\because \mu_r = 1} \right\}$$

Inductance

Self inductance

By faraday's law changing current will produce an induced emf in the circuit to ~~appose~~ apose the change in flux This is called Self inductance.

* self inductance of a circuit is the property of the circuit by which changing in current induced emf in the circuit to oppose the change in current.
* consider a coil having 'n' no. of turns. If change in current is applied the emf is induced in the coil.
* The induced emf is proportional to the rate of change of current.

$$v(\text{or})\,e \propto \frac{di}{dt}$$

$$\boxed{v(\text{or})\,e = L\frac{di}{dt}} \quad \text{—— (1)}$$

where,

L is the self inductance in henry

e or v induced emf is volts

$\frac{di}{dt}$ → Rate of change of current

In Faraday's law

$$e(\text{or})\,V = N\frac{d\phi}{dt} \quad \text{—— (2)}$$

from (1) & (2)

$$\Rightarrow L\frac{di}{\cancel{dt}} = N\frac{d\phi}{\cancel{dt}}$$

$$\boxed{L = \frac{N\phi}{i}}$$

L = self inductance in henry

N = No. of turns

ϕ = flux in wb

i = current in Amps

The inductance is defined as magnetic flux linkage / current through the coil

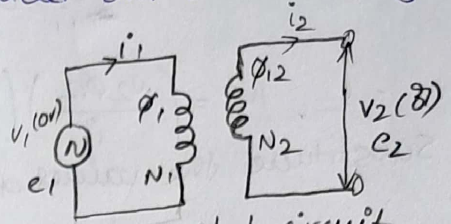
$$L = \frac{N\phi}{i}$$

Mutual inductance :-

Consider 2 coils 1 & 2 magnetically coupled together as shown in fig.

- The change in i_1 produces a flux ϕ_1
- If a 2nd coil is place near the 1st coil
- Some of the flux links coils 2 & 12. The induced emf in coil 2 is given by

$$V_2 = N_2 \frac{d\phi_{12}}{dt} \quad \text{—— (1)}$$



coupled circuit

- Since flux ϕ_{12} is produced by first coil current i_1 the induced emf E_2 in coil is propostional to

$$V_2 \propto \frac{di_1}{dt}$$

$$\boxed{V_2 = M\frac{di_1}{dt}} \quad \text{—— (2)}$$

where,
M is the mutual inductance b/w the two coils.

equating (1) & (2) equations

$$N_2 \frac{d\phi_{12}}{dt} = \frac{M di_1}{dt}$$

$$\boxed{M = \frac{N_2 \phi_{12}}{i_1}}$$

$$V \propto \frac{di_2}{dt}$$

$$\boxed{V_1 = M\, di_2/dt} \quad \text{—— (3)}$$

From faraday's law

$$V_1 = N_1 \frac{d\phi_{21}}{dt} \quad \text{—— (4)}$$

equating (3) & (4)

$$N_1 \frac{\cancel{d}\phi_{21}}{\cancel{dt}} = \frac{M \cancel{d} i_2}{\cancel{dt}}$$

$$\boxed{M = \frac{N_1 \phi_{21}}{i_2}}$$

25/6/22 **coefficient of coupling**

$$k = \frac{\phi_{12}}{\phi_1} = \frac{\phi_{21}}{\phi_2}$$

The mutual inductance is given by

$$M = \frac{N_2 \phi_{12}}{i_1} \text{ and } M = \frac{N_1 \phi_{21}}{i_2}$$

$$M^2 = \left(\frac{N_2 \phi_{12}}{i_1}\right)\left(\frac{N_1 \phi_{21}}{i_2}\right)$$

Substitude the values of ϕ_{12} and ϕ_{21} in terms of k

$$M^2 = \left(\frac{N_2 k \phi_1}{i_1}\right)\left(\frac{N_1 k \phi_2}{i_2}\right)$$

$$= k^2 \left(\frac{N_2 \phi_1}{i_1}\right)\left(\frac{N_1 \phi_2}{i_2}\right) \Rightarrow k^2 \left(\frac{N_2 \phi_2}{i_2}\right)\left(\frac{N_1 \phi_1}{i_1}\right)$$

$$\boxed{M^2 = k^2 L_1 L_2}$$

$$\because L_1 = \left(\frac{N_1 \phi_1}{i_1}\right)$$

$$M = k\sqrt{L_1 L_2}$$

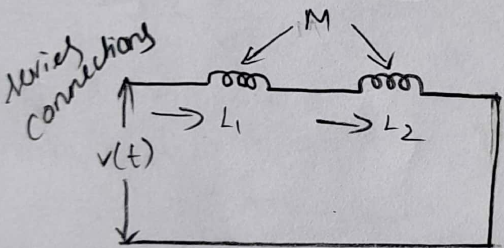
$$L_2 = \left(\frac{N_2 \phi_2}{i_2}\right)$$

$$\boxed{k = \frac{M}{\sqrt{L_1 L_2}}}$$

$$k < 1$$

$\phi_{12} < \phi_1$ & $\phi_{21} < \phi_2$ the $M < \sqrt{L_1 + L_2}$

Coupled coils in series



Series circuits (aiding)

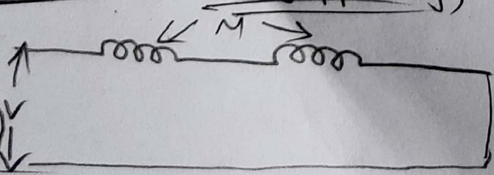
Kirchoff's law:-

$$L_1 \frac{di}{dt} + M \frac{di}{dt} + L_2 \frac{di}{dt} + M \frac{di}{dt} = v(t)$$

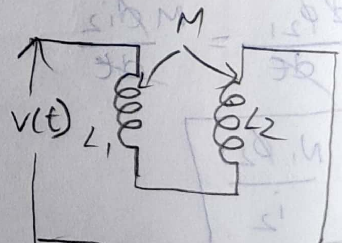
$$(L_1 + L_2 + 2M) \frac{di}{dt} = v(t)$$

equivalent inductance $\boxed{L = L_1 + L_2 + 2M}$

Series connections (opposing)



equ. Ind $\boxed{L = L_1 + L_2 - 2M}$



$$L_1 \frac{di}{dt} - M\frac{di}{dt} + L_2 \frac{di}{dt} - M\frac{di}{dt} = v(t)$$

$$(L_1 + L_2 - 2M)\frac{di}{dt} = v(t)$$

coupled coisine parallel

parallel connection (Aiding)

$$\boxed{L = \frac{L_1L_2 - M^2}{L_1 + L_2 - 2M}}$$

parallel connection (opposing)

$$\boxed{L = \frac{L_1L_2 - M^2}{L_1 + L_2 + 2M}}$$

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Inductance of Solenoid $\{B = \phi/A \Rightarrow \boxed{\phi = BA}\}$

consider a solenoid of 'N' no. of turns carrying current 'I'. If 'B' is the flux density wb/m^2

A is the Area of the cross section of solenoid, in m^2

then flux linkage through the solenoid is $N\phi = NBA$

$$L = \frac{N\phi}{I} = \frac{NBA}{I}$$

But for long solenoid

$$B = \frac{\mu_0 NI}{l}$$

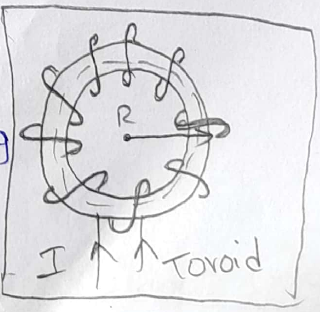
sub the value B in above eqn

$$= \frac{NA}{\cancel{I}}\left(\frac{\mu_0 N\cancel{I}}{l}\right)$$

$$\boxed{L = \frac{\mu_0 N^2 A}{l}}$$

Inductance of Toroid:-

consider a toroid of 'N' no. of turns carrying current 'I' with mean radius 'R' as shown in fig.



→ If 'B' is the flux density in the toroid

$$B = \frac{\mu_0 NI}{l}$$

where 'l' is mean length of the coil

$$l = 2\pi R$$

$$\boxed{B = \frac{\mu_0 N I}{2\pi R}}$$

the flux linkage in the toroid is $N\phi = NBA$

$$= \frac{N \mu_0 N I}{2\pi R}$$

$$\boxed{N\phi = \frac{\mu_0 N^2 I A}{2\pi R}}$$

where 'A' is ~~Area~~ cross sectional area

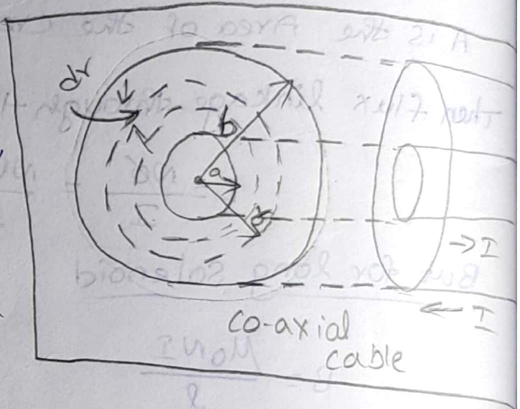
$$N\phi = \frac{\mu_0 N^2 I \cancel{\pi} r^2}{2\cancel{\pi} R}$$

$$N\phi = \frac{\mu_0 N^2 r^2 I}{2R}$$

Inductance of toroid is $\boxed{L = \frac{N\phi}{I} = \frac{\mu_0 N^2 r^2}{2R}}$

Inductance of co-axial cable

consider a co-axial cable of inner radius 'a'. outter radius 'b' as shown in fig.



Let 'I' be the current in inner cylinder and '-I' be the current in outter cylinder.

→ consider an angular ring of thickness dr at a distance 'r' from the center of the cable.

→ the flux density 'B' is given by $B = \frac{\mu_0 I}{2\pi r}$

the Total flux linkage per unit length b/w a & b

$$\phi = \int_a^b \frac{\mu_0 I}{2\pi r} \cdot dr$$

$$= \frac{\mu_0 I}{2\pi} \int_a^b \frac{1}{r} dr$$

$$= \frac{\mu_0 I}{2\pi} \left[\ln r\right]_a^b = \frac{\mu_0 I}{2\pi}\left[\ln b - \ln a\right]$$

$$\boxed{\phi = \frac{\mu_0 I}{2\pi} \ln b/a}$$

Inductance of co-axial per unit length is given by

$$L = \frac{\phi}{I}$$

$$\boxed{L = \frac{\mu_0}{2\pi} \ln\left(\frac{b}{a}\right)}$$

Inductance of co-axial cable with solid Inner conductor

⇒ Consider a co-axial cable with solid inner conductor of radius and outer radius 'b'. Let 'I' be the current in solid conductor & −I be the current in outer ~~same~~ conductor.

The flux density within the solid conductor at a distance 'r' from the axis to the cable is

$$B = \frac{\mu_0 \mu_r I' r}{2\pi a^2}, \quad 0 < r < a$$

The current flowing in solid inner conductor b/w 'a' to 'r' is

$$I' = \frac{I}{\cancel{\pi} a^2} \cancel{\pi} r^2 \Rightarrow \frac{I}{a^2} r^2$$

Now,

$$B = \frac{\mu_0 \mu_r r}{2\pi a^2} \cdot \frac{I}{a^2} r^2 \Rightarrow \frac{\mu_0 \mu_r I r^3}{2\pi a^4}$$

The total flux linkage per unit length b/w 0 and a

$$\phi = \int_0^a \frac{\mu_0 \mu_r I r^3}{2\pi a^4} \cdot dr = \frac{\mu_0 \mu_r I}{2\pi a^4} \left(\frac{r^4}{4}\right)_0^a$$

$$= \frac{\mu_0 \mu_r I}{2\pi a^4} \left(\frac{a^4}{4} - 0\right) = \frac{\mu_0 \mu_r I \cancel{a^4}}{8\pi \cancel{a^4}} = \frac{\mu_0 \mu_r I}{8\pi}$$

Inductance of solid conductor b/w '0' and 'a'.

$$\boxed{L = \frac{\phi}{I} = \frac{\mu_0 \mu_r \cancel{I}}{8\pi \cancel{I}} = \frac{\mu_0 \mu_r}{8\pi}} = L_1$$

Inductance of co-axial cable per unit length a & b

$$L_2 = \frac{\mu_0}{2\pi} \ln[b/a]$$

Inductance of co-axial cable per unit length is

$$L = L_1 + L_2$$

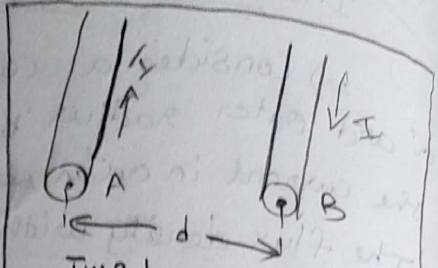
$$L = \frac{\mu_0 \mu_r}{8\pi} + \frac{\mu_0}{2\pi} \ln(b/a)$$

Result :-

$$\boxed{L = \frac{\mu_0}{4\pi}\left[\frac{1}{2}\mu_r + 2\ln(b/a)\right]} \text{ H/m}$$

Inductance of two transmission line :-

consider two conductors (two wires) of A & B of radius 'a' & 'b' respectively & separated by the distance 'd'. The conductor A carries a current of 'I' & conductor B carries a current of '-I' as shown in fig.



Two transmission line

→ the internal flux linkage of the conductor 'A' is given by

$$\phi_1 = \frac{\mu_0 \mu_r I}{8\pi}$$

⇒ the external flux linkage of the conductor 'a' is given by

$$\phi_2 = \frac{\mu_0 I}{2\pi} \ln\left(\frac{d}{a}\right)$$

The total flux linkage of A is

$$\phi = \phi_1 + \phi_2$$

$$= \frac{\mu_0 \mu_r I}{8\pi} + \frac{\mu_0 I}{2\pi} \ln\left(\frac{d}{a}\right).$$

The total inductance of conductor A is

$$L_A = \frac{\phi}{I}$$

$$L_A = \frac{\mu_0}{4\pi}\left[\frac{\mu_r}{2} + 2\ln\left(\frac{d}{a}\right)\right] \text{H/m}$$

Similarly the conductors, the total flux linkage is

$$\phi = \frac{\mu_0 \mu_r I}{8\pi} + \frac{\mu_0 I}{2\pi} \ln\left(\frac{d}{a}\right)$$

The total inductance of conductor B is

$$L_B = \frac{\mu_0}{4\pi}\left[\frac{\mu_r}{2} + 2\ln\left(\frac{d}{a}\right)\right]$$

The loop inductor of transmission line per unit length is given by

$$\boxed{L = L_A + L_B}$$

$$L = \frac{\mu_0}{4\pi}\left[\frac{\mu_r}{2} + 2\ln\left(\frac{d}{a}\right) + \frac{\mu_r}{2} + 2\ln\left(\frac{d}{a}\right)\right]$$

$$= \frac{\mu_0}{4\pi}\left[\mu_r + 2\ln\left(\frac{d}{a}\right)\right]$$

$$= \frac{\mu_0}{4\pi}\left[\mu_r + 2 \times 2\ln\left(\frac{d}{a}\right)\right]$$

Result :- $\boxed{L = \frac{\mu_0}{4\pi}\left[\mu_r + 4\ln\left(\frac{d}{a}\right)\right]}$ H/m

Energy Stored in magnetic field

when current through an inductor is increased from 'o' to 'I' with the potential difference across the inductor is 'v'. Then the energy supplied by the source the time dt is given by

$$\boxed{dW = vi\,dt}$$

energy stored in magnetic field is given by

$$W = \int_0^I vi\,dt = \int_0^I Li\frac{di}{dt}\,dt \qquad \left\{ v = L\cdot\frac{di}{dt} \right\}$$

$$W = L\left[\frac{i^2}{2}\right]_0^I \Rightarrow \boxed{W = \tfrac{1}{2}LI^2}$$

Energy density :-

The energy stored in a magnetic field is given by

$$\boxed{W = \frac{1}{2}LI^2}$$

The inductance of the solinoid is given by

$$\boxed{L = \frac{\mu_0 N^2 A}{l}}$$

substitute the value of 'L' in the above eqn.

$$W = \frac{1}{2}\frac{\mu_0 N^2 A}{l}I^2 \Rightarrow \frac{1}{2}\mu_0\left[\frac{NI}{l}\right]^2 lA \qquad \left\{ \because H = \frac{NI}{l} \right\}$$

$$\boxed{W = \frac{1}{2}\mu_0 H^2 lA}$$

Energy stored per unit volume $\boxed{\because \text{volume} = lA}$

$$\frac{W}{lA} = \frac{1}{2}\mu_0 H^2 \cdot J/m^2$$

$$\frac{W}{V} = \frac{1}{2}(\mu_0 H)\cdot H$$

Magnetic energy density $\boxed{W = \frac{1}{2}BH}$

The energy stored in a magneto static field is

$$W = \int_V w\,dv \Rightarrow \frac{1}{2}\int_V BH\,dv$$

$$W = \frac{1}{2}\int_V \mu H^2\,dV$$

$$\left\{ \begin{aligned} \because B &= \mu H \\ &= \mu_r \mu_0 H \\ B &= \mu_0 H \\ &(\mu_r = 1) \end{aligned} \right.$$

2/7/22

Force on a differential current element :-

$$dF = dQ \; v \times B \quad -(1)$$

$dF \rightarrow$ differential force

$dQ \rightarrow$ differential element of charge

$v \rightarrow$ velocity

$B \rightarrow$ magnetic flux density

$$dQ = \rho_v dv \quad -(2)$$

$\rho_v \rightarrow$ volume charge density

put (2) in (1)

$$dF = \rho_v dv \cdot v \times B \quad -(3)$$

$\boxed{\text{current density } J = \rho_v \cdot v}$ $-(4)$

put (4) in (3)

$$\boxed{dF = J \times B\, dv} \quad -(5)$$

$$J dv = I dl \quad -(6)$$

put (6) in (5)

$$\boxed{dF = I\, dl \times B} \quad -(7)$$

Force due to entire conductor

$$F = \oint F$$

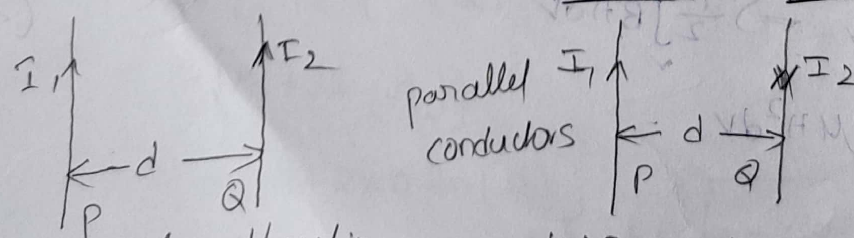
$$\boxed{F = \oint I dl \times B}$$

$$\boxed{F = I l \times B}$$

magnitude of force $\boxed{F = B I l \sin\theta}$

where, θ is the angle between B and I.

Force between the current carrying conductors



a) force of attraction (b) force of repulsion.

Consider a two straight long ||el conductors P & Q seperated by a distance d. Let I_1 & I_2 be the current flowing in the conductor P & Q respectively. Consider a conductor produces a magnetic field whose flux density is P at the conductor Q.

$$B = \frac{\mu_0 I_1}{2\pi d} \quad -(1)$$

$$F = B I_2 l \quad -(2)$$

where $l \rightarrow$ length of the conductor. put eqn (1) in (2)

$$F = \frac{\mu_0 I_1 I_2}{2\pi d} l \quad -(3)$$

$$\boxed{F = \frac{\mu_0 I_1 I_2 l}{2\pi d}} \; N \Rightarrow \frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi d} \; N/m$$

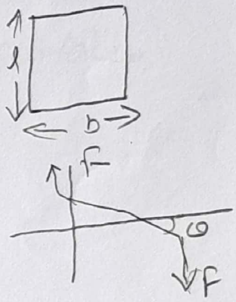
Magnetic Torque and magnetic moment

T = 2 × Torque on each side

$= 2 \times F \times D$

$= \cancel{2} BIl \sin\theta \, \frac{b}{\cancel{2}}$

$$\left\{ \begin{array}{l} F = BIl\sin\theta \\ A = l \times b \end{array} \right\}$$



$$\boxed{T = BIA\sin\theta}$$

magnetic moment of loop $= IA$

$$\boxed{m = IA \cdot \hat{n}}$$

$T = mB\sin\theta \, \hat{n}$

$T = m \times B$

$$\boxed{m = T/B} \Rightarrow \frac{\text{maximum torque on loop}}{\text{magnetic induction}}$$

(or) magnetic flux density).

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11/7/22

unit-4

magnetostatics & applications

faraday's law :-

The total electromagnetic force [EMF] induced in a circuit is equal to the rate of decrease of the total magnetic flux ~~linkage~~ linking the circuit

$$V = \frac{-d\phi}{dt} \quad -(1)$$

→ If N is no. of turns in the circuit then total flux is $N\phi$. where ϕ is magnetic flux

$$V = \frac{-Nd\phi}{dt} \quad -(2)$$

⇒ By definition the emf in the circuit is the line integral of the electric field around a closed path.

$$V = \oint E \cdot dl \quad -(3) \quad \text{where, } E = \text{electric field}$$

⇒ According to gauss law the total flux passing through the surface is equal to the surface integral of magnetic flux density over the surface.

$$\phi = \iint B \cdot ds \quad -(4)$$

Substitute eq (4) in (1)

$$V = \frac{-d}{dt}\iint B \cdot ds \quad -(5)$$

$$= -\iint \frac{\partial B}{\partial t} \cdot ds \quad -(6)$$

But $v = \oint E \cdot dl \quad -(3)$

$$\oint E \cdot dl = -\iint_S \frac{\partial B}{\partial t} ds \quad -(7)$$

Applying Stoke's theorem

$$\oint E \cdot dl = \iint_S \nabla \times E \cdot ds \quad -(8)$$

Then, $\iint_S \nabla \times E \cdot ds = -\iint_S \frac{\partial B}{\partial t} \cdot ds \quad -(9)$

$$\boxed{\nabla \times E = -\frac{dB}{dt}} \quad -(10)$$

This, is referred to as maxwell's equation.

→ faraday's law states that the total electromagnetic force induced in a circuit is equal to the rate of decrease of the total magnetic flux linking the circuit. $V = -\frac{Nd\phi}{dt}$

An EMF ~~is integral~~ can be generated by a closed conducting loop by 3 ways.

1) A stationary loop in a time varying magnetic field
2) A moving loop in a static magnetic field.
3) A moving loop in a time varying magnetic field

Lenz's law :-

lenz's law states that an induced emf ~~at~~ in a circuit produces a circuit which oppose the change in magnetic flux producing it

$$\boxed{EMF = \frac{-d\phi}{dt}}$$

⇒ State faraday's law of electromagnetic induction.

faraday's law states that electromagnetic force induced a circuit is equal to the rate of change of magnetic flux linking the circuit.

$$\boxed{emf\ v = \frac{d\phi}{dt}}$$

$$e \propto \frac{di}{dt} \Rightarrow e = \frac{L\,di}{dt}$$

$$e = 5 \times 10^{-3} \times 100$$

$$= 5 \times \frac{1}{1000} \times 100$$

$$\boxed{e = 0.5V}$$

$l = 1m$

$v = 100 m/sec$

$B = 1 wb/m^2$ (or) T

$\theta = \pi/2$

$e = Blv \sin\theta = 1 \times 1 \times 100 \times \sin \pi/2 \Rightarrow \boxed{e = 100 \text{ volts}}$

stationary loop in a time varying magnetic field

when the conducting loop is stationar. A time varying magnetic field. the Emf induced in the loop.

The induced emf is called transformer ~~loop~~ Emf.

$$V = N\oint \frac{\partial B}{\partial t} ds \text{ --- (1)} \quad (N - \text{No. of turns in a loop})$$

$$V = -\int_S \frac{\partial B}{\partial t} ds \text{ --- (2)}$$

$$V = \oint E \cdot dl \text{ --- (3)}$$

equating eqn (2) & (3)

$$\oint E dl = -\int_S \frac{\partial B}{\partial t} ds \text{ --- (4)}$$

this is the integral form of faradays law

According to stoke's theorem

$$\oint E \cdot dl = \int_S (\nabla \times E) ds \text{ --- (5)}$$

$$\int_S (\nabla \times E) \cdot \cancel{ds} = -\int_S \frac{\partial B}{\partial t} \cancel{ds} \Rightarrow \boxed{\nabla \times E = \frac{-\partial B}{\partial t}} \quad \text{Transformer Emf}$$

this is the differential form of faraday's law

* It ~~st~~ states that a time varying magnetic field induces an electric fied E. whose curl is equal to the -ve of the time deravation of 'B'. (Time derivation of ~~[illegible]~~ magnetic field)

Moving loop in a static magnetic field

consider a conducting loop of length 'l' moving across a static magnetic field 'B' with a velocity 'v'. The magnetic force 'f' acting on any charged partical 'Q' moving with a velocity 'v' in a magnetic field 'B' is given by $f = Q(V \times B)$ --- (1)

* The magnetic force is equivalent to the electrical force that would exerted on the partical by an electric field 'E' given by

$$E = \frac{f}{Q} = V \times B \text{ --- (2)}$$

* The electric field 'E' generated by the motion of charged particles.

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is called a motional electric field E_e is perpendicular to

* The induced voltage is induced Emf and is given by

$$\int E \cdot dl = \int (B \times v) dl \quad \text{---} (3)$$

Ex:- electric genrator motor)
(motional emf)

Moving loop in a time varying magnetic field :-

when a conducting loop is moving in a time varying magnetic field B' the induced Emf is the some of transformer Emf and motional EMF.

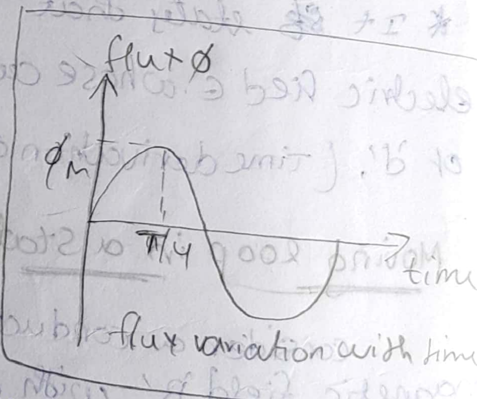
$$\boxed{\text{Total Emf} = \oint E \cdot dl = -\int \frac{dB}{dt} \cdot ds + \oint (v \times B) dl}$$

Transformer EMF

EMF equation of the transformer

let, N_1 = No. of turns in primary winding
N_2 = No. of turns in secondary winding
ϕ_m = maximum flux in the curve (in W_b) = $(B_m \times A)$
f = frequency of the AC Supply (in H_z)

$$\boxed{T = \frac{1}{f}}$$



Average rate of change of flux $= \frac{\phi m}{T/4} = \frac{\phi_M}{1/4f}$

∴ Average rate of change of flux $= 4f\phi_m$ (wb/s)

Now, Induced emf/turn = rate of change of flux per turn.

Average Emf per turn $= 4 f\phi m$ (volts)

Now, we know form factor $= \frac{\text{Rms value}}{\text{Average value}}$

∴ RMF value of Emf per turn = form factor x Average value

$$= 1.11 \times 4 f\phi m$$

$$E = 4.44 f\phi m$$

Rms value of induced Emf in whole primary winding $(E_1) = \left\{ \begin{array}{c} \text{Rms value of} \\ \text{Emf per} \\ \text{turn} \end{array} \right\} \times \left\{ \begin{array}{c} \text{no. of turns} \\ \text{in primary winding} \end{array} \right\}$

$$E_1 = 4.44 f N_1 \phi_m$$

Rms induced Emf in secondary winding $E_2 = 4.44 f N_2 \phi_m$

voltage transformation Ratio(k) $\frac{E_1}{E_2} = \frac{4.44 f N_1 \phi_m}{4.44 f N_2 \phi_m}$

Ideal T/f No load $\frac{E_1}{E_2} = \frac{N_1}{N_2} = k$

$E_1 = V_1$

$E_2 = V_2$

$$\boxed{\frac{E_1}{N_1} = \frac{E_2}{N_2} = k}$$

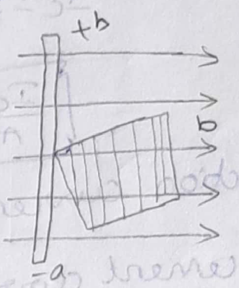
$N_2 > N_1$, $k > 1$ step up T/f

$N_1 > N_2$, $k < 1$ step down T/f

Motional EMF

Force on free electron

$\bar{F} = q(\bar{B} \times \bar{v})$



potential difference = work done per unit charge

$= \frac{\text{work}}{\text{charge}}$

$= \text{force} \times \frac{\text{Distance}}{\text{charge}}$

$e = q(\bar{B} \times \bar{v}) l / q$

$$\boxed{e = l(\bar{B} \times \bar{v})}$$

(or)

$$\boxed{e = Bvl \sin\theta}$$

This is called motional EMF

motional EMF = $Bvl \sin\theta$ $(\theta = 90°)$

motional EMF = Bvl

Maxwell's Equations

Maxwell's equation from Ampere's law

Ampere's circuital law

Conduction current density

Ohm's law $I_C = V/R$

But $R = \frac{\rho l}{A} = \frac{l}{\sigma A}$ where, ρ is Resistivity
σ is conductivity $(\sigma = \frac{1}{\rho})$

l is the length of the conductor
A is the area of cross section of conductor

$$I_C = \frac{V}{R} = \frac{V}{l/\sigma A} = \frac{V\sigma A}{l}$$

If $E \rightarrow$ electric field, then voltage $V = El$
substituting the values of V

$$I_C = \frac{V\sigma A}{l} = \frac{E\cancel{l}\sigma A}{\cancel{l}} = E\sigma A$$

$$\boxed{\frac{I_C}{A} = E\sigma}$$

Conduction current density $\boxed{J_C = \frac{I_C}{A}} \Rightarrow \boxed{J_C = E\sigma}$

Displacement current density $I_D = \frac{dQ}{dt} \Rightarrow \boxed{I_D = C\frac{dV}{dt}}$

But $Q = CV$

$$\boxed{\therefore C = \frac{\varepsilon A}{d}}$$

where, ε is permittivity of the medium
A is area of the parallel plate of capacitor.
d is distance b/w two plates

$$I_D = \frac{\varepsilon A}{d}\frac{dV}{dt} = \frac{\varepsilon A}{\cancel{d}}\cancel{d}\frac{dE}{dt}$$

$$\boxed{V = Ed}$$

$$I_D = \varepsilon A \frac{dE}{dt}$$

$$\frac{I_D}{A} = \varepsilon\frac{dE}{dt} = \frac{dD}{dt} \qquad [\because D = \varepsilon E]$$

$$J_D = \frac{I}{A}$$

$$J_D = \varepsilon \frac{\delta E}{\delta t}$$

(or)

$$J = \frac{\partial D}{\partial t}$$

Ampere's law

$$\oint H \cdot dl = \iint_S (J_C + J_D) ds$$

(or) $\frac{\partial D}{\partial t}$

then $\oint H \cdot dl = \iint_S \left(\sigma E + \frac{\varepsilon dE}{dt}\right) ds$

Integral form of an ampere's law $\oint H \cdot dl = \iint_S \left(J + \frac{dD}{dt}\right) ds$

Stokes theorem

$$\oint H \cdot dl = \iint_S \nabla \times H ds$$

$$\iint \nabla \times H \cdot ds = \iint \left(J + \frac{\partial D}{\partial t}\right) ds$$

maxwell's eqn in (or) point Differential form: $\boxed{\nabla \times H = J + \frac{\partial D}{\partial t}}$

from Amper's law

$$\boxed{\nabla \times H = \nabla E + \varepsilon \frac{dE}{dt}}$$

Statement:- The magneto motive force around a closed path is equal to the sum of the conduction current and displacement current enclosed by the path magnetic voltage around a closed path is equal to the electric current through the path.

14/2/22 Maxwell's Equation from faraday's law:-

faraday's law

Faraday's law states that electromagnetic force (EMF) induced in a circuit is equal to the rate of magnetic flux linkage.

$$V = \frac{-d\phi}{dt} = \frac{-d}{dt} \iint B \cdot ds$$

[∵ $B = \mu H$]

But, $V = \oint E \cdot dl$

$$\oint E \cdot dl = -\frac{d}{dt} \iint_S B \cdot ds.$$

$$\oint E \cdot dl = -\iint_S \frac{\partial B}{\partial t} ds \quad \text{—(1)}$$

$$\oint E \cdot dl = -\mu \iint_S \frac{\partial H}{\partial t} ds$$

This is maxwell eqn in integral form. by applying Stoke's theorem

$$\oint E \cdot dl = \iint_S \nabla \times E \, ds \quad \text{—(2)}$$

Comparing eqn (1) & (2)

$$\boxed{\iint_S \nabla \times E \cdot ds = -\iint_S \frac{\partial B}{\partial t} \cdot ds}$$

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\boxed{\nabla \times E = -\mu \frac{\partial H}{\partial t}}$$ differential (or) point form

Statement:- The electro motive force around a closed path is equal to the magnetic displacement (flux density) through that closed path.

In other words

* The electric voltage is equal to the magnetic current through the path.

Maxwell's Equation from electric gauss's law:-

Gauss's law:-

Gauss' law states that electric flux emerging through any closed surface is equal to the charge enclosed by the surface.

$$\psi = Q$$

$$\iint D \cdot ds = Q \quad \text{—(1)}$$

$$\iiint \rho_v \cdot dv = Q \quad \text{—(2)}$$

Then,

Integral form. $$\boxed{\iint_S D \cdot ds = \iiint_V \rho_v \cdot dv} \quad \text{—(3)}$$

This is the maxwell's equation from electric gauss's law in integral form.

By applying divergence theorem

$$\iint_S D \cdot ds = \iiint_V \nabla \cdot D \, dv \quad \text{—(4)}$$

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Comparing the eqn (2) & (4)

$$\iiint_V \nabla\cdot D\, dv = \iiint_V \rho_v\, dv \quad \text{--- (5)}$$

differential (or) point form $\boxed{\nabla\cdot D = \rho_v}$

Statement:- The total electric displacement to the Surface enclosed by the is equal to the total charge with in the volume.

Maxwell's equation from magnetic gauss's law :-

Magnetic gauss's law

Magnetic gauss's law states that, the total magnetic flux through any closed Surface is equal to zero

$$\phi = 0$$

integral form $\boxed{\iint_S B\cdot ds = 0}$ --- (1)

This is the maxwell's eqn from magnetic gauss's law in integral form

By applying divergence theorem

$$\boxed{\iint_S B\cdot ds = \iiint_V \nabla\cdot B\, dv} \quad \text{--- (2)}$$

comparing eqn (1) & (2)

$$\iiint_V \nabla\cdot B\cdot dv = 0$$

differential (or) point form $\boxed{\nabla\cdot B = 0}$

Statement:- The net magnetic flux is emerging to any closed Surface is zero.

Maxwell's equations are Summarised as follows :-

Differential form	Integral form
1. $\nabla\times H = J + \frac{\partial D}{\partial t}$ (or) $\nabla\times H = \sigma E + \varepsilon\frac{\partial E}{\partial t}$	$\oint H\cdot dl = \iint_S [J + \frac{\partial D}{\partial t}] ds$ $\oint H\cdot dl = \iint_S [\sigma E + \varepsilon\frac{\partial E}{\partial t}] ds$

Differential form	Integral form
2. $\nabla \times E = -\frac{\partial B}{\partial t}$ (or) $\nabla \times E = -\mu \frac{\partial H}{\partial t}$	$\oint E \cdot dl = \iint_S \frac{\partial B}{\partial t} ds$ $\oint E \cdot dl = -\mu \iint_S \frac{\partial H}{\partial t} \cdot ds$
3. $\nabla \cdot D = \rho$	$\oiint D \cdot ds = \iiint \rho \cdot dv$
4. $\nabla \cdot B = 0$	$\oiint B \cdot ds = 0$

In a ~~force~~ free space, there is no charges enclosed & since it is a dielectric ~~there~~ is no conductivity in the medium.

i.e, $\rho = 0$ & $\sigma = 0$ conduction current is zero.

Then maxwell's eqn becomes in free space as follows:-

Differential form	Integral form
1. $\nabla \times H = \frac{\partial D}{\partial t}$	$\oint H \cdot dl = \iint_S \frac{\partial D}{\partial t} \cdot ds$
2. $\nabla \times E = -\frac{\partial B}{\partial t}$	$\oint E \cdot dl = \iint_S \frac{\partial B}{\partial t} \cdot ds$
3. $\nabla \cdot D = 0$	$\oiint_S D \cdot ds = 0$
4. $\nabla \cdot B = 0$	$\oiint_S B \cdot ds = 0$

Unit-5

current Density & wave propogation

Poynting Vector and the flow of power, paynting theorem

the Vector ~~field~~ of electric field intensity and magnetic field intensity product is a machine the rate of energy flow per unit area at any point. the direction of flow p vector is perpendicular to

$$\boxed{\vec{P} = E \times H}$$

the energy flow equation can be obtained from Maxwell's first equation

maxwell equation
$$\nabla \times H = J + \varepsilon \frac{\partial E}{\partial t} \quad (1)$$

$$J = \nabla \times H - \varepsilon \frac{\partial E}{\partial t} \quad (2)$$

$$E \cdot J = E \cdot \nabla \times H - \varepsilon E \frac{\partial E}{\partial t} \quad (3)$$

~~there~~ wave eqn for Conductivity medium $\boxed{B = \mu H}$

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$= -\mu \frac{\partial H}{\partial t}$$

Take curl o.b.s.

$$\nabla \times \nabla \times E = -\mu \nabla \times \frac{\partial H}{\partial t}$$

maxwell eqn from ampere's law,

$$\nabla \times H = J + \frac{\partial D}{\partial t} = \sigma E + \varepsilon \, \partial E / \partial t$$

this is the wave eqn in terms of mag. field H

$$\boxed{\nabla^2 H - \mu \sigma \frac{\partial H}{\partial t} - \mu \varepsilon \frac{\partial^2 H}{\partial t^2} = 0}$$

defind the

Poynting's theorem

$$\boxed{\vec{P} = \vec{E} \times \vec{H}}$$

~~defind~~ It states that P vector product of electric field Intensity E and Magnetic field intensity at any point is a measure of the rate of energy flow per Unit area.

(2) defind the poynting theorem for

Statment :-

Crossing a

Proof :- from Maxwell's equation;

$$\nabla \times \vec{H} = \vec{J_c} + \vec{J_D} = J_c + \varepsilon \frac{\partial \vec{E}}{\partial t}$$

$$\Rightarrow \vec{J_c} = (\nabla \times \vec{H}) - \varepsilon \frac{\partial \vec{E}}{\partial t}$$

~~If~~ it is multiplied by E, it will be the power Per unit Volume.

i.e $\vec{E} \cdot \vec{J} = \vec{E}(\nabla \times \vec{H}) - \varepsilon \vec{E} \cdot \frac{\partial \vec{E}}{\partial t}$ —— (1)

we know that

the vector Identity

$$\nabla(\vec{E} \times \vec{H}) = \vec{H}(\nabla \times \vec{E}) - \vec{E}(\nabla \times \vec{H})$$

(or)

$$\vec{E}(\nabla \times \vec{H}) = \vec{H}(\nabla \times \vec{E}) - \nabla(\vec{E} \times \vec{H}) \quad \text{—— (2)}$$

Subs eqn (2) in (1),

$$\vec{E} \cdot \vec{J} = \vec{H}(\nabla \times \vec{E}) - \nabla(\vec{E} \times \vec{H}) - \varepsilon \vec{E} \frac{\partial \vec{E}}{\partial t} \quad \text{—— (3)}$$

W.K.T

$$\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t}$$

$$\vec{E} \cdot \vec{J} = \vec{H}\left(-\mu \frac{\partial \vec{H}}{\partial t}\right) - \nabla(\vec{E} \times \vec{H}) - \varepsilon \vec{E} \frac{\partial \vec{E}}{\partial t} \quad \text{—— (4)}$$

W.K.T

$$\vec{H} \cdot \frac{\partial \vec{H}}{\partial t} = \frac{1}{2} \frac{\partial H^2}{\partial t}$$

$$\vec{E} \frac{\partial \vec{E}}{\partial t} = \frac{1}{2} \frac{\partial E^2}{\partial t} \Rightarrow \vec{E} \cdot \vec{J} =$$

On integrating Over a Volume :-

$$\int_V \vec{E}\cdot\vec{J} = \frac{-\partial}{\partial t}\int_V \left(\frac{\mu H^2}{2} + \frac{\varepsilon E^2}{2}\right) dv - \int_V \nabla(\vec{E}\times\vec{H})\, ds$$

$$\vec{E}\cdot\vec{J} = -\frac{\mu}{2}\frac{\partial H^2}{\partial t} - \frac{\varepsilon}{2}\frac{\partial E^2}{\partial t} - \nabla(\vec{E}\times\vec{H})$$ Subs.

using divergence theorem

$$\boxed{\int_V \nabla(\vec{E}\times\vec{H})\, dv = \int (\vec{E}\times\vec{H})\, ds}$$

$$\Rightarrow \int_V \vec{E}\cdot\vec{J} = -\frac{\partial}{\partial t}\int_V \left(\frac{\mu H^2}{2} + \frac{\varepsilon E^2}{2}\right) dv - \int_V (\vec{E}\times\vec{H})\, ds.$$

{Rate of energy dissipation in a Volume} = {Rate of which the Stored energy in Volume decrease} + {Rate at which the energy is entering the volume from Outside.}

The term,..

$-\oint(\vec{E}\times\vec{H})\,ds$ (represents the rate of flow of energy through the Surface of Volume)

$\therefore$ The power flow per unit due is

$$\vec{P} = \vec{E}\times\vec{H}$$

Hence proved//.

Transformer EMF

EMF equation of the transformer

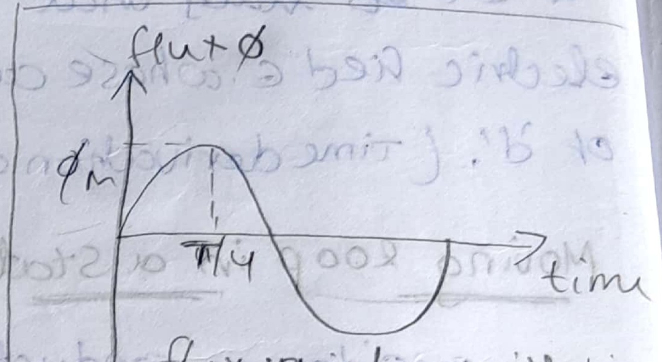
let, N_1 = No. of turns in primary winding

N_2 = No. of turns in secondary winding

ϕ_m = maximum flux in the curve (in wb) = $(B_m \times A)$

f = frequency of the AC Supply (in Hz)

$$\boxed{T = \frac{1}{f}}$$



flux variation with time

Average rate of change of flux $= \dfrac{\phi_m}{T/4} = \dfrac{\phi_m}{1/4f}$

$\therefore$ Average rate of change of flux $= 4f\phi_m$ (wb/s)

Now, Induced emf/turn = rate of change of flux per turn.

Average emf per turn $= 4f\phi_m$ (volts)

Now, we know form factor $= \dfrac{\text{Rms value}}{\text{Average value}}$

$\therefore$ RMS value of emf per turn = form factor × Average value

$= 1.11 \times 4f\phi_m$

$= 4.44 f\phi_m$

Rms value of induced Emf in whole primary winding $(E_1) = \left\{ \begin{array}{c} \text{Rms value of} \\ \text{Emf per} \\ \text{turn} \end{array} \right\} \times \left\{ \begin{array}{c} \text{no. of turns} \\ \text{in primary winding} \end{array} \right\}$

$$E_1 = 4.44 f N_1 \phi_m$$

Rms induced Emf in secondary winding } $E_2 = 4.44 f N_2 \phi_m$

voltage transformation Ratio(k) $\dfrac{E_1}{E_2} = \dfrac{4.44 f N_1 \phi_m}{4.44 f N_2 \phi_m}$

Idea T/f no load $\dfrac{E_1}{E_2} = \dfrac{N_1}{N_2} = k$

$E_1 = V_1$

$E_2 = V_2$

$$\boxed{\frac{E_1}{N_1} = \frac{E_2}{N_2} = k}$$

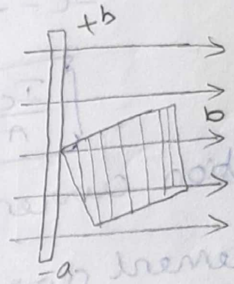
$N_2 > N_1$, $k > 1$ step up T/f

$N_1 > N_2$, $k < 1$ step down T/f

Motional EMF

Force on free electron

$\bar{F} = q(\bar{B} \times \bar{v})$



potential difference = workdone per unit charge

$= \dfrac{\text{work}}{\text{Charge}}$

$= \text{force} \times \dfrac{\text{Distance}}{\text{charge}}$

$e = q(\bar{B} \times \bar{v})\, l/q$

$$\boxed{e = l(\bar{B} \times \bar{v})}$$

(or)

$$\boxed{e = Bvl\sin\theta}$$

This is called motional EMF

motional EMF = $Bvl \sin\theta$ $(\theta = 90°)$

motional EMF = Bvl

Maxwell's Equations

Maxwell's equation from Ampere's law

Ampere's circuital law

$$\oint_l H\cdot dl = I = \iint_S J\,ds$$

Conduction current density

Ohm's law
$$I_C = V/R$$

But
$R = \frac{\rho l}{A} = \frac{l}{\sigma A}$ where, ρ is Resistivity
σ is Conductivity $(\sigma = \frac{1}{\rho})$

l is the length of the conductor
A is the area of cross section of conductor

$$I_C = \frac{V}{R} = \frac{V}{l/\sigma A} = \frac{V\sigma A}{l}$$

If $E \rightarrow$ electric field, then voltage $V = El$
substituting the values of V

$$I_C = \frac{V\sigma A}{l} = \frac{E\cancel{l}\sigma A}{\cancel{l}} = E\sigma A$$

$$\boxed{\frac{I_C}{A} = E\sigma}$$

Conduction current density $\boxed{J_C = \frac{I_C}{A}} \rightarrow \boxed{J_C = E\sigma}$

Displacement current density $I_D = \frac{dQ}{dt} \Rightarrow \boxed{I_D = C\frac{dV}{dt}}$

But $Q = CV$

$$\boxed{\therefore C = \frac{\varepsilon A}{d}}$$

where, ε is permittivity of the medium
A is area of the parallel plate of capacitor.
d is distance b/w two plates

$$I_D = \frac{\varepsilon A}{d}\frac{dV}{dt} \quad (\text{or}) \quad \frac{\varepsilon A}{\cancel{d}}\cancel{d}\frac{dE}{dt}$$

$$\boxed{V = Ed}$$

$$I_D = \varepsilon A \frac{dE}{dt}$$

$$\frac{I_D}{A} = \varepsilon\frac{dE}{dt} = \frac{dD}{dt} \qquad [\because D = \varepsilon E]$$

$$J_D = \frac{I}{A}$$

$$J_D = \varepsilon \frac{\partial E}{\partial t}$$

(or)

$$J = \frac{\partial D}{\partial t}$$

Ampere's law

$$\oint H\cdot dl = \iint_S (J_c + J_D)\, ds$$

(or) $\frac{\partial D}{\partial t}$

then $$\oint H\cdot dl = \iint_S \left(\sigma E + \frac{\varepsilon\, dE}{dt}\right) ds$$

Integral form of an ampere's law $\Big\}$ $$\oint H\cdot dl = \iint_S \left(J + \frac{dD}{dt}\right) ds$$

Stokes theorem

$$\oint H\cdot dl = \iint_S \nabla\times H\, ds$$

$$\iint \nabla\times H\cdot ds = \iint \left(J + \frac{\partial D}{\partial t}\right) ds$$

maxwell's eqn in (or) point Differential form: $$\boxed{\nabla\times H = J + \frac{\partial D}{\partial t}}$$

from Amper's law

$$\boxed{\nabla\times H = \sigma E + \varepsilon \frac{\partial E}{\partial t}}$$

Statement:- The magneto motive force around a closed path is equal to the sum of the conduction current and displacement current enclosed by the path magnetic voltage around a closed path is equal to the electric current through the path.

14/2/22 Maxwell's Equation from faraday's law:-

faraday's law

Faraday's law states that electromagnetic force (EMF) induced in a circuit is equal to the rate of magnetic flux linkage.

$$v = \frac{-d\phi}{dt} = \frac{-d}{dt}\iint_S B\cdot ds$$

But, $v = \oint E \cdot dl$ $[\because B = \mu H]$

$$\oint E \cdot dl = -\frac{d}{dt} \iint_S B \cdot ds$$

$$\oint E \cdot dl = -\iint_S \frac{\partial B}{\partial t} ds \quad \text{—(1)}$$

$$\oint E \cdot dl = -\mu \iint_S \frac{\partial H}{\partial t} ds$$

This is maxwell eqn in integral form. by applying Stoke's theorem

$$\oint E \cdot dl = \iint_S \nabla \times E \cdot ds \quad \text{—(2)}$$

Comparing eqn (1) + (2)

$$\boxed{\iint_S \nabla \times E \cdot ds = -\iint_S \frac{\partial B}{\partial t} \cdot ds}$$

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\boxed{\nabla \times E = -\mu \frac{\partial H}{\partial t}}$$ differential (or) point form

Statement :- The electro motive force around a closed path is equal to the magnetic displacement (flux density) through that closed path.

In other words

* The electric voltage is equal to the magnetic current through the path.

Maxwell's Equation from electric gauss's law :-

Gauss's law :-

Gauss' law states that electric flux emerging through any closed surface is equal to the charge enclosed by the Surface.

$$x = Q$$

$$\iint D \cdot ds = Q \quad \text{—(1)}$$

$$\iiint \rho_v \cdot dv = Q \quad \text{—(2)}$$

Then,

Integral form. $\boxed{\iint_S D \cdot ds = \iiint_V \rho_v \cdot dv}$ —— (3)

This is the maxwell's equation from electric gauss's law in integral form.

By applying divergence theorem

$$\iint_S D \cdot ds = \iiint_V \nabla \cdot D dv \quad \text{—(4)}$$

Comparing the eqn (2) & (4)

$$\iiint_V \nabla \cdot D \, dv = \iiint_V \rho_v \, dv$$

differential (or) point form $\boxed{\nabla \cdot D = \rho_v}$

Statement:- The total electric displacement to the surface enclosed by the is equal to the total charge with in the volume.

Maxwell's equation from magnetic gauss's law :-

magnetic gauss's law

Magnetic gauss's law states that, the total magnetic flux through any closed surface is equal to zero.

$$\phi = 0$$

integral form $\boxed{\iint_S B \cdot ds = 0}$ — (1)

This is the maxwell's eqn from magnetic gauss's law in integral form

By applying divergence theorem

$$\boxed{\iint_S B \cdot ds = \iiint_V \nabla \cdot B \, dv} \quad (2)$$

comparing eqn (1) & (2)

$$\iiint_V \nabla \cdot B \, dv = 0$$

differential (or) point form $\boxed{\nabla \cdot B = 0}$

Statement:- The net magnetic flux is emerging to any closed surface is zero.

Maxwell's equations are summarised as follows :-

Differential form	Integral form
1. $\nabla \times H = J + \frac{\partial D}{\partial t}$ (or) $\nabla \times H = \sigma E + \varepsilon \frac{\partial E}{\partial t}$	$\oint H \cdot dl = \iint_S \left[J + \frac{\partial D}{\partial t}\right] ds$ $\oint H \cdot dl = \iint_S \left[\sigma E + \varepsilon \frac{dE}{\partial t}\right] ds$

Differential form	Integral form
2. $\nabla \times E = -\frac{\partial B}{\partial t}$ (or) $\nabla \times E = -\mu \frac{\partial H}{\partial t}$	$\oint E \cdot dl = \iint_S \frac{\partial B}{\partial t} ds$ $\oint E \cdot dl = -\mu \iint \frac{\partial H}{\partial t} \cdot ds$
3. $\nabla \cdot D = \rho$	$\oiint D \cdot ds = \iiint \rho \cdot dv$
4. $\nabla \cdot B = 0$	$\oiint B \cdot ds = 0$

In a ~~force~~ free space, there is no charges enclosed & since it is a dielectric there is no conductivity in the medium.

i.e, $\rho = 0$ & $\sigma = 0$ conduction current is zero.

Then maxwell's eqn becomes in free space as follows:-

Differential form	Integral form
1. $\nabla \times H = \frac{\partial D}{\partial t}$	$\oint H \cdot dl = \iint \frac{\partial D}{\partial t} \cdot ds$
2. $\nabla \times E = -\frac{\partial B}{\partial t}$	$\oint E \cdot dl = \iint_S \frac{\partial B}{\partial t} \cdot ds$
3. $\nabla \cdot D = 0$	$\oiint_S D \cdot ds = 0$
4. $\nabla \cdot B = 0$	$\oiint_S B \cdot ds = 0$

Ckt theory	Field theory
1. The dependent & independent parameters, I & V are directly obtained	1. Not directly obtained but through E & H
2. this analysis orginated by its own	2. Field theory evolved from transmission theory
3. Applicable only for portion of RF Range (Radio freq)	3. Beyond RF Range i.e, micro wave
4. Laplace T/F is employed	4. maxwell's eqn employed
5. Z, Y & H parameters are used	5. S parameters are used
6. Low power is involved	6. High power involved
7. Simple to understand	7. Need visualization ability
8. Two dimensional analysis	8. Three dimensional analysis
9. freq is used as reference	9. wave length is used as reference
10. lumped components are used & involved	10. Distribution components are involved
11. parameters of medium are not involved	11. parameters of medium involved.