SREENIVASA INSTITUTE OF TECHNOLOGY AND MANAGEMENT STUDIES::CHITTOOR(AUTONOMOUS)

(SPECIAL FUNCTIONS & COMPLEX ANALYSIS)

QUESTION BANK

II- B.TECH / I - SEMESTER REGULATION: R20

DESCRIPTIVE QUESTIONS

UNIT-1: SPECIAL FUNCTIONS

Q.NO	UNIT – I	Blooms Taxonomy
1.	i) Evaluate $\int_0^\infty e^{-3x}x^6dx$ ii) Evaluate $\int_0^\infty e^{-4x}x^{\frac{5}{2}}dx$	L2,L3
2.	Evaluate $\int_0^\infty e^{-x^2}x^4dx$	L2,L3
3.	Evaluate $\int_0^\infty e^{-x^3}x^{\frac{1}{2}}dx$	L2,L3
4.	Evaluate $\int_0^1 x^m\left(\log\frac{1}{x}\right)^n dx$	L2,L3
5.	Evaluate $\int_0^{\frac{\pi}{2}}\sqrt{\tan\theta}\,d\theta$	L2,L3
6.	i)Evaluate $\int_0^1 x^3(1-x)^4dx$ ii) $\int_0^1 x^{\frac{3}{2}}(1-x)^{\frac{5}{2}}dx$	L2,L3
7.	Prove that $\int_0^1\frac{x}{\sqrt{1-x^5}}dx=\frac{1}{5}\beta(\frac{2}{5},\frac{1}{2})$	L2,L3
8.	Evaluate $\int_0^2 x(8-x^3)^{\frac{1}{3}}dx$	L2,L3
9.	Show that $J_{-n}(x)= (-1)^n J_n(x)$, Where n is a positive integer	L2,L3
10.	Show that $J_{1/2}(x)=\sqrt{\frac{2}{\pi x}}\sin x$	L2,L3
11.	Prove that $J_{-1/2}(x)=\sqrt{\frac{2}{\pi x}}\cos x$	L2,L3
12.	Express $J_5(x)$ in terms of $J_0(x)\&J_1(x)$	L2,L3
13.	Prove that $J_{3/2}(x)=\sqrt{\frac{2}{\pi x}}\left[\frac{\sin x}{x}-\cos x\right]$	L2,L3
14.	Show that $J_{\frac{-5}{2}}(x)=\sqrt{\frac{2}{\pi x}}\left(\frac{3\sin x}{x}+\frac{3-x^2}{x^2}\cos x\right)$	L2,L3
15.	Show that $\frac{d}{dx}(x^nJ_n(x))=x^nJ_{n-1}(x)$	L2,L3

Unit-II: Complex Functions

Q.No	UNIT – II	Blooms Taxonomy
1.	If cosh (u+iv) = x+iy, Prove that i) $\frac{x^2}{\cos h^2 u}+\frac{y^2}{\sin h^2 u}=1$ ii) $\frac{x^2}{\cos^2 V}-\frac{y^2}{\sin^2 V}=1$	L2,L3
2.	Separate real and imaginary parts in $\sec(x+iy)$.	L2,L3
3.	Separate real and imaginary parts of tanh z.	L2,L3
4.	Find all the values of 'z' which satisfy sin z = 2.	L2,L3
5.	Find the general and Principal values of $\log(1+i\sqrt{3})$	L2,L3
6.	Find all values of z such that sinh z = i	L2,L3
7.	If Tan (A + i B) = x + iy then Show that (i) $x^2+y^2+2x\cot 2A=1$ (ii) $x^2+y^2-2y\coth 2B=-1$	L2,L3
8.	Prove that $f(z)=z^2$ is analytic everywhere and hence find its derivative.	L2,L3
9.	Test whether or not the function sin x sin y – i cos x cos y is analytic function	L2,L3
10.	If f(z) = u+iv is an analytic function then show that u & v are harmonic functions	L2,L3
11.	Show that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\left|f(z)\right|^2=4\left|f'(z)\right|^2$	L2,L3
12.	Find the analytic function whose real part $u=e^x.\cos y$	L2,L3
13.	If $f(z)=u+iv$ is an analytic function and if $u-v=e^x(\cos y-\sin y)$; find $f(z)$ in terms of z	L2,L3
14.	Show that the function u= 2 x y + 3y is harmonic and find the corresponding analytic function. Find its conjugate	L2,L3
15.	If $w=\emptyset+i\varphi$ represent the complex potential for an electric field and $\varphi=x^2-y^2+\frac{x}{x^2+y^2}$ then find w.	L2,L3

UNIT-III: CONFORMAL MAPPING & BILINEAR TRANSFORMATION

Q.N	UNIT-III	Blooms Taxonomy
1.	Find the image of the circle $|z|=2$ by the transformation w = z+3+2i	L2,L3
2.	Find the image of the circle $|z-2i|=2$ under the transformation $w=\frac{1}{z}$	L2,L3
3.	Find the image of $1<|z|<2$ under the transformation w=2iz+1	L2,L3
4.	Find the image of the line x=4 in z-plane under the transformation $w=z^2$	L2,L3
5.	Find and plot the image of the triangular region with vertices at (0,0),(1,0),(0,1) under the transformation $w=(1-i)z+3$	L2,L3
6.	Find the image of the triangle with vertices at i,1+i,1-i in the z-plane under the transformation w=3z+4-2i	L2,L3
7.	Find the Bilinear transformation which maps the points (-1, 0, 1) into the points (0, i, 3i)	L2,L3
8.	Obtain the bilinear transformations which maps the points $z=\infty,i,0$ into the points $w=0,i,\infty$ respectively	L2,L3
9.	Find the bilinear transformations which maps the points $z=\infty,i,0$ into the points	L2,L3

	w= -1,-i,1.	
10	Find the Bilinear transformation which maps the points z = -1,0,1 into the points w=-1,-i,1	L2,L3
11	Find the image of the circle $\|z+1\|=1$ under the transformation w = 1/z.	L2,L3
12	Obtain the bilinear transformations which maps the points $z=0,1,\infty$ into the points $w=i,-1,-i$ respectively.	L2,L3
13	Find the bilinear transformations which maps the points -1,i,1 into the points 0,i, ∞	L2,L3
14	Find the region in the w-plane in which the rectangle bounded by the lines x=0,y=0,x=2 and y=1 is mapped under the transformation w=z+(2+3i)	L2,L3
15	Find the invariant points of the transformation w = $\frac{2z+4i}{iz+1}$	L2,L3

UNIT-IV: COMPLEX INTEGRATION AND COMPLEX POWER SERIES

Q.No	UNIT - IV	Blooms Taxonomy
1.	Evaluate $\int_0^{2+i} (2x+iy+1)\,dz$ along the straight line 2y = x.	L2,L3
2.	Evaluate $\int_C f(z)\,dz$, C is $z=0$ to $z=1+i$ and $f(z)=x-y+ix^2$ along the real axis from $z=0$ to $z=1$ and then along the line parallel to imaginary axis from $z=1$ to $z=1+i$	L2,L3
3.	Evaluate $\int_c (2xy+y^2)dx+(x^2-2xy)dy$ where C is the boundary of the region bounded by $y=x^2 and\ \ x=y^2$	L2,L3
4.	Use Cauchy's Integral formula to calculate $\int_C \frac{Z^2-1}{Z^2+1}dZ$ C is $\|z-i\|=1$	L2,L3
5.	Evaluate $\int_c \frac{\sin^2 z}{\left(z-\frac{\pi}{6}\right)^2}dz$ where C is the circle $\|z\|=1$	L2,L3
6.	Evaluate $\int_c \frac{\log z\ dz}{(z-1)^2}$ where c : $\|z-1\| = \frac{1}{2}$.	L2,L3
7.	Evaluate . $\oint_C \frac{z+4}{z^2+2z+5}dz$ where c is the circle (i) $\|z+1-i\|=2$ $(ii)\|z+1+i\|=2$	L2,L3
8.	Evaluate $\oint_C \frac{e^z}{(z-1)(z-2)}dz$; where C is the Circle $\|z\|=3$	L2,L3
9.	Use Cauchy's integral formula to evaluate $\oint_C \frac{\sin\pi z^2+\cos\pi z^2}{(z-1)(z-2)}dz$ where c is the circle $\|z\|=3$.	L2,L3
10.	Expand Sin z in Taylor's series about z= $\frac{\pi}{4}$	L2,L3

11.	Obtain the Taylor's series to represent the function $\frac{1}{z^2+4z+3}$, in the region $\|z\|<1$.	L2,L3
12.	Expand $f(z)=\frac{e^{2z}}{(z-1)^3}$ as Laurent's series about $z=1$.	L2,L3
13.	Obtain the Taylor's series to represent the function $\frac{z^2-1}{(z+2)(z+3)}$, in the region $\|z\|<2$.	L2,L3
14.	Find the Laurent expansion $f(z)=\frac{z^2-1}{(z-2)(z-3)}$, $over\ \ 2<\|z\|<3$	L2,L3
15.	Find Laurent expansion of $\frac{1}{z^2-4z+3}$ $over\ 1<\|z\|<3$	L2,L3

UNIT-V: RESIDUE CALCULUS

Q.N	UNIT - V	Blooms Taxonomy
1.	Determine the poles and residues at each pole of $\frac{z+1}{(z-2)(z+3)^2}$	L2,L3
2.	Find the poles and residues at each pole of $\frac{z+1}{z(z-4)^2}$.	L2,L3
3.	Find the poles and residues at each pole of $\frac{z^2-2z}{(z+1)^2(z^2+1)}$.	L2,L3
4.	Find the poles and residues at each pole of $\frac{z\,e^{iz}}{z^2+a^2}$.	L2,L3
5.	Find the poles and residues at each pole $\frac{z\,e^z}{(z-1)^2}$	L2,L3
6.	Find the poles and residues at each pole of $\frac{3z+1}{(z+1)(2z-1)}$	L2,L3
7.	Evaluate $\int_C \frac{z^3}{(z-1)^2(z-3)}dz$ where C is $\|z\|=2$ by residue Theorem	L2,L3
8.	Evaluate $\int_C \frac{z^2-2z}{(z+1)^2(z^2+4)}dz$ c : |z| = 3 using residue theorem	L2,L3
9.	Evaluate $\int_c \frac{\sin^2 z}{\left(z-\frac{\pi}{6}\right)^2}dz$ c : |z| = 2 using residue theorem	L2,L3
10.	Show that $\int_0^{2\pi}\frac{d\theta}{a+b\sin\theta}=\frac{2\pi}{\sqrt{a^2-b^2}}$ $a>0,\ b>0$ using Residue Theorem	L2,L3
11.	Evaluate $\int_0^{2\pi}\frac{d\theta}{2+\cos\theta}$	L2,L3
12.	Show that $\int_0^{2\pi}\frac{\cos 2\theta}{1-2a\cos\theta+a^2}=\frac{2\pi a^2}{1-a^2}$ where $a^2<1$	L2,L3

13.	Evaluate $\int_{-\infty}^{\infty} \frac{x^2\,dx}{(x^2+1)(x^2+4)}$ using Residue theorem.	L2,L3
14.	Prove that $\int_{0}^{\infty} \frac{\sin x}{x}\,dx = \frac{\pi}{2}$	L2,L3

Short Answer Type

UNIT-1: SPECIAL FUNCTIONS & COMPLEX FUNCTIONS- I

Q.N	UNIT - I	Blooms Taxonomy
1.	Define Gamma	L1
2.	Define Beta function	L1
3.	Find $\Gamma\left(\frac{7}{2}\right)$	L2,L3
4.	Evaluate $\int_0^\infty e^{-x}x^8dx$	L2,L3
5.	Evaluate $\int_0^\infty e^{-2x}x^3dx$	L2,L3
6.	Evaluate $\int_0^\infty e^{-x}x^{\frac{1}{2}}dx$	L2,L3
7.	Evaluate $\int_0^\infty e^{-3x}x^4dx$	L2,L3
8.	State Relation between Beta and Gamma function	L1
9.	Find $\beta(3,4)$	L2,L3
10.	Find $\beta\left(\frac{1}{2},\frac{3}{2}\right)$	L2,L3
11.	Evaluate $\int_0^1 x^2(1-x)^4dx$ in terms of Beta function	L2,L3
12.	Evaluate $\int_0^1 x^{\frac{3}{2}}(1-x)^{\frac{5}{2}}dx$ in terms of Beta function	L2,L3
13.	Write two different forms of Beta function	L1
14.	Evaluate $\int_0^{\frac{\pi}{2}} \sqrt{sin\,\theta}\,d\theta$	L2,L3
15.	Define Bessel’s Differential equation	L1
16.	Define $J_n(x)$	L1
17.	Define Generating function of Bessel function	L1
18.	Define Orthogonality of Bessel function	L1
19.	State any two recurrence relations of Bessel functions	L1
20.	Show that $\frac{d}{dx}(xJ_1(x)) = xJ_0(x)$	L1

UNIT-II: COMPLEX FUNCTIONS -II

Q.N	UNIT - II	Blooms Taxonomy
1.	Find real and imaginary parts of $\sin z$	L1
2.	Find real and imaginary parts of $\cos z$	L1
3.	Find real and imaginary parts of $\sinh\ z$	L1
4.	Show that $\sin iz = i \sinh z$	L2,L3
5.	Find real and imaginary parts of $\log\ z$	L1
6.	Find general values of log (1+i)	L2,L3
7.	Find general values of log $(1-\sqrt{3}i)$	L2,L3
8.	Solve $e^z = -2$	L2,L3
9.	Solve $e^z = 1+2i$	L2,L3
10.	Define Analytic function with an example	L1
11.	State Necessary condition of Analytic function	L1
12.	State Sufficient condition of Analytic function	L1
13.	Write C-R equations in Cartesian coordinates	L1
14.	Write C-R equations in Polar coordinates	L1
15.	Show that f(z)=z is analytic for all z	L2,L3
16.	If f(z) is analytic then $f^1(z)$ is ?	L1
17.	If $f(z) = z^2 + 2z + 3$ is analytic then $f^1(z)$ is	L2,L3
18.	Show that $u = x^2 - y^2$ is harmonic function.	L2,L3
19.	What is the formula for evaluation of f(z) when we know u by Milon-Thomson method	L1
20.	What is the formula for evaluation of f(z) when we know v by Milon-Thomson method	L1

UNIT-III: CONFORMAL MAPPING AND BILINEAR TRANSFORMATION

Q.N	UNIT - III	Blooms Taxonomy
1.	Define conformal mapping	L1
2.	The fixed points of the transformation $w = z^2$	L2,L3
3.	The fixed points of the transformation $w = \frac{z}{2-z}$	L2,L3
4.	The fixed points of the transformation $w = \frac{z-1}{z+1}$	L2,L3
5.	The critical points of the transformation $w = \frac{6z-9}{z}$	L2,L3
6.	The fixed points of the transformation $w = \frac{2z-5}{z+4}$	L2,L3
7.	The invariant points of the transformation $w = \frac{1+z}{1-z}$	L2,L3
8.	The cross-ration of four points Z_1, Z_2, Z_3, Z_4	L1
9.	Image of $\lvert Z \rvert = 2$ under w=z+(3+2i) is?	L2,L3
10.	Image of $\lvert Z \rvert = k$ under w=z is?	L2,L3
11.	Image of $\lvert Z - 1 \rvert = 1$ under w=1/z is?	L2,L3

12.	Define Bilinear/Mobius transformation	L1
13.	Define Invariant/fixed points	L1
14.	Find the fixed points of the transformation $w = z^2 + 1$	L2,L3
15.	Find the fixed points of the transformation $w = z^3$	L2,L3
16.	Find the Image of z=1+3i under the transformation w=z+i	L1
17.	Find the Image of z=(5,8) under the transformation w=2z+3	L2,L3
18.	Find the Image of y=c under w=z+i	L2,L3
19.	Find the Image of $\|Z\| = 1$ under w=z+i	L2,L3
20.	Find the Image of y = 1 under w=z+2i	L2,L3

UNIT-IV: COMPLEX INTEGRATION AND COMPLEX POWER SERIES

Q.N	UNIT - IV	Blooms Taxonomy
1.	Define zeros of an Analytic function	L1
2.	Write the zeros of the function $f(z)=z^2+3z+2$	L2,L3
3.	Write the zeros of the function f(z)= sin z	L2,L3
4.	Define singularity of an analytic function	L1
5.	Define Isolated singularity	L1
6.	Define removable singularity	L1
7.	Define Pole of order n	L1
8.	Define essential singularity	L1
9.	State Cauchy's Integral Theorem	L1
10.	State Cauchy's Integral formula	L1
11.	State Generalized Cauchy's Integral formula	L1
12.	State Taylor's Theorem	L1
13.	State Laurent's Theorem	L1
14.	Evaluate $\int_c \frac{z^2-z+1}{z-1} dz$ where c is the circle $\|z\| = 1/2$	L2,L3
15.	Evaluate $\int_c \frac{z^2+5}{z-3} dz$ where c is the circle $\|z\| = 1$	L2,L3
16.	Evaluate $\int_c \frac{e^{-z}}{z+1} dz$ where c is the circle $\|z\| = 1/2$	L2,L3
17.	Evaluate $\int_C \frac{e^{2z}}{z-2} dz$ where C is $\|z\|=1$	L2,L3
18.	Define Taylor series	L1
19.	Define Laurent series	L1
20.	Write the Taylor series of $f(z)=e^z$ about z=1	L2,L3

UNIT-V: RESIDUE CALCULUS

Q.N	UNIT - V	Blooms Taxonomy
1.	Define Residue at a singular point of an analytic function	L1
2.	If f(z) has a simple pole at Z=a then Residue of f(z) at z=a is	L1
3.	If f(z) has a pole of order m at Z=a then Residue of f(z) at z=a is	L2,L3

4.	Write the poles of the function $f(z) = \frac{z^2}{(z-1)(z+2)}$	L2,L3
5.	Write the poles of the function $f(z) = \frac{3z+1}{(z+1)(2z-1)}$	L2,L3
6.	Find the poles of $f(z) = \frac{z}{z^2-4}$	L2,L3
7.	Write the poles of the function $f(z) = \frac{z-3}{z^2+2z+5}$	L2,L3
8.	Find the residue at pole z = -i of $f(z) = \frac{z}{z^2+1}$	L2,L3
9.	Find the residue of the function $f(z) = \frac{z^2}{(z-1)(z+2)}$ at z= -2	L2,L3
10.	Find the residue of $f(z) = \frac{z^3}{z^2-1}$ at z = 1	L2,L3
11.	Find the residue of at z = -1 of $f(z) = \frac{3z+1}{(z+1)(2z-1)}$	L2,L3
12.	Find the residue at z = -2i of $f(z) = \frac{1}{(z^2+4)^2}$	L2,L3
13.	Find the residue at pole of $f(z) = \frac{e^z}{(1+z)^2}$	L2,L3
14.	Find the residue of $f(z) = \frac{e^{-z}}{z^2}$ at $z=0$	L2,L3
15.	Evaluate $\int_c \frac{z+1}{(z-2)(z-3)}\,dz$ where C is $\lvert z \rvert = 1.5$ by residue theorem	L2,L3
16.	Find the residue of the function $f(z) = \frac{z^2}{(z-1)(z+2)}$ at z= 1	L2,L3
17.	Find the poles of f(z)=cot z	L2,L3
18.	Residue of $f(z) = \frac{z^2}{(z-i)}$ at z=i	L2,L3
19.	Residue of $f(z) = \frac{z+1}{z^2(z-3)}$ at z =3	L2,L3
20.	State Cauchy Residue Theorem	L2,L3