Unit - 1

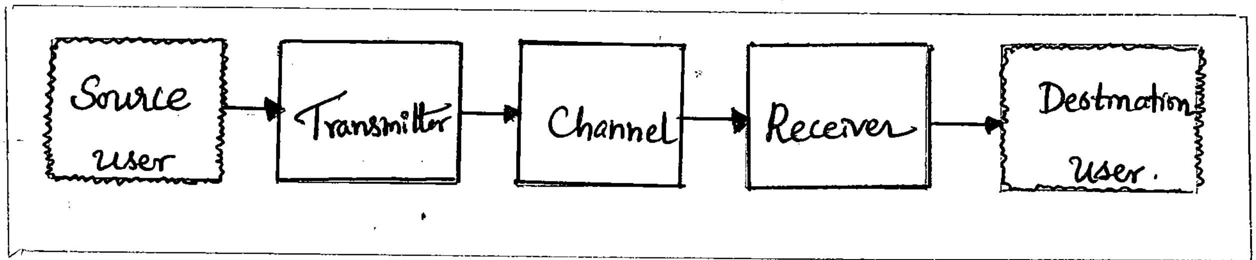
Base Band Data Transmission - I

Syllabus: Overview of Analog modulation, General block diagram of digital communication system, Analog to Digital conversion (ADC): Samplers, Quantizers: Uniform, non-uniform, differential, types of encoders, Pulse code modulation (PCM), Noise in PCM, Differential PCM (DPCM), Noise in DPCM, Delta modulation (DM), noise in DM, Adaptive DM (ADM), continuously variable slope DM (CVSDM), comparision between PCM, DPCM, DM and ADM.

Introduction:

Communication: Exchanging the information from source to destination (or) one user to another user.

A general communication system as.



Communication System.

* Source user : Speech, video etc.
* Transmitter : Conveys information
* Channel : Invariably distorts signals
* Receiver : Extracts information signal
* Destination/user : Utilizes information.

✓ The purpose of a communication system is to transport an information bearing signal from a source to a user destination via a communication channel.

* Basically a communication system is of analog or digital type.

✓ In <u>analog communication</u> system, the information-bearing signal is continuously varying in both amplitude and time and it is used directly to modify some characteristics of a sinusoidal carrier wave such as Amplitude (AM), frequency (FM) or phase (PM).

✓ In a <u>digital communication system</u>, the information bearing signal is processed so that it can be represented by a sequence of discrete messages for <u>higher rate</u> and as <u>accurate</u> as possible.

General block diagram of Digital Communication System :

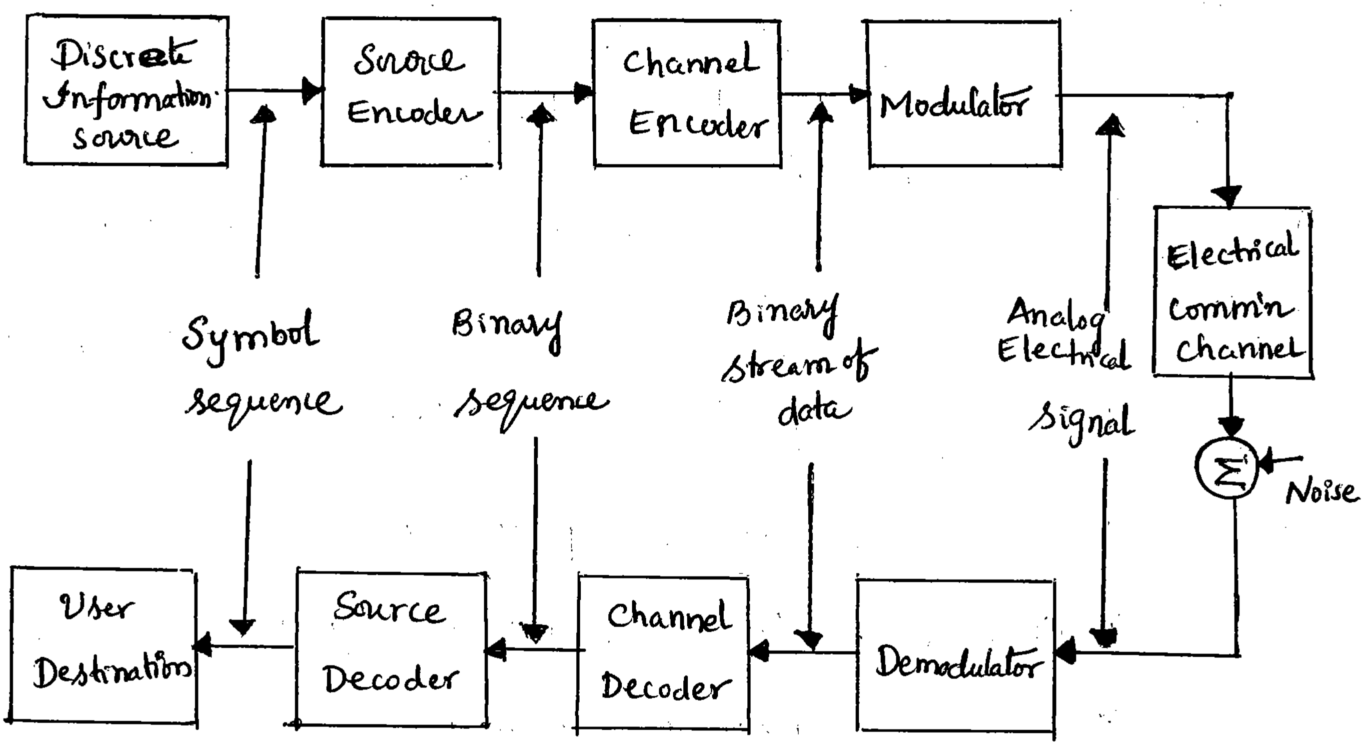


Fig: Blockdiagram Digital Communication System.

The basic building blocks of digital communication systems are

1. Information source
2. Source encoder and decoder.
3. Communication channel
4. Channel encoder and decoder
5. Modulator and Demodulator
6. User destination.

1 Information Source :

~~Usually an information source may be analog or discrete.~~

- If the source is analog, then the analog information can be transformed into discrete information through the process of sampling and quantizing.
- If the source is discrete, it emits the discrete information symbols.

Ex: Analog information

A microphone activated by speech or a TV camera scanning a scene, emit one or more continuous amplitude signals.

Ex: Discrete information.

Teletype or the numerical output of a computer consisting of sequence of discrete symbols or letters.

- Discrete information sources are characterized by the following parameters.

a) Source alphabet (symbols or letters)
b) Symbol rate
c) Source alphabet probabilities.
d) Probabilistic dependance of symbols in a sequence.
e) Entropy (bits/sym) & Source information rate.

② Source encoder and decoder:

- The function of source encoder is to convert the symbol fed at the input into a binary sequence such as 0's & 1's, by assigning codewords to the symbols in the input sequence.
- The input to the source encoder (coder) is a string of symbols occuring at a rate of samples/sec.
- The output of source encoder is binary sequence.

* The function of source decoder converts the binary sequence of the channel decoder into a symbol sequence.
* The decoder for a system using fixed length codeword is simple but, the decoder for a system using variable length codewords will be very complex.
* Decoders for such systems must be able to scope with a no. of problems such as growing memory requirements and loss of synchronization due to bit errors.

③ Communication Channel:

- A communication channel provides the electrical connection between source and destination
- The channel may be a pair of wires & a teletype (telephone) link & free space etc. over which the information signal is radiated.

* Due to the physical limitations of communication channel is have only finite bandwidth and the information bearing signal often suffers amplitude and phase distortion, the signal power also decreases due to the attenuation of the channel.
* Further more the signal is corrupted by unwanted, unpredictable electrical signals refered to as noise.

- One of the way in which the effect of noise can be minimized is to increase the signal power.
- However signal power can not be increased beyond certain levels because of non linear effect that become dominant as the signal amplitude is increased.

For the reason the SNR or (S/N) at the output of channel can be maintained to a proper value which is one of the important parameter.

- If the parameters of communication channel are known the Channel Capacity (C) which represents the max. rate at which nearly errorless data transmission is critically possible & given by

$$\text{channel Capacity} \quad \boxed{C = B \cdot \log\left(1 + \frac{S}{N}\right) \text{ Bits/Sec}}$$

where B - Bandwidth
S/N - Signal to Noise ratio.

@ Channel encoder and decoder:

- Digital channel coding is practical method of realizing high transmission reliability & efficiency that otherwise may be achieved only by the use of signals of longer durations in the modulation, demodulation process.
- Error control is accomplished by the channel coding operation by adding extra bits to the output at the source coder, while these extra bits themselves convey no information make it possible for the receiver to detect and/or correct some of the errors in the information bearing bits.

There are two methods of channel coding

a) Block Coding method.

b) Convolutional Coding method.

* In block coding method the encoder takes a block of 'K' information bits from the source encoder and adds 'r' error control bits.

* In convolution method information bearing message stream is encoded in a continuous fashion by continuously interleaving information bits and error control bits.

✓ The important parameter of channel encoder are

a) Method of coding

b) Rate or efficiency of the coder.

c) Error control capabilities..

d) Complexity of the encoder.

✓ The channel decoder recovers the information bearing bits from the coded binary stream error detection of possible corrections are also performed by the channel decoder.

✓ The complexity of the decoder and the time delay involved in the decoders are important design parameters.

⑤ Modulator and Demodulator:

✓ The modulator accepts a bit stream as it's input and converts to an electrical waveform suitable for transmission over the commn channel.

✓ Modulation can be effectively used to minimize the effect of channel noise, to match the characteristics, to provide the capability to multiplex many signals and to over come equipment limitations.

* The important parameters of the modulation are

a) Type of waveform used.

b) The duration of the waveform

c) The power level.

d) The bandwidth used.

- Demodulation is the reverse process of modulation by which the extraction of message from the modulated signal.
- The characteristics of modulator, demodulator & the channel establish an average bit error rate between channel encoder & decoder.
- The bit error rate and the corresponding symbol error rate will be higher than desired. A lower bit error rate can be accomplished by redesigning the modulator and the demodulator (MODEM) by using control coding

⑥ User Destination:

The extracted symbols are received by the user destination with high data rate and accurately.

Advantages of Digital communication over Analog Communication:

① The digital comm'n provides greater immunity to noise when compared to analog comm'n systems.

② The digital comm'n provides detecting and correcting the errors which are occured during the Txion of the signal.

③ Digital signals which are inherently compatible with computers have a potential to be stored, retrived, processed and manipulated for signal enhancement & improved performance.

④ Using data encryption, only permitted receivers may be allowed to detect the transmitted data i.e. High degree of Security of information. This property is of its most importance in military applications.

⑤ Since the transmitted signal is digital in nature, therefore a large amount of noise interference may be tolerated.

⑥ Since in digital communication, Channel Coding is used, therefore the errors may be detected and corrected in the receiver.

⑦ In digital communication, the speech, video and other data may be merged and transmitted over a common channel using multiplexing.

⑧ Digital communication is adaptive to other advanced branches of data processing such as digital signal processing (DSP), digital image processing (DIP) and data compression etc.

⑨ Digital circuits are more reliable & greater dynamic range is possible.

⑩ The digital communication systems are simpler and cheaper compared to analog communication systems because of the advances made in the IC technologies.

Disadvantages:

① More complex in Circuitary.

② Higher channel bandwidth, ie Due to analog to digital conversion the data rate becomes high.

∴ More transmission bandwidth is required for digital communication.

③ Digital communication needs synchronization in case of synchronous modulation. However, the advantages of digital comm'n overweight the disadvantages.

Telephone channel	Coaxial Cable	Optical fiber
✓ Frequency range : (800 – 3400) Hz.	✓ Bandwidth: 39 GHz	✓ Largest bandwidth : 1000 THz.
✓ Transmission rate : 16.8 Kbps.	✓ Data rate : 274 Mbps.	✓ Free from interference
✓ For voice & data comm'n over long distance	✓ Free from external interference	T - Tera - 10^{12} Hz.
	✓ Repeater spacing (1 km - disadvantage).	

Analog to Digital Conversion (ADC) :

The basic operations are usually performed to convert analog to digital as

1. Samplers
2. Quantizers
3. Encoders.

Sampling :

The process of representing an analog signal by sequence of sampled segments is called sampling.

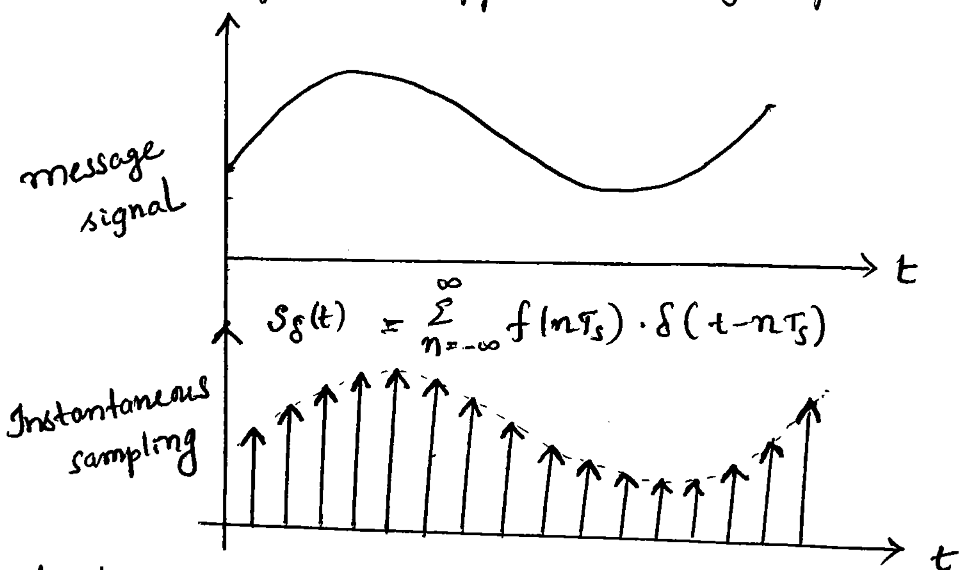
There are three types of sampling.

1. Instantaneous sampling / ideal sampling.
2. Natural sampling.
3. Flat-top sampling.

① Instantaneous Sampling :

Instantaneous sampling signal is a true impulse sequence represented by $\sum_{n=-\infty}^{\infty} \delta(t - nT_s)$.

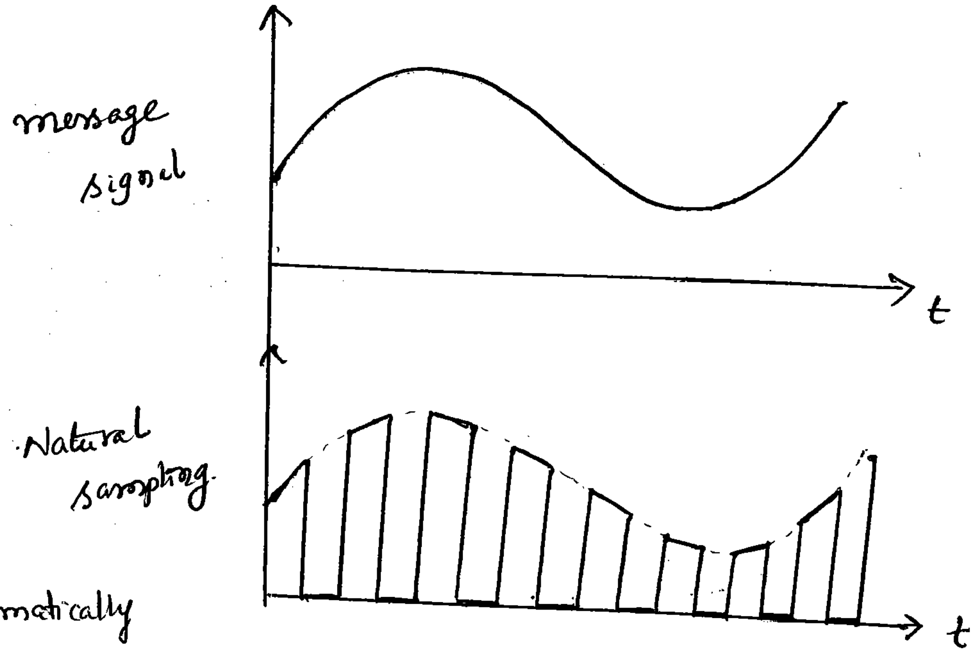
Each impulse is approximated by a pulse of infinite decimal.



Disadvantage: Strength of impulse is very small due to small width when this instantaneously sampled signal is transmitted through the channel. To reduce the effect of noise and the sampled signal the strength of each sample must be increased.

② Natural Sampling :

In natural sampling the samples are rectangular pulses of amplitude 'A' duration 'τ' with a time period of 'T_s'. These pulses are not flat but to follow the natural slope of input waveform.



Mathematically

The sampled signal $s(t) = c(t) \cdot g(t) \longrightarrow ①$

where $g(t) \rightarrow$ message signal

$c(t) \rightarrow$ Complex fourier series as

$$c(t) = f_s T_A \cdot \sum_{n=-\infty}^{\infty} \text{Sinc}(n f_s T) \, e^{j2\pi f_s n t} \longrightarrow ②$$

where $f_s = \frac{1}{T_s}$ - Sampling frequency.

From ① & ②

$$\therefore \quad s(t) = f_s \cdot T_A \cdot \sum_{n=-\infty}^{\infty} \text{Sinc}(n f_s T) \, e^{j2\pi f_s n t} \cdot g(t)$$

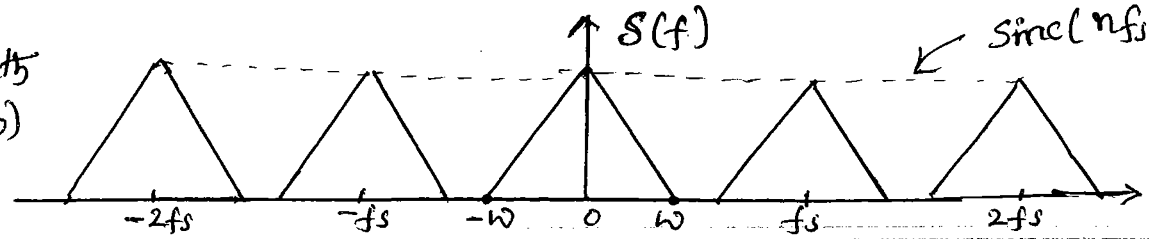
Apply fourier transform on both sides

$$S(f) = f_s \cdot T_A \cdot \sum_{n=-\infty}^{\infty} \text{Sin}(n f_s T) \, \delta(f - n f_s) * G(f).$$

Applying the frequency shifting property of the delta function.

$$\boxed{S(f) = f_s \cdot T_A \cdot \sum_{n=-\infty}^{\infty} \text{Sinc}(n f_s T) \, G(f - n f_s)}$$

Bandwidth
$= W - (-W)$
$= 2W$.



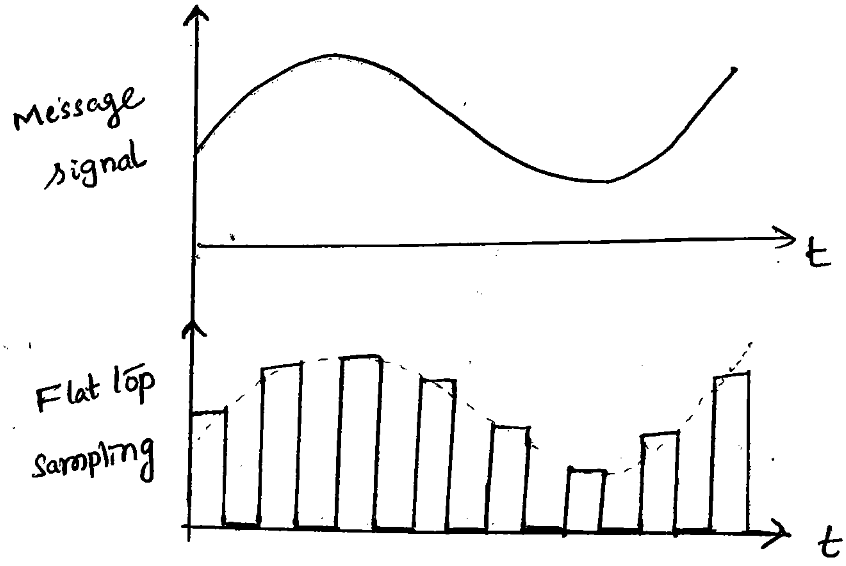
The spectrum is drawn assuming that $g(t)$ is strictly band limited.
ie It contains no frequencies outside the band $-W$ to W and that the sampling rate is greater than $2W$ ($f_s > 2W$) nyquist rate.
So there is no aliasing effect.

Advantage : This is usually used in time division multiplexing.

Disadvantage : The electronic circuitry needed to perform is complicated because the shape of the pulses has to be maintained.

③ Flat top sampling :

In flat top sampling the top of the pulses are flat & that each pulse is lengthened to the duration 'T' for the convience of Txion, to avoid the use of a excessive between pulses.



Disadvantages :

- ✓ Due to flat top sampling amplitude distortion & delay is introduced. The distortion caused by lengthening the sampled pulse is called as apperature effect
- ✓ The appertune effect can be corrected by connecting an equalizer to the low pass reconstructed filter, the equalizer will compensate the aperature effect by increasing the frequency.
- ✓ To ensure perfect reconstruction of the message signal at the Rxer, The sampling rate must be greater than twice the highest frequency

component 'ω' of the message signal in accordance with the sampling theorem.

* In practice a lowpass anti alliasing filter is used at the front end of sampler to exclude frequencies greater than ω before sampling.

Hence the application of sampling permitts the reduction of continuosly varying message signal to a limited no. of discrete values per second.

Quantization:

Continuous sample → $\boxed{\text{Quantizer}}$ → Discrete Sample

Representing the analog sample values by a finite set of levels is called Quantizing.

" The process of converting continuous amplitude, discrete time signal such as sampled version of analog signal into discrete amplitude discrete time signal is called Quantization."

(OR)

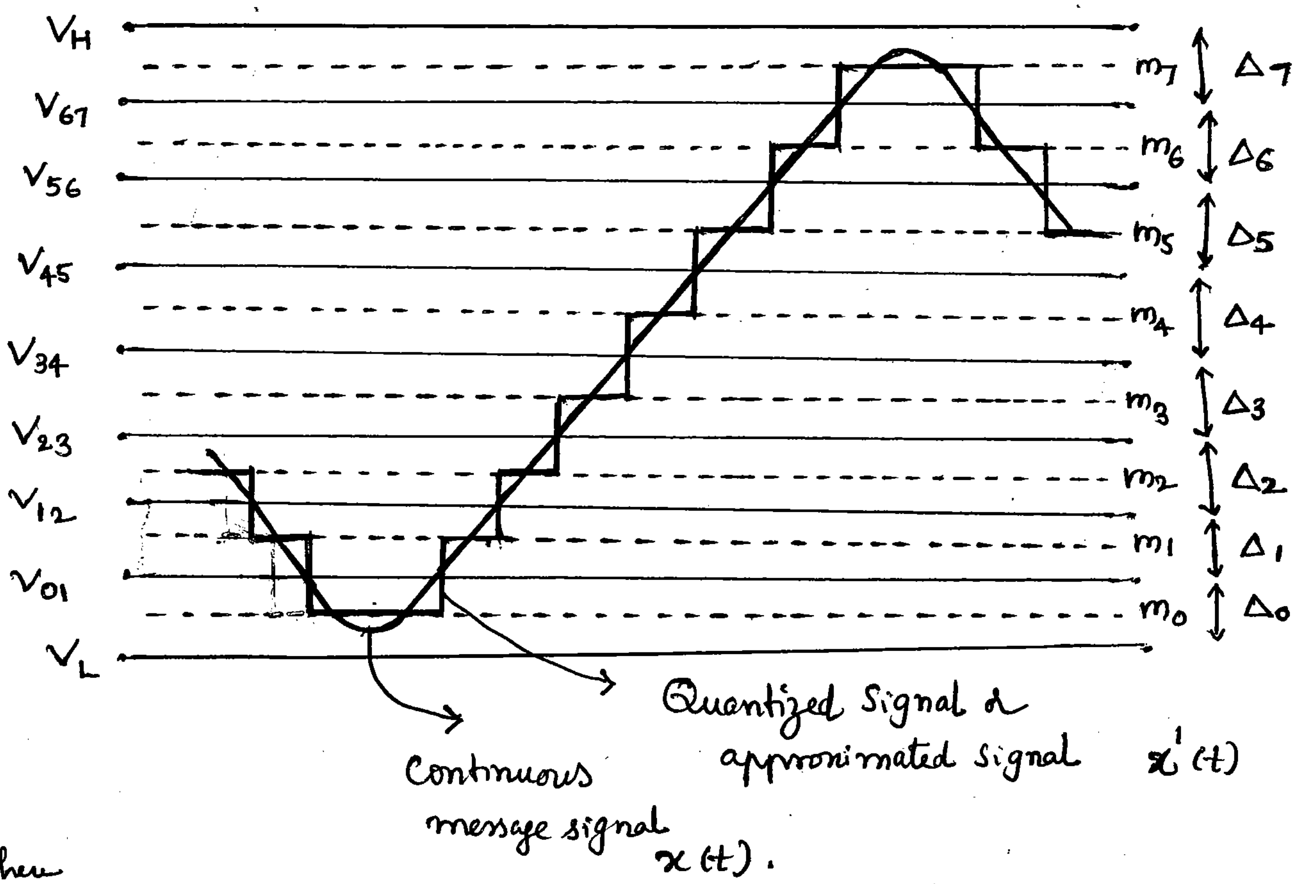
" The approximating the amplitude of each sample to the nearest value from a set of predetermined discrete amplitude levels called Quantization or Representation levels.

Need for quantization:

A continuous signal such as voice or picture signal has continuous range of amplitude when these signals are sampled an infinite no. of amplitude levels will result.

But human sense cannot detect finite differences in the amplitude of the signal. So, continuous amplitude signal may be approximated by a discrete amplitude signal on a min. error basis from a available set of values.

✓ Quantization is referred to breaking down a continuous amplitude range into a finite no. of amplitude values & steps.



where

$$\Delta_0 = \Delta_1 = \Delta_2 = \ldots\ldots = \Delta_7 = \Delta \rightarrow \text{Step Size}$$

$$\boxed{\text{Step Size } (\Delta) = \frac{V_H - V_L}{L}}$$ where L - No. of levels.

$m_0, m_1, m_2 \ldots\ldots m_7$ are quantization levels or representation levels.

Quatization Error :

The difference between the continuous message signal and the quantized signal is known as quantization error. It is denoted as 'e'

$$\boxed{e = x(t) - x'(t)} \; ; \; -\Delta/2 < e < \Delta/2$$

* At every instant of time $x'(t)$ doesn't change with time or it makes discrete quantized jump of step size 'Δ'.

✓ The quantization levels are seperated by an amount of Step Size 'Δ' & the other hand the seperation of the criterion levels V_L and V_H each from their nearest quantization levels is $\Delta/2$.

Thus the quantized signal approximated to the original signal. The approximation can be improved by reducing the size of step their by increasing the no of quantization levels.

Types of Quantizers:

1. Uniform quantizer.
2. Non uniform quantizer.
3. Differential quantizer.

✓ In uniform quantization, step size remains same through out the input range.

ie The representation levels are uniformly placed.

✓ In non uniform quantization, step size varies according to the input signal values.

✓ In differential quantization which is used to reduce the Band-width by taking difference between two samples.

Uniform Quantizer:

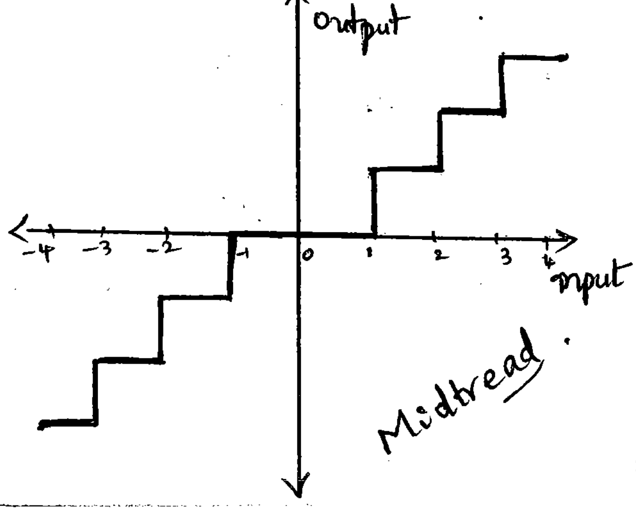
⇒ Quantizer Characteristics are two types which is in staircase fun.

a) Midtread quantization.
b) Midrisor quantization.

} Belogs to Uniform quantization

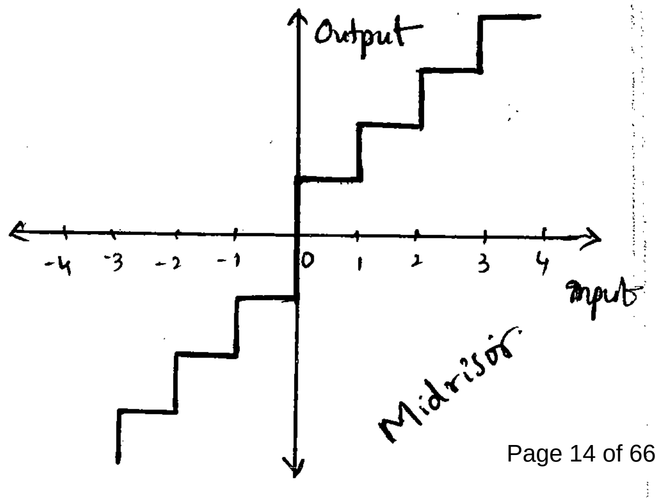
a) Midtread quantization:

Origin is lies middle of the staircase of thread.



b) Mid risor quantization:

Origin is lies middle of the risor of staircase.



** Let the no-of quantization levels can be given as

L (or) $\boxed{Q = 2^N}$ where N - no-of bits/sample

✓ Step size of each quantization level is denoted by 'Δ'

$$\Delta = \frac{V_H - V_L}{L} \text{ (or) } \frac{\text{input peak-to-peak voltage}}{\text{No-of quantization levels}}.$$

$$\therefore \boxed{\Delta = \frac{V_{P\text{-}P}}{2^N}}$$

✓ The bit duration is denoted by 'T_b' & it is given by

$\boxed{T_b = \frac{T_s}{N}}$ where T_s - Sampling period
N - No-of bits per sample

✓ The bit rate & max. bandwidth is calculated as

R_b = bit rate = Sampling rate x No-of bits/sample ↳ sample/sec * bits/sample = bits/sec = bps.

$$= \frac{1}{T_s} \times N$$

$$= \frac{N}{T_s} = \frac{N}{T_s} \Rightarrow \frac{1}{T_b}$$

$\therefore$ Bitrate $\boxed{R_b = \frac{1}{T_b}}$ bps (or) $\boxed{(B.W)_{max} = \frac{1}{T_b} \text{ Hz}}$.

✓ The Quantization noise / error = Actual sample value − Quantized/approximated Value

$$e = x(t) - x'(t).$$

$\therefore$ The maximum quantization error $\boxed{Qe_{max} = \pm \Delta/2}$

where Δ - Step size

Note:

Let No-of bits $\boxed{N = 3}$.

The no-of Quantization levels $Q = 2^N$

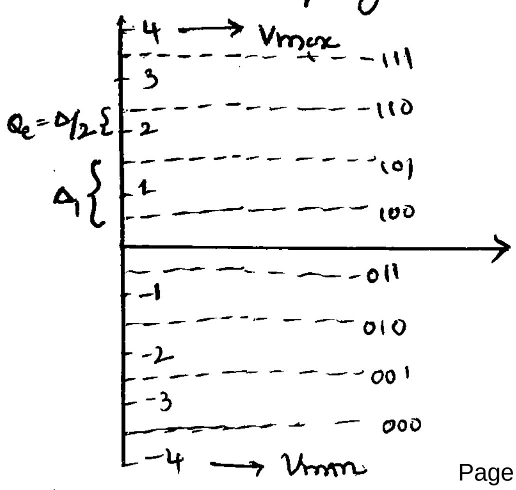
$$Q = 2^3 = 8 \Rightarrow \boxed{Q = 8}.$$

* The step size $\Delta = \frac{V_{PP}}{2^N} = \frac{V_{max} - V_{min}}{2^N}$

$$= \frac{4-(-4)}{2^3} = \frac{8}{8} = 1 \Rightarrow \boxed{\Delta = 1}$$

$\therefore$ The Max. Quantization error

$$Qe_{max} = \pm \Delta/2 \Rightarrow \boxed{Qe_{max} = \pm 1/2}$$



Signal to Quantization Noise Ratio (SQNR):

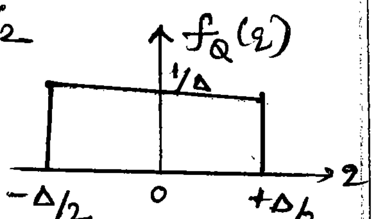
It is defined as the ratio of signal power to the quantization noise power ie $SQNR = \left(S/N_q\right)$.

Quantization Noise power (N_q):

The Quantization error is randomly varying from $-\frac{\Delta}{2}$ to $+\Delta/2$ & it is consider as uniform random variable.

Let 'q' be the random variable its probability density fun PSD as

$$f_Q(q) = \begin{cases} \frac{1}{\Delta}, & -\Delta/2 < q < +\Delta/2 \\ 0, & \text{otherwise.} \end{cases}$$



The mean of the quantization error is being zero.

$\therefore$ The toal quantization noise power is equal to mean square value & variance.

$$\therefore N_q = \sigma_{Q^2} = E[Q^2]$$

$$= \int_{-\infty}^{\infty} q^2 \cdot f_Q(q)\, dq \qquad \left[\because E[x^2] = \int_{x=-\infty}^{\infty} x^2 \cdot f_X(x)\,dx\right]$$

$$= \int_{-\Delta/2}^{+\Delta/2} q^2 \cdot \frac{1}{\Delta} \cdot dq$$

$$= \frac{1}{\Delta} \cdot \frac{q^3}{3}\Bigg|_{-\Delta/2}^{\Delta/2} = \frac{1}{\Delta}\left[\frac{\Delta^3}{24} + \frac{\Delta^3}{24}\right] = \frac{\Delta^3}{\Delta \times 12} = \frac{\Delta^2}{12}$$

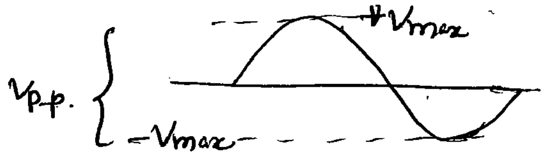
$$\therefore \boxed{N_q = \frac{\Delta^2}{12}}$$

we know Step Size $\Delta = \frac{V_{pp}}{Q_{AL}} = \frac{V_{p\text{-}p}}{Q} = \frac{V_{max} - (-V_{max})}{Q}$

$Q = 2^N$

where N-no of bits/sample

$$\therefore \boxed{\Delta = \frac{2V_{max}}{Q}}$$



$\therefore$ Quantization noise power $N_q = \frac{\Delta^2}{12} = \frac{(2V_{max})^2}{12 \cdot Q^2} = \frac{4V_{max}^2}{12Q^2}$

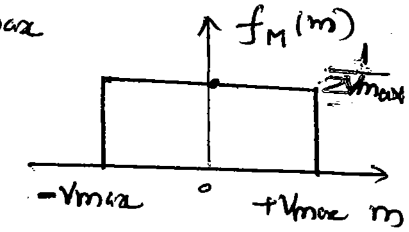
$$\therefore \boxed{N_q = \frac{V_{max}^2}{3Q^2}}$$

where $Q = 2^N$.

Signal power (S) :

Let 'm' be the uniform random variable which represents input signal & probability density function can represented as

$$f_M(m) = \begin{cases} \dfrac{1}{2V_{max}} , & -V_{max} < m < +V_{max} \\ 0 , & \text{Other wise} \end{cases}$$



∴ The total signal power is mean square value

$$S = \int_{-\infty}^{\infty} m^2 . f_M(m) . dm \qquad f_M(m) = \frac{1}{2V_{max}}.$$

$$= \int_{m=-V_{max}}^{V_{max}} m^2 . \frac{1}{2V_{max}} . dm$$

$$= \frac{1}{2V_{max}} . \left[\frac{m^3}{3}\right]_{-V_{max}}^{+V_{max}}$$

$$= \frac{1}{2V_{max}} . \left[\frac{V_{max}^3}{3} + \frac{V_{max}^3}{3}\right]$$

$$S = \frac{\cancel{2} . V_{max}^3}{\cancel{2} . 3 . V_{max}} \Rightarrow \boxed{S = \frac{V_{max}^2}{3}}$$

∴ The signal to Quantization noise ratio $= S/N_q$

$$= \frac{V_{max}^2/3}{V_{max}^2/3Q^2}$$

$$SQNR = Q^2$$

We know $Q = 2^N$

$$= (2^N)^2 = 2^{2N}$$

∴ $\boxed{SQNR = Q^2 = 2^{2N}.}$ where N - no-of bits/sample.

$$(SQNR)_{\text{in dB}} = 10 \log 2^{2N}.$$

If $N = 1 \Rightarrow SQNR = 6\ dB$

$N = 2 \Rightarrow SQNR = 12\ dB$

$N = 3 \Rightarrow SQNR = 18\ dB$

∴ In general

$$\boxed{(SQNR)_{dB} = 6N\ dB}$$

ie As no.of bits increases, Quantization error decreases, So SQNR increases thus bandwidth increases.

Average Signal to Quantization Noise ratio :

It is defined as the ratio of average signal power to the quantization noise power.

∴ Average (or) peak (or) DC signal power

$$S_i = \frac{V_{max}^2}{R} \quad (\text{let } R = 1\Omega)$$

$$\therefore \boxed{S_i = V_{max}^2}$$

We know Quantization noise power $\boxed{N_q = \frac{V_{max}^2}{3Q^2}}$

∴ Average Signal to Quantization noise ratio

$$(SQNR)_i = \frac{S_i}{N_q} = \frac{V_{max}^2}{V_{max}^2/3Q^2} = 3Q^2$$

$$\therefore \boxed{(SQNR)_i = 3Q^2 = 3 \cdot 2^{2N}}$$

In decibels.

$$(SQNR)_{i\,in\,dB} = 10 \log_{10} (3 \cdot 2^{2N})$$

$$= 10 \log_{10} 3 + 10 \log 2^{2N}$$

For uniform Quantization.

$$\boxed{\text{Average}\,(SQNR)_{dB} = 4 \cdot 8 + 6N \text{ dB}}$$

For RMS s/l power

$$S_r = \frac{\left(\frac{V_{max}}{\sqrt{2}}\right)^2}{R=1}$$

$$S_i = \frac{V_{max}^2}{2}$$

$$\text{rms } SQNR = \frac{3Q^2}{2} = 1.5Q^2$$

RMS $(SQNR)_{in\,dB}$

$$= 10\log(1.5) + 20\log Q$$

$$= 1.8 + 6N$$

$$\therefore \boxed{\text{RMS}\,(SQNR)_{dB} = 1.8 + 6N}$$

Non-uniform Quantization:

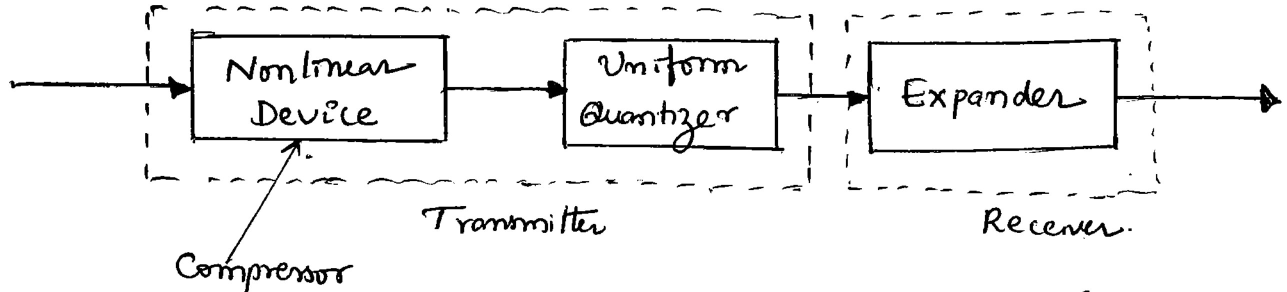


Fig: Model of non uniform quantization.

✓ To increase SQNR and to decrease bit rate (R_b) of the signal non uniform quantization is used.

✓ In non uniform quantization, the step size varies according to the input signal.

* Consider step size is uniform

✓ Small amplitude signal will have high quantization error results in Low SQNR.

✓ High amplitude signal have low quantization error that results in high SQNR.

Companding is the process of maintaining a constant SNR for entire signal. ie Non uniform quantizing is basically a compressor followed by a uniform quantizer and expander.

$$\boxed{\text{Companding} = \text{Compressor} + \text{Expander}}$$

Example:

✓ Quantization of voice signal the signal power corresponding to the voice signal varies from talker to talker.

✓ Even for the same talker the quality of the signal ie the ratio of voltage levels covered by voice signals may range in the order of 1000 : 1.

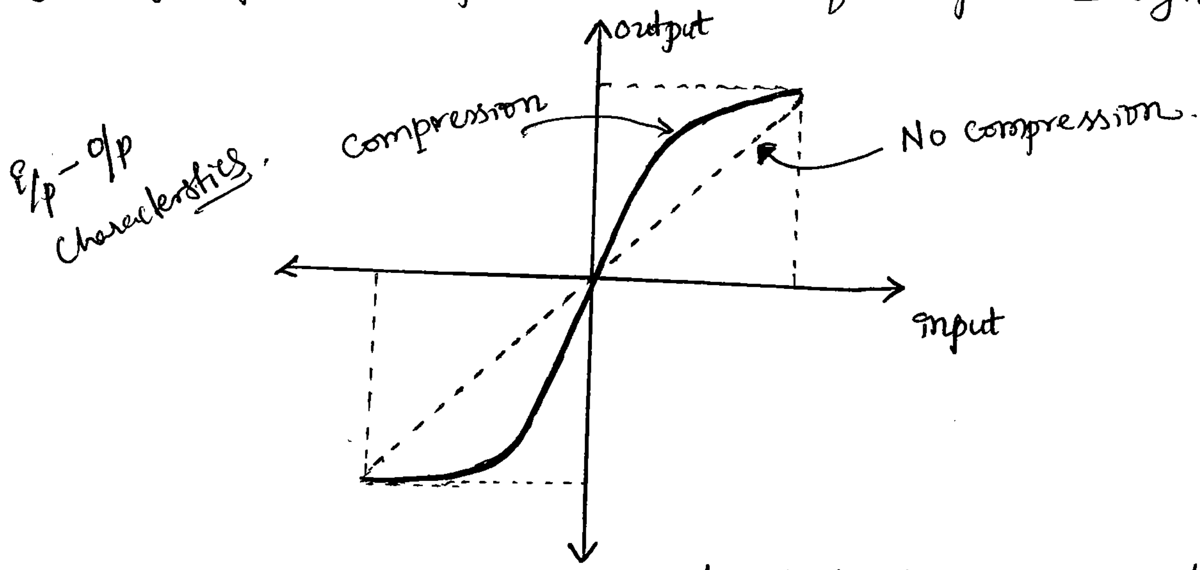
However excursion of voice signal into the large amplitude ranges occurs in practice less frequently such variations are taken care of by making use non uniform quantizing.

This achieved by a non uniform quantizer which operates in such a way that the step size automatically changes.

→ It is difficult to implement non-uniform quantization in practice because there is no prior information about the changes in the signal level.

Compressor: Compressor is used to amplify the weak signal (Low amplitudes) and attenuated strong signal (large amplitude) So, a signal transmitted through such a network will have extremities of its waveform compressed.

A Typically input : output characteristics of compressor is given by

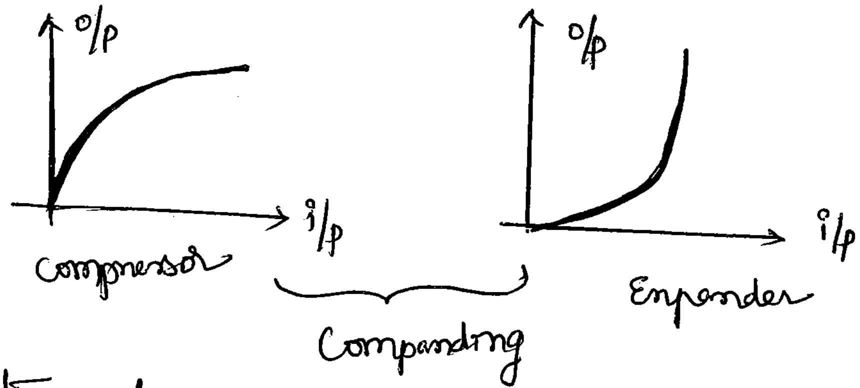


✓ The compression produces signal distortion. To undo the distortion at the receiver we pass the recoverd signal through an expander network.

* An Expander input .... output characteristics which the inverse of the characteristics of the compressor.

Thus The inverse distortions of compressor & expanders generates a signal output signal without distortions.

ie



* There are two types of Companding Law.

1. μ-law Companding.
2. A-law Companding.

1. μ-Law Companding :

μ-law companding used in USA, Canada, Japan.

✓ Low input values → Linear characteristics.

✓ high input values → Logarithmic characteristics.

It is mathematically expressed as

$$\frac{C(|V|)}{V_{max}} = \frac{1}{\ln(1+\mu)} \ln\left(1+\frac{\mu|v|}{V_{max}}\right) \;;\; 0 \le \frac{|V|}{V_{max}} \le 1.$$

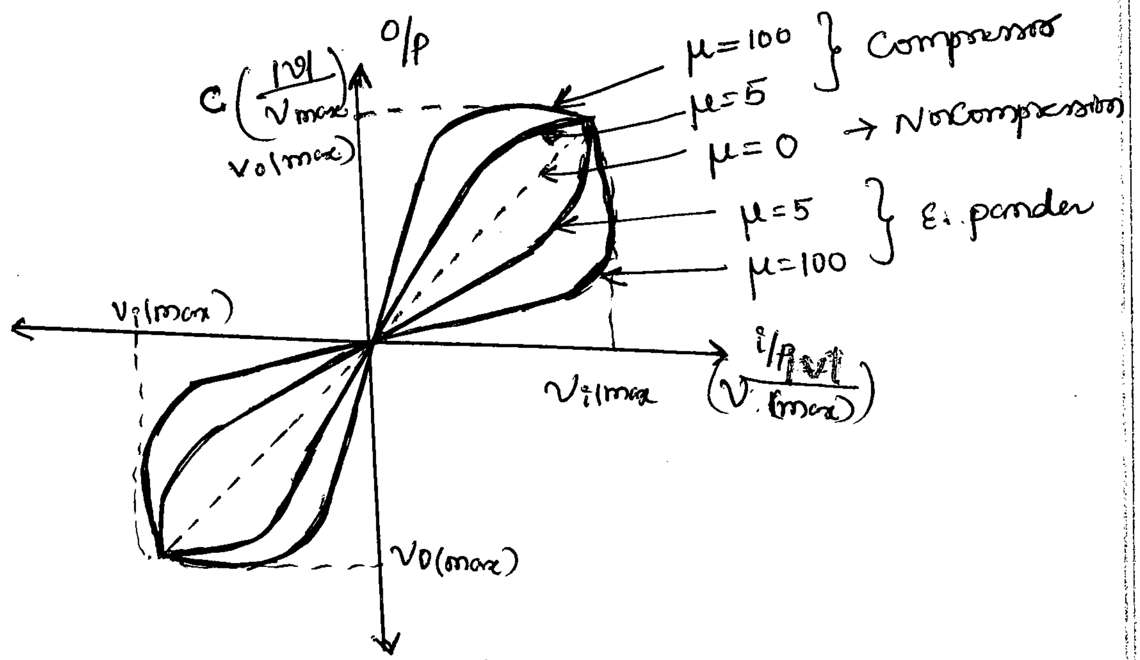
where v is the amplitude of input signal

V_{max} is max. amplitude of i/p signal.

$C(|v|)$ is compression of input signal

μ - Amount of compression.

Fig: μ-law companding



* $\mu = 0$ represents the uniform quantization
* Practically the value of μ is 255. $\boxed{\mu = 255}$

② <u>A-law Companding</u>:

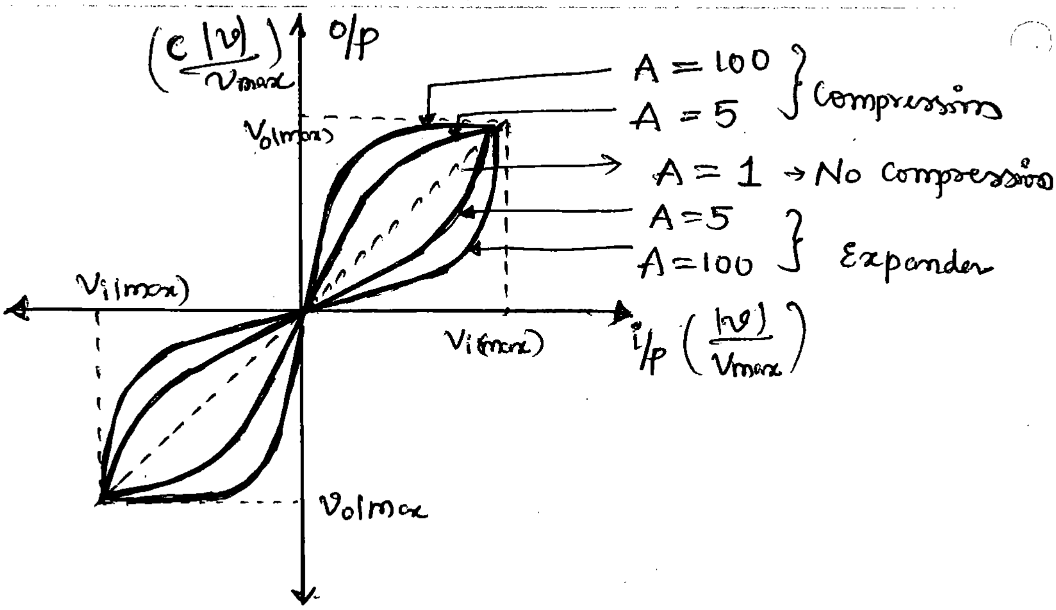
A-law companding used in Europe and rest of the world.
It is mathematically expressed as

$$\frac{C(|v|)}{V_{max}} = \begin{cases} \dfrac{A}{1+\ln(A)} \cdot \dfrac{|v|}{V_{max}} \;; & 0 \le \dfrac{|v|}{V_{max}} \le \dfrac{1}{A} \\ \dfrac{1}{1+\ln(A)}\left[1+\ln\left(A\dfrac{|v|}{V_{max}}\right)\right] \;; & \dfrac{1}{A} \le \dfrac{|v|}{V_{max}} \le 1. \end{cases}$$

A - Defines amount of compression.

$C(|v|)$ - Compression of input signal.

Fig: A-law Companding.



* $A = 1$ represents the uniform quantization
* Practically the value of A is 87.56 $\boxed{A = 87.56}$

Output SQNR

For Uniform Quantization $\frac{S_i}{N_q} = 3Q^2 \left(\frac{m^2(t)}{V_{max}^2} \right)$

for Non uniform quantization $\frac{S_i}{N_q} = 3Q^2 \cdot \frac{1}{\{\ln(1+\mu)\}^2}$, ($\because$ From μ-law companding)

In general signal to Quantization noise ratio SQNR is given by

$$\boxed{\begin{aligned} \frac{S_i}{N_q} &= C \cdot Q^2, \\ \text{where} \quad C &= \begin{cases} 3 \cdot \frac{m^2(t)}{V_{max}^2}, & \rightarrow \text{for uniform quantization} \\ 3 \cdot \frac{1}{\{\ln(1+\mu)\}^2} & \rightarrow \text{for non-uniform quantization} \end{cases} \end{aligned}}$$

SQNR in dB

$$\begin{aligned} (SQNR)_{dB} &= 10 \log (S_i/N_q) \\ &= 10 \log CQ^2 \\ &= 10 \log C + 20 \log 2^N \quad (\because Q = 2^N) \\ &= 10 \log C + 6N \end{aligned}$$

$\therefore \boxed{(SQNR)_{dB} = \alpha + 6N}$, $\alpha = 10 \log C$.

Differential Quantization :

✓ The bit rate of the digitally encoded signal $\boxed{R_b = N f_s}$

where f_s - no.of samples/sec & sampling frequency

N - no.of bits / sample

* To decrease the bit rate of the signal, either N or f_s should be decreased, but f_s cannot be decreased below the nyquist sampling rate & hence 'N' is decreased.

* To decrease 'N' value, the peak-to peak voltage of the message signal is decreased by taking difference b/w the two samples.

"∴ The output of the difference between two samples is quantized and this type of quantization is called Differential Quantization".

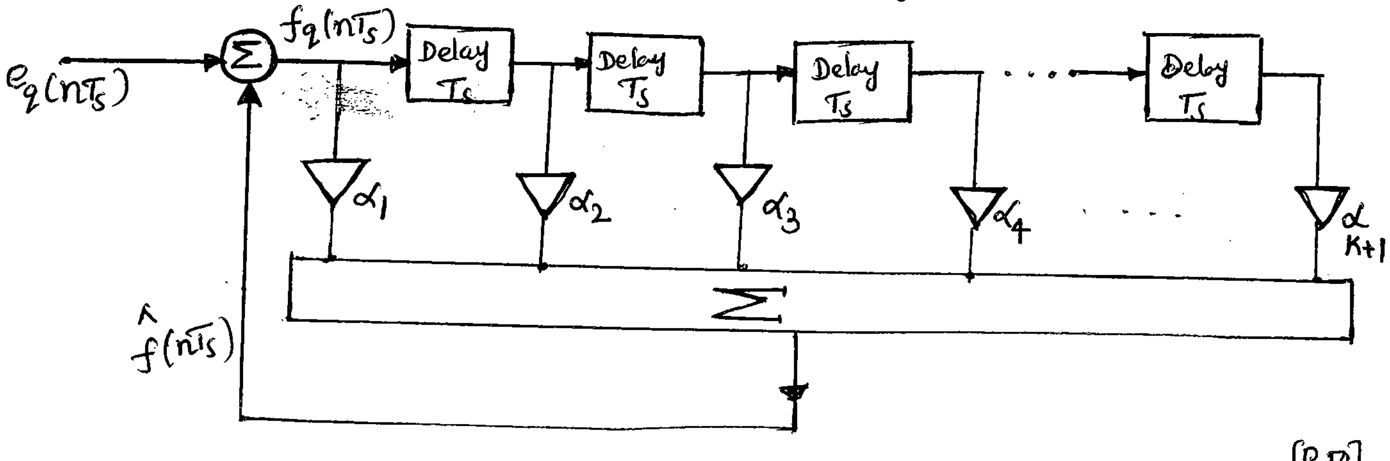
✓ The gap b/w two successive samples ; 'T_s' seconds.

* To find the difference b/w the samples, predictor is employed along with the uniform quantization.

Predictor (or) Prediction filter :

✓ The main purpose of predictor is used to find the difference between the samples and also estimating the future values based on the previous values.

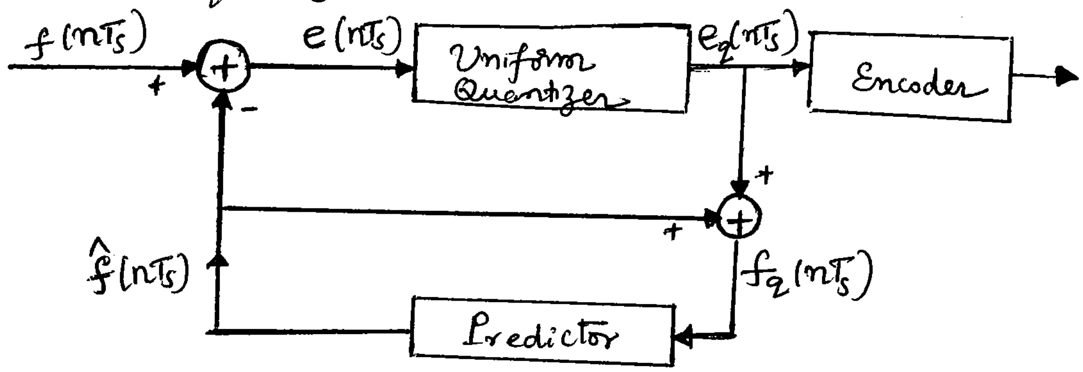
✓ The predictor filter is used in transmitter & Receiver of a DPCM s/m. & can be realized by using tapped delay line filters.



where $\alpha_1, \alpha_2, \alpha_3, \ldots \ldots \alpha_{K+1}$ are tape gains.

$$\hat{f}(nT_s) = \alpha_1 \cdot f_q(nT_s) + \alpha_2 \cdot f_q\{(n-1)T_s\} + \alpha_3 f_q\{(n-2)T_s\} + \cdots + \alpha_{K+1} f_q\{(n-K)T_s\}.$$

✓ A uniform quantizer along with the predictor will realize a differential quantizer as shown in below.



At Transmitter

* Consider a signal 'f(t)' which is sampled at a nyquist rate and produce a sequence of correlated samples of T_s seconds apart & denoted as $f(nT_s)$.

✓ The input to the quantizer is $e(nT_s)$ given by

$$\boxed{e(nT_s) = f(nT_s) - \hat{f}(nT_s)} \longrightarrow (1)$$

where $e(nT_s) \rightarrow$ prediction error & can be reduced by changing the prediction value.

$f(nT_s) \rightarrow$ Unquantized sample

$\hat{f}(nT_s) \rightarrow$ Predicted sample sequence consists of quantized signal

✓ The output of the quantizer $e_q(nT_s)$ is given by

$$\boxed{e_q(nT_s) = e(nT_s) \pm Q_e(nT_s)} \longrightarrow (2)$$

where $e_q(nT_s) \rightarrow$ Quantized output

$Q_e(nT_s) \rightarrow$ Quantization error

* The quantizer output is added to the prediction value $\hat{f}(nT_s)$ to produce the prediction input $f_q(nT_s)$

$$\boxed{f_q(nT_s) = e_q(nT_s) + \hat{f}(nT_s).} \longrightarrow (3)$$

From eqn ② & ③

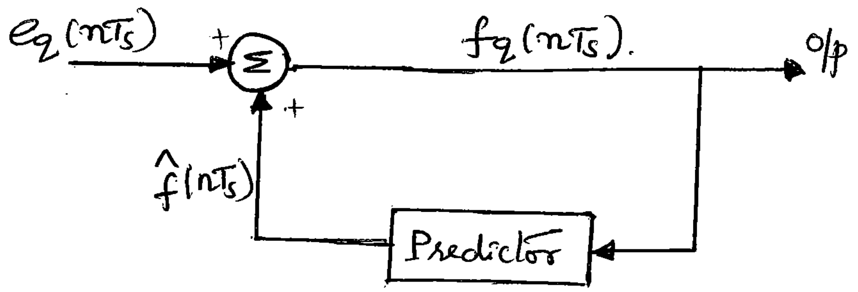
$$f_q(nT_s) = e(nT_s) \pm Q_e(nT_s) + \hat{f}(nT_s) \longrightarrow ④$$

From eqn ① & ④

$$f_q(nT_s) = f(nT_s) - \cancel{\hat{f}(nT_s)} \pm Q_e(nT_s) + \cancel{\hat{f}(nT_s)}$$

$$\therefore \boxed{f_q(nT_s) = f(nT_s) \pm Q_e(nT_s)}$$

ie The input of the prediction filter differs from the input signal ($f(nT_s)$) by quantization error ($Q_e(nT_s)$).



At Receiver

✓ The input in both cases are same, ie: $e_q(nT_s)$

∴ The prediction output must be $\hat{f}(nT_s)$

∴ The receiver output is also same

$$f_q(nT_s) = e_q(nT_s) + \hat{f}(nT_s)$$

From eqn ②

$$f_q(nT_s) = e(nT_s) \pm Q_e(nT_s) + \hat{f}(nT_s)$$

from eqn ①

$$\therefore \boxed{f_q(nT_s) = f(nT_s) \pm Q_e(nT_s).}$$

This shows that we are able to receive the desired signal input plus the quantization noise associated with the difference between the sample $e(nT_s)$, which is generally smaller than $f(nT_s)$.

✓ The quantized version of the original input is reconstructed from the receiver output by using same prediction filter used in transmitter.

Encoding (or) Coding :

In combining the process of sampling & Quantizing, the continuous message & baseband signal becomes limited to a discrete set of values but not in the form best suited for transmission over common channel such as a telephone line (or) radio path or optical fiber etc.

" The process of assigning a binary value & binary code to each discrete set of samples is known as Coding."

✓ Set of symbols is called codeword.

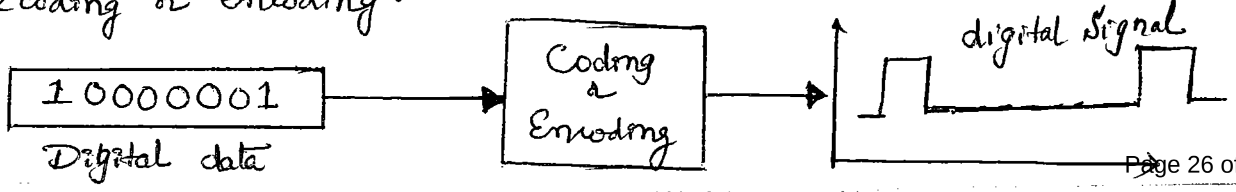
* In binary code, each symbol consists of two distinct values.
* In ternary code, each symbol consists of three distinct values.

✓ Each codeword consists of n-bits, such a code represents a total of 2^n distinct numbers.

* Encoding is also called waveform coding & line coding or transmission coding & binary level representation.

Ordinary no. of representation levels	Level no. expressed as sum of power 2	Binary Numbers
1	2^0	001
2	2^1	010
3	2^1+2^0	011
4	2^2	100
5	2^2+2^0	101
6	2^2+2^1	110
7	$2^2+2^1+2^0$	111

* The first approach converts digital data into digital signal is known as line coding & encoding.



Electrical representation of binary value :

There are different types of coding techniques.

1. Unipolar NRZ Signalling.
2. Polar NRZ Signalling
3. Unipolar RZ Signalling
4. Bipolar RZ Signalling
5. Split phase (or) Manchester Signalling
6. Differential Encoding.

where
NRZ - Non Return to Zero
RZ - Return to Zero.

(1) Unipolar NRZ Signalling :

1 → 1
0 → 0

* Symbol 1 represented by presence of pulse and
* Symbol 0 represented by absence of pulse.
* It is also reffered as ON-OFF signalling.
* It is used for the Transmission of ASK signal (Amplitude Shift Keying)

(2) Polar NRZ Signalling :

1 → +1
0 → −1

* Symbol 1 is represented by '+1' volts and
* Symbol 0 is represented by '−1' volts.
* It is used for PCM, DM, ADM, PSK, FSK Signal.

(3) Unipolar RZ Signalling :

1 → +½ → +1
1 → −½ → 0
0 → 0

* Symbol '1' is Splitting into two half cycles, & 1st half cycles represents +1.
* Symbol '0' & 2nd half cycle of symbol '1' is represented by '0'.
* It is used for fiber optical communication.

(4) Bipolar RZ Signalling :

1 → first, +1 Second, −1 & 0 → No pulse.

* Symbol '1' is represented by alternation of +1 and −1, no pulse for '0'
* It is also referred as AMI (Alternate Mark Inversion)

(5) Split phase (or) Manchester Signalling :

1 → ½ → +1, −½ → −1
0 → −½ → −1, +½ → +1

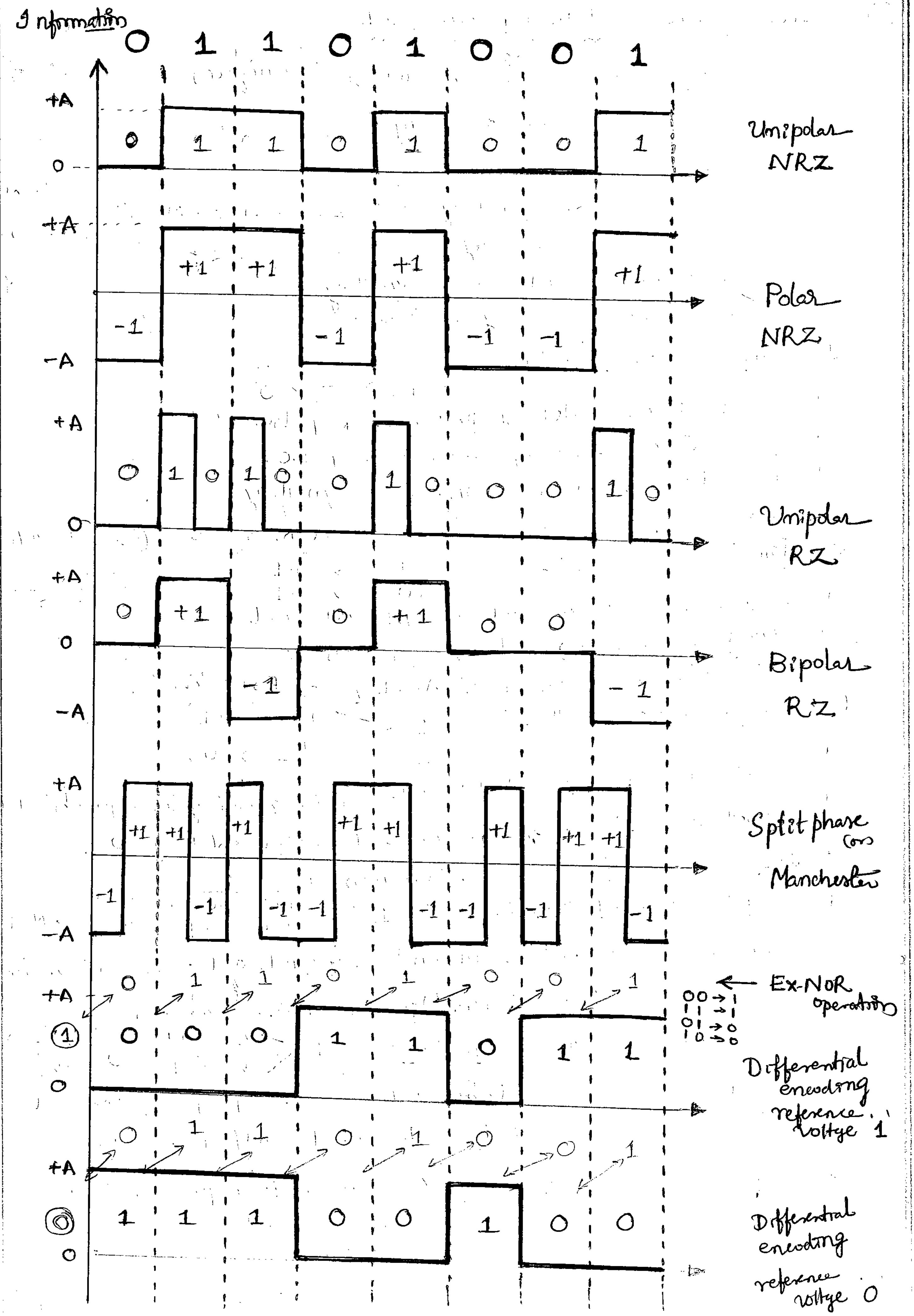
* Symbol '1' is represented by +1 and −1 half cycle pulse and Symbol '0' is represented by −1 and +1 half cycle pulse.

(6) Differential encoding :

0 → complement of previous voltage
1 → previous voltage.

* Symbol '0' is represented by complement of previous voltage
* Symbol 1 is represented by previous voltage. → Exclusive NOR operation

00 - 1
11 - 1
01 - 0
10 - 0


Information
0 1 1 0 1 0 0 1
+A
0
Unipolar NRZ
+1
-1
-A
Polar NRZ
Unipolar RZ
Bipolar RZ
Split phase (or) Manchester
Ex-NOR operation
00 → 1
11 → 1
01 → 0
10 → 0
Differential encoding reference voltage 1
Differential encoding reference voltage 0

Pulse Code Modulation (PCM) :

✓ Pulse digital modulation is a scheme that converts the analog signal to its corresponding digital form.

* The simplest form of pulse digital modulation (PDM) is known as Pulse Code modulation (PCM).

✓ In PCM as message signal is represented by sequence of coded pulses which is accomplished by representing the signal in discrete form in both time and amplitudes.

* PCM system consists of mainly three blocks.

  ⓐ Transmitter ⓑ Transmission path ⓒ Receiver.

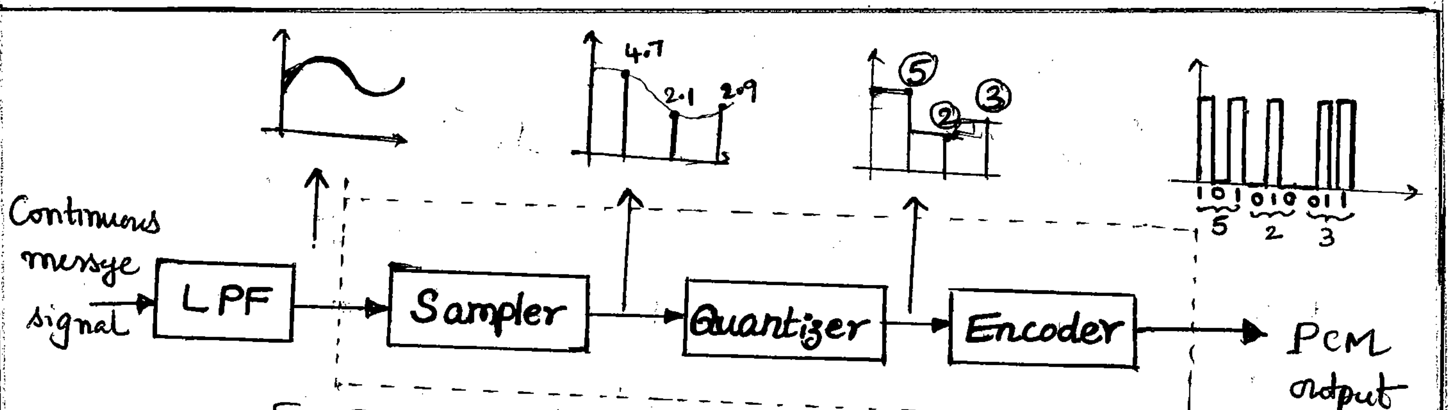
ⓐ 

Fig ⓐ Transmitter

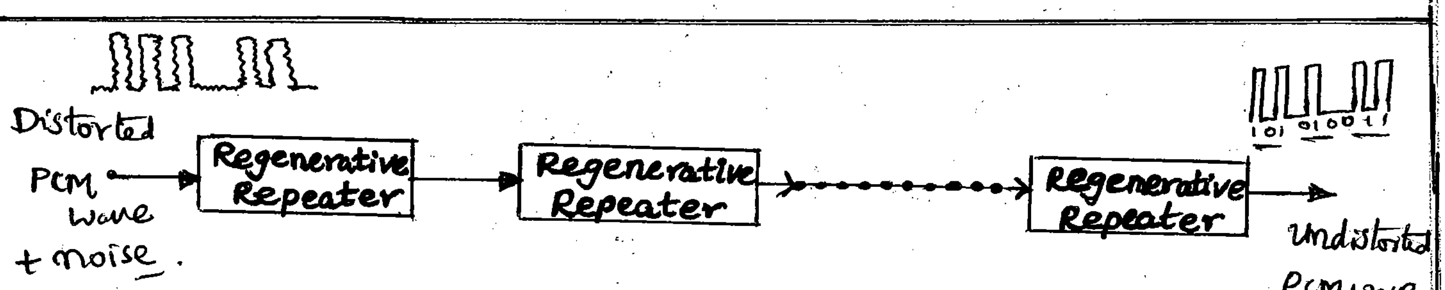
ⓑ 

Fig ⓑ Transmission path → To reduce noise

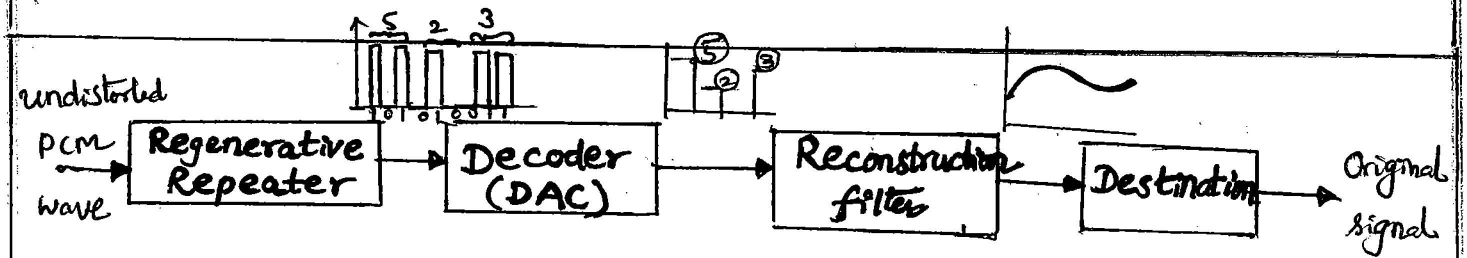
ⓒ 

fig ⓒ Receiver

Principle of operation :

- ✓ The basic operation of PCM system involves the PAM signals are quantized & then coded, thus PCM is obtained and its amplitude and time are represented in discrete values.

- ✓ PCM is not modulation scheme, In the conventional scheme the term 'modulation' usually refers to variation of some characteristics of carrier wave in accordance with an information signal.

* Main use of discrete or digital representation of signals are
  - ✓ Ruggedness for Txion noise and Interference.
  - ✓ Security is obtained.

→ The essential operation in the <u>transmitter</u> of PCM system are Sampling, Quantizing and encoding & these three operations are usually performed in same ckt which is called ADC ie Analog to digital converter as shown in fig ⓐ.

→ Regeneration of impaired signals occurs at intermediate points along the <u>transmission path</u> as in fig ⓑ.

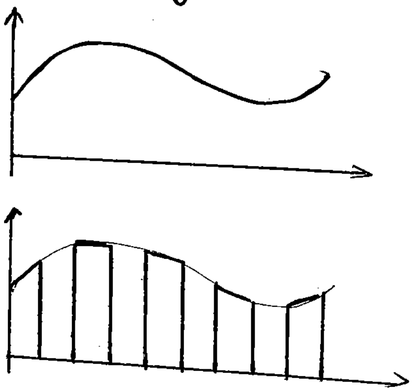
→ At the <u>receiver</u>, the essential operations consists of one last state of regeneration followed by decoding then demodulation of the train of quantized samples ; the operations of decoding and reconstruction are usually performed in the same ckt as digital to Analog converter (DAC) as in fig ⓒ.

ⓐ Transmitter :

- ✓ The LPF prior to sampler is included to prevent aliasing effect (or) foldover effect.
- ✓ LPF is also called as prealiasing filter.

The basic operations performed in transmitter of PCM systems are
(i) Sampling (ii) Quantizing (iii) Encoding.

(i) **Sampling**: The process of representing an analog signal by a sequence of sampled segments is called sampling.



Sampling permits the reduction of continuously varying message signal to a limited no. of discrete values per second.

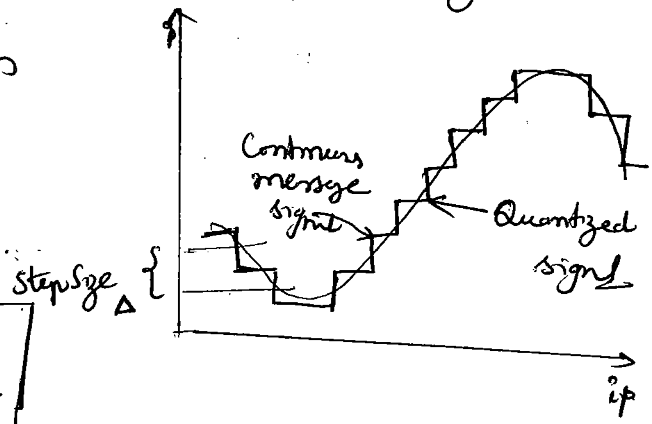
$$\boxed{f_s \geq 2f_m}$$

(ii) **Quantizing**:

The process of converting continuous amplitude discrete time signal into discrete amplitude discrete time signal.

& approximating the amplitude of each sample to the nearest value from a set of discrete amplitude levels called as Quantization.

PCM uses either uniform Quantization (or) non uniform quantization to converts analog to digital signal.



✓ The step size $\boxed{\Delta = \frac{V_{pp}}{Q} = \frac{V_{pp}}{2^N}}$

where

Q - no. of quantization levels $= 2^N$ where N - no. of bits / sample.

✓ The quantization error / noise can be calculated by the difference between continuous message signal and quantized signal.

ie

$$\boxed{e \text{ or } Q_e = x(t) - x'(t) \;;\; -\Delta/2 \leq e \leq +\Delta/2}$$

The maximum quantization error Q_e or $e = \Delta/2$.

(iii) **Encoding**: The process of assigning a binary value to a discrete set of samples known as coding or encoding for the best suited transmission over a communication channel such as telephone line & optical fiber etc.

✓ It mainly converts digital data in digital signal representation.

(b) Transmission path : (Set of Regenerative Repeaters).

* The most important feature of PCM system lies in the ability to control the effect of distortion noise produced by transmitting a PCM wave through a channel.

✓ This capability is accomplished by reconstructing the PCM wave by means of a choice of regenerative repeaters located at sufficiently close space along the transmission path.

Regenerative Repeater :

The three basic functions performed by a regenerative repeater are (i) Equalization (ii) Timing and (iii) Decision making device.

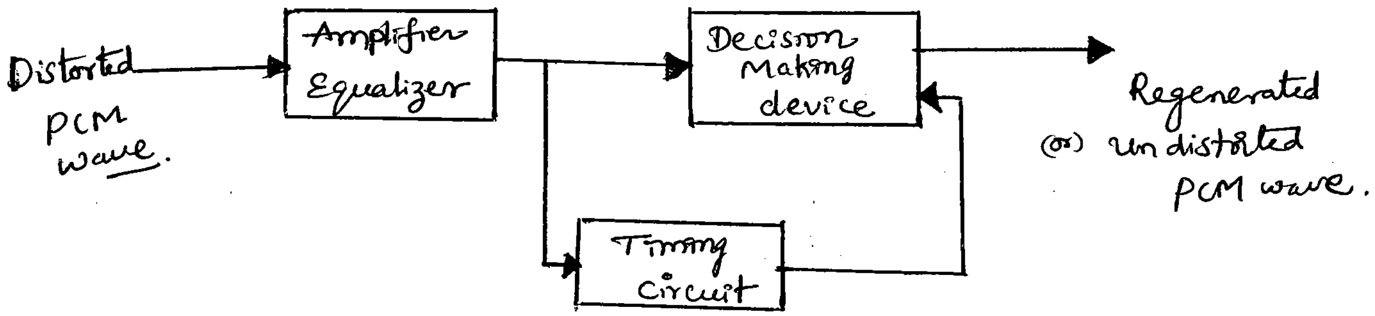


Fig: Regenerative Repeater.

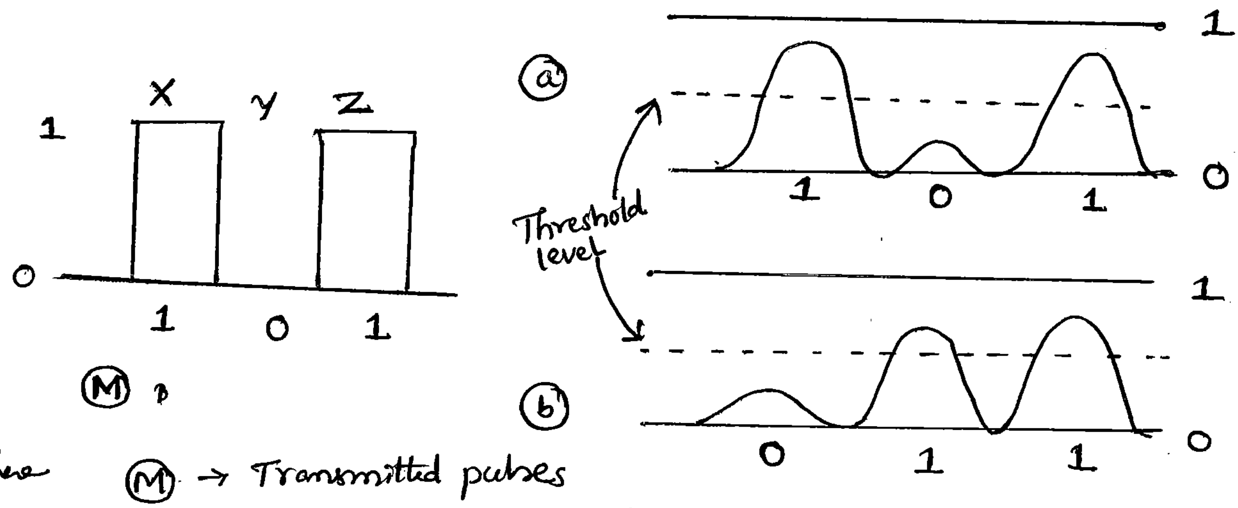
(i) **Equalizer :** It shapes the received pulses so as to compensate for the effect of amplitude and phase distortions produced by the channel.

(ii) **Timing Circuit :** It provides a periodic pulse train derived from received pulses, for sampling the equalized pulses at the instants of time where the signal to quantization noise ratio is maximum.

(iii) **Decision making device :** It enables the sampling time determined by the timing circuitry makes its decision based on whether or not the amplitude of quantized pulse plus noise exceeds a predetermined voltage level.

Example: Consider the case of PCM with unipolar signalling.

The major advantage of the PCM system is that the information does not lie in any property of pulse, but it lies in the presence (or) absence of the pulse.



where (M) → Transmitted pulses
(a) → Received pulse without error
(b) → Received pulse with error.

(a) → when the voltage in a time slot crosses the threshold level (which is the centre of 0 and 1 level), it is received as a '1'.
→ when the voltage in a time slot does not cross the threshold level, it is received as a '0'.

* The received pulses are distorted due to noise, there is no error of decision.

(b) → In the time slot X, the voltage does not cross the threshold level & hence a '0' is received (an error),
→ In the time slot 'y', the voltage crosses the threshold level and hence a '1' is received (an error).
→ There is no error in the time slot 'z'.

* Thus 101 is received as 011, but the probability of the occurrence of such error is extremely small.

Thus the difference between the '0' & '1' levels can be increased, resulting in a reduced effect of noise.
Hence practically error free transmission is possible.

The repeaters in PCM system used to generate pulses in time slots according to the presence of '0' or '1', thus eliminating effect of noise till that point.

© Receiver :

The basic operations in the PCM receiver are

(i) Regenerative repeater last stage.

(ii) Decoding. (DAC)

(iii) Reconstruction filter.

* At the receiver end the first operation is to regenerate the received pulse by cleaning and reshaping.

* The cleaned pulses are then regrouped into codewords and then mapped back into a quantized PAM signal.

" **Decoding:** The mapping back of codewords into a quantized PAM signal is called decoding.

✓ Decoding may be viewed as reverse process of encoding performed in the transmitter.

* The decoding process involves generating a pulse whose amplitude is a linear sum of all the pulses in the code word with each pulse weighted by its place value $(2^0, 2^1, \ldots)$ in the code.

Reconstructing filter :

✓ It reconstructs the original message signal from the PAM signal appeared at the output of the decoder.

✓ Reconstructing filter as a Low pass filter whose cut off frequency equal to the message bandwidth 'w'.

However, the process of quantizing a signal at the transmitter is an "irreversible process". It introduces quantizing noise in the transmitter and results in a loss of information which can not be recovered by any means at the receiver.

Bandwidth in PCM :

- The message bandwidth is 'f_m' when the quantized sample occur at rate of '$f_s \geq 2f_m$' sample/sec.
- If the PCM system uses binary channels to represents the Q-quantization levels then each code word consists of N digits.

where $\boxed{N = \log_2 Q}$ $(\because Q = 2^N)$

Channel symbol / bit rate is given by

$R_b = N \cdot f_s$ where N - no of bits / sample
f_s - Sampling frequency

$\therefore R_b \geq 2Nf_m$ $\therefore f_s \geq 2f_m$.

If a system has a bandwidth of 'B' Hz when we can transmit a min of '2B' samples / sec.

$\therefore$ Transmission rate represented by R a R_b.

$$R_b = 2B \text{ bits/sec.}$$

$$B = R_b/2 \geq \frac{\cancel{2}N \cdot f_m}{\cancel{2}}$$

$\therefore$ $\underline{\underline{B \geq N f_m}}$ $\therefore$ $\boxed{\text{Min. B.W of PCM systems, } B \geq N f_m.}$

Advantages of PCM :

- Used for long distance communication ($\because$ Regenerative repeaters are used).
- High Immunity to noise ie noise can be removed.
- Storage is easy (because PCM signals are digital in nature

Disadvantages of PCM :

- Complexity is high ($\because$ PCM systems includes encoding, decoding, sampling, quantizing and repeaters).
- It requires more bandwidth compare to other systems.

Applications of PCM :

- PCM is used in telephone systems.
- Used for digital audio in computers.
- It is one of the formats for writing CD-ROMs, DVD's, Blue-ray Disc.

Noise in PCM System :

The performance of a PCM system is effected by major sources of noise.

a) Channel noise & thermal noise & transmission noise
b) Quantization noise.

✓ Channel noise is introduced any where between the output of transmitter & input of the Receiver.

✓ Quantized noise is introduced in the transmitter and is carried along to the receiver output and it is a signal dependent in the sence that is disappear when the message signal is switched off.

* The output $\bar{X}(t)$ in a PCM system can be written as

$$\bar{X}(t) = X_0(t) + n_q(t) + n_0(t)$$

where $X_0(t) \rightarrow$ Signal component in the output
$n_q(t) \rightarrow$ Quantization noise
$n_0(t) \rightarrow$ Channel noise.

The overall SNR at the baseband output which is a measure of signal quality can be defined as

$$\boxed{(S/N)_0 = \frac{E\{[X_0(t)]^2\}}{E\{[n_q(t)]^2\} + E\{[n_0(t)]^2\}}}.$$

Quantization noise in PCM :

Let us assume that ideal impulse sample (sampling) is used in PCM system then the output of sample is

$$X_S(t) = X(t) \sum_{K=-\infty}^{\infty} \delta(t - KT_S)$$

T_S - Sampled interval.

The quantized signal is represented by $X_q(t)$ as

$$X_{Sq}(t) = X_q(t) \cdot \sum_{K=-\infty}^{\infty} \delta(t - KT_S)$$

$$x_{Sq}(t) = x(t)\cdot\sum_{k=-\infty}^{\infty}\delta(t-kT_s) + [x_q(t)-x(t)]\sum_{k=-\infty}^{\infty}\delta(t-kT_s).$$

$$= x(t)\sum_{k=-\infty}^{\infty}\delta(t-kT_s) + e_q(t)\sum_{k=-\infty}^{\infty}\delta(t-kT_s)$$

where $e_q(t) = |x_q(t) - x(t)|$

$$\therefore x_{Sq}(t) = x(t)\sum_{k=-\infty}^{\infty}\delta(t-kT_s) + e_q(kT_s)\sum_{k=-\infty}^{\infty}\delta(t-kT_s)$$

where $e_q(kT_s)$ is the error due to the quantization process

✓ The power spectral density of $e_q(kT_s)$ is $G_{eq}(f)$ as

$$G_{eq}(f) = \frac{\text{Variance}}{\text{Timeperiod}} = \frac{E\{e_q^2(kT_s)\}}{T_s}$$

If uniform quantization operating on $x(t)$ having an uniform PDF

$$\sigma_q^2 = E\{e_q^2(kT_s)\} = \frac{\Delta^2}{12} \quad \text{where } \Delta\text{-Step Size}$$

$$\therefore \boxed{G_{eq}(f) = \frac{\Delta^2}{12\cdot T_s}}$$

If we ignore the effects of channel noise temporarily,
where noise components $n_q(t)$ has a PSD of $G_{nq}(f)$ as

$$G_{nq}(f) = G_{eq}(f)\cdot|H_R(f)|^2$$

where $H_R(f)$ - transfer function of ideal LPF (reconstruction filter)

Assume sampling rate $f_s = 2f_m$ & $H_R(f)$ to be ideal LPF with bandwidth 'f_m'

$$\therefore G_{nq}(f) = \begin{cases} G_{eq}(f) & ; -f_m \le f \le f_m \\ 0 & \text{otherwise.} \end{cases}$$

Average quantization noise power is mean square value ie

$$E[n_q^2(t)] = \int_{-f_m}^{f_m} G_{nq}(f)\cdot df = \int_{-f_m}^{f_m} G_{eq}(f)\, df$$

$$= \int_{-f_m}^{f_m} \frac{\Delta^2}{12\cdot T_s}\cdot df$$

$$\therefore\ E[n_q^2(t)] = \frac{\Delta^2}{12}\cdot\frac{1}{T_s}\cdot[f]_{-f_m}^{+f_m}$$

$$= \frac{\Delta^2}{12\cdot T_s}\cdot 2f_m$$

$$= \frac{\Delta^2}{12\cdot T_s^2} \qquad \left(\because f_s = 2f_m\right) \quad \frac{1}{T_s} = f_s$$

Quantization noise power

$$\therefore\ \boxed{E\{[n_q(t)]^2\} = \frac{\Delta^2}{12\cdot T_s^2}}$$

* The output signal component $x_0(t)$ is the response of the LPF to $x(t)\cdot\sum_{K=-\infty}^{\infty}\delta(t-KT_s)$.

$\therefore$ The output signal $x_0(t) = \frac{1}{T_s}\cdot x(t)$

Normalized signal output power is mean square value as

$$E\{[x_0(t)]^2\} = \frac{1}{T_s^2}\cdot\overline{x}^2(t).$$

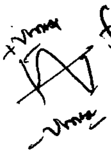
The PSD of the instantaneous value x is $f_X(x)$

$$\therefore\ f_X(x) = \begin{cases} \frac{1}{Q\Delta} & ; -\frac{Q\Delta}{2} < x < \frac{Q\Delta}{2}, \\ 0 & ; \text{otherwise.} \end{cases}$$

$$\therefore\ \text{Variance } \overline{x}^2(t) = \int_{-\infty}^{\infty} x^2\cdot f_X(x)\,dx$$

$$= \int_{-Q\Delta/2}^{Q\Delta/2} x^2\cdot\frac{1}{Q\Delta}\cdot dx$$

$$\frac{1}{Q\Delta} = \frac{1}{Q\cdot\frac{V_{pp}}{Q}} = \frac{1}{V_{pp}}$$



$$f_X(x) = \frac{1}{2V_{max}}$$

$$= \frac{1}{Q\Delta}\cdot\left.\frac{x^3}{3}\right|_{-Q\Delta/2}^{+Q\Delta/2}$$

$$= \frac{1}{3Q\Delta}\left\{\left[\frac{Q\Delta}{2}\right]^3 + \left[\frac{Q\Delta}{2}\right]^3\right\}$$

$$= \frac{1}{3Q\Delta}\cdot\frac{Q^3\cdot\Delta^3}{4} \Rightarrow \boxed{\overline{x}^2(t) = \frac{Q^2\Delta^2}{12}}$$

$\therefore$ Signal output power $E\{x_0^2(t)\} = \frac{1}{T_s^2}\cdot\frac{Q^2\Delta^2}{12}$

$$\therefore\ \boxed{E\{[x_0(t)]^2\} = \frac{Q^2}{12}\cdot\frac{\Delta^2}{T_s^2}}$$

∴ The signal to quantization noise ratio (SQNR) of PCM S/m is

$$\left(S/N\right)_{QN} \text{ a } SQNR = \frac{E[x_0^2(t)]}{E[n_q^2(t)]}$$

$$= \frac{Q^2 \cdot \frac{\cancel{\Delta^2}}{\cancel{12 \cdot T_s^2}}}{\frac{\cancel{\Delta^2}}{\cancel{12 \cdot T_s^2}}}$$

$$\boxed{(S/N)_{QN} = Q^2}$$

$$= (2^N)^2 \quad (\because Q = 2^N)$$

For PCM system $\boxed{SQNR = 2^{2N}}$, $\boxed{(SQNR)\, dB = 6N}$

where N - no-of bits / sample.

Channel noise in PCM:

✓ Channel noise is measured in terms of average probability of symbol error it also called as 'error rate'.

ie The deviation of the recovered receiver output from the original message signal on an average is called average probability of symbol error.

* The main effect of channel noise introduces bit error into the received signal in binary PCM.

✓ The presence of a bit error causes symbol '1' to be mistaken for symbol '0' & viceversa. This deviation is called 'error rate'.

* In order to calculate the effect of bit errors induced by channel noise consider a PCM system using N-bit code word ie 2^N quantization levels.

✓ The most negative level is represented by

00000 000

✓ The next higher quantization level is represented by

00000 001

✓ The most possible positive level is represented by

11111 .... 111 .

* Due to the channel noise bit error will be introduce.

Let us consider bit error is occured as 'Δ'

The error in the next (First) significant bit occurs an error is 2Δ, and the error i^{th} bit position causes an error of $2^{i-1}.\Delta$ ie $\Delta, 2.\Delta, 2^2.\Delta, 2^3\Delta, \ldots, 2^{i-1}\Delta$.

Let the error be represented by Q_Δ as

$$Q_\Delta = 2^{i-1}.\Delta$$

An error may occur any one of the N-bits in the codeword the variance of the error is.

$$E[\bar{Q}_\Delta]^2 = \frac{1}{N}\left[\Delta^2 + (2\Delta)^2 + (2^2\Delta)^2 + (2^3\Delta)^2 + \ldots (2^{N-1}\Delta)^2\right]$$

mean square value.

$$= \frac{1}{N}.\Delta^2 \frac{[(2^2)^N - 1]}{2^2 - 1}$$

Geometric progression

$$r = \frac{a_2}{a_1} = \frac{(2\Delta)^2}{\Delta^2} = 4 = 2^2$$

Sum of the N terms.

$$S_n = a.\frac{r^{n}-1}{r-1}$$

$a = \Delta^2$, $r = 2^2$

$$S_n = \frac{\Delta^2 . 2^{2N} - 1}{2^2 - 1}$$

$$= \frac{\Delta^2}{N} . \frac{2^{2N} - 1}{4 - 1}$$

$$= \frac{\Delta^2}{N} . \frac{2^{2N} - 1}{3}$$

$$= \frac{\Delta^2}{N} . \frac{2^{2N}}{3}$$

(From geometric progression)

$$= \frac{\Delta^2}{N} . \frac{2^{2N}}{3} \quad [\because N \text{ is very large } 2^{2N} >> 1]$$

$$\therefore \boxed{E[Q_\Delta]^2 = \frac{\Delta^2}{N} . \frac{2^{2N}}{3}}$$

✓ The bit errors due to channel noise lead to incorrect values of $X_2(kT_s)$ as impulse sequence;

These errors appears as impulses of random amplitude & random time.

The mean seperation between bit error is $\frac{1}{Pe}$ bits.

Where P_e - Probability of bit error.

Since there are N-bits in codeword.

The mean seperation in code that are error is $\frac{1}{NP_e}$ bits.

$\therefore$ The mean time $T = T_s \times \frac{1}{NP_e}$ where T_s - Sampling period.

$\therefore$ The PSD of the noise power of channel noise

$$G_\Delta(f) \;\&\; G_n(f) = \frac{E[Q_\Delta]^2}{T}$$

$$= \frac{\Delta^2}{\cancel{N}} \cdot \frac{2^{2N}}{3} \cdot \frac{\cancel{N} P_e}{T_s}$$

$$G_\Delta(f) = \frac{\Delta^2 \cdot 2^{2N} \cdot P_e}{3T_s}$$

$\therefore$ At the output of ideal LPF produces an average noise power $\therefore$

$$E\{n_0^2(t)\} = N_0 = \int_{-f_m}^{f_m} G_\Delta(f) \cdot df$$

$$= \int_{-f_m}^{f_m} \frac{\Delta^2 \cdot 2^{2N}}{3T_s} \cdot P_e \cdot df$$

$$= \frac{\Delta^2 \cdot 2^{2N}}{3T_s} \cdot P_e \cdot (f)_{-f_m}^{+f_m}$$

$$= \frac{\Delta^2 \cdot 2^{2N}}{3T_s} \cdot P_e \cdot (2f_m)$$

Channel noise power

$$\boxed{E\{[n_0(t)]^2\} = \frac{\Delta^2 \cdot 2^{2N} \cdot P_e}{3 \cdot T_s^2}} \qquad (\because f_s = 2f_m) \quad \frac{1}{T_s} = f_s.$$

The overall SNR of the PCM system is given by

$$* \quad (SNR)_{PCM} = \frac{E\{[x_0(t)]^2\}}{E[(n_q(t)]^2 + E[(n_0(t))^2]}$$

$\checkmark$ $E\{(x_0(t))^2\} = \frac{Q^2}{12} \cdot \frac{\Delta^2}{T_s^2}$

$\checkmark$ $E\{[n_q(t)]^2\} = \frac{\Delta^2}{12 \cdot T_s^2}$

$\checkmark$ $E\{[n_0(t)]^2\} = \frac{\Delta^2 \cdot 2^{2N}}{3T_s^2} \cdot P_e$.

$$(S/N)_o = (SNR)_o = \frac{\frac{Q^2}{12}\cdot\frac{\Delta^2}{T_s^2}}{\frac{\Delta^2}{12\cdot T_s^2} + \frac{\Delta^2}{T_s^2}\cdot\frac{2^{2N}\cdot Pe}{3}}$$

$$= \frac{\cancel{\frac{\Delta^2}{T_s^2}}\cdot\left(\frac{Q^2}{12}\right)}{\cancel{\frac{\Delta^2}{T_s^2}}\left[\frac{1}{12} + \frac{2^{2N}\cdot Pe}{3}\right)}$$

$$= \frac{Q^2/\cancel{12}}{\frac{1 + 4\cdot 2^{2N}\cdot Pe}{\cancel{12}}}$$

$$= \frac{Q^2}{1 + 4\cdot 2^{2N}\cdot Pe}$$

$$= \frac{Q^2}{[1 + 4\, 2^{2N}\cdot Pe]}$$

$$\boxed{(SNR)_{PCM} = \frac{2^{2N}}{1 + 4\,Pe\cdot 2^{2N}}} \qquad (\because Q = 2^N)$$

The above SNR shows the overall signal to quantization noise plus channel noise for the PCM system.

<u>Case (i)</u>: For small noise case

$$4\,Pe\,2^{2N} << 1$$

& $4\,Pe\,2^{2N}$ can be neglected

$$\therefore (SNR)_o = 2^{2N} = 6N \text{ in dB}$$

$$\therefore \boxed{(SNR)_o = 2^{2N}} \qquad \boxed{(SNR)_{o\,\text{in dB}} = 6N}$$

<u>Case (ii)</u> For large noise case

$$4\,Pe\cdot 2^{2N} >> 1$$

'1' can be neglected.

$$\therefore (SNR)_o = \frac{\cancel{2^{2N}}}{4Pe\,\cancel{2^{2N}}} = \frac{1}{4Pe} \qquad \therefore \boxed{(SNR)_o = \frac{1}{4Pe}}$$

where Pe - Probability of bit error, N - no of bits/sample.

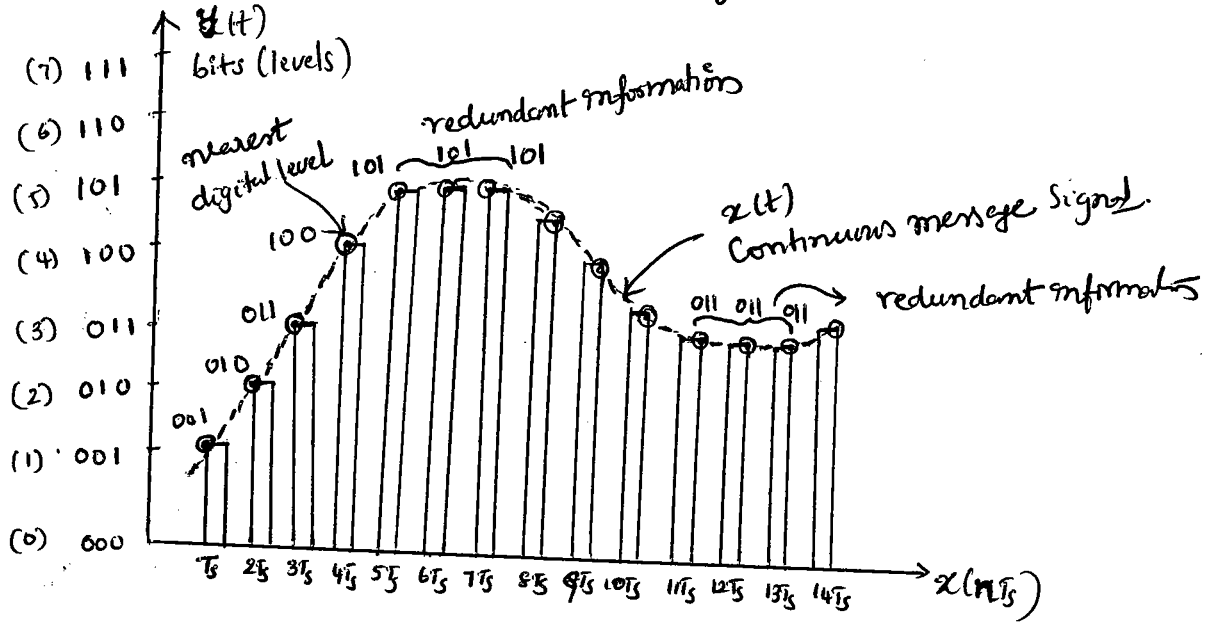
Differential Pulse Code Modulation (DPCM):

- When a voice & Video signals are sampled. It is usually found that adjacent samples are highly correlated (ie close to the same value ie the signal does not change rapidly from one sample to the next sample).

* When these high correlated samples are encoded as in a Standard PCM system, the resultant encoded signal contains "redundant Information"

- Due to redundant information (sample values), consequently the bandwidth of a PCM system is wasted when redundant sample values are retransmitted.

* To minimize the redundant data transmission and to reduce the bandwidth is to transmit samples corresponding to the difference in adjacent sample value, DPCM system is used.



ie Fig: Redundant information in PCM

- The signal is sampled by flat-top sampling at intervals $T_s, 2T_s, \ldots nT_s$
- The samples are encoded by using 3 bit (7 levels) PCM.
- The samples are quantized to the nearest digital level

ie The samples at $5T_s, 6T_s, 7T_s$ are encoded to same value of (101). This information carried only one sample, but these samples are carrying same information means that it is redundant.

* If redundancy is reduced, then overall bit rate will decrease and no. of bits required to transmit one sample will also be reduced and also Bandwidth will be reduced.

This type of digital pulse modulation scheme is called <u>DPCM</u>

ie <u>Differential pulse code Modulation</u>.

✓ In DPCM system uses the difference between the original sample value and the predicted sample value.

ie The value of the present sample is predicted from the past samples.

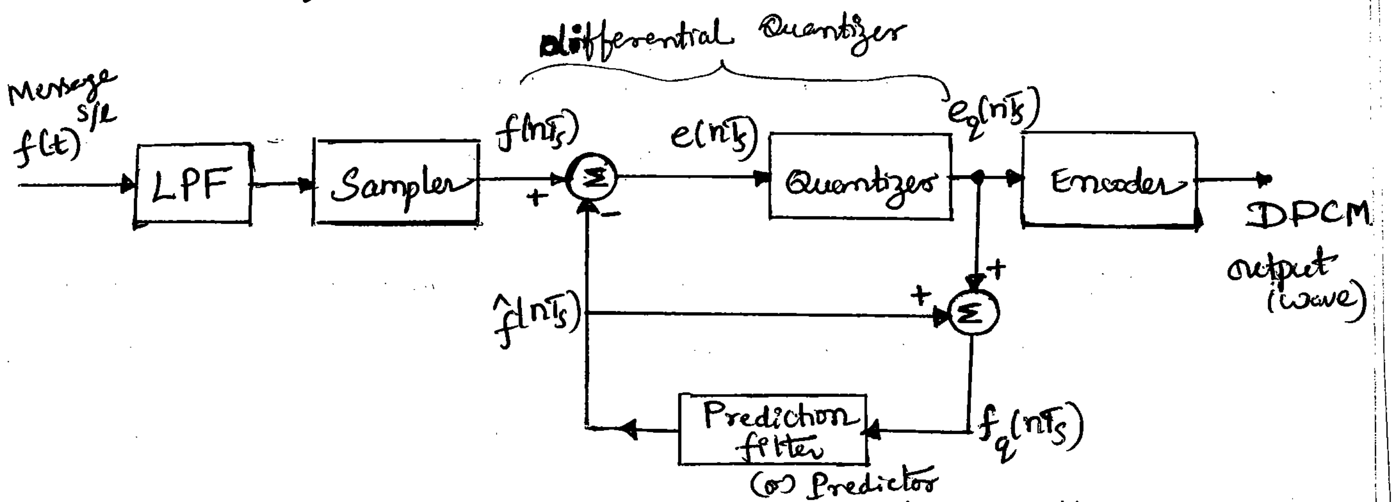


Fig: Block diagram of DPCM -transmitter.

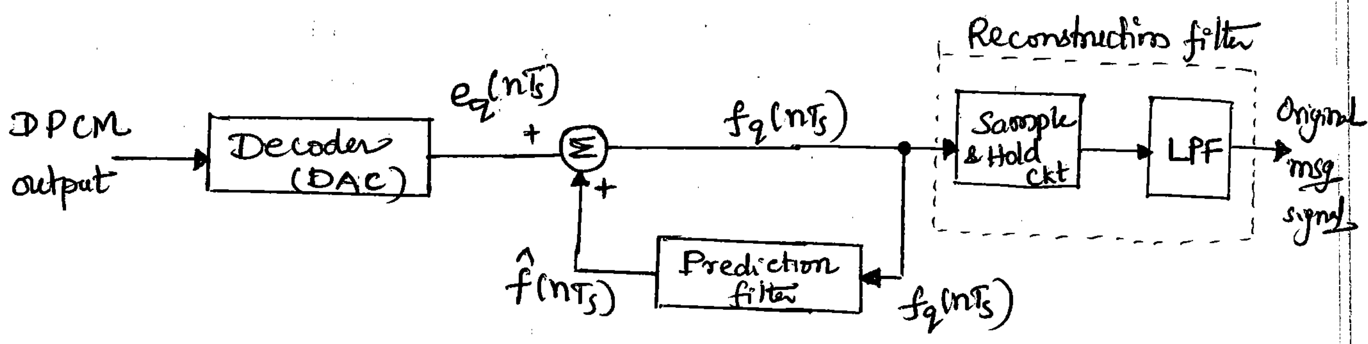
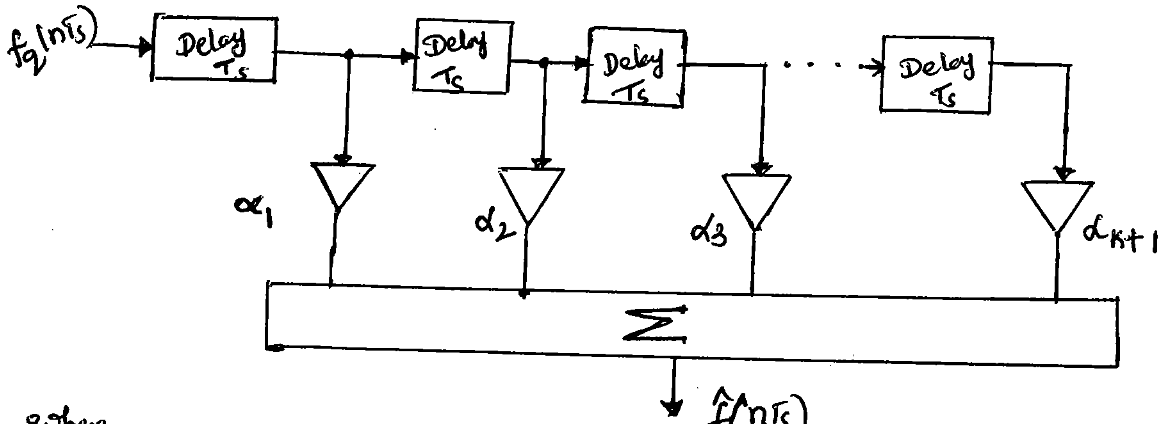


Fig. Block digram of DPCM Receiver.

<u>Predictor</u>: (or) Prediction filter is a moderately sofisticated system it will need to incorporate the facility for storing past differences for carrying out some algorithms to predict next required increament.

* It is used for both transmitter and receiver of the DPCM system.

✓ Predictor can be realized by using tapped delay line filters.
ie The predicted value is modelled as a linear combination of past values of the quantized input.



where

$\alpha_1, \alpha_2, \alpha_3, \ldots \ldots \alpha_{K+1}$ are tape gains.

Fig: Tapped delay line filter used as a predictor.

$$\therefore \hat{f}(nT_s) = \alpha_1 f_q(nT_s) + \alpha_2 f_q[(n-1)T_s] + \alpha_3 f_q[(n-2)T_s] + \ldots$$
$$+ \ldots + \alpha_{K+1} f_q[(n-K)T_s].$$

Thus, Error is the difference between unquantized input signal $f(nT_s)$ and prediction of it $\hat{f}(nT_s)$.

✓ The predicted value is produced by using a prediction filter.

✓ The quantizer output signal gap $e_q(nT_s)$ and previous prediction is added and given as input to the prediction filter. This signal is called $f_q(nT_s)$.

* This makes the prediction more and more close to the actual sampled signal.

We can observe that the quantized error signal $e_q(nT_s)$ is very small and can be encoded by using small no. of bits.

Thus the no. of bits per sample are reduced in DPCM.

The quantizer output can be written as

$$\boxed{e_q(nT_s) = e(nT_s) + Q_e(nT_s)} \longrightarrow (1)$$

where $Q_e(nT_s)$ is the quantization error.

∴ The prediction filter input $f_q(nT_s)$ as

$$f_q(nT_s) = \hat{f}(nT_s) + e_q(nT_s) \longrightarrow (2)$$

From (1) & (2) $$f_q(nT_s) = \hat{f}(nT_s) + e(nT_s) \pm Q_e(nT_s) \longrightarrow (3)$$

∴ The input of the Quantizer is written as

$$e(nT_s) = f(nT_s) - \hat{f}(nT_s) \longrightarrow (4)$$

From (3) & (4) $$f_q(nT_s) = \hat{f}\cancel{(nT_s)} + f(nT_s) - \hat{f}\cancel{(nT_s)} \pm Q_e(nT_s)$$

$$\therefore \boxed{f_q(nT_s) = f(nT_s) \pm Q_e(nT_s)}$$

Hence The quantized version of the signal $f_q(nT_s)$ is the sum/difference of original sample value & Quantization error.

∴ The quantization error can be positive or negative.

Thus $f_q(nT_s)$ doesnot depends on prediction filter characteristics.

Signal to Noise Ratio in DPCM:

✓ The power of message sequence $f(nT_s)$ is given by P_m

✓ The power of quantizing error sequence $q_e(nT_s)$ is given by 'P_q'

The output signal to Quantization noise ratio is

$$(SNR)_o = \frac{P_m}{P_q}$$

$$= \frac{P_m}{P_e} \times \frac{P_e}{P_q}$$

where P_e - prediction error power

$$(SQNR)_0 = G_P \cdot (SQNR)_{uniform}$$

where G_P - prediction gain → produced by the differential quantization.

$$G_P = \frac{P_m}{P_e} \text{ is greater than unity.}$$

$$(SQNR) = \left(\frac{V_{max}}{V_{dmax}}\right)^2 \times (SQNR)_{uniform}$$

where V_{max} - peak amplitude of original message signal

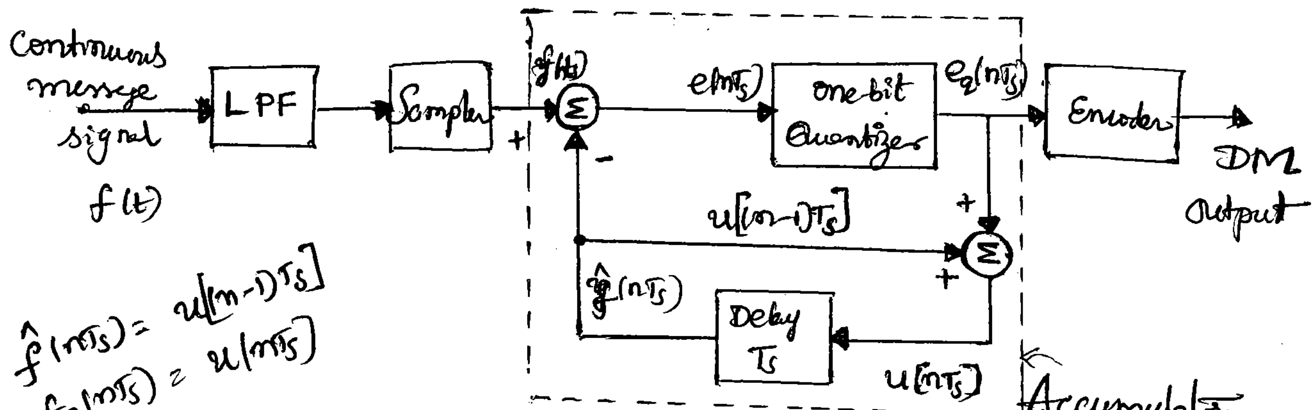
V_{dmax} - peak amplitude of difference signal.

Delta Modulation (DM): [Step Size is fixed]

- Delta modulation is also called Single bit modulation.
- * It is also a 1-bit PCM system (or) 1-bit DPCM system.
- PCM transmits all the bits which are used to code a sample So, bit rate and transmission channel bandwidth are large in PCM. To overcome this problem Delta modulation is used.
- * 1-bit Quantizer is equivalent to a two-level comparator.
- The block diagram is similar to DPCM but only change occured is in the place of a predictor we are using a single delay 'T_s'
- * The difference between the input and delayed value is quantized into only two levels namely '$+\Delta$' and '$-\Delta$'
- If the difference is positive, then approximated signal is increased by one step i.e 'Δ'

  If the difference is negative, then approximated signal reduced by one step 'Δ'.
- * When the step is reduced, '0' is transmitted and when the step is increased '1' is transmitted.

Hence For each sample, only one binary bit is transmitted. & step Size is fixed in delta modulation.



$\hat{f}(nT_s) = u[(n-1)T_s]$

$f_q(nT_s) = u(nT_s)$

Fig : DM - Transmitter

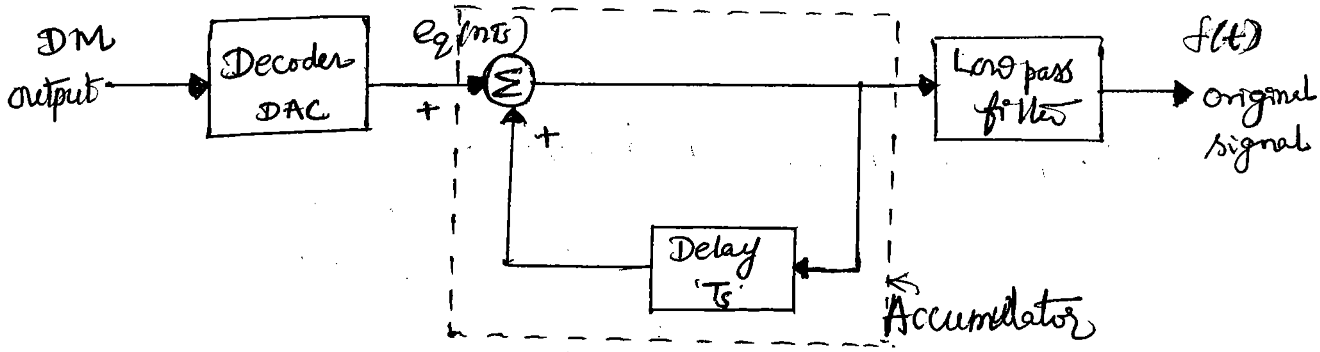


Fig: DM - Receiver

where $f(nT_s)$ - Sampled signal of $f(t)$

$\hat{f}(nT_s)$ - last sample approximation of the staircase waveform

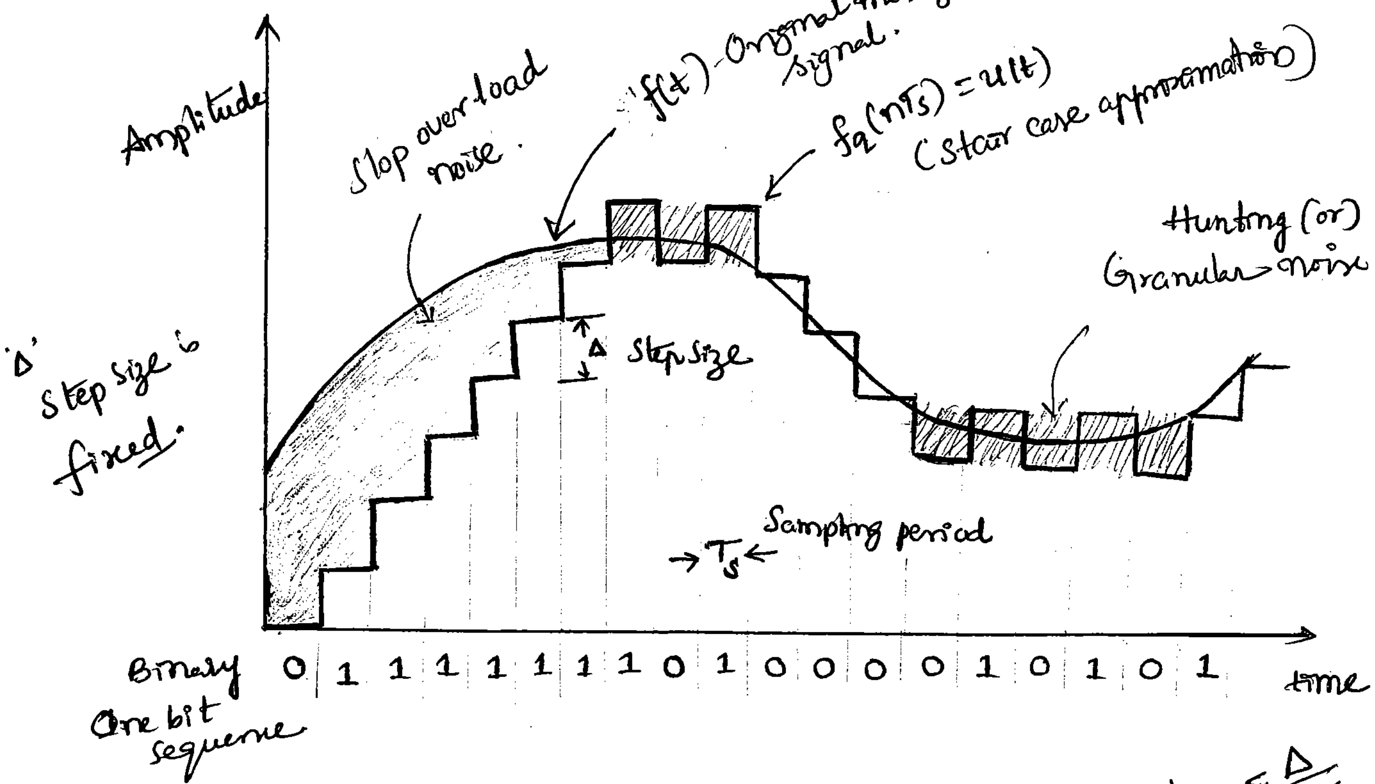


Fig : Delta modulation waveform

$\text{slope} = \frac{\Delta}{T_s}$

✓ In delta modulation, No. of bits/sample $\boxed{N=1}$

$\therefore$ No. of quantization levels $Q = 2^N = 2^1 \quad \therefore \boxed{Q=2}$

Bit rate $R_b = N \times f_s = 1 \cdot f_s \Rightarrow \boxed{f_s = R_b}$

* When the step is reduced, '0' is transmitted, when the step is increased, '1' is transmitted, for each sample, only one bit is transmitted.

If $f(nT_s) \geq \hat{f}(nT_s)$ - $e_q(nT_s) = +\Delta \rightarrow$ '1'

If $f(nT_s) < \hat{f}(nT_s)$, $e_q(nT_s) = -\Delta$, $\rightarrow$ '0'

Hardware Implementation of Deltamodulation:

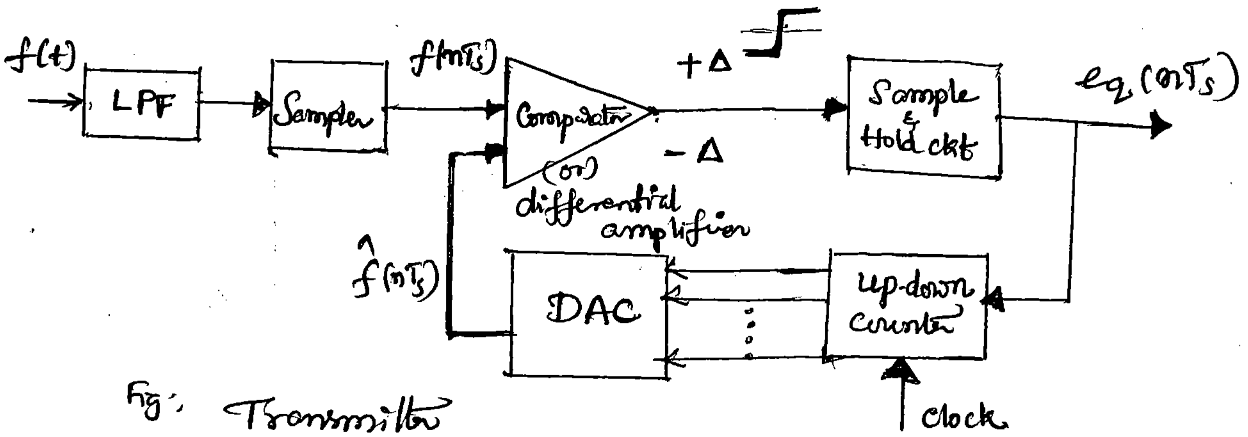


Fig: Transmitter

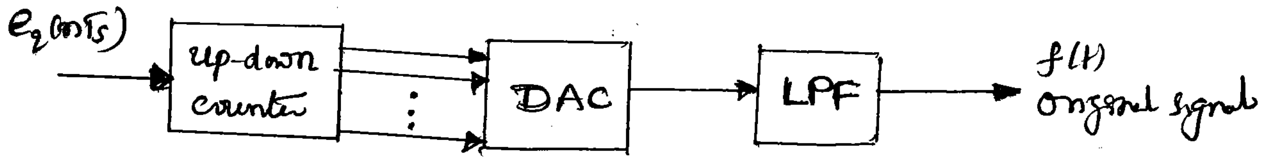


Fig: Receiver.

✓ **Comparator**: If $f(t) \geq \hat{f}(nT_s)$ then output $V(H)$ - logic-1

If $f(nT_s) < \hat{f}(nT_s)$ then output $V(L)$ - logic-0.

✓ **Sample and hold ckt**: It takes the sample of comparator output and maintain (Hold) the same peak value upto the completion of one clock cycle.

✓ **Up down Counter**: It increaments or decrements its count by '1' at each active edge of the clock waveform.

✓ **DAC - Digital to analog Counter :** The output of counter is 'n' bits this n-bits applied to DAC, DAC output is say 5V.

ie If counter is creamented by 1 then output of DAC is incremented by 5V.

If Counter is decremented by 1 then output of DAC is decremented by 5V.

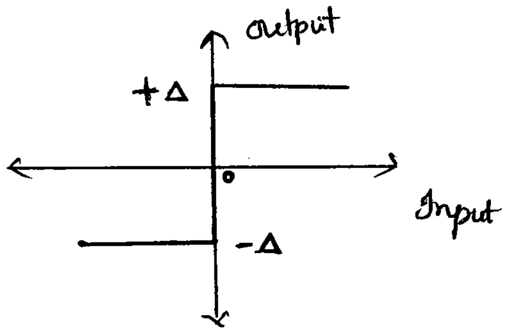
LPF : Low pass filter has the cut off frequency equal to the highest frequency of $f(t)$.

✓ It smoothens the stair case signal to reconstruct original message signal $f(t)$.

ie At transmitter → Sampler, onebit quantizer & accumulator are interconnected (Initially set zero.)

At receiver → Decoder, accumulator, LPF.

The input - output characteristics of two level quantizer as



Advantages of DM :

✓ The DM transmits only one bit for one sample, so the signalling rate and transmission channel Bandwidth is quite small.

✓ The transmitter & receiver implementation is very much simpler.

Disadvantages of DM :

The deltamodulation has two major drawbacks as under

ⓐ Slope over load distortion/noise

ⓑ Granular & idle & Hunting noise/distortion.

Let us consider the message signal $m(t)$ & $f(t)$ and the quantized approximated signal $\hat{f}_q(nT_s)$ & $\bar{m}(t)$ then the distortions as follows.

(a) Slope overload distortion :

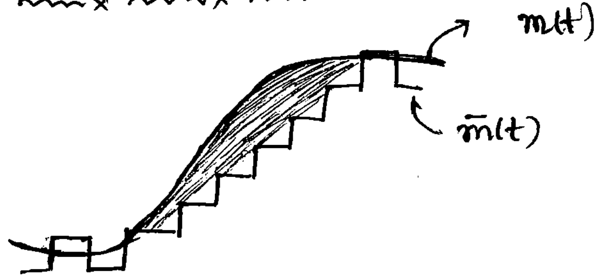


Fig (a) Slope over load (positive)

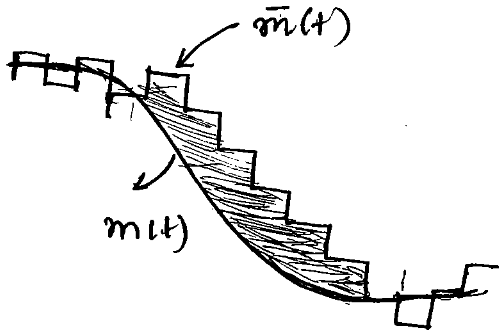


Fig (b) Slope over load (Negative)

✓ The slope of the deltamodulation is Slope = $\frac{\Delta}{T_s}$ & Δf_s.

where Δ - Step size

T_s - Sampling period.

Fig (a). The waveform $\bar{m}(t)$ is unable to follow m(t) because the slope of m(t) is greater than the slope of $\bar{m}(t)$ i.e slope is positive

Fig (b). The slope of m(t) is more negative than the slope of $\bar{m}(t)$.

In both cases the recovered waveform will be distorted, The delta modulation is, then said to be have slope over load distortion.

To avoid slope over load error, condition that the slope of $\bar{m}(t)$ & $\hat{f}_q(nT_s)$ should be greater than or equal equal to maximum slope of message m(t) & f(t).

$\therefore$ The slope of $\bar{m}(t) = \frac{\Delta}{T_s} = \Delta \cdot f_s$.

The slope of $m(t) = \frac{d}{dt}(m(t))$

where $m(t) = A_m \sin \omega_m t$.

$$\therefore \frac{\Delta}{T_s} \geq \max . \frac{d}{dt}(m(t))$$

$$\geq \max . \frac{d}{dt}\left[A_m \sin 2\pi f_m t\right]$$

$$\geq \max . \left[A_m \cdot \cos 2\pi f_m t \cdot 2\pi f_m\right]$$

$$\Delta f_s = \frac{\Delta}{T_s} \geq 2\pi f_m \cdot (A_m)_{max} \qquad (\because \cos [-1, 1]) \quad \uparrow max$$

$$\therefore \boxed{(A_m)_{max} \;\&\; (V_m)_{max} \leq \frac{\Delta f_s}{2\pi f_m}}$$

The permissiable value of the output signal power is

$$P_{max} = \left(\frac{V_{max}^2}{2}\right)\Big/R \;\&\; \frac{(A_{max})^2}{2} = \frac{\Delta^2 f_s^2}{4\pi^2 f_m^2 \times 2} \qquad \left(\because \left[\frac{V_{max}}{\sqrt{2}}\right]^2 \hookrightarrow \text{rms values}\right)$$

$$\therefore \boxed{P_{max} = \frac{\Delta^2 f_s^2}{8\pi^2 \cdot f_m^2}}$$

(b) Granular & Idle & hunting noise :

In both cases the variations in $m(t)$ are such that they are within the step size. Hence the waveform $\bar{m}(t)$ is like a square wave. This will be recovered as d.c whereas the original signal $m(t)$ is not dc. Thus, in this case also distortion is resulted and noise is known as granular & idel & hunting noise.

- To overcome these error Adaptive delta modulation is used.

Noise in Delta Modulation :

The output of the receiver differs from the input of the transmitter because of two noises. They are

(a) Quantization noise $[n_q(t)]$

(2) Channel noise $[n_c(t)$ & $n_o(t)]$

$\therefore$ The output consists at Receiver is $\bar{X}(t) = \underbrace{X_o(t)}_{\text{Signal}} + \underbrace{n_q(t)}_{\text{Quantization noise}} + \underbrace{n_c(t)}_{\text{Channel noise}}$

$\therefore$ The overall SNR of delta modulation is

$$(SNR)_{DM} = \frac{E\{(X_o(t))^2\}}{E\{[n_q(t)]^2\} + E\{[n_c(t)]^2\}} \quad (\text{or}) \quad \frac{S_o}{N_q + N_c}.$$

Signal power (S_0):

For calculating signal power we have a limitation that the slope of the signal waveform must be observed if slope overload error is to be avoided

Consider that there is no slope over load error, condition is

$$\text{slope of } \hat{f}(nT_s) \geq \text{max. slope of } f(t)$$

let message signal $f(t) = V_m \sin 2\pi f_m t$ ($\because V_m$ & A_m)

& Staircase approximated signal is $\hat{f}(nT_s)$

$$\therefore \frac{\Delta}{T_s} \geq \max \frac{d}{dt}(f(t))$$

$$\frac{\Delta}{T_s} \geq \max \frac{d}{dt}(V_m \sin 2\pi f_m t)$$

$$\frac{\Delta}{T_s} \geq (V_m)_{max} \cdot (\cos 2\pi f_m t \cdot)_{max} \cdot 2\pi f_m$$

$$\frac{\Delta}{T_s} = 2\pi f_m \cdot (V_m)_{max} \cdot \qquad (\because \cos \text{ fun } [-1\ 1])$$

$$\therefore (V_m)_{max} = \frac{\Delta / T_s}{2\pi f_m} = \frac{\Delta f_s}{2\pi f_m} = \left(\frac{\Delta}{2\pi}\right)\left(\frac{f_s}{f_m}\right)$$

$\therefore$ R.M.S power of signal component is

$$S_0 = E\{x_0^2(t)\} = \left(\frac{(V_m)_{max}}{\sqrt{2}}\right)^2 \Big/ R = \frac{[V_m(max)]^2}{2} \qquad (R=1)$$

$$= \left(\frac{\Delta}{2\pi}\right)^2 \left(\frac{f_s}{f_m}\right)^2 \times \frac{1}{2}$$

$$= \frac{\Delta^2}{4\pi^2} \times \frac{f_s^2}{f_m^2} \times \frac{1}{2}$$

$$S_0 = \frac{\Delta^2}{8\pi^2}\left(\frac{f_s}{f_m}\right)^2$$

$\therefore$ Signal power $\boxed{S_0 = E\{[X_0^2(t)]\} = \frac{\Delta^2}{8\pi^2}\left(\frac{f_s}{f_m}\right)^2}$

Quantization Noise power: (N_q).

To estimate quantization noise power in delta modulation consider error waveform

∴ Quantization error $e_q(t) = f(t) - \hat{f}_q(t) \leq \Delta$

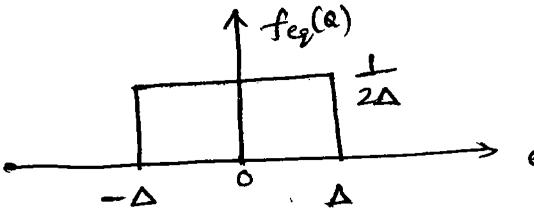
where $f(t)$ - original message signal

$\hat{f}_q(t)$ - staircase approximated signal.

Consider in the absence of slope overload error.

let an uniform PDF of $e_q(t)$ as $f_{e_q}(Q) = \begin{cases} \frac{1}{2\Delta}, & -\Delta \leq e_q \leq \Delta \\ 0, & \text{otherwise.} \end{cases}$

ie



$\left(\frac{1}{b-a} = \frac{1}{\Delta-(-\Delta)} = \frac{1}{2\Delta} \right)$

∴ The power of noise (error component) is mean square value of $e_q(t)$

$$\therefore E\{[e_q^2(Q)]\} = \int_{-\infty}^{\infty} q^2 \cdot f_{e_q}(Q)\, dq$$

$$= \int_{-\Delta}^{\Delta} q^2 \cdot \frac{1}{2\Delta} \cdot dq$$

$$= \frac{1}{2\Delta} \cdot \frac{q^3}{3}\Big|_{-\Delta}^{\Delta}$$

$$= \frac{1}{6\Delta}(\Delta^3 + \Delta^3)$$

$$= \frac{2\Delta^3}{6\Delta} = \frac{\Delta^2}{3}$$

$$\therefore \boxed{E\{[e_q^2(t)]\} = \Delta^2/3.}$$

The normalized total power of the waveform $e_q(t)$ is spread over the bandwidth 'f_s' & it is uniformly distributed over interval $(-f_s, f_s)$.

ie The PSD of $e_q(t)$ is $G_{e_q}(f) = \begin{cases} \frac{\Delta^2}{3} \cdot \frac{1}{2f_s}; & -f_s \leq f \leq f_s \\ 0 & \text{; otherwise.} \end{cases}$

Now at the output of LPF, the Quantization noise power with LPF bandwidth is equal to the input signal bandwidth '$-f_m$ to f_m'

$\therefore$ Quantization noise power N_q as

$$N_q = E\{[n_q(t)]^2\} = \int_{-\infty}^{\infty} G_{eq}(f) \cdot df$$

$$= \int_{-f_m}^{f_m} \frac{\Delta^2}{6f_s} \cdot df$$

$$= \frac{\Delta^2}{6f_s} \cdot (f)_{-f_m}^{f_m}$$

$$= \frac{\Delta^2}{6f_s} \cdot 2f_m$$

$$= \frac{\Delta^2}{\cancel{6}3 f_s} \cancel{2}f_m$$

$$\therefore \left(\frac{\Delta^2}{3}\right)\left(\frac{f_m}{f_s}\right)$$

$\therefore$ Quantization noise power $\boxed{N_q = E\{[n_q^2(t)]\} = \left(\frac{\Delta^2}{3}\right)\left(\frac{f_m}{f_s}\right)}$

Signal to Quantization noise in Delta modulation is

$$(SQNR)_{DM} = \frac{S_o}{N_q}$$

$$= \frac{\frac{\Delta^2}{8\pi^2} \cdot \left(\frac{f_s}{f_m}\right)^2}{\frac{\Delta^2}{3} \cdot \left(\frac{f_m}{f_s}\right)}$$

$$= \frac{3}{8\pi^2} \cdot \left(\frac{f_s}{f_m}\right)^2 \cdot \left(\frac{f_s}{f_m}\right)$$

$$= \frac{3}{8\pi^2} \cdot \left(\frac{f_s}{f_m}\right)^3$$

$$(S/N_q)_{DM} \text{ (or) } \boxed{(SQNR)_{DM} = \frac{3}{8\pi^2} \cdot \left(\frac{f_s}{f_m}\right)^3}$$

where f_s - Sampling Rate

f_m - modulating signal frequency (or) Cutoff freq of LPF.

Channel Noise power N_c (or) N_o :

The channel noise & thermal noise can be occured at the end of transmitter and beginning of receiver.

The mean time seperation between the impulses/samples is assumed to be equal to T_s/P_e ⇒ $\tau = \frac{T_s}{P_e} = \frac{1}{f_s \cdot P_e}$ probability error.

The PSD of the impulse train can be assumed to be white noise with a magnitude of $4\Delta^2 \cdot P_e \cdot f_s$ $\left(\because \frac{(2\Delta)^2}{\tau}\right)$.

The PSD of the channel error noise at the input of the filter (LPF) is given by

$$G_{th}(f) = \frac{4\Delta^2 \cdot P_e \cdot f_s}{\omega^2} = \frac{4\Delta^2 \cdot P_e \cdot f_s}{(2\pi f)^2}$$

$$= \frac{\cancel{4}\Delta^2 \cdot P_e f_s}{\cancel{4}\pi^2 \cdot f^2}$$

$$\boxed{G_{th}(f) = \frac{\Delta^2}{\pi^2} \cdot P_e \cdot \left(\frac{f_s}{f^2}\right)}$$

If $f \to 0$ then $G_{th}(f) \to \infty$ & the integral of $G_{th}(f)$ over a range of frequencies including $f=0$ is infinite.

LPF will have a low frequency cut off 'f_1' > zero and a high frequency cut off 'f_x'. ($f_1 < f_x$).

$\therefore$ Average noise power $= 2\int_{f_1}^{f_x} G_{th}(f) \cdot df$

i.e. Channel noise power

$$N_c = E\{[n_o^2(t)]\} = 2\int_{f_1}^{f_x} \frac{\Delta^2}{\pi^2} \cdot P_e \cdot f_s \cdot \left(\frac{1}{f^2}\right) \cdot df$$

$$= \frac{2\Delta^2}{\pi^2} \cdot P_e \cdot f_s \cdot \left(\frac{f^{-1}}{-1}\right)_{f_1}^{f_x}$$

$$= \frac{2\Delta^2}{\pi^2} \cdot P_e\, f_s \cdot \left(\frac{1}{f_x} - \frac{1}{f_1}\right) \qquad \left(\because \int f^{-2} = \frac{f^{-1}}{-1}\right)$$

$$\therefore N_0 = \frac{2\Delta^2}{\pi^2} \cdot Pe \cdot fs \cdot \left(\frac{1}{f_1} - \frac{1}{fx}\right)$$

If $f_1 << fx$ then $\frac{1}{f_1} >> \frac{1}{fx}$ so, $\frac{1}{fx}$ is neglected.

$\therefore$ Channel noise power $\boxed{N_0 = E\{[n_c(t)]^2\} = \frac{2\Delta^2}{\pi^2} \cdot Pe \cdot fs \cdot \left(\frac{1}{f_1}\right)}$

$\therefore$ The over all SNR of the delta modulation as

$$(SNR)_{DM} = \frac{S_0}{Nq + N_0}$$

$$= \frac{\frac{\Delta^2 fs^2}{8\pi^2 fm^2}}{\frac{\Delta^2}{3} \cdot \frac{fm}{fs} + \frac{2\Delta^2}{\pi^2} \cdot Pe \cdot \frac{fs}{f_1}}$$

$$= \frac{\cancel{\Delta^2} \cdot fs^2 / 8\pi^2 fm^2}{\frac{\cancel{\Delta^2} \cdot fm}{3fs}\left[1 + \frac{2\,\cancel{fs} \cdot Pe}{\pi^2 f_1} \cdot \frac{3 \times fs}{fm}\right]}$$

$$= \frac{\frac{fs^2 \times 3fs}{8\pi^2 fm^2 \cdot fm}}{1 + \frac{6\,Pe}{\pi^2} \cdot \left(\frac{fs^2}{f_1 fm}\right)}$$

$$\boxed{(S/N)_{DM} \;\&\; (SNR)_{DM} = \frac{3}{8\pi^2}\left(\frac{fs}{fm}\right)^3 \cdot \frac{1}{\left[1 + \frac{6\,Pe}{\pi^2} \cdot \left(\frac{fs}{fm}\right)^2\right]}} \quad (\because f_1 = fm)$$

$(SQNR)_{DM} = \frac{3}{8\pi^2} \cdot \left(\frac{fs}{fm}\right)^3 \Rightarrow$ SQNR in dB $= 10 \log\left(\frac{3}{8\pi^2}\right) + 30 \log(fs/fm)$

If $fs = 2fm$ $(SQNR)_{dB} = -5\,dB$ $\downarrow +9\,dB$

fs doubling · If $fs = 4fm$ $(SQNR)_{dB} = +4\,dB$ $\downarrow +9\,dB$ Increment

Hence. If $fs = 8fm$ $(SQNR)_{dB} = +13\,dB$

The max. output signal to noise ratio of a delta modulator is proportional to the sampling rate cubed.

$\therefore$ It indicates a 9 dB improvement with doubling of the sampling rate.

Adaptive Delta Modulation (ADM):

* To overcome the quantization errors due to slope overload distortion, and granular / idle noise.
* The step size (Δ) is made adaptive to variations in the input signal $f(t)$.
* Perticulary in the steep segment of the signal $f(t)$, the step size is increased also if the input is varying slowly, the step size is reduced.

Thus A delta modulation system which adjusts its step size is known as the adaptive delta modulation. (ADM) system.

Transmitter:

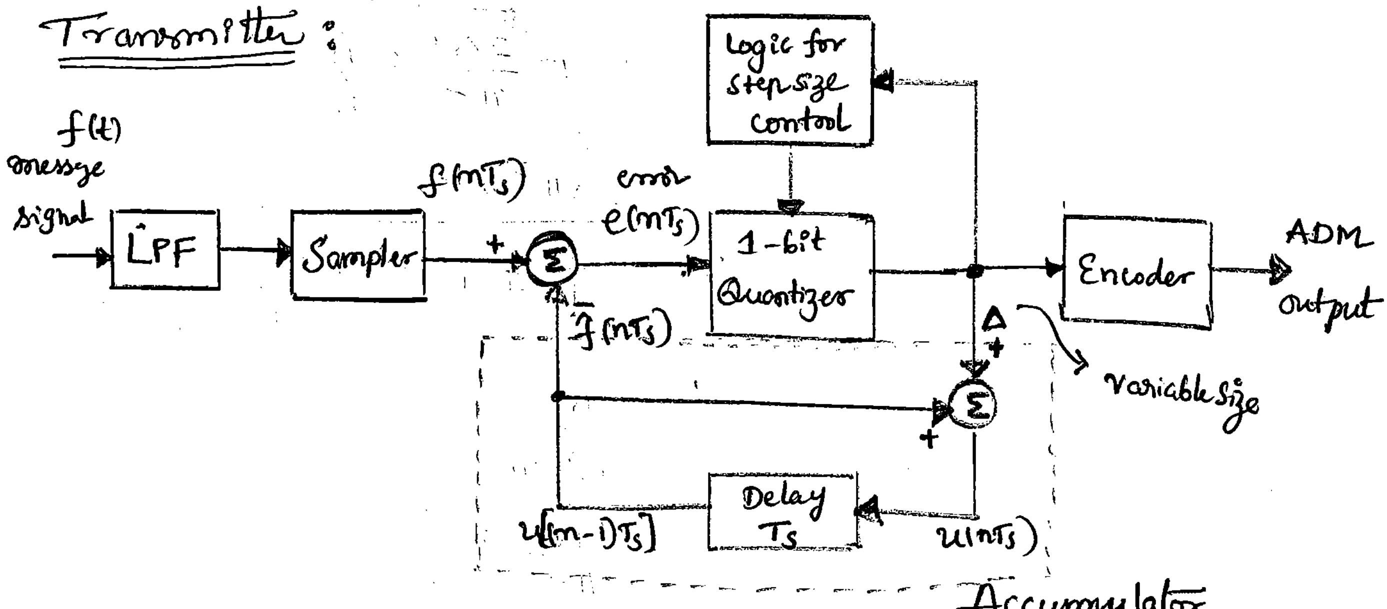


Fig: Adaptive Delta modulation - Transmitter.

Receiver:

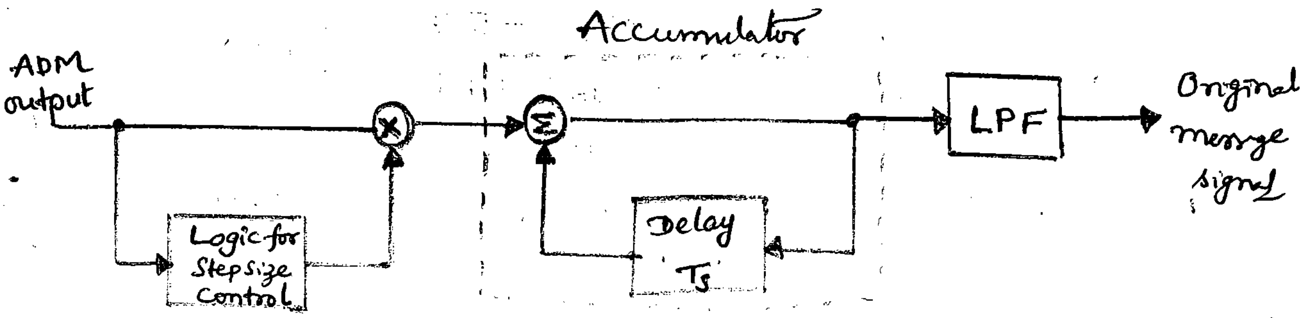


Fig: Adaptive Delta modulation - Receiver

* For both transmitter and receiver diagrams logic for step size control is added.

✓ The step size increases or decreases according to a specified rule depending on one bit quantizer output.

ie
* If one bit quantizer output is high (ie 1), then the step size may be doubled for next sample.
* If one bit quantizer output is low (ie 0), then the step size may be reduced by one step.

✓ The step size is constrained to lie between min. & max. values

ie $$\Delta_{min} \leq e_q(nT_s) \leq \Delta_{max}.$$

The upper limit 'Δ_{max}' controls the amount of slope overload distortion.
The lower limit 'Δ_{min}' controls the amount of granular/Idle noise.

Waveforms:

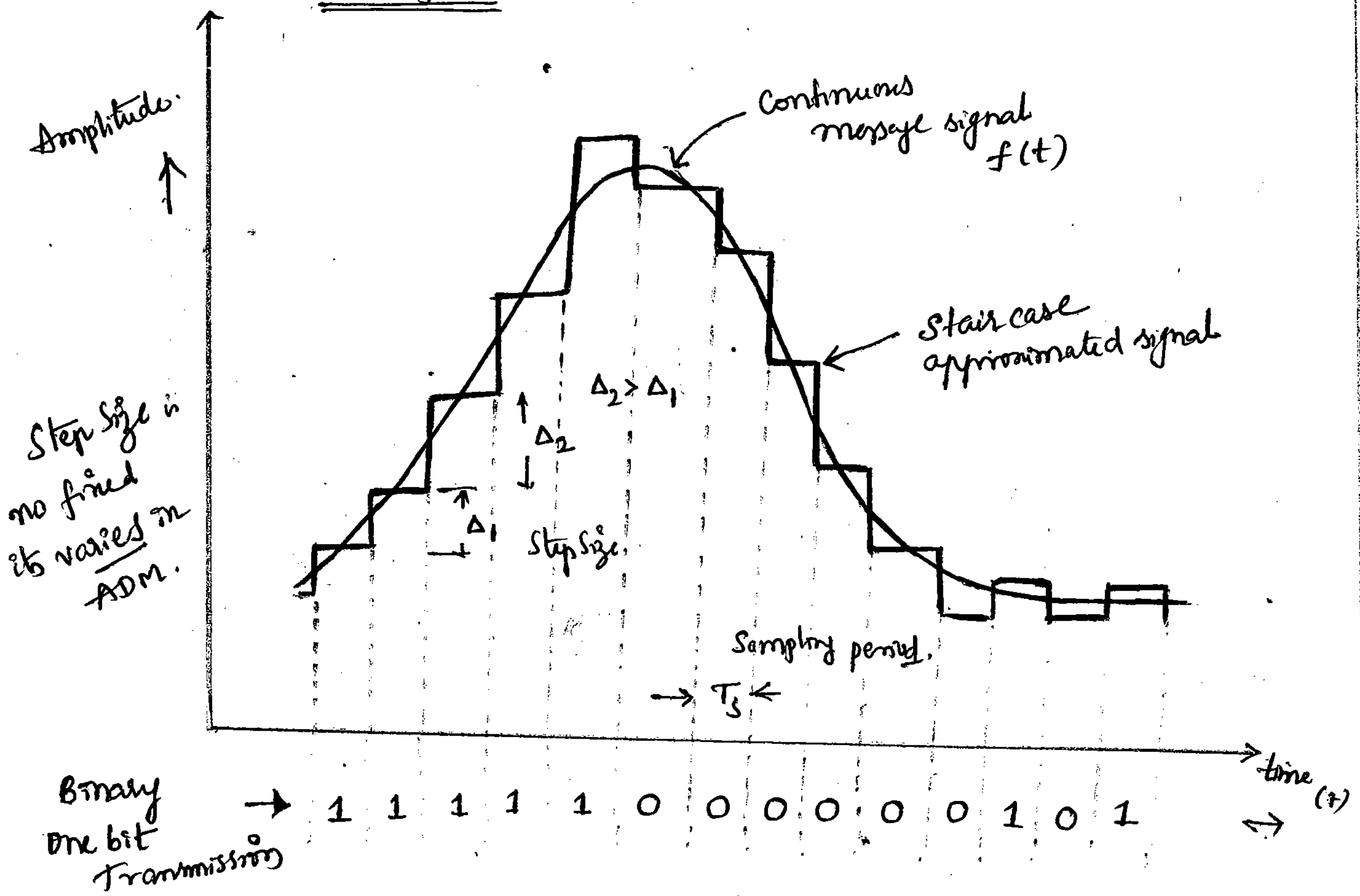
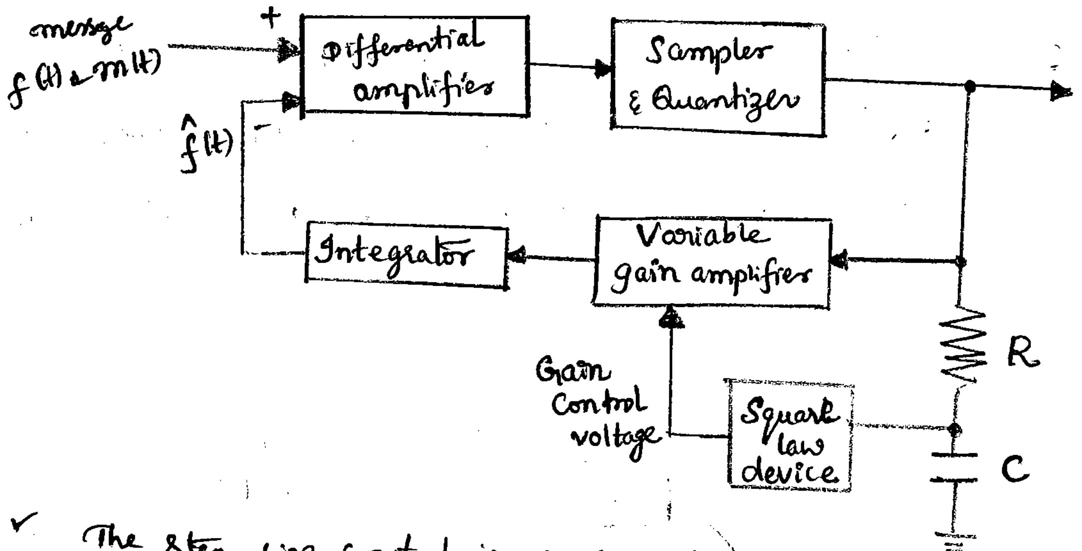


Fig: Waveform for Adaptive delta modulation.

- At the receiver, the first portion produces the step size from each incoming bit, the previous input and present input decides the step size. It is then applied to an accumulator which builds up stair case waveform.
- The Low pass filter then smoothens out the staircase waveform to reconstruct the original signal.

Practical / Hardware Implementation of ADM: (or) CVSDM

Continuously Variable Slope Delta Modulation.



- The step size control is performed by a digital Integrator (accumulator) along with variable gain amplifier.
- The bit size is converted in to a voltage that is fed to a variable gain amplifier.
- The amplification is minimum when the input voltage corresponds to equal no. of 1's and 0's in the period.
- The amplifier controls the step size depending on the voltage feedback to it.

* The step size is varied by controlling the gain of the Integrator.
* The gain control ckt consists of a RC Integrator and a Square law device.
* When the modulator output is a sequence of alternate polarity pulses.
* These pulses when integrated by the RC filter yield an average output almost zero. The gain control input and hence the gain & the step size are small.

Comparision of PCM, DPCM, DM and ADM Systems :

SL No	Parameter	PCM	DM	ADM	DPCM
1	Expansion	Pulse Code Modulation	Delta Modulation	Adaptive delta modulation	Differential pulse Code Modulation
2	No. of bits/sample (N)	4 (or) 8 (or) 16 (or) 32 bits	1-bit	1-bit	More than 1 but less than PCM.
3	Step size 'Δ'	Depends on no. of bits	Fixed.	Varied	Fixed.
4	Bandwidth	high (Since no. of bits are high).	Low	Low	Less than PCM.
5	No. of quantization level. $Q = 2^N$	High (∵ N is high) 2^N	$N = 1$ $Q = 2$	$N = 1$ $Q = 2$	Less than PCM 2^N.
6	Feedback in transmitter (or) Receiver	No feedback	Feedback Exists	feedback Exists	Feedback Exists.
7	Bit rate R_b - bits/sec	Nf_s (7-8)	f_s (N=1)	f_s (N=1)	less than PCM Nf_s. (4-6)
8	SNR	Good	Poor	better	greater than PCM
9	Sampling rate f_s	8 KHz	(64-128) KHz	(48-64) KHz	8 KHz
10	Complexity of implementation	Complex	simple	Simple	Simple
11	Quantization error & distortion	Quantization error present	Slope overload distortion, idle noise.	Errors are absent! only quantization noise	Quantization noise present.
12	Applications	Telephony & TDM	Audio and Speech processing	Audio and speech, telephony systems.	Space commn, telephony.

Problems :

① A ramp signal $m(t) = at$ is applied to a delta modulator which operate with a sampling period T_s and Step size 'Δ' (or) δ.

a) Show that slope overload occurs if $\delta < aT_s$.

b) Sketch the modulator output for the following three values of step size $\delta = 0.75aT_s$, $\delta = aT_s$ & $\delta = 1.25aT_s$.

Sol. Given $m(t) = at$

Sampling period 'T_s'

Step Size = 'Δ' or δ.

a) To avoid slope overload distortion, the condition is

$$\frac{\Delta}{T_s} \geq \max \cdot \frac{d}{dt}(m(t))$$

slope of staircase approximated s/l. ↲ ↳ slope of message signal

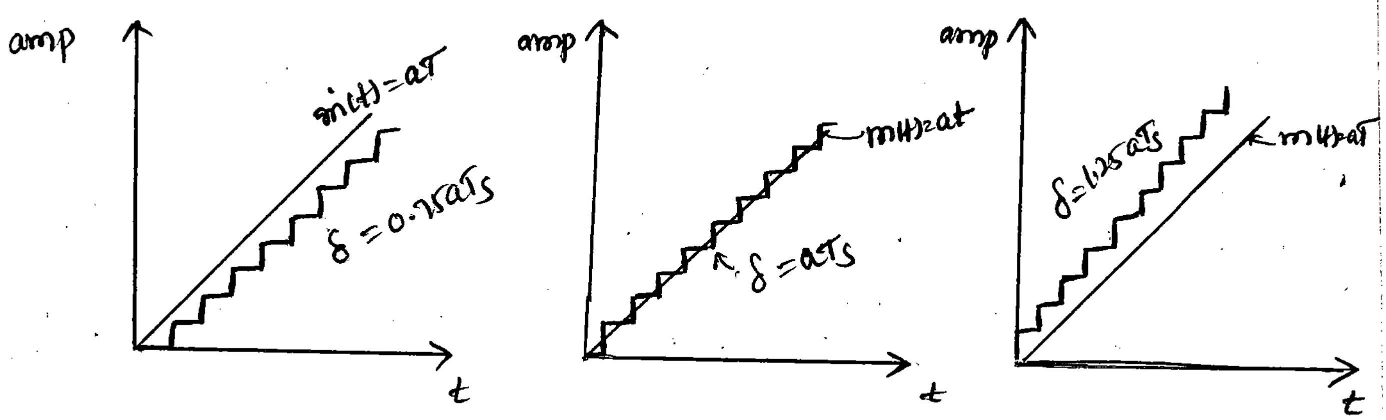
$$\frac{\delta}{T_s} \geq \max \left| \frac{d}{dt} \cdot at \right|$$

$$\frac{\delta}{T_s} \geq a$$

$$\therefore \underline{\delta \geq aT_s}.$$ where a - slope.

∴ ie The slope overload error will occurs if $\underline{\delta < aT_s}$.

b) Step size $\underline{\delta = 0.75aT_s}$ $\underline{\delta = aT_s}$ $\underline{\delta = 1.25aT_s}$.



(2) If the step size is 4V in case of PCM and DM then what is the corresponding quantization noises.

Sol Step Size $\Delta = 4V$.

$\therefore$ PCM $\rightarrow$ Quantization noise $= \frac{\Delta^2}{12} = \frac{4^2}{12} = \frac{16}{12} = 1.33$.

DM $\rightarrow$ Quantization noise $= \frac{\Delta^2}{3} = \frac{4^2}{3} = \frac{16}{3} = 5.33$.

(3) Find out the output signal to quantization noise ratio of delta modulation if the sampling rate 8KHz and the modulating & base band frequency is 2KHz.

Sol. Deltamodulation.

$$f_s = 8KHz$$
$$f_m = 2KHz$$

$$\left(S/N_q\right)_{DM} = \frac{3}{8\pi^2}\left(\frac{f_s}{f_m}\right)^3$$

$$= \frac{3}{8\pi^2} \times \left(\frac{\cancel{8}^{4} \times \cancel{10^3}}{\cancel{2} \times \cancel{10^3}}\right)^3$$

$$= \frac{3}{8\pi^2} \times 64$$

$$= 2.43$$

$$\therefore (SQNR)_{DM} \text{ in dB} = 10\log(2.43) = 3.86 \text{ dB}.$$

(4) Find the sampling rate & step size if signal to quantization noise ratio is 40dB, signal frequency is 2kHz in deltamodulation. (Assume $A_m = 1$).

Sol Given $(SQNR)_{DM} = 40$ dB

$f_m = 2KHz$ $\quad f_s = ?$

$A_m = 1$. $\quad \Delta = ?$

$$(SQNR)_{DM} = 10\log\left(\frac{3}{8\pi^2} \cdot \left(\frac{f_s}{f_m}\right)^3\right).$$

$$40 = 10\log_{10}\left[\frac{3}{8\pi^2} \cdot \left(\frac{f_s}{f_m}\right)^3\right]$$

$$\frac{3}{8\pi^2}\left(\frac{f_s}{f_m}\right)^3 = 10^4$$

$$\left(\frac{f_s}{f_m}\right)^3 = \frac{8\pi^2 \times 10^4}{3}$$

$$\frac{f_s}{f_m} = \left[\frac{8\pi^2 \times 10^4}{3}\right]^{1/3} = 64$$

$$\therefore f_s = 64 \times f_m = 64 \times 2KHz = 128KHz$$

Sampling rate $\therefore \boxed{f_s = 128KHz.}$

$$(A_m)_{max} \leq \frac{\Delta f_s}{2\pi f_m}$$

$$\therefore \Delta = \frac{A_m \times 2\pi f_m}{f_s}$$

$$= \frac{1 \times 2\pi \times 2 \times 10^3}{128 \times 10^3}$$

$$\Delta = 0.098V$$

$\therefore$ Step size $\boxed{\Delta = 0.098V.}$

(5) A.T.V signal with a bandwidth of 4.2MHz is transmitted using PCM system. The no-of quantization levels are 512. Calculate

a) Codeword length

b) Final bitrate

c) Transmission bandwidth

d) SQNR. or (S/N_q).

Sol: Given Bandwidth $B.W = 4.2MHz$.

a) No-of Quantization levels $Q = 2^N = 512$ $(512 = 2^9)$

$$\therefore N = \log_2 Q = \log_2 2^9 = 9$$

$$\therefore \boxed{N = 9}$$

$\therefore$ Codeword length is no-of bits/sample $N = 9$ bits.

c) Transmission bandwidth $B.W_T \geq N f_m = N.W$ B.W of TV signal.

$$= 9 \times 4.2MHz$$

$$\therefore \boxed{B.W_T = 37.8MHz}$$

b) Final bit rate $R_b = N \times f_s$ $\to W \to$ B.W of TV signal

$= N \times 2f_m$

$= 2 \times 9 \times 4.2$ M.

$= 75.6$ MHz

$\therefore \boxed{R_b = 75.6\,MHz}$

d) $(SQNR)_{PCM} = Q^2 = 2^{2N} = 2^{2\times 9} = 2^{18} = 262144$

SQNR of PCM in dB $= 10\log(262144) = 54.2$ dB.

(or) SQNR in dB $= 1.8 + 6N$

$= 1.8 + 6 \times 9$

$= 55.8$ dB.

$\boxed{(SQNR)_{dB} = 55\,dB}$

(6) In a single integration Delta modulation system the voice signal is sampled at a rate of 64 KHz. The max. signal amplitude is 1V, the highest frequency component is 3 KHz.

a) Determine the min value of the step size σ (or) Δ (or) δ to avoid slope overload error.

b) Determine the granular noise power if the voice signal bandwidth is 3.5 KHz.

c) Assuming the voice signal is sinusoidal determine the output signal power and SQNR. (or) (S/N) QN.

d) Determine the min. transmission bandwidth.

Sol. Given Delta modulation. $f_m = 3$ KHz. $A_m = 1$ V.

$f_s = 64$ KHz.

a) To avoid slope overload distortion

$$\Delta f_s = \frac{\Delta}{T_s} \geq \frac{d}{dt}(m(t))$$

$$\Rightarrow (A_m)_{m} \leq \frac{\Delta f_s}{2\pi f_m}.$$

$$\Delta = \frac{A_m \times 2\pi \times f_m}{f_s} = \frac{1 \times 2\pi \times 3 \times 10^3}{64 \times 10^3} = 0.29\,V$$

$\therefore$ Step size $\boxed{\Delta = 0.29\,V.}$

b) Granular noise is quantization noise in delta modulation

$$\therefore \text{Granular noise} = \frac{\Delta^2}{3} \cdot \frac{f_m'}{f_s}$$ signal bandwidth

$$= \frac{(0.29)^2}{3} \times \frac{3.5\times10^3}{64\times10^3} = 0.00153$$

$\therefore$ $\boxed{\text{Granular noise} = 0.00153\,W}$

c) Output signal power $S_o = \dfrac{\Delta^2 f_s^2}{8\pi^2 f_m^2} = \dfrac{\Delta^2}{8\pi^2} \cdot \left(\dfrac{f_s}{f_m}\right)^2$.

$$\therefore S_o = \frac{(0.29)^2}{8\pi^2} \times \left(\frac{64\times10^3}{3\times10^3}\right)^2 = 0.485\,W.$$

$\therefore$ $\boxed{S_o = 0.485\,W}$

SQNR or $(S/N)_{QN}$ in DM $= \dfrac{3}{8\pi^2}\left(\dfrac{f_s}{f_m}\right)^3$

$$= \frac{3}{8\pi^2} \times \left(\frac{64\times10^3}{3\times10^3}\right)^3$$

$$= 368.8$$

$(SQNR)_{DM}$ in dB $= 10\log(368.8) = 25.6$ dB.

$\therefore$ $\boxed{(SQNR)_{dB} = 25.6\text{ dB}}$

d) Min. transmission bandwidth

$$B.W = N f_m$$

For delta modulation $N = 1$

$\therefore$ $B.W = f_m = 3\text{KHz}$

$\therefore$ $\boxed{\text{Band width} = 3\text{KHz}}$

Hence. a) Step size $\Delta = 0.29$

b) Granular noise $= 0.00153$.

c) Output signal power $S_o = 0.485W$; SQNR $= 25.6$ dB.

d) Bandwidth $= 3$KHz.