Base Band Data Transmission - II

Syllabus: Fundamentals of time division multiplexing, T1 digital carrier system, synchronization and signalling of T1, PCM hierarchy, different types of Binary encoding - M-ary encoding, correlative coding - Calculation of PSD, Inter Symbol Interference (ISI), Nyquist criteria, Eye diagram, probability of error in binary encoding & M-ary encoding, Equalizers, baseband signal receiver: Optime filter - white noise, the matched filter, Correlation receiver.

Introduction:

* The term "base band" refers to the band of frequencies representing the original signal as delivered by the source.
* A digital data & message is an ordered sequence of symbols produced by a discrete information source.

Ex: A typical computer terminal is a source of digital data. When the terminal is operated it becomes a source of digital data consisting of a sequence of binary digits 0 & 1 known as bits. Data rates a transfer rates within a computer may be 10^8 bits/sec & more.

✓ The purpose of a digital data communication system is to transfer a digital data (message) from source to destination.

* When a digital data is transmitted over a band limited channel, dispersion in the channel gives rise to an Interference called as "Inter symbol Interference" (ISI).

✓ This effect introduces deviations (errors) between the reconstructed data at the receiver and the original data transmitted.

Fundamentals of Time Division Multiplexing (TDM):

Multiplexing: Several message signals are transmitted through a single channel is called Multiplexing.

There are two types of multiplexing.

(a) FDM (Frequency division Multiplexing)

(b) TDM (Time division Multiplexing).

FDM: In FDM, total bandwidth can be divided into no. of sub channels and each sub channel is assigned to one user.

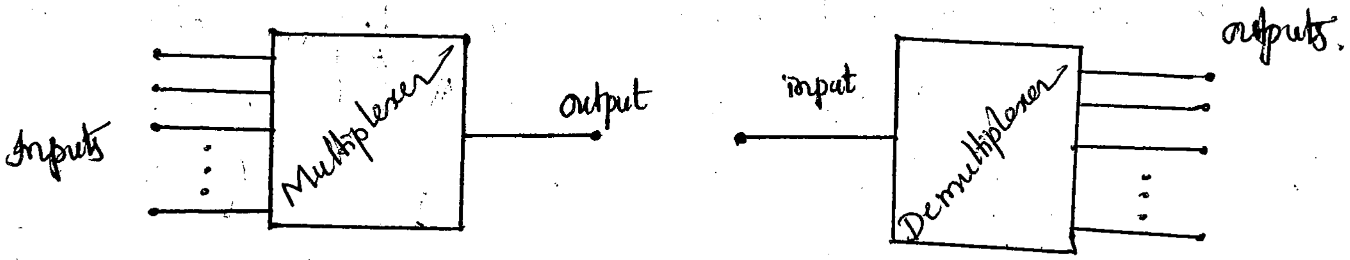
- ✓ It is a Analog Technique.
- ✓ Ex: TV signals and Radio signals.

TDM: In TDM, allots different time slots for different message signals & then transmitted using single channel.

- ✓ It is a Digital technique.
- ✓ Ex: Telephone. (Voice transmission)
  ie 300 Hz to 3.4 KHz.

Digital Multiplexer:

Multiplexer is a device which having more no. of inputs and single output (or) More no. of signals can be transmitted through a single channel.



Ex: Telephone (Voice transmission) ie 300 Hz − 3.4 KHz.

let $N = 8$ bits/sample

∴ No. of quantization levels $Q = 2^N = 2^8 = 256$ levels.

Sampling frequency $f_s \geq 2f_{max} = 2 \times 3.4K = 6.8\ KHz$.

But in standard $f_s = 8KHz = 8000$ samples/sec.

$\therefore$ Sampling time period $T_s = \frac{1}{f_s} = \frac{1}{8 \times 10^3} = 125 \times 10^{-6} = 125\ \mu sec.$

$\therefore$ Bit rate 'R_b' $= N \cdot f_s$

$= 8$ bits/sample $\times$ 8000 Sample/sec

$$\boxed{R_b = 64\ K\ bits/sec}$$

$\therefore$ Each bit time (. bit duration) ie $T_b = \frac{T_s}{N} = \frac{125 \times 10^{-6}}{8} = 15.625\ \mu sec$

$\therefore$ Bit rate $R_b = \frac{1}{T_b} = \frac{1}{15.625 \times 10^{-6}} = 64 \times 10^3$ bits/sec

$\therefore$ $\boxed{R_b = 64K\ bits/sec.}$

Block diagram of TDM:

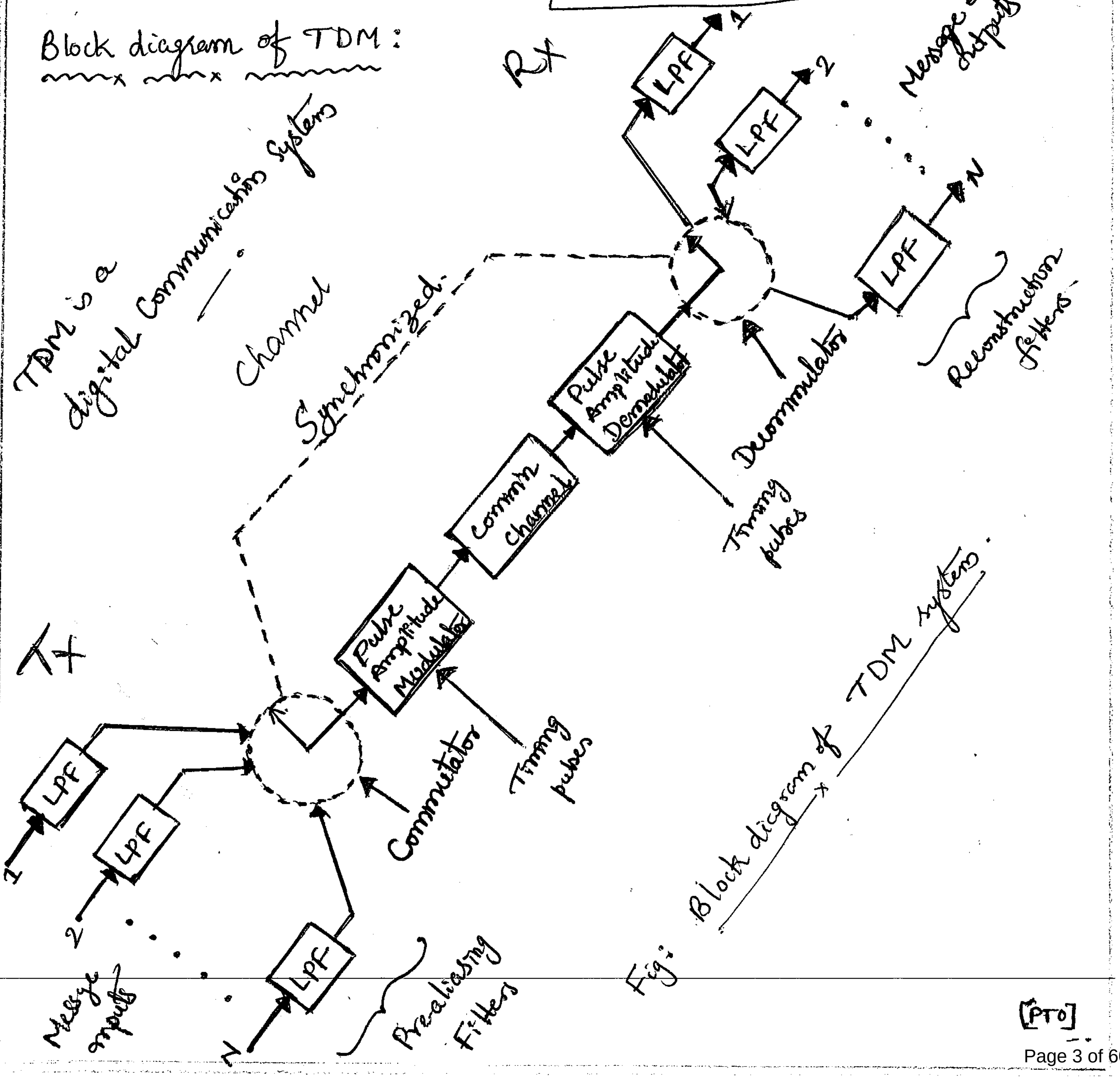
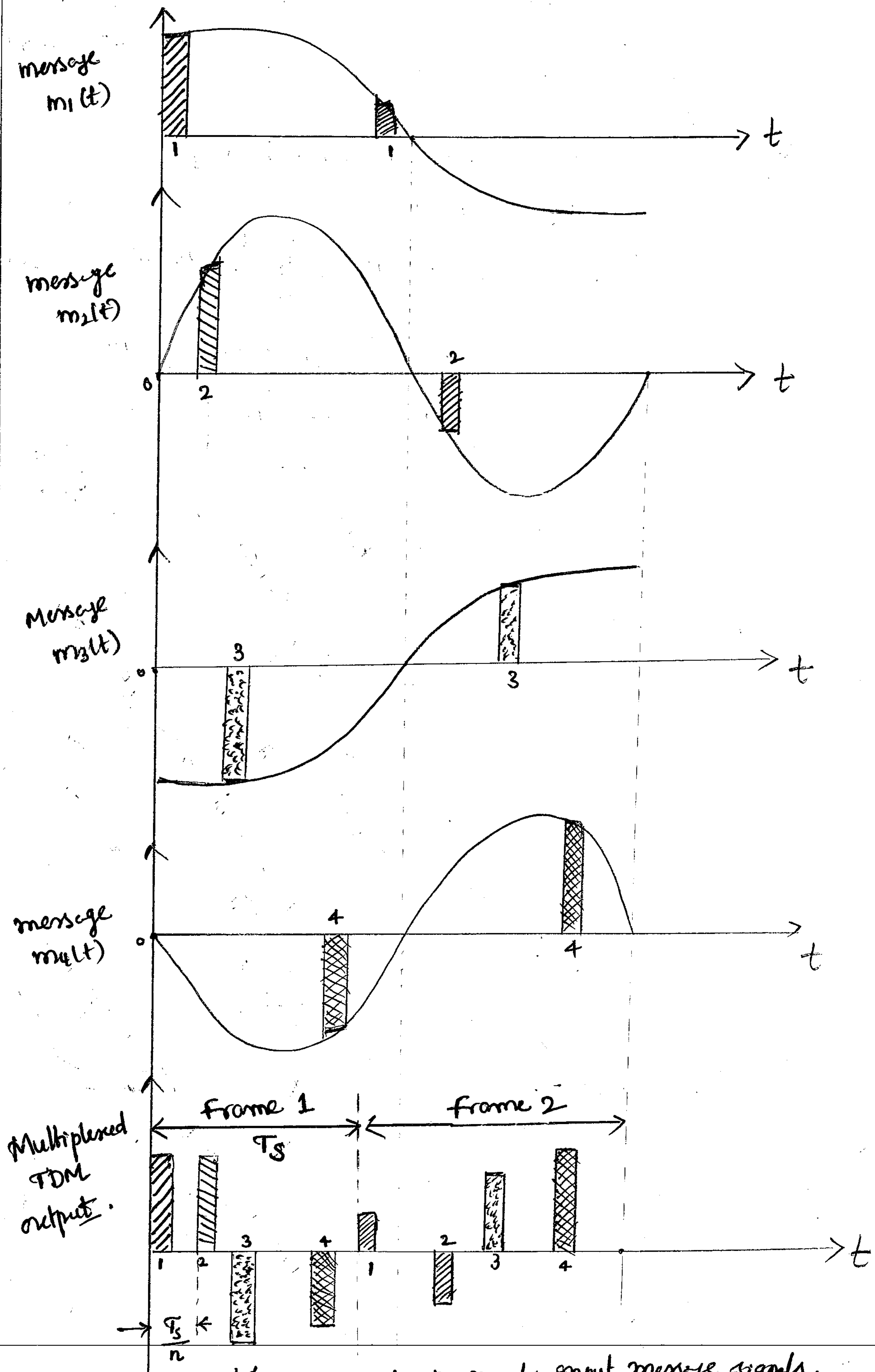


Fig: Block diagram of TDM systems.

[PTO]

Waveforms :



where $n=4$ ie no. of input message signals.

Operation :

- ✓ The input message signals are first converted to bandlimited signals by passing them through Low pass filters. These LPFs eliminate frequency components of the input signals which are not essential for their representation.
- ✓ The LPF outputs are then applied to a Commutator (or) rotating switch.
- ✓ All input signals are sequentially sampled at the transmitter by a commutator (or) Rotating sampling switch.
- ✓ The switch makes revolutions per second and extracts one sample from each input during each revolution.
- ✓ During one revolution, it collects sample data according to its time slots and form a frame.

Frame :

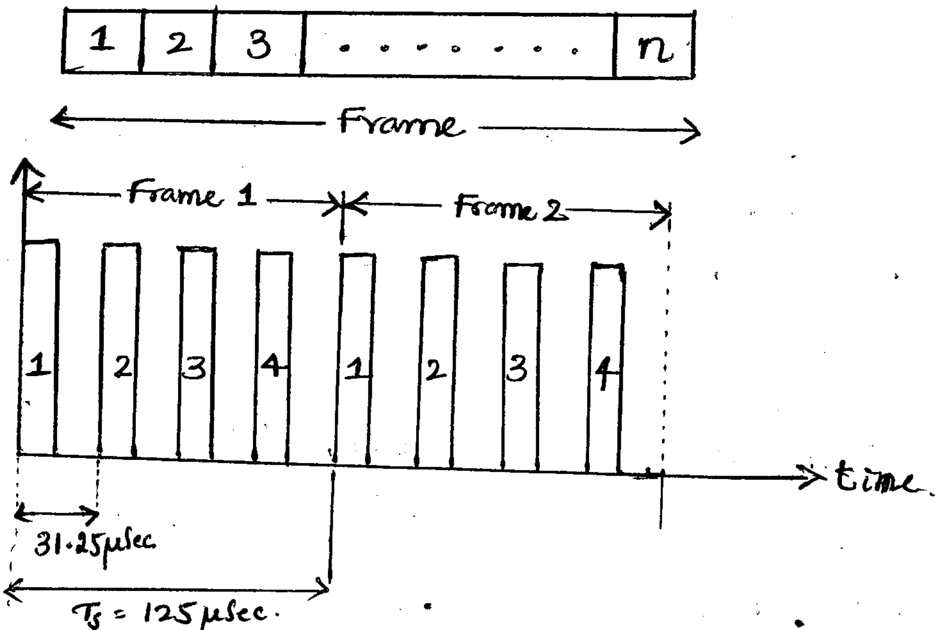
A frame consists of number of time slots one sample from each message signal is alloted in respected time slot.

Let a frame consists of n time slots.

$f_s = 8KHz$
$= 8000$ samples/sec

$T_s = \frac{1}{f_s} = \frac{1}{8\times10^3}$
$T_s = 125\,\mu$sec

$T_{s_1} = \frac{125\mu}{4} = 31.25\,\mu$sec.



voice

- ✓ The output of switch is given to pulse modulator, the pulse modulator may be PAM (or) PCM.
- ✓ The sequence of samples may be transmitted by direct PAM (or)
- ✓ The sample values may quantized and transmitted by using PCM.

- The samples from adjascent input channels are seperated by '$\frac{T_s}{n}$' where n is the no.of message input signals.

ie a set of 'n' samples collected from n-input messages is called as frame.

- At the receiver, the samples from individual channels are seperated and distributed by another rotatary switch called as 'distributor' (or) decommutator.
- The samples from each channel are filtered to reproduce the original message signals.
- TDM system requires careful synchronization between commutator and decommutator.

<u>Synchronization:</u>

Synchronization is a critical consideration in TDM because each sample must be distributed to the correct output line at the appropriate time.

- There are two levels of synchronization in TDM.
  - (a) Frame Synchronization.
  - (b) Sample (or) Word Synchronization.
- Frame synchronization is necessary to seperate one frame to another.
- Sample (or) Word Synchronization is necessary to seperate the samples with each frame.

<u>Applications:</u>

- Time domain multiplexed PCM is used in a variety of application
- The most important use of TDM PCM is in telephone systems, where voice and other signals are multiplexed and transmitted over a variety of transmission media including twin wires, wave guides and optical fibers.
- Applications of TDM in digital multiplexers for <u>Telephony</u>.

T1 digital Carrier System :

Synchronization & Signalling of T_1, PCM Hierarchy :

Let n - no. of message signals

N - no. of bits per frame

f_s - Sampling frequency. then

Bit rate $R_b = n \cdot N \cdot f_s$.

Bit duration $T_b = \frac{T_s}{n \cdot N} = \frac{1}{R_b}$.

In asynchronous TDM, No. of bits / frame = $n \cdot N$.

In Synchronous TDM, No. of bits / frame = $\underline{\underline{n \cdot N + 1}}$ → sync bit.

✓ Digital multiplexing patterns have been adopted for digital telecommunication.

* AT & T ie American Telephone & Telegraph company in north america & japan.

* CCIT in Europe ie CCIT → International telegraph and telephone Consultive Committee for the International Telecommunication unions.

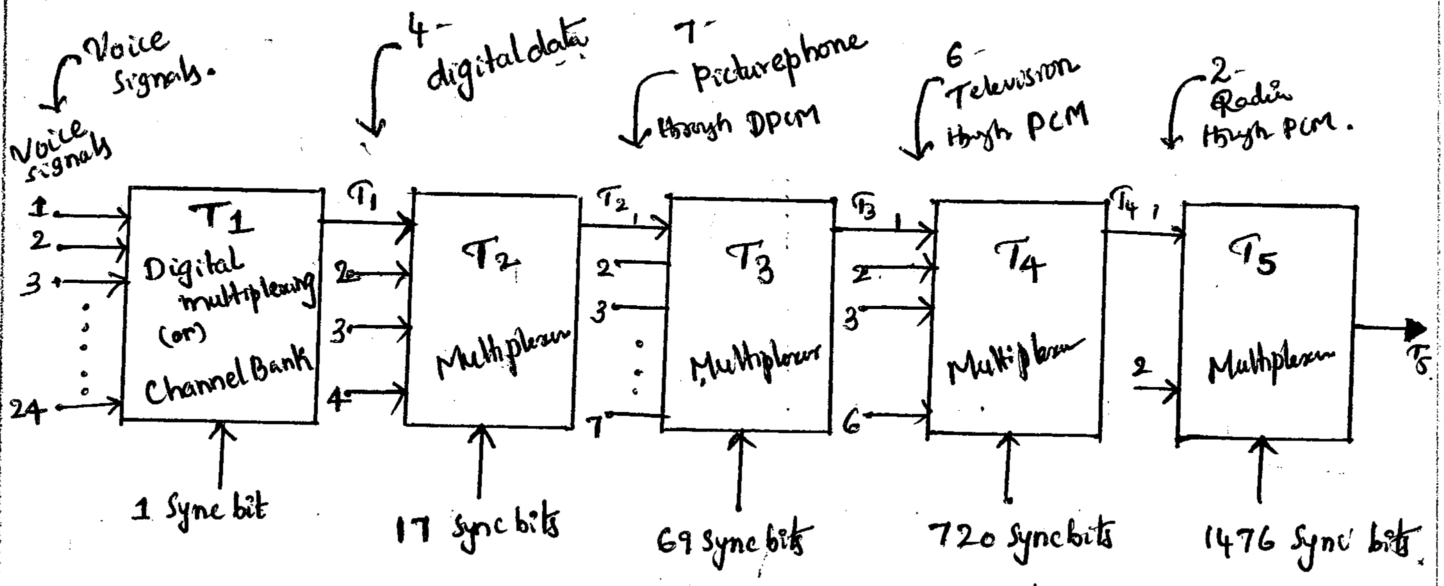


Fig : T - Digital Carrier Systems.

✓ The multiplexed PCM channels are transmitted various T carrier systems such as T_1 carrier system, T_2 carrier system, T_3 carrier system etc. as shown in above fig.

✓ A 24 channel TDM multiplexer is used as the basic system known as T_1 carrier System.

* 24 voice signals are sampled at a rate of 8 kHz and the resulting samples are quantized converted to 7 bit PCM codewords.

* At the end of each 7-bit codeword, an additional binary bit is added for synchronous purpose. ie $7+1 = 8$

* At the end of every frame contains 24 samples with 8 bit code words, another additional bit is inserted to give frame sychronization

ie $(8 \times 24)+1 = 193$ bits

Thus The overall frame size in the T_1 carrier System is 193 bits

We know $f_s = 8$ KHz. $\therefore$ Bit rate $R_b = 193$ bits/frame $\times$ 8000 frames/sec

$$\boxed{R_b = 1.544 \text{ M bits/sec}}$$

* The main purpose of T_1 system is Short distance coverage and high usage in metropolitan area.

✓ The maximum length of T_1 system is limited to 10–100 miles, with a repeaters spacing of 1 Mile.

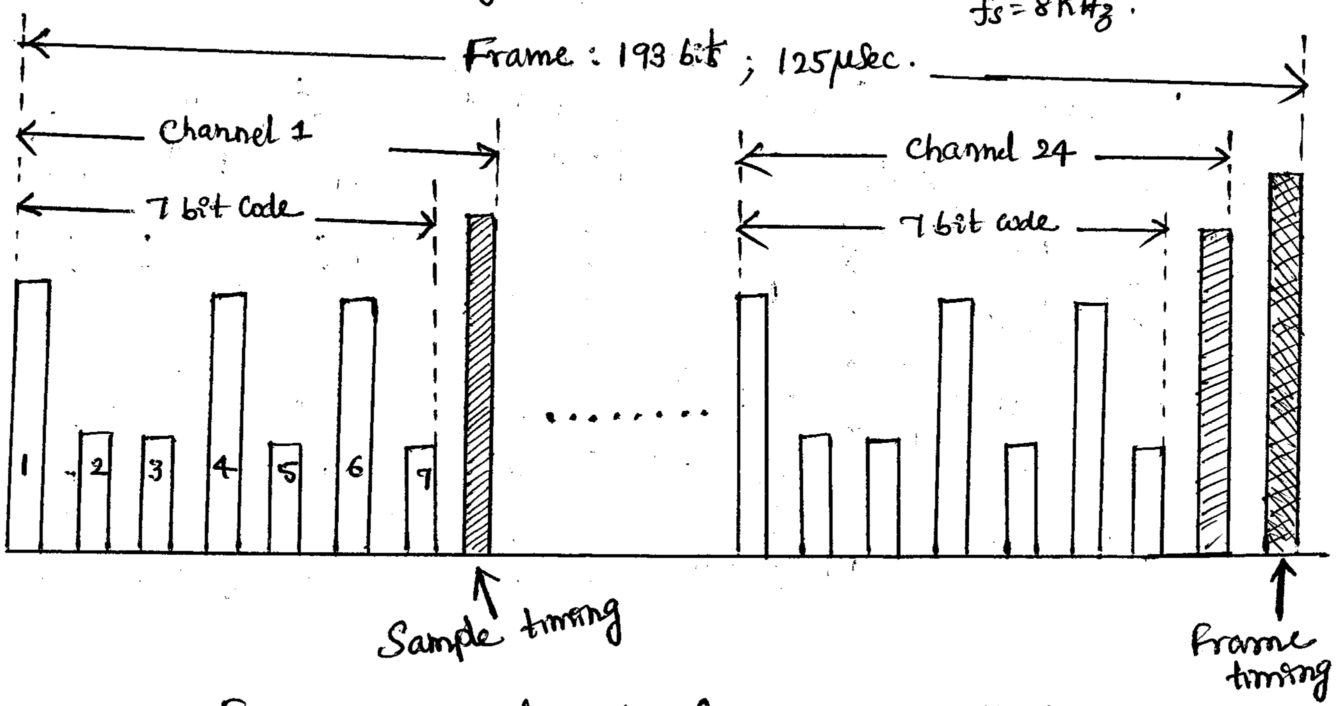


Fig: Frame format for T1 carrier system.

✓ The first level T_1 signals include PCM voice and multiplexed digital data.

- The second level T_2 signals includes multiplexed T_1 signals along with telephone services ie picture phone
- The third level T_3 signals includes multiplexed T_2 signals along with TV (Television) signals. .... etc.

T_1 digital Multiplexer:

A 7 bit code word is added with synchronous bit '1' ie 7+1=8 (Sample Synchronizat)

24 samples with 8-bit code word & another additional bit for frame synchronization

$\therefore$ No. of bit/frames $= (4 \times 24) + 1 = \underline{193}$ bits/frame

$\therefore$ Bit rate of T_1 system $=$ 193 bits/frame $\times$ 8000 frames/sec $= 1.544 \times 10^{6}$ bits/sec

$$\boxed{R_{b_1} = 1.544\,\text{Mbps}}$$

T_2 digital Multiplexer:

It multiplexes 4 T_1 systems with 17 synchronization bits

$\therefore$ No. of bits/frame $= (4 \times 193)$ bits $+ 17$ bits $= \underline{789}$ bits/frame

$\therefore$ The bit rate of T_2 system $=$ 789 bits/frame $\times$ 8000 frames/sec $= 6.312 \times 10^{6}$ bits/sec.

$$\boxed{R_{b_2} = 6.312\,\text{Mbps}}$$

T_3 digital Multiplexer:

It multiplexes 7 T_2 systems with 69 synchronization bits.

$\therefore$ No. of bits/frames $= (7 \times 789) + 69 = \underline{5592}$ bits/frame

$\therefore$ Bit rate of T_3 System $=$ 5592 bits/frame $\times$ 8000 frames/sec $= 44.736 \times 10^{6}$ bits/sec.

$$\boxed{R_{b_3} = 44.736\,\text{Mbps.}}$$

T_4 digital Multiplexer:

It multiplexes 6 T_3 systems with 720 synchronization bits

$\therefore$ No. of bits/frames $= (6 \times 5592) + 720 = \underline{34272}$ bits/frame

$\therefore$ Bit rate of T_4 system $= 34272$ bits/frames $\times$ 8000 frames/sec

$= 274.176 \times 10^6$ bits/sec

$\therefore \boxed{R_{b_4} = 274.176 \text{ Mbps}}$

T_5 digital multiplexer :

It multiplexes $2 T_4$ systems with 1476 Synchronization bits.

$\therefore$ No of bits/frames $= (2 \times 34272 + 1476) = 70020$ bits/frames

$\therefore$ Bit rate of T_5 system $= 70020$ bits/frames $\times$ 8000 frames/sec

$= 560.16 \times 10^6$ bits/sec

$\therefore \boxed{R_{b_5} = 560.16 \text{ Mbps}}$

Hence

AT&T - T-carrier telephony system specifications..

System	Bit rate 'R_b'	Medium	Repeaters Spacing (Miles)	Maximum length (Miles)	System Error rate
T_1	1.544 Mbps	Twin wire	1	50	10^{-6}
T_2	6.312 Mbps	Coaxial Cable	2.5	500	10^{-7}
T_3	44.736 Mbps	Coaxial cable	Multiplexing only	-	-
T_4	274.176 Mbps	Coaxial cable	1	500	10^{-6}
T_5	560.16 Mbps.	Coaxial Cable.	1	500	$(0.4) \times 10^{-8}$.

where Mbps - Mega (10^6) bits per Second

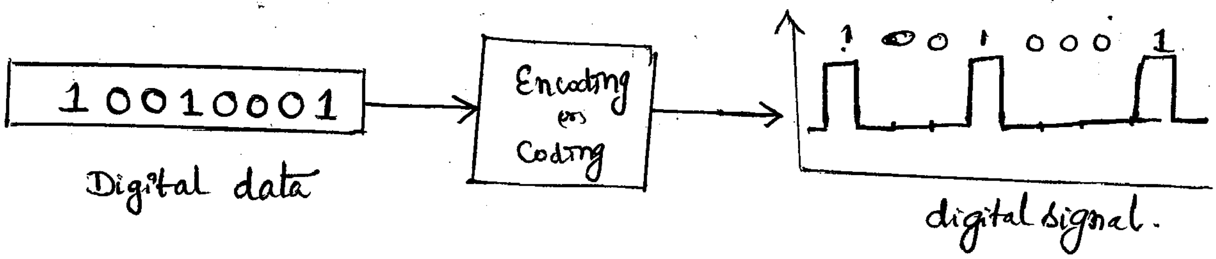
Different types of Binary Encoding:

" The process of assigning a binary value & binary code to each discrete set of samples known as Encoding & coding."

- ✓ Encoding is also called waveform coding & line coding & Txmn coding
- ✓ Set of symbols called code word.
- ✓ Each codeword consists of n-bits, such a code represents a total of 2^n distinct numbers.

* The first approach converts digital data into digital signal is known as line coding & simple encoding.

Ex:



There are different types of binary encoding Techniques.

1. Unipolar non return to zero (UNRZ) Signalling
2. Polar non return to zero (PNRZ) signalling.
3. Unipolar Return to zero (URZ) signalling.
4. Bipolar return to zero (BRZ) signalling.
5. Split phase (or) Manchester signalling.
6. Differential Encoding.

(1) Unipolar NRZ ie UNRZ Signalling

* Symbol 1 represents by presence of pulse
* Symbol 0 represents by absence of pulse.
* It is also Reffered as ON-OFF signalling.

✓ It is used for Txon of ASK (Amplitude Shift Keying) signal.

ie $\boxed{\text{UNRZ} : \begin{matrix} 1 \rightarrow 1 \\ 0 \rightarrow 0 \end{matrix}}$

② Polar NRZ Signalling:

✓ Symbol '1' is represented by +1 & Symbol '0' represents by '-1'

✓ It is used for PCM, DM, ADM, PSK, FSK signal.

ie $\boxed{\text{PNRZ} : \begin{matrix} 1 \rightarrow +1 \\ 0 \rightarrow -1 \end{matrix}}$

③ Unipolar RZ (URZ) signalling:

✓ Symbol '1' is splitting into two half cycles, First half cycle represents +1, symbol '0' & Second half cycle of symbol '1' represented by '0'

✓ It is used for fiber optical Communications.

URZ : 1 → +½ → +1 ; −½ → 0
0 → 0

④ Bipolar RZ ie (BRZ) signalling:

✓ Symbol '1' is represented by alternation of +1 and −1 & no pulse for symbol '0'

✓ It is also reffered as "AMI (Alternate Mark Inversion)"

BRZ : 1 → +1 (first), −1 (second)
0 → 0

⑤ Split phase (or) Manchester Signalling:

✓ symbol '1' is represented by +1 & −1 half cycle pulse and

✓ Symbol '0' is represented by −1 & +1 half cycle pulse.

1 → +½ → +1 ; −½ → −1,   0 → +½ → −1 ; +½ → +1

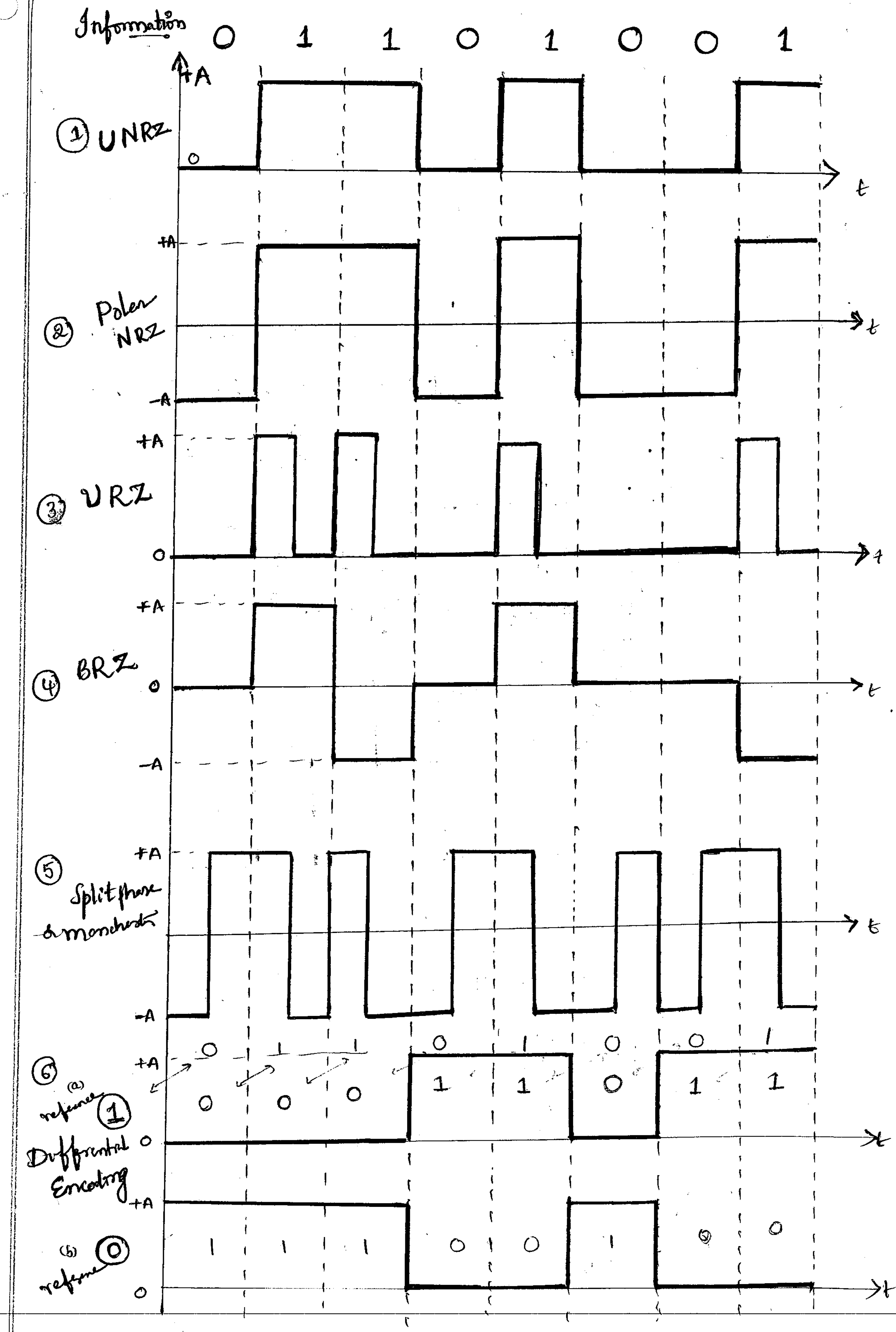
⑥ Differential Encoding:

✓ Symbol '0' is represented by complement of previous voltge

✓ symbol '1' is represented previous voltge.

1 → previous voltge
0 → Complement of previous voltge

Exclusive NOR operation
00 → 1
01 → 0
10 → 0
11 → 1


Information 0 1 1 0 1 0 0 1
+A
0
t
① UNRZ
+A
-A
② Polar NRZ
+A
0
③ URZ
+A
0
-A
④ BRZ
+A
-A
⑤ Split phase & manchester
+A
0
⑥ (a) reference 1 Differential Encoding
(b) reference 0

M-ary Encoding (or) Signalling scheme:

M-ary signalling transmits 'm' symbols with 'm' voltage levels such that the no. of bits in each symbol is

$N = \log_2 M$ → If $M = 2$, $N = \log_2 2 = 1$ called Binary Encoding

If $M = 4$, $N = \log_2 4 = 2$ ie two bits in each symbol.

∴ The voltage levels in M-ary encoding

$$A_m = \begin{cases} 0, \pm 2A, \pm 4A, \ldots \pm (m-1)A; & m\text{-odd} \\ \pm A, \pm 3A, \pm 5A, \ldots \pm (m-1)A; & m\text{-even} \end{cases}$$

For Four symbols ie $m = 4$. Assume A, B, C, D.

voltage levels ∴ $A_m = \pm A$ and $\pm 3A$.

let

$00 \rightarrow A = -A$

$01 \rightarrow B = +A$

$10 \rightarrow C = +3A$

$11 \rightarrow D = -3A$.

let

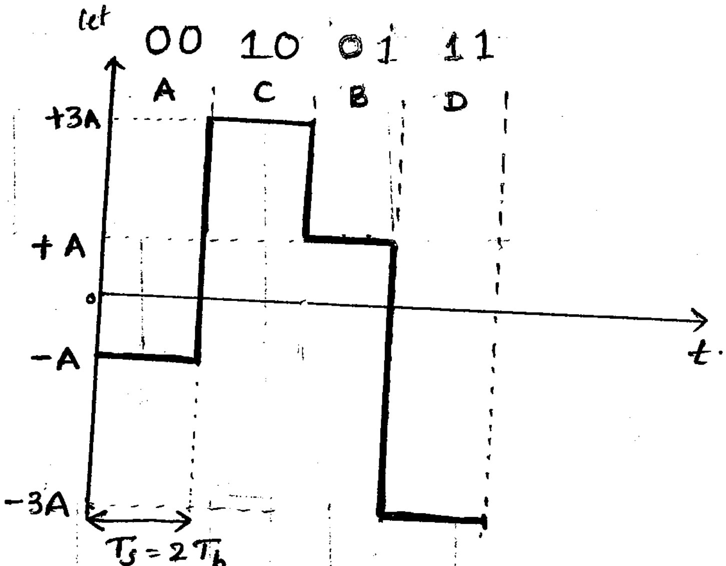


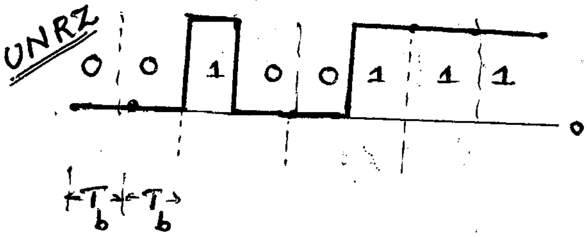
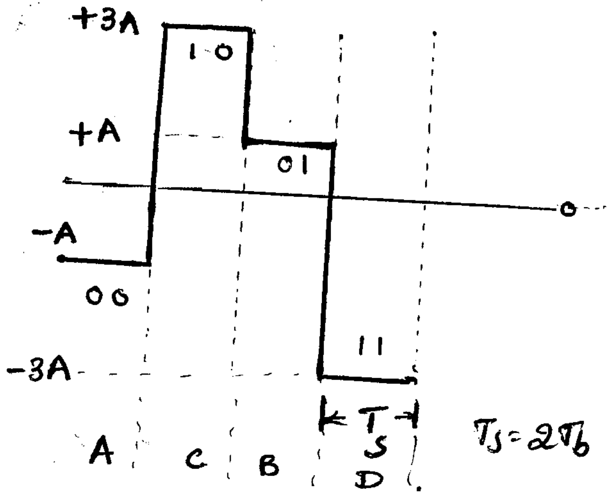
Fig: M-ary signalling.

- ✓ Band width is reduced by 2 ie $R_b/2$.
- ✓ Increase complexity at Receiver.
- ✓ Power level is high.
- ✓ The probability of error for m-ary coding is given by

$$P_e = \frac{2(m-1)}{m} Q(A/\sigma) \quad \text{where } Q(x) \rightarrow Q\text{-function}.$$

If $m = 4$ symbols ∴ $P_e = \frac{2(4-1)}{4} Q(A/\sigma) \Rightarrow \boxed{P_e = \frac{6}{4} Q(A/\sigma)}$

Comparison between Binary & M-ary Signalling Scheme :

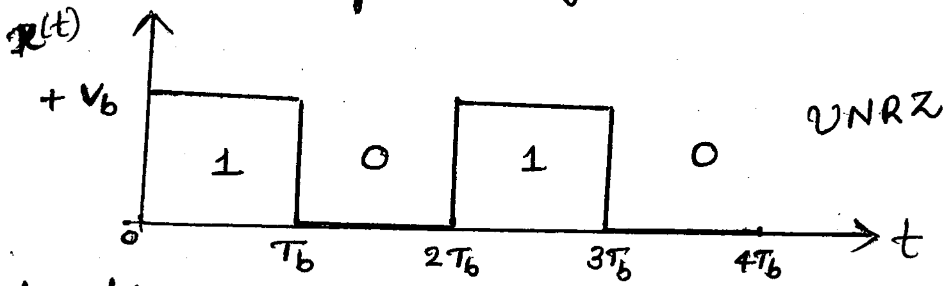
Binary Scheme	M-ary scheme
(1) Representation let 00 10 01 11 data Data is 00100111 For Unipolar NRZ (non Return to Zero) Encoding method  r_s - Symbol rate of source	(1) Representation. let $00 \rightarrow A \rightarrow -A$ $01 \rightarrow B \rightarrow +A$ $10 \rightarrow C \rightarrow +3A$ $11 \rightarrow D \rightarrow -3A$ Sequence $\underset{A}{\underline{00}}\ \underset{C}{\underline{10}}\ \underset{B}{\underline{01}}\ \underset{D}{\underline{11}}$  $T_s = 2T_b$
(2) Bit rate $R_b = N \times f_s$ bit/sec (or) $r_s \times \log_2 M \leftarrow N$ where N - no. of bits /symbol.	(2) Bit rate $R_b = r_s / N$ bits/sec.
(3) No. of threshold required = 1	(3) No. of threshol required = $(m-1)$ levels where m - no. of symbols.
(4) Required transmission power is less	(4) Required transmission power is more.
(5) Hard Complexity is less	(5) Hardware Complexity is more
(6) Probability of error is less $P_e = Q(A/2\sigma)$ for UNRZ	(6) Probability of Error is more $P_e = \frac{2(m-1)}{m} . Q(A/\sigma)$
(7) m-symbols can be represented only in two levels $m=2$ ie logic '0' & logic '1'	m-symbol can be represented with m-voltage levels.

Calculation of PSD :

Unipolar Non Return to Zero

Power Spectrum of Encoded Signal (UNRZ)

Consider UNRZ encoding signalling.



Fourier transform

$$X(\omega) = F\{x(t)\} = \int_{t=-\infty}^{\infty} x(t) \cdot e^{-j\omega t}\, dt$$

$$= \int_{0}^{T_b} (V_b)\, e^{-j\omega t}\, dt + \int_{T_b}^{2T_b} (0) \cdot e^{-j\omega t}\, dt$$

$$= V_b \cdot \left(\frac{e^{-j\omega t}}{-j\omega} \right)_0^{T_b} + 0$$

$$= V_b \left[\frac{e^{-j\omega T_b}}{-j\omega} - \frac{e^{0}}{-j\omega} \right]$$

$$= \frac{V_b}{-j\omega} \left[e^{-j\omega T_b} - 1 \right]$$

$$X(\omega) = j \cdot \frac{V_b}{\omega} \left[e^{-j\omega T_b} - 1 \right]$$

(or)

$$X(f) = \frac{jV_b}{2\pi f} \left[e^{-j2\pi f T_b} - 1 \right] \qquad (\because \omega = 2\pi f)$$

$$= \frac{jV_b}{2\pi f} \left[e^{-j\pi f T_b} \cdot e^{-j\pi f T_b} - e^{-j\pi f T_b} \cdot e^{+j\pi f T_b} \right]$$

$$= \frac{jV_b}{2\pi f} \left[e^{-j\pi f T_b} - e^{+j\pi f T_b} \right] e^{-j\pi f T_b}$$

$$= \frac{\cancel{j}V_b}{\pi f} \cdot \left[\frac{e^{+j\pi f T_b} - e^{-j\pi f T_b}}{2j \cdot \cancel{j}} \right] \cdot e^{-j\pi f T_b} \qquad (\because j^2 = -1)$$

$$= \frac{V_b}{\pi f} \cdot \sin(\pi f T_b) \cdot e^{-j\pi f T_b} \qquad \left(\because \frac{e^{x} - e^{-x}}{2j} = \sin x\right)$$

$$= \frac{V_b}{\cancel{\pi f}} \frac{\cancel{\pi f} T_b}{\pi f T_b} \cdot \sin(\pi f T_b) \cdot e^{-j\pi f T_b}$$

$$X(f) = V_b \cdot T_b \cdot \left[\frac{\sin(\pi f T_b)}{\pi f T_b} \right] \cdot e^{-j\pi f T_b}$$

$$X(f) = V_b \cdot T_b \cdot \mathrm{Sinc}(f T_b) \cdot e^{-j\pi f T_b} \qquad \left(\because \frac{\sin \pi x}{\pi x} = \mathrm{Sinc}(x)\right)$$

$$\therefore \boxed{X(f) = V_b \cdot T_b \cdot e^{-j\pi f T_b} \cdot \mathrm{Sinc}(f T_b)}$$

The magnitude spectrum $|X(f)| = V_b \cdot T_b \cdot \mathrm{Sinc}(f T_b)$.

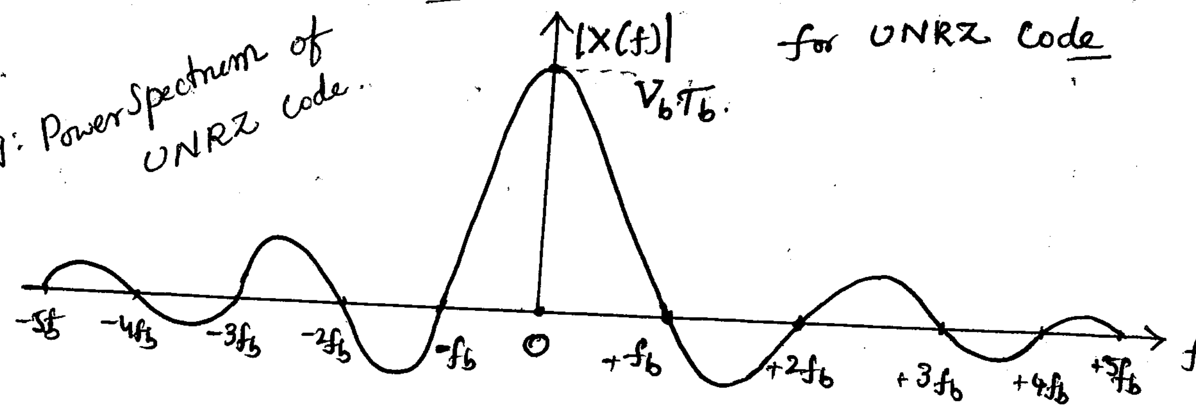


Fig: Power Spectrum of UNRZ code.

The PSD (power spectral density) of signal $x(t)$ for UNRZ is given by

$$\boxed{S_X(f) = \frac{1}{T_b} |X(f)|^2 \cdot \sum_{n=-\infty}^{\infty} R_A(n) \cdot e^{-j2\pi f n T_b}}$$

where $R_A(n) \rightarrow$ Auto correlation function.

Simillary for all encoding techniques the power spectrum as follows.

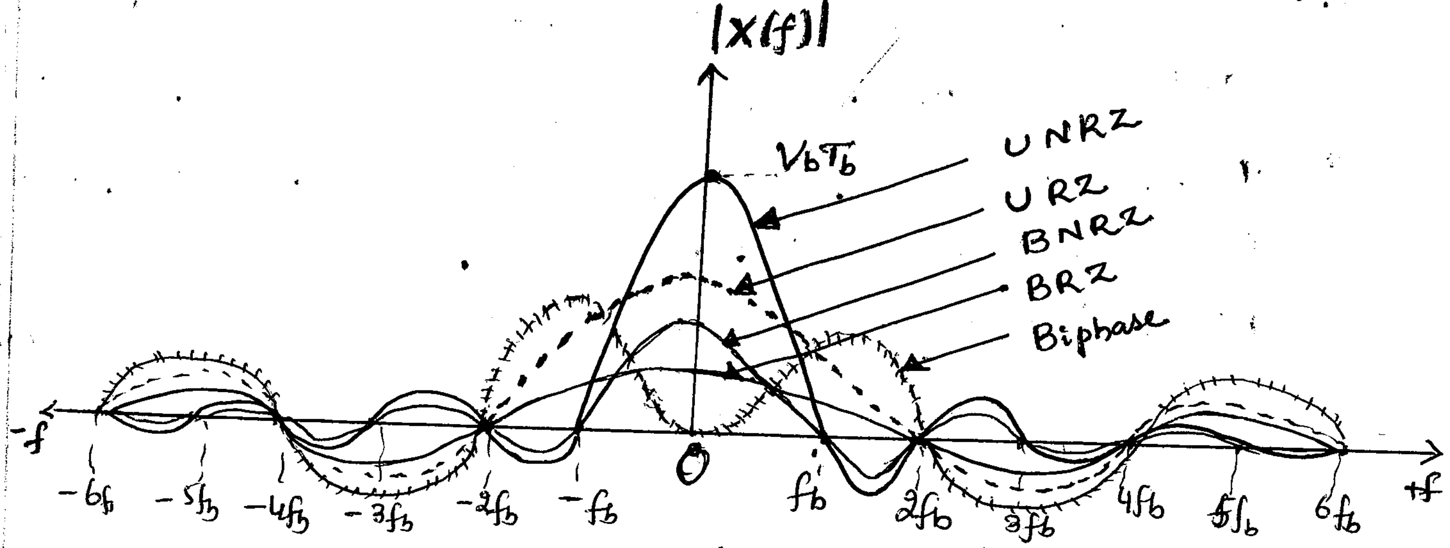


Fig: Power Spectrum of different line codes.

Power Spectral density

PSD $\quad S_X(f) = \frac{1}{T} |X(f)|^2 \sum_{n=-\infty}^{\infty} R_A(n) e^{-j2\pi f n T}$ ↳ Auto correlation function.

Note:

- Unipolar, most of signal power is centered around origin and there is waste of power due to dc component that is present.
- Polar form, most of signal power is centered around origin and they are simple to implement.
- Bipolar format does not have dc component & does not demand more bandwidth, but power requirements is double than other forms.
- Manchester format doesnot have dc component but provides proper clocking.

Desirable Characteristics of Encoded signal:

* ① The transmission of bandwidth of the encoded signal should be less.

* ② The average (or) dc value of encoded signal should be as low as possible.

* ③ The peak transmission power should be as low as possible.

* ④ The noise margin of the encoded signal should be as high as possibility ie noise immunity should be more.

* ⑤ The synchronization problem (or) timing error (or) clock recovery problem should be as low as possible.

ie Transmitter and receiver should be operated same frequencies.

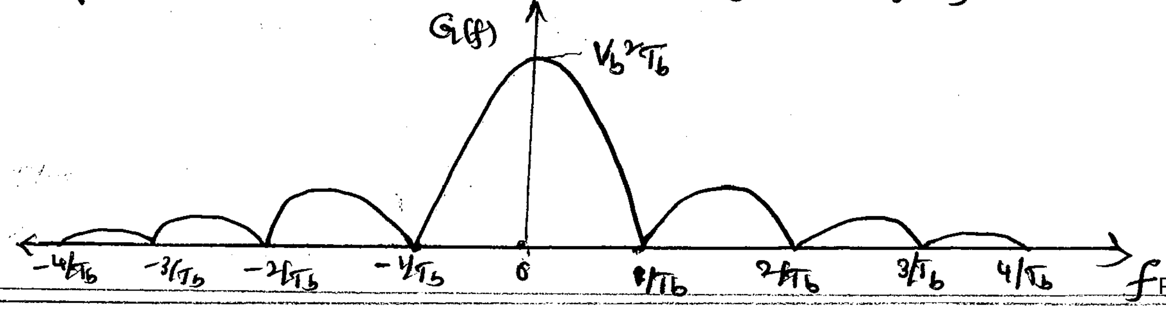
⇒ The PSD for UNRZ code

$$S_x(f) = \frac{1}{T_b} |X(f)|^2$$
$$= \frac{1}{T_b} \cdot \left[V_b \cdot T_b \cdot \text{Sinc}(f T_b)\right]^2$$

(or)

$$G(f) = V_b^2 \cdot T_b \cdot \text{Sinc}^2(f T_b)$$

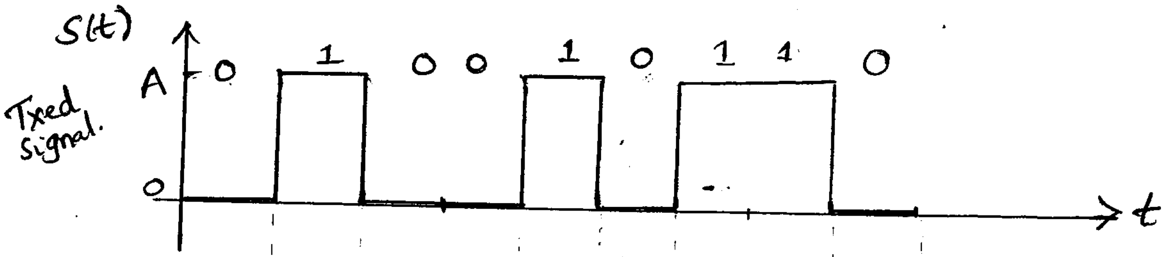
The PSD Spectrum.



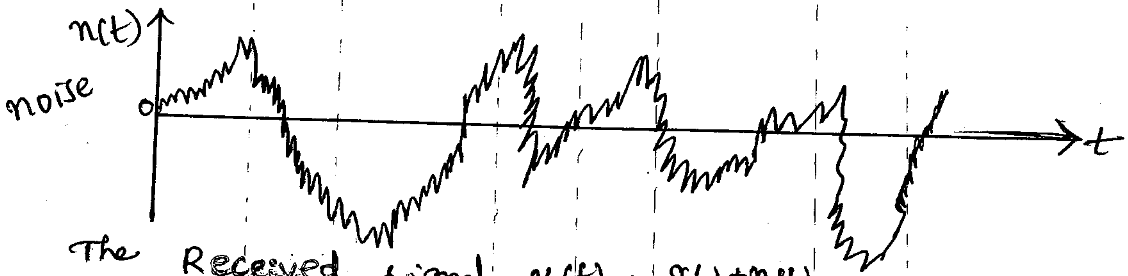
Probability of Error (Pe) :

(a) Binary Encoding :

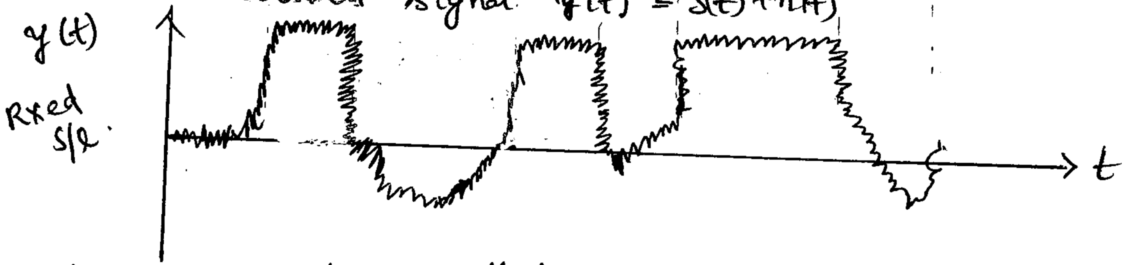
Let us consider the transmitted signal 010010110 using Unipolar Non Return to Zero code (UNRZ) ie.



let us consider $n(t)$ as zero mean Gaussian noise.



The Received signal $y(t) = s(t)+n(t)$



when '0' is transmitted, Received signal $y(t) = n(t)$

$y = n$

$E[y] = 0$ (∵ noise zero mean)

when '1' is transmitted, Received signal

$y(t) = s(t)+n(t)$

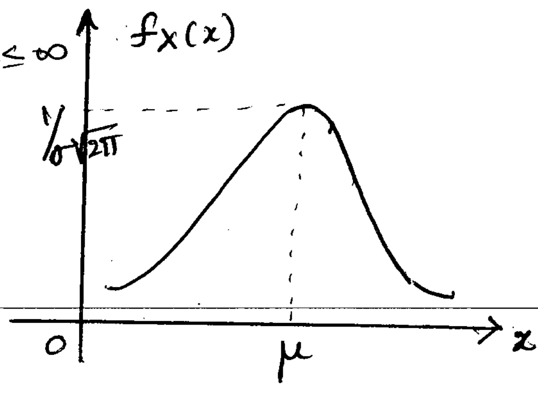
ie $y = s+n$

$n = y - s$ → A

$\boxed{n = y - A}$

Let us consider the bandlimite white Gaussian function as

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{(x-\mu)^2}{2\sigma^2}} \; ; \; -\infty \le x \le \infty$$



where

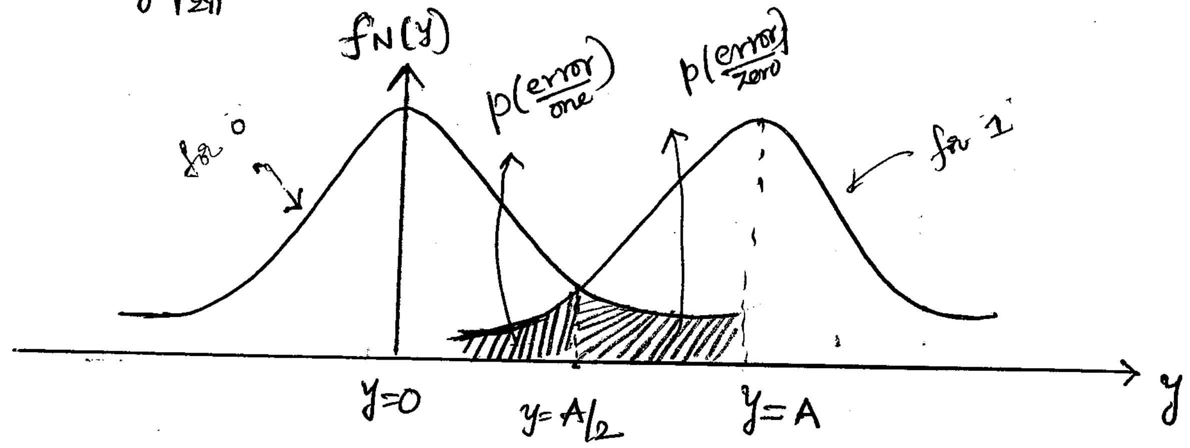
μ - mean (or) Expectation of 'x'

σ - Standard deviation

σ^2 - variance.

$\therefore f_N(y) = \frac{1}{\sigma\sqrt{2\pi}} e^{-y^2/2\sigma^2}$; noise, when '0' is transmitted.

$f_N(y) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{[(y-A)^2 - 0]}{2\sigma^2}}$; Noise, when '1' is transmitted.



Probability of '0' becomes '1', if $y > A/2$

$\therefore \underline{p\left(\frac{\text{error}}{\text{zero}}\right)} = P_{e_0} = \int_{u=A/2}^{\infty} f_N(u)\, du = \int_{u=A/2}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-u^2/2\sigma^2}\, du \longrightarrow ①$

We know that

Q-function $\rightarrow$ $\boxed{Q(x) = \frac{1}{\sqrt{2\pi}} \int_{u=x}^{\infty} e^{-u^2/2}\, du} \longrightarrow ②$

let $z = \frac{u}{\sigma} \Rightarrow dz = \frac{du}{\sigma} \Rightarrow du = \sigma\, dz$

limits if $u = A/2$, $z = u/\sigma = A/2\sigma$

$u = \infty$, $z = \infty$

From eqn ①

$$P_{e_0} = \int_{z=A/2\sigma}^{\infty} \frac{1}{\cancel{\sigma}\cdot\sqrt{2\pi}} \cdot e^{-(z^2\cancel{\sigma^2})/2\cancel{\sigma^2}} \cdot \cancel{\sigma}\cdot dz$$

$$P_{e_0} = \frac{1}{\sqrt{2\pi}} \int_{z=A/2\sigma}^{\infty} e^{-z^2/2}\, dz \longrightarrow ③$$

By comparing eqn ② & ③

$\therefore p\left(\frac{\text{error}}{\text{zero}}\right) = P_{e_0} = Q(A/2\sigma) \longrightarrow ④$

$\therefore \boxed{P_{e_0} = Q(A/2\sigma)}$. where A - Amplitude of logic data

σ - standard deviation.

Similarly probability of 1 becomes 0 ; if $y < A/2$

$$\underline{p\left(\frac{\text{error}}{\text{one}}\right)} = P_{e1} = \int_{u=-\infty}^{A/2} f_N(u)\,du = \int_{u=-\infty}^{A/2} \frac{1}{\sigma\sqrt{2\pi}} e^{-(u-A)^2/2\sigma^2}.du$$

$$P_{e1} = \int_{u=-\infty}^{\infty} f_X(u)\,du - \int_{u=A/2}^{\infty} f_X(u)\,du$$

$$= 1 - \int_{u=A/2}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-(u-A)^2/2\sigma^2}.du$$

Let $z = \frac{u-A}{\sigma}$, $dz = \frac{du}{\sigma} \Rightarrow du = \sigma dz$

limits $u = A/2$, $\Rightarrow z = \frac{A/2 - A}{\cancel{2}\sigma} = -A/2\sigma$

$u = \infty \Rightarrow z = \infty$

$$\therefore \quad P_{e1} = 1 - \int_{z=-A/2\sigma}^{\infty} \frac{1}{\cancel{\sigma}\sqrt{2\pi}} \cdot e^{-z^2/2} \cdot \cancel{\sigma}\,dz$$

$$= 1 - \int_{z=-A/2\sigma}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-z^2/2}.dz.$$

$$= 1 - Q(-A/2\sigma)$$

$$\boxed{P_{e1} = 1 - Q(A/2\sigma)} \longrightarrow (5) \quad (\because Q(x) \text{ is always even for } Q(-x) = Q(x)).$$

from (4) & (5) $\quad P_{e0} + P_{e1} = Q(A/2\sigma) + 1 - Q(A/2\sigma) = 1.$

If the probability of error is equal in both cases ie

$$\boxed{P_{e0} = P_{e1} = Q(A/2\sigma)}. \quad \text{Equally probably}$$

$\Rightarrow$ From conditional probability theorem

Total probability error $P_e = p(\text{zero}) \cdot p\left(\frac{\text{error}}{\text{zero}}\right) + p(\text{one}) \cdot p\left(\frac{\text{error}}{\text{one}}\right)$.

if $p(\text{zero}) = p(\text{one}) = 1/2$

$$\therefore \quad P_e = \frac{1}{2}\left[Q(A/2\sigma) + Q(A/2\sigma)\right] = 1/\cancel{2}\left[\cancel{2}Q(A/2\sigma)\right]$$

$$= Q(A/2\sigma).$$

$\therefore$ $\boxed{\text{Probability of error } P_e = Q(A/2\sigma)}$ for binary system (UNRZ).

Probability of error using error function.

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_{u=0}^{x} e^{-u^2}.du \; ; \quad \text{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_{u=x}^{\infty} e^{-u^2}.du$$

Complementary error function

& $\text{erf}(x) + \text{erfc}(x) = 1.$

$$P_{e0} = P\left(\frac{\text{error}}{\text{zero}}\right) = \int_{u=A/2}^{\infty} f_X(u)\, du$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{u=A/2}^{\infty} e^{-u^2/2\sigma^2}.du$$

$$= \frac{1}{\cancel{\sigma}\sqrt{2\pi}} \int_{z=A/2\sigma\sqrt{2}}^{\infty} e^{-z^2} \cancel{\sigma}\sqrt{2}.dz$$

let $z = \frac{u}{\sigma\sqrt{2}}$

$dz = \frac{du}{\sigma\sqrt{2}} \Rightarrow du = \sigma\sqrt{2}dz$

If $u = A/2$, $z = A/2\sigma\sqrt{2}$

$u = \infty$, $z = \infty$.

$$= \frac{1}{\sqrt{\pi}} \int_{A/2\sigma\sqrt{2}}^{\infty} e^{-z^2}.dz = \frac{1}{2}\left(\frac{2}{\sqrt{\pi}} \int_{z=A/2\sigma\sqrt{2}}^{\infty} e^{-z^2} dz\right)$$

$$\therefore \boxed{P_{e0} = \frac{1}{2}\,\text{erfc}\left(A/2\sigma\sqrt{2}\right)}$$

$$P_{e1} = P\left(\frac{\text{error}}{\text{one}}\right) = \int_{u=-\infty}^{A/2} \frac{1}{\sigma\sqrt{2\pi}} e^{-(u-A)^2/2\sigma^2}.du = \int_{u=-\infty} f_X(u)\,du - \int_{u=A/2} f_X(u)\,du$$

$$= 1 - \int_{u=A/2}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-(u-A)^2/2\sigma^2}.du$$

$$= 1 - \int_{z=-A/\sigma 2\sqrt{2}}^{\infty} \frac{1}{\cancel{\sigma}\cancel{\sqrt{2}}.\sqrt{\pi}} e^{-z^2} dz.\cancel{\sigma}.\cancel{\sqrt{2}}$$

$$= 1 - \left[\int_{z=-A/2\sigma\sqrt{2}}^{\infty} \frac{2}{\sqrt{\pi}}.e^{-z^2} dz\right] \times \frac{1}{2}$$

let $z = \frac{u-A}{\sigma\sqrt{2}}$

$dz = \frac{du}{\sigma\sqrt{2}}$

$\Rightarrow du = \sigma\sqrt{2}dz$

If $u = A/2$, $z = -A/2\sigma\sqrt{2}$

$u = \infty$, $z = \infty$.

$$= 1 - \frac{1}{2}\,\text{erfc}\left(-A/2\sigma\sqrt{2}\right)$$

$$\boxed{P_{e1} = 1 - \frac{1}{2}\,\text{erfc}\left(A/2\sigma\sqrt{2}\right)}$$

($\because$ Error function also always even $\therefore \text{erfc}(x) = \text{erfc}(-x)$)

$$\therefore P_{e0} + P_{e1} = \frac{1}{2}\,\text{erfc}\left(A/2\sigma\sqrt{2}\right) + 1 - \frac{1}{2}\,\text{erfc}\left(A/2\sigma\sqrt{2}\right) = 1.$$

$$P_e \Rightarrow \boxed{P_{e0} = P_{e1} = \frac{1}{2}\,\text{erfc}\left(A/\sigma 2\sqrt{2}\right)}$$

Probability of error

$$P_e = Q(A/2\sigma)$$

$$P_e = \frac{1}{2}\,\text{erfc}\left(\frac{A}{\sigma 2\sqrt{2}}\right)$$

for UNRZ code.

Relation between $Q(x)$, $\text{erfc}(x)$ as $\boxed{\text{erfc}(x) = 2Q(x\sqrt{2})}$

(b) M-ary Encoding: Probability of error (P_e):

M-ary signalling transmits m symbols as voltage levels.

let $m=4$, assume A, B, C, D. Voltage levels $A_m = \pm A, \pm 3A$.

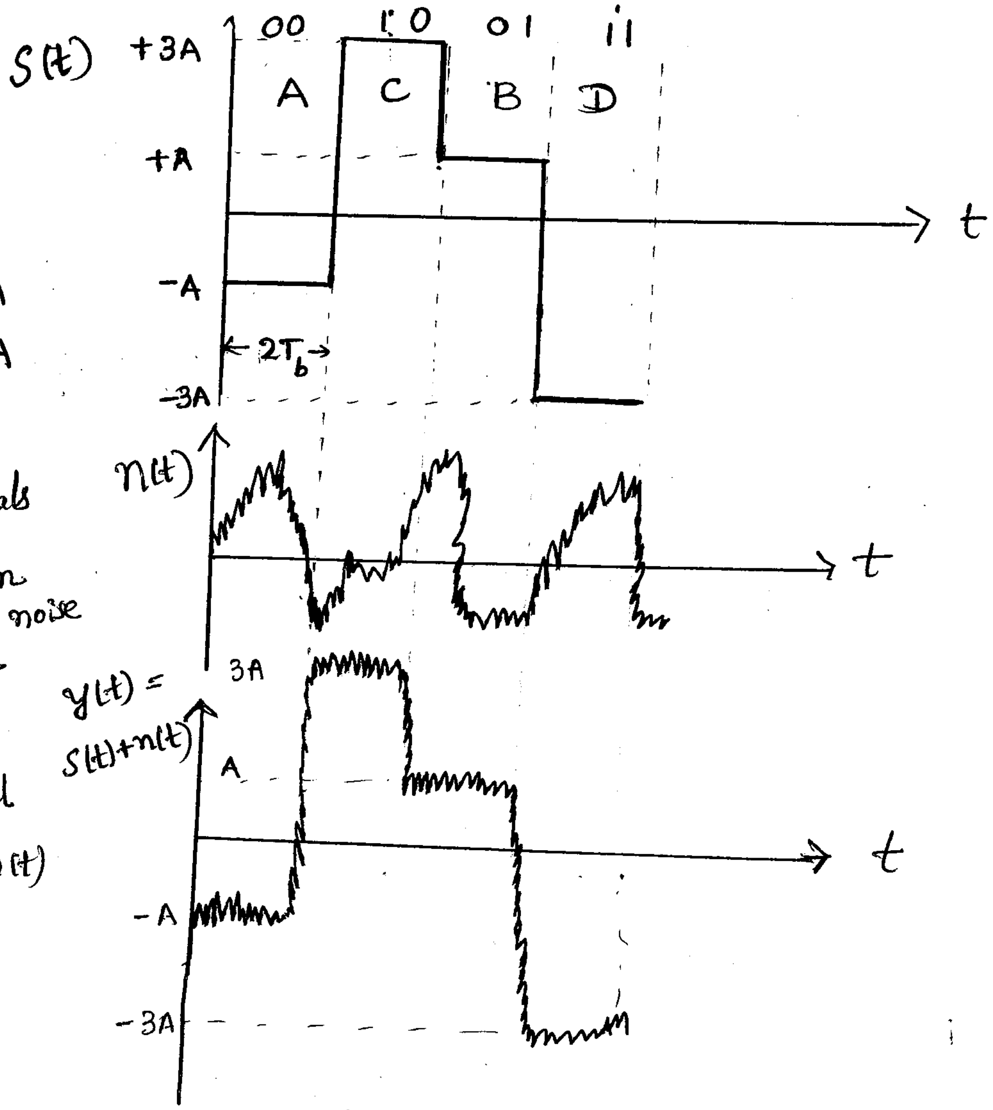
let

$00 \rightarrow A = -A$

$01 \rightarrow B = +A$

$10 \rightarrow C = +3A$

$11 \rightarrow D = -3A$



$S(t)$ = Txed signals

$n(t)$ = zero mean gaussian noise & variance σ^2

$y(t)$ = Rxed signal

$y(t) = S(t) + n(t)$

$y = s + n$.

$y = -A + n$, when symbol 'A' is transmitted $\Rightarrow n = y + A$

$y = A + n$, when symbol B is transmitted $\Rightarrow n = y - A$

$y = 3A + n$, when symbol 'C' is transmitted $\Rightarrow n = y - 3A$.

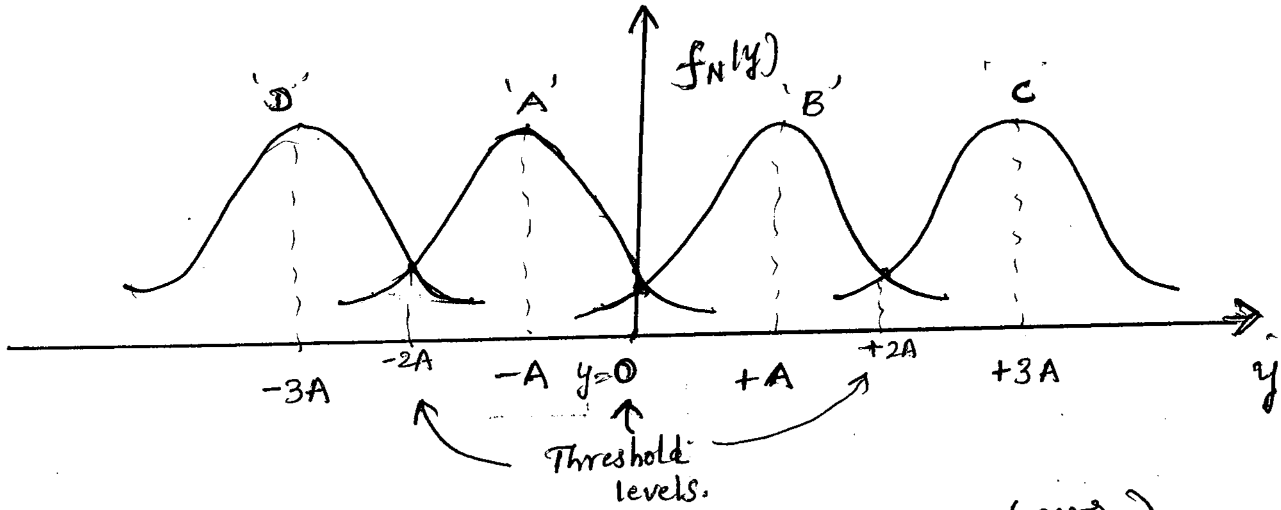
$y = -3A + n$, when symbol 'D' is transmitted $\Rightarrow n = y + 3A$.

* when 'A' is transmitted, $f_y(n) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(y+A)^2/2\sigma^2}$; $-\infty \leq y \leq \infty$

when 'B' is transmitted, $f_y(n) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(y-A)^2/2\sigma^2}$; $-\infty \leq y \leq \infty$

when 'C' is transmitted, $f_y(n) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(y-3A)^2/2\sigma^2}$; $-\infty \leq y \leq -\infty$

when 'D' is transmitted, $f_y(n) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(y+3A)^2/2\sigma^2}$; $-\infty \leq y \leq -\infty$



Total error probability $P_e = P(\text{'A' sent})\, P\left(\frac{\text{error}}{\text{'A' sent}}\right) + P(\text{'B' sent}) \cdot P\left(\frac{\text{error}}{\text{'B' sent}}\right) + P(\text{'C' sent}) \cdot P\left(\frac{\text{error}}{\text{'C' sent}}\right) + P(\text{'D' sent}) \cdot P\left(\frac{\text{error}}{\text{'D' sent}}\right)$.

If $P(\text{'A' sent}) = P(\text{'B' sent}) = P(\text{'C' sent}) = P(\text{'D' sent}) = 1/4$.

$\therefore$ Probability error $P_e = \frac{1}{4}\left\{ P\left(\frac{\text{error}}{\text{'A' sent}}\right) + P\left(\frac{\text{error}}{\text{'B' sent}}\right) + P\left(\frac{\text{error}}{\text{'C' sent}}\right) + P\left(\frac{\text{error}}{\text{'D' sent}}\right) \right\}$. $\longrightarrow$ (1)

Probability error when 'D' transmitted :

The receiver will wrongly decode the symbol when signal+noise voltage is exceeds '$-2A$' when 'D' is transmitted.

$$P\left(\frac{\text{error}}{\text{'D' sent}}\right) = P\{ y \geq -2A \} = \int_{u=-2A}^{+\infty} f_Y(u) \cdot du$$

$$= \int_{u=-2A}^{+\infty} \frac{1}{\sigma\sqrt{2\pi}}\, e^{-(u+3A)^2/2\sigma^2}\, du$$

let $z = \frac{u+3A}{\sigma}$

$dz = \frac{du}{\sigma} \Rightarrow du = \sigma dz$

If $u = -2A$, $z = A/\sigma$
$u = \infty$, $z = \infty$

$$= \int_{z=A/\sigma}^{\infty} \frac{1}{\cancel{\sigma}\sqrt{2\pi}} \cdot e^{-z^2/2} \cdot \cancel{\sigma} \cdot dz$$

$$= \frac{1}{\sqrt{2\pi}} \int_{z=A/\sigma}^{\infty} e^{-z^2/2}\, dz.$$

$$= Q(A/\sigma)$$

$$= Q(z)$$

$\left(\because \text{Q-function } Q(x) = \frac{1}{\sqrt{2\pi}} \int_{u=x}^{\infty} e^{-u^2/2}\, du\right)$

$$\therefore \boxed{P\left(\frac{\text{error}}{\text{'D' sent}}\right) = Q(A/\sigma)} \longrightarrow (2)$$

Probability of error when 'A' is transmitted :

The receiver will wrongly decode the symbol when signal + noise voltage is greater than '0' & less than '$-2A$', when symbol 'A' is transmitted.

$$P\left(\frac{\text{error}}{\text{'A' sent}}\right) = P\{y \geq 0\} + P\{y \leq -2A\}$$

$$= \int_{u=0}^{+\infty} f_y(u)\,du + \int_{u=-\infty}^{-2A} f_y(u)\,du$$

$$= \int_{u=0}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(u+A)^2}{2\sigma^2}}\,du + \int_{u=-\infty}^{-2A} \frac{1}{\sigma\sqrt{2\pi}} e^{-(u+A)^2/2\sigma^2}\,du.$$

let $z = \frac{u+A}{\sigma}$

$dz = \frac{du}{\sigma}$

$\Rightarrow du = \sigma dz$

If $u=0$, $z = A/\sigma$

$u=\infty$, $z=\infty$

If $u=-\infty$, $z=-\infty$

$u=-2A$, $z=-A/\sigma$

$$= \int_{z=A/\sigma}^{\infty} \frac{1}{\cancel{\sigma}\sqrt{2\pi}} e^{-z^2/2}\,dz\,\cancel{\sigma} + \frac{1}{\cancel{\sigma}\sqrt{2\pi}} \int_{z=-\infty}^{-A/\sigma} e^{-z^2/2}\,\cancel{\sigma}\,dz.$$

$$= \frac{1}{\sqrt{2\pi}} \int_{z=A/\sigma}^{\infty} e^{-z^2/2}\,dz + \frac{1}{\sqrt{2\pi}} \int_{z=-\infty}^{-A/\sigma} e^{-z^2/2}\,dz$$

$$= Q(A/\sigma) + Q(-A/\sigma) \qquad (\because Q(-x) = Q(x))$$

$$\therefore \boxed{P\left(\frac{\text{error}}{\text{A' sent}}\right) = 2Q(A/\sigma)} \longrightarrow (3)$$

Probability of error when 'B' is transmitted :

The receiver will wrongly decode the symbol when signal + noise voltye exceeds '$+2A$' and less than '0', when Symbol 'B' is Txed.

$$P\left(\frac{\text{error}}{\text{B' sent}}\right) = P\{y \geq 2A\} + P\{y \leq 0\}$$

$$= \int_{u=2A}^{+\infty} f_y(u)\,du + \int_{u=-\infty}^{0} f_y(u)\,du$$

$$= \int_{u=2A}^{+\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(u-A)^2}{2\sigma^2}}\,du + \int_{u=-\infty}^{0} \frac{1}{\sigma\sqrt{2\pi}} e^{-(u-A)^2/2\sigma^2}\,du$$

let $z = \frac{u-A}{\sigma}$

$dz = \frac{du}{\sigma} \Rightarrow du = \sigma dz$

If $u=2A$, $z=A/\sigma$

$u=\infty$, $z=\infty$

If $u=-\infty$, $z=-\infty$

$u=0$, $z=-A/\sigma$

$$= \int_{z=A/\sigma}^{\infty} \frac{1}{\cancel{\sigma}\sqrt{2\pi}} e^{-z^2/2}\,\cancel{\sigma}\,dz + \int_{z=-\infty}^{-A/\sigma} \frac{1}{\cancel{\sigma}\sqrt{2\pi}} e^{-z^2/2}\,\cancel{\sigma}\,dz.$$

$$= \frac{1}{\sqrt{2\pi}} \int_{z=A/\sigma}^{\infty} e^{-z^2/2}\,dz + \frac{1}{\sqrt{2\pi}} \int_{z=-\infty}^{-A/\sigma} e^{-z^2/2}\,dz$$

$$= Q(A/\sigma) + Q(-A/\sigma)$$
$$= 2Q(A/\sigma) \qquad (\because Q(-x) = Q(x))$$

$$\therefore \boxed{P\left(\frac{\text{error}}{\text{B sent}}\right) = 2Q(A/\sigma)} \longrightarrow ④$$

Probability error when C is transmitted:

The receiver will wrongly decode the symbol when signal + noise voltage is less than +2A, when symbol 'C' is transmitted.

$$P\left(\frac{\text{error}}{\text{'C' sent}}\right) = P\{y \le +2A\}$$
$$= \int_{u=-\infty}^{+2A} f_y(u) = \int_{u=-\infty}^{+2A} \frac{1}{\sigma\sqrt{2\pi}} e^{-(u-3A)^2/2\sigma^2} \, du.$$

$$= \int_{z=-\infty}^{-A/\sigma} \frac{1}{\cancel{\sigma}\sqrt{2\pi}} \cdot e^{-z^2/2} \cdot \cancel{\sigma} \cdot dz$$

$$= \frac{1}{\sqrt{2\pi}} \int_{z=-\infty}^{-A/\sigma} e^{-z^2/2} dz$$

let $z = \frac{u-3A}{\sigma}$

$dz = \frac{du}{\sigma} \Rightarrow du = \sigma dz$

If $u = -\infty$, $z = -\infty$

$u = +2A$, $z = -A/\sigma$.

$$= Q(-A/\sigma)$$
$$= Q(A/\sigma) \qquad (\because Q(-x) = Q(x))$$
even function.

$$\therefore \boxed{P\left(\frac{\text{error}}{\text{'C' sent}}\right) = Q(A/\sigma).} \longrightarrow ⑤$$

$\therefore$ From eqn ①, ②, ③, ④ & ⑤.

$$\therefore P_e = \frac{1}{4}\left\{Q(A/\sigma) + 2Q(A/\sigma) + 2Q(A/\sigma) + Q(A/\sigma)\right\}$$
$$= \frac{1}{4} \cdot 6 \cdot Q(A/\sigma)$$
$$P_e = 6/4 \cdot Q(A/\sigma) \qquad \boxed{\therefore P_e = 6/4 \cdot Q(A/\sigma)} \text{ for M-ary Scheme.}$$

In general probability error

$$\therefore \boxed{P_e = \frac{2(m-1)}{m} Q[A/\sigma)}$$ for 'm' symbols.

Base Band Binary data Transmission System:

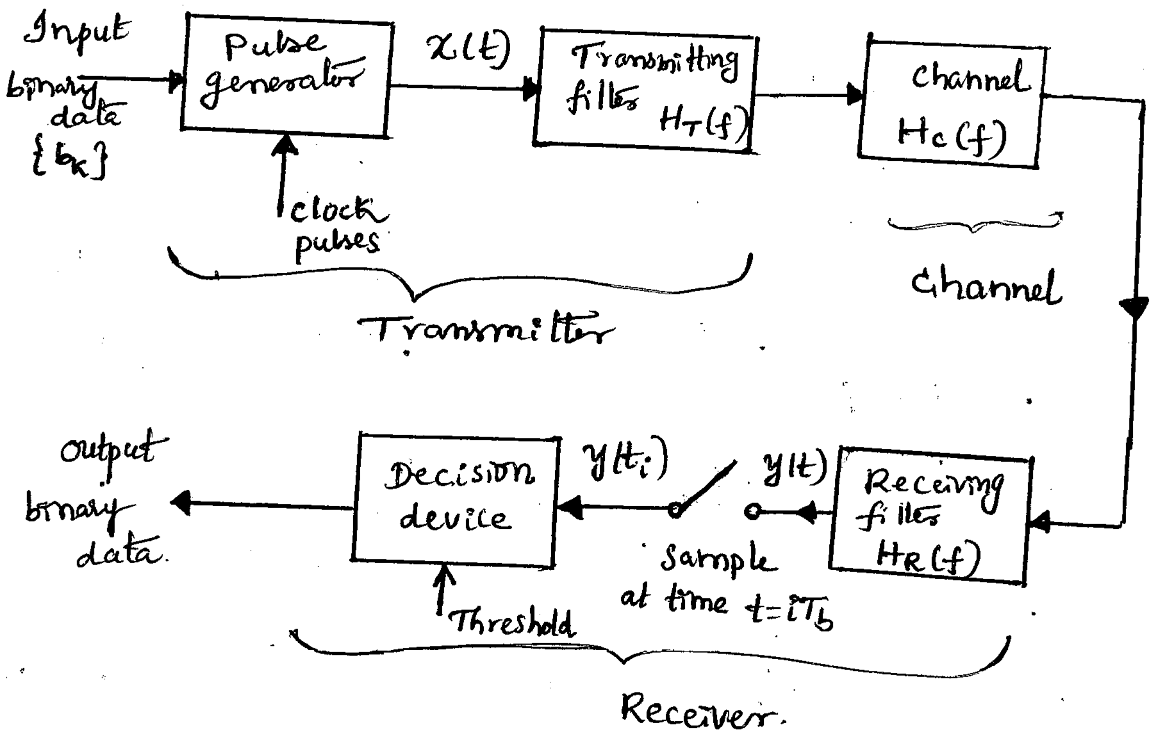


Fig : Baseband binary transmission system.

- Discrete pulse amplitude modulation is the most suitable technique for Txion of base band signal data.
- It is most efficient in terms of power and bandwidth
- Here, the amplitude of the transmitted pulse is varied in a discrete manner in accordance with the given digital data.

* The input to the system is a binary sequences (In the form of '0' and 1's) with a bit rate of r_b and bit duration of T_b

∴ The pulse generator output is a pulse wave form

$$x(t) = \sum_{K=-\infty}^{\infty} a_K \cdot P(t - K \cdot T_b) \longrightarrow (1)$$

where $P(t)$ is the basic pulse & normalize $p(0) = 1$

a_K is represents the coefficient of the pulse & this varies based on the line code & Txion code. and a_K depends on the K^{th} Input bit

ie $p(t) = 1 \quad , t = 0$
$\quad = 0 \quad , t = \pm T_b, 2T_b, \ldots$

amplitude, a_K depends on identity (1 or 0) of b_K

✓ Let us assume Bipolar NRZ scheme

$$a_K = +A \quad ; \text{ if the input bit } b_K \text{ is symbol '1'}$$
$$= -A \quad ; \text{ if the input bit } b_K \text{ is symbol '0'}.$$

✓ $x(t)$ passes through the transmitting filter having a transfer function. $H_T(f)$.

✓ The output of the Txing filter defines the Txed signal which is subsequently passed through the channel having a transfer function $H_c(f)$.

✓ At the receiver side the received signal is passed through a receiving filter of transfer function $H_R(f)$.

✓ The output of the receiving filter is sampled synchronously with the transmitter.
The sampling instants are determined by a clock pulse that is extracted from the receiving filter output.

✓ The sequence of samples obtained at the filter output are then fed to a decision device.

✓ The function of decision device is to reconstruct the original data.

* The decision device compares the amplitude of each sample to a threshold.

✓ If the threshold is exceeded, a decision is taken in favour of symbol '1'.

* If the amplitude of the sample is below the threshold, a decision is made in favour of symbol '0'.

✓ If the amplitude of a sample is exactly equal to the threshold, any one of the symbol '0' & '1' may be chosen without affecting the overall performance.

Inter Symbol Interference : (ISI).

" When a digital data is transmitted over a band limited Channel, dispersion in the channel gives rise to interference called ISI (Inter Symbol Interference)."

This effect introduces deviations (errors) between the reconstructed data at the receiver and the original data at the transmitter.

✓ It should be noted that noise is not only factor that distorts the signal on the channel. In the absence of noise also the channel cause a dispersion (or spreading) of the pulse shaping.

ie

When the channel bandwidth is greater than bandwidth of pulse, Spreading of pulse is very less, but when channel bandwidth is close to signal bandwidth ie if we transmit digital data which demands more bandwidth, spreading will occur and cause signal pulses to overlap.

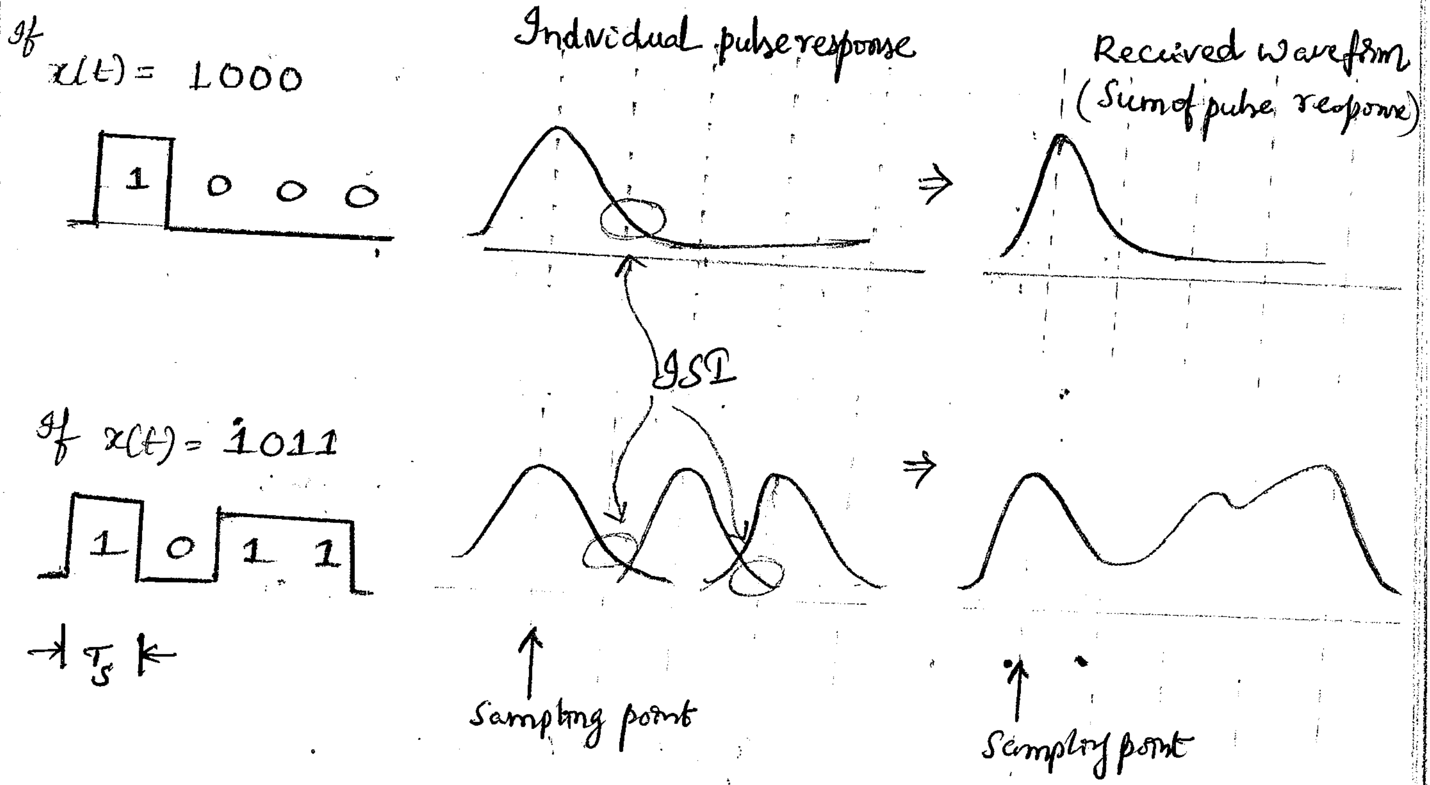


fig : Occurance of Intersymbol Interference.

The receiving filter output may be written as

Mathematically

$$y(t) = \mu \sum_{K=-\infty}^{\infty} a_K \cdot p(t - KT_b - t_d) \longrightarrow (2)$$

where μ - the scaling factor

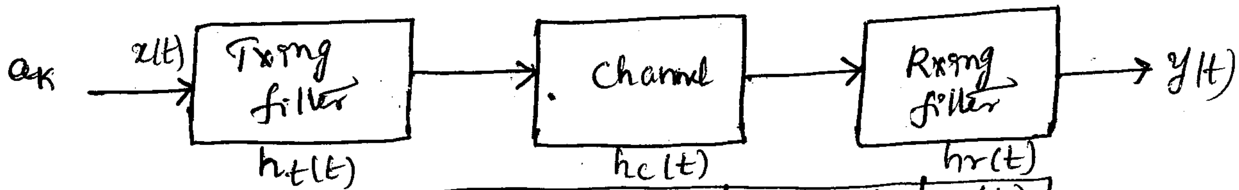
t_d - the time delay introduced by the channel

let us assume $p(t)$ is normalized in such a way that

$$p(0) = 1.$$

The output $y(t)$ is obtained by double convolution involving the impulse response $h_t(t)$ of the Txing filter, the impulse response $h_c(t)$ of the channel & the impulse response $h_r(t)$ of the receiving filter

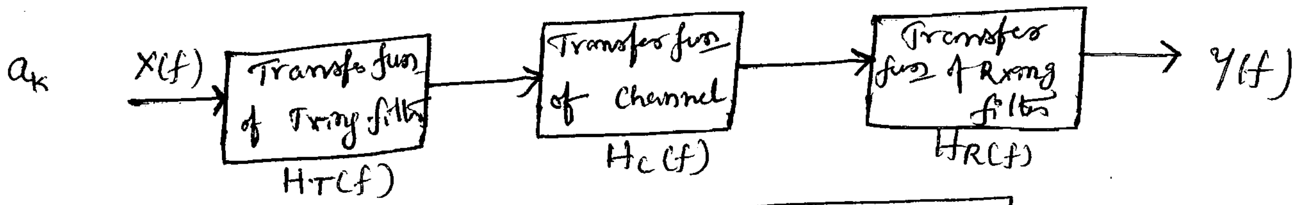
ie



$$\boxed{y(t) = x(t) * h_t(t) * h_c(t) * h_r(t)} \longrightarrow (3)$$

assume $t_d = 0$ (In time domain).

In frequency domain the output of receiving filter is



$$\boxed{Y(f) = X(f) \cdot H_T(f) \cdot H_C(f) \cdot H_R(f)} \longrightarrow (4)$$

(In frequency domain).

The output of Receiving filter is sampled at time $t_i = iT_b$ where i - is an Integer. & $t_d = 0$.

$$\therefore y(t_i) = \mu \sum_{K=-\infty}^{\infty} a_k \cdot p(iT_b - KT_b) + n(t) \longrightarrow (5)$$

$\hookrightarrow t = it_b,\ t_d = 0$ $\quad \hookrightarrow$ noise.

$$\therefore y(t_i) = \mu \sum_{K=-\infty}^{\infty} a_K \cdot p[(i-K)T_b] \qquad i = 0, \pm 1, \pm 2, \pm 3, \ldots$$

If $i = K$ then (In the absense of ISI)

$$y(t_i) = \mu a_i + n(t)$$

where μ - the scaling factor

$n(t)$ = noise added at the channel.

ie If $i \neq K$ then

$$y(t_i) = \underbrace{\mu a_i}_{\text{First}} + \underbrace{\mu \sum_{\substack{K=-\infty \\ i \neq K}}^{\infty} a_K \, p[(i-K)T_b] + n(t)}_{\text{Second}} \longrightarrow (6)$$

* The first term, μa_i is produced by the i^{th} Txed bit, but depends on the contribution of all other Txed bits.
* The second term represents the residual effect of all other txed bits on the decoding of the i^{th} bit. This residual effect is called Intersymbol Interference (ISI).
* The presence of channel introduced ISI & noise in the system however introduces errors in the decision device at the Rxing o/p.
* In order to reduce the errors caused by ISI to a min, it is necessary to design the Txing & Rxing filters suitably.

Nyquist Criterian :

For distortionless baseband data Txion :

* For Reconstructing the original binary data sequence $\{b_K\}$ and need to extract & then decode the corresponding sequence of samples, from the output $y(t)$.

We have to determine the transfer funs of the Txing & receiving filter and also shape of the Txed pulse is required.

* The extration of output Involves smplicity the output $y(t)$ at time $t = iT_b$.

For zero ISI, the condition should be

$$p[(i-K)T_b] = \begin{cases} 1 \,; & i = K \\ 0 \,; & i \neq K \end{cases} \longrightarrow (7)$$

ie The each pulse is zero at the sampling time of other pulses and this will results only when the pulse shape is a <u>Sinc function</u>

∴ The receiver output $y(t)$ is given as

$$\boxed{y(t) = \mu a_i} \quad \forall\, i \longrightarrow (8)$$

Hence above condition ensures perfect reception in the absence of noise.

Consider the sequence of samples $p\{nT_b\}$

where $n = 0, \pm 1, \pm 2, \pm 3, \ldots$

∴ Fourier transform of $p(nT_b)$

$$P_\delta(f) = F\{p(nT_b)\} = \frac{1}{T_b} \sum_{n=-\infty}^{\infty} P(f - nR_b) \longrightarrow (9)$$

The fourier transform of an infinite periodic sequence of delta function of period T_b.

$$\therefore P_\delta(f) = \int_{-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \left[p(mT_b) \cdot \delta(t - mT_b)\right] e^{-j2\pi ft}\, dt \longrightarrow (10)$$

where $m = i-k$ an integer then $i = k,\ m = 0$; $i \neq k,\ m \neq 0$.

According to the conditions of eqn (7), eqn (10) can be written as

ie $i=k$, $m=0$. $\therefore P_\delta(f) = \int_{-\infty}^{\infty} p(0) \cdot \delta(t) \cdot e^{-j2\pi ft}\, dt. \longrightarrow (11)$

As per the property of delta functions. From eqn (9) & (11)

$$\underset{1}{p(0)} = R_b \sum_{n=-\infty}^{\infty} P(f - nR_b)$$

$$\therefore \boxed{\sum_{n=-\infty}^{\infty} P(f - nR_b) = \frac{1}{R_b} = T_b} \longrightarrow (12)$$

Thus,

The nyquist criterian for distortionless baseband Txion in the absence of noise, the frequency fun. $P(f)$ eliminates ISI for samples taken at Intervals T_b provided that is satisfies the eqn (12).

Ideal Nyquist Channel : (or) Ideal solution of ISI :

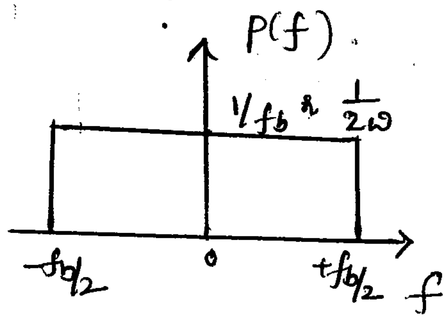
The simplest way of satisfying eqn (12) is to specify the frequency fun $P(f)$ to be in the form of a rectangular fun.

ie

$$P(f) = \begin{cases} \frac{1}{2W} & ; -W \le f < W \\ 0 & ; |f| > W. \end{cases} \longrightarrow (13)$$

The overall system bandwidth W is defined by

$$W = \frac{R_b}{2} = \frac{1}{2T_b} = f_b/_2 \longrightarrow (14)$$



Applying Inverse fourier transform

$$p(t) = F^{-1}\{P(f)\} = \int_{-\infty}^{\infty} P(f) \cdot e^{j2\pi ft} df.$$

$$= \int_{-f_b/2}^{f_b/2} \frac{1}{2W} \cdot e^{j2\pi ft} df \qquad \left(\because \frac{1}{2W} = \frac{1}{2 \cdot f_b/2} = \frac{1}{f_b} = T_b\right.$$

$$= T_b \cdot \left[\frac{e^{j2\pi ft}}{j2\pi t}\right]_{-f_b/2}^{f_b/2}$$

$$= T_b \left[\frac{e^{j2\pi f_b/2 t} - e^{-j2\pi f_b/2 t}}{j2\pi t}\right]$$

$$= \frac{T_b}{\pi t} \left[\frac{e^{j\pi f_b t} - e^{-j\pi f_b t}}{2j}\right]$$

$$= \frac{T_b}{\pi t} \sin(\pi f_b t) \qquad (f_b = 1/T_b)$$

$$p(t) = \frac{\sin(\pi f_b t)}{(\pi f_b t)} \qquad \left(\because \frac{\sin \pi x}{\pi x} = \operatorname{sinc} x\right)$$

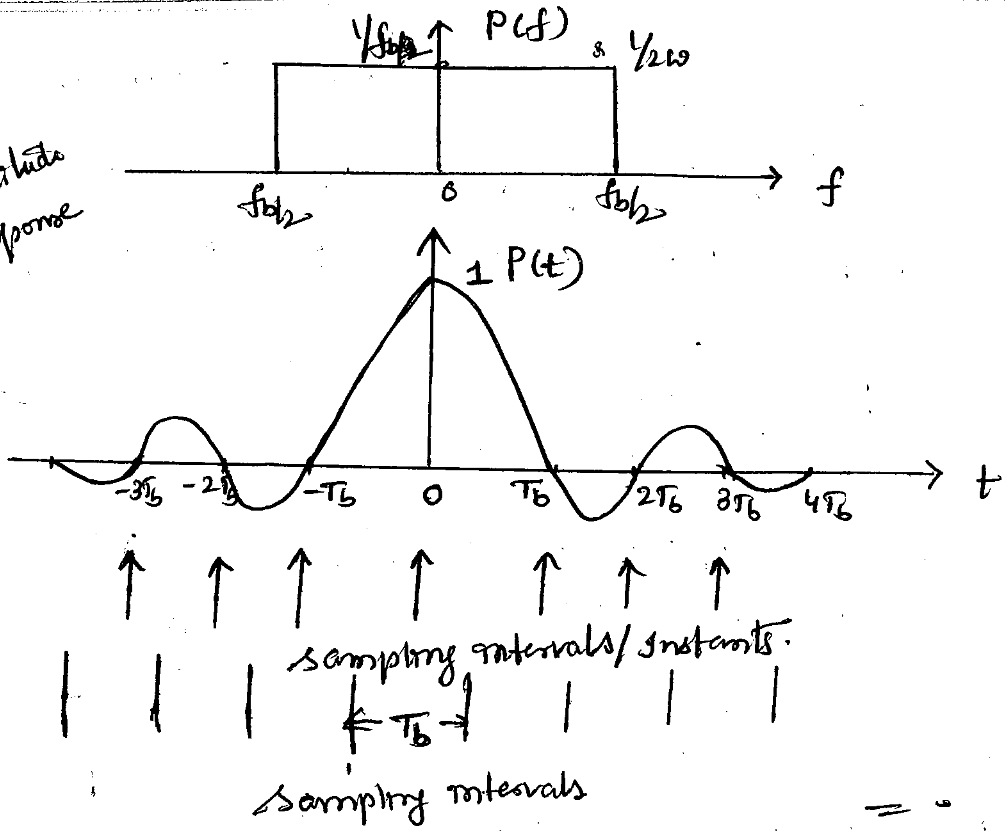
$$p(t) = \operatorname{Sinc}(f_b t) \longrightarrow (15)$$

$$\therefore \boxed{p(t) = \operatorname{Sinc}(f_b t)}$$

The special value of the bit rate R_b & $f_b = 2W$ is called the nyquist rate & Sinc function produces zero ISI.

Fig (a): Ideal amplitude response

Fig (b): Ideal basic pulse shape.



Thus,

Nyquist criterian reduces ISI with min. bandwidth possible, there are two practical difficulties that make it an undesirable objective for system design.

① It requires that the amplitude characteristics of P(f) be flat from $-f_b/2$ to $+f_b/2$ & zero elsewhere. This is physically undesirable because of the abrupt transition at the band edges $\pm f_b/2$.

② The function p(t) decreases as $\frac{1}{|t|}$ for large $|t|$, resulting is a slow rate of decay, this is also caused by the discontinuity of P(f) at $\pm f_b/2$.

To evaluate timing error, consider the sample of y(t) are $t = \Delta t$, where Δt is the timing error.

$$y(\Delta t) = \mu \sum_{k=-\infty}^{\infty} a_k \cdot p(\Delta t - kT_b) \longrightarrow (16) \quad (\because \text{absence of noise})$$

$$= \mu \sum_{k=-\infty}^{\infty} a_k \cdot \frac{\sin \pi f_b (\Delta t - kT_b)}{\pi f_b (\Delta t - kT_b)}$$

Since the definition $f_b \cdot t = 1$ ie $T_b = 1$.

$$y(\Delta t) = \underset{\substack{\uparrow \\ \text{desired symbol}}}{\mu a_0 \cdot \operatorname{Sinc}(f_b \Delta t)} + \underset{\substack{\uparrow \\ \text{ISI caused by} \\ \text{timing error } \Delta t}}{\mu \frac{\sin(\pi f_b \cdot \Delta t)}{\pi} \cdot \sum_{K \neq 0} \frac{(-1)^K \cdot a_K}{(f_b \cdot \Delta t - K)}} \longrightarrow (17)$$

$(\because \sin(x - \pi k) = (-1)^K \cdot \sin x.)$

Practical Nyquist Channel : (or) Practical Solution of ISI : (or) Raised Cosine filter Characteristics :

✓ To overcome practical difficulties encountered with the ideal nyquist channel, here extending the bandwidth from minimum value $W = R_b/2$ to an adjustable value between W and $2W$.

We know $\sum_{n=-\infty}^{\infty} p(f - nR_b) = T_b = 1/f_b$.

The frequency band of Interest $[-f_b/2 ; f_b/2]$ as

$$p(f) + p(f - f_b) + p(f + f_b) = \frac{1}{f_b} \quad ; -f_b/2 \leq f \leq f_b/2$$

✓ The particular form of $P(f)$ that desirable features is provided by a Raised cosine spectrum.

This frequency characteristics consists of a flat portion and a roll-off portion that has a sinusoidal form.

∴ The raised consine function is defined as.

$$\boxed{P(f) = \begin{cases} T_b & ; |f| \leq (f_b/2 - \beta) \\ T_b \cdot \cos^2\left[\frac{\pi}{4\beta}\left(|f| - f_b/2 + \beta\right)\right] & ; (f_b/2 - \beta) \leq f \leq (f_b/2 + \beta) \\ 0 & ; |f| > (f_b/2 + \beta) \end{cases}}$$

where β - roll-off factor " $0 \leq \beta \leq f_b/2$.

$$\boxed{p(t) = \frac{\cos(2\pi\beta t)}{1-(4\beta t)^2} \cdot \frac{\sin \pi f_b t}{\pi f_b t}}$$

at $t = nT_b$

$$\therefore p(nT_b) = \frac{\cos(2\pi\beta nT_b)}{1-(4\beta nT_b)^2} \cdot \frac{\sin n\pi}{n\pi}$$

If Roll-of factor $\beta = 0$ $\therefore$ $\boxed{p(0) = \frac{\sin n\pi}{n\pi}}$ $(\cos 0 = 1)$

If $\beta = f_b/4$ then

$$p(f) = \begin{cases} T_b & ; \ |f| \le f_b/4 \\ T_b \cdot \cos^2\left[\frac{\pi}{f_b}\left(|f| - f_b/4\right)\right] & ; \ f_b/4 \le |f| \le 3f_b/4 \\ 0 & ; \ |f| > 3f_b/4 \end{cases}$$

the roll off factor is a measure of the excess bandwidth of the filter

$$\therefore \boxed{p(nT_b) = \frac{\cos n\pi/2}{1-n^2} \cdot \frac{\sin n\pi}{n\pi}}$$

If $\beta = f_b/2$ then

$$p(f) = \begin{cases} T_b & ; \ |f| = 0 \\ T_b \cdot \cos^2\left(\frac{\pi}{2f_b}|f|\right) & ; \ 0 \le |f| \le f_b \\ 0 & ; \ |f| > f_b \end{cases}$$

$$\therefore \boxed{p(nT_b) = \frac{\cos n\pi}{1-4n^2} \cdot \frac{\sin n\pi}{n\pi}}$$

Thus The frequency response $p(f)$ normalized by multiplying it by 'f_b' for three values of β namely $0, f_b/4, f_b/2$.

✓ The time response $p(t)$ is the inverse fourier transform of function $p(f)$.

* For non zero value of 'β', the function $p(f)$ cuts gradually and it is therefore easier to realize it in practice.

Raised Cosine function in frequency domain.

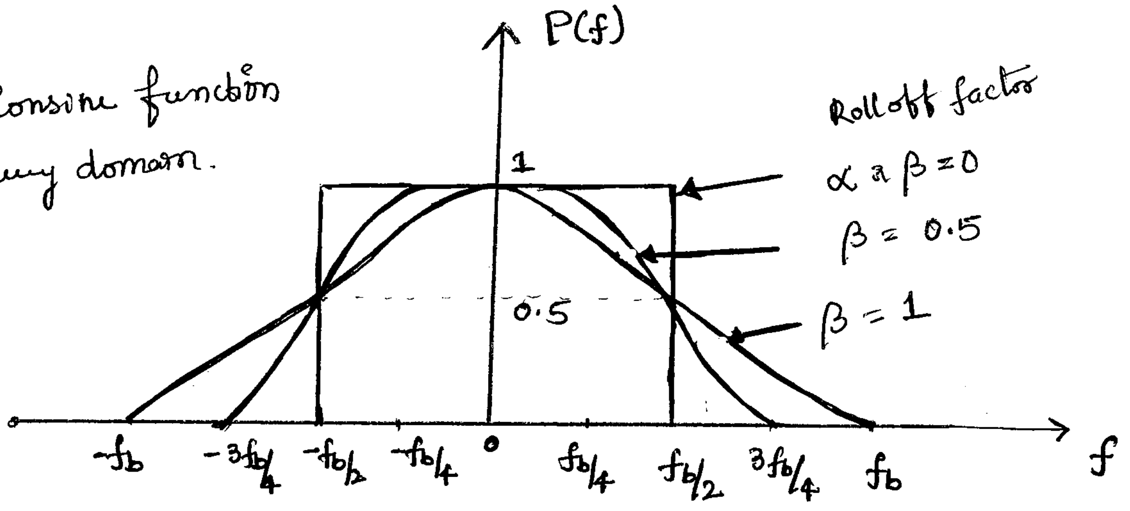
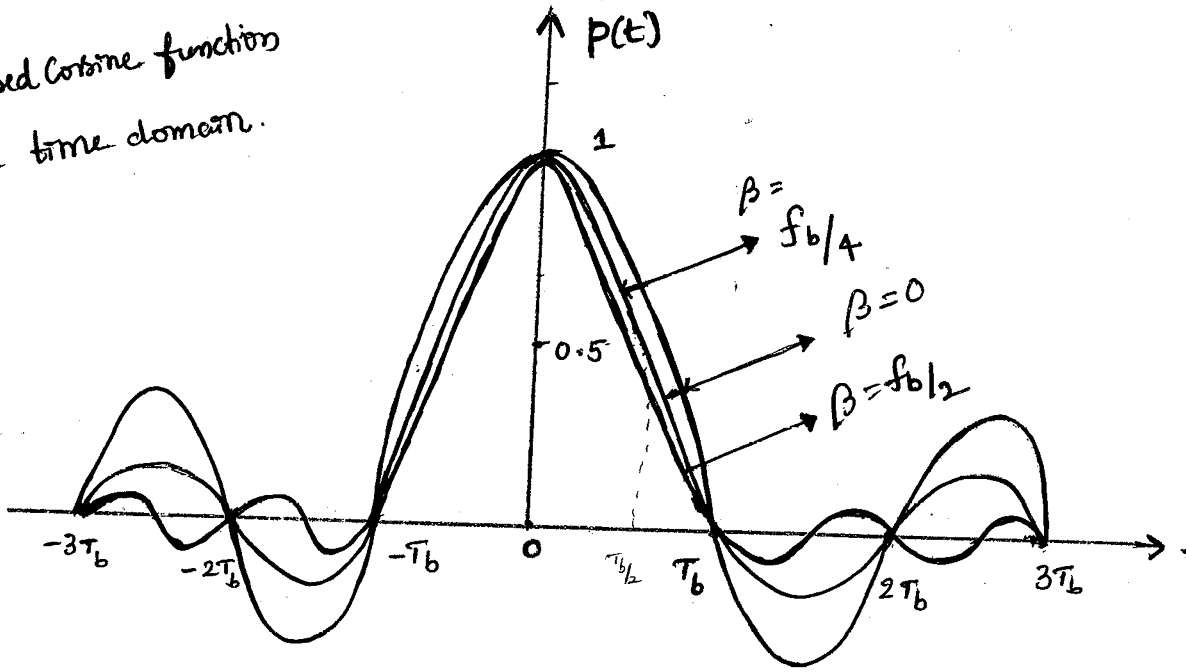


Fig: Frequency response for different roll-off factors.

Raised Cosine function In time domain.



At $t = \pm T_b/2$, we have $p(t) = 0.5$ ie pulse width measured at half amplitude is exactly equal to the bit duration T_b

- Further, there are zero crossing at $t = \pm 3T_b/2$, $\pm 5T_b/2$ ..... in addition to the usual zero crossing at the sampling rate (times) $t = \pm T_b$, $\pm 2T_b$, ... etc.

These two properties of the time response are used for generating timing signals from the received signal for the purpose of Synchronization.

* The transmission bandwidth requirement of raised cosine solution is

$$\boxed{\text{Bandwidth} = B_0(1+\beta)}$$

where $B_0 = W = f_b/2 \rightarrow$ Nyquist bandwidth

β - roll off factor. $\boxed{0 \le \beta \le f_b/2}$

- For large values of β, the bandwidth is increased in frequency domain & faster decaying in the time domain.

Problem

Consider the T1 carrier system designed to accommodate 24 voice channels based on 8 bit PCM word. Calculate the bandwidth of the T1 system assuming.

(a) Ideal Low pass characteristics
(b) A raised cosine spectrum with α & $\beta = 1$ for base band pulse shaping

Sol.

The filtered voice signal in T1 system is usually sampled at 8 KHz - which is the standard sampling rate in digital telegraphy. Each frame of multiplexed signal occupies a period of 125 μsec. Each frame consists of 193 bits.

∴ The bit rate of the T1 system is

$$R_b = 193 \text{ bits/frame} \times 8000 \text{ frames/sec}$$

$$R_b = 1.544 \text{ M bits/sec}$$

$$\therefore \boxed{R_b = 1.544 \text{ M bits/sec}}$$

$$T_b = 1/R_b = 0.647 \mu\text{sec}$$

(i) For an ideal low pass characteristics of the channel, the nyquist bandwidth of the system is

$$B_0 \text{ \& } W = f_b/2 = \frac{1}{2T_b} = 772 \text{ KHz}.$$

$$\therefore \boxed{B_0 \text{ \& } W = 772 \text{ KHz}}$$

Which is the min. transmission bandwidth of T1 system for zero ISI

(ii) for a raised consine spectrum with α & $\beta = 1$

The transmission bandwidth

$$B \cdot W = B_0 (1+\beta)$$
$$= 2 B_0$$
$$= 2 \cdot W = 2 \cdot f_b/2 = f_b = 1/T_b = R_b$$

$$\therefore B \cdot W = 1/T_b = R_b = 1.544 \text{ MHz}.$$

$$\therefore \boxed{\text{Bandwidth} = 1.544 \text{ MHz}.}$$

Eye pattern (or) Eye diagram:

" The Inter symbol Interference (ISI) in data transmission can be studied with the help of a display on the oscilloscope called eye diagram ".

- ✓ The received distorted wave is applied to the vertical deflection plates of an oscilloscope and the saw-tooth wave at a rate equal to the txed symbol rate $1/T$ is applied to the horizontal deflection plates.
- ✓ The waveforms in successive symbol intervals are thus translated into one interval on the oscilloscope display.

Let us consider the distorted but noise free Bi-polar binary symbol s/m.

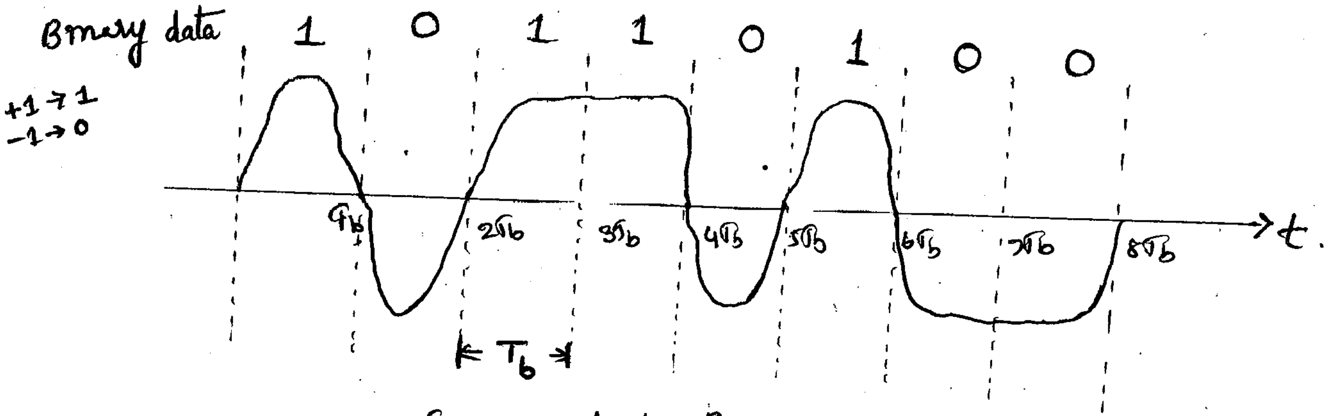


Fig: Distorted Binary wave.

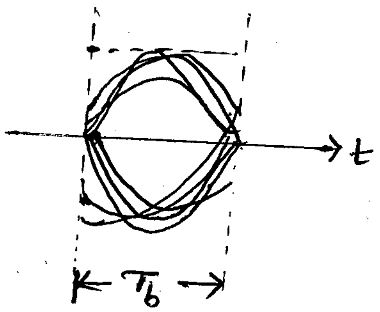


Fig: Eye pattern.

- ✓ When displayed on a long persistence oscilloscope, we get superposition of successive symbol intervals to produce the eye-pattern.

" The pattern is so called because of its resemblance to the human eye. The interior region is called the "eye opening" "

* A generalized binary eye pattern with labels identifying the significant features as follows.

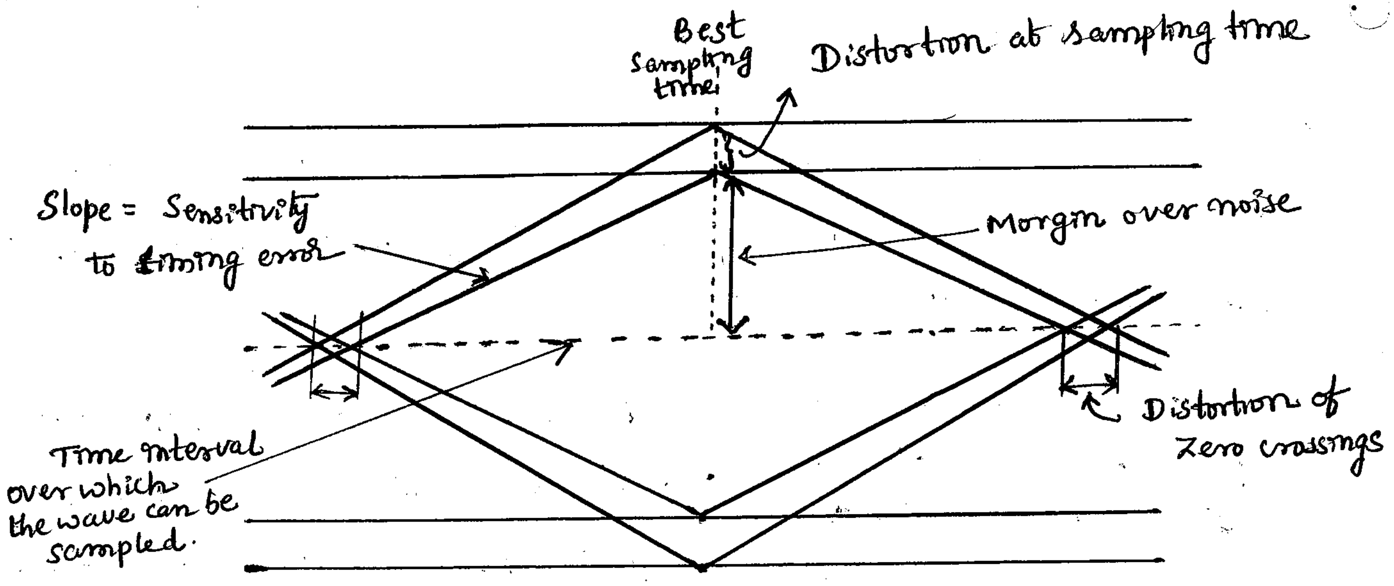


Fig: Interpretation of eye pattern.

* The eye pattern provide the following information about the performance of the system.

1. The width of the eye opening defines the time interval over which the received wave can be sampled without error from ISI. The optimum sampling time corresponds to the maximum eye opening.
2. The height of the eye opening at a specified sampling time is a measure of the margin over channel noise.
3. The sensitivity of the system to timing error is determined by the rate of closure of the eye of the eye pattern in the vicinity of zero crossings as the sampling time is varied.
4. Any non-linear transmission distortion would reveal itself in an asymmetric & Squinted eye.

✓ When the effect of ISI in excessive, traces from the upper portion of the eye pattern cross traces from lower portion with the result that the eye is completed closed. In such a case, It is impossible to avoid error due to combined presence of Intersymbol Interference (ISI) and channel noise in the system.

Equalization :

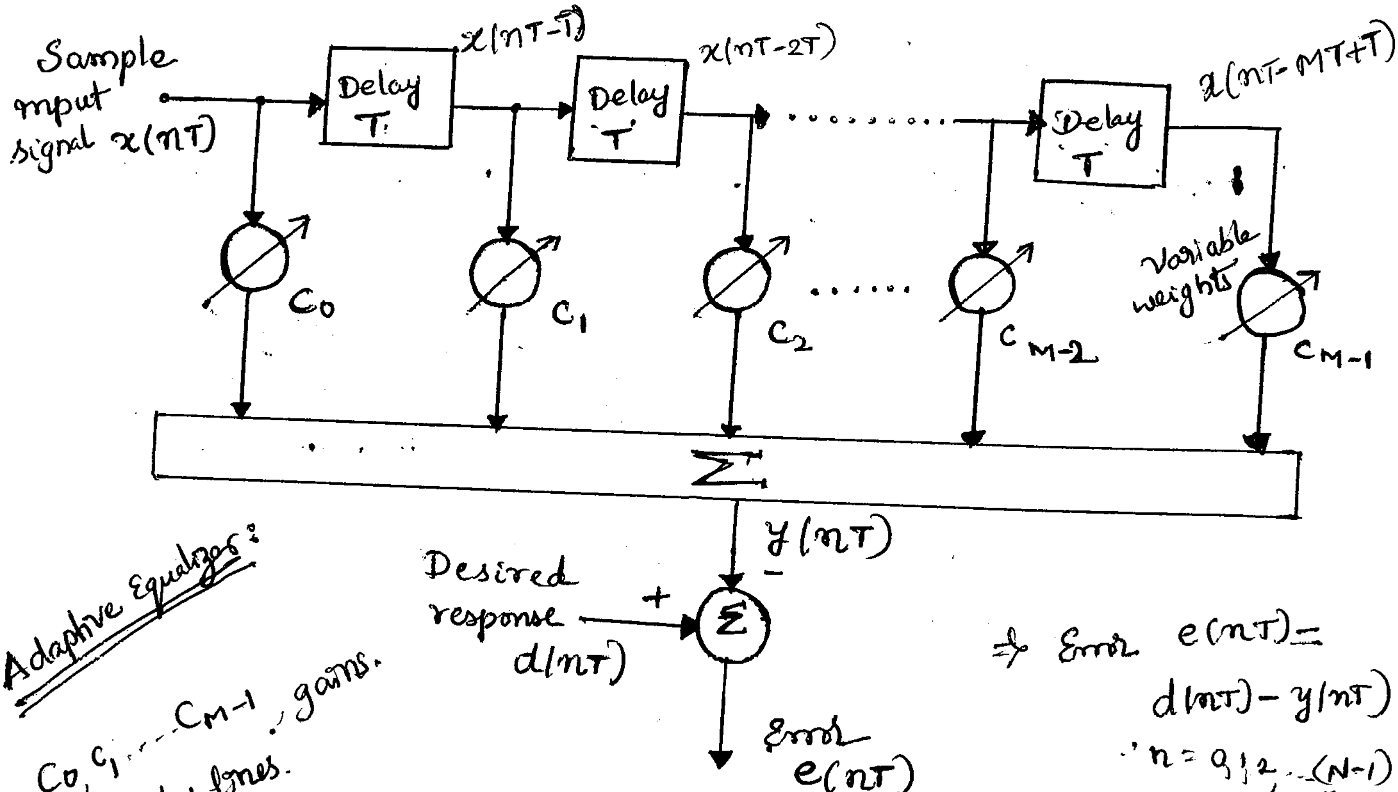
• "Due to an imperfect filter design and changes in the channel characteristics, Intersymbol Interference will be introduced in the received signal, The process of correcting channel induced distortion is called equalization".

There are two types of equalizers.

1. Transversal equalizer
2. ~~Automatic~~ Equalizer
   (a) Preset Equalizer
   (b) Adaptive Equalizers

• The channel introduces signal distortion in the base band data, as a result any real channel needs equalization to approach an ideal frequency response for transmission of modulated digital signals.

* Equalization is very important in the high speed transmission of digital data over a voice grade telephone channel which is essentially linear and bandlimited having high signal-to-noise ratio.



Adaptive Equalizer:

$c_0, c_1, \ldots c_{M-1}$ gains.

tapped delay lines.

⇒ Error $e(nT) = d(nT) - y(nT)$

$n = 0, 1, 2, \ldots (N-1)$

↑ length of the sequence

Fig: Elements of Adaptive Equalizer/filter.

✓ The structure is a tapped delay line filter that consists of a set of delay elements, a set of adjustable multipliers connected to the delay line taps and a summer for adding the multiplier outputs.

Let $x(nT)$ be the sequence is applied to the input of the tapped delay line filter.

∴ The output of the delay line filter will be

$$y(nT) = \sum_{i=0}^{M-1} c_i x(nT-iT) = \sum_{i=0}^{M-1} c_i\, x[nT-iT]$$

where c_i - the multiplying coefficients at the i^{th} tap.

M - the total number of taps.

* The tap spacing along the delay line is chosen equal to the symbol duration T of the Txed signal that is the reciprocal of the sampling rate.

∴ The Error Sequence $e(nT)$ is given by

$$\boxed{e(nT) = d(nT) - y(nT)}$$

where $n = 0, 1, 2, \ldots N-1$

where N - the total length of the sequence.

where

$d(nT)$ - the known Txed sequence as the desired Response.

✓ For optimization is one that minimises the total error Energy defined by

$$\boxed{\mathcal{E} = \sum_{n=0}^{N-1} e^2(nT)}$$

The optimum values of the tap coefficients $c_0, c_1, c_2 \ldots$ result when the total error energy is minimized.

The solution to this optimization problem may be achieved with the help of an algorithm that adjusts the tap weights of the filter in a recursive manner.

From the incoming data, sample by sample to automatically adjust the tap coefficients towards the optimum solution.

Correlative Coding :

Inter symbol Interference (ISI) is an undesirable phenomenon the produces a degradation in system performance.
However, by adding ISI to the transmitted signal in a controlled manner to achieve a bit rate higher than the bandwidth of the channel.

Correlative Coding is transmit signal at a rate of $2R_b$ & $2w$ symbols per second in a channel having a bandwidth of R_b & w & B_0.
It is also known a partial response signalling (or) poly binary (or) Duo binary signalling ".

To avoid the problem associated with the ideal solution
$P(t) = \mathrm{Sinc}(f_b \cdot t)$ ($\because$ Ideal nyquist channel).
The Example of correlating Coding is Duo Binary Signalling scheme.

Duo-Binary Signalling :

✓ It is the simplest type of correlative coding that employs a correlation span of one binary digit.

* Duo - stands to imply doubling of transmission capacity as compared to straight binary system.

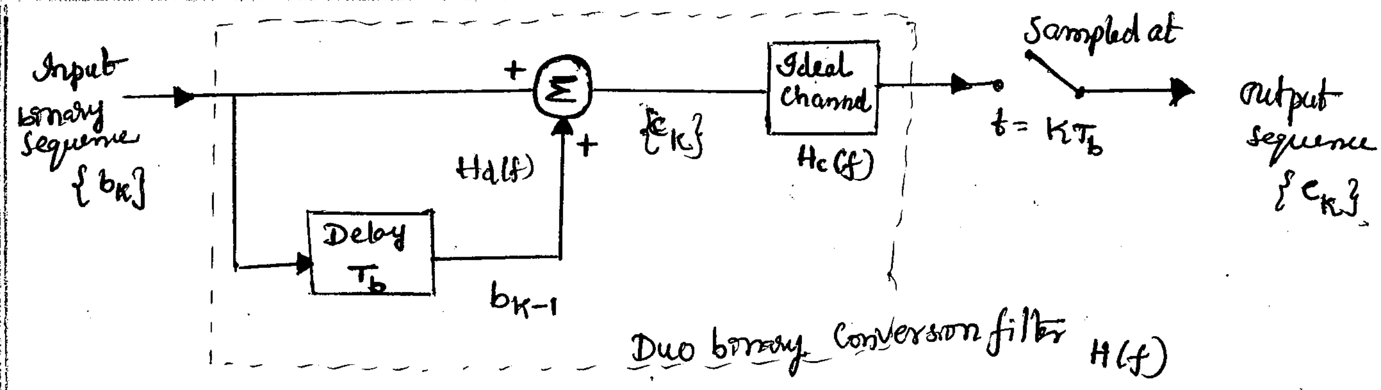
✓ The modified duobinary signalling employes a correlation span of two binary digits.

Let us consider a binary input sequence $\{b_K\}$ consisting of un-correlated digits / symbols 1 and 0 each having duration T_b

ie
$$b_K = \begin{cases} +1 & ; \text{ if symbol } b_K \text{ is } 1 \\ -1 & ; \text{ if symbol } b_K \text{ is } 0 \end{cases} \longrightarrow (1)$$

✓ When the sequence is applied to a duo binary encoder, it is converted into a three level output ie $-2V$, $0V$, & $+2V$.

* The two level sequence $\{b_K\}$ is first passed through a simple filter involving a single delay element, direct path & summer as follows.



Encoding:

Fig: Duobinary Signalling Scheme.

✓ The summer output is denoded as C_K → Correlated output

ie
$$C_K = b_K + b_{K-1} \longrightarrow (2)$$

where
$$\left.\begin{aligned} C_K &= +2V \;;\; b_K = b_{K-1} = 1 \\ &= 0V \;;\; b_K \neq b_{K-1} \\ &= -2V \;;\; b_K = b_{K-1} = 0. \end{aligned}\right\} \longrightarrow (3)$$

" The correlation between the adjacent pulses may be viewed as introducing ISI into the transmitted signal in an artificial manner & this basis is called Correlative Coding."

✓ The transfer function of the Loop filter is

$$H_d(f) = 1 + \exp(-j2\pi f T_b) \longrightarrow (4)$$

✓ The overall transfer function of duobinary signalling filter is

$$\begin{aligned} H(f) &= H_d(f) \cdot H_c(f) \\ &= [1 + \exp(-j2\pi f T_b)] \cdot H_c(f) \\ &= [e^{-j\pi f T_b} \cdot e^{+j\pi f T_b} - e^{-j\pi f T_b} \cdot e^{-j\pi f T_b}] H_c(f). \\ &= [e^{+j\pi f T_b} - e^{-j\pi f T_b}] e^{-j\pi f T_b} \cdot H_c(f) \\ &= 2\cos(\pi f T_b) \cdot e^{-j\pi f T_b} \cdot H_c(f) \end{aligned}$$

$$\boxed{H(f) = 2H_c(f) \cdot \cos(\pi f T_b) \cdot e^{-j\pi f T_b}} \quad \left(\because \frac{e^{j\theta} + e^{-j\theta}}{2} = \cos\theta\right) \longrightarrow (5)$$

For an ideal nyquist channel of bandwidth

$$W = \frac{R_b}{2} = f_b/2 = \frac{1}{2T_b}.$$

$$H_c(f) = \begin{cases} \frac{1}{f_b} \text{ a } \frac{1}{2W} \text{ a } T_b & ; |f| \le f_b/2 \\ 0 & ; \text{otherwise} \end{cases} \longrightarrow (6)$$

From eqn (5) & (6)

The overall transfer function

$$H(f) = \begin{cases} 2T_b \cos(\pi f T_b)\, e^{-j\pi f T_b} & ; |f| \le f_b/2 \\ 0 & ; \text{otherwise.} \end{cases} \longrightarrow (7)$$

The magnitude $|H(f)| = \begin{cases} 2T_b \cos \pi f T_b & ; |f| \le f_b/2 \\ 0 & ; \text{otherwise} \end{cases}$

The phase angle $\underline{|H(f)} = \begin{cases} -\pi f T_b & ; |f| \le f_b/2 \\ 0 & ; \text{otherwise.} \end{cases}$

Representation:

$f = +f_b/2 \Rightarrow \underline{|H(f)} = -\pi/2$

$f = -f_b/2 \Rightarrow \underline{|H(f)} = +\pi/2$

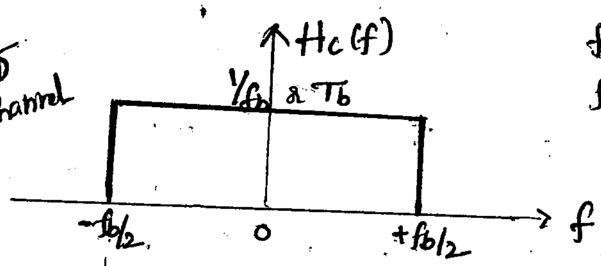


fig: frequency response of Ideal nyquist channel

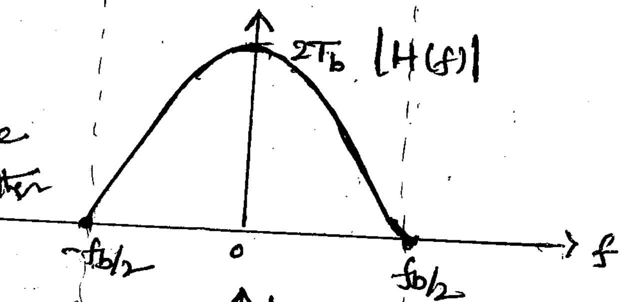


fig. magnitude response of conversion filter

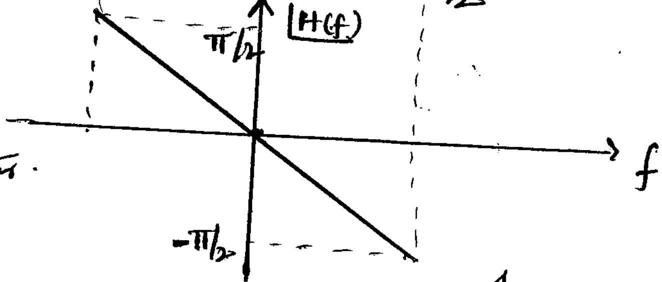


fig: phase response of conversion filter.

The impulse response can be obtained Inverse fourier transform of $H(f)$

$\therefore$ Transfer fun $H(f) = T_b\left[1 + e^{-j2\pi f T_b}\right]$ ie $H_c(f) \cdot H_d(f)$.

$$h(t) = F^{-1}\{H(f)\} = \int_{-\infty}^{\infty} H(f) \cdot e^{j2\pi f t}\, df$$

$$= T_b \int_{-f_b/2}^{f_b/2} \left[1 + e^{-j2\pi f T_b}\right] e^{j2\pi f t}\, df$$

$$h(t) = T_b \int_{-f_b/2}^{+f_b/2} \left[e^{j2\pi f t} + e^{j2\pi f (t-T_b)} \right] df$$

$$= T_b \left\{ \left. \frac{e^{j2\pi f t}}{j2\pi t} \right|_{-f_b/2}^{f_b/2} + \left. \frac{e^{j2\pi f (t-T_b)}}{j2\pi (t-T_b)} \right|_{-f_b/2}^{f_b/2} \right\}$$

$$= T_b \left\{ \frac{e^{j\pi f_b t} - e^{-j\pi f_b t}}{\pi t \cdot 2j} + \frac{e^{j\pi f_b (t-T_b)} - e^{-j\pi f_b (t-T_b)}}{\pi (t-T_b) \cdot 2j} \right\}$$

$$= T_b \left\{ \frac{\sin \pi f_b t}{\pi t} + \frac{\sin \pi f_b (t-T_b)}{\pi (t-T_b)} \right\}$$

$\because \sin(\pi f_b t - \pi) = -\sin \pi f_b t$

$\because \sin(x-\pi) = -\sin(\pi - x) = -\sin x.$

$$= T_b \left\{ \frac{\sin(\pi t/T_b)}{\pi t} - \frac{\sin(\pi t/T_b)}{\pi (t-T_b)} \right\}$$

$(f_b = 1/T_b)$

$$= T_b \cdot \sin(\pi t f_b) \left[\frac{1}{\pi t} - \frac{1}{\pi (t-T_b)} \right]$$

$$= T_b \cdot \sin(\pi t f_b) \cdot \left[\frac{\cancel{\pi t} - \pi T_b - \cancel{\pi t}}{\pi^2 t (t-T_b)} \right]$$

$$h(t) = T_b \cdot \sin(\pi t f_b) \left[\frac{-\cancel{\pi} T_b}{-\pi^2 t (T_b - t)} \right]$$

$$h(t) = \frac{T_b^2 \cdot \sin(\pi f_b t)}{\pi t (T_b - t)} \longrightarrow (8)$$

$\therefore$ Impulse response $\boxed{h(t) = \frac{T_b^2 \sin(\pi t f_b)}{\pi t (T_b - t)} \ \& \ \frac{T_b^2 \sin(\pi t / T_b)}{\pi t (T_b - t)}.}$

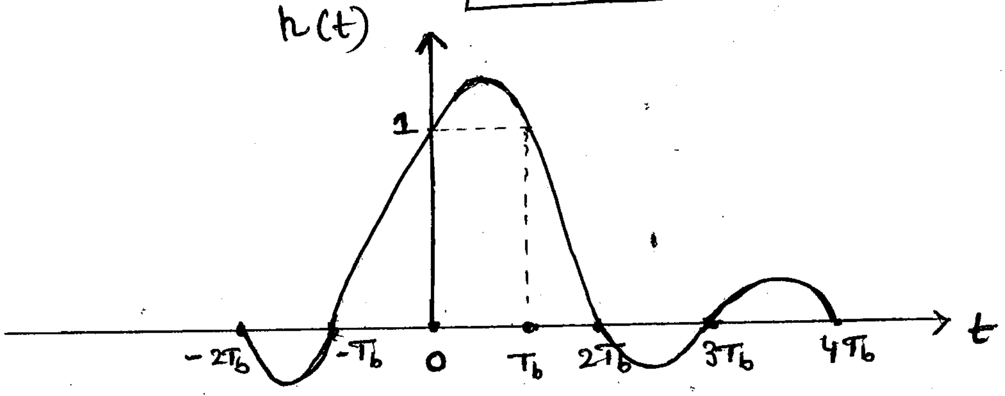


Fig: Impulse response of duobinary conversion filter

$\because$ L-H rule

$$h(t) = \frac{T_b^2 \cdot \cos \pi t f_b (\pi / T_b)}{\pi T_b - 2t\pi}$$

$$h(t) = \frac{T_b \cdot \cos(\pi t f_b)}{T_b - 2t}$$

$t = 0, \ h(t) = 1$

$t = T_b \quad h(t) = \frac{-T_b}{-T_b}$

$h(t) = 1.$

$t = \pm 2T_b, \ h(t) = 0.$

It shows that & explain why we also refer this type of correlative coding as partial signalling. The response to an input pulse is spread over more than on signalling interval.

Decoding :

The original two level sequence $\{b_K\}$ may be detected at receiver from the duo-binary sequence $\{C_K\}$ by substracting previous decoded binary digit from the presently received digit $\{C_K\}$.

The demodulation technique known as nonlinear decision feedback equalization is essentially an inverse of the operation of the digital filter at the transmitter.

ie
$$\hat{b}_K = C_K - \hat{b}_{K-1} \longrightarrow (9)$$

where $\hat{b}_K$ - the estimate of original sequence b_K as conceived by the receiver at $t = KT_b$.

$\hat{b}_{K-1}$ is the estimate at $t = (K-1)T_b$

ie

The detection procedure just described is essentially an inverse of the operation of the simply delay line filter at the transmitter.

Drawback : At $f=0$ $|H(f)| = 2T_b$ ie DC value is high

So more transmitted power is required.

The major drawback of this detection procedure is that once errors are made they tend to propagate through the output known as Error propagation.

A practical method of avoiding the error propagation phenomenon is to use pre coding before the due binary coding as shown in fig below

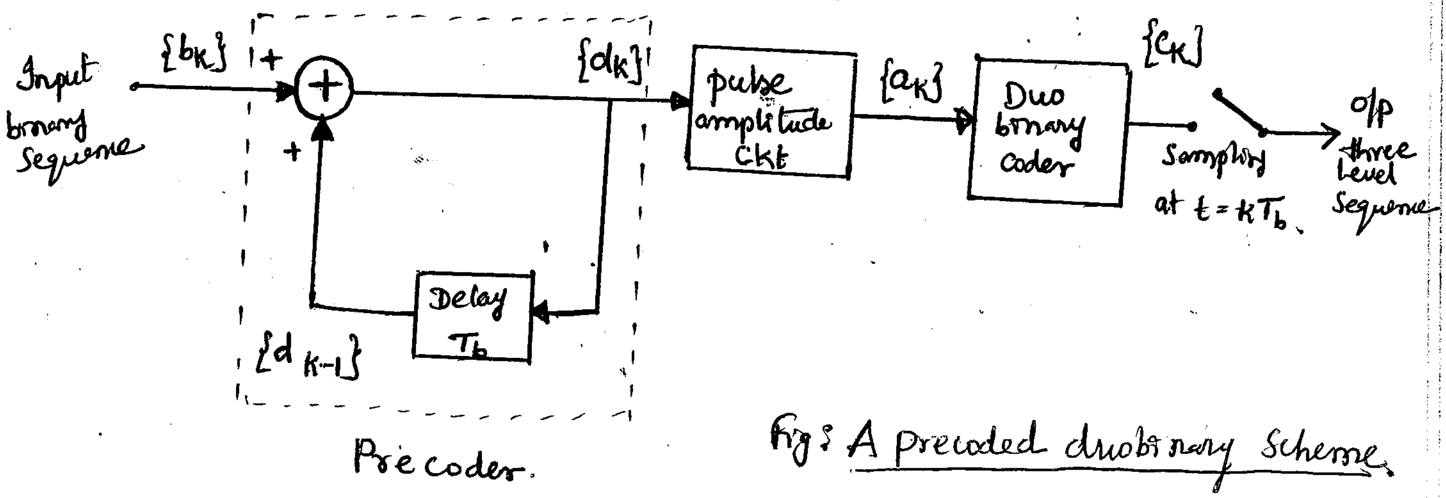


Fig: A precoded duobinary scheme.

- The precoding operation performed on the binary data sequence $\{b_k\}$ converts into another binary sequence $\{d_k\}$, given by

$$d_k = b_k \oplus d_{k-1} \longrightarrow (10)$$

Non linear operation $\rightarrow$ $\boxed{d_k = b_k \oplus d_{k-1}}$

modulo-2 operation.. ie Exclusive OR operation

- Precoder prevents error propagation and makes it possible to recover the input message sequence.

ie The output of precoder is output '1' if exactly one input is a '1' otherwise the output remains '0' ie

Truth Table for precoder.

b_k	d_{k-1}	d_k
0	0	0
0	1	1
1	0	1
1	1	0

X-OR

- This d_k is applied to pulse amplitude ckt produces a corresponding two level sequence of short pulses $\{a_k\}$.

$\therefore$ The sequence c_k as

Linear operation $\rightarrow$ $\boxed{c_k = a_k + a_{k-1}}$ (we know that duobinary scheme o/p).

(or) $c_k = d_k + d_{k-1} \longrightarrow (11)$

The combination of eqn (10) & (11)

$$c_k = \begin{cases} \pm 2V & ; \text{ If data symbol } b_k = 0 \\ 0V & ; \text{ If data symbol } b_k = 1. \end{cases}$$

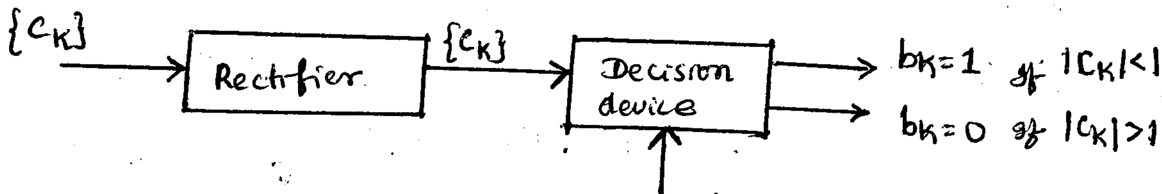


Fig: Detector for recovering original binary sequence from the precoded duobinary sequence.

- The detector consists of Rectifier, the output of which is compared in a decision device to a threshold of 1.

If $|c_k| < 1$ say symbol $b_k = 1$

If $|c_k| > 1$ say symbol $b_k = 0$

~~If $|c_k| = 1$ the receiver simply makes a random gives in favour~~ of symbol 1 and 0.

ie If $c_k = \pm 2V$, $b_k = 0$
$c_k = 0V$, $b_k = 1$.

Problem: For input binary data 1011101 obtain the output of duo-binary encoder and also the output of decoder.

1000101

Sol: The input binary data $\{b_K\} = \{1\ 0\ 1\ 1\ 1\ 0\ 1\}$

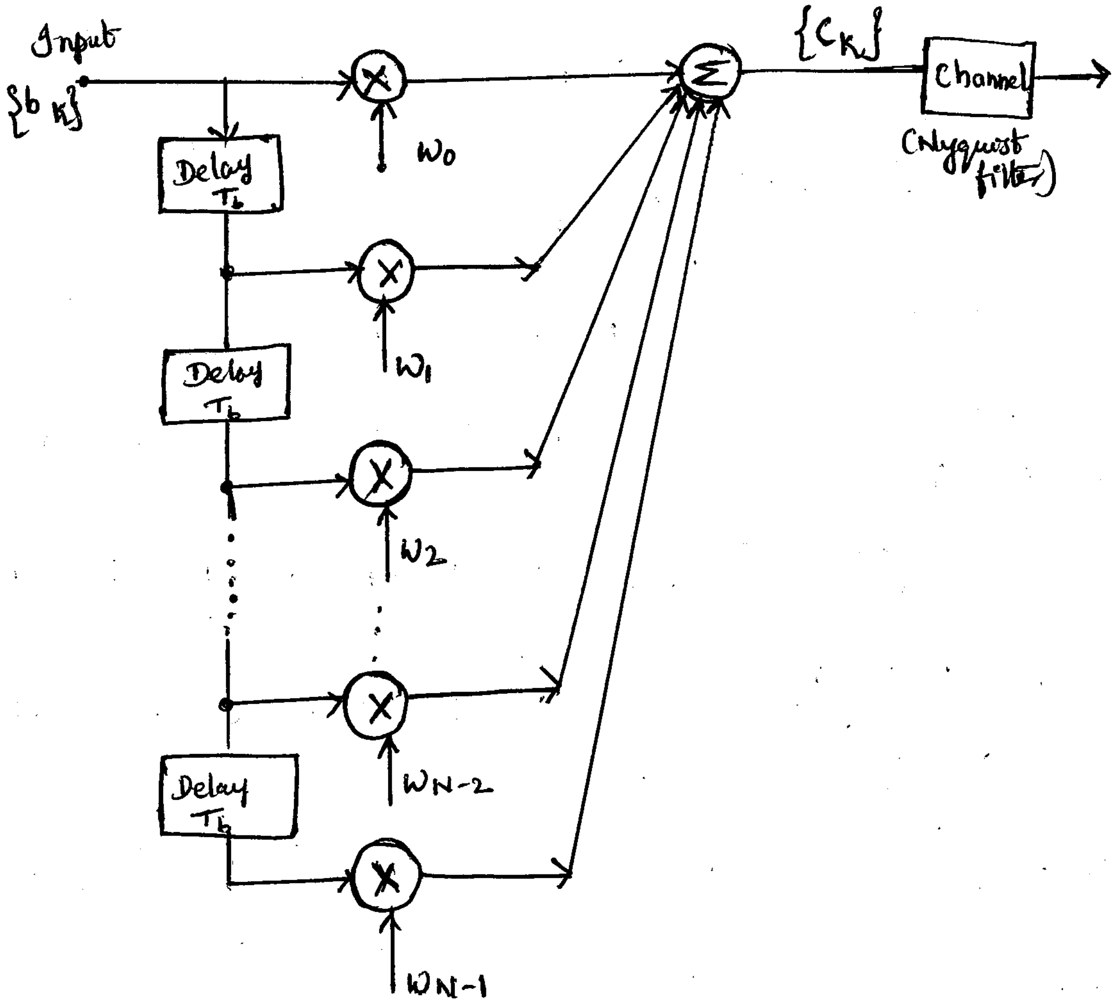
(a) With reference / extra bit '1' ← d_{K-1}

1.	Input binary Sequence $\{b_K\}$	1 0 1 1 1 0 1
2.	Precoded binary sequence $d_K = b_K \oplus d_{K-1}$ → 1	1 (d_{K-1}) 0 0 1 0 1 1 0
3.	Voltage representation of d_K ie $1 \rightarrow +1$, $0 \rightarrow -1$	+1 −1 −1 +1 −1 +1 +1 −1
4.	Voltage representation of Duobinary coder output $\{C_K\}$ ie $C_K = \pm 2V$, $b_K = 0$; $= 0$, $b_K = 1$	0 −2 0 0 0 +2 0
5.	Decoded binary sequence $\{\hat{b}_K\}$ $b_K = 0$, $C_K = \pm 2V$; $b_K = 1$, $C_K = 0$	1 0 1 1 1 0 1

(b) With reference / Extra bit 0 → d_{K-1}

1.	Input binary Sequence $\{b_K\}$	1 0 1 1 1 0 1
2	Precoded binary sequence $d_K = b_K \oplus d_{K-1}$ → 0	0 (d_{K-1}) 1 1 0 1 0 0 1
3	Voltage representation of d_K ie $1 \rightarrow +1$, $0 \rightarrow -1$	−1 +1 +1 −1 +1 −1 −1 +1
4	Voltage representation of Duobinary Coder output $\{C_K\}$; $C_K = \pm 2V$, $b_K = 0$; $= 0$, $b_K = 1$	0 +2 0 0 0 −2 0
5	Decoded binary sequence $\{\hat{b}_K\}$ $b_K = 0$, $C_K = \pm 2V$; $b_K = 1$, $C_K = 0V$	1 0 1 1 1 0 1

Generalized form of Correlative Coding Scheme



✓ The duo-binary and modified duo binary scheme have correlation spans of binary digit and two binary digits respectively.

✓ In the duo binary scheme, the transfer fun H(f) & PSD of the Txed signal is non zero at the origin.
This can be reduced by using the technique called modified duo binary signalling. Here $C_K = a_K - a_{K-2}$.

* The generalization scheme involves the use of tapped delay line filters with tap weights $w_0, w_1, w_2 \ldots w_{N-1}$.

✓ The Correlative samples $\{C_K\}$ can be obtained from a superposition of 'N' successive input sample values $\{b_K\}$.

∴ The output of correlative coding $\boxed{C_K = \sum_{n=0}^{N-1} w_n \cdot b_{K-n}}$

If $N = 2 \Rightarrow C_K = \sum_{n=0}^{1} w_n b_{K-n} = w_0 b_K + w_1 b_{K-1}$

If $w_0 = w_1 = +1$ then $\boxed{C_K = b_K + b_{K-1}}$ ← Duobinary signalling scheme.

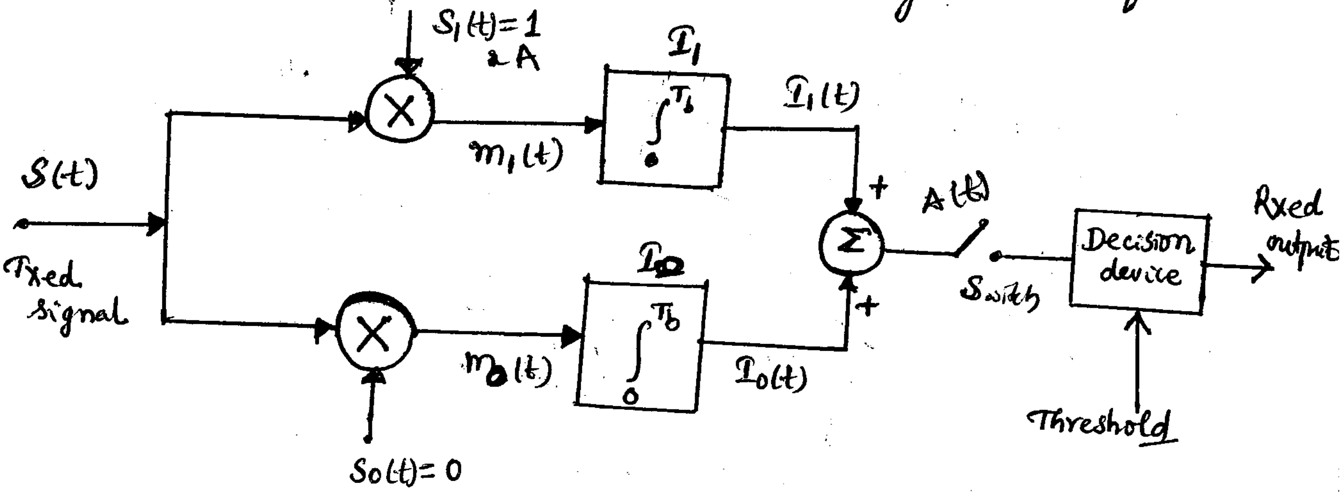
If $N=3$ $\quad C_K = \sum_{n=0}^{2} w_n b_{K-n} = w_0 b_K + w_1 b_{K-1} + w_2 b_{K-2}$

If $w_0 = +1$, $w_1 = 0$, $w_2 = -1$ then $C_K = b_K(1) + 0 + b_{K-2}(-1)$

$\therefore \boxed{C_K = b_K - b_{K-2}}$ ← Modified duo binary signalling scheme.

Correlation Receiver:

- The correlation detection/receiver means synchronous detection. It consists of multiplier and LPF (or) Integrator as follows.



- Let us consider UNRZ Encoding scheme, the txed signal as

$$S(t) = \begin{cases} A + n(t) & ; \text{logic '1'} \\ n(t) & ; \text{logic '0'} \end{cases}$$

Ily for BNRZ scheme $\quad S(t) = \begin{cases} A + n(t) & ; \text{logic 1} \\ -A + n(t) & ; \text{logic '0'} \end{cases}$

Assume noise $n(t)$ has zero mean so, $\int n(t) = 0$.

For UNRZ:

- If logic '1' is received then $S(t) = A + n(t)$

The output of multiplier is $m_1(t) = S(t) \times S_1(t)$

$$= [A + n(t)]\,A$$

$$= A^2 + A \cdot n(t).$$

The output of Integrator is $I_1(t) = \int_0^{T_b} m_1(t)\,dt$

$$= \int_0^{T_b} [A^2 + A \cdot n(t)]\,dt$$

$$I_1(t) = \int_0^{T_b} A^2 dt + \int_0^{T_b} A \cdot n(t)\, dt$$

$$= A^2 \cdot T_b + 0 \qquad (\because \int n(t) = 0) \text{ zero mean}$$

$$\therefore \boxed{I_1(t) = A^2 \cdot T_b}$$

✓ If logic '0' is received then $s(t) = n(t)$

The output of multiplier $m_0(t) = s(t) \times \phi_0(t)$

$$= n(t) \cdot 0$$

$$= 0 \quad \therefore m_0(t) = 0$$

The output of Integrator is $I_0(t) = \int_0^{T_b} m_0(t)\, dt = 0$

$$\therefore \boxed{I_0(t) = 0}.$$

✓ The adder/summer output $A(t)$ is given by

$$\therefore A(t) = I_1(t) + I_0(t)$$

$$= A^2 \cdot T_b + 0$$

$$= A^2 \cdot T_b \quad \therefore A(t) = A^2 \cdot T_b.$$

✓ The threshold value V_{th} can be taken as

$$V_{th} = \frac{A^2 T_b + 0}{2} = \frac{A^2 T_b}{2}$$

ie

If the adder output is larger than the predetermined threshold V_{th} & λ we choose logic '1'

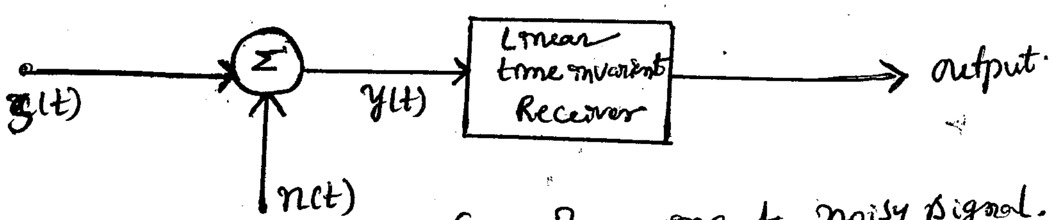
Otherwise the received output, we choose logic '0'.

Note: An Intimate relationship between the two types of criterion

It should be noted that of the two filters (Correlator and matched filter are equivalent only at time $t = T$.

Optimum Receiver :

- Consider a received signal $y(t)$ consisting of a signal $s(t)$ of known form and additive white Gaussian Noise (AWGN) $n(t)$.
- The noise is white in the sense that it has a constant power spectral density $\eta/2$ & $N_0/2$ & Gaussian in the sense that a sample drawn at random from such a process has a gaussian probability distribution of its amplitude & has zero mean.



∴ fig: Processing of noisy signal.

* The purpose of detection is to establish the presence & absense of the signal, we should estimate which one of the two hypotheses noise alone (or) noise plus signal is true.

This can be done by processing the noisy received signal $y(t)$ with the help of a linear time invarient receiver in such a way that the receiver output at some arbitrary time $t = T$ is considerably greater when $s(t)$ is present than that when $s(t)$ is absent.

- There are two approaches to realize a receiver for detection of signal in the presence of noise.

1. Matched filter based on maximisation of SNR at the Receiver. This involves the use of a filter matched to the signal component of the received signal.
2. Correlation Receiver based on probabilistic criteria that is directly related to performance ratings of a particular digital comm'n system of interest. This involves a correlation of the received signal with a stored replica of the transmitted signal.

Matched Filter :

* A matched filter is an optimum filter in the sense that it maximizes the output signal to noise ratio (SNR).

✓ In digital communication matched filters are very useful.

* The main purpose of detection in digital communication is to recognize a pulse signal in presense of noise rather than improving the pulse shape.

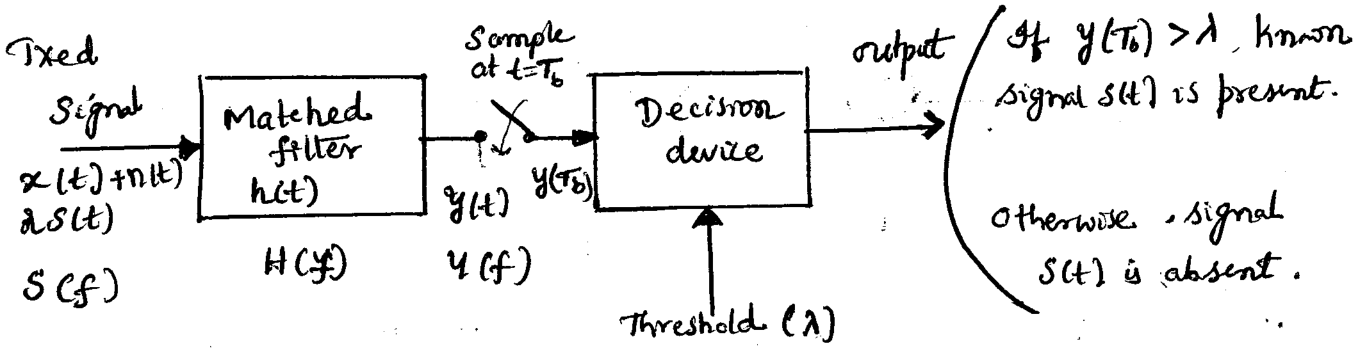


fig: Matched filter receiver.

✓ Let us consider the matched filter with impulse response $h(t)$ and frequency transfer function $H(f)$.

For calculating the SNR value, we have to calculate both signal power and noise power.

The input txed signal $s(t) = \underset{\text{signal}}{x(t)} + \underset{\text{noise}}{n(t)}$ ← AWGN with zero mean.

Consider only input signal $s(t) = x(t)$.

∴ The output of the matched filter $y(t) = x(t) * h(t)$

by taking fourier transform.

$$Y(f) = X(f) \cdot H(f) = \int_{t=-\infty}^{\infty} [x(t) * h(t)]\, e^{-j2\pi ft}\, dt$$

The inverse fourier transform as.

$$y(t) = F^{-1}\{Y(f)\} = \int_{f=-\infty}^{\infty} [X(f) \cdot H(f)] \cdot e^{+j2\pi ft}\, df$$

- The matched filter integrates incoming signal (ie signal + noise) for entire bit duration & then compare with threshold value at the end of bit based on the condition, Decision device decides either '1' or '0'. ie

$$\text{Logic } 1 \rightarrow y(T_b) \geq \lambda \quad \text{Threshold value.}$$
$$\text{Logic } 0 \rightarrow y(T_b) < \lambda$$

$\therefore$ at $t = T_b$

$$y(T_b) = \int_{-\infty}^{\infty} X(f) \cdot H(f) \cdot e^{j2\pi f T_b} \, df$$

The output signal power of matched filter as

$$\boxed{S_o = |y(T_b)|^2 = \left| \int_{-\infty}^{\infty} X(f) \cdot H(f) \cdot e^{j2\pi f T_b} \, df \right|^2} \longrightarrow (1)$$

- Let us consider, the noise generated in the channel is gaussian noise (AWGN) with noise PSD $N_0/2$ & $\eta/2$.

ie Noise PSD at input of matched filter $S_{in}(f) = \frac{N_0}{2}$

The output noise PSD of matched filter can be written as

$$S_o(f) = S_{in}(f) \cdot |H(f)|^2$$
$$= \frac{N_0}{2} \cdot |H(f)|^2$$

$\therefore$ the output noise power of matched filter as

$$N_0 = R_{xx}(0) = \int_{f=-\infty}^{\infty} S_{xx}(f) \, df$$

$$\therefore N_0 = \int_{f=-\infty}^{\infty} \frac{N_0}{2} \cdot |H(f)|^2 \, df$$

$$\therefore \boxed{N_0 = \frac{N_0}{2} \int_{f=-\infty}^{\infty} |H(f)|^2 \, df} \longrightarrow (2)$$

The Signal to noise ratio (SNR) at output of matched filter is given by

$$\boxed{SNR_o = \frac{S_o}{N_o} = \frac{\left| \int_{-\infty}^{\infty} X(f) \cdot H(f) \, e^{+j2\pi f T_b} \, df \right|^2}{\frac{N_0}{2} \cdot \int_{-\infty}^{\infty} |H(f)|^2 \, df}}$$

Using Schwarz inequality ie

$$\left| f_1(t) \cdot f_2(t) \right|^2 \le \left| f_1(t) \right|^2 \cdot \left| f_2(t) \right|^2 \qquad \text{if } f_1(t) = K \cdot f_2^*(t)$$

ie the Squared magnitude of the total area under the product of two such functions is less than & equal to the product of the total area under the squared magnitude of each of the two functions.

$$\Rightarrow \quad \left| \int_{-\infty}^{\infty} f_1(t) \cdot f_2(t)\, dt \right|^2 \le \int_{-\infty}^{\infty} |f_1(t)|^2 dt \cdot \int_{-\infty}^{\infty} |f_2(t)|^2 dt.$$

∴ The output signal to noise ratio of matched filter

$$SNR = \frac{S_0}{N_0} \le \frac{\cancel{\int_{-\infty}^{\infty} |H(f)|^2 df} \cdot \int_{-\infty}^{\infty} \left| X(f) \cdot e^{+j2\pi f T_b} \right|^2 df}{\frac{N_0}{2} \cdot \cancel{\int_{-\infty}^{\infty} |H(f)|^2 df}}$$

$$\le \frac{2}{N_0} \cdot \int_{-\infty}^{\infty} |X(f)|^2 df \qquad \left[\because |e^{j\theta}| = 1 \right]$$

$$\le \frac{2}{N_0} \cdot E \qquad \text{where } E \text{ - Energy of signal} = \int_{-\infty}^{\infty} |X(f)|^2 df$$

$$\therefore \boxed{SNR = \frac{2E}{N_0}}$$

Thus the maximum output SNR depends on the input signal energy, and the power spectral density of the noise, Not on the particular shape of the waveform that is used.

* The mathematic operation of matched filter is convolution, a signal is convolved with the impulse response of the filter.

→ The mathematic operation of correlator is Correlation, a signal is correlated with a replica of itself.

The term matched filter is often used synonmously with "correlator".

→ The Condition for Schwarz inequality is $f_1(x) = K \cdot f_2^*(x)$.

$$\therefore \quad H(f) = K\cdot\left[X(f)\cdot e^{j2\pi f T_b}\right]^*$$

$$H(f) = K\cdot X^*(f)\cdot e^{-j2\pi f T_b}$$

by taking inverse fourier transform on both sides

$$h(t) = F^{-1}\{H(f)\} = \int_{f=-\infty}^{\infty} K\cdot X^*(f)\cdot e^{-j2\pi f T_b}\cdot e^{j2\pi f t}\,df$$

$$= K\cdot\int_{-\infty}^{\infty} X^*(f)\cdot e^{-j2\pi f(T_b-t)}\,df$$

$$= K\cdot\int_{-\infty}^{\infty}\left[X(f)\cdot e^{j2\pi f(T_b-t)}\right]^*\cdot df$$

Replacing $(-f=f)$

$\therefore$ The impulse response of matched filter is

$$h(t) = K\cdot x^*(T_b-t)$$

If $x(t)$ is real function $\therefore \boxed{h(t) = K\cdot x(T_b-t)}$

~ The output of matched filter

$$y(t) = x(t) * h(t)$$

$$= \int_{\tau=-\infty}^{\infty} h(\tau)\cdot x(t-\tau)\,d\tau$$

at $t=0$, $y(0) = \int_{\tau=-\infty}^{\infty} h(\tau)\cdot x(-\tau)\,d\tau = 0$.

at $t=T_b$

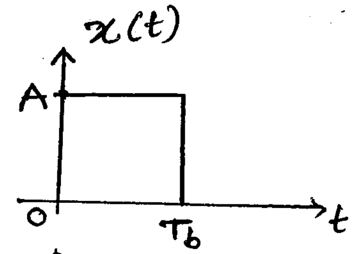
$$\therefore y(T_b) = \int_{\tau=-\infty}^{\infty} h(\tau)\,x(T_b-\tau)\,d\tau.$$

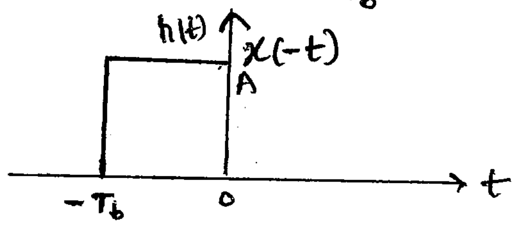
$$\boxed{y(T_b) = \int_{\tau=-\infty}^{\infty} h(\tau)\cdot x(T_b-\tau)\,d\tau}$$

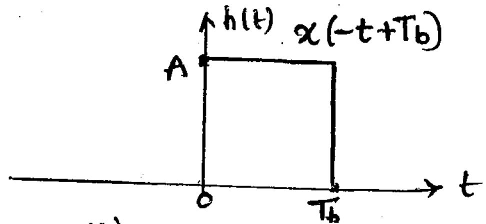
$$y(T_b) = A^2\cdot T_b$$

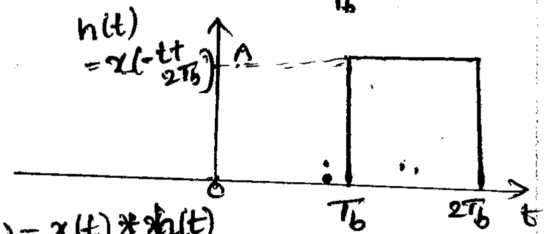
at $t=2T_b$ $\quad y(2T_b) = \int_{-\infty} h(\tau)\cdot x(2T_b-\tau)\,d\tau = 0$

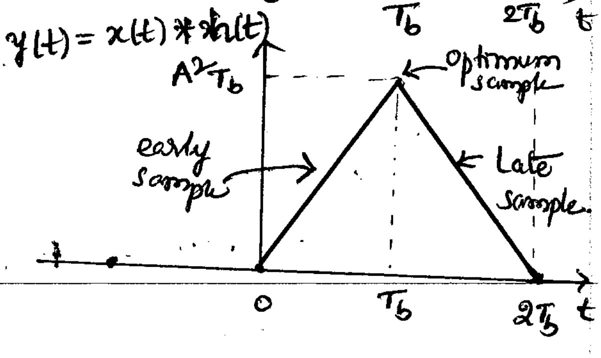
let











Hence the impulse response of a filter that produces the maximum output signal to noise ratio is mirror image of the signal $x(t)$, delayed by the symbol time duration T_b.

—*—

Properties of Matched filter :

Property ①: The spectrum of output signal of a matched filter with the matched signal as the input is proportional to the energy density of the input signal.

Proof : The output of matched filter $Y(f) = X(f) \cdot H(f)$

Consider In frequency domain, the matched filter is characterized by the transfer function

$H(f) = X^*(f) \cdot e^{-j2\pi f T_b}$ which except for a delay factor, is the complex conjugate of the spectrum of $x(t)$.

$$\therefore \quad Y(f) = X(f) \cdot H(f)$$

$$= X(f) \cdot X^*(f) \, e^{-j2\pi f T} \text{ delay.}$$

$$= |X(f)|^2 \cdot e^{-j2\pi f T}$$

$$Y(f) = \Psi_s(f) \cdot e^{-j2\pi f T} \Rightarrow \boxed{Y(f) \propto \Psi_s(f)}$$

where $\Psi_s(f) = |X(f)|^2$ is the Energy spectral density of the input signal $x(t)$.

Thus apart from a time delay factor, the spectrum of the input signal is proportional to the Energy spectral density of the input signal.

Property ②: The output signal of a matched filter is proportional to a shifted version of the autocorrelation function of the input signal to which the filter is matched.

Proof : From the property ①, recognizing that the autocorrelation function and Energy spectral density of a signal form a fourier transform pair.

By taking inverse fourier transform on both sides of $Y(f)$.

$$\therefore \; y(t) = F^{-1}\{Y(f)\} = F^{-1}\{\Psi_s(f) \cdot e^{-j2\pi f T}\}$$

using fourier transform pair $R_s(\tau) \leftrightarrow \Psi_s(f)$

$R_s(\tau)$ being the auto correlation function of the input signal and the time shifting property of fourier transforms

we get $\boxed{y(t) = R_s(t-T)}$,

Hence proved.

Property (3): The output signal to noise ratio of a matched filter depends only on the ratio of the signal energy to the power spectral density of the white noise at the filter output.

Proof: The signal to noise ratio of the matched filter at output is

$$(SNR)_o = \frac{S_o}{N_O} = \frac{2}{N_o} \cdot \int_{-\infty}^{\infty} |X(f)|^2 df$$

where $X(f)$ is the fourier transform of the signal $x(t)$ to which the filter is matched.

Using the Raleigh's Energy theorem we may write the signal Energy $E = \int_{-\infty}^{\infty} x^2(t)dt = \int_{-\infty}^{\infty} |X(f)|^2 df$

∴ PSD of white noise is $N_o/2$

∴ The output signal to noise ratio of the matched filter as

$$\boxed{(SNR)_o = \frac{2E}{N_o} = \frac{(E)}{(N_o/2)}}$$

Hence the SNR at the output of matched filter depends only on the ratio of the signal Energy to PSD of white noise

hence proved.