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UNIT-5

Band Pass Data Transmission

Syllabus: Introduction ASK modulator, bandwidth and frequency spectrum of ASK, Coherent ASK detector, noncoherent ASK detector, Signal space representation & probability error for ASK. FSK - bandwidth & frequency spectrum of FSK, Signal space representation & probability of error for FSK, non coherent FSK detector, Coherent FSK detector, FSK detection using PLL. BPSK - bandwidth & frequency spectrum of BPSK, signal space representation & probability of error for BPSK, Coherent PSK detection, Differential Binary Phase shift keying (DPSK), Quadrature phase shift keying (QPSK), QPSK demodulator, Introduction to M-ary signalling - GMSK and QAM.

Introduction:

* The output of any digital modulator like PCM, DM, DPCM, ADM will be 0's and 1's.
* The 0's & 1's can be transmitted through wire by assigning electrical voltage & it is called "Baseband digital data transmission" because it contains zero frequency components.
* In order to transmit 0's & 1's through free space, electrical energy should converted into electromagnetic energy.
* The device which converts electrical energy into electromagnetic energy called "Antenna" provided that the input to the antenna should be in continuous form.

So, to transmit 0's & 1's through free space it should be converted into continuous form.

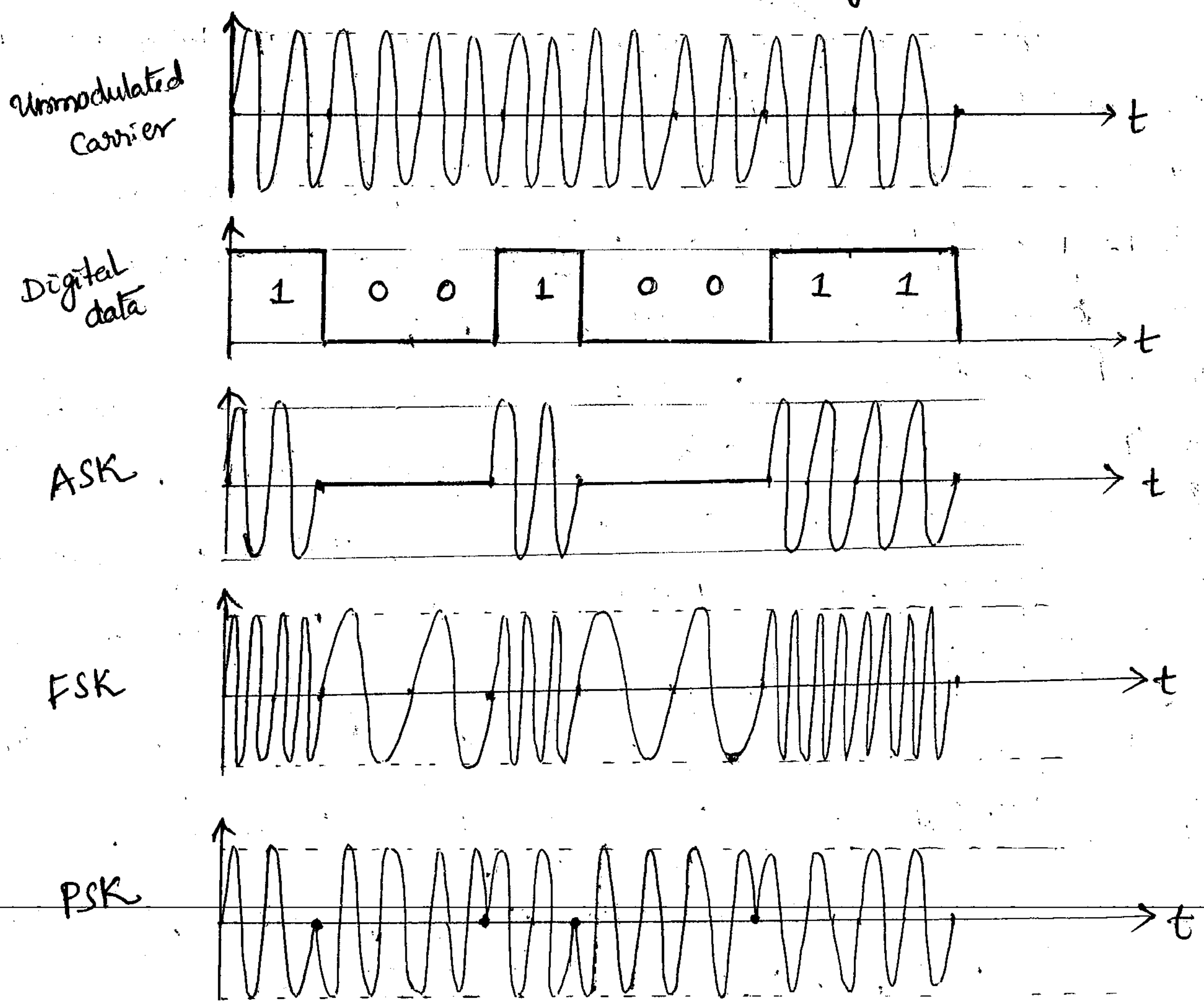
* The modulation scheme used to convert discrete to continuous signals is called "Digital carrier modulation".
* The output of digital carrier modulation is a bandpass signal, hence it is also called "Bandpass digital data transmission".

Digital Modulation Techniques

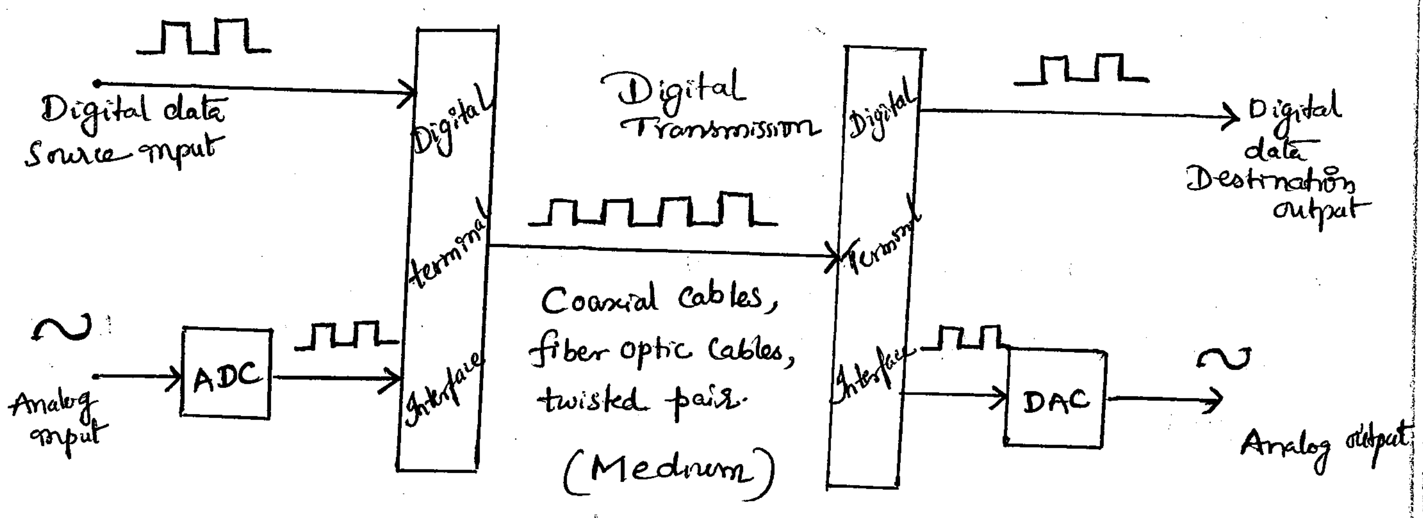
- **Binary** (Encodes only one bit at a time)
  - ASK
    - Coherent
    - Noncoherent
  - FSK
    - Coherent
    - Non Coherent
  - PSK → DPSK, QPSK
    - Coherent
    - Non Coherent.
- **M-ary** (Encodes more than two bits at a time)
  - M-ary ASK
  - M-ary FSK
  - M-ary PSK
  - M-ary QAM → Quadrature Amplitude Modulation

ASK – Amplitude Shift Keying
FSK – Frequency Shift Keying.
PSK – Phase shift Keying

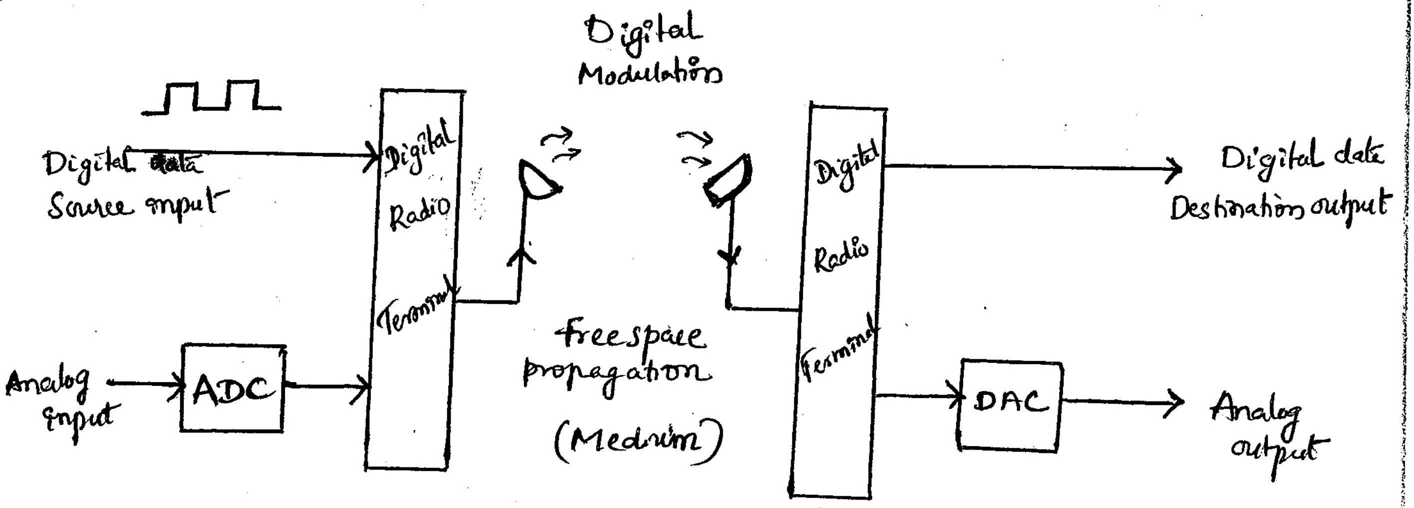
Goals:
1. Maximum data rate
2. Minimum probability of symbol error
3. Minimum Transmitted power.
4. Minimum channel bandwidth
5. Maximum resistance to interfering signals.
6. Minimum Circuit complexity.



1. Digital Transmission:



2. Digital Radio:



Note:

To perform demodulation at the receiver, we have the choice of Coherent (or) Non coherent detection.

- ✓ Coherent detection is performed by cross-correlating the received signal with each one of the replicas & then making a decision based on comparisons with preselected thresholds. Exact knowledge of the carrier wave's phase reference is required.
- ✓ Non coherent detection, knowledge of the carrier wave's phase is not required, the complexity of the receiver is thereby reduced but at the expense of an inferior error performance, compared to a coherent system.

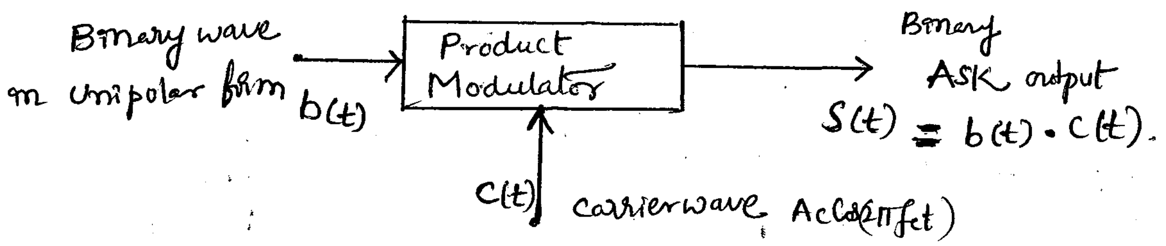
Amplitude Shift Keying (ASK) : (or) OOK
ON-OFF Keying.

- ✓ The binary ASK system is the simplest form of digital modulation
- ✓ ASK is used in Wireless telegraphy.
- ✓ It is no longer used widely in digital communication.

* Symbol 1 represented by transmitting sinusoidal carrier with fixed amplitude A_c and fixed frequency f_c for the bit duration T_b sec.
Symbol '0' represented by switching off the carrier of T_b sec.

∴ The signal can be generated simply by turning the carrier of a sinusoidal oscillator ON & OFF for the prescribed periods indicated by modulating the pulse train.
For this reason ASK is also known as ON-OFF Keying (OOK).

Generation of ASK & ASK Modulator:



let the Sinusoidal carrier be represented by

$$c(t) = A_c \cdot \cos(2\pi f_c t)$$

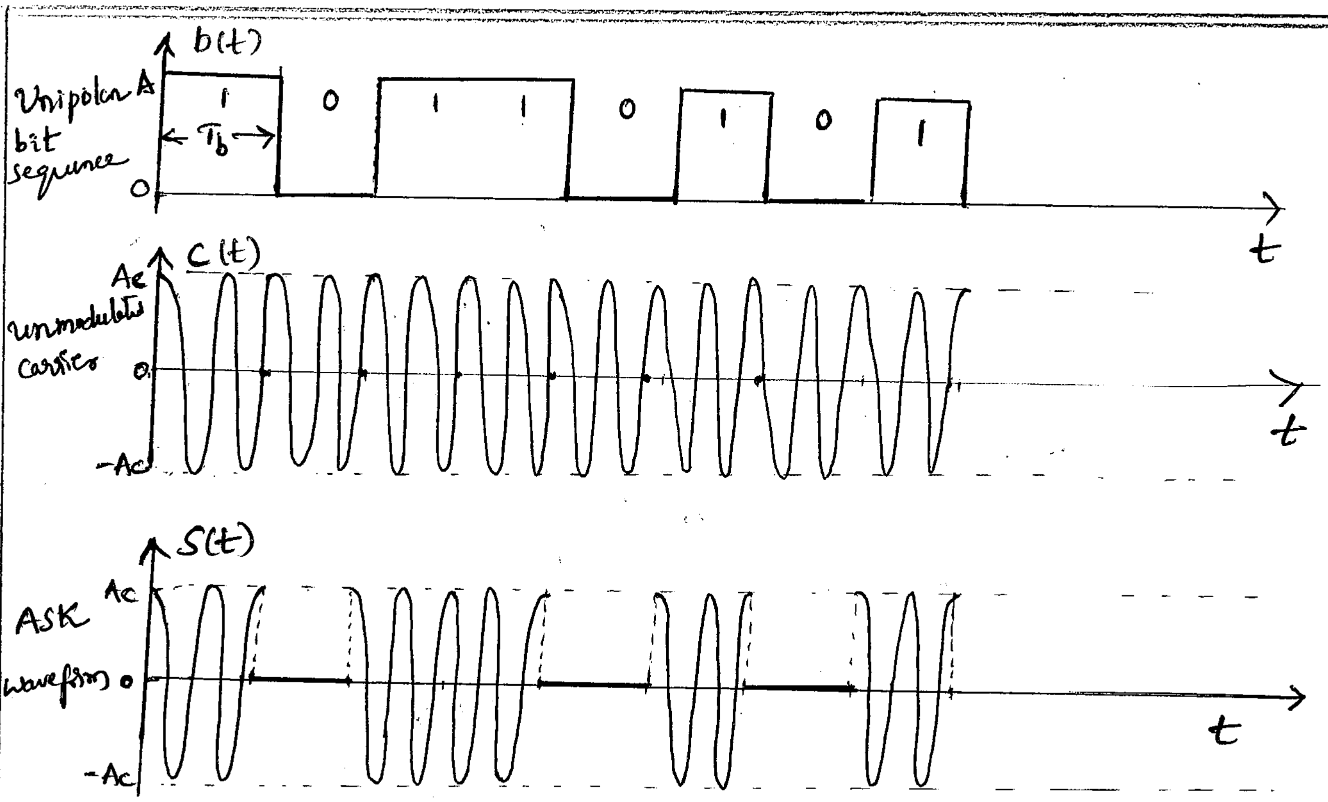
∴ The binary ASK signal can be represented by $s(t)$ given by

$$s(t) = b(t) \cdot (A_c \cos(2\pi f_c t))$$

where $b(t) = 1$; binary 1
$b(t) = 0$ binary '0'

$$\therefore \boxed{S(t) \Rightarrow \begin{cases} A_c \cos(2\pi f_c t) & ;\ \text{binary } 1 \\ 0 & ;\ \text{binary } '0' \end{cases}}$$

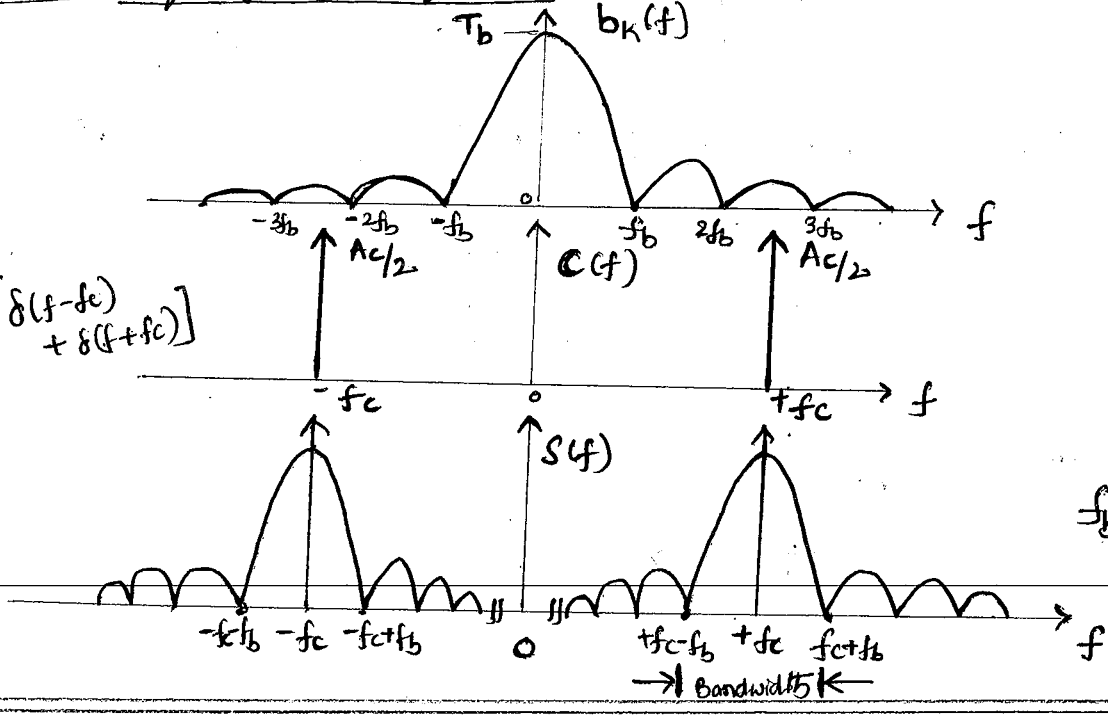
Let us consider the binary data $b(t) = \{1\,0\,1\,1\,0\,1\,0\,1\}$



- ASK signal can be generated by applying the incoming binary data and the sinusoidal carrier to the two input of a product modulator (R) Balanced modulator.
- Modulator causes a shift of the baseband signal spectrum.
- The ASK signal which is basically the product of binary sequence & carrier signal has PSD same as that of baseband ON-OFF signal but shifted in freqiny domain by $\pm f_c$

  ie two impulses will occur at $\pm f_c$.

Bandwidth & Frequency Spectrum of ASK:



$$C(f) = \frac{A_c}{2}\left[\delta(f-f_c) + \delta(f+f_c)\right]$$

$$f_b = 1/T_b$$

$$\text{The bandwidth of ASK} = f_2 - f_1$$
$$= (f_c + f_b) - (f_c - f_b)$$
$$= 2f_b \ \& \ 2/T_b .$$

$\therefore$ Bandwidth $\boxed{B.W = 2f_b \ \& \ 2/T_b \ Hz}$ for ASK

Signal Space representation of ASK:

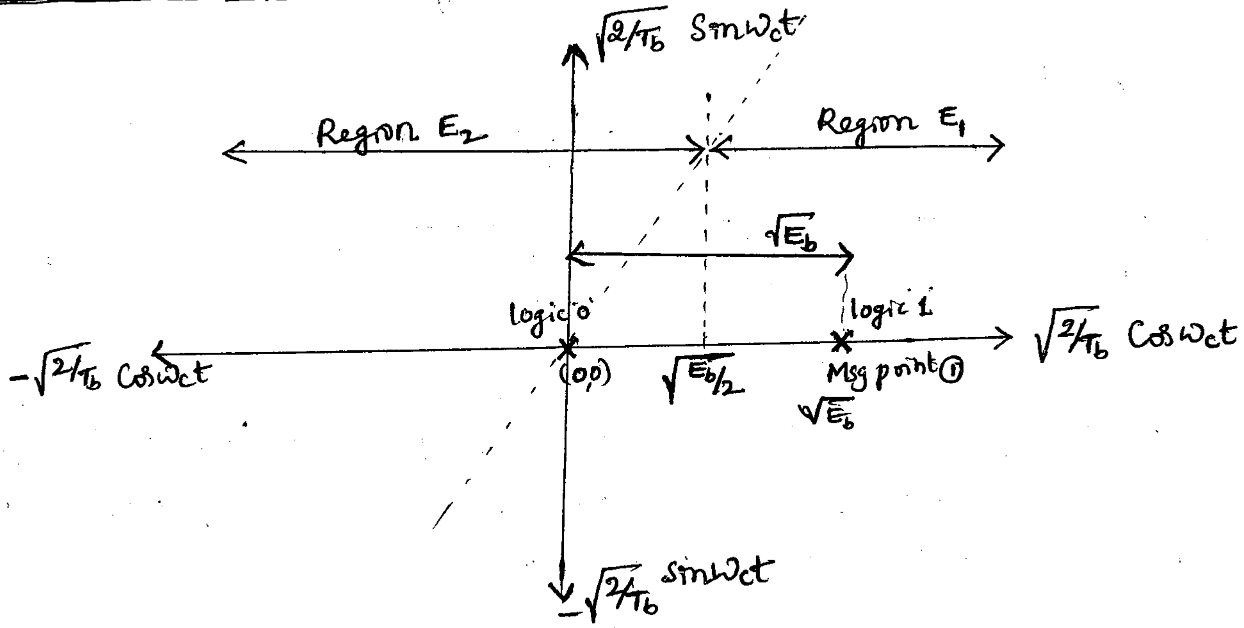


fig. One dimensional signal space with two messages.

✓ ASK output can be represented in power and Energy as follows

In power $S(t) = \begin{cases} \sqrt{2P_c} \cos w_c t & ; \text{logic } 1 \\ 0 & ; \text{logic } 0 \end{cases}$

$P_c = \frac{V_{rms}^2}{R} = \frac{(V/\sqrt{2})^2}{R} = \frac{A_c^2}{2R} = \frac{A_c^2}{2}$ $V = A_c$, $R = 1$

$\therefore A_c^2 = 2P_c \Rightarrow \boxed{A_c = \sqrt{2P_c}}$

in Energy. $S(t) = \begin{cases} \sqrt{E_b} \cdot \sqrt{2/T_b} \cos w_c t & ; \text{logic } 1 \\ 0 & ; \text{logic } 0 \end{cases}$

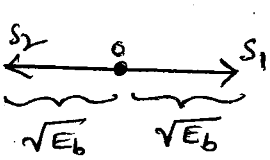
$E = P \cdot T_b$

$\Rightarrow A_c = \sqrt{2P_c} \times \frac{\sqrt{T_b}}{\sqrt{T_b}} = \sqrt{P_c T_b} \cdot \sqrt{2/T_b}$

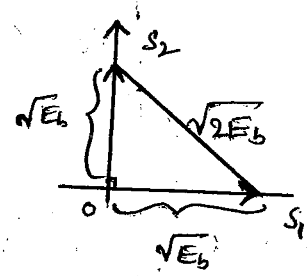
$\boxed{A_c = \sqrt{E_b} \cdot \sqrt{2/T_b}}$

Note: Basics, Consider two signals S_1 & S_2.

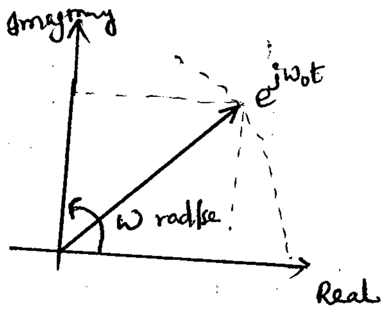
(a) Antipolar binary signal vectors



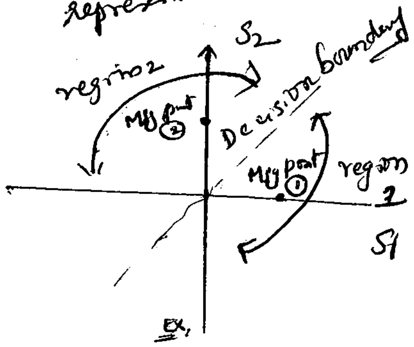
(b) Orthogonal binary signal vectors



(c) phasor representation of



(d) Signal Space representation.



Detection of ASK:

(a) Coherent detection
(b) Non coherent detection.

(a) Coherent Detection of ASK:

Coherent & Synchronous detector.

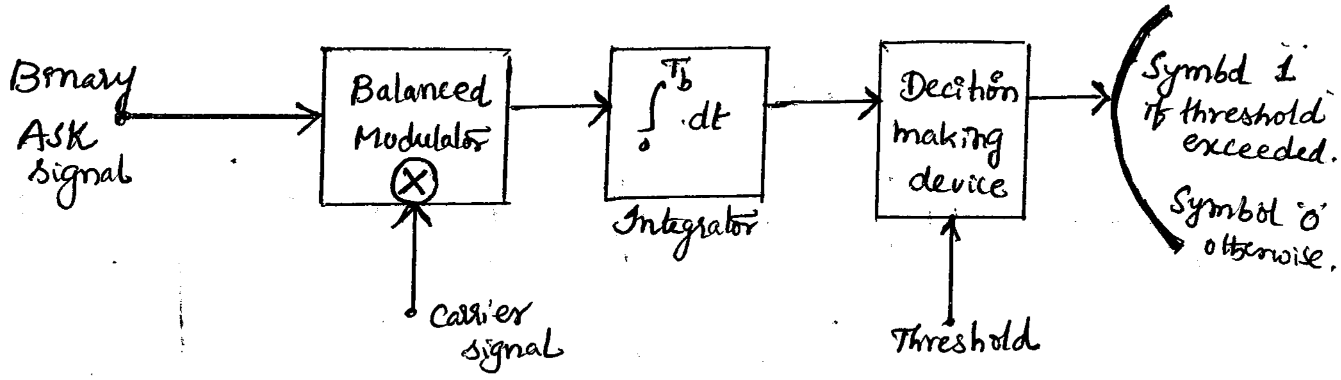


Fig: Coherent detection of ASK.

- Coherent detection consists of product modulator followed by integrator and decision device.
- The incoming ASK signal is applied to one input of the product modulator & second input is sinusoidal carrier.
- The output of product modulator goes to an integrator, the integrator operates on the output of multiplexer for successive bit intervals and essentially performs Low pass filtering.
- The output of Integrator goes to input of decision making device. The decision making device compares the output of integrator with a preset threshold level then produces symbol 1 if threshold is exceeded and symbol '0' if otherwise.
- Coherent detection involves the use of linear operation. here the local carrier is in perfect synchronization with the carriers used in the transmitter.

ie The frequency and phase of the locally generated carrier is same as those carriers used in the transmitter.

(b) Non coherent detection :

- It is also called asynchronous or Envelop detection of ASK.
- Binary ASK signal can also be demodulated non coherently using Envelop detector.

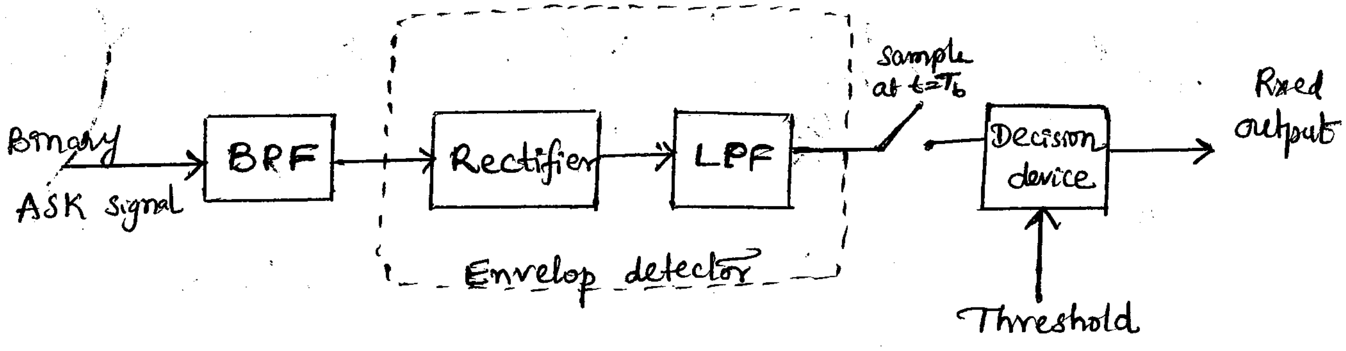
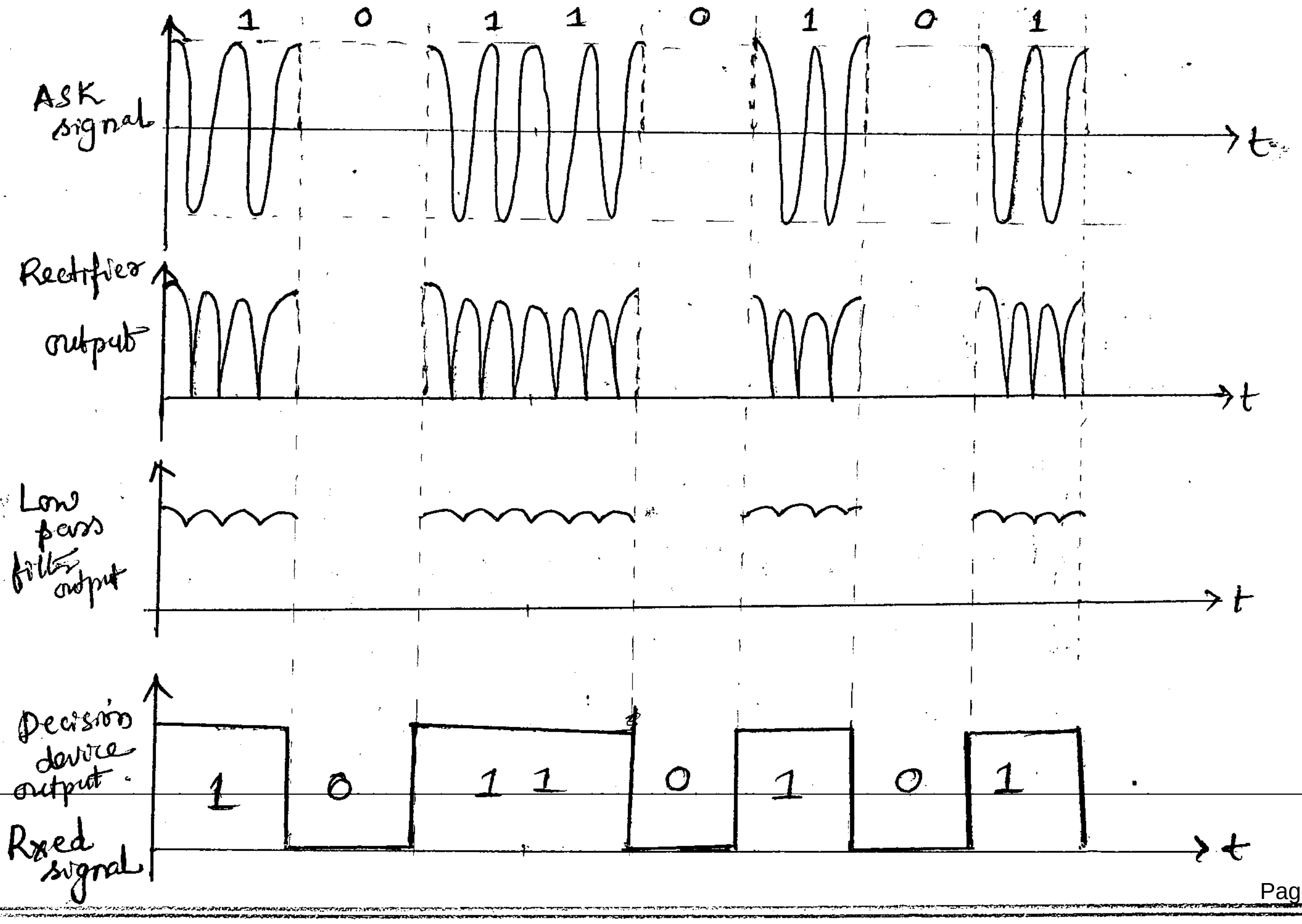


Fig: Non-coherent detection of ASK Signal.

- This greatly simplifies the design consideration needed in detector. (in synchronous detection)
- Non coherent detection schemes do not require a phase-coherent local oscillator.
- This schemes involves some form rectification and low pass filtering at the receiver.



Frequency Shift Keying (FSK):

In binary FSK system, two sinusoidal carrierwaves of the same amplitude A_c but different frequencies f_{c_1} & f_{c_2} are used to represent binary symbol '1' and '0' respectively.

The binary FSK wave $S(t)$ may be written as

$$\boxed{S(t) = \begin{cases} A_c \cdot \cos(2\pi f_{c_1} t) & ; \text{binary } 1 \\ A_c \cos(2\pi f_{c_2} t) & ; \text{binary } 0 \end{cases}}$$

(OR)

$$S(t)_{FSK} = \begin{cases} A_c \cos(\omega_c + \omega_d)t; & \text{logic } 1 \\ A_c \cos(\omega_c - \omega_d)t; & \text{logic } 0 \end{cases} \quad \text{(in voltage)}$$

$$S(t)_{(FSK)} = \begin{cases} \sqrt{2P_c} \cos(\omega_c + \omega_d)t; & \text{logic } 1 \\ \sqrt{2P_c} \cos(\omega_c - \omega_d)t; & \text{logic } 0 \end{cases} \quad \text{(in power)}$$

$P = A_c^2/2$

$A_c = \sqrt{2P_c}$.

In Energy.

$$S(t)_{FSK} = \begin{cases} \sqrt{E_b}\sqrt{2/T_b} \cos(\omega_c + \omega_d)t; & \text{logic } 1 \\ \sqrt{E_b} \cdot \sqrt{2/T_b} \cos(\omega_c - \omega_d)t; & \text{logic } 0 \end{cases}$$

$E = P.T_b$

$A_c = \sqrt{2P_c} = \sqrt{2 P_c} \cdot \frac{\sqrt{T_b}}{\sqrt{T_b}}$

$= \sqrt{P_c . T_b} \cdot \sqrt{2/T_b}$

$A_c = \sqrt{E_b} \cdot \sqrt{2/T_b}$.

where ω_d = a constant frequency offset, from a normalized carrier frequency ω_c.

Generation of FSK:

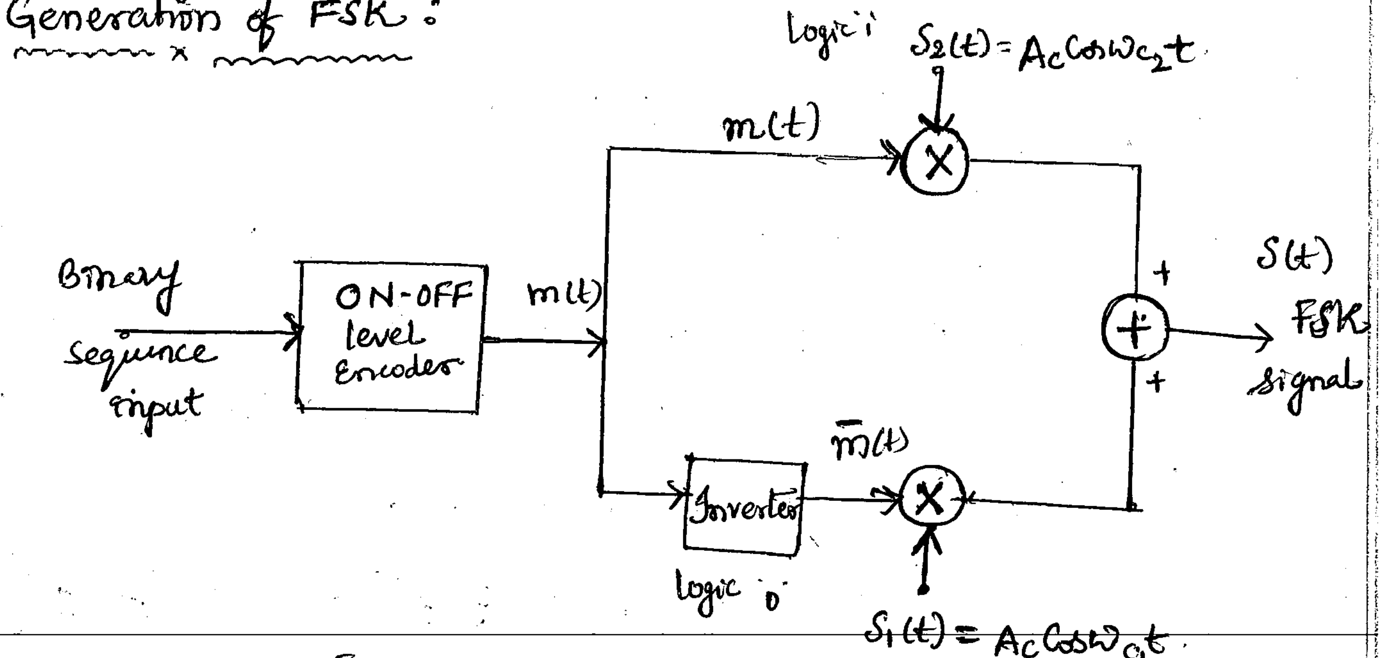
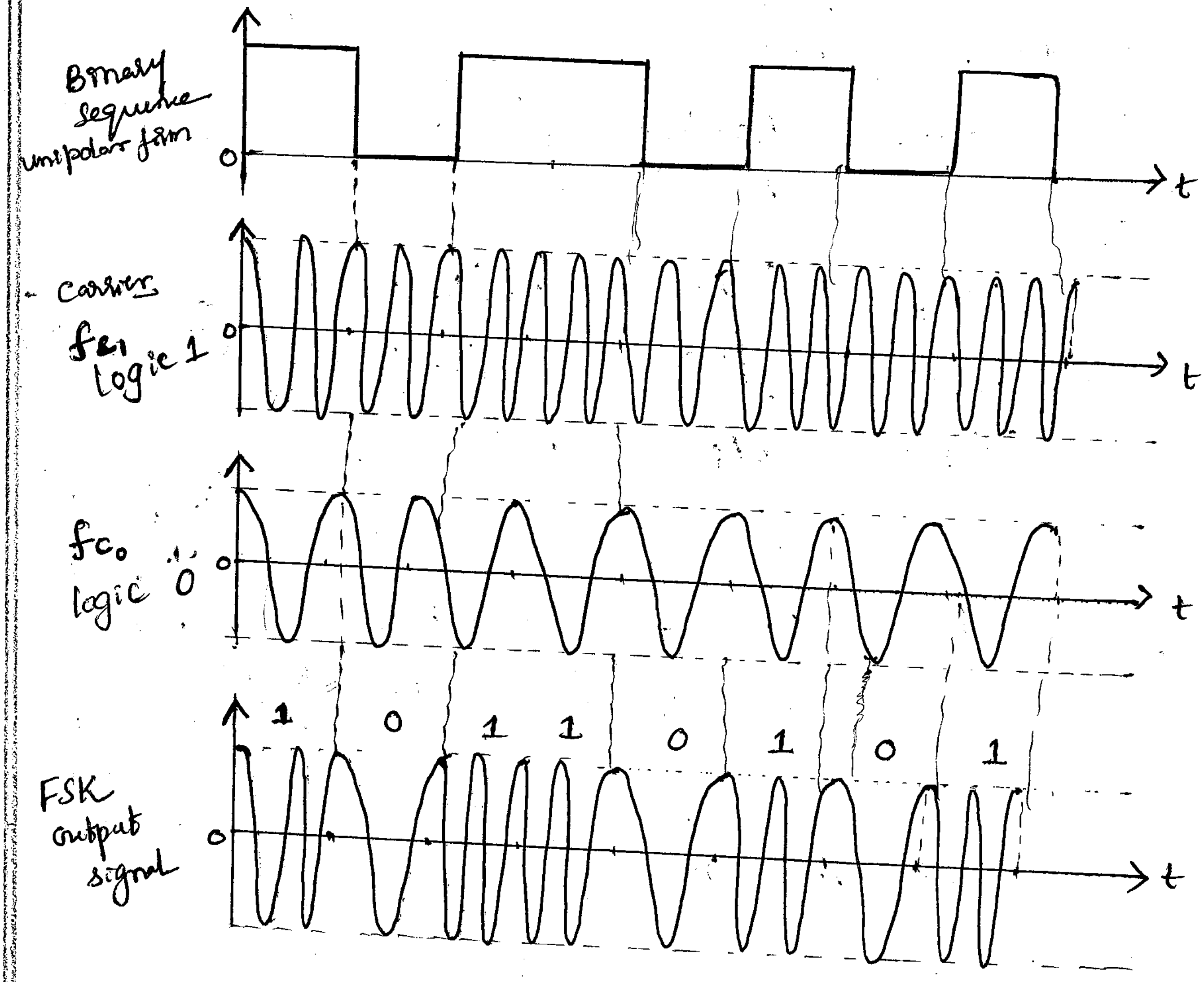


Fig: FSK Modulator.

✓ A binary FSK transmitter has an incoming binary data sequence is applied to on-off level encoder.

✓ The output of encoder is $\sqrt{E_b}$ volts for symbol 1 & 0 for symbol 0.

* Let us consider the binary sequence {1 0 1 1 0 1 0 1}.



Bandwidth and Frequency Spectrum of FSK:

✓ Data signal in frequency domain the spectrum is sinc function

$$S_1(t) = A_c \cos \omega_{c_1} t = A_c \cos[\omega_c - \omega_d] t$$

For logic 0

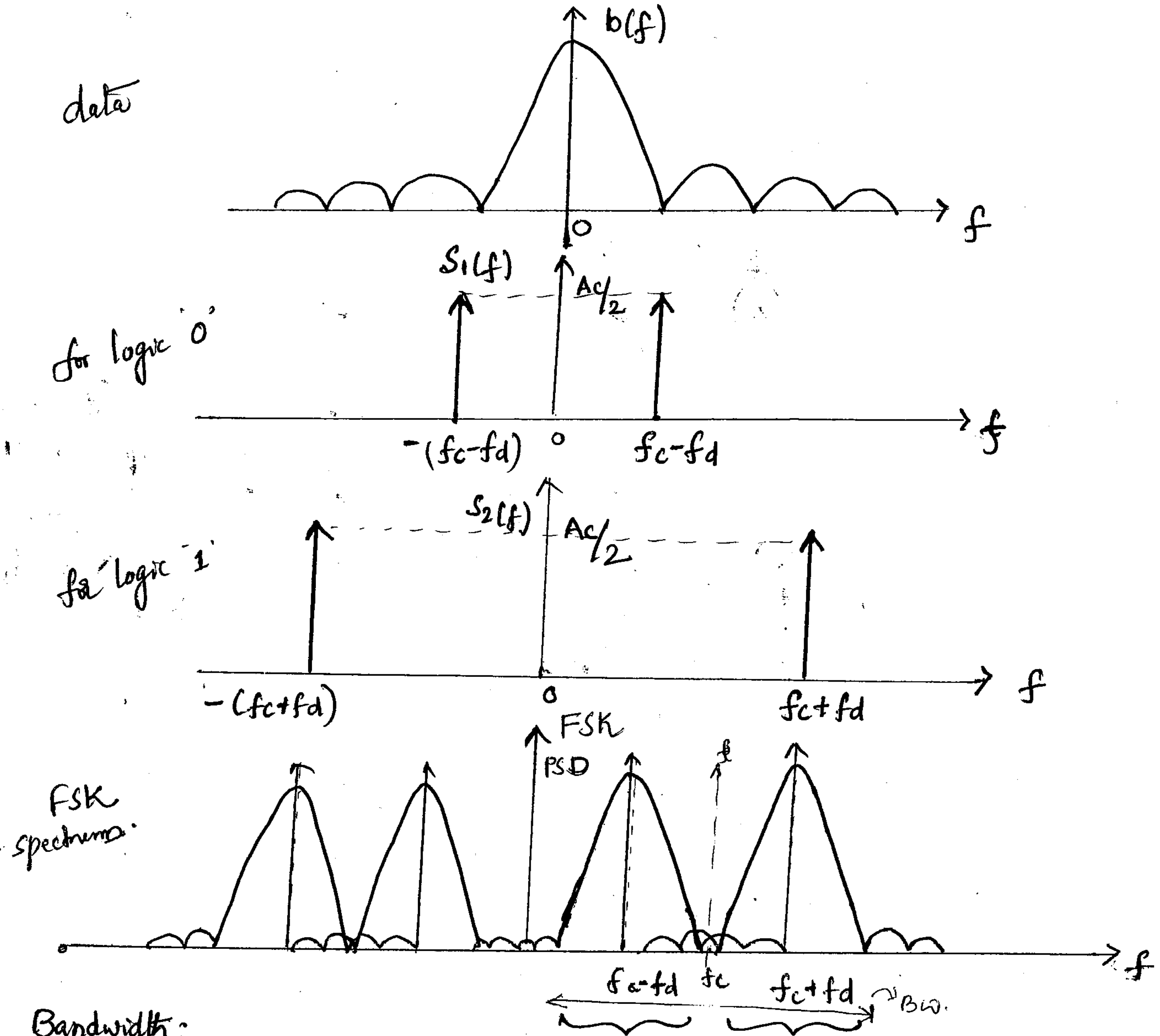
$$S_1(f) = \frac{A_c}{2}\left[\delta(f-(f_c-f_d)) + \delta[f+(f_c-f_d)]\right]$$

lly.

$$S_2(t) = A_c \cos \omega_{c_2} t = A_c \cos[\omega_c + \omega_d] t$$

For logic 1.

$$S_2(f) = \frac{A_c}{2}\left[\delta[f-(f_c+f_d)] + \delta[f+(f_c+f_d)]\right]$$



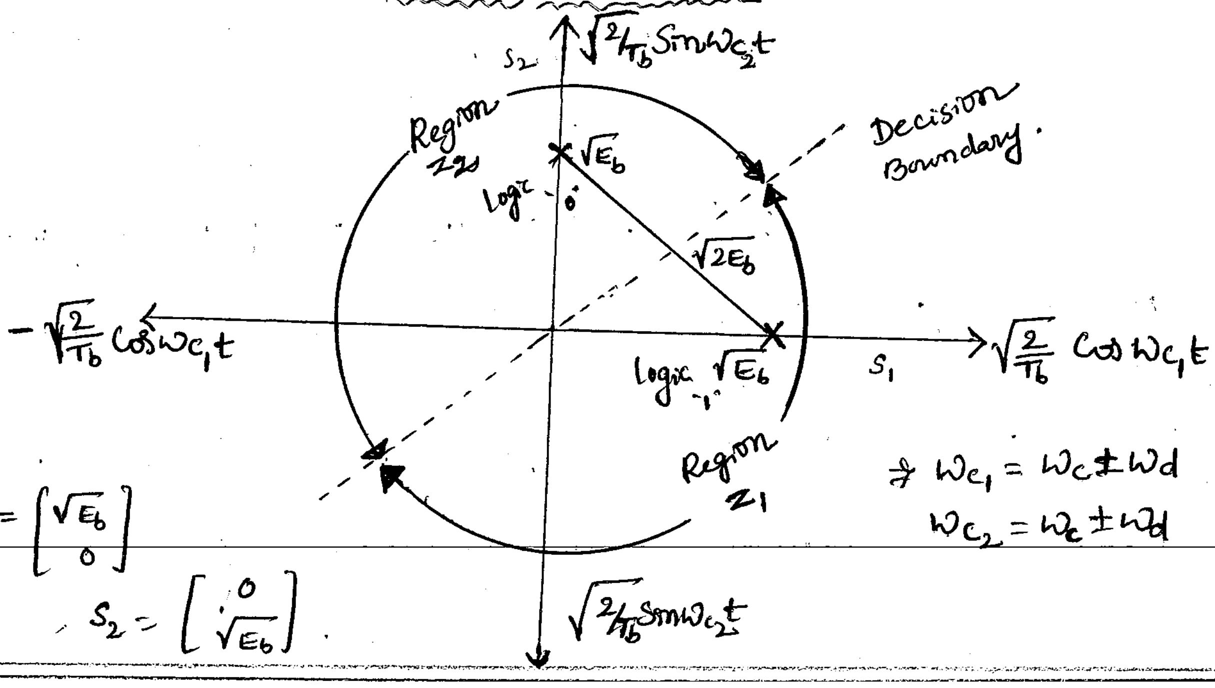
<u>Bandwidth:</u>

Bandwidth of FSK $= 2f_b + 2f_b = 4f_b$ & $4/T_b$.

$\therefore$ $\boxed{B.W = 4f_b \text{ \& } 4/T_b}$ Hz.

BW is very high compared to ASK & PSK.

<u>Signal Space & Vector Representation of FSK:</u>



ϑ $\omega_{c_1} = \omega_c \pm \omega_d$

$\omega_{c_2} = \omega_c \pm \omega_d$

$S_1 = \begin{bmatrix} \sqrt{E_b} \\ 0 \end{bmatrix}$

$S_2 = \begin{bmatrix} 0 \\ \sqrt{E_b} \end{bmatrix}$

Detection of FSK : (a) Coherent Detection.
(b) Non coherent detection.

(a) Coherent detection of FSK :

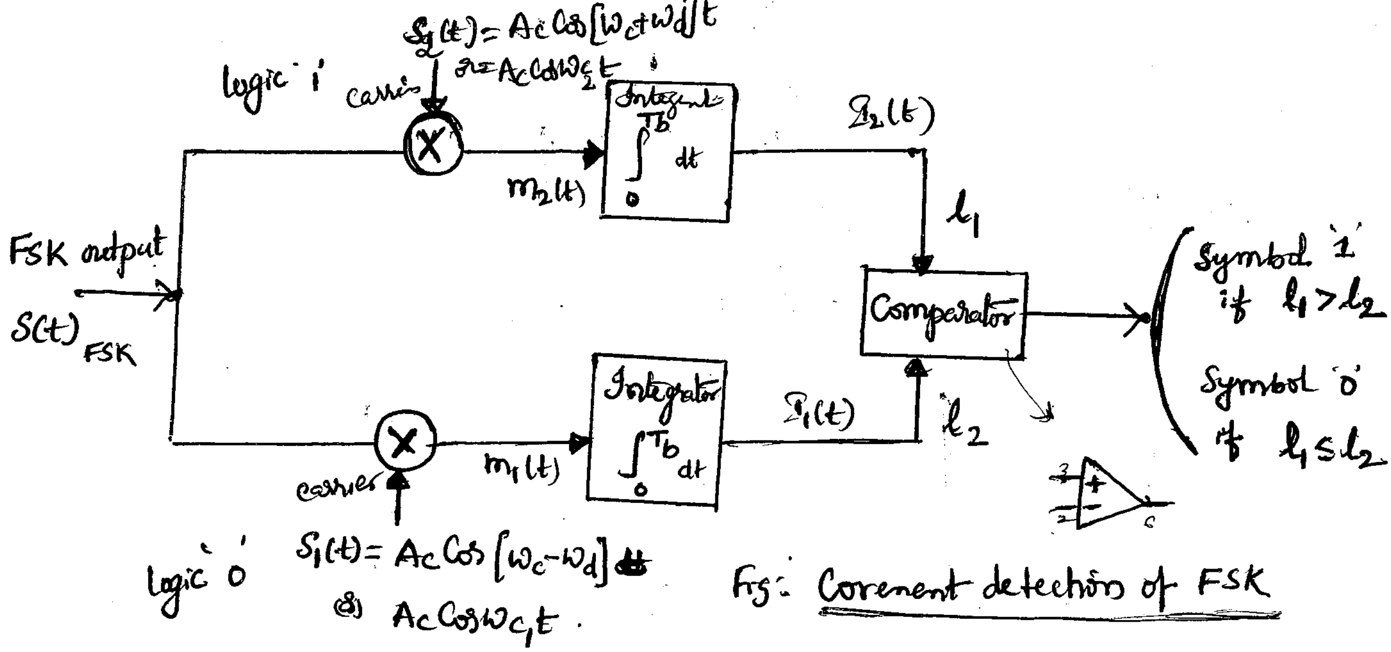


Fig: Corenent detection of FSK

- The incoming FSK signal is multiplied by a received carrier signal that has the exact same frequency and phase as the transmitter reference.

* The multiplexed output of each multiplier is subsequently passed through integrators generating outputs l_1 & l_2 in the two paths.

✓ The output of the two integrators are then fed to the decision making device.

* The decision making device is essentially a comparator which compares the output l_1 (in the upper path) & output l_2 (in the lower path)

✓ If the output $l_1 >$ output l_2, the comparator output is Symbol '1'. If the output $l_1 \le$ output l_2, the comparator output is Symbol '0'.

* This type of digital communication receivers are also called Correlation receiver.

✓ The coherent detection requires phase & timing synchronization.

* FSK coherent detection is rarely used.

(b) Non Coherent detection of FSK:

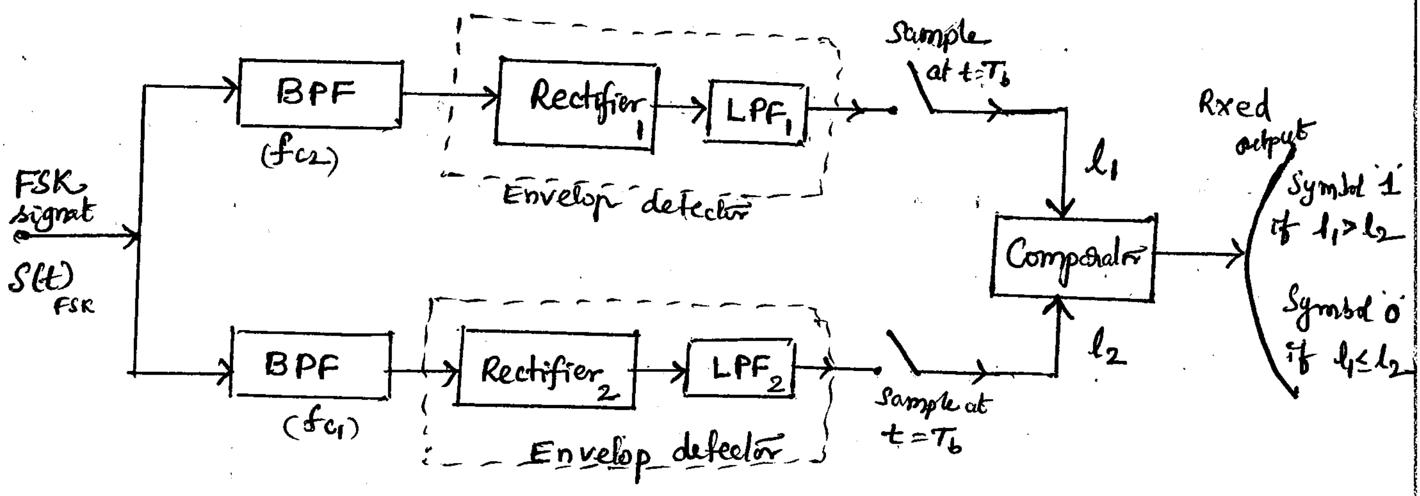
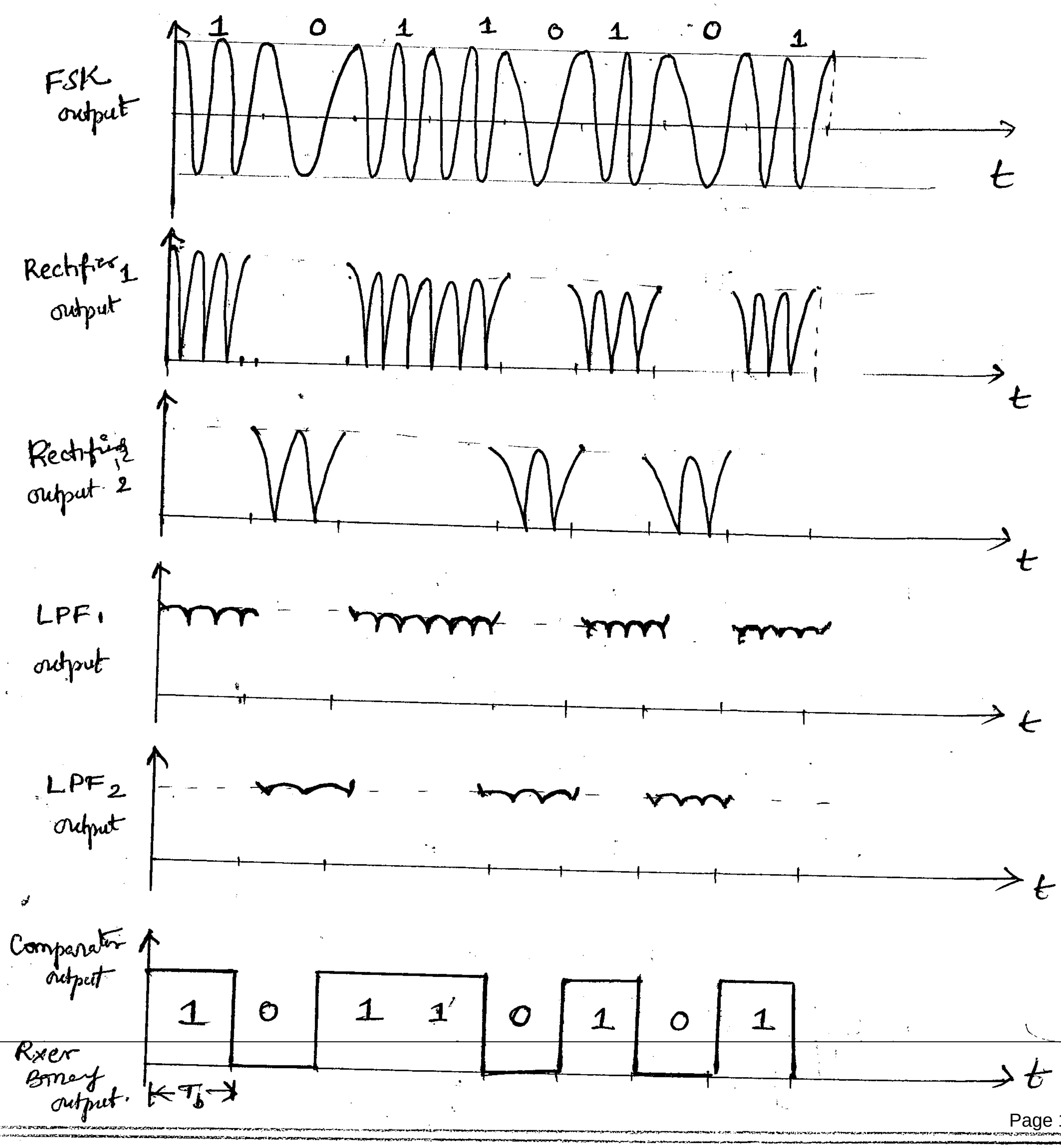


Fig. Non coherent detection of FSK.



Phase Shift Keying (PSK) :

In a binary PSK system, a sinusoidal carrier wave of fixed amplitude A_c and fixed frequency 'f_c' is used to represent both symbol 1 and '0' except that the carrier phase of each symbol differs by a phase of $180°$.

Let the unmodulated carrier as

$$c(t) = A_c \cos(2\pi f_c t).$$

$\therefore$ The binary PSK signal $s(t)$ can be written as

$$s(t) = \begin{cases} A_c \cos(2\pi f_c t) & ; \text{Symbol is binary } 1 \\ A_c \cos(2\pi f_c t + \pi) & \text{symbol is binary '0'}. \end{cases}$$

(or)

$$\boxed{s(t) = \begin{cases} A_c \cos(2\pi f_c t) & ; \text{logic } 1 \\ -A_c \cos(2\pi f_c t) & ; \text{logic '0'} \end{cases}} \quad \text{(In voltage).}$$

$$s(t) = \begin{cases} \sqrt{2P_c} \cos(2\pi f_c t) & ; \text{logic } 1 \\ -\sqrt{2P_c} \cos(2\pi f_c t) & ; \text{logic } 0 \end{cases} \quad \text{(In power).}$$

$$s(t) = \begin{cases} \sqrt{2E_b/T_b} \cos(2\pi f_c t) & ; \text{logic } 1 \\ -\sqrt{2E_b/T_b} \cos 2\pi f_c t & ; \text{logic } 0 \end{cases} \quad \text{(In Energy).}$$

Generation of PSK :

✓ PSK signal can be generated by using the same scheme as used in the generation of 'ASK'.

* The only difference is that the incoming binary data should be in the Bipolar form.

✓ A binary PSK may be also viewed as a DSB-SC (Double-Sideband Suppressed Carrier) modulated wave.

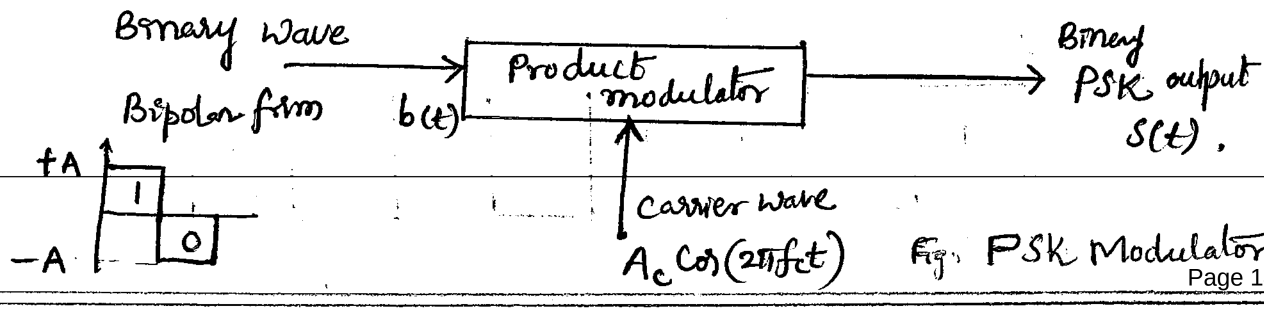
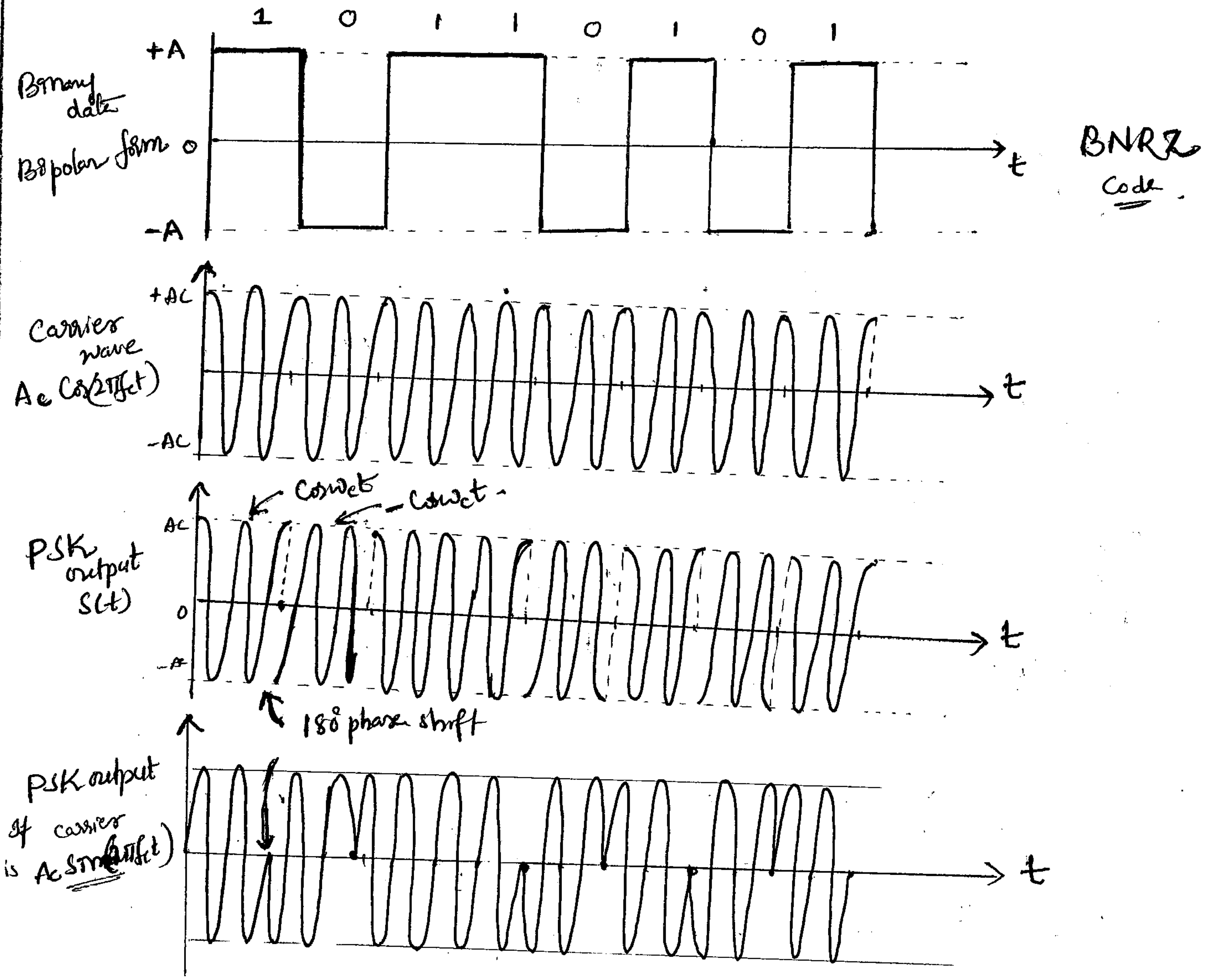
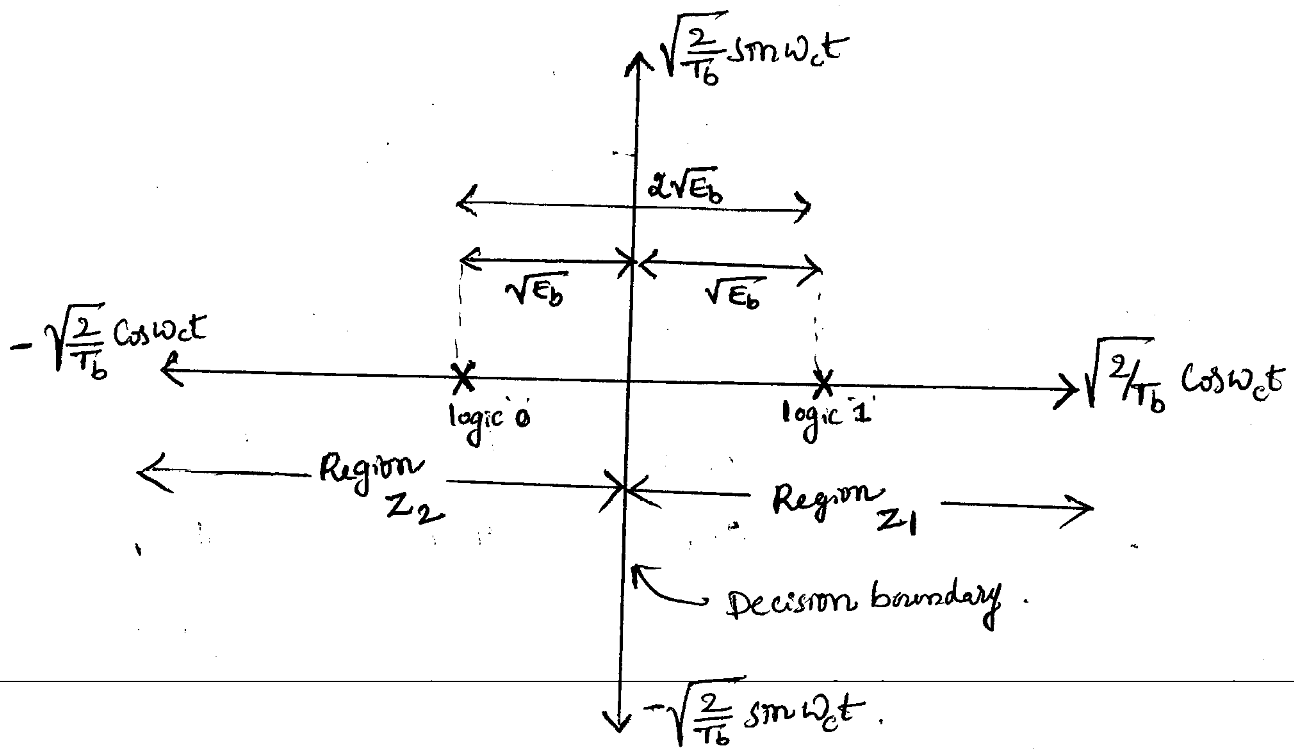


Fig. PSK Modulator.

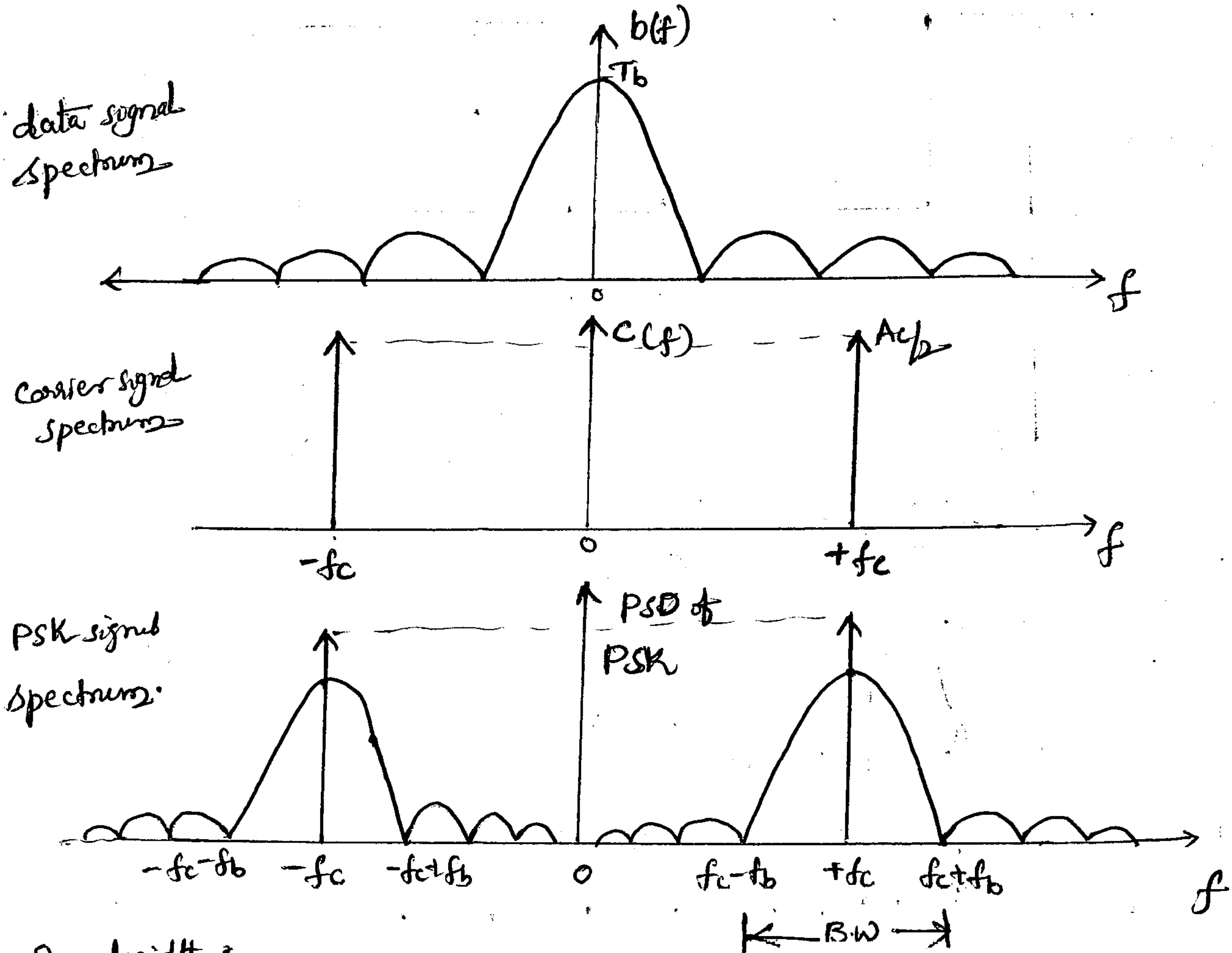
Let us consider as binary sequence $\{1\ 0\ 1\ 1\ 0\ 1\ 0\ 1\}$



Signal Space & Vector Representation of PSK :



Bandwidth & Frequency Spectrum of PSK:



Bandwidth: The bandwidth of binary PSK system is

$$B.W = f_2 - f_1$$
$$= (f_c+f_b) - (f_c-f_b)$$
$$= \cancel{f_c}+f_b-\cancel{f_c}+f_b$$
$$= 2f_b \text{ (or) } 2/T_b$$

$$\therefore \boxed{B.W \text{ of PSK} = 2f_b \text{ (or) } 2/T_b}$$

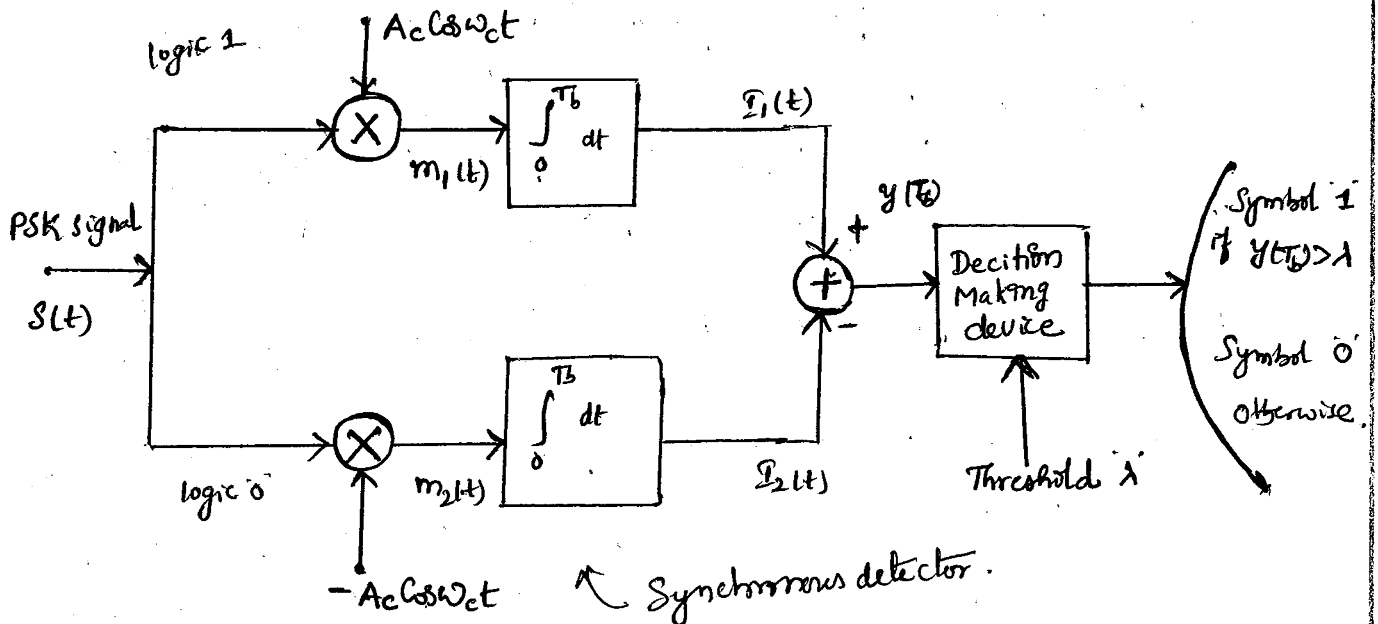
Note: Bandwidth & noise immunity for the digital modulation techniques

		ASK	FSK	PSK
Bandwidth	:	$2f_b$	$\geq 4f_b$	$2f_b$
Noise immunity	:	Less	high	Very high
~~H/w Complexity~~	:			

Detection of PSK :

(a) Coherent detection
(b) Non-Coherent detection - (DPSK).

(a) Coherent detection of PSK :



✓ Synchronous detector output gets only when phase & frequency must be matches otherwise gets zero.

Logic 1 : input as $A_c \cos\omega_c t = s(t)$

$$m_1(t) = (A_c \cos\omega_c t) \cdot (A_c \cos\omega_c t)$$

$$m_1(t) = A_c^2 \cos^2\omega_c t$$

$$\Rightarrow m_2(t) = 0$$

$$I_1(t) = \int_0^{T_b} A_c^2 \cos^2\omega_c t$$

$$I_1(t) = \frac{A_c^2}{2} \cdot T_b .$$

$$\& \quad I_2(t) = 0$$

logic 0 input as $-A_c \cos\omega_c t = s(t)$

$$\therefore m_2(t) = (-A_c \cos\omega_c t)(-A_c \cos\omega_c t) = A_c^2 \cos^2\omega_c t$$

$$m_1(t) = 0$$

$$\Rightarrow I_2(t) = \int_0^{T_b} A_c^2 \cos^2\omega_c t = \frac{A_c^2}{2} T_b .$$

$$I_1(t) = 0 .$$

$\therefore$ Threshold Value $\lambda = \frac{I_1(t) - I_2(t)}{2} = \frac{0-0}{2} = 0 . \quad \boxed{\lambda = 0}$

$\therefore$ The decision making device detected as

If $y(T_b) > 0$ it generates output as symbol '1'

If $y(T_b) \leq 0$ it detects output as symbol '0'.

Non coherent detection of PSK :

There is no non-coherent detection for PSK because there should be phase synchronization since information exists in phase for PSK.

So, we use differential phase shift keying (DPSK) for non coherent detection of PSK.

Differential Phase Shift Keying (DPSK) :

- It is a noncoherent detection of PSK.
- It eliminates the need for a coherent reference signal at the receiver by combining two basic operations at the transmitter.
  - (a) Differential encoding of the input binary wave.
  - (b) Phase-shift Keying

  Hence the name, differential phase-shift keying (DPSK).
- In effect, to send symbol '1' we leave the phase of the current signal waveform unchanged and to sent symbol '0', we phase advance the current signal waveform by 180°.
- The receiver is equipped with a storage capability, so that it can measure the relative phase difference between the waveforms received during two successive bit intervals.
- Provided that the unknown phase θ contained in the received wave varies slowly. The phase difference between waveforms received in two successive bit intervals will be independent of θ.

DPSK Transmitter :

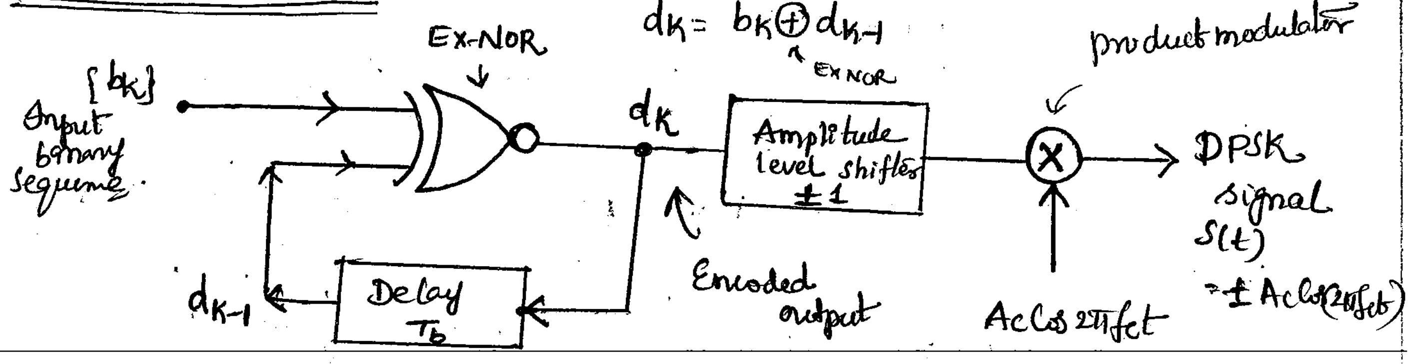


Fig: Block diagram of DPSK transmitter.

DPSK Receiver:

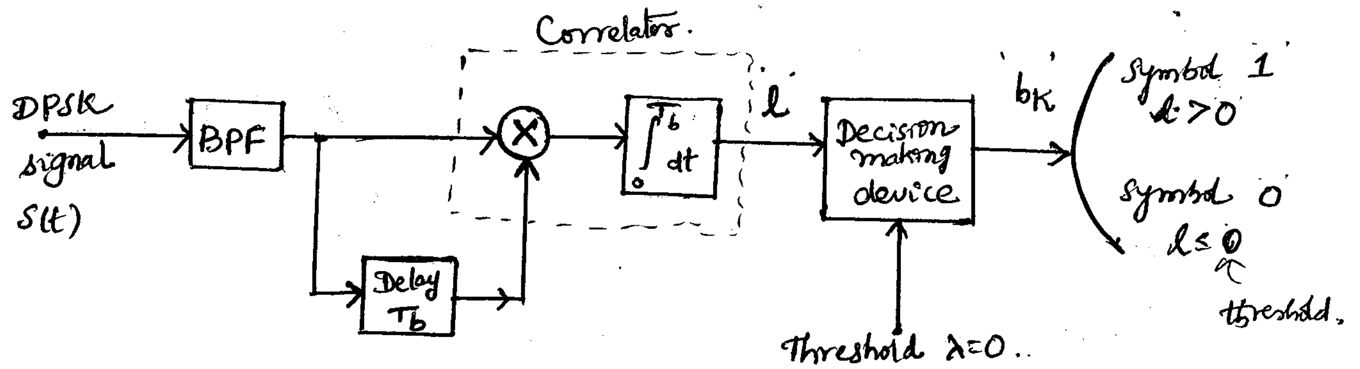


Fig: Block diagram of DPSK receiver.

✓ EX-NOR operation $X_{out} = AB \oplus A'B'$ ← Modulo-2 operation

EX-NOR output

A	B	out
0	0	1
0	1	0
1	0	0
1	1	1

* Let us consider the binary sequence $\{b_K\} = \{0\,0\,1\,0\,0\,1\,0\,0\,1\,1\}$

let an extra bit - Symbol 1 has been arbitrarily added as an initial bit.

*

(a)	Binary data $\{b_K\}$	Ex-NOR 0 0 1 0 0 1 0 0 1 1
(b)	differentially Encoded data at transmitter $\{d_K\}$	1* 0 1 1 0 1 1 0 1 1 1
(c)	Phase of DPSK 1 → no change 0 → 180° phase shift	0 π 0 0 π 0 0 π 0 0 0
(d)	Shifted differentially Encoded data at Rx $\{d_{K-1}\}$	1 0 1 1 0 1 1 0 1 1
(e)	Phase of shifted DPSK.	0 π 0 0 π 0 0 π 0 0
(f)	Phase Comparison output Between (c) & (e).	− − + − − + − − + +
(g)	Detected binary sequence at Receiver $\{b_K\}$	0 0 1 0 0 1 0 0 1 1

where 1 → 0° phaseshift
0 → 180° or π phaseshift

comparison π & 0 = 0
0 & π = 0
π & π = 1
0 & 0 = 1

− → Symbol 0
+ → Symbol 1

Let an extra bit as symbol (0) added as an initial bit.

00 → 1
01 → 0
10 → 0
11 → 1

		Ex-NOR									
(a)	Binary data {bk}		0	0	1	0	0	1	0	0	1	1
(b)	Differentially Encoded data {dk}	0*	1	0	0	1	0	0	1	0	0	0
(c)	phase of DPSK	π	0	π	π	0	π	π	0	π	π	π
(d)	Shifted differential Encoded data {dk-1}		0	1	0	0	1	0	0	1	0	0
(e)	phase of shifted DPSK.		π	0	π	π	0	π	π	0	π	π
(f)	phase Comparison output.		−	−	+	−	−	+	−	−	+	+
(g)	Detected binary data at Receiver {bk}		0	0	1	0	0	1	0	0	1	1

Thus, It is verified that the extra chosen bit 0 changes the phase of the DPSK sequence but the detected sequence remains invariant.

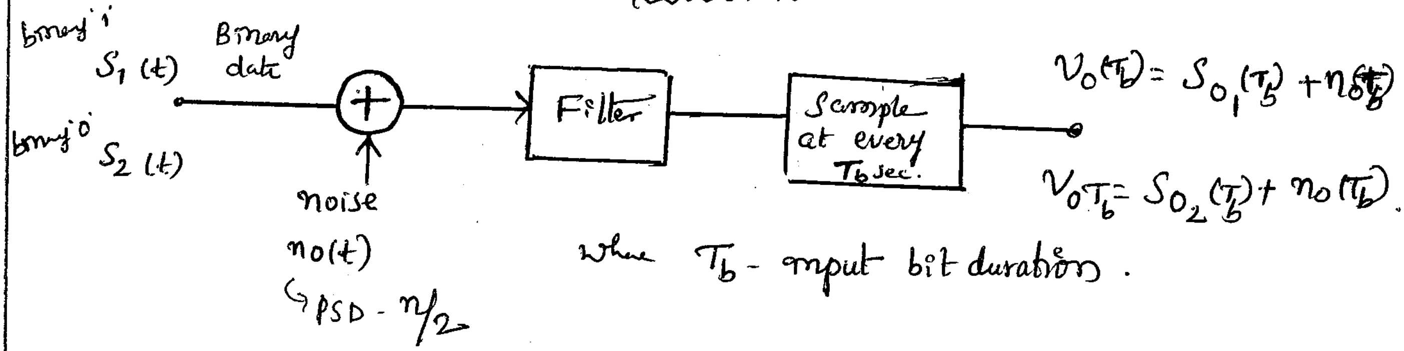
If any error is present

		Let									
(a)	Binary data {bk}		1	1	0	1	1	0	0	1	1	0
(b)	Differentially Encoded data {dk}	1*	1	1	0	1 Error	0	1	0	0	0	1
(c)	phase of DPSK	0	0	0	π	0	π	0	π	π	π	0
(d)	Shifted differential Encoded data		1	1	1	0	1	0	1	0	0	0
(e)	phase of shifted dPSK		0	0	0	π	0	π	0	π	π	π
(f)	phase comparison output		+	+	−	−	−	−	−	+	+	−
(g)	Detected binary data at Rxer.		1	1	0	$\boxed{0}$	$\boxed{0}$	0	0	1	1	0

Dibit Error

Thus. If one error is generated at transmitter, then two errors will be present at the output of receiver. This is called "Dibit Error." & If two error is transmitter then four error to Receiver.

Calculation of probability of Error (P_e):



where T_b - input bit duration.

- ✓ Error will results at $P(t) = S_1(t) - S_2(t)$.
- ✓ The probability of error $\boxed{P_e = \frac{1}{2}\,\text{erfc}\left(\frac{\gamma^2_{max}}{8}\right)^{1/2}}$ ← Complementary error function

where $\gamma_{max} = \dfrac{P_0(T_b)}{\sigma_0}$.

$$\Rightarrow \gamma^2_{max} = \frac{P_0^2(T_b)}{\sigma_0^2}.$$

(OR) $$\boxed{\gamma^2_{max} = \frac{2}{\eta}\int_0^{T_b}[P(t)^2]\,dt}$$

In terms of Q-function

probability error $\boxed{P_e = Q\left(\frac{\gamma_{max}}{2}\right)}$

Error probability of ASK:

In Amplitude shift keying $S(t) = \begin{cases} A_c \cos 2\pi f_c t & ; \text{logic } 1 \\ 0 & ; \text{logic } 0 \end{cases}$

$\therefore S_1(t) = A_c \cos 2\pi f_c t = A_c \cos \omega_c t$.

$S_2(t) = 0$

$\therefore P(t) = S_1(t) - S_2(t)$.

$= A_c \cos 2\pi f_c t - 0 \Rightarrow P(t) = A_c \cos \omega_c t$.

Probability error $P_e = \frac{1}{2}\,\text{erfc}\left(\frac{\gamma^2_{max}}{8}\right)^{1/2}$

where $\gamma^2_{max} = \dfrac{2}{\eta}\displaystyle\int_0^{T_b}[P(t)]^2\,dt$.

(or) $\dfrac{P_0^2(T_b)}{\sigma_0^2} \Rightarrow \gamma^2_{max} = P_0^2(T_b)/(\eta/2)$ → Signal power, → noise power

$$\therefore \gamma^2_{max} = \frac{2}{\eta}\int_0^{T_b}[A_c \cos\omega_c t]^2 dt$$

$$= \frac{2}{\eta}\cdot A_c^2 \cdot \int_0^{T_b} \cos^2\omega_c t\, dt$$

$$= \frac{2A_c^2}{\eta}\int_0^{T_b}\left[\frac{1+\cos 2\omega_c t}{2}\right]dt$$

$$= \frac{A_c^2}{\eta}\left[\int_0^{T_b} 1\, dt + \int_0^{T_b}\cos 2\omega_c t\, dt\right]$$

$$= \frac{A_c^2}{\eta}\left[t\Big|_0^{T_b} + \underbrace{\frac{\sin 2\omega_c t}{2\omega_c}\Big|_0^{T_b}}_{\Downarrow 0}\right] \qquad \omega_c = 2\pi f_c$$

$$= \frac{A_c^2}{\eta}\cdot T_b$$

$$\therefore \boxed{\gamma^2_{max} = \frac{A_c^2 T_b}{\eta}}$$

$$\therefore P_e = \frac{1}{2}\operatorname{erfc}\left(\frac{1}{8}\cdot\frac{A_c^2 T_b}{\eta}\right)^{1/2}$$

$$= \frac{1}{2}\operatorname{erfc}\left[\frac{1}{4\eta}\cdot\left(\frac{A_c^2 T_b}{2}\right)\right]^{1/2}$$

$$= \frac{1}{2}\operatorname{erfc}\sqrt{\frac{E_b}{4\eta}} \qquad \text{where } E_b = P_c T_b = \frac{A_c^2 T_b}{2} \quad \text{where } P_c = \frac{A_c^2}{2}$$

$$\therefore \boxed{P_e = \frac{1}{2}\operatorname{erfc}\sqrt{\frac{E_b}{4\eta}}} \quad \text{For ASK}$$

In terms of Q-functions

$$P_e = Q\left(\frac{\gamma_{max}}{2}\right) = \int_{\gamma_{max}/2}^{\infty}\frac{1}{\sqrt{2\pi}}e^{-x^2/2}\,dx \qquad \left(\because Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-x^2/2}dx\right)$$

$$\gamma^2_{max} = \frac{A_c^2 T_b}{\eta} \Rightarrow \gamma_{max} = \sqrt{\frac{A_c^2 T_b}{\eta}}$$

$$\frac{\gamma_{max}}{2} = \sqrt{\frac{A_c^2 T_b}{4\eta}}$$

$$\therefore P_e = Q\left(\frac{\gamma_{max}}{2}\right) = Q\left(\sqrt{\frac{A_c^2 T_b}{4\eta}}\right).$$

Thus. Error probability for ASK system

$$\boxed{P_e = \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{E_b}{4\eta}}\right) \quad \text{(or)} \quad P_e = Q\left(\sqrt{\frac{A_c^2 T_b}{4\eta}}\right)}$$

$$\Rightarrow Q\left(\sqrt{\frac{E_b}{2\eta}}\right)$$

Error probability in PSK :

In Phase Shift Keying $S(t) = \begin{cases} A_c \cos\omega_c t & ; \text{logic } 1 \\ -A_c \cos\omega_c t & ; \text{logic } 0 \end{cases}$

$S_1(t) = A_c \cos\omega_c t$

$S_2(t) = -A_c \cos\omega_c t \Rightarrow P(t) = S_1(t) - S_2(t)$

$$= A_c\cos\omega_c t + A_c \cos\omega_c t$$

$$\therefore P(t) = 2A_c \cos\omega_c t.$$

Probability error $P_e = \frac{1}{2}\,\text{erfc}\left(\frac{\gamma^2_{max}}{8}\right)^{1/2}$

where

$$\gamma^2_{max} = \frac{2}{\eta}\int_0^{T_b} [P(t)]^2 dt$$

$$= \frac{2}{\eta}\int_0^{T_b}[2A_c\cos\omega_c t]^2 dt$$

$$= \frac{4A_c^2 \cdot 2}{\eta}\int_0^{T_b}\cos^2\omega_c t\, dt$$

$$= \frac{4A_c^2}{\eta} \times \cancel{2}\int_0^{T_b}\left[\frac{1+\cos 2\omega_c t}{\cancel{2}}\right] dt$$

$$= \frac{4A_c^2}{\eta}\cdot\left[(T_b) + \underbrace{\frac{\sin 2\omega_c t}{2\omega_c}\Big|_0^{T_b}}_{0}\right]$$

$$= \frac{4A_c^2 T_b}{\eta} \quad \therefore \boxed{\gamma^2_{max} = \frac{4A_c^2 T_b}{\eta}}$$

Probability error

$$P_e = \frac{1}{2}\,\text{erfc}\left(\frac{1}{\cancel{8}_2}\cdot\frac{\cancel{4}\cdot A_c^2 \cdot T_b}{\eta}\right)^{1/2}$$

$$= \frac{1}{2}\,\text{erfc}\left[\frac{A_c^2}{2}\cdot T_b \cdot \frac{1}{\eta}\right)^{1/2}$$

$$= \frac{1}{2}\,\text{erfc}\left[\frac{E_b}{\eta}\right]^{1/2} \quad \text{when } E_s = P_c \cdot T_b = \frac{A_c^2 T_b}{2}$$

$$P_c = A_c^2/2$$

$$\therefore P_e = \frac{1}{2}\,\text{erfc}\sqrt{(E_b/\eta)}. \qquad \boxed{P_e = \frac{1}{2}\,\text{erfc}\sqrt{\frac{E_b}{\eta}}} \text{ For PSK}$$

In terms of Q-function

probability error $P_e = Q\left(\frac{\gamma_{max}}{2}\right)$. $\quad \gamma^2_{max} = \frac{4A_c^2 T_b}{\eta}$

$$\gamma_{max} = \sqrt{\frac{4A_c^2 T_b}{\eta}} = 2\sqrt{\frac{A_c^2 T_b}{\eta}}$$

$$\therefore \frac{\gamma_{max}}{2} = \sqrt{\frac{A_c^2 T_b}{\eta}}.$$

Probability Error $Q\left(\frac{\gamma_{max}}{2}\right) = Q\left(\sqrt{\frac{A_c^2 T_b}{\eta}}\right)$.

Thus, the probability error in PSK system as

$$\boxed{P_e = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{E_b}{\eta}}\right) \quad \text{or} \quad P_e = Q\left(\sqrt{\frac{A_c^2 T_b}{\eta}}\right) \;\text{ⅲ}\; Q\left(\sqrt{\frac{2E_b}{\eta}}\right)}$$

Error probability in FSK:

In Frequency Shift Keying $S(t) = \begin{cases} A_c \cos(\omega_c + \Omega)t \\ A_c \cos(\omega_c - \Omega)t \end{cases}$ ← constant offset frequency.

$$\therefore S_1(t) = A_c \cos(\omega_c + \Omega)t$$
$$S_2(t) = A_c \cos(\omega_c - \Omega)t \qquad P(t) = S_1(t) - S_2(t)$$
$$\therefore P(t) = A_c\left[\cos(\omega_c + \Omega)t - \cos(\omega_c - \Omega)t\right]$$

Probability error $P_e = \frac{1}{2}\,\mathrm{erfc}\left(\frac{\gamma_{max}^2}{8}\right)^{1/2}$

Where $\gamma_{max}^2 = \frac{2}{\eta}\int_0^{T_b} [p(t)]^2\,dt$

$$\therefore \gamma_{max}^2 = \frac{2}{\eta}\int_0^{T_b} A_c^2\left[\cos(\omega_c+\Omega)t - \cos(\omega_c-\Omega)t\right]^2 dt$$

$$= \frac{2}{\eta}\int_0^{T_b} A_c^2\left\{\cos^2(\omega_c+\Omega)t + \cos^2(\omega_c-\Omega)t - 2\cos(\omega_c+\Omega)t\cdot\cos(\omega_c-\Omega)t\right\}dt$$

$$= \frac{2A_c^2}{\eta}\left\{\int_0^{T_b}\cos^2(\omega_c+\Omega)t\,dt + \int_0^{T_b}\cos^2(\omega_c-\Omega)t\,dt - 2\int_0^{T_b}\cos(\omega_c+\Omega)t\cdot\cos(\omega_c-\Omega)\,dt\right\}$$

$$= \frac{2A_c^2}{\eta}\left[\int_0^{T_b}\left(\frac{1+\cos 2(\omega_c+\Omega)t}{2}\,dt\right) + \int_0^{T_b}\left[\frac{1+\cos 2(\omega_c-\Omega)t}{2}\right]dt\right.$$
$$\left. - \int_0^{T_b}\left[\cos\left[(\omega_c+\cancel{\Omega}+\omega_c-\cancel{\Omega})t\right] + \cos\left[\cancel{\omega_c}+\Omega-\cancel{\omega_c}+\Omega)t\right]\right]dt\right.$$

$\because$ Since $\cos^2\theta = \frac{1+\cos 2\theta}{2}$

$2\cos A \cos B = \cos(A+B) + \cos(A-B)$.

$$\therefore \gamma^2_{max} = \frac{2A_c^2}{\eta}\left\{ \int_0^{T_b} \frac{1}{2}\,dt + \int_0^{T_b} \frac{\cos 2(\omega_c+\Omega)t}{2}\,dt + \int_0^{T_b} \frac{1}{2}\,dt + \int_0^{T_b} \frac{\cos 2(\omega_c-\Omega)t}{2} - \int_0^{T_b} \cos(2\omega_c t)\,dt - \int_0^{T_b} \cos(2\Omega)t\,dt \right\}$$

$$= \frac{2A_c^2}{\eta}\left[\frac{T_b}{2} + 0 + \frac{T_b}{2} + 0 - 0 - \frac{\sin 2\Omega t}{2\Omega}\Big|_0^{T_b} \right]$$

$$= \frac{2A_c^2}{\eta}\left[T_b - \frac{\sin(2\Omega T_b)}{2\Omega} \right]$$

$$\gamma^2_{max} = \frac{2A_c^2 T_b}{\eta}\left[1 - \frac{\sin(2\Omega T_b)}{2\Omega T_b} \right]$$

The quantity γ^2_{max} attains its largest value 'Ω' is selected such that $2\Omega T_b = 3\pi/2$

$$\gamma^2_{max} = \frac{2A_c^2 T_b}{\eta}\left[1 - \frac{\sin(3\pi/2)}{3\pi/2} \right] \qquad \left[\sin 3\pi/2 = -1 \right]$$

$$= \frac{2A_c^2 T_b}{\eta}\left[1 + \frac{2}{3\pi} \right]$$

$$= 2\left(\frac{3\pi+2}{3\pi} \right) \cdot \frac{A_c^2 T_b}{\eta}$$

$$= 2.42 \cdot \frac{A_c^2 \cdot T_b}{\eta} \qquad \therefore \boxed{\gamma^2_{max} = 2.42 \cdot \frac{A_c^2 T_b}{\eta}}$$

Probability Error $P_e = \frac{1}{2}\,\mathrm{erfc}\left(\frac{\gamma^2_{max}}{8} \right)^{1/2}$

$$= \frac{1}{2}\,\mathrm{erfc}\left[\frac{1}{8} \cdot 2.42 \cdot \frac{A_c^2 T_b}{\eta} \right]^{1/2}$$

$$= \frac{1}{2}\,\mathrm{erfc}\left[\frac{1}{4} \cdot \frac{(2.42)}{\eta} \cdot \frac{A_c^2 T_b}{2} \right]^{1/2}$$

$$= \frac{1}{2}\,\mathrm{erfc}\left[\frac{0.6}{\eta} \cdot E_b \right]^{1/2} \qquad \left[\because E_b = \frac{A_c^2 T_b}{2} = P_c T_b \right]$$

$$= \frac{1}{2}\,\mathrm{erfc}\sqrt{\frac{0.6 E_b}{\eta}} \qquad \therefore \boxed{P_e = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{0.6 E_b}{\eta}} \right)} \text{ for FSK}$$

Interms of Q-function Probability error $P_e = Q\left(\frac{\gamma_{max}}{2}\right)$.

$$\therefore \gamma^2_{max} = 2.42 \cdot \frac{A_c^2 \cdot T_b}{\eta}$$

$$\gamma_{max} = \sqrt{2.42 \cdot \frac{A_c^2 \cdot T_b}{\eta}}$$

$$\frac{\gamma_{max}}{2} = \sqrt{\frac{2.42}{2} \cdot \frac{A_c^2 \cdot T_b}{2} \cdot \frac{1}{\eta}}$$

$$= \sqrt{1.21 \cdot \frac{E_b}{\eta}}$$

$\therefore$ Probability error $P_e = Q\left(\sqrt{\frac{1.2\,E_b}{\eta}}\right)$.

Thus the probability of error in FSK systems is

$$\boxed{P_e = \frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{(0.6)E_b}{\eta}}\right) \quad (\text{or}) \quad P_e = Q\left(\sqrt{\frac{(1.2)\cdot E_b}{\eta}}\right)}$$

***

Error probability in QPSK:

In Quadrature Phase shift keying.

$$S(t) = \begin{cases} A_c \cos\omega_c t & ; \text{ logic } 10 \\ -A_c \cos\omega_c t & ; \text{ logic } 00 \\ A_c \sin\omega_c t & ; \text{ logic } 11 \\ -A_c \sin\omega_c t & ; \text{ logic } 01 \end{cases}$$

→ $S_1(t) = A_c \cos\omega_c t$
$S_2(t) = -A_c \cos\omega_c t$ $\quad \therefore P_1(t) = S_1(t) - S_2(t) = 2A_c \cos\omega_c t$.

→ $S_3(t) = A_c \sin\omega_c t$
$S_4(t) = -A_c \sin\omega_c t$ $\quad \therefore P_2(t) = S_3(t) - S_4(t) = 2A_c \sin\omega_c t$.

$\therefore$ Probability Error $P_e = P_{e_1} + P_{e_2}$

$$P_{e_1} = \frac{1}{2}\,\text{erfc}\left(\frac{\gamma_1^2{}_{max}}{8}\right)^{1/2}, \quad P_{e_2} = \frac{1}{2}\,\text{erfc}\left(\frac{\gamma_2^2{}_{max}}{8}\right)^{1/2}.$$

✓ $\gamma_1^2{}_{max} = \frac{2}{\eta}\int_0^{T_b/2} [P_1(t)]^2 dt = \frac{2}{\eta}\int_0^{T_b/2} [A_c^2 \cos\omega_c t]^2 dt$.

✓ $\gamma_2^2{}_{max} = \frac{2}{\eta}\int_0^{T_b/2} [P_2(t)]^2 dt = \frac{2}{\eta}\int_0^{T_b/2} [2A_c \sin\omega_c t]^2 dt$

$$\therefore \gamma_{1\,max}^2 = \frac{2}{\eta}\int_0^{T_b/2}\left[2A_c\cdot\cos w_c t\right]^2 dt$$

$$= \frac{8A_c^2}{\eta}\int_0^{T_b/2}\cos^2 w_c t\,dt$$

$$= \frac{8A_c^2}{\eta}\int_0^{T_b/2}\left(\frac{1+\cos 2w_c t}{2}\right)dt$$

$$= \frac{4A_c^2}{\eta}\left[\frac{T_b}{2} + \underbrace{\left.\frac{\sin 2w_c t}{2w_c}\right|_0^{T_b/2}}_{0}\right]$$

$$= \frac{4A_c^2}{\eta}\cdot\frac{T_b}{2}$$

$$= \frac{2A_c^2 T_b}{\eta} \qquad \therefore \boxed{\gamma_{1\,max}^2 = \frac{2A_c^2 T_b}{\eta}}$$

Probability Error $\quad Pe_1 = \frac{1}{2}\,\text{erfc}\left[\frac{\gamma^2 max}{8}\right]^{1/2}$

$$= \frac{1}{2}\,\text{erfc}\left[\frac{2A_c^2\cdot T_b}{8\cdot\eta}\right]^{1/2}$$

$$= \frac{1}{2}\,\text{erfc}\left[\frac{A_c^2\cdot T_b}{2}\cdot\frac{1}{2\eta}\right]^{1/2}$$

$$= \frac{1}{2}\,\text{erfc}\left[\frac{E_b}{2\eta}\right]^{1/2} \qquad \left(\because E_b = P_c\cdot T_b = \frac{A_c^2}{2}\cdot T_b.\right)$$

$$Pe_1 = \frac{1}{2}\,\text{erfc}\sqrt{\frac{E_b}{2\eta}}$$

Illy

$$\gamma_{2\,max}^2 = \frac{2}{\eta}\int_0^{T_b/2}\left[2A_c\sin w_c t\right]^2 dt$$

$$= \frac{8A_c^2}{\eta}\int_0^{T_b/2}\sin^2 w_c t\,dt$$

$$= \frac{8A_c^2}{\eta}\int_0^{T_b/2}\left[\frac{1-\cos 2w_c t}{2}\right]dt$$

$$= \frac{4A_c^2}{\eta}\left[\frac{T_b}{2} - \underbrace{\left.\frac{\sin 2w_c t}{2w_c}\right|_0^{T_b/2}}_{0}\right]$$

$$= \frac{4A_c^2}{\eta}\cdot\frac{T_b}{2}$$

$$= \frac{2A_c^2 T_b}{\eta} \qquad \therefore \boxed{\gamma_{2\,max}^2 = \frac{2A_c^2 T_b}{\eta}}$$

Similarly Probability error $\boxed{P_{e_2} = \frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{E_b}{2\eta}}\right)}$

$\therefore$ Total probability $P_e = P_{e_1} + P_{e_2}$

$$= \frac{1}{2}\,\text{erfc}\left(\sqrt{E_b/2\eta}\right) + \frac{1}{2}\,\text{erfc}\left(\sqrt{E_b/2\eta}\right)$$

$$= \text{erfc}.\sqrt{E_b/2\eta}$$

$\therefore$ $\boxed{P_e = \text{erfc}\left(\sqrt{\frac{E_b}{2\eta}}\right)}$ For QPSK.

In terms of Q-function. Probability of error $P_e = Q\left(\frac{\gamma_{1max}}{2}\right) + Q\left(\frac{\gamma_{2max}}{2}\right)$

ie $P_e = P_{e_1} + P_{e_2}$.

$\therefore$ $\gamma_{1max}^2 = \frac{2A_c^2 \cdot T_b}{\eta}$

$$\gamma_{1}\,max = \sqrt{\frac{2A_c^2 \cdot T_b}{\eta}}$$

$$\Rightarrow \frac{\gamma_1\,max}{2} = \sqrt{\frac{\cancel{2}\,A_c^2 \cdot T_b}{\cancel{2}4\eta}} = \sqrt{\frac{A_c^2 \cdot T_b}{2} \cdot \frac{1}{\eta}} = \sqrt{E_b/\eta}$$

$\underline{\text{lly}}$ $\frac{\gamma_2\,max}{2} = \sqrt{E_b/\eta}$

$\therefore$ Probability of error $P_{e_1} = Q\left(\frac{\gamma_{1max}}{2}\right) = Q\left(\sqrt{\frac{E_b}{\eta}}\right)$

$$P_{e_1} = Q\left(\frac{\gamma_{2max}}{2}\right) = Q\left(\sqrt{\frac{E_b}{\eta}}\right).$$

$\therefore$ Total probability of error

$$P_e = P_{e_1} + P_{e_2}$$

$$= Q\left(\sqrt{\frac{E_b}{\eta}}\right) + Q\left(\sqrt{\frac{E_b}{\eta}}\right).$$

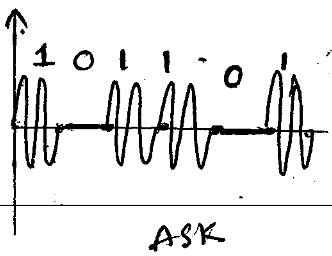
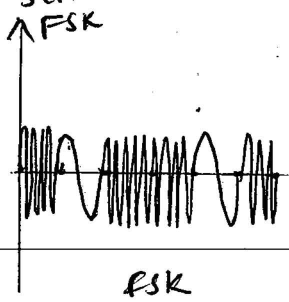
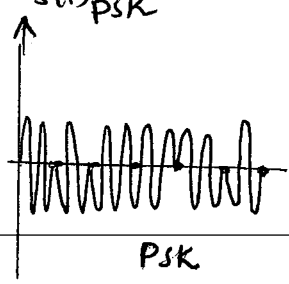
$$= 2Q\left(\sqrt{\frac{E_b}{\eta}}\right)$$

$\therefore$ $\boxed{P_e = 2Q\left(\sqrt{\frac{E_b}{\eta}}\right)}$.

Thus, the probability of error in $\underline{\text{QPSK}}$ system is

$$\boxed{P_e = \text{erfc}\left(\sqrt{\frac{E_b}{2\eta}}\right) \quad (\text{or}) \quad P_e = 2Q\left(\sqrt{E_b/\eta}\right).}$$

Comparison of ASK, FSK & PSK systems:

Parameter	ASK	FSK	PSK
1. Expansion	Amplitude shift Keying & OOK ON-OFF Keying	Frequency shift Keying	Phase shift Keying.
2. Definition 1 - Symbol '1' 0 - Symbol '0'	$S(t) = \begin{cases} A_c \cos\omega_c t \;;\; 1 \\ 0 \;;\; 0 \end{cases}$	$S(t) = \begin{cases} A_c \cos\omega_1 t \;;\; 1 \\ A_c \cos\omega_2 t \;;\; 0 \end{cases}$	$S(t) = \begin{cases} A_c \cos\omega_c t \;;\; 1 \\ -A_c \cos\omega_c t \;;\; 0 \end{cases}$
3 Bandwidth (Hz)	$2f_b$ & $2/T_b$ T_b - Bit duration.	$\geq 4f_b$ & $\geq 4/T_b$	$2f_b$ & $2/T_b$.
4. Distance between '0' & '1'	$\sqrt{E_b}$	$\sqrt{2E_b}$	$2\sqrt{E_b}$.
5 Power	Moderate	Less	More.
6. Noise immunity	less	high	very high
7. Probability error Pe	$\frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{E_b}{4\eta}}\right)$ (or) $Q\left(\sqrt{\frac{E_b}{2\eta}}\right)$.	$\frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{0.6\,E_b}{\eta}}\right)$ (or) $Q\left(\sqrt{\frac{(1.2)E_b}{\eta}}\right)$.	$\frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{E_b}{\eta}}\right)$ (or) $Q\left(\sqrt{\frac{2E_b}{\eta}}\right)$
8. Hardware Complexity ✓ Coherent Non coherent	High Low	High Low	Very High Low.
9 let data 101101 Representation of Digital modulation techniques	S(t) ASK)  ASK	S(t) FSK  FSK	S(t) PSK  PSK

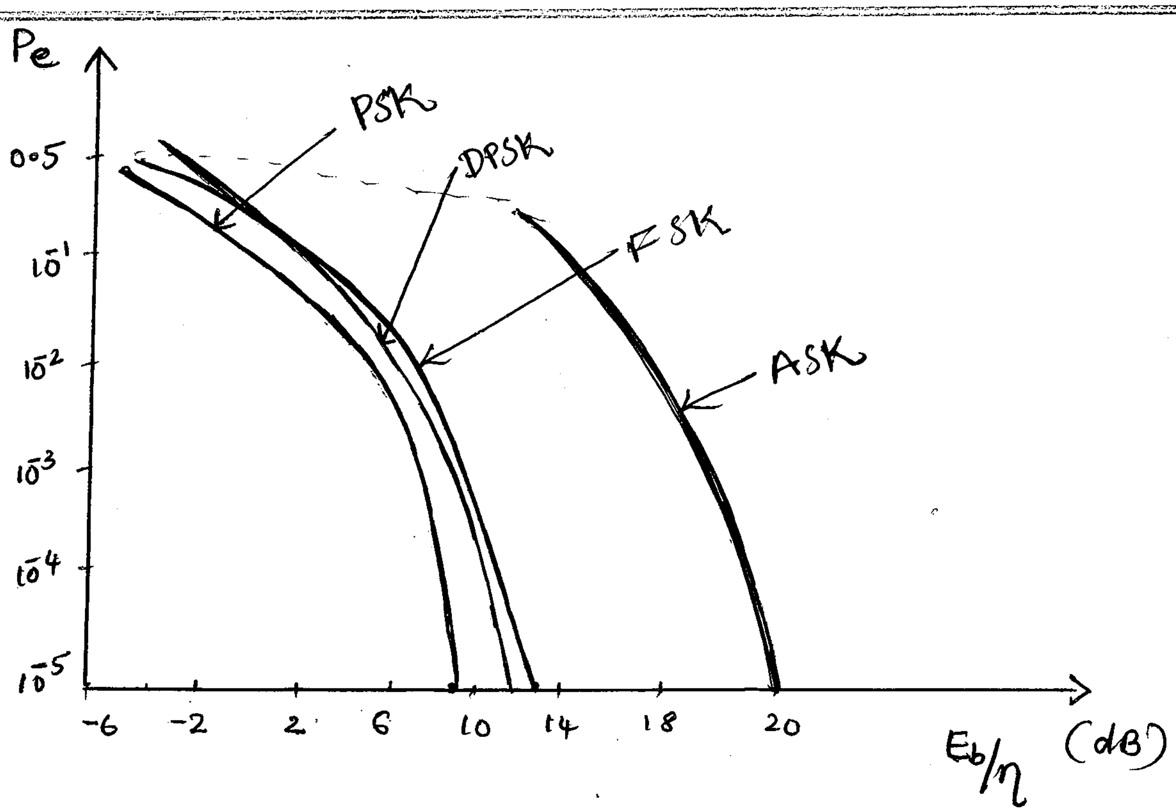


Fig: Probability of error for ASK, FSK, PSK & DPSK.

FSK detection using PLL:

PLL - phase Locked Loop.

$s(t) = b(t)\,(\cos\omega_c t + \theta)$

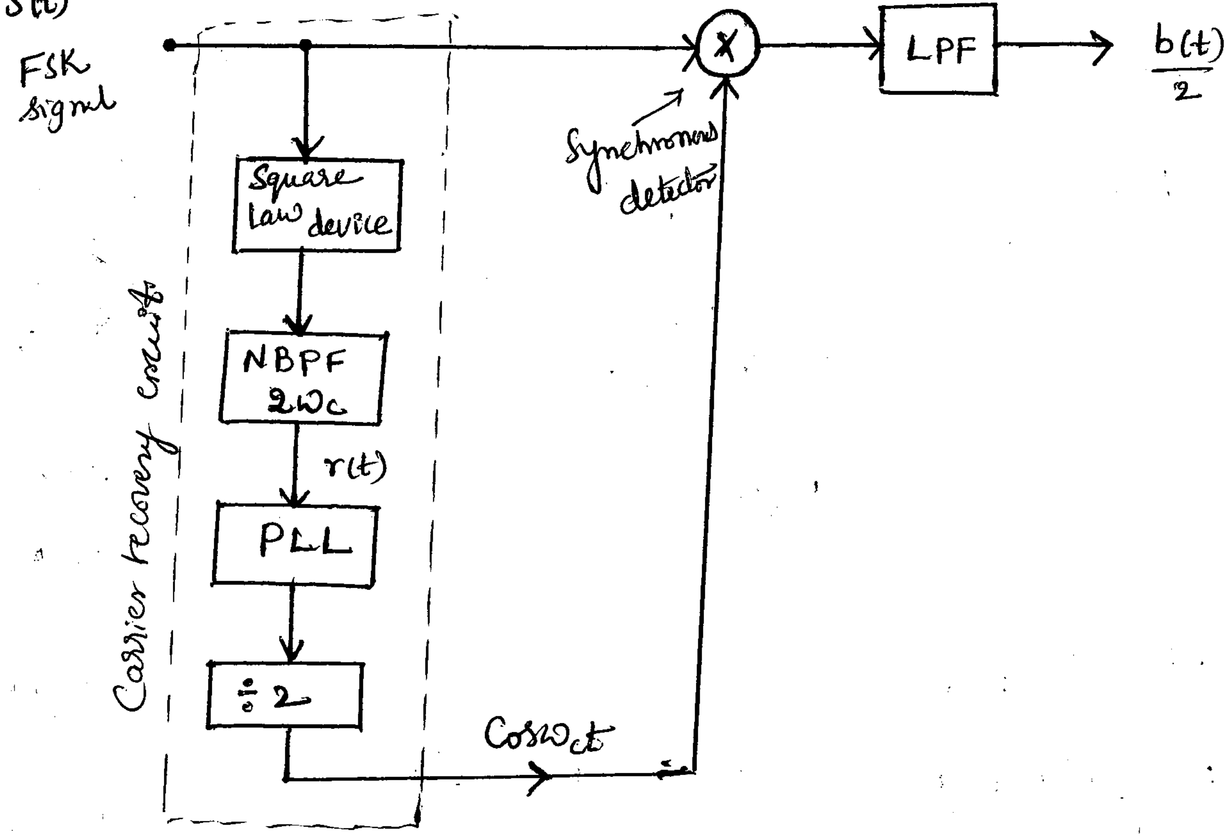


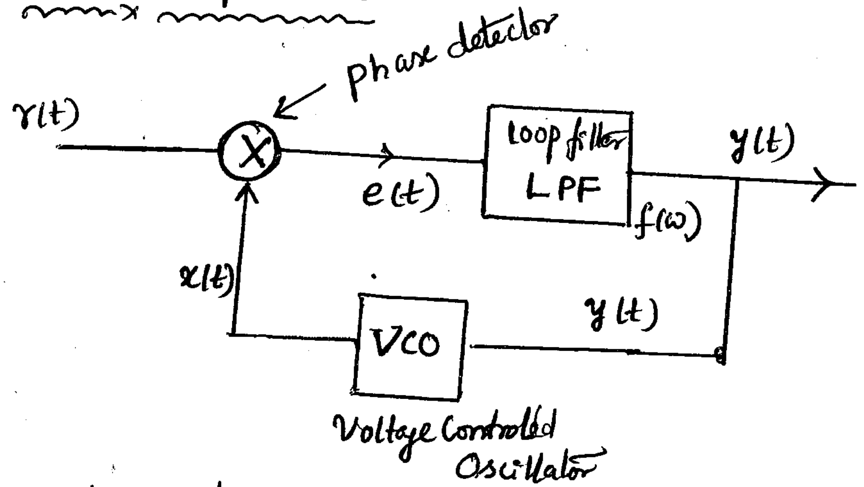
Fig: FSK detection using PLL.

- First the received signal is squared to generate the signal.

$$\cos^2(\omega_c t+\theta) = \frac{1+\cos 2(\omega_c t+\theta)}{2} = \frac{1}{2} + \frac{1}{2}\cos 2(\omega_c t+\theta)$$

- The dc component is removed by the NBPF, whose pass band centered around $2f_c$.
- The frequency divider (Composed of a flipflop & NBPF tuned to f_c) is used to generate the waveform $\cos(\omega_c t+\theta)$.

Phase Locked loop (PLL):



- It is a heart of the all synchronization circuits.
- VCO generated signal is the phase of a locally generated replica of the incoming signal.
- It is nearly used in frequency demodulators.
- PLL have three basic components
  - (a) Phase detector or multiplier
  - (b) Loop filter & LPF
  - (c) Voltage Controlled Oscillator

- multiplier which multiplies FM signal & FSK signal and VCO output.
- LPF, the function of which is to remove high frequency components contained in the multiplier output signal, these variations in the error signal.
- VCO is a device that produce the carrier replica.

VCO is an oscillator whose output frequency is a linear function of its input voltage over some range of input & output.

Introduction to M-ary Signalling:

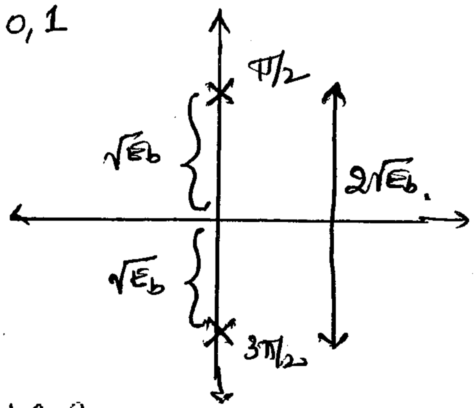
- The main requirements in the communication system is less bandwidth, less transmission power & less hardware complexity.
- In digital modulation scheme, to reduce the bandwidth in binary signalling scheme, M-ary signalling scheme is used.

* General Expression for PSK

$$S(t)_{M\text{-}PSK} = A_c \cos(\omega_c t + \phi_m)$$

where $\phi_m = -(2m+1)\frac{\pi}{M}$; $m = 0, 1, 2, \cdots M-1$

⇒ Let $M = 2$, $\phi_m = -(2m+1)\pi/2$ $m = 0, 1$



BPSK $\therefore$ $m=0$, $\phi_0 = -\pi/2$

$m=1$, $\phi_1 = -3\pi/2$

when $M=4$ ie QPSK.

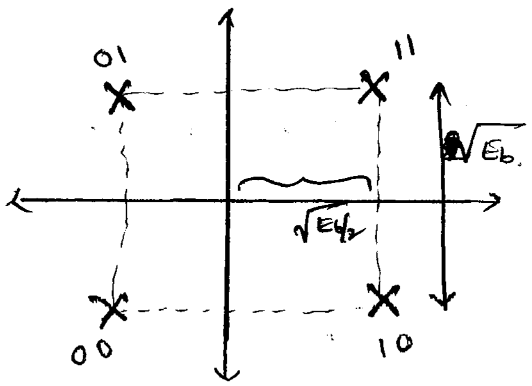
$$\phi_m = -(2m+1)\pi/4 \quad ; m = 0, 1, 2, 3.$$

$m=0$, $\phi_0 = -\pi/4$

$m=1$, $\phi_1 = -3\pi/4$

$m=2$, $\phi_2 = -5\pi/4$

$m=3$, $\phi_3 = -7\pi/4$.



IIly $M=8$ → 8-PSK

$$\phi_m = -(2m+1)\pi/8$$

$m=0$, $\phi_0 = -\pi/8$

$m=1$, $\phi_1 = -3\pi/8$

$m=2$, $\phi_2 = -5\pi/8$

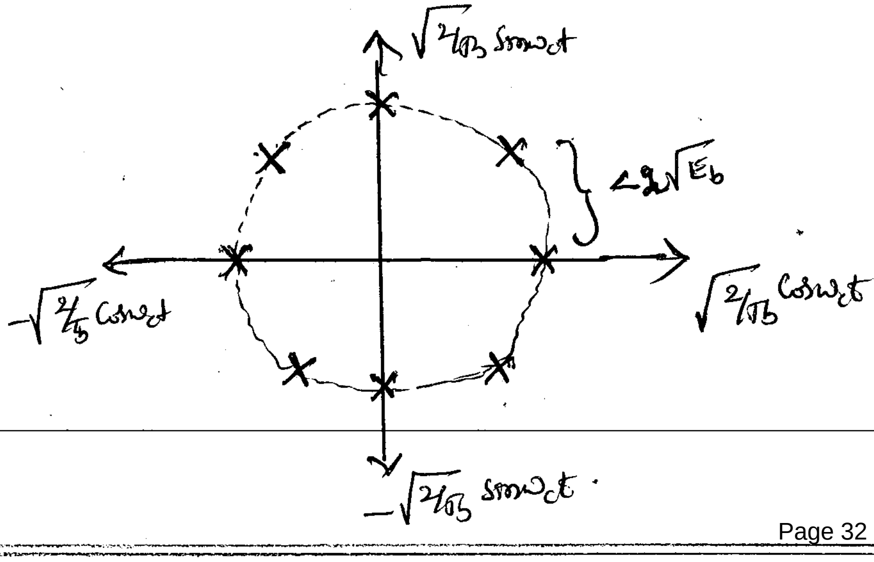
$m=3$, $\phi_3 = -7\pi/8$

$m=4$, $\phi_4 = -9\pi/8$

$m=5$, $\phi_5 = -11\pi/8$

$m=6$, $\phi_6 = -13\pi/8$

$m=7$, $\phi_7 = -15\pi/8$.



Quadrature Phase Shift Keying (QPSK):

- ✓ QPSK is an extension of binary PSK.
- ✓ In binary data transmission, we transmit only one of two possible signals during each bit interval 'T_b',
- ✓ In M-ary data transmission, it is possible to send any one of M-possible signals, during each signalling interval T

ie The no. of possible signal is $M = 2^n$
where n is an integer.

The signalling interval is $T = nT_b$.

* QPSK is an example of M-ary data transmission with $M=4$.
- ✓ In QPSK, one of four possible signals is transmitted during each signalling interval, with each signal uniquely related to a dibit.

ie, The four dibits may be 00, 01, 10, 11 in natural coded form (or) 10, 00, 11, 01 in gray encoded form.

∴ In QPSK system we may represent the four possible dibits in Gray encoded form by transmitting a sinusoidal carrier with one of the four values as

$$S(t) = \begin{cases} A_c \cos(2\pi f_c t - 3\pi/4) & ; \text{ dibit } 00^{(--)} \\ A_c \cos(2\pi f_c t - \pi/4) & ; \text{ dibit } 10^{(+-)} \\ A_c \cos(2\pi f_c t + \pi/4) & ; \text{ dibit } 11^{(++)} \\ A_c \cos(2\pi f_c t + 3\pi/4) & ; \text{ dibit } 01^{(-+)} \end{cases}$$

(OR)

$$S(t) = A_c \cos\left[\omega_c t - (2m+1)\pi/4\right] \quad ; m = 0, 1, 2, 3, \quad M=4$$

$$= A_c\left[\cos\omega_c t \cos(2m+1)\pi/4 + \sin\omega_c t \cdot \sin(2m+1)\pi/4\right]$$

$$S(t) = b_e(t) \cdot A_c \cdot \cos(2\pi f_c t) + b_o(t) \cdot A_c \sin(2\pi f_c t)$$

$$S(t) = b_e(t) \cdot \sqrt{E_b/T_s} \cos 2\omega_c t + b_o(t)\sqrt{\frac{E_b}{T_s}} \sin\omega_c t \qquad (\because T_s = 2T_b)$$

where $b_e(t) = \sqrt{2} \cdot \cos(2m+1)\pi/4$

$b_o(t) = \sqrt{2} \sin(2m+1)\pi/4$.

ie

$b_e(t) = \sqrt{2} \cos(2m+1)\pi/4$

$m=0$, $b_e(t) = \sqrt{2} \times \frac{1}{\sqrt{2}} = +1$

$m=1$, $b_e(t) = \sqrt{2} \times \cos 3\pi/4 = -1$

$m=2$; $b_e(t) = \sqrt{2} \cos 5\pi/4 = -1$

$m=3$, $b_e(t) = \sqrt{2} \cos \frac{7\pi}{4} = +1$

$\Rightarrow$ 00 $\rightarrow$ $- -$ Third Q_3

11 $\rightarrow$ $+ +$ First Q_1

$b_o(t) = \sqrt{2} \sin(2m+1)\pi/4$.

$m=0$. $b_o(t) = \sqrt{2} \sin \pi/4 = +1$

$m=1$ $b_o(t) = \sqrt{2} \sin 3\pi/4 = +1$

$m=2$ $b_o(t) = \sqrt{2} \sin 5\pi/4 = -1$

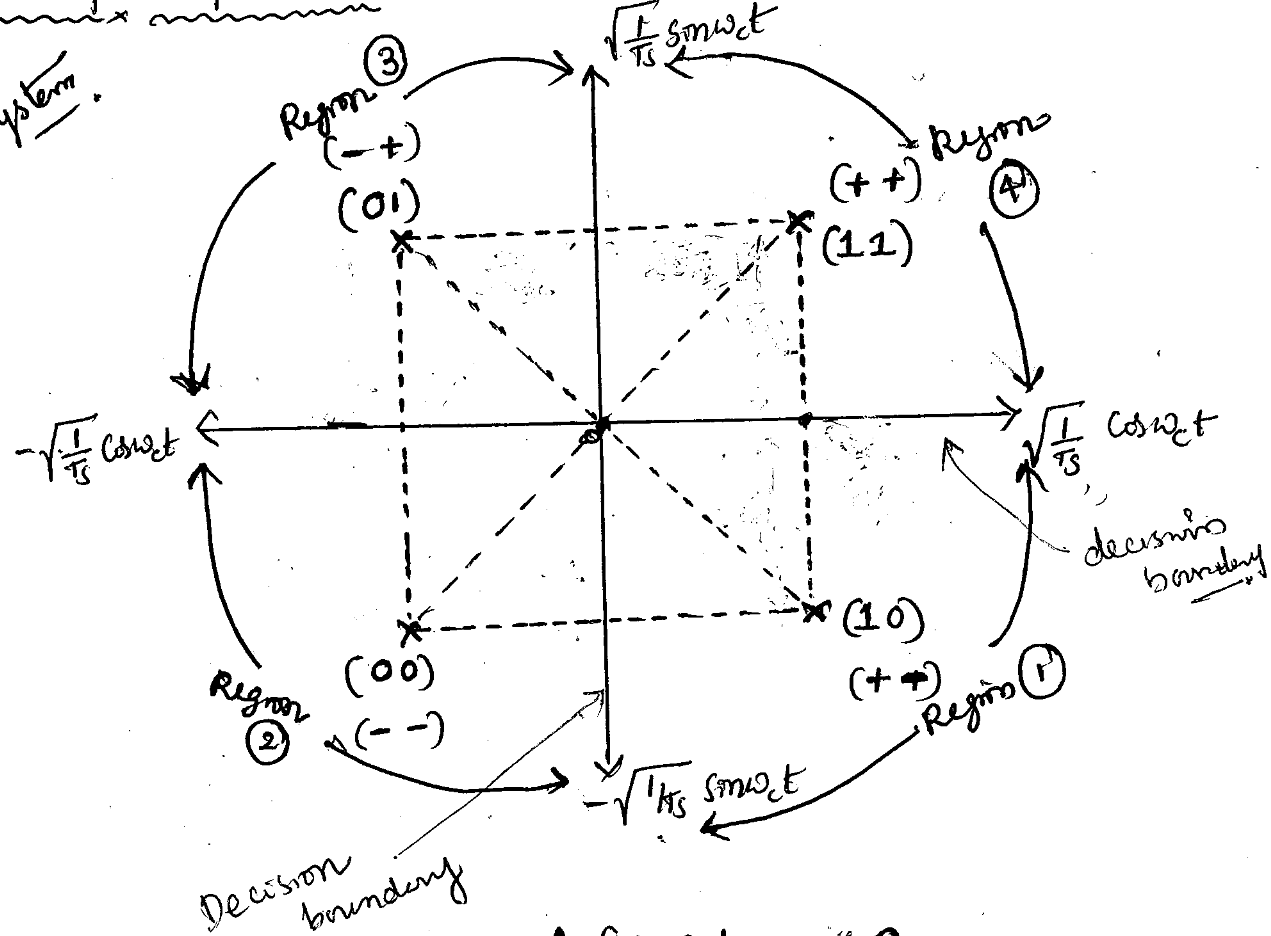
$m=3$ $b_o(t) = \sqrt{2} \sin 7\pi/4 = -1$.

01 $\rightarrow$ $- +$ $\rightarrow$ Second Q_2

10 $\rightarrow$ $+ -$ $\rightarrow$ Fourth Q_4.

Signal Space representation:

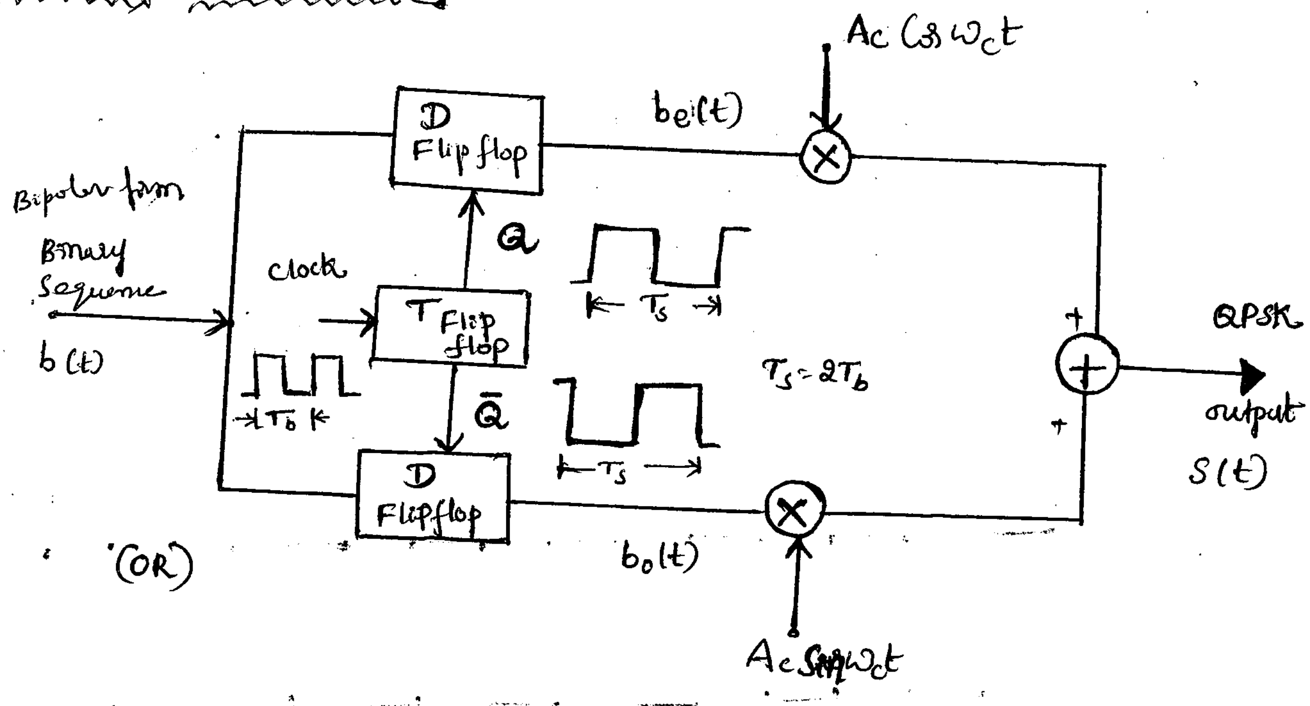
for QPSK system.



$\therefore$ QPSK

$$s(t) = \begin{cases} A_c \cos\omega_c t \ ; & 10 \\ -A_c \cos\omega_c t \ ; & 00 \\ A_c \sin\omega_c t \ ; & 11 \\ -A_c \sin\omega_c t \ ; & 01 \end{cases}$$

QPSK Transmitter:



(OR)

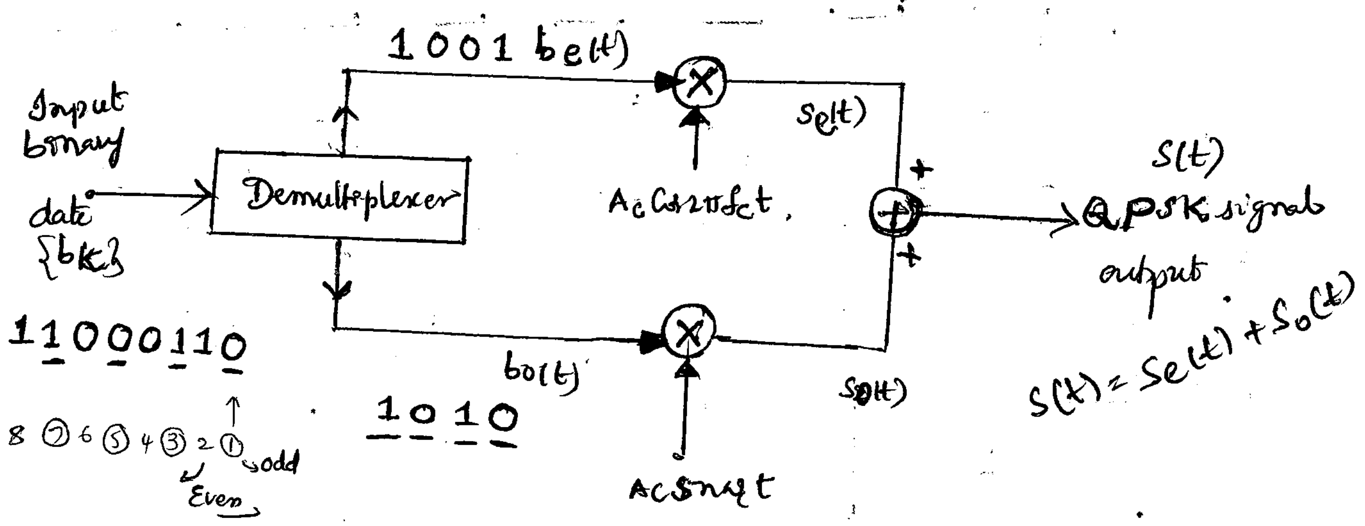


Fig: Block diagram of QPSK transmitter.

Operation:

- ✓ The binary sequence b(t) is represented in bipolar form with symbol '0' and symbol '1'.
- ✓ The binary wave is divided by means of a demultiplexer into two seperate binary waves consisting of the odd & Even numbered input bits.
- ✓ These two binary waves are denoted by $b_e(t)$ and $b_o(t)$ are used to modulate a pair of quadrature carrier or orthogonal basis functions $A_c \cos 2\pi f_c t$ and $A_c \sin \omega_c t$.
- ✓ The result is a pair of binary PSK waves which may be detected independently due to the orthogonality of $A_c \cos \omega_c t$ & $A_c \sin \omega_c t$.

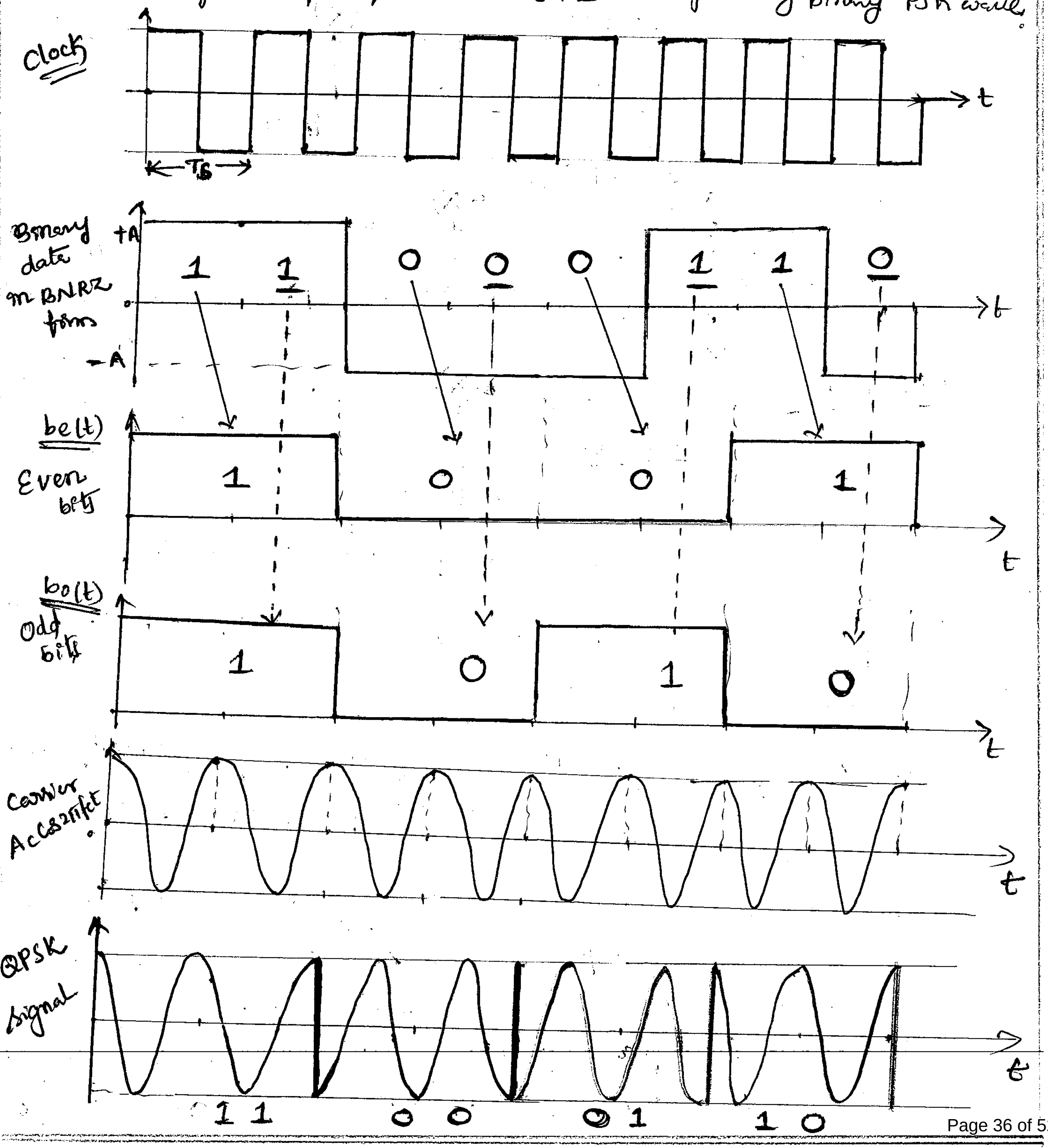
✓ Finally the two binary PSK waves are added to produce the desired QPSK wave.

* The symbol duration T_s of a QPSK wave is twice as long as the bit duration T_b of the input binary wave $\therefore T_s = 2T_b$.

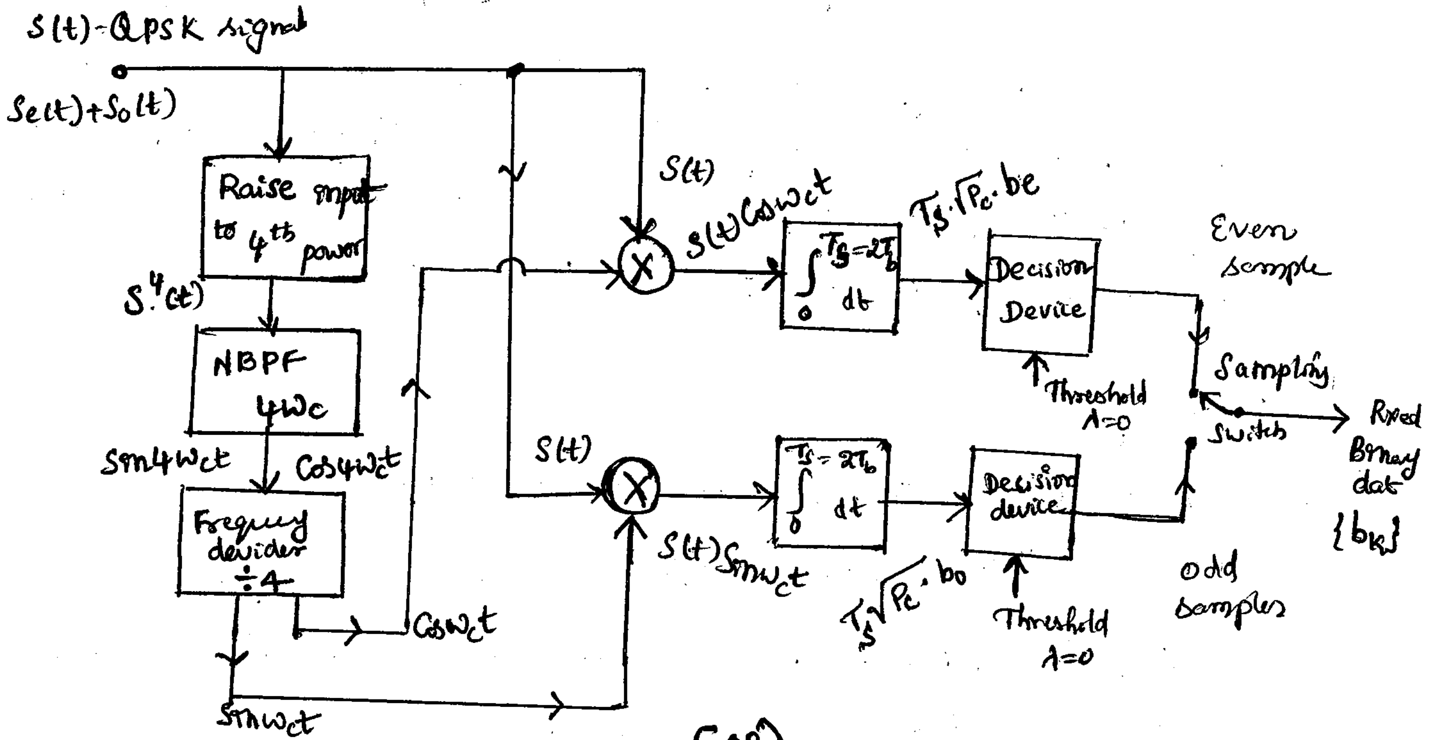
ie For a given bit rate $1/T_b$ a QPSK wave requires half the transmission bandwidth of the corresponding binary PSK wave.

(or)

For a given transmission bandwidth, a QPSK wave carries twice as many bits of information as the corresponding binary PSK wave.



QPSK Receiver :



(OR)

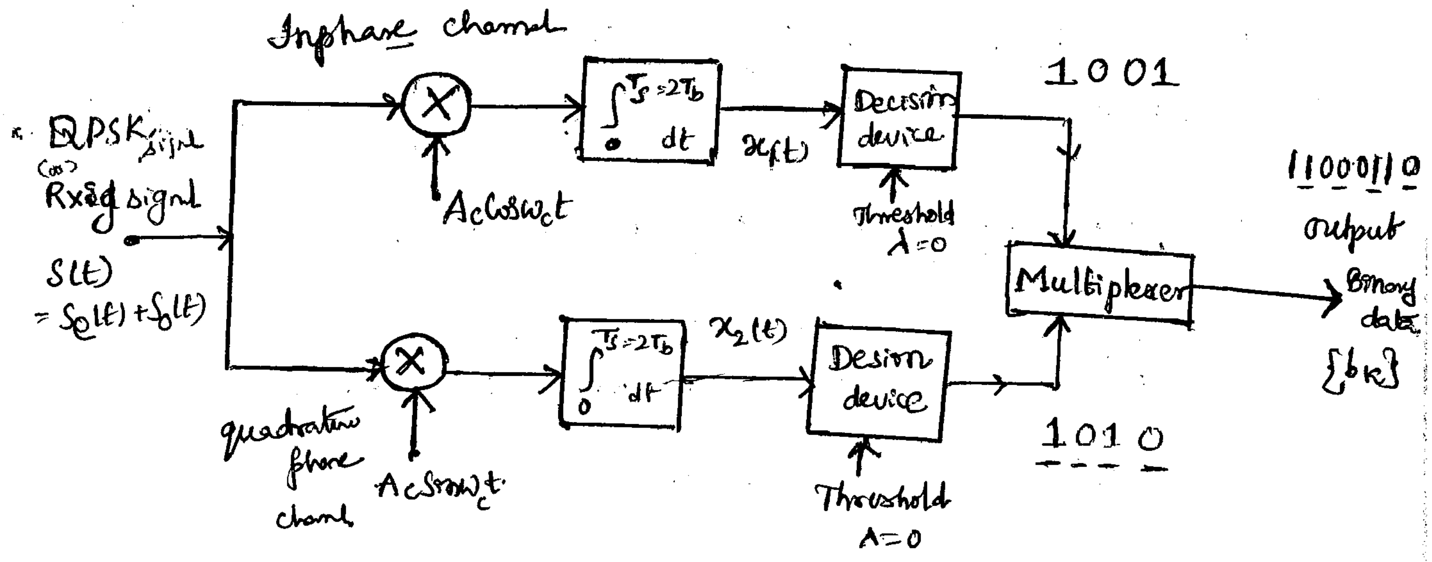


fig: Block diagram of QPSK Receiver.

Coherent

Operation:

✓ The QPSK receiver consists of a pair of correlators with a common input '$S_o(t)+S_e(t)=S(t)$' and supplied with a locally generated pair of reference carrier signals $A_c \cos\omega_c t$ & $A_c \sin\omega_c t$.

✓ The correlator outputs $x_1(t)$ & $x_2(t)$ are each compared with a threshold of zero.

ie If $x_1(t) > 0 \rightarrow$ Symbol 1

$x_1(t) < 0 \rightarrow$ Symbol 0 at Inphase channel. & upper channel

&

$x_2(t) > 0 \rightarrow$ Symbol '1'
$x_2(t) < 0 \rightarrow$ Symbol '0'   at Quadrature phase Channel & Lower channel.

✓ These two binary sequences at the inphase and quadrature channel outputs are combined in a multiplexer to produce the original binary sequence at the transmitter input with the minimum probability of symbol error.

Summary:

✓ QPSK transmits two bits at a time by assigning four (4) phases $\pi/4, 3\pi/4, 5\pi/4, 7\pi/4$ ie $45°, 135°, 225°, 315°$.

✓ QPSK divides bit duration 'T_b' into '$2T_b$' bit duration such that even bit, odd bit with '$2T_b$' duration.

✓ PSK & DPSK transmits with duration T_b' & the bandwidth is '$2f_b$' Hz

✓ QPSK transmits with duration $2T_b$' & the bandwidth $\frac{2f_b}{2} = f_b$ Hz

✓ The probability of error in QPSK system is

$$\boxed{P_e = \text{erfc}\left(\sqrt{\frac{E_b}{2\eta}}\right)}$$ where $E_b = \frac{A_c^2 T_b}{2}$

$$\gamma_{1\,max}^2 = \gamma_{2\,max}^2 = \frac{2A_c^2 T_b}{\eta}$$

In terms of Q-function the probability of error

$$\boxed{P_e = 2Q\left(\sqrt{\frac{E_b}{\eta}}\right)}$$ where $E_b = \frac{A_c^2 T_b}{2}$

✓ The main disadvantage of QPSK system is it requires more hardware complex circuits.

✓ $$S(t) = A_c \cos\omega_c t\,(b_e) + A_c \sin\omega_c t\,(b_o)$$

QPSK definition

$$S(t) = \begin{cases} A_c \sin\omega_c t & b_o = 1 & 11 \\ -A_c \sin\omega_c t & b_o = -1 & 01 \\ A_c \cos\omega_c t & b_e = 1 & 10 \\ -A_c \cos\omega_c t & b_e = -1 & 00 \end{cases}$$

Minimum Shift Keying : 'MSK' → (CPFSK).

In the coherent detection of binary FSK, the phase information contained in the received signal was not fully exploited other than to provide for synchronization of the receiver to the transmitter.

By proper utilization of the phase when performing detection, it is possible to improve the noise performance of the Rx significantly. It is acheived by a "continuous-phase frequency shift keying" (CPFSK)

ie
<u>Minimum Shift Keying</u> is a special form of binary CPFSK with the change in the carrier frequency from symbol '0' to symbol '1' & symbol '1' to symbol '0' is equal to one half the bit rate of the incoming data

ie
The CPFSK signal $s(t)$ can be expressed as

$$\boxed{s(t) = A_c \cos[2\pi f_c t + \theta(t)]}$$

where $\theta(t)$ - phase - continuous function of time.

f_c - nominal carrier frequency is choosen as the arithmetic mean of two frequencies f_1 & f_2

$$\therefore \quad f_c = \frac{f_1+f_2}{2} \qquad \begin{array}{l} f_1 - \text{for logic 1} \\ f_2 - \text{for logic 0} \end{array}$$

The phase $\theta(t)$ increases or decreases linearly with time during each bit period of T_b sec.

$$\text{ie} \quad \theta(t) = \theta(0) \pm \frac{\pi h}{T_b} t \qquad 0 \le t \le T_b.$$

($+$ → logic '1', $-$ → logic '0')

where the parameter $h = T_b(f_1 - f_2)$ → '<u>deviation ratio</u>'

$\therefore$ at $t = T_b$

$$\theta(T_b) - \theta(0) = \begin{cases} +\pi h & ; \text{logic } 1 \\ -\pi h & ; \text{logic } 0 \end{cases}$$

ie The sending of symbol 1 increases the phase of the CPFSK signal $s(t)$ by πh radians, where as the sending of symbol 0 reduces by πh radians.

$$\therefore \quad S(t) = A_c \cos\left[2\pi f_c t + \theta(t)\right]$$

$$S(t) = A_c\left\{\underbrace{\cos[\theta(t)]\cos(2\pi f_c t)}_{\text{Inphase Component}} - \underbrace{\sin[\theta(t)]\cdot\sin(2\pi f_c t)}_{\text{Quadrature component}}\right\}$$

The deviation ratio $h = 1/2$

then $\boxed{\theta(t) - \theta(0) = \pm\frac{\pi}{2T_b}t}$

The phase tree is as follows.

ie the phase of the CPFSK signal is an odd or even multiple of πh radians at odd or even multiples of the bit duration T_b

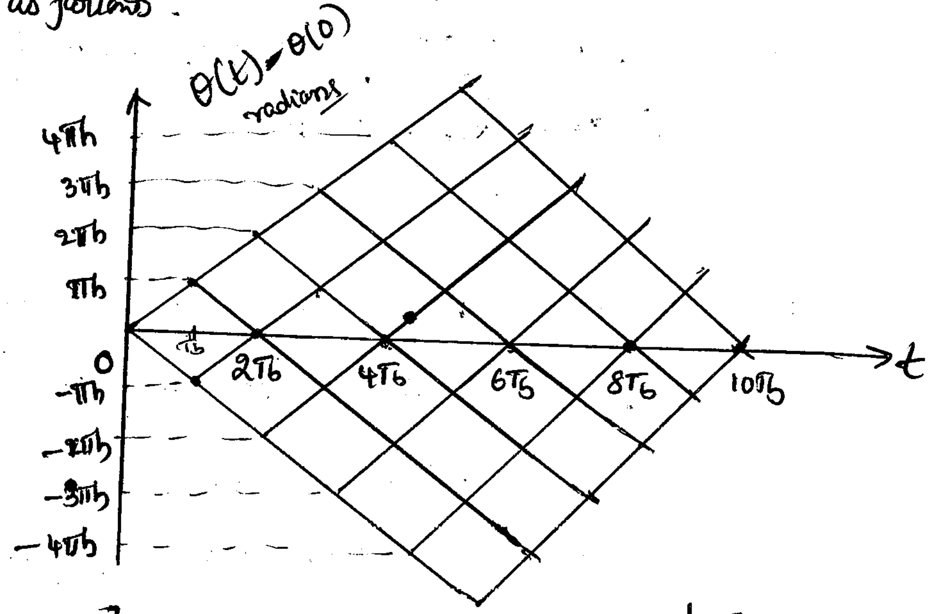


Fig: Phase tree

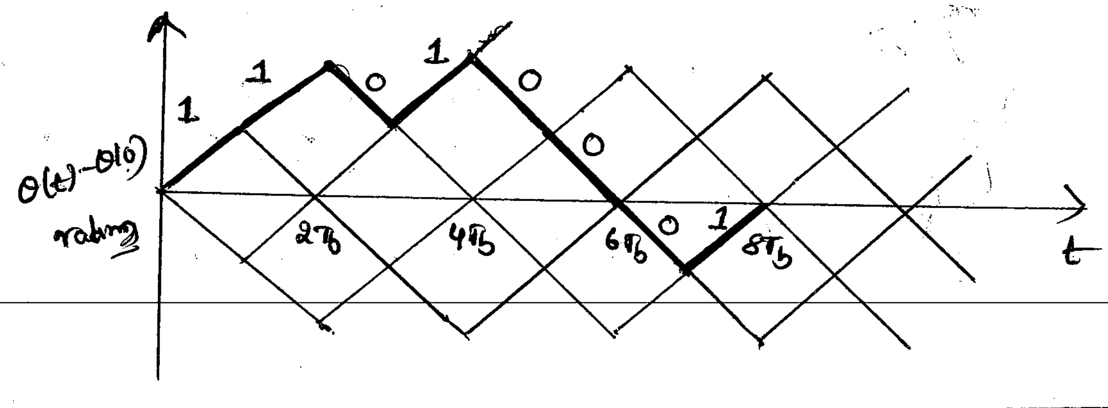
$$\therefore \quad S_I(t) = A_c[\cos\theta(t)] \qquad \theta(0)=0.$$

Inphase $\boxed{S_I(t) = \pm A_c\cos\left(\frac{\pi}{2T_b}t\right)}$ $\quad + \rightarrow \theta(0)=0$, $- \rightarrow \theta(0)=\pi$ $\quad$ 0 to T_b

Ily, $\boxed{S_Q(t) = \pm A_c \sin\left(\frac{\pi}{2T_b}\right)t}$ $\quad + \rightarrow \theta(T_b) = \pi/2$, $- \rightarrow \theta(T_b) = -\pi/2$ $\quad$ T_b to $2T_b$.

Example: the binary sequence 1101000 1, with $\theta(0)=0$

$h=1/2$



ie A CPFSK signal with a deviation ratio of one-half is reffered as Minimum Shift Keying (MSK)

It is also referred as fast FSK.

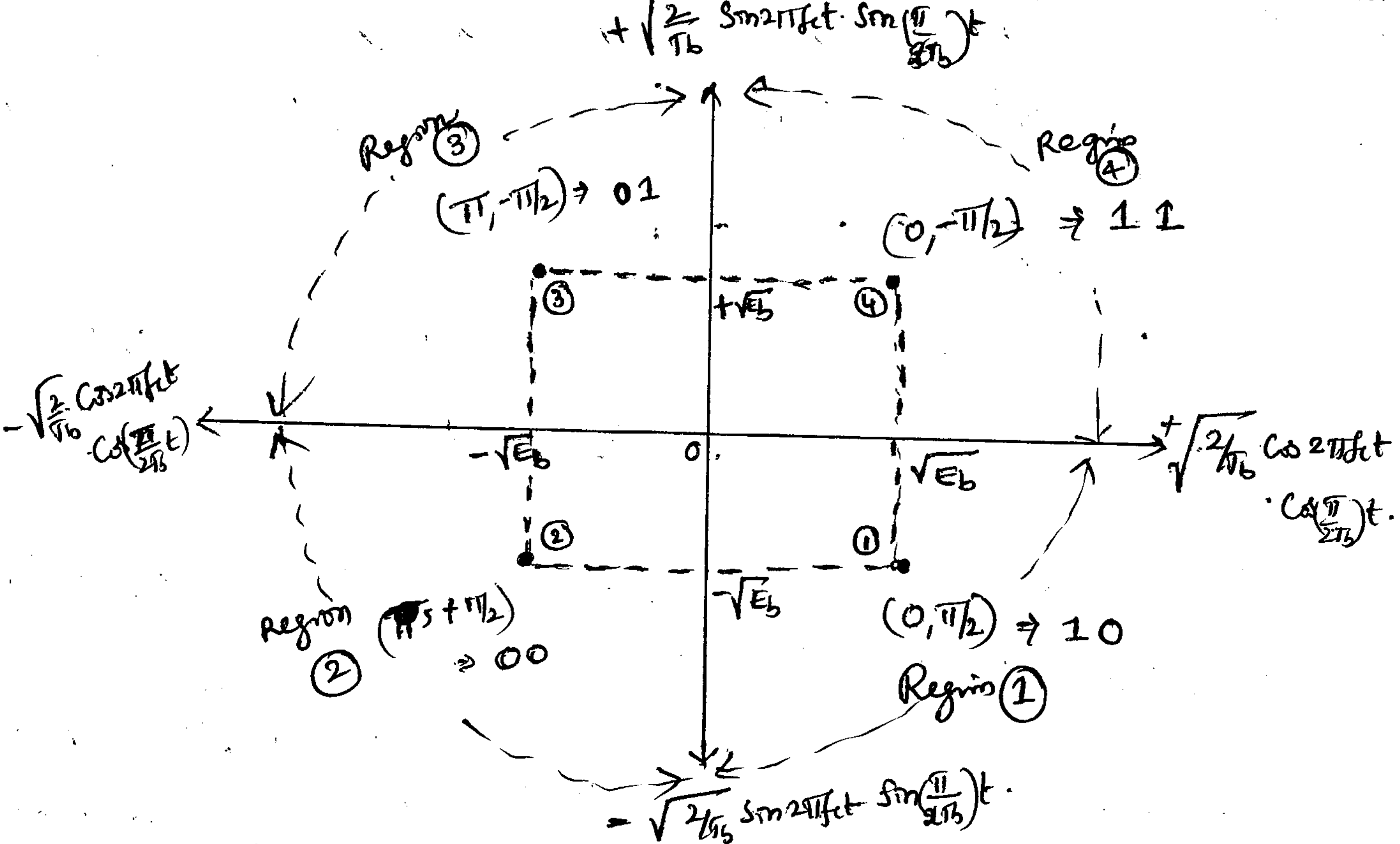
ie The coordinates of the message points for the QPSK signal is expressed in terms of signal energy per symbol 'E'.

But for the MSK signal expressed in terms of the signal energy per bit E_b with $E_b = E/2$.

Signal Space Representation:

ie

Txed Binary Symbol $0 \le t \le T_b$	phase states (radians) $\theta(0)$	$\theta(T_b)$	Coordinates of message points s_1	s_2	gray coded form
1	0	$+\pi/2$	$+\sqrt{E_b}$	$-\sqrt{E_b}$	1 0
0	π	$+\pi/2$	$-\sqrt{E_b}$	$-\sqrt{E_b}$	0 0
1	π	$-\pi/2$	$-\sqrt{E_b}$	$+\sqrt{E_b}$	0 1
0	0	$-\pi/2$	$+\sqrt{E_b}$	$+\sqrt{E_b}$	1 1



The probability of error of MSK system is

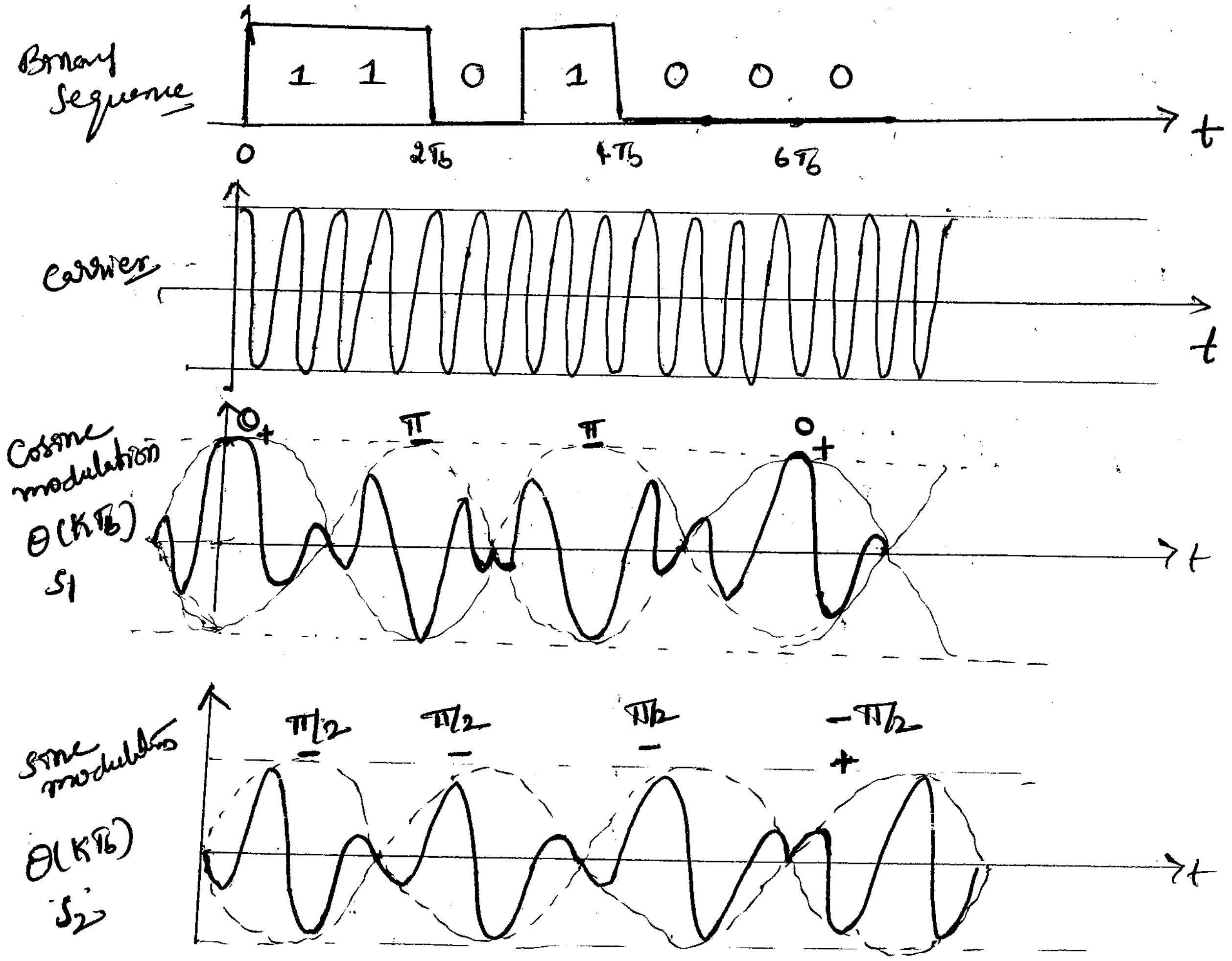
$$P_e = \operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right) - \frac{1}{4}\operatorname{erfc}^2\left(\sqrt{\frac{E_b}{N_0}}\right) \Rightarrow E_b/N_0 >> 1 \text{ then}$$

$$\boxed{P_e = \operatorname{erfc}\sqrt{\frac{E_b}{N_0}}}$$

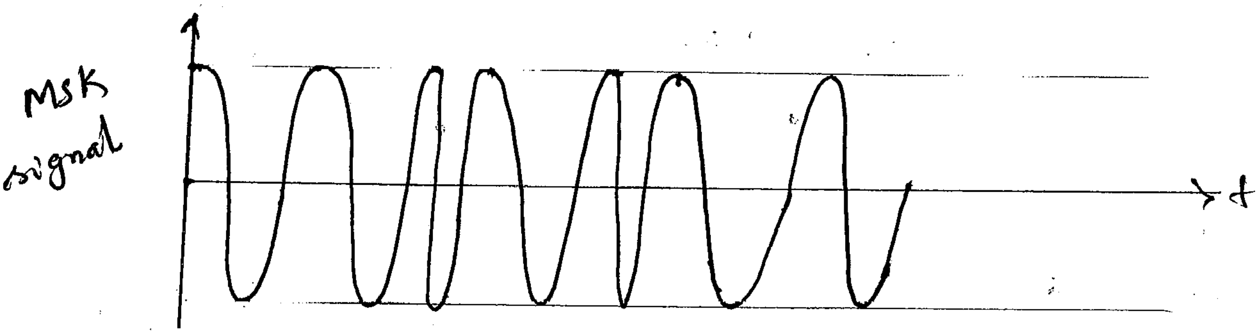
$N_0 = \eta/2$

Same as PSK

Waveform Representation:



The sum of the signals of S_1 and S_2 we will get MSK signal $S(t)$.



Thus, MSK may be viewed as a quadrature multiplexed frequency modulated wave.

The basic difference between QPSK and MSK is that in QPSK the phase shift $\theta(t)$ assume a constant distinct value for a entire duration of symbol depending on dibit Txed. Where as MSK the phase shift $\theta(t)$ varies ~~with~~ time along a distinct straight path depending on the debit being ~~sent~~ Txed.

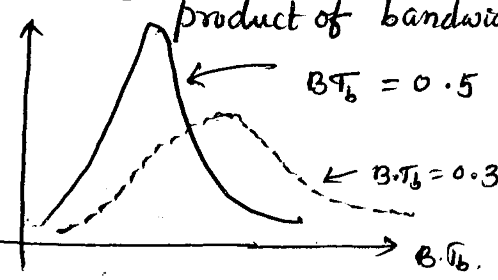
Gaussian Minimum Shift Keying (GMSK):

- ✓ The modified MSK using a gaussian filter is called "Gaussian filtered MSK & simply Gaussian MSK ie GMSK.
- ✓ In GMSK, a premodulation low pass Gaussian filter is used as a pulse shaping filter to reduce the bandwidth of the baseband signal before it is applied to the MSK modulator.
- ✓ The use of a filter with gaussian characteristics with MSK are
  - * Reduction in the transmitted bandwidth of the signal
  - * Uniform Envelop
  - * Reduction of side lope levels of the power spectrum.
  - * Reduction of adjacent channel interference
  - * Suppression of out-of band noise.
- ✓ The relationship between the premodulation filter bandwidth $f_{3\text{-dB}}$ and bit period T_b defines the bandwidth of the system.

$$B.T_b = \frac{f_{3\text{-dB}}}{\text{Bit rate}}$$ → *Change the shape of the response of pulse shaping filter

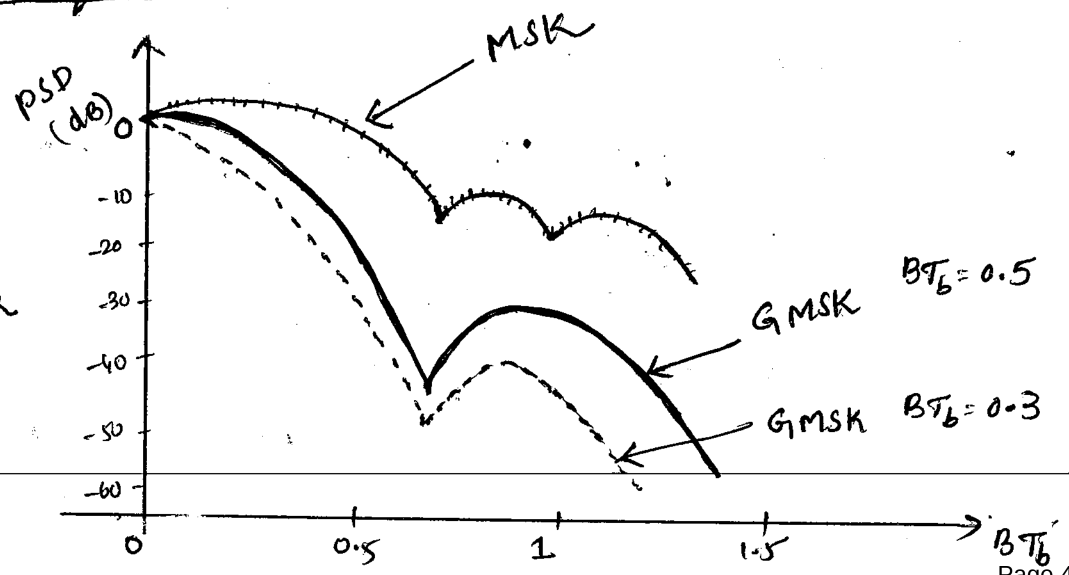
↑ product of bandwidth and bit duration

ie Gaussian filter impulse response for BT_b



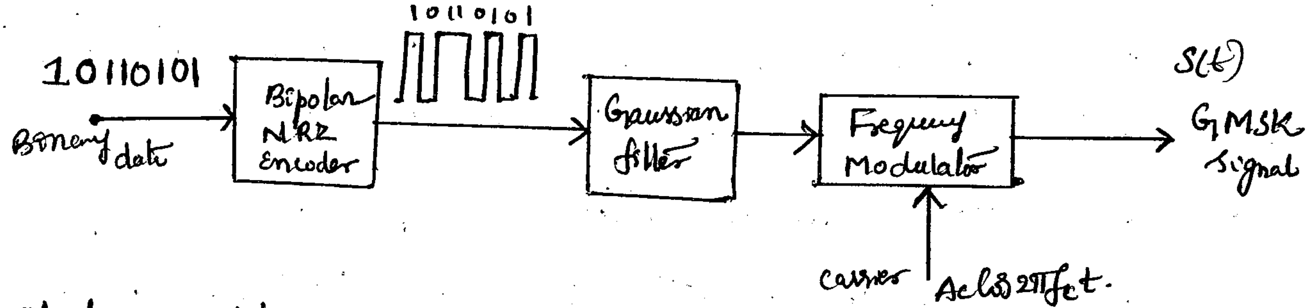
If $f = 2880$ Hz
$R_b = 9.6$ Kbps
$B.T_b = 0.3$

PSD comparison of MSK & GMSK:



ie The channel spacing can be tighter for GMSK when compared to MSK for same adjacent channel interference.

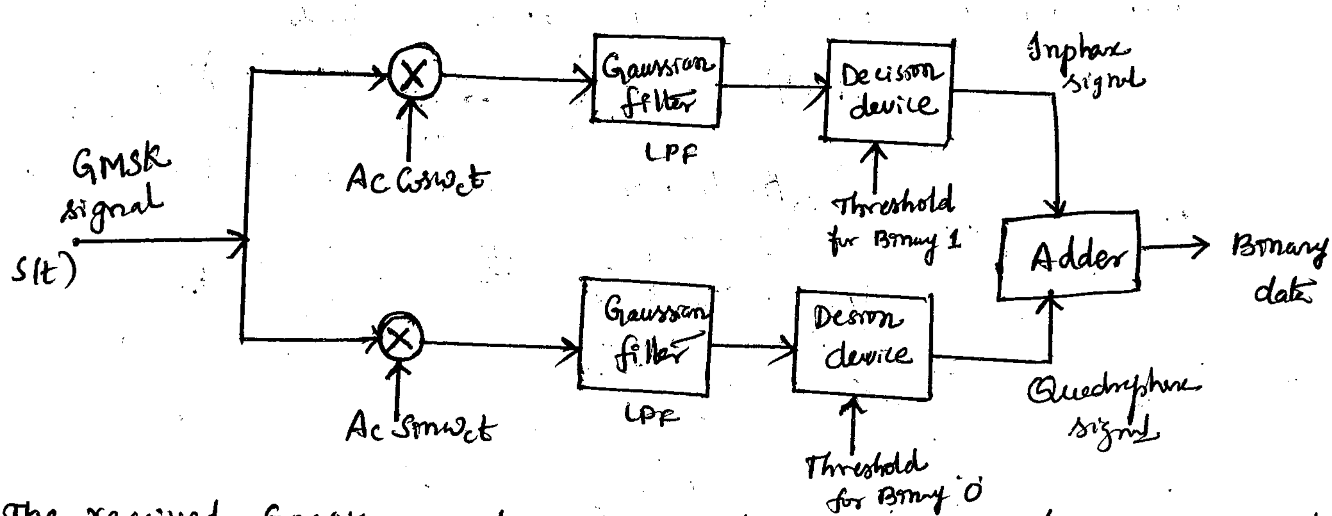
Generation of GMSK:



- ✓ The binary data sequence is encoded using a bipolar NRZ encoder
- ✓ The resulting data stream is then applied through a gaussian lowpass filter whose characteristics are gaussian in nature.
- ✓ The filtered signal acts as modulating signal which modulates the carrier signal in frequency modulator.
- ✓ The output of the frequency modulator is a GMSK signal.

Degeneration & Detection of GMSK:

- ✓ GMSK can be non-coherently detected as in FSK demodulator or coherently detected as in MSK demodulator.



- ✓ The received GMSK signal is applied to two product / balanced modulators whose carrier signals have a phase shift of 90° with each other.
- ✓ The output of product modulator is applied to Gaussian Low pass filter.
- ✓ The detection of binary data is done by the decision device.

Hence GMSK provides high spectrum efficiency, excellent power efficiency and a constant amplitude envelope.

* GMSK is widely used in the GSM cellular radio and PCS systems.

↳ Global System for Mobile Communication

M-ary Quadrature Amplitude Modulation : (QAM)

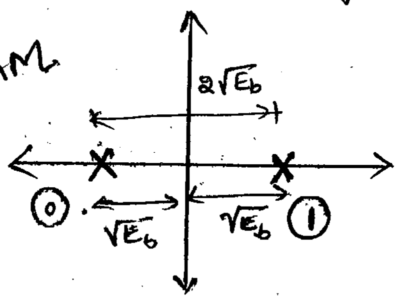
It is also called as Quadrature Amplitude Shift Keying (QASK).

(or) QAM = ASK + PSK.

- In this modulation scheme the carrier experiences amplitude as well as phase modulation.
- QAM is logical extension of QPSK, since it also consists of two independently amplitude modulated carriers in quadrature.
- QAM is also called as Amplitude phase shift Keying (APK).

(a) 2-QAM

ie BPSK



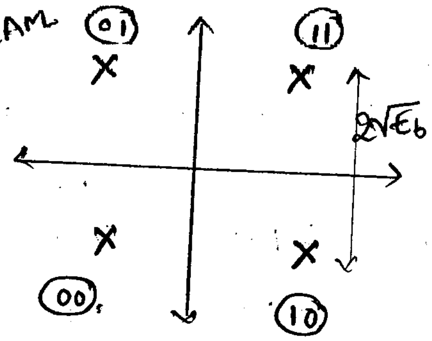
$B.W = \frac{2f_b}{N}$

1 block = 1 bit

$\boxed{B.W = 2f_b}$

(b) 4-QAM

ie. QPSK



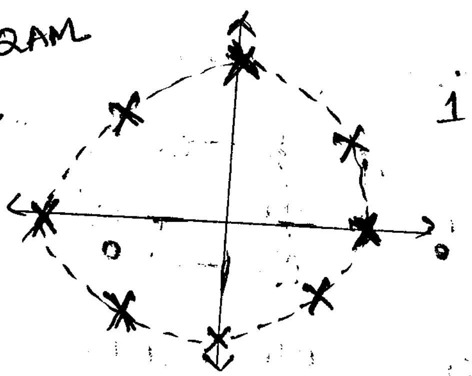
1 block = 2 bits

$BW = \frac{2f_b}{2} = f_b$

$\therefore \boxed{B.W = f_b}$

(c) 8-QAM

& 8-PSK



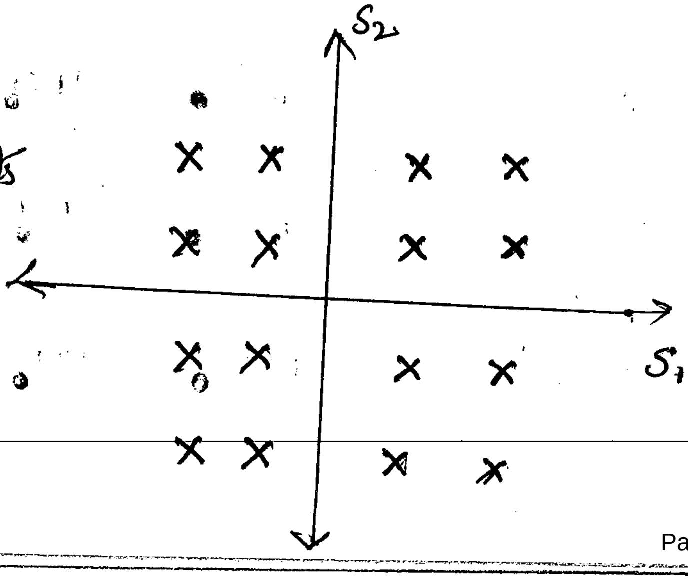
1 block = 3 bits $\therefore \boxed{B.W = \frac{2f_b}{3}}$

(d) 16-QAM $\Rightarrow$

1 block = 4 bits

Bandwidth $= \frac{2f_b}{4} = f_b/2$

$\therefore \boxed{B.W = \frac{f_b}{2}}$



✓ The general form of M-ary QAM is defined by the Txed sig.

$$S(t) = a_i \cdot A_c \cos(2\pi f_c t) + b_i \cdot A_c \sin(2\pi f_c t)$$

(or)

$$\boxed{S(t) = \sqrt{\frac{2E_b}{T_b}}\, a_i \cos(2\pi f_c t) + \sqrt{\frac{2E_b}{T_b}}\, b_i \sin(2\pi f_c t)}\ ;\ 0 \le t \le T_b$$

where E_b → the energy of the signal

a_i, b_i – a pair of independent integers whose chosen in accordance with the location of the message points.

T_b – bit duration.

* $S(t)$ consists of two phase quadrature carriers, each of which is modulated by a set of discrete amplitudes, hence the name called Quadrature amplitude Modulation (QAM).

$\phi_1(t) = \sqrt{2/T_b}\cos 2\pi f_c t \qquad \phi_2(t) = \sqrt{2/T_b}\sin 2\pi f_c t.$

The coordinates of message points $a_i\sqrt{E_b}$ and $b_i\sqrt{E_b}$.

let $L=4$, then $M = L^2 = 16$ QAM. & $M = \sqrt{L}$

$$[a_i\ b_i] = \begin{bmatrix} (-3a, 3a) & (-a, +3a) & (+a, 3a) & (3a, 3a) \\ (-3a, a) & (-a, a) & (a, a) & (3a, a) \\ (-3a, -a) & (-a, -a) & (a, -a) & (3a, -a) \\ (-3a, -3a) & (-a, -3a) & (a, -3a) & (3a, -3a) \end{bmatrix}$$

Constellation diagram: (or) Signal Space Representation.

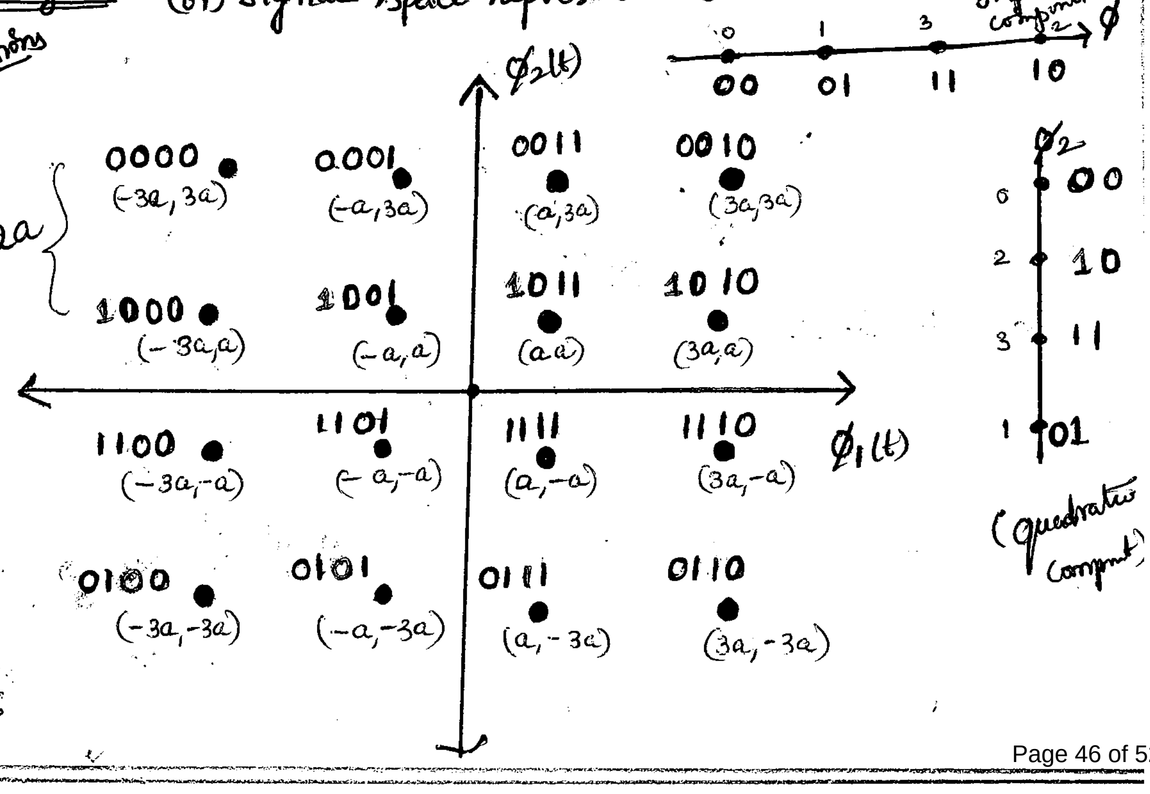
$L=4$

$2^4 = 16$ combinations

0000 → 1111.

16 QAM.

M=16.



0	1	3	2
8	9	11	10
12	13	15	14
4	5	7	6

The average normalized energy

$$E_s = \frac{1}{4}\left\{ (a^2+a^2) + (9a^2+a^2) + (a^2+9a^2) + (9a^2+9a^2) \right\}$$

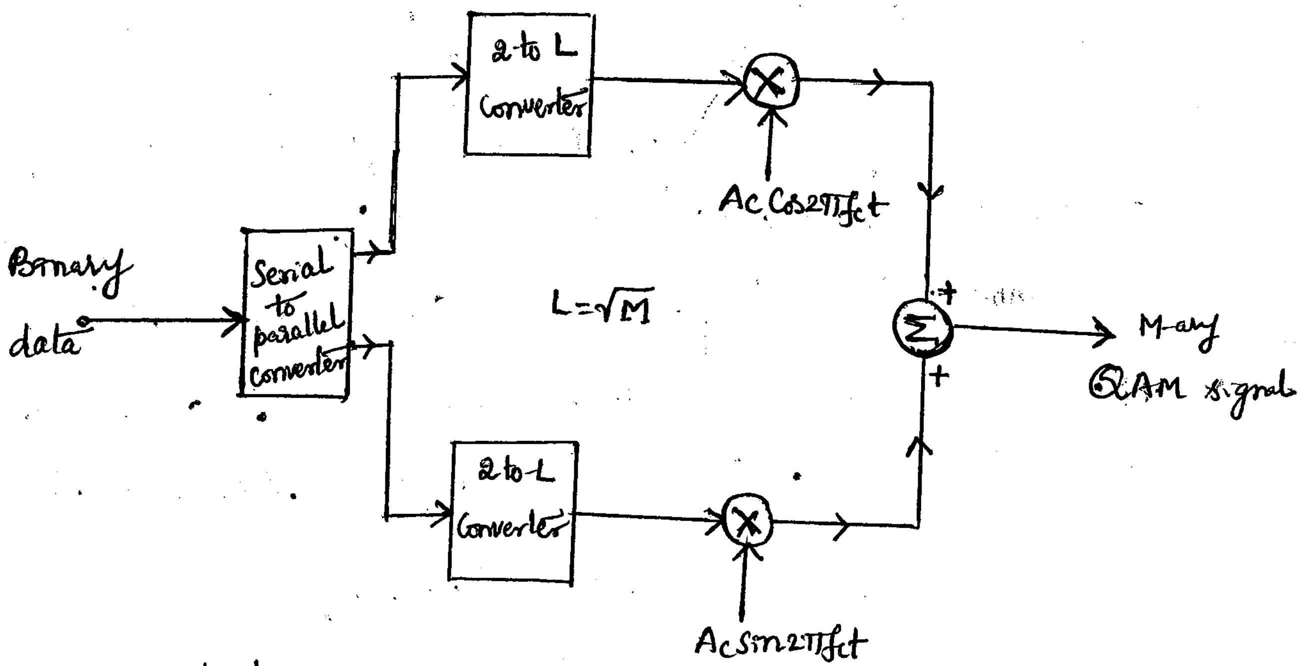
$$= \frac{1}{4}\, 40a^2 = 10a^2$$

$$E_s = 10a^2 \Rightarrow a^2 = 0.1\, E_s \Rightarrow a = \sqrt{0.1 E_s}$$

The signal represents 4-bits so $E_s = 4 E_b$

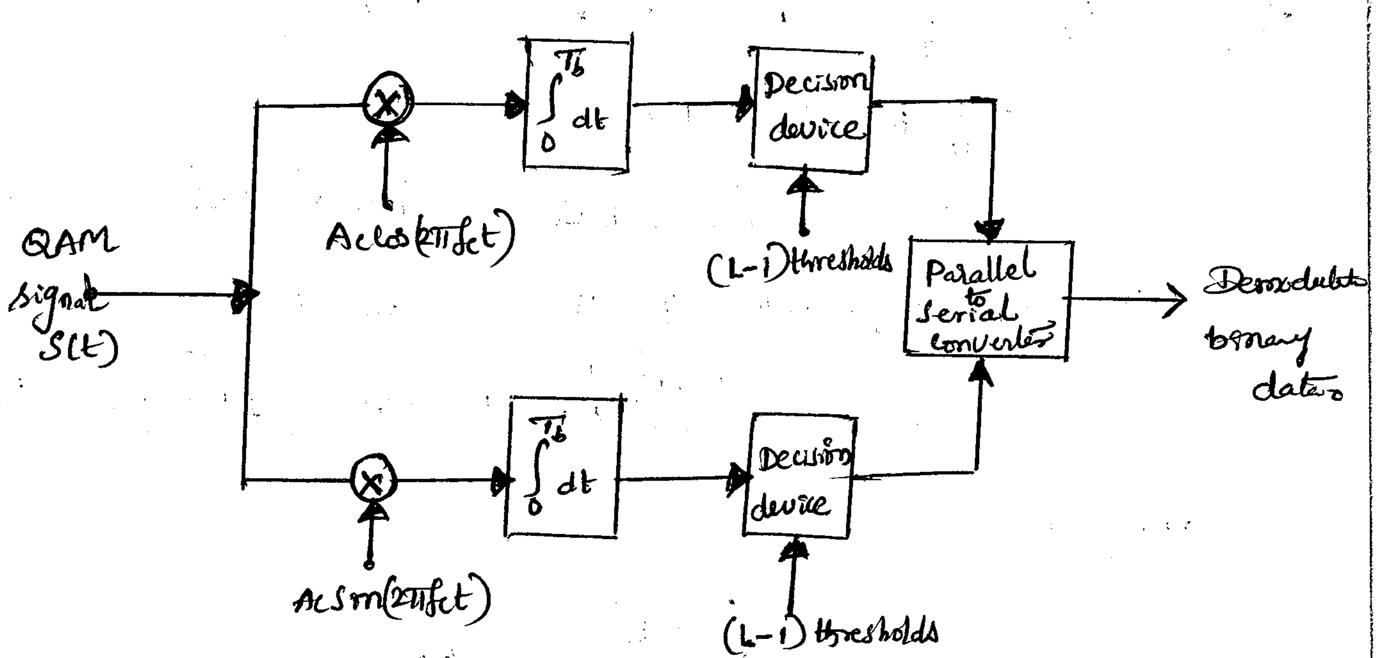
$$\therefore \boxed{a = \sqrt{0.4 E_b}}, \quad d = 2a \Rightarrow \boxed{d = 2\sqrt{0.4 E_b}}$$

QAM Transmitter:



- The serial to parallel converter accepts a binary sequence at a bit rate $R_b = 1/T_b$ and produces two parallel binary sequences whose bit rates are $R_b/2$ each.
- The 2-L level converters, where $L = \sqrt{M}$, generate polar L-level signals in response to the respective in-phase and quadrature channel inputs.
- Quadrature carrier multiplexing of the two polar L-level signals so generated produces the desired M-ary QAM signal.

QAM Receiver :



- The decoding of each baseband channel is accomplished at the output of decision device circuit, which is designed to compare the L-levels signals against (L-1) decision thresholds.
- The two binary sequences so detected are combined in the parallel to serial converter to reproduce the original binary sequence.

* The probability of error in QAM system is

$$P_e = 2\left(1 - \frac{1}{L}\right) \operatorname{erfc}\sqrt{\frac{E_b}{N_0}} \quad \& \; N_0 = \eta/2.$$

$$L = \sqrt{M}$$

$$\boxed{P_e = 2\left(1 - \frac{1}{\sqrt{M}}\right) \operatorname{erfc}\left(\sqrt{\frac{2E_b}{\eta}}\right)}$$

Ily we have

32-QAM

64-QAM ✓

128 QAM

256 QAM ✓

More bandwidth efficient and used high data rate transmission in Terestrial micro wave digital radio

digital video broadcast cable & modems.

Pe The waveform MSK signal exhibits phase continuity, that is, there are no abrupt phase change as in QPSK.

Problems:

(1) Binary data is transmitted over an RF bandpass channel with a usable bandwidth of 10 MHz at a rate of 4.8×10^6 bits/sec using a ASK signalling method. The carrier amplitude at the receiver antenna is 1 mV and the noise PSD at the receiver input is 10^{-15} watts/Hz. Find the error probability of the receiver.

Sol Given data ASK system

$$B \cdot W = 10 MHz$$

$$r_b = 4.8 \times 10^6 \text{ bits/sec}$$

$$A_c = 1 mV$$

$$N_0 = \eta/2 = 10^{-15} \text{ watts/Hz} \quad \Rightarrow \quad \eta = 2 \times 10^{-15} \text{ watts/Hz}$$

$$\therefore T_b = \frac{1}{f_b} = \frac{1}{r_b} = \frac{1}{4.8 \times 10^6} = 0.208 \mu Sec.$$

$\therefore$ The probability of error in ASK is

$$P_e = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{E_b}{4\eta}} \quad \text{or} \quad P_e = Q\left(\sqrt{\frac{E_b}{2\eta}}\right)$$

where $E_b = \frac{A_c^2 T_b}{2} = \frac{(1 \times 10^{-3})^2 \times 0.208 \times 10^{-6}}{2}$

$$E_b = 1.04 \times 10^{-3}$$

$$P_e = Q\left(\sqrt{\frac{1.04 \times 10^{-3}}{2 \times 2 \times 10^{-15}}}\right) = Q(\sqrt{26}) \quad \therefore \boxed{P_e = Q(\sqrt{26})}$$

(or) $\boxed{P_e = 2 \times 10^{-7}}$

(2) A bandpass data transmitter scheme uses a PSK signalling scheme with

$S_1(t) = A_c \cos \omega_c t \;; 0 \le t \le T_b \;; T_b = 0.2 msec.$

$S_2(t) = -A_c \cos \omega_c t \;; 0 \le t \le T_b \;; \omega_c = 10\pi/T_b.$

The carrier amplitude at the receiver input is 1 mV and the PSD of AWGN at input is 10^{-11} watts/Hz. Calculate the average bit error rate of the receiver.

Sol Given PSK system

$$S_1(t) = A_c \cos \omega_c t$$

$$S_2(t) = -A_c \cos \omega_c t$$

$$P(t) = S_1(t) - S_2(t) = 2 A_c \cos \omega_c t$$

$T_b = 0.2\ m\,sec$

$\omega_c = \frac{10\pi}{T_b}$

$2\pi f_c = \frac{10 \times \pi}{0.2 \times 10^{-3}}$

$\Rightarrow \boxed{f_c = 25\ KHz}$

& AWGN PSD.

$N_0 = \eta/2 = 10^{-11}$ watts/Hz

$\therefore \eta = 2 \times 10^{-11}$.

& $f_b = \frac{1}{T_b} = r_b = \frac{1}{0.2 \times 10^{-3}} = 5000$

$\therefore \boxed{r_b = 5000\ bits/sec.}$

$\therefore$ Probability of error in PSK scheme is

$$P_e = Q\left(\frac{\gamma_{max}}{2}\right)$$

where
$$\gamma_{max}^2 = \frac{2}{\eta}\int_0^{T_b} p^2(t)\cdot dt$$

$$= \frac{2}{\eta}\int_0^{T_b} [2A_c \cos\omega_c t]^2 dt$$

$$= \frac{2}{\eta} \times 4A_c^2 \cdot \int_0^{T_b} \cos^2\omega_c t\ dt$$

$$= \frac{8A_c^2}{\eta} \cdot \int_0^{T_b}\left[\frac{1+\cos 2\omega_c t}{2}\right] dt$$

$$= \frac{4A_c^2}{\eta}\left[\int_0^{T_b} 1\cdot dt + \int_0^{T_b} \cos 2\omega_c t\ dt\right]$$

$$= \frac{4A_c^2}{\eta} T_b + \underbrace{\frac{\sin 2\omega_c t}{2\omega_c}\Big|_0^{T_b}}_{0}$$

$$\gamma_{max}^2 = \frac{4A_c^2 T_b}{\eta} \Rightarrow \gamma_{max} = \sqrt{\frac{4A_c^2 T_b}{\eta}}$$

$$\frac{\gamma_{max}}{2} = \sqrt{\frac{4A_c^2 T_b}{4\cdot\eta}} = \sqrt{\frac{A_c^2 T_b}{\eta}}$$

$$= \sqrt{\frac{(1 \times 10^{-3})^2 \times (0.2 \times 10^{-3})}{2 \times 10^{-11}}}$$

$$= \sqrt{\frac{\cancel{2} \times 10^{-10}}{\cancel{2} \times 10^{-11}}}$$

$$= \sqrt{10}$$

$\therefore$ Probability of error $\boxed{P_e = Q(\sqrt{10}) = 0.0008}$

• The average bit error rate $= r_b \times P_e = 5000 \times 0.0008 = 4$

$\therefore$ $\boxed{\text{Average bit error rate} = 4\ bits/sec.}$

(3) Binary data has to be transmitted over a telephone link that has a usable bandwidth of 3000 Hz and a maximum achievable signal to noise ratio of 6 dB at its output.

(a) Determine the maximum signalling rate and P_e if a coherent ASK scheme is used for tx'ing binary data through the channel.

(b) If the data rate is maintained at 300 bits/sec, calculate the error probability.

Sol: Given data ASK system.

let ASK signal requires a bandwidth of $3r_b$ Hz & bits/sec.

ie $B.W = 3000$ Hz

$3r_b = 3000$

$r_b = 1000$ bits/sec $\qquad (SNR)_o = 6$ dB.

(a) $T_b = 1/r_b = 1$ msec.

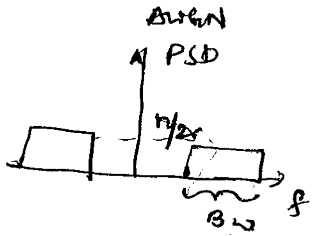
ASK: $S(t) = \begin{cases} A_c \cos w_c t & ; \text{logic } 1 \\ 0 & ; \text{logic } 0 \end{cases}$

$\therefore$ Average signal power $S_o = \dfrac{A_c^2/2}{2} = \dfrac{A_c^2}{4}$

Average Noise power $N_o = 2 \times PSD \times B.W$

$= 2 \times \dfrac{\eta}{2} \times 3000$

$\boxed{N_o = 3000\eta}$



$\therefore$ Signal to noise ratio $(SNR)_o = 6$ dB

$\Rightarrow \dfrac{\text{Avg signal power}}{\text{Noise power}} = 6$ dB $\qquad 6\text{ dB} = 10\log_{10}\dfrac{P_o}{P_i}$

$\log_{10}\dfrac{P_o}{P_i} = 0.6$

$\dfrac{P_o}{P_i} = 10^{0.6}$

$= 3.98$

$\simeq 4$

$\Rightarrow \dfrac{\frac{A_c^2}{4}}{3000\eta} = 4$

$\Rightarrow \dfrac{A_c^2}{12000\eta} = 4$

$\Rightarrow \dfrac{A_c^2}{\eta} = 48000$

$\Rightarrow \dfrac{A_c^2}{4\eta} \times T_b = \dfrac{48000}{4} \times 1\times10^{-3} = \dfrac{48}{4} = 12$

$\therefore$ The probability of error in ASK is

$$P_e = Q\left(\sqrt{\frac{E_b}{2\eta}}\right) \text{ \& } Q\left(\sqrt{\frac{A_c^2 T_b}{4\eta}}\right)$$

$$\therefore P_e = Q(\sqrt{12}) = Q(3.464) \cong 0.0003$$

$$\boxed{P_e = 0.0003} \Rightarrow \boxed{P_e = 3\times10^{-4}}$$

The maximum signalling rate $r_b = 1000$ bits/sec.

(b) If the bit rate is reduced to 300 bits/sec.

$\therefore r_b = 300$ bits/sec

$$T_b = \frac{1}{r_b} = 3.33 \text{ msec.}$$

$$\frac{A_c^2 T_b}{4\eta} = \frac{48000}{4} \times 3.33\times10^{-3}$$

$$= 12 \times 3.33$$

$$\cong 40$$

$\therefore$ Probability of error $P_e = Q\sqrt{\frac{A_c^2 T_b}{4\eta}}$

$$P_e = Q(\sqrt{40})$$

$$= Q(6.326)$$

$$\cong 10^{-10}$$

$$\therefore \boxed{P_e = 10^{-10}.}$$

All The Best
ssssssss

Digital Communications

THE END

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S. Gopal. M.Tech.