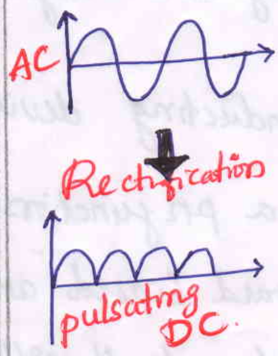
Unit - 2

Rectifiers, Filters and Voltage regulators.

Syllabus:



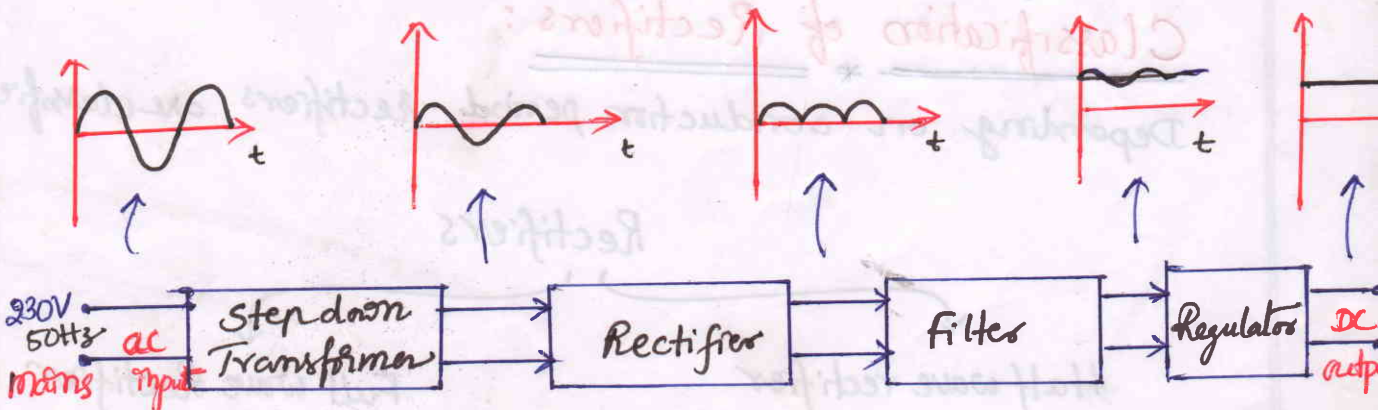
The p-n junction as a rectifier. A Half wave rectifier - ripple factor, A Full wave rectifier - A bridge rectifier, Harmonic components in a rectifier ckt, Inductor filters, Capacitor filters, L-section filters. π-section filters, use of zener diode as a regulator (Simple circuit).

Introduction to Rectifiers:

All electronic ckts need dc power supply either from battery or power pack units. It may not be economical and convenient to depend upon battery power supply. Hence many electronic equipment contain ckts which converting the ac supply voltage into dc supply voltage. The unit containing these circuits is called Linear Mode Power Supply. A power supply unit that converts dc into dc or dc into ac is called Switched Mode power Supply. → Converter → inverter.

Block Diagram of LMPS:

Linear Mode Power Supply.



By using LMPS the power supply should be protected in the event of short ckt on the load side & Temperature changes should be minimum.

p-n junction Diode as a Rectifier:

Rectifier: A rectifier is an electronic device which converts the bidirectional flow of current into unidirection, and it is the device which converts a.c voltage to pulsating d.c voltage using one or more p-n junction diodes.

PN junction diode is a unidirectional conducting device.

ie If an alternating voltage is applied across a pn junction diode during +ve half cycle the diode will be forward biased and will conduct successfully. While during -ve half cycle it will be reverse biased and will not conduct at all.

Thus conduction occurs only during positive half cycle.

If the resistance is connected in series with the diode, the output voltage across the resistance will be unidirectional ie d.c.

Hence pn junction diode acts as a rectifier converting alternating voltage to a pulsating d.c voltage.

ie Rectifiers offers low resistance current in one direction and high resistance in opposite direction.

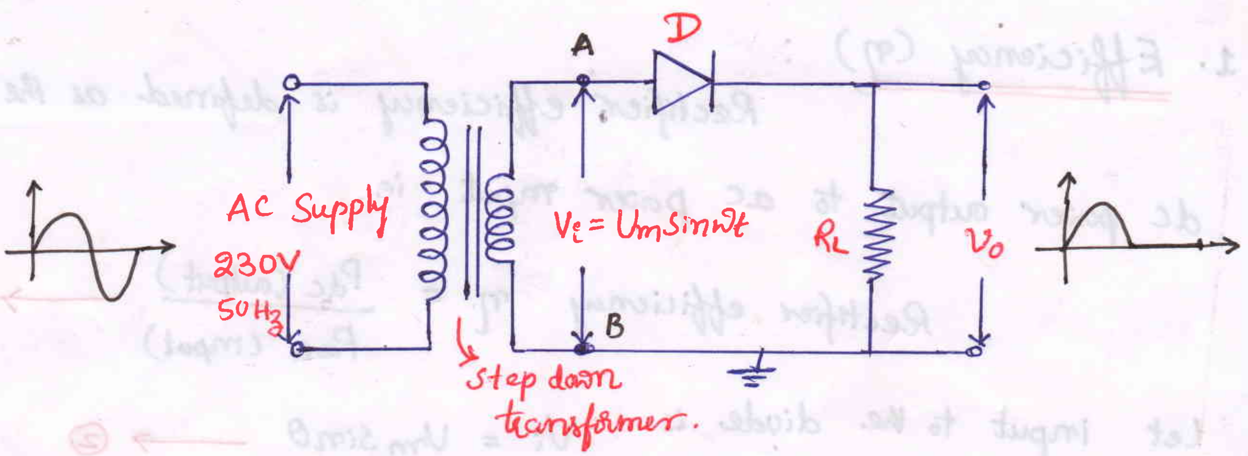
The output of rectifier is pulsating that consists of desired dc components and unwanted ac components.

Classification of Rectifiers:

Depending on conduction period rectifiers are classified as

- Rectifiers
  - Half wave rectifier (using only one diode).
  - Full wave Rectifier
    - Centre Tapped (using two diodes)
    - Bridge (using 4 diodes)

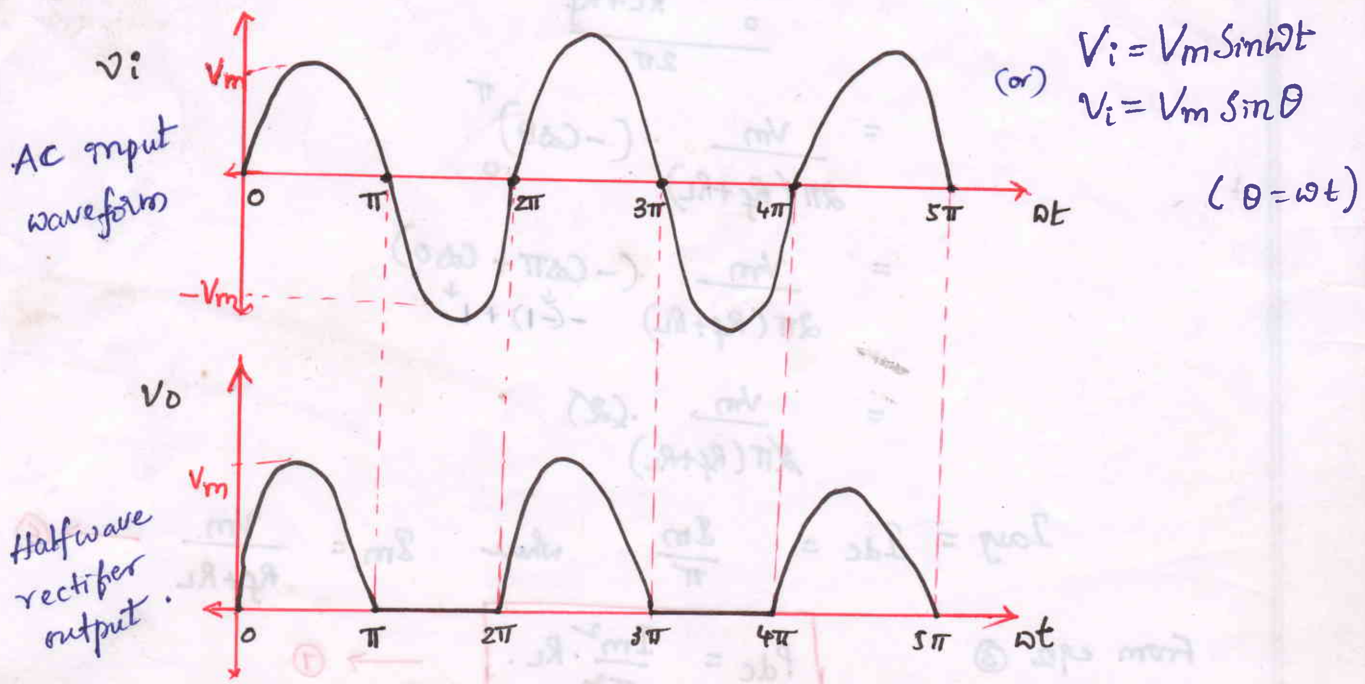
Halfwave Rectifier:



The half wave rectifier circuit consists of a step down transformer and a diode with a load resistance connected in series.

Operation:

1. During +Ve half cycle of input a.c supply, point A becomes +Ve w.r.t. B the diode becomes forward biased and it allows the current to flow through it & hence we get output across R_L.
2. During −Ve half cycle of input a.c supply the point A becomes −Ve w.r.t. B, the diode becomes reverse biased and it blocks the current flow. hence no output across R_L.
3. i.e. The halfwave rectifier ckt conducts for only half of input cycle ie from 0 to π as shown in below.



Halfwave Rectifier Parameters :

1. Efficiency (η) :

Rectifier efficiency is defined as the ratio of dc power output to ac power input ie.

$$\text{Rectifier efficiency } \eta = \frac{P_{dc}\ (\text{output})}{P_{ac}\ (\text{input})} \longrightarrow ①$$

Let input to the diode is $V_i = V_m \sin\theta \longrightarrow ②$

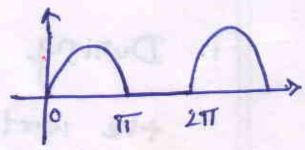
where V_m - maximum or peak value of input

R_f - diode forward resistance

R_L - load resistance.

Power $P = I^2 R$

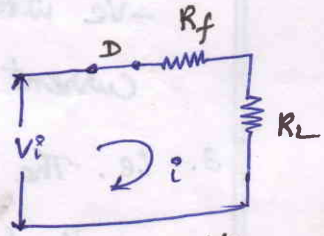
ie $P_{dc} = I_{dc}^2 R_L \longrightarrow ③$



But $I_{dc} = I_{avg} \longrightarrow ④$

I_{avg} can be evaluated as $I_{avg} = \dfrac{\text{Area of the Curve}}{\text{Base}}$.

ie $$I_{avg} = \frac{\int_0^{\pi} i\, d\theta}{2\pi} \longrightarrow ⑤$$



$$i = \frac{V_i}{R_f + R_L}$$

$$= \frac{\int_0^{\pi} \frac{V_m \sin\theta}{R_L + R_f}}{2\pi} \quad (\because \text{From } ②)$$

$$= \frac{V_m}{2\pi(R_f + R_L)} \cdot (-\cos\theta)_0^{\pi}$$

$$= \frac{V_m}{2\pi(R_f + R_L)} (-\underset{-(-1)}{\cos\pi} + \underset{+1}{\cos 0})$$

$$= \frac{V_m}{\cancel{2}\pi(R_f + R_L)} \cdot (\cancel{2})$$

$$I_{avg} = I_{dc} = \frac{I_m}{\pi} \quad \text{where} \quad I_m = \frac{V_m}{R_f + R_L} \longrightarrow ⑥$$

From eqn ③ $$\boxed{P_{dc} = \frac{I_m^2}{\pi^2} \cdot R_L .} \longrightarrow ⑦$$

$$P_{ac} = I_{RMS}^2 \cdot (R_f + R_L) \rightarrow (8)$$

where The RMS means Root mean square. ie finding mean and then finding square root. Hence RMS value of load current can be obtained as

$$I_{RMS} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i^2 \, d\theta}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{\pi} \left(\frac{V_m \sin\theta}{R_f + R_L}\right)^2 d\theta} \qquad \left(\because I = \frac{V_i}{R_f+R_L},\ V_i = V_m \sin\theta\right)$$

$$= \sqrt{\frac{1}{2\pi} \cdot \frac{V_m^2}{(R_f+R_L)^2} \cdot \int_0^{\pi} \sin^2\theta \, d\theta}$$

$$= \sqrt{\frac{V_m^2}{2\pi (R_f+R_L)^2} \cdot \int_0^{\pi} \left(\frac{1-\cos 2\theta}{2}\right) d\theta} \qquad \left(\because \sin^2\theta = \frac{1-\cos 2\theta}{2}\right)$$

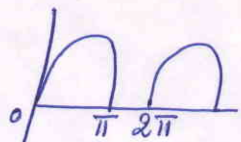
$$= \sqrt{\frac{I_m^2}{2\pi} \cdot \left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{\pi}} \qquad \left(\because I_m = \frac{V_m}{R_f+R_L}\right)$$

$$= \sqrt{\frac{I_m^2}{2\pi} \cdot \left(\frac{\pi}{2} - 0\right)}$$

$$= \sqrt{\frac{I_m^2}{2\cancel{\pi}} \times \frac{\cancel{\pi}}{2}}$$

$$= \sqrt{\frac{I_m^2}{4}}$$

$$\boxed{I_{RMS} = \frac{I_m}{2}} \rightarrow (9)$$



For π to 2π the output is zero so take the limits only from 0 to π

$$P_{ac} = I_{RMS}^2 (R_f + R_L) = \frac{I_m^2}{4} \cdot (R_f + R_L)$$

$$\therefore \boxed{P_{ac} = \frac{I_m^2}{4}(R_f + R_L)} \rightarrow (10)$$

$$\text{Efficiency } \eta = \frac{P_{dc}}{P_{ac}} = \frac{\frac{\cancel{I_m^2}}{\pi^2} \cdot R_L}{\frac{\cancel{I_m^2}}{4} \times (R_f + R_L)} = \frac{4}{\pi^2} \cdot \left(\frac{R_L}{R_L + R_f}\right)$$

Efficiency of Halfwave rectifier.

$$\therefore \boxed{\eta = \frac{4}{\pi^2} \cdot \left(\frac{R_L}{R_L + R_f}\right) = \frac{0.406 R_L}{R_L + R_f}}$$

$$\boxed{\eta = 40.6\,\%}$$

* $R_f << R_L$ or $R_L >> R_f$ must condition.

ie 59.4 % of the input power will be wasted.

Ripple Factor:

The output of the rectifier ckt is not pure d.c but it is pulsating d.c (ie ac+dc components) is called ripples. The measure of such ripples present in the output with the help of factor is called ripple factor.

Definition: The ratio of R.M.S value of the ac components in the output to the average or d.c components present in the output.

It is denoted as $\gamma = \frac{I_{ac}}{I_{dc}}$.

I_{RMS} = RMS value of total output current

$$I_{RMS} = \sqrt{I_{ac}^2 + I_{dc}^2}$$

$$\Rightarrow I_{ac} = \sqrt{I_{RMS}^2 - I_{dc}^2}$$

$\therefore$ Ripple factor $\gamma = \frac{\text{RMS value of a.c components of output.}}{\text{average or dc components of output.}}$

$$\gamma = \frac{I_{ac}}{I_{dc}}$$

$$\gamma = \frac{\sqrt{I_{RMS}^2 - I_{dc}^2}}{I_{dc}}$$

General Expression for ripple factor.

$$\boxed{\gamma = \sqrt{\left(\frac{I_{RMS}}{I_{DC}}\right)^2 - 1}}$$

For Half wave rectifier $I_{RMS} = \frac{I_m}{2}$, $I_{dc} = \frac{I_m}{\pi}$

$$\gamma = \sqrt{\frac{\left(\frac{I_m}{2}\right)^2}{\left(\frac{I_m}{\pi}\right)^2} - 1}$$

$$= \sqrt{\frac{\pi^2}{4} - 1}$$

$$= \sqrt{1.4674} = 1.211$$

$$\boxed{\text{Ripple factor } \gamma = 1.21}$$

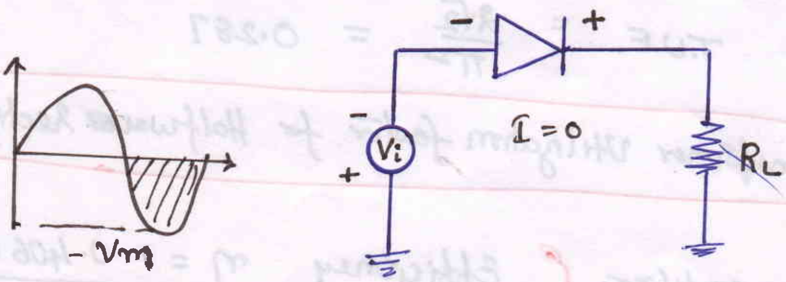
ie The amount of ac components present in the output is 1.21 times of dc components. ie ac components exceeds dc components.

Peak inverse Voltage (PIV):

The peak inverse Voltage is the peak voltage across the diode in the reverse direction. ie when the diode is reverse biased.

In half wave rectifier, the load current is ideally zero when the diode is reverse biased & hence maximum value of the voltage that can exist across the diode is V_m.

$\therefore$ PIV of diode = max. value of Secondary voltage V_m.



$\boxed{PIV = V_m}$

Fig: PIV rating of diode.

Transformer Utilization Factor (TUF):

The factor which indicates how much is the utilization of the transformer in the ckt is called Transformer Utilization factor.

Definition: The ratio of dc power delivered to the load to the ac power rating of the transformer.

$$\text{AC power rating of transformer} = V_{RMS} I_{RMS}$$
$$= \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{2}$$
$$P_{ac\,(rated)} = \frac{V_m \cdot I_m}{2\sqrt{2}}$$

$$\text{DC power delivered to the load} = I_{dc}^2 \cdot R_L$$
$$= \left(\frac{I_m}{\pi}\right)^2 \cdot R_L \qquad \left(\because I_{dc} = \frac{I_m}{\pi}\right)$$
$$P_{dc} = \frac{I_m^2}{\pi^2} \cdot R_L$$

$\therefore$ ie Secondary voltage is purely sinusoidal hence its rms value is $\sqrt{2}$ times ie $V_{rms} = V_m/\sqrt{2}$.

$$\therefore \quad T.U.F = \frac{\text{DC power delivered to the load}}{\text{AC power rating of the transformer}}$$

$$= \frac{\frac{I_m^2}{\pi^2} \cdot R_L}{\frac{V_m \cdot I_m}{2\sqrt{2}}}$$

& $I_m = \frac{V_m}{R_L + R_f} = \frac{V_m}{R_L}$

↳ R_f is Very small

$$= \frac{V_m^2}{\pi^2 \cdot R_L^2} \cdot R_L \times \frac{2\sqrt{2}}{\frac{V_m \cdot V_m}{R_L}}$$

$$= \frac{2\sqrt{2} \cdot R_L}{\pi^2 \cdot R_L}$$

$$T.U.F. = \frac{2\sqrt{2}}{\pi^2} = 0.287$$

$$\boxed{\text{Transformer Utilization factor for Halfwave Rectifier} = 0.287}$$

Hence:

For Half wave rectifier.

Efficiency $\eta = \frac{0.406 R_L}{(R_L + R_f)}$ or 40.6%

$I_{dc} = I_m/\pi$, $I_{rms} = \frac{I_m}{2}$

Ripple factor $\gamma = 1.21$

Peak inverse Voltage $PIV = V_m$.

Transformer Utilization factor TUF = 0.287.

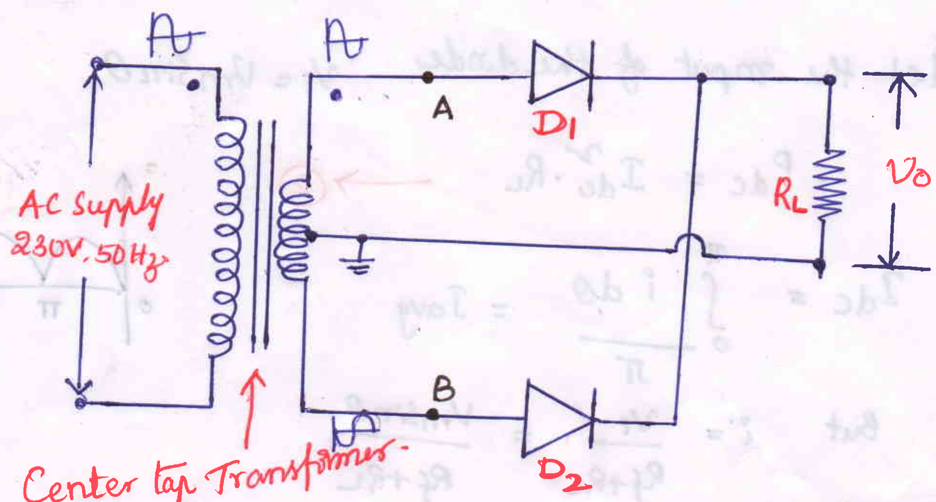
Disadvantages of Half Wave Rectifier:

1. The ripple factor of halfwave rectifier is 1.21 which is quite high. So the output contains lot of varying components.
2. The ckt has low transformer utilization factor (0.287), showing that the transformer is not fully utilized.
3. The max. theoretical rectification efficiency is found to be 40%. The practical value will be less than this. So, the halfwave rectifier ckt is quite inefficient.
4. The rectifier ckt produces output only in half cycle of input. Hence half wave rectifier ckt is not used in any electronic ckts.

Full wave Rectifier :

It is of two types. 1. Centre Tapped.
2. Bridge.

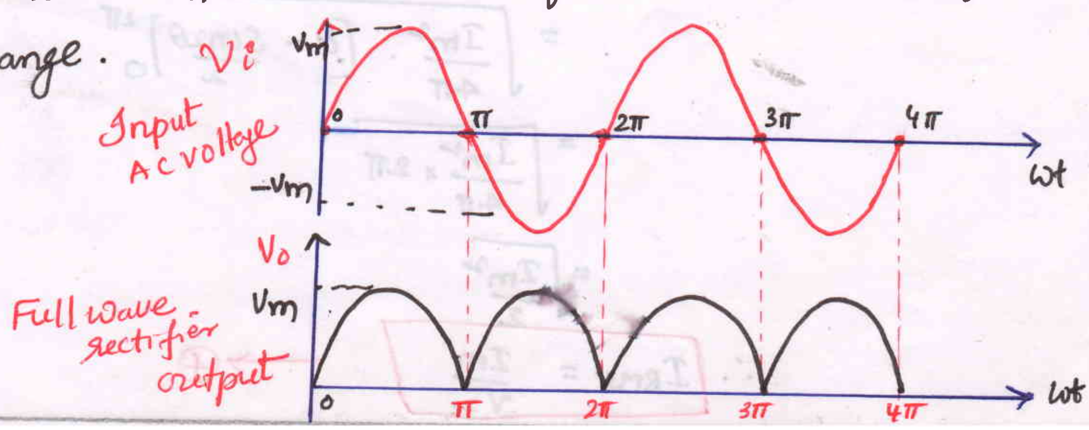
Center Tapped Full wave Rectifier :



The full wave rectifier conducts during both positive and negative half cycles of the input A.C supply. using two diodes. these diodes feed a common load R_L with the help of centre tap transformer.

Operation :

1. During the +Ve half cycle of a.c input voltage, point A becomes +Ve w.r.t. B hence diode D_1 will be forward biased & will conduct while diode D_2 will be reverse biased & will not conduct & it is open ckted.
2. During the -Ve half cycle of a.c input voltage point A becomes -Ve & point B becomes +Ve. The diode D_2 conducts being forward biased while D_1 does not conduct being reverse biased.
3. In either the case the current through load resistor R_L is the same. Hence direction of Vo and polarity of Vo does not change.



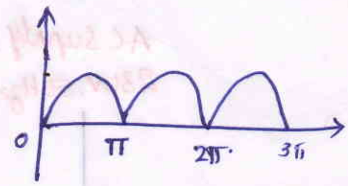
Fullwave Rectifier Parameters :

Efficiency (η) :

Rectifier efficiency $\eta = \dfrac{P_{dc}\,(\text{output})}{P_{ac}\,(\text{input})}$ → (1)

let the input of the diode $V_i = V_m \sin\theta$

$$P_{dc} = I_{dc}^2 \cdot R_L \rightarrow (2)$$



$$I_{dc} = \frac{\int_0^{\pi} i \, d\theta}{\pi} = I_{avg}$$

But $i = \dfrac{V_i}{R_f + R_L} = \dfrac{V_m \sin\theta}{R_f + R_L}$

$$I_{dc} = \int_0^{\pi} \frac{V_m \sin\theta}{\pi (R_f + R_L)}$$

$$= \frac{V_m}{\pi (R_f + R_L)} \cdot \int_0^{\pi} \sin\theta .$$

$$= \frac{V_m}{\pi (R_f + R_L)} \cdot (-\cos\theta)_0^{\pi}$$

$$= \frac{I_m}{\pi} (1+1) \quad \left(\because \frac{V_m}{R_f + R_L} = I_m .\right)$$

$$\boxed{I_{dc} = \frac{2 I_m}{\pi}} \Rightarrow \boxed{P_{dc} = \frac{4 I_m^2}{\pi^2} \cdot R_L .} \rightarrow (3)$$

$$P_{ac} = I_{RMS}^2 \cdot (R_f + R_L) .$$

$$I_{Rms} = \sqrt{\frac{1}{2\pi} \int_0^{2\pi} i^2 \, d\theta} = \sqrt{\frac{1}{2\pi} \int_0^{2\pi} \frac{V_m^2 \sin^2\theta}{(R_f + R_L)^2} \cdot d\theta .}$$

$$= \sqrt{\frac{I_m^2}{2\pi} \int_0^{2\pi} \left[\frac{1 - \cos 2\theta}{2}\right] d\theta .}$$

$$= \sqrt{\frac{I_m^2}{4\pi} \cdot \left[\theta - \frac{\sin 2\theta}{2}\right]_0^{2\pi}}$$

$$= \sqrt{\frac{I_m^2}{4\pi} \times 2\pi}$$

$$= \sqrt{\frac{I_m^2}{2}}$$

$$\therefore \boxed{I_{RMS} = \frac{I_m}{\sqrt{2}}} \rightarrow (4)$$

$$P_{ac}\,(\text{input}) = I_{RMS}^2 \cdot (R_f + R_L).$$

$$P_{ac} = \frac{I_m^2}{2}(R_f + R_L) \longrightarrow (5)$$

$$\text{Efficiency } \eta = \frac{P_{dc}\,(\text{output})}{P_{ac}\,(\text{input})}$$

$$\eta = \frac{\frac{4 I_m^2}{\pi^2} \cdot R_L}{\frac{I_m^2}{2} \cdot (R_f + R_L)}$$

$$\eta = \frac{8}{\pi^2} \cdot \left(\frac{R_L}{R_f + R_L}\right) = 0.812 \left(\frac{R_L}{R_L + R_f}\right).$$

$$\boxed{\text{Efficiency } \eta = 0.812 \cdot \left(\frac{R_L}{R_L + R_f}\right) \text{ or } 81.2\%.}$$

Ripple factor:

$$\text{Ripple factor } \gamma = \frac{\text{RMS value of ac components of output}}{\text{average or dc components of output}}.$$

$$I_{RMS} = \sqrt{I_{ac}^2 + I_{dc}^2}$$

$$I_{ac} = \sqrt{I_{RMS}^2 - I_{dc}^2}$$

$$\gamma = \frac{\sqrt{I_{RMS}^2 - I_{dc}^2}}{I_{dc}}$$

$$\gamma = \sqrt{\left(\frac{I_{RMS}}{I_{dc}}\right)^2 - 1}$$

For Full wave rectifier $I_{RMS} = \frac{I_m}{\sqrt{2}}$, $I_{dc} = \frac{2 I_m}{\pi}$

$$\gamma = \sqrt{\frac{\frac{I_m^2}{2}}{\frac{4 I_m^2}{\pi^2}} - 1}$$

$$\gamma = \sqrt{\frac{\pi^2}{8} - 1}$$

$$\gamma = 0.481 \quad \therefore \quad \boxed{\text{Ripple factor } \gamma = 0.481}$$

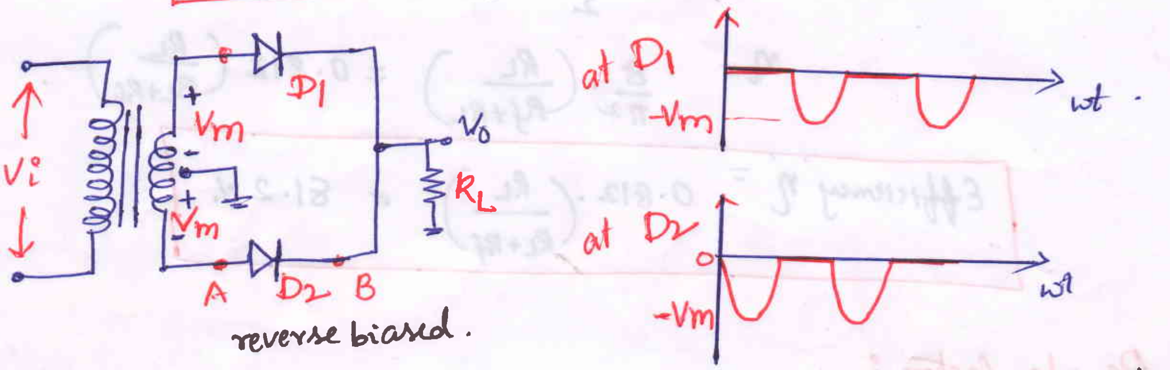
$\therefore$ The DC components are more compared to ac components.

Peak inverse Voltage (PIV) :

It can be observed when the diode is reverse biased.

ie when diode D_2 is reverse biased point A is at $-V_m$ with respect to ground while point B is at $+V_m$ with respect to Gnd. neglecting drop of diode. Thus total peak voltage across diode is $2V_m$.

ie $\boxed{\text{Peak inverse voltage} = 2V_m}$ for full wave rectifier.



reverse biased.

where V_m is max. value of ac voltage across half the secondary of transformer.

Transformer Utilization Factor (T.U.F):

In fullwave rectifier, the secondary current flows through each half seperately in every half cycle. Hence TUF is calculated for primary and secondary windings seperately and then the average TUF is determined.

$$\text{Secondary T.U.F} = \frac{\text{DC power to the load}}{\text{AC power ratings of Secondary}}$$

$$= \frac{I_{dc}^2 \cdot R_L}{V_{RMS} \cdot I_{RMS}} = \frac{\left(\frac{2I_m}{\pi}\right)^2 \cdot R_L}{\frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}}} \qquad \left(\because V_{RMS} = \frac{V_m}{\sqrt{2}},\ I_{RMS} = \frac{I_m}{\sqrt{2}}\right)$$

$$= \frac{4V_m^2}{\pi^2 \cdot R_L^2} \cdot R_L \times \frac{\sqrt{2} \times \sqrt{2}}{V_m \cdot V_m} \times R_L \qquad (\because V_m = I_m R_L)$$

$$= \frac{8}{\pi^2} = 0.812$$

$$\boxed{\text{Secondary T.U.F} = 0.812}$$

The primary of the transformer is feeding two halfwave rectifiers.

ie T.U.F for primary winding = 2x T.U.F of half wave rectifier.

$= 2 \times 0.287$

$\boxed{\text{Primary T.U.F.} = 0.574}$

Average T.U.F for full wave rectifier ckt = $\frac{\text{Primary T.UF} + \text{Secondary T.UF}}{2}$

$= \frac{0.574 + 0.812}{2}$

$= 0.693$

$\therefore$ $\boxed{\text{Average TUF for full wave rectifier} = 0.693}$.

Hence for Fullwave rectifier

Efficiency $\eta = 0.812\left(\frac{R_L}{R_L+R_f}\right)$ or 81.2%.

Ripple factor $\gamma = 0.481$, $I_{dc} = \frac{2I_m}{\pi}$, $I_{RMS} = \frac{I_m}{\sqrt{2}}$

Peak inverse voltage $= 2V_m$.

Transformer utilization factor. Primary $= 0.574$
Secondary $= 0.812$
Average $= 0.693$.

Advantages of Full wave Rectifier:

1. The dc load voltage and current are more than half wave.
2. No dc current through transformer windings hence no possibility of saturation.
3. T.U.F is better as transformer losses are less.
4. The efficiency is higher.
5. The large d.c power output.
6. The ripple factor is less.

Disadvantages of Full wave rectifier:

1. The PIV rating of diode is higher.
2. Higher PIV diodes are larger in size and costlier.
3. The cost of centre tap transformer is higher.

Voltage Regulation:

The voltage regulation is the factor which tells us about the change in the d.c output voltage as load changes from no load to full load condition.

If $(V_{dc})_{NL}$ = dc voltage on no load.

$(V_{dc})_{FL}$ = d.c voltage on Full load then

$$\boxed{\text{Voltage regulation} = \frac{(V_{dc})_{NL} - (V_{dc})_{FL}}{(V_{dc})_{FL}}}$$

* Less the value of voltage regulation better is the performance of rectifier ckt.

For Half wave Rectifier:

$$(V_{dc})_{NL} = \frac{V_m}{\pi}$$

$$(V_{dc})_{FL} = I_{DC} \cdot R_L$$

$$= \frac{I_m}{\pi} \cdot R_L$$

$$\& \quad I_m = \frac{V_m}{R_f + R_L}$$

$$\boxed{V_{dc} = \frac{V_m}{\pi} - I_{DC} R_f}$$

Regulation

$$\%R = \frac{\frac{V_m}{\pi} - \frac{V_m \cdot R_L}{\pi (R_f + R_L)}}{\frac{V_m}{\pi}\left(\frac{R_L}{R_f + R_L}\right)}$$

$$= \frac{1 - \frac{R_L}{R_f + R_L}}{\frac{R_L}{R_f + R_L}} \times 100$$

$$\boxed{\%R = \frac{R_f}{R_L} \times 100}$$

For Full wave rectifier:

$$(V_{dc})_{NL} = \frac{2V_m}{\pi}$$

$$(V_{dc})_{FL} = I_{DC} \cdot R_L$$

$$= \frac{2I_m}{\pi} \cdot R_L$$

$$\& \quad I_m = \frac{V_m}{R_f + R_L}$$

$$\boxed{V_{dc} = \frac{2V_m}{\pi} - I_{DC} R_f}$$

Regulation

$$\%R = \frac{\frac{2V_m}{\pi} - \frac{2V_m}{\pi (R_f + R_L)} \cdot R_L}{\frac{2V_m}{\pi} \cdot \left(\frac{R_L}{R_f + R_L}\right)}$$

$$= \frac{1 - R_L/R_f + R_L}{R_L/R_f + R_L}$$

$$= \frac{R_f}{R_L} \times 100$$

$$\therefore \boxed{\%R = \frac{R_f}{R_L} \times 100}$$

where R_f - diode forward resistance

R_L - Load resistance

Hence As output voltage decreases as load increases from no load to full load.

Bridge Rectifier :

The bridge rectifier ckt is essentially a full-wave rectifier using four (4) diodes, forming the four arms of an electrical brid[ge]. The main advantage of this ckt is that it does not require a centr[e] tap on the secondary winding of the transformer.

Hence wherever possible, ac voltage can be directly applied to the bridge.

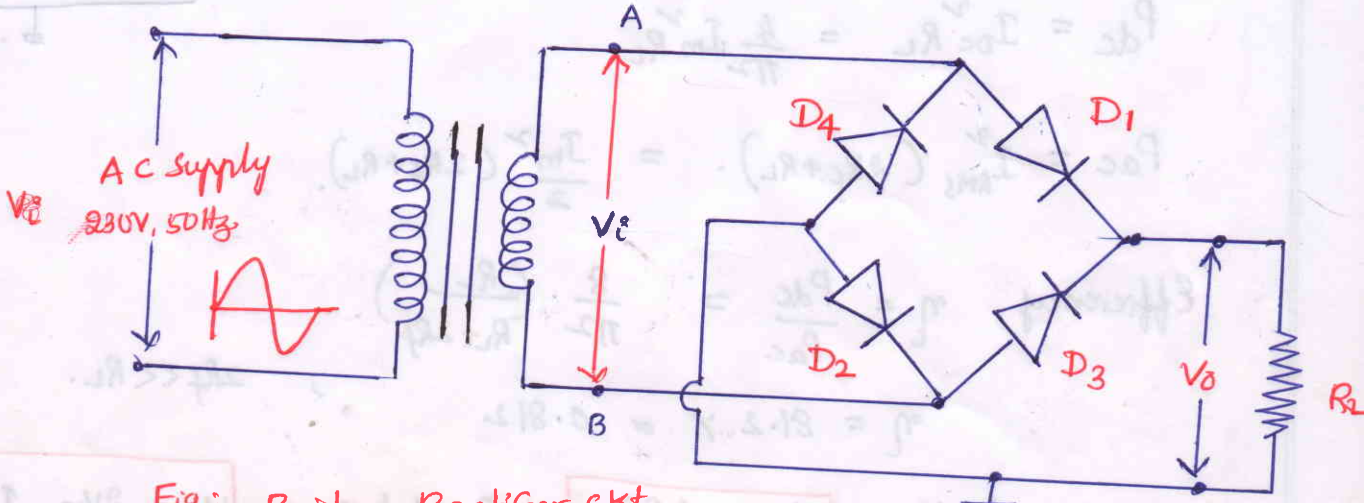
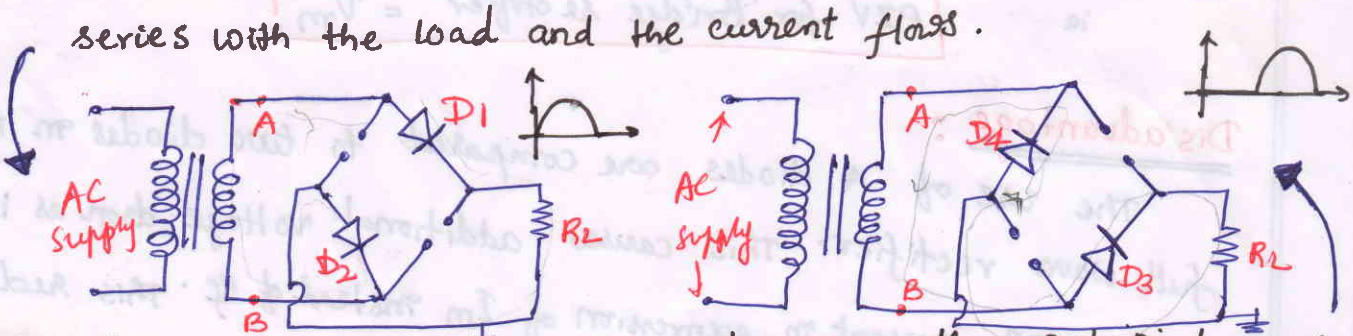


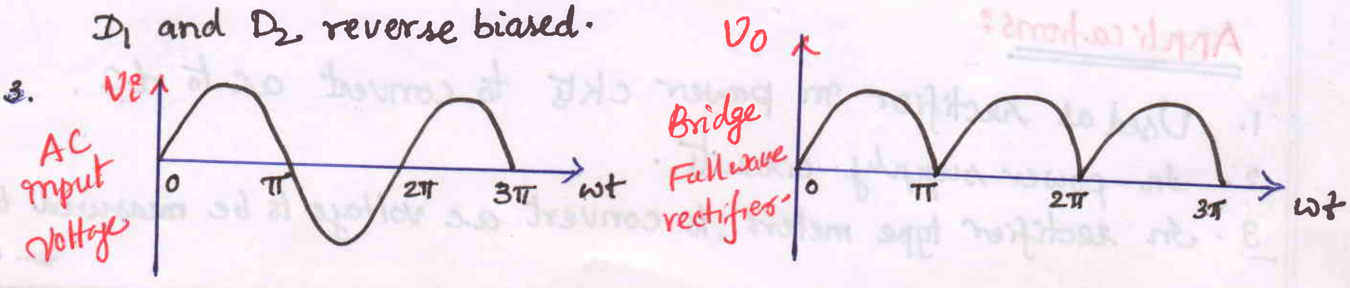
Fig: Bridge Rectifier ckt.

Operation :

1. During the +Ve half of ac input voltage. The point A of Second[ary] becomes positive. The diodes D_1 and D_2 will be forward biased. while D_3 and D_4 reverse biased. The two diodes D_1 & D_2 conduct i[n] series with the load and the current flows.



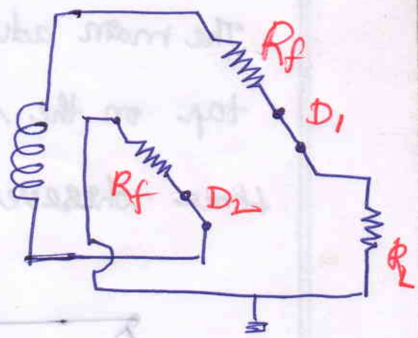
2. During the -Ve half of a-c input voltage the point B becomes positive diodes D_3 and D_4 are forward biased & Conduct. while D_1 and D_2 reverse biased.

3. 

I_{dc} and I_{RMS} are same as derived for the full wave rectifier.

ie $I_{dc} = \frac{2 I_m}{\pi}$ and $I_{RMS} = \frac{I_m}{\sqrt{2}}$

Max. value of the load current $\boxed{I_m = \frac{V_m}{R_L + 2R_f}}$



$V_{dc} = I_{dc} \cdot R_L = \frac{2V_m}{\pi}$.

$P_{dc} = I_{DC}^2 R_L = \frac{4}{\pi^2} I_m^2 R_L$.

$P_{ac} = I_{RMS}^2 (2R_f + R_L)$. $= \frac{I_m^2}{2} (2R_f + R_L)$.

Efficiency $\eta = \frac{P_{dc}}{P_{ac}} = \frac{8}{\pi^2} \cdot \left(\frac{R_L}{R_L + 2R_f}\right)$, $2R_f << R_L$.

$\eta = 81.2\%$ or 0.812

Ripple factor $\boxed{\gamma = 0.482}$ Regulation $\boxed{V_{dc} = \frac{2V_m}{\pi} - I_{DC} \cdot 2R_f.}$

Peack Inverse Voltage : The reverse voltage appearing across the reverse biased diodes is $2V_m$ but two diodes are sharing it. Hence PIV rating of the diode is V_m. not $2V_m$.

ie $\boxed{\text{PIV for Bridge rectifier} = V_m.}$

Dis advantage :

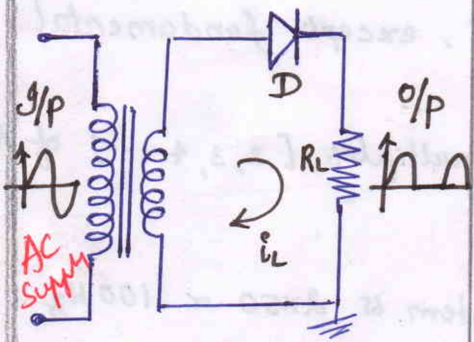
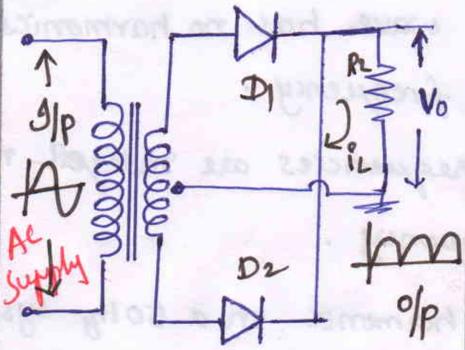
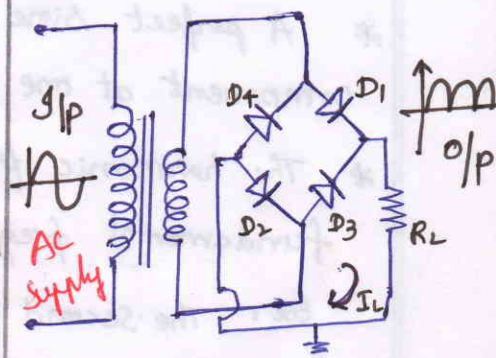
The use of 4 diodes are compared to two diodes in normal full wave rectifier. This causes additional voltage drop as indicated by term $2R_f$ present in expression of I_m instead of R_f. This reduces the output voltage.

Applications:

1. Used as rectifier in power ckts to convert ac to d.c.
2. In power supply circuits.
3. In rectifier type meters, to convert a.c voltage to be measured to d.c.

Comparision of Rectifier Ckts (Halfwave, Fullwave, Bridge):

Circuit Diagrams.

Half wave	Full wave	Bridge
		

SL NO	Parameter	Half wave	Full wave	Bridge
1.	Number of Diodes	1	2	4
2	Average DC Current (I_{DC})	$\frac{I_m}{\pi}$	$\frac{2I_m}{\pi}$	$\frac{2I_m}{\pi}$
3	Average DC Voltage (V_{dc})	$\frac{V_m}{\pi}$	$\frac{2V_m}{\pi}$	$\frac{2V_m}{\pi}$
4	R.M.S. Current (I_{RMS})	$\frac{I_m}{2}$	$\frac{I_m}{\sqrt{2}}$	$\frac{I_m}{\sqrt{2}}$
5	D.C power output (P_{dc})	$\frac{I_m^2 R_L}{\pi^2}$	$\frac{4}{\pi^2} I_m^2 R_L$	$\frac{4}{\pi^2} I_m^2 R_L$
6	A.C power input (P_{ac})	$\frac{I_m^2}{4}(R_f+R_L)$	$\frac{I_m^2}{2}(R_f+R_L)$	$\frac{I_m^2}{2}(2R_f+R_L)$
7	Efficiency (η)	40.6 %	81.2 %	81.2 %
8	Ripple factor (γ)	1.21	0.482	0.482
9	Max. Load Current (I_m)	$\frac{V_m}{R_f+R_L}$	$\frac{V_m}{R_f+R_L}$	$\frac{V_m}{2R_f+R_L}$
10	PIV rating of diode	V_m	$2V_m$	V_m
11	Ripple frequency	50 Hz	100 Hz	100 Hz.
12	Transformer utilization factor (TU.F)	0.287	0.693	0.812.
13	Regulation V_{dc}	$\frac{V_m}{\pi} - I_{dc}.R_f$	$\frac{2V_m}{\pi} - I_{dc}.R_f$	$\frac{2V_m}{\pi} - I_{dc}2R_f$.

Harmonic Components in a Rectifier Circuit :

* The term "harmonic" is defined as " a sinusoidal component of a periodic wave form or quantity possessing a frequency, which is an integral multiple of the fundamental frequency".
* A perfect sine wave has no harmonics, except fundamental component at one frequency.
* The harmonic frequencies are integer multiples [2, 3, 4 ....] of the fundamental frequency.

  Ex: The second harmonic on a 50 Hz system is 2x50 or 100 Hz.
* The amount of the harmonic Voltage and current levels that a System can tolerate is dependent on the equipment and the source.
* The sum of the fundamental and all the harmonics is called as Fourier Series: $i = \frac{I_m}{\pi} + \frac{I_m}{2}\sin\omega t - \frac{2I_m}{3\pi}\cos 2\omega t - \frac{2I_m}{15\pi}\cos 4\omega t - \cdots$
* This series can be viewed as a spectrum analysis where the fundamental frequency and the harmonic component are identified.
* The current waveform of a half wave rectifier ckt using single diode is given by

$I_{dc} = \frac{I_m}{\pi}$

$$\boxed{i = I_m\left[\frac{1}{\pi} + \frac{1}{2}\sin\omega t - \frac{2}{\pi}\sum_{K=2,4,6..}\frac{\cos K\omega t}{(K+1)(K-1)}\right]}$$

The angular frequency of the power supply is the lowest angular frequency. All other terms are the even harmonics of the power frequency.

* The full wave rectifier consists of two half wave rectifier ckts.

The currents $i_1(\alpha) = i_2(\alpha+\pi)$, So the total current of the full wave rectifier is $i = i_1 + i_2$ ie

$$\boxed{i = I_m\left[\frac{2}{\pi} - \frac{4}{\pi}\sum_{\substack{K=even\\K\neq 0}}\frac{\cos K\omega t}{(K+1)(K-1)}\right]}$$

$I_{dc} = \frac{2I_m}{\pi}$

ie

* The fundamental angular frequency is eliminated and the lowest frequency is the second harmonic term 2ω.

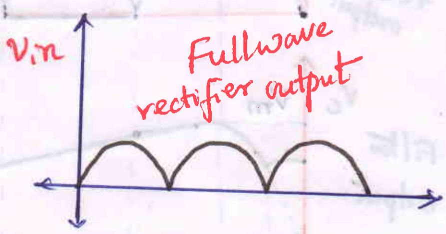
It avoids any de saturation of the transformer core that could give rise to additional harmonics at the output.

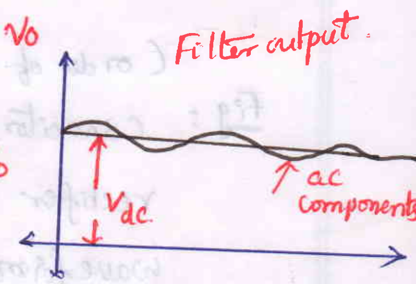
$$i = \frac{2I_m}{\pi} - \frac{4I_m}{3\pi}\cos 2\omega t - \frac{4I_m}{15\pi}\cos 4\omega t - \cdots$$

Filters :

The output of the rectifier contains dc components as well as ac components. Filters are used to minimise the undesirable a.c. ie ripple leaving only the d.c components to appear at the output of filter.

ie Filter is an electronic ckt composed of capacitor, inductor or combination of both and connected between the rectifier and the load so as to convert pulsating d.c to pure d.c.



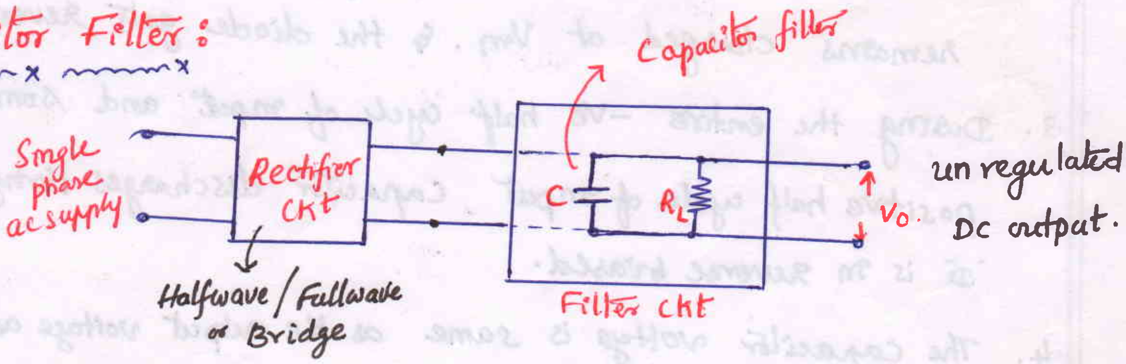


The output of filter is not pure dc but it also contains a small amount of ac components.

Types of Filters:

1. Capacitor Filter
2. Inductor or Choke filter.
3. LC or L-Section filter.
4. CLC or π-section filter.

① Capacitor Filter :



A capacitor 'C' is connected between the rectifier and the load as in shunt (parallel) as shown in above fig.

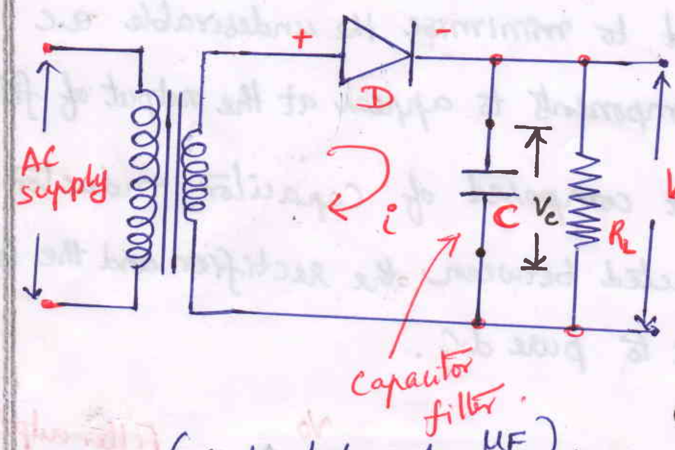
Reactance of Capacitor $X_C = \frac{1}{j\omega C} = \frac{1}{j2\pi f C} \Rightarrow \boxed{X_C \propto \frac{1}{f}}$

where ω - angular frequency $= 2\pi f$

f - frequency

C - capacitor $\quad 2\pi$ - constant.

Capacitor Filter with Halfwave rectifier:



(order of tens of μF).

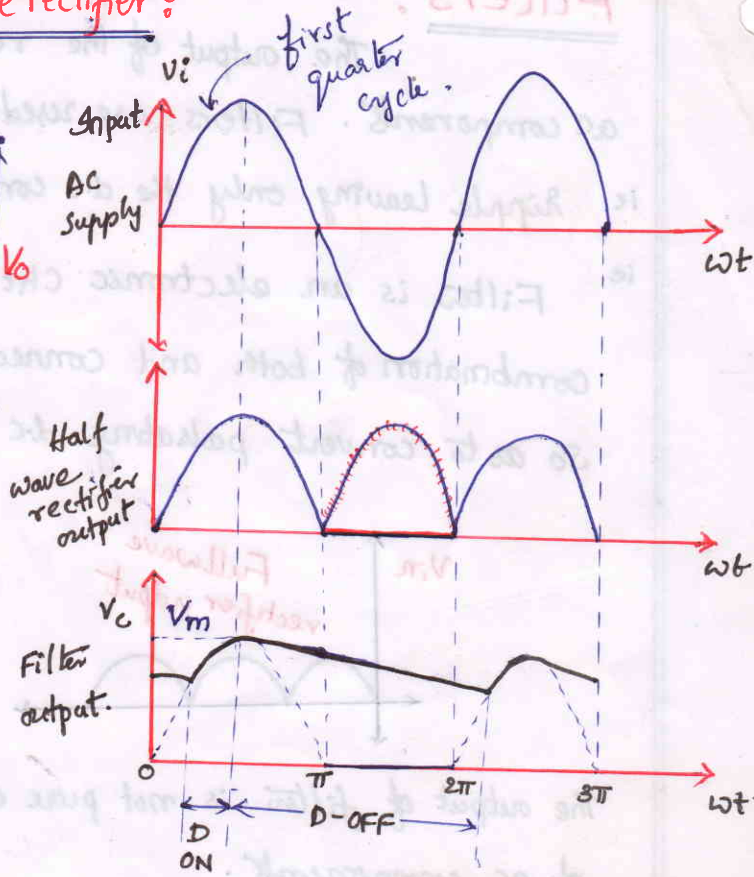


Fig: Capacitor filter with Halfwave rectifier and input & output waveforms.

Operation:

1. During the +ve quarter cycle of the input signal, the diode is forward biased, so capacitor charges to peak value of the input V_m. Practically capacitor C charges to $(V_m - 0.7)V$ due to forward voltage drop of diode. This initial charging happens only once, immediately when the power is turned ON.

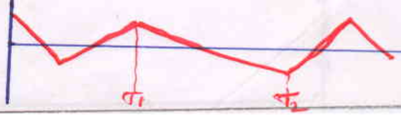
2. When the input starts decreasing below its peak value, the capacitor remains charged at V_m. & the diode gets reverse biased.

3. During the entire -Ve half cycle of input and some part of the next positive half cycle of input, capacitor discharges through R_L and diode 'D' is in reverse biased.

4. The capacitor voltage is same as the output voltage as it is in parallel with R_L.

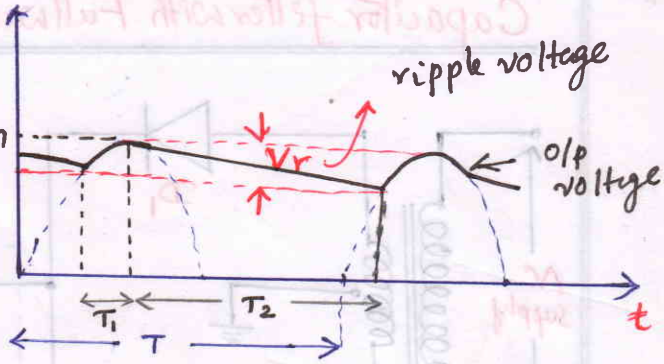
ie When the diode is non conducting the capacitor supplies the load current.

As the time required by the capacitor is very small to charge while its discharging time constant is very large, the ripple in the output gets reduced considerably. Output of the filter is approximately a triangular wave.



Expression for Ripple factor:

Consider the output waveform of Half wave rectifier with capacitor filter.

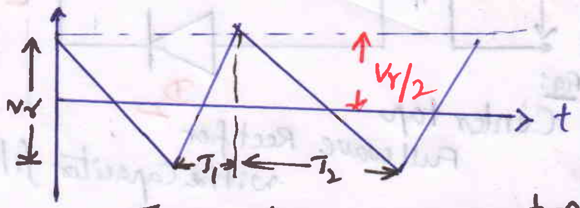


let T - Time period of the a.c i/p voltage.

T_1 - Time for which diode is conducting

T_2 - Time for which diode is nonconducting.

let V_r - The peak to peak value of the ripple voltage



Triangular approximation of ripple voltage.

RMS value of the triangular waveform is

$$V_{rRMS} = \frac{V_r}{2\sqrt{3}} \rightarrow (1)$$

During the time interval T_2, the capacitor C is discharging through R_L

The charge lost $\quad Q = C V_r \rightarrow (2)$

But $\quad i = \frac{dQ}{dt} \Rightarrow Q = \int_0^{T_2} i\,dt = I_{DC} \cdot T_2 \rightarrow (3)$

as integration gives average value or d.c value.

From (2) & (3)

$\therefore \quad I_{DC} \cdot T_2 = C \cdot V_r$

$$V_r = \frac{I_{DC} \cdot T_2}{C} \rightarrow (4)$$

Now $\quad T_1 + T_2 = T \quad , \quad T_2 >> T_1 \quad \therefore \quad T = T_2$

From (4) $\quad V_r = \frac{I_{DC} \cdot (T)}{C} \quad$ & $\quad T = 1/f \rightarrow$ frequency.

$$\therefore V_r = \frac{I_{DC}}{f C}$$

But $\quad I_{DC} = \frac{V_{dc}}{R_L} \quad \therefore \boxed{V_r = \frac{V_{dc}}{f \cdot C \cdot R_L}} \rightarrow (5)$

Ripple factor $\gamma = \frac{V_{rRMS}}{V_{dc}} = \frac{V_r}{2\sqrt{3} \cdot V_{dc}} = \frac{\cancel{V_{dc}}}{2\sqrt{3} \cdot f \cdot C \cdot R_L \cdot \cancel{V_{dc}}}$

$$= \frac{1}{2\sqrt{3}\, f \cdot C \cdot R_L}$$

$\therefore$ For Half wave rectifier with capacitor filter $\quad \boxed{\text{Ripple factor } \gamma = \frac{1}{2\sqrt{3} \cdot f \cdot C \cdot R_L}}$

Capacitor filter with Fullwave Rectifier:

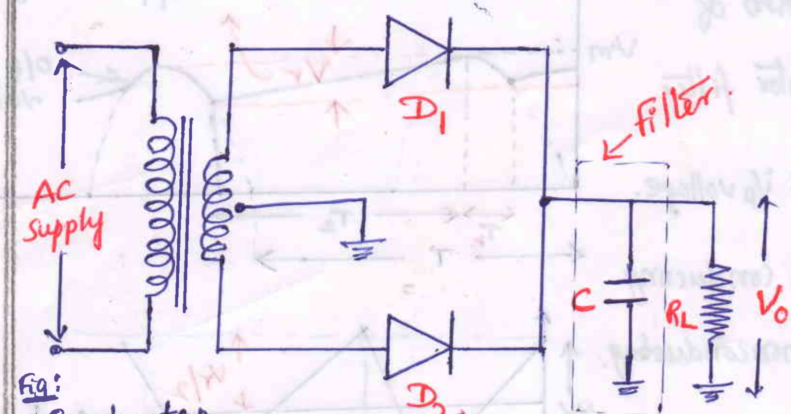
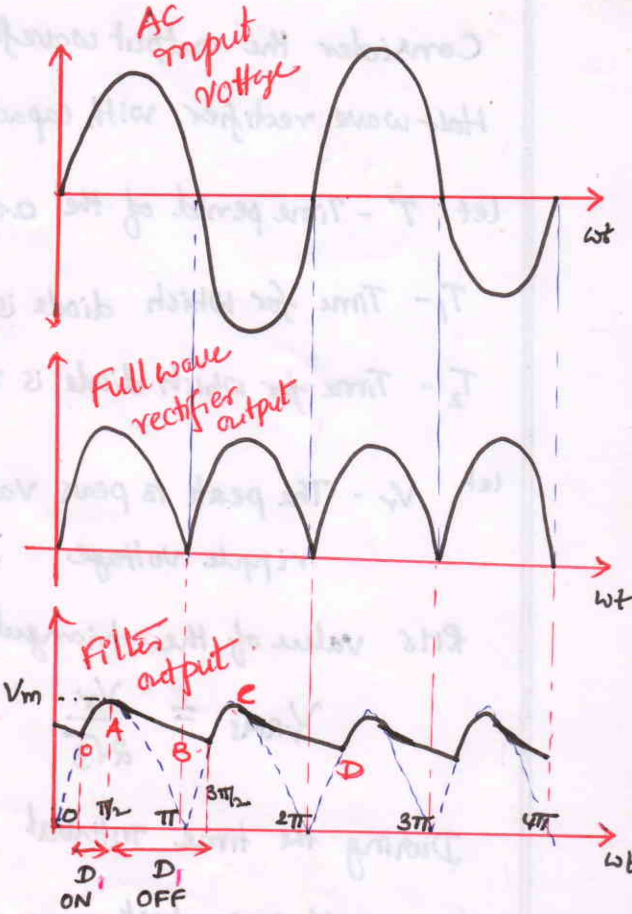


Fig: Center tap Fullwave Rectifier with the Capacitor filter



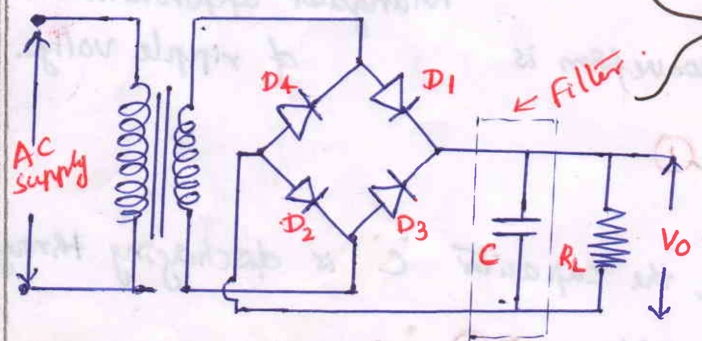
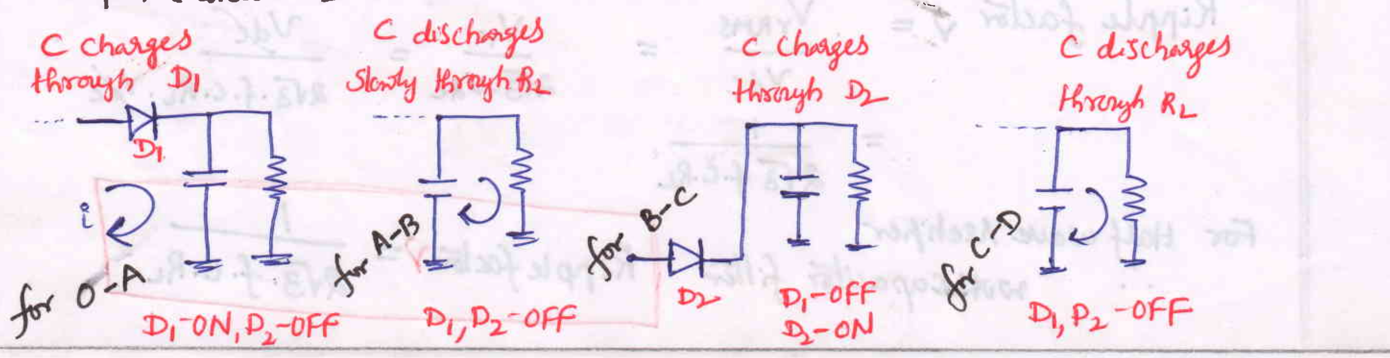


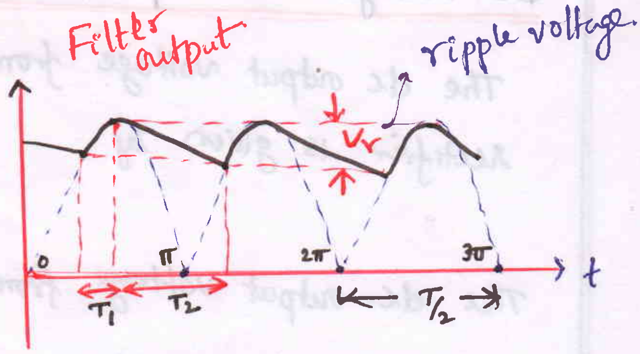
fig: Bridge Fullwave rectifier with capacitor filter.

Operation:

1. When power is ON, the capacitor C gets charged through forward biased diode ON. ie D_1 is ON to V_m during the first quarter cycle of the rectified output.
2. The next quarter cycle from $\pi/2$ to π, the capacitor starts discharging through R_L. One capacitor gets charged to V_m the diode D_1 becomes reverse biased. it lying in the quarter π to $3\pi/2$ of the rectified output voltage.
3. The input voltage exceeds capacitor voltage making Diode D_2 is forward biased. It charges capacitor back to V_m at point C.
4. The next quarter cycle ie point C to D capacitor discharges through R_L & the diode D_2 becomes reverse biased.



Expression for Ripple factor :



Consider the output wave of Fullwave rectifier with capacitor filter.

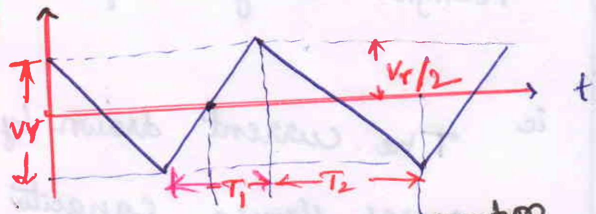
let T - time period of ac i/p voltage

T_1 - time for which diode conducting

T_2 - time for which diode non conducting

$T/2$ - half of the time period of i/p voltage

let V_r - The peak-to peak value of ripple voltage.



Triangular approximation of ripple voltage.

RMS value of the triangular waveform is

$$V_{r\,RMS} = \frac{V_r}{2\sqrt{3}} \longrightarrow (1)$$

During the time interval T_2, the capacitor 'C' discharging through R_L

The charge lost $\quad Q = C V_r \longrightarrow (2)$

But $\quad i = \frac{dQ}{dt} \rightarrow Q = \int_0^{T_2} i\,dt = I_{dc} \cdot T_2 \longrightarrow (3)$

as integration gives avarage of dc value.

From (2) & (3) $\quad I_{dc} \cdot T_2 = C \cdot V_r$

$$V_r = \frac{I_{dc} \cdot T_2}{C} \longrightarrow (4)$$

now $\quad T_1 + T_2 = T \quad , \quad T_2 >> T_1 \quad \therefore \; T_2 = \frac{T}{2} \quad$ ie $\quad \frac{T}{2} = T_2$

From (4) $\quad V_r = \frac{I_{dc}}{C}\left(\frac{T}{2}\right)$

$$V_r = \frac{I_{dc}}{2fC} \quad \text{when} \quad T = 1/f$$

But $\quad I_{dc} = \frac{V_{dc}}{R_L} \Rightarrow \boxed{V_r = \frac{V_{dc}}{2 \cdot f \cdot C \cdot R_L}} \longrightarrow (5)$

Ripple factor $\quad \gamma = \frac{V_{r\,RMS}}{V_{dc}} = \frac{V_r}{2\sqrt{3} \cdot V_{dc}} = \frac{V_{dc}}{2\sqrt{3} \cdot 2 f \cdot C \cdot R_L \cdot V_{dc}} = \frac{1}{4\sqrt{3} f C R_L}$

$\therefore$ For Full wave rectifier with capacitor filter $\quad \boxed{\text{Ripple factor } \gamma = \frac{1}{4\sqrt{3} f \cdot C \cdot R_L}}$

DC Voltage with Capacitor Filter:

The dc output voltage from a capacitor filter fed from a half wave rectifier is given by $\boxed{V_{dc} = V_m - I_{dc} \cdot \left(\frac{1}{2fC}\right)}$.

The d.c. output voltage from a capacitor filter fed from a Full wave rectifier is given by $\boxed{V_{dc} = V_m - I_{dc}\left(\frac{1}{4fC}\right)}$.

ie The current drawn by the load increases, the dc output voltage decreases. Hence capacitor filter ckt having poor regulation.

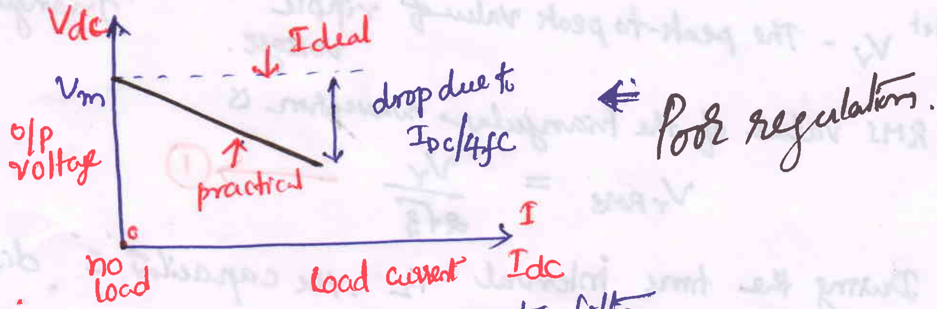


Fig: Load regulation for capacitor filter.

To decrease ripple factor:

1. Increase the value of filter capacitor.
2. Increase the value of load resistance.

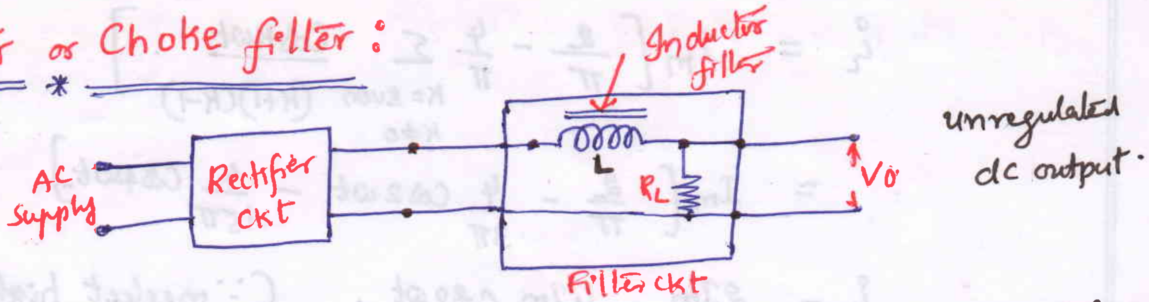
Advantages of Capacitor filter:

1. Less no. of components.
2. Low ripple factor hence low ripple voltage.
3. Suitable for high voltage at small load currents.

Disadvantages of capacitor filter:

1. Ripple factor depends on load resistance.
2. Regulation is poor.
3. Not suitable for variable loads as ripple content increases as R_L decreases.
4. Diode are subjected to high currents hence must be selected accordingly.

Inductor or Choke filter :



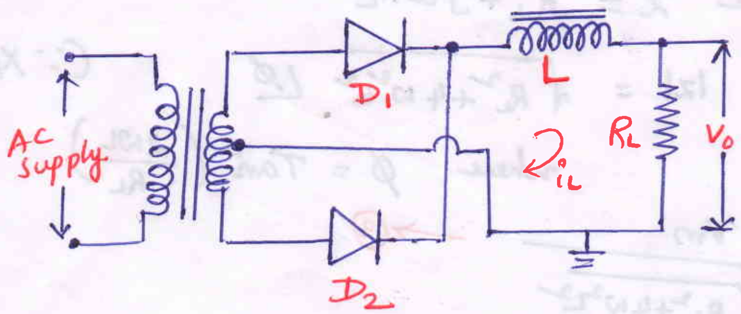
An Inductor or Choke 'L' is connected in series between rectifier ckt and Load.

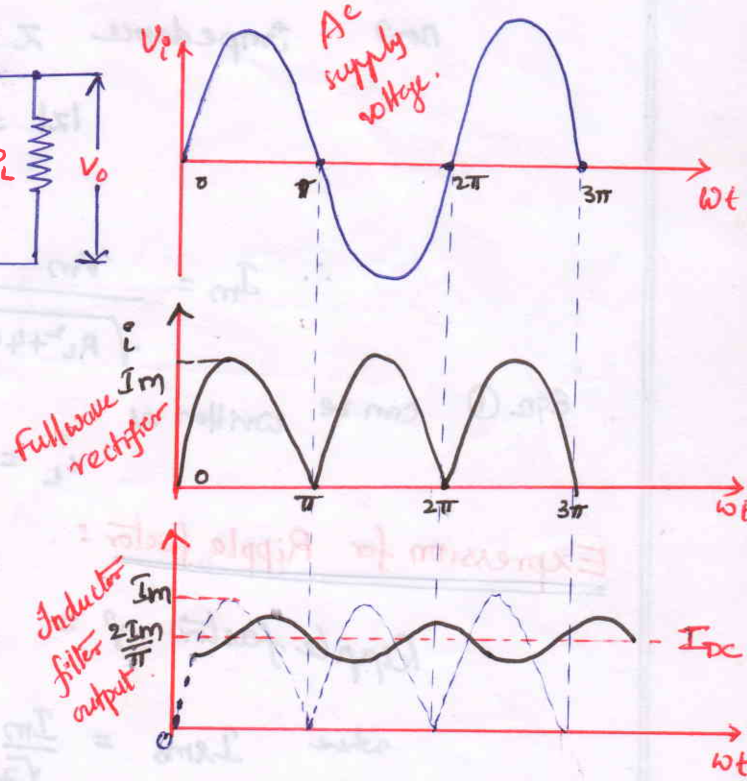
The Reactance of Inductor $X_L = j\omega L = j2\pi f L$.

$\Rightarrow \boxed{X_L \propto f}$

The inductance acts as a short ckt for d.c but it has a large impedance for a.c. ie Inductor blocks a.c & alows only dc.

Inductor filter with fullwave rectifier :





Operation:

1. During the +ve half cycle of the secondary voltage of the transformer the diode D_1 is forward biased. Hence current flows through D_1, L & R_L.
2. During the -ve half cycle of the input the Diode D_1 is reverse biased while D_2 is forward biased. Hence current flows through D_2, L and R_L.
3. Hence we get unidirectional current through R_L due to Inductor 'L' which opposes change in current. it tries to make the output smooth by opposing the ripple content in the output.

Consider the fourier series for a load current for full wave rectifier as follows.

$$i_L = I_m\left[\frac{2}{\pi} - \frac{4}{\pi}\sum_{\substack{K=\text{even}\\K\neq 0}} \frac{\cos K\omega t}{(K+1)(K-1)}\right]$$

$$= I_m\left[\frac{2}{\pi} - \frac{4}{3\pi}\cos 2\omega t - \frac{4}{15\pi}\cos 4\omega t\ldots\right]$$

$$i_L = \frac{2I_m}{\pi} - \frac{4I_m}{3\pi}\cos 2\omega t. \quad (\because \text{neglect higher terms}) \longrightarrow (1)$$

Neglecting R_f, resistance of choke & secondary transformer.

The dc component of the current is

$$\boxed{I_{dc} = \frac{2I_m}{\pi} = \frac{2V_m}{\pi R_L}} \longrightarrow (2) \qquad \because I_m = \frac{V_m}{R_L}.$$

While secondary harmonic component represents a.c component or ripple present. ie $I_m = \frac{V_m}{Z}$ for a.c components.

now impedance $Z = R_L + j2X_L$

$$|Z| = \sqrt{R_L^2 + 4\omega^2 L^2}\ \angle\phi \qquad (\because X_L = \omega L)$$

where $\phi = \tan^{-1}\left(\frac{2\omega L}{R_L}\right)$.

$$\therefore I_m = \frac{V_m}{\sqrt{R_L^2 + 4\omega^2 L^2}} \longrightarrow (3)$$

Eqn (1) can be written as

$$i_L = \frac{2V_m}{\pi R_L} - \frac{4V_m}{3\pi\sqrt{R_L^2 + 4\omega^2 L^2}}\cdot\cos(2\omega t - \phi). \longrightarrow (4)$$

Expression for Ripple factor:

$$\text{Ripple factor}\ \gamma = \frac{I_{RMS}}{I_{dc}}$$

where $I_{rms} = \frac{I_m}{\sqrt{2}}$ of a.c components present in i_L

from eqn (4) ie

$$I_{rms} = \frac{4V_m}{3\sqrt{2}\pi\sqrt{R_L^2 + 4\omega^2 L^2}}$$

$$I_{dc} = \frac{2V_m}{\pi \cdot R_L}$$

$$\text{Ripple factor} = \frac{\dfrac{4V_m}{3\sqrt{2}\pi\sqrt{R_L^2 + 4\omega^2 L^2}}}{\dfrac{2V_m}{\pi R_L}} = \frac{2}{3\sqrt{2}}\cdot\frac{1}{\sqrt{1 + \dfrac{4\omega^2 L^2}{R_L^2}}}$$

Initially on no load condition $R_L \to \infty \Rightarrow \frac{4\omega^2L^2}{R_L^2} \to 0$

$$\therefore \text{Ripple factor} = \frac{2}{3\sqrt{2}} = 0.472$$

This is very close to normal full wave rectifier without filtering. ($\gamma = 0.481$)

But as Load increases, R_L decreases hence $\frac{4\omega^2L^2}{R_L^2} >> 1$, So neglect '1'

$$\therefore \text{Ripple factor} = \frac{2}{3\sqrt{2}} \cdot \frac{1}{\sqrt{\frac{4\omega^2L^2}{R_L^2}}} = \frac{R_L}{3\sqrt{2}\omega L}$$

$\therefore$ For Fullwave rectifier with inductor filter

$$\boxed{\text{Ripple factor} = \frac{R_L}{3\sqrt{2}\omega L}}$$ where $\omega = 2\pi f$.

γ depends on R_L & L

If L is high, γ is Low & if R_L is large, γ is high.

So, Inductor filter should be used where R_L is low.

Regulation: $\boxed{V_{dc} = I_{dc} \times R_L}$

$$= \frac{2I_m}{\pi} \times R_L$$

$$\boxed{V_{dc\,(\text{noload})} = \frac{2V_m}{\pi}} \quad \left(\because I_m = \frac{V_m}{R_L}\right).$$

ie $V_{dc} = 0.637\, V_m$.

$V_{rms} = \frac{\text{Peak}}{\sqrt{2}} = \frac{V_m}{\sqrt{2}} \Rightarrow V_m = \sqrt{2}\cdot V_{rms}$.

$$\therefore V_{dc} = 0.637 \cdot \sqrt{2} \cdot V_{rms}$$

$$\therefore \boxed{V_{dc} = 0.9\, V_{rms}}$$

The output voltage is constant independent of load.

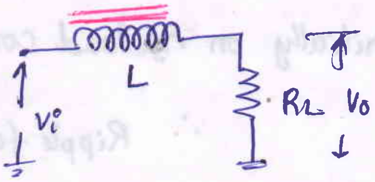
So, perfect regulation exists.

choke or Inductor resistance

$$\therefore \boxed{V_{dc\,(\text{Load})} = \frac{2V_m}{\pi} - I_{dc}(R_L + R_c + R_f)}.$$

(11)

Inductor filter with Halfwave Rectifier :



For Half wave rectifier the instantaneous current

$$i = i_m \sin\omega t \quad , 0 \le \omega t \le \pi \longrightarrow (1)$$
$$= 0 \qquad \pi \le \omega t \le 2\pi$$

The fourier Series Expansion for waveform in eq (1)

$$i = I_m \left[\frac{1}{\pi} + \frac{1}{2}\sin\omega t - \frac{2}{\pi} \sum_{K=2,4,6} \frac{\cos K\omega t}{(K+1)(K-1)} \right]$$

$$i = \frac{I_m}{\pi} - \frac{I_m}{2}\sin\omega t - \frac{2I_m}{3\pi}\cos 2\omega t \cdots\cdots$$

An average d.c value $\boxed{I_{dc} = \frac{I_m}{\pi} = \frac{V_m}{\pi \cdot R_L}}$

The first ripple component is $\frac{I_m}{2}\sin\omega t$ in which $I_{max} = \frac{I_m}{2}$

$$I_{rms} = \frac{I_{max}}{\sqrt{2}} = \frac{I_m}{2\sqrt{2}} = \frac{V_m}{2\sqrt{2}\,(z)} \quad , \quad I_m = V_m/z = \frac{V_m}{(R_L + j\omega L)}$$

$$\boxed{I_{rms} = \frac{V_m}{2\sqrt{2}\,(R_L + j\omega L)}}$$

$$I_{rms} = \frac{V_m}{2\sqrt{2}\sqrt{R_L^2 + \omega^2 L^2}}$$

Ripple factor :

$$\text{Ripple factor } \gamma = \frac{I_{rms}}{I_{dc}}$$

$$= \frac{V_m}{2\sqrt{2}\cdot\sqrt{R_L^2+\omega^2L^2}} \times \frac{\pi \cdot R_L}{V_m}$$

$$= \frac{\pi}{2\sqrt{2}} \cdot \frac{R_L}{\sqrt{1+\frac{\omega^2L^2}{R_L^2}}} \cdot \frac{1}{R_L}$$

$$= \frac{\pi}{2\sqrt{2}} \cdot \frac{R_L}{\omega L} \quad , \quad \frac{\omega^2L^2}{R_L^2} \gg 1$$

$$= \frac{\pi \cdot R_L}{2\sqrt{2}\cdot\omega L}$$

For Halfwave rectifier with inductor filter

$$\boxed{\text{Ripple factor} = \frac{\pi \cdot R_L}{2\sqrt{2}\cdot\omega L} = \frac{1.13\,R_L}{\omega L}} \quad \text{where } \omega = 2\pi f$$

L-Section or LC filter :

It is also called choke input filter as the filter element looking from the rectifier side is an inductor 'L'.

The d.c winding resistance of the choke is R_X.

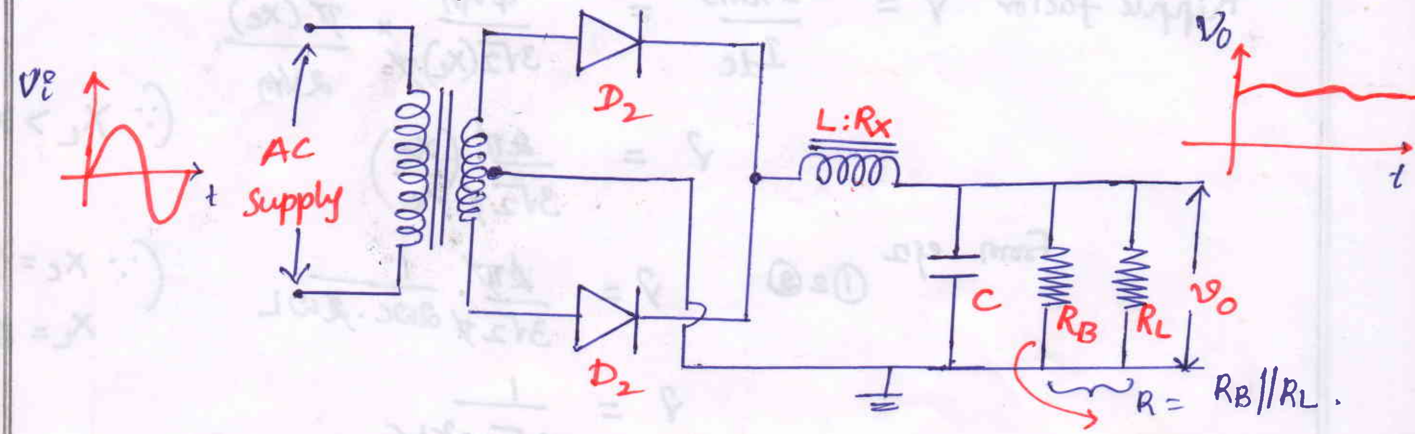


Fig : Choke input filter with full wave rectifier.

* The ripple factor is directly proportional to the load resistance R_L in the inductor filter and inversely proportional to R_L in the capacitor filter. If these two filters are combined as LC filter or L-Section filter.

* If the value of the inductance is increased, it will increase the time of conduction. As some critical value of inductance, one diode either D_1 or D_2 will always be conducting.

Expression for Ripple factor :

The filter elements L and C are having reasonably large values, the reactance X_L of the inductance L at 2ω ie $X_L = 2\omega L$ → (1)

the reactance X_C of the capacitor C at 2ω ie $X_C = \frac{1}{2\omega C}$ → (2)

∵ { The angular frequency will be ω rad/sec ie $\omega = 2\pi f$, then the lowest ripple angular frequency will be '2ω' rad/sec.}

Consider the fourier series of load current for fullwave rectifier.

$$i = \frac{2I_m}{\pi} - \frac{4I_m}{3\pi}\cos 2\omega t - \frac{4I_m}{15\pi}\cos 4\omega t \cdots$$

The dc current $\boxed{I_{dc} = \frac{2I_m}{\pi} = \frac{2V_m}{\pi\cdot(X_C)}}$. → (3)

The max. a.c Current $(I_{ac})_{max} = \dfrac{4 I_m}{3\pi} = \dfrac{4 V_m}{3\pi (X_L)} \longrightarrow ④$

$$I_{rms} = \frac{(I_{ac})_{max}}{\sqrt{2}} = \frac{4 V_m}{3\sqrt{2} \cdot (X_L) \cdot \pi} \longrightarrow ⑤$$

Ripple factor $\gamma = \dfrac{I_{rms}}{I_{dc}} = \dfrac{4 V_m}{3\sqrt{2}(X_L)\cancel{\pi}} \times \dfrac{\cancel{\pi}(X_C)}{2 V_m}$

$$\gamma = \frac{2\cancel{\pi}}{3\sqrt{2}\cancel{\pi}}\left(\frac{X_C}{X_L}\right) \qquad (\because X_L >> X_C)$$

From eqn ① & ②

$$\gamma = \frac{2\cancel{\pi}}{3\sqrt{2}\cancel{\pi}} \cdot \frac{1}{2\omega C \cdot 2\omega L} \qquad \left(\because X_C = \tfrac{1}{2\omega C},\ X_L = 2\omega L\right)$$

$$\gamma = \frac{1}{6\sqrt{2}\,\omega^2 LC}$$

For Fullwave rectifier with LC filter $\boxed{\text{Ripple factor} = \dfrac{1}{6\sqrt{2}\,\omega^2 LC}.}$

It is independent of 'R_L'.

Ripple factor for Halfwave rectifier with LC filter :

Consider the fourier series of Current for Halfwave rectifier.

$$i = \frac{I_m}{\pi} + \frac{I_m}{2}\sin\omega t - \frac{2 I_m}{3\pi}\cos 2\omega t \ldots$$

The d.c current $I_{dc} = \dfrac{I_m}{\pi} = \dfrac{V_m}{\pi (X_C)}$

The max. a.c current is $I_{ac(max)} = \dfrac{I_m}{2}$

$$I_{rms} = \frac{I_{ac(max)}}{\sqrt{2}} = \frac{I_m}{2\sqrt{2}} = \frac{V_m}{2\sqrt{2}\cdot(X_L)}$$

Ripple factor $\gamma = \dfrac{I_{rms}}{I_{dc}}$ $\left(\because X_C = \dfrac{1}{\omega C},\ X_L = \omega L\right)$

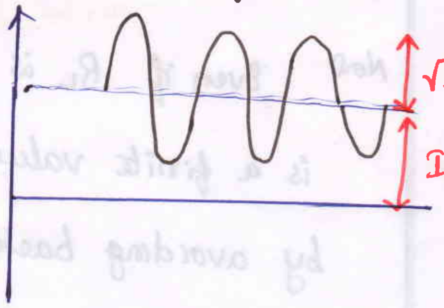
$$= \frac{\cancel{V_m}}{2\sqrt{2}\cdot X_L} \times \frac{\pi (X_C)}{\cancel{V_m}}$$

$$\gamma = \frac{\pi}{2\sqrt{2}} \cdot \left(\frac{X_C}{X_L}\right) = \frac{\pi}{2\sqrt{2}} \cdot \frac{1}{\omega C \cdot \omega L}$$

For Halfwave rectifier with LC filter Ripple factor $\boxed{\gamma = \dfrac{\pi}{2\sqrt{2}\,\omega^2 LC}}$

Critical Inductance (L_c) :

The inductance at which current flows continiuously in the ckt is called Critical Inductance.



If the rectifier ckt is to pass current through out the entire ckt the peak ($\sqrt{2}I_{rms}$) of ac component of current must not exceed the d.c current (I_{dc}).

The current to filter ckt is $i = \frac{2I_m}{\pi} - \frac{4I_m}{3\pi}\cos 2\omega t$.

$$I_{dc} = \frac{2I_m}{\pi} = \frac{2V_m}{\pi \cdot R_L}$$

$$I_{ac} = \frac{4I_m}{3\pi} = \frac{4V_m}{3\pi \cdot X_L}$$

$$I_{rms} = \frac{I_{ac}}{\sqrt{2}} = \frac{4V_m}{3\pi\sqrt{2}X_L}.$$

Here I_{dc} should be greater than or equal to $\sqrt{2}\,I_{rms}$

ie $$I_{dc} \geq \sqrt{2}\cdot I_{rms}.$$

$$\frac{\cancel{2}V_m}{\cancel{\pi} R_L} \geq \cancel{\sqrt{2}}\cdot\frac{\overset{2}{\cancel{4}}V_m}{3\cancel{\pi}\cancel{\sqrt{2}}\cdot X_L}$$

$$\frac{1}{R_L} \geq \frac{2}{3X_L} \qquad (\because X_L = 2\omega L)$$

$$\frac{1}{R_L} \geq \frac{1}{3\omega L}$$

$$\therefore \boxed{L_c \geq \frac{R_L}{3\omega}} \quad \text{where } \omega = 2\pi f.$$

Bleeder Resistance :

A basic requirement of LC filter is that current through choke (inductor) must be continuous & should not interrupted. If the current flow is interrupted, a large back e.m.f will be developed across the choke & exceeds PIV rating of diodes & voltage rating of capacitor resulting in damage of diode & capacitor.

According to critical Inductance $L_c = \frac{R_L}{3\omega}$, If $R_L = \infty \Rightarrow L_c = \infty$

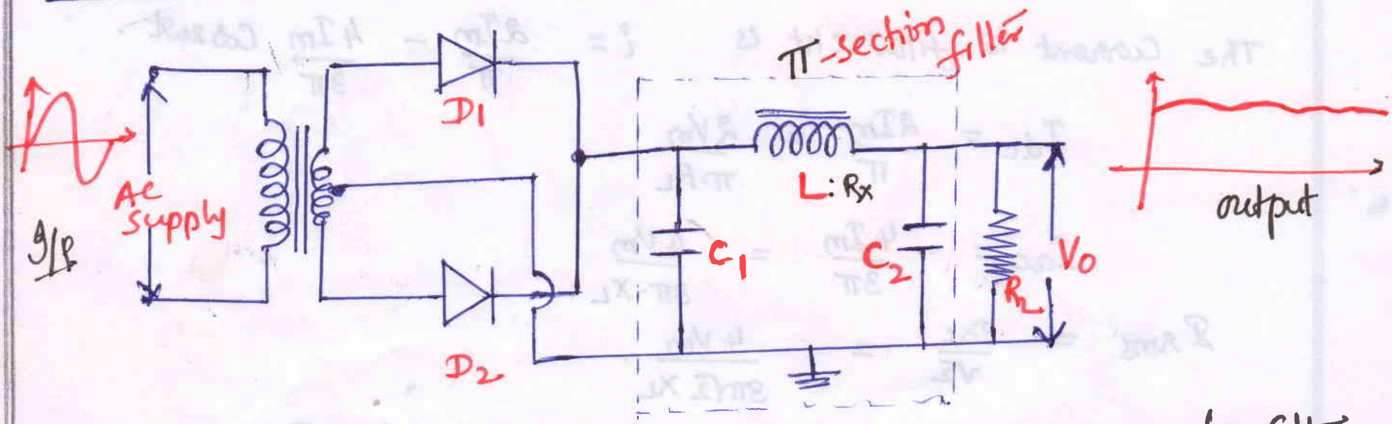
ie As R_L is infinite, the output is open ckted resulting in the interruption of current flow through choke.

To avoid the above two situation a resistor is called Bleeder resistor (R_B) is connected in parallel across load. $R_B = \frac{3X_L}{2}$

(or) $\boxed{R_B = 3\omega L}$

Now even if R_L is ∞ the effective resistance $R = R_B // R_L = \frac{R_B \cdot R_L}{R_B + R_L}$ is a finite value by allowing current to pass in the ckt, thus by avoiding back emf and damage to capacitor & diode.

π-Section or CLC filter :



* CLC & π-section filter is basically consists of a capacitor filter followed by an LC & L-section filter.

* The output of C_1 is triangular wave superimposed over dc. The ripple contains in the output of capacitor filter is reduced by L-section filter.

Expression for Ripple factor :

The fourier series analysis of the output of the filter is

$$V = V_{dc} - \frac{V_{r(p-p)}}{\pi}\left[\sin 2\omega t - \frac{\sin 4\omega t}{2} + \frac{\sin 6\omega t}{3} - \dots\right] \rightarrow (1)$$

For Capacitor filter C_1 the ripple voltage (peak to peak) $= \frac{V_{dc}}{(2fC_1)R_L}$

$$V_{r(p-p)} = \frac{I_{dc}}{2fC_1} \rightarrow (2)$$

The r.m.s value of second harmonic from eqn (1) can be written as

$$V_{rRMS} = \frac{V_{r(p-p)}}{\pi\sqrt{2}} = \frac{I_{dc}}{2fC_1 \cdot \pi\sqrt{2}} \times \frac{2}{2} \quad \left(\because X_{C_1} = \frac{1}{2\omega C_1}\right)$$

$$V_{rRMS} = 2I_{dc} / 4\pi f C_1 \cdot \sqrt{2} = \frac{\sqrt{2}\, I_{dc}}{2\omega C_1} = \sqrt{2}\, I_{dc} \cdot X_{C_1} \rightarrow (3)$$

ie $V_{rRMS} = \sqrt{2}\, I_{dc} \cdot X_{C_1} \rightarrow ③$ $\left(\because X_{C_1} = \frac{1}{2\omega C_1}\right)$

V_{rRMS} of eqn ③ is impressed on L-Section filter.

now the ripple voltage

$$(V_{rRMS})' = \frac{V_{rRMS}}{X_L} \cdot X_{C_2} \rightarrow ④$$

Substituting eqn ③ & ④, we get.

$$(V_{rRMS})' = \frac{\sqrt{2}\, I_{dc} \cdot X_{C_1} \cdot X_{C_2}}{X_L}$$

$$= \frac{\sqrt{2} \cdot V_{dc} \cdot X_{C_1} X_{C_2}}{R_L \cdot (X_L)}$$

Ripple factor $\gamma = \frac{(V_{rRMS})'}{V_{dc}} = \frac{\sqrt{2} \cdot X_{C_1} \cdot X_{C_2}}{X_L \cdot R_L}$

We know $X_{C_1} = \frac{1}{2\omega C_1}$

$X_{C_2} = \frac{1}{2\omega C_2}$

$X_L = 2\omega L$

$$\therefore \gamma = \frac{\sqrt{2} \cdot \frac{1}{2\omega C_1} \cdot \frac{1}{2\omega C_2}}{2\omega L \cdot R_L}$$

$$= \sqrt{2} \cdot \frac{1}{8\omega^3 L C_1 C_2 \cdot R_L}$$

$$\gamma = \frac{1}{4\sqrt{2}\, \omega^3 L C_1 C_2 R_L}$$

$\therefore$ For Fullwave rectifier with π-Section filter

$$\boxed{\text{Ripplefactor } \gamma = \frac{1}{4\sqrt{2}\, \omega^3 L C_1 C_2 R_L}}$$

where $\omega = 2\pi f$.

Regulation:

The output voltage is given by

$$V_{dc} = V_m - \frac{V_r}{2} - I_{dc} \cdot R_x.$$

where V_r - peak to peak ripple voltage

R_x - dc resistance of choke.

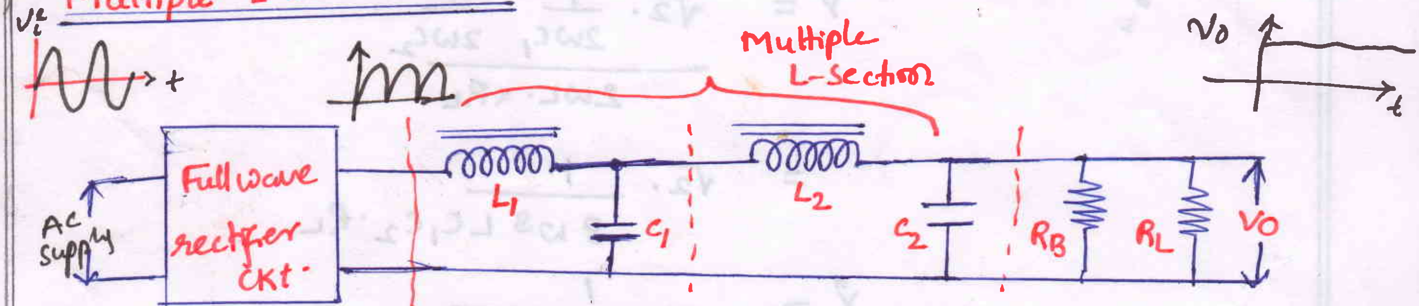
$V_r = I_{dc}/2fC$, for Fullwave, $V_r = \frac{I_{dc}}{fC}$, For Half wave

Comparision of Filters:

SL NO.	Parameter	Without Filter	With Filter L	C	LC	π-Section
1.	V_{dc} at no load	$0.636 V_m$	$0.636 V_m$	V_m	V_m	V_m
2	V_{dc} at load I_{dc}	$0.636 V_m$	$0.636 V_m$	$V_m - \frac{I_{dc}}{4fC}$	$0.636 V_m$	$V_m - \frac{I_{dc}}{4fC}$
3	Ripple factor (γ)	0.481	$\frac{R_L}{3\sqrt{2}\omega L}$	$\frac{1}{4\sqrt{3} f R_L C}$	$\frac{1}{6\sqrt{2}\omega^2 LC}$	$\frac{1}{4\sqrt{2}\omega^3 L C_1 C_2 R_L}$
4	Peak inverse voltage (PIV)	$2V_m$	$2V_m$	$2V_m$	$2V_m$	$2V_m$

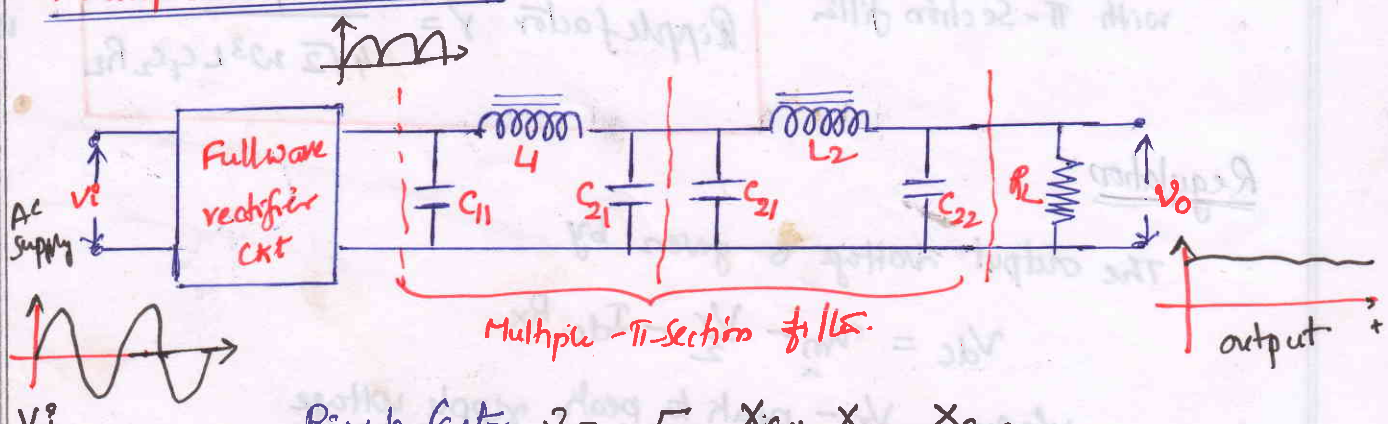
Above comparision is for full wave rectifiers with & without filters and diode, transformer and filter element resistances are neglected.

Multiple L-Section Filter :



Ripple factor $\gamma = \frac{\sqrt{2}}{3} \frac{X_{C_1} X_{C_2} \ldots}{X_{L_1} X_{L_2}}$ or $\gamma = \frac{\sqrt{2}}{3} \frac{1}{(4\omega^2 LC)^n}$

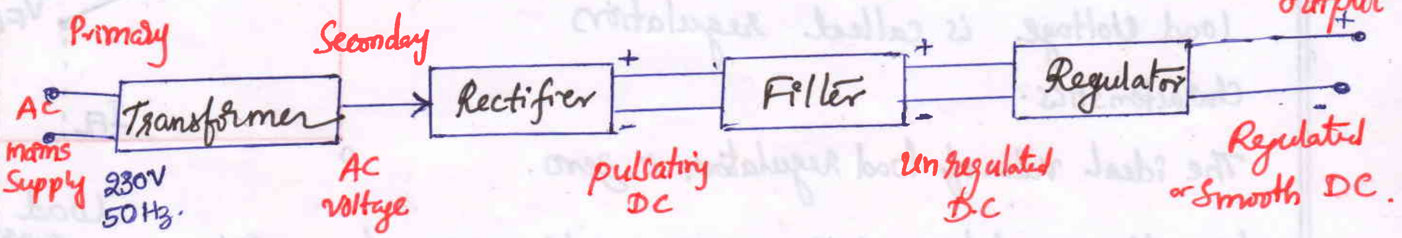
Multiple-π-Section Filter :

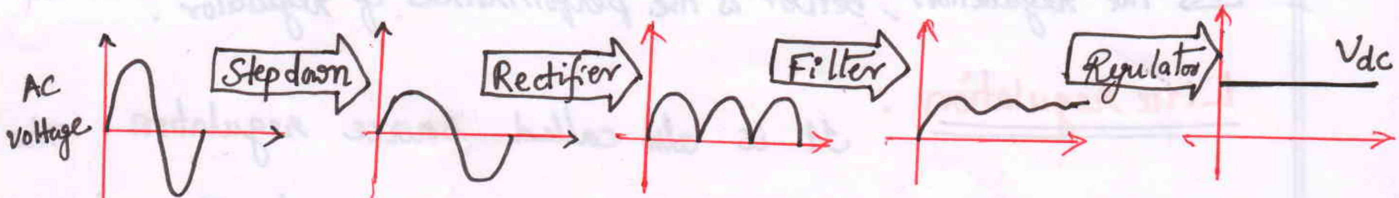


Ripple factor $\gamma = \sqrt{2} \cdot \frac{X_{C_{11}}}{R_L} \frac{X_{C_{21}} \cdot X_{C_{22}}}{X_{L_1} \cdot X_{L_2}} \ldots$

Regulated Power Supply (RPS):

A typical d.c power supply consists of various stages as follows.



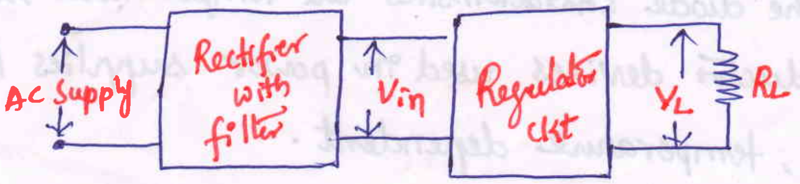


Regulator: It is an electronic device that maintains nearly constant output voltage under varying input voltage conditions and varying load conditions. It converts unregulated d.c to pure d.c.

Factors affecting load voltage & power supply.

1. Load Current / Load regulation.
2. Line voltage / input supply voltage. / Line regulation.
3. Temperature.

Load Regulation: The load regulation is the change in the regulated output voltage when the load current is changed from minimum (no load) to maximum (full load). & it is denoted as LR.



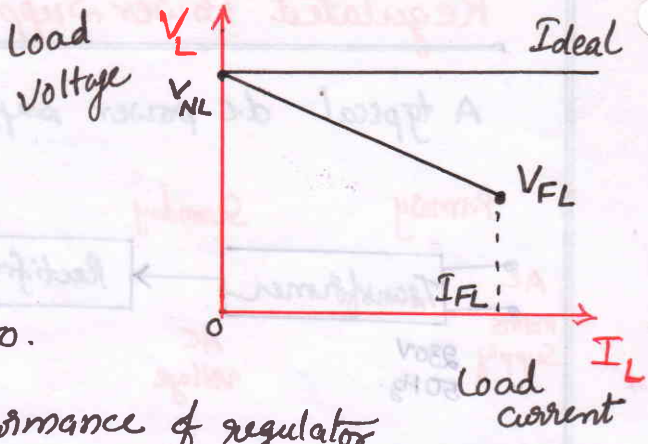
Load regulation $\boxed{\% LR = \frac{V_{NL} - V_{FL}}{V_{FL}} \times 100}$

where V_{NL} – Load voltage with no load current.

V_{FL} – Load voltage with Full load current.

Regulation Characteristics:

The graph of load current Vs load voltage is called regulation characteristics.



The ideal value of load regulation is zero.

Less the regulation, better is the performance of regulator.

Line regulation:

It is also called source regulation. The input to the unregulated power supply is 230V ac supply. This is line voltage.

Source regulation is defined as the change in regulated load voltage for a specified range of line voltage (typically ±230V) & it is denoted as SR.

Source regulation $\boxed{SR = V_{HL} - V_{LL}}$

where V_{HL} - load voltage with high line voltage
V_{LL} - load voltage with low line voltage.

$$\boxed{\% SR = \frac{V_{HL} - V_{LL}}{V_{nom}} \times 100}$$

where V_{nom} - nominal load voltage.

Temperature:

In a power supply, the rectifier ckt uses the p-n jun. diodes, the diode characteristics are temperature sensitive. The other semiconductor devices used in power supplies have their characteristics, temperature dependent.

Hence the temperature is an important factor responsible for the changes in the load voltage.

Ripple Rejection:

$$RR = \frac{\text{Ripple content in output}}{\text{Ripple content in input}} = \frac{V_R(out)}{V_R(in)}$$

In decibels (dB) unit

$$RR' = 20 \log_{10}(RR) \text{ dB}.$$

ie.
In practice the output voltage of an unregulated power supply varies due to the following reasons.

1. Change in input supply voltage.
2. Change in load resistance.
3. Change in temperature.

Voltage regulation :

It is the ability of a power supply to maintain to a constant output voltage inspite of a.c input voltage fluctuations, change in load resistance and temperature.

The change in total output voltage can be expressed as

$$\boxed{\Delta V_0 = \frac{\partial V_0}{\partial V_i} \cdot \Delta V_i + \frac{\partial V_0}{\partial I_L} \Delta I_L + \frac{\partial V_0}{\partial T} \cdot \Delta T}$$

This can be written as $\Delta V_0 = S_V \Delta V_i + S_L \Delta I_L + S_T \Delta T$

where S_V, S_L, S_T are stability coefficients defined as.

(i) **Voltage Stability Factor (S_V) :** It is the % change in the output voltage which occurs per volt change in the input line voltage with load current & temperature are constant. (Voltage regulation)

ie $S_V = \frac{\Delta V_0}{\Delta V_i} \Big|$ at $\Delta I_L = 0$ & $\Delta T = 0$. It has no units.

(ii) **Output resistance (S_L) :** It is defined as the change in output voltage with change in the load current at input voltage and temperature are constant. (Load regulation).

$S_L = \frac{\Delta V_0}{\Delta I_L} \Big|$ at $\Delta V_i = 0$ & $\Delta T = 0$ ∴ It units are Ohms (Ω)

(iii) **Temperature Stability Factor (S_T) :** It is defined as the change in the output voltage with change in the temperature at input voltage and load current are constant. (Temp-regulation).

ie $S_T = \frac{\Delta V_0}{\Delta T} \Big|$ at $\Delta V_i = 0$ & $\Delta I_L = 0$

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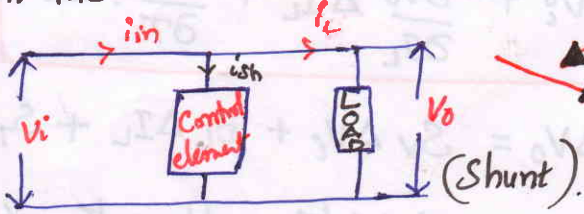
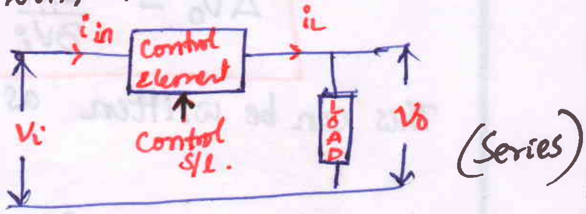
Hence the three coefficients (S_V, S_L, S_T) must be as small as possible. ie. Smaller the values of three cofficients better the regulation of power supply.

Types of Voltage regulators :

Basically regulators are two types.

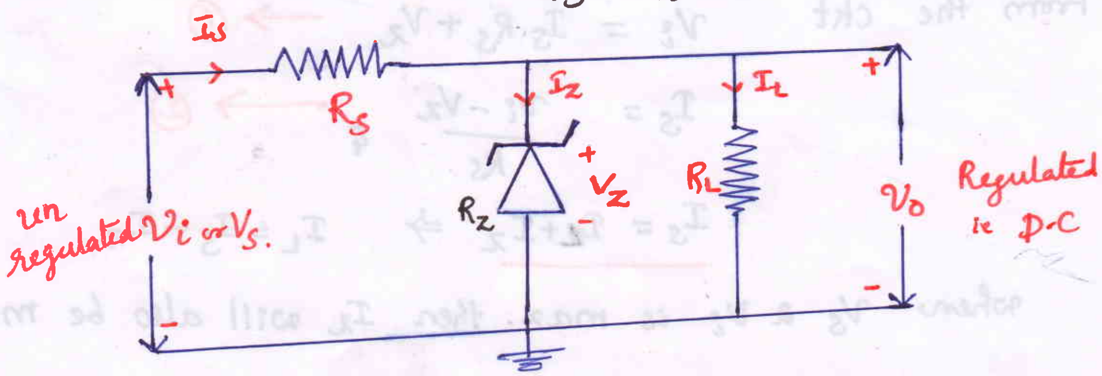
1. Series Voltage regulator.
2. Shunt voltage regulator.

SL NO	Series Voltage regulator	Shunt Voltage Regulator.
1.	The control element is in Series with the load. 	1. The Control element is in parallel with the load. 
2.	The entire load current i_L always passes through the Control element.	2. Only small current passess through the control element which is required to be diverted to keep output constant.
3	The control element is high current low voltage rating component.	3. The control element is low current high voltage rating component.
4.	The regulation is good	4. The regulation is poor.
5	Efficiency depends on the output voltage	5. Efficiency depends on the load current.
6	Complicated to design as compared to shunt regulators.	6. Simple to design.
7	Preferred for fixed as well as variable voltage applications.	7. Not suitable for varying load conditions preferred for fixed voltage applications.
8.	Example : Zener Shunt regulator, transistorized Shunt regulator, etc.	8. Example : Series feedback type regulator, Series regulator with pre regulator and foldback limiting etc.

Zener diode as a Regulator :

The simplest shunt voltage regulator ckt uses a Zener diode, to regulate the load voltage.

$I_S = I_Z + I_L$.



Operation:

→ The zener diode is used in its reverse biased region.

→ Under reverse biased condition the current through the diode is very small of the order of few 'μA' upto certain limit.

→ When the sufficient reverse bias is applied, electrical breakdown of the zener diode occurs. The large current flows through the zener diode, such a breakdown occurs at a voltage called Zener Voltage (V_Z).

→ Under this condition whatever may be the current, the voltage across the zener diode is constant equal to V_Z.

→ The large current due to breakdown is limited by connecting the resistance in the ckt.

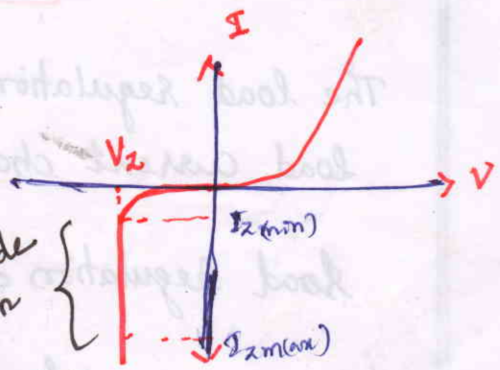
Hence the load voltage V_O is equal to the zener voltage V_Z.

Thus Zener diode acts as an ideal voltage source which maintains a constant load voltage, independent of the current.

ie When $V_i \geq V_Z$ Zener diode is ON

Output $V_O = V_Z$ (Breakdown voltage).

When $V_i < V_Z$ zener diode is OFF.



Current $I_S = I_Z + I_L$

$I_S = \dfrac{V_i - V_Z}{R_S}$, $I_L = \dfrac{V_O}{R_L}$

$I_Z = I_S - I_L$

Zener diode is operated in this reverse biased region.

Design of Zener Diode:

The zener diode should be connected such that

$$I_{Z(min)} < I_Z < I_{Z(max)} \quad \& \quad V_i \geq V_Z .$$

From the ckt $\quad V_i = I_S \cdot R_S + V_Z \longrightarrow (1)$

$$I_S = \frac{V_i - V_Z}{R_S} \quad \& \longrightarrow (2)$$

$$I_S = I_L + I_Z \Rightarrow \quad I_L = I_S - I_Z$$

when V_S & V_i is max. then I_Z will also be max.

$$I_{Z(max)} = I_{S(max)} - I_{L(min)}$$

$$I_{Z(max)} = \frac{V_{i(max)} - V_Z}{R_{S(min)}} - I_{L(min)} \quad (\text{From } (2))$$

$$R_{S(min)} = \frac{V_{i(max)} - V_Z}{I_{Z(max)} + I_{L(min)}} \quad \text{ie} \quad \boxed{R_{S(min)} = \frac{V_{i(max)} - V_0}{I_{Z(max)} + I_{L(min)}}}$$

when V_S & V_i is min. then I_Z will also be min.

$$I_{Z(min)} = I_{S(min)} - I_{L(max)}$$

$$I_{Z(min)} = \frac{V_{i(min)} - V_Z}{R_{S(max)}} - I_{L(max)}$$

$$R_{S(max)} = \frac{V_{i(min)} - V_Z}{I_{Z(min)} + I_{L(max)}} \quad \text{ie} \quad \boxed{R_{S(max)} = \frac{V_{i(min)} - V_0}{I_{Z(min)} + I_{L(max)}}}$$

The values of R_S must be selected such that

$$\boxed{R_{S(min)} < R_S < R_{S(max)}}$$

The load regulation is the change in the regulated output when load current changes from min to max. ie % of load

load regulation of zener diode $\quad \boxed{\%R = \frac{V_{NL} - V_{FL}}{V_{FL}} \times 100} \;\%$

where V_{NL} – voltage with no load

V_{FL} – voltage with full load.

Show that the voltage stability factor (S_V) of shunt zener voltage regulator $S_V \cong \frac{R_Z}{R_S}$ where R_Z - zener diode resistance
R_S - Source resistance or current limiting resistance.

Proof:

We know $V_i = I_S \cdot R_S + V_0 \rightarrow ①$

& $I_S = I_Z + I_L$ $\quad I_Z = \frac{V_0}{R_Z}$

$V_i = (I_Z + I_L) R_S + V_0$ $\quad I_L = V_0/R_L$

$$V_i = \left(\frac{V_0}{R_Z} + \frac{V_0}{R_L}\right) R_S + V_0$$

$$V_i = V_0\left[\frac{1}{R_Z} + \frac{1}{R_L}\right] R_S + V_0$$

or

$$V_i = V_0\left[\frac{R_S}{R_Z} + \frac{R_S}{R_L} + 1\right] \rightarrow ②$$

Voltage stability factor $S_V = \frac{\Delta V_0}{\Delta V_i}$

From eqn ② $\quad \frac{V_0}{V_i} = \frac{1}{\left[\frac{R_S}{R_Z} + \frac{R_S}{R_L} + 1\right]} \rightarrow ③$

If R_S is selected such that $R_S << R_L$ and $R_S >> R_Z$

ie $R_L >> R_S >> R_Z$ then from eqn ③

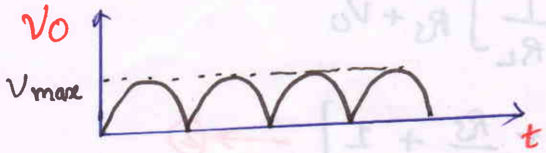
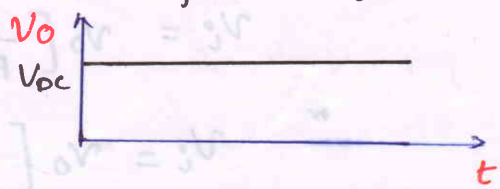
$$S_V = \frac{V_0}{V_i} = \frac{1}{R_S/R_Z} = \frac{R_Z}{R_S}$$

$\therefore \boxed{S_V = \frac{R_Z}{R_S}}$ where R_Z - zener diode resistance
R_S - Source resistance.

Dis advantages of Zener shunt regulator:

1. The max. load current that can be supplied is limited to $[I_{Z(max)} - I_{Z(min)}]$.
2. A large amount of power is wasted in the zener diode and the series resistance R_S in comparision with the load power.
3. The voltage stability factor and output resistance are not very low, as desired for a regulated circuit.

Comparision of Rectifier and Regulator :

Rectifier	Regulator .
1. Rectifier converts pure sinusoidal input into pulsating d.c output.	1. Regulator converts the pulsating d.c input into constant (pure) d.c output.
2. The output contains ripples	2. The output is ripple free.
3. The output waveform is 	3. The output waveform is 
4. Output voltage changes with respect to load current input voltage and temperature.	4. Output voltage does not change with respect to load current, input voltage and temperature.
5. Uses the devices which are diodes.	5. Uses the devices such as transistors, op-amps etc.
6. The examples are half wave, full wave and Bridge rectifiers.	6. The examples are zener diode regulator, transistorized regulator etc.
7. Not provided with over load protection, Short ckt protection, thermal shutdown. etc.	7. Provided will all sorts of protection circuits.

Some type numbers of Diodes.

1. OA79 → O → Semiconductor Device.
   A → Denotes Ge-device.
2. BY127 → B → Denotes Si-device.
   Y → Rectifying diode.
   127 → number.
3. 1N4153 → N - Bipolar Device
   1 - single polar function
4. BY100 → 800V, 1 A Si diode.

Form factor = $\frac{\text{rms value}}{\text{average value}}$

For HWR } $\frac{I_m/2}{I_m/\pi} = \frac{2}{\pi}$ $= 1.57$

For FWR } $\frac{I_m/\sqrt{2}}{2I_m/\pi} = \pi/2\sqrt{2}$ $= 1.11$

Peak factor = $\frac{\text{Peak value}}{\text{rms value}}$

For HWR } $I_m/\frac{I_m}{2} = 2$

For FWR } $I_m/\frac{I_m}{\sqrt{2}} = \sqrt{2}$

Simple Problems:

Half wave Rectifier:

(1) A HWR has a load of $3.5\,K\Omega$. If the diode resistance and the secondary coil resistance together have a resistance of 800Ω and the input voltage has a signal voltage of peak value 240V. Calculate

a) Peak, average and rms value of current flowing.

b) dc power output.

c) ac power input

d) Efficiency of the rectifier.

Sol Given data For Halfwave Rectifier (HWR).

$R_L = 3.5\,K\Omega$

$R_s + R_f = 800\Omega$ & $V_m = 240V$

a) Peak value of Current $I_m = \dfrac{V_m}{R_s + R_f + R_L} = \dfrac{240}{800 + 3.5\times10^3} = 55.81\,mA$

ie $\boxed{I_m = 55.81\,mA}$

Average value of current $I_{dc} = \dfrac{I_m}{\pi} = \dfrac{55.81\,mA}{\pi} = 17.77\,mA$

$\boxed{I_{dc} = 17.77\,mA}$

RMS value of current $I_{rms} = \dfrac{I_m}{2} = \dfrac{55.81\,mA}{2} = 27.905\,mA$

$\boxed{I_{rms} = 27.905\,mA}$

b) DC power output is

$P_{dc} = (I_{dc})^2 \times R_L = (17.77\,mA)^2 \times 3.5\times10^3 = 1.105\,W$

$\boxed{P_{dc} = 1.105\,W}$

c) ac power input is

$P_{ac} = (I_{rms})^2 \cdot (R_f + R_L) = (27.905\,mA)^2 \cdot (4300) = 3.348\,W$

$\boxed{P_{ac} = 3.348\,W}$

d) Efficiency of the rectifier

$\eta = \dfrac{P_{dc}}{P_{ac}} = \dfrac{1.105}{3.348} = 0.33 \Rightarrow \boxed{\%\eta = 33\%}$

(2) A diode has an internal resistance of 20Ω and 1000Ω load from a 110V rms source of supply. Calculate

a) Peak, average & rms value of current.

b) Efficiency of rectification

c) The percentage of regulation from no load to full load.

Sol Given data $R_f = 20\Omega$ $\quad V_{rms} = 110V$

$R_L = 1000\Omega$

A single diode means → Half wave rectifier.

$$V_{rms} = \frac{V_m}{\sqrt{2}} \Rightarrow V_m = \sqrt{2} \times 110V = 155.5\ V.$$

a) Peak Current $I_m = \frac{V_m}{R_f + R_L} = \frac{155.5}{20+1000} = 0.1525A$

$\boxed{I_m = 152.5\ mA}$

Average Current $I_{dc} = \frac{I_m}{\pi} = \frac{0.1525}{\pi} = 0.0485\ A$

$\boxed{I_{dc} = 48.5\ mA}$

rms Current $I_{rms} = \frac{I_m}{2} = \frac{0.1525}{2} = 0.07625$

$\boxed{I_{rms} = 76.25\ mA.}$

b) Efficiency $\eta = \frac{P_{dc}}{P_{ac}}$

$$P_{dc} = (I_{dc})^2 \cdot R_L = (48.5\ mA)^2 \times (1000\Omega) = 2.35W$$

$$P_{ac} = (I_{rms})^2 \cdot (R_f + R_L) = (76.25 \times 10^{-3})^2 (20+1000) = 5.93W$$

$$\%\eta = \frac{2.35}{5.93} \times 100 = 39.629\%$$

$\boxed{\%\eta = 39.63\%.}$

c) Percentage of Regulation $\%R = \frac{V_{NL} - V_{FL}}{V_{FL}} \times 100$

$$V_{NL} = \frac{V_m}{\pi} = \frac{155.5}{\pi} = 49.497\ V$$

$$V_{FL} = V_{dc} = I_{dc} \cdot R_L = 0.0485 \times 1000 = 48.5\ V$$

$$\%R = \frac{49.497 - 48.5}{48.5} \times 100 = 2\%$$

$\boxed{\therefore \%R = 2\%.}$

(3) A voltage of $200\cos\omega t$ is applied to HWR with load resistance of $5K\Omega$. Find the max. dc current component, rms current, ripple factor, TUF and rectifier Efficiency.

Sol Given data Applied voltage $= 200\cos\omega t$ for HWR

ie $V_m = 200\ V$, $R_L = 5K\Omega$

a) The Max. dc Current $I_{dc} = \frac{I_m}{\pi}$, $I_m = \frac{V_m}{R_L} = \frac{200}{5K} = 40mA$

$$= \frac{40 \times 10^{-3}}{\pi}$$

$$= 12.73mA$$

$\boxed{I_{dc} = 12.73\ mA}$

b) RMS Current $I_{rms} = \frac{I_m}{2} = \frac{40 \times 10^{-3}}{2} = 20mA$

$\boxed{I_{rms} = 20\ mA}$

c) Ripple factor $\gamma = \sqrt{\left(\frac{I_{rms}}{I_{dc}}\right)^2 - 1} = \sqrt{\left(\frac{20\times10^{-3}}{12.73\times10^{-3}}\right)^2 - 1} = 1.21$

$\boxed{\text{Ripple factor } \gamma = 1.21}$

d) TUF - Transformer Utilization Factor

$= \frac{P_{dc}}{P_{ac\,(rated)}}$, $P_{dc} = (I_{dc})^2 \cdot R_L = (12.73m)^2 \times 5K$

$\boxed{P_{dc} = 0.81W}$

$P_{ac\,(rated)} = V_{rms} \cdot I_{rms}$

$= \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{2} = \frac{200 \times 20m}{\sqrt{2}}$

$\boxed{P_{ac\,(rated)} = 2.828W}$

$\therefore TUF = \frac{0.81}{2.828} = 0.286$

$\therefore TUF = 0.286$

e) Efficiency $\eta = \frac{P_{dc}}{P_{ac}}$, $P_{ac} = (I_{rms})^2 \cdot R_L = (20m)^2 \times 5K = 2W$

$\boxed{P_{ac} = 2W}$

$\% \eta = \frac{0.81}{2} \times 100$

$= 40.5\%$ Efficiency $\boxed{\eta = 40.5\%}$

(4) Show that max. DC output power $P_{dc} = V_{dc} \cdot I_{dc}$ in a HWR occur when the load resistance equals to diode resistance R_f. (ie $R_f = R_L$)

Proof. For a Halfwave Rectifier.

$V_{dc} = I_{dc} R_L$, $I_{dc} = \frac{I_m}{\pi}$, $I_m = \frac{V_m}{R_f + R_L}$

$P_{dc} = V_{dc} \cdot I_{dc} = (I_{dc})^2 \cdot R_L = \frac{V_m^2 R_L}{\pi^2 (R_f + R_L)^2}$

For this power to be maximum $\frac{dP_{dc}}{dR_L} = 0$

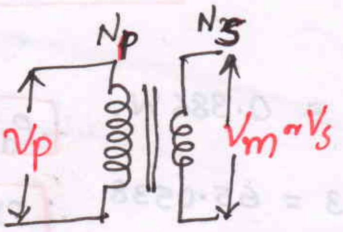
$\Rightarrow \frac{d}{dR_L}\left(\frac{V_m^2 R_L}{\pi^2 (R_f+R_L)^2}\right) = \frac{V_m^2}{\pi^2}\left\{\frac{(R_f+R_L)^2 - R_L \times 2[R_f + R_L]}{(R_f+R_L)^2}\right\} = 0$

$\Rightarrow R_f^2 + 2R_fR_L + R_L^2 - 2R_fR_L - 2R_L^2 = 0$

$R_f^2 - R_L^2 = 0$

$\Rightarrow \boxed{R_f = R_L}$

Note :



The primary voltage is always given is RMS Value

Ex: If primary voltage of transformer is 230 V then $(V_p)_{max} = \sqrt{2} \times 230 \Rightarrow V_{rms} = \frac{V_m}{\sqrt{2}}$.

$\frac{V_p}{V_s} = \frac{N_P}{N_S} \Rightarrow \boxed{V_s \sim V_m = V_p\left[\frac{N_S}{N_P}\right]}$

Ex: The supply voltage to a step down transformer is 220V, Turns ratio is 8 : 2 (N_P : N_S) then V_m = ?

$(V_p)_{max} = \sqrt{2} \times 220 = 311.12V$

$V_m = 311.12 \times \left(\frac{2}{8}\right) = 77.8V$.

Full Wave Rectifier : (Center tap).

(5) A 230V, 60 Hz voltage is applied to the primary of a 5:1 step-down center tap transformer used in a full wave rectifier having a load of 900Ω. If the diode resistance and secondary coil resistance together has a resistance of 100Ω. Determine

a) d.c voltage across the load.

b) d.c current flowing through the load.

c) d.c power delivered to the load

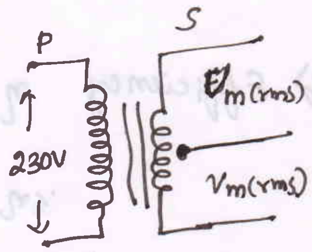
d) PIV across each diode.

e) Ripple voltage and its frequency.

f) Rectification Efficiency.

Sol. Given. $V_{p(rms)} = 230V$, $f = 60\,Hz$. $\frac{N_1}{N_2} = \frac{5}{1}$.

$R_L = 900\Omega$, $R_S + R_f = 100\Omega$.



The voltage across the two ends of Secondary.

$$V_{p(rms)} \cdot \frac{N_2}{N_1} = 230 \times \frac{1}{5} = 46V.$$

Voltage from center tapping to one end $V_{rms} = \frac{46}{2} = 23V$.

ie $V_m = \sqrt{2} \cdot V_{rms} = \sqrt{2} \times 23V$.

a) d.c voltage across the load.

$$V_{dc} = \frac{2V_m}{\pi} = \frac{2 \times \sqrt{2} \times 23}{\pi} = 20.7V. \quad \text{at no-load.}$$

$$V_{dc} = I_{dc} \cdot R_L, \quad I_{dc} = \frac{2I_m}{\pi}, \quad I_m = \frac{V_m}{[R_f + R_s + R_L]} = \frac{\sqrt{2} \times 23}{900 + 100}$$

$$\boxed{I_m = 32.53\,mA}$$

$$I_{dc} = \frac{2 \times 32.53m}{\pi}, \quad \boxed{I_{dc} = 20.71\,mA}$$

$$V_{dc} = 20.71 \times 10^{-3} \times 900 \Rightarrow \boxed{V_{dc} = 18.64\,V} \text{ at Full load.}$$

b) $$I_{dc} = \frac{2I_m}{\pi} \Rightarrow I_{dc} = 20.71\,mA. \quad \therefore \boxed{I_{dc} = 20.71\,mA}$$

c) D.C power delivered to the load.

$$P_{dc} = (I_{dc})^2 \cdot R_L = (20.71\,mA)^2 \times 900\Omega = 0.386\,W. \quad \therefore \boxed{P_{dc} = 0.386\,W}$$

d) PIV across each diode $= 2V_m = 2 \times \sqrt{2} \times 23 = 65.0538$ $\therefore \boxed{PIV = 65\,V.}$

e) Ripple factor $\gamma = \frac{V_{r(rms)}}{V_{dc\,(FL)}} = 0.482$ (Full wave)

Ripple voltage $V_{r(rms)} = 0.482 \times 18.64 = 8.98\,V$ $\therefore \boxed{V_{r\,RMS} = 8.98V.}$

Frequency of ripple voltage $= 2f = 2 \times 60 = \underline{120\ Hz}$.

f) Rectification Efficiency $\eta = \dfrac{P_{dc}}{P_{ac}} = \dfrac{(V_{dc})^2/R_L}{(V_{rms})^2/R_L} = \left(\dfrac{V_{dc}}{V_{rms}}\right)^2$

$\eta = \left(\dfrac{18.64}{23}\right)^2 = 0.656 \Rightarrow \boxed{\% \eta = 66\%}$

6 A Full wave rectifier ckt uses two Si diodes with a forward resistance of 20Ω each. A dc voltmeter connected across the load of $1k\Omega$ reads 55.4 volts. Calculate

a) I_{rms}.
b) average value across each diode.
c) ripple factor.
d) Transformer secondary voltage rating.

Sol
Given data $V_{dc} = 55.4V$, $R_L = 1k\Omega = 1000\Omega$, $R_f = 20\Omega$

$I_{dc} = \dfrac{V_{dc}}{R_f + R_L} = \dfrac{55.4}{20 + 1000} = 54.31\ mA$

We know $I_{dc} = \dfrac{2I_m}{\pi}$, $I_m = \dfrac{I_{dc} \cdot \pi}{2} = 85.31\ mA$

a) $I_{rms} = \dfrac{I_m}{\sqrt{2}} = \dfrac{85.31 \times 10^{-3}}{\sqrt{2}} = 60.32\ mA$ $\boxed{I_{rms} = 60.32\ mA}$

b) The average value across the each Si diode will be $\underline{0.7\ V}$.

c) Ripple factor $\gamma = \sqrt{\left(\dfrac{I_{rms}}{I_{dc}}\right)^2 - 1} = \sqrt{\left(\dfrac{60.32}{54.31}\right)^2 - 1} = 0.4833$.

$\therefore \boxed{\gamma = 0.4833}$.

d) Transformer secondary voltage rating. ie V_{rms}.

ie $V_{dc} = \dfrac{2V_m}{\pi} - I_{dc}(R_s + R_f)$

$55.4 = \dfrac{2V_m}{\pi} - 54.31 \times 10^{-3}(20) = \dfrac{2V_m}{\pi} - 1.0862$

$\Rightarrow V_m = (55.4 + 1.0862) \cdot \dfrac{\pi}{2} = 88.73\ V$

$V_{rms} = \dfrac{V_m}{\sqrt{2}} = \dfrac{88.73}{\sqrt{2}} = \underline{62.74\ V}$.

Hence
Transformer secondary voltage rating is $65V - 0 = \underline{\underline{65\ V}}$.

Bridge Rectifier :

(7) In a Bridge rectifier, the transformer is connected to 220V, 60 Hz. mains and the turns ratio of the step-down transformer is 11:1. Assuming the diode to be ideal. Find.

a) I_{dc} b) Voltage across the load c) PIV.

Sol Given data $V_{rms}(\text{primary}) = 220V.$ $f = 60\,Hz.$

$\frac{N_1}{N_2} = \frac{11}{1}.$

Assume $R_L = 1K\Omega$

$V_{rms}(\text{secondary}) = \frac{V_{rms}(\text{primary})}{\text{Turns Ratio}} = \frac{220}{11/1} = \frac{220}{11} = 20V.$

$V_m = \sqrt{2}\,V_{rms(s)} = \sqrt{2}\times 20 = 28.28\,V.$

& $V_{dc} = \frac{2V_m}{\pi} = \frac{2\times 28.28}{\pi} = 18\,V$

a) $I_{dc} = \frac{V_{dc}}{R_L} = \frac{18}{1K} \Rightarrow \boxed{I_{dc} = 18\,mA}$

(or)

$I_m = \frac{V_m}{R_L} = \frac{28.28}{1K} = 28.28\,m$

$I_{dc} = \frac{2I_m}{\pi} = 18\,mA.$

b) Voltage across the load. $V_{dc} = I_{dc}.R_L = 18\times10^{-3}\times 1\times10^{3} = 18V$

$\boxed{V_{dc} = 18V}$

c) $PIV = V_m\,(\text{Bridge}) = 28.28V$ $\boxed{PIV = 28.28\,V.}$

(8) A Bridge rectifier uses four identical diodes having forward resistance of 5Ω each. Transformer secondary resistance is 5Ω and the secondary voltage of 30 V (rms). Determine the d.c voltage (output) for $I_{dc} = 200m$ and the value of the output ripple voltage.

Sol Given data. $V_{rms} = 30V$, $R_s = 5\Omega$, $R_f = 5\Omega$, $I_{dc} = 200\,mA$

a) $V_{dc} = ?$ $V_{dc} = I_{dc}.R_L$, $R_L = ?$, $V_m = \sqrt{2}.V_{rms} = \sqrt{2}\times 30$

where $I_{dc} = \frac{2I_m}{\pi} \Rightarrow I_m = \frac{I_{dc}.\pi}{2} = 0.314\,A$

But $I_m = \frac{V_m}{(R_s + 2R_f + R_L)} \Rightarrow \frac{\sqrt{2}\times 30}{5+10+R_L} = 0.314$

$\boxed{R_L \cong 120\Omega}$

$V_{dc} = 200\,m\times 120 = 24V$

$\therefore \boxed{V_{dc} = 24V.}$ (or) $V_{dc} = \frac{2V_m}{\pi} - I_{dc}(R_s + 2R_f).$

b) Ripple factor $\gamma = \frac{V_{r(rms)}}{V_{dc}} = 0.482$ (Bridge)

$$\Rightarrow V_{r(rms)} = 0.482 \times 24 = 11.568\,V.$$

$$\therefore \boxed{V_{r(rms)} = 11.568\,V}$$

9 In a full wave rectifier the required d.c voltage is 9V. and the diode drop of 0.8V. Calculate a.c. rms input voltage required in case of bridge rectifier ckt & Center tapped full wave rectifier ckt?

Sol Given data $V_{dc} = 9V$, V_B ie drop of diode $= 0.8\,V.$, $V_{rms} = ?$

For Full wave centre tap rectifier :

$$V_{dc} = \frac{2}{\pi}(V_m - V_B) = \frac{2}{\pi}[V_m - 0.8]$$

$$\Rightarrow V_m = \frac{9 \times \pi}{2} + 0.8 = 14.937\,V$$

$$V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{14.937}{\sqrt{2}} = 10.56V \quad \therefore \boxed{V_{rms} = 10.56\,V}$$

This is across the half of the secondary.

For Bridge rectifier: (∵ Since 2 diodes $2V_B$ for 1 cycle)

$$V_{dc} = \frac{2}{\pi}[V_m - 2V_B]$$

$$V_m = \frac{9 \times \pi}{2} + 2(0.8) = 15.74\,V$$

$$V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{15.74}{\sqrt{2}} = 11.13\,V \quad \therefore \boxed{V_{rms} = 11.13\,V}$$

This is across the entire secondary.

Capacitor Filter:

10 A 15-0-15 V (rms) ideal transformer is used with a full wave rectifier ckt with diodes having forward drop of 1V. The load is a resistance of 100Ω & a capacitor of 10,000 μF is used as a filter across the load resistance. Calculate the d.c load current and voltage?

Sol Given data. $V_{rms} = 15\,V$, $V_B = 1V$ (diode drop), $R_L = 100\Omega$

Capacitor $C = 10,000\,\mu F$, $V_m = \sqrt{2}.V_{rms} = 21.21\,V.$

$$I_m = \frac{V_m - V_B}{R_L} = \frac{21.21 - 1}{100} = 0.202\,A$$

$$I_{dc} = \frac{2 I_m}{\pi} = \frac{2 \times 0.202}{\pi} = 0.1286\,A \quad \text{without filter.}$$

The d.c output voltage from a capacitor filter fed from a Fullwave rectifier is given by $V_{dc} = V_m - I_{dc}\left(\frac{1}{4fC}\right) - V_B$.

Assume $f = 50\,Hz$.

$\therefore$ & $I_{dc} = \frac{V_{dc}}{R_L}$

$$V_{dc} = \left[\frac{4fR_LC}{4fR_LC+1}\right] \cdot V_m - V_B$$

$$= \left[\frac{4\times 50\times 100\times 10{,}000\times 10^{-6}}{4\times 50\times 100\times 10{,}000\times 10^{-6}+1}\right]\times 15\times\sqrt{2} - 1$$

$$= 21.104 - 1 \qquad \therefore \boxed{V_{dc} = 20.104\,V}$$

$$V_{dc} = 20.104$$

$$\& \quad I_{dc} = \frac{V_{dc}}{R_L} = \frac{20.104}{100} \Rightarrow \boxed{I_{dc} = 0.201\,A.} \text{ with filter.}$$

(11) A Full wave rectified voltage of 18 V peak is applied across a $500\mu F$ filter capacitor. Calculate the ripple & dc voltages if the load takes a current of 100mA.

Sol Given data $V_m = 18\,V$ $C = 500\,\mu F$ & $I_{dc} = 100\,mA$, let $f = 50\,Hz$

D.C voltage $V_{dc} = V_m - I_{dc}\left(\frac{1}{4fC}\right) = 18 - \frac{100\times 10^{-3}}{4\times 50\times 500\times 10^{-6}}$

$$\boxed{V_{dc} = 17\,V.}$$

$$V_{r(rms)} = \frac{I_{dc}}{4\sqrt{3}\cdot f\cdot C} = \frac{100\times 10^{-3}}{4\sqrt{3}\times 50\times 500\times 10^{-6}} = 0.577\,V$$

$$\boxed{V_{r(rms)} = 0.577\,V}$$

$$\therefore \text{Ripple} = \frac{V_{r(rms)}}{V_{dc}}\times 100 = \frac{0.577}{17}\times 100 = 3.39 \Rightarrow \boxed{r = 3.39\,\%}$$

Inductor or Choke Filter:

(12) Calculate the value of inductance to use in the inductor filter connected to a full-wave rectifier operating at 60 Hz to provided a dc output with 4 % ripple for a 100Ω load.

Sol Ripple factor for Inductor filter $r = \frac{R_L}{3\sqrt{2}\,\omega L}$, $f = 60\,Hz$, $R_L = 100\,\Omega$, $\omega = 2\pi f$, $r = 4\% = 0.04$

$$0.04 = \frac{100}{3\sqrt{2}\times 2\pi\times 60\times L}$$

$$L = \frac{0.0625}{0.04} = 1.5625\,H \qquad \therefore \boxed{L = 1.5625\,H.}$$

L-Section & LC filter:

(13) A Full wave rectifier (FWR) supplies a load requiring 300V at 200 mA. Calculate the transformer secondary voltage for

a) a capacitor input filter using a capacitor of 10 μF and

b) a choke input filter using a choke of 10 H and a capacitance of 10 μF

neglect the resistance of choke?

Sol Given data $V_{dc} = 300V$, $I_{dc} = 200\,mA$. & let $f = 50\,Hz$.

a) Capacitor $C = 10\mu F$ - For capacitor filter.

$$V_{dc} = V_m - \frac{I_{dc}}{4fC} \Rightarrow 300 = V_m - \frac{200\times10^{-3}}{4\times50\times10\times10^{-6}}$$

$$V_m = 400\,V.$$

$$V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{400}{\sqrt{2}} = 282.84V \quad \therefore \boxed{V_{rms} = 282.84V}$$

ie Transformer secondary voltage.

b) Choke input filter $L = 10H$ & $C = 10\mu F$

$$V_{dc} = \frac{2V_m}{\pi(R_x+R)} \cdot R \text{, but } R_x = 0.$$ ← Resistance of choke

$$300 = \frac{2}{\pi}\times V_m \Rightarrow V_m = \frac{300\times\pi}{2} = 471.24V$$

$$V_{rms} = \frac{V_m}{\sqrt{2}} = 333.22V \quad \therefore \boxed{V_{rms} = 333.22V}$$

(14) Determine the ripple factor of a L-type choke input filter comprising a 10 H choke & 8 μF capacitor used with a FWR. Compare with a simple 8 μF capacitor i/p filter & a load current of 50 mA & also at 150 mA. Assume the d.c voltage of 50V.

Sol Given data. $V_{dc} = 50V$, $L = 10H$, $C = 8\mu F$. let $f = 50Hz$

$\omega = 2\pi f = 100\pi$ rad/sec.

For LC filter

$$\text{Ripple factor} = \frac{1}{6\sqrt{2}\omega^2 LC} = \frac{1}{6\sqrt{2}\times(100\pi)^2\times10\times8\times10^{-6}} = 0.01492$$

$$\cong 1.492\,\%$$

For any load current, for choke input filter the ripple factor is 1.492%

For simple capacitor filter $C = 8\mu F$

a) $I_L = 50mA \Rightarrow R_L = \frac{V_{dc}}{I_L} = \frac{50}{50\times10^{-3}} = 1000\Omega$

$$\text{Ripple factor } r = \frac{1}{4\sqrt{3}fC\cdot R_L} = \frac{1}{4\sqrt{3}\times50\times8\times10^{-6}\times1000} = 0.3608, \quad \boxed{r = 36.08\%}$$

b) $I_L = 150mA$ $\quad R_L = \frac{V_{dc}}{I_L} = \frac{50}{150\times10^{-3}} = 333.33\Omega$

Ripple factor $r = \frac{1}{4\sqrt{3}fCR_L} = 1.0825 \Rightarrow \boxed{r = 108.25\%}$

Hence

LC choke input filter is more effective than capacitor input filter.

π-Section or CLC filter:

(15) A full wave single phase rectifier employs a π-section filter consisting of two 4 μF capacitances and a 20H choke. The Transformer voltage to the center tap is 300 V(rms). The load current is 500 mA. Calculate the d.c output voltage and the ripple voltage. The resistance of the choke is 200Ω.

Sol. Given data. $C_1 = C_2 = 4\mu F$, $L = 20H$. $\quad$ let $f = 50Hz$.

$I_{dc} = 500mA$, $R_x = 200\Omega$

$V_{rms} = 300V$, $V_m = \sqrt{2}.V_{rms} = \sqrt{2}\times300 = 424.2V$.

$V_{dc} = V_m - \frac{V_r}{2} - I_{dc}.R_x$ $\quad$ (full wave)

where V_r - peak to peak ripple voltage $= \frac{I_{dc}}{2fC}$

$V_r = \frac{500\times10^{-3}}{2\times50\times4\times10^{-6}} = 1.25mV$.

$V_{dc} = 424.2 - \frac{1.25\times10^{-3}}{2} - 500\times10^{-3}\times200 = 324.2V$

$\therefore \boxed{V_{dc} = 324.2V}$

Ripple factor $= \frac{\sqrt{2}}{8\omega^3 LC_1C_2R_L}$, $\quad R_L = \frac{V_{dc}}{I_{dc}} = \frac{324.2}{500\times10^{-3}}$

$R_L = 648.4\Omega$

$= \frac{\sqrt{2}}{8\times(2\pi\times50)^3\times20\times(4\mu\times4\mu)\times648.4}$, $\quad \omega = 2\pi f$.

$r = 0.0275$

$\therefore$ Ripple voltage $V_{r(rms)}$ = Ripple factor × V_{dc} = 0.0275×324.2 $= 8.92V$

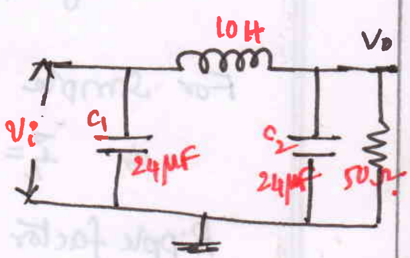
$\therefore \boxed{V_{r(rms)} = 8.92V.}$

(16) Design a CLC or π-section filter for $V_{dc} = 10V$, $I_{dc} = 200mA$ and $r = 2\%$.

Sol. $R_L = \frac{V_{dc}}{I_{dc}} = \frac{10}{200\times10^{-3}} = 50\Omega$ $\quad \boxed{R_L = 50\Omega}$ $\quad$ let $L = 10H$ & $C_1 = C_2 = C$.

$r = \frac{5700}{L.C_1C_2\times R_L} \Rightarrow \frac{114}{LC_1C_2} = 0.02 \Rightarrow C^2 = 570$

0.02 $\quad r = \frac{\sqrt{2}}{8\omega^3 LC_1C_2R_L}$, $f = 50Hz$. $\quad \boxed{L = 10H}$ & $\boxed{C \cong 24\mu F}$



Voltage Regulator:

(17) In a zener regulator, the d.c input is $10V \pm 20\%$. The output requirements are $5V$, $20mA$. Assume $I_{z(min)}$ & $I_{z(max)}$ as $5mA$ & $80mA$. Design the zener regulator.

Sol Given data. $V_0 = 5V$, $I_L = 20mA$, $I_{z(min)} = 5mA$, $I_{z(max)} = 80mA$

$V_{in(min)} = 10 - (0.2 * 10) = 8V$

$V_{in(max)} = 10 + (0.2 \times 10) = 12V$.

& $I_L = I_{L(min)} = I_{L(max)} = 20mA$.

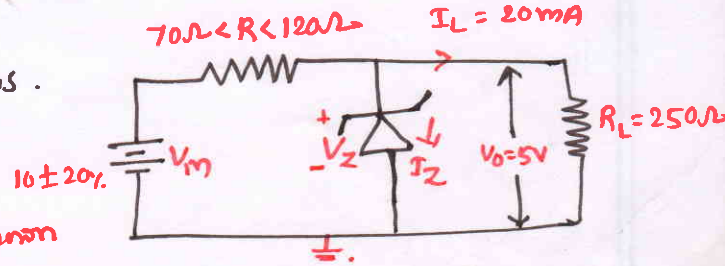
$R_L = \frac{V_0}{I_L} = \frac{5}{20 \times 10^{-3}} = 250\Omega \Rightarrow R_L = 250\Omega$.

Series resistance $R_{(max)} = \frac{V_{in(min)} - V_0}{I_{L(max)} + I_{z(min)}} = \frac{8-5}{20m + 5m} = 120\Omega$.

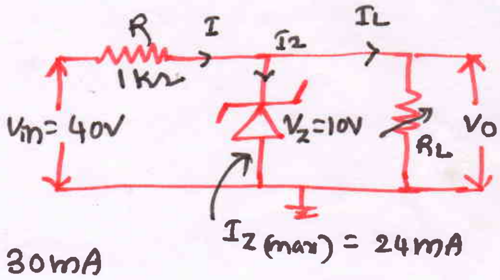
$R_{(min)} = \frac{V_{in(max)} - V_0}{I_{L(min)} + I_{z(max)}} = \frac{12-5}{20m + 80mA} = 70\Omega$.

$\therefore$ $70\Omega < R < 120\Omega$.

The designed zener regulator ckt as.



(18) For the zener voltage regulator shown determine the range of R_L and I_L that gives the stabilizer voltage of $10V$.



Sol. From the ckt $I = I_z + I_L$

But From R, $I = \frac{V_{in} - V_0}{R} = \frac{40-10}{1 \times 10^3} = 30mA$

$\therefore I = 30mA$.

When I_L is minimum, I_z is maximum ie

$I = I_{z(max)} + I_{L(min)} \Rightarrow 30m = 24m + I_{L(min)}$

$\Rightarrow \boxed{I_{L(min)} = 6mA.}$

But $I_{L(min)} = \frac{V_0}{R_{L(max)}} \Rightarrow R_{L(max)} = \frac{V_0}{6m} = 1.667k\Omega$ $\boxed{R_{L(max)} = 1.667k\Omega}$

When I_L is maximum, I_z is minimum ie

$I = I_{z(min)} + I_{L(max)} \Rightarrow 30m = 5m + I_{L(max)}$

let $I_{z(min)} = 5mA$

$\Rightarrow \boxed{I_{L(max)} = 25mA}$

But $R_{L(min)} = \frac{V_0}{I_{L(max)}} = \frac{10}{25 \times 10^{-3}} = 400\Omega$, $\boxed{R_{L(min)} = 400\Omega.}$

$\therefore$ So range of I_L is $6mA$ to $25mA$ & R_L is 400Ω to $1.667k\Omega$.