Wave Optics

Introduction

Basically optics is the branch of science which deals with the study of light. It is also known as the branch of physics, which deals with the study of properties and nature of light. Optics is mainly divided into two parts.

Geometrical optics

It deals with the image formation by optical systems that is the Geometrical optics concerns with the formation of images, when light rays passes through an optical system, such as a lens and a prism.

Physical optics

It deals with the nature of light, that is the physical optics deals with the nature of light, such as Interference, Diffraction and polarization.

Superposition Principle:

Superposition principle enables us to find the **resultant of two or more wave motions**. According to this principle, when two or more waves travelling through a medium superimpose upon one another and a new wave will be formed, whose resultant displacement at any instant is equal to the **vector sum of the displacements due to individual waves** at that instant

For example,

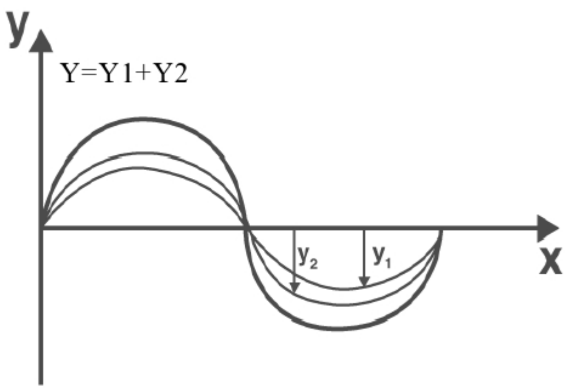
- when crest (or trough) of one wave falls on crest (or trough) of the other, the amplitude of the resultant wave is sum of the amplitudes of two waves.
- When crest of one wave falls on trough of the other, the amplitude of resultant wave is difference of the amplitude of the two waves.

Y 1 (X, t) represents the displacement of the first wave.

Y 2 (X, t) represents the displacement of the second wave.

Then the resultant displacement of the two waves mathematically given as

Y (X, t) = Y 1 (X, t) + Y 2 (X, t)



Interference:

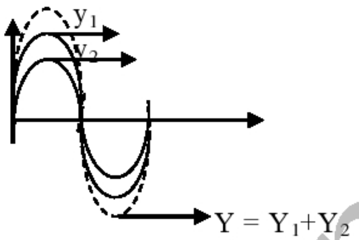
If two (or) more waves of same frequency **superimpose** one upon another in a certain region or space, the total energy is redistributed in such a way that at certain points, the intensity will be **maximum** and at other points it will be **minimum**.

Constructive interference:

If y_1 is the amplitude of 1st wave and y_2 is the amplitude of the 2nd wave, then the resultant displacement is

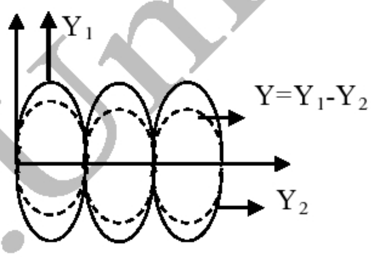
$$y = y_1 + y_2$$

if the 2 waves reach at a point in phase (ie., the crest of 1st wave exactly coincides with the crest of 2nd wave), then the resultant displacement "y" will be maximum and the waves are said to be exhibit constructive interference.



Destructive interference

If the two waves reach a particular point with a phase difference 180^0 (ie., the crest of the 1st wave falls on the trough of 2nd wave and vice –versa) then the waves are said to exhibit destructive interference.

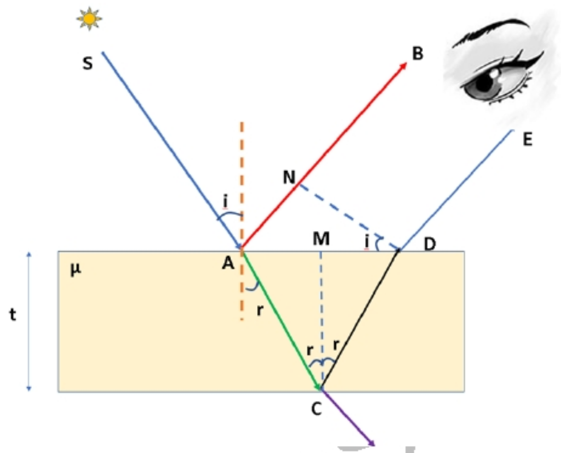


Interference in thin films

The colours of thin films, soap bubbles and oil slicks can be explained as due to the phenomena of interference. Let a plane wave front be allowed to incident normally on a thin film of uniform thickness "t". The plane wave front is obtained with the help of a partially reflecting a glass plate G inclined at an angle 45^0 with the parallel monochromatic beam of light. The plane wave front is partly reflected at the upper surface of the film and partly transmitted into the film. The transmitted wave front is reflected again from the bottom surface of the film and emerges through the first surface. **The wavefront reflected from the upper surface and the lower surface interfere with each other.** The resultant interference pattern can be observed with eye.

Consider a thin film of **thickness "t" and the refractive index of "μ".**

- Let a single ray of monochromatic light of wavelength "λ" be incident on the upper surface of the film at an angle "I". This ray is **partly reflected along AB and partly refracted along AC**. The angle of reflection is "r".
- At point "C" the ray is again partly reflected from the second surface **CD** and partly refracted along **DE**.



- **AB** and **DE** are the first two rays due to reflection comes from the thin film.

The two rays will superimpose with each other and produces interference; the interference may be constructive or destructive depending upon the path difference.

It is seen from the figure that the 2 rays are following the same path after "DN"

Therefore the path difference between the two rays (δ) is given by

δ = path ACD in film – path AN in air

$\delta = \mu(AC+CD) - AN(1)$ **(Since the refractive index of air=1)**

<u>Calculation of AC and CD</u>

Consider the triangle ACM

$\cos r = \frac{CM}{AC} = \frac{t}{AC}$

$AC = \frac{t}{\cos r}$

Consider the triangle MCD

$\cos r = \frac{CM}{CD} = \frac{t}{CD}$

$CD = \frac{t}{\cos r}$

i) <u>Calculation of AN:</u>

In triangle ADN

$\sin i = \frac{AN}{AD}$

AN= 2 t tan r (sin i)

But AD =(AM+MD)

Consider the triangle ACM

$\tan r = \frac{AM}{CM} = \frac{AM}{t}$

AM= t tan r

Consider the Δle CDM

$\tan r = \frac{MD}{CM} = \frac{MD}{t}$

MD = t tan r

$\delta = \mu(AC+CD) - AN$

$= \mu\left(\frac{t}{\cos r} + \frac{t}{\cos r}\right) - 2t \tan r (\sin i)$

$$= \frac{2\mu t}{\cos r} - 2t\left(\frac{\sin r}{\cos r}\right)(\mu \sin r)$$ Since, Snell's law (μ) = $\frac{\sin i}{\sin r}$

$$\delta = \frac{2\mu t}{\cos r} - \frac{2\mu t \sin^2 r}{\cos r}$$

$$\delta = \frac{2\mu t}{\cos r}(1 - \sin^2 r)$$

$$\delta = \frac{2\mu t}{\cos r}(\cos^2 r)$$

$$\boxed{\delta = 2\mu t \cos r}$$

w.k.t

i) The light ray along "AB" is the reflected ray from a <u>rarer to denser medium</u>. Hence it undergoes a path difference $\frac{\lambda}{2}$

(**<u>Note:</u>** When light is traveling from the rarer to denser medium, crests get reflected as troughs and troughs get reflected as crests. The wave is said to undergo a path difference of $\frac{\lambda}{2}$.

Therefore, the **total path difference is** $\delta = 2\mu t \cos r - \frac{\lambda}{2}$

i) For constructive interference the path difference should be equal to integral multiple of "λ"

$$\boxed{\delta = 2\mu t \cos r + \frac{\lambda}{2} = n\lambda} \qquad 2\mu t \cos r = (2n-1)\frac{\lambda}{2}$$

ii) For destructive interference the path difference should be equal to

$$\delta = 2\mu t \cos r + \frac{\lambda}{2} = (2n+1)\,\lambda/2$$

$$\boxed{2\mu t \cos r = (n\lambda)}$$ Where n=0, 1, 2…..

Newton's Rings

Newton's rings:

Newton observed **"Circular interference fringes"** in a very thin of air or some other transparent medium of thickness "t" is enclosed in between a plane glass plate or a Plano-convex lens. These rings are known as **Newton's rings.**

Experimental set-up:

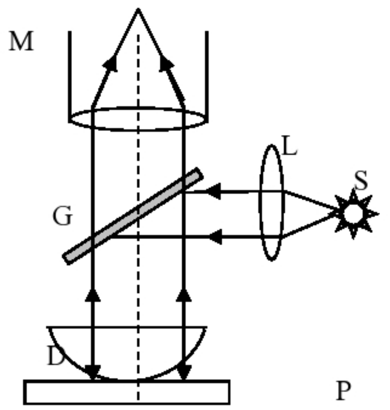
i) Light from a monochromatic source is made parallel by a convex lens "L" and then it is made to fall on a glass plate "G" inclined at an angle 45^0.

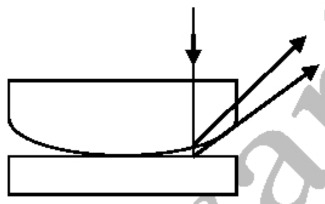
ii) This beam is made to **incident normally** on to a Plano-convex lens placed on a glass plate "P" as shown below.

iii) Light rays reflected from the **top and bottom surfaces of the air film** super impose upon each other and produces interference

iv) Depending upon the path difference in between them they produce circular dark and bright

S-Source
G-Glass plate
P-Plane glass plate
D-Plano-convex lens
M-Microscope





Theory of Newton's rings

Now the path difference between two rays, one reflected from the top and the bottom surface of the air film is $2\mu t\cos r = 2t$ (Since the **incidence is normal, so r=0 and cos 0^0=1,** and also **μ=1 for air**) . As one of the rays travels from denser to rarer medium, so an additional path difference of $\frac{\lambda}{2}$ is introduced.

Total path difference= $2t+\frac{\lambda}{2}$

Condition for getting bright ring is

$$2t+\frac{\lambda}{2} = n\lambda$$

$$2t = n\lambda-\frac{\lambda}{2}$$

$$\boxed{2t = (2n-1)\frac{\lambda}{2}} \quad \text{-----------------------(2)}$$

Condition for getting dark ring is

$$2t+\frac{\lambda}{2} = (2n+1)\ \frac{\lambda}{2}$$

$$\boxed{2t = n\lambda} \quad \text{------------------------ (3)}$$

Calculation of wavelength of a monochromatic source

The diameter of n^{th} dark ring is

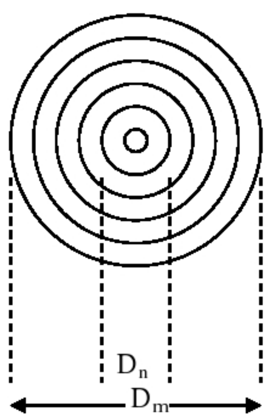
$$D_n^{\ 2} = 4Rn\lambda$$

The diameter of m^{th} dark ring is m>n

$$D_m^{\ 2} = 4Rm\lambda$$

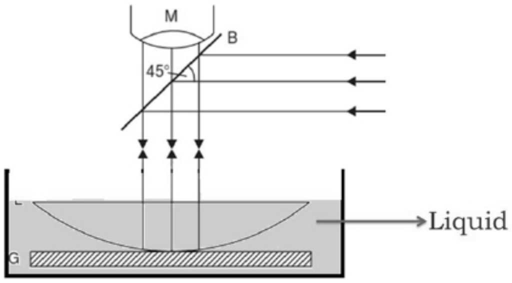
$$D_m^{\ 2}-D_n^{\ 2} = 4R\lambda\ (m-n)$$

$$\boxed{\lambda = \frac{D_m^{\ 2}-D_n^{\ 2}}{4R(m-n)}}$$



m, n order of dark rings
R-radius of curvature of the Plano-convex lens

Determination of refractive index of liquid by newtons rings



Now the Newton's Rings system is placed into a containers containing a liquid of refractive index μ .

Now we have to find the value of refractive index of the liquid.

Now the air film is replaced by the liquid film.

Now again the experiment is repeated. The diameters of m^{th} and n^{th} dark Rings are now obtained.

Then we have

$$D_m'^2 - D_n'^2 = \frac{4(m-n)\lambda R}{\mu} \quad \text{--- (1)}$$

Also for air film, we have

$$D_m^2 - D_n^2 = 4(m-n)\lambda R \quad \text{---- (2)}$$

From equations (1) and (2), we get

Using this formulae, we can calculate μ .

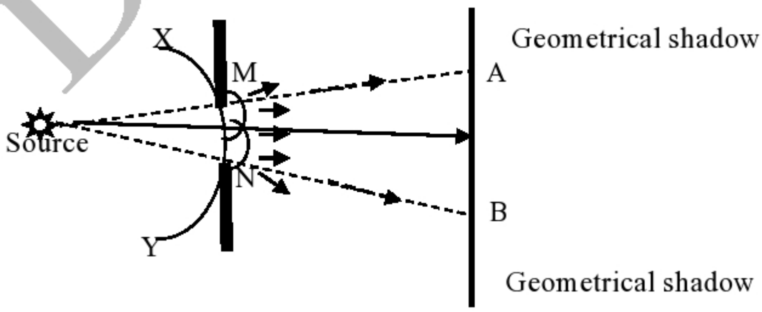
$$\mu = \frac{D_m^2 - D_n^2}{D_m'^2 - D_n'^2}$$

Diffraction

Diffraction:

The phenomenon of **bending of light around** the corners of obstacles and **spreading of light into the geometrical shadow** region of an obstacle placed in the path of light is called diffraction.

We can explain the diffraction based on Huygen's wave theory of light. Acc. to Huygen's wave theory, every point on the primary wavefront acts as a source of secondary waves.

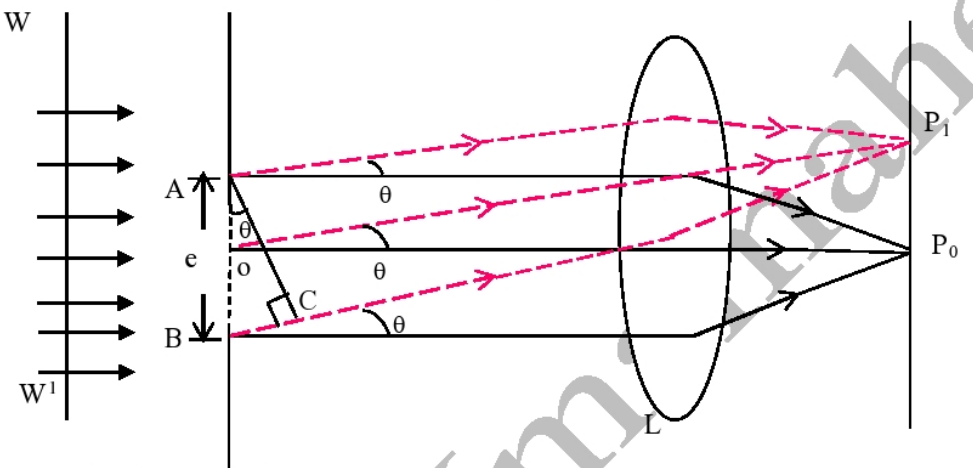


Let as consider a primary wavefront from the source reaches an aperture MN, each point on the primary wavefront acts as the source of secondary waves and emits waves. These waves travel in all possible direction and enter the geometrical shadow region and interfere with each other and produces interference fringes (bright and dark fringes)

Fresnel and Fraunhofer diffraction:

Fresnel diffraction	Fraunhofer diffraction
A point source at finite distance	A point source at infinite distance
The wavefront undergoing diffraction is cylindrical or spherical wavefront	The wavefront undergoing diffraction is plane wavefront
The source and the screen are at finite distance from the obstacle	The source and the screen are at infinite distance from the obstacle
No lens is needed to focus the rays	Converging lens is needed to focus the rays

Fraunhofer Diffraction at a Single slit



WW¹=Plane wave front

AB= Rectangular slit

L=Lens

Fraunhofer diffraction at a single

Consider a slit AB of width 'e', ww' is a plane wavefront of monochromatic light of wavelength λ is incident normally on the slit. The diffracted light through the slit is focused by using a convex lens on to a screen placed in the focal plane of the lens. According to Huygens principle, every point on the primary wavefront on the slits is a source of secondary wavelet. These secondary wavelets spread out in all directions.

The secondary wavelets traveling normal to the slit, along the direction OP_0 are brought to focus at P_0 by the convex lens L. **Thus P_0 is a central bright image**. The central bright image is formed because there is **no path difference** for the Ray traveling normal to the slit.

The secondary wavelets traveling at an angle θ are focused at a point p_1 on the screen. The **intensity of point p_1 depends upon the path difference** between the secondary waves originating from the corresponding points of the wavefront. To find intensity at p_1, draw a normal AC from A to the light ray at B. Now the path difference between the secondary wavelets from A and B in the direction θ is given by

Path difference = BC

From the figure triangle ABC is a right angled triangle.

$$\therefore \sin\theta = \frac{BC}{AB}$$

$\Rightarrow BC = AB\sin\theta$ But AB = e

$$\therefore BC = e\sin\theta \qquad \text{------ (1)}$$

Now the phase difference $\phi = \frac{2\pi}{\lambda} \times$ path difference.

$$\therefore \phi = \frac{2\pi}{\lambda} \times e\sin\theta \qquad \text{---- (2)}$$

Now let the width of the slit is divided into 'n' equal parts. The amplitude of the wave from each part is 'a'. The phase difference between any two successive waves from these parts will be given by

$$\frac{1}{n}[\text{total phase}] = \frac{1}{n}\left[\frac{2\pi}{\lambda}e\sin\theta\right] = d \qquad \text{----- (3)}$$

By the method of vector addition of amplitudes, the Resultant amplitude R is given by

$$R = \frac{a\sin\left(\frac{nd}{2}\right)}{\sin\left(\frac{d}{2}\right)} \qquad \text{---- (4)}$$

From equations (3) and (4)

$$R = \frac{a\sin\left(\cancel{n}\frac{1}{\cancel{n}}\frac{\cancel{2}\pi}{\lambda\cancel{2}}e\sin\theta\right)}{\sin\left(\frac{\cancel{2}\pi}{n\lambda}\frac{e\sin\theta}{\cancel{2}}\right)}, \quad R = \frac{a\sin\left(\frac{\pi e\sin\theta}{\lambda}\right)}{\sin\left(\frac{\pi e\sin\theta}{n\lambda}\right)}$$

Now let $\frac{\pi e\sin\theta}{\lambda} = \alpha$ ----- (5)

$$R = \frac{a\sin\alpha}{\sin\left(\frac{\alpha}{n}\right)}$$

In the above expression $\left(\frac{\alpha}{n}\right)$ is very small

Hence $\sin\left(\frac{\alpha}{n}\right) = \frac{\alpha}{n}$.

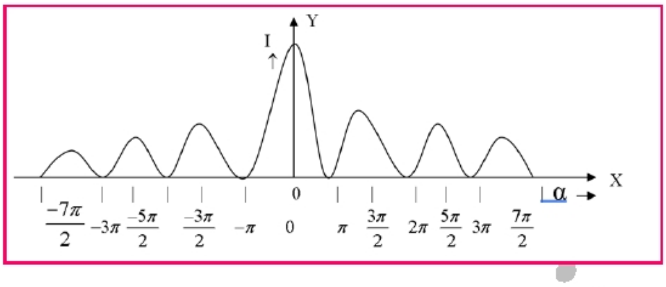
$$\therefore R = \frac{a\sin\alpha}{\left(\frac{\alpha}{n}\right)}, \quad R = \frac{na\sin\alpha}{\alpha}$$

$$\Rightarrow R = \frac{A\sin\alpha}{\alpha}, \text{ Here } A = na \qquad \text{---- (6)}$$

We know that intensity of light is proportional to square of the amplitude.

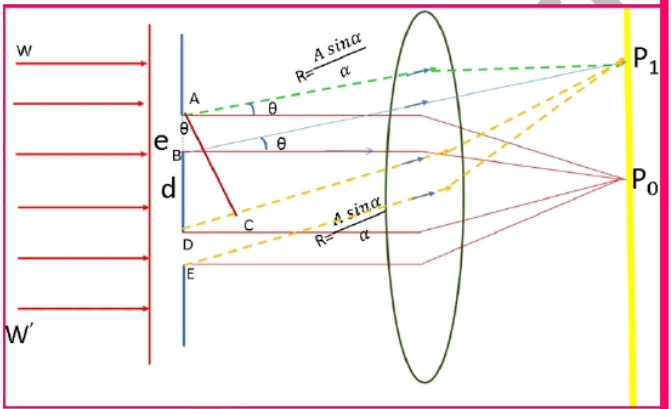
Intensity $I = R^2$

$$\Rightarrow I = A^2\left(\frac{\sin\alpha}{\alpha}\right)^2 \quad ---- (7)$$



Fraunhofer Diffraction at Double slit

Let S_1 and S_2 be two slits of equal widths e, the two slits are repeated by a distance d. A monochromatic light of wavelength λ is incident normally on the two slits. The light diffracted from these slits is focused by a lens of the screen placed in the focal plane of the lens. The diffraction of two slits is the combination of diffraction as well as interference.



Let a plane wave front is incident normally on both slits, all points within the slits became the sources of secondary wavelets. These secondary wavelets from the slits travel uniformly in all directions. The secondary wavelets traveling in the direction of incident light come to a focus at P_o. The secondary wavelets traveling in a direction making an angle θ with the incident direction come to a focus at P_1.

From the theory of diffraction due to a single slit, the resultant amplitude R due to all wavelets diffracted from each slit in a direction θ is given by

$$R = \frac{A\sin\alpha}{\alpha}$$

Now let us consider the two slits are acting as source and sending a wavelet of amplitude $\left(\frac{A\sin\alpha}{\alpha}\right)$ in the direction θ.

$\therefore$ **The resultant amplitudes at a point p_1 on the screen will be the result of interference between two waves of amplitude $\left(\frac{A\sin\alpha}{\alpha}\right)$ and having a phase difference of δ.**

Calculation of the phase difference :

The waves from the two slits travel equal path after AD, so, the path difference between the rays coming the two slits can be calculated

by considering the Δ^{le} ADC

$$Sin\,\theta = \frac{DC}{AD}$$

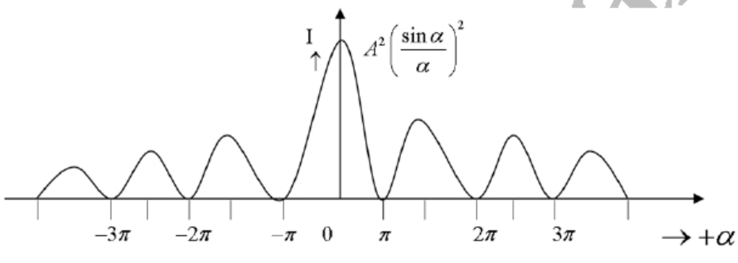
$$Sin\,\theta = \frac{DC}{(e+d)}$$

DC = (e+d) Sin θ

Path difference =(e+d) Sin θ

$$Phase\ differnce(\delta) = \frac{2\pi}{\lambda}(e+d)sin\theta$$

Therefore the resultant amplitude at P_1 due to the waves from the two slits with resultant amplitude of $\left(\frac{A\sin\alpha}{\alpha}\right)$ and a phase differnce δ is



Diffraction effect

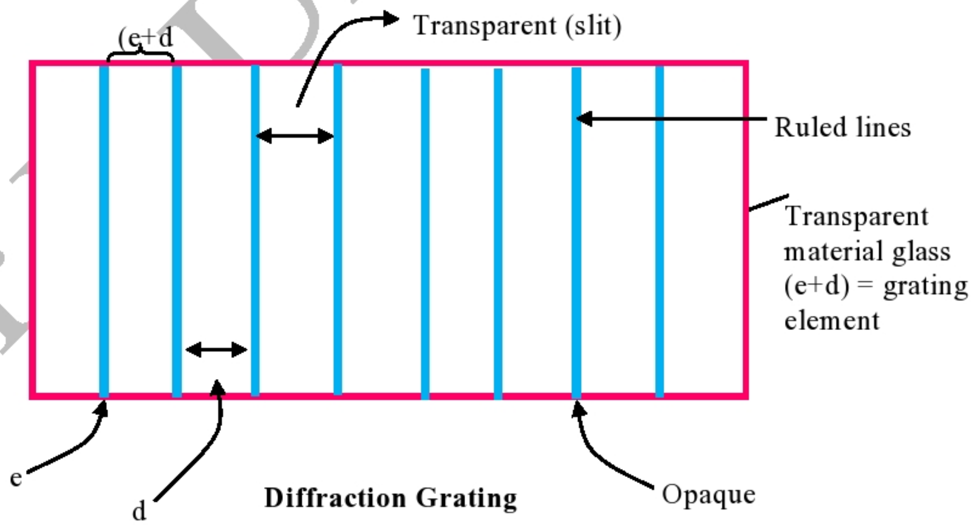
R= 2 ccos δ

Therefore the Intensity at P_1 is

$$I = R^2 = \frac{4A^2Sin^2\alpha}{\alpha^2}cos^2\delta$$

Diffraction Grating

Diffraction grating is an arrangement which consists of a large number of parallel slits of the same width. These parallel slits are separated by equal and opaque spacings, known as diffraction grating. Fraunhofer used the first grating consisting of large number of parallel wires placed side by side very closely at regular intervals. The gratings are designed by ruling equidistant parallel lines on a transparent material such as Glass with a fine diamond tip. The ruled lines are opaque to light while the space between any two lines is transparent to light and act as a slit. This is shown in figure. Usually gratings are designed by taking the cost of an actual grating on a transparent film like that of cellulose acetate.



Diffraction Grating

Now solution of cellulose acetate is poured on the ruled surface and allowed to dry, for the formation of a thin film. This thin film is easily detachable from the surface. These impressions of a grating are preserved by mounting the film between two glass plate thin.

Let e be the width of each line.

Let d be the width of the slit.

Now $(e+d)$ is known as grating element.

If 'N' is the number of lines per inch on the grating, then

$N(e+d)$ grating elements are there per inch.

i.e. $N(e+d) = 1" = 2.54cms$

$$(e+d) = \frac{2.54}{N}cm$$

Usually there will be 15,000 lines per inch (or) 30,000 lines per inch on the grating. When light falls on the grating, the light is diffracted through each slit.

As a result, both diffraction and interference of diffracted light gets enhanced and forms a diffraction pattern. This pattern is known as Diffraction pattern.