Maximum overshoot / peak overshoot (M_p):

→ It is the difference between peak value of time response and steady state value.

$$\% \; M_p = \frac{c(t_p) - c(\infty)}{c(\infty)} \times 100$$

where
$c(\infty) \rightarrow$ final value of $c(t)$
$c(t_p) \rightarrow$ Max value of $c(t)$
$\because c(\infty) = 1$

→ $$M_p = \frac{c(t_p) - 1}{1} \times 100$$

$\cancel{M_p = c(t_p)} \Rightarrow$

$$\boxed{M_p = c(t_p) - 1}$$

$t = t_p$

$$c(t) = 1 - \frac{e^{-\xi \omega_n t}}{\sqrt{1-\xi^2}} \left[\sin(\omega_d t + \theta)\right]$$

$$c(t_p) = 1 - \frac{e^{-\xi \omega_n \pi/\omega_d}}{\sqrt{1-\xi^2}} \times \sin\left(\pi/\cancel{\omega_d} \times \cancel{\omega_d} + \theta\right)$$

$$c(t_p) = 1 - \frac{e^{-\xi \omega_n \pi/\omega_d}}{\sqrt{1-\xi^2}} \cdot \sin(\pi + \theta)$$

$$c(t_p) = 1 + \frac{e^{-\xi\omega_n \pi/\omega_d}}{\sqrt{1-\xi^2}} \sin\theta$$

$$c(t_p) = 1 + \frac{e^{-\xi\omega_n \pi/\omega_d}}{\cancel{\sqrt{1-\xi^2}}} \times \cancel{\sqrt{1-\xi^2}}$$

$$c(t_p) = 1 + e^{-\xi\cancel{\omega_n}\pi/\cancel{\omega_n}\sqrt{1-\xi^2}}$$

$$c(t_p) = 1 + e^{\frac{-\xi\pi}{\sqrt{1-\xi^2}}}$$

$$\% M_p = (\cancel{1} + e^{-\xi\pi/\sqrt{1-\xi^2}} - \cancel{1}) \times 100$$

Setteling time(t_s):

The setteling time 't_s' is defind as the time required for the response to reach final value with in the specified tolerance band i.e; 2% or 5%.

For 2% $\quad t_s = \frac{4}{\xi\omega_n}$

For 5% $\quad t_s = \frac{3}{\xi\omega_n}$

Steady state error:

Steady state error is defined as the difference between input r(t) and output c(t) of the system as time goes to infinity (i.e., when response has reached to steady state).

→ It is denoted as $\boxed{e_{ss}}$

~~→ Steady state error can be calculated by using final value theorem.~~

~~Final value theorem~~ ~~e_{ss} or $e_\infty = \lim_{t \to \infty} e(t)$~~

~~Laplace transform~~

~~$e_{ss} = \lim_{t \to \infty} s\,E(s)$~~

~~$$\boxed{e_{ss} \text{ or } e_\infty = \lim_{t \to \infty} e(t)}$$~~

~~Apply laplace transform~~

~~$$\boxed{e_{ss} = \lim_{t \to \infty} s\,E(s)}$$~~

→ Steady state error can be calculated by using final value theorem.

$\cancel{f}(t) = F(s)$

Final value theorem for laplace Transform.

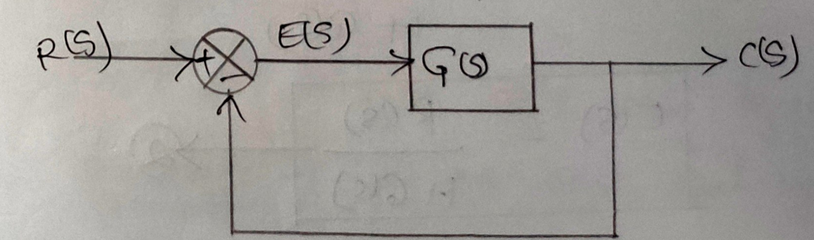
$$\boxed{L[f(t)] = F(s), \text{ then } \quad \lim_{t \to \infty} f(t) = \lim_{s \to 0} s\,F(s)}$$

For steady state error e_{ss} is given by

$$e_{ss} = \lim_{t \to \infty} e(t)$$

$$\boxed{e_{ss} = \lim_{s \to 0} s\,E(s)} \rightarrow (1)$$

To calculate $E(s)$, consider closed loop feedbook system with unity feedback.



$$\frac{C(S)}{R(S)} = \frac{G(S)}{1+G(S)H(S)}$$

where $H(S) = 1$

$$C(S) = \frac{R(S)\,G(S)}{1+G(S)}$$

Error $E(S) = R(S) - C(S)$

$$E(S) = R(S) - \frac{R(S)G(S)}{1+G(S)}$$

$$= \frac{R(S)(1+G(S)) - R(S)\,G(S)}{1+G(S)}$$

$$= \frac{R(S) + \cancel{R(S)G(S)} - \cancel{R(S)G(S)}}{1+G(S)}$$

$$\boxed{E(S) = \frac{R(S)}{1+G(S)}} \rightarrow (2)$$

Substitute eq (2) in eq (1)

$$\boxed{e_{ss} = \lim_{S \to 0} \frac{s\,R(S)}{1+G(S)}} \rightarrow (3)$$

Causes of steady state error

- Nature of inputs
- Types of system
- Non - linearity of system components

Static error constants;

- When input is applied to control system, the steady state error may zero, constant or infinity.
- Value of steady state error depends on type of system and input signal.

→ There are three types of static error constants.

- positional error constant
- velocity error constant
- Acceleration error constant

Unit step Input:

Consider unit step input $r(t)$

$$r(t) = u(t) = 1$$

$$L\{r(t)\} = R(S) = \frac{1}{S}$$

Substitute $R(S)$ in eq ③

$$e_{ss} = \underset{S \to 0}{Lt} \frac{\cancel{S} \times \frac{1}{\cancel{S}}}{1 + G(S)}$$

$$\boxed{e_{ss} = \frac{1}{1 + \underset{S \to 0}{Lt}\, G(S)}}$$

$$e_{ss} = \frac{1}{1 + K_p}$$

$$\boxed{K_p = \underset{S \to 0}{Lt}\, G(S)}$$

where

K_p – position error constant.

Consider poles and zero equation

$$G(S) = \frac{K \cdot (S+Z_1)(S+Z_2) + (S+Z_3) + \cdots}{S^N (S+P_1)(S+P_2) + (S+P_3) + \cdots}$$

Type 0 system:

$$K_p = \underset{S \to 0}{Lt}\, G(S)$$

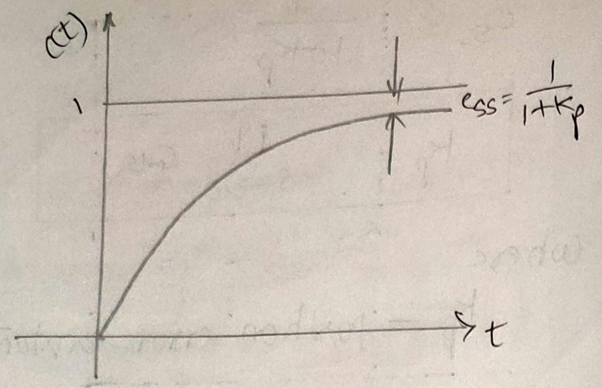
$$= \underset{S \to 0}{Lt}\, K \frac{(S+Z_1)(S+Z_2)(S+Z_3)}{(S+P_1)(S+P_2)(S+P_3)}$$

$$= K \frac{Z_1 Z_2 Z_3}{P_1 P_2 P_3}$$

$$\boxed{K_p = K}$$

In this case, the steady state error is constant

$$e_{ss} = \underset{S \to 0}{Lt} \frac{1}{1+K_p}$$



Type 1 system:

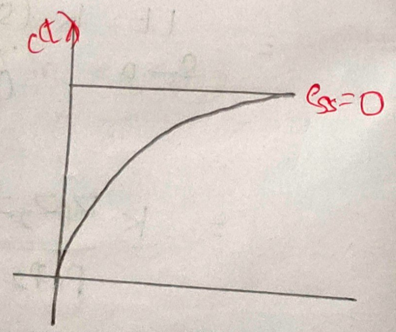
$$G(s) = \frac{K(s+z_1)(s+z_2)(s+z_3)}{s(s+p_1)(s+p_2)(s+p_3)}$$

$$K_p = \underset{s \to 0}{Lt} \frac{K z_1 z_2 z_3}{0 (p_1 p_2 p_3)}$$

$$\boxed{K_p = \infty}$$

$$e_{ss} = \frac{1}{1+K_p}$$

$$\boxed{e_{ss} = 0}$$



$K_p = \infty$ and $e_{ss} = 0$ for step input 4 for higher order systems

N	K_p	e_{ss}
0	K	$\frac{1}{1+K}$
1	∞	0
2	∞	0

Ramp Input:

Ramp input

$r(t) = t$

$$L\{r(t)\} = R(s) = \frac{1}{s^2}$$

Consider, steady state error equation

$$e_{ss} = \underset{s \to 0}{Lt} \frac{R(s)}{1+G(s)}$$

$$e_{ss} = \underset{s \to 0}{Lt} \frac{\cancel{s} \cdot \frac{1}{s^{\cancel{2}}}}{1+G(s)}$$

$$e_{ss} = \underset{s \to 0}{Lt} \frac{1}{s(1+G(s))}$$

$$e_{ss} = \lim_{s \to 0} \frac{1}{s + sG(s)}$$

Now,

$$e_{ss} = \frac{1}{0 + \lim_{s \to 0} sG(s)}$$

$$\boxed{\therefore e_{ss} = \frac{1}{\lim_{s \to 0} sG(s)}}$$

$$\boxed{e_{ss} = \frac{1}{K_v}}$$

$$K_v = \lim_{s \to 0} sG(s)$$

where

$K_v \to$ called as Velocity error constant

Type 0 system:

$$G(s) = \frac{K(s+z_1)(s+z_2)(s+z_3)}{(s+p_1)(s+p_2)(s+p_3)}$$

$$K_v = \lim_{s \to 0} sG(s)$$

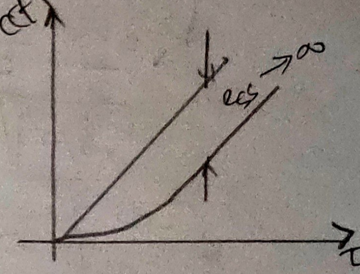
$$K_v = \lim_{s \to 0} s\,\frac{K(s+z_1)(s+z_2)(s+z_3)}{(s+p_1)(s+p_2)(s+p_3)}$$

If $s=0$

$$\boxed{K_v = 0}$$

$$e_{ss} = \frac{1}{K_v}$$

$$\boxed{e_{ss} = \infty}$$



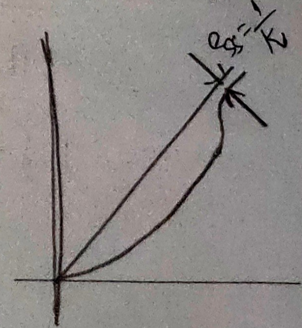
Type 1 system:

$$K_v = \lim_{s \to 0} \cancel{s} \times \frac{K(s+z_1)(s+z_2)(s+z_3)}{\cancel{s}(s+p_1)(s+p_2)(s+p_3)}$$

If $s=0$

then $\boxed{K_v = K}$

$$\boxed{e_{ss} = \frac{1}{K_v} = \frac{1}{K}}$$



~~Std~~ Steady state for type 1 system ~~ss~~ for ramp input is constant.

Type 2 system:

$$K_v = \lim_{s \to 0} s \frac{K(s+z_1)(s+z_2)\cdots}{s^2(s+p_1)(s+p_2)\cdots}$$

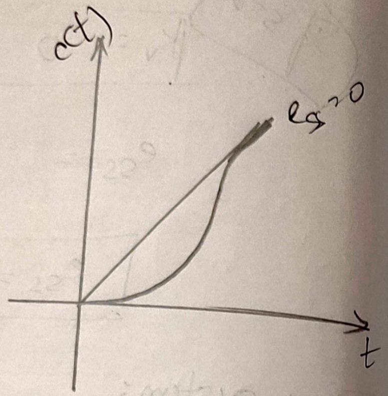
If $s=0$

$$K_v = \frac{1}{0}$$

$$\boxed{K_v = \infty}$$

$$e_{ss} = \frac{1}{K_v} = \frac{1}{\infty}$$

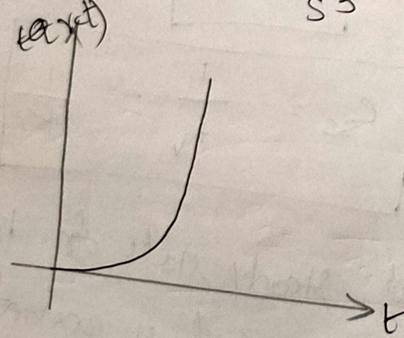
$$\boxed{e_{ss} = 0}$$



Parabolic Input:

$$r(t) = t^2$$

$$L\{r(t)\} = R(s) = \frac{1}{s^3}$$



$$e_{ss} = \lim_{s \to 0} \frac{\cancel{s} \times \frac{1}{s^{\cancel{3}}}}{1+G(s)}$$

$$e_{ss} = \lim_{s \to 0} \frac{1}{s^2(1+G(s))}$$

$$e_{ss} = \lim_{s \to 0} \frac{1}{s^2 + s^2 G(s)}$$

If $s=0$, then

$$e_{ss} = \frac{1}{0 + \lim_{s \to 0} s^2 G(s)}$$

$$e_{ss} = \frac{1}{\lim_{s \to 0} s^2 G(s)}$$

$$\boxed{e_{ss} = \frac{1}{K_a}}$$

$$\boxed{K_a = \lim_{s \to 0} s^2 G(s)}$$

Where
K_a - called as acceleration error constant.

Type 'o' system:

$$K_a = \lim_{s\to 0} s^2 \frac{K(s+z_1)(s+z_2)(s+z_3)}{(s+p_1)(s+p_2)(s+p_3)}$$

if $s=0$

$$\boxed{K_a = 0}$$

$$e_{ss} = \frac{1}{0}$$

$$\boxed{e_{ss} = \infty}$$

Type 1 system:

$$K_a = \lim_{s\to 0} s^{\cancel{2}} \frac{K(s+z_1)(s+z_2)}{\cancel{s}(s+p_1)(s+p_2)}$$

$$K_a = \lim_{s\to 0} 0$$

$$\boxed{K_a = 0}$$

$$\boxed{e_{ss} = \frac{1}{K_a} = \frac{1}{0} = \infty}$$

Type 2 system:

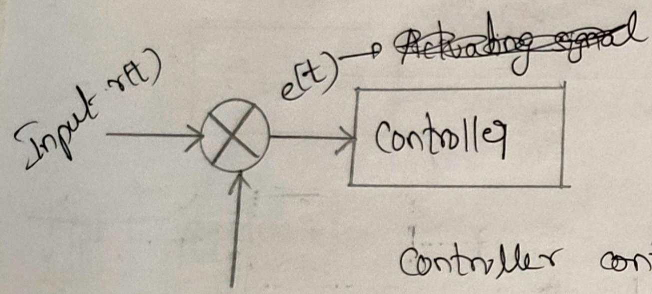
$$K_a = \lim_{s\to 0} \cancel{s^2} \frac{K(s+z_1)(s+z_2)}{\cancel{s^2}(s+p_1)(s+p_2)}$$

$$\boxed{K_a = K}$$

$$\boxed{e_{ss} = \frac{1}{K}}$$

Controllers:

- A controller is the most important component or device of algorithm of the control system.
- Controller makes error to zero or to lowest values.

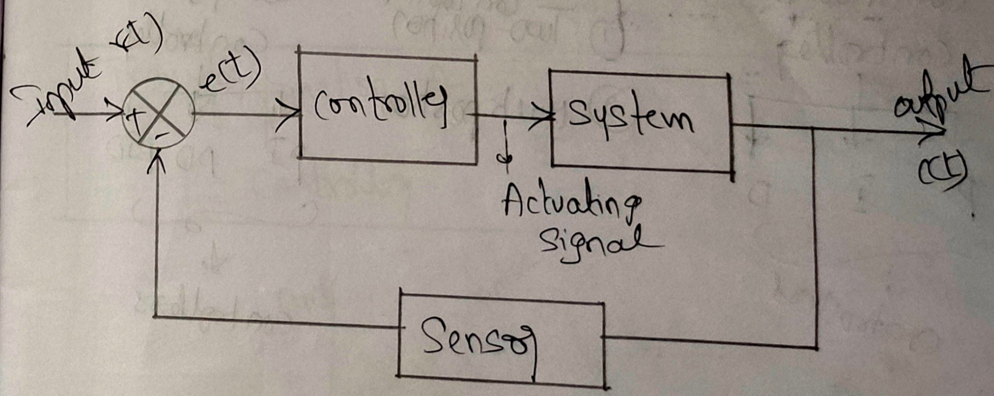


Controller controls the system characteristic of system behavior.

- Controller receives error signal based on this, it generates a control action to make error to zero or lowest value
- With the help of controller accurate output can be obtained.

Uses of controllers;

- Decreasing steady state error
  - If e_{ss} decreases, the stability also improves.
  - helps to speed up the slow response of overdamped system.
  - reducing noise signals.



Types of Controllers;

There are two main types of controllers
- Discontinuous controllers.
- continuous controllers

~~Controller~~ types

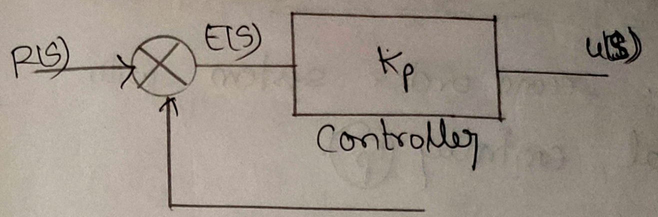
→ Controllers are classified based on their ~~a~~control actions.

→ Controller

- Continuous controller
  - P
  - I
  - D
  - (controllers)
- Discontinuous controller
  1. Two-position
  2. Multi-position
- Composite controller
  - PI
  - PD
  - PID
  - (controllers)

Proportional (P) controller:

→ The control signal is proportional to error signal.

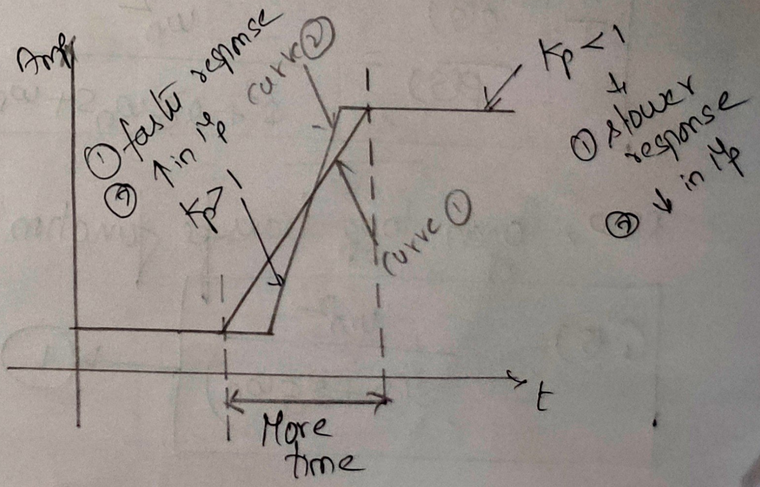


$U(S) \propto E(S)$

$$\boxed{U(S) = K_p E(S)}$$

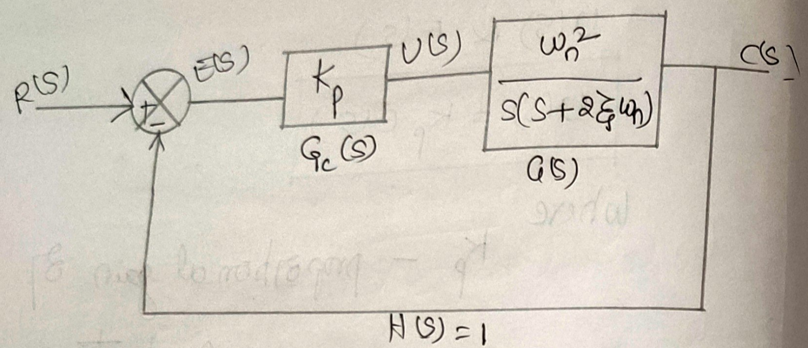
where K_p – proportional gain or constant

Transfer function of P-Controller $\dfrac{U(S)}{E(S)} = K_p$



Effect of proportional (p) controller:

Consider, second order system with proportional controller (K_p).



Without controller:

The transfer function is

$$\boxed{T = \frac{C(S)}{R(S)} = \frac{\omega_n^2}{S^2 + 2\xi\omega_n S + \omega_n^2}}$$

Now, open loop transfer function.

$$\boxed{G(S) = \frac{\omega_n^2}{S(S+2\xi\omega_n)}} \rightarrow (1)$$

The system is type 1,

$$K_V = \lim_{S \to 0} S\,G(S)$$

$$K_V = \lim_{S \to 0} \cancel{S} \times \frac{\omega_n^2}{\cancel{S}(S+2\xi\omega_n)}$$

$$K_V = \frac{\omega_n^2}{0 + 2\xi\omega_n}$$

$$K_V = \frac{\omega_n^{\cancel{2}}}{2\xi\cancel{\omega_n}}$$

$$\boxed{K_V = \frac{\omega_n}{2\xi}} \rightarrow (2)$$

$$e_{ss} = \frac{1}{K_V}$$

$$e_{ss} = \frac{1}{\frac{\omega_n}{2\xi}}$$

$$\boxed{e_{ss} = \frac{2\xi}{\omega_n}} \rightarrow (3)$$

With controller

open loop transfer function $= \dfrac{K_p \omega_n^2}{s(s+2\xi\omega_n)}$

Here also type 1 system

$$K_v = \lim_{s\to 0} s \times G_c(s)\,G_p(s)$$

$$= \lim_{s\to 0} \cancel{s} \times \frac{K_p \omega_n^2}{\cancel{s}(s+2\xi\omega_n)}$$

$$K_v = \frac{K_p \omega_n^{\cancel{2}}}{2\xi\cancel{\omega_n}}$$

$$\boxed{K_v = \frac{K_p \omega_n}{2\xi}}$$

$$e_{ss} = \frac{1}{K_v}$$

$$e_{ss} = \frac{1}{\frac{K_p \omega_n}{2\xi}}$$

$$\boxed{e_{ss} = \frac{2\xi}{K_p \omega_n}}$$

$\therefore$ Steady state error depends on K_p

$$\boxed{e_{ss} \propto \frac{1}{K_p}}$$

$$\boxed{\text{If } K_p \uparrow \downarrow \text{ Then } e_{ss} \downarrow \uparrow}$$

$$\boxed{s^2 + 2\xi\omega_n s + \omega_n^2} = \boxed{s^2 + 2\xi\omega_n s + K_p\omega_n^2}$$

$$(\omega_n')^2 = K_p \omega_n^2$$

$$\boxed{\omega_n' = \sqrt{K_p}\,\omega_n}$$

$$2\xi\omega_n' s = 1$$

$$\boxed{2\xi\sqrt{K_p}\,\omega_n = 2\xi\omega_n s}$$

compare

$$2\xi\sqrt{K_p}\,\omega_n = 1 \qquad\qquad 2\xi\omega_n s = 1$$

with K_p, the value of $\xi \downarrow$

$$\boxed{\xi = \frac{1}{2\sqrt{K_p}\,\omega_n}} \qquad\qquad \boxed{\xi = \frac{1}{2\xi\omega_n}}$$

value constant

The effects of proportional controller are

- ① type and order of the system donot changed.
- ② damping ratio decreases hence overshoot increases.
- ③ T.F remains same.
- ④ steady state error ~~de~~ reduces which indicates that accuracy is improved.

→ P-controller cannot eliminate complete e_{ss}.