measurement was made, or the conditions assumed in calculating it, if any possible question could exist.

An antenna mounted so that earth-reflection interference effects occur will have its vertical-plane pattern drastically affected, as Fig. 1–15 indicates. Horizontal-plane patterns may be affected as to absolute values, but relative values will not be affected if the earth in the vicinity is "smooth." That is, the *shape* of the horizontal pattern will not be affected, unless the earth is irregular.

#### 3.3. Directivity and Gain

In discussions of directivity and gain, the concept of an isotropic radiator, or isotrope, is fundamental. This concept was introduced briefly in sec. 1.1.4. Essentially an isotrope is an imaginary, lossless antenna that radiates uniformly in all directions. Its pattern is a perfect spherical surface in space; that is, if the electric intensity of the field radiated by an isotrope is measured at all points on an imaginary spherical surface with the isotrope at the center (in free space), the same value will be measured everywhere. Therefore, if an isotrope radiates a total power  $P_t$  in watts and is located at the center of a transparent (or imaginary) far field sphere of radius R meters, the power density over the spherical surface is

$$p_{isotrope} = \frac{P_t}{4\pi R^2}$$
 watts per square meter (3–11)

Equation (3–11) is true because  $P_t$  is distributed uniformly over the surface area of the sphere, which is  $4\pi R^2$  square meters.

Actually an isotropic radiator is not physically realizable; all actual antennas have some degree of nonuniformity in their radiation patterns. A nonisotropic antenna will radiate more power in some directions than in others and therefore has a directional pattern. A directional antenna will radiate more power in its direction of maximum radiation than an isotrope would, with both radiating the same total power. Thus, since the directional antenna sends less power in some directions than an isotrope does, it follows that it must send more power in other directions, if the total powers radiated are the same.

Directivity D is a quantitative measure of an antenna's ability to concentrate radiated power per unit solid angle in a certain direction, and thus D is highly dependent on the three-dimensional pattern of an antenna. D will be explicitly defined in materials that follow. On the other hand, gain G is the ratio of the power radiated per unit solid angle to the power per unit solid angle radiated by a lossless isotrope, each having the same input power. Unless otherwise specified, the direction applicable to D and G is that for which maximum radiation occurs.

Over the years there have been several gain-related terms used. In fact, gain once was not defined in terms of signal strength relative to any specific antenna type (Terman 1943). Gain is sometimes specified relative to the gain of a one-half wavelength dipole, whose gain exceeds the isotrope by the factor 1.64 (2.15 dB). Then, gain is specified in

 $dB_d$  (gain above the gain of a lossless one-half wavelength dipole). However, almost universally, gain is expressed relative to the lossless isotrope and when specificity is desired, it is referred to as absolute gain or expressed in terms of  $dB_i$ .

### 3.3.1. Definitions of Directivity and Gain

The IEEE definitions of directivity and gain are expressed in terms of radiation intensity  $U(\theta, \phi)$ , which is a range-independent quantity that is the product of power density  $p(\theta, \phi, r)$  and range squared  $r^2$ , that is,  $U(\theta, \phi) = p(\theta, \phi, r)r^2$ . Specifically,

- Directivity is the ratio of the radiation intensity in a given direction from the antenna to the radiation intensity averaged over all directions (IEEE 1993, p. 362). *Notes:* (1) The average radiation intensity is equal to the total power radiated by the antenna divided by  $4\pi$  (area of sphere in steradians). (2) If the direction is not specified, the direction of maximum radiation intensity is implied.
- Gain is the ratio of the radiation intensity, in a given direction, to the radiation intensity that would be obtained if the power accepted by the antenna were radiated isotropically (IEEE 1993, p. 547). *Notes:* (1) Gain does not include losses arising from impedance and polarization mismatches. (2) If the antenna is without dissipative  $(I^2R)$  losses; then, in any given direction its gain is equal to its directivity. (3) If the direction is not specified, the direction of maximum radiation intensity is implied.

Sections 3.3.2 through 3.3.5 that follow discuss the mathematics of directivity and gain, and relationships between them. To accomplish this, the concepts of solid angle and radiation intensity are first introduced.

#### 3.3.2. Solid Angle

Solid angle is the angle that, seen from the center of a sphere, includes a given area on the surface of the sphere. The value of the solid angle is numerically equal to the size of that area divided by the square the radius of the sphere (Jurgenson and Brown 2000). Mathematically the solid angle is unitless, but for practical reasons the steradian (s.r.) is assigned so that 1 steradian = 1 radian<sup>2</sup>. Since radians are dimensionless, steradians are also dimensionless.

Figure 3–4 shows a cross-hatched area dA in spherical coordinates that is bisected by the solid angle  $d\Omega$ . Since  $dA = r^2 \sin \theta \, d\theta \, d\phi = r^2 d\Omega$ , the value of the cross-hatched solid angle in Fig. 3–4 is  $d\Omega = \sin \theta \, d\theta \, d\phi$ . In general, however, solid angles can have other shapes. The solid angle of area  $A/r^2$  that subtends all of a sphere may be determined as follows:

$$\Omega = \frac{A}{r^2} = \int_0^{2\pi} \left[ \int_0^{\pi} \sin \theta d\theta \right] d\phi = \int_0^{2\pi} 2d\phi = 4\pi$$
(3-12)



#### FIGURE 3-4.

The solid angle  $d\Omega$  subtends area dA, and consequently the solid angle  $d\Omega = r^2 \sin \theta \, d\theta$ .

#### 3.3.3. Radiation Intensity

The power *P* radiated by an antenna is equal to  $p(\theta, \phi, r)dA$  integrated over a surface enclosing the antenna, where

- r = distance from origin to surface of sphere
- $p(\theta, \phi, r) =$  power density at  $\theta, \phi$ , and r
- $dA = r^2 \sin \theta d\theta d\phi$  = incremental area at  $\theta$ ,  $\phi$ , and r normal to the propagation direction
- $p(\theta, \phi, r)dA$  = incremental power in area dA at  $\theta$ ,  $\phi$ , and r

Now, using the coordinates of Fig. 3–4, let an antenna be at the center of the sphere. Then, the total radiated power

*P* can be determined by summing the incremental power  $p(\theta, \phi, r)dA$  over the surface, as follows:

$$P = \oint_{\text{surface}} p(\theta, \phi, r) dA = \int_{0}^{2\pi\pi} \int_{0}^{\pi} p(\theta, \phi, r) r^2 \sin\theta \, d\theta \, d\phi$$
(3-13)

The power density varies as  $1/r^2$ , and thus the product  $p(\theta, \phi, r)r^2$  is independent of distance *r* from antenna. This range-independent product is defined as the radiation intensity *U*. Therefore

$$U(\theta, \phi) = p(\theta, \phi, r)r^{2}$$
(3-14)

and the incremental power that crosses surface area dA is

$$dP = p(\theta, \phi, r) dA = p(\theta, \phi, r) r^{2} \sin \theta \, d\theta \, d\phi = U(\theta, \phi) d\Omega$$

where  $d\Omega = \sin \theta d\theta d\phi$ . Thus, the radiation intensity  $U(\theta, \phi)$  is  $dP/d\Omega$ , that is, the incremental power per unit solid angle in direction  $(\theta, \phi)$ .  $U(\theta, \phi)$  is usually expressed in units of watts per steradian (i.e., watts per square radian). Then, from (3–13) and (3–14), total radiated power *P* is

$$P = \int_{0}^{2\pi\pi} \int_{0}^{\pi} p(\theta, \phi, r) r^2 \sin\theta \, d\theta \, d\phi = \int_{0}^{2\pi\pi} \int_{0}^{\pi} U(\theta, \phi) d\Omega \tag{3-15}$$

In other words, total radiated power equals the sum of all radiation intensity that encloses the antenna.

#### 3.3.4. Directivity

Directivity D is a quantitative measure of an antenna's ability to concentrate energy in a certain direction. Specifically, D is the ratio of the maximum radiation intensity  $U_{max}$  to the average radiation intensity  $U_{av}$ . Then

Directivity = 
$$U_{max}/U_{av}$$
 (dimensionless) (3–16)

If the radiation is isotropic, the radiation intensity in every direction is  $U_{av}$ . Thus, from (3–12) and (3–15), total radiated power P is a sphere's solid angle  $4\pi$  times  $U_{av}$  and is given by (3–17)

$$P = 4\pi U_{av} \tag{3-17}$$

Now, by using (3-15), (3-16) and (3-17), directivity can be expressed as

$$\frac{U_{max}}{U_{av}} = \frac{4\pi U_{max}}{P} = \frac{4\pi U_{max}}{\int\limits_{0}^{2\pi\pi} \int\limits_{0}^{\pi} \int\limits_{0}^{U} U(\theta, \phi) \sin \theta \, d\theta \, d\phi}$$
(3-18)

Now, by shifting the constant  $U_{max}$  to the denominator, it is seen from (3–19) that D is a function the relative value on  $U(\theta, \phi)$  (the bracketed term) that has a maximum of unity:

$$D = \frac{4\pi}{\int\limits_{0}^{2\pi\pi} \int\limits_{0}^{\pi} \left[ \frac{U(\theta, \phi)}{U_{max}} \right] \sin \theta \, d\theta \, d\phi}$$
(3-19)

The bracketed term of (3–19) can be replaced by functions, at a fixed distance, of power density  $p(\theta, \phi, r)$  or electric field strength  $E(\theta, \phi, r)$ . This is because, with r constant,

$$U(\theta, \phi)/U_{max} = p(\theta, \phi, r)/p(\theta, \phi, r)_{max} = |E(\theta, \phi, r)/E(\theta, \phi, r)_{max}|^2$$
(3-20)

Therefore, with r fixed, D can be determined with relative values of either  $U(\theta, \phi)$ ,  $p(\theta, \phi, r)$ , or  $|E(\theta, \phi, r)|^2$ .

It is also to be noted that directivity D of (3–19) can be expressed as

$$D = \frac{4\pi}{\Omega_A} \tag{3-21}$$

where  $\Omega_A$  is known as the beam solid angle. For the special case of an isotrope, since an isotrope radiates equally in all directions,  $\Omega_A = 4\pi$  and D = 1. In general, however,

$$\Omega_A = \int_{0}^{2\pi} \int_{0}^{\pi} \left[ \frac{U(\theta, \phi)}{U_{max}} \right] \sin \theta \, d\theta \, d\phi \tag{3-22}$$

and with *r* constant,  $U(\theta, \phi)/U_{max}$  can be replaced with either  $p(\theta, \phi, r)/p(\theta, \phi, r)_{max}$  or  $|E(\theta, \phi, r)/E(\theta, \phi, r)_{max}|^2$ . Use of the term  $\Omega_A$  in (3–21) helps to emphasize that directivity is a measure of an antenna's ability to concentrate energy in a certain direction. Finally, it is important to again underscore that directivity is calculated by integrating relative values of an antenna's radiation pattern, and this does not require knowledge of an absolute value.

### 3.3.5. Gain

Gain, previously defined in sec. 3.3.1, is the ratio of the radiation intensity in a given direction to the radiation intensity that would be obtained if the power were radiated isotropically. If the direction is not specified, the direction of maximum radiation intensity  $U_{max}$  is implied. In addition, for isotropically radiated power, the radiation intensity is  $U_{av}$ . Therefore, for a lossless antenna, gain G equals directivity as defined previously by (3–16).

Gain is determined, in principle, by comparing the radiation intensity if both the actual test antenna and an isotrope have the same input power. The isotrope is assumed to radiate all of its input power, but some of the power delivered to the actual antenna may be dissipated in ohmic resistance (i.e., converted to heat). Thus, gain takes into account the antenna efficiency as well as its directional properties. The efficiency factor k is the ratio of the power radiated by the antenna to the total input power. Thus, the relationship between gain G and directivity D is

$$G = kD \tag{3-23}$$

where  $0 \le k \le 1$  to account for dissipative  $(I^2 R)$  losses.

It is apparent that the quantity of greatest significance to a system designer is gain. The antenna theorist, on the other hand, finds the concept of directivity convenient, for it depends only upon the pattern of the antenna. The theorist can compute the directive gain of a half-wave dipole, for example, by application of Maxwell's equations and the assumption that the dipole has no ohmic loss. The directivity of an isotrope, incidentally, is by definition unity, and the gain of an isotrope with efficiency factor k is (also by definition) equal to k.

Antenna gain is a power ratio. Values of gain of practical antennas may range from near zero (total loss) to as much as  $10^6$  or more. As with any power ratio, antenna gain may be expressed in decibels. The antenna gain *G* expressed in decibels is  $G_{dB} = 10 \log G$ ,

where the word log denotes the common logarithm (base 10). Directivity in decibels is calculated using the same formula, with D substituted for G. Gain is expressed in decibels more commonly than as a power ratio.

#### 3.4. Effective Area and Friis Transmission Equation

Although there is a reciprocal relationship between the transmitting and receiving properties of *reciprocal* antennas, it is sometimes more convenient to describe the receiving properties in a somewhat different way. Whereas the gain is the natural parameter to use for describing the increased power density of the transmitted signal due to the directional properties of the antenna, a related quantity called the *effective area*, is a more natural parameter for describing the reception properties of the antenna.

The effective area of an antenna  $A_e$  is defined as follows. Suppose that a distant transmitter radiates a signal that, at the receiving-antenna location, has a power density  $p_r$ . This is the amount of power incident on, or passing through, each *unit area* of any imaginary surface perpendicular to the direction of propagation of the waves.

A receiving antenna exposed to this field will have radio-frequency current and voltage induced in it, and at the antenna terminals this voltage and current represent radio-frequency power that can be delivered to a load (e.g., the input circuit of a receiver). In principle the power available at these terminals can be measured (although in practice it may be so small a power that amplifying equipment will be needed to measure it). This power, however small, is the *received signal power*,  $P_r$ . It can be deduced that in order for this amount of power to appear at the antenna output terminals, the antenna had to "capture" power from the field over a surface in space (oriented perpendicularly to the direction of the wave) of area  $A_e$  such that

$$P_r = p_r A_e \tag{3-24}$$

The area  $A_e$ , which bears this relationship to  $P_r$  and  $p_r$ , is the effective area of the antenna. (The conception of the capture process thus conveyed is a much oversimplified one, but it has validity for the present purposes.) Since received power depends on the antenna polarization, effective area is a function of the polarization of the incident field.

As might be supposed, there is a connection between the effective area of an antenna and its physical area as viewed from the direction of the incoming signal. The two areas are not equal, however, although for certain types of high-gain antennas they may be nearly equal. But some antennas of physically small cross section may have considerably larger effective areas. It is as though such an antenna has the ability to "reach out" and capture power from an area larger than its physical size, as in the case of a dipole antenna.

As is discussed in sec. 5.7.6, there is also a relationship between the gain of a lossless antenna and its physical size. This relationship suggests that there may also be a connection between the gain and the effective area, and this indeed turns out to be true. The equation relating the two quantities is

$$A_e = \frac{G\lambda^2}{4\pi} \tag{3-25}$$

where  $\lambda$  is the wavelength corresponding to the frequency of the signal. (This relationship may be proved theoretically and verified experimentally.)

Because of this connection between the effective area and the gain, (3-24) may be rewritten, with the right-hand side of (3-25) substituted for  $A_e$ . Thus:

$$P_r = \frac{p_r G \lambda^2}{4\pi} \tag{3-26}$$

Therefore the concept of the effective area of an antenna is not a necessary one. As (3-26) shows, it is possible to calculate the received-signal power without knowing  $A_e$ . As seen, the effective area definition has a conceptual value, however, and is a convenient quantity to employ in some types of problems. We now consider the relationship between transmit and receive powers with two antennas separated by far-field distance R. This relationship is highly useful, and it is known as the Friis Transmission Equation (Friis 1946). The following abbreviations are used:

- $P_r$  = received power
- $P_t$  = input power of transmit antenna
- $A_{et}$  = effective aperture of transmit antenna
- $A_{er}$  = effective aperture of receive antenna
- $G_t$  = gain of transmit antenna
- $G_r$  = gain of receive antenna

Now, from equation (3–11) for a lossless isotropic radiator and the definition of gain, the power density  $p_r$  at distance R from a transmit antenna may be expressed as  $(P_t/4\pi R^2)G_t$ . Recall that another antenna in the presence of  $p_r$  will receive power  $P_r$  that equals  $p_rA_{er}$ . Thus  $P_r$  becomes  $(P_t/4\pi R^2)A_{er}G_t$ . Finally, we use relationships, based on (3–25), between  $G_r$  and  $A_{er}$  and between  $G_t$  and  $A_{et}$  to express  $P_r$  as functions of  $A_{et}A_{er}$  and of  $G_rG_t$ . The result is the Friis Transmission Equation which is given in (3–27) that follows.

$$P_{r} = \left(\frac{P_{t}}{4\pi R^{2}}\right) A_{er} G_{t} = P_{t} \frac{A_{et} A_{er}}{R^{2} \lambda^{2}} = P_{t} \frac{G_{t} G_{r} \lambda^{2}}{\left(4\pi\right)^{2} R^{2}}$$
(3-27)

When reciprocity applies, as is usually so, the effective transmit aperture area  $A_{et}$  of an antenna equals its effective receive aperture area  $A_{er}$ . Similarly, transmit gain  $G_t$  and receive gain  $G_r$  of an antenna are equal. Then, because of reciprocity, the transmitter and receiver locations can be interchanged. Thus, either antenna can be designated as the transmit antenna and the other as the receive antenna.

Equation (3–27) underscores the practical significance of gain. For example, a transmit power of 1,000 watts and a transmit antenna gain of 10 (10 dB) will provide the same received power as will a transmit power of 500 watts and a transmit antenna gain of 20

(13 dB). Obviously, this relationship has great economic significance. Sometimes it may be much less expensive to double antenna gain (add 3 dB) than it would be to double transmit power (though in other cases the converse may be true). Generally, it is desirable

# 3.5. Beamwidth

When the radiated power of an antenna is concentrated into a single major "lobe," as exemplified by the pattern of Fig. 3–2, the angular width of this lobe is the *beamwidth*. The term is applicable only to antennas whose patterns are of this general type. Some antennas have a pattern consisting of many lobes, for example, all of them more or less comparable in their maximum power density, or gain, and not necessarily all of the same angular width. One would not speak of the beamwidth of such an antenna, but a large class of antennas do have patterns to which the beamwidth parameter may be appropriately applied.

to use as much antenna gain as feasible, because it increases received signals.

# 3.5.1. Practical Significance of Beamwidth

If an antenna has a narrow beam and is used for reception, it can be used to determine the direction from which the received signal is arriving, and consequently it provides information on the direction of the transmitter. To be useful for this purpose, the antenna beam must be "steerable," that is, capable of being pointed in various directions. It is intuitively apparent that for this direction-finding application, a narrow beam is desirable and the accuracy of direction determination will be inversely proportional to the beamwidth, assuming no errors in other parts of the system (although in practice this is not always a good assumption). Its relation to direction-finding accuracy is one significant aspect of the beamwidth parameter. This, however, is by no means its only practical significance.

In some applications a receiver may be unable to discriminate completely against an unwanted signal that is either at the same frequency as the desired signal or on nearly the same frequency. In such a case, pointing a narrow receiving-antenna beam in the direction of the desired signal is helpful; the resulting greater gain of the antenna for the desired signal, and reduced gain for the undesired one, may provide the necessary discrimination. If the directions of the desired and undesired signals are widely separated, even a relatively wide beam will suffice. But the closer the two signals are in direction, the narrower the beam must be to provide effective discrimination. (In radar systems the analogous problem is the ability to distinguish two targets that are close together in bearing, that is, in direction angle; this ability is called angular resolving power or resolution.)

Finally, as the foregoing discussion of gain and directivity (sec. 3.3) indicated, there is a relationship between the solid-angle width of an antenna beam and its directivity (also see sec. 3.11). Thus, with some exceptions, generally a narrower beam implies a greater gain. Since gain is usually a desirable property, this relationship constitutes an additional virtue of narrow beamwidth.

On the other hand, there are situations that call for a wide beam. For example, a broadcasting station must radiate a signal simultaneously to listeners in many different directions—typically, over a 360 degree azimuth sector. Any narrowing of the beam to obtain gain must therefore be done in the vertical plane. At the low frequencies of the AM broadcast band (535–1,705 kHz), and at lower frequencies, such vertical-plane beam narrowing is not feasible because it would require an impractically large (high) antenna. In television and FM broadcasting in the VHF and UHF bands, however, this means of obtaining gain while preserving 360 degree azimuthal coverage is much used. A situation in which an antenna beam must be moderately broad in the vertical plane is that of a search radar antenna on a ship that rolls and pitches. It is usually required that the beam of such antenna be directed at the horizon. But if the beam has, say, a 2-degree vertical beamwidth, and the ship rolls 20 degrees (which is not unusual), it is evident that at times no part of the beam will remain directed at the horizon. The vertical beamwidth in this case must be of size comparable to the maximum roll angle of the ship, unless some method of stabilizing the beam is employed (as is sometimes done).

## 3.5.2. Beamwidth Definition

As illustrated in Fig. 3–2, an antenna beam is typically round-nosed. Defining beamwidth, therefore, is a problem. As seen clearly in the rectangular plot, Fig. 3–2b, it would be possible in this case to cite the width of the beam between the two nulls (zero values) on either side of the maximum, since these are two definitely measurable points. Not all beams have such nulls, however, although they are present ordinarily. Moreover, it is logical to define the width of a beam in such a way that it indicates the angular range within which radiation of useful strength is obtained, or over which good reception may be expected. From this point of view the convention has been adopted of measuring beamwidth between the points on the beam pattern at which the power density is half the value at the maximum. (On a plot of the electric-intensity pattern, the corresponding points are those at which the intensity is equal to  $1/\sqrt{2}$  or 0.707 of the maximum value.)



FIGURE 3–5.

Determination of half-power (3 dB down) beamwidth.

The angular width of the beam between these points is called the *half-power beamwidth*. When a beam pattern is plotted with the ordinate scale in decibels, as is frequently done, the half-power points correspond to the minus 3 dB points. For this reason the half-power beamwidth is often referred to as the 3 dB *beamwidth*. Figure 3–5 illustrates the procedure of determining the 3 dB beamwidth on a rectangular pattern plot. Only the nose region of the beam pattern is shown. As indicated, this beamwidth is approximately 10 degrees. This criterion of beamwidth, although adequate and convenient in many situations, does not always provide a sufficient description of the beam characteristics. Beams have different shapes. An additional description may be given by measuring the width of the beam at several points, for example, at -3 dB, -10 dB, and at the nulls (if they are present). Some beams may have an asymmetric shape, for some special reason, and the specific nature of the asymmetry may be important. Special methods of describing such beams can be employed. In the final analysis the best description of a beam is a plot of its pattern.

### 3.5.3. Principal-Plane Beamwidths

An antenna beam occupies a solid angle, and a single beamwidth figure refers only to the pattern in a particular plane. It is apparent that the beam may have different widths in different planes through the beam axis. (The axis is the direction of maximum radiation, a line from the antenna passing through the nose of the beam.) Therefore it is customary to give the widths of the beam in two planes at right angles, usually the principal planes of the coordinate system. The beamwidths in planes at intermediate angles will generally have intermediate values, so that giving just these two beamwidth figures conveys considerable information concerning the solid-angular shape of the beam. When the beam is linearly polarized, the beamwidth in the plane containing the *E*-vector (plane of polarization) is sometimes called the *E*-plane beamwidth, and that in the perpendicular plane the *H*-plane beamwidth.

#### 3.6. Minor Lobes

A directional antenna usually has, in addition to a main beam or major lobe of radiation, several smaller lobes in other directions; they are *minor lobes* of the pattern. Those adjacent to the main lobe are *sidelobes*, and those that occupy the hemisphere in the direction opposite to the main-beam direction are *back lobes*. Minor lobes ordinarily represent radiation (or reception) in undesired directions, and the antenna designer therefore attempts to minimize their level relative to that of the main beam. This level is expressed in terms of the ratio of the power densities in the mainbeam maximum and in the strongest minor lobe. This ratio is often expressed in decibels.

Since the sidelobes are usually the largest of the minor lobes, this ratio is often called the sidelobe ratio or sidelobe level. A typical sidelobe level, for an antenna in which some attempt has been made to reduce the sidelobe level, is 20 dB, which means that the power density in the strongest sidelobe is one percent of the power density in the main beam. Sidelobe levels of practical well-designed directional antennas typically range from about 13 dB (power-density ratio 20) to about 40 dB (power-density ratio  $10^4$ ). Attainment of a sidelobe level better than 30 dB requires very careful design and construction, but better than 50 dB ( $10^5$ ) has sometimes been accomplished.

Figure 3–6 shows a typical antenna pattern with a main beam and minor lobes, plotted on a decibel scale to facilitate determination of the sidelobe level, which is here seen to be 25 dB.



FIGURE 3–6.

Decibel pattern plot, indicating sidelobe level.

In some applications sidelobes are not especially harmful unless their level becomes comparable to the main-beam level; in other applications it may be important to hold the sidelobe level to an absolute minimum. In most radar systems, for example, a low sidelobe level is important. Since signals received in a sidelobe are ordinarily indistinguishable from signals received from the main beam, a large target located in the direction of one of the antenna sidelobes (or even a back lobe) may appear to the radar as though it were a target in the main beam. Such sid-

elobe echoes create "clutter" on the radar output that may mask main-beam echoes from smaller targets and overload the data-processing personnel or equipment. Therefore radar antennas are often designed for sidelobe levels of -25 dB or better in the horizontal-plane patterns. Sidelobes in the vertical-plane patterns may also be harmful in height finding radars but do no harm in search radars that provide angle information only in the azimuth plane, except to the extent that they represent wasted power.

# 3.7. Radiation Resistance and Efficiency

In a large class of antennas the radiation is associated with a flow of rf current in a conductor or conductors. Thus, when a current *I* flows in a resistance *R*, an amount of power  $P = RI^2$  will be dissipated and converted into heat. In an antenna, even if there were no resistance in the conductors, the electrical energy supplied by the transmitter is radiated and it is in a sense "lost." It is customary to associate this "loss" of power through radiation with a fictitious "radiation resistance," that bears the same relationship to the current and the radiated power as an actual resistance bears to the current and dissipated power. If the power radiated by the antenna is *P* and the antenna rms current is *I*, the radiation resistance is

$$R_r = \frac{P}{I^2} \tag{3-28}$$

The concept of radiation resistance is applicable only to antennas in which the radiation is associated with a definite current in a single linear conductor. Even then, the definition of radiation resistance is ambiguous. This is because of standing waves, the current is not the same everywhere along a linear conductor. It is therefore necessary to specify the point along the conductor at which the current will be measured. Two points sometimes specified are where the current has its maximum value and the feed point (input terminals). These two points are sometimes at the same place, as in a center-fed dipole, but they are not always the same. And in principle, *any* point may be specified. The value then obtained for the radiation resistance of the antenna depends on what point is specified; this value is the radiation resistance *referred to that point*.

The above words "maximum current" refer to the rms current in that part of the antenna where the current is maximum. It does *not* mean the *peak* value of the current located where the current is greatest. The reader will recall that, in accordance with equation (1-3), by definition the *amplitude*  $I_0$  of a current sine wave is actually the peak value of current during its cycle. Furthermore, the reader is cautioned that, in some texts, formulas for radiation resistance are written as a function of current "amplitude"  $I_0$ , instead of rms current. Thus, use of the wrong combination of formula and current level will cause the radiation resistance to be incorrectly calculated.

The radiation resistance of some types of antennas can be calculated, and yet not for others. Sometimes radiation resistance can be obtained by measurement (see section 9.10, chapter 9). Typical values of the input radiation resistance of actual antennas range from a fraction of an ohm to several hundred ohms. The very low values are undesirable, because they imply large antenna currents (i.e.,  $P = I^2 R$ ). Therefore, there exists the possibility of considerable ohmic loss of power, that is, dissipation of power as heat rather than as radiation. An excessively high value of radiation resistance is also undesirable, because this requires that a very high voltage (i.e.,  $P = E^2/R$ ) be applied to the antenna.

Antennas always have ohmic resistance, although sometimes it may be so small as to be negligible. The ohmic resistance is usually distributed over the antenna; and since the antenna current varies, the resulting loss may be quite complicated to calculate. In general, however, the actual loss can be considered to be equivalent to the loss in a fictitious lumped resistance placed in series with the radiation resistance. If this equivalent ohmic loss resistance is denoted by  $R_0$ , the full power (dissipated plus radiated) is  $I^2(R_0 + R_r)$ , whereas the radiated power is  $I^2R_r$ . Hence the antenna radiation efficiency k of (3–23) is given by

$$k_r = \frac{R_r}{R_0 + R_r} \tag{3-29}$$

It must be acknowledged, however, that this definition of the efficiency is not really very useful even though it may occasionally be convenient. The fact is that both  $R_0$  and  $R_r$  are fictitious quantities, derived from measurements of current and power;  $R_r$  is given in these terms by (3–28), and  $R_0$  is correspondingly equal to  $P_0/I^2$ . Making these substitutions into (3–29) gives the more basic definition of the efficiency:

$$k_r = \frac{P_r}{P_0 + P_r} \tag{3-30}$$

where  $P_r$  is the power radiated, and  $P_0$  is the power dissipated.