

Signals and Systems :-

→ Signal is a physical quantity that varies with time, Space, or any other independent variables. (Mathematically)

(or)

→ Any physical phenomenon that carries or conveys information is called a Signal.

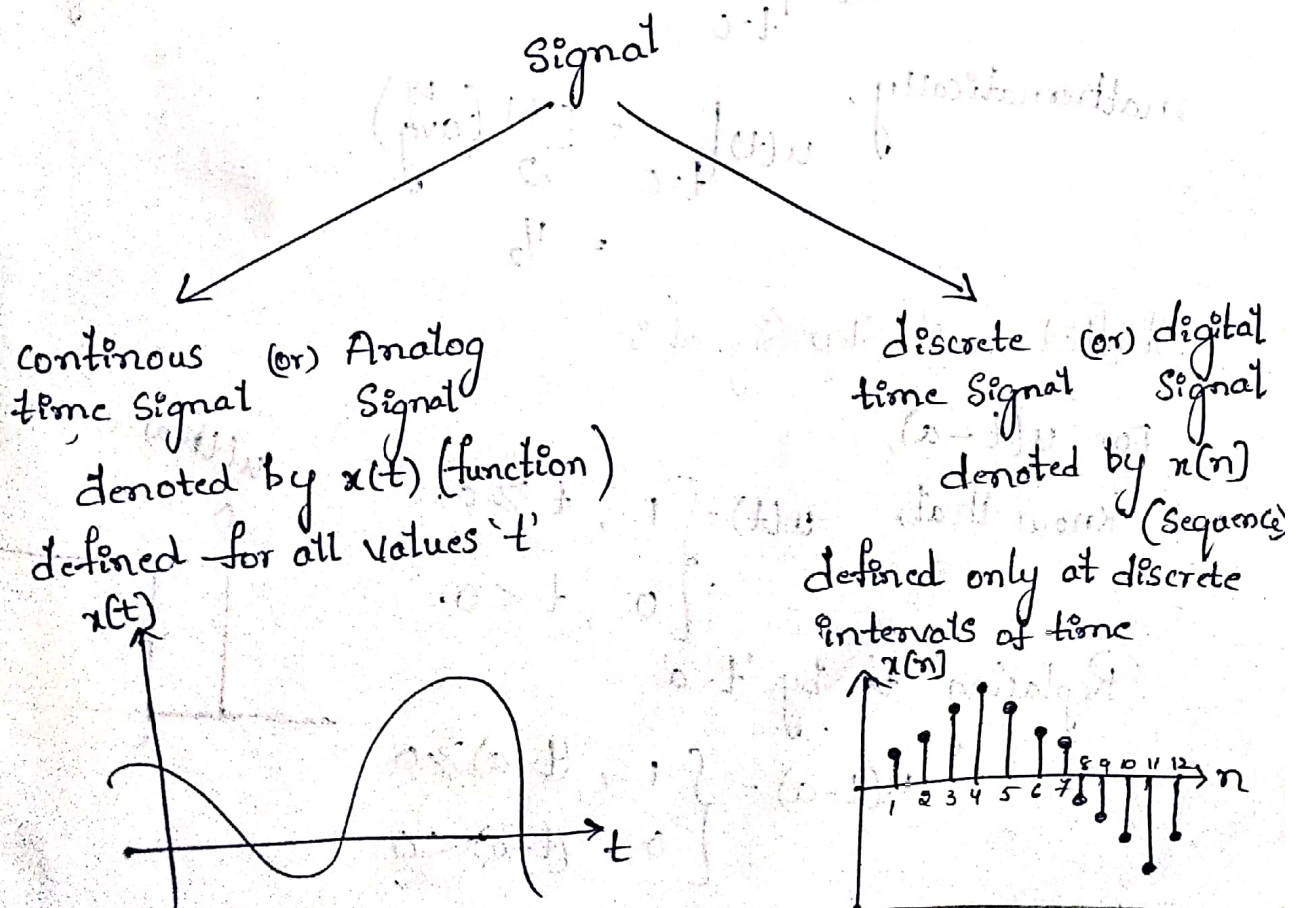
Eg: $f(x_1, x_2, x_3, \dots, x_n)$

Here $x_1, x_2, \dots, x_n$ are independent variables.

→ One dimensional Signals and Multi dimensional Signals

If a Signal varies with only one independent variable is called as one dimensional Signals.

If a Signal varies with more than one independent variable is called as Multi-dimensional Signals.

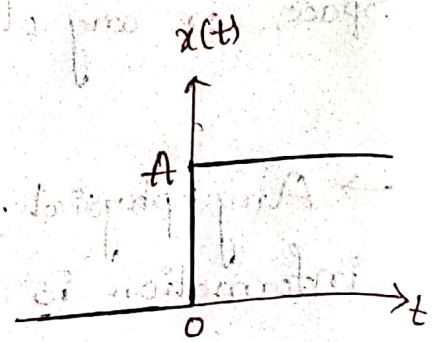


Standard Signals or Elementary Signals for Continuous time Signals :-

→ Step Signal :-

It is defined for

$$x(t) = \begin{cases} A, & t \geq 0 \\ 0, & t < 0 \end{cases}$$



Step Signal will become unit step signal, when the amplitude 'A' = 1.

Then the condition for unit step signal is,

$$x(t) = u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

but according to gyp's phenomenon,

$u(t)|_{t=0}$ = not defined. ($\because$ it is discontinuity).

mathematically, $u(t)|_{t=0} = \frac{0+1}{2}$ (by avg)
= $\frac{1}{2}$

• Shifted unit Step Signal :-

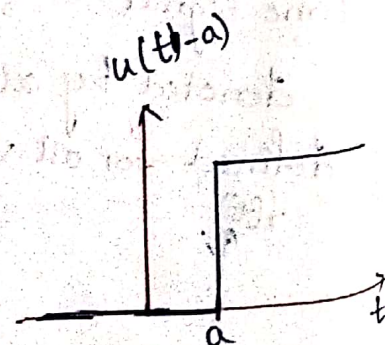
For $u(t-a)$,

we know that,

$$u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

Replacing 't' by 't-a'

$$u(t-a) = \begin{cases} 1, & (t-a) \geq 0 \\ 0, & (t-a) < 0 \end{cases}$$



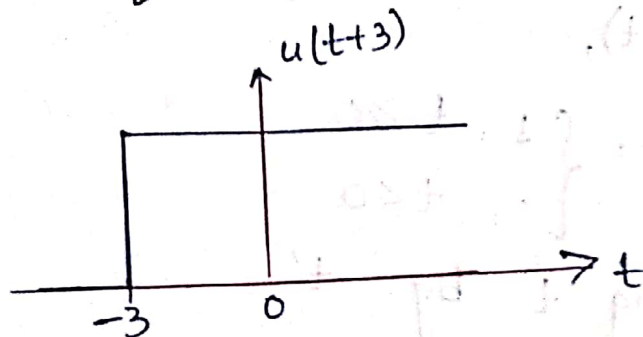
$$u(t-a) = \begin{cases} 1, & t \geq a \\ 0, & t < a \end{cases}$$

Eg:- $u(t+3)$:

$$u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

Replacing 't' by 't+3'

$$u(t+3) = \begin{cases} 1, & (t+3) \geq 0 \\ 0, & (t+3) < 0 \end{cases} \Rightarrow u(t+3) = \begin{cases} 1, & t \geq -3 \\ 0, & t < -3 \end{cases}$$

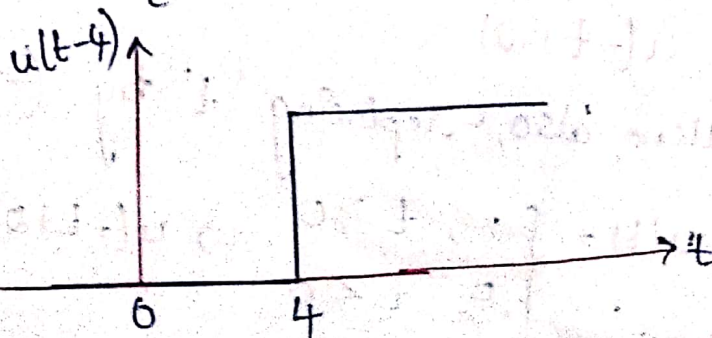


$u(t-4)$

$$u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

Replacing 't' by 't-4'

$$u(t-4) = \begin{cases} 1, & (t-4) \geq 0 \\ 0, & (t-4) < 0 \end{cases} \Rightarrow \begin{cases} 1, & t \geq 4 \\ 0, & t < 4 \end{cases}$$



In Simple words, for unit step signal,

$u(t-a) \Rightarrow u(t)$ will shift to R.H.S side by 'a' units

$u(t+a) \Rightarrow u(t)$ will shift to L.H.S side by 'a' units

Similarly, for Step Signal,

$x(t-t_0) \Rightarrow x(t)$ will shift to R.H.S side by ' t_0 ' units

$x(t+t_0) \Rightarrow x(t)$ will shift to L.H.S side by ' t_0 ' units

Eg:- in case of time Scaling:-

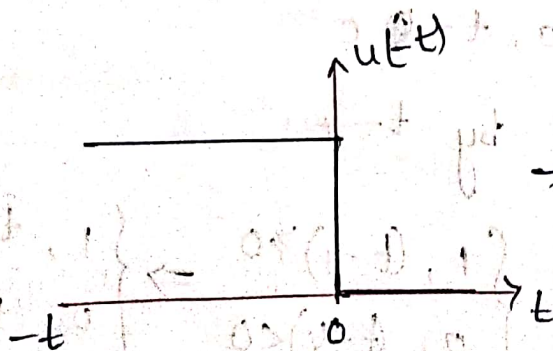
For $u(-t)$,

$$u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

Replacing 't' by '-t'.

$$u(-t) = \begin{cases} 1, & -t \geq 0 \\ 0, & -t < 0 \end{cases} \Rightarrow \begin{cases} 1, & t \leq 0 \\ 0, & t > 0 \end{cases}$$

($\because$ inequality of signs are changed
' $\geq$ ' to ' $\leq$ ' & vice versa)
then
' $\leq$ ' becomes ' $\geq$ '
& vice versa)



$\rightarrow$ this is Scaled version of $u(t)$.

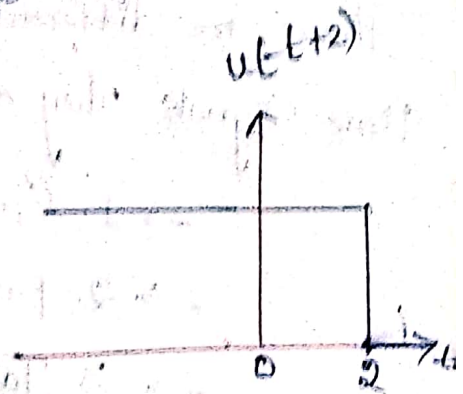
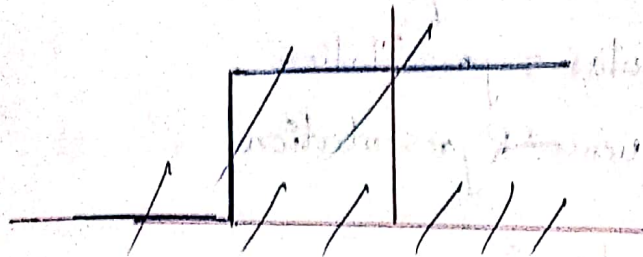
Eg: $u(-t+2)$

here also, replacing 't' by ' $(-t+2)$ '

$$u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases} \Rightarrow u(-t+2) = \begin{cases} 1, & (-t+2) \geq 0 \\ 0, & (-t+2) < 0 \end{cases}$$

$$\Rightarrow u(-t+2) = \begin{cases} 1, & -(t-2) \geq 0 \\ 0, & -(t-2) < 0 \end{cases}$$

$$= \begin{cases} 1, & t \leq 2 \\ 0, & t > 2 \end{cases}$$



i.e. $-t + 2 = 0$

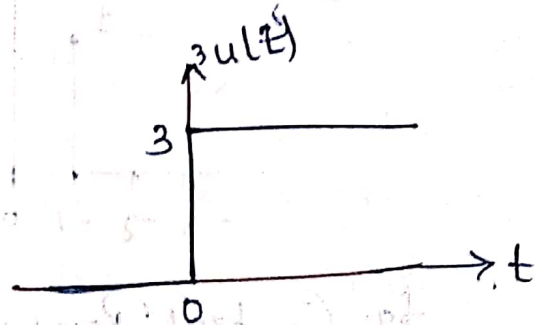
$$-t = -2$$

$t = -2$, $t = 2$, so discontinuity occurs at $t = 2$ (i.e. rise in signal)

eg:- $3u(t)$.

$$u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

$$3u(t) = \begin{cases} 3, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

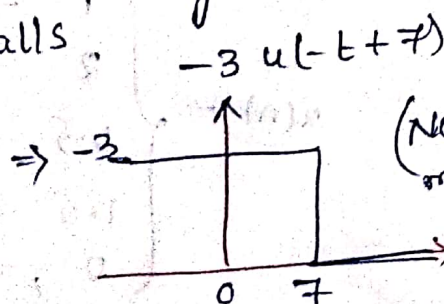
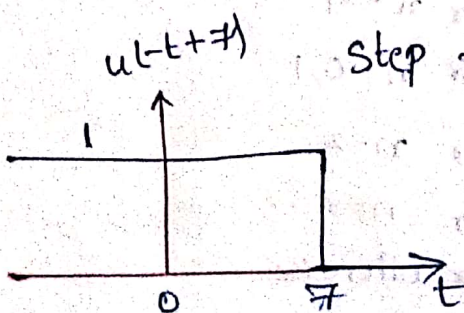


eg. $-3u(-t+7)$:
 $t+7=0$

$$-t + 7 = 0$$

$+7 = 0$
 $t = 7$ Here step occurs at '7' &

Here step occurs as t is having -ve sign in bracket
step falls: $-3u(-t+7)$



(Now by multiplying
-3 by
function
magnitude)

Representation of discrete time Signals :-
There are different types of representation for discrete time Signals. They are

- 1. Graphical representation
- 2. Functional representation
- 3. Tabular representation
- 4. Sequence Representation

Graphical Representation :-

Let us consider a signal $x(n]$ with values $x(-1)=1$; $x(0)=2$; $x(1)=2$; $x(2)=0.5$ and $x(3)=1.5$. This discrete time signal can be represented graphically as shown in fig below

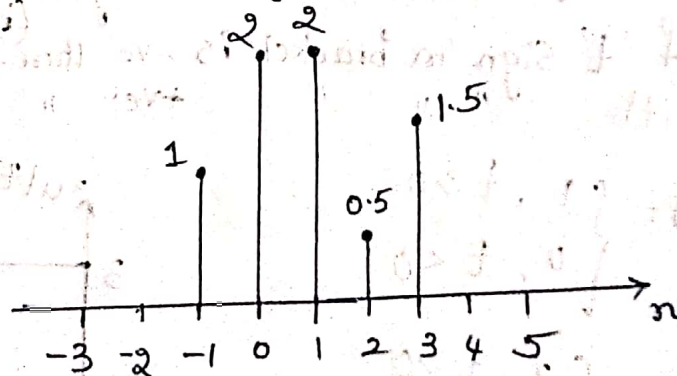


Fig: Graphical Representation of a discrete time signal.

Functional Representation :-

The discrete-time signal shown in above fig can be represented using functional representation as below.

$$x(n] = \begin{cases} 1 & \text{for } n = -1 \\ 2 & \text{for } n = 0, 1 \\ 0.5 & \text{for } n = 2 \\ 1.5 & \text{for } n = 3 \\ 0 & \text{for otherwise} \end{cases}$$

Tabular Representation :-

This discrete time signal can also be represented as

n	-1	0	1	2	3
$x(n)$	1	2	2	0.5	1.5

Sequence Representation :-

A finite duration sequence with time origin ($n=0$) indicated by the symbol $\uparrow$ is represented as,

$$x(n) = \{1, 2, 2, 0.5, 1.5\}$$

$\uparrow$

A finite duration sequence that satisfies the condition $x(n) = 0$ for $n < 0$ can be represented as

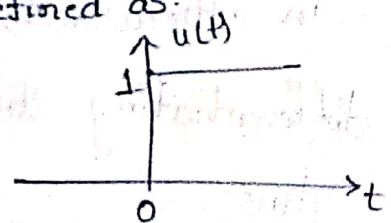
$$x(n) = \{2, 4, 6, 8, -3\}$$

Elementary continuous time signals :-

→ 1. unit step function :-

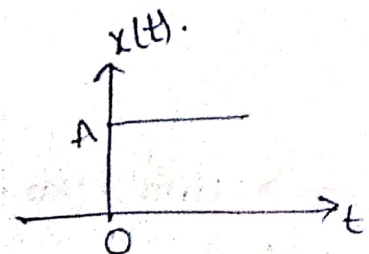
The unit step function is defined as:

$$u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$



A step function is defined as

$$x(t) = \begin{cases} A, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

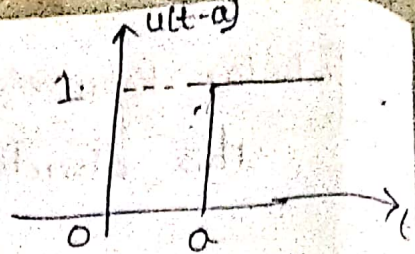


If step function has unity magnitude then it is called unit step function.

delayed unit step signal $u(t-a)$:

$$u(t-a) = \begin{cases} 1, & (t-a) \geq 0 \\ 0, & (t-a) < 0 \end{cases}$$

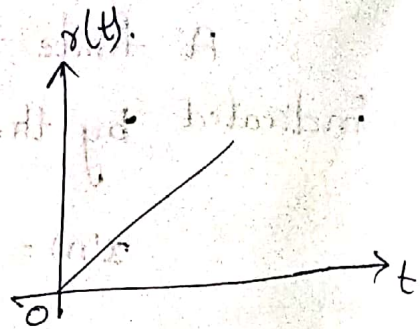
$$= \begin{cases} 1, & t \geq a \\ 0, & t < a \end{cases}$$



→ unit ramp function:-

The unit ramp function is defined as,

$$r(t) = \begin{cases} t, & t \geq 0 \\ 0, & t < 0 \end{cases}$$



(or)

$$r(t) = t u(t).$$

The ramp function can be obtained by applying unit step function to an integrator.

$$r(t) = \int u(t) \cdot dt = \int 1 \cdot dt = t$$

In other words, the unit step function can be obtained by differentiating the unit ramp function.

Thus,

$$r(t) = \int u(t) \cdot dt.$$

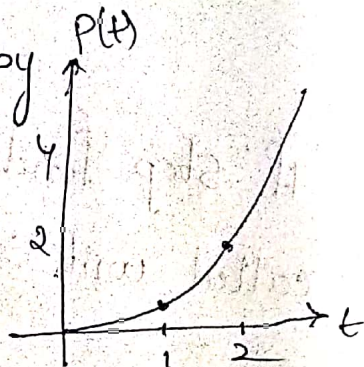
$$\frac{d}{dt} r(t) = u(t) \quad (\because \text{by differentiating on both sides}).$$

→ unit parabolic function:-

The unit parabolic function is given by

$$p(t) = \frac{t^2}{2}, \quad t \geq 0$$

$$= 0, \quad t < 0$$



(Or)

$$p(t) = \frac{t^2}{2} u(t)$$

The unit parabolic function can be obtained by integrating the ramp function.

$$p(t) = \int r(t) \cdot dt = \int t \cdot dt = \frac{t^2}{2} \quad (\because \text{for } t \geq 0).$$

In other words, the ramp function is derivative of the parabolic function,

Thus

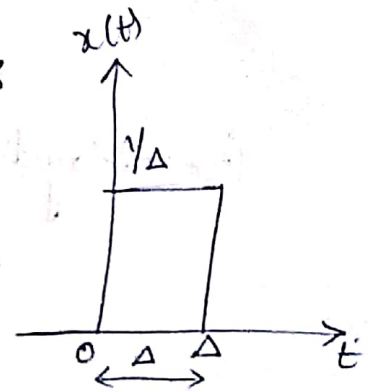
$$r(t) = \frac{d p(t)}{dt}$$

→ Impulse function :-

the impulse function is defined as

$$\int_{-\infty}^{\infty} \delta(t) \cdot dt = 1 \quad \&$$

$$\delta(t) = 0, \text{ for } t \neq 0.$$



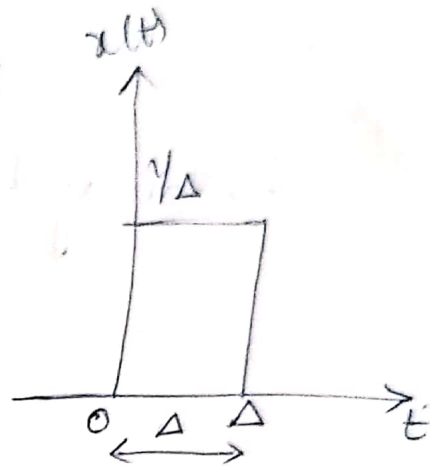
at

→ Impulse function :-

the impulse function is defined as

$$\int_{-\infty}^{\infty} \delta(t) \cdot dt = 1 \quad \&$$

$$\delta(t) = 0, \text{ for } t \neq 0.$$



Properties of ^{unit} impulse function :-

$$\rightarrow \int_{-\infty}^{\infty} x(t) \delta(t) dt = x(0). \rightarrow (1)$$

Let us consider the product of $x(t)$ & $\delta(t)$,
which is $x(t) \cdot \delta(t)$

Let the signal $x(t)$ is continuous at $t=0$; the value of $x(t)$ at $t=0$ is $x(0)$. The impulse exists only at $t=0$.
Therefore we can write

$$x(t) \cdot \delta(t) = x(0) \cdot \delta(t) \rightarrow (2)$$

Sub eq (2) in eq (1)

$$\begin{aligned} \int_{-\infty}^{\infty} x(t) \cdot \delta(t) \cdot dt &= \int_{-\infty}^{\infty} x(0) \cdot \delta(t) \cdot dt \\ &= x(0) \cdot \int_{-\infty}^{\infty} \delta(t) \cdot dt \\ &= x(0) \cdot (1) \end{aligned}$$

$$\boxed{\int_{-\infty}^{\infty} x(t) \cdot \delta(t) \cdot dt = x(0)}$$

$$\rightarrow 2) \quad x(t) \delta(t-t_0) = x(t_0) \cdot \delta(t-t_0) \rightarrow (3)$$

Let the signal $x(t)$ is continuous at $t=t_0$ & the value of $x(t)$ at $t=t_0$ is $x(t_0)$. The impulse exists only at $t=t_0$.
Therefore,

$$x(t) \cdot \delta(t-t_0) = x(t_0) \cdot \delta(t-t_0)$$

$$\rightarrow 3) \quad \int_{-\infty}^{\infty} x(t) \cdot \delta(t-t_0) \cdot dt = x(t_0) \rightarrow (4)$$

Sub eq (3) in eq (4), we get

$$\int_{-\infty}^{\infty} x(t) \cdot \delta(t-t_0) \cdot dt = \int_{-\infty}^{\infty} x(t_0) \cdot \delta(t-t_0) \cdot dt$$

$$= x(t_0) \cdot \int_{-\infty}^{\infty} \delta(t-t_0) \cdot dt$$

$$\begin{aligned} &= x(t_0) \int_{-\infty}^{\infty} \delta(\lambda) \cdot d\lambda \quad \text{Let } t-t_0 = \lambda \\ & \quad \quad \quad dt = d\lambda \\ &= x(t_0) \end{aligned}$$

$$\int_{-\infty}^{\infty} x(t) \cdot \delta(t - t_0) \cdot dt = x(t_0)$$

$$\rightarrow 4) \quad \delta(at) = \frac{1}{|a|} \cdot \delta(t) \rightarrow 5$$

Let us consider the signal,

$$\int_{-\infty}^{\infty} x(t) \cdot \delta(at) \cdot dt \text{ for } a > 0$$

$$\text{Let } at = \lambda$$

$$a \, dt = d\lambda$$

$$dt = \frac{1}{a} \cdot d\lambda$$

$$\text{Now } \int_{-\infty}^{\infty} x(t) \cdot \delta(at) \, dt = \int_{-\infty}^{\infty} x\left(\frac{\lambda}{a}\right) \cdot \delta(\lambda) \cdot \frac{d\lambda}{a}$$

$$= \frac{1}{a} \int_{-\infty}^{\infty} x\left(\frac{\lambda}{a}\right) \cdot \delta(\lambda) \cdot d\lambda$$

using eq ①, we can write, if $a \geq 0$

$$\int_{-\infty}^{\infty} x(t) \cdot \delta(at) \cdot dt = \frac{1}{a} x(0) \rightarrow 6$$

if $a < 0$, then

$$\int_{-\infty}^{\infty} x(t) \cdot \delta(at) \cdot dt = \frac{1}{|a|} \cdot x(0) \rightarrow 7$$

$$\text{we know that } x(0) = \int_{-\infty}^{\infty} x(t) \cdot \delta(t) \cdot dt$$

Therefore we can write,

$$\frac{1}{|a|} \cdot x(0) = \frac{1}{|a|} \cdot \int_{-\infty}^{\infty} x(t) \cdot \delta(t) \cdot dt$$

$$= \int_{-\infty}^{\infty} x(t) \cdot \frac{1}{|a|} \cdot \delta(t) \cdot dt \rightarrow (8)$$

by equating eq (7) & (8)

$$\boxed{\delta(at) = \frac{1}{|a|} \cdot \delta(t)}$$

$$\rightarrow 5) \int_{-\infty}^{\infty} x(\tau) \cdot \delta(t-\tau) \cdot d\tau = x(t)$$

by replacing τ by ' t ' & t_0 by ' t ' in property -3, we get

$$\int_{-\infty}^{\infty} x(\tau) \cdot \delta(\tau-t) \cdot d\tau = x(t)$$

Using the even property of impulse function, we can write

$$\delta(\tau-t) = \delta(t-\tau)$$

$$\Rightarrow \int_{-\infty}^{\infty} x(\tau) \cdot \delta(t-\tau) = x(t)$$

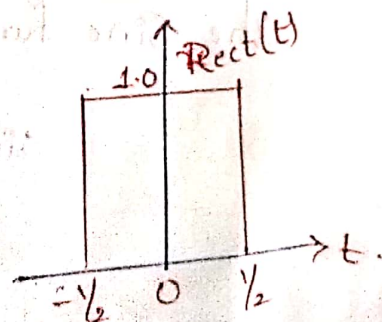
This eqn is called as convolution integral.

Rectangular Pulse Function:-

The Rectangular Pulse function is defined as

$$\text{rect}\left(\frac{t}{1}\right) = 1 \text{ for } -\frac{1}{2} < t < \frac{1}{2}$$

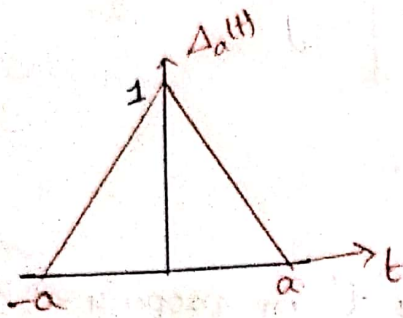
0, otherwise.



Triangular pulse Function :-

The unit triangular pulse function is denoted as

$$\Delta_a(t) = \begin{cases} 1 - \frac{|t|}{a}, & |t| \leq a \\ 0, & |t| > a \end{cases}$$



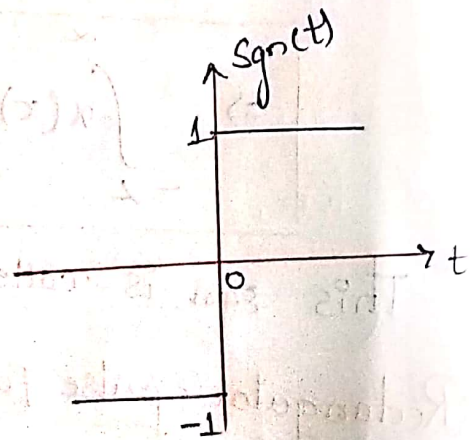
Signum function :-

The unit signum function is defined by

$$\text{sgn}(t) = \begin{cases} 1, & t > 0 \\ -1, & t < 0 \end{cases}$$

This function can be expressed in terms of unit step function as

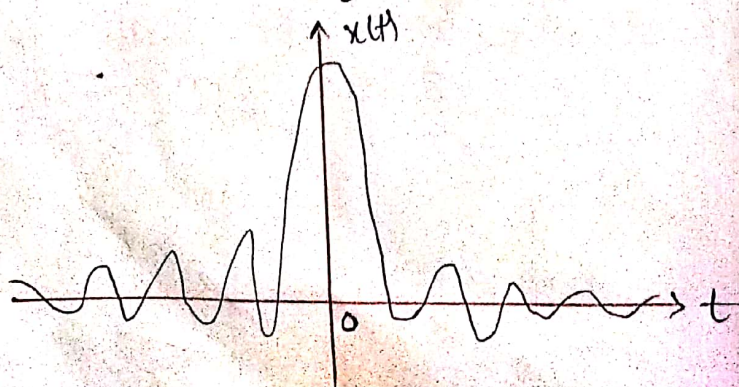
$$\text{sgn}(t) = -1 + 2u(t)$$



Sinc function :-

The sinc function defined by the expression,

$$\text{sinc}(t) = \frac{\sin t}{t}, \quad -\infty < t < \infty$$



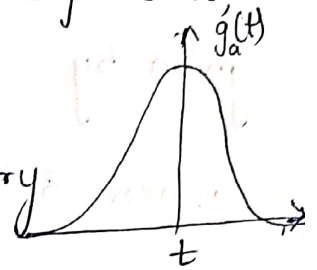
The Sinc function oscillates with period 2π and decays with increasing 't'. Its value is zero at $n\pi$, $n = \pm 1, \pm 2, \dots$. It is an even function of 't'.

Gaussian function:-

The Gaussian function is defined by the expression

$$g_a(t) = e^{-at^2}, -\infty < t < \infty.$$

This function is used in Probability theory.



Sinusoidal Signal:-

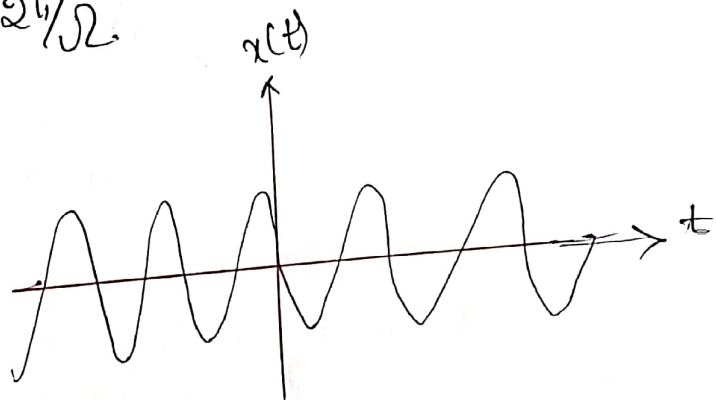
A continuous time Sinusoidal Signal is given by

$$x(t) = A \sin(\Omega t + \theta).$$

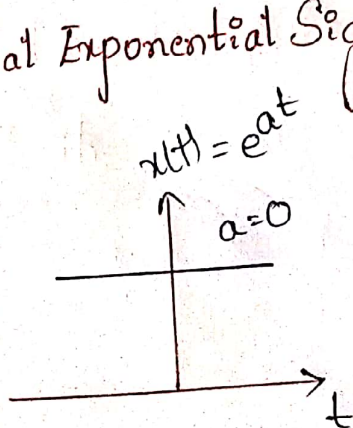
where A = Amplitude Signal

Ω = frequency, θ = Phase angle

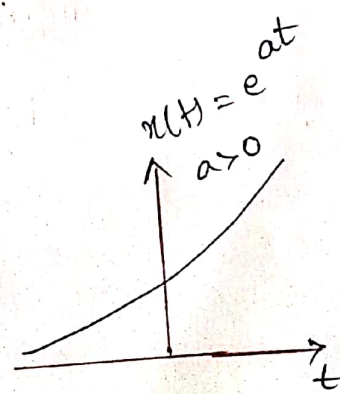
& Time period = $2\pi/\Omega$.



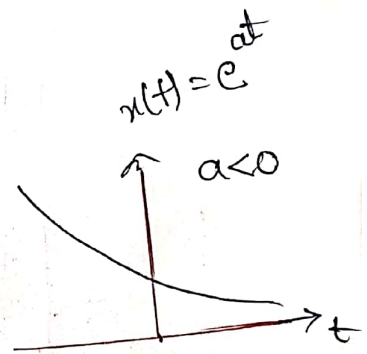
Real Exponential Signals:



a) DC-Signal



b) exponentially growing signal



c) exponentially decaying signal

A real exponential Signal is defined as

$$x(t) = A \cdot e^{at}, \text{ where both } A \text{ \& } a \text{ are real.}$$

Complex Exponential Signal :-

The most general form of complex Exponential is given by

$$x(t) = e^{st}$$

where s is a complex variable defined as $s = \sigma + j\Omega$.

A real exponential signal is defined as

$$x(t) = A \cdot e^{at}, \text{ where both } A \text{ \& } a \text{ are real.}$$

Complex Exponential Signal :-

The most general form of complex Exponential is given by

$$x(t) = e^{st} \rightarrow (1)$$

where s is a complex variable defined as $s = \sigma + j\Omega$

Therefore, $x(t) = e^{st} = e^{(\sigma + j\Omega)t}$

$$= e^{\sigma t + j\Omega t}$$

$$= e^{\sigma t} \cdot e^{j\Omega t} \rightarrow (2)$$

by using Euler's identity we can expand,

$$e^{j\Omega t} = \cos \Omega t + j \sin \Omega t \rightarrow (3)$$

Sub Eq (3) in Eq (2)

$$x(t) = e^{\sigma t} (\cos \Omega t + j \sin \Omega t) \rightarrow (4)$$

Depending on the values of σ & Ω , we get different signals.

If $\sigma = 0$ & $\Omega = 0$; $x(t) = 1$

Then, that signal $x(t)$ is a pure DC signal

if $\Omega = 0$ &, then $s = \sigma + j\Omega$
 $= \sigma + j(0)$

$$s = \sigma$$

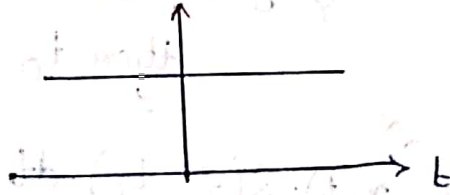
then $x(t) = e^{\sigma t}$, decays exponentially for $\sigma < 0$
grows exponentially for $\sigma > 0$

If $\sigma = 0$ then $s = j\Omega$ gives $x(t) = e^{j\Omega t}$

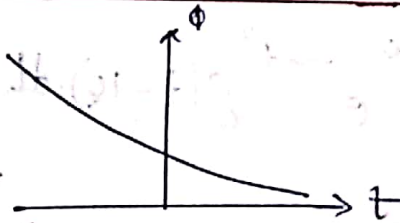
If $\sigma < 0$ with finite Ω , we get a exponentially decaying Sinusoidal Signal.

If $\sigma > 0$ with finite Ω , we get a exponentially Sinusoidal Signal.

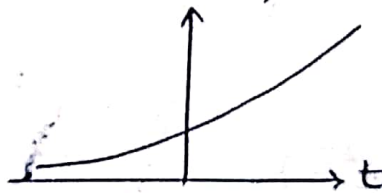
$$s = 0$$



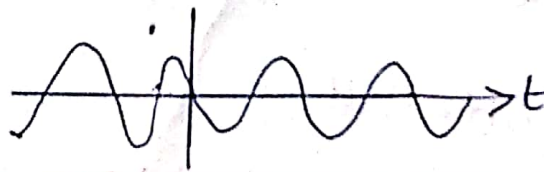
$$s = -\sigma$$



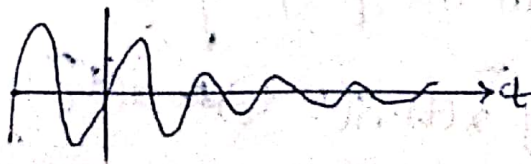
$$s = \sigma$$



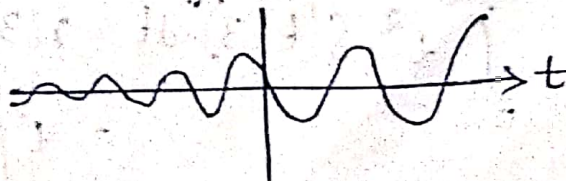
$$s = \pm j\Omega$$



$$s = \sigma \pm j\Omega, \sigma < 0$$



$$s = \sigma \pm j\Omega, \sigma > 0$$



Evaluate the following integrals:-

i) $\int_{-\infty}^{\infty} e^{-\alpha t^2} \delta(t-10) \cdot dt$

From the property (3), $\int_{-\infty}^{\infty} x(t) \cdot \delta(t-t_0) \cdot dt = x(t_0)$.

Here by comparing this with above property,
then we get

$$x(t) = e^{-\alpha t^2} \quad \& \quad \delta(t-t_0) = \delta(t-10)$$

then $t_0 = 10$.

$$\int_{-\infty}^{\infty} x(t) \cdot \delta(t-t_0) \cdot dt = x(t_0)$$

$$\int_{-\infty}^{\infty} e^{-\alpha t^2} \delta(t-10) \cdot dt = x(10)$$

$$= e^{-\alpha t^2} \Big|_{t=10}$$

$$= e^{-\alpha(10)^2} = e^{-100\alpha}$$

$$\Rightarrow \int_{-\infty}^{\infty} e^{-\alpha t^2} \cdot \delta(t-10) \cdot dt = e^{-100\alpha}$$

ii). $\int_0^{\infty} t^2 \cdot \delta(t-3) \cdot dt$

Similarly, from the property (3)

$$\int_{-\infty}^{\infty} x(t) \cdot \delta(t-t_0) \cdot dt = x(t_0)$$

$$\int_0^{\infty} t^2 \cdot \delta(t-3) \cdot dt = x(3)$$

$$= t^2 \Big|_{t=3}$$

$$= 3^2 = 9$$

iii) $\int_{-\infty}^{\infty} [\delta(t) \cdot \cos t + \delta(t-\pi) \sin t] dt$

$$= \int_{-\infty}^{\infty} \delta(t) \cdot \cos t \, dt + \int_{-\infty}^{\infty} \delta(t-1) \cdot \sin t \, dt$$

Here also we can apply Property - ① to individual terms

$$\Rightarrow \textcircled{1} \rightarrow \int_{-\infty}^{\infty} g(t) \cdot \cos t \cdot dt \rightarrow$$

$$\int_{-\infty}^{\infty} x(t) \cdot \delta(t) dt = x(0)$$

Here $x(t) = \cos t$

then $\int_0^{\pi} g(t) \cdot \cos t \cdot dt = x(0)$
 $= \cos 0 = 1$

$$\Rightarrow \textcircled{2} \rightarrow \int_{-\infty}^{\infty} \delta(t-1) \cdot \sin t \, dt$$

by comparing with Property - ③

$$f(x) \cdot g(x) = \int_0^x f(t) \cdot g(x-t) dt = \int_0^x f(t) \cdot g(x-t) dt$$

Here $x(t) = \sin t$ then $\int_{t_0}^t \sin t \cdot \delta(t-t_0) dt = x(t_0)$

$$t_0 = \frac{1}{\omega} \sin \omega t = \frac{1}{\omega} \sin \omega t$$

*Term ① + Term ②

$$\int [\delta(t) \cos t + \delta(t-1) \sin t] dt =$$

$$\Rightarrow \int_{-\infty}^{\infty} S(t) \cos t \, dt + \int_{-\infty}^{\infty} S(t-1) \cdot \sin t \, dt$$

— 1 + 501

Solve the following Integrals :-

i) $\int_{-\infty}^{\infty} \delta(t+3) \cdot e^{-t} dt$

ii) $\int_{-\infty}^{\infty} (t-3)^2 \delta(t-3) \cdot dt$

iii) $\int_{-\infty}^{\infty} \delta(t) \cdot e^{-j\omega t} dt$

iv) $\int_{-\infty}^{\infty} e^{-t} \cdot \delta(2t-2) \cdot dt$

v) $\int_{-\infty}^{\infty} \delta(t) \cdot \cos \pi t dt =$

Q) $\int_{-\infty}^{\infty} x(2-t) \cdot \delta(3-t) \cdot dt$

here let us consider, $2-t = \lambda$

$$-dt = d\lambda$$

$$dt = -d\lambda$$

then $3-t \Rightarrow 1 + \underline{2-t}$

$$3-t = 1 + \lambda$$

Sub $(2-t)$, $(3-t)$, dt values in above eqn, then we get

$$= \int_{-\infty}^{\infty} x(\lambda) \cdot \delta(1+\lambda) (-d\lambda)$$

$$= - \int_{-\infty}^{\infty} x(\lambda) \cdot \delta(1+\lambda) \cdot d\lambda$$

$$= - \int_{-\infty}^{\infty} x(\lambda) \delta(\lambda - (-1)) \cdot d\lambda$$

This is in the form of property - ③

i.e: $\int_{-\infty}^{\infty} x(t) \cdot \delta(t-t_0) \cdot dt = x(t_0)$

$$= - \int_{-\infty}^{\infty} x(\lambda) \cdot \delta(\lambda - (-1)) \cdot d\lambda = -x(-1)$$

Elementary Discrete-Time Signals :-

→ 1. unit Step Sequence :-

The unit Step Sequence is defined as

$$u(n) = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

The graphical representation of $u(n)$ is given as

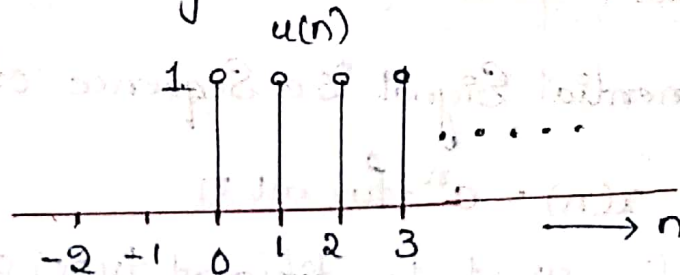


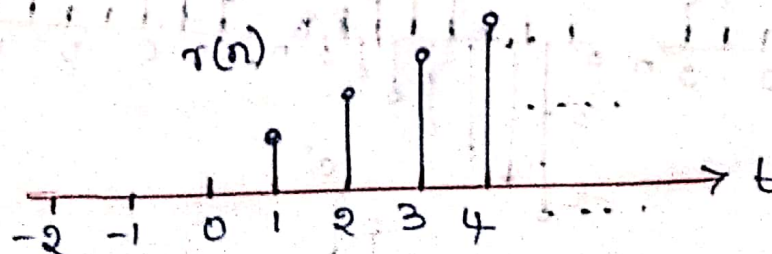
fig: unit Step Sequence.

→ unit ramp Sequence :-

The unit ramp Sequence is defined as

$$r(n) = \begin{cases} n, & \text{for } n \geq 0 \\ 0, & \text{for } n < 0 \end{cases}$$

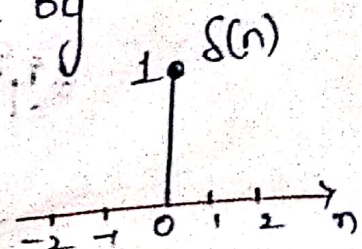
The graphical representation of $r(n)$ is given as:



→ unit Sample Sequence (unit impulse Sequence) :-

The unit Sample Sequence is given by

$$\delta(n) = \begin{cases} 1, & \text{for } n = 0 \\ 0, & \text{for } n \neq 0 \end{cases}$$



The unit impulse function has the following properties

$$\delta(n) = u(n) - u(n-1)$$

$$u(n) = \sum_{k=-\infty}^n \delta(k)$$

$$\sum_{n=-\infty}^{\infty} x(n) \cdot \delta(n-n_0) = x(n_0)$$

→ Exponential Sequence :-

The exponential signal is a sequence of the form

$$x(n] = a^n \text{ for all } n$$

The below fig, illustrates different types of discrete time Exponential Signals.

when the value of $a > 1 \Rightarrow$ the sequence grows exponentially

$a < 1 \Rightarrow$ the sequence decays exponentially

$a < 0 \Rightarrow$ the discrete time signal takes alternating signs.

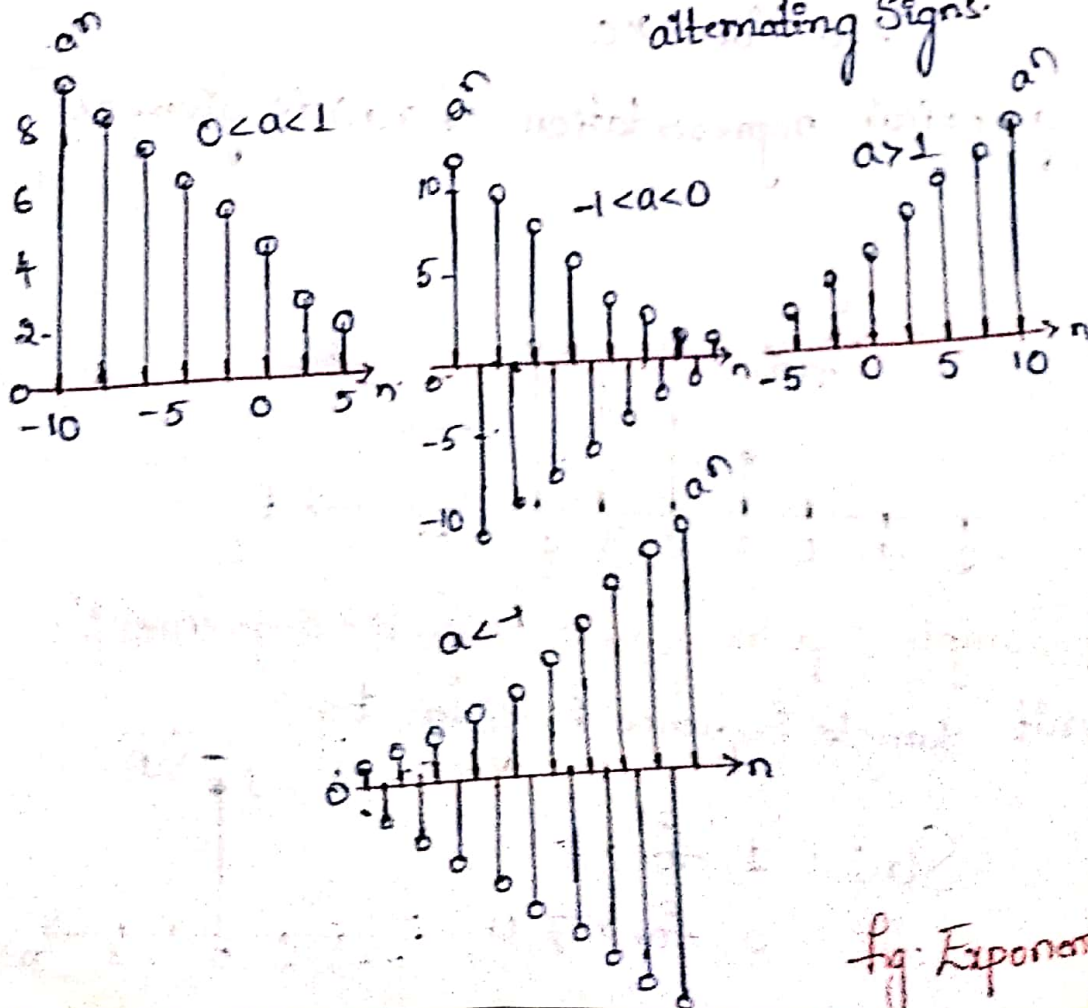


Fig. Exponential Signals

→ Sinusoidal Signal :-

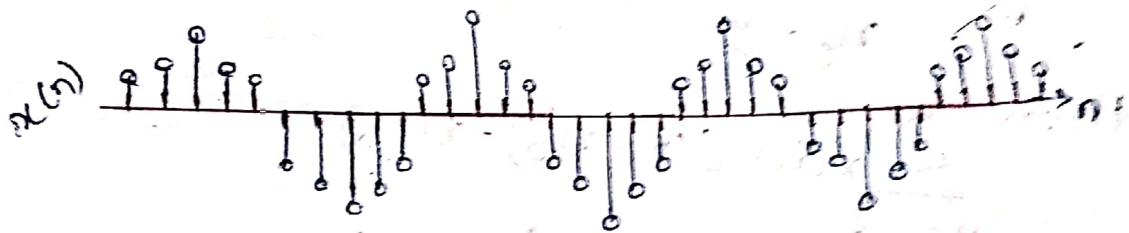
The discrete time Sinusoidal Signal is given by

$$x(n) = A \cos(\omega_0 n + \phi)$$

where ω_0 is the frequency and ϕ is the phase
using Euler's identity, we can write

$$A \cos(\omega_0 n + \phi) = \frac{A}{2} e^{j\phi} e^{j\omega_0 n} + \frac{A}{2} e^{-j\phi} e^{-j\omega_0 n}$$

fig: Sinusoidal Sequence,



→ Complex Exponential Signal :-

The discrete time exponential Signal is given by

$$x(n) = a^n e^{j(\omega_0 n + \phi)}$$

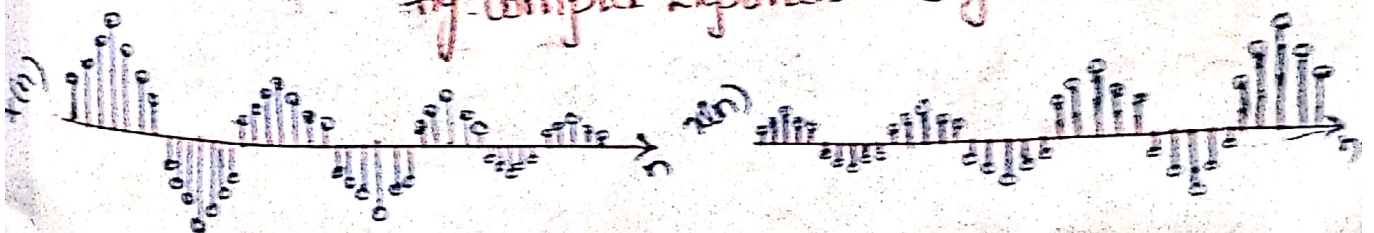
$$= a^n (\cos(\omega_0 n + \phi) + j \sin(\omega_0 n + \phi))$$

For $|a| = 1$, the real and imaginary parts of complex exponential sequence are sinusoidal

For $|a| < 1$, the amplitude of the sinusoidal sequence decays exponentially

For $|a| > 1$, the amplitude of the sinusoidal sequence grows exponentially.

fig: Complex Exponential Signals.



Prob: Find the following Summations :-

(i) $\sum_{n=-\infty}^{\infty} e^{2n} \delta(n-2)$

By using the property (3) of unit impulse Sequence

i.e. $\sum_{n=-\infty}^{\infty} x(n) \cdot \delta(n-n_0) = x(n_0)$

by comparing given Summation, with this property we get $x(n) = e^{2n}$
 $n_0 = 2$

then $\sum_{n=-\infty}^{\infty} x(n) \cdot \delta(n-n_0) = x(n_0)$

$\sum_{n=-\infty}^{\infty} e^{2n} \cdot \delta(n-2) = x(2)$
 $= e^{2n} \Big|_{n=2} = e^4$

(ii) $\sum_{n=0}^{\infty} \delta(n+1) e^{n^2}$

By using Property (3), of unit Sample Sequence

i.e. $\sum_{n=-\infty}^{\infty} x(n) \cdot \delta(n-n_0) = x(n_0)$

$\sum_{n=0}^{\infty} e^{n^2} \cdot \delta(n-1) = x(1)$

$= e^{n^2} \Big|_{n=1}$

$= e^{1^2} = e$

Assignment

(iii) $\sum_{n=-\infty}^{\infty} \delta(n+1) \sin 2n$

(v) $\sum_{n=2}^{\infty} \delta(n+1) \sin 2n$

(iv) $\sum_{n=0}^{\infty} \delta(n+1) 2^n$

(vi) $\sum_{n=0}^{\infty} x(n) \cdot \delta(n-2)$

Basic operation on Signals :-

The basic set of operations on Signals are

- 1. Time Shifting
- 2. Time Reversal
- 3. Time Scaling
- 4. Amplitude Scaling
- 5. Signal Multiplication
- 6. Signal Addition

Time Shifting :-

Let us consider a Signal $x(t)$ shown in fig. (a).
The time Shifting of $x(t)$ may delay or advance the Signal in time.

Mathematically this can be represented by

$$y(t) = x(t - T)$$

If ' T ' is positive, the Shifting delays the Signal as in fig (b).

If ' T ' is negative, then $y(t) = x(t - (-T))$

$$= x(t + T),$$

the Shifting advances the Signal, as in fig (c).

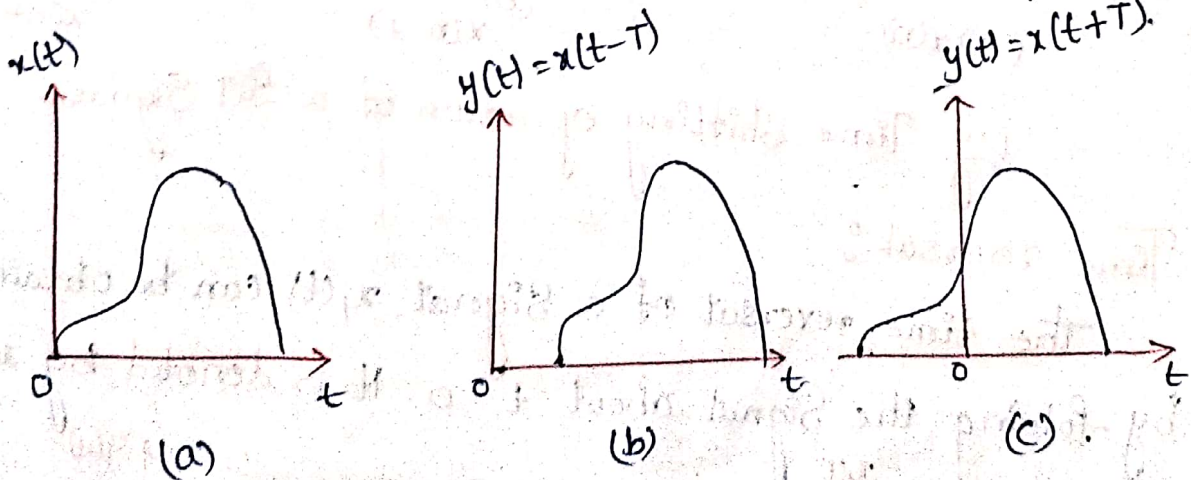


Fig: Time Shifting operation of Continuous time Signal

Similarly, the time Shifting operation of a discrete time Signal is represented by

$$y(n) = x(n-k)$$

which shows that the Signal $y(n)$ can be obtained by Shifting $x(n)$ by 'k' units.

If 'k' is positive, the shift is to the right i.e. delay.

If 'k' is negative, the shift is to the left i.e. advance.

Let us consider, a Signal $x(n)$ shown in fig(a) below.

Now, $x(n-2)$ is obtained by shifting $x(n)$ right by 2 units of time as shown in fig(b).

$x(n+3)$ is obtained by Shifting $x(n)$ left by 3 units of time as shown in fig(c).

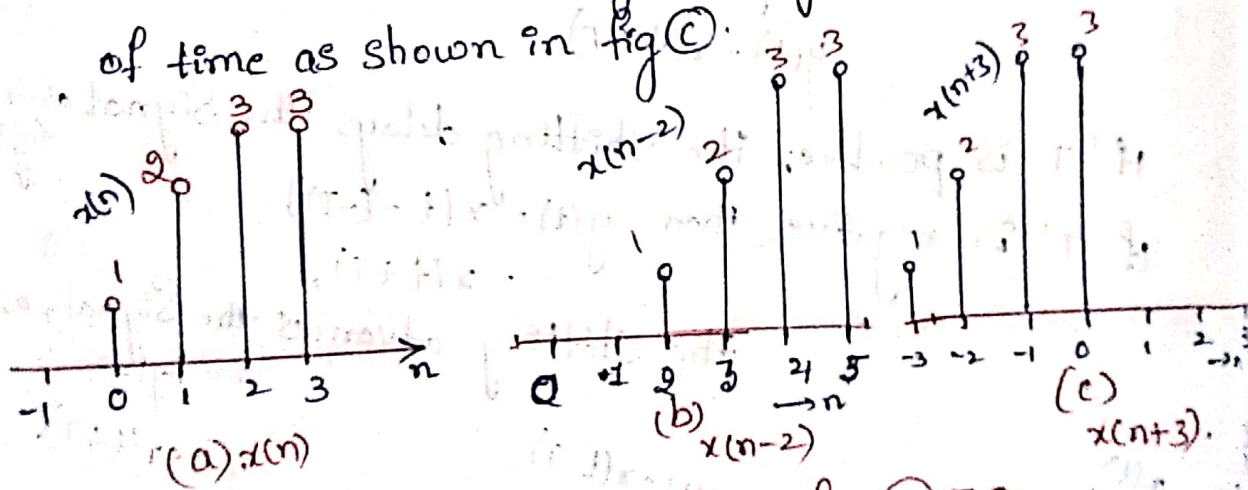
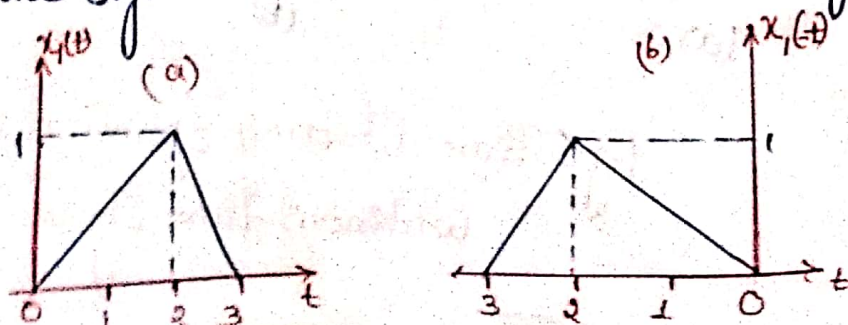


Fig: Time Shifting operation of a DT Signal.

Time reversal :-

The time reversal of a Signal $x(t)$ can be obtained by folding the Signal about $t=0$. It is denoted by $x(-t)$.



The Signal $x_1(-t+2)$ is obtained by delaying $x_1(-t)$ by 2 units of time.

The Signal $x_1(-t-2)$ is obtained by advancing $x_1(-t)$ by 2 units of time.

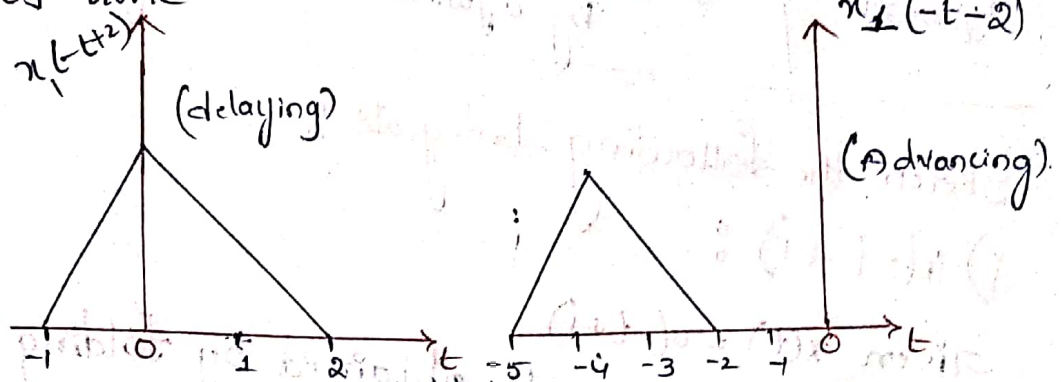


Fig: Time Reversal operation:-

The time reversal of a discrete time Signal $x(n)$ can be obtained by folding the Sequence $x(n)$ about $n=0$.

For the Signal $x(n)$, the time reversal Signal is shown below.

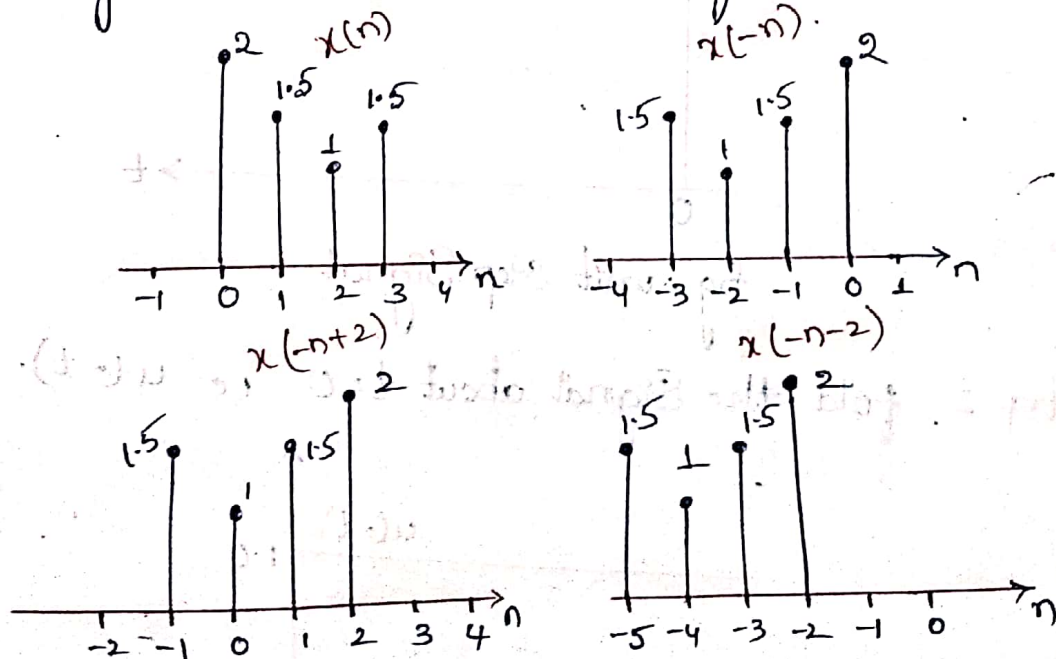


Fig: Time Reversal operation on DT Signals.

The Signal $x(-n+2)$ is obtained by delaying $x(-n)$ by 2 units of time.

The Signal $x(-n-2)$ is obtained by advancing $x(-n)$ by 2 units of time.

continuous/discrete	$+a$ units	$-a$ units
t/n	shifts left (adv) by a units	shifts right (delay) by a units
$-t/n$	shifts right (delay) by a units	shifts left (adv) by a units

Sketch the following Integrals :-

i) $u(-t+1)$:-

Given $x(t) = u(-t+1)$

The signal $x(t)$ can be obtained by folding the unit step signal $u(t)$ about $t=0$ and shifting the signal

Step-1: First draw the unit step signal.

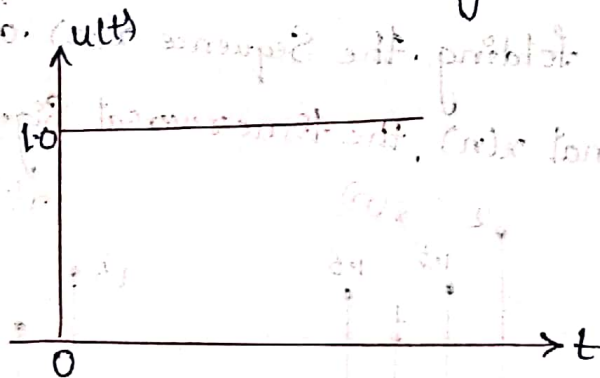


Fig: unit step signal.

Step 2: fold the signal about $t=0$. i.e. $u(-t)$.

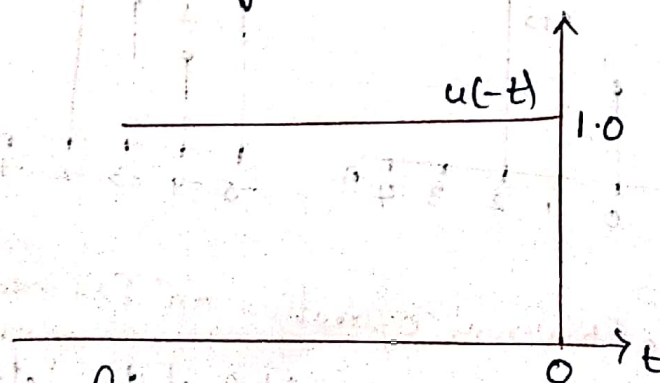
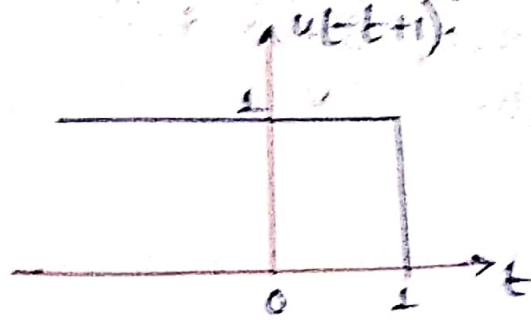


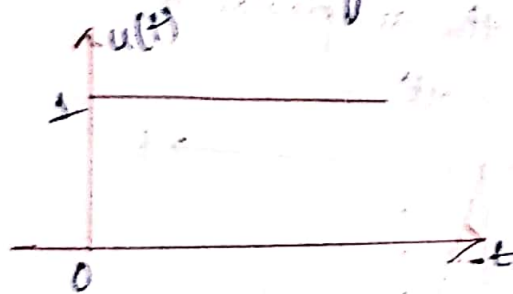
Fig: Time reversal of unit step signal.

Step 2:- Shift the Signal $u(t)$ right by one unit of time as shown in below fig.

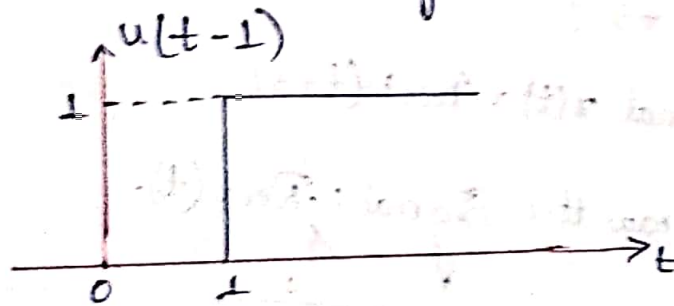


ii) $x(t) = -2u(t-1)$

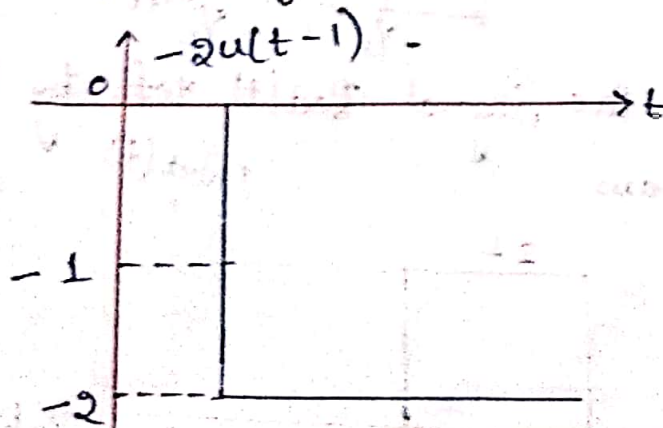
Step I:- Draw the unit Step Signal.



Step II:- Shift the signal $u(t)$ right by one unit of time as shown in below fig.



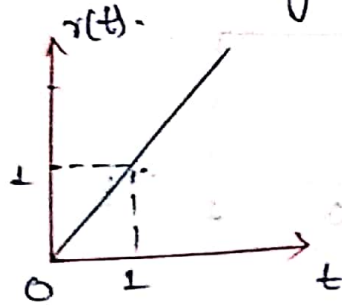
Step III:- Multiply the Signal $u(t-1)$ with -2.



III) $x(t) = -2r(t)$.

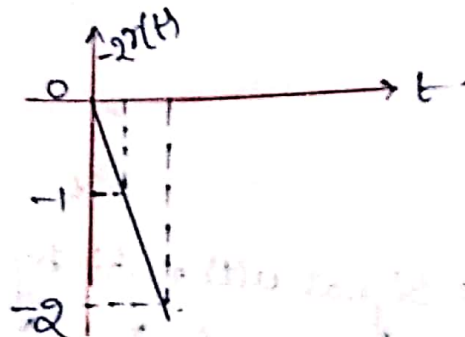
The Signal $x(t)$ is a ramp Signal.

Step I: Draw the ramp Signal $r(t)$.



Step II: ~~Plot the signal at t=0, for r(t)~~

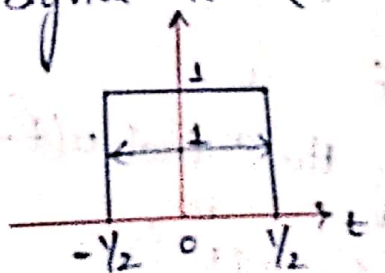
Here $-2r(t)$, i.e. the amplitude will become -2



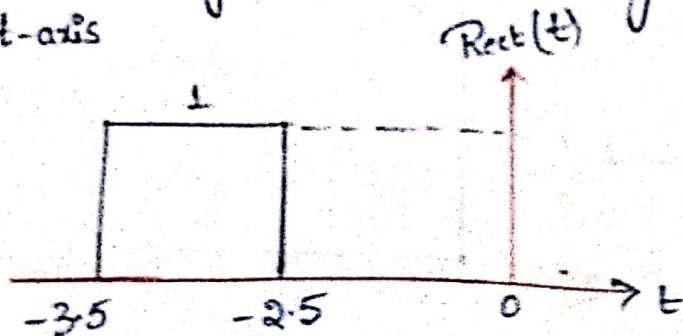
IV) $\text{Rect}(t+3)$:

Given Signal $x(t) = \text{Rect}(t+3)$

Step I: Draw the Signal: $\text{Rect}(t)$.



Step II: Shift the Signal $\text{Rect}(t)$ left by 3 units of time
i.e. at t-axis



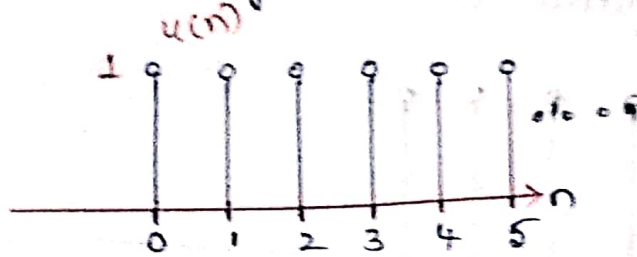
H.W: Sketch the following Signals.

i) $x(-t+2)$ ii) $u(t-3)$ iii) $\text{Rect}(t-4)$

Sketch the following Signals in discrete form.

(i) $u(n+2) - u(n-3)$

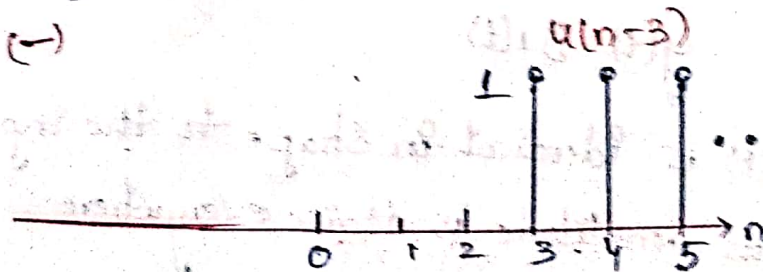
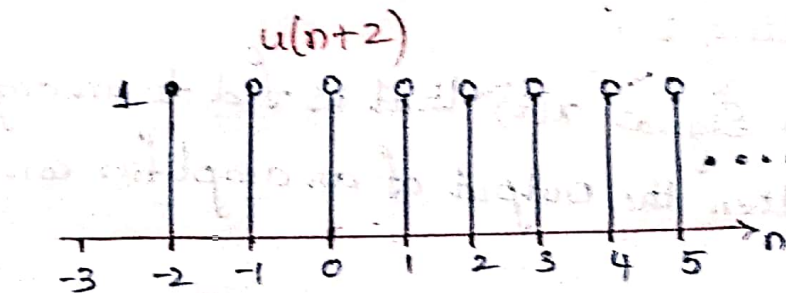
first, draw the unit step Sequence $u(n)$



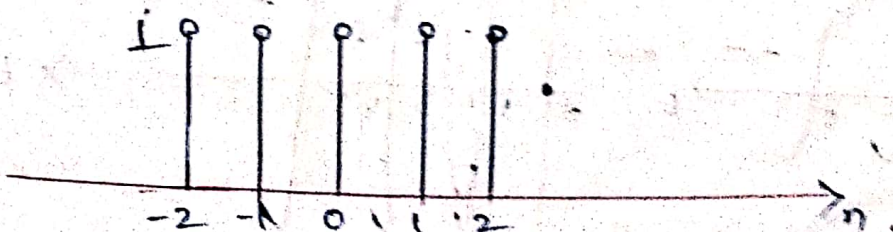
Step-I: sketch the Signal $u(n+2)$, by shifting $u(n)$ left by two Samples as shown in fig(a)

Step-II: Sketch the Signal $u(n-3)$ by shifting $u(n)$ right by three Samples as shown in fig(b).

Step-III: Subtract $u(n-3)$ from $u(n+2)$

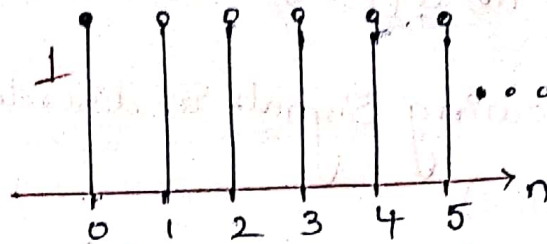


$\Rightarrow u(n+2) - u(n-3)$



i) $u(-n+2) \cdot u(n)$.

Draw the Sequence $u(n)$.

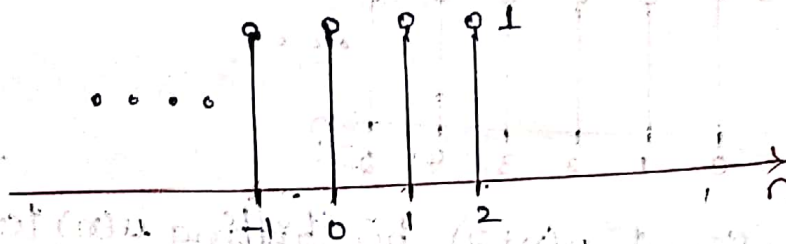


Draw the Sequence $u(-n+2)$

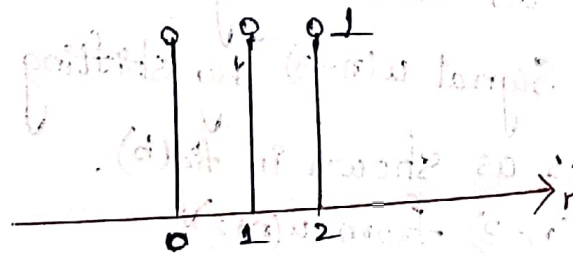
$$(-n+2) \geq 0$$

$$-n \geq -2$$

$$n \leq 2$$



$$\Rightarrow u(-n+2) \cdot u(n)$$

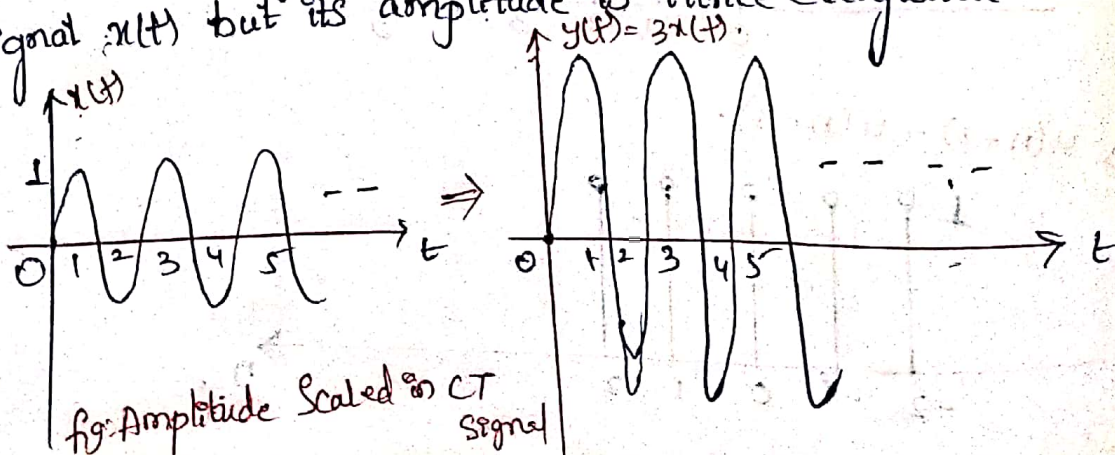


Amplitude Scaling :-

Consider a Signal $x(t)$ that is fed to an amplifier with a gain 3, then the output of an amplifier can be represented by

$$y(t) = 3x(t).$$

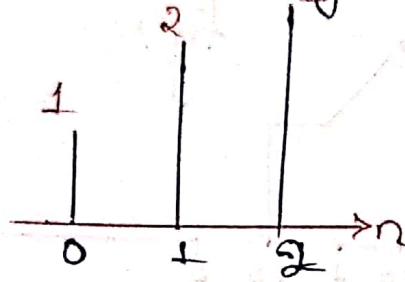
i.e. The o/p $y(t)$ is identical in shape to the input Signal $x(t)$ but its amplitude is thrice everywhere



from the above fig, we find that the amplitude is rescaled, hence the name amplitude scaling.

Similarly, the amplitude scaling of a discrete-time signal can be represented by $y(n) = ax(n)$.

If $x(n]$ is shown in below fig.



Let $a = \frac{2}{1}$, then $y(n) = 2x(n)$

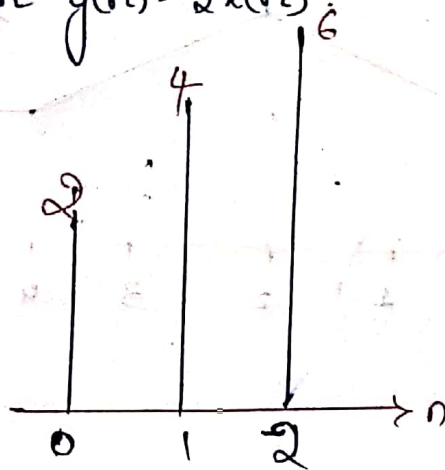


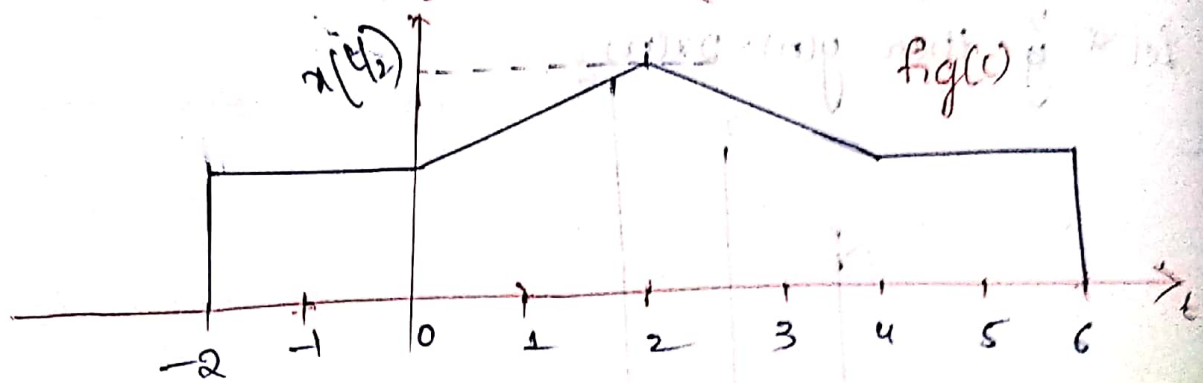
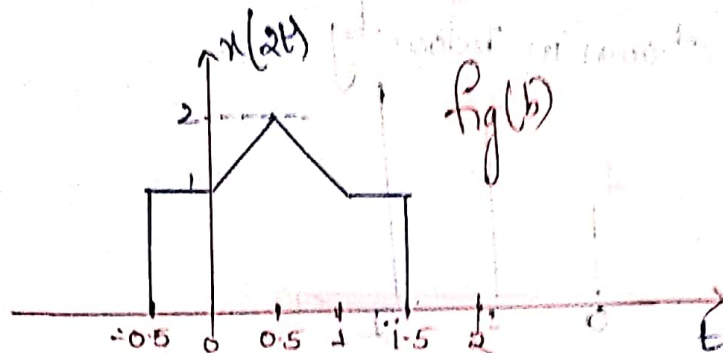
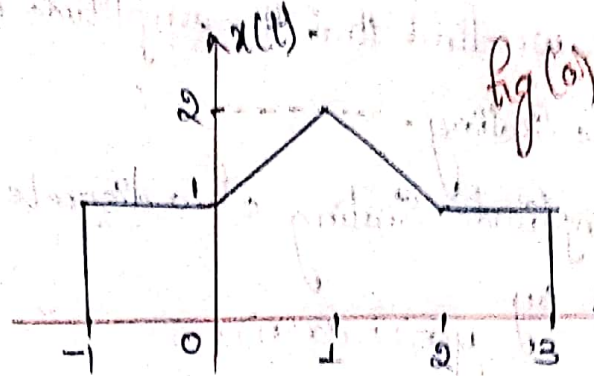
fig. Amplitude Scaling in DT Signal.

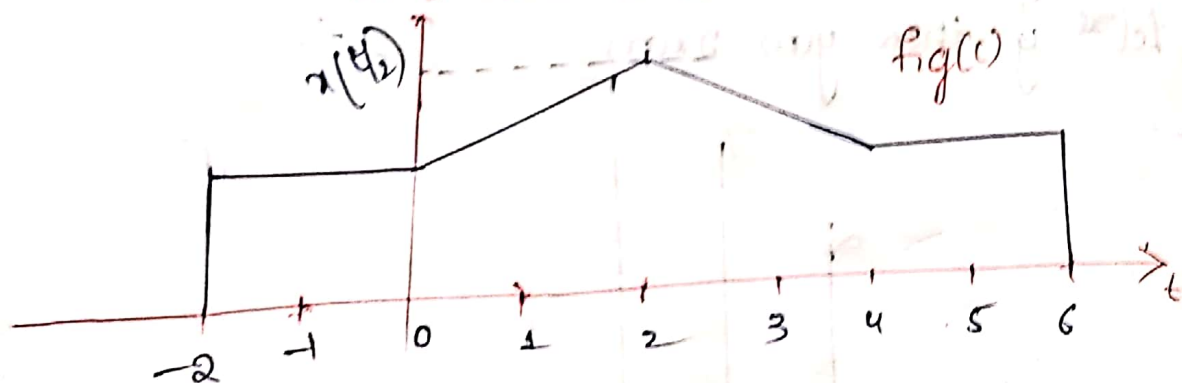
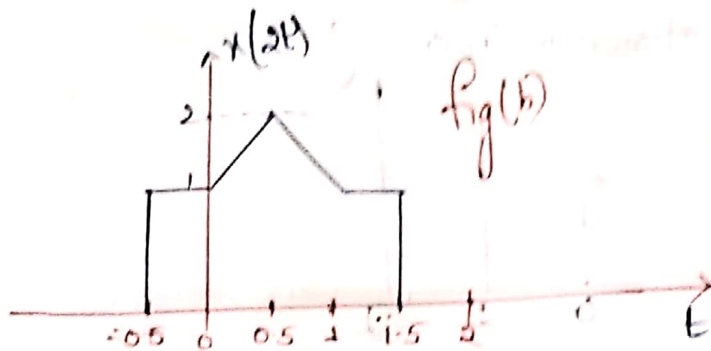
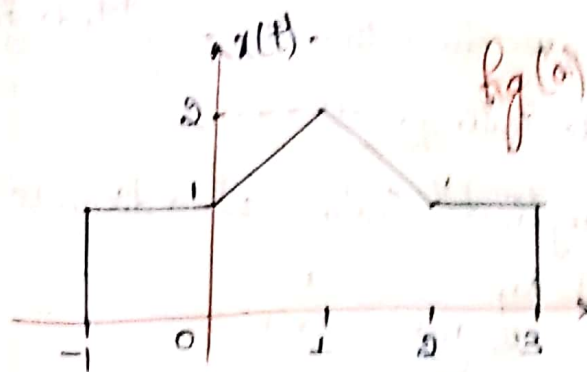
Time Scaling:-

Let us consider a signal $x(t)$ shown in fig(a). The time scaling of a signal can be accomplished by replacing 't' by at in the signal $x(t)$.

$$y(t) = x(at).$$

If $a=2$, then $y(t) = x(2t)$.





In the discrete-time case, we write

$$y(n) = x(an).$$

where $a > 0$,

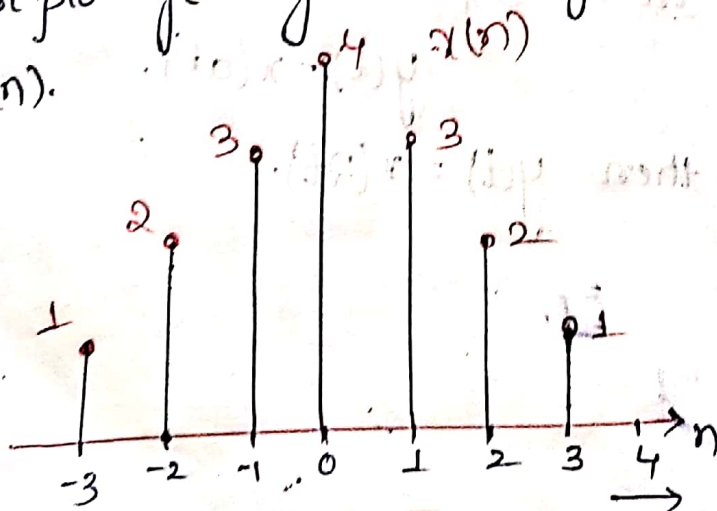
if $a=2$

$$y(n) = x(2n).$$

Let $x(n)$ be a sequence shown in fig(d).

Now, we can plot $y(n)$ by substituting different values for n in $x(n)$.

For



Here $x(n) = \{1, 2, 3, 4, 3, 2, 1\}$

i.e. n -value exists at $n = -3, -2, -1, 0, 1, 2, 3$. from fig.

For n $y(n) = x(2n)$.

So, here $y(n)$ exists only at $n = -1, 0, 1$.

$$\text{if } n = -1 \Rightarrow y(n) = x(2(-1)) = x(-2) = 2$$

$$n = 0 \Rightarrow y(0) = x(2(0)) = x(0) = 4$$

$$n = 1 \Rightarrow y(1) = x(2(1)) = x(2) = 2$$

$\therefore$ Plot the Sequence $y(n) = \{2, 4, 2\}$

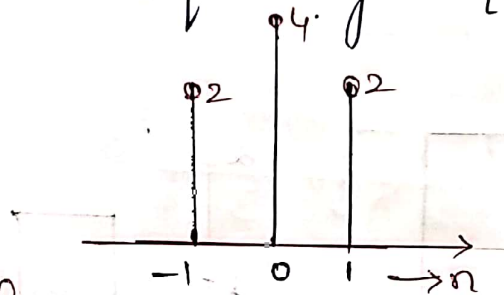
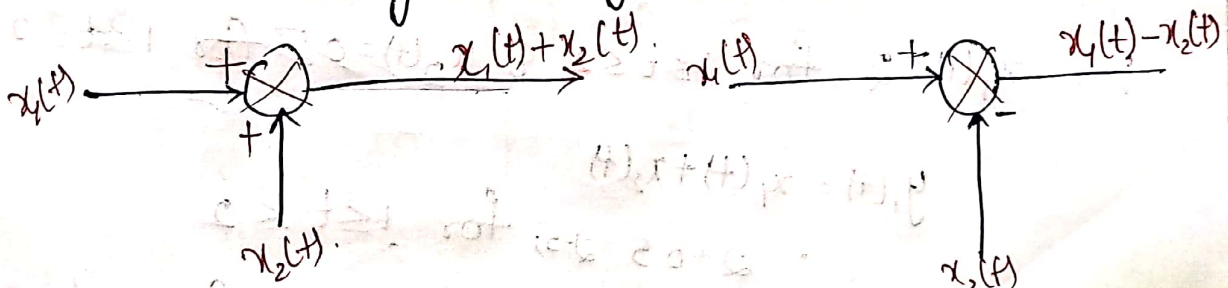


fig: Time Scaling in DT Signal

Signal Addition:-

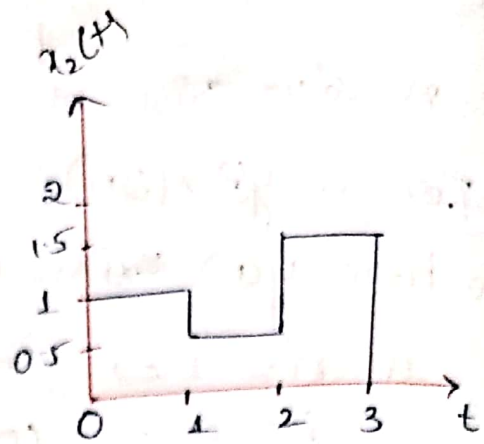
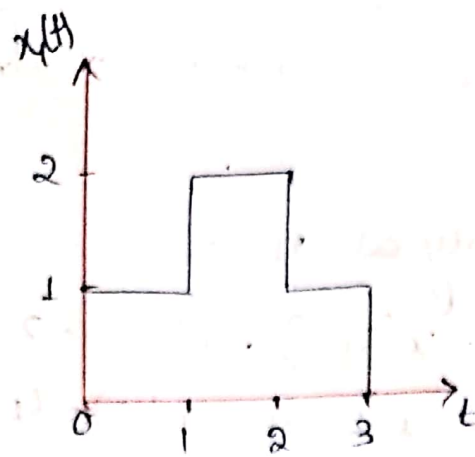
The Sum of two continuous time Signals can be obtained by adding their values at every instant.

Similarly, the Subtraction of two continuous-time Signals can be obtained by Subtracting their values at every instant.



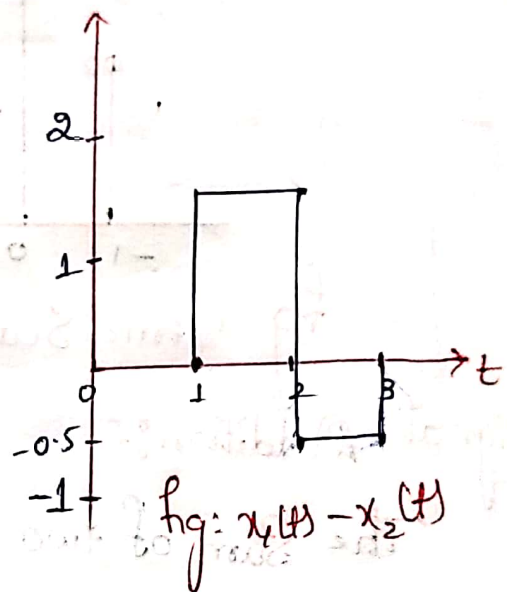
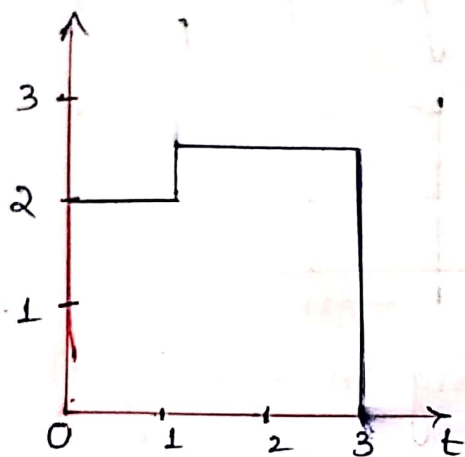
a) Addition of CT Signals (b) Subtraction of CT Signals

Let us consider addition and Subtraction of two Signals $x_1(t)$ & $x_2(t)$ as shown in below fig



$$y_1(t) = x_1(t) + x_2(t)$$

$$y_2(t) = x_1(t) - x_2(t)$$



$$f_{y_1}: x_1(t) + x_2(t)$$

$$f_{y_2}: x_1(t) - x_2(t)$$

Im

For $x_1(t) = 1$ for $0 \leq t \leq 1$ & $x_2(t) = 1$ for $0 \leq t \leq 1$

$$\text{Hence } y_1(t) = x_1(t) + x_2(t) = 1 + 1 = 2, \text{ for } 0 \leq t \leq 1$$

for $x_1(t) = 2$, for $1 \leq t \leq 2$ & $x_2(t) = 0.5$ for $1 \leq t \leq 2$

$$y_1(t) = x_1(t) + x_2(t) = 2 + 0.5 = 2.5, \text{ for } 1 \leq t \leq 2$$

for $x_1(t) = 1$, for $2 \leq t \leq 3$ & $x_2(t) = 1.5$ for $2 \leq t \leq 3$

$$y_1(t) = x_1(t) + x_2(t) = 1 + 1.5 = 2.5, \text{ for } 2 \leq t \leq 3$$

Similarly,

$$y_2(t) = x_1(t) - x_2(t)$$

$$= 1 - 1 = 0, \text{ for } 0 \leq t \leq 1$$

$$= 2 - 0.5 = 1.5, \text{ for } 1 \leq t \leq 2$$

$$= 1 - 1.5 = -0.5, \text{ for } 2 \leq t \leq 3$$

For discrete time Signals, the Sum of two Sequences can be obtained by adding the Sequence values at same index 'n'.

Similarly, the difference of two Sequences can be obtained by subtracting the Sequence values at index 'n'.

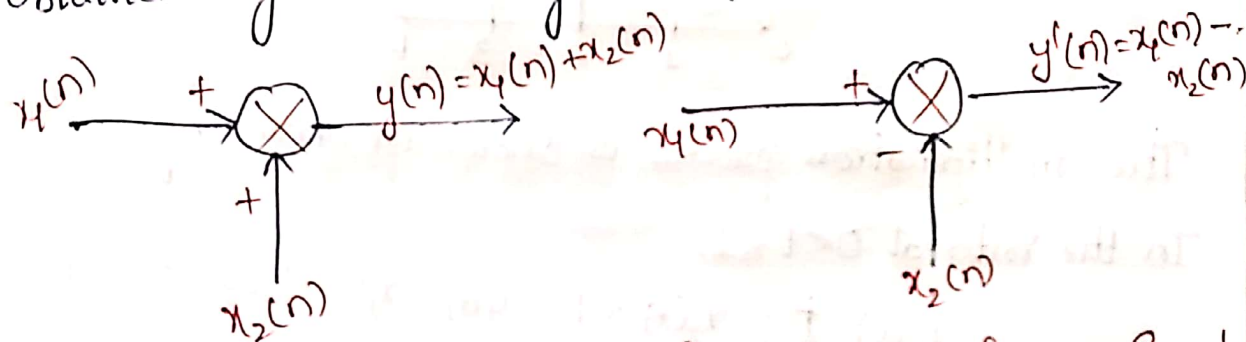


fig: Addition of two DT Signals fig: Subtraction of 2 DT Signals

If $x_1(n) = \{1, 3, 2, 1\}$ &

$x_2(n) = \{1, -2, 3, 2\}$

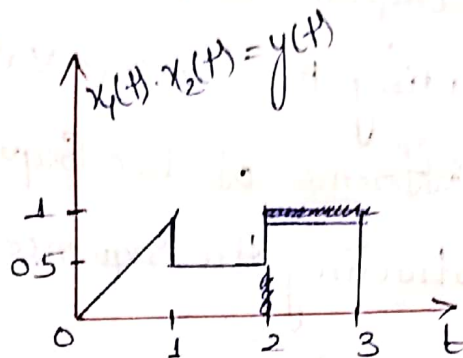
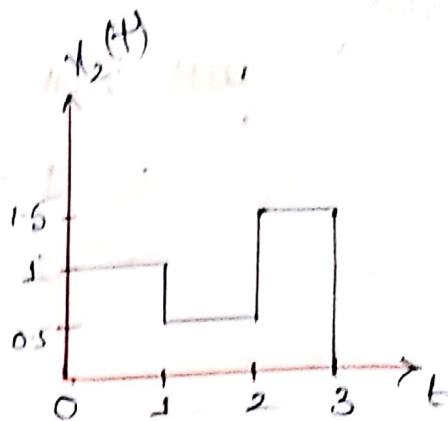
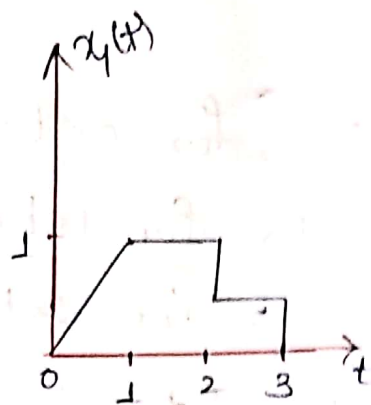
then

$$x_1(n) + x_2(n) = \{1+1, 3-2, 2+3, 1+2\} = \{2, 1, 5, 3\}.$$

$$x_1(n) - x_2(n) = \{1-1, 3-(-2), 2-3, 1-2\} = \{0, 5, -1, -1\}.$$

Signal Multiplication:-

The multiplication of two Signals can be obtained by multiplying their values at every instant.



The multiplication process is shown in above fig :-

In the interval $0 \leq t \leq 1$,

$$x_1(t) = t; \quad x_2(t) = 1; \quad y(t) = x_1(t) \cdot x_2(t) = t \cdot 1 = t.$$

In the interval $1 \leq t \leq 2$,

$$x_1(t) = 1; \quad x_2(t) = 0.5; \quad y(t) = x_1(t) \cdot x_2(t) = 1 \times 0.5 = 0.5$$

In the interval $2 \leq t \leq 3$,

$$x_1(t) = 0.5; \quad x_2(t) = 1.5; \quad y(t) = x_1(t) \cdot x_2(t) = 0.5 \times 1.5 = 0.75$$

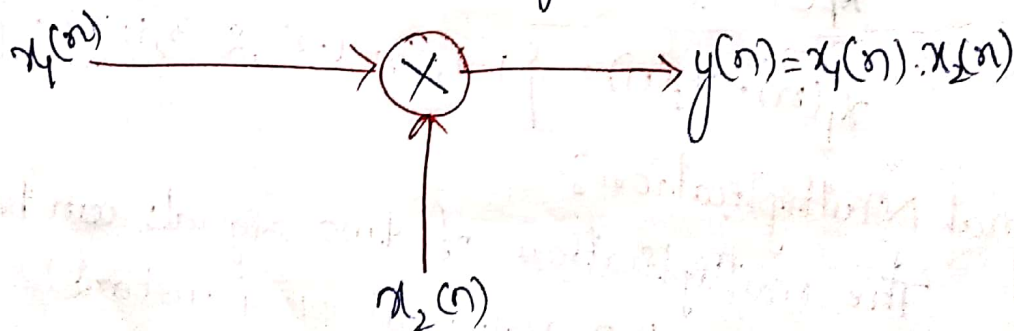
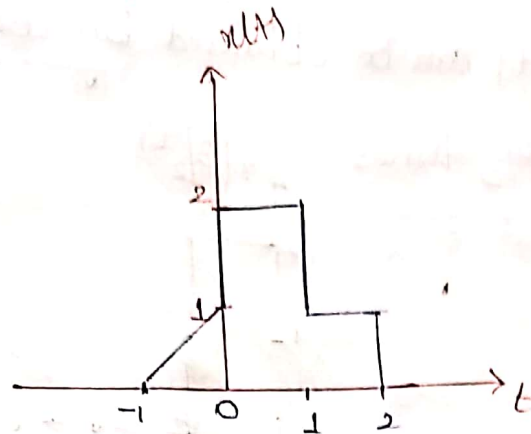


Fig: Multiplication process in DT Signals

$x_1(n) = \{1, 2, -2, 3\}$ & $x_2(n) = \{1, 0.5, 0.5, 1\}$ then

$$y(n) = x_1(n) \cdot x_2(n) = \{1 \times 1, 2 \times 0.5, -2 \times 0.5, 3 \times 1\} = \{1, 1, -1, 3\}$$

Find the following Signals for the Signal $x(t)$.



i) $x(2(t+3))$

iv) $x(t+1)$

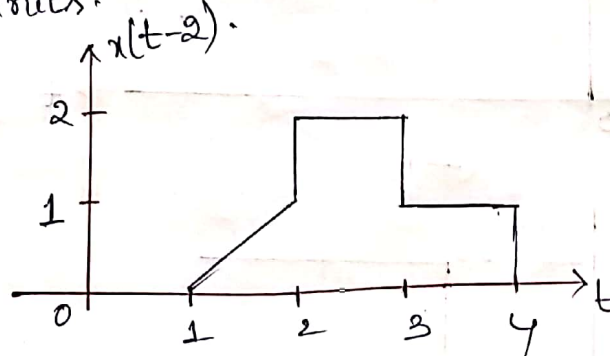
ii) $x(t-2)$

v) $x(3t-1)$

iii) $x(3/2 t)$

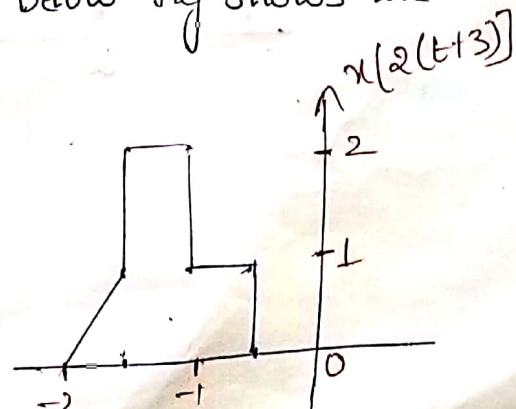
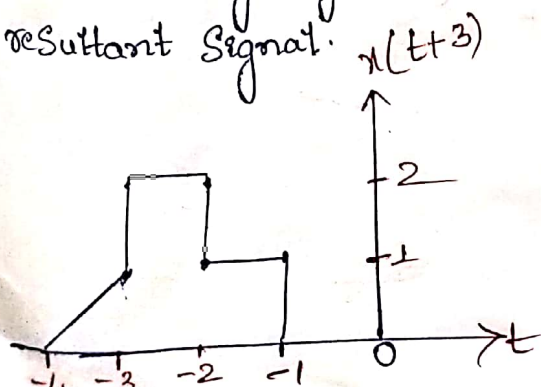
ii) $x(t-2)$

The Signal $x(t-2)$ can be obtained by delaying the Signal $x(t)$ by 2 units.



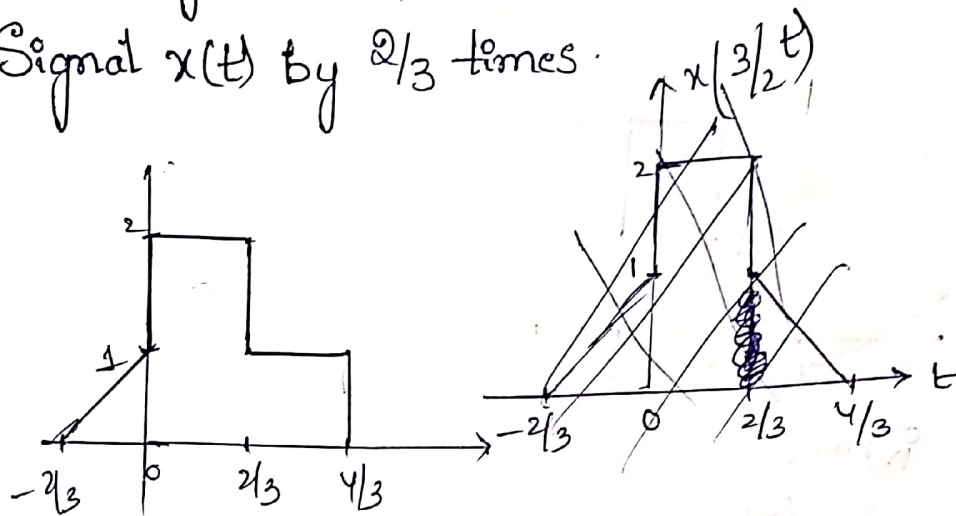
i) $x(2(t+3))$

The Signal $x(2(t+3))$ can be obtained by first shifting the Sequence $x(t)$ left by 3 units of time and then applying time Scaling by factor 2. The below fig shows the resultant Signal.



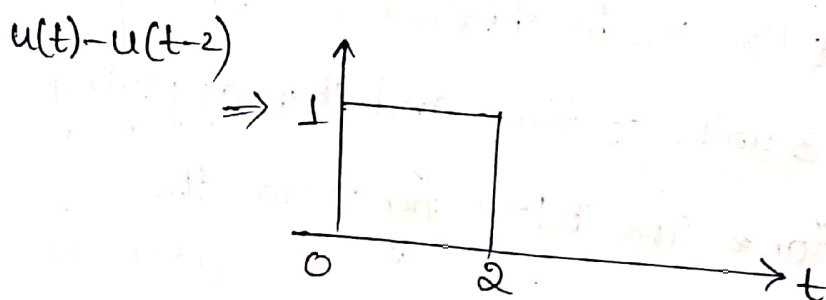
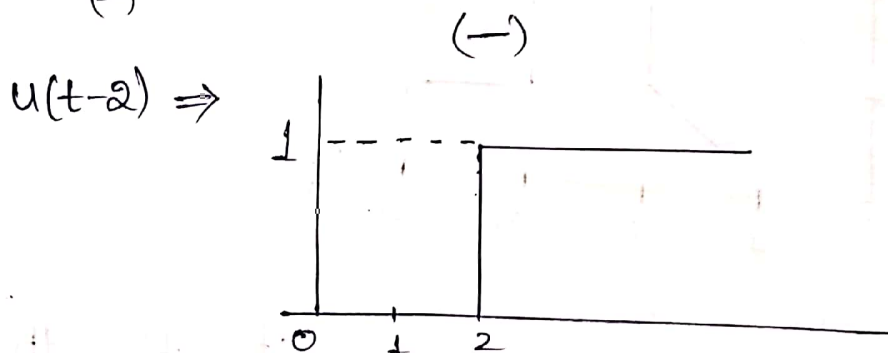
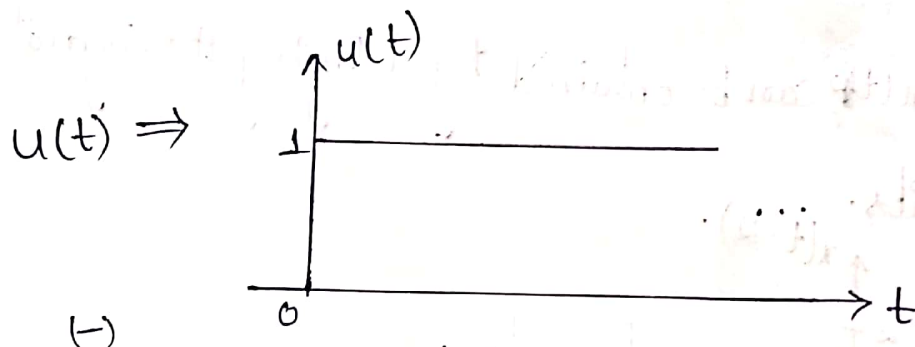
(11) $x(\frac{3}{2}t)$

The signal $x(\frac{3}{2}t)$ can be obtained by compressing the signal $x(t)$ by $\frac{2}{3}$ times.



Sketch the following signals :-

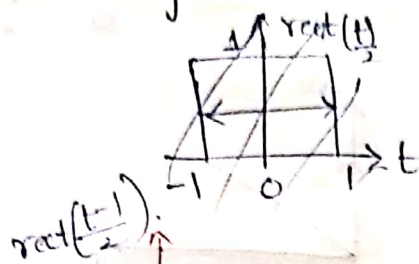
1) $u(t) - u(t-2)$



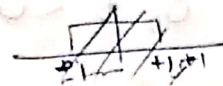
$$(3) \text{rect}\left(\frac{t-1}{2}\right) + \text{rect}(t-1) \quad (\text{or}) \quad \Pi\left(\frac{t-1}{2}\right) + \Pi(t-1)$$

1st term $\Rightarrow \text{rect}\left(\frac{t-1}{2}\right)$

Time period = $\frac{2}{2} = 1$, i.e. t is in b/w $+1$ & -1 on x -axis



but here t is shifting right by 1 unit on x -axis
then t is in b/w 0 & 2 on x -axis

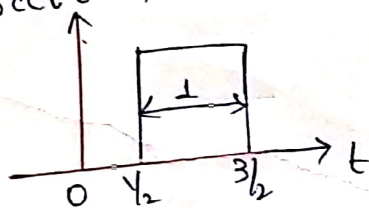


2nd term $\Rightarrow \text{rect}(t-1)$

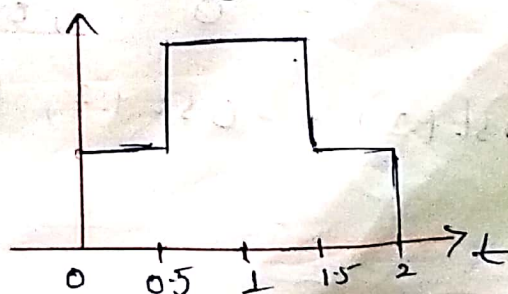
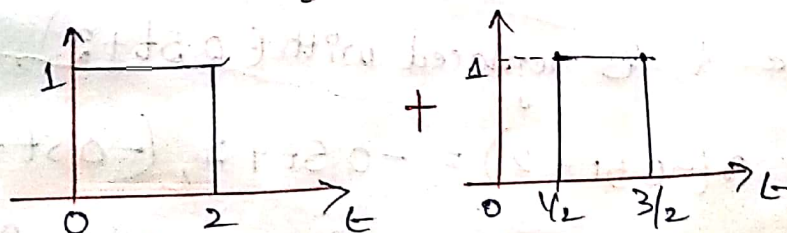
Time period = $\frac{1}{2} \Rightarrow$ on x -axis $+1/2$ & $-1/2$

but here in function we have $t-1$, i.e. t is shifting right by 1 on x -axis

Then on x -axis at $-1/2$ & $+1/2$ it is changing
 $\text{rect}(t-1)$ to $1/2$ & $3/2$.



1st term + 2nd term $\Rightarrow \text{rect}\left(\frac{t-1}{2}\right) + \text{rect}(t-1)$



Q) $r(t) \cdot u(2-t)$

$$u(2-t) = 1, t \leq 2$$

$$2-t \geq 0$$

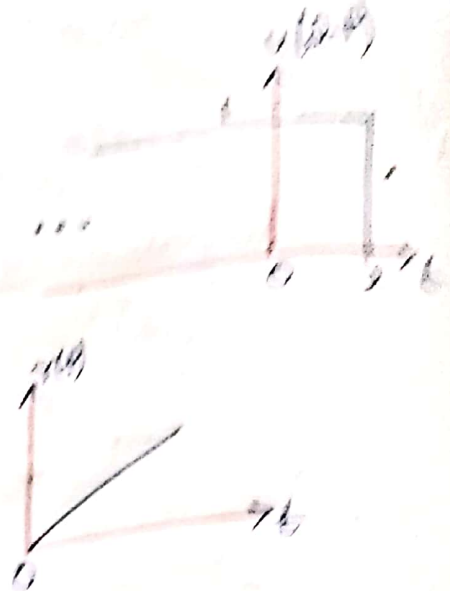
$$t \leq 2$$

$$t \leq 2$$

$$u(2-t) = 0, t > 2$$

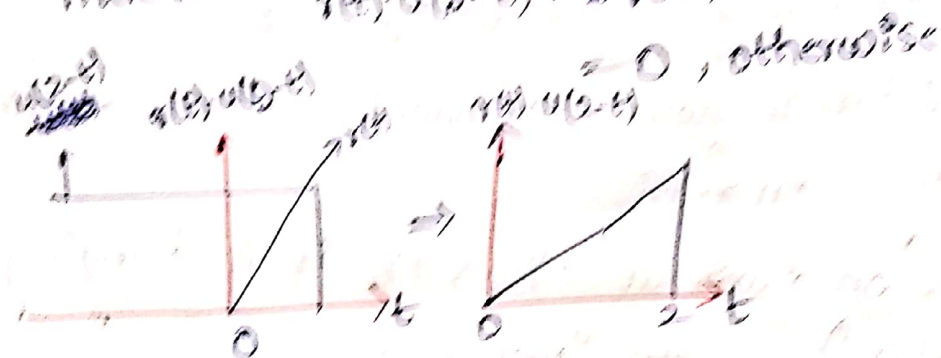
$$r(t) = t, t \geq 0$$

$$= 0, t < 0$$



From the above 3 conditions, the graph will be present (if common in both conditions) are 0s.

Therefore $r(t) \cdot u(2-t) = t, 0 \leq t \leq 2$



Q) $r(-0.5t+2)$

Let $r(t) = r(-0.5t+2)$.

$$r(t) = t, t \geq 0$$

$$= 0, \text{ otherwise}$$

Here 't' is replaced with $(-0.5t+2)$, then

$$r(-0.5t+2) = -0.5t+2, (-0.5t+2) \geq 0$$

$$= 0, \text{ otherwise}$$

$$r(-0.5t+2) = -0.5t+2, (-0.5t+2) \geq 0$$

$$-0.5t \geq -2$$

$$0.5t \leq 2 \Rightarrow \frac{1}{2}t \leq 2$$

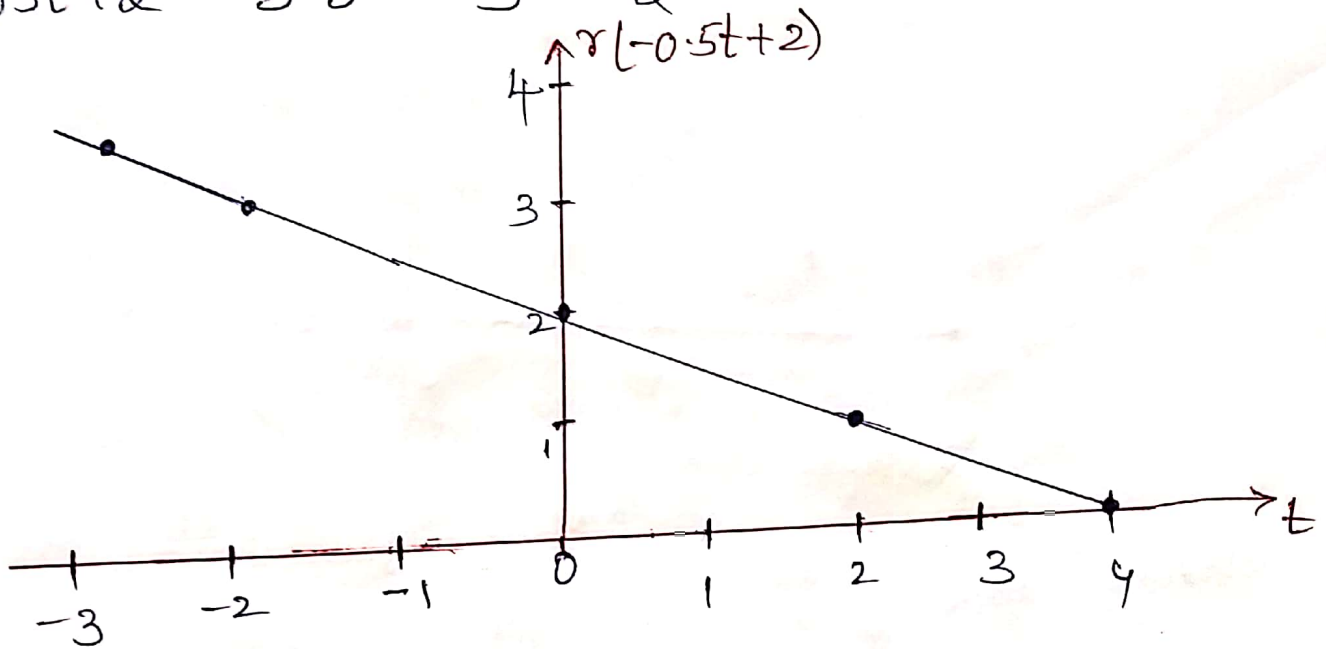
$$t \leq 4$$

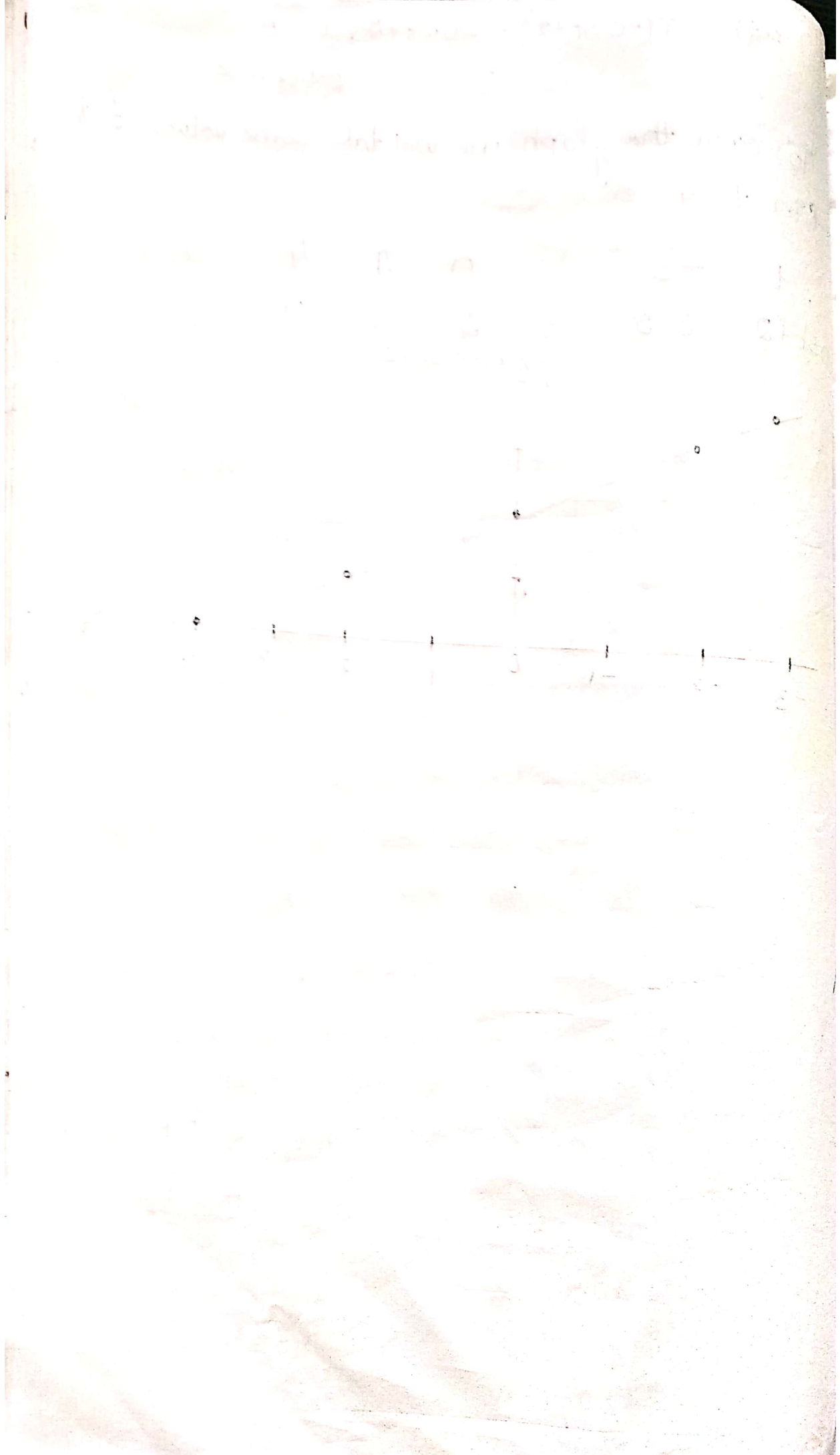
$$r(t) \Rightarrow r(-0.5t+2) = -0.5t+2, t \leq 4$$

$$= 0, \text{ otherwise.}$$

To draw the graph, we will take some values of 't'
 in $t \leq 4$

t	-3	-2	0	2	4
$-0.5t+2$	3.5	3	2	1	0





Classification of Signals :-

The Signals are classified according to their characteristics. Some of them are

- Continuous-time and discrete time Signals
- Deterministic and random Signals
- Periodic and aperiodic Signals
- Even and odd Signals
- Energy and Power Signals
- Causal and non-causal Signals

a) Continuous time and discrete time Signals :-

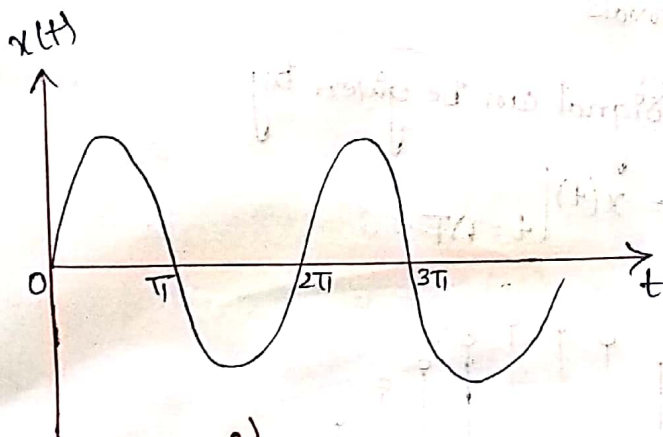
→ Continuous time Signals :-

Continuous time Signals are defined for all values of time 't' and is represented by $x(t)$.

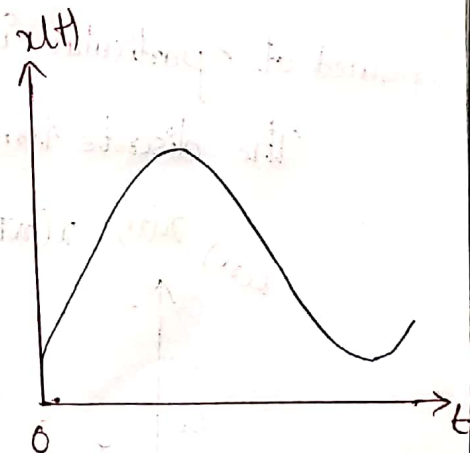
A continuous time Signal is also called analog Signal.

It is defined at every time instant.

The $x(t)$, is called as function.



a)



b)



c)

Ex's of continuous time signals.

→ Discrete Time Signals :-

The discrete-time Signals are defined at discrete instant of time and is represented by $x(n)$, where n is integer.

Some Signals are discrete in nature and Some Signals may be discrete representation of continuous time signal.

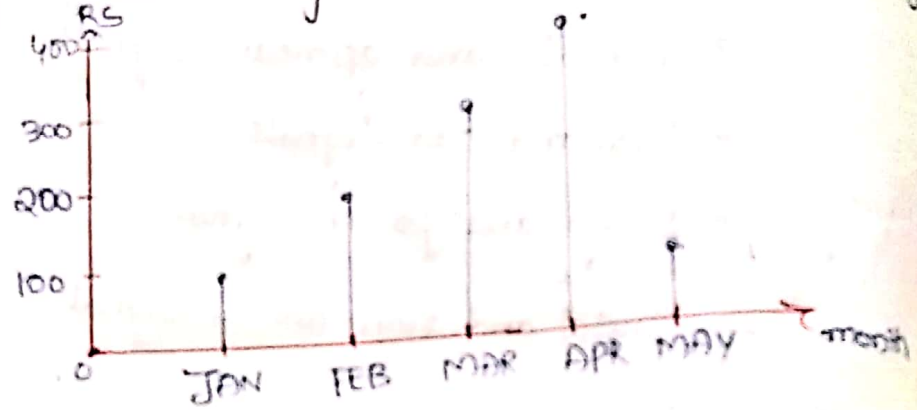


Fig: Discrete time Signal.

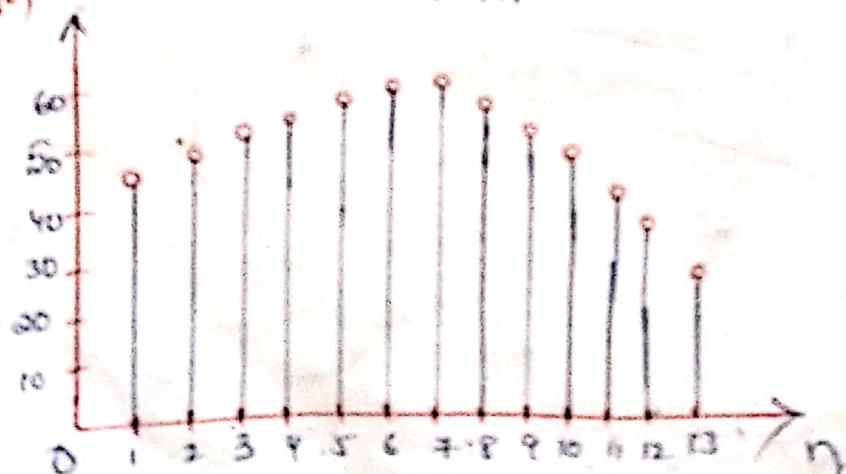
In the above Fig, the amount deposited every month in Savings account is discrete.

The other type of discrete time Signal is obtained by Sampling a continuous time Signal at regular intervals.

That is, the amplitude of the continuous time Signal is defined at particular intervals.

The discrete time Signal can be given by

$$x(n) = x(nT) = x(t) \big|_{t=nT}$$



In the above fig shows the temperature in a room which is continuous in time

$$x(n) = x(nT) = x(t) \Big|_{t=nT}$$

$T \rightarrow$ Sampling period

$n \rightarrow$ Integer called time index

We call $x(nT)$ as the Sampled Signal or Sample Sequence of $x(t)$. The instants at which the signal appears are called Sampling instants.

for convenience, we can write $x(nT) = x(n)$, $n=0, \pm 1, \pm 2, \dots$

Thus a discrete time signal is represented by the sequence numbers $\dots x(-2), x(-1), x(0), x(1), x(2), x(3), \dots$

Q) Sketch the continuous time signal $x(t) = 2 \sin \pi t$ for an interval $0 \leq t \leq 2$. Sample the continuous time signal with a Sampling period $T = 0.2 \text{ sec}$ & sketch the discrete time signal.

Given $x(t) = 2 \sin \pi t$.

To sketch the signal, we have to find the values of $x(t)$ for different values of t , that is.

$$x(0) = 2 \sin \pi(0) = 2 \sin 0 = 2(0) = 0$$

$$x(0.25) = 2 \sin \pi(1/4) = 2 \sin \frac{\pi}{4} = 2 \left(\frac{1}{\sqrt{2}} \right) = \sqrt{2}$$

$$x(0.5) = 2 \sin \pi(1/2) = 2 \sin \frac{\pi}{2} = 2(1) = 2$$

$$x(0.75) = 2 \sin \pi(3/4) = 2 \sin \frac{3\pi}{4} = 2 \left(\frac{1}{\sqrt{2}} \right) = \sqrt{2}$$

$$x(1) = 2 \sin \pi(1) = 2 \sin \pi = 2(0) = 0$$

$$x(1.25) = 2 \sin \pi(5/4) = 2 \sin \frac{5\pi}{4} = 2 \left(-\frac{1}{\sqrt{2}} \right) = -\sqrt{2}$$

$$x(1.5) = 2 \sin \pi(3/2) = 2 \sin \frac{3\pi}{2} = 2(-1) = -2$$

$$x(1.75) = 2 \sin \pi(7/4) = 2 \sin \frac{7\pi}{4} = 2 \left(-\frac{1}{\sqrt{2}} \right) = -\sqrt{2}$$

$$x(2) = 2 \sin \pi(2) = 2 \sin 2\pi = 2(0) = 0$$

Take 't' on x-axis and $x(t)$ on y-axis.

Mark the above points and join all the points by a curve to get a continuous time signal as shown in below fig.

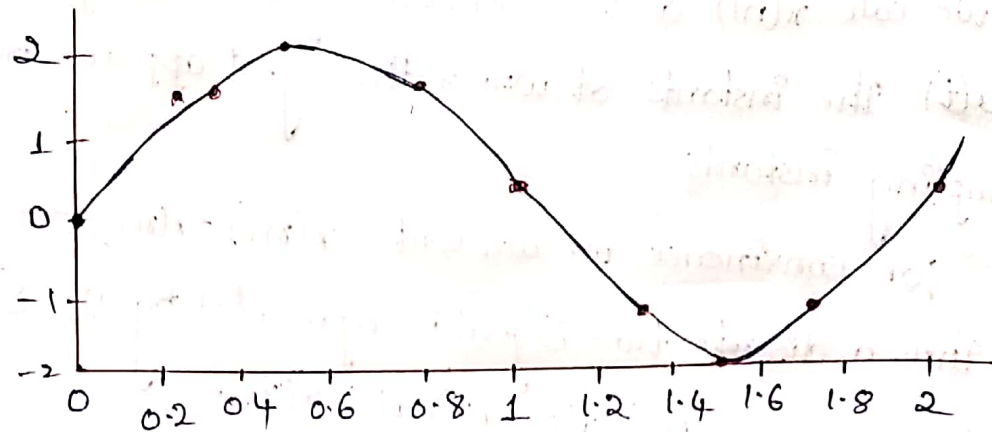


fig: Continuous time signal.

Now,

Given a Sampling period $T = 0.2$ sec.

we know that,

$$x(n) = x(nT) = x(t) \Big|_{t=nT}$$

$$= x(t) \Big|_{t=0.2n}$$

$$= x(0.2n)$$

$$x(n) = 2 \sin \pi(0.2n) = 2 \sin(0.2\pi n)$$

$x(0) = 2 \sin(0.2\pi(0)) = 2 \sin 0 = 0$	$x(5) = 2 \sin(0.2\pi(5)) = 2 \sin \pi = 0$
$x(1) = 2 \sin(0.2\pi) = 1.175$	$x(6) = 2 \sin(0.2\pi(6)) = -1.175$
$x(2) = 2 \sin(0.2\pi(2)) = 1.902$	$x(7) = 2 \sin(0.2\pi(7)) = -1.902$
$x(3) = 2 \sin(0.2\pi(3)) = 1.902$	$x(8) = 2 \sin(0.2\pi(8)) = -1.902$
$x(4) = 2 \sin(0.2\pi(4)) = 1.175$	$x(9) = 2 \sin(0.2\pi(9)) = -1.175$
	$x(10) = 2 \sin(0.2\pi(10)) = 2 \sin 2\pi = 0$

The Sequence $x(n)$ can be written as

$$x(n) = \{0, 1.175, 1.902, 1.902, 1.175, 0, -1.175, -1.902, -1.902, -1.175, 0\}.$$

Take the index 'n' on x-axis and $x(n)$ on y-axis and draw the Sequences as Shown in below fig.

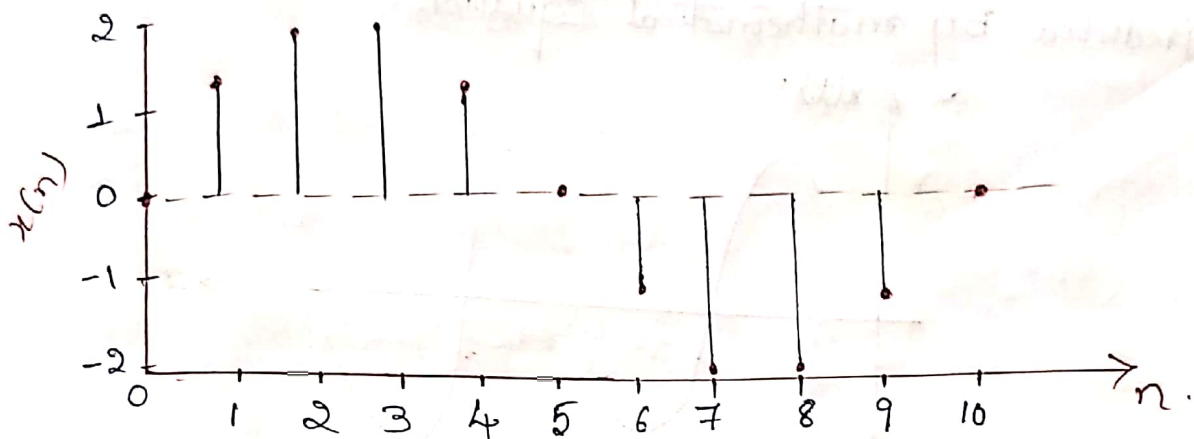


fig. Sampled Signal of $2 \sin \pi t$.

Q) Sketch the Signal $x(t) = e^{-t}$ for an interval $0 \leq t \leq 2$.
Sample the Signal with a Sampling Period $T = 0.2 \text{ sec}$ and sketch the discrete time Signal.

Q) $x(n) = 0.1 \cos\left(\frac{\pi}{2}n + \frac{\pi}{3}\right)$

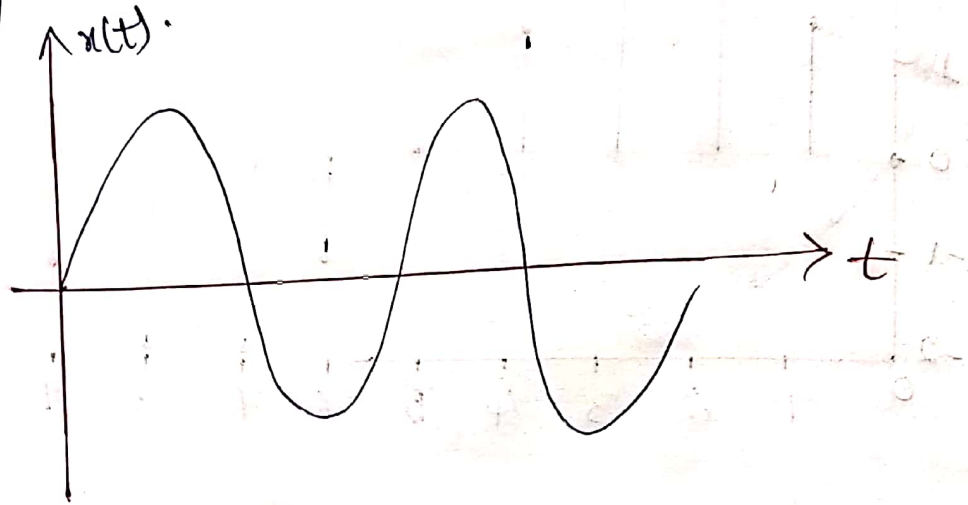
Q) $x(n) = (0.8)^n$, for $0 \leq n \leq 10$
0, otherwise.

b) Deterministic and Random Signals :-

Deterministic Signal :-

A deterministic Signal is a Signal exhibiting no uncertainty of, value at any given instant of time. Its instantaneous value can be accurately

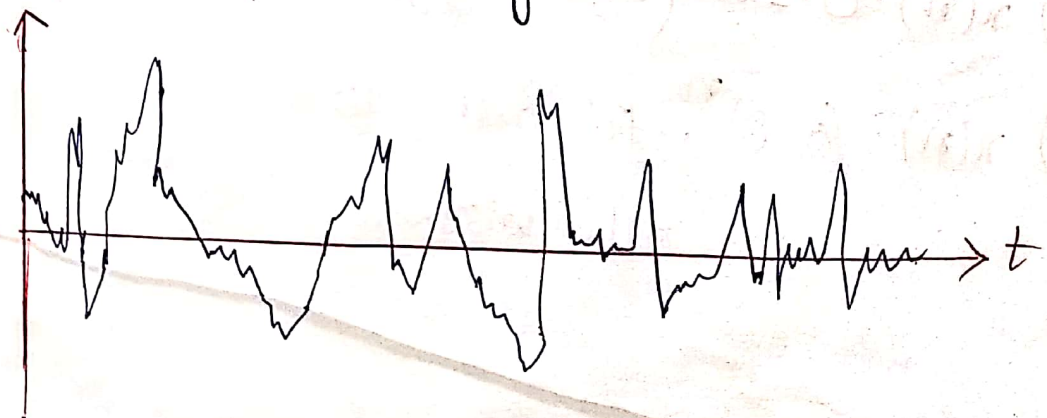
Predicted by mathematical equation.



Eg: $x(t) = A \sin \omega t$

Random Signal :-

A Random Signal is a Signal characterized by uncertainty before its actual occurrence. Some random Signals are shown in below fig.



c) Periodic and aperiodic Signals :

A continuous time Signal $x(t)$ is said to be periodic if it satisfies the condition;

$$x(t+T) = x(t), \text{ for all values of 't'}$$

The Smallest Value of 'T' that satisfies the above condition is called fundamental period.

If a continuous time Signal $x(t)$ doesnot satisfies the above condition, then the Signal is said to be aperiodic.

$$x(t+T) \neq x(t), \text{ for all values of 't'}$$

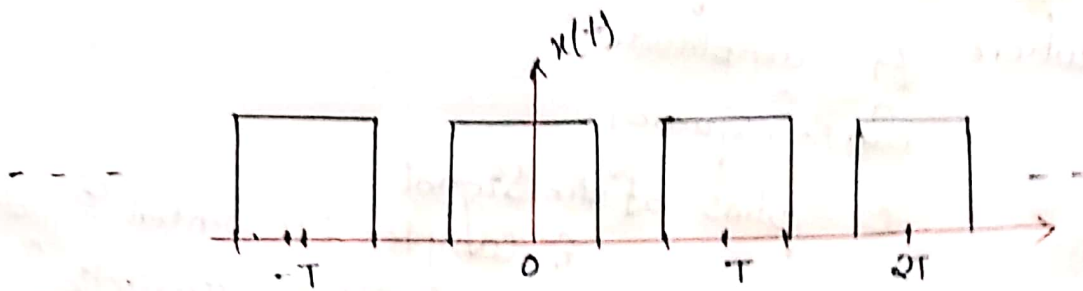


fig:- Periodic Signal

A discrete time Signal $x(n)$ is said to be periodic if it satisfies the condition.

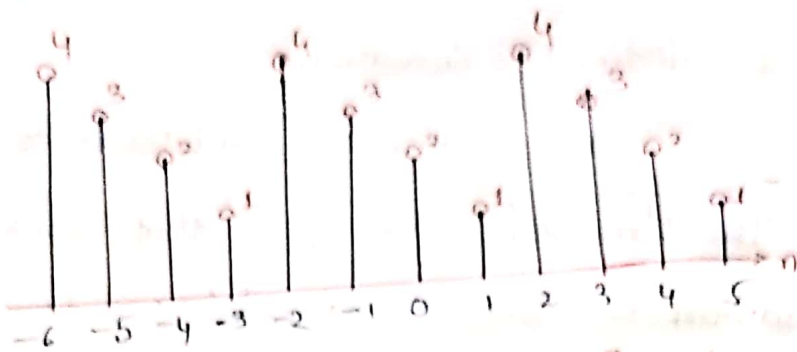
$$x(n) = x(n+N), \text{ for all 'n'}$$

The Smallest Value of 'N' for which above Eqn has is called fundamental period.

If a discrete time Signal $x(n)$ doesnot satisfies the above condition, then the Signal is said to be aperiodic

$$x(n) \neq x(n+N), \text{ for all values of 'n'}$$

An Example for Periodic ~~Signal~~ discrete-time Signal is shown below.



by: A discrete-time periodic Signal

Here, the period of the Signal $N=4$.

Consider a Signal $x(t) = A \sin(\Omega_0 t + \theta)$,

where A = amplitude.

Ω_0 = frequency.

θ = phase of the Signal.

& complex Exponential Signals

All Sinusoidal Signals are continuous periodic Signals.

So, the condition for periodic Signals are

$$x(t) = x(t+T).$$

For, $x(t) = A \sin(\Omega_0 t + \theta)$.

$$x(t+T) = A \sin(\Omega_0(t+T) + \theta)$$

$$= A \sin[\Omega_0 t + \Omega_0 T + \theta].$$

$$x(t) = A \sin[\Omega_0 t + 2\pi + \theta] \quad (\because \sin(2\pi + \theta) = \sin \theta)$$

So, by comparing $x(t)$ & $x(t+T)$.

$$\Omega_0 T = 2\pi$$

$$T = \frac{2\pi}{\Omega_0}$$

The term 'T' is known as Time period 'T' & Ω_0 is

Similarly, we consider

$$x(t) = e^{j\Omega_0 t}$$

for a periodic Signal,

$$x(t+T) = e^{j\Omega_0(t+T)}$$

$$\text{as } x(t) = x(t+T),$$

Eqn's we get

$$e^{j\Omega_0 t} = e^{j\Omega_0(t+T)}$$

$$e^{j\Omega_0 t} = e^{j\Omega_0 t} e^{j\Omega_0 T}$$

The Sum of the

Periods T_1 & T_2 must

on the relation between

If the Sum

Periods $T = T_1/T_2$ must

integers. Otherwise

For eg, let

$$T_1 = 5 \text{ \& } T_2 = 7.$$

then the ratio

rational number

The term 'T' is known as fundamental period or Time period 'T' & ω_0 is referred as fundamental frequency. Similarly, we consider for complex exponential signal.

$$x(t) = e^{j\omega_0 t} \quad (1)$$

for a periodic signal, $x(t) = x(t+T)$

$$x(t+T) = e^{j\omega_0(t+T)}$$

$$= e^{j\omega_0 t + j\omega_0 T} = e^{j\omega_0 t} \cdot e^{j\omega_0 T}$$

as $x(t) = x(t+T)$, for periodic signal, by comparing Eqn's we get

$$e^{j\omega_0 T} = 1$$

$$e^{j\omega_0 T} = e^{j2\pi}$$

$$\omega_0 T = 2\pi \Rightarrow T = \frac{2\pi}{\omega_0}$$

The Sum of two periodic signals $x_1(t)$ & $x_2(t)$ with periods T_1 & T_2 may or may not be periodic depending on the relation between T_1 & T_2 .

If the sum to be periodic, then the ratio of periods $T = T_1/T_2$ must be a rational number or ratio of two integers. Otherwise, the sum is not periodic.

For eg, let us take 2 signals with time periods,

$$T_1 = 5 \text{ \& } T_2 = 7$$

Then the ratio of two periods $= \frac{T_1}{T_2} = \frac{5}{7}$, is a rational number.

Therefore, the addition of two signals is periodic and the period is given by $T = 7T_1 = 5T_2$.

on the other hand, if the time periods $\frac{T_1}{T_2} = \frac{3}{2}$ is not a rational number.

Therefore, the Sum of two signals is not periodic.

Find the fundamental period 'T' of the following continuous time signals & find if they are periodic.

1) $x_1(t) = 4 \cos 5\pi t$

2) $x_2(t) = \sin 10\pi t \cdot u(t)$

3) $x_3(t) = j e^{5j t}$

4) $x_4(t) = e^{-|t|}$

5) $x_5(t) = 20 \cos\left(10\pi t + \frac{\pi}{6}\right)$

① $x_1(t) = 4 \cos 5\pi t$

From above equation, $\Omega_0 = 5\pi$. ($\therefore$ by comparing equation $\cos \Omega_0 t$).

Fundamental time period $T = \frac{2\pi}{\Omega_0} = \frac{2\pi}{5\pi} = \frac{2}{5} = 0.4 \text{ sec}$.

② $x_2(t) = \sin 10\pi t \cdot u(t)$

In this equation, we ~~can find~~ cannot find the Ω_0 value. So, this is a aperiodic signal.

③ $x_3(t) = e^{-|t|}$

~~From the above equation, $\Omega_0 = 5\pi$~~

Here, we are comparing eqn with $e^{j\Omega_0 t}$

So, by comparing eqn, the given $x_3(t)$ is not in the form of $e^{j\omega_0 t}$.

So, it is aperiodic signal.

⑤ $x_5(t) = 20 \cos(10\pi t + \frac{\pi}{6})$.

Here, we are comparing the given equation $x_5(t)$ with $\cos \omega_0 t$.

So, by comparing, we get $\omega_0 = 10\pi$.

Fundamental time period $T = \frac{2\pi}{\omega_0} = \frac{2\pi}{10\pi} = \frac{1}{5} = 0.2 \text{ sec}$.

This is a rational number. So, the given function $x_5(t)$ is periodic.

⑥ $x(t) = u(t) - u(t-10)$.

As we know that complex exponential signals and sinusoidal signals are the examples of periodic signals.

Here, the given equation $x(t)$ is not in the form of either complex exponential signals or sinusoidal signals.

So, we cannot find the fundamental time period.

This is not a periodic signal.

⑦ $x(t) = 2 \cos(10t + 1) - \sin(4t - 1)$.

Here, we will take individual term

1st term $\Rightarrow 2 \cos(10t + 1)$ & 2nd term $\Rightarrow \sin(4t - 1)$

Here $\omega_0 = 10$

Here $\omega_0 = 4$.

Fundamental of 1st term $\Rightarrow T_1 = \frac{2\pi}{\omega_0} = \frac{2\pi}{10} = \frac{\pi}{5}$

$T_2 = \frac{2\pi}{\omega_0} = \frac{2\pi}{4} = \frac{\pi}{2}$

Total Fundamental period $T = \frac{T_1}{T_2} = \frac{\frac{\pi}{5}}{\frac{\pi}{2}} \Rightarrow \frac{\pi}{5} \times \frac{2}{\pi} = \frac{2}{5}$

Here $\frac{2}{5}$ is rational number.

So, the given $x(t)$ is periodic signal.

$$T = \frac{T_1}{T_2} = \frac{2}{5}$$

$$T = 5T_1 = 2T_2$$

$$T = 5T_1 = 5 \times \frac{T_1}{5} = T_1 \text{ sec (or) } T = 2T_2 = 2 \times \frac{T_2}{2} = T_2 \text{ sec}$$

$\therefore$ The total fundamental period $T = T_1 \text{ sec}$.

8. Find the Fundamental period T for the following Continuous time Signals & find whether the following Signals are periodic or not.

Q) $2u(t) + 2\sin 2t$

Q) $3\cos 4t + 2\sin 2\pi t$

Q) $3\cos 4t + 2\sin 2\pi t$

Q) $3\cos\left(17\pi t + \frac{\pi}{3}\right) + 2\sin\left(19\pi t - \frac{\pi}{3}\right)$

Q) $x(t) = \sin^2 t$

$$= \frac{1 - \cos 2t}{2}$$

The period of the Signal $T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi$ sec.

The Signal is Periodic.

Note:- (important).

In periodic continuous time Signals, the time period should not be in the ratio of ' π '. only it should be in the multiple of ' π '.

Eg: If $T = \frac{\pi}{4}$, then the Signal is aperiodic.

If $T = 2\pi$ (or) 3π (or) $\dots$, then the Signal is periodic.

Q) $12e^{j(4t + \frac{\pi}{2})} + 9e^{j(3\pi t + \frac{\pi}{3})}$

Here also, we will take individual terms to find the total time period

1st term $\Rightarrow \cos\left(\frac{1}{2}\right) 12e^{j(4t + \frac{\pi}{2})} \Rightarrow 12e^{j4t + \frac{\pi}{2}}$

Here $\omega = 4$ then $T_1 = \frac{2\pi}{\omega} = \frac{2\pi}{4} = \frac{\pi}{2}$

$$2^{\text{nd}} \text{ term} \Rightarrow q e^{j(3\pi t + \frac{\pi}{3})}$$

$$\Rightarrow q e^{(j3\pi t + \frac{\pi}{3}j)}, \text{ Here } \Omega = 3\pi, \text{ then } T_2 = \frac{2\pi}{\Omega} = \frac{2\pi}{3\pi}$$

$$\therefore T_2 = 2/3$$

$$\therefore \text{The fundamental time period} = T = \frac{T_1}{T_2} = \frac{\frac{\pi}{2}}{2/3} = \frac{\pi}{2} \times \frac{3}{2}$$

$$T = \frac{3\pi}{4}$$

Here, the fundamental time period, $T = \frac{3\pi}{4}$ is not a rational number.

So, the signal is Aperiodic.

Condition for discrete-time Sinusoidal Signal to be Periodic:-

In case of continuous time signals, all sinusoidal signals are periodic. But in discrete time signals, not all the sinusoidal sequences are periodic. Consider a discrete-time signal given by

$$x(n) = A \sin(\omega_0 n + \theta) \quad \text{--- (1)}$$

A discrete time signal is periodic if and only if

$$x(n+N) = x(n), \text{ for all } n.$$

Sub $x(n+N)$ then

$$x(n+N) = A \sin(\omega_0(n+N) + \theta)$$

$$= A \sin(\omega_0 n + \omega_0 N + \theta) \quad \text{--- (2)}$$

from eq (1) & (2), $\omega_0 N = 2\pi m$

that is there must be an integer m such that

$$\omega_0 = 2\pi \frac{m}{N}$$

$$N = 2\pi \left(\frac{m}{\omega_0} \right)$$

From this Eqn, we find that for the discrete time signal to be periodic, the fundamental frequency ω_0 , must be a rational multiple of 2π . Otherwise, the discrete time signal is aperiodic.

The Smallest value of positive integer N , for some integer m , is the fundamental period.

Find whether the following are periodic or not?

Q) $\cos(0.1\pi n)$

Here $\omega_0 = 0.1\pi = \omega_0$

the fundamental period $N = \frac{2\pi m}{\omega_0} = \frac{2\pi m}{0.1\pi} = 20 \times 10 = 20$

N takes Smallest positive integer value, when $m=1$.

$\therefore N = 20(1) = 20$ & the sequence is periodic.

Q) $\frac{3}{5} e^{j3\pi(n + \frac{1}{2})}$

$$= \frac{3}{5} e^{(j3\pi n + j\frac{3\pi}{2})}$$

Here $\omega_0 = 3\pi$, by comparing eqn with $e^{j\omega_0 n}$.

Fundamental period $N = \frac{2\pi m}{\omega_0} = \frac{2\pi \times m}{3\pi} = \frac{2}{3} m$

N takes Smallest positive integer value when $m=3$.

$N = \frac{2}{3} \times 3 = 2$; the signal is periodic.

Q) $\cos(\frac{1}{4}n)$

$\Omega_0 = \frac{1}{4}$

But in discrete time signals, Ω_0 must be multiple of 2π .
But here Ω_0 is not multiple of 2π .

So, the given signal is aperiodic.

Q) $e^{j(\frac{2\pi}{3}n)} + e^{j(\frac{3\pi}{4}n)}$

Here, we will consider two individual terms,

1st term $\Rightarrow e^{j(\frac{2\pi}{3}n)}$

2nd term $\Rightarrow e^{j(\frac{3\pi}{4}n)}$

$\Omega_{01} = \frac{2\pi}{3}$

$\Omega_{02} = \frac{3\pi}{4} \Rightarrow 2\pi \times \frac{3}{8}$

$N_1 = \frac{2\pi m}{\Omega_{01}} = \frac{2\pi m}{2\pi/3} = 3m$

$N_2 = \frac{2\pi m}{\Omega_{02}} = \frac{2\pi m}{3\pi/4} = 2 \times \frac{4}{3} m$

Here $m=3$, $N_1=3$

Here $m=3$, then $N_2=8 = 8/3$

Fundamental period $N = \frac{N_1}{N_2} = \frac{3}{8/3} = \frac{9}{8}$

$N = \frac{N_1}{N_2} = \frac{3}{8}$

$N = 8N_1 = 3N_2$

$N = 8N_1$ or $N = 3N_2$

$= 8 \times 3$
 $= 24$

$= 3 \times 8$
 $= 24$

$N=24$, the signal is periodic

Q) $12 \cos(20n)$

Q) $e^{j6\pi n}$

Q) $\sin(\frac{6\pi}{7}n+1)$

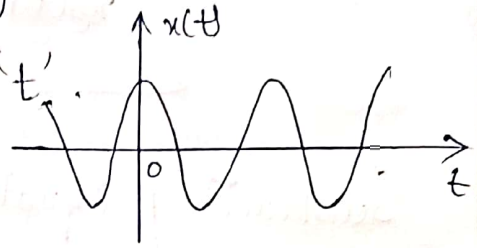
Q) $\sin 2\pi n + \sin 6\pi n$

Symmetric (Even) and Anti-Symmetric (Odd) Signals :-

Symmetric (Even Signals) :- A continuous time signal $x(t)$ is said to be Symmetric (even) signal if it satisfies the condition

$$x(-t) = x(t) \text{ for all } 't'.$$

→ (1)



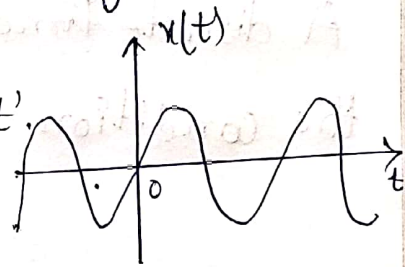
Eg of a Symmetric signal is

$$x(t) = A \cos t \text{ as shown in fig.}$$

Anti-Symmetric (Odd Signals) :- A continuous time signal $x(t)$ is said to be anti-Symmetric (odd) signal if it satisfies the condition

$$x(-t) = -x(t) \text{ for all } 't'.$$

→ (2)



Eg of anti-Symmetric signal is

$$x(t) = A \sin t$$

$$x(-t) = A \sin(-t) \Rightarrow -A \sin t \Rightarrow -x(t)$$

$$x(-t) = -x(t)$$

Any signal $x(t)$ can be expressed as Sum of Even and odd components. That is given by

$$x(t) = x_e(t) + x_o(t) \rightarrow (3)$$

Where $x_e(t)$ is Even component of signal and $x_o(t)$ is odd component of signal.

Replacing 't' by '-t' in eq (3)

$$x(-t) = x_e(-t) + x_o(-t) \rightarrow (4)$$

= $x_e(t) - x_o(t)$ ($\because$ by eq'n's (1) & (2))

Adding Equations (3) & (4).

$$2x_e(t) = x(t) + x(-t)$$

$$x_e(t) = \frac{1}{2} [x(t) + x(-t)] \rightarrow (5)$$

Subtracting Equations (3) & (4).

$$2x_o(t) = x(t) - x(-t)$$

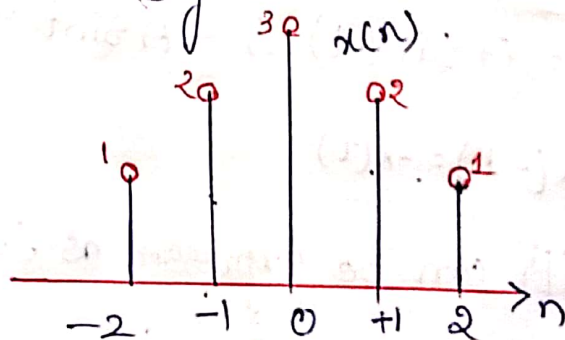
$$x_o(t) = \frac{1}{2} [x(t) - x(-t)] \rightarrow (6)$$

A discrete time Signal $x(n)$ is Even, if it satisfies the condition

$$x(n) = x(-n) \text{ for all 'n'}$$

$\rightarrow (7)$

An eq of Even Signal is shown in fig below.



Here, $x(1) = 2 = x(-1)$

$$x(2) = 1 = x(-2)$$

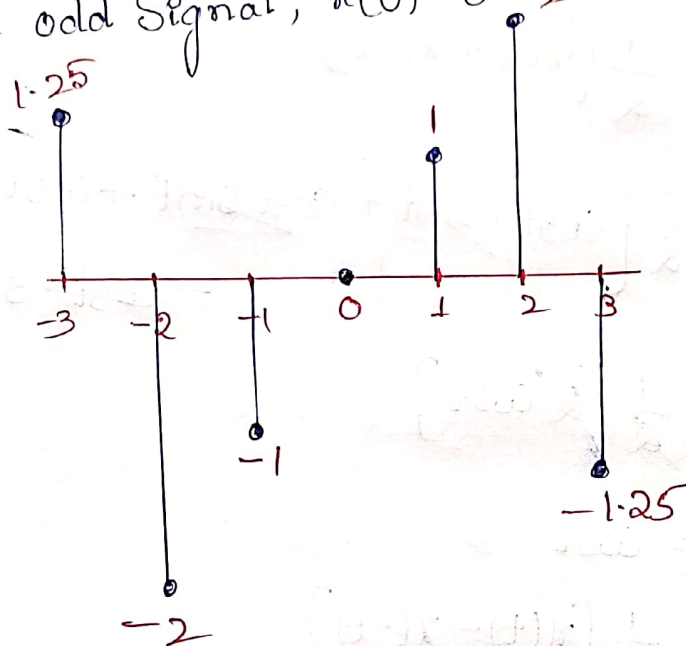
∴ The given Signal satisfies the Even Signal Condition.

The Signal $x(n)$ is Said to be an odd Signal if

$$x(-n) = -x(n) \text{ for all 'n'} \rightarrow (8)$$

An Eg of odd Signal is Shown in fig below

For an odd Signal, $x(0) = 0$.



Here

$$x(1) \Rightarrow 1 \Rightarrow -x(1)$$

$$x(2) \Rightarrow 2 \Rightarrow -x(2)$$

$$x(3) \Rightarrow -1.25 \Rightarrow -x(3)$$

$\therefore$ The above Signal satisfies the condition of odd Signal

$\therefore$ The even and odd parts of a Signal $x(n)$ can be obtained using the following conditions.

$$x_e(n) = \frac{1}{2} [x(n) + x(-n)]$$

$$x_o(n) = \frac{1}{2} [x(n) - x(-n)]$$

Find the even and odd components of the following Signals

i) $x(t) = \sin t + \cos t + \sin t \cos t$

Given $x(t) = \cos t + \sin t + \cos t \cdot \sin t$

$$x(-t) = \cos(-t) + \sin(-t) + \cos(-t) \cdot \sin(-t)$$

$$= \cos t - \sin t - \cos t \cdot \sin t$$

Now

$$x_e(t) = \frac{1}{2} (x(t) + x(-t))$$

$$= \frac{1}{2} (\cancel{\cos t + \sin t} + \cancel{\cos t \cdot \sin t} + \cos t - \sin t - \cancel{\cos t \cdot \sin t})$$

$$= \frac{1}{2} (2 \cos t)$$

$$= \cos t$$

$$x_o(t) = \frac{1}{2} (x(t) - x(-t))$$

$$= \frac{1}{2} (\cancel{\cos t + \sin t} + \cancel{\cos t \cdot \sin t} - \cancel{\cos t + \sin t} + \cancel{\cos t \cdot \sin t})$$

$$= \frac{1}{2} (2 (\sin t + \cos t \cdot \sin t))$$

$$x_o(t) = \sin t + \cos t \cdot \sin t$$

Q) $x(n) = \{1, 0, -1, 2, 3\}$

$n = -2, -1, \underset{\uparrow}{0}, 1, 2$

$$x_e(n) = \frac{1}{2} (x(n) + x(-n))$$

$$x_e(0) = \frac{1}{2} (x(0) + x(-0)) = \frac{1}{2} (x(0) + x(0))$$

$$= \frac{1}{2} (2x(0)) = 1$$

$$x_e(1) = \frac{1}{2} (x(1) + x(-1)) = \frac{1}{2} (x(1) + x(-1)) = \frac{1}{2} (2 + 0) = 1$$

$$x_e(2) = \frac{1}{2} [x(2) + x(-2)] = \frac{1}{2} [3 + 1] = \frac{4}{2} = 2$$

$$x_e(-1) = \frac{1}{2} [x(-1) + x(-(-1))] = \frac{1}{2} [x(-1) + x(1)] = \frac{1}{2} [-1 + 1] = 0$$

$$x_e(-2) = \frac{1}{2} [x(-2) + x(-(-2))] = \frac{1}{2} [x(-2) + x(2)] = \frac{1}{2} [1 + 3] = \frac{4}{2} = 2$$

$$x_e(n) = \{2, 0, -1, 0, 2\}.$$

$$x_o(n) = \frac{1}{2} [x(n) - x(-n)]$$

$$x_o(-2) = \frac{1}{2} [x(-2) - x(2)] = \frac{1}{2} [1 - 3] = \frac{-2}{2} = -1$$

$$x_o(-1) = \frac{1}{2} [x(-1) - x(1)] = \frac{1}{2} [0 - 2] = \frac{-2}{2} = -1$$

$$x_o(0) = \frac{1}{2} [x(0) - x(0)] = 0$$

$$x_o(1) = \frac{1}{2} [x(1) - x(-1)] = \frac{1}{2} [2 - 0] = \frac{2}{2} = 1$$

$$x_o(2) = \frac{1}{2} [x(2) - x(-2)] = \frac{1}{2} [3 - 1] = \frac{2}{2} = 1$$

$$\therefore x_o(n) = \{-1, -1, 0, 1, 1\}.$$

Find the Even and odd components of the following signals.

$$x(n) = \{-2, 1, 2, -1, 3\}$$

$$x(t) = 4 \sin t + 4 \sin t \cos^2 t$$

Energy and Power Signals :-

Consider a voltage $v(t)$ across a resistance ' R ' producing a current $i(t)$. The instantaneous power dissipated in the resistance R is defined by.

$$P(t) = v(t) \cdot i(t).$$

$$= v(t) \cdot \frac{v(t)}{R} = \frac{v^2(t)}{R} \rightarrow (1)$$

(Or)

$$P(t) = i^2(t) \cdot R \rightarrow (2)$$

For a resistance of 1Ω , the above 2 eqn's shows that the instantaneous power $p(t)$ is equal to the square of the amplitude of the signal.

Integrating eq (2) over the interval $|t| \leq T$, we can express the total energy and average power of a signal as.

$$E = \lim_{T \rightarrow \infty} \int_{-T}^T i^2(t) \cdot dt \text{ joules} \rightarrow (3)$$

$$P = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T i^2(t) \cdot dt \text{ watts} \rightarrow (4)$$

For any signal $x(t)$, the total energy is defined as

$$E = \lim_{T \rightarrow \infty} \int_{-T}^T x^2(t) \cdot dt \text{ joules}$$

and the average power is defined as.

$$P = T \xrightarrow{Lt} \frac{1}{2T} \int_{-T}^T |x(t)|^2 dt \text{ watts.}$$

The Square root of P is called as r.m.s. value of Signal.
 $x(t)$.

For a discrete time Signal, $x(n)$ the Energy E is defined as

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2.$$

The average power of a discrete time Signal $x(n)$ is defined as

$$P = N \xrightarrow{Lt} \frac{1}{2N+1} \sum_{n=-N}^N |x(n)|^2.$$

Definitions of power and Energy Signals :-
For as

A Signal is an Energy Signal if and only if the total energy of the Signal is finite ($0 < E < \infty$), & for an energy Signal $P = 0$.

A Signal is said to be Power Signal if the average Power of the Signal is finite ($0 < P < \infty$) & for a Power Signal $E = \infty$.

Determine the power and R.M.S value of the Signal

$$x(t) = A \cos(\omega_0 t + \theta).$$

$$\text{Given } x(t) = A \cos(\omega_0 t + \theta).$$

The power

$$P = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T |x(t)|^2 dt$$

$$= \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T [A \cos(\omega_0 t + \theta)]^2 dt$$

$$\cos^2 \theta = \frac{1 + \cos 2\theta}{2} \quad (\because \text{by formula}).$$

$$= \lim_{T \rightarrow \infty} \frac{A^2}{2T} \int_{-T}^T \left[\frac{1 + \cos 2(\omega_0 t + \theta)}{2} \right] dt$$

$$= \lim_{T \rightarrow \infty} \frac{A^2}{2T} \int_{-T}^T 1 dt + \lim_{T \rightarrow \infty} \frac{A^2}{4T} \int_{-T}^T \cos 2(\omega_0 t + \theta) dt$$

Here $\int_{-T}^T \cos 2(\omega_0 t + \theta) dt$

$$\text{Let } x = (\omega_0 t + \theta)$$
$$\int_{-T}^T \cos 2x dx = \left[\frac{\sin 2x}{2} \right]_{-T}^T$$

$$= \frac{1}{2} [\sin 2T + \sin 2(-T)]$$

$$= \frac{1}{2} [\cancel{\sin 2T} - \cancel{\sin 2T}]$$

$$= \frac{1}{2} (0) = 0$$

$$\therefore \int_{-T}^T \cos 2(\omega_0 t + \theta) dt = 0$$

$$= \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T}^T \frac{A^2}{4T} dt + \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T}^T \frac{A^2}{4T} dt$$

$$= \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T}^T \frac{A^2}{4T} dt$$

$$= \lim_{T \rightarrow \infty} \frac{1}{T} \cdot \frac{A^2}{4T} \cdot 2T$$

$$P = \frac{A^2}{2}$$

Here 'A' is amplitude of given $x(t)$.
 When ever the given Signal $x(t)$ is in the form of $\sin$ or $\cos \omega t$, then to find the power of those Signal ~~$x(t)$~~ $P = \frac{(\text{amplitude of given signal})^2}{2} = \frac{A^2}{2}$.

The R.M.S value of Signal $= \sqrt{P} = \sqrt{\frac{A^2}{2}} = \frac{A}{\sqrt{2}}$.

Determine the power and R.M.S value of Signal.

$$x(t) = A \cdot e^{j\omega t}$$

$$\text{Given Signal } x(t) = A \cdot e^{j\omega t}$$

$$\text{The power of the Signal } P = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T |x(t)|^2 dt$$

$$= \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T |A \cdot e^{j\omega t}|^2 dt$$

$$= \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T A^2 \cdot e^{2j\omega t} dt$$

$$= \lim_{T \rightarrow \infty} \frac{1}{2T} \cdot \frac{A^2}{2j\omega} [e^{2j\omega t}]_{-T}^T$$

$$|e^{j2\omega_0 t}| = \sqrt{\cos^2(2\omega_0 t) + \sin^2(2\omega_0 t)} = \sqrt{1} = 1.$$

$$\therefore \text{by } (e^{j0}) = \sqrt{\cos^2 0 + \sin^2 0} = \sqrt{1} = 1$$

$$= \lim_{T \rightarrow \infty} \frac{A^2}{2T} \int_{-T}^T 1 \cdot dt$$

$$= \lim_{T \rightarrow \infty} \frac{A^2}{2T} [t]_{-T}^T = \lim_{T \rightarrow \infty} \frac{A^2}{2T} (T - (-T))$$

$$= \lim_{T \rightarrow \infty} \frac{A^2}{2T} (2T)$$

$$P = A^2 = (\text{Amp of the signal})^2$$

$$\therefore \text{R.M.S value of the Signal} = \sqrt{P} = \sqrt{A^2} = A$$

If the Signal is in the form of complex exponential Signal $Ae^{j\omega_0 t}$, then the power of the Signal = $(\text{Amp}^2 \text{ of signal})^2$.

$$\text{R.M.S value of Signal} = \sqrt{\text{Power}} = \sqrt{\text{Amp}^2} = \text{Amp} = A$$

Determine the power and R.M.S value of the following Signals

$$1) x_1(t) = 5 \cos\left(50t + \frac{\pi}{3}\right)$$

$$2) x_2(t) = 10 \sin\left(50t + \frac{\pi}{4}\right) + 16 \cos\left(100t + \frac{\pi}{3}\right)$$

$$111) x_3(t) = 10 \cos 5t \cos 10t$$

$$112) x_4(t) = e^{j\alpha t} \cos \beta_0 t$$

$$111) x_3(t) = 10 \cos 5t \cos 10t = \frac{10}{2} \cdot 2 \cos 5t \cos 10t$$

$$= 5 \times (2 \cos 5t \cos 10t)$$

$$= 5 \left[\right.$$

$$2 \cos A \cos B = \cos(A+B) + \cos(A-B)$$

$$= 5 [\cos(5t+10t) + \cos(5t-10t)]$$

$$= 5 [\cos(15t) + \cos(5t)]$$

$$= 5 \cos 15t + 5 \cos 5t$$

$\therefore$ The signal is in the form of $A \cos \beta_0 t$ / $A \sin \beta_0 t$
then power of signal = $\frac{A^2}{2}$

$$\therefore \text{The power of the signal} = \frac{A^2}{2} + \frac{A^2}{2}$$

$$= \frac{5^2}{2} + \frac{5^2}{2} = \frac{25+25}{2} = \frac{50}{2}$$

$$= 25W$$

$$\therefore \text{R.M.S value} = \sqrt{P} = \sqrt{25} = 5$$

$$112) x_4(t) = e^{j\alpha t} \cos \beta_0 t$$

$$\text{Power of signal} = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T |e^{j\alpha t} \cos \beta_0 t|^2 dt$$

$$= \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T |e^{2j\alpha t}|^2 |\cos^2 \beta_0 t| dt$$

$$|e^{2j\theta}| = \sqrt{\cos^2 2\theta + \sin^2 2\theta} = \sqrt{1} = 1$$

$$= T \xrightarrow{\infty} \frac{1}{2T} \int_{-T}^T (1) |\cos^2 \omega_0 t| dt$$

$$= T \xrightarrow{\infty} \frac{1}{2T} \int_{-T}^T \frac{1 + \cos 2\omega_0 t}{2} dt$$

$$= T \xrightarrow{\infty} \frac{1}{2T} \int_{-T}^T dt + \cancel{\frac{1}{4T} \int_{-T}^T \cos 2\omega_0 t dt}$$

$$= T \xrightarrow{\infty} \frac{1}{4T} [t]_{-T}^T + 0$$

$$= T \xrightarrow{\infty} \frac{1}{4T} (T - (-T))$$

$$= T \xrightarrow{\infty} \frac{1}{4T} (2T) = \frac{1}{2} = 0.5$$

$$P = 0.5W.$$

R.M.S value of given Signal = $\sqrt{P} = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$

Sketch the following Signals and calculate their Energies.

1) $e^{-10t} \cdot u(t)$.

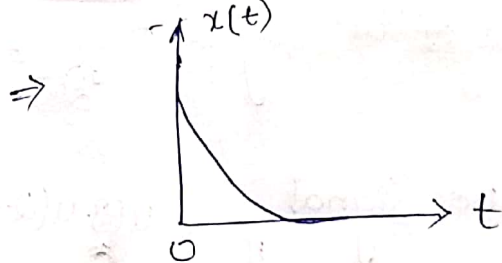
2) $u(t) - u(t-15)$.

3) $\cos(2\pi t) \cdot u(t) \cdot u(2-t)$.

1) $e^{-10t} \cdot u(t) = x(t) \Rightarrow$ Here the cond of $u(t) = 1, t \geq 0$
 $= 0, t < 0$.

$$\begin{aligned} t=0 & \xrightarrow{e^{-10t}} e^{-10(0)} \Rightarrow 1 \\ =1 & \xrightarrow{e^{-10t}} e^{-10(1)} \Rightarrow e^{-10} \\ =2 & \xrightarrow{e^{-10t}} e^{-10(2)} \Rightarrow e^{-20} \\ =3 & \xrightarrow{e^{-10t}} e^{-10(3)} \Rightarrow e^{-30} \\ =4 & \xrightarrow{e^{-10t}} e^{-10(4)} \Rightarrow e^{-40} \end{aligned}$$

So, the signal $x(t) = e^{-10t} \cdot u(t)$
 will be only at the time period, $t \geq 0$ & signal is 0 when $t < 0$.



the energy $E = \lim_{T \rightarrow \infty} \int_{-T}^T (x(t))^2 dt$

$= \lim_{T \rightarrow \infty} \int_0^T e^{-2\alpha t} dt$ (because $u(t)=1, t \geq 0$
 $=0, t < 0$)

$= \lim_{T \rightarrow \infty} \left[\frac{e^{-2\alpha t}}{-2\alpha} \right]_0^T$

$= \lim_{T \rightarrow \infty} \left[\frac{e^{-2\alpha T}}{-2\alpha} - \frac{e^{-2\alpha(0)}}{-2\alpha} \right]$

$= \lim_{T \rightarrow \infty} \frac{-1}{2\alpha} \left[e^{-2\alpha T} - e^{-2\alpha(0)} \right]$

$= \frac{-1}{2\alpha} \left[e^{-2\alpha T} - 1 \right]$

$= \frac{-1}{2\alpha} \left[e^{-\infty} - 1 \right] = \frac{-1}{2\alpha} [0 - 1] = \frac{1}{2\alpha}$

$E = \frac{1}{2\alpha}$

($\because e^{-\infty} = \frac{1}{e^{\infty}} = \frac{1}{\infty} = 0$)

③ $\cos(\alpha \pi t) \cdot u(t) \cdot u(2-t)$

First Solve $u(t) \cdot u(2-t)$.

$u(t)=1, t \geq 0$; $u(2-t)=1, 2-t \geq 0$

$-t \geq -2$

$=0, t < 0$

$t \leq 2$

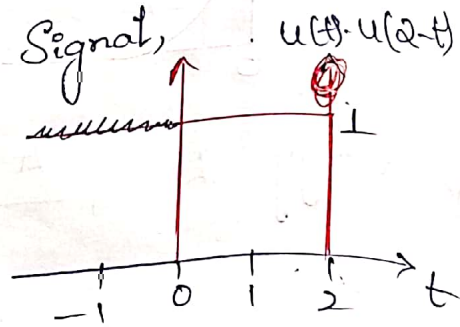
$=0, t > 2$

$u(t) \cdot u(2-t)=1, 0 \leq t \leq 2$ ($\because$ by combining the eqn cond's we get '1' value at these 't' values).

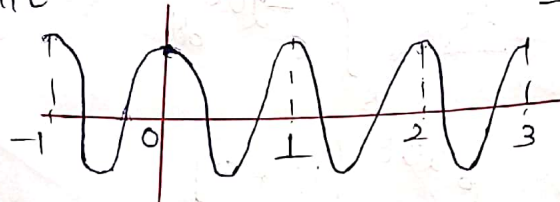
$\cos(2\pi t)$ $\rightarrow$ the period of the signal $= \frac{2\pi}{2\pi} = 1$.

So, Now, for drawing the signal, $u(t) \cdot u(2-t)$

$$u(t) \cdot u(2-t) = 1, 0 \leq t \leq 2$$

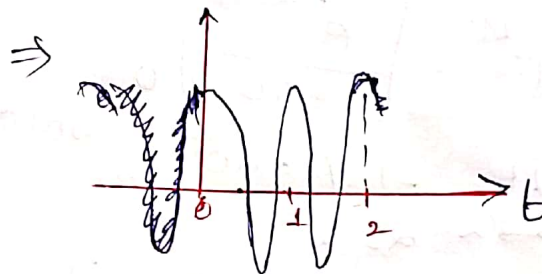


$\cos 2\pi t$



$\Rightarrow$ here period of $\cos 2\pi t$ $= 1$

$$\Rightarrow \cos 2\pi t \cdot u(t) \cdot u(2-t)$$



Now, for the energy of the signal $E = \lim_{T \rightarrow \infty} \int_0^T (x(t))^2 dt$

$$= \lim_{T \rightarrow \infty} \int_0^T \cos^2(2\pi t) \cdot dt$$

$$= \lim_{T \rightarrow \infty} \int_0^T \left(\frac{1 + \cos 4\pi t}{2} \right) dt$$

$$= \lim_{T \rightarrow \infty} \frac{1}{2} \int_0^T dt + \lim_{T \rightarrow \infty} \frac{1}{2} \int_0^T \cos 4\pi t dt$$

$$= \frac{1}{2} [2T] + \lim_{T \rightarrow \infty} \frac{1}{2} \left[\frac{\sin 4\pi t}{4\pi} \right]_0^T$$

$$= 1 + \lim_{T \rightarrow \infty} \frac{1}{8\pi} [\cos 8\pi - \cos 0]$$

$$E = 1 + 0$$

$$E = 1 \text{ J}$$

Refer to the

which of the following signals are energy signals, power signals, neither energy nor power signals.

1) $u(t) - u(t-1)$

2) $(1 + e^{-5t}) \cdot u(t)$

3) $e^{-3t} u(t)$

4) $e^{j(2t + \frac{\pi}{4})}$

5) $\cos t$

6) $e^{-5t} u(t)$

7) $x_1(n) = \left(\frac{1}{3}\right)^n \cdot u(n)$

8) $x_2(n) = e^{j\left(\frac{\pi}{2}n + \frac{\pi}{8}\right)}$

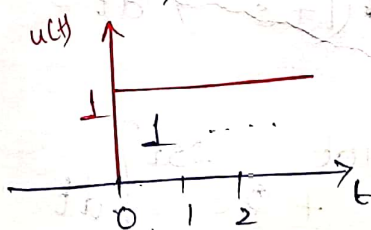
9) $x_3(n) = \cos\left(\frac{\pi}{4}n\right)$

10) $\pi\left(\frac{t+1}{2}\right)$

① $u(t) - u(t-1)$

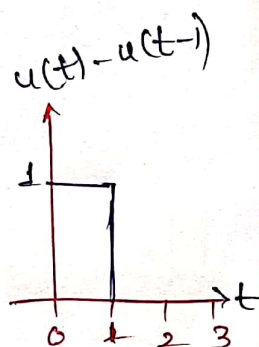
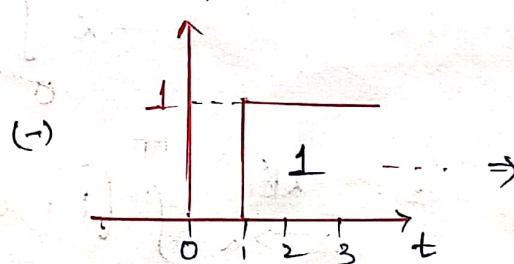
$u(t) = 1, t \geq 0$

$= 0, t < 0$



$u(t-1) = 1, t-1 \geq 0$
 $t \geq 1$

$= 0, t < 1$



Here the signal is defined only in between 0 & 1

∴ Energy of signal = $E = \int_{-\infty}^{\infty} |x(t)|^2 dt$

$= \int_0^1 (u(t) - u(t-1))^2 dt$

$= \int_0^1 1 dt = \left[t\right]_0^1 = 1 - 0 = 1 \text{ J}$

Energy of the Signal $E = 10$

$$\text{Power of the Signal } P = \lim_{T \rightarrow \infty} \frac{1}{2T} \left[\int_0^T (1)^2 dt \right]$$

$$= \lim_{T \rightarrow \infty} \frac{1}{2T} [t]_0^T$$

$$= \lim_{T \rightarrow \infty} \frac{1}{2T} (1+0)$$

$$= \lim_{T \rightarrow \infty} \frac{1}{2T} = \frac{1}{\infty} = 0$$

Here, the Energy of the Signal is finite and power is zero. Therefore, the given signal is Energy Signal.

$$2) (1 + e^{-5t}) \cdot u(t)$$

$$u(t) = 1, t \geq 0 \\ = 0, t < 0$$

therefore, given Signal exists only at $t \geq 0$.

$$\therefore \text{Energy of the Signal} = \lim_{T \rightarrow \infty} \int_{-T}^T |x(t)|^2 dt$$

$$= \lim_{T \rightarrow \infty} \int_0^T (1 + e^{-5t})^2 dt$$

$$= \lim_{T \rightarrow \infty} \int_0^T (1 + e^{-10t} + 2e^{-5t}) dt$$

$$= \lim_{T \rightarrow \infty} \left[\int_0^T 1 dt + \int_0^T e^{-10t} dt + 2 \int_0^T e^{-5t} dt \right]$$

$$= \lim_{T \rightarrow \infty} \left[\left[t \right]_0^T + \left[\frac{e^{-10t}}{-10} \right]_0^T + 2 \left[\frac{e^{-5t}}{-5} \right]_0^T \right]$$

$$= \lim_{T \rightarrow \infty} \left[(T+0) - \frac{1}{10} [e^{-10T} + e^{-10(0)}] - \frac{2}{5} [e^{-5T} + e^{-5(0)}] \right]$$

$$= \lim_{T \rightarrow \infty} \left[T - \frac{1}{10} [e^{-10T} + 1] - \frac{2}{5} [e^{-5T} + 1] \right]$$

$$= \infty$$

Power of the Signal $P = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T (x(t))^2 dt$

$$= \lim_{T \rightarrow \infty} \frac{1}{2T} \int_0^T (1 + e^{-5t})^2 dt$$

$$= \lim_{T \rightarrow \infty} \frac{1}{2T} \int_0^T (1 + e^{-10t} + 2 \cdot e^{-5t}) dt$$

$$= \lim_{T \rightarrow \infty} \frac{1}{2T} \left[\int_0^T dt + \int_0^T e^{-10t} dt + 2 \int_0^T e^{-5t} dt \right]$$

$$= \lim_{T \rightarrow \infty} \frac{1}{2T} \left[\left(\frac{t}{1} \right)_0^T + \left(\frac{e^{-10t}}{-10} \right)_0^T + 2 \left(\frac{e^{-5t}}{-5} \right)_0^T \right]$$

$$= \lim_{T \rightarrow \infty} \frac{1}{2T} \left[T - \frac{1}{10} [e^{-10T} + 1] - \frac{2}{5} [e^{-5T} + 1] \right]$$

$$= \lim_{T \rightarrow \infty} \left[\frac{T}{2} - \frac{1}{20T} [e^{-10T} + 1] - \frac{2}{10T} [e^{-5T} + 1] \right]$$

$$= \frac{1}{2} - \frac{1}{\infty} - \frac{2}{\infty} = \frac{1}{2} - 0 - 0 = \frac{1}{2} W.$$

Here the Energy of the Signal is infinite and
Power of the Signal is finite.
Hence the given signal is power signal.

$$x_1(n) = \left(\frac{1}{3}\right)^n u(n).$$

$u(n)=1, n \geq 0$, So, the given signal is defined only at $n \geq 0$.

$$\text{Energy of the Signal } E = \lim_{N \rightarrow \infty} \int_{-N}^N |x(t)|^2 dt$$

$$E = \lim_{N \rightarrow \infty} \sum_{n=-N}^N |x(n)|^2$$

$$= \lim_{N \rightarrow \infty} \sum_{n=0}^N \left(\frac{1}{3}\right)^{2n}$$

$$= \lim_{N \rightarrow \infty} \sum_{n=0}^N \left(\frac{1}{9}\right)^n = \lim_{N \rightarrow \infty} \left[1 + \left(\frac{1}{9}\right) + \left(\frac{1}{9}\right)^2 + \dots \right]$$

$$\left(1 + x + x^2 + x^3 + \dots = \frac{1}{1-x} \right)$$

$$\left[1 + \dots \right] = \lim_{N \rightarrow \infty} \left[\frac{1}{1 - 1/9} \right] = \frac{9}{8}$$

$$\text{Power of the Signal } P = \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=-N}^N |x(n)|^2$$

$$P = \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=0}^N \left(\frac{1}{3}\right)^{2n}$$

$$= \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=0}^N \left(\frac{1}{9}\right)^n$$

$$= \lim_{N \rightarrow \infty} \frac{1}{2N+1} \left[1 + \frac{1}{9} + \left(\frac{1}{9}\right)^2 + \left(\frac{1}{9}\right)^3 + \dots \right]$$

by formula $(1+x+x^2+x^3+\dots = \frac{1}{1-x})$

$$= N \xrightarrow{Lt} \infty \cdot \frac{1}{2N+1} \left(\frac{1}{1-\frac{1}{9}} \right) = N \xrightarrow{Lt} \infty \cdot \frac{1}{2N+1} \left[\frac{8}{9} \right]$$

$$= \frac{1}{\infty} \left[\frac{8}{9} \right] = 0$$

Energy of the Signal = $\frac{9}{8}$ finite.

Power of the Signal = 0.

The given Signal is ~~finite~~ Energy Signal.

$$x_2(n) = e^{j\left(\frac{\pi}{2}n + \frac{\pi}{8}\right)}.$$

$$\text{Energy of the Signal} = N \xrightarrow{Lt} \infty \sum_{n=-N}^N |x(n)|^2$$

$$= N \xrightarrow{Lt} \infty \sum_{n=-N}^N \left| e^{j\left(\frac{\pi}{2}n + \frac{\pi}{8}\right)} \right|^2$$

$$\left| e^{j\theta} \right| = \sqrt{\cos^2\theta + \sin^2\theta} = 1 \quad (\because \text{Here } \theta = \frac{\pi}{2}n + \frac{\pi}{8}).$$

$$= N \xrightarrow{Lt} \infty \sum_{n=-N}^N (1)^2$$

$$\text{Power of the Signal} = N \xrightarrow{Lt} \infty \frac{1}{2N+1} \sum_{n=-N}^N |x(n)|^2$$

$$= N \xrightarrow{Lt} \infty \frac{1}{2N+1} \sum_{n=-N}^N \left| e^{j\left(\frac{\pi}{2}n + \frac{\pi}{8}\right)} \right|^2$$

$$= N \xrightarrow{Lt} \infty \cdot \frac{1}{2N+1} \sum_{n=-N}^N (1)$$

$$= \frac{1}{2(\infty)+1} \cdot 1 = \frac{1}{\infty} = 0$$

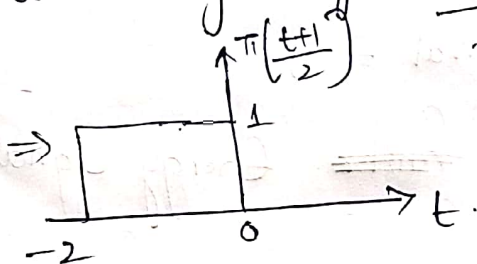
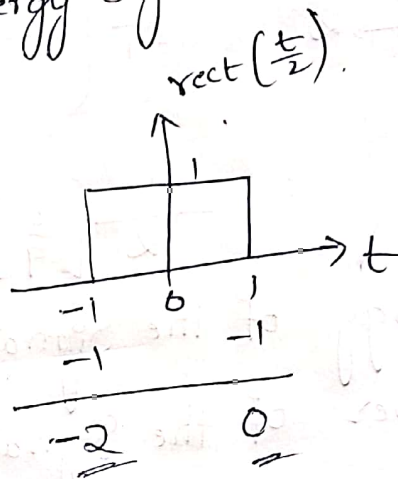
Here, Energy of the Signal is finite & power is 0.

∴ The given Signal is Energy Signal.

$$10) \pi\left(\frac{t+1}{2}\right)$$

$$\text{Time period} = \frac{2}{2} = 1, \text{ } \oplus$$

Here $a = -1$ in given Signal



$$\text{Energy of the Signal} = \lim_{N \rightarrow \infty} \sum_{n=-N}^N |x(n)|^2$$

$$= \lim_{N \rightarrow \infty} \sum_{n=-2}^0 \left| \pi\left(\frac{t+1}{2}\right) \right|^2$$

$$= \lim_{N \rightarrow \infty} \sum_{n=-2}^0 |1|^2 = 1$$

$$\text{Power of the Signal} = \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=-N}^N |x(n)|^2$$

$$= \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=-2}^0 |1|^2$$

$$= \frac{1}{2(\infty)+1} \cdot 1 = \frac{1}{\infty} = 0$$

∴ The given Signal is Energy Signal.

Causal and non-causal Signals :-

A continuous time Signal $x(t)$ is said to be causal if $x(t) = 0$, for $t < 0$, otherwise the Signal is non-causal. For an anti-causal Signal $x(t) = 0$, for $t > 0$.

Similarly a discrete time Signal $x(n)$ is said to be causal if $x(n) = 0$, for $n < 0$; otherwise the Signal is non-causal. For an anti-discrete time Signal $x(n) = 0$, $n > 0$.

Find which of the following Signals are causal or non-causal?

$$x_1(t) = e^{at} \cdot u(t)$$

$$x_4(n) = \left(\frac{1}{2}\right)^n \cdot u(n+2)$$

$$x_2(t) = e^{-2t} \cdot u(-t)$$

$$x_3(n) = x(n+3) - u(n-2)$$

$$x_1(t) = e^{at} \cdot u(t)$$

Here, for $u(t) = 1$, $t \geq 0$
 $= 0$, $t < 0$

$\therefore x_1(t)$ signal is also defined only at values $t \geq 0$

$\therefore$ So, for causal Signal the cond is $t \geq 0$, so the given Signal is causal Signal.

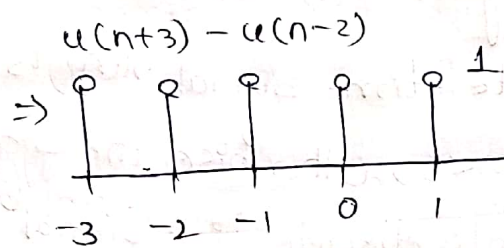
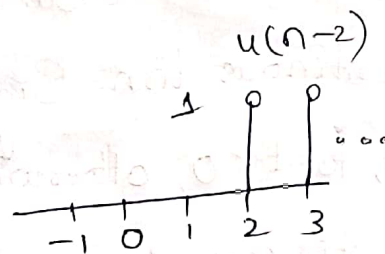
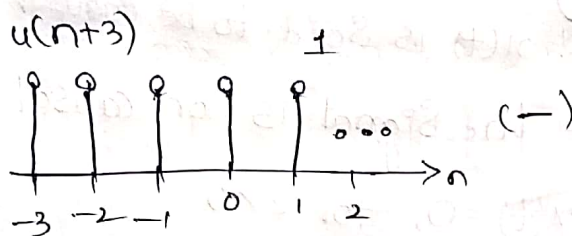
$$x_2(t) = e^{-2t} \cdot u(-t)$$

for $u(-t) = 1$, $t \leq 0$
 $= 0$, $t > 0$

So, the $x_2(t)$ is defined only at $t \leq 0$ values

$\therefore$ it is a non-causal Signal

$$x_3(n) = u(n+3) - u(n-2).$$



Here, the Signal is defined at $t = -3$, i.e. neg value
So, the given Signal is non-causal Signal.

$$x_u(n) = \left(\frac{1}{2}\right)^n \cdot u(n+2).$$

$$u(n+2) = 1, (n+2) \geq 0 \\ n \geq -2 \\ = 0, n < -2.$$

Here $u(n+2)$ is defined at $n \geq -2$, i.e. the Signal is defined at negative value starting.

So, the given Signal is non-causal Signal.

Classification of Systems :-

The Systems may be classified as follows

- Continuous time & discrete time Systems
- Lumped-parameter & distributed-parameter Systems
- Static & dynamic Systems
- Causal & non-causal Systems
- Linear & non-Linear System
- Time variant & time invariant Systems

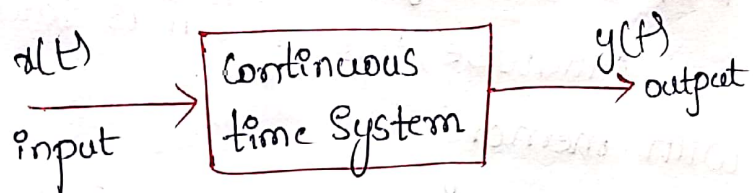
→ Stable and unstable Systems.

→ 1. Continuous time & discrete-time Systems.

Continuous time Systems :- A continuous time System is one which operates on a continuous time input signal & produces a continuous time output signal.

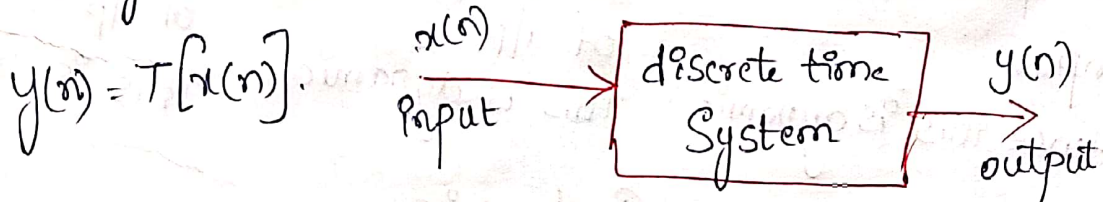
If the input & output of continuous time system are $x(t)$ & $y(t)$, then we can say that $x(t)$ is transformed to $y(t)$. That is

$$y(t) = T[x(t)]$$



Discrete Time Systems :- A discrete time system is one which operates on a discrete time input signal & produces a discrete time output signal.

If the input & output of discrete time input-signal and produces a discrete time output signal. Systems are $x(n)$ & $y(n)$, then we can write



⇒ **Lumped-parameter & distributed-parameter Systems:-**

In lumped parameter systems, each & every component is lumped at one-point in space. These systems are described by ordinary differential eqns.

In distributed parameter Systems, the Signals are functions of Space as well as time. These Systems are described by partial differential Equations.

Static & dynamic Systems :-

A System is called Static or Memoryless if its o/p at any instant depends on the input at that instant but not on past or future values of input.

Otherwise, the System is said to be dynamic or with memory.

Eg :- Static Systems

$$y(t) = x^2(t)$$

$$y(n) = nx(n)$$

Dynamic Systems

$$y(t) = \frac{dx(t)}{dt}$$

$$y(n) = x(n-1)$$

Find whether the following Systems are dynamic or not

i) $y(t) = x(t-2)$

Here $y(t)$ depends on past value of input

Hence, this is dynamic

ii) $y(n) = x(n+2)$

Here $y(n)$ depends on future values of i/p

This is dynamic

iii) $y(t) = x^2(t)$

Here $y(t)$ depends on present value of i/p

This is static

Causal & non-causal Systems :-

A Causal System is one for which the output at any time 't' depends on present & past i/p's but not future inputs. These Systems are also called as non-anticipative Systems.

A non-causal System is one whose output depends on future values.

eg: Causal System

$$y(t) = x(t) + x(t-1)$$

$$y(n) = nx(n) + x(n-3)$$

Non-causal System

$$y(t) = x(t+3) + x^2(t)$$

$$y(t) = x(2n)$$

check out whether the following SLM are causal or not

1) $y(n) = x(n) + \frac{1}{x(n-1)}$

4) $y(t) = \int_{-2}^{2t} x(\tau) \cdot d\tau$

2) $y(t) = x^2(t) + x(t-2)$

5) $y(n) = x(-n)$

3) $y(t) = x(t-2) + x(2-t)$

6) $y(n) = x^2(n^2)$

1) $y(n) = x(n) + \frac{1}{x(n-1)}$

Let $n=0 \Rightarrow y(0) = x(0) + \frac{1}{x(-1)}$ Here, just substituting

$n=1 \Rightarrow y(1) = x(1) + \frac{1}{x(0)}$ 'n' values in the

$n=2 \Rightarrow y(2) = x(2) + \frac{1}{x(1)}$ given signal

$n=-1 \Rightarrow y(-1) = x(-1) + \frac{1}{x(-2)}$

$\therefore$ by observing all these values of signal $y(t)$ is depending only present & past values.

So, this is a Causal signal.

2) $y(t) = x^2(t) + x(t-2)$

Let $t=-1 \Rightarrow y(-1) = x^2(-1) + x(-3)$

$t=0 \Rightarrow y(0) = x^2(0) + x(-2)$

$t=1 \Rightarrow y(1) = x^2(1) + x(-1)$

Here also, the signal sequence signal $y(t)$ depends on past & present values

∴ So, the given Signal is causal Signal.

$$a) y(t) = \int_{-\infty}^t x(\tau) \cdot d\tau$$

$$\text{Let } \int x(\tau) \cdot d\tau = z(t)$$

$$\text{Let } t=0, \quad y(0) = \int_{-\infty}^0 x(\tau) \cdot d\tau = z(0) - z(-\infty)$$

$$\text{Let } t=1 \quad y(1) = \int_{-\infty}^1 x(\tau) \cdot d\tau = z(1) - z(-\infty)$$

$$\text{Let } t=-1 \quad y(-1) = \int_{-\infty}^{-1} x(\tau) \cdot d\tau = z(-1) - z(-\infty)$$

Here o/p signal $y(t)$ depends on present & past values

∴ the given Signal is causal Signal.

$$b) y(n) = x(n^2)$$

$$\text{Let } n=-2 \Rightarrow y(-2) = x((-2)^2) = x(4)$$

$$n=-1 \Rightarrow y(-1) = x((-1)^2) = x(1)$$

$$n=0 \Rightarrow y(0) = x(0)$$

$$n=1 \Rightarrow y(1) = x(1^2) = x(1)$$

$$n=2 \Rightarrow y(2) = x(2^2) = x(4)$$

∴ Here in the -ve values of n , the o/p sequence $y(n)$ depends on future values
So, the given Signal is non-causal Signal.

$$y(t) = x(t-1)$$

$$y(t) = x(t)$$

$$t = -1 \Rightarrow y(-1) = x(-1-1) = x(-2)$$

$$t = 0 \Rightarrow y(0) = x(0)$$

$$t = 1 \Rightarrow y(1) = x(1-1) = x(0)$$

Here -ve values of Signal $y(t)$ depends on +ve values, i.e. future values.

So, this Signal is ~~not~~ non-causal Signal System

Linear and non-linear Systems :-

Superposition principle states that the response to a weighted sum of i/p signals be equal to the weighted sum of the outputs corresponding to each of the individual input signals. A system that satisfies the Superposition principle is said to be a linear system.

If an arbitrary input $x_1(t)$ produces output $y_1(t)$ and an arbitrary input $x_2(t)$ produces output $y_2(t)$ then the system is linear if the input $ax_1(t) + bx_2(t)$ produces an output $ay_1(t) + by_2(t)$, where a & b are constants.

That is,

$$F[ax_1(t) + bx_2(t)] = aF[x_1(t)] + bF[x_2(t)]$$

Similarly for discrete-time systems :-

$$F[ax_1(n) + bx_2(n)] = aF[x_1(n)] + bF[x_2(n)]$$

Check whether the following system are linear or not?

$$Q) \frac{dy}{dt} + 3ty(t) = t^2 x(t).$$

For two input signals $x_1(t)$ & $x_2(t)$, the respective op signals will be $y_1(t)$ & $y_2(t)$.

$$t^2 x_1(t) = \frac{dy_1}{dt} + 3ty_1(t) \rightarrow (1)$$

$$t^2 x_2(t) = \frac{dy_2}{dt} + 3ty_2(t) \rightarrow (2)$$

~~Eq (1) x a~~

$$y_3 [ax_1(t) + bx_2(t)] = \cancel{ax_1}$$

$$y_3(t) = \cancel{T[ax_1(t) + bx_2(t)]}$$

$$= \cancel{\frac{d}{dt} [ax_1(t) + bx_2(t)] + 3t [ax_1(t) + bx_2(t)]}$$

$$= \cancel{[ax_1(t) + bx_2(t)]}$$

multiply Eq (1) with 'a'

$$\Rightarrow at^2 x_1(t) = a \frac{dy_1}{dt} + 3at y_1(t) \Rightarrow a F(x_1(t)) \rightarrow (3)$$

multiply Eq (2) with 'b'

$$\Rightarrow bt^2 x_2(t) = b \frac{dy_2}{dt} + 3bt y_2(t) \Rightarrow b F(x_2(t)) \rightarrow (4)$$

$$F[ax_1(t) + bx_2(t)]$$

$$t^2[ax_1(t) + bx_2(t)] = \frac{d[ay_1(t) + by_2(t)]}{dt} + 3t[ay_1(t) + by_2(t)]$$

$$t^2[ax_1 + bx_2(t)] = a \frac{d}{dt} y_1(t) + b \frac{d}{dt} y_2(t) + 3at y_1(t) + 3bt y_2(t)$$

$$= \cancel{at^2 x_1(t)}$$

$$= a \frac{d}{dt} y_1(t) + 3at y_1(t) + b \frac{d}{dt} y_2(t) + 3bt y_2(t)$$

$$= a t^2 x_1(t) + b t^2 x_2(t)$$

$$F[ax_1(t) + bx_2(t)] = a F[x_1(t)] + b F[x_2(t)]$$

∴ The given System is Linear System.

$$11) \frac{dy(t)}{dt} + 2y(t) = x^2(t)$$

$$x_1^2(t) = \frac{dy_1(t)}{dt} + 2y_1(t) \rightarrow (1)$$

$$x_2^2(t) = \frac{dy_2(t)}{dt} + 2y_2(t) \rightarrow (2)$$

multiply Eq (1) by 'a' & Eq (2) with 'b'

$$a x_1^2(t) = a \frac{dy_1(t)}{dt} + 2a y_1(t) \rightarrow (3) \Rightarrow a F[x_1(t)]$$

$$b x_2^2(t) = b \frac{dy_2(t)}{dt} + 2b y_2(t) \rightarrow (4) \Rightarrow b F[x_2(t)]$$

Eq (3) + (4)

$$a \frac{dy_1(t)}{dt} + 2a y_1(t) + b \frac{dy_2(t)}{dt} + 2b y_2(t) = a x_1^2(t) + b x_2^2(t)$$

→ (5)

$$F(ax_1(t) + bx_2(t))$$

$$\Rightarrow \frac{d[ay_1(t) + by_2(t)]}{dt} + 2[ay_1(t) + by_2(t)] = [ax_1(t) + bx_2(t)]^2$$

$$\Rightarrow a \frac{d}{dt} y_1(t) + b \frac{d}{dt} y_2(t) + 2a y_1(t) + 2b y_2(t) = [ax_1(t) + bx_2(t)]^2$$

$$\Rightarrow a \frac{d}{dt} y_1(t) + 2a y_1(t) + b \frac{d}{dt} y_2(t) + 2b y_2(t) = [ax_1(t) + bx_2(t)]^2 \rightarrow (8)$$

$$\Rightarrow aF[x_1(t)] + bF[x_2(t)] = [ax_1(t) + bx_2(t)]^2$$

Eq (5) & (6) are not equal.

$$\text{i.e.: } aF[x_1(t)] + bF[x_2(t)] \neq F[ax_1(t) + bx_2(t)].$$

Check whether the following System are linear or not

$$1) \frac{dy}{dt} + 2y(t) = x(t) \cdot \frac{dx(t)}{dt}$$

$$2) y(n) = Ax(n) + B$$

$$3) y(n) = n \cdot x(n)$$

$$y(t) = e^{x(t)}$$

$$y_1(t) = e^{x_1(t)} \rightarrow (1)$$

$$y_2(t) = e^{x_2(t)} \rightarrow (2)$$

multiply Eq (1) with 'a' & Eq (2) with 'b'

$$a y_1(t) = a e^{x_1(t)} \rightarrow (3) \Rightarrow a f(x_1(t))$$

$$b y_2(t) = b e^{x_2(t)} \rightarrow (4) \Rightarrow b f(x_2(t))$$

$$\text{Eq (3) + Eq (4)}$$

$$\Rightarrow a y_1(t) + b y_2(t) = a e^{x_1(t)} + b e^{x_2(t)} \rightarrow (5)$$

$$f[a x_1(t) + b x_2(t)]$$

$$\Rightarrow a y_1(t) + b y_2(t) = e^{a x_1(t) + b x_2(t)}$$

$$\Rightarrow a y_1(t) + b y_2(t) = e^{a x_1(t)} \cdot e^{b x_2(t)} \rightarrow (6)$$

Eq (5) & (6) are not equal

$\therefore$ the given System is non-linear

Time-invariant & Time-variant Systems:-

A System is said to be time-invariant if its input-output characteristics do not change with time.

$$y(t) = e^{x(t)}$$

$$y_1(t) = e^{x_1(t)} \rightarrow (1)$$

$$y_2(t) = e^{x_2(t)} \rightarrow (2)$$

multiply eq (1) with 'a' & eq (2) with 'b'

$$a y_1(t) = a e^{x_1(t)} \rightarrow (3) \Rightarrow a f(x_1(t))$$

$$b y_2(t) = b e^{x_2(t)} \rightarrow (4) \Rightarrow b f(x_2(t))$$

$$\text{eq (3) + eq (4)}$$

$$\Rightarrow a y_1(t) + b y_2(t) = a e^{x_1(t)} + b e^{x_2(t)} \rightarrow (5)$$

$$f(ax_1(t) + bx_2(t))$$

$$\Rightarrow a y_1(t) + b y_2(t) = e^{ax_1(t) + bx_2(t)}$$

$$\Rightarrow a y_1(t) + b y_2(t) = e^{ax_1(t)} \cdot e^{bx_2(t)} \rightarrow (6)$$

eq (5) & (6) are not equal

$\therefore$ the given System is non-linear

Time-invariant & Time-variant Systems:-

A System is said to be time-invariant if its input-output characteristics do not change with time. Suppose that we applied a signal $x(t)$ to a system and obtained an output $y(t)$ as shown in fig.)

If we delay the input by 'T' seconds, then for a time-invariant System, the o/p will be delayed by 'T' s.

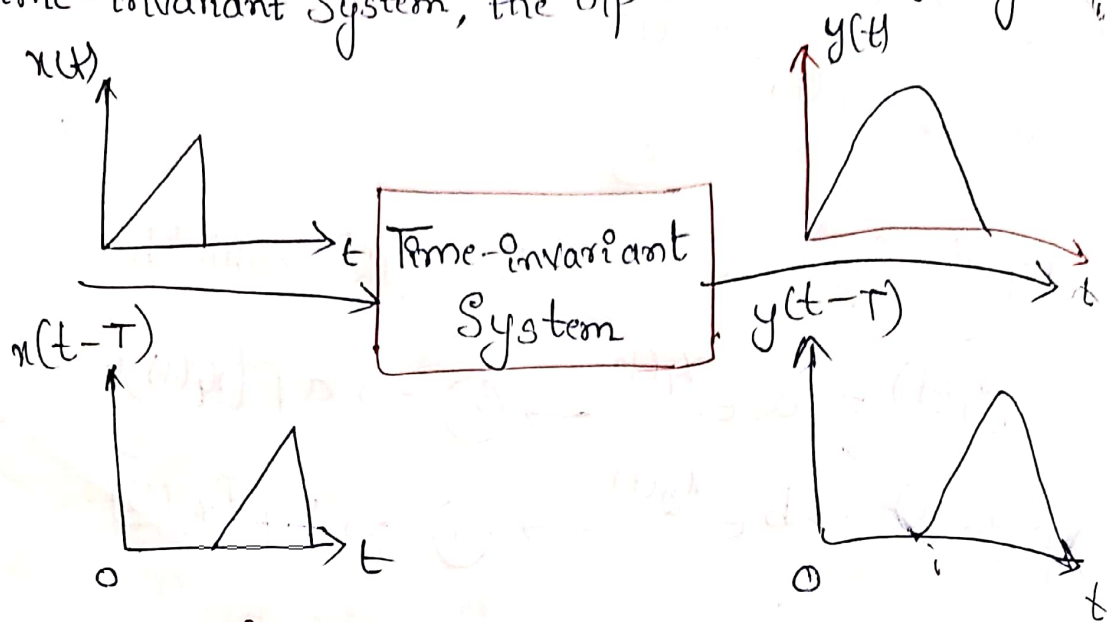


Fig: Time Invariant System

The output due to the delayed input $x(t-T)$ for a time invariant System is given by

$$y(t-T) = T[x(t-T)].$$

If the output, due to input $x(t-T)$, is not equal to $y(t-T)$, then the System is time-Variant

For a linear Time Invariant System (LTI) the coefficients of differential equations describing the System are constants. If the coefficients are function of time, then the System is a linear time-Variant System

Eg:- $2 \frac{d^2 y(t)}{dt^2} + 4 \frac{dy(t)}{dt} + 5y(t) = 5x(t)$

→ all coeff's are constants

Since all coefficients are constants, the given s/m is an LTI

eg²:- $\frac{d^2 y(t)}{dt^2} + 4t \frac{dy(t)}{dt} + 5y(t) = x(t).$

↑ Coeff is a function of time.

It is a linear time Varying System, Since all Coeff's are not constant.

& Similarly for the discrete-time Systems.

To test the time invariant Property of a System apply a Signal $x(t)$ & find $y(t)$. Now delay the input Signal by T and find the o/p Signal & denote it as

$$y(t, T) = F[x(t-T)].$$

Delay the output Signal by T and denote it as $y(t-T)$. If $y(t, T) = y(t-T)$ for any value of T , then the System is time-invariant. on the other hand, if the output

$$y(t, T) \neq y(t-T)$$

i.e. $y(t-T) \neq F[x(t-T)].$

eg ~~for discrete~~:-

$$y(n) - 4y(n-1) + 5y(n-2) = x(n) \rightarrow \text{LTI S/m.}$$

$$y(n) - 3ny(n-1) + n^2 y(n-2) = x(n) + x(n)$$

→ Linear time variant S/m

For discrete time s/m:

$$y(n, k) = F[x(n-k)]$$

delay the o/p Sequence by k Samples denote it as $y(n-k)$.

$$\text{if } y(n-k) \neq F[x(n-k)]$$

Check whether the following Systems are time-variant or time-invariant?

$$(1) \frac{dy(t)}{dt} + 5ty(t) = x(t)$$

Here $y(t, T) = F[x(t-T)]$, Here ^{Now} we are delaying i/p

$$\Rightarrow \frac{dy(t)}{dt} + 5ty(t) = x(t-T) \quad \text{--- (1)}$$

$$y(t-T) \Rightarrow \frac{dy(t-T)}{dt} + 5(t-T) \cdot y(t-T) = x(t-T) \quad \text{--- (2)}$$

Here Eq (1) & (2) are not Equal.

So, the given function is Time-Variant System

$$2) y(t) = x(t+10) + x^2(t)$$

Here $y(t, T) = F[x(t-T)]$, Now here we are delaying i/p

$$\Rightarrow x(t-T+10) + x^2(t-T) = y(t, T) \quad \text{--- (2)}$$

$$y(t-T) = x(t-T+10) + x^2(t-T) \quad \text{--- (2)}$$

Q 3) $y(t) = x(t^2) = F[x(t)]$

$y(t, T) = F[x(t-T)]$, Here we are delaying i/p signal.

$y(t, T) = F[x(t-T)] = x(t^2 - T) \rightarrow (1)$

$y(t-T) = x((t-T)^2) \rightarrow (2)$

Here Eq (1), & (2) are not equal,

Therefore, the given eqn is Linear Time Variant System

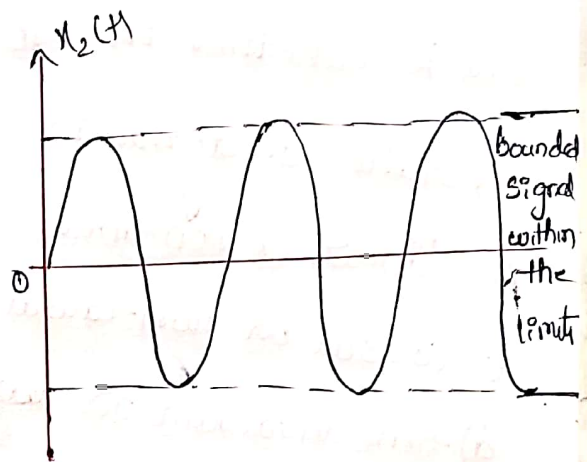
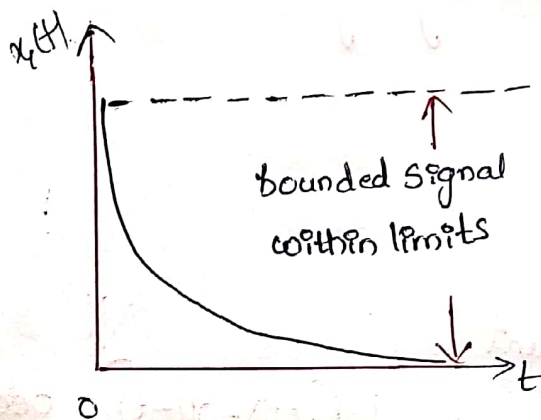
HW
i) $y(n) = x^2(n-1)$ iv) $y(n) = x(n) + n \cdot x(n-1)$

ii) $y(t) = x(-t)$

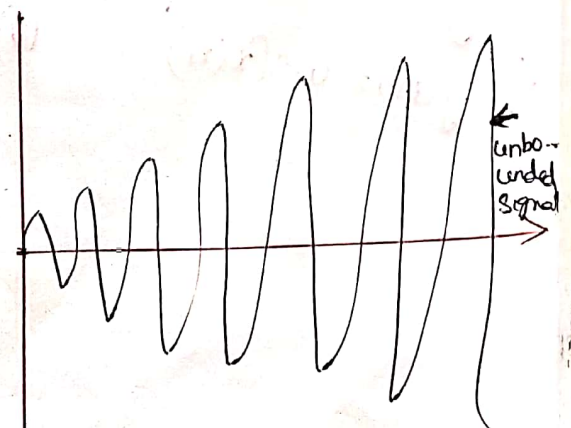
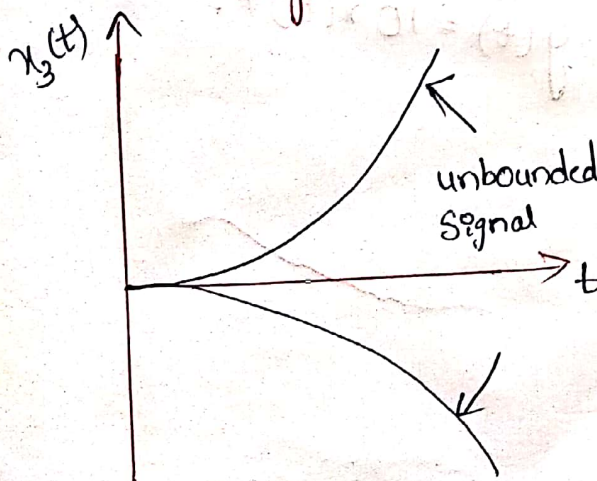
v) $y(t) = x(t) \cdot \cos 50\pi t$

iii) $y(t) = e^{x(t)}$

Stable and unstable Systems :-



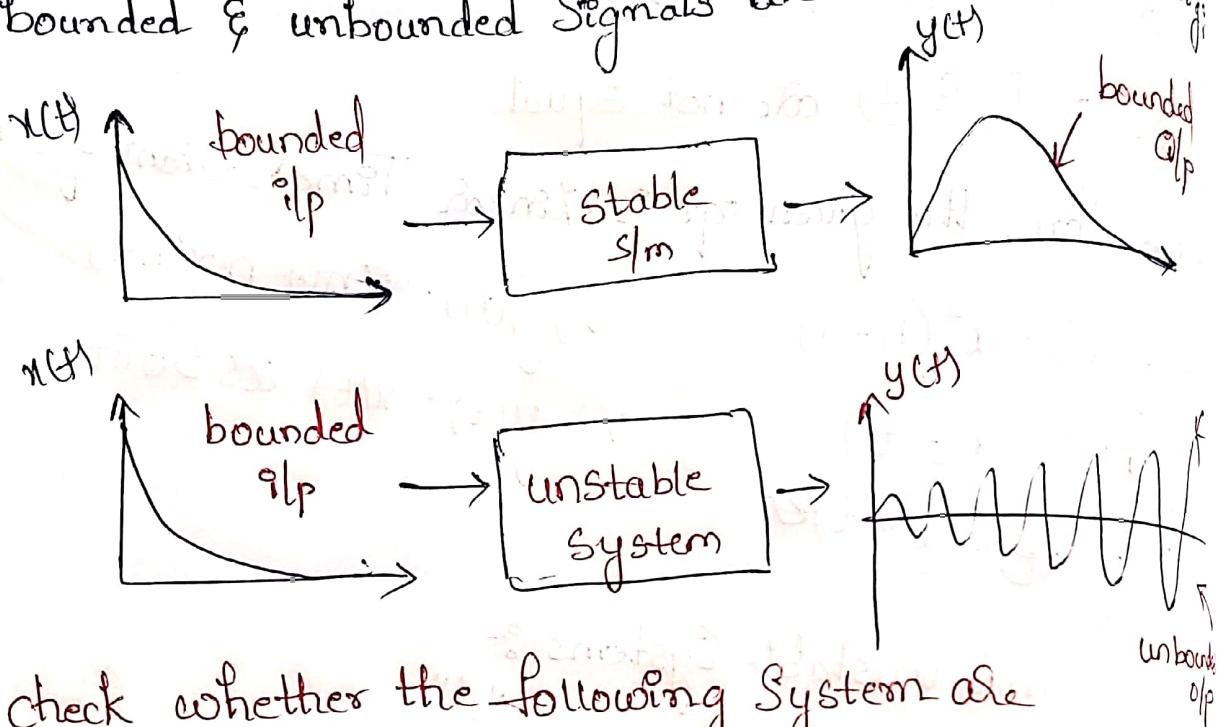
eg for Bounded Signals.



eg for unbounded Signals.

An input signal $x(t)$ is said to be bounded if it satisfies the condition $|x(t)| \leq M_x < \infty$ for all 't'.

Similarly, the output signal is bounded if it satisfies the condition $|y(t)| \leq M_y < \infty$. The eg's of bounded & unbounded signals are shown in below fig.



check whether the following systems are

a) static or dynamic.

b) linear or non-linear

c) causal or non-causal

d) time invariant or variant

i) $y(n) = x(n) \cdot u(n)$

ii) $y(n) = \cos(x(n))$

iii) $y(t) = \frac{1}{2} [x(t) - x(-t)] = \text{odd}(x(t))$

iv) $y(t) = 10x(t) + 5$

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ANALOGY BETWEEN VECTORS AND SIGNALS:

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1) p.T the two functions $\sin n\omega_0 t$ and $\sin m\omega_0 t$ are orthogonal over an interval (t_0, t_0+T) for integral values of n and m .

$$f_1(t) = \sin n\omega_0 t$$

$$f_2(t) = \sin m\omega_0 t$$

Consider $\int_{t_1}^{t_2} f_1(t) f_2(t) dt$

$$= \int_{t_0}^{t_0 + \frac{2\pi}{\omega_0}} \sin n\omega_0 t \sin m\omega_0 t dt$$

$$= \int_{t_0}^{t_0 + \frac{2\pi}{\omega_0}} \frac{1}{2} (\cos(n-m)\omega_0 t - \cos(n+m)\omega_0 t) dt$$

$$= \frac{1}{2} \int_{t_0}^{t_0 + \frac{2\pi}{\omega_0}} \cos(n-m)\omega_0 t dt - \frac{1}{2} \int_{t_0}^{t_0 + \frac{2\pi}{\omega_0}} \cos(n+m)\omega_0 t dt$$

$$= \frac{1}{2} \left[\frac{\sin(n-m)\omega_0 t}{(n-m)\omega_0} \right]_{t_0}^{t_0 + \frac{2\pi}{\omega_0}} - \frac{1}{2} \left[\frac{\sin(n+m)\omega_0 t}{(n+m)\omega_0} \right]_{t_0}^{t_0 + \frac{2\pi}{\omega_0}}$$

$$= \frac{1}{2(n-m)\omega_0} \left[\sin(n-m)\omega_0 \left(t_0 + \frac{2\pi}{\omega_0} \right) - \sin(n-m)\omega_0 t_0 \right]$$

$$- \frac{1}{2(n+m)\omega_0} \left[\sin(n+m)\omega_0 \left(t_0 + \frac{2\pi}{\omega_0} \right) - \sin(n+m)\omega_0 t_0 \right]$$

$$= \frac{1}{2(n-m)\omega_0} \left[\sin(n-m)(\omega_0 t_0 + 2\pi) - \sin(n-m)\omega_0 t_0 \right]$$

$$- \frac{1}{2(n+m)\omega_0} \left[\sin(n+m)(\omega_0 t_0 + 2\pi) - \sin(n+m)\omega_0 t_0 \right]$$

$\therefore f_1(t)$ & $f_2(t)$ are orthogonal signals.

$$2) \text{ p.T } \int_{t_0}^{t_0+T} \cos m\omega_0 t \cos n\omega_0 t \, dt = 0 \text{ if } m \neq n$$

$$= T/2 \text{ if } m = n$$

$$3) \int_{t_0}^{t_0+T} \sin m\omega_0 t \sin n\omega_0 t \, dt = 0 \text{ if } m \neq n$$

$$= T/2 \text{ if } m = n$$

$$4) \int_{t_0}^{t_0+T} \sin m\omega_0 t \cos n\omega_0 t \, dt = 0 \text{ for all values of } m, n.$$

$$2) \text{ consider } \int_{t_0}^{t_0+T} \cos m\omega_0 t \cos n\omega_0 t \, dt$$

$$1) \frac{1}{2} \int_{t_0}^{t_0+T} (\cos(m+n)\omega_0 t + \cos(m-n)\omega_0 t) \, dt$$

$$2) \frac{1}{2} \left[\int_{t_0}^{t_0+T} \cos(m+n)\omega_0 t \, dt \right] + \frac{1}{2} \left[\int_{t_0}^{t_0+T} \cos(m-n)\omega_0 t \, dt \right]$$

$$3) \frac{1}{2} \left[\frac{\sin(m+n)\omega_0 t}{(m+n)\omega_0} \right]_{t_0}^{t_0+T} + \frac{1}{2} \left[\frac{\sin(m-n)\omega_0 t}{(m-n)\omega_0} \right]_{t_0}^{t_0+T}$$

$$= \frac{1}{2(m+n)\omega_0} [\sin(m+n)\omega_0(t_0+T) - \sin(m+n)\omega_0(t_0)]$$

$$+ \frac{1}{2(m-n)\omega_0} [\sin(m-n)\omega_0(t_0+T) - \sin(m-n)\omega_0(t_0)]$$

$$= \frac{1}{2(m+n)\omega_0} [\sin(m+n)(\omega_0 t_0 + \omega_0 T) - \sin(m+n)\omega_0 t_0]$$

$$+ \frac{1}{2(m-n)\omega_0} [\sin(m-n)(\omega_0(t_0) + \omega_0 T) - \sin(m-n)\omega_0 t_0]$$

$$T = \frac{2\pi}{\omega_0}$$

$$\Rightarrow \frac{1}{2(m+n)\omega_0} [\sin(m+n)(\omega_0 t_0 + 2\pi) - \sin(m+n)\omega_0 t_0]$$

$$+ \frac{1}{2(m-n)\omega_0} [\sin(m-n)(\omega_0 t_0 + 2\pi) - \sin(m-n)\omega_0 t_0]$$

$$= 0$$

$\therefore \int \cos m\omega_0 t, \cos n\omega_0 t$ are orthogonal
to signals for $m \neq n$

if $m = n$ then

$$\Rightarrow \int_{t_0}^{t_0+T} \cos m\omega_0 t \cos m\omega_0 t dt$$

$$\Rightarrow \int_{t_0}^{t_0+T} \cos^2 m\omega_0 t dt$$

$$\Rightarrow \int_{t_0}^{t_0+T} \frac{1 + \cos 2m\omega_0 t}{2} dt$$

$$\Rightarrow \frac{1}{2} \left[\int_{t_0}^{t_0+T} dt \right] + \frac{1}{2} \left[\int_{t_0}^{t_0+T} \cos 2m\omega_0 t dt \right]$$

$$= \frac{1}{2} (t_0+T - t_0) + \frac{1}{2} \left[\frac{\sin 2m\omega_0 t}{2m\omega_0} \right]_{t_0}^{t_0+T}$$

$$= \frac{1}{2} (T) + \frac{1}{4m\omega_0} [\sin 2m(\omega_0(t_0+T)) - \sin 2m(\omega_0 t_0)]$$

$$\Rightarrow \frac{T}{2} + \frac{1}{4m\omega_0} [\sin 2m(\omega_0 t_0 + 2\pi) - \sin 2m(\omega_0 t_0)]$$

$$= \underline{\underline{\frac{T}{2}}}$$

$$3) \int_{t_0}^{t_0+T} \sin m\omega_0 t \sin n\omega_0 t \, dt$$

if $m=n$ then

$$\int_{t_0}^{t_0+T} \sin^2 m\omega_0 t \, dt$$

$$\int_{t_0}^{t_0+T} \left(\frac{1 - \cos 2m\omega_0 t}{2} \right) dt$$

$$= \frac{1}{2} \left[\int_{t_0}^{t_0+T} dt \right] - \frac{1}{2} \left[\int_{t_0}^{t_0+T} \cos 2m\omega_0 t \, dt \right]$$

$$= \frac{1}{2} [t_0+T - t_0] - \frac{1}{2} \left[\frac{\sin 2m\omega_0 t}{2m\omega_0} \right]_{t_0}^{t_0+T}$$

$$= \frac{1}{2} (T) - \frac{1}{4m\omega_0} [\sin 2m\omega_0 (t_0+T) - \sin 2m\omega_0 t_0]$$

$$= \frac{T}{2} - \frac{1}{4m\omega_0} [\sin 2m(\omega_0 t_0 + 2\pi) - \sin 2m\omega_0 t_0]$$

$$= \underline{\underline{\frac{T}{2}}}$$

$$4) \int_{t_0}^{t_0+T} \sin m\omega_0 t \cos n\omega_0 t \, dt$$

$$= \int_{t_0}^{t_0+T} \frac{1}{2} (\sin(m+n)\omega_0 t + \sin(m-n)\omega_0 t) \, dt$$

$$= \frac{1}{2} \left[\int_{t_0}^{t_0+T} \sin(m+n)\omega_0 t \, dt \right] + \frac{1}{2} \left[\int_{t_0}^{t_0+T} \sin(m-n)\omega_0 t \, dt \right]$$

$$= \frac{1}{2} \left[\frac{-\cos(m+n)\omega_0 t}{(m+n)\omega_0} \right]_{t_0}^{t_0+T} + \frac{1}{2} \left[\frac{-\cos(m-n)\omega_0 t}{(m-n)\omega_0} \right]_{t_0}^{t_0+T}$$

$$= \frac{1}{2(m+n)\omega_0} [-\cos(m+n)(\omega_0 t_0 + 2\pi) + \cos(m+n)t_0] + \frac{1}{2(m-n)\omega_0} [-\cos(m-n)(\omega_0 t_0 + 2\pi) + \cos(m-n)t_0]$$

vector space is

$$= 0$$

$\therefore \sin m\omega t, \cos n\omega t$ are orthogonal signals for $m \neq n$

If $m = n$

$$1) \int_{t_0}^{t_0+T} \sin m\omega t \cos m\omega t dt$$

$$2) \frac{1}{2} \int_{t_0}^{t_0+T} 2 \sin m\omega t \cos m\omega t dt$$

$$3) \frac{1}{2} \int_{t_0}^{t_0+T} \sin 2m\omega t dt$$

$$4) \frac{1}{2} \left(-\frac{\cos 2m\omega t}{2m\omega} \right)_{t_0}^{t_0+T}$$

$$= -\frac{1}{2} \left[\frac{\cos 2m\omega(t_0+T) - \cos 2m\omega t_0}{2m\omega} \right]$$

$$5) \frac{1}{4m\omega} \left[-\cos 2m(\omega t_0 + 2\pi) + \cos 2m\omega t_0 \right]$$

$$6) \frac{1}{4m\omega} \left[-\cos 2m\omega t_0 + \cos 2m\omega t_0 \right]$$

$$= 0 \quad (\text{for } m = n)$$

$\therefore \sin m\omega t, \cos n\omega t$ are orthogonal signals $\forall$ values of m, n .

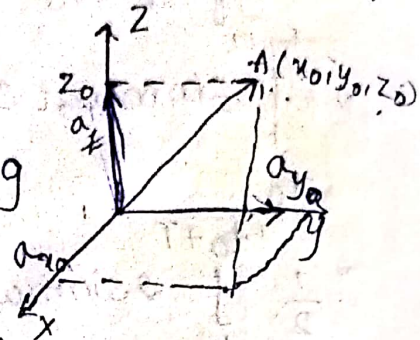
Orthogonal Vector Space :-

Analogy b/w vectors and signals may be extended further. Let us consider a 3-dimensional vector space described by a rectangular coordinates as shown in fig.

* We shall designate a vector of unit length along the x-axis by a_x

a_y is unit vector along y-axis

a_z " " " " z axis



$\therefore$ the magnitude of vectors a_x, a_y, a_z is "1" (unity), which follows that for any general vector A

* the component of A along x-axis = $A \cdot a_x$

" " " " " " y-axis = $A \cdot a_y$

" " " " " " z-axis = $A \cdot a_z$

* A vector A drawn from the origin through a general point in space has components (x_0, y_0, z_0) along x, y, z axes respectively.

* We can express this vector A in terms of its components along the three mutually $\perp$ axes

$$A = x_0 a_x + y_0 a_y + z_0 a_z$$

* Any vector in this space can be expressed in terms of three vectors a_x, a_y, a_z

* Any vector can be expressed as sum of its components along n mutually $\perp$ vectors

* Since the three vectors a_x, a_y, a_z are mutually $\perp$, it follows that $a_x a_y, a_y a_z, a_x a_z = 0$
 $a_x a_x = a_y a_y = a_z a_z = 1$

generally

$$a_m \cdot a_n = 1 \quad \text{for } m=n$$

$$0 \quad \text{for } m \neq n, \text{ Here } a_m, a_n$$

are unit vectors.

* If unit vectors along n mutually $\perp$ coordinates are designated as $x_1, x_2, x_3, \dots, x_n$ and a general vector A in this n dimensional space has components $c_1, c_2, c_3, \dots, c_n$ along these n coordinates then

$$A = c_1 x_1 + c_2 x_2 + c_3 x_3 + \dots + c_n x_n$$

* The condition of orthogonality implies that the dot product of any two vectors x_m, x_n must be $x_m \cdot x_n = 0$ ^{$m \neq n$} and the dot product of any vector with itself must be $x_m \cdot x_n = 1$ ^{$m=n$}

* The component $C_r = A \cdot x_r$

Proof:

$$A = c_1 x_1 + c_2 x_2 + c_3 x_3 + \dots + c_r x_r + \dots + c_n x_n$$

$$A \cdot x_r = c_1 x_1 \cdot x_r + c_2 x_2 \cdot x_r + \dots + c_r x_r \cdot x_r + \dots + c_n x_n \cdot x_r$$

$$A \cdot x_r = 0 + 0 + \dots + c_r + 0$$

$$\therefore \boxed{C_r = A \cdot x_r}$$

(A = vector ' A ' $\rightarrow$ point in space.)

x_r = Co-ordinate in space

C_r = Co-efficients

* We call the set of vectors $(x_1, x_2, \dots, x_n)$ are orthogonal vector space.

* In general the dot product of $x_n \cdot x_n$ can be some constant k_n instead of unity, when k_n is unity the set is called normalised orthogonal set (or) orthonormal vector space

$$x_m \cdot x_n = 0 \quad \text{for } m \neq n$$

$$x_m \cdot x_m = k_m \quad \text{for } m=n \quad \text{then } c_r = \frac{A \cdot x_r}{k_r}$$

proof

$$A = c_1 x_1 + c_2 x_2 + \dots + c_r x_r + \dots + c_n x_n$$

$$A \cdot x_r = c_1 x_1 \cdot x_r + c_2 x_2 \cdot x_r + \dots + c_r x_r \cdot x_r + \dots + c_n x_n \cdot x_r$$

$$A \cdot x_r = c_r (k_r)$$

$$c_r = \frac{A \cdot x_r}{k_r}$$

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Orthogonal signal space:-

Approximation of a function by a set of mutually orthogonal functions:-

Let $g_1(t), g_2(t) \dots g_n(t)$ be the 'n' mutually orthogonal functions then $\int_{t_1}^{t_2} g_i(t) g_k(t) dt = 0$ for $i \neq k$

$$f_1(t) = c_1 g_1(t) + c_2 g_2(t) + \dots + c_k g_k(t) + \dots + c_n g_n(t)$$

$$f(t) = \sum_{r=1}^n c_r g_r(t)$$

$$f_e(t) = f(t) - \sum_{r=1}^n c_r g_r(t)$$

Let E be the average of the square of the error

$$E = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} f(t) dt$$

$$E = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} \left(f(t) - \sum_{r=1}^n c_r g_r(t) \right)^2 dt$$

$$\frac{\partial E}{\partial c_1} = \frac{\partial E}{\partial c_2} = \dots = \frac{\partial E}{\partial c_j} = \frac{\partial E}{\partial c_n} = 0$$

$\therefore E$ is a function of $c_1, c_2, \dots, c_n$

Consider

$$\frac{\partial E}{\partial c_j} = 0$$

$$\frac{\partial}{\partial c_j} \left[\frac{1}{t_2 - t_1} \int_{t_1}^{t_2} \left(f(t) - \sum_{r=1}^n c_r g_r(t) \right)^2 dt \right] = 0$$

$$\Rightarrow \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} \frac{\partial}{\partial c_j} \left(f(t) - \sum_{r=1}^n c_r g_r(t) \right)^2 dt = 0$$

Here by differentiating w.r.t to c_j

$$\int_{t_1}^{t_2} \left(-2 \sum_{r=1}^n f(t) \cdot g_r(t) + 2 c_j g_j(t) \cdot g_j(t) \right) dt = 0$$

$$- \int_{t_1}^{t_2} f(t) g_j(t) dt + \int_{t_1}^{t_2} c_j g_j^2(t) dt = 0$$

$$\int_{t_1}^{t_2} f(t) g_j(t) dt = \int_{t_1}^{t_2} c_j g_j^2(t) dt$$

$$c_j = \frac{\int_{t_1}^{t_2} f(t) g_j(t) dt}{\int_{t_1}^{t_2} g_j^2(t) dt}$$

$$c_j = \frac{1}{K_j} \left[\int_{t_1}^{t_2} f(t) g_j(t) dt \right]$$

Evaluation of mean square error :-

$$C_j = \frac{\int_{t_1}^{t_2} f(t) g_j(t) dt}{\int_{t_1}^{t_2} g_j^2(t) dt} \rightarrow (1)$$

$$C_j = \frac{1}{K_j} \int_{t_1}^{t_2} f(t) g_j(t) dt \rightarrow (2)$$

By definition

$$E = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} f^2(t) dt$$

$$E = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} \left[f(t) - \sum_{r=1}^n C_r g_r(t) \right]^2 dt$$

$$E = \frac{1}{t_2 - t_1} \left[\int_{t_1}^{t_2} f^2(t) dt - 2 \sum_{r=1}^n C_r \int_{t_1}^{t_2} f(t) g_r(t) dt + \sum_{r=1}^n \int_{t_1}^{t_2} C_r^2 g_r^2(t) dt \right]$$

from Eqⁿ (2)

$$\int_{t_1}^{t_2} f(t) g_r(t) dt = C_r K_r$$

$$E = \frac{1}{t_2 - t_1} \left[\int_{t_1}^{t_2} f^2(t) dt - 2 \sum_{r=1}^n C_r (K_r) + \sum_{r=1}^n C_r^2 (K_r) \right]$$

$-2a + a = -a$
 $a = \sum_{r=1}^n C_r K_r$

$$E = \frac{1}{t_2 - t_1} \left[\int_{t_1}^{t_2} f^2(t) dt - \sum_{r=1}^n C_r K_r \right]$$

$$E = \frac{1}{t_2 - t_1} \left[\int_{t_1}^{t_2} f^2(t) dt - (C_1 K_1 + C_2 K_2 + \dots + C_n K_n) \right]$$

Ex consider the rectangular function

$$f(t) = \begin{cases} 1 & \text{for } 0 < t < \pi \\ -1 & \text{for } \pi < t < 2\pi \end{cases}$$

we shall now show how the approximation improves when a large no. of mutually orthogonal functions are used. Approximate $f(t)$ with a set of functions $\sin t$, $\sin 2t$, $\sin 3t$, ... which are mutually orthogonal over the interval $(0, 2\pi)$

The rectangular function is approximated by a finite series of sinusoidal functions.

$$f_1(t) = C_1 \sin t + C_2 \sin 2t + \dots + C_n \sin nt$$

where
$$C_r = \frac{\int_0^{2\pi} f(t) \sin rt \, dt}{\int_0^{2\pi} \sin^2 rt \, dt}$$

$$C_r = \frac{\int_0^{\pi} \sin rt \, dt + \int_{\pi}^{2\pi} -\sin rt \, dt}{\int_0^{2\pi} \left(\frac{1 - \cos 2rt}{2} \right) dt}$$

$$C_r = \frac{\left(-\frac{\cos rt}{r} \right)_0^{\pi} + \left(\frac{\cos rt}{r} \right)_{\pi}^{2\pi}}{\frac{1}{2} (t)_0^{2\pi} - \frac{1}{2} \left[\frac{\sin 2rt}{2r} \right]_0^{2\pi}}$$

$$C_r = \frac{-\frac{1}{r} [-1 - 1] + \frac{1}{r} [1 - (-1)]}{\frac{1}{2} (2\pi) - \frac{1}{4r} (0 - 0)}$$

$$C_r = \frac{\frac{4}{r}}{\frac{\pi}{2}} = \frac{8}{\pi r}$$

$$C_r = \frac{-\frac{1}{r} [(-1)^r - 1] + \frac{1}{r} [1 - (-1)^r]}{\frac{1}{2} (2\pi) - \frac{1}{2} (0/2r)}$$

$$C_r = \frac{-\frac{2}{r} (-1)^r + \frac{2}{r}}{\pi} = \frac{2}{r} \frac{[1 - (-1)^r]}{\pi}$$

$$C_r = \frac{2}{\pi r} [1 - (-1)^r]$$

$$C_r = \frac{4}{\pi r} \quad \text{if } r \text{ is odd}$$

$$C_r = 0 \quad \text{if } r \text{ is even.}$$

$$\therefore f(t) = \sum C_r \sin r t = \frac{4}{\pi} \sin t, \quad r \text{ is odd}$$

$$0, \quad r \text{ is even}$$

$$\therefore f(t) = \frac{4}{\pi} \sin t + \frac{4}{3\pi} \sin 3t + \frac{4}{5\pi} \sin 5t + \dots$$

As we ↑ the no. of terms the approximation improves and the mean square error diminishes.

Let us evaluate the error E in these approximations

$$E = \frac{1}{t_2 - t_1} \left[\int_{t_1}^{t_2} f^2(t) dt - (C_1^2 K_1 + C_2^2 K_2 + \dots + C_n^2 K_n) \right]$$

$$\begin{aligned} t_2 - t_1 &= 2\pi \\ \int_{t_1}^{t_2} f^2(t) dt &= \int_0^{2\pi} f^2(t) dt \\ &= \int_0^{\pi} 1 dt + \int_{\pi}^{2\pi} (-1)^2 dt \\ &= (\pi - 0) + (2\pi - \pi) \\ &= 2\pi \end{aligned}$$

$$\begin{aligned} K_j &= \int_{t_1}^{t_2} g_j^2(t) dt \\ &= \int_0^{2\pi} \sin^2 r t dt = \pi, \quad j \in 1, 2, 3, \dots \end{aligned}$$

$\therefore$ one term approximation (when $f(t)$ is approximated by single sinusoidal term)

$$E_1 = \frac{1}{t_2 - t_1} \left[\int_{t_1}^{t_2} f^2(t) dt - C_1^2 K_1 \right]$$

$$E_1 = \frac{1}{2\pi} \left[2\pi - \frac{16}{\pi^2} (\pi) \right] = 0.19$$

2 term approximation

$$E_2 = \frac{1}{2\pi} \left[2\pi - \left(\frac{4}{\pi}\right)^2 \pi - \left(\frac{4}{3\pi}\right)^2 \pi \right] = 0.079$$

3 term approximation

$$E_3 = \frac{1}{t_2 - t_1} \left[\int_{t_1}^{t_2} f^2(t) dt - (c_1^2 K_1) - c_2^2 K_2 - c_3^2 K_3 \right]$$

$$= \frac{1}{2\pi} \left[2\pi - \left(\frac{4}{\pi}\right)^2 \pi - \left(\frac{4}{3\pi}\right)^2 \pi - \left(\frac{4}{5\pi}\right)^2 \pi \right]$$

$$= \frac{1}{2\pi} \left[2\pi - \frac{16}{\pi} - \frac{16}{9\pi} - \frac{16}{25\pi} \right] = 0.066$$

5 term approximation

$$E_5 = \frac{1}{2\pi} \left[2\pi - \left(\frac{4}{\pi}\right)^2 \pi - \left(\frac{4}{3\pi}\right)^2 \pi - \left(\frac{4}{5\pi}\right)^2 \pi - \left(\frac{4}{7\pi}\right)^2 \pi - \left(\frac{4}{9\pi}\right)^2 \pi \right]$$

$$E_5 = 0.0403$$

$\therefore$ we observe that as the number of terms in approximation are increasing then the mean square error is $\downarrow$. Hence as the no of terms $n \rightarrow \infty \Rightarrow E \rightarrow 0$

Orthogonality in complex functions:

If $f_1(t)$ and $f_2(t)$ are complex functions of real variable 't' then it can be shown that

$$f_1(t) \approx c_{12} \cdot f_2^*(t) \quad | \quad t_1, t_2$$

The optimum value of c_{12} to minimize the mean square error is given by

$$c_{12} = \frac{\int_{t_1}^{t_2} f_1(t) f_2^*(t) dt}{\int_{t_1}^{t_2} f_2(t) f_2^*(t) dt} \rightarrow (1)$$

where $f_2^*(t)$ is a complex conjugate of $f_2(t)$. It is evident from eqⁿ(1) that two complex functions $f_1(t)$ and $f_2(t)$ are orthogonal over t_1, t_2 i.e., $c_{12} = 0$

$$\int_{t_1}^{t_2} f_1(t) f_2^*(t) dt = 0$$

$$\text{(or)} \int_{t_1}^{t_2} f_1^*(t) f_2(t) dt = 0 \rightarrow (2)$$

for a set of complex functions $\{g_r(t)\}$, $r = 1, 2, \dots, n$ are mutually orthogonal over the interval t_1, t_2

$$\int_{t_1}^{t_2} g_m(t) g_n^*(t) dt = 0 \rightarrow (3) \text{ if } m \neq n$$

$$= K_m \text{ if } m = n$$

If this set of functions is complete, then any function $f(t)$ can be expressed as $f(t) = c_1 g_1(t) + c_2 g_2(t) + \dots + c_r g_r(t) + \dots$ $\rightarrow (4)$

If the set of functions are real $g_r^*(t) = g_r(t)$

Representation of a function by a closed or complete set of mutually orthogonal functions:

The mean square error approaches zero as the no. of terms $(n) \rightarrow \infty$. Under this condition we can write

$$0 = \frac{1}{t_2 - t_1} \left[\int_{t_1}^{t_2} f^2(t) dt - \sum_{n=1}^{\infty} c_n^2 K_n \right]$$
$$\int_{t_1}^{t_2} f^2(t) dt = \sum_{n=1}^{\infty} c_n^2 K_n$$

$f(t)$ can be expressed as

$$c_1 x_1(t) + c_2 x_2(t) + \dots + c_n x_n(t) + \dots$$

$x_1(t), x_2(t), \dots$ are a set of mutually orthogonal functions. It is said to be complete on closed set if $\nexists$ no function $p(t)$ for which

$$\int_{t_1}^{t_2} p(t) x_n(t) dt = 0$$

If $p(t)$ exists and above integral is zero then obviously $p(t)$ must be a member of $x_n(t)$

for a set of mutually orthogonal signals $x_n(t)$ over an interval (t_1, t_2)

$$\int_{t_1}^{t_2} x_m(t) x_n(t) dt = 0 \quad \text{if } m \neq n$$
$$= K_m \quad \text{if } m = n$$

for this complete set the function

$f(t)$ is expressed as $c_1 x_1(t) + c_2 x_2(t) + \dots$

here $c_i = \frac{\int_{t_1}^{t_2} f(t) x_i(t) dt}{\int_{t_1}^{t_2} x_i^2(t) dt}$

1) Normalize the signal set $e^{jn\omega_0 t}$ where $n=0, \pm 1, \pm 2, \dots$ over a period T_0

$$\int_{-T_0/2}^{T_0/2} f_1(t) f_2(t) dt = 0 \quad \text{for } m \neq n$$
$$= K_m \quad \text{for } m = n$$

If $K_m = 1$ then it is said to be orthonormal set.

An orthogonal set can be converted to orthonormal by dividing $x_n(t)$ by $\sqrt{K_m}$ (K_m = the energy of a signal over a period).

$$E = \int_0^{T_0} |e^{jn\omega_0 t}|^2 dt$$

$$E = \int_0^{T_0} dt = T_0$$

Orthogonal set = $e^{jn\omega_0 t}$

Orthonormal set = $\frac{e^{jn\omega_0 t}}{\sqrt{T_0}}$

2) If $x(t)$ and $y(t)$ are orthogonal then s.t the energy of the signal $x(t) + y(t)$ is identical to the energy of the signal $x(t)$ + energy of the signal $y(t)$.

3) s.t the set of exponential $1, e^{\pm j\omega_0 t}, e^{\pm 2j\omega_0 t}, e^{\pm 3j\omega_0 t}, \dots$ is orthogonal over any interval T_0 .

$$2) \quad E = \int_{t_1}^{t_2} |x(t)|^2 dt$$

$$E = \int_{t_1}^{t_2} (x(t) + y(t))^2 dt$$

$$E = \int_{t_1}^{t_2} x^2(t) + y^2(t) + 2x(t)y(t) dt$$

$$E = \int_{t_1}^{t_2} x^2(t) dt + \int_{t_1}^{t_2} y^2(t) dt + 2 \int_{t_1}^{t_2} x(t)y(t) dt$$

given that both $x(t)$ and $y(t)$ are orthogonal
 i.e., $x(t) \cdot y(t) = 0$ [$\therefore$ Angle b/w them is 90°
 $x(t)y(t)\cos 90^\circ = 0$]

L.H.S Energy of the signal $x(t) + y(t)$ is

$$\int_{t_1}^{t_2} x^2(t) dt + \int_{t_1}^{t_2} y^2(t) dt = \text{R.H.S}$$

Energy of the signal $x(t)$ is $\int_{t_1}^{t_2} x^2(t) dt$

Energy of the signal $y(t)$ is $\int_{t_1}^{t_2} y^2(t) dt$

The sum of energy of both the signal $x(t)$ and $y(t)$ is $\int_{t_1}^{t_2} x^2(t) dt + \int_{t_1}^{t_2} y^2(t) dt$

$\therefore$ L.H.S = R.H.S