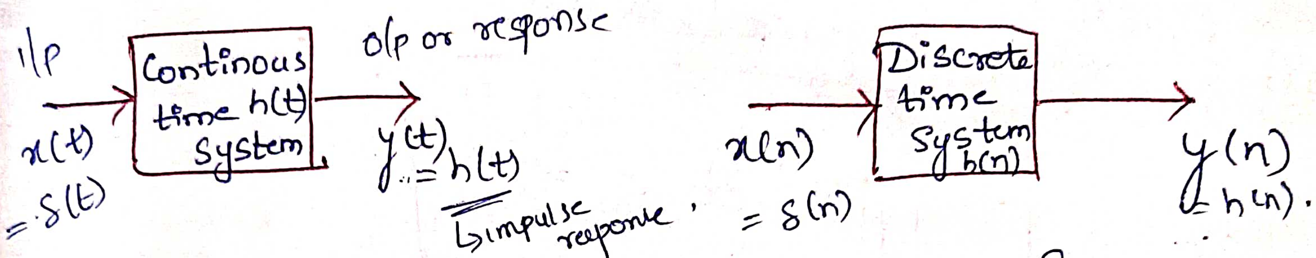
UNIT-III :

Analysis of Linear Systems :-

System:- Any physical device which performs certain operation on the input signal and produces an output

Ex:- Amplifier, filters... etc



Impulse response :- The response of the system to an applied impulses is called "impulse response" of the system. It is represented with h(t) or h(n)

Both continous time and discrete time systems are further classified into

→ Time invariant or variant system

→ Linear or Non-linear system

→ Static or Dynamic system

→ Causal or Non-causal system

→ Stable or unstable system

Impulse Response for causal System :-

Causal System
$$\begin{cases} \text{continous time System : } h(t)=0 \text{ for } t<0 \\ \text{discrete time System : } h(n)=0 \text{ for } n<0 \end{cases}$$

Impulse Response for Stable System :-

A continous time System is Said to be stable if its impulse response Satisfies :

$$\boxed{\int_{-\infty}^{\infty} |h(t)|\, dt < \infty}$$

A discrete time System is Said to be stable System, if its Impulse response satisfies :

$$\boxed{\sum_{n=-\infty}^{\infty} |h(n)| < \infty}$$

Q) $y(t) = e^{x(t)}$; $|x(t)| \le 8$.

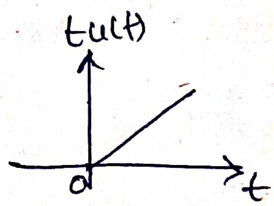
$x(t)$ Varies from $-8 \le x(t) \le 8 \Rightarrow$ Bounded i/p.

then $y(t) = e^{-8} \le y(t) \le e^{8} \Rightarrow$ Bounded o/p

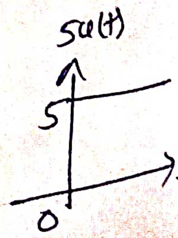
hence it is a Stable System.

Q) $y(t) = (t+5)\, u(t)$.

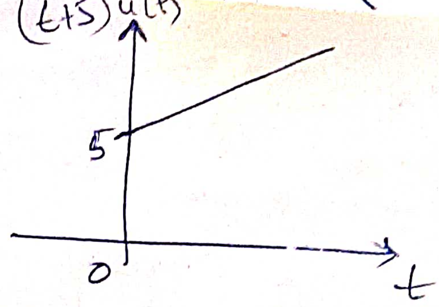
$= \underbrace{tu(t)}_{\text{ramp}} + \underbrace{5u(t)}_{\text{unit}}$



+



$(t+5)u(t) \Rightarrow$



It is an unstable System.

Q) $h(t) = (2+e^{-3t})u(t)$.

For impulse response stability cond:

$$\int_{-\infty}^{\infty} |h(t)|\, dt < \infty.$$

$$\Rightarrow \int_{-\infty}^{\infty} |(2+e^{-3t})u(t)| \cdot dt$$

$$\Rightarrow \int_{-\infty}^{\infty} |[2u(t) + e^{-3t}u(t)]|\, dt$$

$$\Rightarrow 2\int_{0}^{\infty} dt + \int_{0}^{\infty} |e^{-3t}|\, dt \quad -\frac{1}{3}[e^{-\infty} - 0].$$

$$= 2[\infty] + [\ ] = \infty.$$

It is unstable System

Q) $h(t) = \frac{1}{RC} e^{-t/RC} \cdot u(t).$

$$\int_{-\infty}^{\infty} h(t) \cdot dt \Rightarrow \int_{-\infty}^{\infty} |[\frac{1}{RC} \cdot e^{\frac{-t}{RC}}]u(t)|\, dt$$

$$= \int_{0}^{\infty} \frac{1}{RC} |e^{-\frac{t}{RC}}|\, dt$$

$$\Rightarrow \frac{1}{RC}\int_0^{\infty} e^{-t/RC}\,dt$$

$$\frac{1}{\cancel{RC}}\left(\frac{1}{\frac{-1}{\cancel{RC}}}\left[e^{-t/RC}\right]_0^{\infty}\right)$$

$$= -1\left[e^{-\infty} - e^{0}\right] \Rightarrow -1\left[0-1\right] = 1$$

It is a stable System

Q) $h(n) = u(n)$

~~for~~ impulse response stability condition:

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty$$

$$\Rightarrow \sum_{n=-\infty}^{\infty} |u(n)| \Rightarrow \sum_{n=0}^{\infty} u(n)$$

$$= u(0) + u(1) + u(2) + \dots + u$$

$$= 1+1+1+ \dots \dots$$

It is unstable $= \infty$

Q) $y(n) = x(n) + \frac{1}{2}x(n-1) + \frac{1}{4}x(n-2)$

For an impulse response

i/p $\Rightarrow$ $\delta(n) \Rightarrow$ i/p impulse Signal

o/p $\Rightarrow$ $h(n) \Rightarrow$ impulse response of the System

$$h(n) = \delta(n) + \frac{1}{2}\delta(n-1) + \frac{1}{4}\delta(n-2)$$

For stability, condition is

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty.$$

$$\Rightarrow \sum_{n=-\infty}^{\infty} \left| \delta(n) + \frac{1}{2}\delta(n-1) + \frac{1}{4}\delta(n-2) \right|.$$

$$\Rightarrow \sum_{n=-\infty}^{\infty} \delta(n) + \frac{1}{2}\sum_{n=-\infty}^{\infty} \delta(n-1) + \frac{1}{4}\sum_{n=-\infty}^{\infty} \delta(n-2)$$

$$\Rightarrow 1 + \frac{1}{2} + \frac{1}{4} = 1\frac{3}{4} = \frac{7}{4}.$$

It is a stable System as $\sum_{n=-\infty}^{\infty} |h(n)| < \infty$

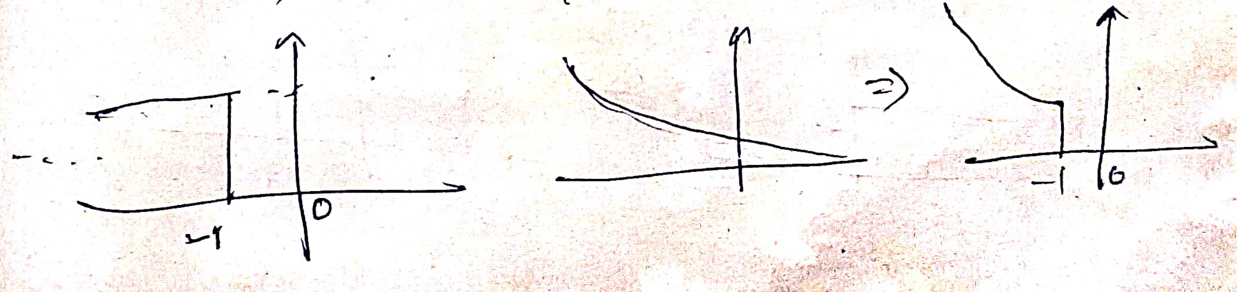
Q) Find whether the following Systems are causal or not?

i) $h(t) = e^{-3t}u(-t-1)$ ii) $h(t) = (t-1)\,u(t-1)$

iii) $h(t) = t^{-2}u(4-t)$.

Q) $h(t) = e^{-3t}u(-t-1)$.

$u(-t-1)$ & e^{-3t} $\Rightarrow$ $h(t) = e^{-3t}u(-t-1)$



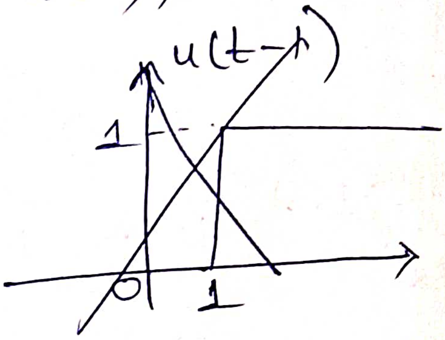
For impulse response causality cond:-

$$h(t) = 0, \quad t < 0$$

Hence this is non-causal System.

ii) $h(t) = (t-1).u(t-1).$

~~$h(t) = t\,u(t-1) - u(t-1)$~~

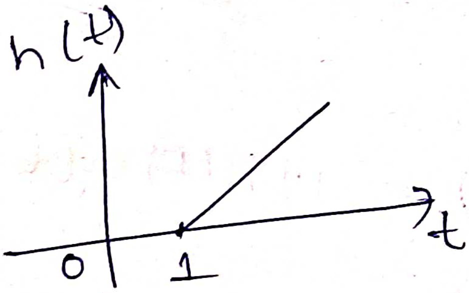


we know that

$$r(t) = t\,u(t).$$

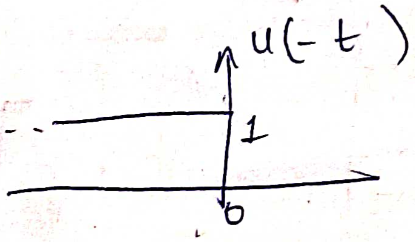
then $(t-1)\,u(t-1) \Rightarrow r(t-1).$

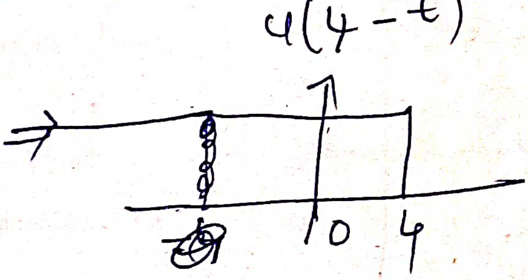
$h(t) = (t-1)u(t-1) \Rightarrow r(t-1)$



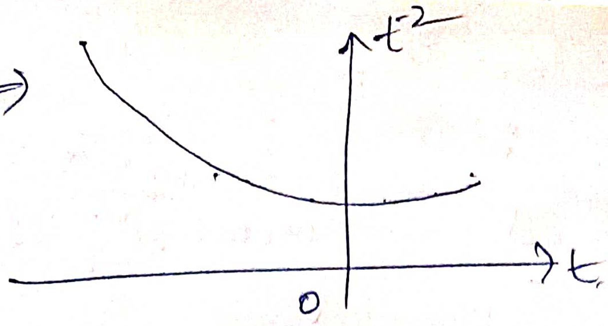
$\therefore$ as $h(t)=0,\ t<0 \Rightarrow$ It is a causal System

iii) $h(t) = \cancel{te}^{-2}\,u(4-t)$



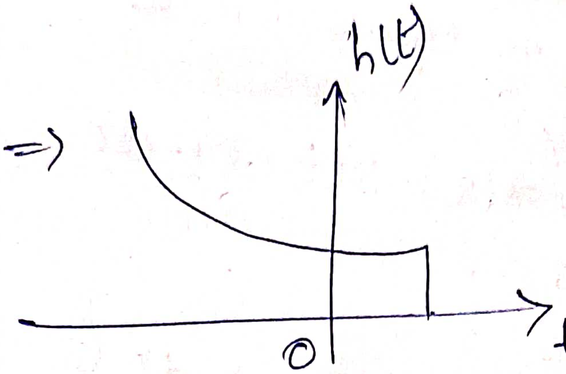


$t^{-2} \Rightarrow$



$t^{-2}u(4-t)$

$\Rightarrow$



as It doesnot Satisfy causal cond $h(t)=0$, for $t<0$

It is non-causal System

Derivation of convolution Integral :-

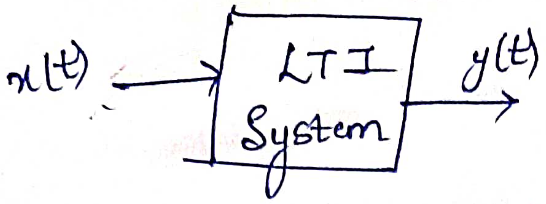
LTI :- A continous or discrete time System is Said to be Linear time invariant System, if it Satisfies ~~the con~~ both linearity and time invariant Properties.

Impulse Response :- The response of the System to an applied impulses is called "Impulse Response of the System

Continous time System : $h(t)$

discrete time System : $h(n)$

Consider an LTI System :



$\Rightarrow y(t) = T\{x(t)\}$

Here $x(t)$ can be represented as

$$x(t) = \int_{-\infty}^{\infty} x(\tau) \cdot \delta(t-\tau) \cdot d\tau.$$

$$y(t) = T\left\{\int_{-\infty}^{\infty} x(\tau) \cdot \delta(t-\tau) \cdot d\tau\right\}.$$

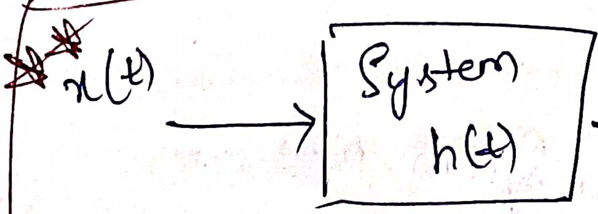
$$= \int_{-\infty}^{\infty} x(\tau) \cdot T\{\delta(t-\tau)\} \cdot d\tau.$$

$T\{\delta(t-\tau)\} \Rightarrow$ indicates, it is a ~~fu~~ i/p func of $\delta(t-\tau)$ then the o/p function $\Rightarrow h(t-\tau)$.

$$y(t) = \int_{-\infty}^{\infty} x(\tau) \cdot h(t-\tau) \cdot d\tau.$$

$\Rightarrow$ It is defined as convolution Integral

$$y(t) = x(t) * h(t)$$



$y(t) = x(t) * h(t)$. $\longrightarrow$ ①

Taking Fourier transform on both Sides of eq ①

$$y(\omega) = X(\omega) \cdot H(\omega)$$

$$H(\omega) = \frac{Y(\omega) \rightarrow o/p}{X(\omega) \rightarrow i/p}$$

This is called transfer function of the System

i.e: If $h(t)$ is given, if we take fourier transform of $h(t)$, we get the transfer function of the System

For discrete time System :-

$$x(n) \longrightarrow \boxed{\text{LTI System}} \longrightarrow y(n)$$

$$\therefore \; y(n) = T\{x(n)\}.$$

any Signal $x(n)$ can be written as

$$x(n) = \sum_{K=-\infty}^{\infty} x(K) \cdot \delta(n-k).$$

$$y(n) = T\left\{\sum_{K=-\infty}^{\infty} x(k) \cdot \delta(n-k)\right\}$$

$$= \sum_{K=-\infty}^{\infty} x(k) \cdot T\{\delta(n-k)\}$$

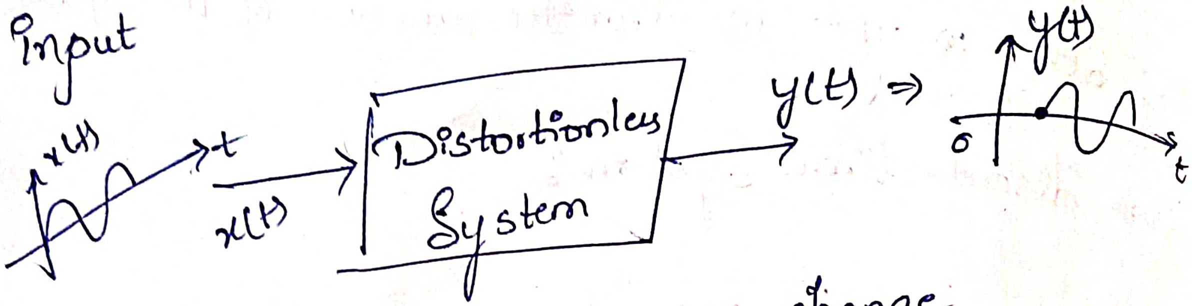
$$y(n) = \sum_{K=-\infty}^{\infty} x(k) \cdot h(n-k). = x(n) * h(n)$$

$\Rightarrow$ convolution integral for discrete time Signal

$x(n) \longrightarrow \boxed{\begin{array}{c}LTI \\ h(n)\end{array}} \longrightarrow y(n) = x(n) * h(n)$

Distortionless System :-

A System is Said to be distortionless if the output of the System is faithful reproduction of input



i) the o/p amplitude may change

ii) the o/p is delayed with respect to input

The o/p of distortionless System is given by

$$y(t) = K \cdot x(t - t_0)$$

here $K \Rightarrow$ constant

$t_0 \Rightarrow$ delay.

$$y(t) = K \cdot x(t - t_0)$$

Taking fourier transform on both Sides

$$Y(\omega) = K \cdot FT\{x(t - t_0)\}$$

$$Y(\omega) = K \cdot e^{-j\omega t_0} X(\omega)$$

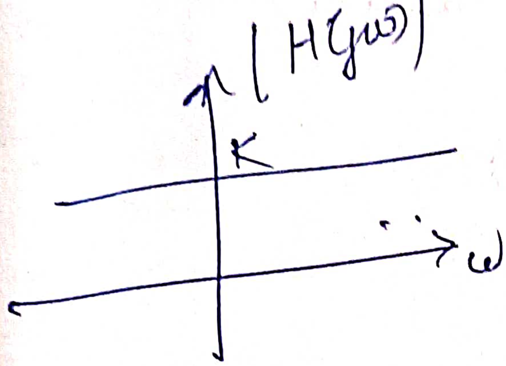
$$\boxed{H(\omega) = \frac{Y(\omega)}{X(\omega)} = K\cdot e^{-j\omega t_0}}$$ ⇒ transfer function

$$|H(j\omega)| = |K\cdot e^{-j\omega t_0}|$$

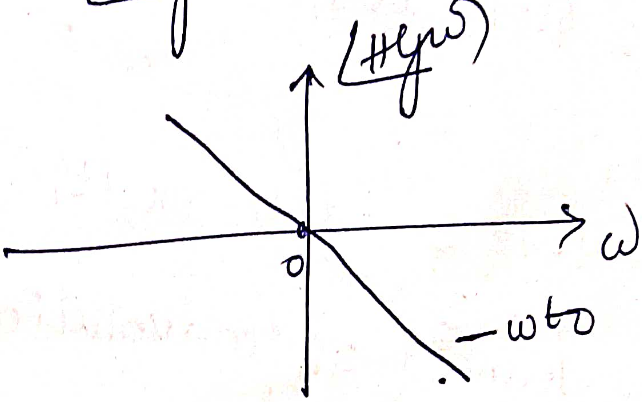
$$= K$$

$$\angle H(j\omega) = \cancel{K(\cos)}$$
$$= \cancel{K}(\cos\omega t_0 - j\sin\omega t_0)$$
$$= \cancel{K}\left[\tan^{-1}\left(\frac{-\sin\omega t_0}{\cos\omega t_0}\right)\right]$$
$$= \cancel{\tan^{-1}}\left(-\cancel{\tan}(\omega t_0)\right)$$

$$\angle H(j\omega) = -\omega t_0$$



a) Magnitude response



b) phase response

~~Graphical procedure~~

~~time domain :-~~

Find the convolution of following Signals in time domain

i) $x_1(t) = e^{-3t} u(t)$

$x_2(t) = e^{-4t} u(t)$

ii) $x_1(t) = t\, u(t)$.

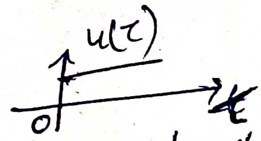
$x_2(t) = t\, u(t)$

iii) $x_1(t) = \cos t\, u(t)$ $\quad x_2(t) = u(t)$.

Q) $x_1(t) = e^{-3t} u(t)$. $x_2(t) = e^{-4t} u(t)$

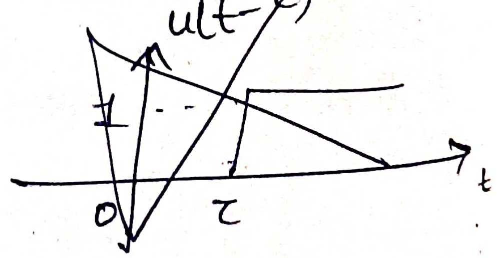
by formula of convolution in time domain,

$$y(t) = x_1(t) * x_2(t) = \int_{-\infty}^{\infty} x_1(\tau) \cdot x_2(t-\tau) \cdot d\tau$$



$x_1(\tau) = e^{-3\tau} u(\tau) \Rightarrow$ here $u(\tau)$ lies from 0 to ∞

$x_2(t-\tau) = e^{-4(t-\tau)} \cdot u(t-\tau)$.

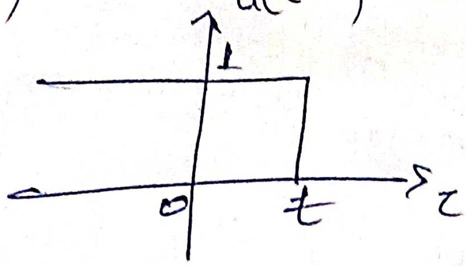


~~i.e: u(t-τ) lies from τ to ∞~~

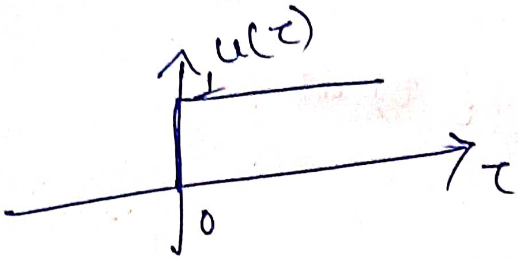
~~then~~

$$\cancel{y(t) = \int_{\tau}^{\infty} e^{-3\tau}}$$

$u(t-\tau)$



$u(\tau)$



as in both cases, the Signal is present in b/w 0 to t

then $y(t) = x_1(t) * x_2(t)$

$$= \int_0^t e^{-3\tau} u(\tau) \cdot e^{-4(t-\tau)} u(t-\tau) \cdot d\tau$$

$$= \int_0^t e^{-3\tau - 4t + 4\tau} \cdot d\tau.$$

$$= \int_0^t e^{-4t + 4\tau} \cdot d\tau$$

$$= e^{-4t} \int_0^t e^{4\tau} \cdot d\tau.$$

$$= e^{-4t} \frac{1}{4} \left[e^{4\tau} \right]_0^t$$

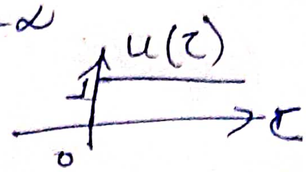
$$= e^{-4t} \left[e^{4t} - 1 \right] = \boxed{e^{-3t} - e^{-4t}}$$

11) $x_1(t) = t\,u(t)$. $x_2(t) = t\,u(t)$.

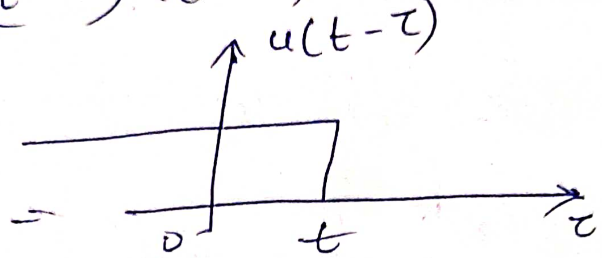
by convolution in time domain,

$$y(t) = x_1(t) * x_2(t) = \int_{-\infty}^{\infty} x_1(\tau)\cdot x_2(t-\tau)\,.d\tau$$

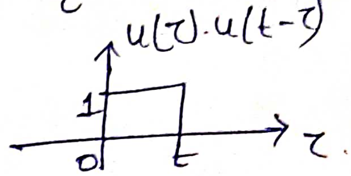
$x_1(\tau) = \tau\,u(\tau)$



$x_2(t-\tau) = (t-\tau)\,u(t-\tau)$



The Signal is common only from



then, the convolution in time domain,

$$y(t) = \int_0^t \tau.(t-\tau)\,.d\tau$$

$$= \int_0^t (\tau t - \tau^2)\,d\tau \Rightarrow \int_0^t \tau t\,d\tau - \int_0^t \tau^2 d\tau$$

$$= t\left[\frac{\tau^2}{2}\right]_0^t - \left[\frac{\tau^3}{3}\right]_0^t$$

$$= \frac{t}{2}\left[t^2\right] - \frac{1}{3}\left(t^3\right)$$

$$= t^3\left[\frac{1}{2} - \frac{1}{3}\right)$$

$$= t^3\left[\frac{1}{6}\right) = \frac{t^3}{6};\ t>0$$

$$\therefore\ y(t) = x_1(t) * x_2(t) = \frac{t^3}{6}\,u(t).$$

Q) $x_1(t) = \cos t \, u(t)$ $\quad x_2(t) = u(t)$.

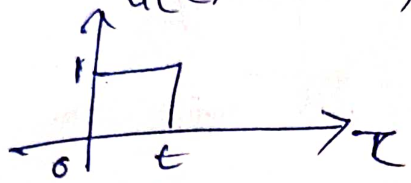
$$y(t) = x_1(t) * x_2(t)$$

$$= \int_{-\infty}^{\infty} x_1(\tau) \cdot x_2(t-\tau) \cdot d\tau.$$

$x_1(\tau) = \cos\tau \, u(\tau)$

$x_1(t-\tau) = u(t-\tau)$

$u(t) \cdot u(t-\tau) \Rightarrow$ $\quad u(\tau) \cdot u(t-\tau)$



$$y(t) = \int_0^t \cos\tau \cdot d\tau = [\sin\tau]_0^t = \sin t - \sin 0$$

$\therefore$ $y(t) = \sin t \; ; \; t \geq 0$

$y(t) = \sin t \, u(t)$.

Properties of Convolution :-

Consider two Signals $x_1(t)$ & $x_2(t)$. the Convolution of these 2 Signals is given by

$$x_1(t) * x_2(t) = \int_{-\infty}^{\infty} x_1(\tau) \cdot x_2(t-\tau) \cdot d\tau.$$

i). Commutative property : $x_1(t) * x_2(t) = x_2(t) * x_1(t)$

ii) Distributive property : $x_1(t) * [x_2(t) + x_3(t)]$

$$= [x_1(t) * x_2(t)] + [x_1(t) * x_3(t)]$$

iii) Associative Property:

$$x_1(t) * [x_2(t) * x_3(t)] = [x_1(t) * x_2(t)] * x_3(t).$$

iv) ~~width~~/property:

~~Let $x_1(t)$ is duration~~

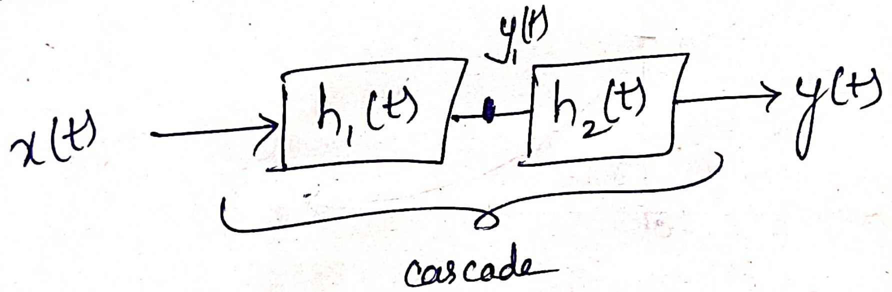
Convolution with impulse:

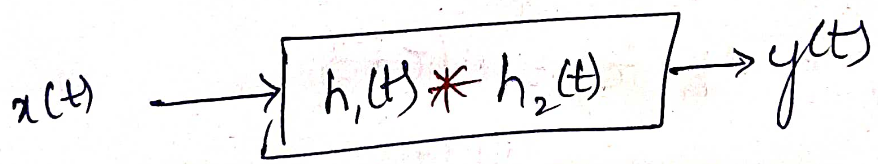
$$x(t) * \delta(t) = x(t)$$

$$x(t) * \delta(t) = \int_{-\infty}^{\infty} x(\tau) \cdot \delta(t-\tau) \cdot d\tau$$

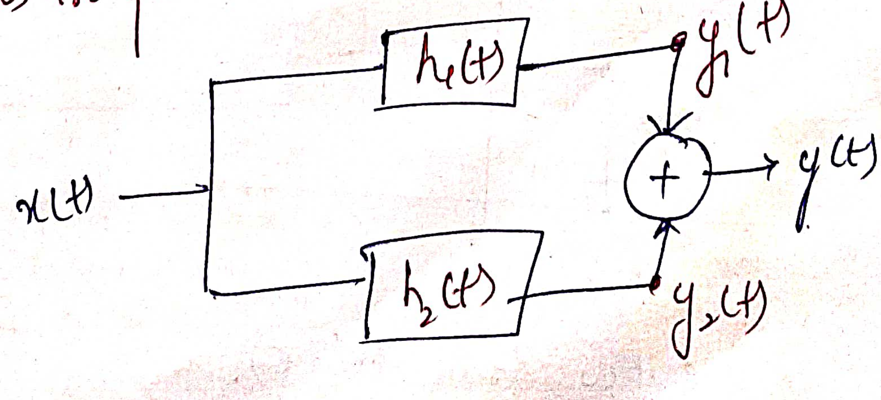
$$= x(t).$$

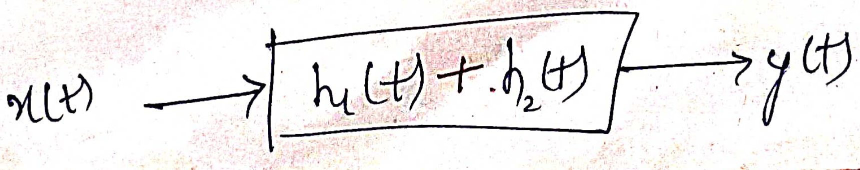
Systems in cascade:





Systems in parallel:





Graphical procedure to perform convolution, in time domain

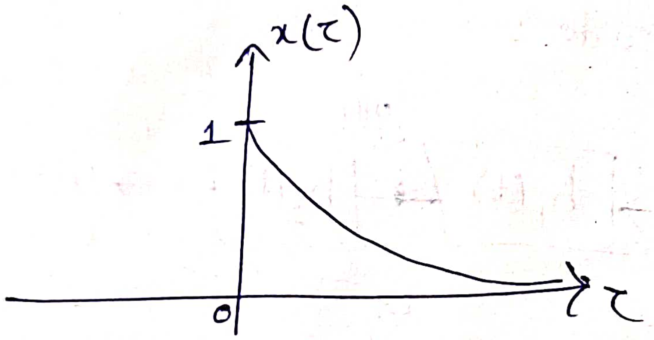
Q) $x(t) = e^{-3t} u(t)$; $h(t) = u(t+3)$.

To find the convolution of 2 signals,

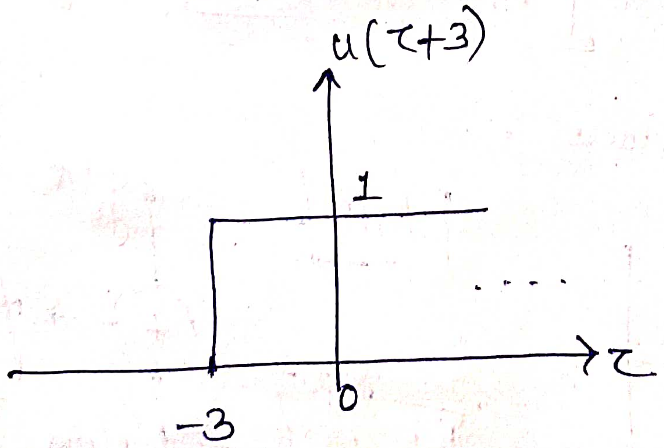
$$y(t) = \int_{-\alpha}^{\alpha} x(\tau) \cdot h(t-\tau) \cdot d\tau = x(t) * h(t)$$

Step I: Write in terms of $x(\tau)$

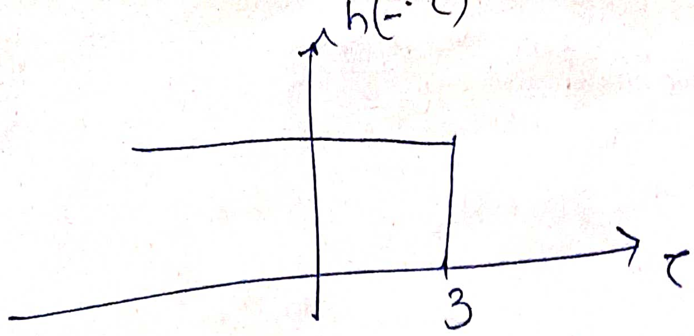
$$x(\tau) = e^{-3\tau} \cdot u(\tau).$$



Step II :- $h(\tau) = u(\tau+3)$

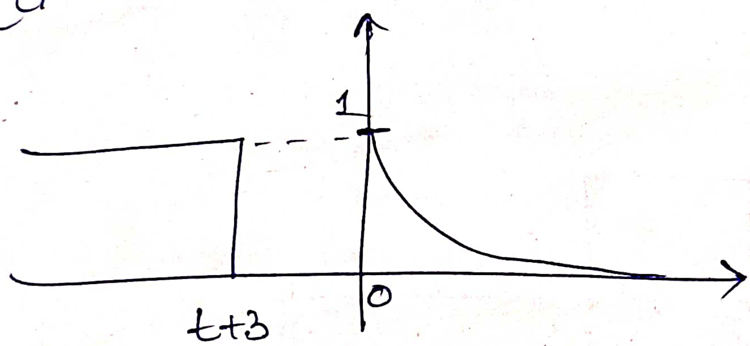


Step III :- Now draw $h(-\tau)$.

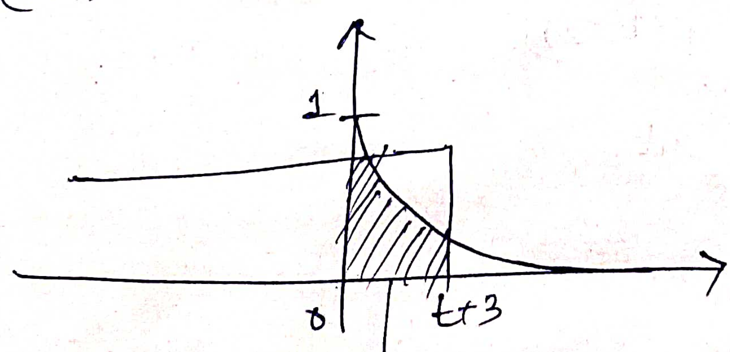


Step IV: Now draw the signal $h(t-\tau)$

$x(\tau)\cdot h(t-\tau)$



if $(t+3) \geq 0$



overlapped part

$$\Rightarrow \int_0^{t+3} x(\tau)\cdot h(t-\tau)\cdot d\tau \Rightarrow \int_0^{t+3} e^{-3\tau}\cdot(1)\cdot d\tau$$

$$= \frac{-1}{3}\left[e^{-3\tau}\right]_0^{t+3} \Rightarrow \frac{-1}{3}\left[e^{-3(t+3)} - 1\right] = \frac{1}{3}\left[1 - e^{-3(t+3)}\right]$$

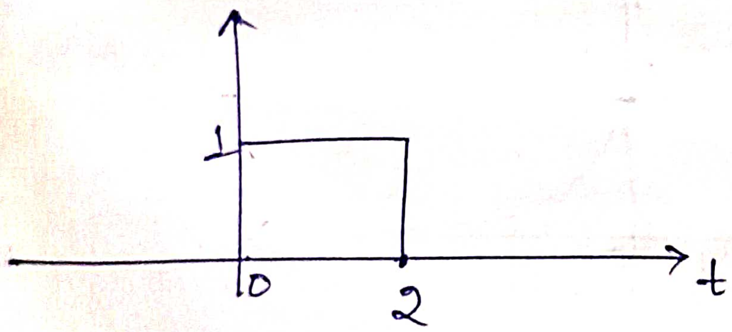
$$y(t) = \begin{cases} 0, & \text{when } t+3 < 0 \\ & t < -3 \\ \frac{1}{3}\left[1 - e^{-3(t+3)}\right], & t+3 \geq 0 \\ & t \geq -3 \end{cases}$$

Q) Find the Convolution of the following Signals by graphical method.

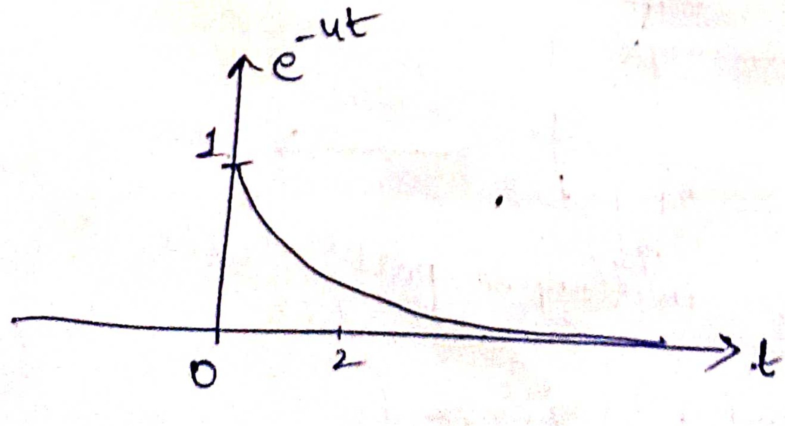
$h(t) = e^{-2t} u(t)$. $\quad x(t) = e^{-4t}\left[u(t) - u(t-2)\right]$

$$x(t) = e^{-4t}\left[u(t) - u(t-2)\right]$$

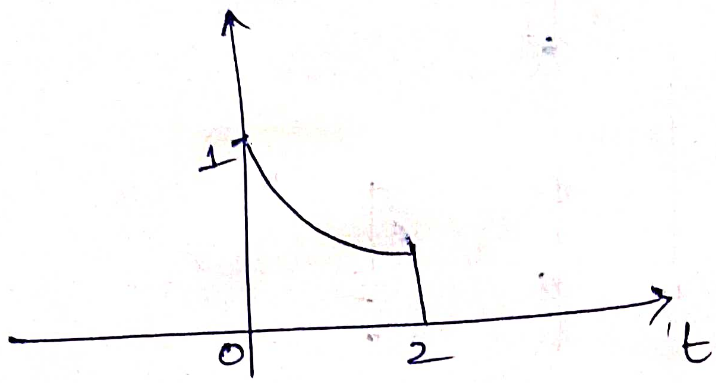
$u(t) - u(t-2)$



$e^{-4t}\cancel{\left[u(t) - u(t-2)\right]}$

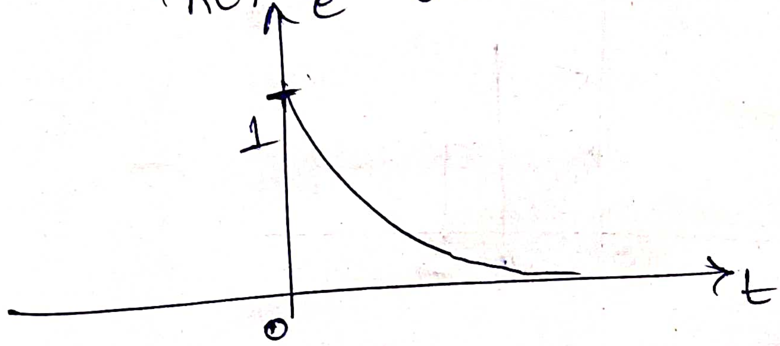


$\Rightarrow e^{-4t}\left[u(t) - u(t-2)\right]$

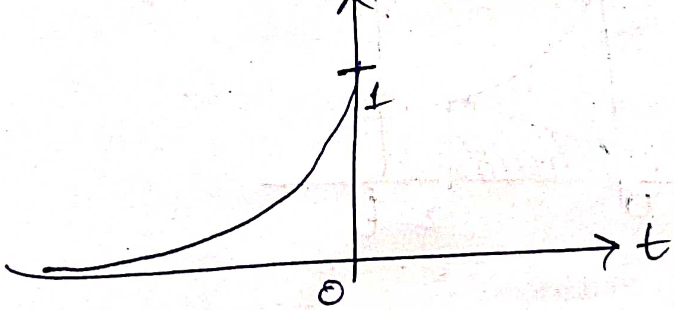


$h(\tau) = e^{-2\tau} u(\tau)$

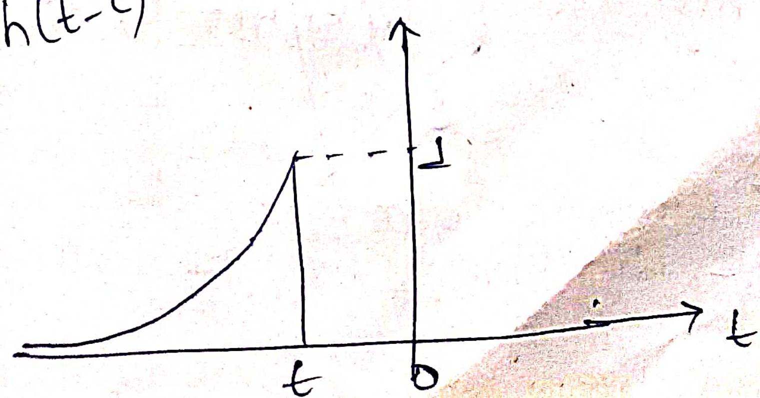
$h(\tau) \Rightarrow e^{-2\tau} u(\tau)$



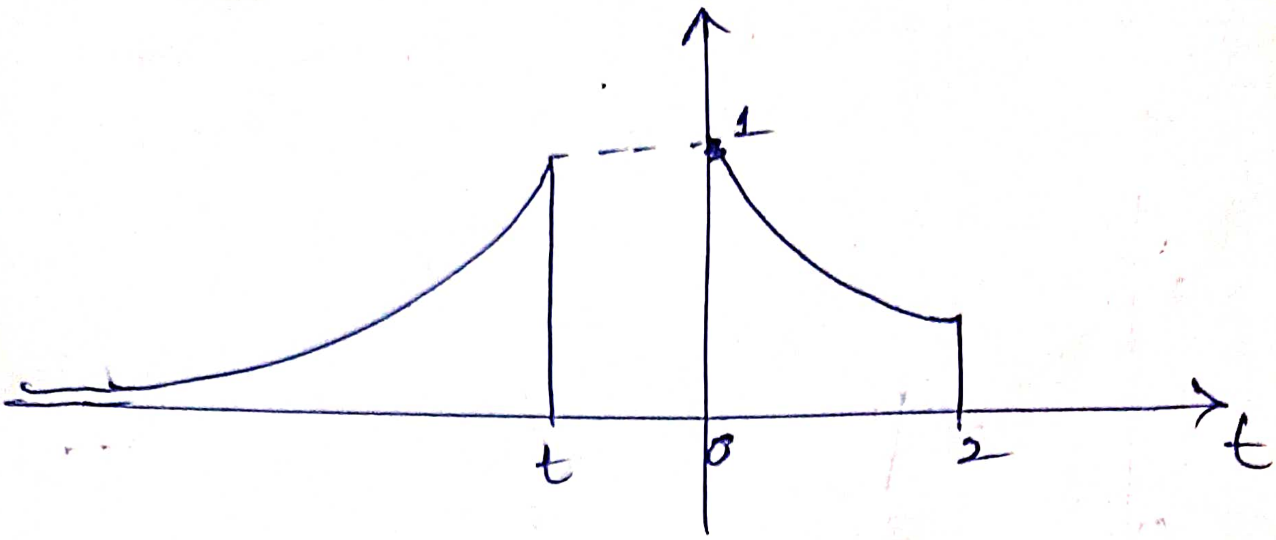
$h(-\tau)$



$h(t-\tau)$

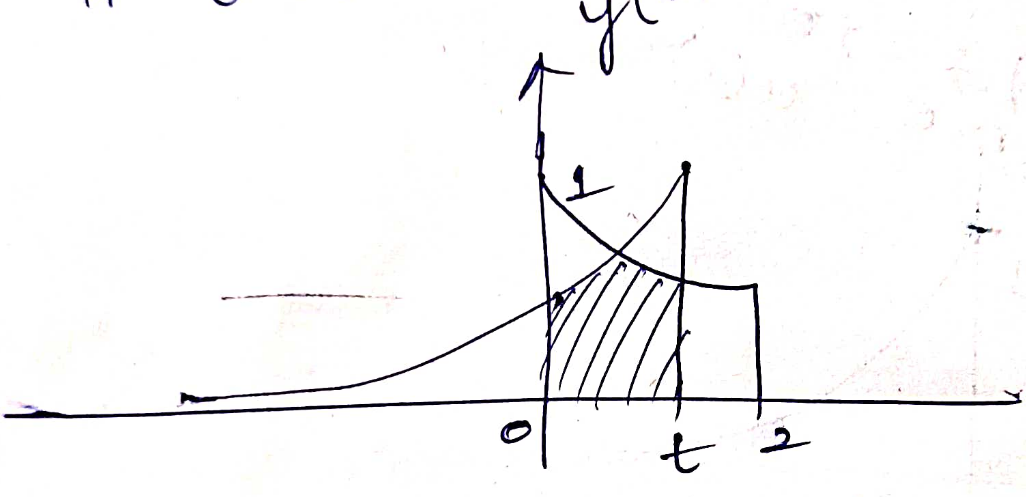


$x(\tau) \cdot h(t-\tau)$

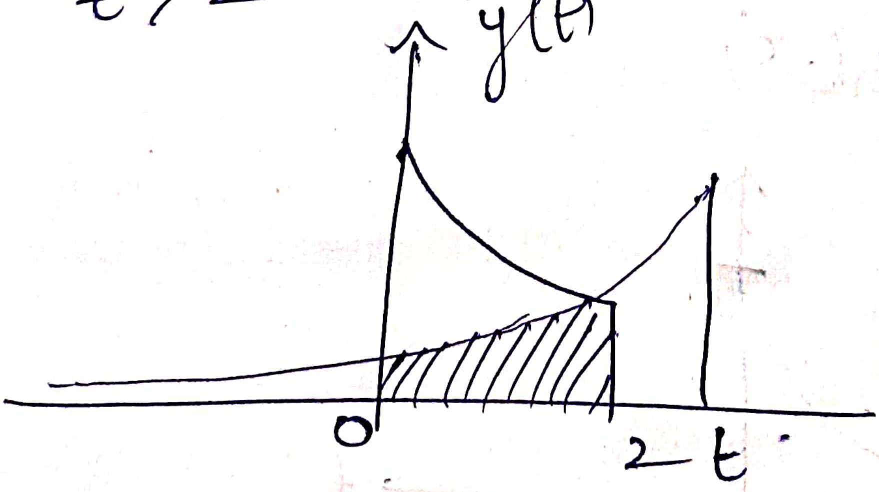


$\underline{\underline{x(\tau) \cdot h(t-\tau)}}$

if $0 < t < 2$



if $t > 2$



$$y(t) = \int_{-\infty}^{\infty} x(\tau) \cdot h(t-\tau) \cdot d\tau$$

$0 < t < 2$

$$\Rightarrow \int_0^t e^{-4\tau} \cdot e^{-2(t-\tau)} \cdot d\tau$$

$$= \int_0^t e^{-4\tau - 2t + 2\tau} \, d\tau$$

$$= e^{-2t} \int_0^t e^{-2\tau} \, d\tau \Rightarrow \frac{-e^{-2t}}{2} \left[e^{-2\tau} \right]_0^t$$

$$= \frac{-1}{2} e^{-2t} \left[e^{-2t} - 1 \right]$$

$$= \frac{1}{2} \left[e^{-2t} - e^{-4t} \right].$$

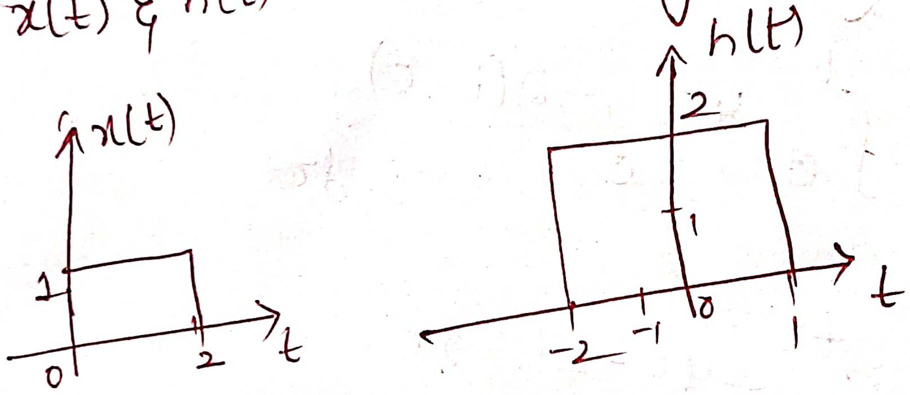
$t > 2$

$$\Rightarrow \int_0^2 e^{-4\tau} \cdot e^{-2(t-\tau)} \, d\tau \Rightarrow e^{-2t} \int_0^2 e^{-2\tau} \, d\tau \Rightarrow \frac{-e^{-2t}}{2} \left[e^{-2t} \right]_0^2$$

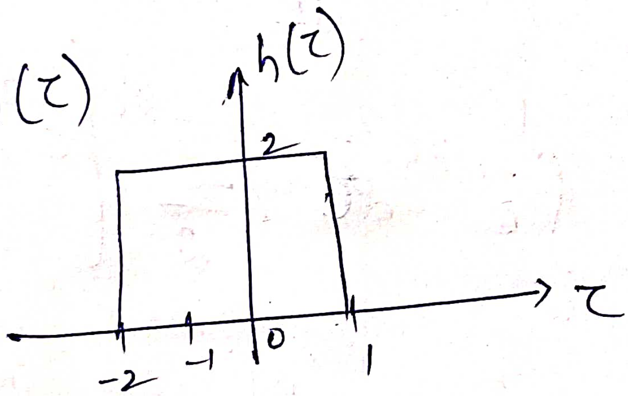
$$= -\frac{e^{-2t}}{2}\left[e^{-4}-1\right] = \frac{e^{-2t}}{2}\left[1-e^{-4}\right]$$

$$y(t) = \begin{cases} 0 & \text{when } t<0 \\ \frac{1}{2}\left[e^{-3t}-e^{-4t}\right]; & 0<t<2 \\ \frac{e^{-2t}}{2}\left(1-e^{-4}\right); & t>2 \end{cases}$$

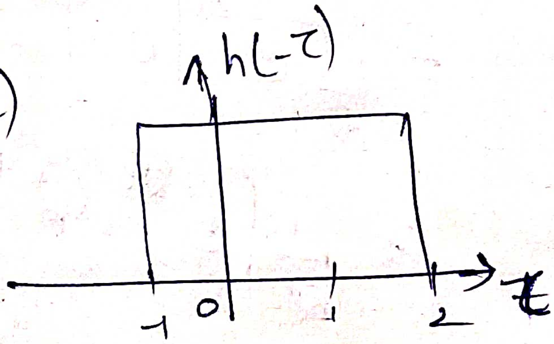
Q) Find the response $y(t)$ of an LTI System whose $x(t)$ & $h(t)$ are shown in fig. using Convolution integral



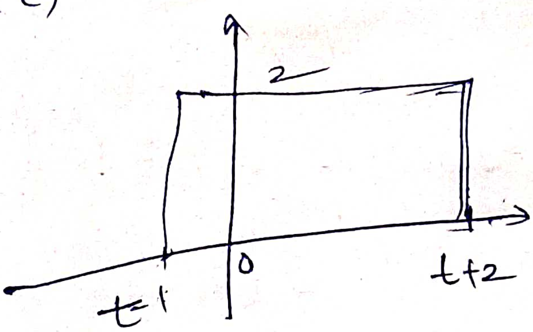
$h(\tau)$



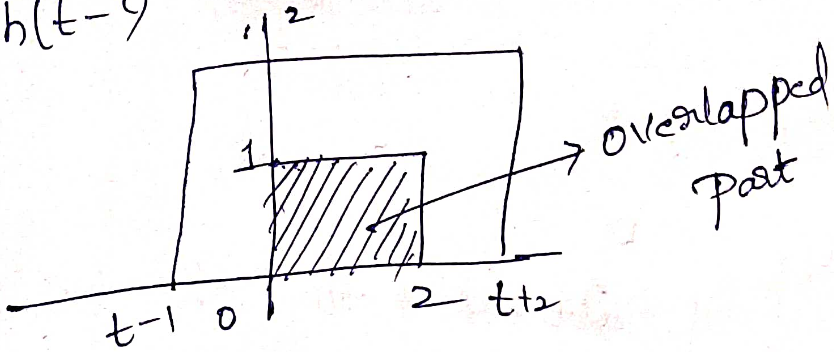
$h(-\tau)$



$h(t-\tau)$

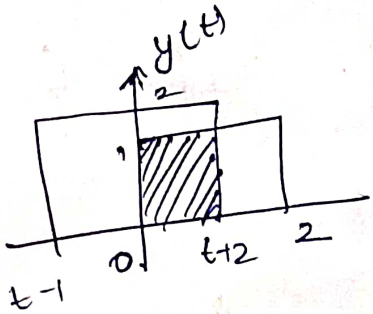


$x(\tau)\cdot h(t-\tau)$



If ~~t-1~~ $t-1<0$ ~~t+2>2~~

$0 \leq t \leq 1$

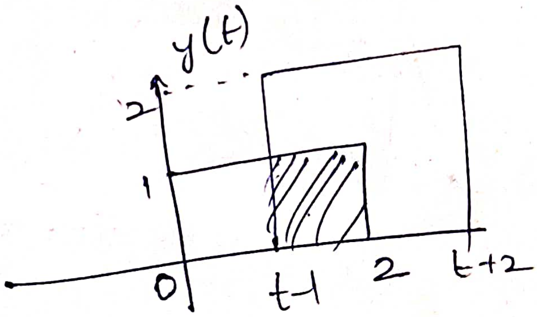


If $0 \leq t+2 \leq 2 \Rightarrow$

$-2 \leq t \leq 0$

If $0 \leq t-1 \leq 2 \Rightarrow$

$1 \leq t \leq 3$



for $0<t<1$

$$= \int_0^2 (1)(2)\, d\tau = 2[2-0] = 4$$

If $-2 \leq t \leq 0$

$$= \int_0^{t+2} 1(2)\, d\tau \Rightarrow 2[t+2-0]$$

$$= 2t+4$$

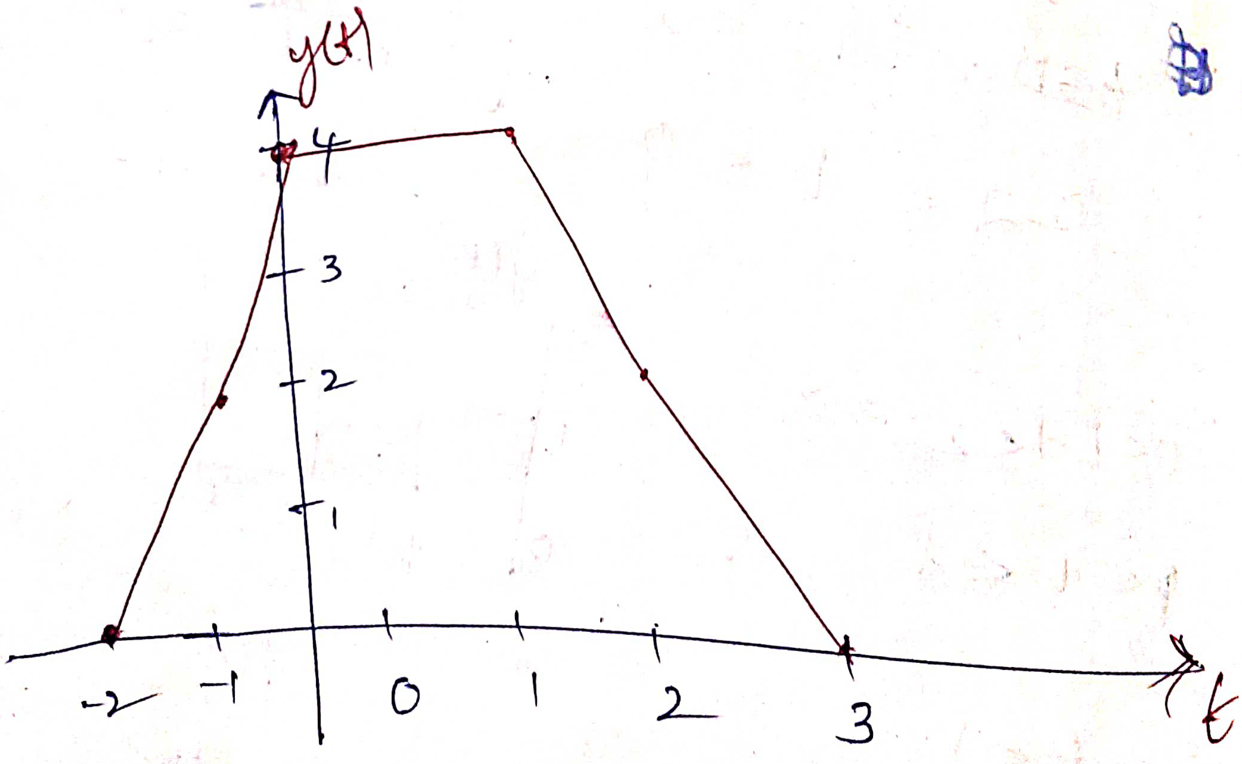
for $1 \le t \le 3$

$$= \int_{t-1}^{2} 2(1)\,dt = 2[2-t+1]$$

$$= 2[3-t]$$

$$= 6-2t.$$

$$y(t) = \begin{cases} 2t+4, & \text{for } -2 \le t \le 0 \\ 4; & \text{for } 0 \le t \le 1 \\ 6-2t; & \text{for } 1 \le t \le 3 \end{cases}$$

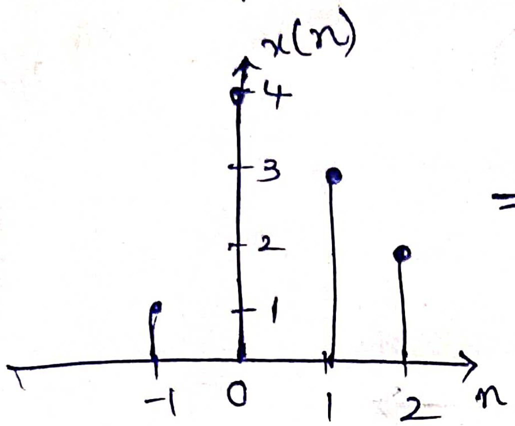


Convolution of 2 Sequences in the graphical method.

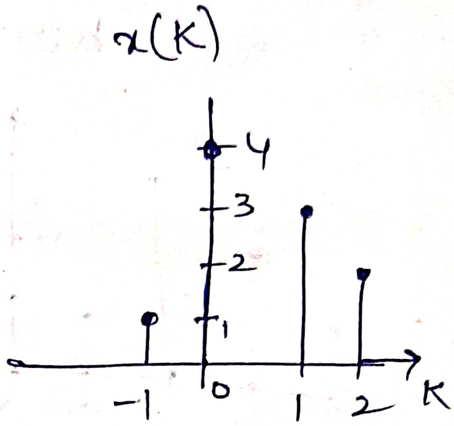
Q) Determine the Convolution of 2 Sequences.

$x(n) = \{1, \underset{\uparrow}{4}, 3, 2\}$; $h(n) = \{1, 3, 2, 1\}$

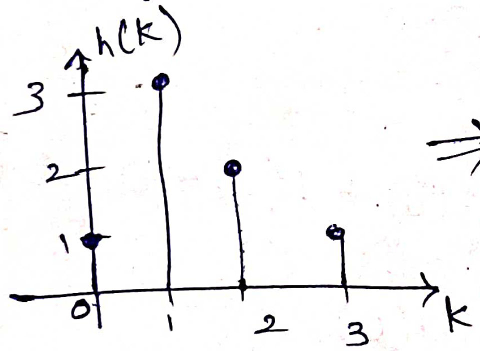
Given $x(n) = \{1, \underset{\uparrow}{4}, 3, 2\}$



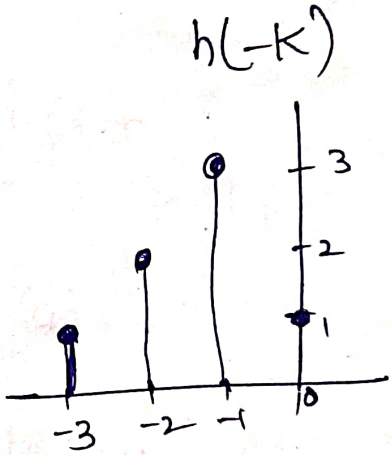
$\Rightarrow$



$h(n) = h(K) = \{1, 3, 2, 1\}$



$\Rightarrow$



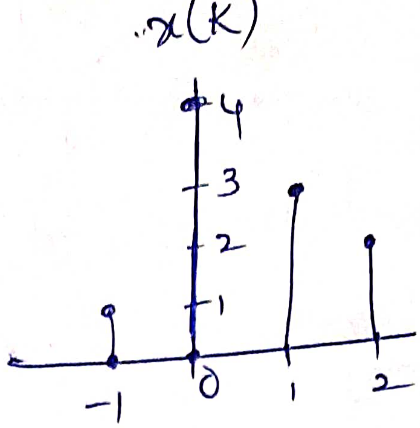
the Sequence, $x(n)$ is Starting at $n_1 = -1$ & $h(n)$ Sequence is Starting at $n_2 = 0$.

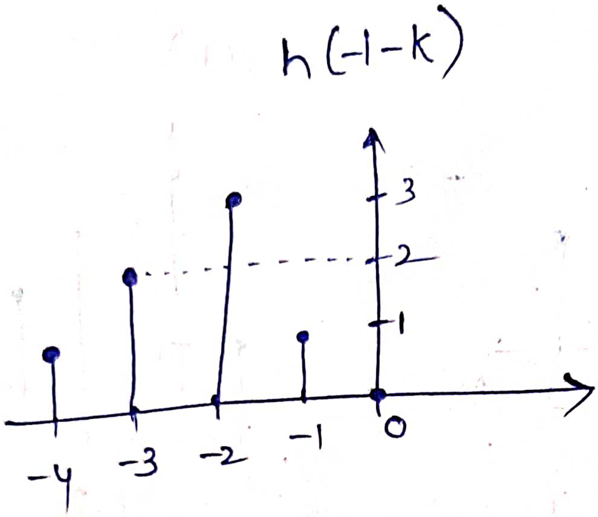
therefore, the starting time to Evaluate $y(n)$ is at $y(n) = n_1 + n_2 \Rightarrow -1 + 0 = -1$.

n value of $y(n)$ starts from '-1'.

$$y(n) = \sum_{K=-\infty}^{\infty} x(K) \cdot h(n-K).$$

$$y(-1) = \sum_{K=-\infty}^{\infty} x(K)\, h(-1-K)$$



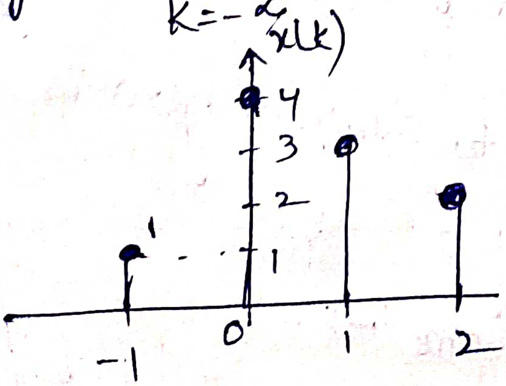


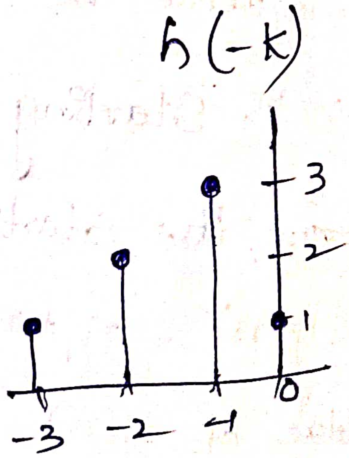
$$y(-1) = \sum_{k=-\infty}^{\infty} x(k) \cdot h(-1-k)$$

$$= 1(0) + 2(0) + 3(0) + 1(1) + 0 + 0 + 0 = 1$$

Next value is at $y(0)$

$$y(0) = \sum_{k=-\infty}^{\infty} x(k)\, h(-k)$$

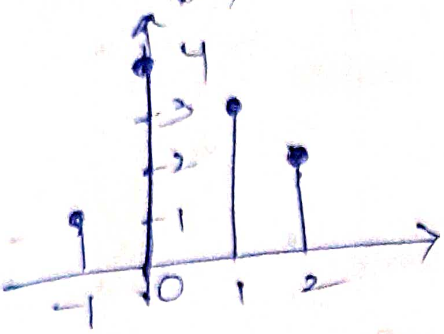


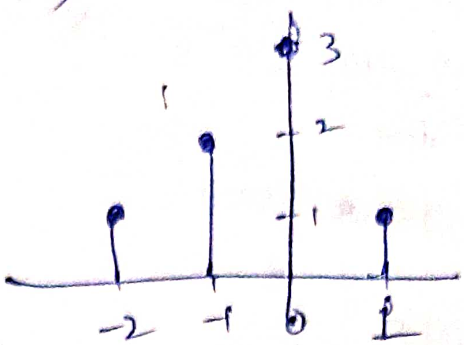


$$y(0) = 1(3) + 4(1) = 7$$

Next value is at $y(1)$

$$y(1) = \sum_{K=-\infty}^{\infty} x(k)\,h(1-k)$$

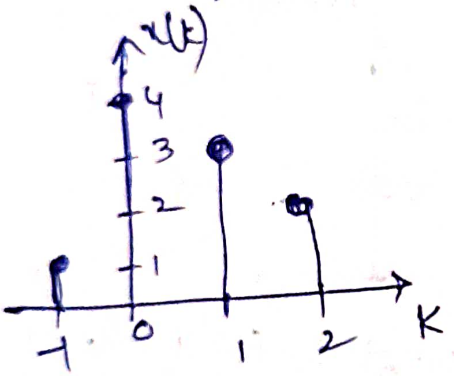


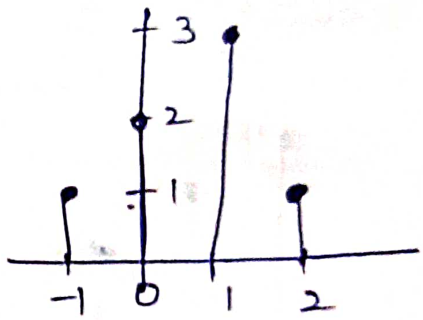


$$y(1) = 1(2) + 4(3) + 3(1) = 2+12+3 = 17.$$

Next value is at $y(2)$

$$y(2) = \sum_{K=-\infty}^{\infty} x(k)\cdot h(2-k)$$

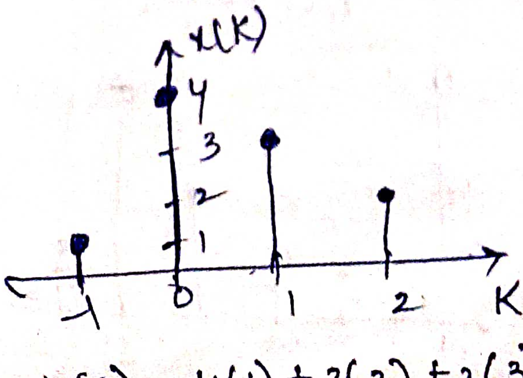


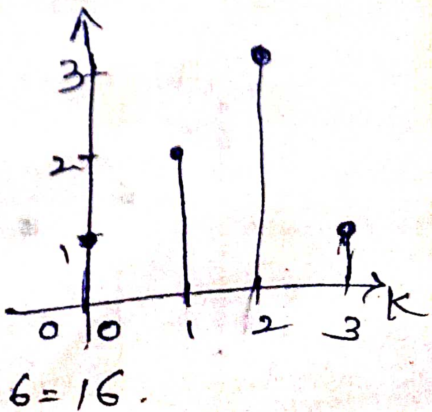


$$y(2) = 1(1) + 4(2) + 3(3) + 2(1)$$

$$= 1+8+9+2 = 20$$

$$y(3) = \sum_{K=-\infty}^{\infty} x(K)\,h(3-K).$$

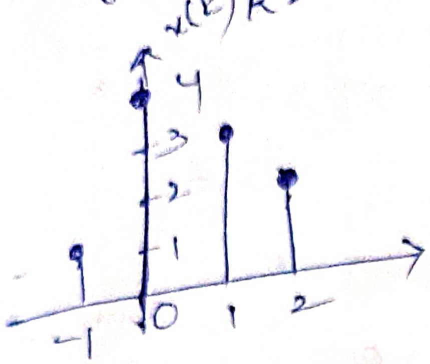


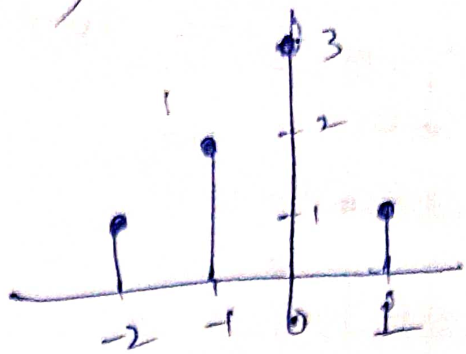


$$y(3) = 4(1) + 3(2) + 2(3) = 4+6+6 = 16.$$

Next value is at $y(1)$

$$y(1) = \sum_{k=-\infty}^{\infty} x(k)\, h(1-k)$$



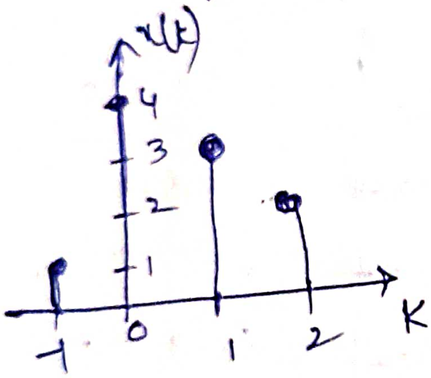


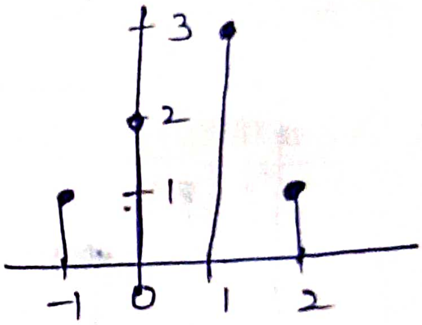
$$y(1) = 1(2) + 4(3) + 3(1) = 2 + 12 + 3 = 17.$$

Next value is at $y(2)$

$$y(2) = \sum_{k=-\infty}^{\infty} x(k) \cdot h(2-k)$$

$h(2-k)$



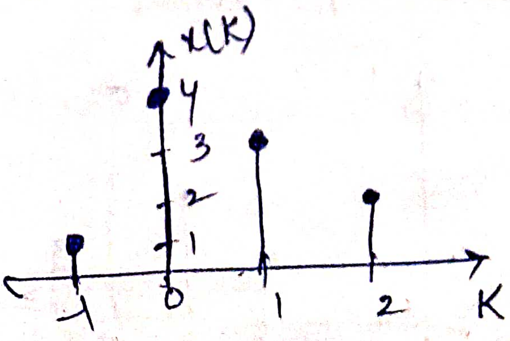


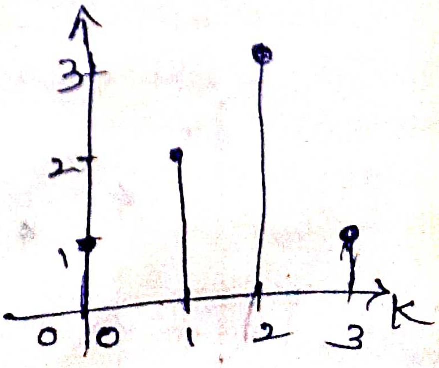
$$y(2) = 1(1) + 4(2) + 3(3) + 2(1)$$

$$= 1 + 8 + 9 + 2 = 20$$

$$y(3) = \sum_{k=-\infty}^{\infty} x(k)\, h(3-k).$$

$h(3-k)$



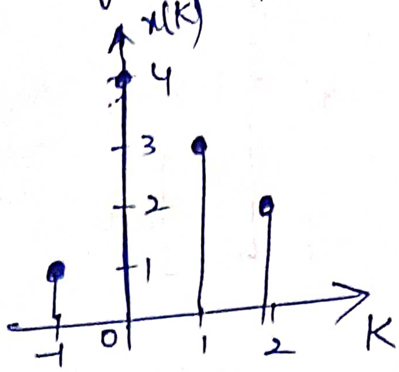


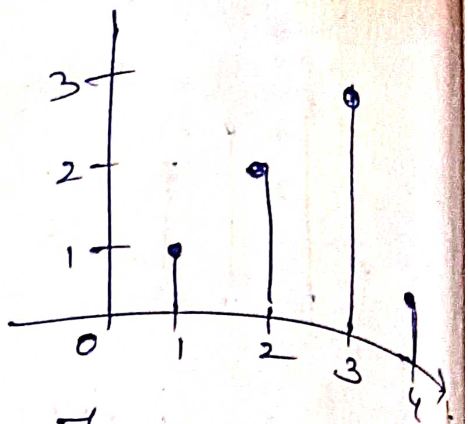
$$y(3) = 4(1) + 3(2) + 2(3) = 4 + 6 + 6 = 16.$$

Next Value is at $y(4)$

$$y(4) = \sum_{K=-\infty}^{\infty} x(K) \cdot h(4-K)$$

$h(4-k)$



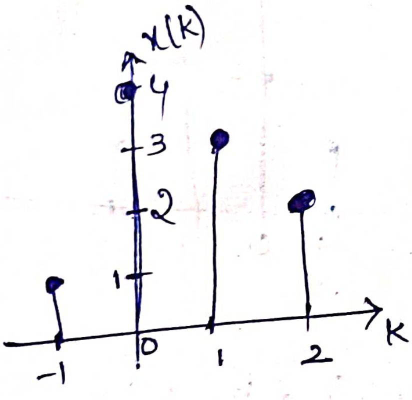


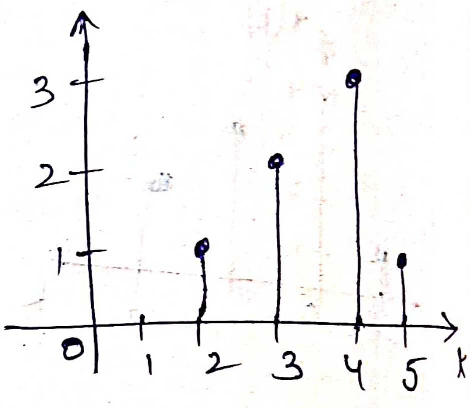
$$y(4) = 3(1) + 2(2) = 7$$

Next value is at $y(5)$.

$$y(5) = \sum_{K=-\infty}^{\infty} x(K) \cdot h(5-K).$$

$h(5-k)$



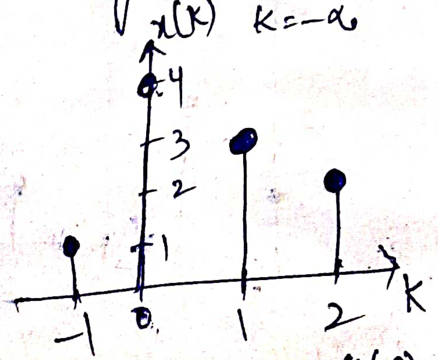


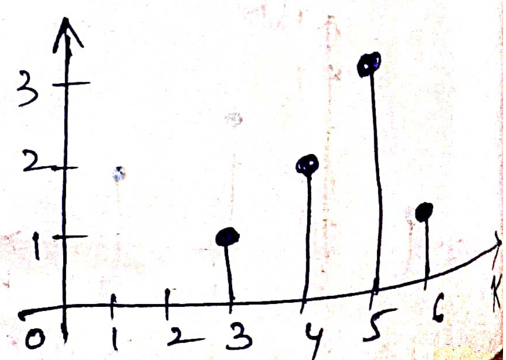
$$y(5) = 2(1) = 2$$

Next value is at $y(6)$

$$y(6) = \sum_{K=-\infty}^{\infty} x(k) \cdot h(6-K)$$

$h(6-k)$

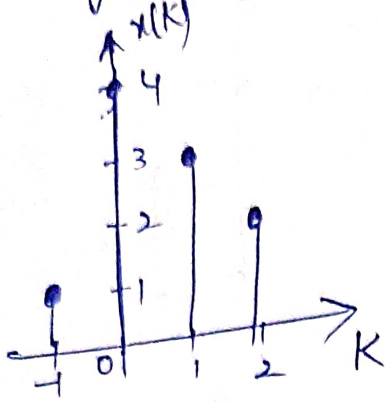




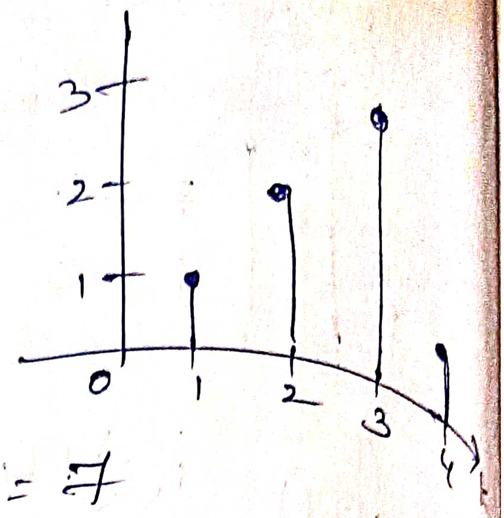
$y(6) = 0.$

Next value is at $y(4)$

$$y(4) = \sum_{k=-\infty}^{\infty} x(k)\cdot h(4-k)$$



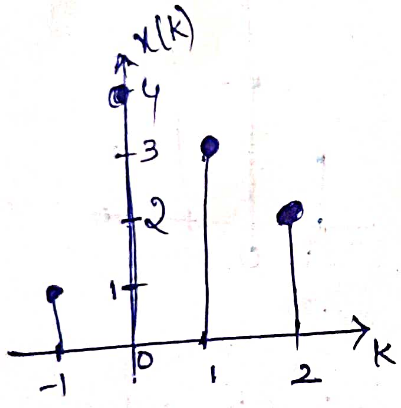
$h(4-k)$



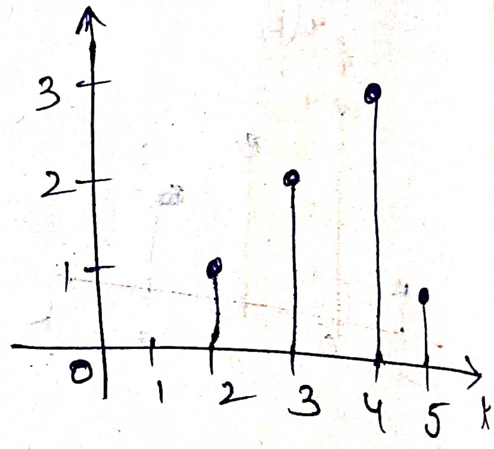
$$y(4) = 3(1) + 2(2) = 7$$

Next value is at $y(5)$.

$$y(5) = \sum_{k=-\infty}^{\infty} x(k)\cdot h(5-k).$$



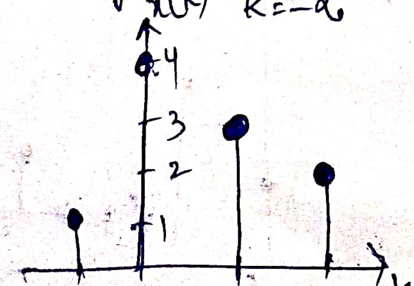
$h(5-k)$



$$y(5) = 2(1) = 2$$

Next value is at $y(6)$

$$y(6) = \sum_{k=-\infty}^{\infty} x(k)\cdot h(6-k)$$



$h(6-k)$

Fourier Series.

$y(n) = \{1, \underset{\uparrow}{7}, 17, 20, 16, 7, 2\}$.

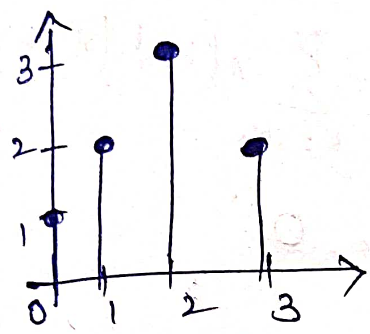
Q) Determine the output response $y(n)$ if

$x(n) = \{1, 2, 3, 2\}$; $h(n) = \{1, 2, 2\}$.

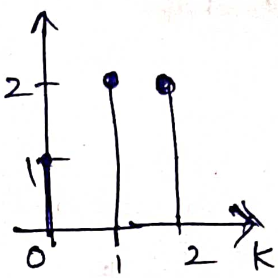
Both $x(n)$ & $h(n)$ starts at $n=0$.

then $y(n)$ starts at $n_1+n_2 = 0+0 = 0$.

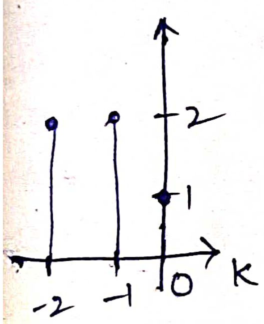
$x(k)$



$h(k)$



$h(-k)$

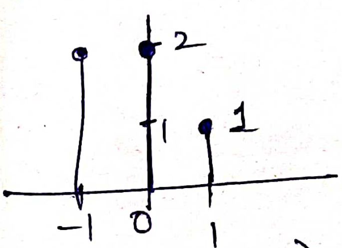


$$y(0) = \sum_{k=-\infty}^{\infty} x(k) \cdot h(0-k)$$

$$= \sum_{k=-\infty}^{\infty} x(k)\, h(-k)$$

$$= 1(1) = 1$$

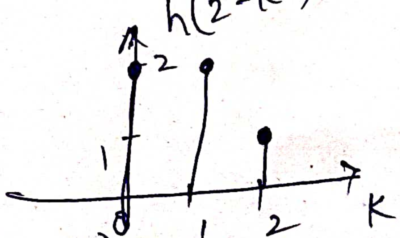
$h(1-k)$



$$y(1) = \sum_{k=-\infty}^{\infty} x(k) \cdot h(1-k)$$

$$= 1(2) + 1(2) = 4$$

$h(2-k)$



$$y(2) = \sum_{k=-\infty}^{\infty} x(k) \cdot h(2-k)$$

$2(1) + 2(2) + 1(3) = 2+4+3$

$= \cancel{2(1)+3(1)}$ $= 9$

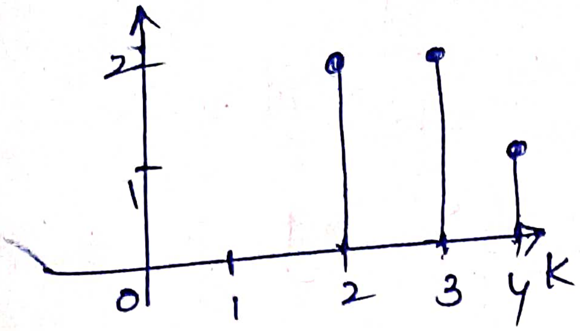
$h(3-k)$

$$y(3) = \sum_{k=-\infty}^{\infty} x(k) \cdot h(3-k)$$

$= 2(2) + 3(2) + 2(1) = 4+6+2$

$= 12$

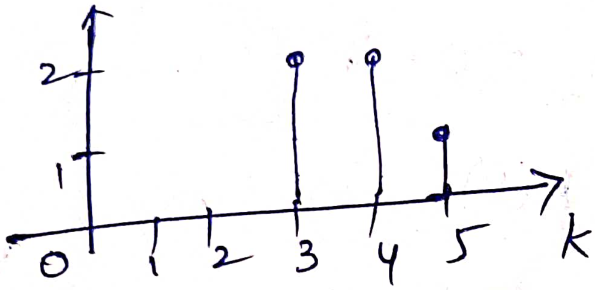
$h(4-k)$



$$y(4) = \sum_{k=-\infty}^{\infty} x(k)\,h(4-k)$$

$$= 3(2) + 2(2) = 6+4 = 10$$

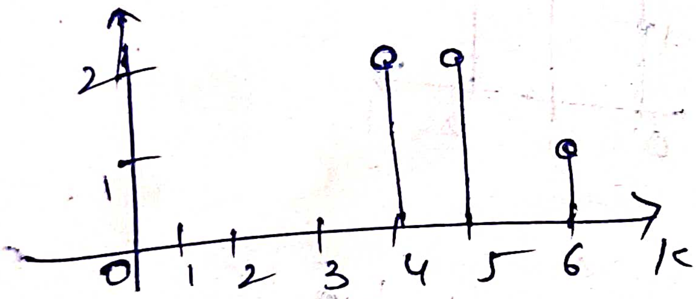
$h(5-k)$



$$y(5) = \sum_{k=-\infty}^{\infty} x(k).h(5-k)$$

$$= 2(2) = 4.$$

$h(6-k)$



$$y(6) = \sum_{k=-\infty}^{\infty} x(k).h(6-k)$$

$$= 0.$$

$$y(n) = \{1, 4, 9, 12, 10, 4\}$$