

Assignment - II

1) Design a CFG for

$$L = \{a^n b^n, \text{ where } n \geq 1\}$$

Given

$$L = \{a^n b^n, \text{ where } n \geq 1\}$$

$$L = \{ab, aabb, aaabbb, \dots\}$$

CFG:-

$$S \rightarrow asb / ab$$

$$S \rightarrow asb / ab / \epsilon$$

2) Design a CFG for

$$L = \{a^n b^n c^m / \text{where } n, m \geq 1\}$$

Given

$$L = \{a^n b^n c^m, \text{ where } n, m \geq 1\}$$

$$L = \{abc, aabbcc, aabbccc, aaabbbccc, \dots\}$$

CFG:-

$$S \rightarrow AB$$

$$A \rightarrow aAb / ab$$

$$B \rightarrow CB / c$$

3) Design a CFG for

$$L = \{a^n b^{2n} / \text{where } n \geq 1\}$$

Given

$$L = \{a^n b^{2n}, \text{ where } n \geq 1\}$$

$$L = \{abb, aabbbb, aaabbbbbbb, \dots\}$$

CFG:-

$$S \rightarrow asbb / abb$$

4) Design a CFG for

$$L = \{a^n b^m c^n \mid \text{where } n, m \geq 1\}$$

Given

$$L = \{a^n b^m c^n, \text{ where } n, m \geq 1\}$$

$$L = \{abc, abbc, aabbcc, \dots\}$$

CFG:-

$$S \rightarrow asc/A$$

$$A \rightarrow bA/b$$

5) Consider the following CFG

$$S \rightarrow AIB$$

$$A \rightarrow 0A/\epsilon$$

$$B \rightarrow 0B/1B/\epsilon$$

for the string $w = 00101$ find the Left derivative, Right derivative and parse tree.

Given CFG

$$S \rightarrow AIB$$

$$A \rightarrow 0A/\epsilon$$

$$B \rightarrow 0B/1B/\epsilon$$

Left Derivative:-

$$S \rightarrow \underline{A}IB$$

$$S \rightarrow 0\underline{A}IB$$

$$S \rightarrow 00\underline{A}IB$$

$$S \rightarrow 00\underline{\epsilon}IB$$

$$S \rightarrow 001\underline{B}$$

$$S \rightarrow 0010\underline{B}$$

$$S \rightarrow 00101\underline{B}$$

$$S \rightarrow 00101$$

$$w = 00101$$

$$\therefore S \rightarrow AIB$$

$$\therefore A \rightarrow 0A$$

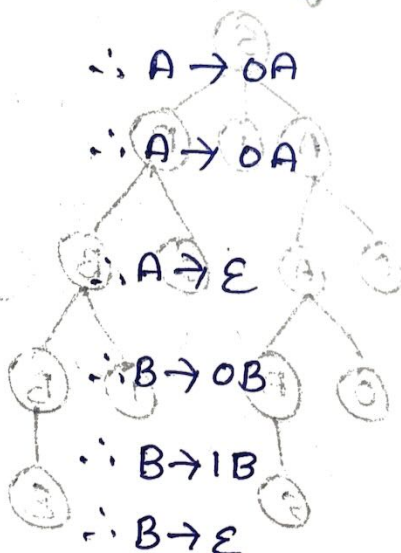
$$\therefore A \rightarrow 0A$$

$$\therefore A \rightarrow \epsilon$$

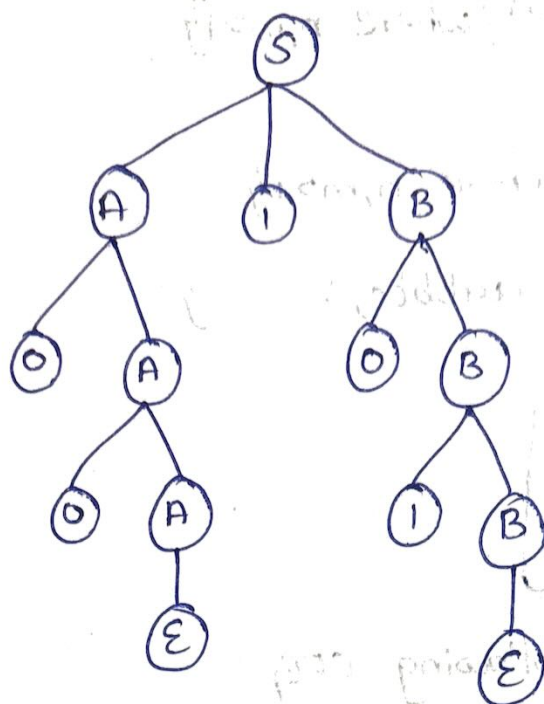
$$\therefore B \rightarrow 0B$$

$$\therefore B \rightarrow 1B$$

$$\therefore B \rightarrow \epsilon$$



parse tree for left derivative:-



Right Derivative:-

$$S \rightarrow AIB$$

$$\therefore S \rightarrow AIB$$

$$S \rightarrow AIOB$$

$$\therefore B \rightarrow OB$$

$$S \rightarrow AIOIB$$

$$\therefore B \rightarrow IB$$

$$S \rightarrow AIOI$$

$$\therefore B \rightarrow \epsilon$$

$$S \rightarrow OAIOI$$

$$\therefore A \rightarrow OA$$

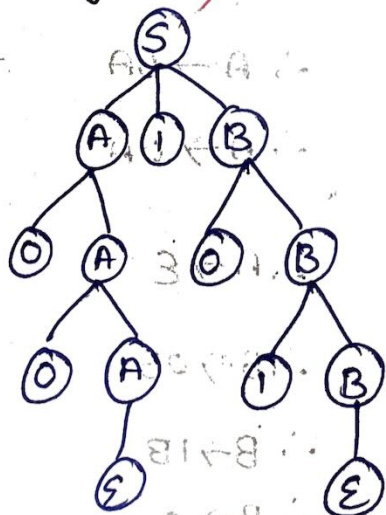
$$S \rightarrow OOAIOI$$

$$\therefore A \rightarrow OA$$

$$S \rightarrow OOOIOI$$

$$\therefore A \rightarrow \epsilon$$

parse tree for Right derivative:-



6. Consider the following CFG

$$S \rightarrow bB/aA$$
$$A \rightarrow b | bs | aAA$$
$$B \rightarrow a/as/bBB$$

for the string $w = bbaababa$ find Left derivative, Right derivative and parse tree.

Given CFG

$$S \rightarrow bB/aA$$

$A \rightarrow b / bs / aAA$

$$w = bbaababa$$
$$B \rightarrow a/as/bBB$$

Left derivative:-

$$S \rightarrow bB$$
$$\therefore S \rightarrow bB$$
$$S \rightarrow b b \underline{B B}$$
$$\therefore B \rightarrow bBB$$

$S \rightarrow bba \underline{B}$

$$\therefore B \rightarrow a$$

$s \rightarrow bbaas$

$$\therefore B \rightarrow as$$

$S \rightarrow bbaabB$

$$\therefore S \rightarrow bB$$

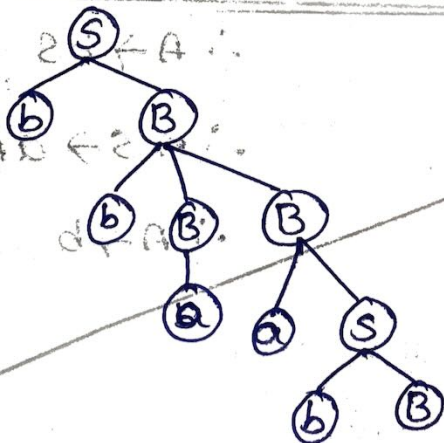
$s \rightarrow bbaabas$

$$\therefore B \rightarrow aS$$

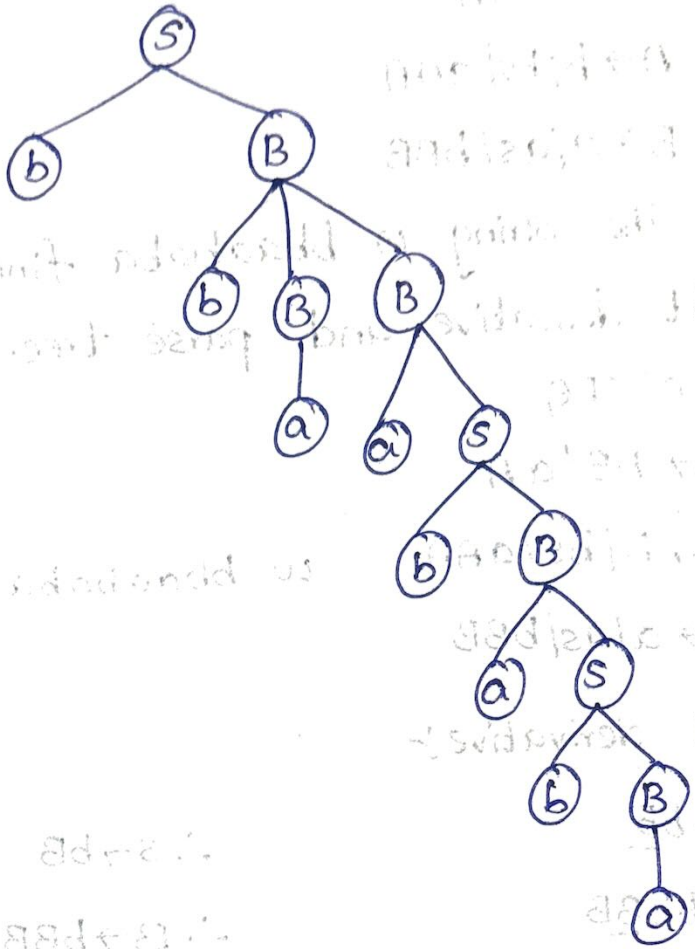
$S \rightarrow bbaababB$

$$\therefore S \rightarrow bB$$

$S \rightarrow bbaababa$

$$\therefore B \rightarrow a$$


Parse tree for Left derivative:-



Right derivative:-

$$S \rightarrow bB$$
$$\therefore S \rightarrow bB$$
$$S \rightarrow bbBB$$
$$\therefore B \rightarrow bBB$$
$$s \rightarrow bb\underline{B}a$$
$$\therefore B \rightarrow a$$

$s \rightarrow bba\underline{s}a$

$$\therefore B \rightarrow aS$$

$S \rightarrow bbaaAa$

$$S \rightarrow \alpha A$$

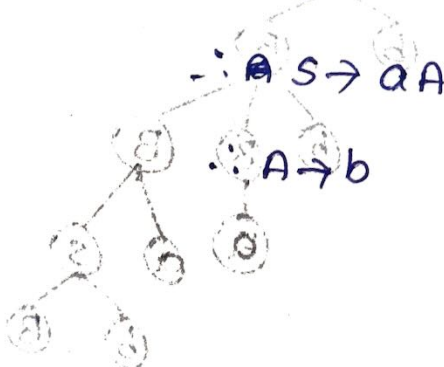
$S \rightarrow bbaabSa$

$$\therefore A \rightarrow bS$$

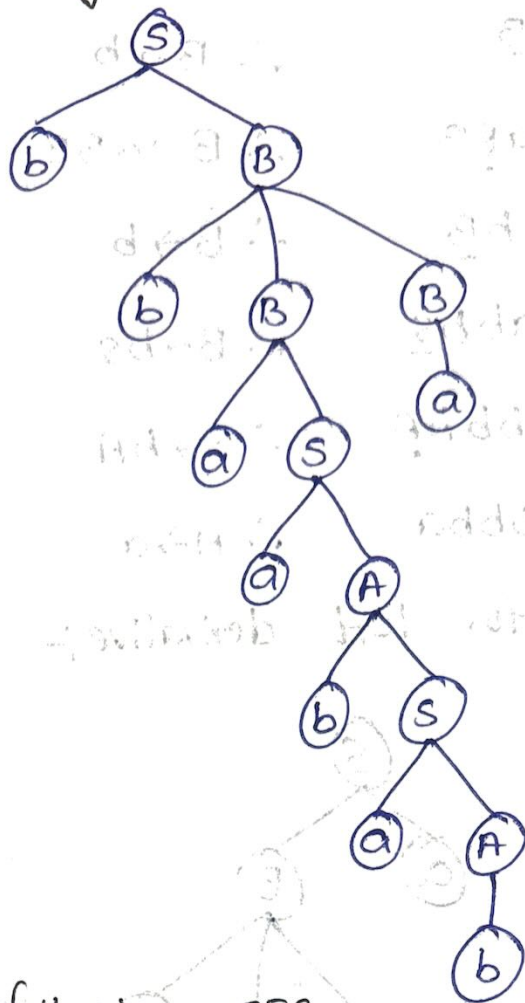
$S \rightarrow bbaabaa$

$$\therefore A \rightarrow QA$$

$S \rightarrow bbaababa$

$$A \rightarrow b$$


parse tree for Right Derivative:-



7) Consider the following CFG

$$S \rightarrow aB / bA$$

$$A \rightarrow as / bAA / a$$

$$B \rightarrow bs / aBB / b$$

for the string $w = aaabbbabbba$ find Left derivative
Right derivative and parse

Given:-

$$S \rightarrow aB / bA$$

$$A \rightarrow as / bAA / a$$

$$B \rightarrow bs / aBB / b$$

Left derivative:-

$$S \rightarrow a\underline{B}$$

$$S \rightarrow a\underline{a}BB$$

$$S \rightarrow aa\underline{a}BBB$$

$$w = aaabbbabbba$$

$$\therefore S \rightarrow aB$$

$$\therefore B \rightarrow aBB$$

$$\therefore B \rightarrow aBB$$

$S \rightarrow aabBB$

$$AB \rightarrow b$$
$$S \rightarrow aaabbb \underline{B}$$
$$\therefore B \rightarrow b$$
$$S \rightarrow aaabbaBB$$
$$\therefore B \rightarrow aBB$$
$$S \rightarrow aaabbaabB$$
$$\therefore B \rightarrow b$$
$$S \rightarrow aaabbaabbs$$
$$B \rightarrow bs$$

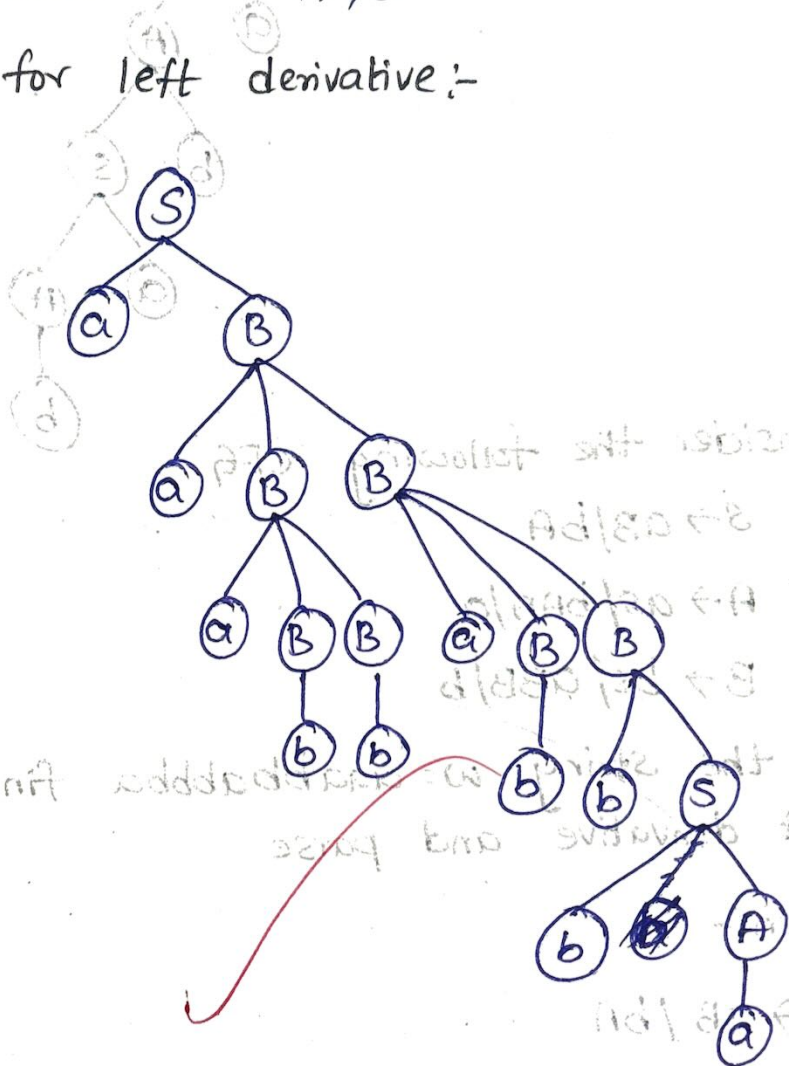
$S \rightarrow a a a b b a b b b A$

$$\therefore S \rightarrow bA$$

$S \rightarrow aaabbbabbbba$

$$\therefore A \rightarrow a$$

parse tree for left derivative:-



Right derivative:-

$$S \rightarrow aB$$
$$\therefore S \rightarrow aB$$
$$S \rightarrow a a B B$$
$$\therefore B \rightarrow aBB$$

$S \rightarrow aABbs$

$$\therefore B \rightarrow bs$$

$S \rightarrow aaBbbbA$

$$\therefore S \rightarrow bA$$

$S \rightarrow aa\underline{B}bba$

$\therefore A \rightarrow a$

$S \rightarrow aaa\underline{B}\underline{B}bba$

$\therefore B \rightarrow aBB$

$S \rightarrow aaaa\underline{B}Bbba$

$\therefore B \rightarrow b$

$S \rightarrow aaaa\underline{B}bba$

$\therefore B \rightarrow b$

$S \rightarrow aaaa\underline{b}Bbba$

$\therefore B \rightarrow bs$

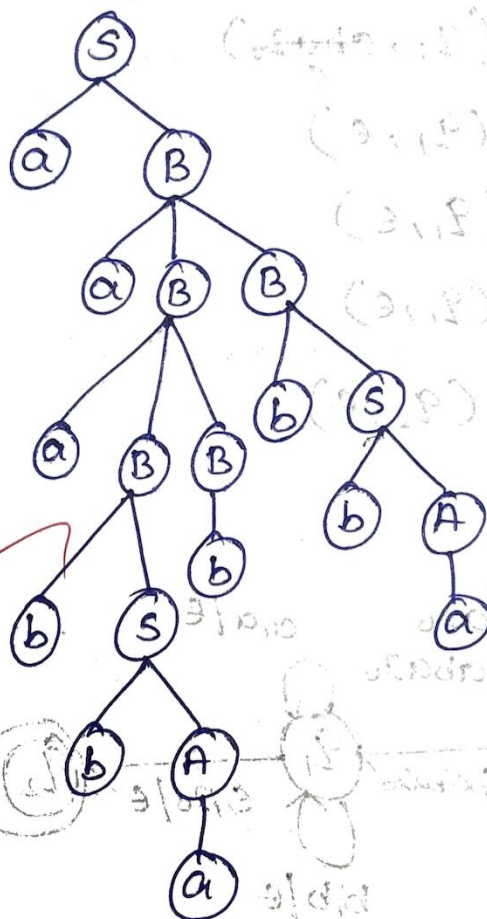
$S \rightarrow aaabb\underline{A}bba$

$\therefore S \rightarrow bA$

$S \rightarrow aaabbabbba$

$\therefore A \rightarrow a$

parse tree for Right derivative:-



8) Construct a PDA for following language

$L = \{wcw^r \mid \text{where } w = (a+b)^*\}$

Given

$L = \{wcw^r \mid \text{where } w = (a+b)^*\}$

$L = \{ abacaba, abcba, aabaaCaabaa, \dots \}$

$L = ababcbabac$

$$S(q_0, a, z_0) = (q_0, a z_0)$$

$$S(q_0, b, a) = (q_0, b a z_0)$$

$$S(q_0, a, b) = (q_0, a b a z_0)$$

$$S(q_0, b, a) = (q_0, b a b a z_0)$$

$$S(q_0, c, b) = (q_1, b a b a z_0)$$

$$S(q_1, b, b) = (q_1, a b a z_0)$$

$$S(q_1, a, a) = (q_1, \epsilon)$$

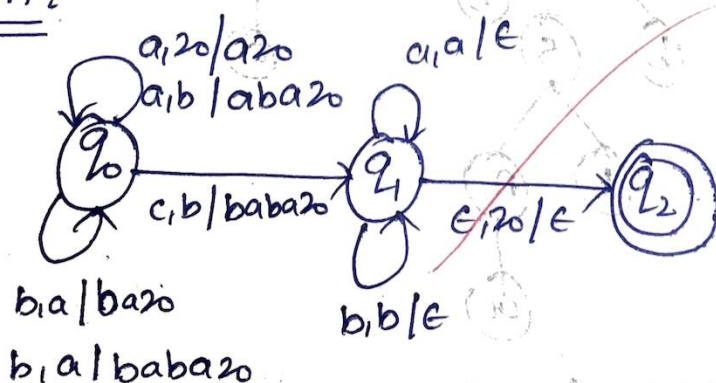
$$S(q_1, b, b) = (q_1, \epsilon)$$

$$S(q_1, a, a) = (q_1, \epsilon)$$

$$S(q_1, \epsilon, z_0) = (q_2, \epsilon)$$

b
a
b
a
z

PDA:-



⑨ Simplify the following CFG

$$S \rightarrow AB | a | \epsilon$$

$$A \rightarrow BC | b | c$$

$$B \rightarrow aB | \epsilon$$

$$C \rightarrow aC | B$$

Given

$$S \rightarrow AB/a/e$$

$$A \rightarrow Bc/b/c$$

$$B \rightarrow aB/e$$

$$C \rightarrow aD/B$$

1) Removal of unwanted Non-terminals

consider,

$$C \rightarrow aD/B$$

'D' is no productivity remove 'aD'.

$$C \rightarrow B$$

Since, C does not have terminals remove the productivity of 'C'.

consider,

$$A \rightarrow Bc/b/c$$

Since C does not have productivity remove BC, C

$$A \rightarrow b$$

$$\begin{aligned} S &\rightarrow AB/a/e \\ A &\rightarrow b \\ B &\rightarrow aB/e \end{aligned}$$

2) Removal of 'ε':-

$$S \rightarrow AB/a/e$$

$$A \rightarrow b$$

$$B \rightarrow aB/e$$

$$S \rightarrow \epsilon$$

$$S \rightarrow AB/a$$

$$A \rightarrow b$$

$$B \rightarrow aB/e$$

$$B \rightarrow e$$

$$S \rightarrow AB/a/A$$

$$A \rightarrow b$$

$$B \rightarrow aB/a$$

3) Removal of unit productivity :-

$$S \rightarrow AB/a/A$$

$$A \rightarrow b$$

$$B \rightarrow aB/a$$

Substitute $A \rightarrow b$ in $S \rightarrow A$.

$$S \rightarrow AB/a/b$$

$$A \rightarrow b$$

$$B \rightarrow aB/a$$

10) Can simplify the following CFG

$$S \rightarrow AB/AC/e$$

$$A \rightarrow bDA$$

$$A \rightarrow aAb/bAa/a/e$$

$$B \rightarrow bDA/acB/ABD$$

$$C \rightarrow abDA/acb/e$$

$$D \rightarrow bd/ac$$

Given

$$S \rightarrow AB/AC/e$$

$$A \rightarrow aAb/bAa/a/e$$

$$B \rightarrow bDA/acB/ABD$$

$$C \rightarrow abDA/acb/e$$

$$D \rightarrow bd/ac$$

i) Removal of unwanted Non-terminals

$$S \rightarrow AC/E$$

$$A \rightarrow aAb/bAa/a/e$$

$$C \rightarrow acb/e$$

ii)

ii) Removal of 'ε':-

$$\underline{S \rightarrow \epsilon}$$

$$S \rightarrow AC$$

$$A \rightarrow aAb/bAa/a/e$$

$$C \rightarrow acb/e$$

$$\underline{A \rightarrow \epsilon}$$

$$S \rightarrow AC/c$$

$$A \rightarrow aAb/bAa/a/ab/ba$$

$$C \rightarrow acb/e$$

$$\underline{C \rightarrow \epsilon}$$

$$S \rightarrow AC/c/A$$

$$A \rightarrow aAb/bAa/a/ab/ba$$

$$C \rightarrow acb/ab$$

iii) Removal of unit productivity

$$S \rightarrow AC/acb/ab/aAb/bAa/a/ba$$

$$A \rightarrow aAb/bAa/a/ab/ba$$

$$C \rightarrow acb/ab$$

11) Convert the following CFG into CNF

$$S \rightarrow AB/a/b$$

$$A \rightarrow b$$

$$B \rightarrow aB/a$$

The conditions for CNF

$$\boxed{\begin{array}{l} S \rightarrow AA \\ S \rightarrow a \end{array}}$$

$$S \rightarrow AB/a/b$$

$$A \rightarrow b$$

$$B \rightarrow aB/a$$

$$B \rightarrow aB$$

$$\boxed{D \rightarrow a}$$

$$\boxed{B \rightarrow DB}$$

$$\boxed{\begin{array}{l} S \rightarrow AB/a/b \\ A \rightarrow b \\ B \rightarrow DB/a \\ D \rightarrow a \end{array}}$$

12) Convert the following CFG into CNF

$$S \rightarrow AC/acb/ab/aAb/bAa/a/ba$$

$$A \rightarrow aAb/bAa/a/ab/ba$$

$$C \rightarrow acb/ab$$

The conditions for CNF

$$\boxed{\begin{array}{l} S \rightarrow AA \\ S \rightarrow a \end{array}}$$

Given

$S \rightarrow AC/acb/ab/aAb/bAa/a/ba$

$A \rightarrow aAb/bAa/a/ab/ba$

$C \rightarrow acb/ab$

$S \rightarrow acb$

$D \rightarrow a$

$E \rightarrow b$

$S \rightarrow DCE$

$F \rightarrow DC$

$S \rightarrow FE$ ✓

$S \rightarrow ab$

$S \rightarrow DE$ ✓

$S \rightarrow aAb$

$S \rightarrow DAE$

$G \rightarrow DA$

$S \rightarrow GE$ ✓

$S \rightarrow bAa$

$S \rightarrow EAD$

$I \rightarrow EA$

$S \rightarrow ID$ ✓

$S \rightarrow ba$

$S \rightarrow ED$

$A \rightarrow aAb$

$A \rightarrow DAE$

$A \rightarrow GE$

$A \rightarrow bAa$

$A \rightarrow EAD$

$A \rightarrow ID$

$A \rightarrow ab$

$A \rightarrow DE$

$A \rightarrow ba$

$A \rightarrow ED$

$C \rightarrow acb$

$C \rightarrow DCE$

$C \rightarrow FE$

$C \rightarrow ab$

$C \rightarrow DE$

The CNF is:-

$S \rightarrow AC/FE/DE/GE/ID/a/ED$

$A \rightarrow GE/ID/a/DE/ED$

$C \rightarrow FE/DE$

$D \rightarrow a$

$E \rightarrow a$

$F \rightarrow DC$

$G \rightarrow DA$

$$I \rightarrow EA$$

13) Convert the following CFG into GNF

$$S \rightarrow AB/a/b$$

$$A \rightarrow b$$

$$B \rightarrow DB/a$$

$$D \rightarrow a$$

let,

$$S \rightarrow A_1$$

$$A \rightarrow A_2$$

$$B \rightarrow A_3$$

$$D \rightarrow A_4$$

The resultant CFG is:-

$$A_1 \rightarrow A_2 A_3 / a / b \quad \text{--- ①}$$

$$A_2 \rightarrow b \quad \text{--- ②}$$

$$A_3 \rightarrow A_4 A_3 / a \quad \text{--- ③}$$

$$A_4 \rightarrow a \quad \text{--- ④}$$

$$A_1 \rightarrow A_2 A_3 / a / b$$

$i < j$ - leave as it is

$$A_3 \rightarrow A_4 A_3 / a$$

$i < j$ - leave as it is

sub equation '2' in '1'

$$A_1 \rightarrow b A_3 / a / b$$

sub equation '4' in '3'

$$A_3 \rightarrow a A_3 / a$$

The GNF is:-

$$A_1 \rightarrow bA_3/a/b$$

$$A_2 \rightarrow b$$

$$A_3 \rightarrow aA_3/a$$

$$A_4 \rightarrow a$$

14) Convert the following CFG into GNF

$$S \rightarrow AC/FE/DE/GE/ID/a/ED$$

$$A \rightarrow GE/ID/a/DE/ED$$

$$C \rightarrow FE/DE$$

$$D \rightarrow a$$

$$E \rightarrow a$$

$$F \rightarrow DC$$

$$G \rightarrow DA$$

$$I \rightarrow EA$$

The condition for GNF is

$$S \rightarrow a$$

$$S \rightarrow aBBAA \text{ ---}$$

let,

$$S \rightarrow A_1, A \rightarrow A_2, C \rightarrow A_3, D \rightarrow A_4, E \rightarrow A_5, F \rightarrow A_6, \\ G \rightarrow A_7, I \rightarrow A_8$$

The resultant CFG is:-

$$A_1 \rightarrow A_2 A_3 / A_6 A_5 / A_4 A_5 / A_7 A_5 / A_8 A_4 / a / A_5 A_4 \text{ --- (1)}$$

$$A_2 \rightarrow A_7 A_5 / A_8 A_4 / a / A_4 A_5 / A_5 A_4 \text{ --- (2)}$$

$$A_3 \rightarrow A_6 A_5 / A_4 A_5 \text{ --- (3)}$$

$$A_4 \rightarrow a$$

$$A_5 \rightarrow b$$

$$A_6 \rightarrow A_4 A_3 \quad \text{--- (4)}$$

$$A_7 \rightarrow A_4 A_2 \quad \text{--- (5)}$$

$$A_8 \rightarrow A_5 A_2 \quad \text{--- (6)}$$

$$A_1 \rightarrow A_2 A_3 / A_6 A_5 / A_4 A_5 / A_7 A_5 / A_8 A_4 / a / A_5 A_4$$

$i < j$ j j j j j j (leave as it is)

$$A_2 \rightarrow A_7 A_5 / A_8 A_4 / a / A_4 A_5 / A_5 A_4$$

$i < j$ j j j j (leave as it is)

$$A_3 \rightarrow A_6 A_5 / A_4 A_5$$

$i < j$ j (leave as it is)

$$A_6 \rightarrow A_4 A_3$$

$i > j$ (substitute)

→ substitute A_4 in equation (4)

$$(4) \Rightarrow A_6 \rightarrow A_4 A_3$$

$$\boxed{A_6 \rightarrow a A_3} \quad \text{--- (7)}$$

$$A_7 \rightarrow A_4 A_2$$

$i > j$ (substitute)

→ substitute A_4 in eq'n (5)

$$(5) \Rightarrow A_7 \rightarrow A_4 A_2$$

$$\boxed{A_7 \rightarrow a A_2} \quad \text{--- (8)}$$

$$A_8 \rightarrow A_5 A_2$$

$i > j$ (substitute)

$\rightarrow$ substitute A_5 in eq'n (6)

$$A_8 \rightarrow A_5 A_2$$

$$\boxed{A_8 \rightarrow b A_2}$$

$$\textcircled{3} \Rightarrow A_3 \rightarrow A_6 A_5 / A_4 A_5$$

$$\boxed{A_3 \rightarrow a A_3 A_5 / a A_5}$$

$$\left(\begin{array}{l} \because A_6 \rightarrow a A_3 \\ A_4 \rightarrow a \end{array} \right)$$

$$\textcircled{2} \Rightarrow A_2 \rightarrow A_7 A_5 / A_8 A_4 / a / A_4 A_5 / A_5 A_4$$

$$A_2 \rightarrow a A_2 A_5 / b A_2 A_4 / a / a A_5 / b A_4 \quad [\because A_7 \rightarrow a A_2$$

$$A_8 \rightarrow b A_2$$

$$\textcircled{1} \Rightarrow A_1 \rightarrow A_2 A_3 / A_6 A_5 / A_4 A_5 / A_7 A_5 / A_8 A_4 / a / A_5 A_4 \quad \left. \begin{array}{l} A_4 \rightarrow a \\ A_5 \rightarrow b \end{array} \right]$$

$$A_1 \rightarrow a A_2 A_5 A_3 / b A_2 A_4 A_3 / a A_3 / a A_5 A_3 / b A_4 A_3 / a A_3 A_5 / a A_5 / a A_2 A_5 / b A_2 A_4 / a / b A_4$$

The GNF is:-

$$A_1 \rightarrow a A_2 A_5 A_3 / b A_2 A_4 A_3 / a A_3 / a A_5 A_3 / b A_4 A_3 / a A_3 A_5 / a A_5 / a A_2 A_5 / b A_2 A_4 / a / b A_4$$

$$A_2 \rightarrow a A_2 A_5 / b A_2 A_4 / a / a A_5 / b A_4$$

$$A_3 \rightarrow a A_3 A_5 / a A_5$$

$$A_4 \rightarrow a$$

$$A_5 \rightarrow b$$

$$A_6 \rightarrow aA_3$$

$$A_7 \rightarrow aA_2$$

$$A_8 \rightarrow bA_2$$

SA

$$A \rightarrow aA$$

$$(aA \rightarrow aA) \quad (bA \rightarrow bA)$$

$$(aA \rightarrow aA) \quad (bA \rightarrow bA)$$

$$A \rightarrow aA$$

$$A \rightarrow aA$$

$$aA \rightarrow aA \quad bA \rightarrow bA$$

$$(aA \rightarrow aA)$$

$$aA \rightarrow aA \quad bA \rightarrow bA$$