SREENIVASA INSTITUTE OF TECHNOLOGY AND MANAGEMENT STUDIES
(Autonomous)
DEPARTMENT of MECHANICAL ENGINEERING

KINEMATICS OF MACHINERY (23MEC342T)

Class: II year IV Semester **Branch:** Mechanical Engg.

Prepared by

Dr. N. SATHISH KUMAR
Associate Professor
Department of Mechanical Engineering,
SITAMS, Chittoor

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Unit-1

Basics of Mechanism

* **Theory of Machines:-**

It is a branch of Science which deals with the Study of the relative motion of Various parts of a machine and also forces which acts on them.

It is classified into two types, they are

1) Kinematics of Motion
2) Dynamics of Motion

1) **Kinematics of Motion:-**

It is a study of relative motion b/w the Various parts of the machine. Here the Various forces involved in the motion are not Considered.

Kinematics is the Study to known the displacement, Velocity & Acceleration of a part of a machine.

2) **Dynamics of Motion:-**

It is a Study of relative motion b/w the Various parts of the machine. It involves with Considering forces. There are two types,

a) Kinetics
b) Statics

a) Kinetics:-

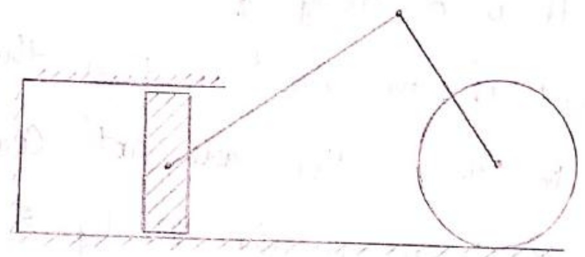
Various forces are Considered on a machine when a body is under motion

b) Statics:-

Various forces are Considered on a machine when a body is under rest (or) Stationary.

* Mechanism:-

It is a Combination of rigid and restraining links (or) bodies which are so shaped and Connected that they move upon each other with definite relative motion.



* Force:-

It is an External Agent which produces or tends to produce destroy or tends to destroy motion

* Resultant Force:-

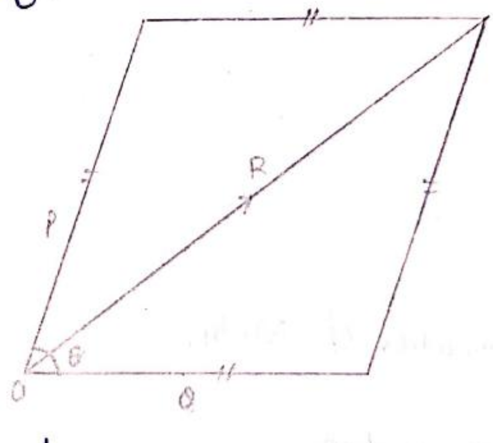
If a number of forces acting Simultaneously on a particle (or) Body then a single force which will produce a same effect as that of all the given forces.

* Composition of Forces:-

The process of getting resultant forces by Component forces [P, Q, R, etc.,] is Called as Composition of forces.

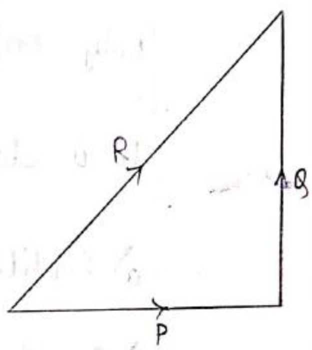
* Parallelogram law of forces:-

It states that if two forces acting Simultaneously on a particle be represented in magnitude and direction by the two adjacent Sides of a parallelogram their resultant may be represented in magnitude and direction by the diagonal of the parallelogram passing through the point 'O'.



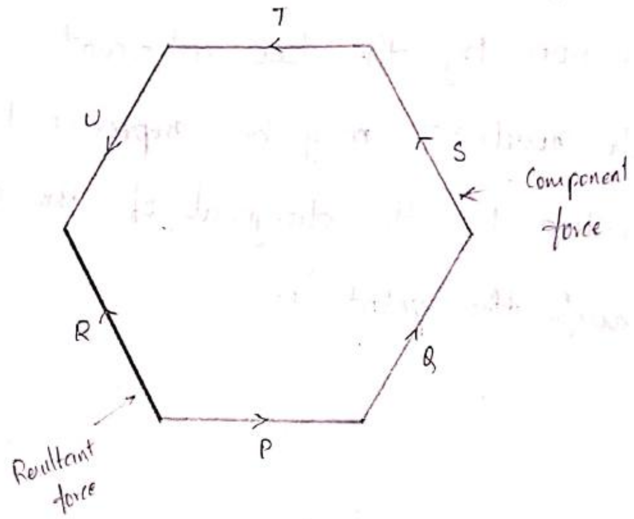
* Triangular law of forces:-

It states that if two forces acting Simultaneously on a particle be represented in magnitude and direction by the two Sides of a triangle their resultant may be represented in magnitude and direction by the third Side of a triangle taken in Opposite Order.



* polygon law of forces:-

It states that the number of forces acting simultaneously on a particle be represented in magnitude and direction by the side of polygon then the resultant may be represented in magnitude and direction by the closing side of polygon taken in opposite order.



* Kinematics of Motion:-

1) Plane motion:-

When the motion of a body is confined to only one plane then it is known as plane motion. It is classified into two types, they are

a) Rectilinear Motion

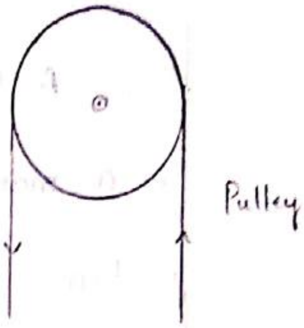
b) Curvilinear Motion

a) Rectilinear Motion:-

It is a straight line path. It is also known as Translatory motion

b) Curvilinear Motion:-

Moving along a curved path



* Kinematic Link:-

Each part of a machine which moves relative to the some other part is known as kinematic link

Ex:- Reciprocating Steam Engine

* Machine:-

It is a device which receives energy and transforms it into some useful work.

Machine Consists of number of parts.

* Types of Links:-

There are three types of links, They are

1) Rigid Link

2) Flexible Link

3) Fluid Link

1) Rigid Link:-

A Rigid link is one which does not undergo any deformation while transmitting motion.

Ex:- Connecting Rod, Crank of a Reciprocating steam Engine etc.,

2) Flexible Link:-

A Flexible Link is one which is partly deformed in a manner not to affect the transmission of motion

Ex:- Belts System, chain System, etc.,

3) Fluid Link:-

A Fluid Link is one which is formed by having a fluid in a receptacle and the motion is transmitted through the fluid by pressure or Compression only

Ex:- Hydraulic presses, Hydraulic brakes, Jacks, etc.,

* Kinematic pair:-

★★★

"Pair" is Any two links (or) elements of a machine when it Contact with each other is known as Pair.

A Joint of two links having relative motion between them is known as kinematic pair.

(or)

If the Relative motion b/w two links Completely (or) Succesfully Constrained motion [In a definite direction] that pair is known as kinematic pair.

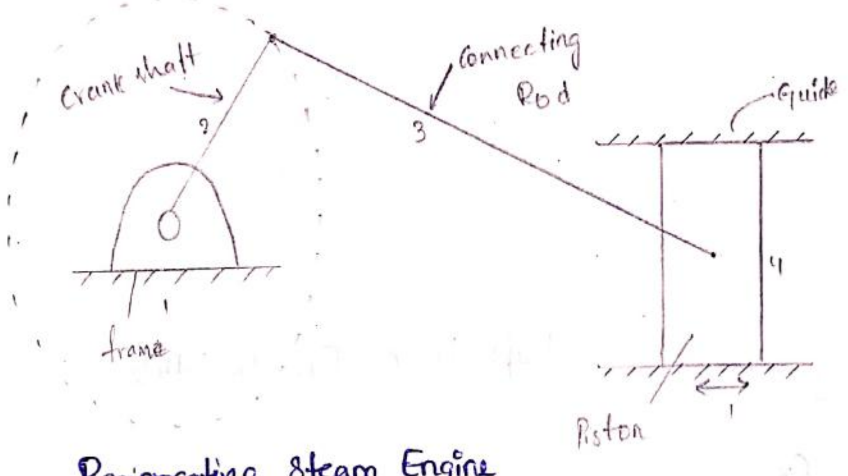
* Constrained Motions:-

Imp- There are classified into three types, they are

1) Complete Constrained motion [CCM]
2) Incomplete Constrained motion [ICM]
3) Succesfully Constrained motion [SCM]

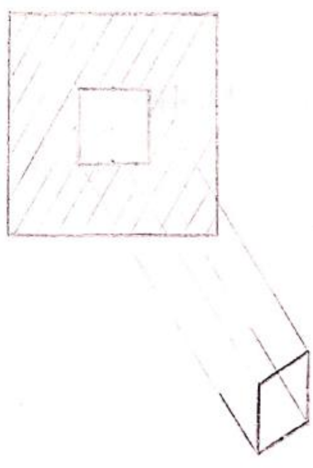
1) Complete Constrained Motion:-

1)



Reciprocating Steam Engine

2)



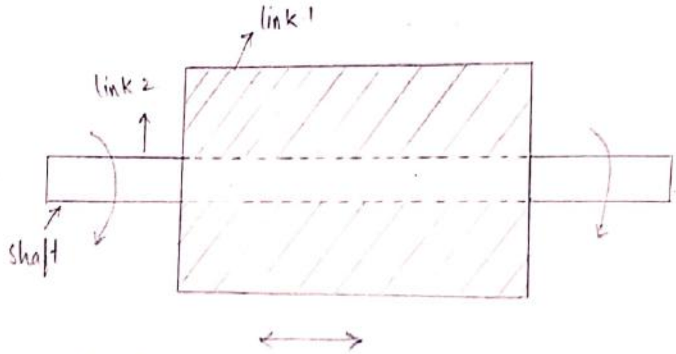
Square Bar in a Square Hole

When the Motion between a pair is limited to a definite direction irrespective of the direction of force applied. Then it is known as Completely Constrained motion

Ex:- 1) The piston & Cylinder in a Steam Engine
2) Motion of a Square bar in a Square plate (or) Hole

2) Incomplete Constrained Motion:-

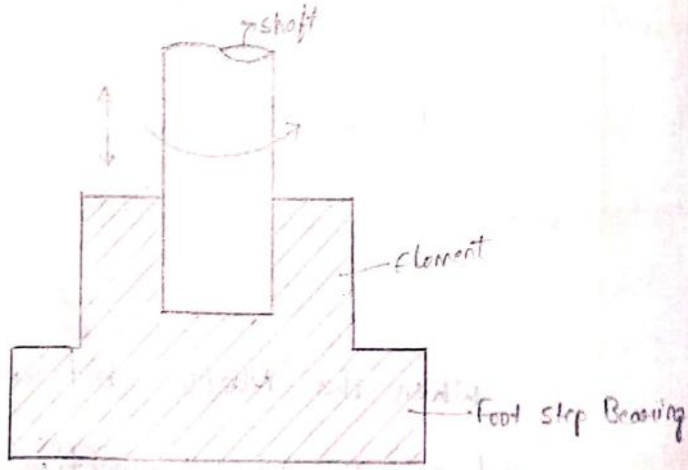
When the Motion between a pair Can takes Place more than one direction then it is known as Incomplete Constrained Motion.



Shaft in a Circular Hole

Ex:-
1) A Circular Shaft in a Circular Hole

3) Successfully Constrained Motion:-



Shaft on a foot step Bearing

When the motion between the elements forming the pair is such that the constrained motion is not completed by itself but by some other means then the motion is said to be successfully constrained motion.

Ex:- Shaft on a foot step bearing, The shaft may rotate in a bearing or it may move upwards in that case that type of motion is Incomplete constrained motion. But if the load is placed on the shaft to prevent axial upward moment. Then the motion of the pair is said to be successfully constrained motion

* Classification of kinematics pair:-

★★★

1) According to the relative motion b/w two links

- → Sliding pair
- → Turning pair
- → Rolling pair
- → Screw pair
- → Spherical pair

2) According to the type of Contact

- → Higher pair
- → Lower pair

3) According to type of closure

- → Self closed pair
- → Forced closed (or) Open pair

* Sliding pair:- [Completely Constrained Motion]

When the two elements of a pair are Connected in such a way that one can only slide relative to the other.

Ex:- 1) The piston and Cylinder

2) Tail Stock on a lathe bed

3) A square bar in a square hole

Sliding pair has a Completely Constrained motion

* Turning pair:-

When the two elements of a pair are Connected in such a way that one can only turn or revolve about a fixed axis of another link.

Ex:- 1) A circular shaft with Collars at both ends fitted in a Circular hole

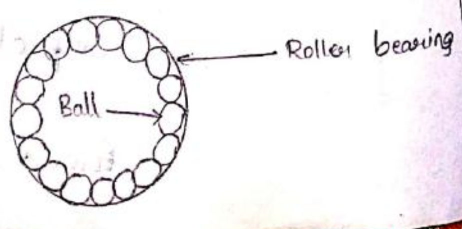
2) Spindle with lathe machine

Turning pair is also known as Completely Constrained motion

* Rolling pair:-

When the two elements of a pair are Connected in such a way that one rolls over the another fixed link

Ex:- Ball and Roller Bearings



* **Screw pair:-**

When the two elements of a pair are Connected in such a way that one element Can turn about the Other by Screw threads

Ex:- 1) Bolt & Nut

2) Water bottle & Cap

* **Spherical pair:-**

When the two elements of a pair are Connected in such a way that one element [Spherical shape] turns about the other fixed element

Ex:- 1) Attachment of a Car mirror

2) Pen stand

* **Lower pair:-**

When the two elements of a pair have a Surface Contact when relative motion takes place and the surface of one element slides over the surface of the Other.

Ex:- A Square bar in a Square hole

Sliding pair is known as lower pair & Screw pair, Turning pair is also known as lower pair

* **Higher pair:-**

When the two elements of a pair have a line (or) Point Contact when the relative motion takes place and the motion between the two elements is partly turning and partly sliding.

Ex:- 1) Ball and roller bearings

2) Cam & followers

3) Toothed gearings

* Self closed pair:-

When the two elements of a pair are Connected together mechanically in such a way that only a required kind of relative motion occurs.

Ex:-1) lower pairs

* Open pair:-

When the two elements of a pair are not Connected mechanically but are kept in Contact by the action of External force.

Ex:- Cam & follower

* Types of joints:-

They are classified into 3 types, they are

1) Binary joint → 

2) Ternary joint → 

3) Quaternary joint → 

According to A.W. Klein,

$$J + \frac{h}{2} = \frac{3}{2} l - 2$$

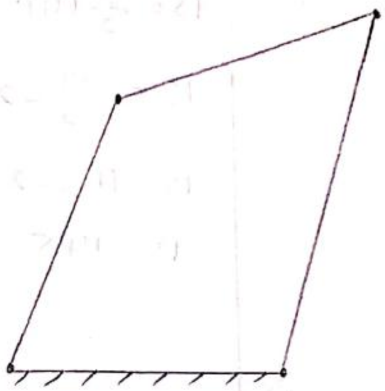
where,

J = No. of joints

h = No. of higher pairs

l = No. of links

1)



links = 4

Joints = 4

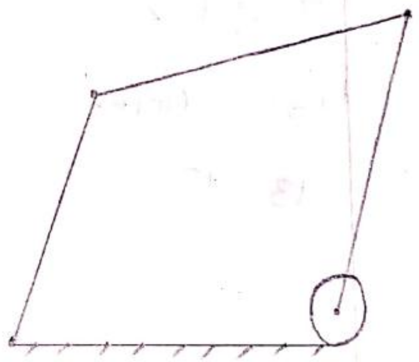
$$J + \frac{h}{2} = \frac{3}{2} l - 2$$

$$4 + \frac{0}{2} = \frac{3}{2}(4) - 2$$

$$4 = 6 - 2$$

$$4 = 4$$

2)



links = 4

Joints = 3

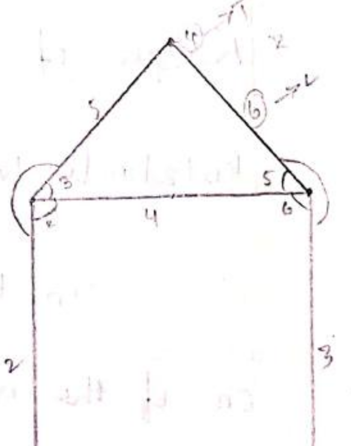
Higher pair = 1

$$J + \frac{h}{2} = \frac{3}{2} l - 2$$

$$3 + \frac{1}{2} = \frac{3}{2}(4) - 2$$

$$3.5 = 4$$

3) $l = 6$, $J = 7$,



$$J = \frac{3}{2} l - 2$$

$$7 = \frac{3}{2} \times 6 - 2$$

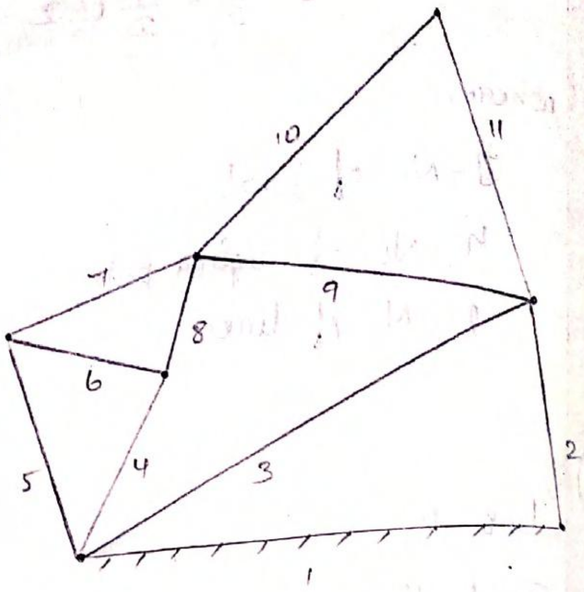
$$7 = 7$$

4) 2 Binary → 2

2 Terenary → 4

3 Quatenary → 9

$\overline{15}$



$J = 15$

$L = 11$

$J = \frac{3}{2}l - 2$

$15 = \frac{3}{2}(11) - 2$

$15 = \frac{33}{2} - 2$

$15 = 16.5 - 2$

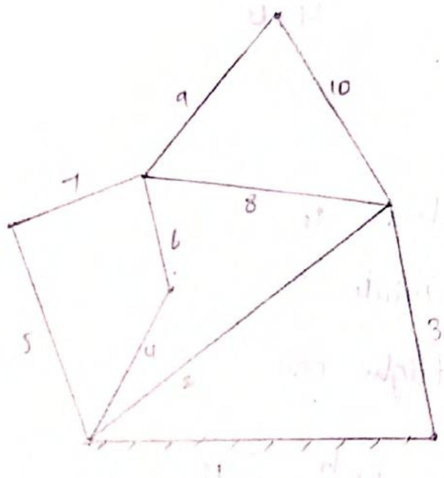
$15 = 14.5$

5)

4 binary → 4

3 quatenary → 9

$\overline{13}$



$J = \frac{3}{2}l - 2$

$13 = \frac{3}{2}(10) - 2$

$13 = 13$

* Degrees of freedom for a plane mechanism (or) Kutzbach Mechanism :-

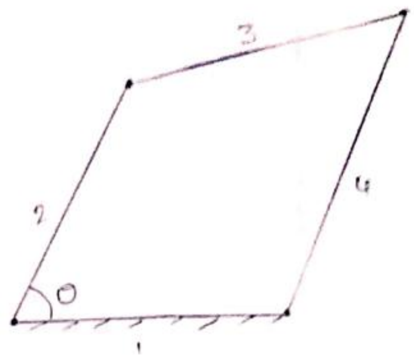
In the design (or) Analysis of a mechanism one of the most Concern is the no. of degrees of freedom

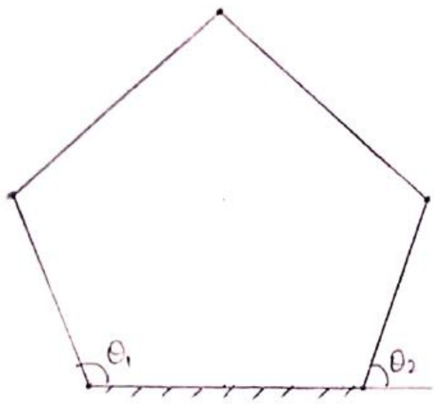
→ Degrees of freedom:-

The No. of input parameters [pair variables] which must be independently control in order to bring the mechanism into useful engineer purpose is called degrees of freedom

Ex:- 1) Four Bar Mechanism

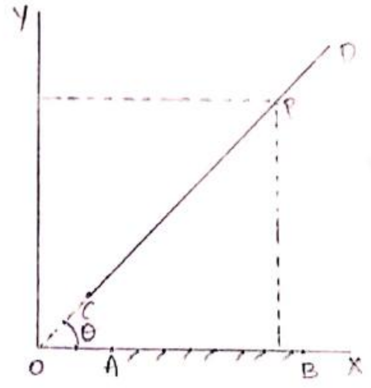
2) Five Bar Mechanism

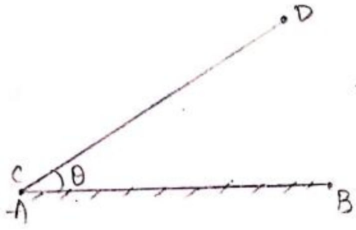




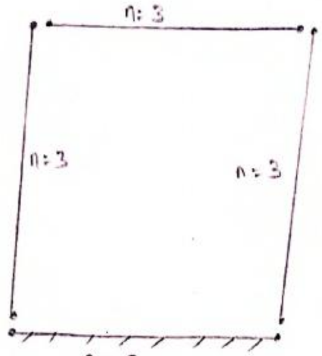
Proof:-

The link AB with Co-ordinate System Oxy as the reference link [Fixed link]. The position of point P on a movable link CD can be completely specified by 3 variables i.e., the Co-ordinates of point 'P' is denoted by 'x' and 'y' and the inclination 'θ' with the x-axis

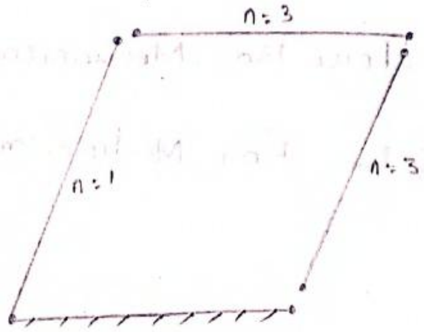




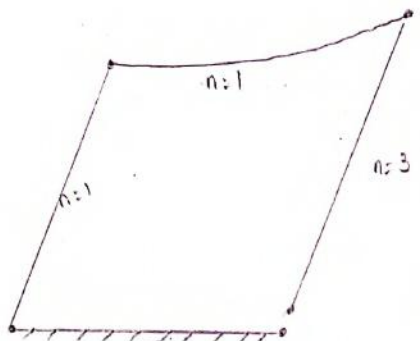
Each link of a mechanism has three degrees of freedom before it is connected to any other link. But when the link is connected to other link by a turning pair by a single variable 'θ'.



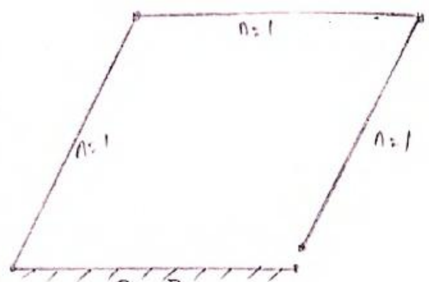
n = 9



n = 7



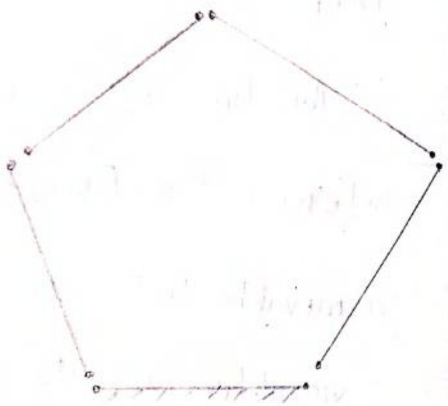
n: 5

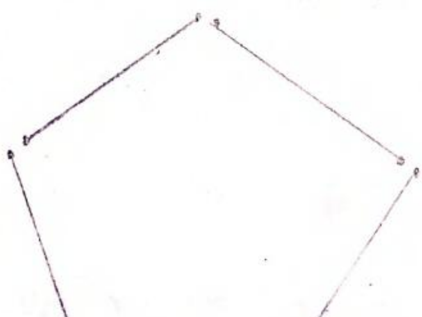


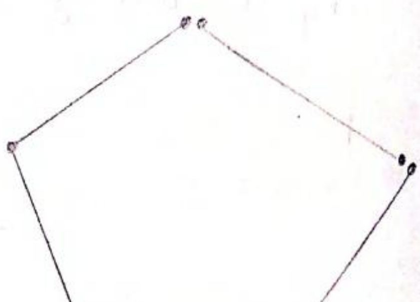
n = 3

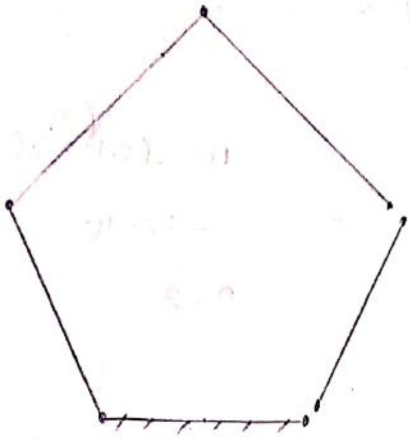
$$n = 3(l-1) - 2j - h$$
$$= 3(4-1) - 2(4) - 0$$
$$= 9 - 8$$
$$n = 1$$

n = 1

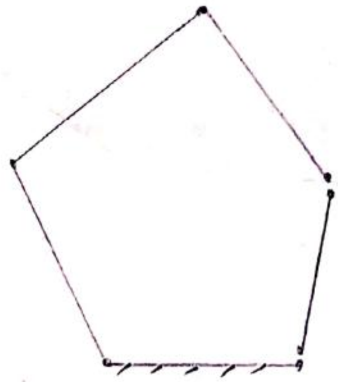

n = 12


n = 10

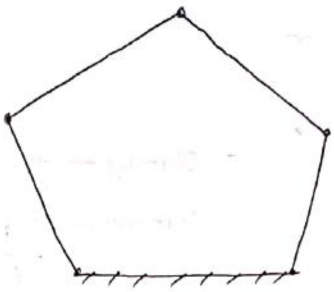

n = 8



$n = 6$



$n = 4$



$n = 2$

$$n = 3(l-1) - 2j - h$$
$$= 3(5-1) - 2(5)$$
$$= 12 - 10$$
$$n = 2$$

* Applications of Kutzbach Criterion:-

With no higher pair

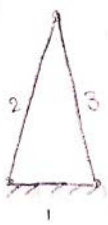
With higher pair

$$n = 3(l-1) - 2j - h$$

(h → Higher pair)

With No Higher Pair:-

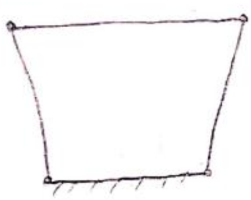
3 Bar Mechanism:-



$$n = 3(3-1) - 2(3) - 0$$
$$= 6 - 6$$
$$n = 0$$

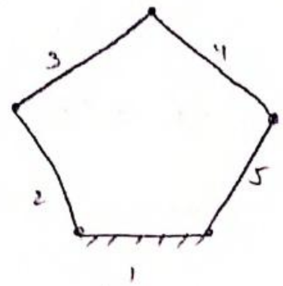
This mechanism is locked chain (or) Structure

4 Bar Mechanism:-



$$n = 3(4-1) - 2(4) - 0$$
$$= 9 - 8$$
$$n = 1$$

5 Bar Mechanism:-

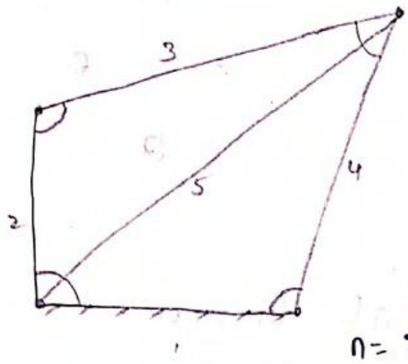


$n = 3(5-1) - 2(5) - 0$

$= 12 - 10$

$n = 2$

Two Seperate input motions to produce Constrained motion for the mechanism



2 Binary ⟶ 2

2 Terenary ⟶ 4

$J = 6$

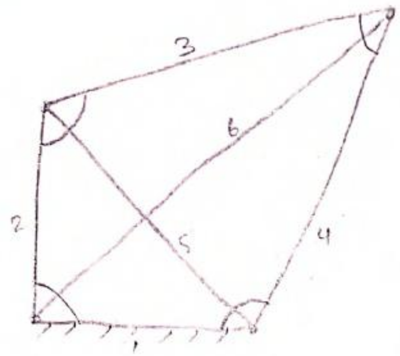
$L = 5$

$n = 3(l-1) - 2j - h$

$= 3(5-1) - 2(6) - 0$

$= 12 - 12$

$n = 0$ (Structure)



4 Terenary ⟶ 8

$J = 8$

$L = 6$

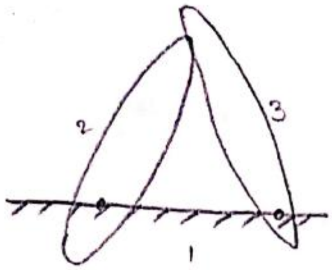
$n = 3(6-1) - 2(8) - 0$

$= 15 - 16$

$n = -1$ (Indeterminate Structure)

It is Reduntant Constrain (or) Indeterminate Structure

Now Considering a Higher pair Mechanism,



$l = 3$

$J = 2$

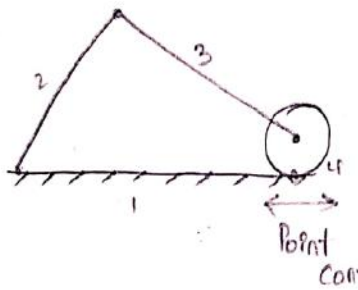
$H = 1$

$n = 3(l-1) - 2j - h$

$= 3(3-1) - 2(2) - 1$

$= 6 - 4 - 1$

$n = 1$



$L = 4$

$J = 3$

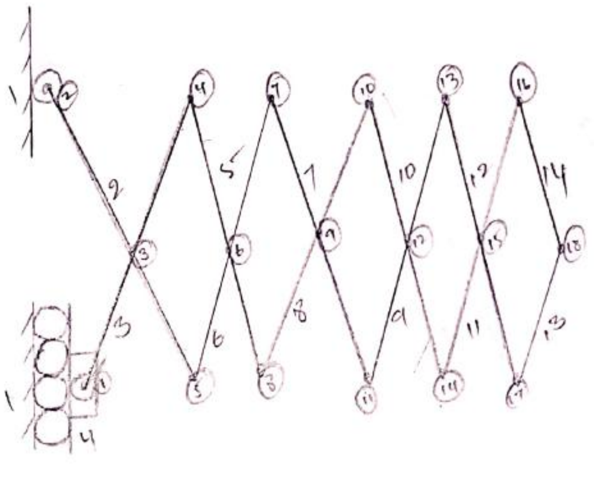
$H = 1$

$n = 3(4-1) - 2(3) - 1$

$= 9 - 6 - 1$

$= 9 - 7$

$n = 2$



$L = 14$

$J = 18$

$H = 1$

$n = 3(14-1) - 2(18) - 1$

$= 3(13) - 2(18) - 1$

$= 39 - 36 - 1$

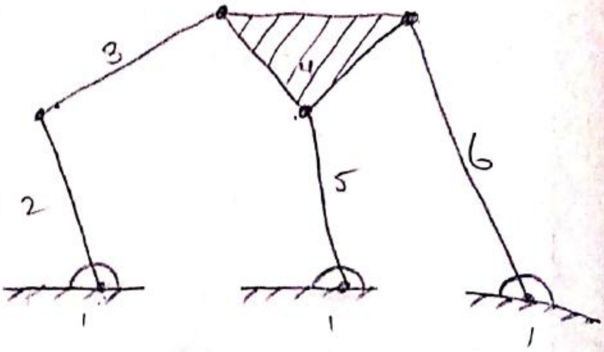
$= 39 - 37$

$n = 2$

$l=6$

$J=7$

$H=0$



$n=3(6-1)-2(7)-0$

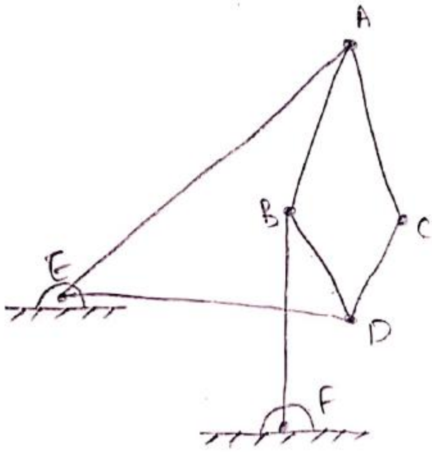
$=15-14$

$n=1$

$l=8$

$J=10$

$H=0$

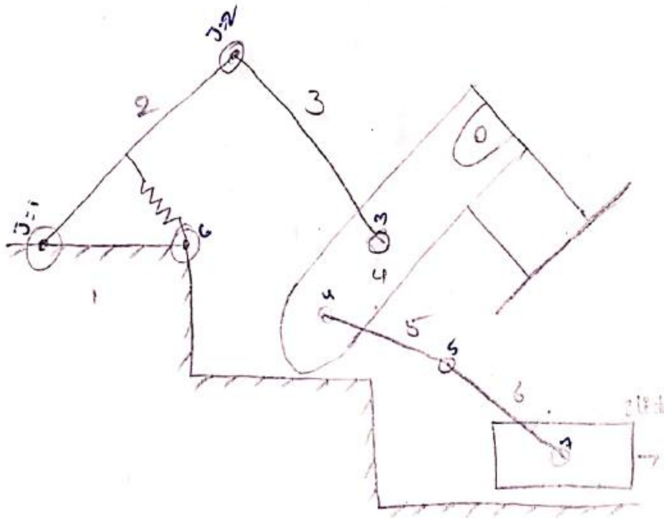


$n=3(l-1)-2j-h$

$=3(8-1)-2(10)-0$

$=21-20$

$n=1$



$L=7$

$J=$

$H=1$

$n=3(l-1)-2j-h$

$=3(7-1)-2(7))-1$

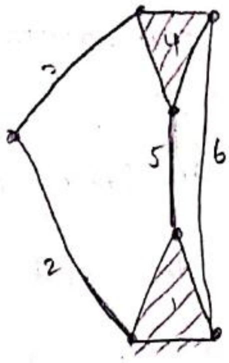
$n=18-15$

$n=3$

$L = 6$

$J = 7$

$H = 0$

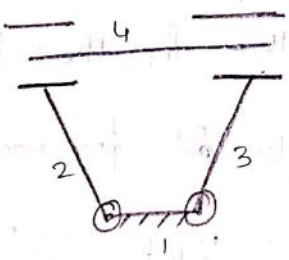


$n = 3(l-1) - 2j - h$

$= 3(6-1) - 2(7) - 0$

$= 15 - 14$

$n = 1$



In this mechanism, there are 4 links all are lower pairs only. But, link '4' is Capable of sliding without the help of remaining links. Therefore, This mechanism is a locked System. and also link '4' has redundant degree of freedom

* Grubler's Criterion:-

Grubler's Criterion applys to the mechanism with only Single degree of freedom joints. Therefore, the overall movability of mechanism is unity. Therefore, Substitute in Kutzbach Criterion

$n = 1, \; h = 0$

$n = 3(l-1) - 2j - h$

$1 = 3(l-1) - 2j - 0$

$3l - 3 - 2j - 1 = 0$

$$3l-2j-4=0$$

* Space mechanism:-

For space mechanism each link has 6 Degree of freedom and in a mechanism one link is fixed. Therefore, No. of movable links are 'l-1'. Then 'l-1' movable link will have '6(l-1)' Degree of freedom. But, Some of the joints will have only one D.O.F. Hence, for these joints 5 degree of freedom [D.O.F] will be lost for each joint. Similarly, some joints will have 2 D.O.F which means 4 D.O.F will be lost for each joint

$$n=6(l-1)-5\times g-4C-3S$$

where,

g = Total No. of sliding pairs [n=1]

C = Total No. of cylindrical pairs [n=2]

S = Total No. of spherical pairs [n=3]

* Inversion of Mechanism:-

In any mechanism, there is a one fixed link in 'n' number of mechanisms there is 'n' number of fixed links then inversion of mechanism is nothing but in this mechanism for finding different links fixed in a kinematic change is known as Inversion of mechanism.

It may be noted that the relative motion b/w the links is not changed in any manner through the Process of inversion but their absolute motion [measured w.r.to fixed link]. May be changed drastically.

Note:-

The part of the mechanism which initially moves w.r. to frame (or) fixed link is called as driver. and that part of the mechanism to which motion is transmitted. is called as follower. Most of the mechanism are reversible. So that the same link can play the role of driver as well as follower.

Ex:-1) In a reciprocating steam Engine → Piston - driver
Crankshaft - Follower

2) Reciprocating Air Compressor → Crankshaft - driver
Piston - Follower

→ classification [Inversion of M/m]:-

1) Four Bar Chain (or) quadric Cycle chain
- Beam Engine
- Coupling Rod
- Watt's Indicator M/m

2) Single slider Crank chain
- Bull Engine
- Oscillating Engine
- Crank & Slotted lever quick return motion m/m
- Whitworth quick return motion M/m
- Rotary I.C. Engine (or) Gnome Engine

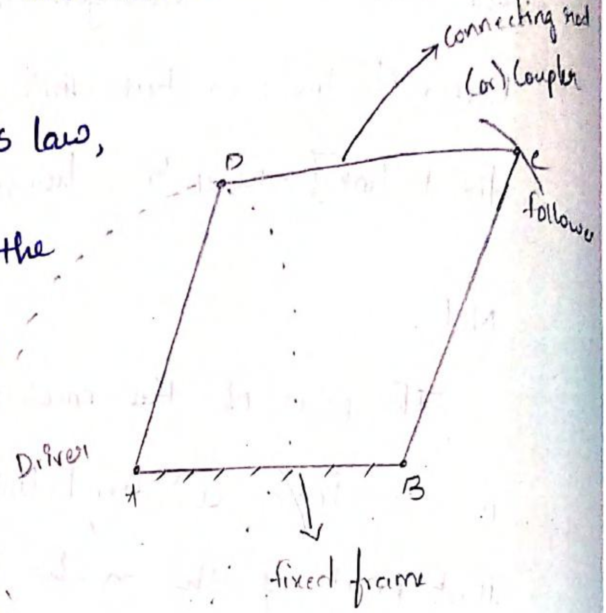
3) Double Slider Crank chain
- Oldham's Coupling
- Scotch yoke M/m
- Elliptical Trammels

* four Bar chain Mechanism :-

In a four bar chain mechanism, it is the simplest mechanism. It consists of only 4 links all are turning pairs. One of the link is fixed and the remaining links are movable

According to Grashof's law,

In any mechanism, the sum of smallest and longest link is not greater than the sum of the remaining links.



→ Inversion of four bar chain M/m :-

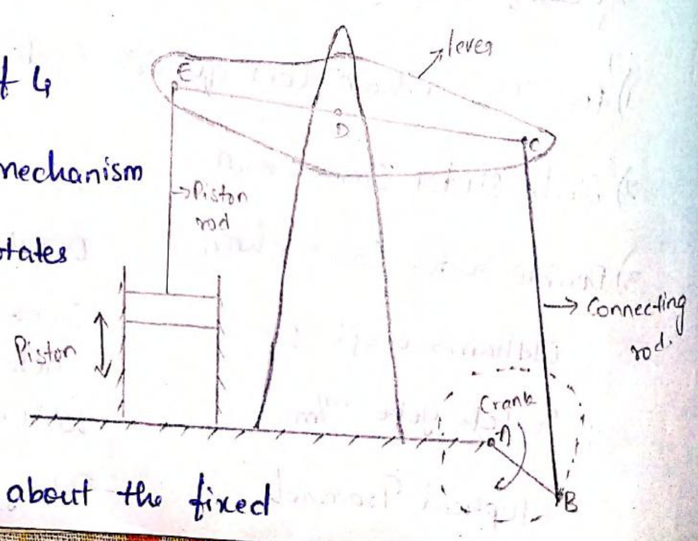
1) Beam Engine [Crank & lever m/m]

2) Coupling rod of locomotive [Double Crank m/m]

3) watt's M/m [straightened line motion m/m] (or) [Double lever m/m]

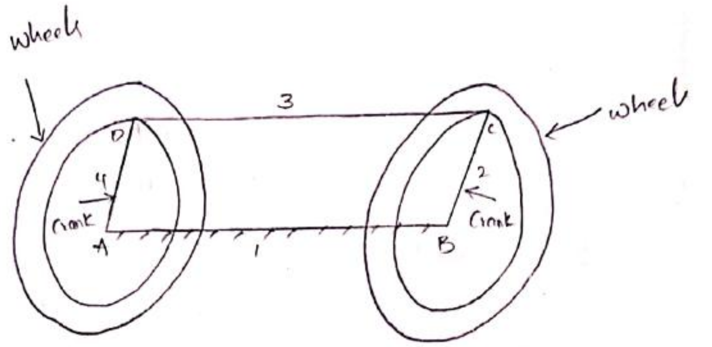
1) Beam Engine :-

It consists of 4 links. In this mechanism when the crank rotates about the fixed centre 'A'. The lever oscillates about the fixed



Centre at 'D'. The End 'E' of the lever CDE is Connected to a piston rod which reciprocates due to the rotation of the Crank. This mechanism is to Convert rotary motion to reciprocating motion.

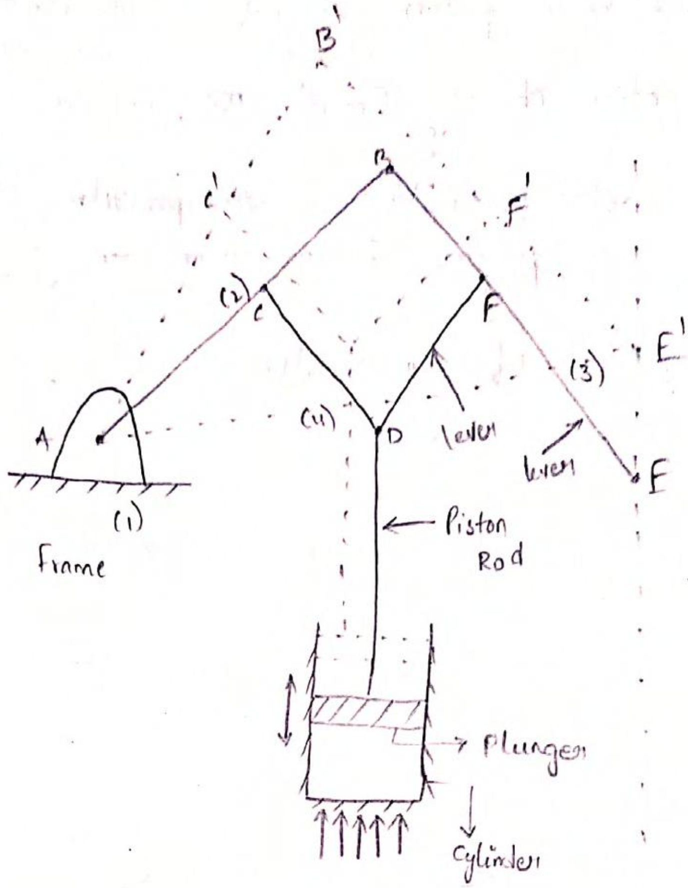
2) Coupling Rod of Locomotive :-



The mechanism of a Coupling rod of locomotive which Consists of 4 links. In this mechanism AD and BC are having equal lengths act as Cranks and are Connected to the respective wheels. The link CD acts as a Coupling rod. and the link AB is fixed in Order to maintain a Constant Centre to Centre distance between them. This mechanism is transmitting rotary motion from wheel to the other wheel.

(P.T.O)

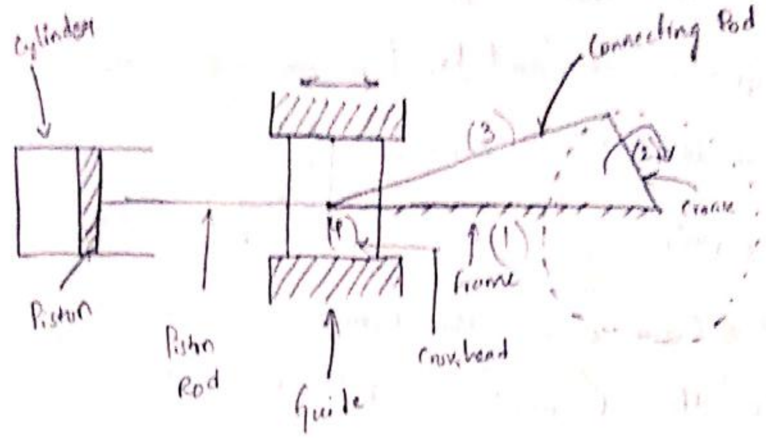
* Watt's indicator m/m :-



1) It consists of 4 links, fixed link at 'A', link AB, BE, COF 2) It may be noted that CD and DF form one link because these two links have no relative motion between them 3) The links BE and COF act as levers

4) The displacement of the link CDF is directly proportional to the pressure of gas or steam which acts on the indicator plunger. on any small displacement of the m/m the tracing point 'E' at the end of the link 'BE' traces out approximately a straight line

* Single Slider Crank chain mechanism :-



1) It is modification of the basic 4 bar chain.

2) It Consists of 1 sliding (4-1) pair b/w Crosshead & guide and having 3 turning pair (1-2, 2-3, 3-4)

3) It is mostly used in reciprocating steam engine

4) This mechanism Converts from rotary motion to reciprocating motion

* Inversion of Single Slider Crank chain m/m :-

1) Bull Engine

2) Oscillating Engine

3) Crank & Slotted lever quick return motion m/m

4) whitworth quick return motion m/m

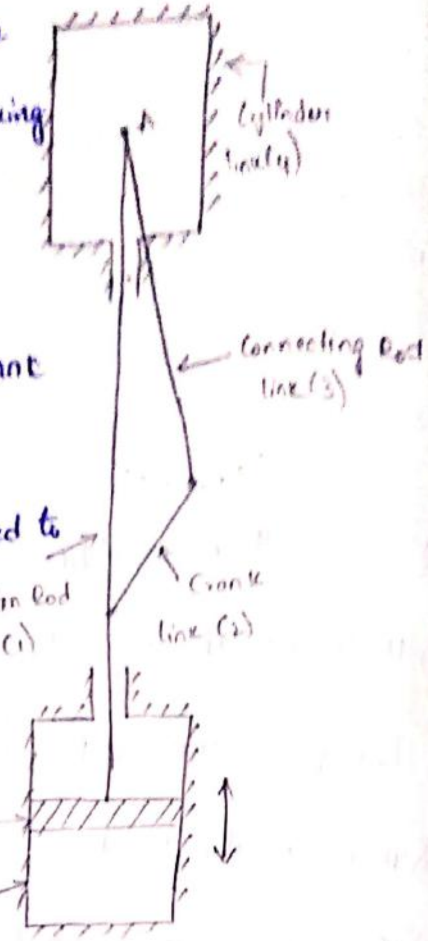
5) Rotary I.C Engine (or) Gnome Engine

In a 4 link mechanism.

(P.T.O)

1) Bull Engine:-

1) In this mechanism, the inversion is obtained by fixing the cylinder (or) link 4 i.e., Sliding pair.

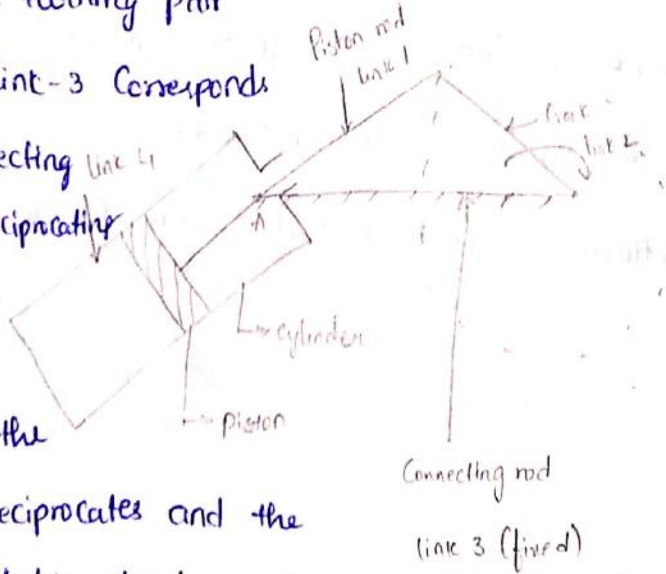


2) In this Case, when the Crank rotates the Connecting rod Oscillates about a pin pivoted to a fixed link at 'A'

3) The piston attached to the Piston rod reciprocates.

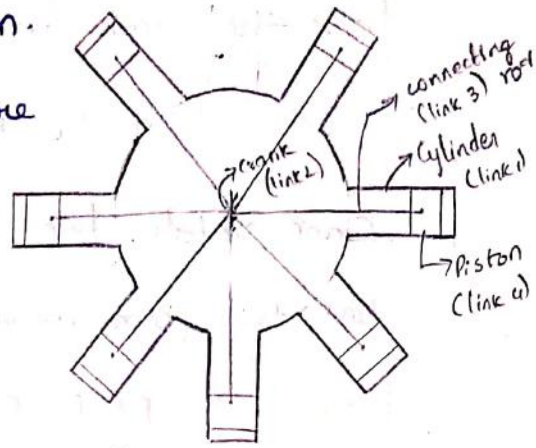
4) The duplex pump which is used to supply a feed water to boilers have two pistons attached to link-1.

2) Oscillating Engine:- It is used to Convert reciprocating motion to rotary motion. In this mechanism link 3 forming the turning pair fixed. The link-3 Corresponds to the Connecting rod of a reciprocating Steam Engine. when the Crank rotates the Piston rod reciprocates and the Cylinder oscilates about a pin pivoted to the fixed link at Point 'A'.



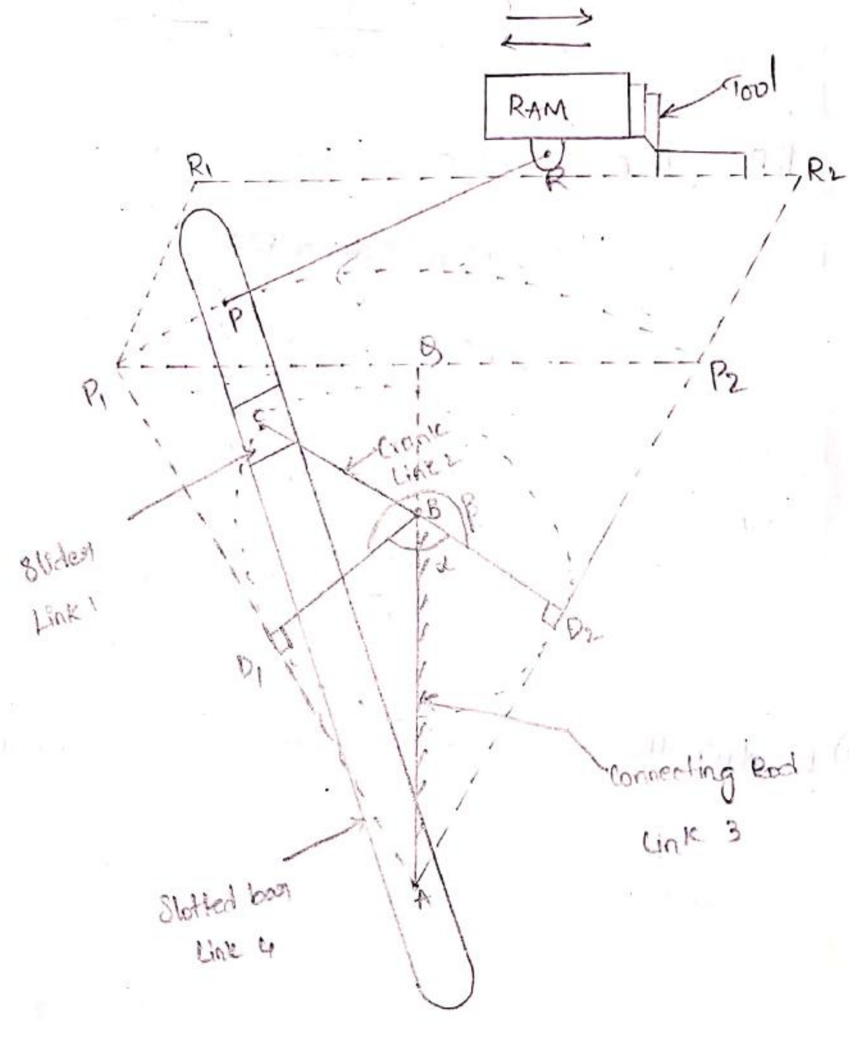
3) 7 Cylinders [Gnome Engine]

It is used in Navigation. Now a days, gas turbine are used in that place. It Consists of 7 Cylinders in one place and all revolves about the fixed Centre 'O'.



The Crank Link-2 is fixed in this mechanism. when the Connecting rod links rotates the piston link-4 reciprocates inside the Cylinder.

4) Crank and Slotted lever Quick return motion Mechanism:-



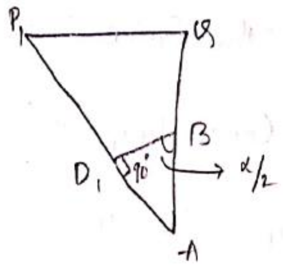
Crank rotates from BD_1 to BD_2 in clockwise direction and the ram moves

R_1 to R_2 $\Rightarrow$ Cutting Stroke

Crank rotates from BD_2 to BD_1 in Anti clockwise direction and the ram moves

R_2 to R_1 $\Rightarrow$ Return Stroke

$\triangle AQP_1 \parallel \triangle ABD_1$



$$\frac{\text{Time of Cutting stroke}}{\text{Time of return Stroke}} = \frac{\beta}{360-\beta}$$

$$= \frac{\beta}{\alpha} = \frac{360-\alpha}{\alpha}$$

$$P_1P_2 = R_1R_2 = 2P_1Q$$

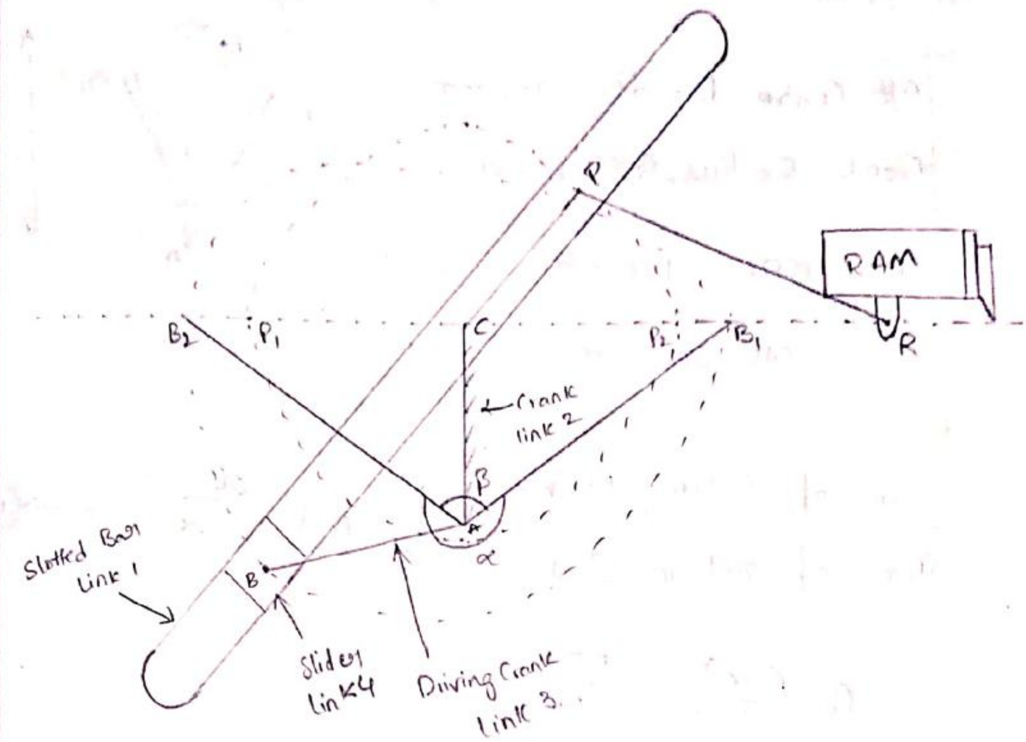
$$= 2 \times \sin \angle P_1AQ \times P_1A$$

$$= 2 \times \sin\left(90^\circ - \frac{\alpha}{2}\right) \times P_1A$$

$$= 2 \times \cos\frac{\alpha}{2} \times P_1A$$

$$= 2 \times \frac{BD_1}{AB} \times P_1A$$

5) Whitworth quick return motion mechanism:-



$$\frac{\text{Time of Cutting Stroke}}{\text{Time of return Stroke}} = \frac{\alpha}{\beta} = \frac{\alpha}{360-\alpha}$$

$$= \frac{360-\beta}{\beta}$$

AB_1 to AB_2 (α)

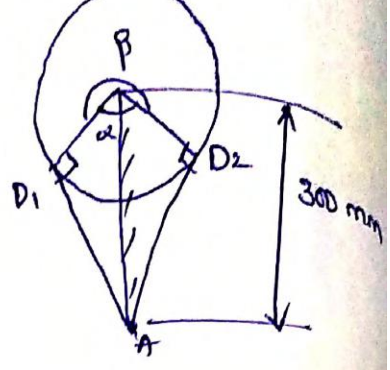
AB_2 to AB_1 (β)

Problems:-

1) A Crank & Slotted lever mechanism used in a shaper has a Centre distance of 300 mm between the Centre of oscillation of the sloted lever and the Centre of rotation of the Crank. The radius of the Crank is

120 mm. Find the ratio of the time of Cutting to time of return stroke.

A. Given,



AB Centre distance: 300 mm

Crank Radius, R = 120 mm

$BD_1 = BD_2 = 120$ mm

AB = 300 mm

$$\frac{\text{Time of Cutting stroke}}{\text{Time of return stroke}} = \frac{\beta}{\alpha} \text{ (or) } \frac{360-\alpha}{\alpha} \longrightarrow (1)$$

$$\cos\left(\frac{\alpha}{2}\right) = \frac{BD_1}{AB}$$

$$= \frac{120}{300}$$

$$\frac{\alpha}{2} = 66.421$$

$$\alpha = 132.84$$

From (1),

$$= \frac{360 - 132.84}{132.84}$$

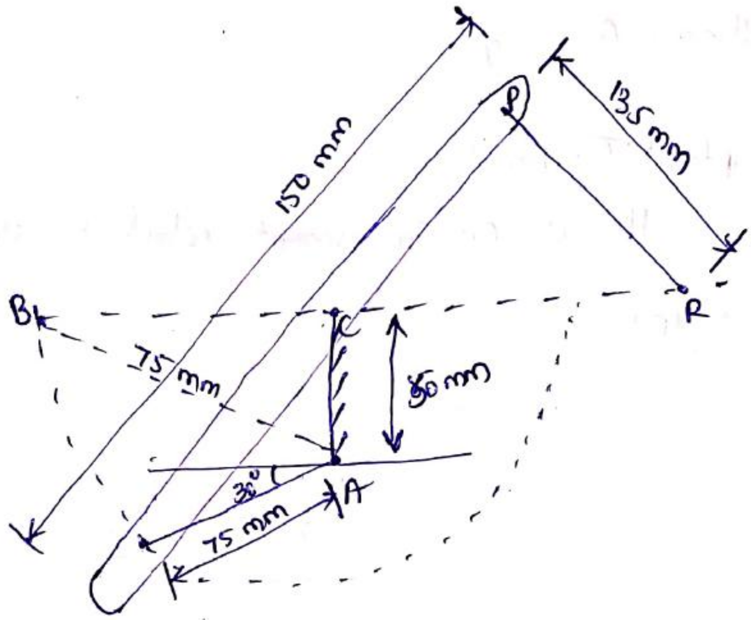
$$= 1.71$$

2) In a whitworth quick return motion mechanism The distance b/w the fixed Centres is 50 mm and the length of the driving Crank is 75 mm the length of the slotted lever is 150 mm and the

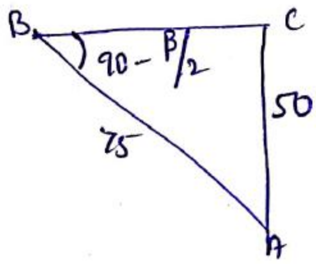
length of the Connecting rod is 135 mm. find the ratio of the time of cutting to the time of return stroke and also the length of the effective stroke.

Angle = 30°

A



$$\sin\left[90 - \frac{\beta}{2}\right] = \frac{AC}{AB_1}$$



$$= \frac{50}{75}$$

$\beta = 96.37$

$\alpha = 263.63$

* Double Slider Crank chain Mechanism:-

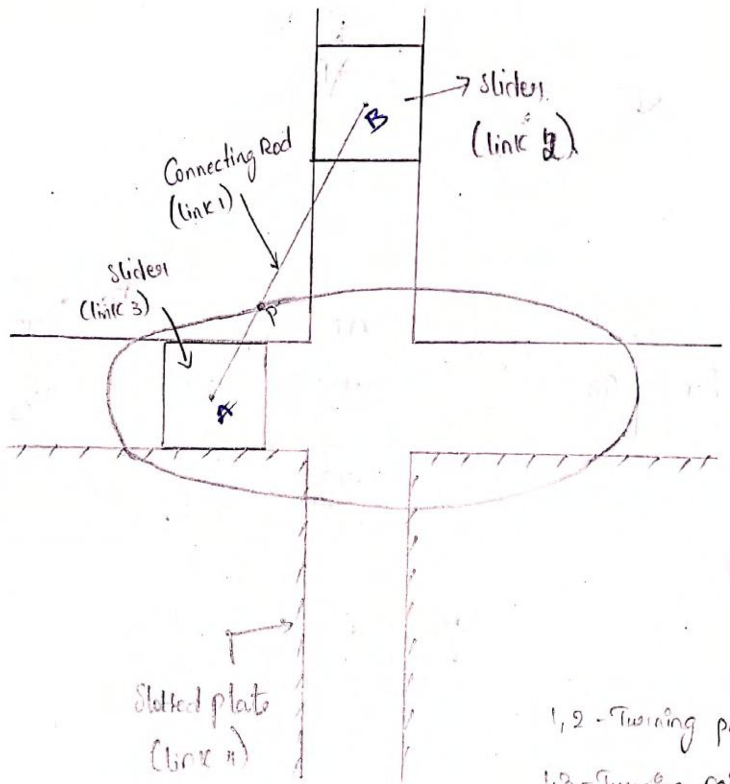
It Consists of 2 turning pairs and 2 Sliding pairs

→ Types:-

1) Elliptical Trammel
2) Scotch yoke m/m
3) Oldham's Coupling

1) Elliptical Trammel:-

It is an instrument which is used for drawing an ellipse



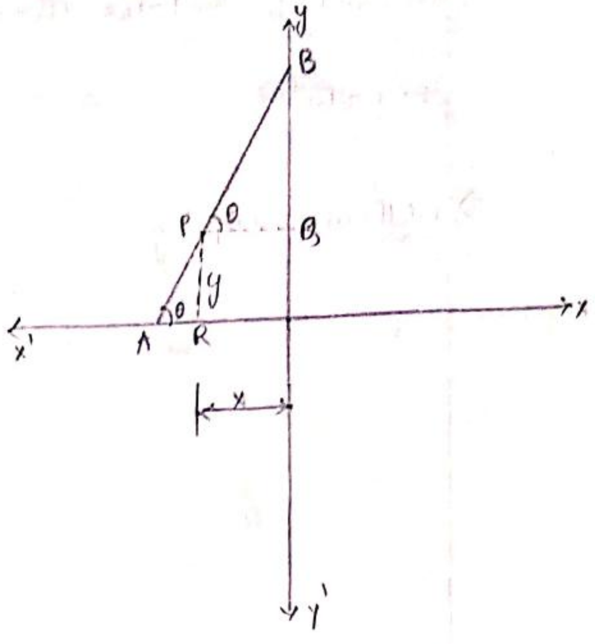
1,2 - Turning pair
1,3 - Turning pair
2,4 - Sliding pair
3,4 - Sliding pair

$$\cos\theta = \frac{PQ}{BP}$$

$$\Rightarrow PQ = BP\cos\theta = (x)$$

$$\sin\theta = \frac{PR}{PA}$$

$$\Rightarrow PR = PA\sin\theta = (y)$$



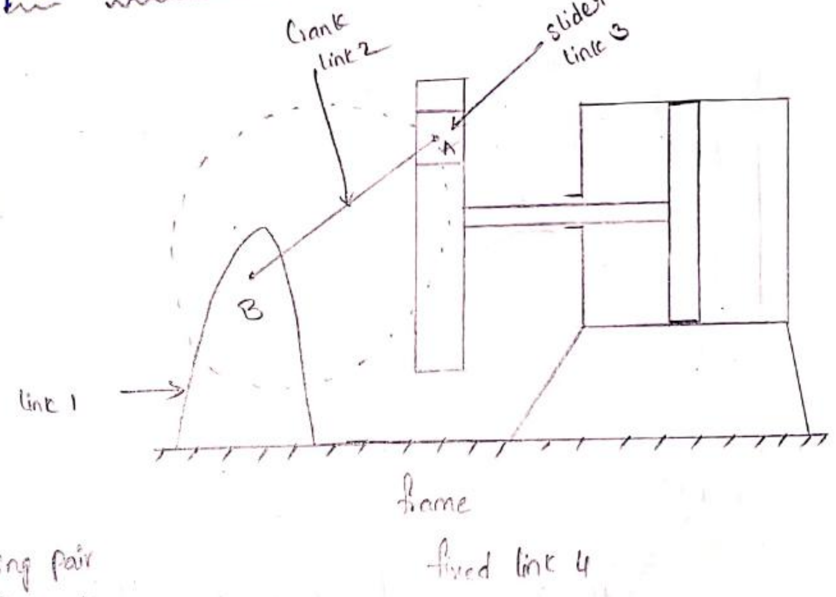
$$\frac{x}{BP} = \cos\theta,\ \frac{y}{PA} = \sin\theta$$

Squaring and Adding,

$$\frac{x^2}{(BP)^2} + \frac{y^2}{(PA)^2} = \cos^2\theta + \sin^2\theta$$

$$\frac{x^2}{(BP)^2} + \frac{y^2}{(PA)^2} = 1 \Rightarrow \text{Ellipse}$$

2) Scotch yoke mechanism:-



1,2 - Turning pair

2,3 - Turning pair

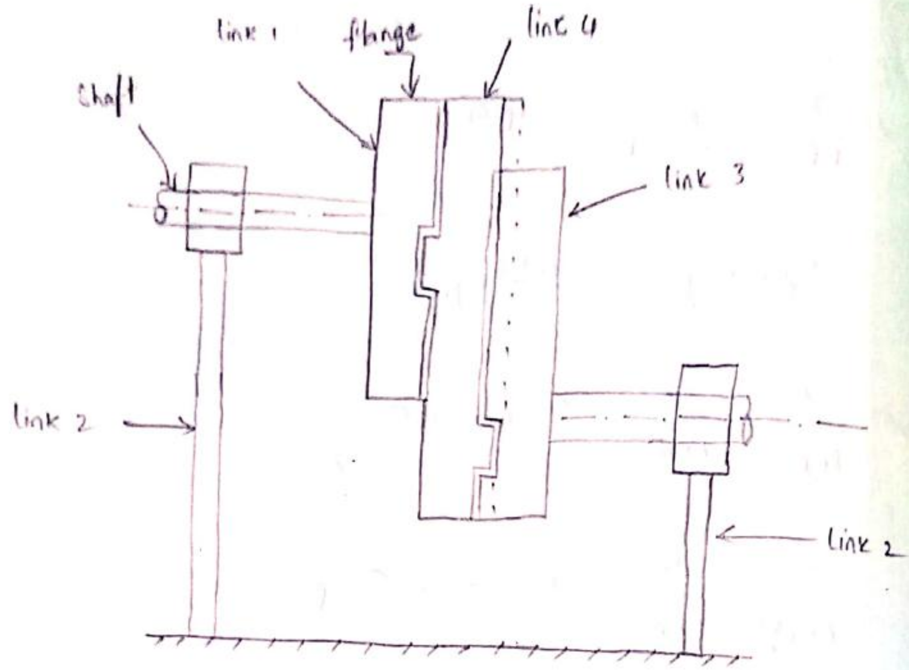
3,4 - Sliding pair

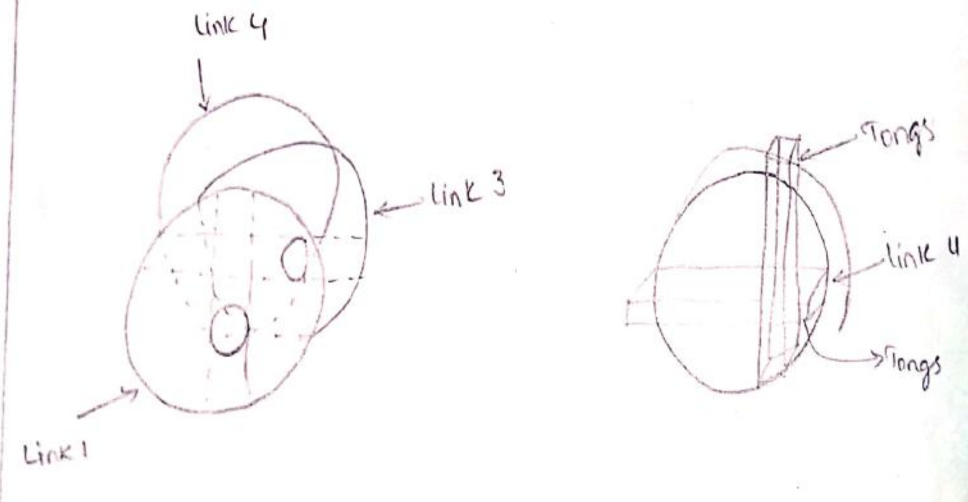
4,1 - Sliding pair

4 is used for Converting rotary motion to reciprocating motion. This inversion is obtained by fixing

either the link 1 (or) link 3. Here, link 1 is fixed when the link 2 rotates about 'B' as a Centre the link 4 reciprocates

3) Oldham's Coupling:-





It is used for Connecting two parallel shafts whose axis are at a small distance apart. The shafts are Coupled in such a way that if one shaft rotates the Other shafts also rotates at the same speed. Therefore as shown in figure the shafts should be Connected have two flanges rigidly fastened [joined] at their ends by forging Operation. The link '1' and '3' forms turning Pair with link '2'. These flanges have diametrical slots on their inner face. The intermediate piece link '4' which is a Circular disc have two tongs on each face at right angles to each other. The tongs of link '4' closely fit into the slots in the two flanges. Therefore, link '4' can slide and reciprocates in b/w link '1' & '3'.

* Straight line motion mechanism:-

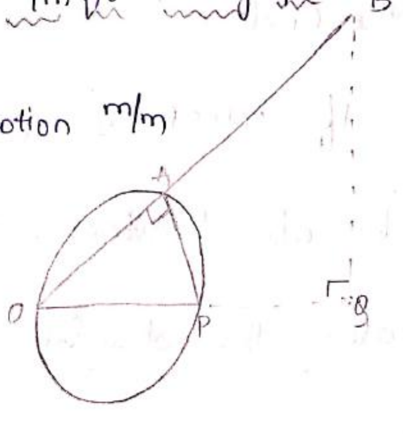
1) Exact Straight line motion m/m < Peaucellier m/m, Harts m/m
2) Approximate Straight line motion m/m

The Straightened mechanism are of two types

1) Only turning pairs are used and
2) One Sliding pair is used.

→ Exact Straight line motion m/m for Turning pair:-

In Exact straight line motion m/m



$OA \times OB = \text{Constant}$

Δ^{les} OAP & OBQ,

$$\frac{OA}{OP} = \frac{OQ}{OB}$$

$$\Rightarrow OA \times OB = OP \times OQ$$
$$= \text{Const.} \times \text{Const.}$$

let, OB be a ~~chord~~ point on the Circumference of a Circle of a diameter OB.

let, OA be a any chord and 'B' is a Point on a parallel such that

$OA \times OB = $ Constant

Then the locus of point 'B' will be a Straight line $\perp^r$ to the diameter OB.

This may be proved as follows,

1, Draw PQ $\perp^r$ to OB

2 Join AP.

Now ΔOAP & ΔOBQ are Similar

$$\therefore \frac{OA}{OP} = \frac{OQ}{OB}$$

$$OA \times OB = OP \times OQ$$

OP' is Constant because it is a diameter of the Circle

∴ if $OA \times OB$ is Constant then OQ is (also) will be also Constant. Then the point 'P' meves along the straight path BQ' which is $\perp^r$ to OP

*** * Peaucellier Mechanism :-

It Consists of 8 links all are turning pairs only. It Consists of a fixed link O, O_1 and

Other straight links O_1A, OE, OD, AD, DB, BE, EA.

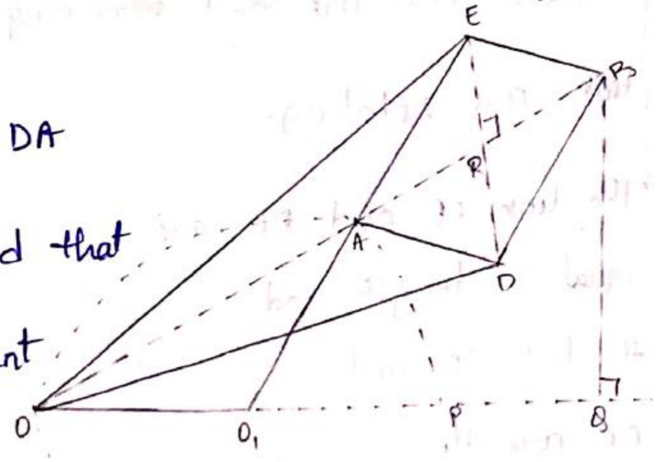
The pin at 'A' is constrained to move along the circumference of a circle with the fixed diameter OP. By means of the link O_1A

$OO_1 = O_1A$

$OE = OD$

$AD = DB = BE = EA$

$AE = EB = BD = DA$



Now, we proved that

$OA \times OB$ = Constant

when the link O_1A rotates.

Join ED to bisect AB at 'R'. Now from right angled triangles ORE & BRE we have,

$$OE^2 = OR^2 + RE^2 \longrightarrow ①$$

$$BE^2 = BR^2 + RE^2 \longrightarrow ②$$

Substracting Eq. ② from Eq. ①

$$OE^2 - BE^2 = (OR^2 + RE^2) - (BR^2 + RE^2)$$

$$OE^2 - BE^2 = OR^2 - BR^2$$

$$= (OR + BR)(OR - BR)$$

$$= OB \times OA$$

$$\boxed{OA \times OB = \text{Constant}}$$

* Hart's Mechanism:-

It Consists of 6 Links and all are turning Pairs only

OO_1 are fixed link and remaining links are rotating.

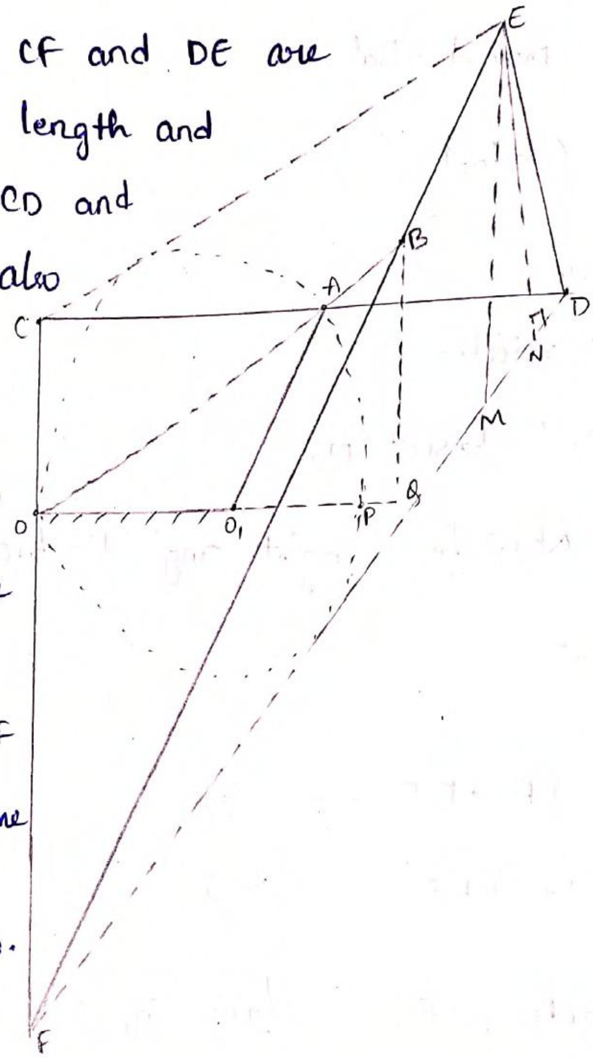
$OO_1 = O_1A$

$CF = ED$

$CD = EF$ are of Equal lengths

The link CF and DE are equal in length and the link CD and EF are also equal.

The points 'O', 'A' & 'B' divides the links CF, CD and EF in the same ratio (or) equal ratio.



A little consideration will show that BOCE is a trapezium. Similarly, OA and OB are parallel to CE and FD

Now, OAB is a straight line. It is proved that the Product OA×OB is Constant.

In triangle FCE, 'O' and 'B' divides 'FC' and 'EF' in the same ratio. therefore,

$$\frac{CO}{CF} = \frac{EB}{EOF}$$

Therefore, 'OB' is parallel to 'CE'

Similarly, In Δ^{le} FCD, 'O' and 'A' divides 'CF' and 'CD' in the same ratio then

$$\frac{CO}{CF} = \frac{CA}{CD}$$

Therefore, 'OA' is parallel to 'FD'.

from, Similar Δ^{les} CFE and OFB,

$$\frac{CE}{CF} = \frac{OB}{OF}$$

$$OB = \frac{CE \times OF}{CF} \longrightarrow (1)$$

from, Similar Δ^{les} FCD and OCA,

$$\frac{FD}{FC} = \frac{OA}{OC}$$

$$OA = \frac{FD \times OC}{FC} \longrightarrow (2)$$

(1) × (2),

$$OA \times OB = \frac{FD \times OC}{FC} \times \frac{CE \times OF}{FC}$$

$$OA \times OB = \frac{FD \times OC \times CE \times OF}{FC^2}$$

$$= FD \times CE \times \left(\frac{OC \times OF}{FC^2}\right)$$

Since the lengths OC, OF and FC are fixed. Therefore,

OA × OB = Constant.

Now, from Point 'E' Draw EM $\parallel^{el}$ to CF and EN $\perp^{r}$ FD

Therefore, $FD \times CE = FD \times FM$

Here,

$FD = FN + ND$

$FM = FN - NM$ | $NM = ND$

$FM = FN - ND$

$FD \times CE = (FN + ND)(FN - ND)$

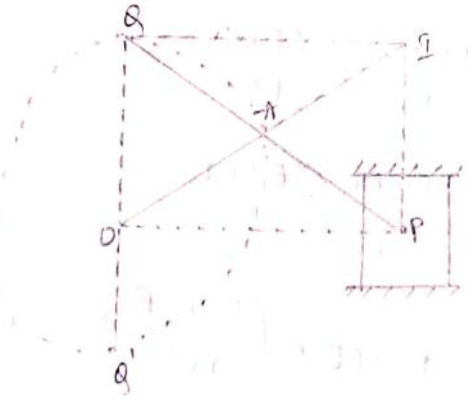
$= FN^2 - ND^2$

$= (FE^2 - EN^2) - (ED^2 - EN^2)$

$FD \times CE = FE^2 - ED^2 = $ Constant

$FD \times CE = $ Constant

* Exact Straight line motion mechanism with one Sliding pair by Scott russells mechanism :-



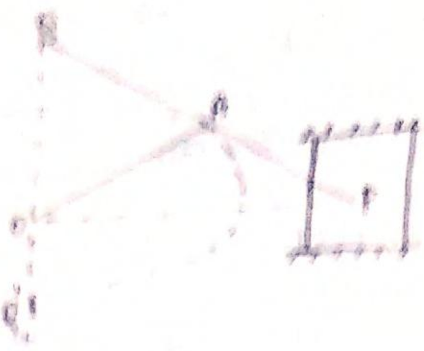
It Consists of a fixed link and a movable link 'P' of a sliding pair as shown in figure. The straight link PAQ is Connected by turning pairs to the link 'OA' and the link 'P'. The link 'OA' rotates about 'O'.

A little Consideration will show that the mechanism OAP is same as that of reciprocating steam engine. in which OA is Crank & AP is Connecting rod. In this mechanism the straight line motion is not generated but it is nearly Copied. In this mechanism, 'A' is the midpoint of 'QP' and OA = AP = AQ. The instantaneous Centre for the link PAQ lies at 'I' in OA produced and is such that 'IP' is ⊥r to 'OP' and 'IQ' is ⊥r to 'OQ'. Therefore, OQIP is a rectangle. 'Q' moves along the Vertical line 'OQ' for all positions of 'QP'. Hence, 'Q' traces a straight line 'OQ'.

* Approximate Straight Line motion mechanism:-

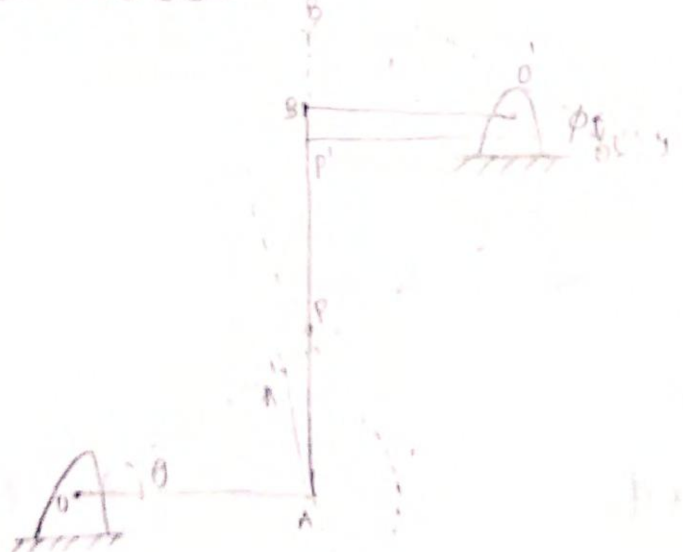
1) Watts
2) Modified Scott Russell
3) Grasshoper
4) Tchebichef's
5) Robert's

* Modified Scott Russell :-



It is similar to Scott Russell mechanism. But In this Case, AB is not equal to AQ. And the Points 'P' & 'Q' are Constrained to move in the horizontal & Vertical direction. It is similar to elliptical trammel. So, that any point 'A' on 'PQ' traces an Ellipse with Semi major axis 'AQ' & Semi minor axis 'AP'

* Watt's Mechanism:-



This mechanism Consists of 4 bar chain. In this figure $OABO'$ is a Crossed 4 bar chain. in which OO' are fixed in the mean position of the m/m. links OA and BO' are parallel and the Coupling rod AB is $\perp^r$ to OA and BO'. The tracing point 'P' traces out an approximate straight line over certain positions of its moment.

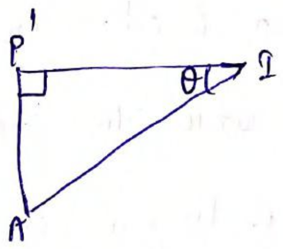
If $\frac{PA}{PB} = \frac{O'B}{OA}$.

A little consideration will show that the initial Position of the mechanism. The instantaneous Centre of the link AB lies at infinity distance

Let $OA'B'O'$ be the new position of the m/m after the links OA & $O'B$ are displaced through an angle 'θ' & 'ϕ'. The instantaneous Centre lies at 'I'

Since The Angles 'θ' & 'ϕ' are very Small

Arc AA' = Arc BB'



$\therefore OA \times \theta = O'B \times \phi$

$$\frac{OA}{O'B} = \frac{\phi}{\theta}$$

Then,

$A'P' = IP' \times \tan\theta$

Similarly,

$B'P' = IP' \times \tan\phi$

$$\frac{B'P'}{A'P'} = \frac{\phi}{\theta} \longrightarrow (1)$$

Hence,

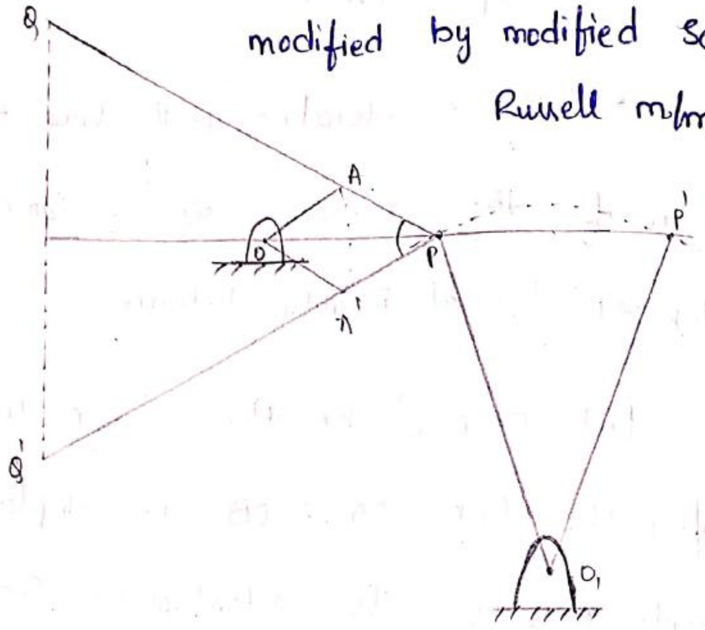
$$\frac{OA}{O'B} = \frac{\phi}{\theta} \longrightarrow (2)$$

from (1) & (2),

$$\boxed{\frac{B'P'}{A'P'} = \frac{OA}{O'B} = \frac{BP}{AP}}$$

* Grasshoper Mechanism :-

It is also 4 Bar chain m/m but it is slightly modified by modified Scott Russell m/m.

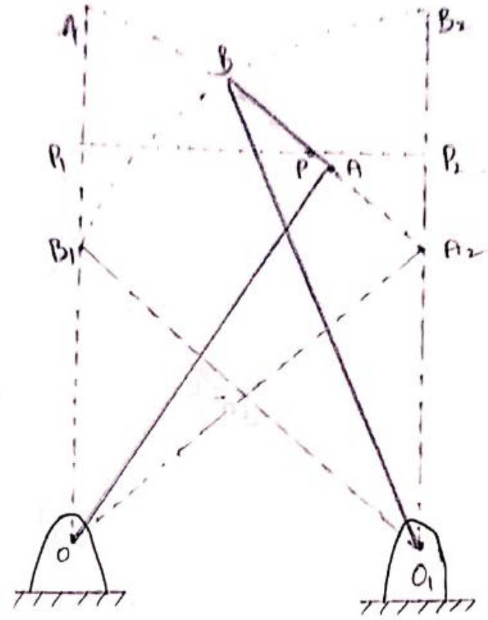


The link OA Osicilates about 'O' through an angle which Causes the pin 'p' to move along a Circular one with O_1 as Centre O_1P as radius. The small angular displacement at OP on each side of the Horizontal the point Q on the Extension of the Link PA traces out an approximately a straight path Q, Q' with length

$$OA = \frac{(AP)^2}{AQ}$$

* Tchebichef's m/m :-

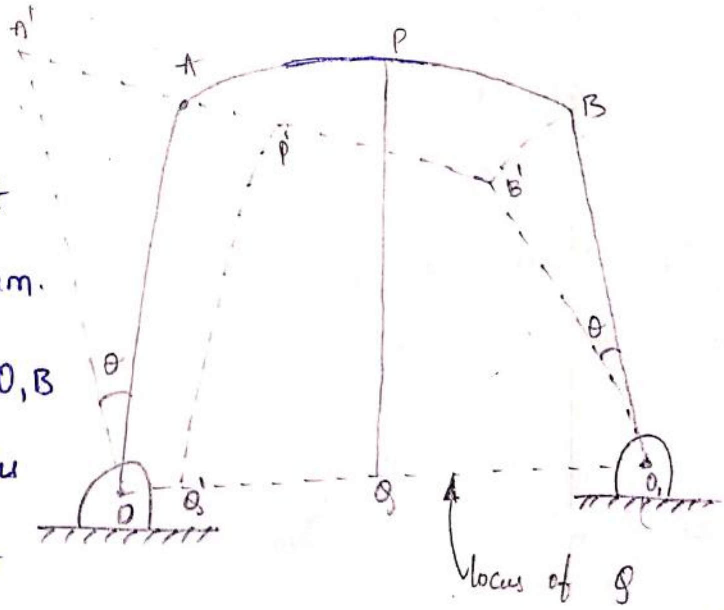
It Consists of 4 links. All are turning pairs only.



It is a Crossed links OA and O₁B are of equal length and the point 'P' is midpoint of AB traces out the approximate straight line parallel to OO₁ if the lengths of the links are in proportions AB : OO₁ : OA

* Robert's m/m :-

In this mechanism, totally 4 links in its mean position it look likes a trapezium. The links OA and O₁B are of equal lengths and OO₁ is fixed. A



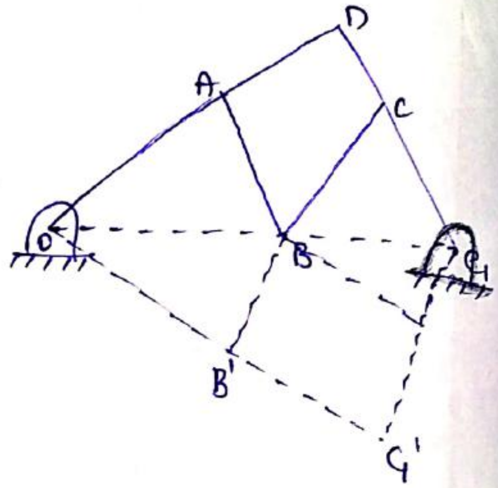
bar PQ is rigidly attached to the link AB at its middle

Point 'P'. A little bit displacement as shown by the dotted lines the point 'Q' traces out an approximate straight line.

* Pantograph:-

OAB, ODC

$$\frac{OA}{OB} = \frac{AB}{C_1D}$$



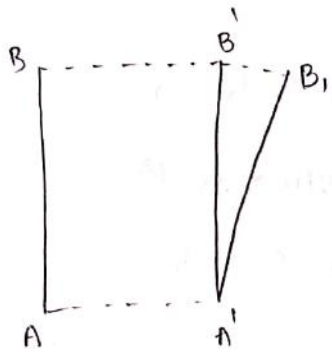
23/01/20

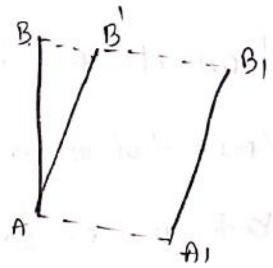
Unit - II

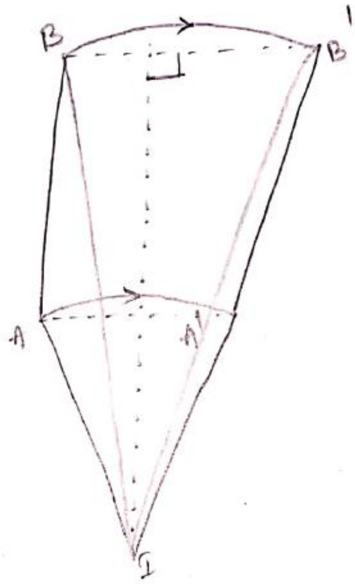
Velocity in Mechanisms

Velocity in mechanisms

- Instantaneous Centre
- Relative velocity





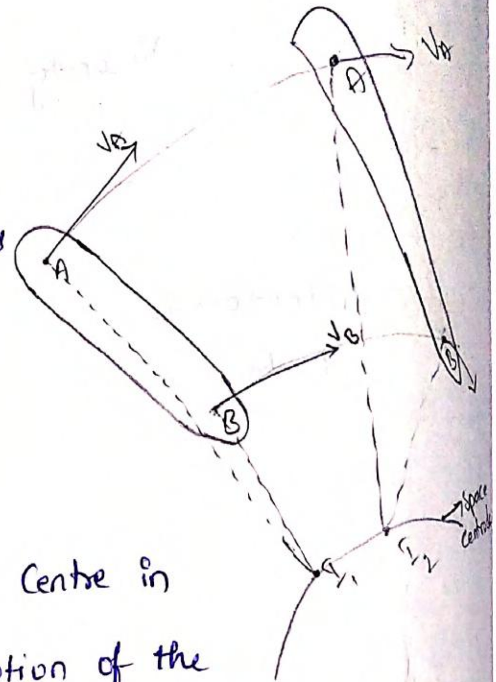


In actual practice, the motion of line AB is so gradual that it is difficult to see the two seperate motions. But, we see the two seperate motions the point 'B' moves faster than 'A'. This combination of rotation & Translation of the link AB may be assumed to be a motion of pure rotation about some Centre 'I' is known as instantaneous Centre of rotation.

The locus of all instantaneous Centres is known as Centrode. A line drawn through an instantaneous

Centre ⊥r to the plane of motion is called instantaneous axis

* Space & Body Centrodes:-

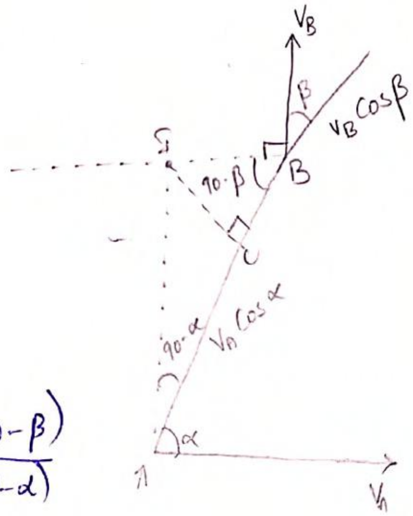


Locus of the instantaneous ~~Point~~ centre relative to the body itself is known as Body Centrode.

Locus of the instantaneous Centre in Space during a definite motion of the body is known as Space Centrode

* velocity of a body:-

Resolving the forces of the link AB,



$$V_A \cos\alpha = V_B \cos\beta$$

$$\frac{V_A}{V_B} = \frac{\cos\beta}{\cos\alpha} = \frac{\sin(90-\beta)}{\sin(90-\alpha)}$$

Now, Applying Lami's theorem,

$$\frac{AI}{\sin(90-\beta)} = \frac{BI}{\sin(90-\alpha)}$$

$$\frac{AI}{\cos\beta} = \frac{BI}{\sin\alpha}$$

$$\frac{AI}{BI} = \frac{\cos\beta}{\sin\alpha}$$

$$\frac{V_A}{V_B} = \frac{AI}{BI}$$

$$\frac{V_A}{AI} = \frac{V_B}{BI} \quad \left[\omega = \frac{V}{Radius}\right]$$

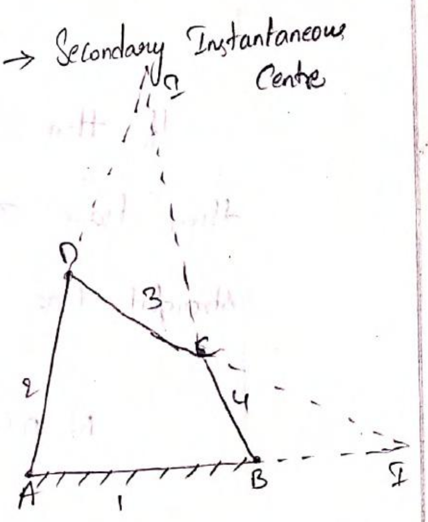
No. of instantaneous Centre in m/m, $N = \frac{n(n-1)}{2}$

* Types of Instantaneous Centre:-

AAA (2m)

1) fixed Instantaneous Centre } Primary Instantaneous Centre
2) Permanent " " } Primary Instantaneous Centre
3) Neither fixed nor permanent I.C → Secondary Instantaneous Centre

for a four bar mechanism



AB - fixed I.C

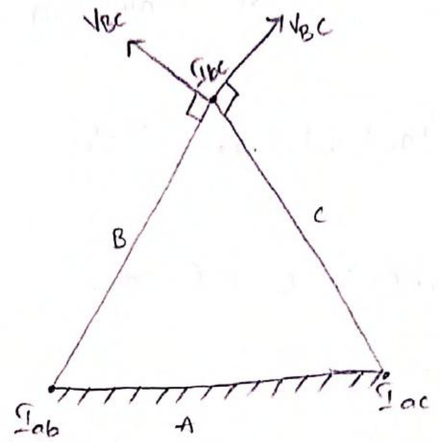
BC, CD, DA } Permanent I.C

DI, CI, BI, CI } Neither fixed nor permanent I.C

links	1	2	3	4
Instantaneous Centre	12 13 14	23 24	34	

* Aronhold's Kennedy's Theorem (or)

Three Centres inline Theorem :-



If three bodies moves relatively to each other they have 3 instantaneous Centre and lie on a straight line

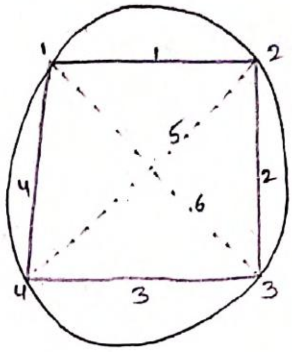
$$N = \frac{n(n-1)}{2} = \frac{3(3-1)}{2}$$

$$= \frac{6}{2} = 3 \text{ (Instantaneous Centre)}$$

The two instantaneous Centres at the pin joints of 'B' with 'A' and 'C' with 'A' and the two I.C is called as fixed I.C

According to Kennedy's Theorem, the 3rd Instantaneous Centre I_{BC} must lie on the line joining the I_{ab} and I_{AC}

* Circle diagram [To find I.C]:-

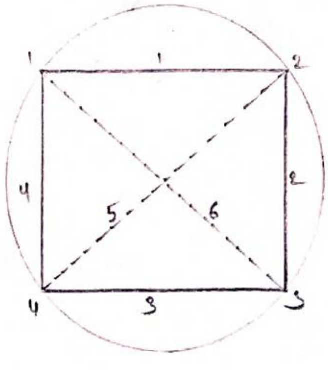


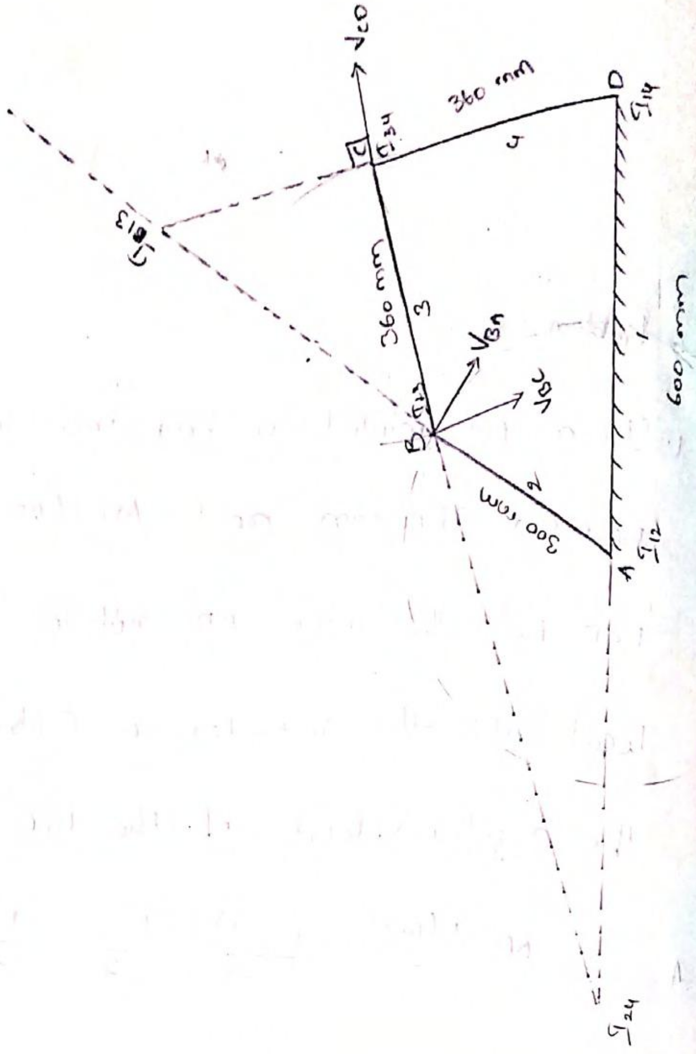
→ Problem:-

1. In a pin jointed 4 bar mechanism AB = 300 mm BC & CD = 360 mm and AD = 600 mm. The angle BAD = 60°. The crank AB rotates uniformly at 100 RPM locate all the instantaneous Centres and also find the angular velocity of the link BC?

A. $N = \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = \frac{4(3)}{2} = \frac{12}{2} = 6$

Links	1	2	3	4
Instantaneous Centre	12 ✓ 13 ✓ 14 ✓	23 ✓ 24 ✓	34 ✓	





$\omega_{BC} = ?$

$$\omega_{AB} = \frac{2\pi N}{60} = \frac{2\pi(100)}{60} = 10.47 \text{ rad/s}$$

$$\omega = \frac{V}{R}$$

$$V = \omega \times R$$

$$V_B = \omega_{AB} \times AB$$

$$= 10.47 \times 300$$

$$= \text{Ans}/1000$$

$$V_B = 3.141 \text{ m/s}$$

$$V_B = \omega_{BC} \times I_{13} \times B$$

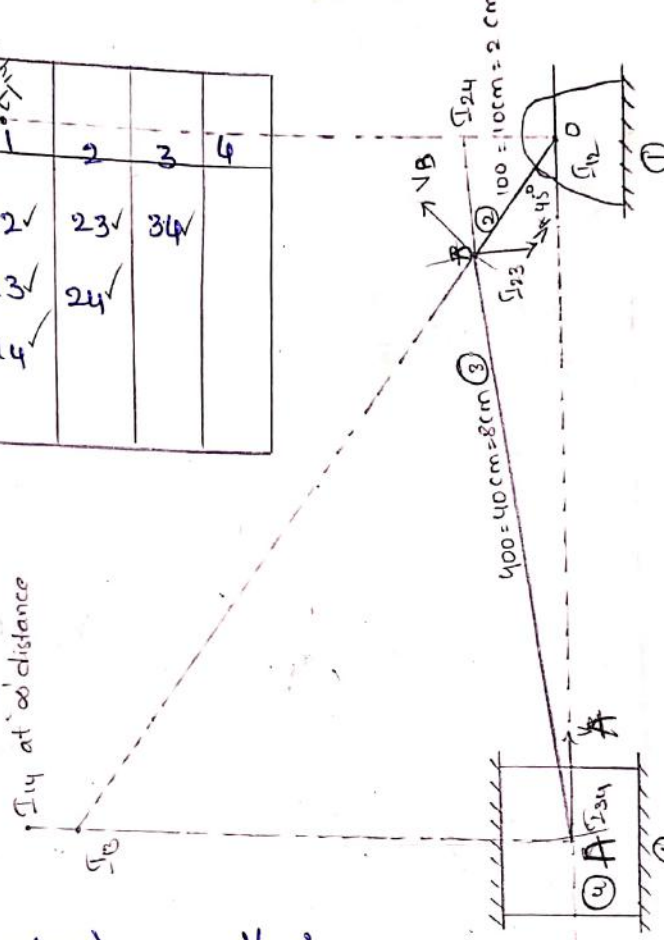
$$\omega_{BC} = \frac{V_B}{I_{13} \times B} = \frac{3.141}{0.54} = 5.81 \text{ rad/s}$$

2) ... the I.C of the Slider Crank m/m as shown in figure. The lengths of Crank OB and Connecting rod AB are 100 mm & 400 mm. Will the Crank rotates clockwise with an Angular velocity of 10 rad/s. find

i) Velocity of the slider A.

ii) Angular Velocity of the Connecting rod AB

Links	1	2	3	4
Instantaneous Centre	12✓	23✓	34✓	
	13✓	24✓		
	14✓			



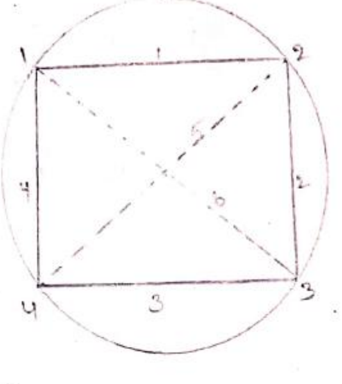
$$N = \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$$

$V_A = ?$

$$\frac{V_A}{I_{13}A} = \frac{V_B}{I_{13}B}$$

$$V_B = \omega_{OB} \times OB = 10 \times 0.1 = 1$$

$$V_A = \frac{V_B \times I_{13}A}{I_{13}B} = 1 \times \frac{0.46}{0.56} = 0.82 \text{ m/s.}$$

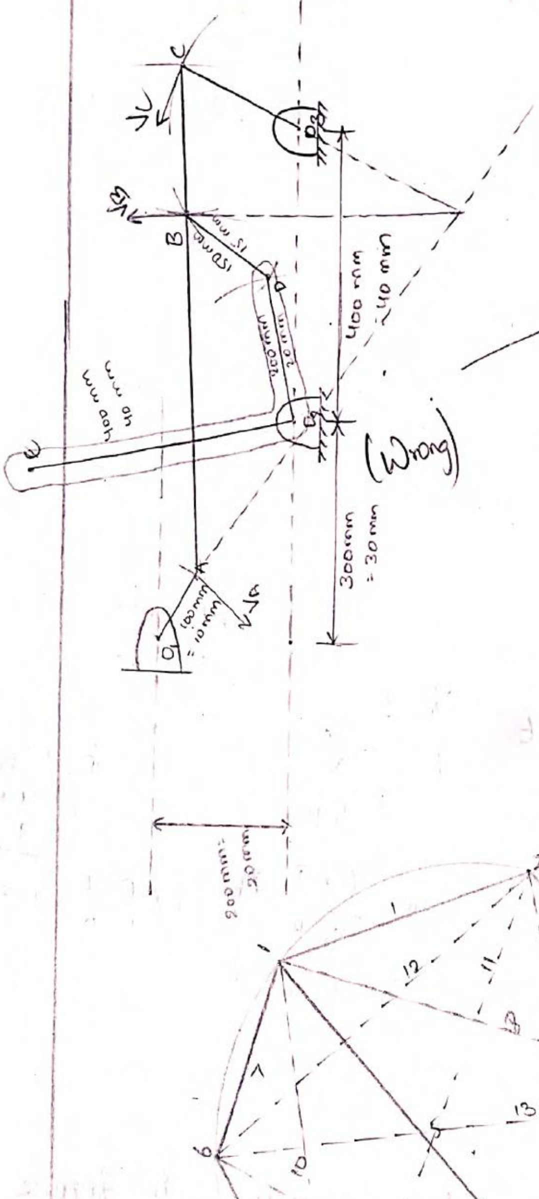


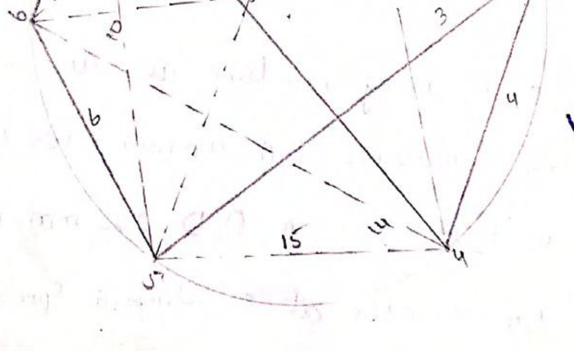
3) A m/m of a wrapping machine as shown in figure has the following dimensions. $O_1A = 100$ mm, $AC = 700$ mm, $BC = 200$ mm, $O_3C = 200$ mm, $O_2E = 400$ mm, $O_2D = 200$ mm and $BD = 150$ mm. The Crank O_1A rotates at a Uniform Speed of 100 rad/s

find the velocity of the point 'E' of the belt Crank lever by Instantaneous Centre method.

links	1	2	3	4	5	6
I.C	12	23	34	45	56	
	13	24	35	46		
	14	25	36			
	15	26				
	16					

$N = \frac{n(n-1)}{2} = \frac{6(6-1)}{2} = \frac{30}{2} = 15$





$V_B = V_A \times \frac{I_{13}B}{I_{13}A}$

$V_A = \omega_{OA} \times OA$

$= 100 \times 0.1$

$= 10$

$= 10 \times \frac{0.86}{0.95}$

$= 9.05 \text{ m/s}$

$V_C = V_A \times \frac{I_{13}C}{I_{13}A}$

$= 10 \times \frac{0.94}{0.95} = 9.89 \text{ m/s}$

$V_D = V_A \times \frac{I_{15}D}{I_{15}B}$

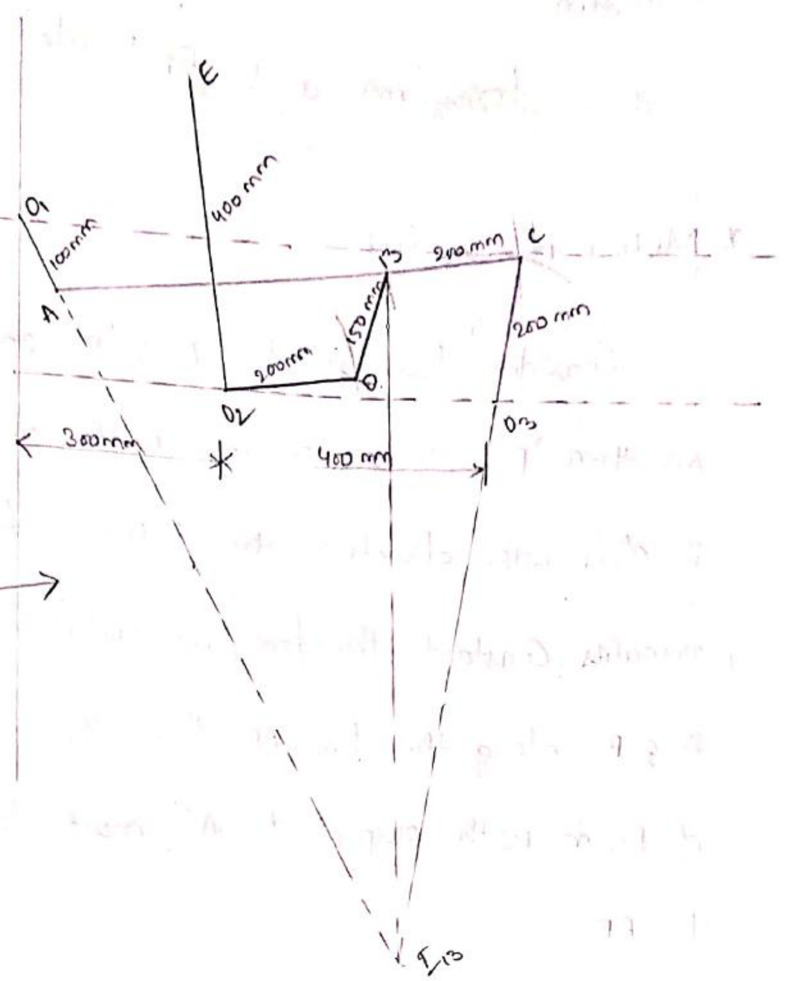
$= 10 \times \frac{0.65}{0.13}$

$= 3.84 \text{ m/s}$

$V_E = V_D \times \frac{I_{16}E}{I_{16}D}$

$= 3.84 \times \frac{0.004}{0.002}$

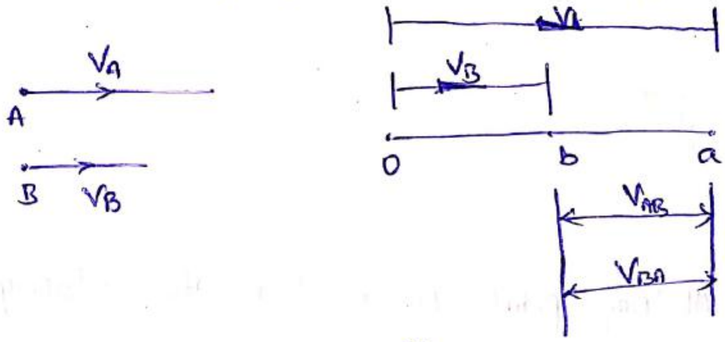
$= 7.68$



* Relative velocity of motion:-

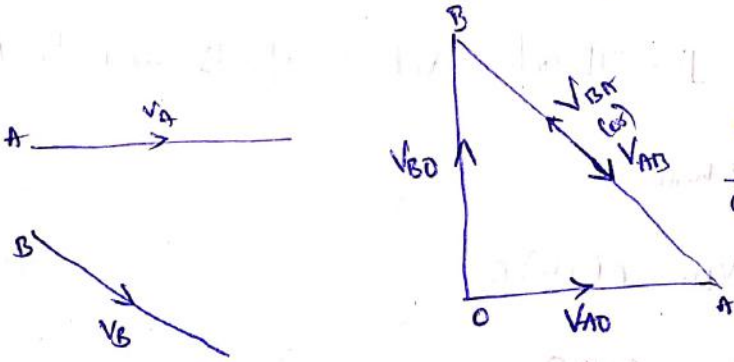
Relative velocity of 2 bodies moving in straight line,

1)



$\overline{V_{AB}} = \overline{V_A} - \overline{V_B}$

2)



$\overline{ba} = \overline{Oa} - \overline{Ob}$

$\overline{ab} = \overline{Ob} - \overline{Oa}$

$V_{AB} = -V_{BA}$

(Statement is writtened Back side)

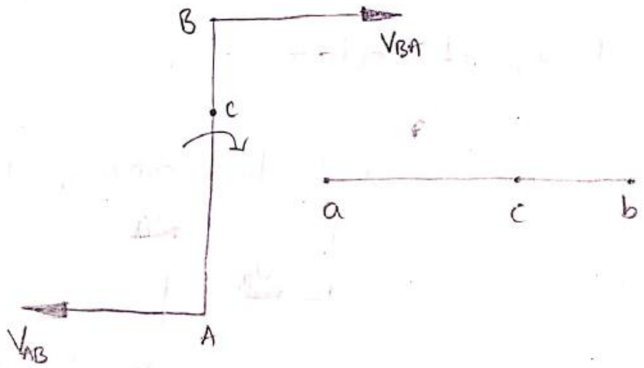
When the two bodies moving in inclined Condition,

(Diagram is in front side)

* **Motion of a link:-**

Consider two points 'A' & 'B' on a rigid link AB then 'P' will be moving with relative to 'A' in Clock wise direction the distance from 'A' to 'B' remains Constant. Therefore, no relative motion between A & B along the line AB then the relative motion of beam with respect to 'A' must be perpendicular to AB.

$V_{BA} = -V_{AB}$



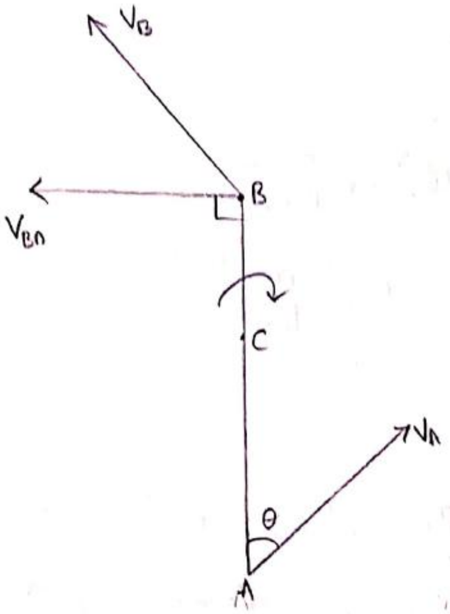
At any point on a link the velocity will be always ⊥r. Therefore, Velocity of 'B' w.r. to 'A' (V_{BA}) is $AB \times \omega_{AB}$

$$V_{BA} = AB \times \omega_{AB}$$

$$V_{CA} = AC \times \omega_{AC}$$

$$\Rightarrow \frac{V_{CA}}{V_{BA}} = \frac{AC \times \omega_{AC}}{AB \times \omega_{AB}}$$

* Velocity of a point on a link by relative velocity method:-

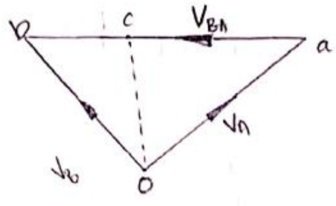


$\frac{bc}{ba} = \frac{BC}{BA}$

$bc = ba \times \frac{BC}{BA}$

$bc = 4.1 \times \frac{1.35}{5}$

$bc = 1.107$



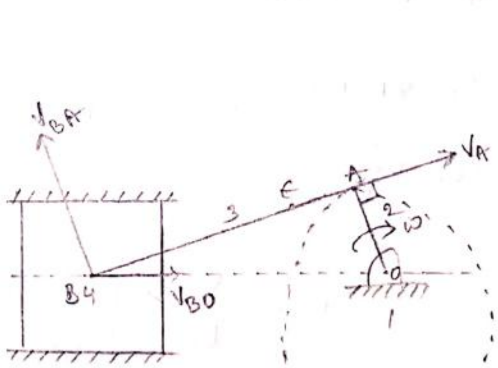
* Velocities in Slider Crank m/m.-

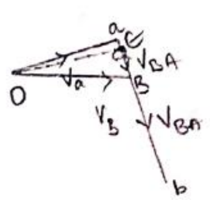
$\frac{ae}{ab} = \frac{AE}{AB}$

$ae = ab \times \frac{AE}{AB}$

$= 0.55 \times \frac{1}{4}$

$ae = 0.1375$



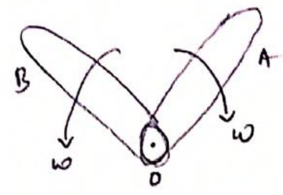


* Rubbing velocity at a pin joint :-

It is the algebraic sum of the Angular Velocity of the two Links joined by pin and multiplied by the pin radius.

$V = (\omega_1 - \omega_2) \times r$ [same direction]

$V = (\omega_1 + \omega_2) \times r$ [opposite direction]



1) In a four Bar chain mechanism ABCD. AD is a fixed link and it is 150 mm long. The Crank AB is 40 mm long and rotates at 120 R.P.M Clockwise. while the link CD is 80 mm Oscillates about D. 'BC' and 'AD' are of equal length. find the Angular Velocity of the link CD. when an angle BAD is 60°

A.

$\omega_{AB} = \frac{V_{BA}}{AB}$

$\omega_{AB} = \frac{2\pi N}{60}$

$= \frac{2\pi(120)}{60}$

$= 4\pi$

$\omega_{AB} = 12.566$ rad/s

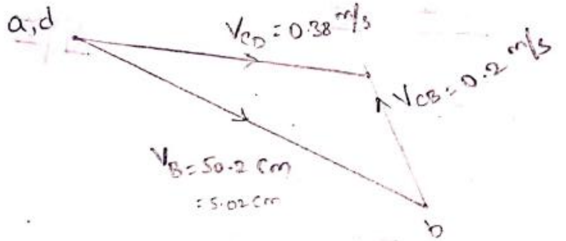
$12.566 = \frac{V_{BA}}{40\,mm = 0.04\,m}$

$V_B = V_{BA} = 12.566 \times 0.04$

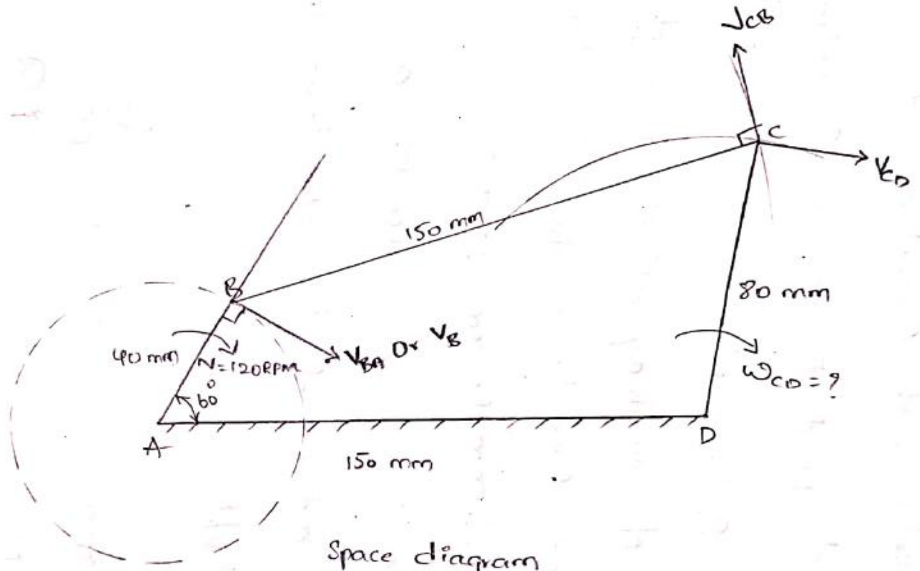
$V_B = 0.502$ m/s

$\omega_{CD} = \frac{V_{CD}}{DC} = \frac{0.38}{0.08} = 4.75$ rad/s

$\omega_{CB} = \frac{V_{CB}}{CB} = \frac{0.2}{0.15} = 1.33$ rad/s



Scale - 1:10



Space diagram

Scale - 1:2

2) The Crank and Connecting rod of a steam Engine are 0.5 m and 2m long. The Crank rotates 180 RPM in the clockwise when it has turned 45° from Inner Dead Centre. Determine

1) Velocity of piston
2) Angular velocity of Connecting rod
3) Velocity of point 'E' on the Connecting rod 1.5 m from the gudgeon pin.
4) Velocities of rubbing at the pins of the Crank shaft, Crank and Cross head. when the diameters of the Pins are 50 mm, 60 mm, 30 mm
5) Position and linear velocity of any point G, on the Connecting rod which has the least velocity relative to Crankshaft

A.

$N_{OB} = 180$ rpm

$\omega_{OB} = \frac{2\pi N}{60}$

$= \frac{2\pi(180)}{60}$

$= 6\pi$

$\omega_{OB} = 18.84$ rad/sec

$V_{BO} = \omega_{OB} \times BO$

$= 18.84 \times 0.5$

$V_{BO} = 9.42$ m/s

ii) $\omega_{BP} = \frac{V_{PB}}{BP}$

$= \frac{7.2}{2}$

$= 3.6$ rad/s

iii) $\frac{be}{bp} = \frac{BE}{BP}$

$be = bp \times \frac{BE}{BP}$

$= 7.2 \times \frac{0.5}{2}$

$= 1.8$m

$\frac{bg}{bp} = \frac{BG}{BP}$

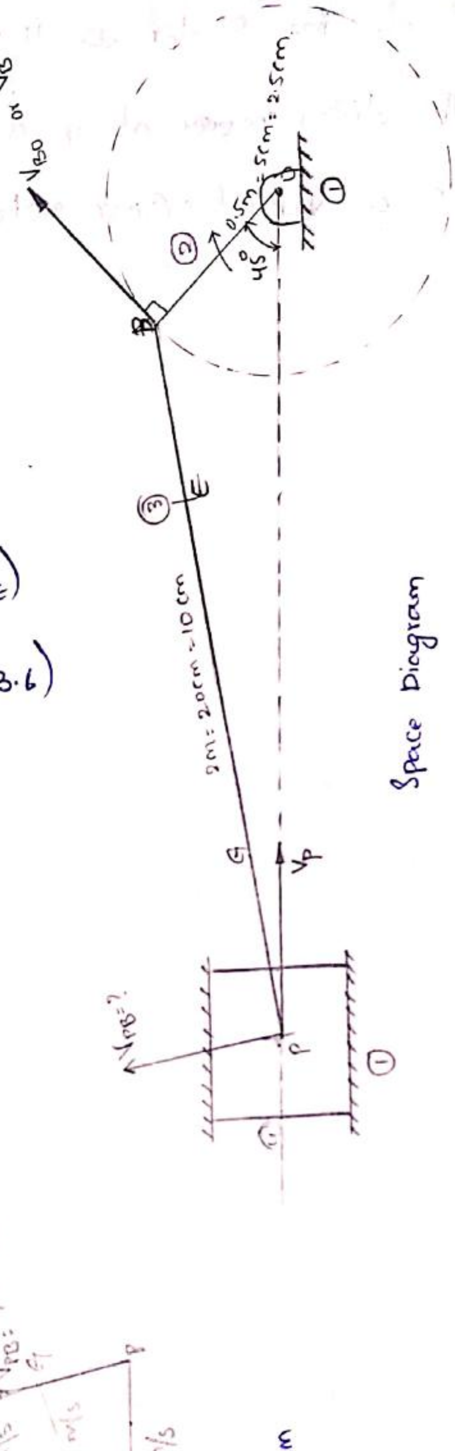
$bg = bp \times \frac{BG}{BP} = \frac{7.2 \times 1.5}{2} = 5.4$ m/s

Diameter of Crankshaft = 50 mm = 0.05 m

" " Crank = 60 mm = 0.06 m

" " Crosshead = 30 mm = 0.03 m

Velocity of rubbing at the pin of Crankshaft,



$V_O = r_o \times \omega_{OB}$

$= \frac{0.05}{2} \times 18.84$

$V_O = 0.471$ m/s

$V_B = r_B \times (\omega_{OB} + \omega_{BE})$

$= \frac{0.06}{2} \times (18.84 + 3.6)$

$= 0.03 (22.44)$

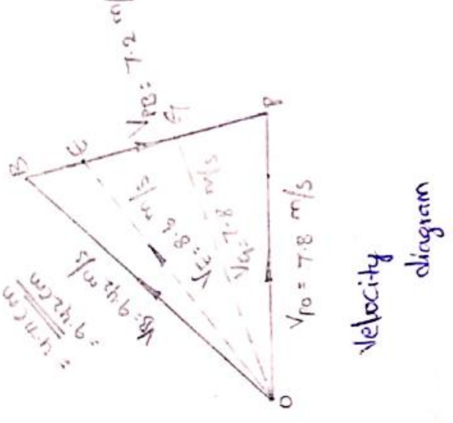
$= 0.6732$ m/s

$V_P = r_P \times \omega_{PB}$

$= \frac{0.03}{2} \times 3.6$

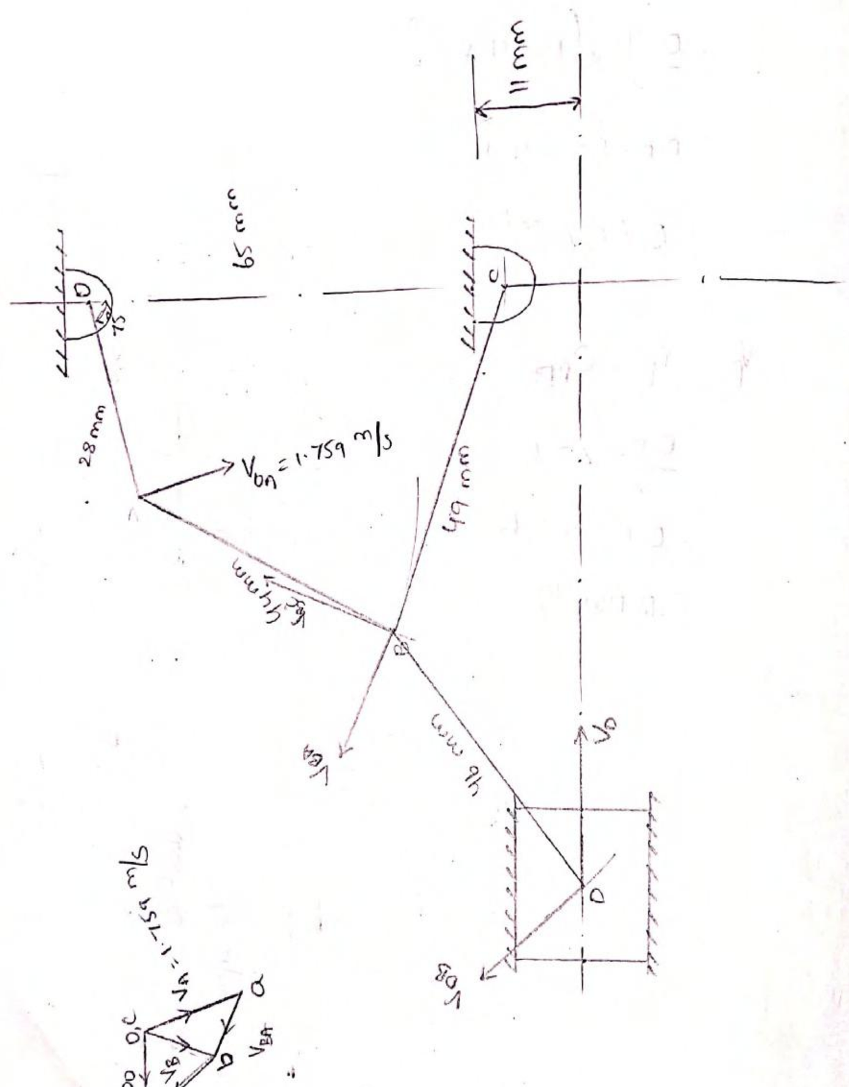
$= 0.015 \times 3.6$

$= 0.054$ m/s



3) As shown in the figure the angular velocity of the Crank OA is 600 rpm. Determine the linear velocity of the slider 'D' and the angular velocity of the link BD when the Crank is inclined at an angle of 75° to the Vertical

The dimensions of various links ... , m3 = 44.
BC = 49 mm, BD = 46 mm. The Centre distance b/w the Centres of rotation 'O' and 'C' is 65 mm. The path of travel of the slider is 11 mm below the fixed point 'C'. The slider moves along a horizontal path and 'O' and 'C' is vertical. Crank rotates in Anticlockwise direction



$\omega_{OA} = \frac{2\pi N}{60}$

$= \frac{2\pi(600)}{60}$

$= 20\pi$

$= 62.83$ rad/s

$V_{DB} = 1.7$ m/s

$V_{DO} = 1.6$ m/s

$V_{BA} = 1.1$ m/s

$V_{BD} = 1.5$ m/s

$V_{AO} = OA \times \omega_{OA}$

$= 0.028 \times 62.83$

$= 1.759$ m/s

$V_D = 1.6$ m/s

$\omega_{BD} = ?$

$V_{BD} = DB \times \omega_{BD}$

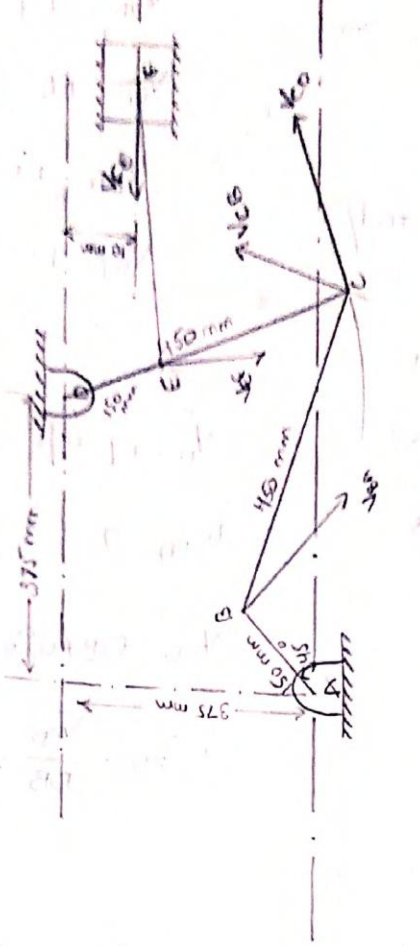
$\omega_{BD} = \frac{V_{BD}}{DB} = \frac{1.7}{0.046} = 36.95$ rad/sec

4) The mechanism as shown in figure has a dimensions of various links as follows AB = DE = 150 mm and BC & CD = 450 mm & EF = 375 mm. The crank AB makes an angle of 45° with the horizontal and rotates about 'A' in the ~~Cnt~~ clock wise direction at a uniform speed of 120 rpm. The lever DC ossicilates about the fixed point 'D' which is connected to AB by the coupler BC. The Block-F or slider-F moves in the horizontal direction. Determine.

i) Velocity of slider F

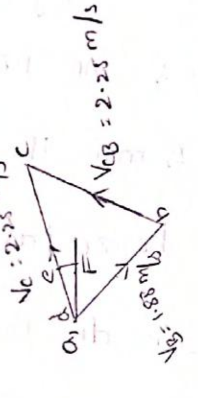
ii) 'ω' of DC

iii) Rubbing speed at the pin 'C' which is 50 mm in diameter.



Scale- 1:100

All dimensions are in mm



$V_c = 2.25$ m/s

$V_{CB} = 2.25$ m/s

$\frac{Ce}{Cd} = \frac{CE}{CD}$

$Ce = Cd \times \frac{CE}{CD}$

$= 2.25 \times \frac{0.3}{0.45}$

$Ce = 1.5$ m

$EF = 0.3$ m/s

$DF = 0.7$ m/s

ii) $\omega_{DC} = \frac{V_{CD}}{DC}$

$= \frac{2.25}{0.45}$

$\omega_{DC} = 5$ rad/s

$\omega_{CB} = \frac{V_{CB}}{BC}$

$= \frac{2.25}{0.45}$

$= 5$ rad/s

N= 120 rpm

$\omega_{AB} = \frac{2\pi N}{60}$

$= 4\pi$

$\omega_{AB} = 12.56$ rad/sec

V_{BA} or $V_B = \omega_{AB} \times BA$

$= 12.56 \times 0.15$

$= 1.884$ m/s

iii) $V = r \times \omega$

$V = r \times (\omega_1 - \omega_2)$

$= r \times (\omega_{CB} - \omega_{CD})$

$= 0.025 \times (5-5)$

$V = 0$

5) In a mechanism as a shown in figure. The Various dimensions are AB = 125 mm, BP = 500 mm, PC = 125 mm CD = 250 mm, DE = 125 mm. The slider 'p' translates along an axis which is 25 mm vertically. Below the point 'A'. The Crank AB rotates at 120 rpm in the Anticlockwise direction. The Belt Crank lever CDE about fixed Centre 'D' draw the velocity diagram and Calculate the Velocity of point 'E' of the lever.

A. $\omega_{BA} = \frac{2\pi N_{AB}}{60}$

$= \frac{2\pi(120)}{60}$

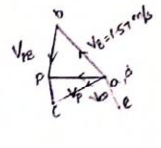
$= 12.56$ rad/s

$V_{BA} = r \times \omega$

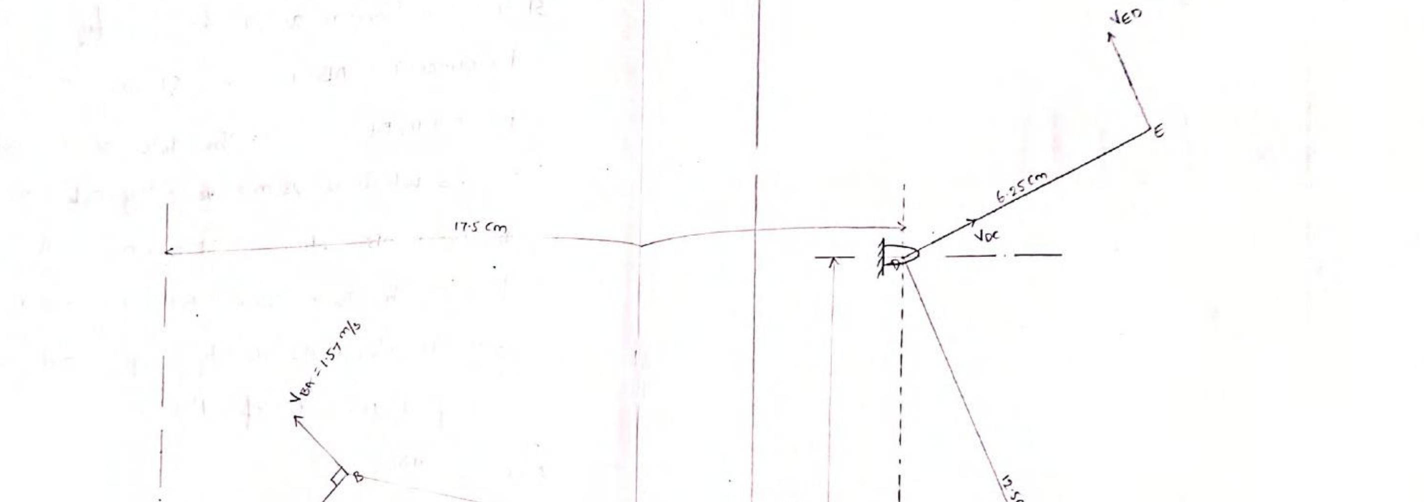
$= AB \times \omega_{BA}$

$= 0.125 \times 12.56$

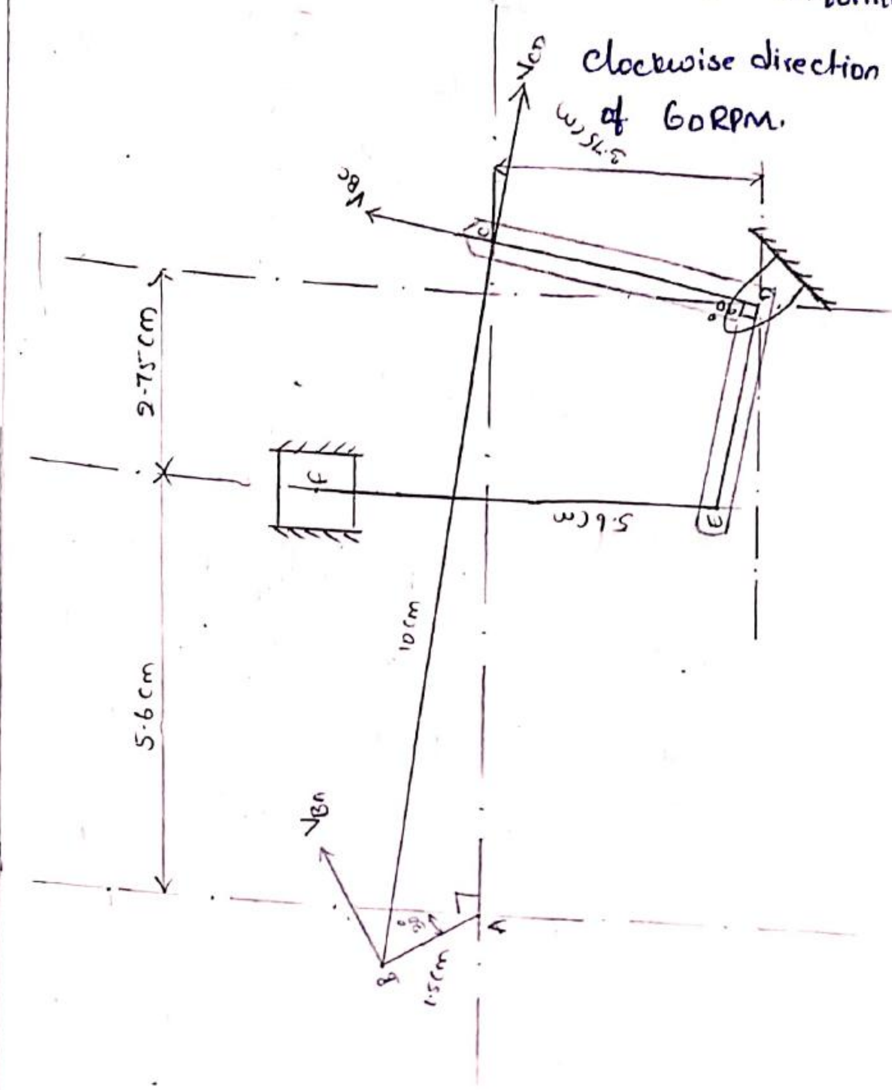
$V_{BA} = V_B = 1.57$ m/s


b
V_{PE}
$V_B = 1.57$ m/s
p
o
c
e

$V_{CD} = 1.3$


V_{ED}
E
6.25 cm
V_{DC}
17.5 cm
D
$V_{BA} = 1.57$ m/s
B
10 cm
12.5 cm
25 cm
6.25 cm
45°
A
V_{PB}
V_{PA}
P
C
6.25 cm
V_{CP}

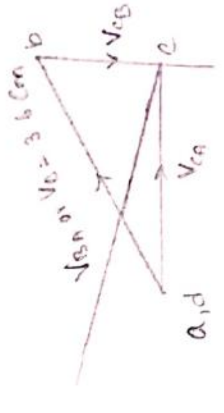
6) The dimensions of a Various links in a m/m as shown in figure, AB = 60 mm, BC = 400 mm, CD = 150 mm, DE = 115 mm and EF = 225 mm. find the Velocity of the Slider 'F', when the Crank AB rotates Uniformly in clockwise direction at a Speed of 60RPM.



$V_B = W_{AB} \times AB$

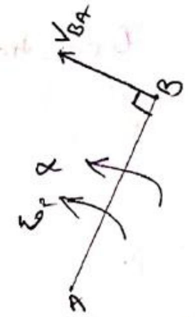
$= \frac{2\pi(60)}{60} \times \frac{60}{1000}$

$V_B = 3.6$ cm



* Acceleration in Mechanism:-

Consider a link 'AB'. let the Point 'B' moves w.r. to 'A'. with an Angular velocity 'ω' $\left(\frac{rad}{s}\right)$ and α $\left(\frac{rad}{s^2}\right)$



for Acceleration, there is a two Component for which acceleration

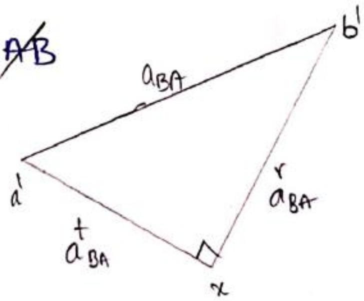
1) Radial Component → which is $\perp^r$ to the velocity of particle
2) Tangential Component → which is $\parallel^{al}$ to the " " "

Therefore, Radial Acceleration,

$$a^r_{BA} = \omega^2 \times \text{length of the link } AB$$

$$= \frac{(V_{BA})^2}{(AB)^2} \times AB$$

$$\boxed{a^r_{BA} = \frac{V_{BA}^2}{AB}}$$



Tangential Acceleration,

$$a^t_{BA} = \alpha \times \text{length of the link } AB$$

$$\boxed{a^t_{BA} = \alpha \times AB}$$

1) The Crank of a Slider Crank m/m rotates clockwise at a speed of 300 RPM. The Crank is 150 mm and the Connecting rod is 600 mm long. Determine

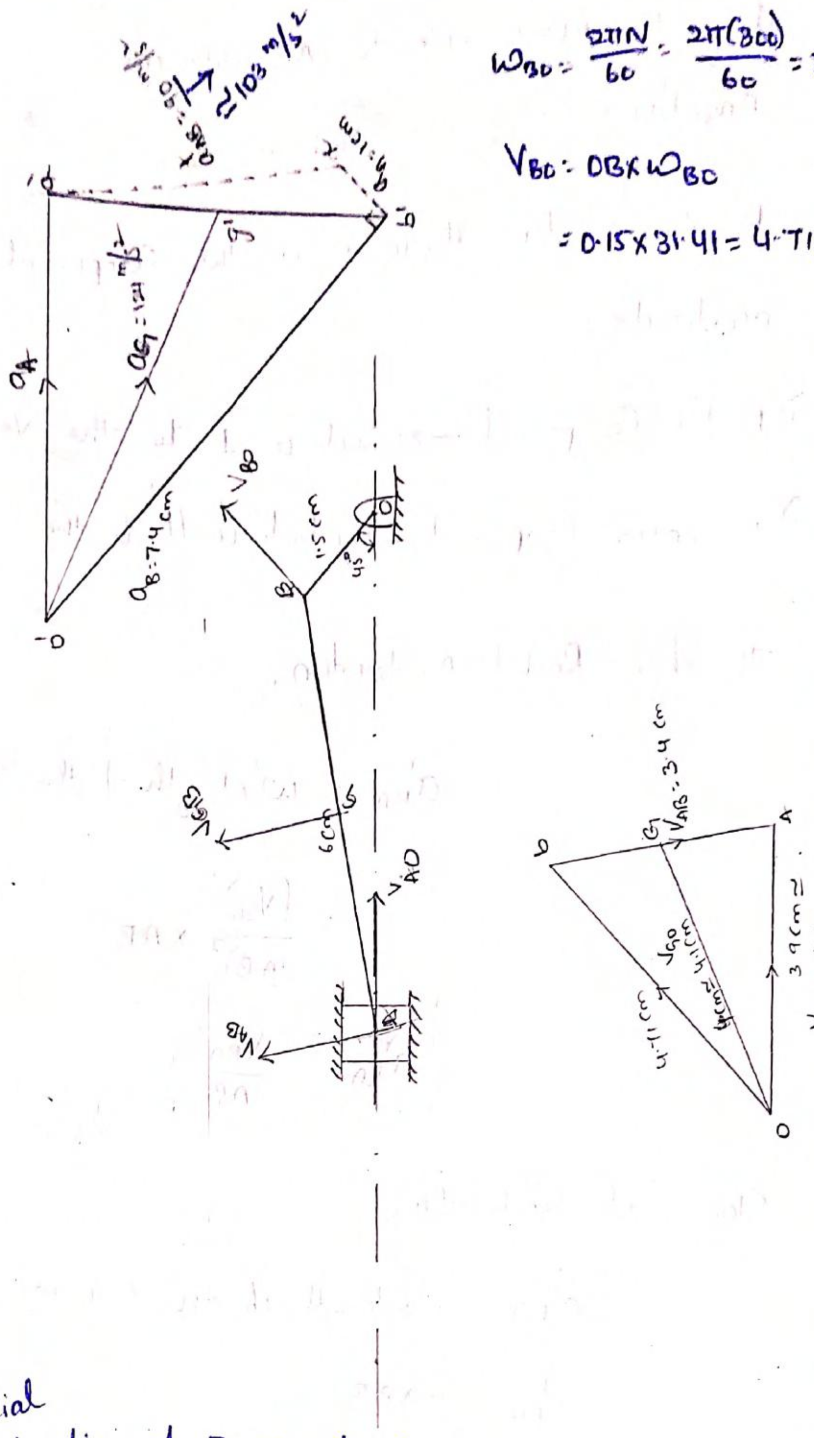
i) Linear velocity and Acceleration of the mid point of the Connecting rod

ii) Angular Velocity and Angular acceleration of the Connecting rod at a Crank angle of 45° from the IDC

$\omega_{BO} = \frac{2\pi N}{60} = \frac{2\pi(300)}{60} = 31.41\ rad/s$

$V_{BO} = OB \times \omega_{BO}$

$= 0.15 \times 31.41 = 4.713\ m/s$



Radial

Acceleration of B w.r.t to O,

$$a^r_{BO} = a_B = \frac{V_{BO}^2}{OB} = \frac{(4.713)^2}{0.15} = 148.1\ m/s^2$$

$$a^r_{AB} = a_A = \frac{V_{AB}^2}{BA} = \frac{(3.4)^2}{0.6} = 19.26\ m/s^2$$

(ii) Angular velocity of Connecting rod,

$$\omega_{AB} = \frac{V_{BA}}{AB} = \frac{3.4}{0.6} = 5.67 \text{ rad/sec}^2$$

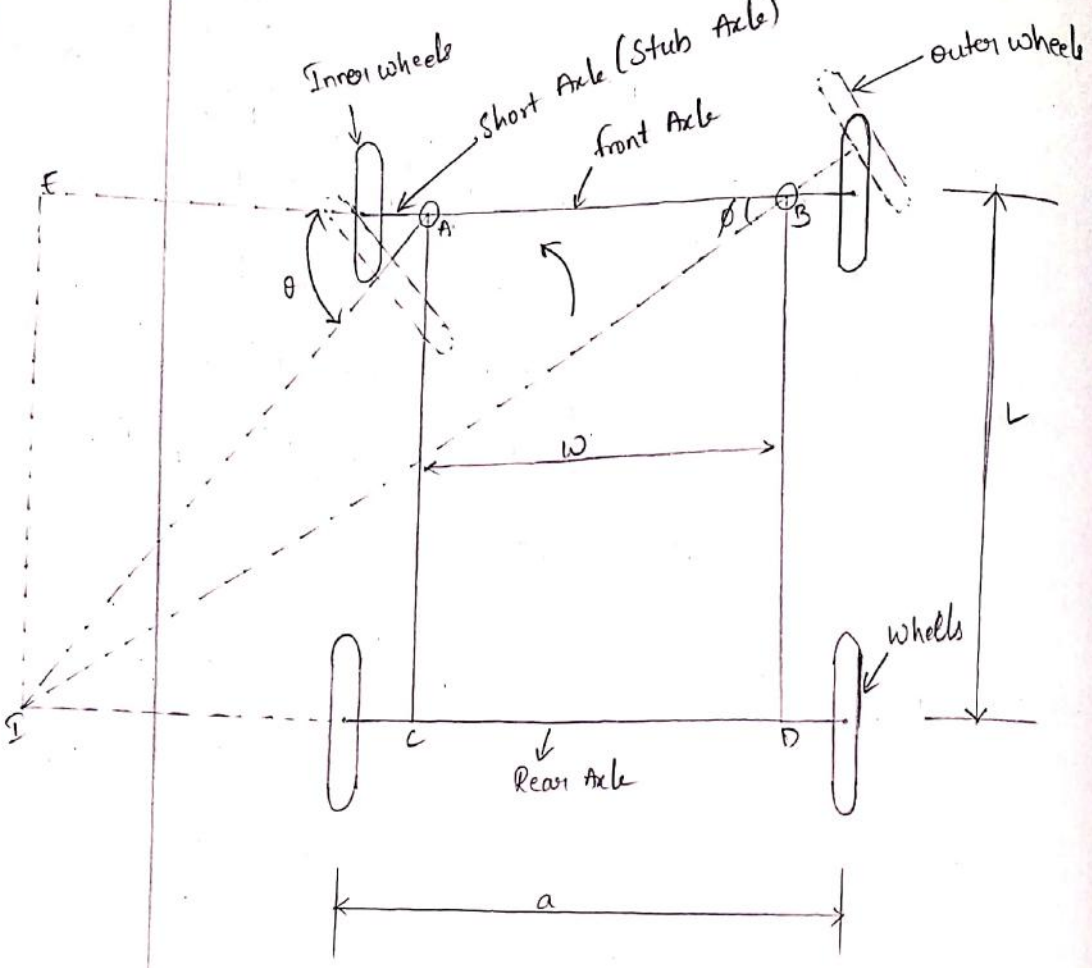
Angular acceleration of Connecting rod,

$$\alpha_{AB} = \frac{a^{t}_{AB}}{BA} = \frac{103}{0.6} = 171.66 \text{ rad/s}^2$$

10/03/2020

Unit-III

Steering Mechanism



* Derivation for Correct Steering m/m :-

Here,

W = Distance between the pivots of front Axle

L = wheel base

a = wheel track

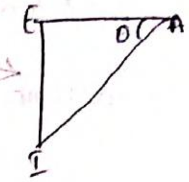
→ Definition for steering m/m :-

The m/m which is used for changing the direction of two (or) more of the wheel axles so as to move the automobile in any desired path is known as steering gear.

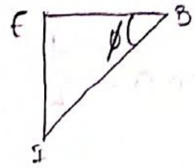
'θ' and 'ϕ' = Angle turned by the Stub Axle

Then,

$$\cot\theta = \frac{AE}{EI}$$



$$\cot\phi = \frac{EB}{EI}$$ | EB = EA + AB (As per fig)



$$\cot\phi = \frac{EA+AB}{EI}$$

$$\cot\phi - \cot\theta = \frac{EA+AB}{EI} - \frac{EA}{EI}$$

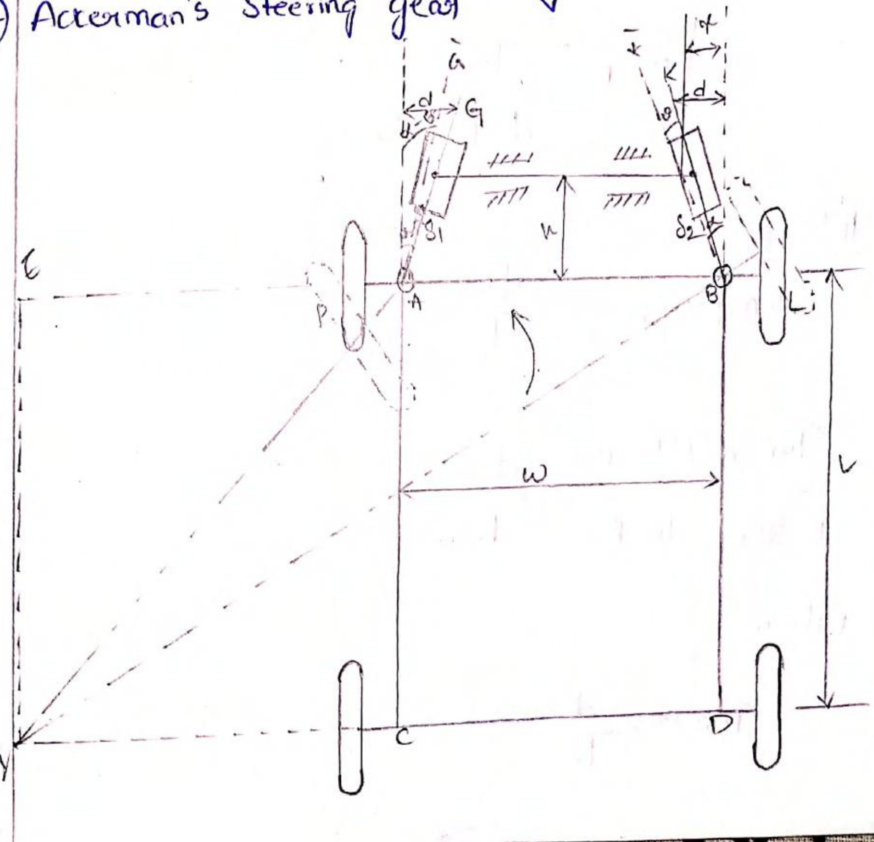
$$= \frac{EA}{EI} + \frac{AB}{EI} - \frac{EA}{EI}$$

$$\boxed{\cot\phi - \cot\theta = \frac{AB}{EI} = \frac{W}{L}}$$

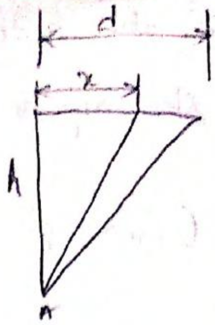
* Types of Steering gear :-

1) Davis Steering gear

2) Ackerman's Steering gear



$$\tan(\alpha-\theta) = \frac{d-x}{h}$$



$$\frac{\tan\alpha - \tan\theta}{1+\tan\alpha.\tan\theta} = \frac{d-x}{h}$$

where,

$$\tan\alpha = \frac{d}{h}$$

$$\frac{\frac{d}{h} - \tan\theta}{1+\frac{d}{h}(\tan\theta)} = \frac{d-x}{h}$$

$$\frac{\frac{d-h\tan\theta}{h}}{\frac{h+d\tan\theta}{h}} = \frac{d-x}{h}$$

$$\frac{d-h\tan\theta}{h+d\tan\theta} = \frac{d-x}{h}$$

$$h(d-h\tan\theta) = (d-x)(h+d\tan\theta)$$

$$\boxed{\tan\theta = \frac{hx}{d^2+h^2-dx}}$$

||ly

$$\tan(\alpha+\phi) = \frac{d+x}{h}$$

$$\frac{\tan\alpha + \tan\phi}{1-\tan\alpha.\tan\phi} = \frac{d+x}{h}$$

where,

$$\tan\alpha = \frac{d}{h}$$

$$\frac{\frac{d}{h}+\tan\phi}{1-\frac{d}{h}\cdot\tan\phi} = \frac{d+x}{h}$$

$$\frac{\frac{d+h\tan\phi}{h}}{\frac{h-d\tan\phi}{h}} = \frac{d+x}{h}$$

$$h(d+h\tan\phi) = (d+x)(h-d\tan\phi)$$

$$hd+h^2\tan\phi = dh-d^2\tan\phi+xh-xd\tan\phi$$

$$\boxed{\tan\phi = \frac{hx}{d^2+h^2-dx}}$$

Correct Steering equation,

$$\cot\phi-\cot\theta = \frac{w}{L}$$

$$\frac{d^2+h^2+dx}{hx} - \frac{d^2+h^2-dx}{hx} = \frac{w}{L}$$

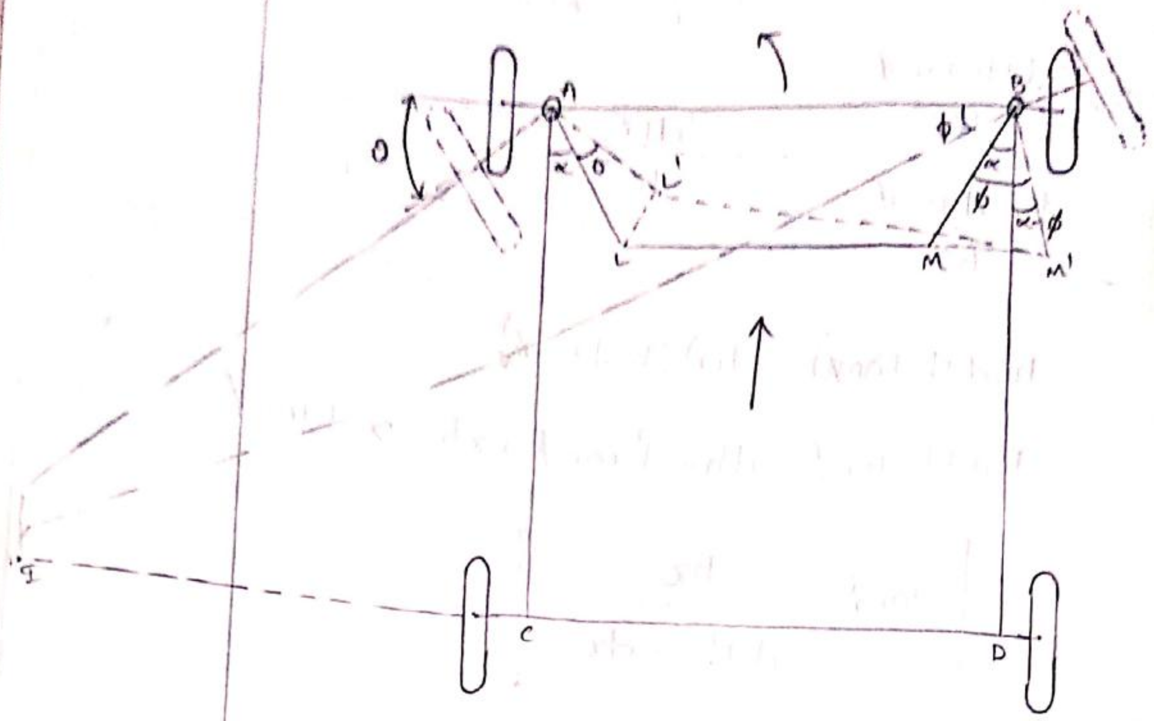
$$\frac{d^2+h^2+dx-d^2-h^2+dx}{hx} = \frac{w}{L}$$

$$\frac{2dx}{hx} = \frac{w}{L}$$

$$\frac{d}{h} = \frac{w}{2L}$$

$$\boxed{\tan\alpha = \frac{w}{2L}}$$

2) Ackerman's Steering gear:-



Projection of LL' on AB = Projection of MM' on AB

$$AL' \cos(90-(\alpha+\theta)) - AL\cos(90-\alpha) =$$

$$BM\cos(90-\alpha) + BM'\sin(\phi-\alpha)$$

$$AL'\sin(\alpha+\theta) - AL\sin\alpha = BM\sin\alpha + BM'\sin(\phi-\alpha)$$

$$[AL' = AL = BM = BM']$$

$$\sin(\alpha+\theta) - \sin\alpha = \sin\alpha + \sin(\phi-\alpha)$$

$$\sin\alpha\cos\theta + \cos\alpha\sin\theta - \sin\alpha = \sin\alpha + \sin\phi\cos\alpha - \cos\phi\sin\alpha$$

$$\sin\alpha\cos\theta - \sin\alpha - \sin\alpha = \sin\phi\cos\alpha - \cos\phi\sin\alpha - \cos\alpha\sin\theta$$

$$\sin\alpha\cos\theta - \sin\alpha - \sin\alpha + \cos\phi\sin\alpha = \sin\phi\cos\alpha - \cos\alpha\sin\theta$$

$$\sin\alpha(\cos\phi + \cos\theta - 2) = \cos\alpha(\sin\phi - \sin\theta)$$

$$\frac{\sin\alpha}{\cos\alpha} = \frac{\sin\phi - \sin\theta}{\cos\phi + \cos\theta - 2}$$

$$\boxed{\tan\alpha = \frac{\sin\phi - \sin\theta}{\cos\phi + \cos\theta - 2}}$$

* Universal Hook's Joint:-

→ What is Hook's Joint?

A) 1) It is used to 2 shafts which intersect at a small angle

2) It transmits power from the gear box of the Engine of the rear axle.

3) To transmit power to different spindles of multiple drilling machine.

4) The driving shaft rotates at a uniform angular speed where as the driven shaft rotates at a continuously varying angular speed

* Analysis of Hook's Joint:-

$$\tan\theta = \frac{EG_1}{EO}$$

$$\tan\phi = \frac{EG_2}{EO}$$

$$\frac{\tan\phi}{\tan\theta} = \frac{EG_2}{EO} \times \frac{EO}{EG_1}$$

$$\frac{\tan\theta}{\tan\phi} = \frac{EG_1}{EG_2}$$

$$= \frac{ON = ON_1 \cos\alpha}{OM = ON_1}$$

$$= \frac{ON_1 \cos\alpha}{ON_1}$$

$$\boxed{\tan\theta = \cos\alpha \times \tan\phi} \longrightarrow (1)$$

Ratio of Angular Velocity of Shaft:-

$$\frac{\omega_1}{\omega_2} = ?$$

let,

ω_1 = Angular velocity of driving shaft

$$\omega_1 = \frac{d\theta}{dt}, \quad \omega_2 = \frac{d\phi}{dt}$$

∴ Differentiating Eq. (1) w.r.to time 't',

$$\frac{d}{dt}(\tan\theta) = \frac{d}{dt}(\cos\alpha \times \tan\phi)$$

$$\sec^2\theta \times \frac{d\theta}{dt} = \cos\alpha \times \sec^2\phi \frac{d\phi}{dt}$$

$$\sec^2\theta \times \omega_1 = \cos\alpha \times \sec^2\phi \times \omega_2$$

$$\frac{\omega_1}{\omega_2} = \frac{\cos\alpha \times \sec^2\phi}{\sec^2\theta}$$

$$\frac{\omega_2}{\omega_1} = \frac{\sec^2\theta}{\cos\alpha \times \sec^2\phi}$$

$$= \frac{1/\cos^2\theta}{\cos\alpha \times \sec^2\phi}$$

$$\frac{\omega_2}{\omega_1} = \frac{1}{\cos^2\theta \times \cos\alpha \times \sec^2\phi} \longrightarrow ②$$

Now,

$$\sec^2\phi = 1 + \tan^2\phi$$

$$= 1 + \frac{\tan^2\theta}{\cos^2\alpha}$$

$$= \frac{\cos^2\alpha + \tan^2\theta}{\cos^2\alpha}$$

$$= \frac{\cos^2\alpha + \frac{\sin^2\theta}{\cos^2\theta}}{\cos^2\alpha}$$

$$= \frac{\cos^2\alpha \cos^2\theta + \sin^2\theta}{\cos^2\theta \times \cos^2\alpha \cos^2\theta \times \cos^2\alpha}$$

$$= 1 + \frac{\sin^2\theta}{\cos^2\theta \times \cos^2\alpha}$$

$$= \frac{\cos^2\theta(1-\sin^2\alpha) + \sin^2\theta}{\cos^2\theta \times \cos^2\alpha}$$

$$= \frac{\cos^2\theta - \cos^2\theta\sin^2\alpha + \sin^2\theta}{\cos^2\theta \times \cos^2\alpha}$$

from ②

$$\frac{\omega_2}{\omega_1} = \frac{1}{\cos^2\theta \cdot \cos\alpha \times \left[\frac{\cos^2\theta - \cos^2\theta\sin^2\alpha + \sin^2\theta}{\cos^2\theta \times \cos^2\alpha}\right]}$$

$$= \frac{\cos\alpha}{\cos^2\theta - \cos^2\theta\sin^2\alpha + \sin^2\theta}$$

$$\frac{\omega_2}{\omega_1} = \frac{\cos\alpha}{1-\cos^2\theta\sin^2\alpha} \longrightarrow (3) = \frac{N_2}{N_1}$$

* Conditions for equal speeds of driven & driving shafts:-

If $\omega_2 = \omega_1$

from eq.-(3)

$$\frac{\omega_2}{\omega_2} = \frac{\cos\alpha}{1-\cos^2\theta\sin^2\alpha}$$

$$\cos\alpha = 1-\cos^2\theta\sin^2\alpha$$

$$\cos^2\theta\sin^2\alpha = 1-\cos\alpha$$

$$\cos^2\theta = \frac{1-\cos\alpha}{\sin^2\alpha}$$

$$= \frac{1-\cos\alpha}{1-\cos^2\alpha}$$

$$\cos^2\theta = \frac{1-\cos\alpha}{(1-\cos\alpha)(1+\cos\alpha)}$$

$$\cos^2\theta = \frac{1}{1+\cos\alpha}$$

$$= \frac{\sin^2\theta+\cos^2\theta}{1+\cos\alpha}$$

$$\cos^2\theta = \frac{\cos^2\theta\left[\frac{\sin^2\theta}{\cos^2\theta}+1\right]}{1+\cos\alpha}$$

$$\cancel{1} + \cos\alpha = \tan^2\theta + \cancel{1}$$

$$\tan^2\theta = \cos\alpha$$

$$\tan\theta = \pm\sqrt{\cos\alpha}$$

* **Condition for maximum & minimum of driven Shaft :-**

W.K.T, eq. (3)

driven → ω_2, driving → ω_1

$$\frac{\omega_2}{\omega_1} = \frac{\cos\alpha}{1 - \cos^2\theta \sin\alpha}$$

for maximum speed of ω_2, the denominator's is maximum,

$$(1 - \cos^2\theta \sin^2\alpha) = \text{minimum}$$

$$\theta = 0^\circ \text{ or } 180^\circ$$

$$\frac{\omega_2}{\omega_1} = \frac{\cos\alpha}{1 - 1(\sin^2\alpha)} = \frac{\cos\alpha}{\cos^2\alpha} = \frac{1}{\cos\alpha}$$

$$\boxed{\omega_2 = \frac{\omega_1}{\cos\alpha}}$$

for minimum speed of ω_2, the denominator is maximum.

$$1 - \cos^2\theta \sin^2\alpha = \text{maximum}$$

$$\theta = 90^\circ \text{ or } 270^\circ$$

$$\frac{\omega_2}{\omega_1} = \frac{\cos\alpha}{1 - \cos^2\theta \times \sin^2\alpha}$$

$$\frac{\omega_2}{\omega_1} = \frac{\cos\alpha}{1-\cos^2(90)\times\sin^2\alpha}$$

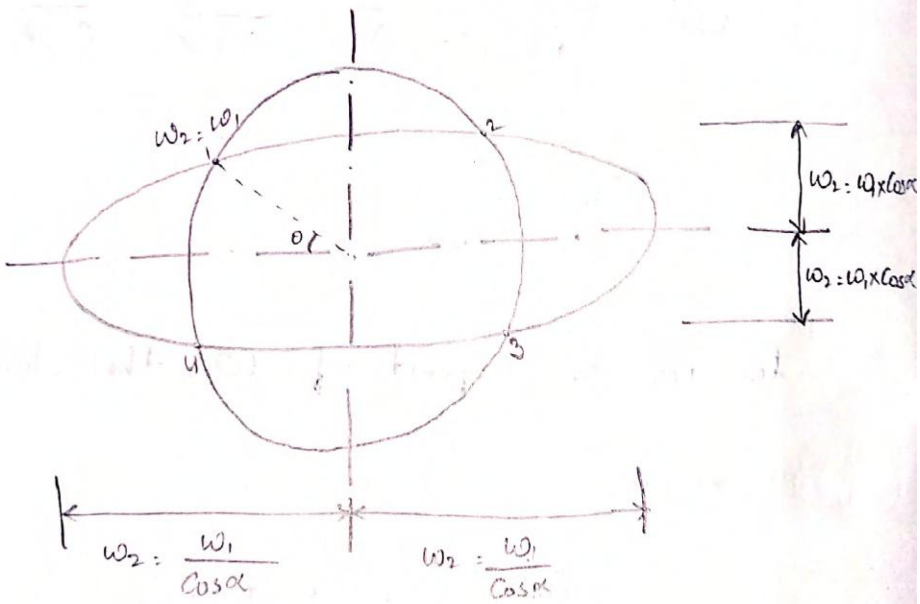
$$\frac{\omega_2}{\omega_1} = \frac{\cos\alpha}{1-0}$$

$$\frac{\omega_2}{\omega_1} = \cos\alpha$$

$$\boxed{\omega_2 = \omega_1 \times \cos\alpha}$$

(5M). * **Polar velocity diagram:-**

The diagram which shows the Variation of Angular velocities of the driven shaft and driving shaft for one Complete revolution is known as Polar velocity diagram.



1) The Speed of the driven shaft is maximum when $\theta = 0^\circ$ or 180°, where as the speed is minimum then $\theta = 90^\circ$ or 270°

2) The speed of the driven shaft is equal to the speed of the driving shaft at 4 points.

3) The speed of the driving shaft is constant and hence it is represented by a circle of radius = ω_1

4) The speed of the driven shaft is not constant. Therefore, the maximum value is $\frac{\omega_1}{\cos\alpha}$ and minimum value is $\omega_1 \times \cos\alpha$. It is represented by an ellipse of semi major axis = $\frac{\omega_1}{\cos\alpha}$ and semi minor axis = $\omega_1 \times \cos\alpha$

* <u>Condition for maximum fluctuation speed of the driven shaft</u>:-

Maximum fluctuation of the speed = max. speed − Min. speed

$$= \frac{\omega_1}{\cos\alpha} - \omega_1 \times \cos\alpha$$

$$= \omega_1 \left[\frac{1}{\cos\alpha} - \cos\alpha\right]$$

$$= \omega_1 \left[\frac{1-\cos^2\alpha}{\cos\alpha}\right]$$

$$= \omega_1 \left[\frac{\sin^2\alpha}{\cos\alpha}\right]$$

$$= \omega_1 \left[\frac{\sin\alpha}{\cos\alpha} \times \sin\alpha\right]$$

$$= \omega_1 \left[\tan\alpha \times \sin\alpha\right]$$

where,

α is very small, so, $\sin\alpha = \tan\alpha = \alpha$

$$= \omega_1 [\alpha \times \alpha]$$

$= \omega_1 \times \alpha^2$

Fluctuation of the Speed $= \omega_1 \times \alpha^2$

* Condition for Angular Acceleration of the driven Shaft:

We know that the eq. (8) can be written as

$$\frac{\omega_2}{\omega_1} = \frac{\cos\alpha}{1-\cos^2\theta \cdot \sin^2\alpha}$$

The Angular velocity of the driven shaft is given by,

$$\omega_2 = \frac{\omega_1 \cos\alpha}{1-\cos^2\theta \cdot \sin^2\alpha}$$

The Angular acceleration of the driven shaft is,

$$\frac{d\omega_2}{dt} = \frac{\omega_1 \cos\alpha}{1-\cos^2\theta \cdot \sin^2\alpha}$$

$$= \omega_1 \times \frac{d}{dt}\left[\frac{\cos\alpha}{1-\cos^2\theta \cdot \sin^2\alpha}\right]$$

$$= \omega_1 \times \cos\alpha \cdot \frac{d}{dt}\left[1-\cos^2\theta \cdot \sin^2\alpha\right]^{-1}$$

$$= \omega_1 \cos\alpha \cdot (-1)\left[1-\cos^2\theta \sin^2\alpha\right]^{-2} \times$$

$$\left[0-\sin^2\alpha \times 2\cos\theta(-\sin\theta) \times \frac{d\theta}{dt}\right]$$

$$= \frac{-\omega_1 \times \cos\alpha}{\left[1-\cos^2\theta \sin^2\alpha\right]^2} \times \left[-\sin^2\alpha \times -\sin 2\theta \times \frac{d\theta}{dt}\right]$$

$$= \frac{-\omega_1 \times \cos\alpha}{\left[1-\cos^2\theta \sin^2\alpha\right]^2} \times \left[\sin^2\alpha \times \sin 2\theta \times \omega_1\right]$$

$$= \frac{-\omega_1^2 \times \cos\alpha \times \sin^2\alpha \cdot \sin 2\theta}{\left[1-\cos^2\theta \sin^2\alpha\right]^2}$$

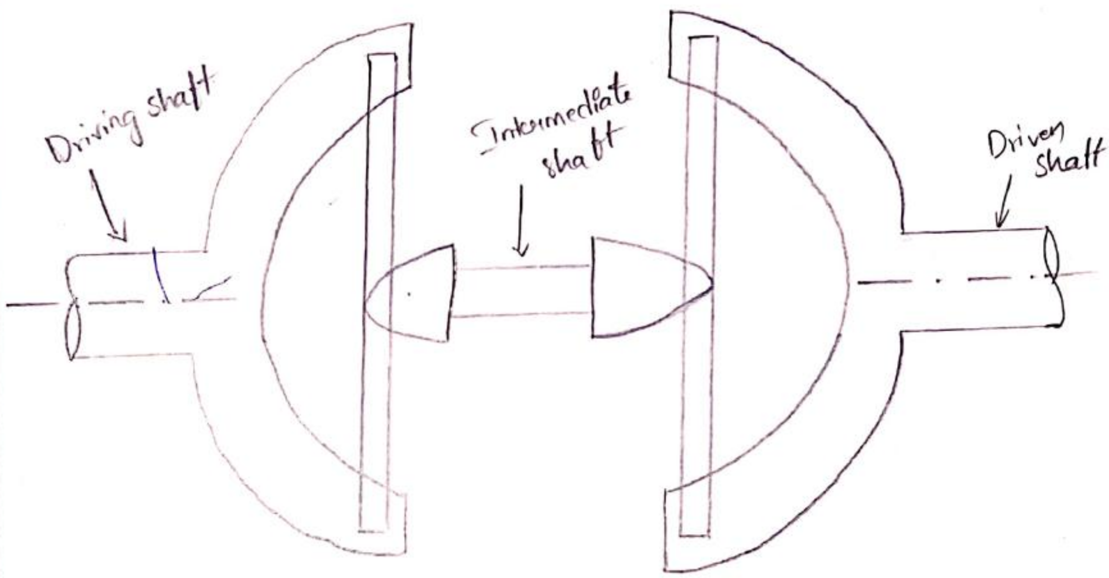
* Double Hooke's Joint:-

In a single Hooke's Joint, A driving shaft rotates at a constant speed whereas a driven shaft is varying speed. In order to have a constant velocity ratio of the driving and driven shaft a double hooke's Joint is used.

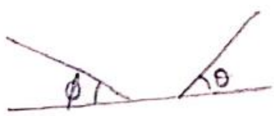
In a double Hook joint, two Hook Joints and intermediate shafts are used the speed of the driving & driven shafts will be equals, if.

1) The driving & driven shafts must be equally inclined to the intermediate shaft.

2) The two forces on the intermediate shaft lying in the same plane.



where,



θ = Angle turned by driving shaft

ϕ = Angle turned by driven shaft

γ = Angle turned by Intermediate shaft

α = Angle of inclination of the driving shaft with intermediate shaft

$$\tan\theta = \cos\alpha \cdot \tan\gamma$$

$$\tan\phi = \cos\alpha \cdot \tan\gamma$$

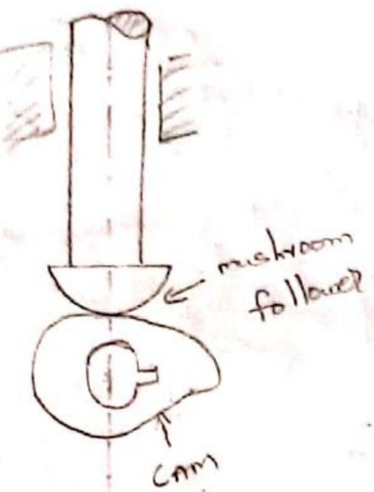
$$\therefore \tan\theta = \tan\phi$$

$$\boxed{\theta = \phi}$$

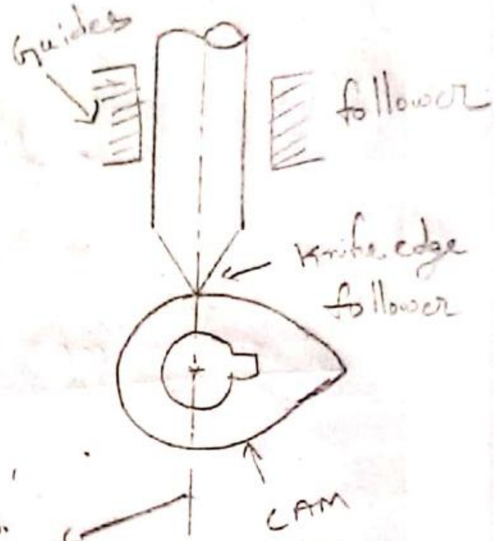
Unit - IV

CAMS

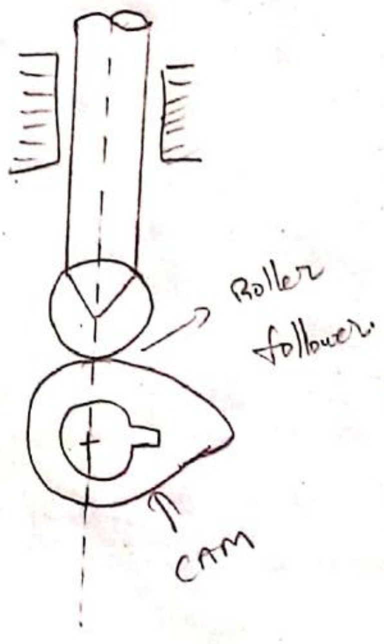
* mushroom follower;-



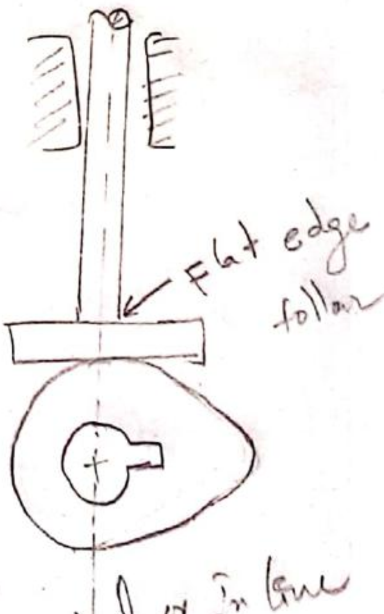
Knife edge follower;-



Roller follower;-

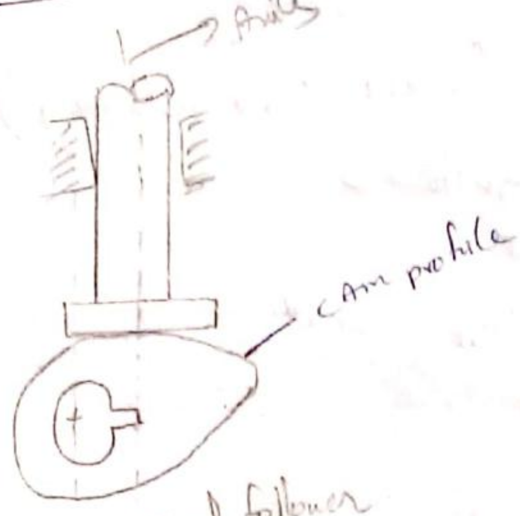


flat edge follower;-



radial or In line

offset follower;



* CAM & Follower:- A rotating machine element which gives reciprocating or oscillating motion to a second element is known as 'CAM', The second element is known as Follower.

⇒ The CAM rotates at uniform (or) constant speed and drives the follower whose motion depends upon the shape of the CAM.

⇒ The CAM's are commonly used in IC Engines in printing machineries, Automatic machines in machine tools.

* Types of followers;- There are 3 types;

① According to the shape of the part which is in contact with the CAM

i) knife edge follower

ii) Roller follower

iii) flat edge follower

② According to the motion of the follower

i) Reciprocating follower (Translatory)

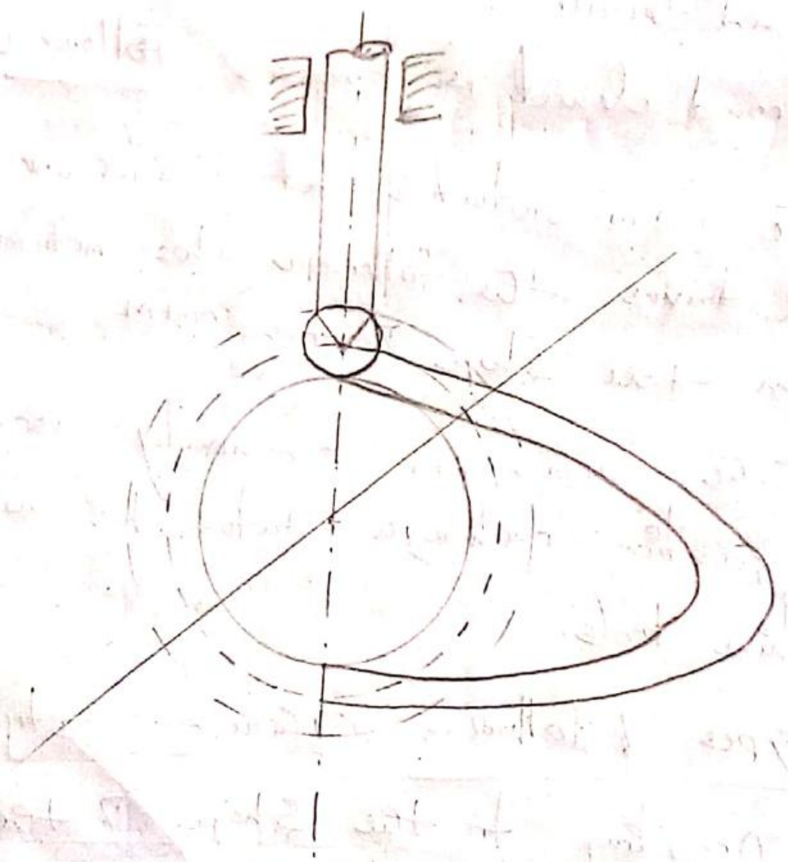
ii) oscillating follower

③ According to the location of the axis of the follower

i) ~~Infile~~ Inline or Radial follower

ii) OFF set follower -

2Q* Nomenclature of the cam profile :-

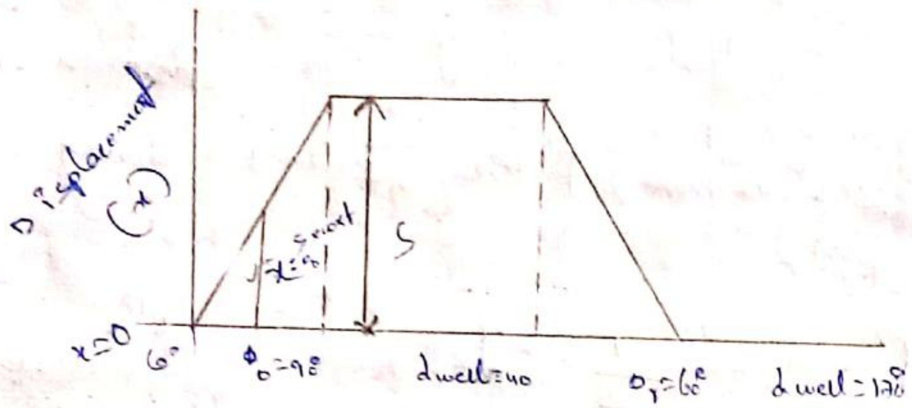


* motions of followers:-

1. uniform motion or uniform velocity
2. simple harmonic motion
3. uniform Acceleration & Retardation.
4. cycloidal
5. Any other desired shape of motion.

① uniform motion or uniform velocity:-

Displacement Diagram:-



θ

x-axis - abscissa (Horizontal)

y-axis - ordinate (vertical)

where s = stroke of the follower

$x =$

w.k.T $\theta = \omega \times t \Rightarrow$ angular velocity × time.

$\theta_0 =$

$\therefore \frac{x}{\theta} = \frac{S}{\theta_0}$

$x = \frac{S}{\theta_0} \times \theta$

$\frac{S}{\theta_0} \times \omega \times t$ dis.

velocity = $\frac{\text{displacement}}{\text{time}}$

$= \frac{dx}{dt}$

$= \frac{d}{dt}\left(\frac{S}{\theta_o} \times \omega \times t\right)$

$$\boxed{V_o = \frac{S}{\theta_o} \times \omega \times 1}$$

V_o = velocity of outward stroke

$$\boxed{V_r = \frac{S}{\theta_r} \times \omega}$$

V_r = velocity of Return stroke

acceleration of the follower during outward stroke

f_o (or) $a_o = \frac{dv}{dt}$ = const

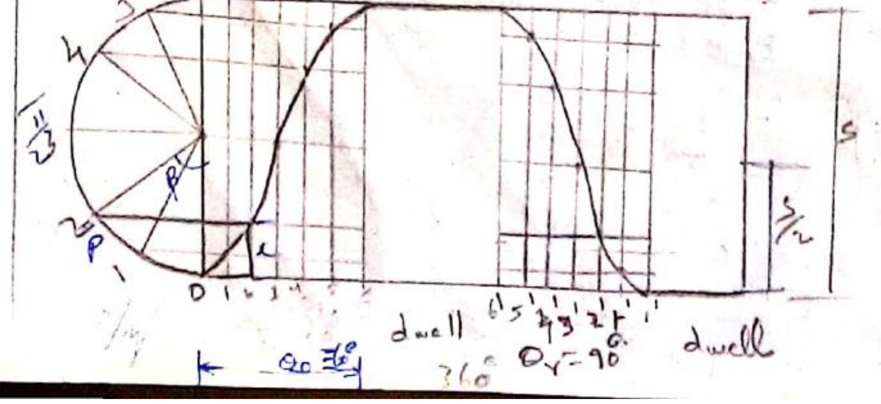
acceleration = $\frac{\text{velocity}}{\text{Time}}$

$a_o = 0$ or $\boxed{f_o = 0}$

$$\boxed{f_r = 0}$$

10M ② Simple harmonic motion:-

Displacement diagram:-



wkt $\theta = \omega \times t$

$$\frac{x}{\theta} = \frac{S}{\theta_0}$$

$$x = \frac{S}{\theta_0} \times \theta$$

$$\boxed{\beta = \frac{S}{\theta_0} \times \theta}$$

$$\beta = \frac{S}{\theta_0} \times \theta = \frac{\pi}{\theta_0} \times \theta$$

now the displacement $x = \frac{S}{2} - \frac{S}{2} \cos\beta$

$$x = \frac{S}{2} - \frac{S}{2} \cos\left(\frac{\pi}{\theta_0} \times \theta\right)$$

$$V_0 = \frac{dx}{dt} = \frac{d}{dt}\left(\frac{S}{2} - \frac{S}{2} \cos\left(\frac{\pi}{\theta_0} \times \theta\right)\right)$$

~~$V_0 = \frac{S}{2} - \frac{S}{2} \sin \frac{\pi}{\theta_0} \times \theta$~~

$$V_0 = \frac{d}{dt}\left(\frac{S}{2}\left(1 - \cos \frac{\pi}{\theta_0} \times \omega \times T\right)\right)$$

$$\boxed{\because \theta = \omega \times t}$$

$$V_0 = \frac{S}{2}\left(0 - \left(-\sin \frac{\pi\omega t}{\theta_0}\right) \times \frac{\pi\omega}{\theta_0}\right)$$

$$= \frac{S}{2}\left(\sin \frac{\pi\omega t}{\theta_0} \times \frac{\pi\omega}{\theta_0}\right)$$

$$V_0 = \frac{S}{2}\left(\sin \frac{\pi\theta}{\theta_0}\right) \cdot \frac{\pi\omega}{\theta_0}$$

$$\boxed{V_0 = \frac{S}{2} \times \frac{\pi \times \omega}{\theta_0} \times \sin\left(\frac{\pi \times \theta}{\theta_0}\right)}$$

$$v_{o(max)} = \sin\left(\frac{\pi\theta}{\theta_0}\right) = \sin\frac{\pi}{2}$$

$$\frac{\pi\theta}{\theta_0} = \frac{\pi}{2}$$

$$\theta = \frac{\theta_0}{2}$$

$$v_{o(max)} = \frac{s}{2} \times \frac{\pi \times \omega}{\theta_0} \times \sin\left(\frac{\pi \times \theta_0}{2 \times \theta_0}\right)$$

$$\boxed{v_{o(max)} = \frac{s}{2} \times \frac{\pi \times \omega}{\theta_0}}$$ outward stroke

ly $$\boxed{v_{r(max)} = \frac{s}{2} \times \frac{\pi \times \omega}{\theta_r}}$$ return stroke.

ly <u>acceleration (a_o)</u> :- $\frac{dv_o}{dt}$

$$= \frac{d}{dt}\left(\frac{s}{2} \times \frac{\pi \times \omega}{\theta_0} \times \sin\frac{\pi\omega \times t}{\theta_0}\right)$$

$$= \left(\frac{s}{2} \times \frac{\pi \times \omega}{\theta_0} \times \cos\frac{\pi\omega t}{\omega_0} \times \frac{\pi\omega}{\theta_0}\right)$$

$$= \frac{s}{2}\left(\frac{\pi \times \omega}{\theta_0}\right)^2 \cos\frac{\pi\theta}{\theta_0}$$

max. acceleration $a_{o(max)} = \cos\frac{\pi\theta}{\theta_0} = \cos 0$

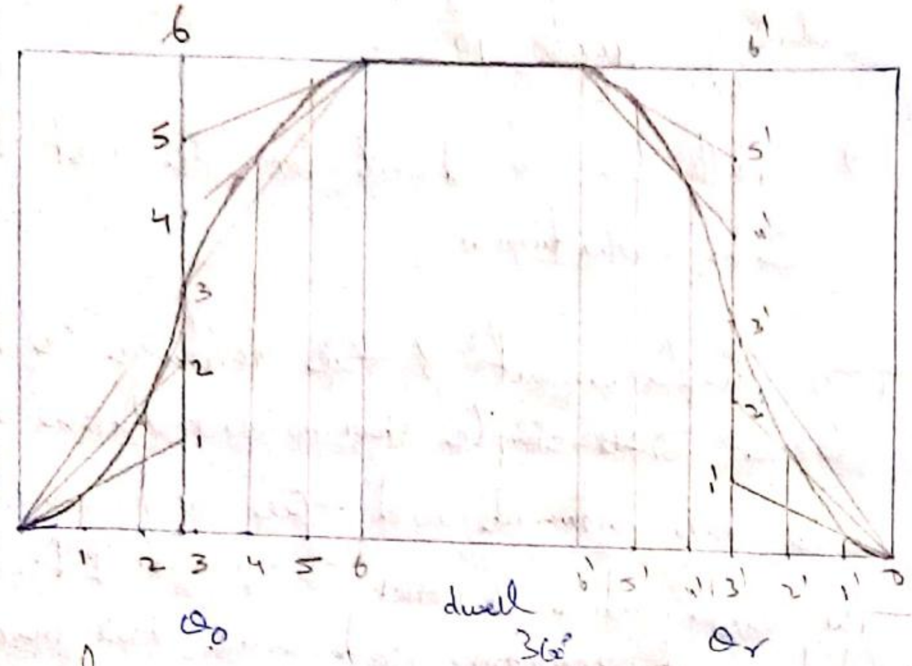
$$a_{o(max)} = \frac{s}{2}\left(\frac{\pi \times \omega}{\theta_0}\right)^2 \times \cos 0$$

$$\boxed{a_{o(max)} = \frac{s}{2}\left(\frac{\pi \times \omega}{\theta_0}\right)^2}$$ outward

$$\boxed{a_{r(max)} = \frac{s}{2}\left(\frac{\pi \times \omega}{\theta_r}\right)^2}$$ return

3. Uniform Acceleration & Retardation:—

Displacement diagram:



Formulas

$$V_{o(max)} = \frac{2 \times s \times \omega}{\theta_o}$$

$$V_{r(max)} = \frac{2 \times s \times \omega}{\theta_r}$$

$$a_{o(max)} = \frac{4 \times s \times \omega^2}{\theta_o^2}$$

$$a_{r(max)} = \frac{4 \times s \times \omega^2}{\theta_r^2}$$

Problems:— Draw the profile of a cam operating a knife edge follower when the axis of the follower passes through the axis of the cam shaft from the following data.

(i) follower to move outwards through 40mm during 60° of cam rotations

② Follower to dwell for the next 45°.

③ Follower to return to the original position during next 90°.

④ Follower to dwell for the rest of the cam rotation.

The displacement of the follower is to take place with simple Harmonic motion during both the outward and the return stroke. The least radius of the CAM is 50mm. If the cam rotates at 300rpm. Determine the max velocity and acceleration of the follower during outward stroke & Return stroke.

Solution: Given data

~~S = 40 mm~~ stroke of the follower (S) = 40mm

least radius of the CAM = 50mm

Cam rotates at N = 300rpm

velocity at outward:-

$\omega = \frac{2\pi N}{60} = 31.41$

① $V_0(max) = \frac{S}{2} \times \frac{\pi \times \omega}{\theta_0} = \frac{40}{2} \times \frac{\pi \times \frac{2\pi N}{60}}{60 \times \frac{\pi}{180}}$

~~1.884~~ m/sec

$\omega = \frac{2\pi N}{60}$

$= \frac{2\pi\, 300}{60} = 31.41$

$= 1884.6mm$

$= 1.8846 m/sec$

Scanned by CamScanner

note 2up

horizontal 10mm = 20°

vertical 10mm = 10mm

① velocity at Return stroke

$v_{r(max)} = \frac{s}{2} \times \frac{\pi \times \omega}{\theta_r}$

$= \frac{40}{2} \times \frac{\pi \times \frac{2\pi N}{60}}{90 \times \frac{\pi}{180}}$

$= 1256.4\ mm$

$= 1.256\ m/sec$

~~velocity~~

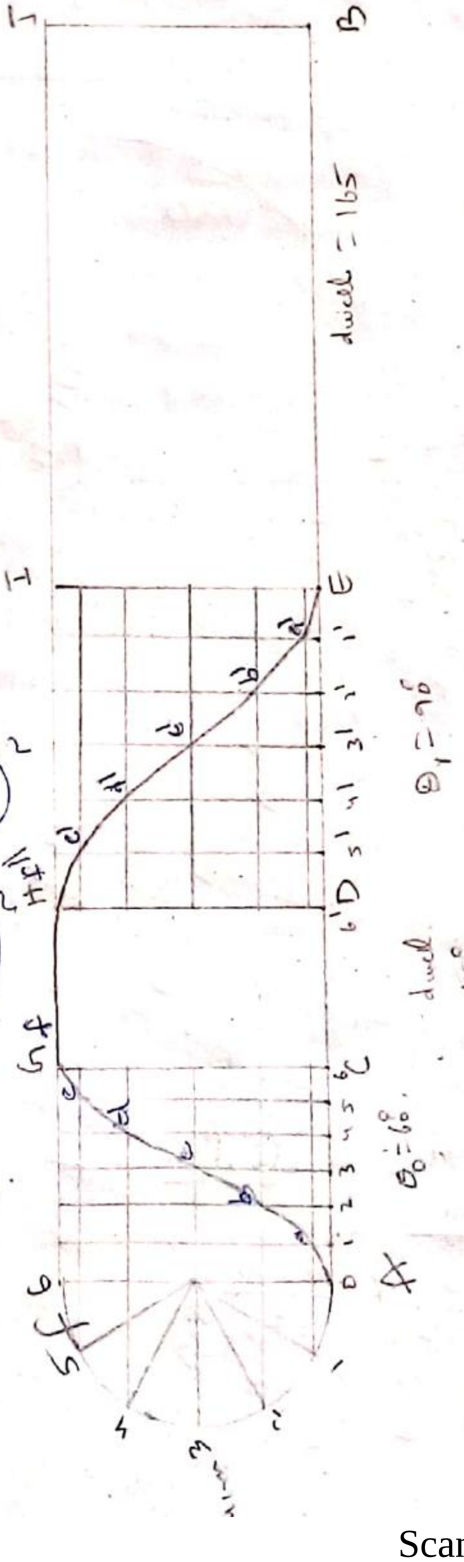
② acceleration at outward:

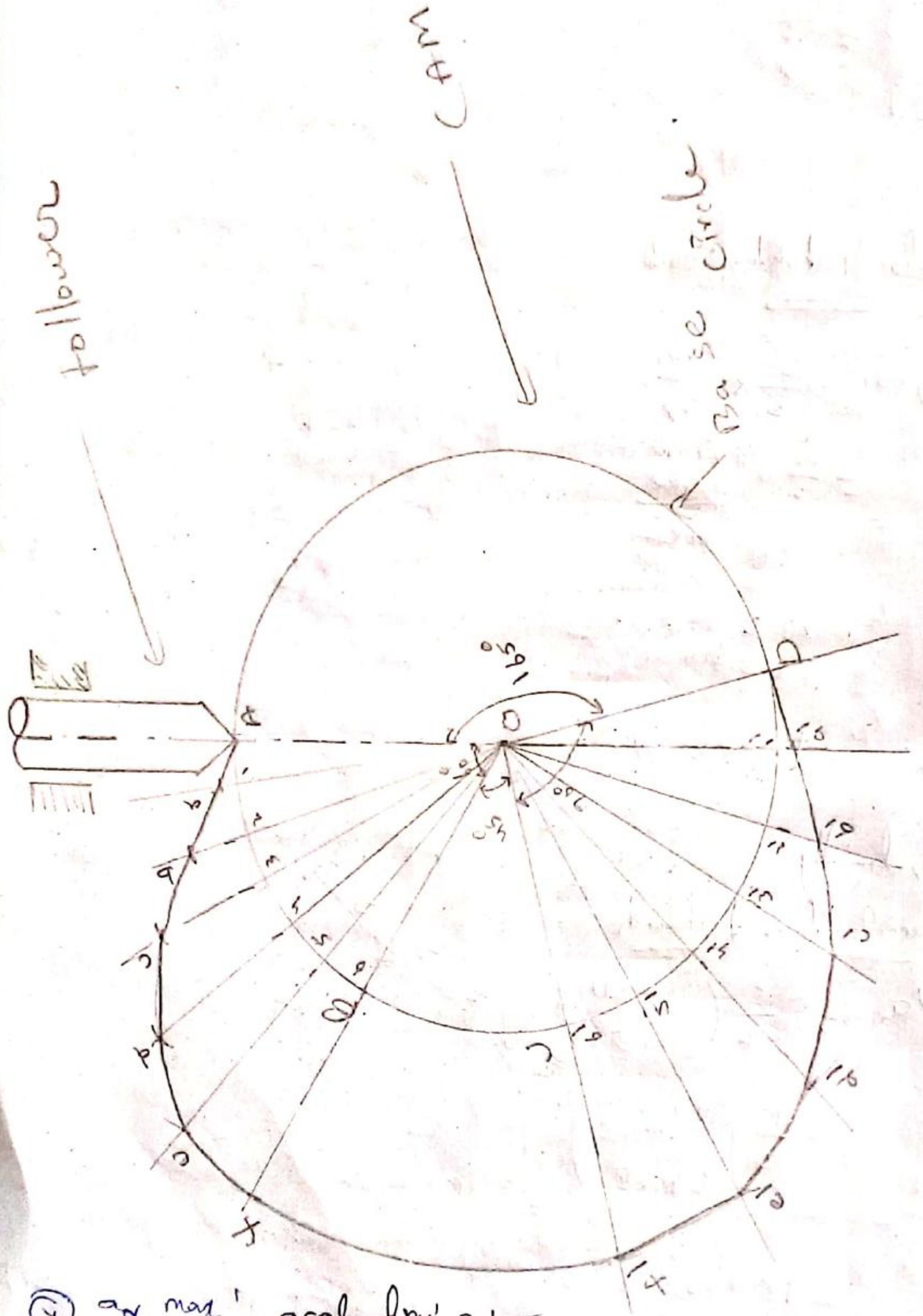
$a_{o(max)} = \frac{s}{2}\left(\frac{\pi \times \omega}{\theta_o}\right)^2$

$= \frac{40}{2}\left(\frac{\pi \times \frac{2\pi N}{60}}{60 \times \frac{\pi}{180}}\right)^2$

$= 177585.85$

$= 17.758585\ m$





(4) a_r max: acceleration a_o:-

$$a_r = \frac{S}{2}\left(\frac{\pi \times \omega}{\theta_r}\right)^2$$

$$= \frac{40}{2}\left(\frac{\pi \times \frac{2\pi N}{60}}{90 \times \frac{\pi}{180}}\right)^2$$

$= 78927.048$

$= 789.27$

② Draw the profile of a cam operating a knife edge follower when the axis of the follower is not passing through the axis of cam shaft, but is offset by 20mm from the axis of the cam shaft, Then draw the profile of the cam

① follower to move outwards through 40mm during 60° of cam rotation

② Follower to dwell the next 45°

③ Follower to return to the original position during next 90°.

④ Follower to dwell for the next angle of cam rotation

The displacement of the follower is simple harmonic motion. Both the outward & return, The least radius is 50mm, cam rotates at 300rpm.

$v_o = \frac{S}{2} \times \frac{\pi \times \omega}{\theta_o}$,

$v_r = \frac{S}{2} \times \frac{\pi \times \omega}{\theta_r}$

$a_o = \frac{S}{2} \left(\frac{\pi \omega}{\theta_o}\right)^2$

$a_r = \frac{S}{2} \left(\frac{\pi \omega}{\theta_r}\right)^2$

Sol

Displacement Diagram

Same as 1st question.

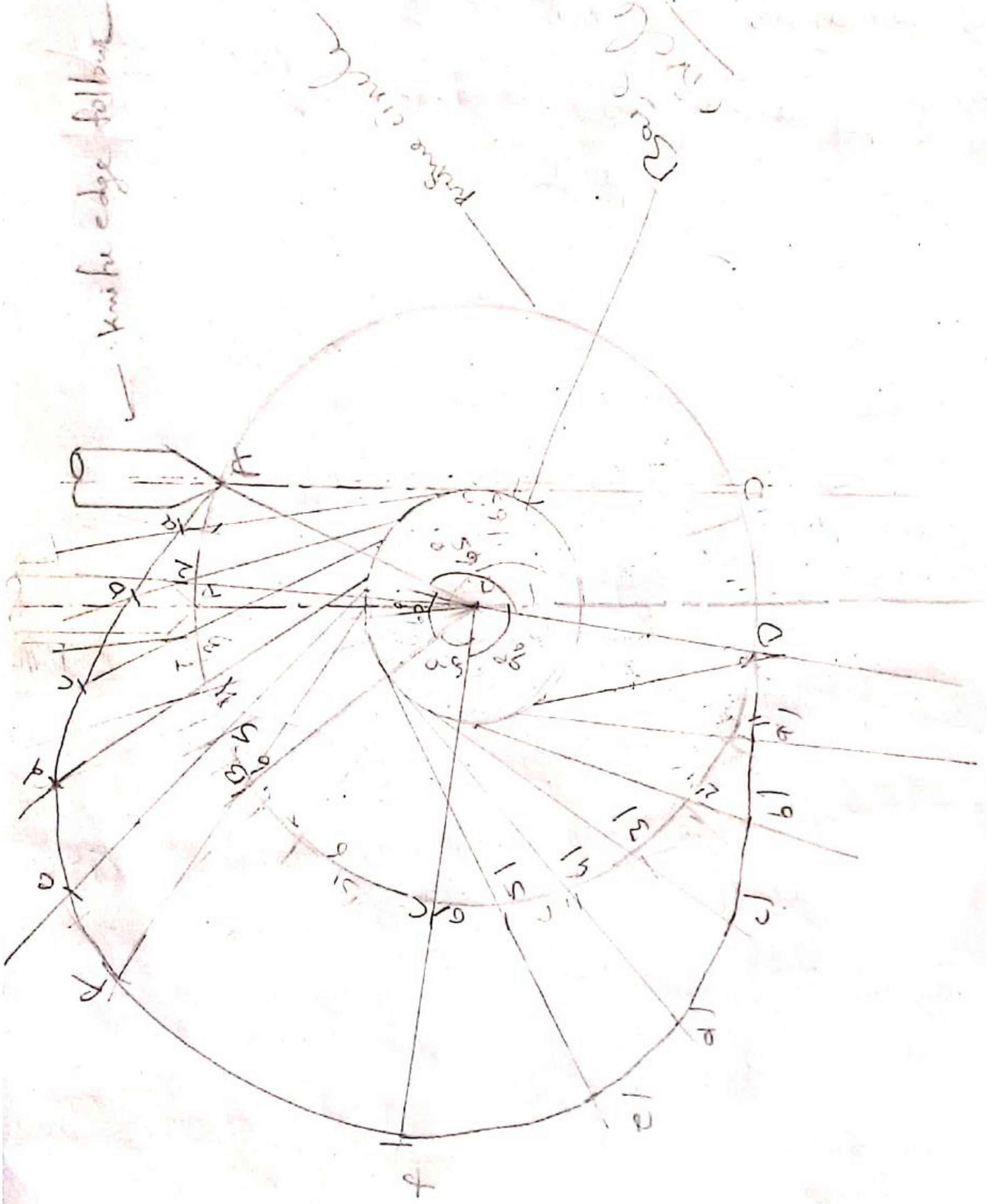
Calculations:—

velocity v_o = 1.886 m/sec

v_r = 1.256 m/sec

a_o = 17.75 m/s

a_r = 7.59.27 m/s



(3) Draw the profile of a cam operating when the axis of the follower passes through the axis of cam

(1) Follower to move outward through 30mm with simple harmonic motion during 120° of cam rotation.

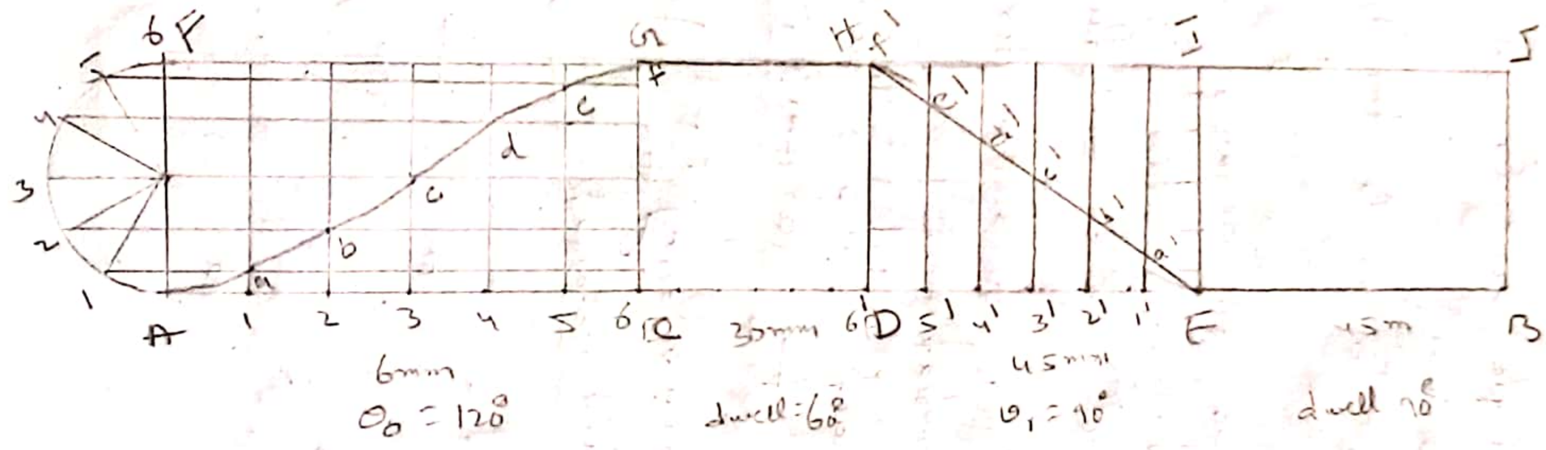
(2) follower to dwell for the next 60°.

(3) Follower to move return through its original ~~condition~~ position with uniform velocity during 90° of cam rotation.

(4) Follower to dwell for the rest of the cam rotation.

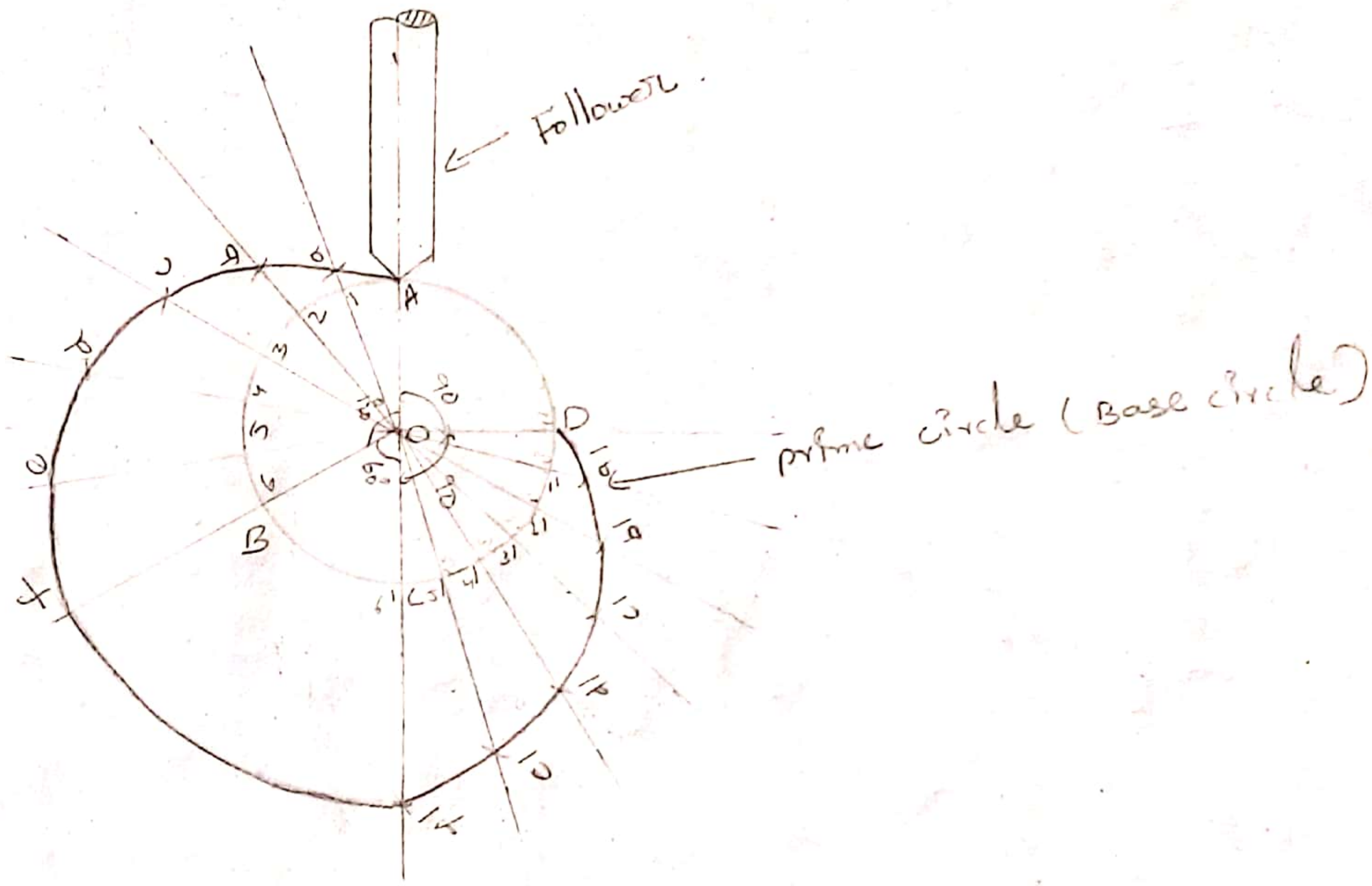
The least radius of the cam is 20mm and cam rotates at 240 r.p.m, Determine :-

(1) maximum velocity for outstroke, returnstroke and maximum acceleration to ~~outst~~, returnstroke
outward & return stroke



16 mm = 20°

60mm = 120°



Follower
prime circle (Base circle)
A
B
D
O

(4) Draw the profile of a cam, operating a knife edge follower from the following data.

(1) follower to move outward stroke through a distance of 20mm during 120° of cam rotation

(2) Follower to dwell for the next 60° of cam rotation

(3) Follower to return stroke to the original position during 90° of cam rotation.

(4) Follower to dwell for the remaining 90° of cam rotation

The cam is rotating clockwise at a uniform speed of 500 r.p.m the minimum radius of the cam is 40mm and the line of stroke of the follower is off set 15 mm from the axis of the cam & the displacement of the follower with uniform acceleration and retardation for both outward and return strokes (1) Determine max. velocity of the follower during outward and return stroke.

(2) The max. acceleration during outward and return stroke.

Calculation

$V_{o(max)} = \frac{2 \times S \times \omega}{\theta_o}$

$= \frac{2 \times 20 \times 52.35}{120 \times \frac{\pi}{180}} = 999.81$ mm $\Rightarrow 0.999$ m/sec

$\therefore \omega = \frac{2\pi N}{60} = \frac{2 \times \pi \times 500}{60} = 52.35$

$V_{r(max)} = \frac{2 \times S \times \omega}{\theta_r}$

$= \frac{2 \times 20 \times 52.35}{90 \times \frac{\pi}{180}}$

$= 1333.08$ mm

$=$ ~~13.33~~ m/sec.

1.333 m/sec.

$a_o = \frac{4 \times S \times \omega^2}{\theta_o^2}$

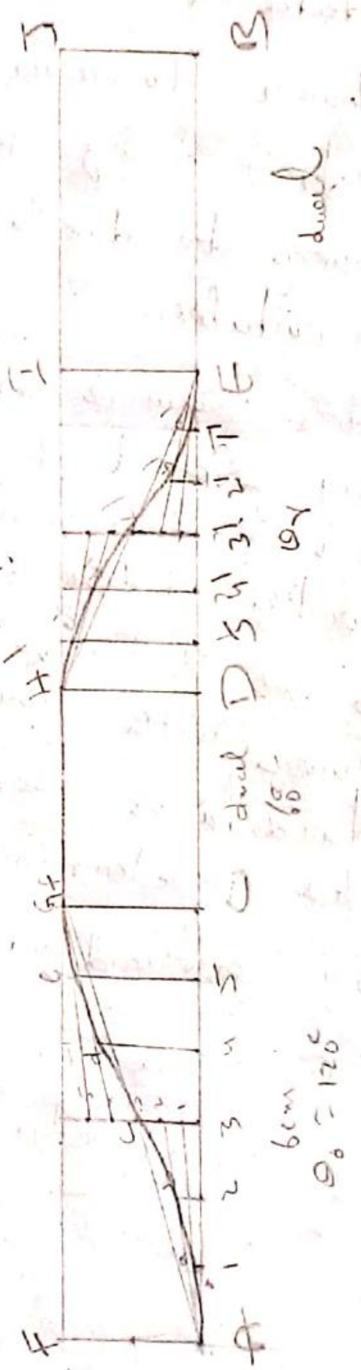
$= 49981.1$ mm

$= 49.981$ m/sec

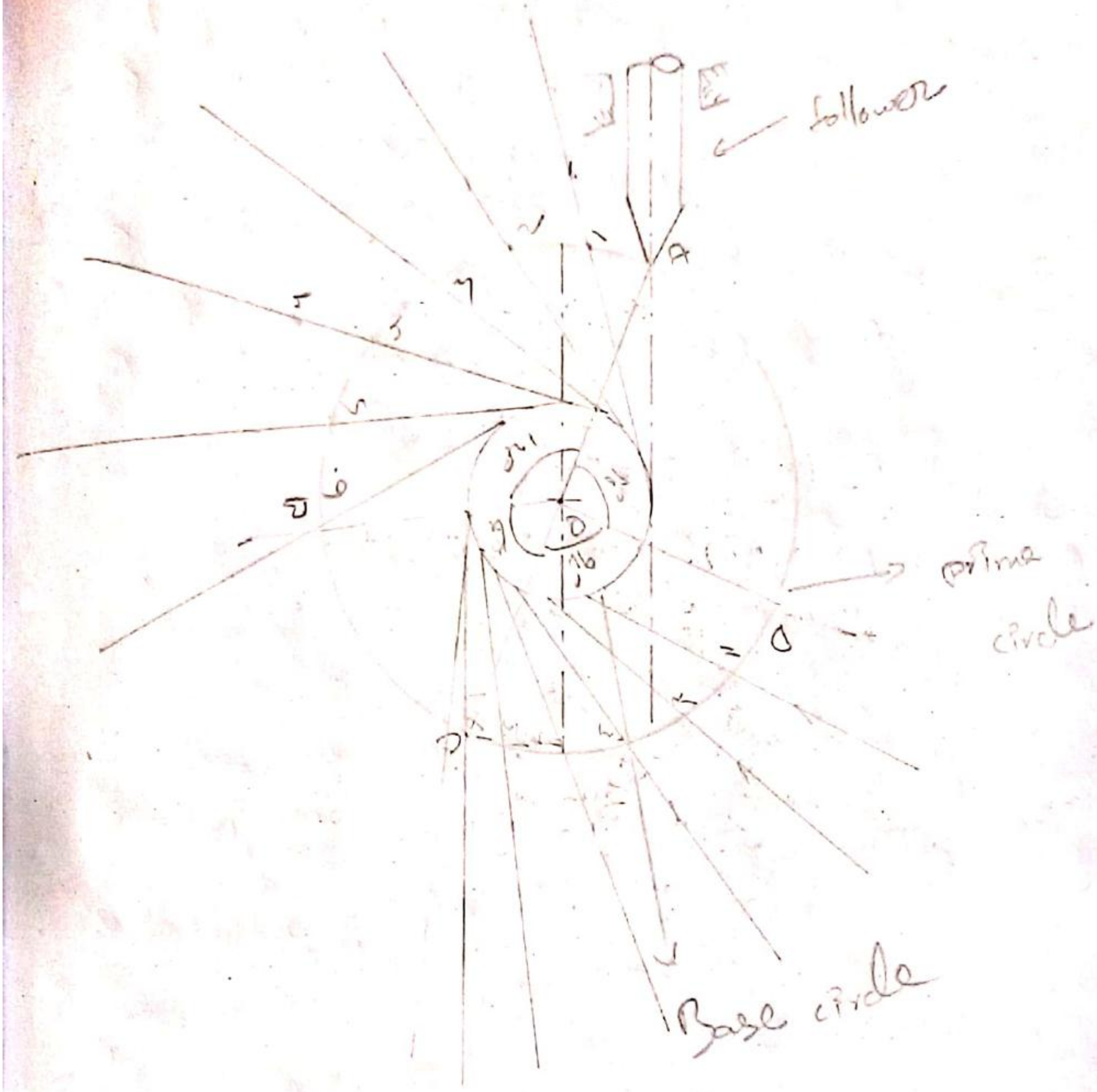
$a_r = \frac{4 \times S \times \omega^2}{\theta_r^2}$

$= 88855.25$ mm

$= 88.85$ m/sec



Draw follower displacement of 20 mm for 120°, dwell for 60°, return for 90° & dwell.


follower
A
prime circle
Base circle

5 A cam ~~with~~ is 30mm diameter is rotating clock wise at a uniform speed of 1200 rpm & motion of a roller follower is 10mm diameter

(1) follower to outward stroke of 35 mm during 120° of cam rotation with uniform acceleration and Retardation.

(2) follower to dwell 60° of cam rotation

(3) follower to return stroke 90° with uniform acceleration and Retardation

(4) Follower to dwell for the remaining 90° of cam rotation

Draw the cam profile if the axis of roller follower passes through the axis of the cam. Determine $V_{o\,max}$, $V_{r\,max}$, $a_{o\,max}$, $a_{r\,max}$.

Calculations:

$v_{o(max)} = \frac{2 \times S \times \omega}{\theta_o}$

$\omega = \frac{2\pi N}{60} = \frac{2 \times \pi \times 1200}{\frac{120 \times \pi}{180}}$ (125.66)

$= \frac{2 \times \pi \times 1200}{60}$ = ~~3600~~ mm

= 125.66 = ~~2.6~~ m/sec.

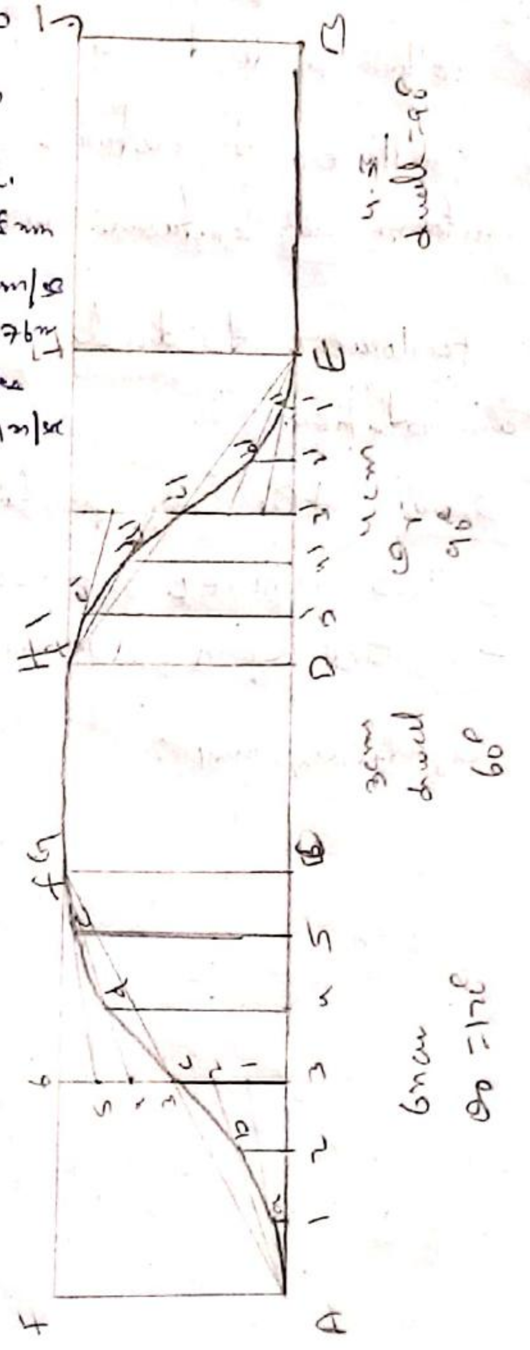
= ~~376.98~~ mm

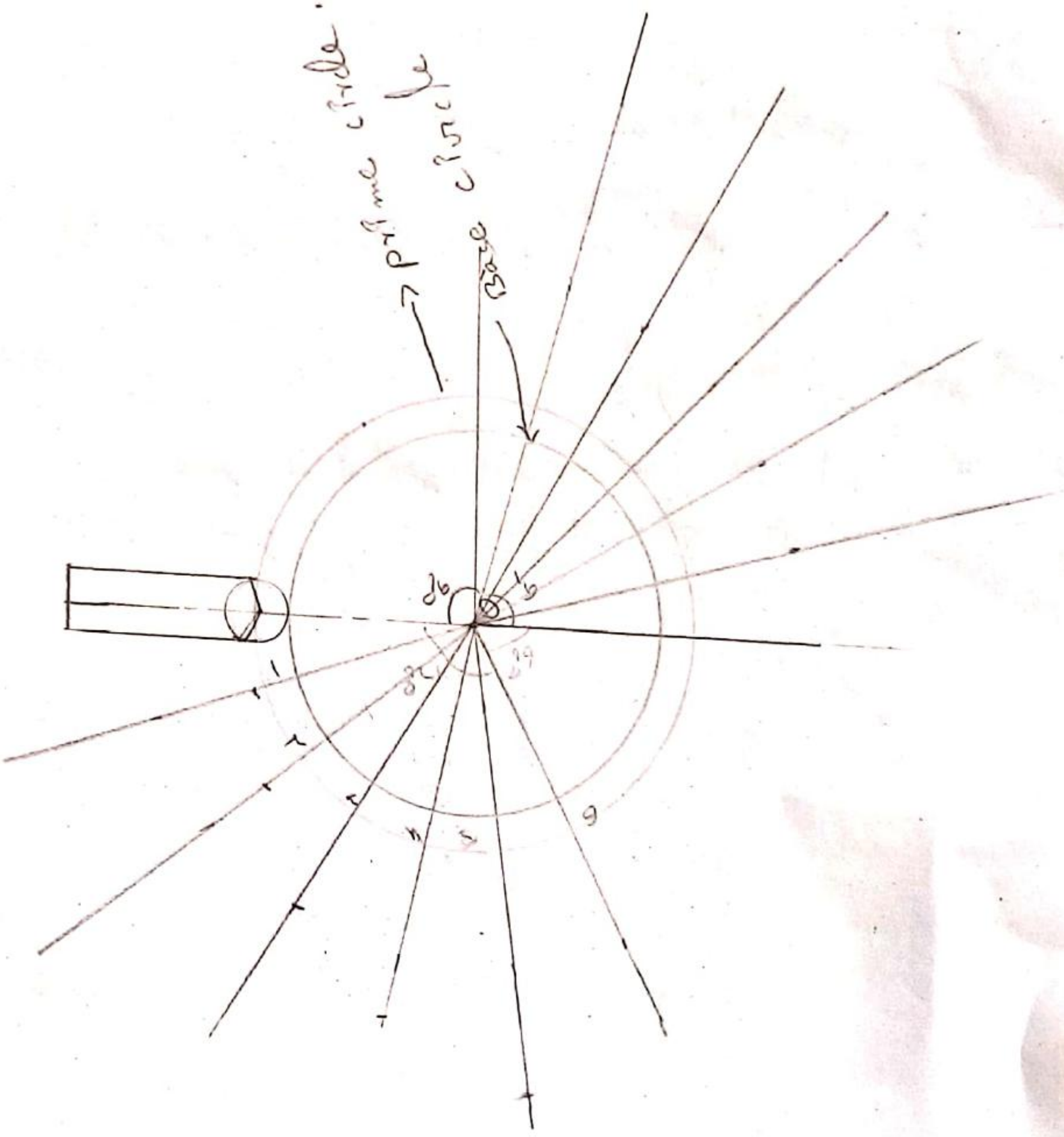
= 0.576 m/s

= 4199.876 m

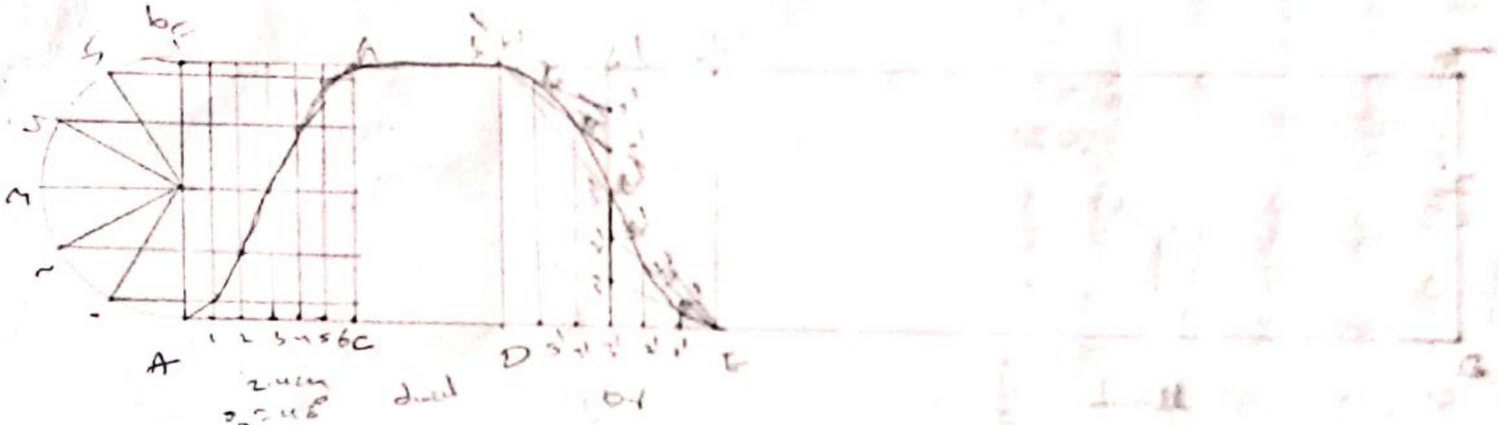
= ~~41.99~~

$V_{r(max)} = 2\times$ = 4.199 m/sec




prime circle
Base circle

(6) Draw the profile of a cam, the follower moves with simple harmonic motion during ascent while it moves with uniform accelerated and decelerated during descent. least radius of the cam is 15mm, angle of ascent $\theta_o = 48°$ angle of dwell 42°, angle of descent $\theta_d = 60°$ and lift of the follower is 40mm and diameter of the roller is 30mm. Distance b/w line of action of the follower and axis of cam is 20mm, The cam rotates at 360 Rpm. Find the maximum velocity and acceleration of the follower during descent period.


A
D
E

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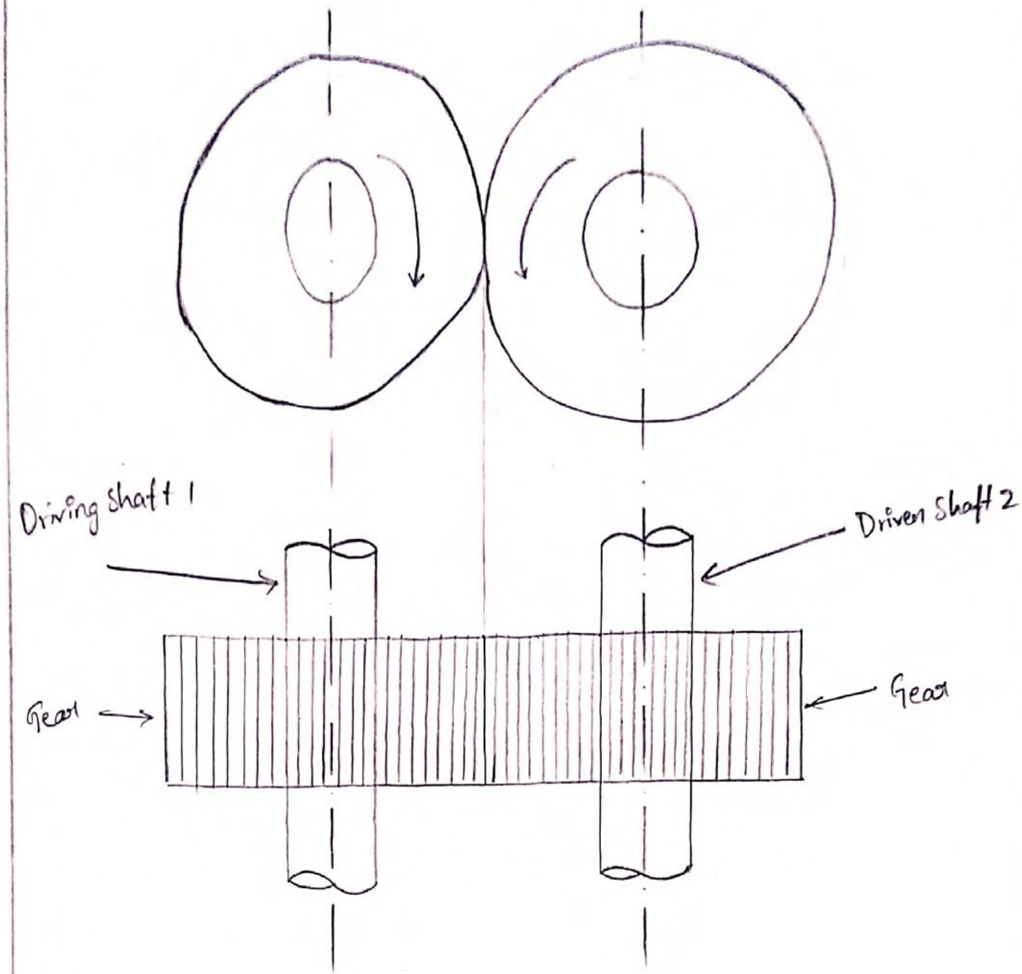
Unit-V

Gear Trains



* Gear Trains:-

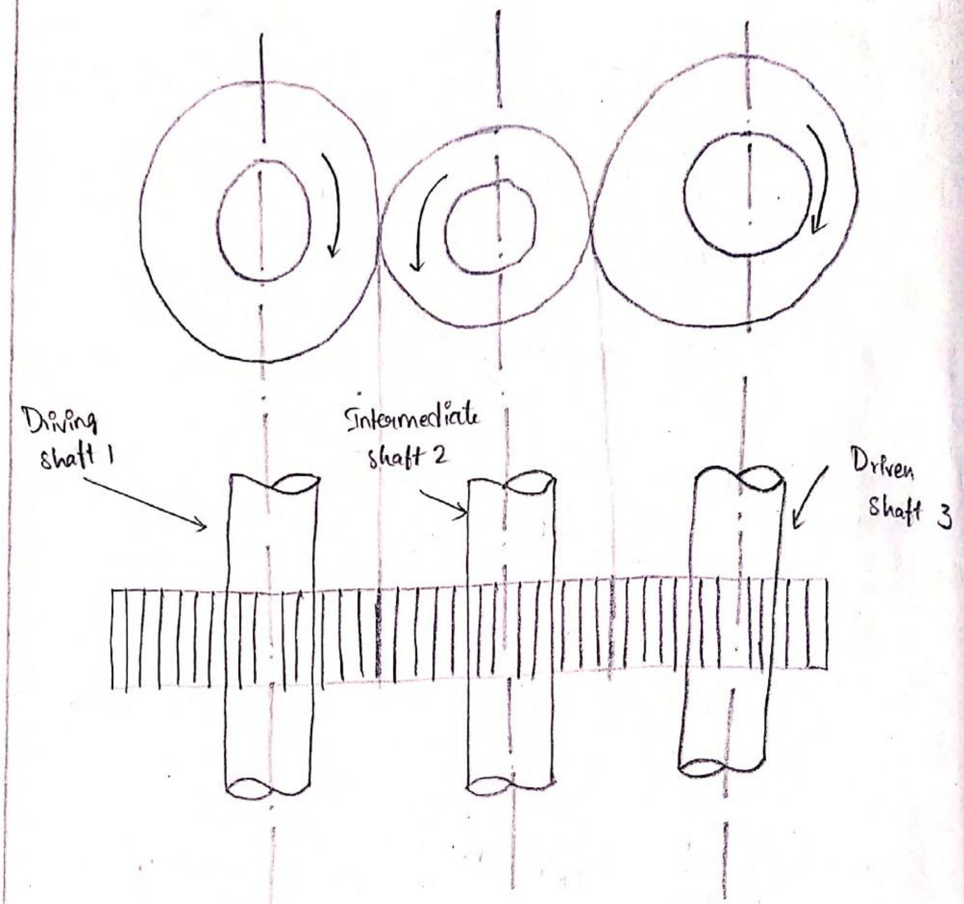
A Combination of two (or) more gears which are arranged in such a way that power is transmitted from a driving shaft to a driven shaft is known Gear Train. The Gear Train may Consist as spur gear, Bevel gear (or) Spiral Gear.

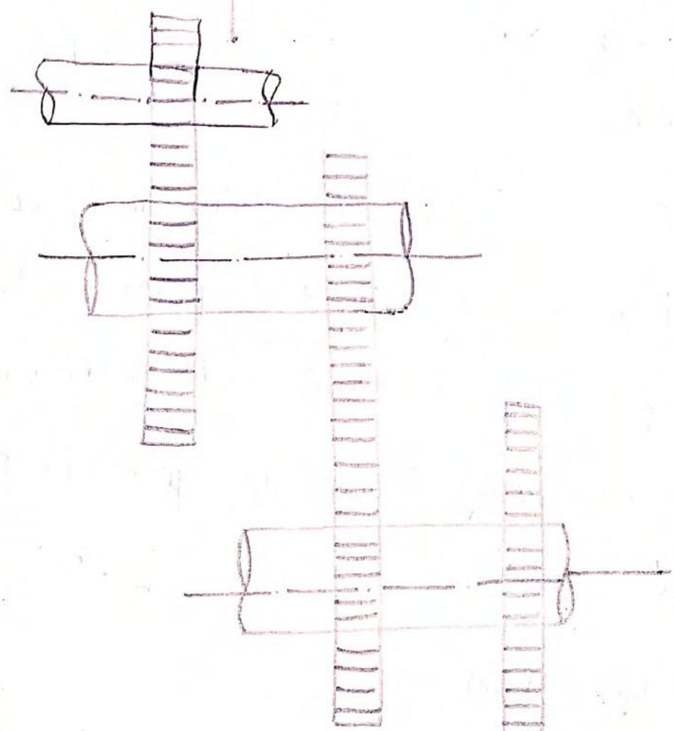
* Types of Gear Train:-

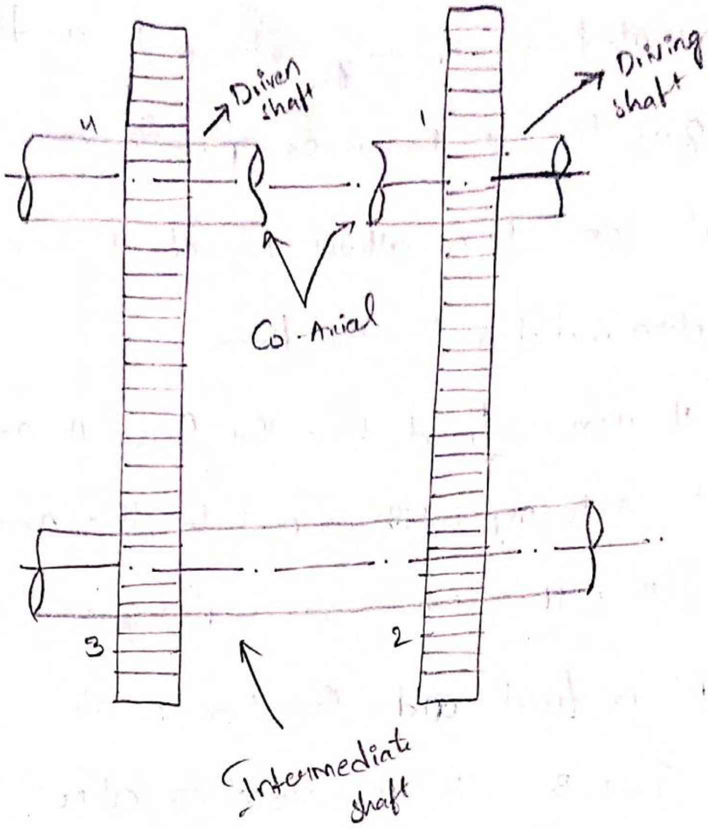
1) Simple Gear Train
2) Compound Gear Train

3) Reverted Gear Train

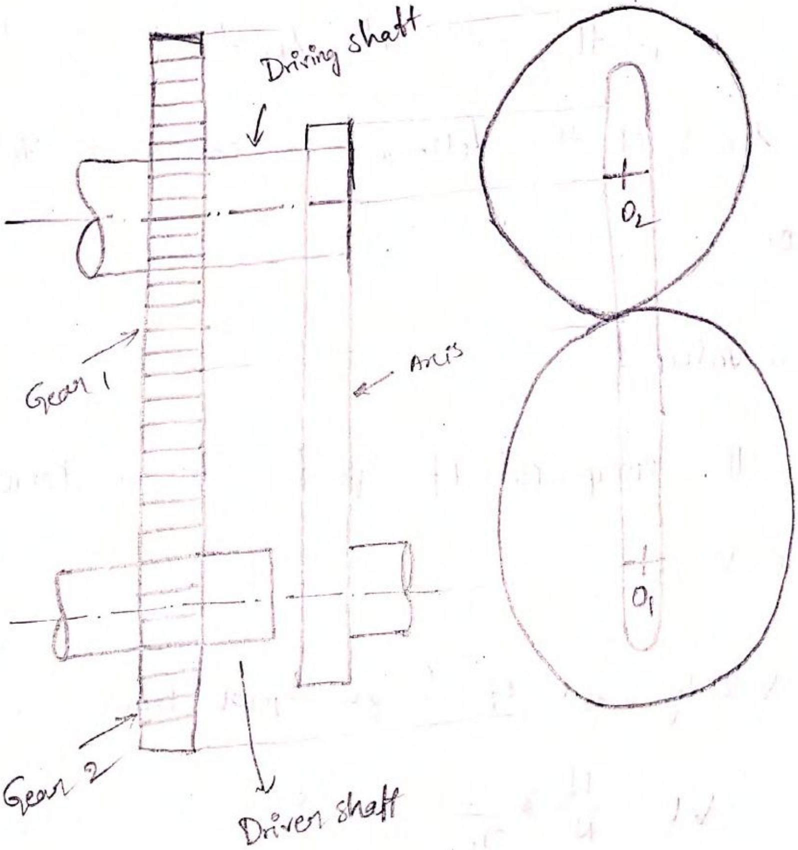
4) Epicyclic Gear Train







* Epicyclic Gear Train:-



If the axis of the shafts over which the gears are mounted are moving relative to a fixed axis, The gear train is known as epicyclic gear train. In an epicyclic gear train atleast one of the gear axis is in motion relative to the frame.

If arm is fixed, then the Gear 'A' and Gear 'B' will be rotating with respect to the axis of the shaft. Then it is known as Simple gear train. But, if gear 'A' is fixed and the Arm is rotated about 'a' then Gear 'B' will be rotating about Gear 'A' and we get Epicyclic Gear Train.

* Velocity Ratio of Gear Trains:-

It is the ratio of speed of the driver to the speed of the follower is known as Velocity ratio.

* Train Value:-

The reciprocal of Speed ratio is known as Train Value.

* The Velocity ratio of Simple Gear Train:-

$$V.R = \frac{N_1}{N_2} = \frac{T_2}{T_1}$$

$$\text{Train value} = \frac{N_2}{N_1} = \frac{T_1}{T_2}$$

} for two gears

$$V.R = \frac{N_1}{N_2} \times \frac{N_2}{N_3} = \frac{T_2}{T_1} \times \frac{T_3}{T_2}$$ } for three gears

$$\Rightarrow \frac{N_1}{N_3} = \frac{T_3}{T_1}$$

Problems

1) A Simple Gear Train Consist of 2 gears only. Each gear mounted on Seperate shafts. The shafts are Parallel The Gear '1' is driving. the gear '2'. The Speed of the gear '1' is 1000 RPM. The no. of teeth on Gear '1' and '2' are 24 and 60. Determine

i) Speed Ratio of Gear train

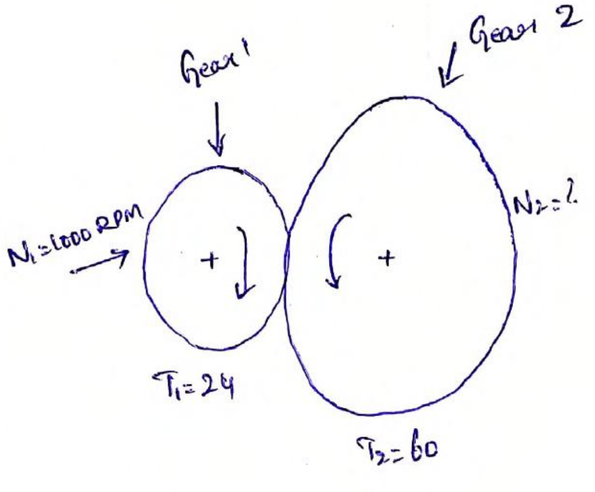
ii) Train value of " "

iii) Speed of the 2nd gear

iv) Direction of rotation of the 2nd gear if 1st gear is rotating clockwise

A:

i) Speed ratio $= \frac{N_1}{N_2} = \frac{T_2}{T_1}$



$$= \frac{60}{24}$$

$$= 2.5$$

$$\frac{N_1}{N_2} = 2.5$$

$$\frac{1000}{2.5} = N_2$$

$N_2 = 400$ RPM.

UNIT - III

Chapter: Power Transmission Systems Belt and chain Drives

Q.1 Why belt drives are used? What are types of belt drives?

The belts are used to transmit power from one shaft to another by means of pulleys which rotate at the same speed or at different speeds.

Types of Belt Drives

The belt drives are usually classified into the following three groups :

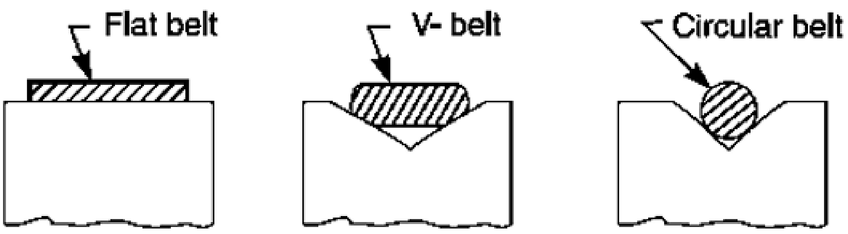
1. Light drives. These are used to transmit small powers at belt speeds upto about 10 m/s, as in agricultural machines and small machine tools.

2. Medium drives. These are used to transmit medium power at belt speeds over 10 m/s but up to 22 m/s, as in machine tools.

3. Heavy drives. These are used to transmit large powers at belt speeds above 22 m/s, as in compressors and generators.

Q.2 What are types of belts used in belt drives?

Types of Belts:



(a) Flat belt. (b) V-belt. (c) Circular belt.

1. Flat belt. The flat belt, as shown in Fig. 11.1 (a), is mostly used in the factories and

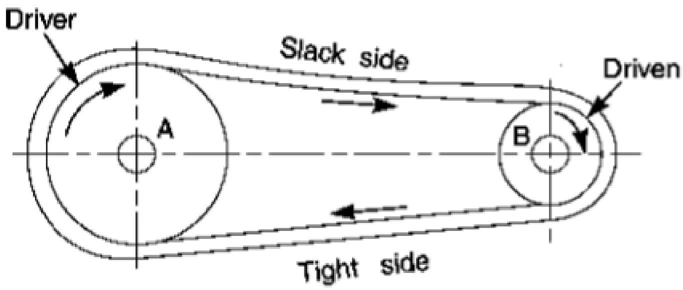
workshops, where a moderate amount of power is to be transmitted, from one pulley to another when the two pulleys are not more than 8 metres apart.

2. V-belt. The V-belt, as shown in Fig. 11.1 (b), is mostly used in the factories and workshops,where a moderate amount of power is to be transmitted, from one pulley to another, when the two pulleys are very near to each other.

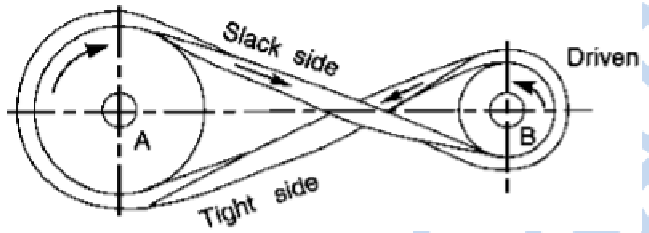
3. Circular belt or rope. The circular belt or rope, as shown in Fig. 11.1 (c), is mostly used in the factories and workshops, where a great amount of power is to be transmitted, from one pulley to another, when the two pulleys are more than 8 meters apart.

Q.3 What are types of flat belt drives?

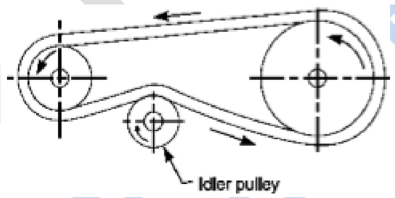
1. Open belt drive. The open belt drive, as shown in Fig., is used with shafts arranged parallel and rotating in the same direction.



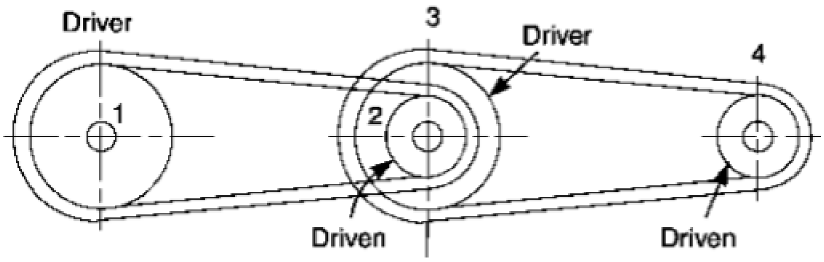
2. Crossed or twist belt drive. The crossed or twist belt drive, as shown in Fig., is used with shafts arranged parallel and rotating in the opposite directions.



3. Belt drive with idler pulleys. A belt drive with an idler pulley, as shown in Fig. , is used with shafts arranged parallel and when an open belt drive cannot be used due to small angle of contact on the smaller pulley.



4. Compound belt drive. A compound belt drive, as shown in Fig, is used when power is transmitted from one shaft to another through a number of pulleys.



Q.4 What are different materials used for belts?

Material used for Belts:

1. Leather belts. 2. Cotton or fabric belts 3. Rubber belt. 4. Balata belts.

Q.5 Define slip in the belt, and write equation for velocity ratio considering slip.

SLIP: The motion of belts and shafts assuming a firm frictional grip between the belts and the shafts. But sometimes, the frictional grip becomes insufficient. This may cause some forward motion of the driver without carrying the belt with it. This may also cause some forward motion of the belt without carrying the driven pulley with it. This is called slip of the belt and is generally expressed as a percentage.

The result of the belt slipping is to reduce the velocity ratio of the system. As the slipping of the belt is a common phenomenon, thus the belt should never be used where a definite velocity ratio is of importance.

V.R considering slip:

$$\frac{N_2}{N_1} = \frac{d_1}{d_2}\left(1 - \frac{s}{100}\right)$$

Q.6 Define creep, in the belt, and write equation for velocity ratio considering creep.

CREEP:

When the belt passes from the slack side to the tight side, a certain portion of the belt extends and it contracts again when the belt passes from the tight side to slack side. Due to these changes of length, there is a relative motion between the belt and the pulley surfaces. This relative motion is termed as creep.

The total effect of creep is to reduce slightly the speed of the driven pulley or follower.

Considering creep, the velocity ratio is given by:

$$\frac{N_2}{N_1} = \frac{d_1}{d_2} \times \frac{E + \sqrt{\sigma_2}}{E + \sqrt{\sigma_1}}$$

σ_1 and σ_2 = Stress in the belt on the tight and slack side respectively, and

E = Young's modulus for the material of the belt.

Q.7 Derive equation for Length of an Open Belt Drive.

Length of an Open Belt Drive:

r1 and r2 = Radii of the larger and smaller pulleys,

x = Distance between the centres of two pulleys (i.e. O1 O2), and

L = Total length of the belt.

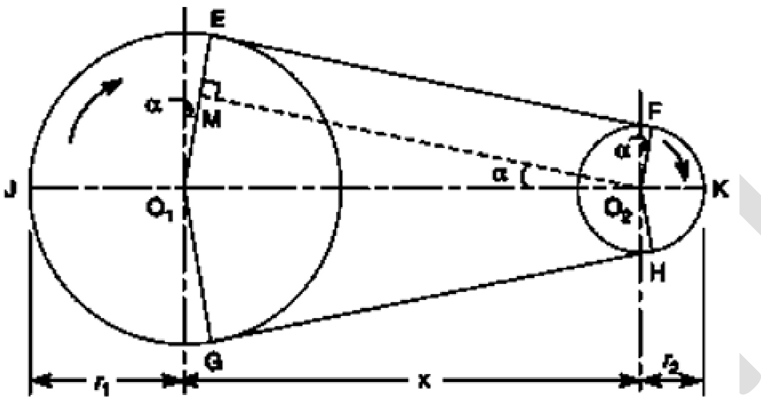


Fig. Length of an open belt drive.

Length of the belt,

L = Arc GJE + EF + Arc FKH + HG

= 2 (Arc JE + EF + Arc FK)

From the geometry of the figure, we find that

$$\sin\alpha = \frac{O_1 M}{O_1 O_2} = \frac{O_1 E - EM}{O_1 O_2} = \frac{r_1 - r_2}{x}$$

Since α is very small, therefore putting

$$\sin\alpha = \alpha \text{ (in radians)} = \frac{r_1 - r_2}{x}$$

∴ $$\text{Arc } JE = r_1\left(\frac{\pi}{2} + \alpha\right)$$

Similarly $$\text{Arc } FK = r_2\left(\frac{\pi}{2} - \alpha\right)$$

$$EF = MO_2 = \sqrt{(O_1 O_2)^2 - (O_1 M)^2} = \sqrt{x^2 - (r_1 - r_2)^2}$$

$$= x\sqrt{1-\left(\frac{r_1-r_2}{x}\right)^2}$$

Expanding this equation by binomial theorem,

$$EF = x\left[1-\frac{1}{2}\left(\frac{r_1-r_2}{x}\right)^2+....\right] = x-\frac{(r_1-r_2)^2}{2x} \qquad ...(v)$$

Substituting the values of arc *JE* from equation (*iii*), arc *FK* from equation (*iv*) and *EF* from equation (*v*) in equation (*i*), we get

$$L = 2\left[r_1\left(\frac{\pi}{2}+\alpha\right)+x-\frac{(r_1-r_2)^2}{2x}+r_2\left(\frac{\pi}{2}-\alpha\right)\right]$$

$$= 2\left[r_1\times\frac{\pi}{2}+r_1.\alpha+x-\frac{(r_1-r_2)^2}{2x}+r_2\times\frac{\pi}{2}-r_2.\alpha\right]$$

$$= 2\left[\frac{\pi}{2}(r_1+r_2)+\alpha(r_1-r_2)+x-\frac{(r_1-r_2)^2}{2x}\right]$$

$$= \pi(r_1+r_2)+2\alpha(r_1-r_2)+2x-\frac{(r_1-r_2)^2}{x}$$

Substituting the value of $\alpha = \frac{r_1-r_2}{x}$ from equation (*ii*),

$$L = \pi(r_1+r_2)+2\times\frac{(r_1-r_2)}{x}\times(r_1-r_2)+2x-\frac{(r_1-r_2)^2}{x}$$

$$= \pi(r_1+r_2)+\frac{2(r_1-r_2)^2}{x}+2x-\frac{(r_1-r_2)^2}{x}$$

$$= \pi(r_1+r_2)+2x+\frac{(r_1-r_2)^2}{x} \qquad \text{...(In terms of pulley radii)}$$

$$= \frac{\pi}{2}(d_1+d_2)+2x+\frac{(d_1-d_2)^2}{4x} \qquad \text{...(In terms of pulley diameters)}$$

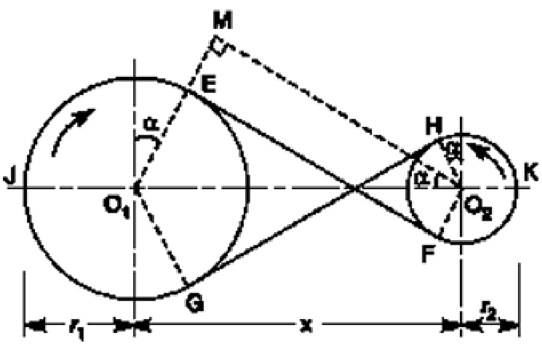
Q.8 Derive equation for Length of an Cross Belt Drive.

Length of an Cross Belt Drive:

r1 and r2 = Radii of the larger and smaller pulleys,

x = Distance between the centers of two pulleys (i.e. O1 O2), and

L = Total length of the belt.



Let the belt leaves the larger pulley at E and G and the smaller pulley at F and H, as shown in Fig. 11.12. Through O_2, draw O_2M parallel to FE.

From the geometry of the figure, we find that O_2M will be perpendicular to O_1E.

Let the angle $MO_2O_1 = \alpha$ radians.

We know that the length of the belt,

$$L = \text{Arc } GJE + EF + \text{Arc } FKH + HG$$

$$= 2\,(\text{Arc } JE + EF + \text{Arc } FK) \qquad \ldots(i)$$

From the geometry of the figure, we find that

$$\sin\alpha = \frac{O_1M}{O_1O_2} = \frac{O_1E + EM}{O_1O_2} = \frac{r_1 + r_2}{x}$$

Since α is very small, therefore putting

$$\sin\alpha = \alpha \text{ (in radians)} = \frac{r_1 + r_2}{x} \qquad \ldots(ii)$$

$$\therefore \quad \text{Arc } JE = r_1\left(\frac{\pi}{2} + \alpha\right) \qquad \ldots(iii)$$

Similarly $\quad \text{Arc } FK = r_2\left(\frac{\pi}{2} + \alpha\right) \qquad \ldots(iv)$

and

$$EF = MO_2 = \sqrt{(O_1O_2)^2 - (O_1M)^2} = \sqrt{x^2 - (r_1 + r_2)^2}$$

$$= x\sqrt{1 - \left(\frac{r_1 + r_2}{x}\right)^2}$$

$$EF = x\left[1 - \frac{1}{2}\left(\frac{r_1 + r_2}{x}\right)^2 + \ldots\right] = x - \frac{(r_1 + r_2)^2}{2x} \qquad \ldots(v)$$

Substituting the values of arc JE from equation (iii), arc FK from equation (iv) and EF from equation (v) in equation (i), we get

$$L = 2\left[r_1\left(\frac{\pi}{2} + \alpha\right) + x - \frac{(r_1 + r_2)^2}{2x} + r_2\left(\frac{\pi}{2} + \alpha\right)\right]$$

$$= 2\left[r_1 \times \frac{\pi}{2} + r_1.\alpha + x - \frac{(r_1 + r_2)^2}{2x} + r_2 \times \frac{\pi}{2} + r_2.\alpha\right]$$

$$= 2\left[\frac{\pi}{2}(r_1 + r_2) + \alpha(r_1 + r_2) + x - \frac{(r_1 + r_2)^2}{2x}\right]$$

$$= \pi(r_1 + r_2) + 2\alpha(r_1 + r_2) + 2x - \frac{(r_1 + r_2)^2}{x}$$

Substituting the value of $\alpha = \frac{r_1 + r_2}{x}$ from equation (ii),

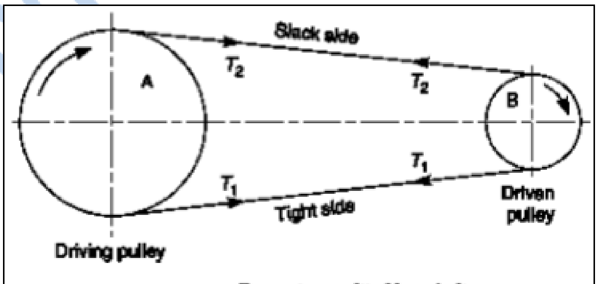
$$L = \pi(r_1 + r_2) + \frac{2(r_1 + r_2)}{x} \times (r_1 + r_2) + 2x - \frac{(r_1 + r_2)^2}{x}$$

$$= \pi(r_1 + r_2) + \frac{2(r_1 + r_2)^2}{x} + 2x - \frac{(r_1 + r_2)^2}{x}$$

$$= \pi(r_1 + r_2) + 2x + \frac{(r_1 + r_2)^2}{x} \qquad \ldots\text{(In terms of pulley radii)}$$

Q.9 Derive equation for Power Transmitted by a Belt:

Power Transmitted by a Belt Fig. shows the driving pulley (or driver) A and the driven.



Power transmitted by a belt.

Let T1 and T2 = Tensions in the tight and slack side of the belt respectively in Newtons.

r1 and r2 = Radii of the driver and follower respectively, and

v = Velocity of the belt in m/s.

The effective turning (driving) force at the circumference of the follower is the difference between the two tensions (i.e. T1 . T2).

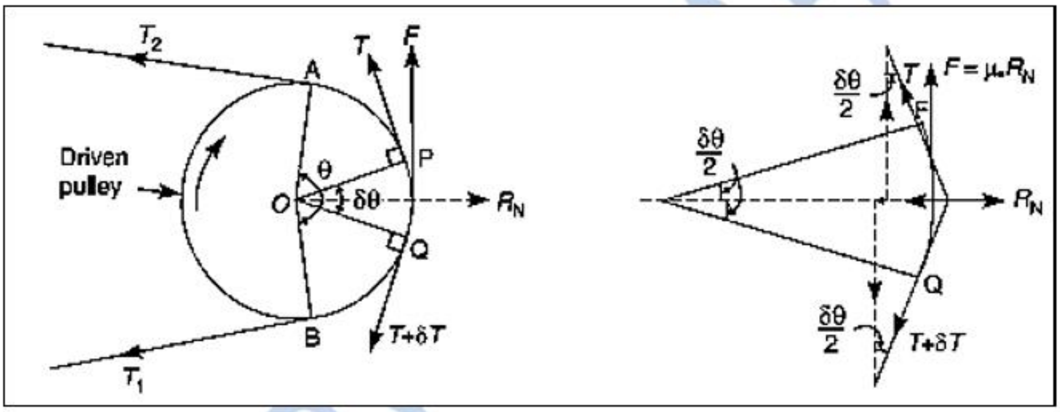
Work done per second = (T1 - T2) v   N-m/s

and power transmitted, P = (T1 -T2) v

Q.10 Derive equation for Ratio of Driving Tensions For Flat Belt Drive.

Ratio of Driving Tensions For Flat Belt Drive

Consider a driven pulley rotating in the clockwise direction as shown in Fig.



Let T_1 = Tension in the belt on the tight side,

T_2 = Tension in the belt on the slack side, and

θ = Angle of contact in radians (*i.e.* angle subtended by the arc *A B*, along which the belt touches the pulley at the centre).

Now consider a small portion of the belt *PQ*, subtending an angle δθ at the centre of the pulley as shown in Fig. 11.15. The belt *PQ* is in equilibrium under the following forces :

1. Tension *T* in the belt at *P*,
2. Tension $(T + \delta T)$ in the belt at *Q*,
3. Normal reaction R_N, and
4. Frictional force, $F = \mu \times R_N$, where μ is the coefficient of friction between the belt and pulley.

Resolving all the forces horizontally and equating the same,

$$R_N = (T + \delta T)\sin\frac{\delta\theta}{2} + T\sin\frac{\delta\theta}{2} \qquad ...(i)$$

Since the angle δθ is very small, therefore putting $\sin \delta\theta/2 = \delta\theta/2$ in equation (*i*),

$$R_N = (T + \delta T)\frac{\delta\theta}{2} + T \times \frac{\delta\theta}{2} = \frac{T.\delta\theta}{2} + \frac{\delta T.\delta\theta}{2} + \frac{T.\delta\theta}{2} = T.\delta\theta \qquad ...(ii)$$

$$...\left(\text{Neglecting } \frac{\delta T.\delta\theta}{2}\right)$$

Now resolving the forces vertically, we have

$$\mu \times R_N = (T + \delta T)\cos\frac{\delta\theta}{2} - T\cos\frac{\delta\theta}{2} \qquad ...(iii)$$

Since the angle δθ is very small, therefore putting $\cos \delta\theta/2 = 1$ in equation (*iii*),

$$\mu \times R_N = T + \delta T - T = \delta T \quad \text{or} \quad R_N = \frac{\delta T}{\mu} \qquad ...(iv)$$

Equating the values of R_N from equations (*ii*) and (*iv*),

$$T.\delta\theta = \frac{\delta T}{\mu} \quad \text{or} \quad \frac{\delta T}{T} = \mu.\delta\theta$$

Integrating both sides between the limits T_2 and T_1 and from 0 to θ respectively,

$$i.e. \qquad \int_{T_2}^{T_1}\frac{\delta T}{T} = \mu\int_0^{\theta}\delta\theta \qquad \text{or} \qquad \log_e\left(\frac{T_1}{T_2}\right) = \mu.\theta \quad \text{or} \quad \boxed{\frac{T_1}{T_2} = e^{\mu.\theta}} \qquad ...(v)$$

Q.11 Define Centrifugal Tension and derive equation.

Centrifugal Tension

Since the belt continuously runs over the pulleys, therefore, some centrifugal force is caused, whose effect is to increase the tension on both, tight as well as the slack sides. The tension caused by centrifugal force is called centrifugal tension..

Consider a small portion PQ of the belt subtending an angle d□□□the centre of the pulley as shown in Fig.

Let m = Mass of the belt per unit length in kg, v = Linear velocity of the belt in m/s, r = Radius of the pulley over which the belt runs in metres, and TC = Centrifugal tension acting tangentially at P and Q in newtons.

We know that length of the belt PQ

$$= r.d\theta$$

and mass of the belt PQ $= m.r.d\theta$

∴ Centrifugal force acting on the belt PQ,

$$F_C = (m.r.d\theta)\frac{v^2}{r} = m.d\theta.v^2$$

The centrifugal tension T_C acting tangentially at P and Q keeps the belt in equilibrium.

Now resolving the forces (*i.e.* centrifugal force and centrifugal tension) horizontally and equating the same, we have

$$T_C \sin\left(\frac{d\theta}{2}\right) + T_C \sin\left(\frac{d\theta}{2}\right) = F_C = m.d\theta.v^2$$

Since the angle $d\theta$ is very small, therefore, putting $\sin\left(\frac{d\theta}{2}\right) = \frac{d\theta}{2}$, in the above expression,

$$2T_C\left(\frac{d\theta}{2}\right) = m.d\theta.v^2 \quad \text{or} \quad \boxed{T_C = m.v^2}$$

Q.12 Define Initial Tension in the Belt and derive equation for it.

Initial Tension in the Belt:

When a belt is wound round the two pulleys (i.e. driver and follower), its two ends are joined together ; so that the belt may continuously move over the pulleys, since the motion of the belt from the driver and the follower is governed by a firm grip, due to friction between the belt and the pulleys. In order to increase this grip, the belt is tightened up. At this stage, even when the pulleys are stationary, the belt is subjected to some tension, called initial tension.

Let T_0 = Initial tension in the belt,

T_1 = Tension in the tight side of the belt,

T_2 = Tension in the slack side of the belt, and

α = Coefficient of increase of the belt length per unit force.

A little consideration will show that the increase of tension in the tight side

$$= T_1 - T_0$$

and increase in the length of the belt on the tight side

$$= \alpha (T_1 - T_0) \quad \text{...(i)}$$

Similarly, decrease in tension in the slack side

$$= T_0 - T_2$$

and decrease in the length of the belt on the slack side

$$= \alpha (T_0 - T_2) \quad \text{...(ii)}$$

Assuming that the belt material is perfectly elastic such that the length of the belt remains constant, when it is at rest or in motion, therefore increase in length on the tight side is equal to decrease in the length on the slack side. Thus, equating equations (i) and (ii),

$$\alpha (T_1 - T_0) = \alpha (T_0 - T_2) \quad \text{or} \quad T_1 - T_0 = T_0 - T_2$$

$$\therefore \quad T_0 = \frac{T_1 + T_2}{2} \quad \text{...(Neglecting centrifugal tension)}$$

$$= \frac{T_1 + T_2 + 2T_C}{2} \quad \text{...(Considering centrifugal tension)}$$

Q.13 Write Advantages and Disadvantages of V-belt Drive Over Flat Belt Drive

Advantage:

1. The V-belt drive gives compactness due to the small distance between the centres of pulleys. 2. The drive is positive, because the slip between the belt and the pulley groove is negligible. 3. Since the V-belts are made endless and there is no joint trouble, therefore the drive is smooth. 4. It provides longer life, 3 to 5 years.

5. It can be easily installed and removed.

6. The operation of the belt and pulley is quiet.

Disadvantages

1. The V-belt drive cannot be used with large centre distances.

2. The V-belts are not so durable as flat belts.

3. The construction of pulleys for V-belts is more complicated than pulleys for flat belts.

4. Since the V-belts are subjected to certain amount of creep, therefore these are not suitable

for constant speed application such as synchronous machines, and timing devices.

5. The belt life is greatly influenced with temperature changes, improper belt tension

Q.14 Write the Ratio of Driving Tensions for V-belt.

Ratio of Driving Tensions for V-belt

A V-belt with a grooved pulley is shown in Fig. 11.20.

Let R_1 = Normal reaction between the belt and sides of the groove.

R = Total reaction in the plane of the groove.

2β = Angle of the groove.

μ = Coefficient of friction between the belt and sides of the groove.

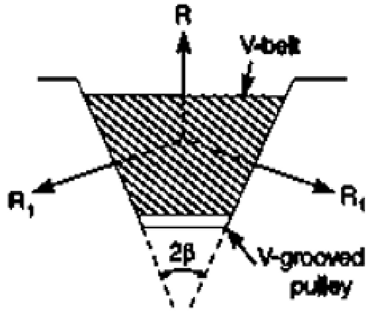


Fig. 11.20.

Resolving the reactions vertically to the groove,

$$R = R_1 \sin\beta + R_1 \sin\beta = 2 R_1 \sin\beta$$

or

$$R_1 = \frac{R}{2\sin\beta}$$

We know that the frictional force

$$= 2\mu . R_1 = 2\mu \times \frac{R}{2\sin\beta} = \frac{\mu R}{\sin\beta} = \mu . R \operatorname{cosec}\beta$$

Consider a small portion of the belt, as in Art. 11.14, subtending an angle $\delta\theta$ at the centre. The tension on one side will be T and on the other side $T + \delta T$. Now proceeding as in Art. 11.14, we get the frictional resistance equal to $\mu . R \operatorname{cosec}\beta$ instead of $\mu . R$. Thus the relation between T_1 and T_2 for the V-belt drive will be

$$\left(\frac{T_1}{T_2}\right) = e^{\mu . \theta \operatorname{cosec}\beta}$$

Q.15 What are Chain Drives?

Belt and rope drives that slipping may occur. In order to avoid slipping, steel chains are used. The chains are made up of rigid links which are hinged together in order to provide the necessary flexibility for warping around the driving and driven wheels. The wheels have projecting teeth and fit into the corresponding recesses, in the links of the chain as shown in Fig. The wheels and the chain are thus constrained to move together without slipping and ensures perfect velocity ratio. The toothed wheels are known as sprocket wheels or simply sprockets. These wheels resemble to spur gears.

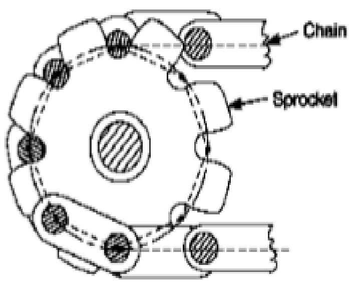


Fig. Sprocket and chain.

Q.16 Advantages and Disadvantages of Chain Drive Over Belt or Rope Drive

Following are the advantages and disadvantages of chain drive over belt or rope drive:

Advantages:

1. As no slip takes place during chain drive, hence perfect velocity ratio is obtained.

2. Occupy less space

3. The chain drives may be used when the distance between the shafts is less.

4. The chain drive gives a high transmission efficiency (upto 98 per cent).

.Disadvantages:

1. The production cost of chains is relatively high.

2. The chain drive needs accurate mounting and careful maintenance.

3. The chain drive has velocity fluctuations especially when unduly stretched.

Sr No	Belt drive	Chain drive
1	It is a non positive drive	It is a positive drive
2	Space required is more	Space required is less or compact in design
3	Weight is less	Weight is more
4	No need of lubrication	Lubrication necessary for better and noise free operation
5	Velocity ratio is not constant due to slip phenomenon	Velocity ratio is constant because it is a positive drive

Sr No	Flat belt	V- belt
1	In flat belt, less power transmit due to less frictional grip	More power transmission, due to more frictional grip
2	It is not a positive drive, because slip occurs between belt and pulley	Drive is positive because the slip between the belt and pulley groove is negligible
3	More space is required as distance between two shaft is more	Less space required because distance between centre of two shaft is less
4	Rectangular cross section and sit on a flat pulley rim	Trapozoidal in cross section and requires V groove pulley
5	Cost is less	Cost is more

Sr No	Open belt	Cross belt
Velocity ratio	High velocity ratio	Low velocity ratio
Direction of driven pulley	Driven pulley is rotated in same direction to the driving pulley	Driven pulley is rotated in opposite direction to the driving pulley
Application	Sawmills, floor mills, cooling fan of engine etc	Conveyors, electrical generator etc
Length of belt drive	$L = \pi (r_1 + r_2) + 2x + \frac{(r_1 - r_2)^2}{x}$	$L = \pi (r_1 + r_2) + 2x + \frac{(r_1 - r_2)^2}{x}$

Sr No	Gear drive	Belt drive
1	Perfect (Constant) velocity ratio obtained	Perfect (constant) velocity ratio may not be obtained due to slip
2	It is a positive drive	It is not a positive drive
3	Suitable for high power transmission when distance between the centre of shaft is less	Suitable for moderate power transmission when the central distance between the shaft is suitably more
4	These are compact required less space due to small central distance between shafts	Required more space due to large central distance between shafts
5	Cost is more	Cost is less

Gears

Gears are toothed members which transmit power / motion between two shafts by meshing without any slip. Hence, gear drives are also called **Positive Drives.** In any pair of gears, the smaller one is called **Pinion** and the larger one is called **Gear** immaterial of which is driving the other.

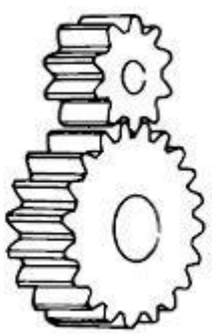
When pinion is the driver, it results in step down drive in which the output speed decreases and the torque increases. On the other hand, when the gear is the driver, it results in step up drive in which the output speed increases and the torque decreases.

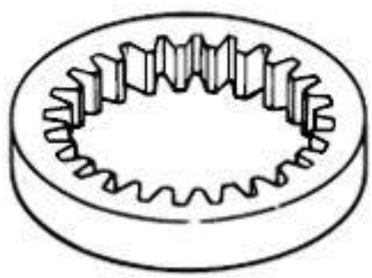
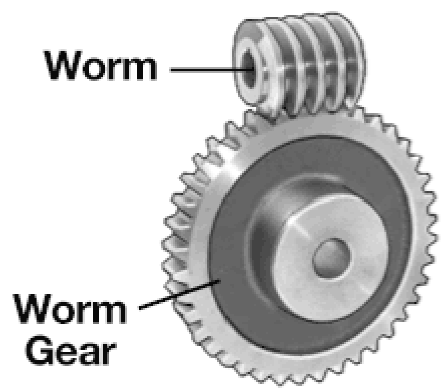
Classification of Gears

Gears are classified according to the shape of the tooth pair and disposition into spur, helical, double helical, straight bevel, spiral bevel and hypoid bevel, worm and spiral gears.

Spur Gears

Spur gears have their teeth parallel to the axis and are used for transmitting power between two parallel shafts. They are simple in construction, easy to manufacture and cost less. They have highest efficiency and excellent precision rating. They are used in high speed and high load application in all types of trains and a wide range of velocity ratios. Hence, they find wide applications right from clocks, household gadgets, motor cycles, automobiles, and railways to aircrafts.



(a) Spur Gear (b) Helical Gear (c) Internal Gear (d) Worm and Worm Gear

Helical Gears

Helical gears are used for parallel shaft drives. They have teeth inclined to the axis as shown in Fig. 1.9. Hence for the same width, their teeth are longer than spur gears and have higher load carrying capacity. Their contact ratio is higher than spur gears and they operate smoother and quieter than spur gears. Their precision rating is good. They are recommended for very high speeds and loads. Thus, these gears find wide applications in automotive gearboxes. Their efficiency is slightly lower than spur gears. The helix angle also introduces axial thrust on the shaft.

Double Helical Gear or Herringbone Gear

Double helical or Herringbone gears used for transmitting power between two parallel shafts. They have opposing helical teeth with or without a gap depending on the manufacturing method adopted. Two axial thrusts oppose each other and nullify. Hence the shaft is free from any axial force. Though their load capacity is very high, manufacturing difficulty makes them costlier than single helical gear. Their applications are limited to high capacity reduction drives like that of cement mills and crushers.

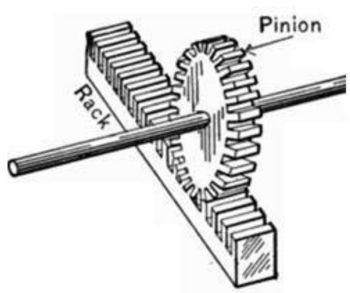
Internal Gear

Internal gears are used for transmitting power between two parallel shafts. In these gears, annular wheels are having teeth on the inner periphery. This makes the drive very compact. In these drives, the meshing pinion and annular gear are running in the same direction. Their precision rating is fair. They are useful for high load and high speed application with high reduction ratio. Applications of these gears can be seen in planetary gear drives of automobile automatic transmissions, reduction gearboxes of cement mills, step-up drives of wind mills.

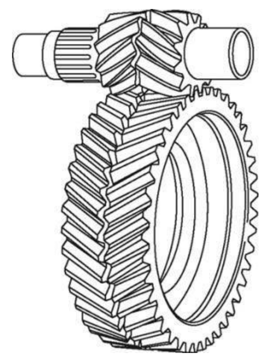
They are not recommended for precision meshes because of design, fabrication, and inspection limitations. They should only be used when internal feature is necessary. However, today precision machining capability has led to their usage even in position devices like antenna drives.

Rack and Pinion

Rack is a segment of a gear of infinite diameter. The tooth can be spur or helical. This type of gearing is used for converting rotary motion into translatory motion or visa versa.



Rack and Pinion



Double Helical Gear

Straight Bevel Gear

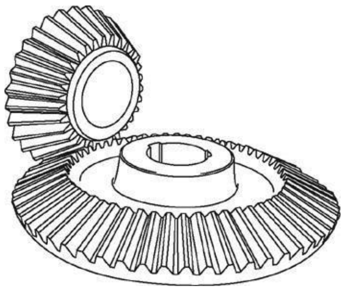
Straight bevel gears are used for transmitting power between intersecting shafts. They can operate under high speeds and high loads. Their precision rating is fair to good. They are suitable for 1:1 and higher velocity ratios and for right-angle meshes to any other angles. Their good choice is for right angle drive of particularly low ratios. However, complicated both form and fabrication limits achievement of precision. They should be located at one of the less critical meshes of the train. Wide application of the straight bevel drives is in automotive differentials, right angle drives of blenders and conveyors.

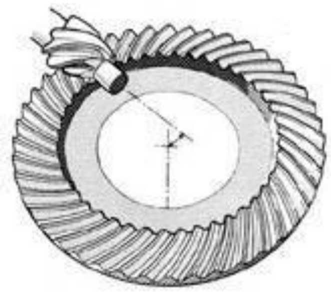
Spiral Bevel Gear

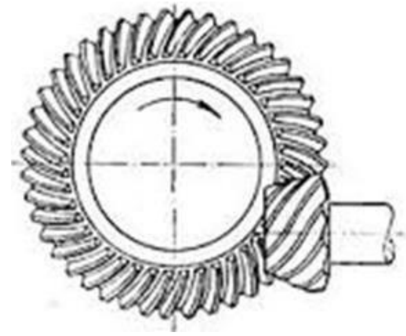
Spiral bevel gears are also used for transmitting power between intersecting shafts. Because of the spiral tooth, the contact length is more and contact ratio is more. They operate smoother than straight bevel gears and have higher load capacity. But, their efficiency is slightly lower than straight bevel gear.

Hypoid Bevel Gear

These gears are also used for right angle drive in which the axes do not intersect. This permits the lowering of the pinion axis which is an added advantage in automobile in avoiding hump inside the automobile drive line power transmission. However, the non – intersection introduces a considerable amount of sliding and the drive requires good lubrication to reduce the friction and wear. Their efficiency is lower than other two types of bevel gears. These gears are widely used in current day automobile drive line power transmission.







Straight Bevel Gear **Hypoid Bevel Gear** **Spiral Gear**

Worm Gear

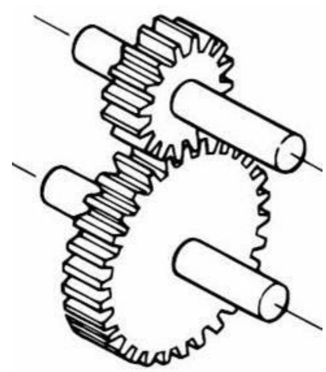
Worm and worm gear pair consists of a worm, which is very similar to a screw and a worm gear, which is a helical gear. They are used in right-angle skew shafts. In these gears, the engagement occurs without any shock. The sliding action prevalent in the system while resulting in quieter operation produces considerable frictional heat. High reduction ratios 8 to 400 are possible. Efficiency of these gears is low anywhere from 90% to 40 %. Higher speed ratio gears are non-reversible. Their precision rating is fair to good. They need good lubrication for heat dissipation and for improving the efficiency. The drives are very compact. Worm gearing finds wide application in material handling and transportation machinery, machine tools, automobiles etc.

Spiral Gear

Spiral gears are also known as crossed helical gears. They have high helix angle and transmit power between two non-intersecting non-parallel shafts. They have initially point contact under the conditions of considerable sliding velocities finally gears will have line contact. Hence, they are used for light load and low speed application such as instruments, sewing machine etc. Their precision rating is poor.

Characteristics of Gears

Spur Gear:



- The teeth are straight and parallel to the shaft axis.
- Transmits power and motion between rotating two parallel shafts.

Features:

- Easy to manufacture
- There are no axial force
- Relatively easy to produce high-quality gears
- The most common type of gear

Applications:

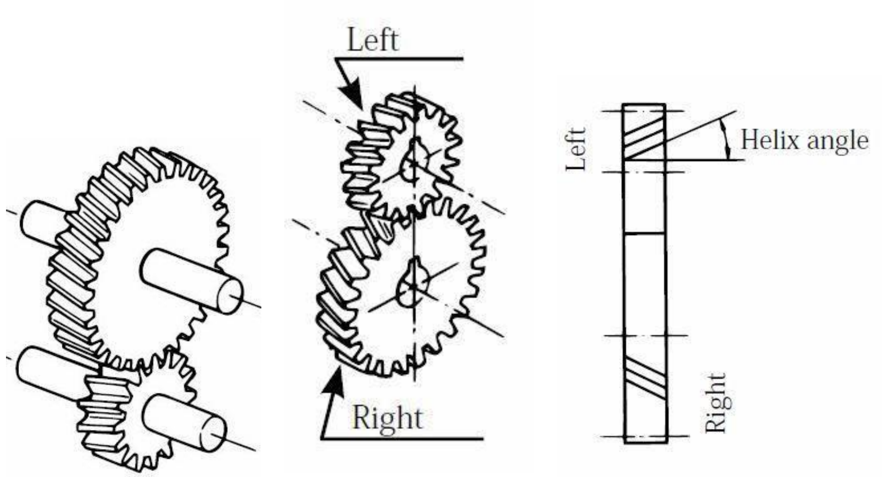
- Transmission components

Helical Gear:

- The teeth are twisted oblique to the gear axis.

Helix Direction (Helix Hand):

- The hand of helix is designated as either left or right
- Right-hand and left-hand helical gears mate as a set. But they must have the same helix angle



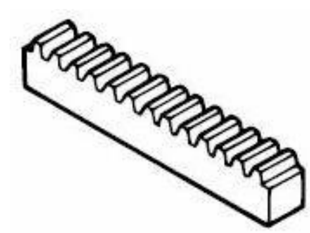
Features:

- Has higher strength compared with a spur gear
- More effective in reducing noise and vibration when compared with a spur gear
- Gears in mesh produce thrust forces in the axial direction

Applications:

- Transmission components, automobile, speed reducers, etc.

Gear Rack:



- The rack is a bar containing teeth on one face for meshing with a gear.
- The basic rack form is the profile of a gear of infinite diameter.
- Racks with machined ends can be joined together to make any desired length.

Features:

- Changes a rotary motion into a rectilinear motion and vice versa

Applications:

- A transfer system for machine tools, printing presses, robots, etc.

Internal Gear:

- An annular gear having teeth on the inner surface of its rim.
- The internal gear always meshes with an external gear.

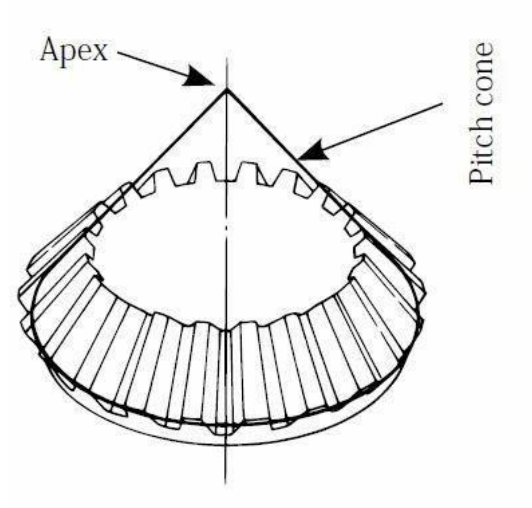
Features:

- When meshing two external gears, the rotation occurs in the opposite directions. When meshing an internal gear with an external gear the rotation occurs in the same direction
- Care should be taken with regard to the number of teeth on each gear when meshing a large (internal) gear with a small (external) gear, since three types of interference can occur
- Usually internal gears are driven by small external gears
- Allows for a compact design of the machine

Applications:

- Planetary gear drive of high reduction ratios, clutches, etc.

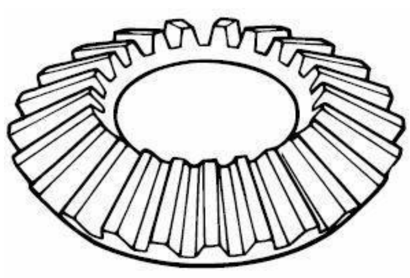
Bevel Gear:

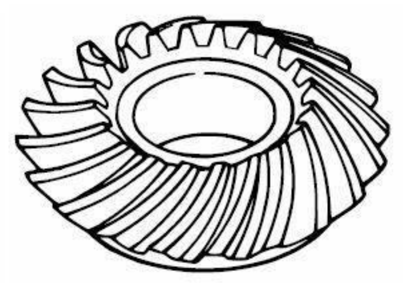


Picture: Bevel gear, apex and pitch cone

One of a pair of gears used to connect two shafts whose axes intersect, and the pitch surfaces are cones. Teeth are cut along the pitch cone. Depending on tooth trace, bevel gear is classified as:

1. Straight Bevel Gear
2. Spiral Bevel Gear





Straight Bevel Gear **Spiral Bevel Gear** **Miter Gear**

Straight Bevel Gear:

- A simple form of bevel gear having straight teeth which, if extended inward, would come together at the intersection of the shaft axes.

Features:

- Relatively easy to manufacture
- Provides reduction ratios up to approx. 1:5

Applications:

- Machine tools, printing presses, etc. Especially suitable for use as a differential gear unit

Spiral Bevel Gear:

- Bevel gear with curved, oblique teeth to provide gradual engagement and larger contact surface at a given time than an equivalent straight bevel gear.

Features:

- Has higher contact ratio, higher strength and durability than an equivalent straight bevel gear
- Allows a higher reduction ratio

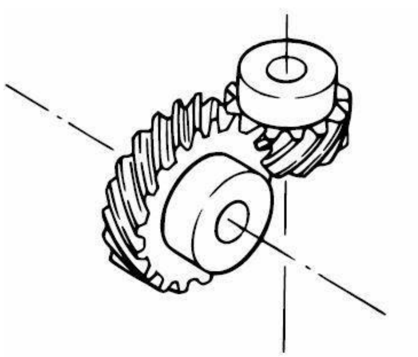
- Has better efficiency of transmission with reduced gear noise
- Involves some technical difficulties in manufacturing

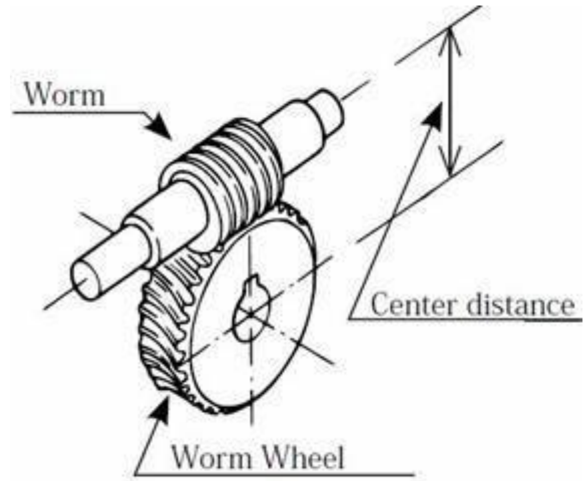
Applications:

- Automobiles, tractors, vehicles, final reduction gearing for ships. Especially suitable for high-speed, heavy load drives

Miter Gears:

- A special class of bevel gear where the shafts intersect at 90° and the gear ratio is 1:1.
- It is used to change the direction of shaft rotation without change in speed.





Screw Gear **Worm, Worm Gear and Center distance**

Screw Gear:

A helical gear that transmits power from one shaft to another, nonparallel, nonintersecting shaft

Features:

- Can be used as a speed reducer or as a speed increaser
- Due to sliding contact, has higher friction
- Not suitable for transmission of high horsepower

Applications:

- Driving gear for automobile. Automatic machines that require intricate movement

Worm Gear:

- Worm is a shank having at least one complete tooth (thread) around the pitch surface and is the driver of a worm wheel.
- Worm Gear (worm wheel) is a gear with teeth cut on an angle to be driven by a worm.
- The worm gear pair is used to transmit motion between two shafts which are at 90° to each other and lie on a plane.

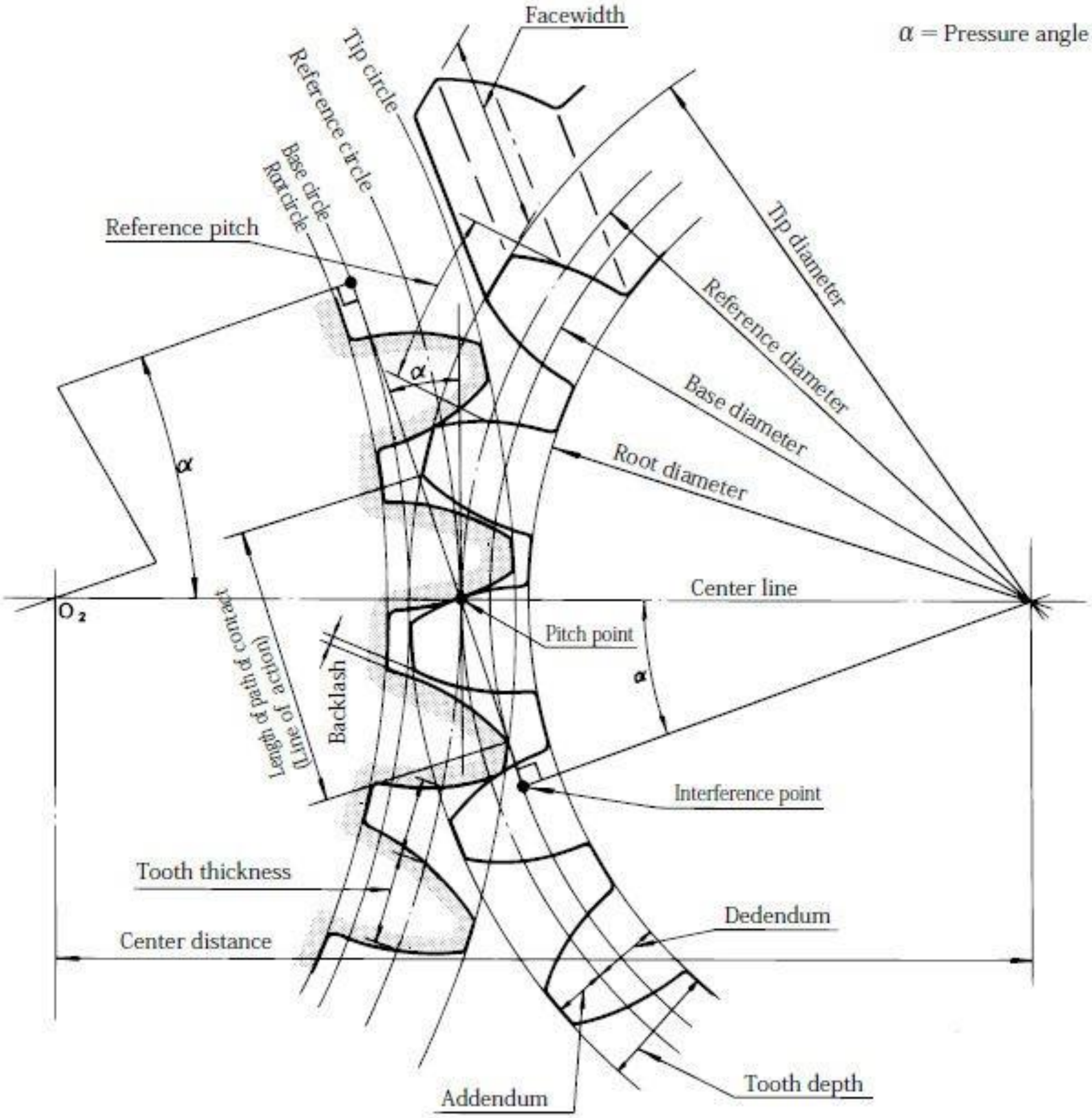
Features:

- Provides large reduction ratios for a given center distance
- Quiet and smooth meshing action
- It is not possible for a worm wheel to drive a worm unless certain conditions are met

Applications:

- Speed reducers, anti reversing gear devices making the most of its self-locking features, machine tools, indexing devices, chain blocks, portable generators, etc.

Nomenclature



The **Pitch Circle** is a theoretical circle upon which all calculations are usually based; **its diameter is the pitch diameter**. The pitch circles of a pair of mating gears are tangent to each other. A pinion is the smaller of two mating gears. **The larger is often called the gear.**

The **Circular Pitch p** is the distance, measured on the pitch circle, from a point on one tooth to a corresponding point on an adjacent tooth. Thus **the circular pitch is equal to the sum of the tooth thickness and the width of space**.

The **Module m** is the ratio of the pitch diameter to the number of teeth. The customary unit of length used is the millimeter. The module is the index of tooth size in SI.

The **Diametral Pitch P** is the ratio of the number of teeth on the gear to the pitch diameter. Thus, it is the reciprocal of the module. Since diametral pitch is used only with U.S. units, it is expressed as teeth per inch.

The **Addendum 'a'** is the radial distance between the top land and the pitch circle.

The **Dedendum 'b'** is the radial distance from the bottom land to the pitch circle. The whole **depth h_t** is the sum of the addendum and the dedendum.

The **Clearance Circle** is a circle that is tangent to the addendum circle of the mating gear. The clearance c is the amount by which the dedendum in a given gear exceeds the addendum of its mating gear.

The **Backlash** is the amount by which the width of a tooth space exceeds the thickness of the engaging tooth measured on the pitch circles.

$$P = \frac{N}{d} \qquad (1-1)$$

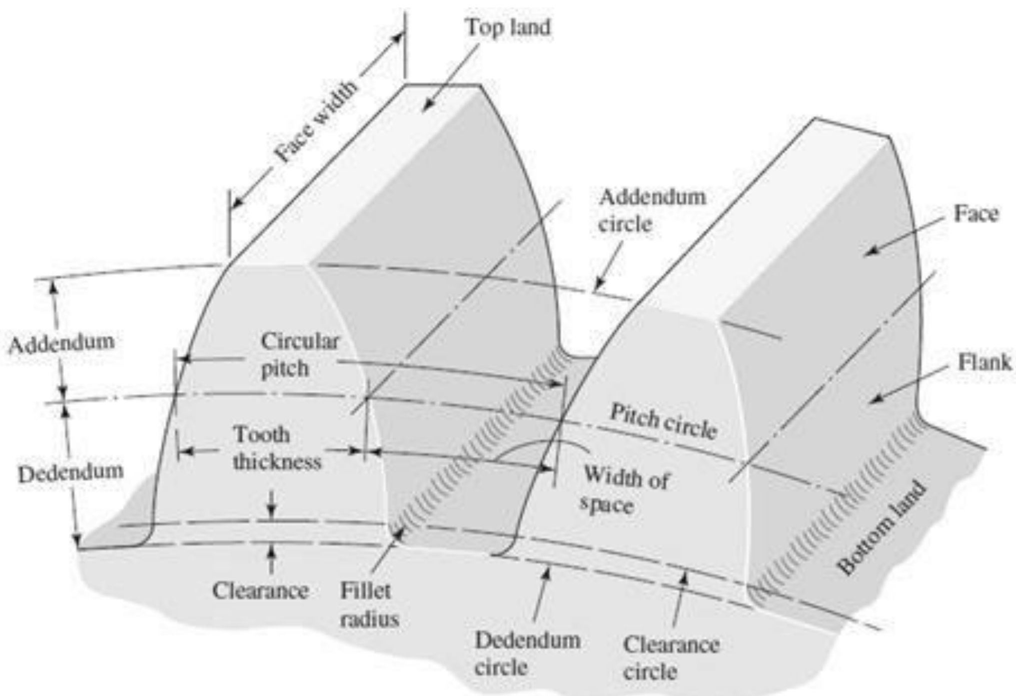
$$m = \frac{d}{N} \qquad (1-2)$$

$$p = \frac{\pi d}{N} = \pi m \qquad (1-3)$$

$$pP = \pi \qquad (1-4)$$

where

P = diametral pitch, teeth per inch; **N =** number of teeth; **d =** pitch diameter, in; **m =** module, mm; **d =** pitch diameter, mm; **p =** circular pitch



Involute Gear

This figure indicates how two involute teeth in mesh are moving to transmit rotary motion. Rotational sequence of the gears: P1- P2- P3 When Gear 1 drives Gear 2 by acting at the instantaneous contact point, the contact point moves on the common tangent in the order of P1- P2- P3. The contact point rolls along the involute curves of the gears. Moreover, the points P1, P2 and P3 lie on the common tangent to the two base circles. It is similar to the point, P, on a criss-crossed belt as the disks rotate. In effect, the involute shape of the gear teeth allows the contact point to move smoothly, transmitting the motion. Therefore, the involute curve is the ideal shape for gear teeth.

Features:

- Conjugate action is relatively independent of small errors in center distance
- Can be manufactured at low cost since the tooth profile is relatively simple
- Its root thickness makes it strong
- A typical tooth profile used almost exclusively for gears.

Law of Gearing

The fundamental law of gearing states that the angular velocity ratio between the gears of a gear set must remain constant throughout the mesh. This amounts to the following relationship:

$$\frac{\omega_1}{\omega_2}=\frac{n_1}{n_2}=\frac{d_2}{d_1}=\frac{Z_2}{Z_1}$$

In order to maintain constant angular velocity ratio between two meshing gears, the common normal of the tooth profiles, at all contact points with in mesh, must always pass through a fixed point on the line of centers, called pitch point.

Profiles Satisfying Law of Gears

Profiles which can satisfy the law of gearing are shown in Fig. a to c. These are (a) involute (b) cycloidal and (c) circular arc or Novikov. Among these, cycloidal was the first to be evolved. This is followed by the invention of involute profile which replaced many of the other profiles due to several technological advantages. Circular arc or Novikov profile has some advantages over the other profiles. But due to manufacturing difficulties, it did not become popular. However with powder metallurgy process it is slowly getting into industry now for specific application.

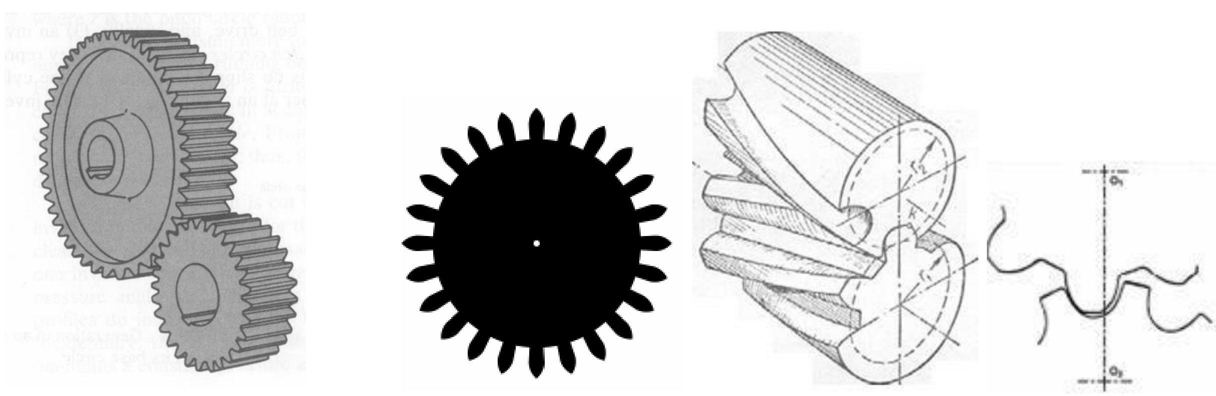
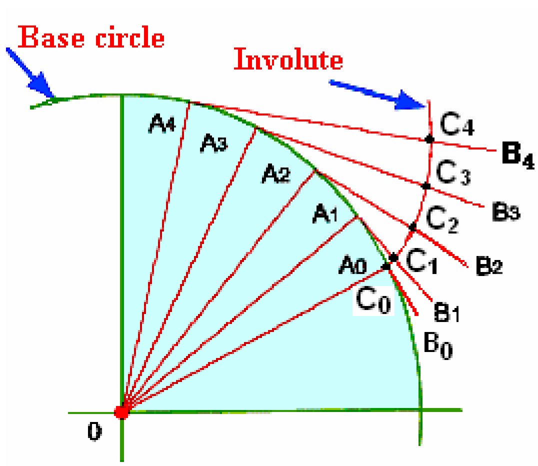


Fig. Profiles satisfying the law of gearing, (a) involute (b) cycloidal and (c) Circular arc



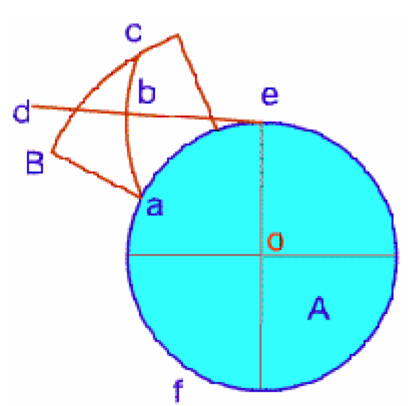


Fig.1.23 The generation of involute profile on right side **Fig.1.24** The generation of involute profile on left side

Involute Gear Tooth Profile

Involute is the path generated by the end of a thread as it unwinds from a reel. In order to understand what is involute, imagine a reel with thread wound in the clockwise direction as in Fig. Tie a knot at the end of the thread. In the initial position, the thread is at B0 with knot on the reel at C0. Keeping the reel stationary, pull the thread and unwind it to position B1. The knot now moves from C0 to C1. If the thread is unwound to position B2 the knot moves to C2 position. In repeated unwinding, the taut thread occupies position B3, B4 while the knot moves to C3, C4 positions. Connect these points C0 to C4 by a smooth curve, the profile obtained is nothing but an involute, the illustration of which is given below. This forms the left side part of the tooth profile. If similar process is repeated with thread wound on the reel in anticlockwise direction in the same position, it forms the right side part of the same tooth. The completely formed involute tooth is shown in Fig.

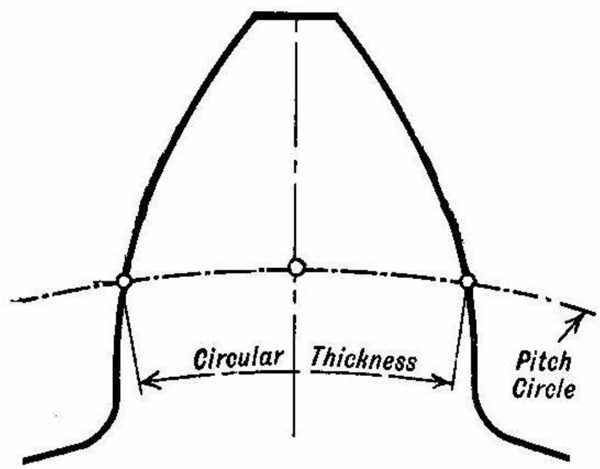


Fig.1.25 Involute gear tooth profile appearance after generation

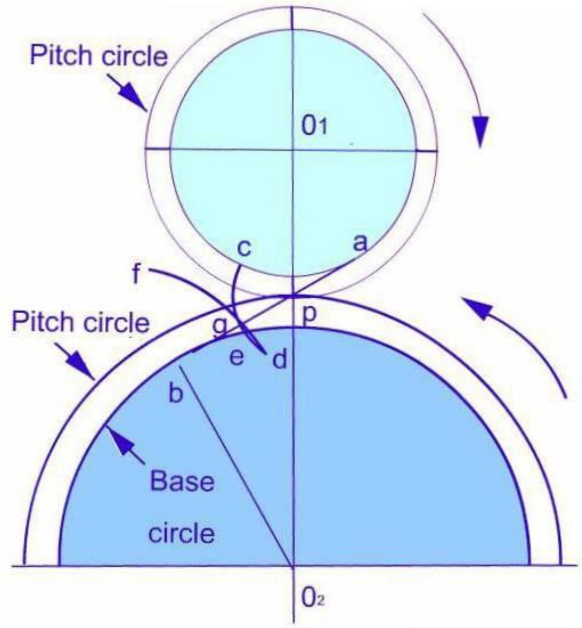


Fig. 1.26 Gear meshing

Involute profiles have special properties. Imagine two involute teeth in contact as shown in Fig.1.26. If a normal is drawn at the contact point to the involute profile, it will be tangent to the generating circles. This can be visualized better from Fig. 1.23 where the taut thread is normal to the profile as well as tangent to the reel which forms the generating or the base circle. The profile will be involute above the base circle only. Below the base circle the profile will not be involute.

The common normal to the profile at the contact point will be tangent to the base circles. It passes through a fixed point lying at the intersection of the tangent to the rolling/pitch circles and the line connecting the centres of the gear wheels. This point is known as the pitch point. As the gears rotate the contact point travels along the common tangent to the base circle. Hence this line is also known as the line of action. The movement of the contact point along the line of action can be seen in the gear meshing later on.

Advantages of Involute Gears

1. Variation in centre distance does not affect the velocity ratio.
2. Pressure angle remains constant throughout the engagements which results in smooth running.
3. Straight teeth of basic rack for involute admit simple tools. Hence, manufacturing becomes simple and cheap.

Cycloidal Gear Tooth Profile

Cycloid is the locus of a point on the circumference of a circle when it rolls on a straight line without slipping. If the circle rolls on the outside of another circle or inside of another circle gives rise to epicycloid and hypocycloid respectively. This is illustrated in Fig. 1.27. The profile of a cycloidal tooth consists of two separate curves or double curvature. This tooth form also satisfies the law of gearing or conjugate action similar to an involute gear.

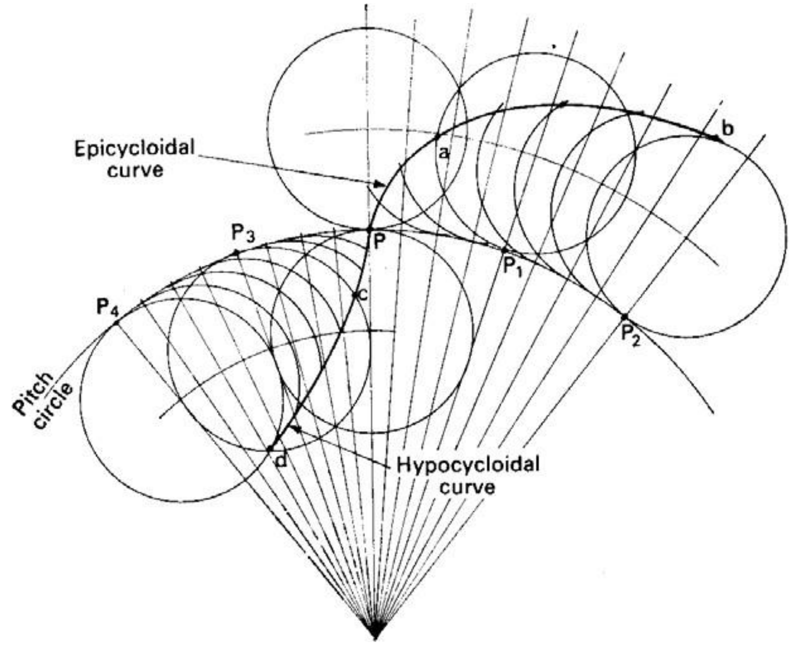


Fig.1.27 Figure illustrating the generation of cycloidal tooth

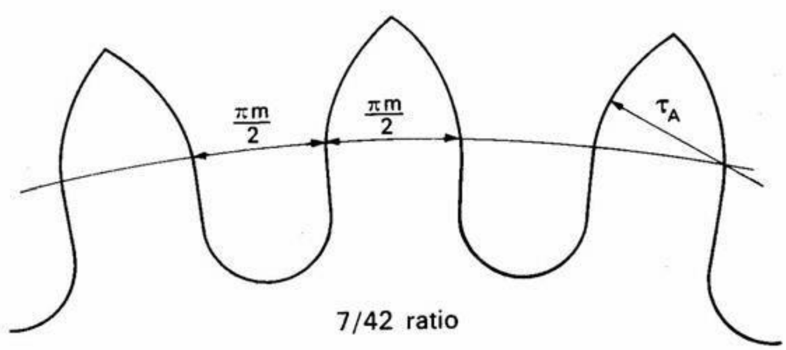


Fig.1.28 Cycloidal tooth form

Advantages of Cycloidal Gears

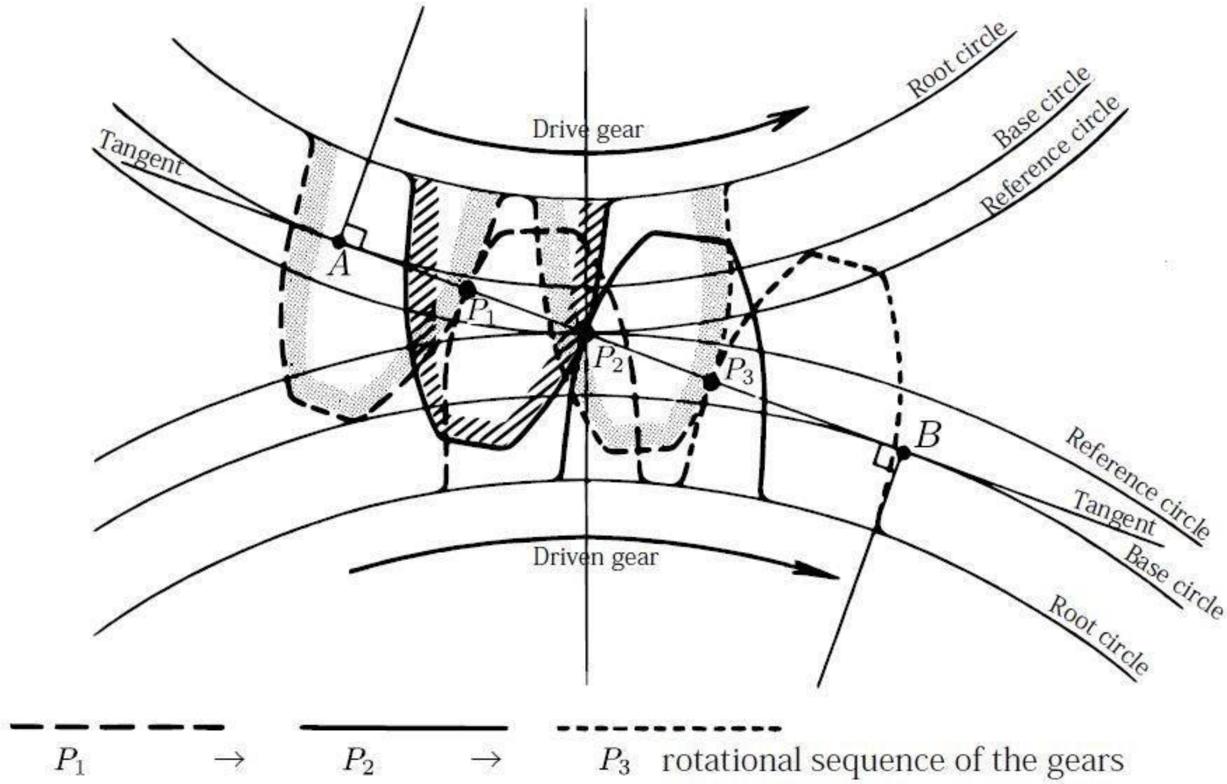
1. Cycloidal gears do not have interference.
2. Cycloidal tooth is generally stronger than an involute tooth owing to spreading flanks in contrast to the radial flanks of an involute tooth.
3. Because of the spreading flanks, they have high strength and compact drives are achievable.
4. Cycloidal teeth have longer life since the contact is mostly rolling which results in low wear.

Disadvantages of Cycloidal Gears

1. For a pair of Cycloidal gears, there is only one theoretically correct center distance for which a constant angular-velocity ratio is possible.
2. The hob of Cycloidal gear has curved teeth unlike involute rack teeth. Hence hob manufacture is difficult and costly.
3. Cycloidal gear will cost more.

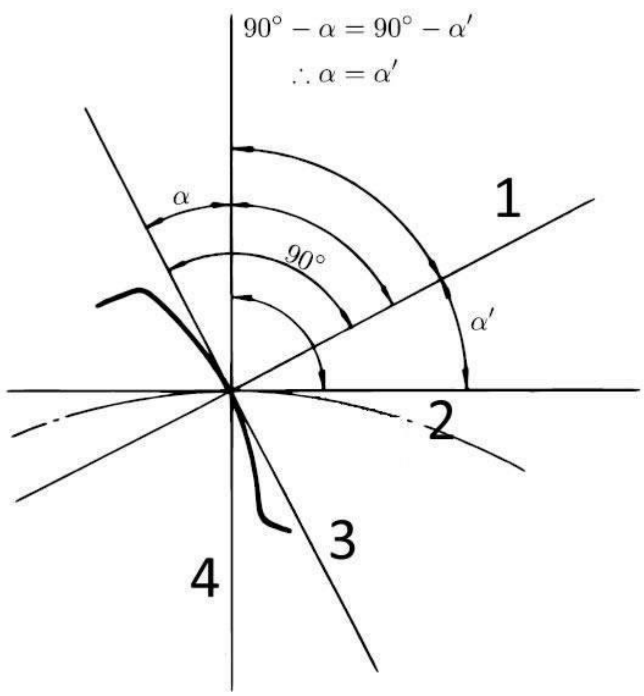
Applications of Cycloidal Gears

1. Cycloidal gears are extensively used in watches, Clocks, and instruments where strength and interference are prime considerations.
2. Cast bull gears of paper mill machinery.
3. Crusher drives in sugar mills.



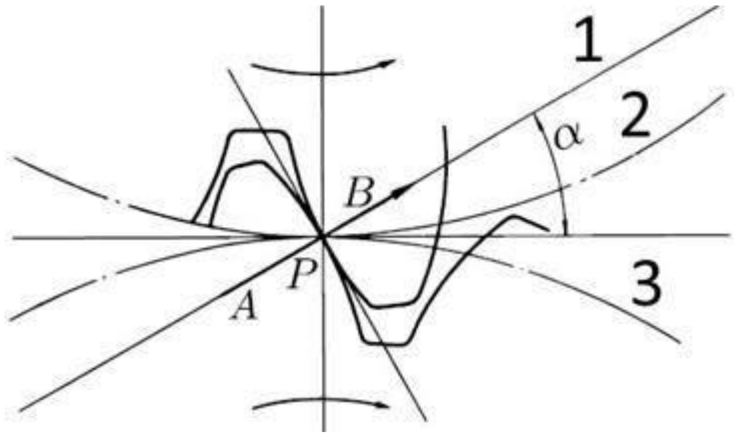
Pressure Angle

The pressure angle exists between the **tooth profile and a radial line to its pitch point**. In involute teeth, it is defined as the angle formed by the radial line and the line tangent to the profile at the pitch point. Here **α = α'.** Therefore, **α'** is also the pressure angle.



1. Normal to the profile
2. Tangent to the reference circle
3. Tangent to the profile
4. Radial line

This figure indicates the meshing of a gear A and a gear B at the pitch point.



1. Common normal
2. Reference circle
3. Reference circle

At the pitch point, the gear A is pushing the gear B. The pushing force acts along the common normal of the gear A and the gear B. The pressure angle can be described as the angle between the common normal and the line tangent to the reference circle. The most common pressure angle is 20°. Formerly, a pressure angle of 14.5° was also used.

Forms of Teeth

In actual practice, following are the two types of teeth commonly used.

1. Cycloidal Teeth; and **2.** Involute Teeth

Cycloidal Teeth

A **cycloid** is the curve traced by a point on the circumference of a circle which rolls without slipping on a fixed straight line. When a circle rolls without slipping on the outside of a fixed circle, the curve traced by a point on the circumference of a circle is known as **epicycloid**. On the other hand, if a circle rolls without slipping on the inside of a fixed circle, then the curve traced by a point on the circumference of a circle is called **hypocycloid.**

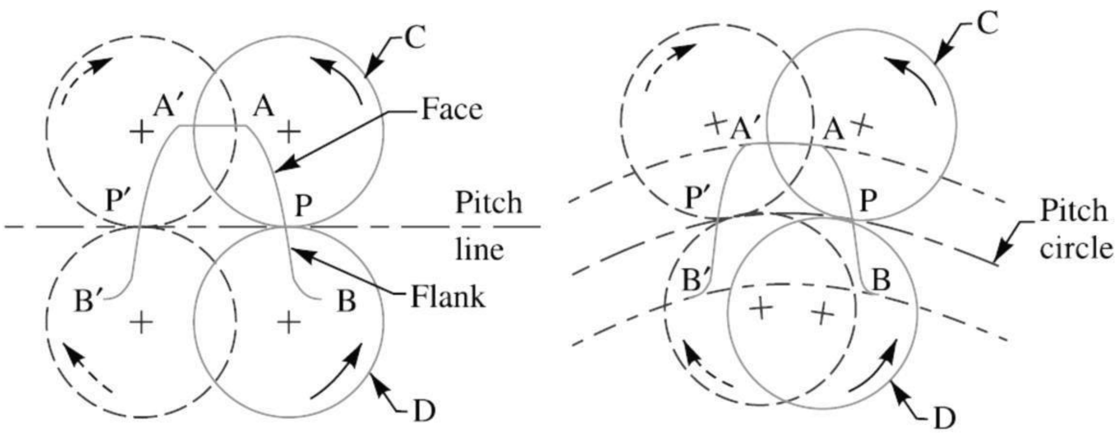


Figure 1.8 Construction of cycloidal teeth of a gear

In Fig. 1.8 (a), the fixed line or pitch line of a rack is shown. When the circle **C** rolls without slipping above the pitch line in the direction as indicated in Fig. 1.8 (a), then the point **P** on the circle traces the **epicycloid PA**. This represents the face of the cycloidal tooth profile. When the circle D rolls without slipping below the pitch line, then the point P on the circle **D** traces **hypocycloid PB** which represents the flank of the cycloidal tooth. The profile **BPA** is one side of the cycloidal rack tooth. Similarly, the two curves **P' A'** and **P' B'** forming the opposite side of the tooth profile are traced by the point **P'** when the circles **C** and **D** roll in the opposite directions.

In the similar way, the cycloidal teeth of a gear may be constructed as shown in Fig. 1.8 (b). The circle **C** is rolled without slipping on the outside of the pitch circle and the point P on the circle C traces epicycloid **PA**, which represents the face of the cycloidal tooth. The circle D is rolled on the inside of pitch circle and the point P on the circle D traces hypocycloid PB, which represents the flank of the tooth profile. The profile BPA is one side of the cycloidal tooth.

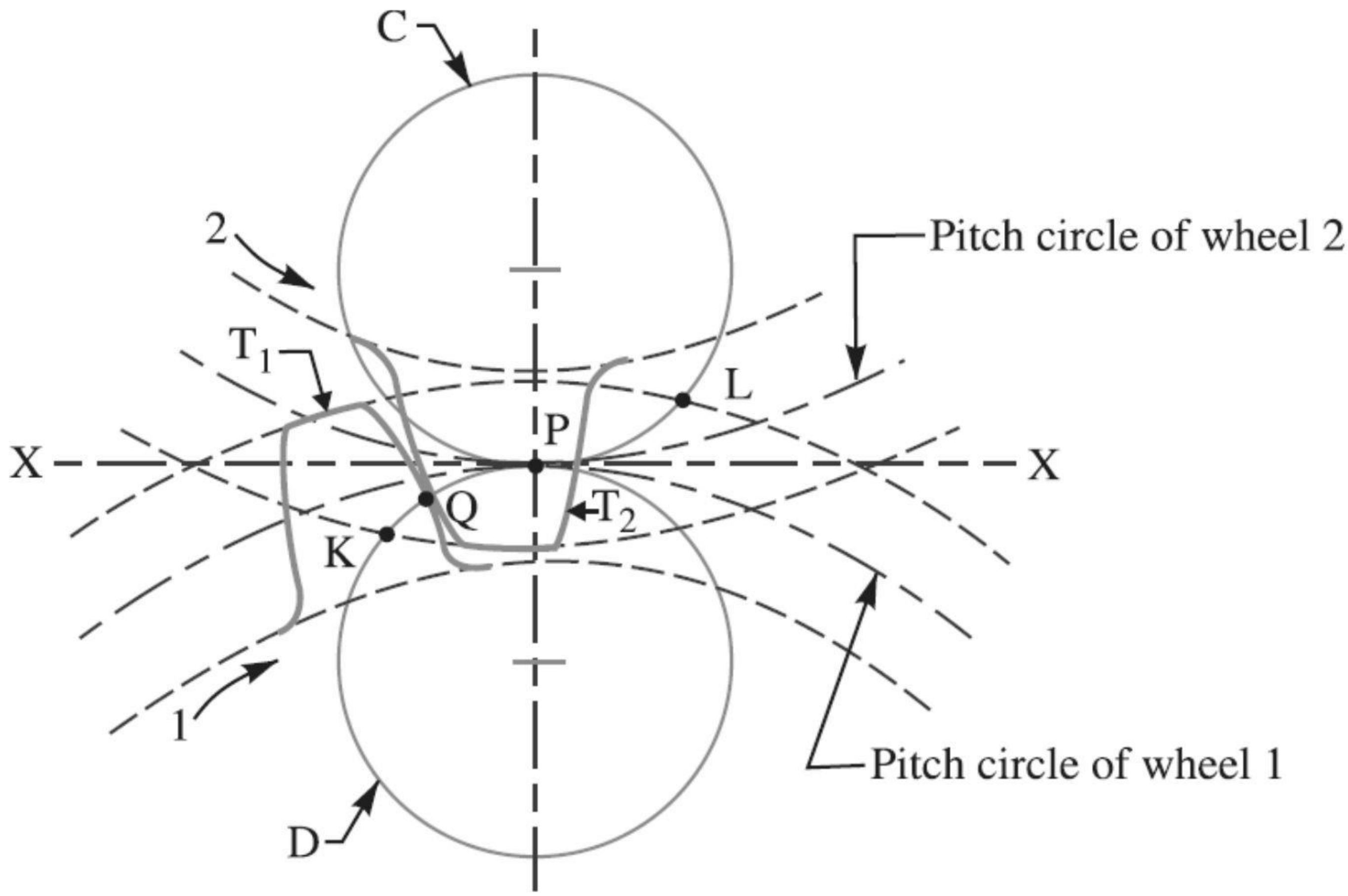


Figure 1.9 Construction of two mating cycloidal teeth

The construction of the two mating cycloidal teeth is shown in Fig. 1.9. A point on the circle D will trace the flank of the tooth $\mathbf{T_1}$ when circle D rolls without slipping on the inside of pitch circle of wheel 1 and face of tooth $\mathbf{T_2}$ when the circle D rolls without slipping on the outside of pitch circle of wheel 2. Similarly, a point on the circle C will trace the face of tooth $\mathbf{T_1}$ and flank of tooth $\mathbf{T_2}$. The rolling circles C and D may have unequal diameters, but if several wheels are to be interchangeable, they must have rolling circles of equal diameters. A little consideration will show **that the common normal XX at the point of contact between two cycloidal teeth always passes through the pitch point, which is the fundamental condition for a constant velocity ratio.**

Involute Teeth

An involute of a circle is a plane curve generated by a point on a tangent, which rolls on the circle without slipping or by a point on a taut string which is unwrapped from a reel as shown in Fig. 1.10 (a). In connection with toothed wheels, the circle is known as **base circle.** The involute is traced as follows:

Let **A** be the starting point of the involute. The base circle is divided into equal number of parts e.g. $\mathbf{AP_1}$, $\mathbf{P_1 P_2}$, $\mathbf{P_2 P_3}$ etc. The tangents at $\mathbf{P_1}$, $\mathbf{P_2}$, $\mathbf{P_3}$ etc., are drawn and the lengths $\mathbf{P_1A_1}$, $\mathbf{P_2A_2}$, $\mathbf{P_3A_3}$ equal to the arcs $\mathbf{AP_1}$, $\mathbf{AP_2}$ and $\mathbf{AP_3}$ are set off. Joining the points **A**, $\mathbf{A_1}$, $\mathbf{A_2}$, $\mathbf{A_3}$ etc., we obtain the involute curve **AR**. A little consideration will show that at any instant $\mathbf{A_3}$, the tangent $\mathbf{A_3}$T to the involute is perpendicular to $\mathbf{P_3A_3}$ and $\mathbf{P_3A_3}$ is the normal to the involute. In other words, normal at any point of an involute is a tangent to the circle.

Now, let $\mathbf{O_1}$ and $\mathbf{O_2}$ be the fixed centres of the two base circles as shown in Fig. 1.10(b). Let the corresponding involutes **AB** and **A'B'** be in contact at point **Q. MQ** and **NQ** are normals to the involute at **Q** and are tangents to base circles. Since the normal for an involute at a given point is the tangent drawn from that point to the base circle, therefore the common normal **MN** at **Q** is also the common tangent to the two base circles. We see that the common normal MN intersects the line of centres $\mathbf{O_1O_2}$ at the fixed point P (called pitch point). Therefore the involute teeth satisfy the fundamental condition of constant velocity ratio.

From similar triangles $\mathbf{O_2NP}$ and $\mathbf{O_1MP}$,

$$\frac{O_1M}{O_2N} = \frac{O_1P}{O_2P} = \frac{\text{�}}{\text{�}} \qquad (\text{�}$$

which determines the ratio of the radii of the two base circles. The radii of the base circles is given by

$O_1M = O_1P \cos$ � an� $\quad O_2N = O_2P \cos$ �

where **ϕ** is the pressure angle or the angle of obliquity.

Centre distance bet�en the base circles $= O_1P + O_2P = \dfrac{O_1M}{\cos \text{�}} + \dfrac{O_2N}{\cos \text{�}} = \dfrac{O_1M + O_2N}{\cos \text{�}}$

If the centre distance is changed, then the radii of pitch circles also changes. But their ratio remains unchanged, because it is equal to the ratio of the two radii of the base circles. The common normal, at the point of contact, still passes through the pitch point. As a result of this, the wheel continues to work correctly (It is not the case with cycloidal teeth). However, the pressure angle increases with the increase in centre distance.

Systems of Gear Teeth

The following four systems of gear teeth are commonly used in practice.

1. 14 1/2° Composite System,

2. 14 1/2° Full Depth Involute System

3. 20° Full Depth Involute

4. 20° Stub Involute System

The **14 1/2°** composite system is used for general purpose gears. It is stronger but has no interchangeability. The tooth profile of this system has cycloidal curves at the top and bottom and involute curve at the middle portion. The teeth are produced by formed milling cutters or hobs. **The tooth profile of the 14 1/2° full depth involute system was developed for use with gear hobs for spur and helical gears.**

The tooth profile of the 20° full depth involute system may be cut by hobs. The increase of the pressure angle from 14 1/2° to 20° results in a stronger tooth, because the tooth acting as a beam is wider at the base. **The 20° stub involute system has a strong tooth to take heavy loads.**

Condition for Constant Velocity Ratio of Gears–Law of Gearing

Consider the portions of the two teeth, one on the wheel 1 (or pinion) and the other on the wheel 2, as shown by thick line curves in Fig. 1.7. Let the two teeth come in contact at point Q, and the wheels rotate in the directions as shown in the figure.

Let **TT** be the common tangent and **MN** be the common normal to the curves at point of contact **Q.** From the centres **O_1** and **O_2**, draw **O_1M** and **O_2N** perpendicular to **MN.** A little consideration will show that the point **Q** moves in the direction QC, when considered as a point on wheel 1, and in the direction **QD** when considered as a point on wheel 2. Let **v_1** and **v_2** be the velocities of the point **Q** on the wheels 1 and 2 respectively. If the teeth are to remain in contact, then the components of these velocities along the common normal **MN** must be equal.

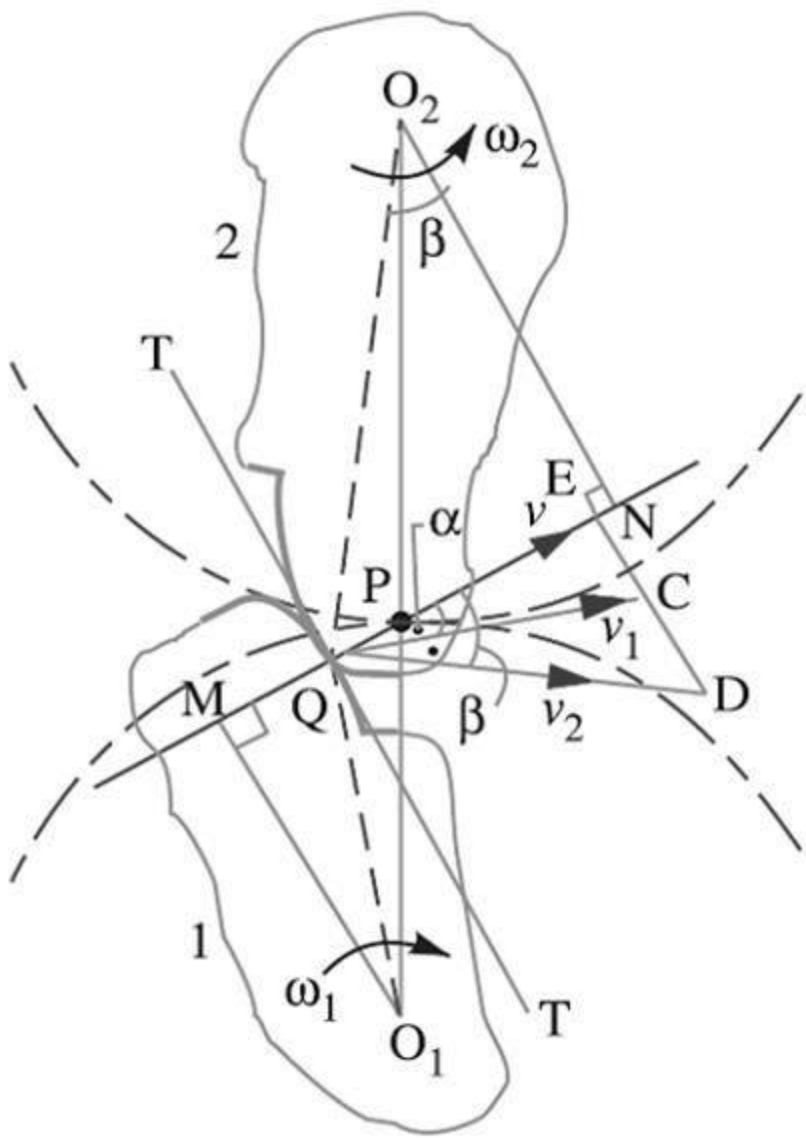


Figure 1.7 Law of Gearing

$\therefore v_1 \cos a = v_2 \cos Q$

$(\omega_1 \times O_1Q) \cos a = (\omega_2 \times O_2Q) \cos Q$

$(\omega_1 \times O_1Q) \dfrac{O_1M}{O_1Q} = (\omega_2 \times O_2Q) \dfrac{O_2N}{O_2Q}$

$\therefore \omega_1 O_1M = \omega_2 O_2N$

$$\therefore \frac{\omega_1}{\omega_2} = \frac{O_2N}{O_1M} \qquad \text{(i)}$$

Also from similar triangles **O_1MP** and **O_2NP**,

$$\frac{O_2N}{O_1M} = \frac{O_2P}{O_1P} \qquad \text{(ii)}$$

Combining equations (i) and (ii), we have

$$\frac{\omega_1}{\omega_2} = \frac{O_2N}{O_1M} = \frac{O_2P}{O_1P} \qquad \text{(iii)}$$

We see that the angular velocity ratio is inversely proportional to the ratio of the distance of **P** from the centres **O_1** and **O_2**, or **the common normal to the two surfaces at the point of contact Q intersects the line of centres at point P which divides the centre distance inversely as the ratio of angular velocities.**

Therefore, in order to have a constant angular velocity ratio for all positions of the wheels, P must be the fixed point (called **Pitch Point**) for the two wheels. In other words, **the common normal at the point of contact between a pair of teeth must always pass through the pitch point.** This is fundamental condition which must be satisfied while designing the profiles for the teeth of gear wheels. It is also known as **Law of Gearing.**

Notes: 1. The above condition is fulfilled by teeth of involute form, provided that the root circles from which the profiles are generated are tangential to the common normal.

2. If the shape of one tooth profile is arbitrary chosen and another tooth is designed to satisfy the above condition, then the second tooth is said to be conjugate to the first. The conjugate teeth are not in common use because of difficulty in manufacture and cost of production.

3. If **D_1** and **D_2** are pitch circle diameters of wheel 1 and 2 having teeth **T_1** and **T_2** respectively, then velocity ratio,

$$\frac{\omega_1}{\omega_2} = \frac{O_2P}{O_1P} = \frac{D_2}{D_1} = \frac{T_2}{T_1}$$

Interference in Involute Gears

A pinion gearing with a wheel is shown in Fig. 1.11. **MN** is the common tangent to the base circles and **KL** is the path of contact between the two mating teeth. A little consideration will show that if the radius of the addendum circle of pinion is increased to **O_1N**, the point of contact L will move from **L** to N. When this radius is further increased, the point of contact L will be on the inside of base circle of wheel and not on the involute profile of tooth on wheel. The tip of tooth on the pinion will then **undercut the tooth on the wheel at the root** and remove part of the involute profile of tooth on the wheel. This effect is known as **Interference** and occurs when the teeth are being cut. In brief, the phenomenon when **the tip of a tooth undercuts the root on its mating gear is known as interference.**

Similarly, if the radius of the addendum circle of the wheel increases beyond **O_2M**, then the tip of tooth on wheel will cause interference with the tooth on pinion. The points **M** and **N** are called interference points. Obviously **interference may be avoided if the path of contact does not extend beyond interference points.** The limiting value of the radius of the addendum circle of the pinion is **O_1N** and of the wheel is **O_2M.** From the above discussion, we conclude that the interference may only be avoided, if the point of contact between the two teeth is always on the involute profiles of both the teeth. In other words, interference may only be prevented, if the addendum circles of the two mating gears cut the common tangent to the base circles between the points of tangency.

Note: In order to avoid interference, the limiting value of the radius of the addendum circle of the pinion **(O_1N)** and of the wheel **(O_2M),** may be obtained as follows:

From Fig. 1.11, we see that

$$O_1N = \sqrt{(O_1M)^2 + (MN)^2} = \sqrt{(r_b)^2 + [(R + r)\sin\phi]^2}$$

where r_b = Radius of base circle of the pinion = $O_1P \cos\phi = r\cos\phi$

$$O_2M = \sqrt{(O_2N)^2 + (MN)^2} = \sqrt{(R_b)^2 + [(R + r)\sin\phi]^2}$$

where R_b = Radius of base circle of the wheel = $O_2P \cos\phi = R\cos\phi$

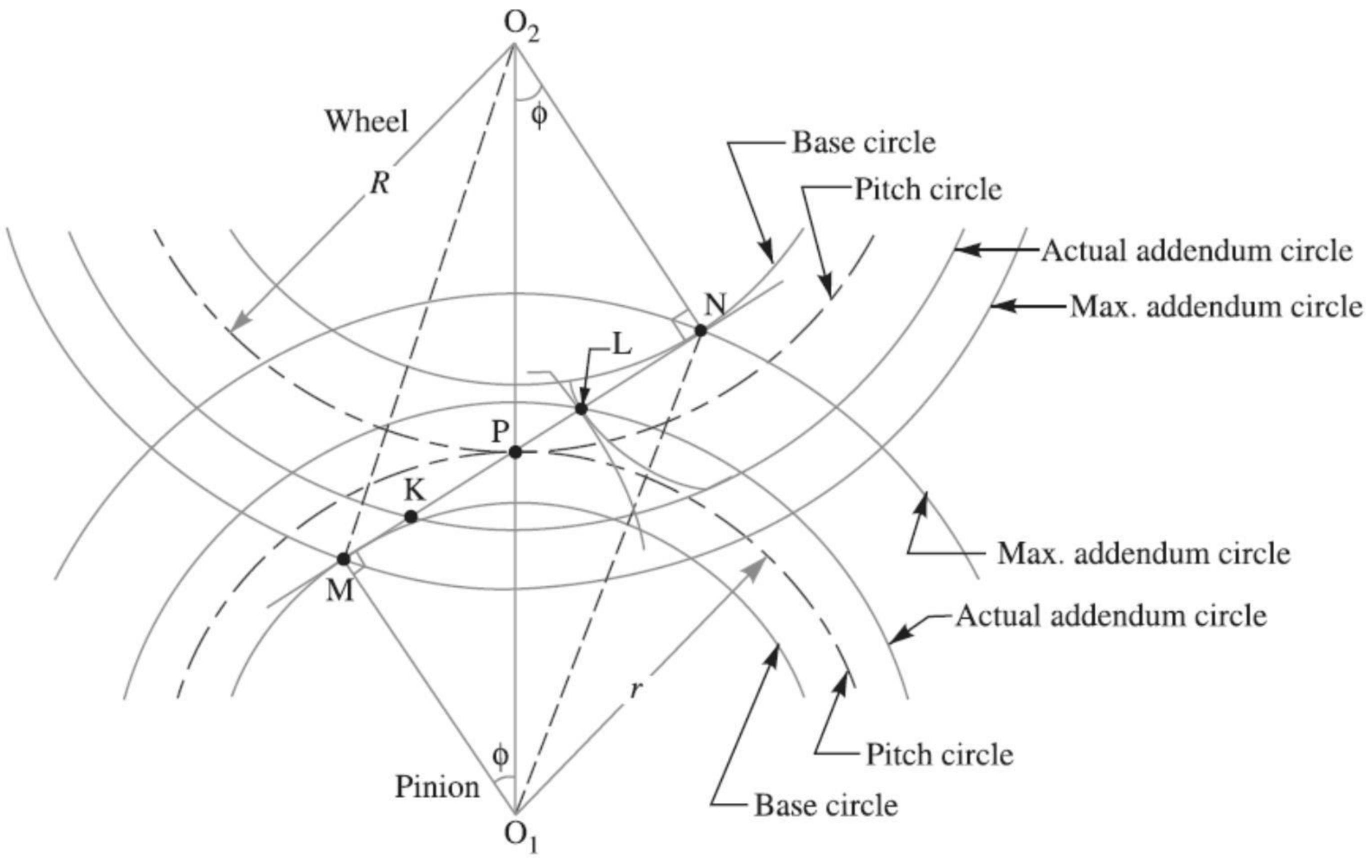


Figure 1.11 Interference in involute gears

Minimum Number of Teeth on the Pinion in Order to Avoid Interference

Interference may only be avoided, if the point of contact between the two teeth is always on the involute profiles of both the teeth. The number of teeth on the pinion (**T_P**) in order to avoid interference may be obtained from the following relation:

$$T_P = \frac{2A_W}{G\left[\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2\phi} - 1\right]}$$

where **A_W =** Fraction by which the standard addendum for the wheel should be multiplied,

G = Gear ratio or velocity ratio **= T_G / T_P = D_G / D_P,**

ϕ = Pressure angle or angle of obliquity.

Contact Ratio

The zone of action of meshing gear teeth is shown in Fig. 1–15. We recall that tooth contact begins and ends at the intersections of the two addendum circles with the pressure line. In Fig. 1–15 initial contact occurs at ***a*** and final contact at ***b***. Tooth profiles drawn through these points intersect the pitch circle at **A** and **B**, respectively. As shown, the distance **AP** is called **the arc of approach** q_a , and the distance **PB**, the arc of recess $\mathbf{q_r}$. The sum of these is the arc of action $\mathbf{q_t}$.

Now, consider a situation in which the arc of action is exactly equal to the circular pitch, that is, $\mathbf{q_t = p}$. This means that one tooth and its space will occupy the entire arc **AB.** In other words, when a tooth is just beginning contact at ***a***, the previous tooth is simultaneously ending its contact at ***b***. Therefore, during the tooth action from ***a*** to ***b***, there will be exactly one pair of teeth in contact.

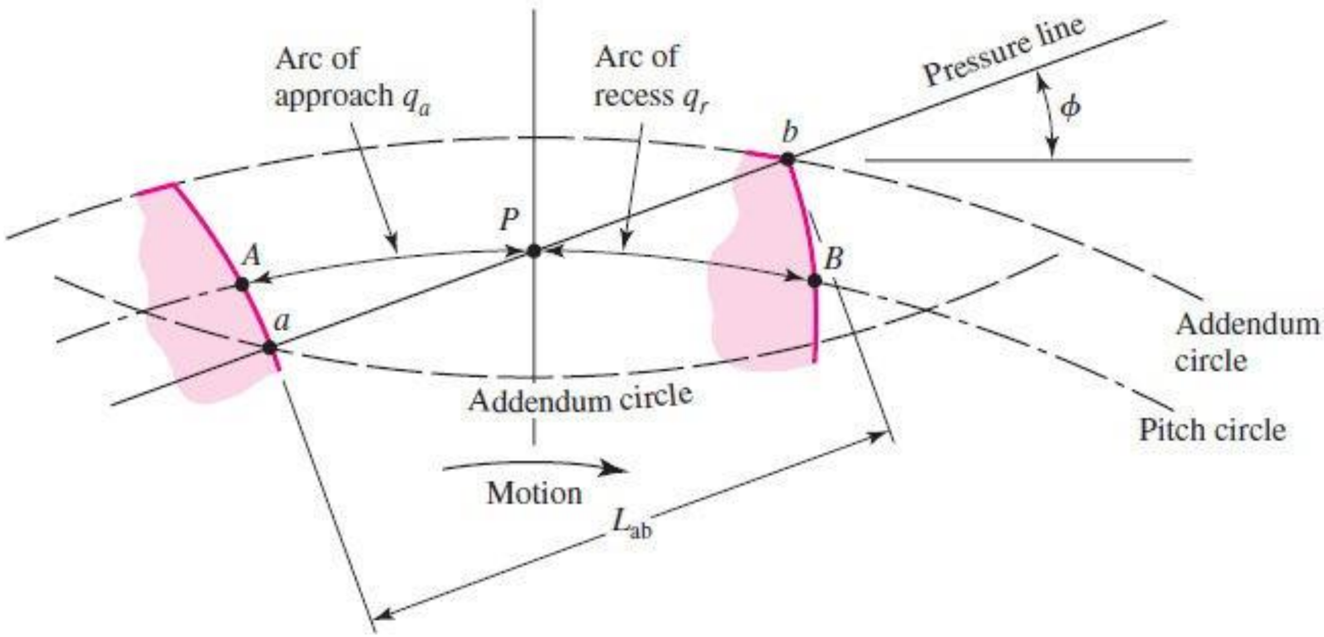


Figure 1-15 Definition of contact ratio

Next, consider a situation in which the arc of action is greater than the circular pitch, but not very much greater, say, $\mathbf{q_t = 1.2p}$. This means that when one pair of teeth is just entering contact at ***a***, another pair, already in contact, will not yet have reached ***b***. Thus, for a short period of time, there will be two teeth in contact, one in the vicinity of **A** and another near **B**. As the meshing proceeds, the pair near **B** must cease contact, leaving only a single pair of contacting teeth, until the procedure repeats itself. Because of the nature of this tooth action, either one or two pairs of teeth in contact, it is convenient to define the term contact ratio $\mathbf{m_c}$ as

$$m_c = \frac{q_t}{p} \qquad (1-8)$$

a number that indicates the average number of pairs of teeth in contact. Note that this ratio is also equal to the length of the path of contact divided by the base pitch**. Gears should not generally be designed having contact ratios less than about 1.20, because inaccuracies in mounting might reduce the contact ratio even more, increasing the possibility of impact between the teeth as well as an increase in the noise level.**

An easier way to obtain **the contact ratio is of action *ab* instead of the arc distance AB.** Since ***ab*** in Fig. 1–15 is tangent to the base circle when extended, the base to measure the line pitch $\mathbf{p_b}$ must be used to calculate $\mathbf{m_c}$ instead of the circular pitch as in Eq. (1–8). If the length of the line of action is L_{ab}, the contact ratio is

$$m_c = \frac{L_{ab}}{p \cos �} \qquad (1-9)$$

in which Eq. (1–7) was used for the base pitch.

Interference

The contact of portions of tooth profiles that are not conjugate is called **Interference.** Consider Fig. 1–16. Illustrated are two 16-tooth gears that have been cut to the now obsolete $14½^0$ pressure angle. The driver, gear 2, turns clockwise. The initial and final points of contact are designated A and B, respectively, and are located on the pressure line. Now notice that the points of tangency of the pressure line with the base circles C and D are located inside of points A and B. Interference is present.

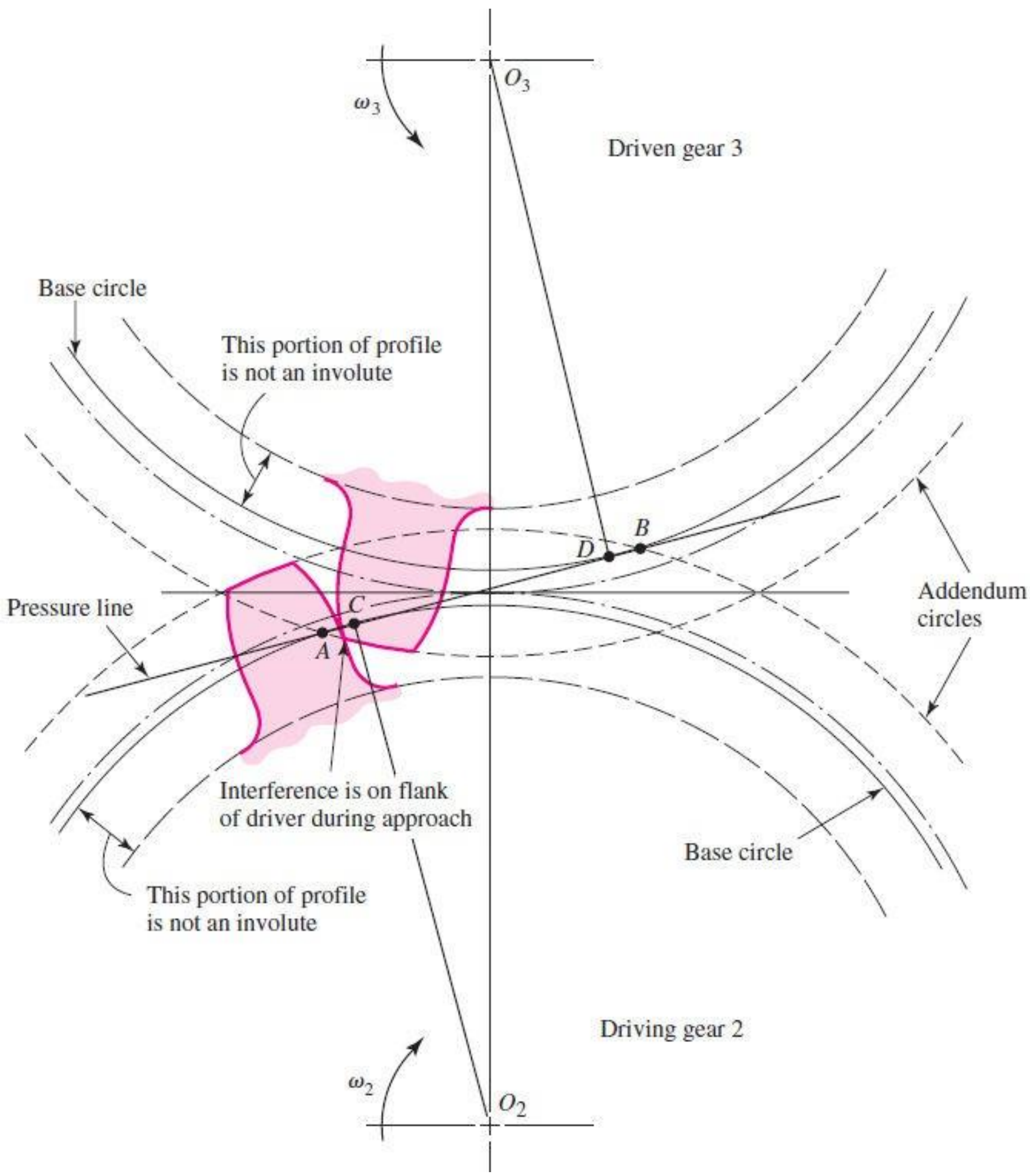


Figure 1-16

The interference is explained as follows. Contact begins when the tip of the driven tooth contacts the flank of the driving tooth. In this case the flank of the driving tooth first makes contact with the driven tooth at point **A**, and this occurs before the involute portion of the driving tooth comes within range. In other words, contact is occurring below the base circle of gear 2 on the non-involute portion of the flank. The actual effect is that the involute tip or face of the driven gear tends to dig out the non-involute flank of the driver.

In this example the same effect occurs again as the teeth leave contact. Contact should end at point **D** or before. Since it does not end until point **B**, the effect is for the tip of the driving tooth to dig out, or interfere with, the flank of the driven tooth.

When gear teeth are produced by a generation process, interference is automatically eliminated because the cutting tool removes the interfering portion of the flank. This effect is called **Undercutting**; if undercutting is at all pronounced, the undercut tooth is considerably weakened. Thus the effect of eliminating interference by a generation process is merely to substitute another problem for the original one.

The smallest number of teeth on a spur pinion and gear, one-to-one gear ratio, which can exist without interference is **N_P** . This number of teeth for spur gears is given by

$$N_P = \frac{2k}{3\sin^2\phi}\left(1 + \sqrt{1 + 3\sin^2\phi}\right) \qquad (1-10)$$

where **k = 1** for full-depth teeth, 0.8 for stub teeth and **ϕ = pressure angle.**

For a 20° pressure angle, with **k = 1**,

$$N_P = \frac{2(1)}{3\sin^2 20°}\left(1 + \sqrt{1 + 3\sin^2 20°}\right) = 12.3 = 13 \text{ teeth}$$

Thus 13 teeth on pinion and gear are interference-free. Realize that 12.3 teeth is possible in meshing arcs, but for fully rotating gears, 13 teeth represents the least number. For a **14½^0 pressure angle**, **N_P = 23 teeth**, so one can appreciate why few 14½^0 tooth systems are used, as the higher pressure angles can produce a smaller pinion with accompanying smaller center-to-center distances.

If the mating gear has more teeth than the pinion, that is, **$m_G = N_G/N_P = m$** is more than one, then the smallest number of teeth on the pinion without interference is given by

$$N_P = \frac{2k}{(1+2m)\sin^2\phi}\left(m + \sqrt{m^2 + (1+2m)\sin^2\phi}\right) \qquad (1-11)$$

For example, if **m = 4, ϕ = 20**°

$$N_P = \frac{2(1)}{(1+2(4))\sin^2 20°}\left(4 + \sqrt{4^2 + (1+2(4))\sin^2 20°}\right) = 15.4 = 16 \text{ teeth}$$

Thus a 16-tooth pinion will mesh with a 64-tooth gear without interference.

The largest gear with a specified pinion that is **interference-free** is

$$N_G = \frac{N_P^2\sin^2\phi - 4(1)^2}{4(1) - 2(13)\sin^2 20°} = 16.45 = 16 \text{ teeth}$$

For example, for a 13-tooth pinion with a pressure angle ϕ of 20°,

$$N_G = \frac{13^2\sin^2 20° - 4k^2}{4k - 2N_P\sin^2\phi} \qquad (1-12)$$

For a 13-tooth spur pinion, the maximum number of gear teeth possible without interference is 16.

The smallest spur pinion that will operate with a rack without interference is

$$N_P = \frac{2(�)}{\sin^2 �} \qquad (1-13)$$

For a 20◦ pressure angle full-depth tooth the smallest number of pinion teeth to mesh with a rack is

$$N_P = \frac{2(1)}{\sin^2 20°} = 17.1 = 18 \text{ teeth}$$

Since gear-shaping tools amount to contact with a rack, and the gear-hobbing process is similar, the minimum number of teeth to prevent interference to prevent undercutting by the hobbing process is equal to the value of $\mathbf{N_P}$ when $\mathbf{N_G}$ is infinite.

The importance of the problem of teeth that have been weakened by undercutting cannot be overemphasized. Of course, **interference can be eliminated by using more teeth on the pinion**. However, **if the pinion is to transmit a given amount of power, more teeth can be used only by increasing the pitch diameter.**

Interference can also be reduced by using a larger pressure angle. This results in a smaller base circle, so that more of the tooth profile becomes involute. The demand for smaller pinions with fewer teeth thus favors the use of a 25^0 pressure angle even though the frictional forces and bearing loads are increased and the contact ratio decreased.

Undercutting

Interference can also be avoided by removing the material on the gear tooth between the base circle and dedendum circle. This is the portion of the gear tooth that is not an involute and would interfere with the mating tooth. An undercut gear tooth is shown in Figure. Undercutting obviously reduces the strength of the gear, thus reducing the power that can be safely transmitted. In addition, it also reduces the length of contact, which reduces the contact ratio and results in rougher and noisier gear action. Therefore, undercutting should be avoided unless the application absolutely requires a compact gearset. In these cases, advanced kinematic and strength analyses and experiments are necessary to verify proper operation.

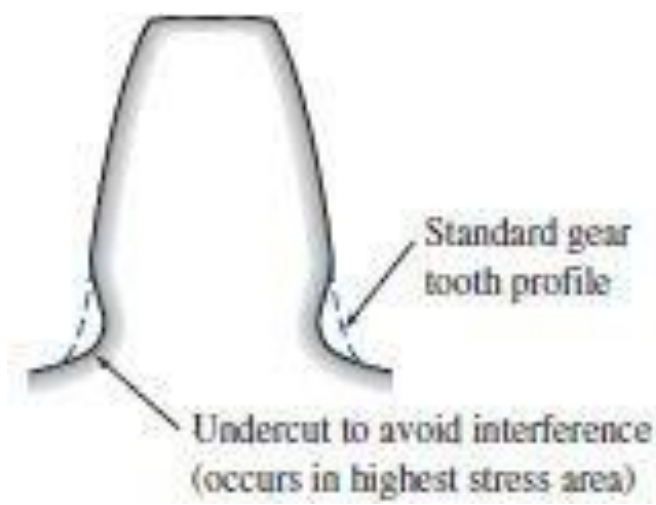


Fig. Undercut gear tooth

Backlash

Backlash is the amount that the width of a tooth space exceeds the thickness of a gear tooth, measured on the pitch circle. In more practical terms, it is the amount that a gear can turn without its mating gear turning. Although backlash may seem undesirable, some backlash is necessary to provide for lubrication on the gear teeth. Gears that run continuously in one direction can actually have considerable backlash. Gears that frequently start/stop or reverse direction should have closely controlled backlash. A nominal value of backlash is designed into a gear tooth profile. The amount of backlash determines the thickness of a gear tooth because backlash is a measure of the tooth thickness to the tooth space. Backlash values are strongly influenced by any variation in the center distance of the gears.

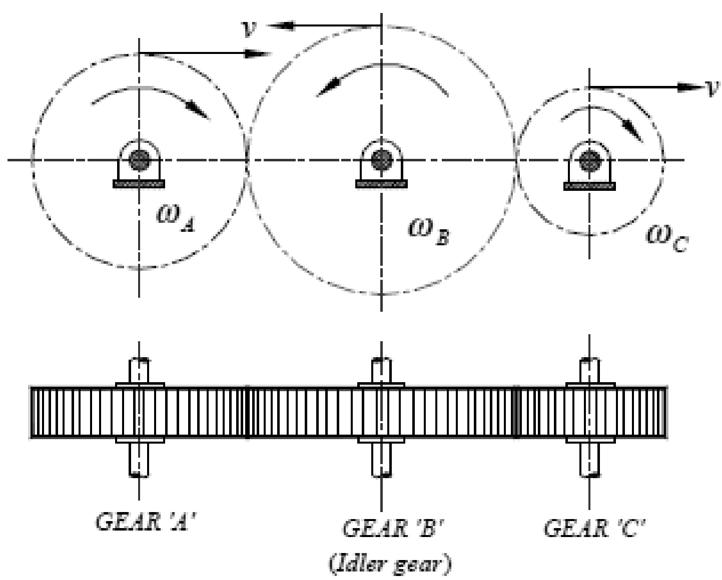
Gears Trains

A gear train is two or more gear working together by meshing their teeth and turning each other in a system to generate power and speed. It **reduces speed** and **increases torque**. To **create large gear ratio**, gears are connected together to form gear trains. They often consist of multiple gears in the train. The most common of the gear train is the gear pair connecting parallel shafts. The teeth of this type can be spur, helical or herringbone. The angular velocity is simply the reverse of the tooth ratio.

Any combination of gear wheels employed to transmit motion from one shaft to the other is called a **Gear Train**. The meshing of two gears may be idealized as two smooth discs with their edges touching and no slip between them. This ideal diameter is called the **Pitch Circle Diameter (PCD)** of the gear.

Simple Gear Trains

The typical spur gears as shown in diagram. The direction of rotation is reversed from one gear to another. It has no affect on the gear ratio. The teeth on the gears must all be the same size so if gear A advances one tooth, so does B and C.



t = number of teeth on the gear; **N =** speed in rpm;

Module must be the same for all gears otherwise they would not mesh.

$$m = \frac{D_A}{t_A} = \frac{D_B}{t_B} = \frac{D_C}{t_C}$$

Linear velocity on pitch circle, $v = \omega \frac{D}{2} = \omega r$

The velocity ***v*** of any point on the circle must be the same for all the gears, otherwise they would be slipping.

$$v = \omega_A \frac{D_A}{2} = \omega_B \frac{D_B}{2} = \omega_C \frac{D_C}{2}$$

$$\omega_A D_A = \omega_B D_B = \omega_C D_C$$

$�mt_A = �mt_B = �mt_C$

$�t_A = �t_B = �t_C$ o�� ���n�o��� $N_A t_A = N_B t_B = N_C t_C$

Application

a) To connect gears where a large center distance is required

b) To obtain desired direction of motion of the driven gear (CW or CCW)

c) To obtain high speed ratio

Torque & Efficiency

The power transmitted by a torque T N-m applied to a shaft rotating at N rev/min is given by:

$$P = \frac{2\pi NT}{60}$$

In an ideal gear box, the input and output powers are the same so;

$$P = \frac{2\pi N_1 T_1}{60} = \frac{2\pi N_2 T_2}{60}$$

$$N_1 T_1 = N_2 T_2 \quad \Rightarrow \frac{T_2}{T_1} = \frac{N_1}{N_2} = GR$$

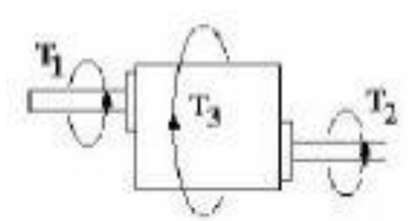
It follows that if the speed is reduced, the torque is increased and vice versa. In a real gear box, power is lost through friction and the power output is smaller than the power input. The efficiency is defined as:

$$\eta = \frac{\text{Power Out}}{\text{Power In}} = \frac{2\pi N_2 T_2}{2\pi N_1 T_1} = \frac{N_2 T_2}{N_1 T_1}$$

Because the torque in and out is different, a gear box has to be clamped in order to stop the case or body rotating. A holding torque $\mathbf{T_3}$ must be applied to the body through the clamps.

The total torque must add up to zero.

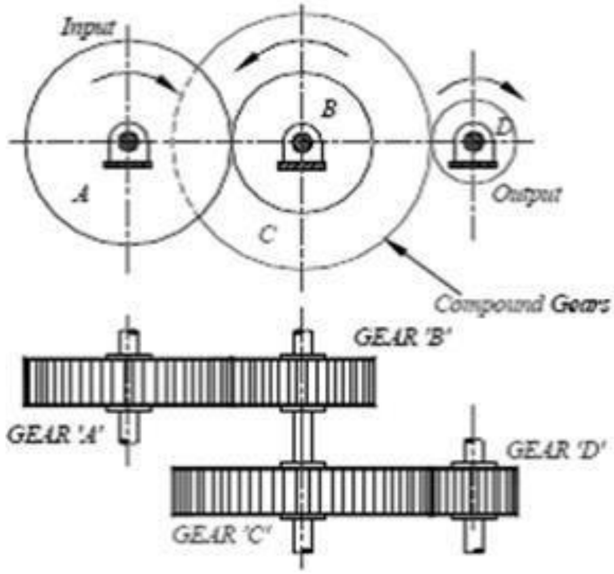
$\mathbf{T_1 + T_2 + T_3 = 0}$



If we use a convention that anti-clockwise is positive and clockwise is negative we can determine the holding torque. The direction of rotation of the output shaft depends on the design of the gear box.

Compound Gear Train

Compound gears are simply a chain of simple gear trains with the input of the second being the output of the first. A chain of two pairs is shown below. Gear B is the output of the first pair and gear C is the input of the second pair. Gears B and C are locked to the same shaft and revolve at the same speed. For large velocities ratios, compound gear train arrangement is preferred.



The velocity of each tooth on A and B are the same so: $\omega_A t_A = \omega_B t_B$ -as they are simple gears.

Likewise for C and D, $\omega_C t_C = \omega_D t_D$.

$$\frac{\omega_A}{t_B} = \frac{\omega_B}{t_A} \text{ and } \frac{\omega_C}{t_D} = \frac{\omega_D}{t_C}$$

$$\frac{\omega_A \times \omega_C}{\omega_B \times \omega_D} = \frac{t_B}{t_A} \times \frac{t_D}{t_C}$$

Since gears B and C are on the same shaft $\omega_B = \omega_C$

$$\frac{\omega_A}{\omega_D} = \frac{t_B}{t_A} \times \frac{t_D}{t_C} = GR$$

Since $\omega = 2 \times \pi \times N$ the gear ratio may be written as $\frac{N(In)}{N(Out)} = \frac{t_B}{t_A} \times \frac{t_D}{t_C} = GR$

Reverted Gear train

The driver and driven axes lies on the same line. These are used in speed reducers, clocks and machine tools.

$$GR = \frac{N_A}{N_D} = \frac{t_B \times t_D}{t_A \times t_C}$$

If **R** and **T=** Pitch circle radius & number of teeth of the gear

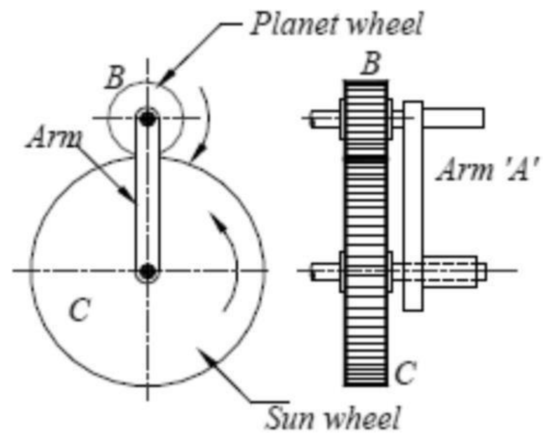
$$R_A + R_B = R_C + R_D \quad \text{and} \quad t_A + t_B = t_C + t_D$$

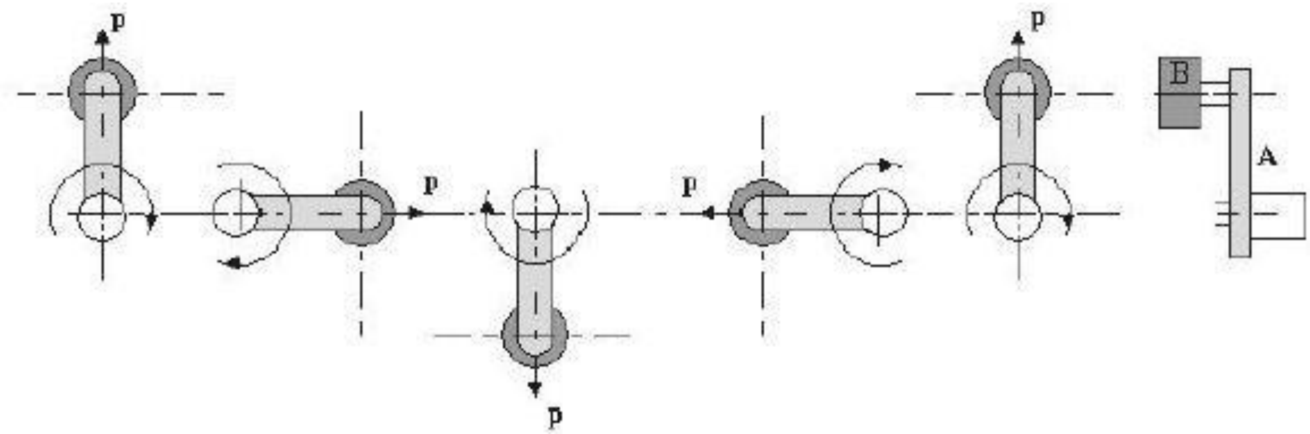
Epicyclic Gear Train

Epicyclic means one gear revolving upon and around another. The design involves planet and sun gears as one orbits the other like a planet around the sun. This design can produce large gear ratios in a small space and are used on a wide range of applications from marine gearboxes to electric screwdrivers.

Basic Theory: The diagram shows a gear B on the end of an arm. Gear B meshes with gear C and revolves around it when the arm is rotated. B is called the planet gear and C the sun. First consider what happens when the planet gear orbits the sun gear.

Observe point **p** and you will see that gear B also revolves once on its own axis. Any object orbiting around a center must rotate once. Now consider that B is free to rotate on its shaft and meshes with C. Suppose the arm is held stationary and gear C is rotated once. B spins about its own center and the number of revolutions it makes is the ratio **t_C /t_B** B will rotate by this number for every complete revolution of C.





Now consider that C is unable to rotate and the arm A is revolved once. Gear B will revolve **(1+ t_C/t_B)** because of the orbit. It is this extra rotation that causes confusion. One way to get round this is to imagine that the whole system is revolved once. Then identify the gear that is fixed and revolve it back one revolution. Work out the revolutions of the other gears and add them up. The following tabular method makes it easy.

Suppose gear C is fixed and the arm A makes one revolution. Determine how many revolutions the planet gear B makes.

Step 1 is to revolve everything once about the center.

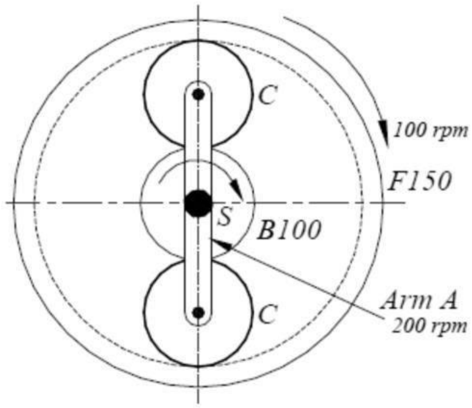
Step 2 identify that C should be fixed and rotate it backwards one revolution keeping the arm fixed as it should only do one revolution in total. Work out the revolutions of B.

Step 3 is simply add them up and we find the total revs of C is zero and for the arm is 1.

Step	*Action*	*A*	*B*	*C*
1	Revolve all once	1	1	1
2	Revolve *C* by –1 revolution, keeping the arm fixed	0	$+\frac{t_C}{t_B}$	-1
3	Add	1	$1+\frac{t_C}{t_B}$	0

[illegible] $(1 + \frac{t_C}{t_B})$

Problem 1: In an epicyclic gear train shown in figure, the arm A is fixed to the shaft S. The wheel B having 100 teeth rotates freely on the shaft S. The wheel F having 150 teeth is driven separately. If the arm rotates at 200 rpm and wheel F at 100 rpm in the same direction; find (a) number of teeth on the gear C and (b) speed of wheel B.



Solution:

T_B =100; T_F =150; N_A = 200rpm; N_F =100rpm:

Since the module is same for all gears the number of teeth on the gears is proportional to the pitch circle:

$d_F = d_B + 2d_C$; $\quad T_F = T_B + 2T_C$

$150 = 100 + 2 \times T_C$ or $T_C = 25$

The gear B and gear F rotates in the opposite directions:

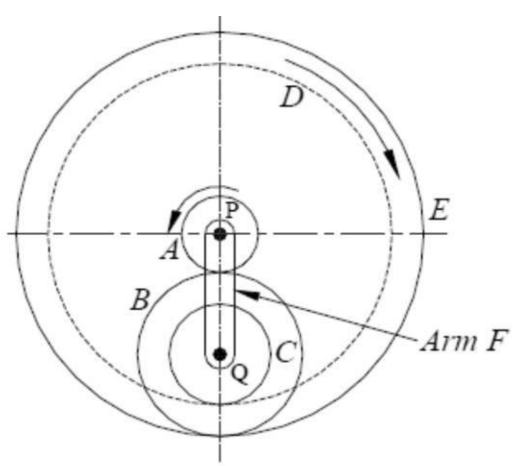
$\text{Train Value} = -\frac{T_B}{T_F}$

$\text{Also Train Value} = \frac{N_L - N_{Arm}}{N_F - N_{Arm}} = \frac{N_F - N_A}{N_B - N_A}$

$-\frac{T_B}{T_F} = \frac{N_F - N_A}{N_B - N_A}$; $-\frac{100}{150} = \frac{100 - 200}{N_B - 200} \Rightarrow N_B = 350$

The Gear B rotates at 350 rpm in the same direction of gears F and Arm A.

Problem 2: In a compound epicyclic gear train as shown in the figure, has gears A and an annular gears D & E free to rotate on the axis P. B and C is a compound gear rotate about axis Q. Gear A rotates at 90 rpm CCW and gear D rotates at 450 rpm CW. Find the speed and direction of rotation of arm F and gear E. Gears A,B and C are having 18, 45 and 21 teeth respectively. All gears having same module and pitch.



Solution:

T_A=18 ; T_B=45; T_C=21; N_A = -90rpm; N_D=450rpm

Since the module and pitch are same for all gears; the number of teeth on the gears is proportional to the pitch circle:

$r_D = r_A + r_B + r_C \Rightarrow T_D = T_A + T_B + T_C$; $T_D = 18 + 45 + 21 = 84$

Gears A and D rotates in the opposite directions.

$\text{Train Value} = -\frac{T_A}{T_B} \times \frac{T_C}{T_D}$

$\text{Also Train Value} = \frac{N_L - N_{arm}}{N_F - N_{arm}} = \frac{N_D - N_F}{N_A - N_F}$

$-\frac{T_A}{T_B} \times \frac{T_C}{T_D} = \frac{N_D - N_F}{N_A - N_F}$; $-\frac{18 \times 21}{45 \times 84} = \frac{450 - N_F}{-90 - N_F} \Rightarrow N_F$ = speed of arm = 400. 9 rpm CW

Now consider gears A, B and E:

$r_E = r_A + 2r_B \Rightarrow T_E = T_A + 2T_B$; $T_E = 18 + 2 \times 45 = 108$

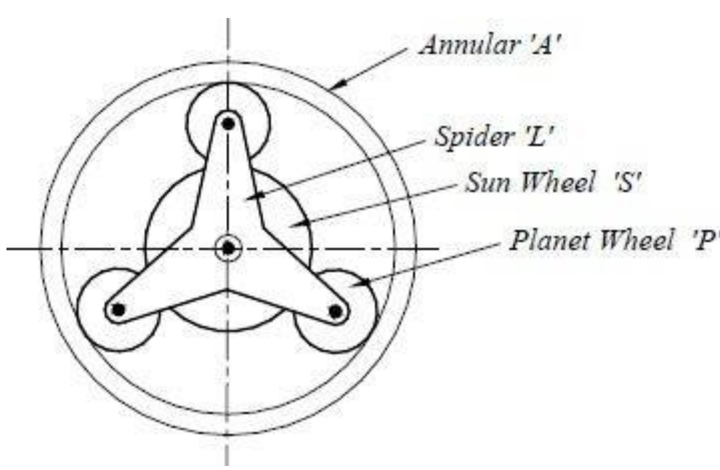
Gears A and E rotates in the opposite directions:

$\text{Train Value} = -\frac{T_A}{T_E}$

$\text{Also Train Value} = \frac{N_E - N_F}{N_A - N_F}$

$-\frac{T_A}{T_E} = \frac{N_E - N_F}{N_A - N_F}$; $-\frac{18}{108} = \frac{N_E - 400.9}{-90 - 400.9} \Rightarrow N_E$ = 482. 72 rpm CW

Problem 3: In an epicyclic gear of sun and planet type shown in figure, the pitch circle diameter of the annular wheel A is to be nearly 216mm and module 4mm. When the annular ring is stationary, the spider that carries three planet wheels P of equal size, to make, one revolution for every five revolution of the driving spindle carrying the sun wheel. Determine the number of teeth for all the wheels and the exact pitch circle diameter of the annular wheel. If an input torque of 20 N-m is applied to the spindle carrying the sun wheel, determine the fixed torque on the annular wheel.



Solution: Module being the same for all the meshing gears:

$T_A = T_S + 2T_P$

$T_A = \frac{\text{PCD of A}}{m} = \frac{216}{4} = 54$ teeth

Operation	*Spider arm L*	Sun Wheel S T_S	Planet wheel P T_P	Annular wheel A $T_A = 54$
Arm L is fixed & Sun wheel *S* is given +1 revolution	0	+1	$-\frac{T_S}{T_P}$	$-\frac{T_S}{T_P}\times\frac{T_P}{T_A} = -\frac{T_S}{T_A}$
Multiply by *m* (*S* rotates through *m* revolution)	*0*	*m*	$-\frac{T_S}{T_P}m$	$-\frac{T_S}{T_A}m$
Add *n* revolutions to all elements	*n*	*m+n*	$n-\frac{T_S}{T_P}m$	$n-\frac{T_S}{T_A}m$

If L rotates +1 revolution: so n = 1 (1)

The sun wheel S to rotate +5 revolutions correspondingly:

n + m = 5 (2)

From (1) and (2) m = 4

When A is fixed:

$$n - \frac{T_S}{T_A}m = 0 \Rightarrow T_A = 4T_S \therefore T_S = \frac{54}{4} = 13.5 \text{ teeth}$$

But fractional teeth are not possible; therefore T_S should be either 13 or 14 and T_A correspondingly 52 and 56.

Trial 1: Let $T_A = 52$ and $T_S = 13$

$$T_P = \frac{T_A - T_S}{2} = \frac{52-13}{4} = 19.5 \text{ teeth} \quad \text{not practical}$$

Trial 2: Let $T_A = 56$ and $T_S = 14$

$$T_P = \frac{T_A - T_S}{2} = \frac{56-14}{4} = 21 \text{ teeth} \quad \text{acceptable}$$

Therefore, $\mathbf{T_A = 56, T_S = 14}$ and $\mathbf{T_P = 21}$

PCD of A = 56 × 4 = **224 mm**

Also, torque on

$$T_S \times N_S = \text{torque on } L \times N_L = 20 \times \frac{5}{1} = 100 \text{ N} - \text{m}$$

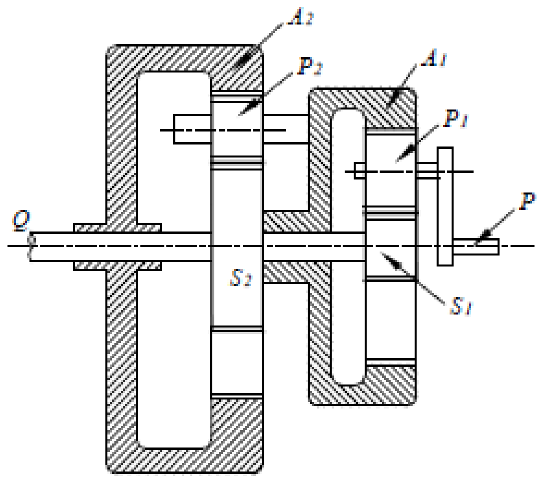
Holding torque on A $= T_L - T_S = 100 - 20 = 80$ N − m

Problem 4: Figure 5 shows a compound epicyclic gear train, gears S_1 and S_2 being rigidly attached to the shaft Q. If the shaft P rotates at 1000 rpm clockwise, while the annular A_2 is driven in counter clockwise direction at 500 rpm, determine the speed and direction of rotation of shaft Q. The number of teeth in the wheels are $S_1 = 24$; $S_2 = 40$; $A_1 = 100$; $A_2 = 120$.

Solution: Consider the gear train PA_1S_1:

If A_1 is fixed: n+ m; gives **n = - m**

$$\frac{N_L}{N_S} = \frac{n}{n + \frac{25}{6}n} = \frac{6}{31} \therefore N_L = \frac{6}{31}N_{S1}$$



Operation	*Arm P*	*A_1 (100)*	*S_1 (24)*
Arm P is fixed & wheel A_1 is given +1 revolution	0	+1	$+\frac{100}{P_1}\times-\frac{P_1}{24}$ $=-\frac{25}{6}$
Multiply by m (A_1 rotates through m revolution)	*0*	*+m*	$-\frac{25}{6}m$
Add n revolutions to all elements	*n*	*n+ m*	$n-\frac{25}{6}m$

OR

Operation	*Arm P*	*A_1 (100)*	*S_1 (24)*
Arm P is fixed & wheel A_1 is given -1 revolution	0	-1	$-\frac{A_1}{P_1}\times-\frac{P_1}{S_1}$ $=+\frac{A_1}{S_1}$
	0	-1	$\frac{100}{24}=\frac{25}{6}$
Add +1 revolutions to all elements	+1	0	$\frac{25}{6}+1=\frac{31}{6}$

Now consider whole gear train:

Operation	*A_1 (100)*	*A_2 (120)*	S_1 (24), S_2 (40) and Q	Arm P
A_1 is fixed & wheel A_2 is given +1 revolution	0	+1	$+\frac{120}{P_2}\times-\frac{P_2}{40}$ $=-3$	$-3\times\frac{6}{31}$ $=-\frac{18}{31}$
Multiply by m (A_1 rotates through m revolution)	*0*	*+m*	$-3m$	$-\frac{18}{31}m$
Add n revolutions to all elements	*n*	*n+ m*	$n-3m$	$n-\frac{18}{31}m$

Arm P makes 1000 rpm: $n-\frac{18}{31}N=1000$ (i)

And A_2 makes − 500 rpm: $n+N=-500$ (ii)

from (i) and (ii):

$-500-N-\frac{18}{31}N=1000$; $\therefore m=-949$ rpm and $n=949-500=449$ rpm

$N_Q=n-3N=449-(3\times-949)=3296$ rpm