Unit-4

Vector differentiation

Scalar and vector pt fns, vector operator. del, del applies to scalar point fns, gradient, DD, del applied fto vecto " " , div and curl, vector identities.

Vector function:- It is of the form $\overline{A} = a_1\overline{i} + a_2\overline{j} + a_3\overline{k}$ where a_1, a_2, a_3 function is said to be vector function. Eg:- $\overline{A} = t^2\overline{i} + t\overline{j} - \overline{k}$

Scalar function:-

Any real function is said to be scalar function. Eg: xyz, $\sin t$, t^3

Partial derivative:- consider $\overline{F}(t) = f_1(t)\overline{i} + f_2(t)\overline{j} + f_3(t)\overline{k}$ change dependent variable w.r.to any one of independent variable is said to be Partial variable, it is denoted by $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$.

operations on vector function.

Vector differential operator:

It is denoted by ∇ (del or Nabla). & defined as $\nabla = \overline{i}\frac{\partial}{\partial x} + \overline{j}\frac{\partial}{\partial y} + \overline{k}\frac{\partial}{\partial z}$

Laplacian operator:- It is denoted by ∇^2 & defined as $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

Differential operator on scalar function:-

Consider a scalar function $\phi(x, y, z)$. The gradient of ϕ (or) grad ϕ (or) $\nabla\phi$ is defined as $\nabla\phi = \overline{i}\frac{\partial\phi}{\partial x} + \overline{j}\frac{\partial\phi}{\partial y} + \overline{k}\frac{\partial\phi}{\partial z}$

Problems:-

1. Find grad of the function $f = x^2 - y^2 + 2z^2$

$$\text{grad } f = \vec{i}\,\frac{\partial f}{\partial x} + \vec{j}\,\frac{\partial f}{\partial y} + \vec{k}\,\frac{\partial f}{\partial z}$$

$$= \vec{i}(2x) + \vec{j}(-2y) + \vec{k}(4z)$$

2. Find grad f for $f = xy^2 + yz^2 + zx^2$

$$\text{grad } f = \vec{i}\,(y^2 + 2zx) + \vec{j}\,(2xy + z^2) + \vec{k}\,(2yz + x^2)$$

3. Find grad f for $f = xy + 2yz - 8$

$$\text{grad } f = (y)\vec{i} + (x + 2z)\vec{j} + (2y)\vec{k}$$

4. If $\phi = x^3 + y^3 + z^3 - 3xyz$. Find $\nabla\phi$ at $(1,1,1)$

$$\nabla\phi = (3x^2 - 3yz)\vec{i} + (3y^2 - 3xz)\vec{j} + (3z^2 - 3xy)\vec{k}$$

at $(1,1,1)$

$$\nabla\phi = 0\vec{i} + 0\vec{j} + 0\vec{k}$$

$$= 0 \,//$$

5. If $\phi = \log(x^2+y^2+z^2)$ Find $\nabla\phi$ at $(1,1,1)$

$$\nabla\phi = \frac{1}{x^2+y^2+z^2} 2x\vec{i} + \frac{1}{x^2+y^2+z^2} 2y\vec{j} + \frac{1}{x^2+y^2+z^2} 2z\vec{k}$$

at $(1,1,1)$

$$= \frac{2}{3}\vec{i} + \frac{2}{3}\vec{j} + \frac{2}{3}\vec{k}$$

6. Find grad f for the function $f = x+y+z$

$$\nabla f = \vec{i} + \vec{j} + \vec{k}$$

Normal vector

It is given by Normal vector of $f = |\nabla\phi|$

1). Find normal vector to the surface $x^2+y^2+2z^2=26$ at $(2,2,3)$

soln: Let $\phi = x^2+y^2+2z^2-26$

$$\nabla\phi = 2x\vec{i} + 2y\vec{j} + 4z\vec{k}$$

At $(2,2,3)$

$$= 4\vec{i} + 4\vec{j} + 12\vec{k}$$

$$|\nabla\phi| = \sqrt{16+16+144} = \sqrt{276} =$$

2. Find normal vector to the surface $z = x^2+y^2$ at $(-1,-2,5)$

Let $\phi = z - x^2 - y^2$

$$\nabla\phi = -2x\vec{i} - 2y\vec{j} + \vec{k}$$

At $(-1,-2,5)$

$= 2\vec{i} + 4\vec{j} + \vec{k}$

$|\nabla\phi| = \sqrt{4+16+1} = \sqrt{21}$ //

③ unit normal vector :- unit normal vector of f is given by $\frac{\nabla f}{|\nabla f|}$

1) Find unit normal vector to the surface $z = x^2 + y^2$ at $(-1,-2,5)$

Soln: Let $f = z - x^2 - y^2$

$\nabla f = -2x\vec{i} - 2y\vec{j} + \vec{k}$

At $(-1,-2,5)$, $= 2\vec{i} + 4\vec{j} + \vec{k}$

$|\nabla f| = \sqrt{4+16+1} = \sqrt{21}$

$\therefore$ unit vector of $f = \frac{\nabla f}{|\nabla f|} = \frac{2\vec{i}+4\vec{j}+\vec{k}}{\sqrt{21}}$ //

2. Find unit normal vector to the surface $x^2 + y^2 + 2z^2 = 26$ at $(2,2,3)$

Soln: Let $\phi = x^2 + y^2 + 2z^2 - 26$

$\nabla\phi = 2x\vec{i} + 2y\vec{j} + 4z\vec{k}$

At $(2,2,3)$, $= 4\vec{i} + 4\vec{j} + 12\vec{k}$ $\quad |\nabla\phi| = \sqrt{16+16+144} = \sqrt{276}$

unit vector $= \frac{4\vec{i}+4\vec{j}+12\vec{k}}{\sqrt{276}}$

3. Find unit normal for z

Angle b/w 2 surfaces:- $\cos\theta = \frac{\nabla f \cdot \nabla g}{|\nabla f| |\nabla g|}$

1. Find angle b/w the surfaces $x^2+y^2+z^2=9$ and $z = x^2+y^2-3$ at $(2,-1,2)$

let $f = x^2+y^2+z^2-9$, $g = z - x^2 - y^2 + 3$

$\nabla f = 2x\vec{i} + 2y\vec{j} + 2z\vec{k}$

At $(2,-1,2)$

$= 4\vec{i} - 2\vec{j} + 4\vec{k}$

$\nabla g = -2x\vec{i} - 2y\vec{j} + \vec{k}$

At $(2,-1,2)$

$= -4\vec{i} + 2\vec{j} + \vec{k}$

$|\nabla f| = \sqrt{16+4+16}$
$= \sqrt{36}$
$= 6$

$|\nabla g| = \sqrt{16+4+1}$
$= \sqrt{21}$

$\therefore \cos\theta = \frac{-16-\cancel{4}+\cancel{4}}{\sqrt{21}} = \frac{-16}{21}$ //

2. ~~x^2~~ $xy^2 = 3x+z^2$, $3x^2-y^2+2z=4$ at $(1,-2,1)$

Soln:

Let $f = xy^2 - 3x - z^2$, $g = 3x^2 - y^2 + 2z - 4$

$\nabla f = \cancel{y^2 \vec{i}}\ (y^2-3)\vec{i} - 2xy\vec{j} + (-2z)\vec{k}$

At $(1,-2,1)$

$\nabla f = \vec{i} + 4\vec{j} - 2\vec{k}$

$|\nabla f| = \sqrt{1+16+4}$
$= \sqrt{21}$

$\nabla g = 6x\vec{i} - 2y\vec{j} + 2\vec{k}$

At $(1,-2,1)$

$\nabla g = 6\vec{i} + 4\vec{j} + 2\vec{k}$

$|\nabla g| = \sqrt{36+16+4}$
$= \sqrt{56}$

$\therefore \cos\theta = \frac{6+16-4}{\sqrt{21}\sqrt{56}} = \frac{18}{\sqrt{21}\sqrt{56}}$

Directional Derivative:-

The D.D of f in the direction of $\vec{a}$ $= \nabla f \cdot \frac{\vec{a}}{|\vec{a}|}$

1. Find DD of $f(x,y,z) = xy + yz + zx$ in the direction of the vector $\vec{i} + 2\vec{j} + 2\vec{k}$ at $(1,2,0)$

Soln:

$\nabla f = (y+z)\vec{i} + (x+z)\vec{j} + (x+y)\vec{k}$

at $(1,2,0)$

$= 2\vec{i} + \vec{j} + 3\vec{k}$

Gn $\vec{a} = \vec{i} + 2\vec{j} + 2\vec{k}$

$|\vec{a}| = \sqrt{1+4+4} = \sqrt{9} = 3$

$\therefore$ D.D $= (2\vec{i} + \vec{j} + 3\vec{k}) \cdot \frac{\vec{i} + 2\vec{j} + 2\vec{k}}{3}$

$= \frac{1}{3}(2+2+6) = 10/3$ //

2. Find DD of $f = 2xy + z^2$ at $(1,-1,3)$ in the direction of $\vec{i} + 2\vec{j} + 3\vec{k}$.

$f = 2xy + z^2$

$\nabla f = 2y\vec{i} + 2x\vec{j} + 2z\vec{k}$

$\vec{a} = \vec{i} + 2\vec{j} + 3\vec{k}$

at $(1,-1,3)$

$|\vec{a}| = \sqrt{1+4+9}$

$= -2\vec{i} + 2\vec{j} + 6\vec{k}$

$= \sqrt{14}$

D.D $= \nabla f \cdot \frac{\vec{a}}{|\vec{a}|}$

$= (-2\vec{i} + 2\vec{j} + 6\vec{k}) \cdot \frac{(\vec{i} + 2\vec{j} + 3\vec{k})}{\sqrt{14}}$

$= \frac{1}{14}(-2+4+18)$

$= \frac{20}{14} = \frac{10}{7}$ //

3. Find DD of $f(x,y,z) = 2x^2+3y^2+z^2$ at $(2,1,3)$ in the direction of $\vec{i}-2\vec{k}$.

$$f = 2x^2+3y^2+z^2$$

$$\nabla f = 4x\vec{i}+6y\vec{j}+2z\vec{k}$$

$\vec{a} = \vec{i}-2\vec{k}$

$|\vec{a}| = \sqrt{1+4} = \sqrt{5}$

At $(2,1,3)$, $\nabla f = 8\vec{i}+6\vec{j}+6\vec{k}$

$$D.D = \nabla f \cdot \frac{\vec{a}}{|\vec{a}|}$$

$$= (8\vec{i}+6\vec{j}+6\vec{k}) \cdot \frac{\vec{i}-2\vec{k}}{\sqrt{5}}$$

$$= \frac{1}{5}\left[8-12\right] = 4/5$$

4. Find DD of $f = x^2+y^2+2z^2$ at the point $P(1,2,3)$ in the direction of $\vec{PQ}$ where $Q=(5,0,4)$

$$f = x^2+y^2+2z^2$$

$\vec{PQ} = \vec{OQ}-\vec{OP}$

$$\nabla f = 2x\vec{i}+2y\vec{j}+4z\vec{k}$$

$= (5,0,4)-(1,2,3)$

At $(1,2,3)$

$= (4,-2,1)$

$$\nabla f = 2\vec{i}+4\vec{j}+12\vec{k}$$

$\vec{a} = 4\vec{i}-2\vec{j}+\vec{k}$

$|\vec{a}| = \sqrt{16+4+1} = \sqrt{21}$

$$D.D = \frac{(2\vec{i}+4\vec{j}+12\vec{k})\cdot(4\vec{i}-2\vec{j}+\vec{k})}{\sqrt{21}}$$

$$= \frac{1}{\sqrt{2}}(8-8+12) = \frac{12}{\sqrt{2}}$$

5. ~~Find DD of $f = x^2-y^2+2z^2$ along $\vec{PQ}$ where $P=(1,2,3)$, $Q=(5,0,4)$~~ X

Divergence of a vector:-

If $\bar{f}$ be any continuously differentiable vector point function then,

$\operatorname{div}\bar{f} = \bar{i}\cdot\frac{\partial \vec{f}}{\partial x} + \bar{j}\cdot\frac{\partial \bar{f}}{\partial y} + \bar{k}\cdot\frac{\partial f}{\partial z}$ is called divergence of a vector

curl of a vector:-

Let $\vec{f}$ be any vector point fn and

$\operatorname{curl} F = \vec{i}\times\frac{\partial f}{\partial x} + j\times\frac{\partial f}{\partial y} + k\times\frac{\partial f}{\partial z}$ or $\begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ f_1 & f_2 & f_3 \end{vmatrix}$.

solenoidal vector:-

A vector point function $\vec{f}$ is said to be solenoidal vector if $\operatorname{div} F = 0$

Irrorational vector:-

A vector point function $\bar{f}$ is said to be irrotational if $\operatorname{curl}\bar{f} = 0$.

1. Find $\operatorname{div}\bar{F}$ for $\bar{F} = x^2yz\bar{i} + xy^2z\bar{j} + xyz^2\bar{k}$

Soln:

$\operatorname{div}F = \left(\bar{i}\frac{\partial}{\partial x} + \bar{j}\frac{\partial}{\partial y} + \bar{k}\frac{\partial}{\partial z}\right)(x^2yz\vec{i} + xy^2zj + xyz^2\vec{k})$

$= \frac{\partial}{\partial x}(x^2yz) + \frac{\partial}{\partial y}(xy^2z) + \frac{\partial}{\partial z}(xyz^2)$

$= 2xyz + 2xyz + 2xyz = 6xyz$ //

2. Find div $\bar{F}$, $\bar{F} = x^2\bar{i} + y^2\bar{j} + z^2\bar{k}$

$\operatorname{div}\vec{F} = \cancel{2x\,\bar{i}\,\partial}\; 2x + 2y + 2z$ //

3. Find div $\bar{F}$ for $\bar{F} = xy^2\bar{i} + 2x^2yz\bar{j} - 3yz^2\bar{k}$, $(1,-1,1)$

$\operatorname{div}\bar{F} = y^2 + 2x^2z - 6yz$ //

at $(1,-1,1)$ $= 1 + 2 + 6 = 9$

4 a) Find div $\bar{F} = x\bar{i} + y\bar{j} + z\bar{k}$, ~~b)~~

soln: 3 //

5. If $\phi = 2x^3y^2z^4$. Find div(grad ϕ)

$\operatorname{grad}\phi = 6x^2y^2z^4\,\vec{i} + 4x^3yz^4\,\vec{j} + 8x^3y^2z^3\,\vec{k}$

$\operatorname{div}\operatorname{grad}\phi = \frac{\partial}{\partial x}(6x^2y^2z^4) + \frac{\partial}{\partial y}(4x^3yz^4) + \frac{\partial}{\partial z}(8x^3y^2z^3)$

$= 12xy^2z^4 + 4x^3z^4 + 24x^3yz^2$ //

6. Find curl $\bar{F}$ for $\bar{F} = 2xz^2\bar{i} - yz\bar{j} + 3xz^3\bar{k}$

$$\text{curl } \vec{F} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 2xz^2 & -yz & 3xz^3 \end{vmatrix} = \vec{i}[0+y] - \vec{j}[3z^3 - 4xz] + \vec{k}[0 - \tfrac{0}{xz}]$$

$$= y\vec{i} - (3z^3 - 4xz)\vec{j} - \cancel{2xz}\overset{0}{}\vec{k}$$

7. Find curl $\bar{F}$, $\bar{F} = z\bar{i} + x\bar{j} + y\bar{k}$

$$\text{curl } \vec{F} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ z & x & y \end{vmatrix} = \vec{i}(1) - \vec{j}(-1) + \bar{k}(1)$$

$$= \vec{i} + \vec{j} + \vec{k}$$

8. S.t curl $\bar{r} = 0$ where $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$

$$\text{curl } \vec{r} = \begin{vmatrix} \vec{i} & \bar{j} & \bar{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x & y & z \end{vmatrix} = \vec{i}(0-0) - \vec{j}(0) + \bar{k}(0)$$

$$= 0$$

9. Find div $\bar{F}$, curl $\bar{F}$ where $\bar{F} = \text{grad}(x^3 + y^3 + z^3 - 3xyz)$

Soln:

$$\vec{f} = \text{grad}(x^3 + y^3 + z^3 - 3xyz)$$

$$= \vec{i}\frac{\partial}{\partial x}(\ \text{"}\ ) + \vec{j}\frac{\partial}{\partial y}(\ \text{"}\ ) + \bar{k}\frac{\partial}{\partial z}(\ \text{"}\ )$$

$$\vec{f} = \vec{i}(3x^2 - 3yz) + \vec{j}(3y^2 - 3xz) + \bar{k}(3z^2 - 3xy)$$

$$\text{grad } \vec{f} = \cancel{\vec{i}(6x) + \vec{j}(6y) + \bar{k}(6z)}$$

$$= 6x\vec{i} + 6y\vec{j} + 6z\vec{k}$$

$$\text{div } \vec{F} = 6x + 6y + 6z$$

$$\text{curl } \vec{F} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 3x^2-yz & 3y^2-xz & 3z^2-xy \end{vmatrix}$$

$$= \vec{i}\,(-x+x) + \vec{j}\,(-y+y) + \bar{k}\,(-z+z)$$

$$= 0 //$$

⑩ Find $\text{div}\,\bar{F}$, $\text{curl}\,\bar{F}$ for $\bar{F} = (x^2-y^2)\,\bar{i} + 4xy\,\bar{j} + (x^2-xy)\,\bar{k}$

$$\text{div}\,\vec{F} = 2x + 4x + 0$$

$$= 6x$$

$$\text{curl}\,F = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2-y^2 & 4xy & x^2-xy \end{vmatrix}$$

$$= \vec{i}\,(-x-0) - \vec{j}\,(2x-y) + \bar{k}\,(4y+2y)$$

$$= -x\,\vec{i} - (2x-y)\,\vec{j} + 6y\,\vec{k} //$$

⑪ Find $\text{curl}\,\bar{F}$, $\text{div}\,\bar{F}$ where $\bar{F} = (x^2+yz)\,\bar{i} + (y^2+zx)\,\bar{j} + (z+xy)\,\bar{k}$

$$\text{div}\,\vec{F} = 2x + 2y + 1$$

$$\text{curl}\,\vec{F} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2+yz & y^2+zx & z+xy \end{vmatrix}$$

$$= \vec{i}\,[x-x] + \vec{j}\,[y-y] + \vec{k}\,[z-z]$$

$$= 0 //$$

(12) S.t $\bar{F} = (y+z)\bar{i} + (z+x)\bar{j} + (x+y)\bar{k}$ is irrotational

$$\text{curl } \vec{F} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ y+z & z+x & x+y \end{vmatrix}$$

$= \bar{i}(0) + \bar{j}(0) + \bar{k}(0)$

$= 0.$ $\therefore$ $\vec{F}$ is irrotational.

(13) S.t $\bar{F} = 3y^4z^2\bar{i} + z^3x^2\bar{j} - 3x^2y^2\bar{k}$ is solenoidal

$\text{div } \vec{F} = 0$

$\therefore$ $\vec{F}$ is solenoidal //.

(14) S.t find P, $\bar{F} = (x+3y)\bar{i} + (y-2z)\bar{j} + (x+Pz)$ is solenoidal.

$\text{div } \vec{F} = 1+1+P$

$0 = 2+P$

$\boxed{P = -2}$

Fi

(15) S.t $V = (\sin y + z)\bar{i} + (x\cos y - z)\bar{j} + (x-y)\bar{k}$ is irrotational.

$\text{curl } \vec{F} = 0$ $\therefore$ $\vec{F}$ is irrotational.

(16) Find a, b, c if $\bar{F} = (2x+3y+az)\bar{i} + (bx+2y+3z)\bar{j} + (2x+cy+3z)\bar{k}$ is irrotational. $a=2, b=3, c=3$

Theorems! -

1. Evaluate $\nabla\left(\frac{\bar{r}}{r^3}\right)$ where $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$, $r = |\bar{r}|$

let $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$

$r = |\bar{r}| = \sqrt{x^2+y^2+z^2}$

$r^2 = x^2+y^2+z^2$

diff P.w.r.to x

$\cancel{2}r\frac{\partial r}{\partial x} = \cancel{2}x$

$\frac{\partial r}{\partial x} = \frac{x}{r}$, $\frac{\partial r}{\partial y} = \frac{y}{r}$, $\frac{\partial r}{\partial z} = \frac{z}{r}$

No. $\nabla\left(\frac{\bar{r}}{r^3}\right) = \left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right)\left(\frac{x\bar{i}+y\bar{j}+z\bar{k}}{r^3}\right)$

$\frac{\partial}{\partial x}\left(\frac{x}{r^3}\right) + \frac{\partial}{\partial y}\left(\frac{y}{r^3}\right) + \frac{\partial}{\partial z}\left(\frac{z}{r^3}\right)$

$= \Sigma \frac{\partial}{\partial x}\left(\frac{x}{r^3}\right) = \Sigma \frac{r^3 - 3r^2\frac{x^2}{r}}{r^6} = \Sigma \frac{r^3 - 3x^2 r}{r^6}$

$\Sigma\left(\frac{1}{r^3} - \frac{3x^2}{r^5}\right)$

$= \left(\frac{1}{r^3} + \frac{1}{r^3} + \frac{1}{r^3}\right) - \frac{3}{r^5}(x^2+y^2+z^2)$

$= \frac{3}{r^3} - \frac{3}{r^5} r^2$

$= \frac{3}{r^3} - \frac{3}{r^3} = 0$ //

2. ~~Find S.t~~ Find curl $\bar{F}$, $\bar{F} = r^n \bar{r}$, where $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$, $r = |\bar{r}|$

$$\text{curl } \bar{F} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ r^n x & r^n y & r^n z \end{vmatrix}$$

$= \Sigma \bar{i}\left[z n r^{n-1}\frac{y}{r} - y n r^{n-1}\frac{z}{r}\right]$

$= 0$ //

3. S.t $\operatorname{curl}(r^n \bar{r}) = 0$

$$r^n \bar{r} = r^n x \bar{i} + r^n y \bar{j} + r^n z \bar{k}$$

$$\operatorname{curl} r^n \vec{r} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ r^n x & r^n y & r^n z \end{vmatrix}$$

$$= 0 //$$

4. S.t $\nabla^2\left(\frac{1}{r}\right) = 0$

5. S.t $\nabla^2(\log r) = \frac{1}{r^2}$

6. S.t $\nabla r^n = n r^{n-2} \bar{r}$, where $\bar{r} = r = x\bar{i} + y\bar{j} + z\bar{k}$

$$\vec{r} = x\bar{i} + y\bar{j} + z\bar{k}$$

$$\frac{\partial r}{\partial x} = \frac{x}{r}, \quad \frac{\partial r}{\partial y} = \frac{y}{r}, \quad \frac{\partial r}{\partial z} = \frac{z}{r}$$

$$\nabla r^n = \sum \bar{i} \frac{\partial}{\partial x}(r^n)$$

$$= \sum \bar{i}\, n r^{n-1} \frac{\partial r}{\partial x}$$

$$= \sum \bar{i}\, n r^{n-1} \frac{x}{r}$$

$$= \sum \bar{i}\, n r^{n-2} x$$

$$= n r^{n-2} \sum x\bar{i}$$

$$= \cancel{n r^{n-2}}\; n r^{n-2} \bar{r} //$$

(14) Soln:-

$\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$

$|\bar{r}| = r^2 = x^2+y^2+z^2$

D.P. w.r.to x

$\cancel{2}r\frac{\partial r}{\partial x} = \cancel{2}x$

$\frac{\partial r}{\partial x} = \frac{x}{r}$

$||^{ly}$ $\frac{\partial r}{\partial y} = \frac{y}{r}$

$\frac{\partial r}{\partial z} = \frac{z}{r}$

$\nabla^2\left(\frac{1}{r}\right) = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\left(\frac{1}{r}\right)$

$= \Sigma \frac{\partial^2}{\partial x^2}\left(\frac{1}{r}\right)$

$= \Sigma \frac{\partial}{\partial x} \frac{-1}{r^2}\left(\frac{\partial r}{\partial x}\right)$

$= \Sigma \frac{\partial}{\partial x} \frac{-1}{r^2} \frac{x}{r}$

$= \Sigma \frac{\partial}{\partial x} \frac{-x}{r^3}$

$= \Sigma - \left[\frac{r^3(1) - x\, 3r^2 \frac{\partial r}{\partial x}}{r^6}\right]$

$= -\Sigma \frac{r^3 - 3r^2 x \frac{x}{r}}{r^6}$

$= -\Sigma\left(\frac{1}{r^3} - \frac{3rx^2}{r^6}\right)$

$= -\left(\frac{1}{r^3} - \frac{3rx^2}{r^6} + \frac{1}{r^3} - \frac{3ry^2}{r^6} + \frac{1}{r^3} - \frac{3rz^2}{r^6}\right)$

$= -\left[\frac{3}{r^3} - \frac{3r(x^2+y^2+z^2)}{r^6}\right]$

$= -\left[\frac{3}{r^3} - \frac{3r^3}{r^6}\right] = 0$ //