FINANCIAL DERIVATIVES

Chapter 1: Intro.. to Derivatives

- **What is a derivative?**
  - A financial instrument that has a value derived from the value of something else

Chapter 1: Intro. to Derivatives

- **Uses of Derivatives**
  - Risk management
    - o Hedging (e.g. farmer with corn forward)
  - Speculation
    - o Essentially making bets on the price of something
  - Reduced transaction costs
    - o Sometimes cheaper than manipulating cash portfolios
  - Regulatory arbitrage
    - o Tax loopholes, etc

Chapter 1: Intro. to Derivatives

- **Perspectives on Derivatives**
  - The end-user
    - o Use for one or more of the reasons above
  - The market-maker
    - o Buy or sell derivatives as dictated by end users
    - o Hedge residual positions
    - o Make money through bid/offer spread
  - The economic observer
    - o Regulators, and other high-level participants

Chapter 1: Intro. to Derivatives

- **Financial Engineering and Security Design**
  - Financial engineering
    - The construction of a given financial product from other products
    - o Market-making relies upon manufacturing payoffs to hedge risk
    - o Creates more customization opportunities
    - o Improves intuition about certain derivative products because they are similar or equivalent to something we already understand
  - Enables regulatory arbitrage

Chapter 1: Intro. to Derivatives

- **The Role of the Financial Markets**
  - Financial markets impact the lives of average people all the time, whether they realize it or not
    - o Employer's prosperity may be dependent upon financing rates
    - o Employer can manage risk in the markets
    - o Individuals can invest and save
    - o Provide diversification
    - o Provide opportunities for risk-sharing/insurance
    - o Bank sells off mortgage risk which enables people to get mortgages

Chapter 1: Intro. to Derivatives

- **Risk-Sharing**
  - Markets enable risk-sharing by pairing up buyers and sellers
    - Even insurance companies share risk
      - Reinsurance
      - Catastrophe bonds
  - Some argue that even more risk-sharing is possible
    - Home equity insurance
    - Income-linked loans
    - Macro insurance
  - Diversifiable risk vs. non-diversifiable risk
    - Diversifiable risk can be easily shared
  - Non-diversifiable risk can be held by those willing to bear it and potentially earn a profit by doing so

Chapter 1: Intro. to Derivatives

- **Derivatives in Practice**
  - Growth in derivatives trading
    - o The Intro.duction of derivatives in a given market often coincides with an increase in price risk in that market (i.e. the need to manage risk isn't prevalent when there is no risk)
  - Volumes are easily tracked in exchange-traded securities, but volume is more difficult to transact in the OTC market

Chapter 1: Intro. to Derivatives

- **Derivatives in Practice**
  - How are derivatives used?
    - o Basic strategies are easily understood
    - o Difficult to get information concerning:
      - o What fraction of perceived risk do companies hedge
      - o Specific rationale for hedging
  - Different instruments used by different types of firms

Chapter 1: Intro. to Derivatives

- **Buying and Short-Selling Financial Assets**
  - Buying an asset
    - Bid/offer prices
  - Short-selling
    - Short-selling is a way of borrowing money; sell asset and collect money, ultimately buy asset back ("covering the short")
    - Reasons to short-sell:
      - Speculation
      - Financing
      - Hedging
    - Dividends (and other payments required to be made) are often referred to as the "lease rate"
    - Risk and scarcity in short-selling:
      - Credit risk (generally requires collateral)
      - Scarcity

Chapter 2: Intro. to Forwards / Options

- **Forward Contracts**
  - A forward contract is a binding agreement by two parties for the purchase/sale of a specified quantity of an asset at a specified future time for a specified future price

Chapter 2: Intro. to Forwards / Options

- **Forward Contracts**
  - Spot price
  - Forward price
  - Expiration date
  - Underlying asset
  - Long or short position
  - Payoff
  - No cash due up-front

Chapter 2: Intro. to Forwards / Options

- **Gain/Loss on Forwards**
  - Long position:
    - The payoff to the long is S – F
    - The profit is also S – F (no initial deposit required)

  - Short position:
    - The payoff to the short is F – S
    - The profit is also F – S (no initial deposit required)

Chapter 2: Intro. to Forwards / Options

- Comparing an outright purchase vs. purchase through forward contract
  - Should be the same once the time value of money is taken into account

Chapter 2: Intro. to Forwards / Options

- **Settlement of Forwards**
  - Cash settlement
  - Physical delivery

Chapter 2: Intro. to Forwards / Options

- **Credit risk in Forwards**
  - Managed effectively by the exchange
  - Tougher in OTC transactions

Chapter 2: Intro. to Forwards / Options

- **Call Options**
  - The holder of the option owns the right but not the obligation to purchase a specified asset at a specified price at a specified future time

Chapter 2: Intro. to Forwards / Options

- **Call option terminology**
  - Premium
  - Strike price
  - Expiration
  - Exercise style (European, American, Bermudan)
  - Option writer

Chapter 2: Intro. to Forwards / Options
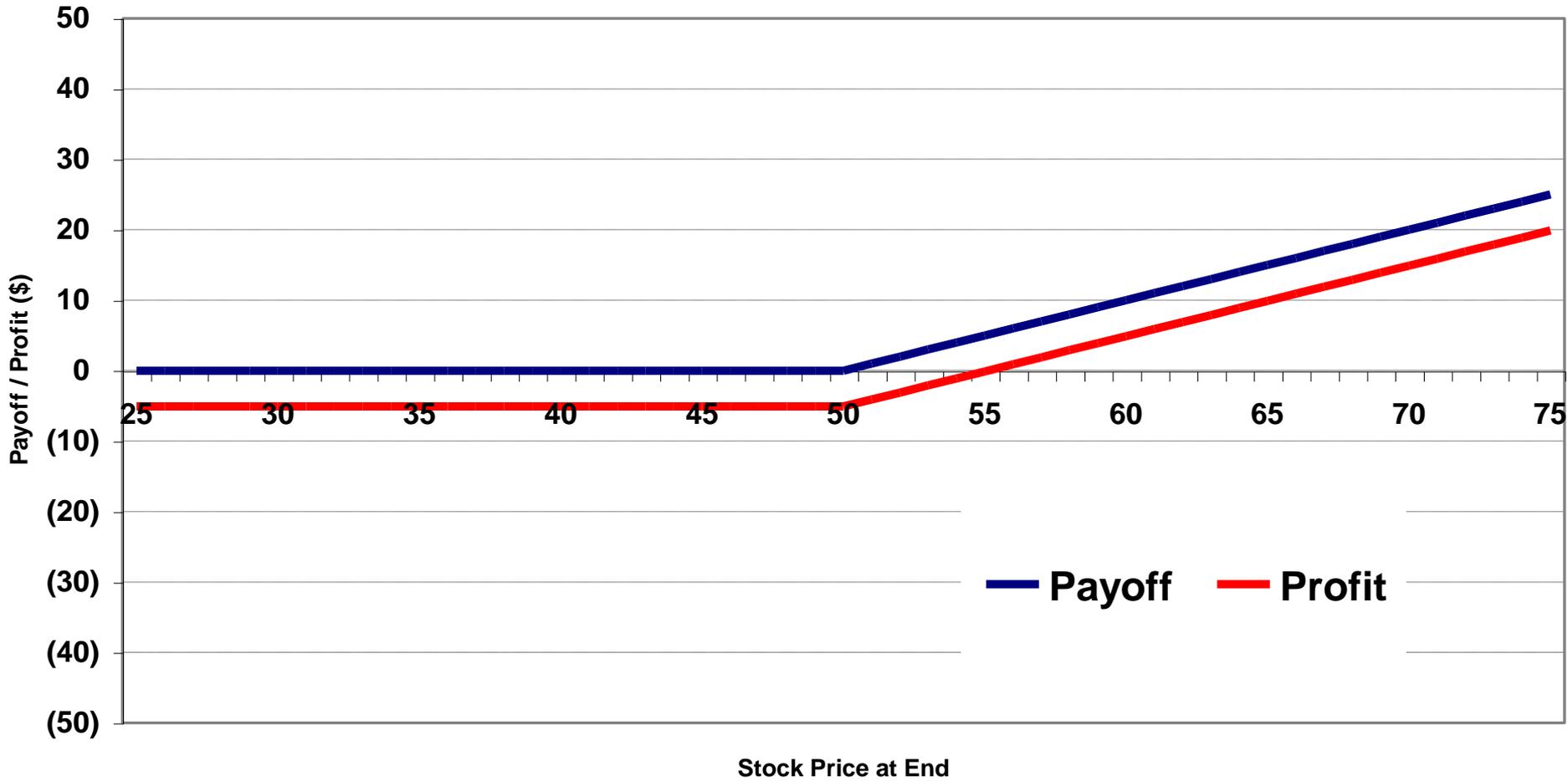
- **Call option economics**
  - For the long:
    - Call payoff = max(0, S-K)
    - Call profit = max(0, S-K) – future value of option premium

  - For the writer (the short):
    - Call payoff = -max(0, S-K)
    - Call profit = -max(0, S-K) + future value of option premium

Payoff and Profit for Long Call

Chapter 2: Intro. to Forwards / Options

- **Put Options**
  - The holder of the option owns the right but not the obligation to sell a specified asset at a specified price at a specified future time

Chapter 2: Intro. to Forwards / Options

- **Put option terminology**
  - Premium
  - Strike price
  - Expiration
  - Exercise style (European, American, Bermudan)
  - Option writer

Chapter 2: Intro. to Forwards / Options

- **Put option economics**
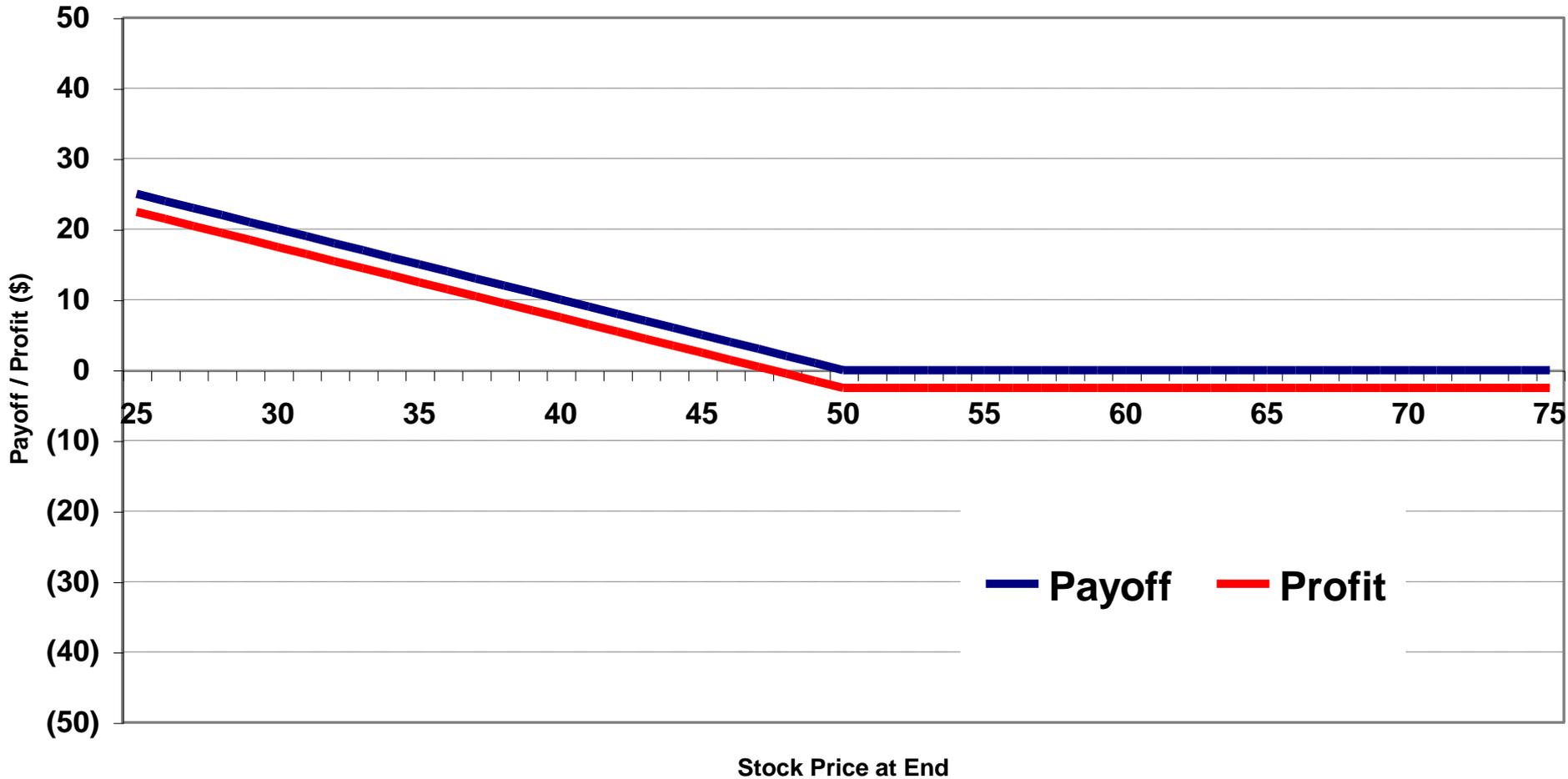 - For the long:
    - Put payoff = max(0, K-S)
    - Put profit = max(0, K-S) – future value of option premium

  - For the writer (the short):
    - Put payoff = -max(0, K-S)
    - Put profit = -max(0, K-S) + future value of option premium

Payoff and Profit for Long Put

Chapter 2: Intro. to Forwards / Options

- **Moneyness terminology for options:**
  - In the Money ("ITM")
  - Out of the money ("OTM")
  - At the money ("ATM ")

Chapter 2: Intro. to Forwards / Options

Summary of Forward and Option Positions

Position	Max Loss	Max Gain
Long forward	-Forward price	Unlimited
Short forward	Unlimited	Forward price
Long call	-FV(premium)	Unlimited
Short call	Unlimited	FV(premium)
Long put	-FV(premium)	Strike – FV(premium)
Short put	FV(premium) - Strike	FV(premium)

Chapter 2: Intro. to Forwards / Options

- **Options are Insurance**
  - Homeowner's insurance is a put option
    - o Pay premium, get payoff if house gets wrecked (requires that we assume that physical damage is the only thing that can affect the value of the home)
  - Often people assume insurance is prudent and options are risky, but they must be considered in light of the entire portfolio, not in isolation (e.g. buying insurance on your neighbor's house is risky)
  - Calls can also provide insurance against a rise in the price of something we plan to buy

Chapter 2: Intro. to Forwards / Options

- **Financial Engineering: Equity-Linked CD Example**
  - 3yr note
  - Price of 3yr zero is 80
  - Price of call on equity index is 25
  - Bank offers ROP + 60% participation in the index growth

Chapter 2: Intro. to Forwards / Options

- **Other issues with options**
  - Dividends
    - o The OCC may make adjustments to options if stocks pay "unusual" dividends
    - o Complicate valuation since stock generally declines by amount of dividend
  - Exercise
    - o Cash settled options are generally automatic exercise
    - o Otherwise must provide instructions by deadline
    - o Commission usually paid upon exercise
    - o Might be preferable to sell option instead
    - o American options have additional considerations
  - Margins for written options
    - o Must post when writing options
  - Taxes

Exercise 2.4(a)

- **You enter a long forward contract at a price of 50. What is the payoff in 6 months for prices of $40, $45, $50, $55?**
  - $40 - 50 = -10$
  - $45 - 50 = -5$
  - $50 - 50 = 0$
  - $55 - 50 = 5$

Exercise 2.4(b)

- **What about the payoff from a 6mo call with strike price 50. What is the payoff in 6 months for prices of $40, $45, $50, $55?**
  - Max(0, 40 − 50) = 0
  - Max(0, 45 − 50) = 0
  - Max(0, 50 − 50) = 0
  - Max(0, 55 − 50) = 5

Exercise 2.4(c)

- **Clearly the price of the call should be more since it never underperforms the long forward and in some cases outperforms it**

Exercise 2.9(a)

- **Off-market forwards (cash changes hands at inception)**
  - Suppose 1yr rate is 10%
  - $S(0) = 1000$
  - Consider 1y forwards
  - Verify that if $F = 1100$ then the profit diagrams are the same for the index and the forward
    - Profit for index $= S(1) - 1000(1.10) = S(1) - 1100$
    - Profit for forward $= S(1) - 1100$

Exercise 2.9(b)

- **Off-market forwards (cash changes hands at inception)**
  - What is the "premium" of a forward with price 1200
    - o Profit for forward = S(1) − 1200
    - o Rewrite as S(1) −1100 − 100
    - o S(1) − 1100 is a "fair deal" so it requires no premium
    - o The rest is an obligation of $100 payable in 1 yr
    - o The buyer will need to receive 100 / 1.10 = 90.91 up-front

Exercise 2.9(c)

- **Off-market forwards (cash changes hands at inception)**
  - What is the "premium" of a forward with price 1000
    - o Profit for forward = S(1) − 1000
    - o Rewrite as S(1) −1100 + 100
    - o S(1) − 1100 is a "fair deal" so it requires no premium
    - o The rest is a payment of $100 receivable in 1 yr
    - o This will cost 100 / 1.10 = 90.91 to fund

Chapter 3: Options Strategies

- **Put/Call Parity**
  - Assumes options with same expiration and strike

	S(T) < K	S(T) > K
Portfolio 1		
Long Stock	S(T)	S(T)
Long Put	K - S(T)	0
Total	**K**	**S(T)**
Portfolio 2		
Long Call	0	S(T) - K
PV(K) in Cash	K	K
Total	**K**	**S(T)**

* Portfolios must have the same price!!

Chapter 3: Options Strategies

- **Put/Call Parity**
  - So for a non-dividend paying asset, $S + p = c + PV(K)$

Chapter 3: Options Strategies

- **Insurance Strategies**
  - Floors: long stock + long put
  - Caps: short stock + long call
  - Selling insurance
    - o Covered writing, option overwriting, selling a covered call
    - o Naked writing

Chapter 3: Options Strategies

- **Synthetic Forwards**
  - Long call + short put = long forward
  - Requires up-front premium (+ or -), price paid is option strike, not forward price

Chapter 3: Options Strategies

- **Spreads and collars**
  - Bull spreads (anticipate growth)
  - Bear spreads (anticipate decline)
  - Box spreads
    - o Using options to create synthetic long at one strike and synthetic short at another strike
    - o Guarantees a certain cash flow in the future
    - o The price must be the PV of the cash flow (no risk)
  - Ratio spreads
    - o Buy m options at one strike and selling n options at another
  - Collars
    - o Long collar = buy put, sell call (call has higher price)
    - o Can create a zero-cost collar by shifting strikes

Chapter 3: Options Strategies

- **Speculating on Volatility**
  - Straddles
    - o Long call and long put with same strike, generally ATM strikes
  - Strangle
    - o Long call and long put with spread between strikes
    - o Lower cost than straddle but larger move required for breakeven
  - Butterfly spreads
    - o Buy protection against written straddle, or sell wings of long straddle

Exercise 3.9

- **Option pricing problem**
  - S(0) = 1000
  - F = 1020 for a six-month horizon
  - 6mo interest rate = 2%
  - Subset of option prices as follows:

Strike	Call	Put
950	120.405	51.777
1000	93.809	74.201
1020	84.470	84.470

  - Verify that long 950-strike call and short 1000-strike call produces the same profit as long 950-strike put and short 1000-strike put

Exercise 3.9

		Time 6mos		
	Time 0	**S(T) < 950**	**S(T) > 1000**	**Else**
Long 950 call	-120.405	0	S(T) - 950	S(T) - 950
Short 1000 call	93.809	0	-(S(T) - 1000)	0
Total	**-26.596**	0	50	S(T) - 950
With Interest	**-27.128**	**0**	**50**	**S(T) - 950**
Profit		**-27.128**	**22.872**	**S(T) - 977.128**

		Time 6mos		
	Time 0	**S(T) < 950**	**S(T) > 1000**	**Else**
Long 950 put	-51.777	950 - S(T)	0	0
Short 1000 put	74.201	-(1000-S(T))	0	-(1000-S(T))
Total	**22.424**	-50	0	-(1000-S(T))
With Interest	**22.872**	**-50**	**0**	**-(1000-S(T))**
Profit		**-27.128**	**22.872**	**S(T) - 977.128**

Chapter 4: Risk Management

- **Risk management**
  - Using derivatives and other techniques to alter risk and protect profitability

Chapter 4: Risk Management

- **The Producer's Perspective**
  - A firm that produces goods with the goal of selling them at some point in the future is exposed to price risk
  - Example:
    - Gold Mine
    - Suppose total costs are $380
    - The producer effectively has a long position in the underlying asset
    - Unhedged profit is $S - 380$

Chapter 4: Risk Management

- **Potential hedges for producer**
  - Short forward
  - Long put
  - Short call (maybe)
  - Can tweak hedges by adjusting "insurance"
    - o Lower strike puts
    - o Sell off some upside

Chapter 4: Risk Management

- **The Buyer's Perspective**
  - Exposed to price risk
  - Potential hedges:
    - o Long forward
    - o Call option
    - o Sell put (maybe)

Chapter 4: Risk Management

- **Why do firms manage risk?**
  - As we saw, hedging shifts the distribution of dollars received in various states of the world
  - But assuming derivatives are fairly priced and ignoring frictions, hedging does not change the expected value of cash flows
  - So why hedge?

Chapter 4: Risk Management

Company produces for $10, can sell for either 11.20 or 9

	Price = 9	**Price = 11.20**
Pre-tax income	-1	1.20
Taxable income	0	1.20
Tax (40%)	0	0.48
After-tax income	-1	0.72

Expected after-tax profit = -0.14

Chapter 4: Risk Management

Suppose company hedges with forward contract F =10.10

	Price = 9	**Price = 11.20**
Pre-tax income	-1	1.20
Gain on short forward	1.10	-1.10
Taxable income	0.10	0.10
Tax (40%)	0.04	0.04
After-tax income	**0.06**	**0.06**

Because of differential tax treatment between gains and losses, hedging has actually increased the expected value of future cash flows

Chapter 4: Risk Management

- **Reasons to hedge:**
  - Taxes
    - o Treatment of losses
    - o Capital gains taxation (defer taxation of capital gains)
    - o Differential taxation across countries (shift income across countries)
  - Bankruptcy and distress costs
  - Costly external financing
  - Increase debt capacity
    - o Reducing riskiness of future cash flows may enable the firm to borrow more money
  - Managerial risk aversion
  - Nonfinancial risk management
    - o Incorporates a series of decisions into the business strategy

Chapter 4: Risk Management

- **Reasons not to hedge:**
  - Transactions costs in derivatives
  - Requires derivatives expertise which is costly
  - Managerial controls
  - Tax and accounting consequences

Chapter 4: Risk Management

- **Empirical evidence on hedging**
  - FAS133 requires derivatives to be bifurcated and marked to market (but doesn't necessarily reveal alot about hedging activity)
  - Tough to learn alot about hedging activity from public info
  - General findings
    - o About half of nonfinancial firms use derivatives
    - o Less than 25% of perceived risk is hedged
    - o Firms with more investment opportunities more likely to hedge
    - o Firms using derivatives have higher MVs and more leverage

Chapter 5: Forwards and Futures

- **Alternative Ways to Buy a Stock**
  - Outright purchase (buy now, get stock now)
  - Fully leveraged purchase (borrow money to buy stock now, repay at T)
  - Prepaid forward contract (buy stock now, but get it at T)
  - Forward contract (pay for and receive stock at T)

Chapter 5: Forwards and Futures

- **Prepaid Forwards**
  - Prepaid forward price on stock = today's price (if no dividends)
  - Prepaid forward price on stock = today' price – PV of future dividends:

$$S_0 - \sum_{i=1}^{n} P(0, t_i) \cdot D_{t_i}$$

Chapter 5: Forwards and Futures

– For prepaid forwards on an index, assume the dividend rate is δ, then the dividend paid in any given day is δ/365 x S

  o If we reinvest the dividend into the index, one share will grow to more than one share over time

  o Since indices pay dividends on a large number of days it is a reasonable approximation to assume dividends are reinvested continuously

  o Therefore one share grows to exp(δT) shares by time T

  o So the price of a prepaid forward contract on an index is

$$S_0 e^{-\delta T}$$

Chapter 5: Forwards and Futures

- **Forwards**
  - The forward price is just the future value of the prepaid forward price

  - Discrete or no dividends: $F_{0,T} = S_0 e^{rT} - \sum_{i=1}^{n} e^{r(T-t_i)} D_{t_i}$

  - Continuous dividends: $F_{0,T} = S_0 e^{(r-\delta)T}$

Chapter 5: Forwards and Futures

- **Other definitions**

- Forward premium:
$$\frac{F_{0,T}}{S_0}$$

- Annualized forward premium:
$$\frac{1}{T}\ln\left(\frac{F_{0,T}}{S_0}\right)$$

Chapter 5: Forwards and Futures

Synthetic Forwards

Transaction	Time 0 Cash Flows	Time T Cash Flows
Buy $e^{-\delta T}$ units of the index	$-S_0 e^{-\delta T}$	S_T
Borrow $S_0 e^{-\delta T}$	$S_0 e^{-\delta T}$	$-S_0 e^{(r-\delta)T}$
Total	0	$S_T - S_0 e^{(r-\delta)T}$

And so Forward = Stock – zero-coupon bond

Chapter 5: Forwards and Futures

- **Theoretically arbitrage is possible if the forward price is too high or too low relative to the stock/bond combination:**
  - If forward price is too high, sell forward and buy stock (cash-and-carry arbitrage)
  - If forward price is too low, buy forward and sell stock (reverse-cash-and-carry arbitrage)

Chapter 5: Forwards and Futures

- **No-Arbitrage Bounds with Transaction Costs**
  - In practice there are transactions costs, bid/offer spreads, different interest rates depending on whether borrowing or lending, and the possibility that buying or selling the stock will move the market
  - This means that rather than a specific forward price, arbitrage will not be possible when the forward price is inside of a certain range

Chapter 5: Forwards and Futures

Assume some notation:

- Stock prices are S^b and S^a
- Forward prices are F^b and F^a
- Interest rates for borrowing and lending are r^b and r^l

Fixed commission of k to execute forward or buy stock

Chapter 5: Forwards and Futures

Derive F^- (the trader believes forward price is too low)

- Go long the forward contract at a price of F

- Short stock, receive $S^b - k$

- Invest cash, it grows to $\left(S^b - k\right)e^{r^l T}$

Arbitrage if $F < F^- = \left(S^b - k\right)e^{r^l T}$

Chapter 5: Forwards and Futures

Does the Forward Price Predict the Future Price?

Forward price is $S_0(1+r)$

Expected future value of the stock is $S_0(1+\alpha)$

Difference is $S_0(\alpha - r)$

Chapter 5: Forwards and Futures

- **An Interpretation of the Forward Pricing Formula**

  - "Cost of carry" is r-δ since that is what it would cost you to borrow money and buy the index

  - The "lease rate" is δ

  - Interpretation of forward price = spot price + interest to carry asset – asset lease rate

Chapter 5: Forwards and Futures

- **Futures Contracts**
  - Basically exchange-traded forwards
  - Standardized terms
  - Traded electronically or via open outcry
  - Clearinghouse matches buys and sells, keeps track of clearing members
  - Positions are marked-to-market daily
    - o Leads to difference in the prices of futures and forwards
  - Liquid since easy to exit position
  - Mitigates credit risk
  - Daily price limits and trading halts

Chapter 5: Forwards and Futures

- **S&P 500 Futures**
  - Multiplier of 250
  - Cash-settled contract
  - Notional = contracts x 250 x index price
  - Open interest = total number of open positions (every buyer has a seller)
  - Costless to transact (apart from bid/offer spread)
  - Must maintain margin; margin call ensues if margin is insufficient
  - Amount of margin required varies by asset and is based upon the volatility of the underlying asset

Chapter 5: Forwards and Futures

- Since futures settle every day rather than at the end (like forwards), gains/losses get magnified due to interest/financing:

  - If rates are positively correlated with the futures price then the futures price should be higher than the forward price

  - Vice versa if the correlation is negative

Chapter 5: Forwards and Futures

- **Arbitrage in Practice**
  - Textbook examples demonstrates the uncertainties associated with index arbitrage:
    - o What interest rate to use?
    - o What will future dividends be?
    - o Transaction costs (bid/offer spreads)
    - o Execution and basis risk when buying or selling the index

Chapter 5: Forwards and Futures

- **Quanto Index Contracts**
  - Some contracts allow investors to get exposure to foreign assets without taking currency risk; this is referred to as a *quanto*
  - Pricing formulas do not apply, more work needs to be done to get those prices

Chapter 5: Forwards and Futures

- **Daily marking to market of futures has the effect of magnifying gains and losses**
  - If we desire to use futures to hedge a cash position in the underlying instrument, matching notionals is not sufficient:
    - o A $1 change in the asset price will result in a $1 change in value for the cash position but a change in value of exp(rT) for the futures
    - o Therefore we need fewer futures contracts to hedge the cash position
    - o We need to multiple the notional by to account for the extra volatility

Exercise 5.10(a)

- **Index price is 1100**
- **Risk-free rate is 5% continuous**
- **9m forward price = 1129.257**
- **What is the dividend yield implied by this price?**

$$1129.257 = 1100e^{(.05-\delta).75}$$

$$\ln\left(\frac{1129.257}{1100}\right) = (.05-\delta).75$$

$$\delta = .05 - \frac{\ln\left(\dfrac{1129.257}{1100}\right)}{0.75} = 1.5\%$$

Exercise 5.10(b)

- If we though the dividend yield was going to be only 0.5% over the next 9 months, what would we do?

$$F^* = 1100e^{(.05-.005).75} = 1137.76$$

- Forward price is too low relative to our view

- Buy forward price, short stock

- In 9 months, we will have 1100*exp(.05(.75)) = 1142.033

- Buy back our short for 1129.257

- We are left with 12.7762 to pay dividends

Chapter 8: Swaps

- The examples in the previous chapters showed examples of pricing and hedging single cash flows that were to take place in the future
  - But it may be the case that payment streams are expected in the future, as opposed to single cash flows
    - o One possible solution is to execute a series of forward contracts, one corresponding to each cash flow that is to be received
  - A swap is a contract that calls for an exchange of payments over time; it provides a means to hedge a stream of risky cash flows

Chapter 8: Swaps

- Consider this example in which a company needs to buy oil in 1 year and then again in 2 years
  - The forward prices of oil are 20 and 21 respectively

Chapter 8: Swaps

Pricing of Swap   **Swap Level Payment**   20.483

Time	Discount Rate	Discount Factor	Forward Price	Prepaid Swap PV	Buyer Swap PV
0					
1	6.000%	0.9434	20	18.87	19.324
2	6.500%	0.8817	21	18.51	18.059
			Total	**37.383**	**37.383**

Chapter 8: Swaps

Example of Swap Cash Flows   **Swap Pmnt**   20.483

Time	Realized Oil Price	Buyer Payment	Ctpy Payment	Net CF
0				
1	25	20.48	25.00	4.52
2	18	20.48	18.00	(2.48)

*Cash flows are on a per-barrel basis; in actuality these would be multiplied by the notional amount

The swap price is not $20.50 (the average of the forward prices) since the cash flows are made at different times and therefore is a time-value-of-money component. The equivalency must be on a PV basis and not an "absolute dollars" basis

Chapter 8: Swaps

- The counterparty to the swap will typically be a dealer
  - In the dealer's ideal scenario, they find someone else to take the other side of the swap; i.e. they find someone who wishes to sell the oil at a fixed price in the swap, and match buyer and seller (price paid by buyer is higher than price received by the seller, the dealer keeps the difference)
  - Otherwise the dealer must hedge the position
    - o The hedge must consist of both price hedges (the dealer is short oil) and interest rate hedges

Chapter 8: Swaps

Consider the dealer's position after a price hedge but before an interest rate hedge:

Year	Payment from Oil Buyer	Long Forward	Net Cash Flow
1	$20.483 – Oil Price	Oil Price – 20	$0.483
2	$20.483 – Oil Price	Oil Price - 21	-$0.517

Chapter 8: Swaps

- **The Market Value of a Swap**
  - Ignoring commissions and bid/offer spreads, the market value of a swap is zero at inception (that is why no cash changes hands)
  - The swap consists of a strip of forward contracts and an implicit interest rate loan, all of which are executed at fair market levels

Chapter 8: Swaps

- **But the value of the swap will change after execution:**
  - Oil prices can change
  - Interest rates can change
  - Swap has level payments which are fair in the aggregate; however after the first payment is made this balance will be disturbed

Chapter 8: Swaps

Swap Market Value at Inception   Swap Level Payment   20.483

Time	Discount Rate	Discount Factor	Forward Price	Buyer Payment	Net CF to Buyer	PV of Net CF
0						
1	6.000%	0.9434	20	20.483	(0.483)	(0.456)
2	6.500%	0.8817	21	20.483	0.517	0.456
						0.000

Chapter 8: Swaps

Swap Market Value after Oil Prices Rise   Swap Level Payment   20.483

Time	Discount Rate	Discount Factor	Forward Price	Buyer Payment	Net CF to Buyer	PV of Net CF
0						
1	6.000%	0.9434	22	20.483	1.517	1.431
2	6.500%	0.8817	23	20.483	2.517	2.219
						3.650

Chapter 8: Swaps

- **Interest rate swaps**
  - Interest rate swaps are similar to the commodity swap examples described above, except that the pricing is based solely upon the levels of interest rates prevailing in the market. They are used to hedge interest rate exposure
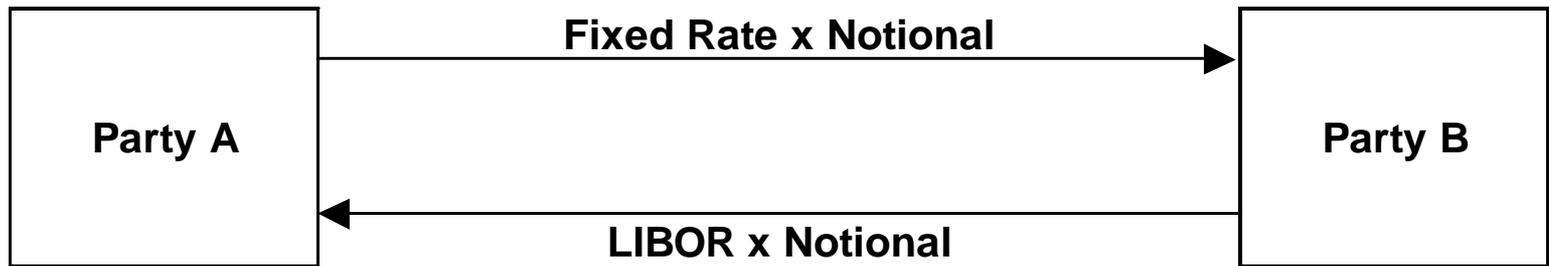
Chapter 8: Swaps

- **LIBOR**
    - LIBOR stands for "London Interbank Offered Rate" and is a composite view of interest rates required for borrowing and lending by large banks in London
    - LIBOR are the floating rates most commonly referenced by an interest rate swap

Chapter 8: Swaps

- ## **Interest rate swap schematic**

  A typical interest rate swap is one in which Part A pays a fixed rate to Party B and receives a floating rate (to be paid by Party B)

Party A	→ Fixed Rate x Notional →	Party B
	← LIBOR x Notional ←	

The amount of time for which the arrangement holds is called the swap term or tenor.

Chapter 8: Swaps

Computing the Swap Rate

Time	Zero-Coupon Yield	Discount Factor	Forward Rate (t-1,t)	Net Pmnt Received	PV of Net CF
0					
1	6.000%	0.9434	6.000%	R - 6.0000%	0.9434 x (R - 6.0000%)
2	6.500%	0.8817	7.002%	R - 7.0024%	0.8817 x (R - 7.0024%)
3	7.000%	0.8163	8.007%	R - 8.0071%	0.8163 x (R - 8.0071%)
				Total	**0.000%**

The fair swap rate satisfies

$$0.9434[R - 6\%] + 0.8817[R - 7.0024\%] + 0.8163[R - 8.0071\%] = 0$$

Chapter 8: Swaps

Computing the Swap Rate   **Swap Rate**   6.9548%

Time	Zero-Coupon Yield	Discount Factor	Forward Rate (t-1,t)	Fixed Rate Payment	Net Pmnt Received	PV of Net CF
0						
1	6.000%	0.9434	6.000%	6.9548%	-0.9548%	-0.9008%
2	6.500%	0.8817	7.002%	6.9548%	0.0476%	0.0419%
3	7.000%	0.8163	8.007%	6.9548%	1.0523%	0.8590%

                                                                                        0.000%

Chapter 8: Swaps

In general we can see that the swap rate is the rate that satisfies:

$$\sum_{i=1}^{n} P(0, t_i)\left[R - r_0(t_{i-1}, t_i)\right] = 0$$

Chapter 8: Swaps

Can be rewritten as

$$R = \frac{\sum_{i=1}^{n} P(0, t_i) \cdot r_0(t_{i-1}, t_i)}{\sum_{i=1}^{n} P(0, t_i)}$$

Which can be again rewritten as

$$R = \sum_{i=1}^{n} \left[\frac{P(0, t_i)}{\sum_{j=1}^{n} P(0, t_j)} \right] r_0(t_{i-1}, t_i)$$

Chapter 8: Swaps

Computing the Swap Rate - Weighted Average Formula

Time	Zero-Coupon Yield	Discount Factor	Forward Rate (t-1,t)	Weight of Forward	Weight x Forward
0					
1	6.000%	0.9434	6.000%	35.72%	2.143%
2	6.500%	0.8817	7.002%	33.38%	2.337%
3	7.000%	0.8163	8.007%	30.90%	2.475%
Sum		**2.6414**		**100.00%**	**6.9548%**

Chapter 8: Swaps

- One more way to write the swap rate

Recall that the annual forward rate is calculated such that

$$P(0, t_2) = P(0, t_1) \cdot \frac{1}{1 + r_0(t_1, t_2)}$$

This means that $r_0(t_1, t_2) = \dfrac{P(0, t_1)}{P(0, t_2)} - 1$

Chapter 8: Swaps

$$\sum_{i=1}^{n} P(0,t_i)\left[R - r_0(t_{i-1},t_i)\right]$$

$$= \sum_{i=1}^{n} P(0,t_i)\left[R - \left(\frac{P(0,t_{i-1})}{P(0,t_i)} - 1\right)\right]$$

$$= \sum_{i=1}^{n} \left[R \cdot P(0,t_i) - P(0,t_{i-1}) + P(0,t_i)\right]$$

$$= \sum_{i=1}^{n} R \cdot P(0,t_i) - \sum_{i=1}^{n} P(0,t_{i-1}) + \sum_{i=1}^{n} P(0,t_i)$$

$$= \sum_{i=1}^{n} R \cdot P(0,t_i) - 1 + P(0,t_n) = 0$$

Chapter 8: Swaps

Therefore

$$\sum_{i=1}^{n} R \cdot P(0, t_i) + P(0, t_n) = 1$$

• The swap rate is just the par rate on a fixed bond

• In fact the swap can be viewed as the exchange of a fixed rate bond for a floating rate bond

Chapter 8: Swaps

- **The Swap Curve**
  - The Eurodollar futures contract is a futures contract on 3m LIBOR rates
  - It can used to infer all the values of R for up to 10 years, and therefore it is possible to calculate fixed swap rates directly from this curve
  - The difference between a swap rate and a Treasury rate for a given tenor is known as a swap spread

Chapter 8: Swaps

- **Swap implicit loan balance**
  - In an upward sloping yield curve the fixed swap rate will be lower than forward short-term rates in the beginning of the swap and higher than forward short-term rates at the end of the swap
  - Implicitly therefore, the fixed rate payer is lending money in the beginning of the swap and receiving it back at the end

Chapter 8: Swaps

- **Deferred swaps**
  - Also known as forward-starting swaps, these are swaps that do not begin until k periods in the future

$$R = \frac{\sum_{i=k}^{n} P(0,t_i) \cdot r_0(t_{i-1}, t_i)}{\sum_{i=k}^{n} P(0,t_i)}$$

Chapter 8: Swaps

- **Why Swap Interest Rates?**
  - Swaps permit the separation of interest rate and credit risk
  - A company may want to borrow at short-term interest rates but it may be unable to do that in enough size
  - Instead it can issue long-term bonds and swap debt back to floating, financing its borrowing at short-term rates

Chapter 8: Swaps

- **Amortizing and Accreting Swaps**
  - These are just swaps where the notional value declines (amortizing) or expands (accreting) over time

$$R = \frac{\sum_{i=1}^{n} Q_{t_i} \cdot P(0, t_i) \cdot r_0(t_{i-1}, t_i)}{\sum_{i=1}^{n} Q_{t_i} \cdot P(0, t_i)}$$

Exercise 8.2(a,b)

- Interest rates are 6%, 6.5%, and 7% for years 1, 2, and 3
- Forward oil prices are 20, 21, and 22 respectively
- What is the 3yr swap price?
- What is the 2yr swap price beginning in 1 year?

Exercise 8.2(a)

Swap Payment $ 20.95

Time	Discount Rate	Discount Factor	Forward Price	Forward Weight	Net CF	Net PV CF
0						
1	6.00%	0.943	20.00	35.72%	(0.95)	(0.90)
2	6.50%	0.882	21.00	33.38%	0.05	0.04
3	7.00%	0.816	22.00	30.90%	1.05	0.86
Sum		2.641				0.000

Exercise 8.2(b)

Swap Payment $ 21.48

Time	Discount Rate	Discount Factor	Forward Price	Forward Weight	Net CF	Net PV CF
0						
1	6.00%	0.943	20.00	0.00%		
2	6.50%	0.882	21.00	51.92%	(0.48)	(0.42)
3	7.00%	0.816	22.00	48.08%	0.52	0.42
Sum		**1.698**				**(0.000)**