1. Define viscosity? And derive an expression for coefficient of viscosity.

▶ 1.3 VISCOSITY

Viscosity is defined as the property of a fluid which offers resistance to the movement of one layer of fluid over another adjacent layer of the fluid. When two layers of a fluid, a distance '*dy*' apart, move one over the other at different velocities, say *u* and *u* + *du* as shown in Fig. 1.1, the viscosity together with relative velocity causes a shear stress acting between the fluid layers.

The top layer causes a shear stress on the adjacent lower layer while the lower layer causes a shear stress on the adjacent top layer. This shear stress is proportional to the rate of change of velocity with respect to *y*. It is denoted by symbol τ (Tau).

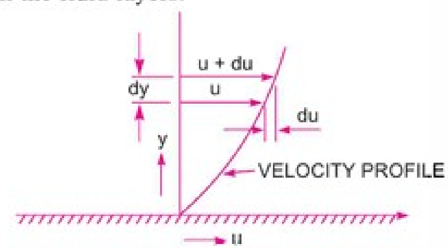

Fig. 1.1 *Velocity variation near a solid boundary.*

Mathematically, $\tau \propto \dfrac{du}{dy}$

or $$\tau = \mu \frac{du}{dy} \qquad ...(1.2)$$

where μ (called *mu*) is the constant of proportionality and is known as the co-efficient of dynamic viscosity or only viscosity. $\dfrac{du}{dy}$ represents the rate of shear strain or rate of shear deformation or velocity gradient.

From equation (1.2), we have $$\mu = \frac{\tau}{\left(\dfrac{du}{dy}\right)} \qquad ...(1.3)$$

Thus viscosity is also defined as the shear stress required to produce unit rate of shear strain.

1.3.1 Units of Viscosity. The units of viscosity is obtained by putting the dimensions of the quantities in equation (1.3)

$$\mu = \frac{\text{Shear stress}}{\dfrac{\text{Change of velocity}}{\text{Change of distance}}} = \frac{\text{Force/Area}}{\left(\dfrac{\text{Length}}{\text{Time}}\right) \times \dfrac{1}{\text{Length}}}$$

$$= \frac{\text{Force/(Length)}^2}{\dfrac{1}{\text{Time}}} = \frac{\text{Force} \times \text{Time}}{(\text{Length})^2}$$

In MKS system, force is represented by kgf and length by metre (m), in CGS system, force is represented by dyne and length by cm and in SI system force is represented by Newton (N) and length by metre (m).

$\therefore$ MKS unit of viscosity $= \dfrac{\text{kgf-sec}}{\text{m}^2}$

CGS unit of viscosity $= \dfrac{\text{dyne-sec}}{\text{cm}^2}$

In the above expression N/m^2 is also known as Pascal which is represented by Pa. Hence N/m^2 = Pa = Pascal

$\therefore$ SI unit of viscosity = Ns/m^2 = Pa s.

SI unit of viscosity $= \dfrac{\text{Newton-sec}}{\text{m}^2} = \dfrac{\text{Ns}}{\text{m}^2}$

The unit of viscosity in CGS is also called Poise which is equal to $\dfrac{\text{dyne-sec}}{\text{cm}^2}$.

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2. A plate 0.025 mm distant from a fixed plate, moves at 60 cm/s and requires a force of 2 N per unit area i.e., 2 N/m² to maintain this speed. Determine the fluid viscosity between the plates.

Solution. Given :

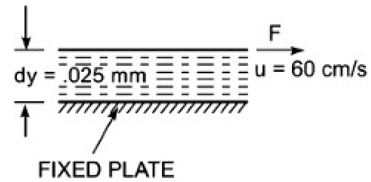


Fig. 1.3

Distance between plates, $dy = .025$ mm

$= .025 \times 10^{-3}$ m

Velocity of upper plate, $u = 60$ cm/s $= 0.6$ m/s

Force on upper plate, $F = 2.0 \dfrac{\text{N}}{\text{m}^2}$.

This is the value of shear stress *i.e.*, τ

Let the fluid viscosity between the plates is μ.

Using the equation (1.2), we have $\tau = \mu \dfrac{du}{dy}$.

where du = Change of velocity $= u - 0 = u = 0.60$ m/s

dy = Change of distance $= .025 \times 10^{-3}$ m

τ = Force per unit area $= 2.0 \dfrac{\text{N}}{\text{m}^2}$

$$\therefore \quad 2.0 = \mu \frac{0.60}{.025 \times 10^{-3}} \quad \therefore \quad \mu = \frac{2.0 \times .025 \times 10^{-3}}{0.60} = 8.33 \times 10^{-5} \frac{\text{Ns}}{\text{m}^2}$$

$= 8.33 \times 10^{-5} \times 10$ poise $=$ **8.33×10^{-4} poise. Ans.**

3.

The space between two square flat parallel plates is filled with oil. Each side of the plate is 60 cm. The thickness of the oil film is 12.5 mm. The upper plate, which moves at 2.5 metre per sec requires a force of 98.1 N to maintain the speed. Determine :

(i) the dynamic viscosity of the oil in poise, and

(ii) the kinematic viscosity of the oil in stokes if the specific gravity of the oil is 0.95.

Solution. Given :

Each side of a square plate $= 60$ cm $= 0.60$ m

$\therefore$ Area, $A = 0.6 \times 0.6 = 0.36$ m^2

Thickness of oil film, $dy = 12.5$ mm $= 12.5 \times 10^{-3}$ m

Velocity of upper plate, $u = 2.5$ m/sec

$\therefore$ Change of velocity between plates, $du = 2.5$ m/sec

Force required on upper plate, $F = 98.1$ N

$$\therefore \quad \text{Shear stress,} \quad \tau = \frac{\text{Force}}{\text{Area}} = \frac{F}{A} = \frac{98.1\ \text{N}}{0.36\,\text{m}^2}$$

(*i*) Let μ = Dynamic viscosity of oil

$$\text{Using equation (1.2),} \quad \tau = \mu \frac{du}{dy} \text{ or } \frac{98.1}{0.36} = \mu \times \frac{2.5}{12.5 \times 10^{-3}}$$

$$\therefore \qquad \mu = \frac{98.1}{0.36} \times \frac{12.5 \times 10^{-3}}{2.5} = 1.3635\ \frac{\text{Ns}}{\text{m}^2} \qquad \left(\because \frac{1\ \text{Ns}}{\text{m}^2} = 10\ \text{poise}\right)$$

$$= 1.3635 \times 10 = \mathbf{13.635\ poise.\ Ans.}$$

(*ii*) Sp. gr. of oil, $S = 0.95$

Let ν = kinematic viscosity of oil

Using equation (1.1*A*),

Mass density of oil, $\rho = S \times 1000 = 0.95 \times 1000 = 950\ \text{kg/m}^3$

Using the relation, $\nu = \frac{\mu}{\rho}$, we get $\nu = \frac{1.3635\left(\frac{\text{Ns}}{\text{m}^2}\right)}{950} = .001435\ \text{m}^2\text{/sec} = .001435 \times 10^4\ \text{cm}^2\text{/s}$

$$= \mathbf{14.35\ stokes.\ Ans.} \qquad (\because\ \text{cm}^2\text{/s} = \text{stoke})$$

4.

Find the kinematic viscosity of an oil having density 981 kg/m³. The shear stress at a point in oil is 0.2452 N/m² and velocity gradient at that point is 0.2 per second.

Solution. Given :

Mass density, $\rho = 981\ \text{kg/m}^3$

Shear stress, $\tau = 0.2452\ \text{N/m}^2$

Velocity gradient, $\frac{du}{dy} = 0.2\ \text{s}$

Using the equation (1.2), $\tau = \mu \frac{du}{dy}$ or $0.2452 = \mu \times 0.2$

$$\therefore \qquad \mu = \frac{0.2452}{0.200} = 1.226\ \text{Ns/m}^2$$

Kinematic viscosity ν is given by

$$\therefore \qquad \nu = \frac{\mu}{\rho} = \frac{1.226}{981} = .125 \times 10^{-2}\ \text{m}^2\text{/sec}$$

$$= 0.125 \times 10^{-2} \times 10^4\ \text{cm}^2\text{/s} = 0.125 \times 10^2\ \text{cm}^2\text{/s}$$

$$= 12.5\ \text{cm}^2\text{/s} = \mathbf{12.5\ stoke.\ Ans.} \qquad (\because\ \text{cm}^2\text{/s} = \text{stoke})$$

5.

The dynamic viscosity of an oil, used for lubrication between a shaft and sleeve is 6 poise. The shaft is of diameter 0.4 m and rotates at 190 r.p.m. Calculate the power lost in the bearing for a sleeve length of 90 mm. The thickness of the oil film is 1.5 mm.

Solution. Given :

Viscosity $\mu = 6$ poise

$$= \frac{6}{10}\frac{\text{N s}}{\text{m}^2} = 0.6\ \frac{\text{N s}}{\text{m}^2}$$

Dia. of shaft, $D = 0.4$ m

Speed of shaft, $N = 190$ r.p.m

Sleeve length, $L = 90\ \text{mm} = 90 \times 10^{-3}\ \text{m}$

Thickness of oil film, $t = 1.5\ \text{mm} = 1.5 \times 10^{-3}\ \text{m}$

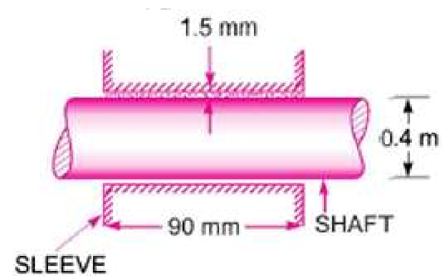


Fig. 1.5

Tangential velocity of shaft, $u = \frac{\pi D N}{60} = \frac{\pi \times 0.4 \times 190}{60} = 3.98\ \text{m/s}$

Using the relation $\tau = \mu \dfrac{du}{dy}$

where du = Change of velocity = $u - 0 = u = 3.98$ m/s

dy = Change of distance = $t = 1.5 \times 10^{-3}$ m

$$\tau = 10 \times \frac{3.98}{1.5 \times 10^{-3}} = 1592 \text{ N/m}^2$$

This is shear stress on shaft

∴ Shear force on the shaft, F = Shear stress × Area

$$= 1592 \times \pi D \times L = 1592 \times \pi \times .4 \times 90 \times 10^{-3} = 180.05 \text{ N}$$

Torque on the shaft, $T = \text{Force} \times \dfrac{D}{2} = 180.05 \times \dfrac{0.4}{2} = 36.01$ Nm

∴ *Power lost $= \dfrac{2\pi NT}{60} = \dfrac{2\pi \times 190 \times 36.01}{60}$ = **716.48 W. Ans.**

6.

Calculate the capillary rise in a glass tube of 2.5 mm diameter when immersed vertically in (a) water and (b) mercury. Take surface tensions $\sigma = 0.0725$ N/m for water and $\sigma = 0.52$ N/m for mercury in contact with air. The specific gravity for mercury is given as 13.6 and angle of contact = 130°.

Solution. Given :

Dia. of tube, $d = 2.5 \text{ mm} = 2.5 \times 10^{-3}$ m

Surface tension, σ for water $= 0.0725$ N/m

σ for mercury $= 0.52$ N/m

Sp. gr. of mercury $= 13.6$

∴ Density $= 13.6 \times 1000$ kg/m^3.

(*a*) **Capillary rise for water ($\theta = 0°$)**

Using equation (1.20), we get $h = \dfrac{4\sigma}{\rho \times g \times d} = \dfrac{4 \times 0.0725}{1000 \times 9.81 \times 2.5 \times 10^{-3}}$

= .0118 m = **1.18 cm. Ans.**

(*b*) **For mercury**

Angle of contact between mercury and glass tube, $\theta = 130°$

Using equation (1.21), we get $h = \dfrac{4\sigma \cos\theta}{\rho \times g \times d} = \dfrac{4 \times 0.52 \times \cos 130°}{13.6 \times 1000 \times 9.81 \times 2.5 \times 10^{-3}}$

= –.004 m = – **0.4 cm. Ans.**

The negative sign indicates the capillary depression.

7.

Calculate the capillary effect in millimetres in a glass tube of 4 mm diameter, when immersed in (i) water, and (ii) mercury. The temperature of the liquid is 20°C and the values of the surface tension of water and mercury at 20°C in contact with air are 0.073575 N/m and 0.51 N/m respectively. The angle of contact for water is zero and that for mercury is 130°. Take density of water at 20°C as equal to 998 kg/m³.

Solution. Given :

Dia. of tube, $d = 4 \text{ mm} = 4 \times 10^{-3} \text{ m}$

The capillary effect (*i.e.*, capillary rise or depression) is given by equation (1.20) as

$$h = \frac{4\sigma \cos\theta}{\rho \times g \times d}$$

where σ = surface tension in N/m

θ = angle of contact, and ρ = density

(*i*) **Capillary effect for water**

$\sigma = 0.073575$ N/m, $\theta = 0°$

$\rho = 998$ kg/m³ at 20°C

$$\therefore \quad h = \frac{4 \times 0.073575 \times \cos 0°}{998 \times 9.81 \times 4 \times 10^{-3}} = 7.51 \times 10^{-3} \text{ m} = \mathbf{7.51 \text{ mm. Ans.}}$$

(*ii*) **Capillary effect for mercury**

$\sigma = 0.51$ N/m, $\theta = 130°$ and

$\rho = \text{sp. gr.} \times 1000 = 13.6 \times 1000 = 13600$ kg/m²

$$\therefore \quad h = \frac{4 \times 0.51 \times \cos 130°}{13600 \times 9.81 \times 4 \times 10^{-3}} = -2.46 \times 10^{-3} \text{ m} = \mathbf{-2.46 \text{ mm. Ans.}}$$

The negative sign indicates the capillary depression.

Unit-II

Fluid Kinematics, Fluid Dynamics and Closed Conduit Flow

1 Discuss in detail of different types of fluid flows.

▶ 5.3 TYPES OF FLUID FLOW

The fluid flow is classified as :

(*i*) Steady and unsteady flows ;
(*ii*) Uniform and non-uniform flows ;
(*iii*) Laminar and turbulent flows ;
(*iv*) Compressible and incompressible flows ;
(*v*) Rotational and irrotational flows ; and
(*vi*) One, two and three-dimensional flows.

5.3.1 Steady and Unsteady Flows. Steady flow is defined as that type of flow in which the fluid characteristics like velocity, pressure, density, etc., at a point do not change with time. Thus for steady flow, mathematically, we have

$$\left(\frac{\partial V}{\partial t}\right)_{x_0, y_0, z_0} = 0, \left(\frac{\partial p}{\partial t}\right)_{x_0, y_0, z_0} = 0, \left(\frac{\partial \rho}{\partial t}\right)_{x_0, y_0, z_0} = 0$$

where (x_0, y_0, z_0) is a fixed point in fluid field.

Unsteady flow is that type of flow, in which the velocity, pressure or density at a point changes with respect to time. Thus, mathematically, for unsteady flow

$$\left(\frac{\partial V}{\partial t}\right)_{x_0, y_0, z_0} \neq 0, \left(\frac{\partial p}{\partial t}\right)_{x_0, y_0, z_0} \neq 0 \text{ etc.}$$

5.3.2 Uniform and Non-uniform Flows. Uniform flow is defined as that type of flow in which the velocity at any given time does not change with respect to space (*i.e.*, length of direction of the flow). Mathematically, for uniform flow

$$\left(\frac{\partial V}{\partial s}\right)_{t = \text{constant}} = 0$$

where ∂V = Change of velocity
∂s = Length of flow in the direction S.

Non-uniform flow is that type of flow in which the velocity at any given time changes with respect to space. Thus, mathematically, for non-uniform flow

$$\left(\frac{\partial V}{\partial s}\right)_{t = \text{constant}} \neq 0.$$

Unit-II

Fluid Kinematics, Fluid Dynamics and Closed Conduit Flow

5.3.3 Laminar and Turbulent Flows. Laminar flow is defined as that type of flow in which the fluid particles move along well-defined paths or stream line and all the stream-lines are straight and parallel. Thus the particles move in laminas or layers gliding smoothly over the adjacent layer. This type of flow is also called stream-line flow or viscous flow.

Turbulent flow is that type of flow in which the fluid particles move in a *zig-zag* way. Due to the movement of fluid particles in a *zig-zag* way, the eddies formation takes place which are responsible for high energy loss. For a pipe flow, the type of flow is determined by a non-dimensional number $\frac{VD}{\nu}$ called the Reynold number,

where D = Diameter of pipe

V = Mean velocity of flow in pipe

and ν = Kinematic viscosity of fluid.

If the Reynold number is less than 2000, the flow is called laminar. If the Reynold number is more than 4000, it is called turbulent flow. If the Reynold number lies between 2000 and 4000, the flow may be laminar or turbulent.

5.3.4 Compressible and Incompressible Flows. Compressible flow is that type of flow in which the density of the fluid changes from point to point or in other words the density (ρ) is not constant for the fluid. Thus, mathematically, for compressible flow

$$\rho \neq \text{Constant}$$

Incompressible flow is that type of flow in which the density is constant for the fluid flow. Liquids are generally incompressible while gases are compressible. Mathematically, for incompressible flow

$$\rho = \text{Constant.}$$

5.3.5 Rotational and Irrotational Flows. Rotational flow is that type of flow in which the fluid particles while flowing along stream-lines, also rotate about their own axis. And if the fluid particles while flowing along stream-lines, do not rotate about their own axis then that type of flow is called irrotational flow.

5.3.6 One-, Two- and Three-Dimensional Flows. **One-dimensional flow** is that type of flow in which the flow parameter such as velocity is a function of time and one space co-ordinate only, say x. For a steady one-dimensional flow, the velocity is a function of one-space-co-ordinate only. The variation of velocities in other two mutually perpendicular directions is assumed negligible. Hence mathematically, for one-dimensional flow

$$u = f(x),\ v = 0 \text{ and } w = 0$$

where u, v and w are velocity components in x, y and z directions respectively.

Two-dimensional flow is that type of flow in which the velocity is a function of time and two rectangular space co-ordinates say x and y. For a steady two-dimensional flow the velocity is a function of two space co-ordinates only. The variation of velocity in the third direction is negligible. Thus, mathematically for two-dimensional flow

$$u = f_1(x, y),\ v = f_2(x, y) \text{ and } w = 0.$$

Three-dimensional flow is that type of flow in which the velocity is a function of time and three mutually perpendicular directions. But for a steady three-dimensional flow the fluid parameters are functions of three space co-ordinates (x, y and z) only. Thus, mathematically, for three-dimensional flow

$$u = f_1(x, y, z),\ v = f_2(x, y, z) \text{ and } w = f_3(x, y, z).$$

Unit-II

Fluid Kinematics, Fluid Dynamics and Closed Conduit Flow

2. Obtain an expression for continuity equation for two dimensional flow.

▶ 5.5 CONTINUITY EQUATION

The equation based on the principle of conservation of mass is called continuity equation. Thus for a fluid flowing through the pipe at all the cross-section, the quantity of fluid per second is constant. Consider two cross-sections of a pipe as shown in Fig. 5.1.

Let V_1 = Average velocity at cross-section 1-1

ρ_1 = Density at section 1-1

A_1 = Area of pipe at section 1-1

and V_2, ρ_2, A_2 are corresponding values at section, 2-2.

Then rate of flow at section 1-1 = $\rho_1 A_1 V_1$

Rate of flow at section 2-2 = $\rho_2 A_2 V_2$

According to law of conservation of mass

Rate of flow at section 1-1 = Rate of flow at section 2-2

or $$\rho_1 A_1 V_1 = \rho_2 A_2 V_2 \quad ...(5.2)$$

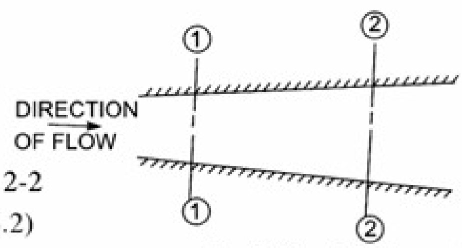


Fig. 5.1 *Fluid flowing through a pipe.*

Equation (5.2) is applicable to the compressible as well as incompressible fluids and is called **Continuity Equation.** If the fluid is incompressible, then $\rho_1 = \rho_2$ and continuity equation (5.2) reduces to

$$A_1 V_1 = A_2 V_2 \quad ...(5.3)$$

3 *The diameters of a pipe at the sections 1 and 2 are 10 cm and 15 cm respectively. Find the discharge through the pipe if the velocity of water flowing through the pipe at section 1 is 5 m/s. Determine also the velocity at section 2.*

Solution. Given :

At section 1, $D_1 = 10 \text{ cm} = 0.1 \text{ m}$

$$A_1 = \frac{\pi}{4}(D_1^2) = \frac{\pi}{4}(.1)^2 = 0.007854 \text{ m}^2$$

$V_1 = 5$ m/s.

At section 2, $D_2 = 15 \text{ cm} = 0.15 \text{ m}$

$$A_2 = \frac{\pi}{4}(.15)^2 = 0.01767 \text{ m}^2$$

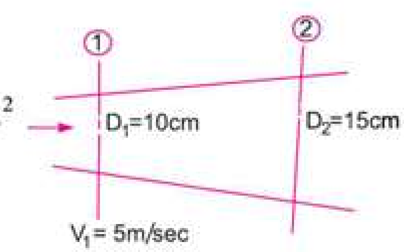


Fig. 5.2

(*i*) Discharge through pipe is given by equation (5.1)

or $$Q = A_1 \times V_1$$

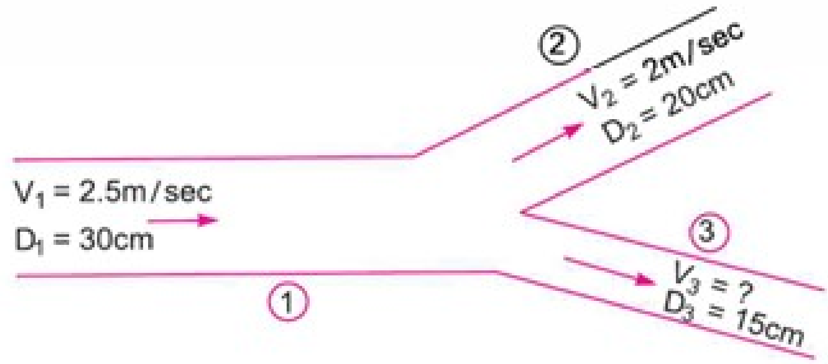
$$= 0.007854 \times 5 = \mathbf{0.03927\ m^3/s.\ Ans.}$$

Using equation (5.3), we have $A_1 V_1 = A_2 V_2$

(*ii*) ∴ $$V_2 = \frac{A_1 V_1}{A_2} = \frac{0.007854}{0.01767} \times 5.0 = \mathbf{2.22\ m/s.\ Ans.}$$

4 *A 30 cm diameter pipe, conveying water, branches into two pipes of diameters 20 cm and 15 cm respectively. If the average velocity in the 30 cm diameter pipe is 2.5 m/s, find the discharge in this pipe. Also determine the velocity in 15 cm pipe if the average velocity in 20 cm diameter pipe is 2 m/s.*

Solution. Given :



$$D_1 = 30 \text{ cm} = 0.30 \text{ m}$$

$$\therefore \quad A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} \times .3^2 = 0.07068 \text{ m}^2$$

$$V_1 = 2.5 \text{ m/s}$$

$$D_2 = 20 \text{ cm} = 0.20 \text{ m}$$

$$\therefore \quad A_2 = \frac{\pi}{4} (.2)^2 = \frac{\pi}{4} \times .4 = 0.0314 \text{ m}^2,$$

$$V_2 = 2 \text{ m/s}$$

$$D_3 = 15 \text{ cm} = 0.15 \text{ m}$$

$$\therefore \quad A_3 = \frac{\pi}{4} (.15)^2 = \frac{\pi}{4} \times 0.225 = 0.01767 \text{ m}^2$$

Find (*i*) Discharge in pipe 1 or Q_1

(*ii*) Velocity in pipe of dia. 15 cm or V_3

Let Q_1, Q_2 and Q_3 are discharges in pipe 1, 2 and 3 respectively.

Then according to continuity equation

$$Q_1 = Q_2 + Q_3$$

(*i*) **The discharge Q_1** in pipe 1 is given by

$$Q_1 = A_1V_1 = 0.07068 \times 2.5 \text{ m}^3/\text{s} = \mathbf{0.1767 \text{ m}^3/\text{s}. \text{ Ans.}}$$

(*ii*) **Value of V_3**

$$Q_2 = A_2V_2 = 0.0314 \times 2.0 = 0.0628 \text{ m}^3/\text{s}$$

Substituting the values of Q_1 and Q_2 in equation (1)

$$0.1767 = 0.0628 + Q_3$$

$\therefore$ $$Q_3 = 0.1767 - 0.0628 = 0.1139 \text{ m}^3/\text{s}$$

But $$Q_3 = A_3 \times V_3 = 0.01767 \times V_3 \quad \text{or} \quad 0.1139 = 0.01767 \times V_3$$

$\therefore$ $$V_3 = \frac{0.1139}{0.01767} = \mathbf{6.44 \text{ m/s. Ans.}}$$

5 *Water is flowing through a pipe of 5 cm diameter under a pressure of 29.43 N/cm² (gauge) and with mean velocity of 2.0 m/s. Find the total head or total energy per unit weight of the water at a cross-section, which is 5 m above the datum line.*

Solution. Given :

Diameter of pipe $= 5 \text{ cm} = 0.5 \text{ m}$

Pressure, $p = 29.43 \text{ N/cm}^2 = 29.43 \times 10^4 \text{ N/m}^2$

Velocity, $v = 2.0 \text{ m/s}$

Datum head, $z = 5 \text{ m}$

Total head $=$ pressure head + kinetic head + datum head

Pressure head $= \dfrac{p}{\rho g} = \dfrac{29.43 \times 10^4}{1000 \times 9.81} = 30 \text{ m}$ $\quad \left\{\rho \text{ for water} = 1000 \dfrac{\text{kg}}{\text{m}^3}\right\}$

Kinetic head $= \dfrac{v^2}{2g} = \dfrac{2 \times 2}{2 \times 9.81} = 0.204 \text{ m}$

$\therefore$ Total head $= \dfrac{p}{\rho g} + \dfrac{v^2}{2g} + z = 30 + 0.204 + 5 = \mathbf{35.204 \text{ m. Ans.}}$

6 *A pipe, through which water is flowing, is having diameters, 20 cm and 10 cm at the cross-sections 1 and 2 respectively. The velocity of water at section 1 is given 4.0 m/s. Find the velocity head at sections 1 and 2 and also rate of discharge.*

Solution. Given :

$D_1 = 20 \text{ cm} = 0.2 \text{ m}$

∴ Area, $A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} (.2)^2 = 0.0314 \text{ m}^2$

$V_1 = 4.0 \text{ m/s}$

$D_2 = 0.1 \text{ m}$

∴ $A_2 = \frac{\pi}{4} (.1)^2 = .00785 \text{ m}^2$

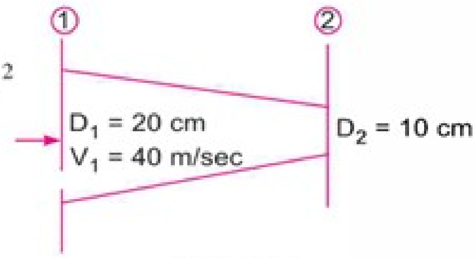


Fig. 6.2

(*i*) Velocity head at section 1

$$= \frac{V_1^2}{2g} = \frac{4.0 \times 4.0}{2 \times 9.81} = \mathbf{0.815 \text{ m. Ans.}}$$

(*ii*) Velocity head at section 2 = $V_2^2/2g$

To find V_2, apply continuity equation at 1 and 2

$$\therefore \quad A_1V_1 = A_2V_2 \quad \text{or} \quad V_2 = \frac{A_1V_1}{A_2} = \frac{.0314}{.00785} \times 4.0 = 16.0 \text{ m/s}$$

∴ Velocity head at section 2 = $\frac{V_2^2}{2g} = \frac{16.0 \times 16.0}{2 \times 9.81} = \mathbf{83.047 \text{ m. Ans.}}$

(*iii*) Rate of discharge $= A_1V_1 \quad \text{or} \quad A_2V_2$

$= 0.0314 \times 4.0 = 0.1256 \text{ m}^3\text{/s}$

= **125.6 litres/s. Ans.** $\{\because 1 \text{ m}^3 = 1000 \text{ litres}\}$

7 What is venturimeter? Derive an expression for the discharge through a venturimeter.

▶ 6.7 PRACTICAL APPLICATIONS OF BERNOULLI'S EQUATION

Bernoulli's equation is applied in all problems of incompressible fluid flow where energy considerations are involved. But we shall consider its application to the following measuring devices :

1. Venturimeter.
2. Orifice meter.
3. Pitot-tube.

6.7.1 Venturimeter. A venturimeter is a device used for measuring the rate of a flow of a fluid flowing through a pipe. It consists of three parts :

(*i*) A short converging part, (*ii*) Throat, and (*iii*) Diverging part. It is based on the Principle of Bernoulli's equation.

Expression for rate of flow through venturimeter

Consider a venturimeter fitted in a horizontal pipe through which a fluid is flowing (say water), as shown in Fig. 6.9.

Let d_1 = diameter at inlet or at section (1),

p_1 = pressure at section (1)

v_1 = velocity of fluid at section (1),

a = area at section (1) = $\frac{\pi}{4} d_1^2$

and d_2, p_2, v_2, a_2 are corresponding values at section (2).

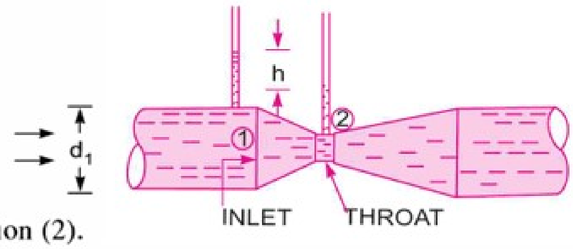


Fig. 6.9 *Venturimeter.*

Applying Bernoulli's equation at sections (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

As pipe is horizontal, hence $z_1 = z_2$

$$\therefore \quad \frac{p_1}{\rho g} + \frac{v_1^2}{2g} = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} \quad \text{or} \quad \frac{p_1 - p_2}{\rho g} = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$$

But $\frac{p_1 - p_2}{\rho g}$ is the difference of pressure heads at sections 1 and 2 and it is equal to h or $\frac{p_1 - p_2}{\rho g} = h$

Substituting this value of $\frac{p_1 - p_2}{\rho g}$ in the above equation, we get

$$h = \frac{v_2^2}{2g} - \frac{v_1^2}{2g} \qquad \text{...(6.6)}$$

Now applying continuity equation at sections 1 and 2

$$a_1 v_1 = a_2 v_2 \quad \text{or} \quad v_1 = \frac{a_2 v_2}{a_1}$$

Substituting this value of v_1 in equation (6.6)

$$h = \frac{v_2^2}{2g} - \frac{\left(\frac{a_2 v_2}{a_1}\right)^2}{2g} = \frac{v_2^2}{2g}\left[1 - \frac{a_2^2}{a_1^2}\right] = \frac{v_2^2}{2g}\left[\frac{a_1^2 - a_2^2}{a_1^2}\right]$$

or

$$v_2^2 = 2gh \frac{a_1^2}{a_1^2 - a_2^2}$$

$$\therefore \qquad v_2 = \sqrt{2gh\frac{a_1^2}{a_1^2 - a_2^2}} = \frac{a_1}{\sqrt{a_1^2 - a_2^2}}\sqrt{2gh}$$

$\therefore$ Discharge, $Q = a_2 v_2$

$$= a_2 \frac{a_1}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh} = \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh} \qquad ...(6.7)$$

Equation (6.7) gives the discharge under ideal conditions and is called, theoretical discharge. Actual discharge will be less than theoretical discharge.

$$\therefore \qquad Q_{act} = C_d \times \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh} \qquad ...(6.8)$$

where C_d = Co-efficient of venturimeter and its value is less than 1.

8 *A horizontal venturimeter with inlet and throat diameters 30 cm and 15 cm respectively is used to measure the flow of water. The reading of differential manometer connected to the inlet and the throat is 20 cm of mercury. Determine the rate of flow. Take C_d = 0.98.*

Solution. Given :

Dia. at inlet, $d_1 = 30$ cm

$\therefore$ Area at inlet, $a_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4}(30)^2 = 706.85 \text{ cm}^2$

Dia. at throat, $d_2 = 15$ cm

$\therefore$ $a_2 = \frac{\pi}{4} \times 15^2 = 176.7 \text{ cm}^2$

$C_d = 0.98$

Reading of differential manometer = x = 20 cm of mercury.

$\therefore$ Difference of pressure head is given by (6.9)

$$\text{or} \qquad h = x\left[\frac{S_h}{S_o} - 1\right]$$

where S_h = Sp. gravity of mercury = 13.6, S_o = Sp. gravity of water = 1

$$= 20\left[\frac{13.6}{1} - 1\right] = 20 \times 12.6 \text{ cm} = 252.0 \text{ cm of water.}$$

The discharge through venturimeter is given by eqn. (6.8)

$$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

$$= 0.98 \times \frac{706.85 \times 176.7}{\sqrt{(706.85)^2 - (176.7)^2}} \times \sqrt{2 \times 9.81 \times 252}$$

$$= \frac{86067593.36}{\sqrt{499636.9 - 31222.9}} = \frac{86067593.36}{684.4}$$

$$= 125756 \text{ cm}^3/\text{s} = \frac{125756}{1000} \text{ lit/s} = \mathbf{125.756 \text{ lit/s. Ans.}}$$

9 *An oil of sp. gr. 0.8 is flowing through a venturimeter having inlet diameter 20 cm and throat diameter 10 cm. The oil-mercury differential manometer shows a reading of 25 cm. Calculate the discharge of oil through the horizontal venturimeter. Take $C_d = 0.98$.*

Solution. Given :

Sp. gr. of oil, $S_o = 0.8$

Sp. gr. of mercury, $S_h = 13.6$

Reading of differential manometer, $x = 25$ cm

$\therefore$ Difference of pressure head, $h = x \left[\frac{S_h}{S_o} - 1\right]$

$$= 25 \left[\frac{13.6}{0.8} - 1\right] \text{ cm of oil} = 25 \, [17 - 1] = 400 \text{ cm of oil.}$$

Dia. at inlet, $d_1 = 20$ cm

$\therefore$ $a_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4} \times 20^2 = 314.16 \text{ cm}^2$

$d_2 = 10$ cm

$\therefore$ $a_2 = \frac{\pi}{4} \times 10^2 = 78.54 \text{ cm}^2$

$C_d = 0.98$

$\therefore$ The discharge Q is given by equation (6.8)

or
$$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

$$= 0.98 \times \frac{314.16 \times 78.54}{\sqrt{(314.16)^2 - (78.54)^2}} \times \sqrt{2 \times 981 \times 400}$$

$$= \frac{21421375.68}{\sqrt{98696 - 6168}} = \frac{21421375.68}{304} \text{ cm}^3/\text{s}$$

$$= 70465 \text{ cm}^3/\text{s} = \mathbf{70.465 \text{ litres/s. Ans.}}$$

10 *An orifice meter with orifice diameter 10 cm is inserted in a pipe of 20 cm diameter. The pressure gauges fitted upstream and downstream of the orifice meter gives readings of 19.62 N/cm² and 9.81 N/cm² respectively. Co-efficient of discharge for the orifice meter is given as 0.6. Find the discharge of water through pipe.*

Solution. Given :

Dia. of orifice, $d_0 = 10$ cm

$\therefore$ Area, $a_0 = \frac{\pi}{4}(10)^2 = 78.54 \text{ cm}^2$

Dia. of pipe, $d_1 = 20$ cm

$\therefore$ Area, $a_1 = \frac{\pi}{4}(20)^2 = 314.16 \text{ cm}^2$

$p_1 = 19.62 \text{ N/cm}^2 = 19.62 \times 10^4 \text{ N/m}^2$

$\therefore$ $$\frac{p_1}{\rho g} = \frac{19.62 \times 10^4}{1000 \times 9.81} = 20 \text{ m of water}$$

Similarly $$\frac{p_2}{\rho g} = \frac{9.81 \times 10^4}{1000 \times 9.81} = 10 \text{ m of water}$$

$\therefore$ $$h = \frac{p_1}{\rho g} - \frac{p_2}{\rho g} = 20.0 - 10.0 = 10 \text{ m of water} = 1000 \text{ cm of water}$$

$C_d = 0.6$

The discharge, Q is given by equation (6.13)

$$Q = C_d \frac{a_0 a_1}{\sqrt{a_1^2 - a_0^2}} \times \sqrt{2gh}$$

$$= 0.6 \times \frac{78.54 \times 314.16}{\sqrt{(314.16)^2 - (78.54)^2}} \times \sqrt{2 \times 981 \times 1000}$$

$$= \frac{20736838.09}{304} = 68213.28 \text{ cm}^3/\text{s} = \textbf{68.21 litres/s. Ans.}$$

1. A jet of water of diameter 75 mm moving with a velocity of 30 m/s, strikes a curved fixed plate tangentially at one end at an angle of 30° to the horizontal. The jet leaves the plate at an angle of 20° to the horizontal. Find the force exerted by the jet on the plate in the horizontal and vertical direction.

Solution. Given :

Diameter of the jet, $d = 75 \text{ mm} = 0.075 \text{ m}$

∴ Area, $a = \frac{\pi}{4} (.075)^2 = .004417 \text{ m}^2$

Velocity of jet, $V = 30 \text{ m/s}$

Angle made by the jet at inlet tip with horizontal, $\theta = 30°$

Angle made by the jet at outlet tip with horizontal, $\phi = 20°$

The force exerted by the jet of water in the direction of x is given by equation (17.8) and in the direction of y by equation (17.9),

∴ $F_x = \rho a V^2 [\cos\theta + \cos\phi]$

$= 1000 \times .004417 [\cos 30° + \cos 20°] \times 30^2 =$ **7178.2 N. Ans.**

and $F_y = \rho a V^2 [\sin\theta - \sin\phi]$

$= 1000 \times .004417 [\sin 30° - \sin 20°] \times 30^2 =$ **628.13 N. Ans.**

2. A jet of water of 30 mm diameter strikes a hinged square plate at its centre with a velocity of 20 m/s. The plate is deflected through an angle of 20°. Find the weight of the plate. If the plate is not allowed, to swing, what will be the force required at the lower edge of the plate to keep the plate in vertical position.

Solution. Given :

Diameter of the jet, $d = 30 \text{ mm} = 3 \text{ cm} = 0.03 \text{ m}$

∴ Area, $a = \frac{\pi}{4} d^2 = \frac{\pi}{4}(.03)^2 = .0007068 \text{ m}^2$

Velocity of jet, $V = 20 \text{ m/s}$

Angle of swing, $\theta = 20°$

Using equation (17.10) for angle of swing,

$$\sin\theta = \frac{\rho a V^2}{W}$$

or $$\sin 20° = 1000 \times \frac{.0007068 \times 20^2}{W} = \frac{282.72}{W}$$

∴ $$W = \frac{282.72}{\sin 20°} = 826.6 \text{ N}$$

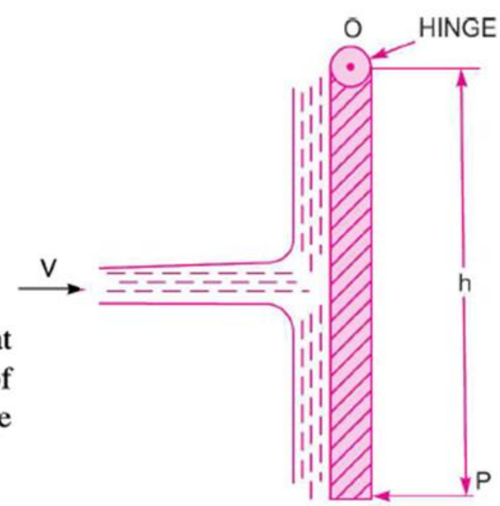


Fig. 17.7

If the plate is not allowed to swing, a force P will be applied at the lower edge of the plate as shown in Fig. 17.7. The weight of the plate is acting vertically downward through the C.G. of the plate.

Let F = Force exerted by jet of water

h = Height of plate

= Distance of P from the hinge.

The jet strikes at the centre of the plate and hence distance of the centre of the jet from hinge = $\frac{h}{2}$.

Taking moments* about the hinge, O, $P \times h = F \times \frac{h}{2}$.

$$\therefore \quad P = \frac{F \times h}{2 \times h} = \frac{F}{2} = \frac{\rho a V^2}{2} \qquad (\because \quad F = \rho a V^2)$$

$$= 1000 \times \frac{.0007068 \times 20^2}{2} = \textbf{141.36 N. Ans.}$$

3. A nozzle of 50 mm diameter delivers a stream of water at 20 m/s perpendicular to a plate that moves away from the jet at 5 m/s. Find :

(i) the force on the plate,

(ii) the work done, and

(iii) the efficiency of jet.

Dia. of jet $= 50 \text{ mm} = 0.05 \text{ m}$

$\therefore$ Area, $a = \frac{\pi}{4}(0.05^2) = 0.0019635 \text{ m}^2$

Velocity of jet, $V = 20$ m/s, Velocity of plate, $u = 5$ m/s

(*i*) The force on the plate is given by equation (17.11) as,

$$F_x = \rho a (V - u)^2$$

$$= 1000 \times 0.0019635 \times (20 - 5)^2 = \textbf{441.78 N. Ans.}$$

(*ii*) The work done by the jet

$$= F_x \times u = 441.78 \times 5 = \textbf{2208.9 Nm/s. Ans.}$$

(*iii*) The efficiency of the jet, $\eta = \frac{\text{Output of jet}}{\text{Input of jet}}$

$$= \frac{\text{Work done/s}}{\text{K.E. of jet/s}} = \frac{F_x \times u}{\frac{1}{2} m V^2}$$

$$= \frac{F_x \times u}{\frac{1}{2}(\rho A V) \times V^2}$$

$$= \frac{2208.9}{\frac{1}{2}(1000 \times 0.0019635 \times 20) \times 20^2} = \frac{2208.9}{6540}$$

$$= 0.3377 = \textbf{33.77\%. Ans.}$$

4. Obtain an expression for the force exerted by a jet of water on a fixed vertical plate in the direction the jet.

17.2 FORCE EXERTED BY THE JET ON A STATIONARY VERTICAL PLATE

Consider a jet of water coming out from the nozzle, strikes a flat vertical plate as shown in Fig. 17.1

Let V = velocity of the jet, d = diameter of the jet,

a = area of cross-section of the jet = $\frac{\pi}{4} d^2$.

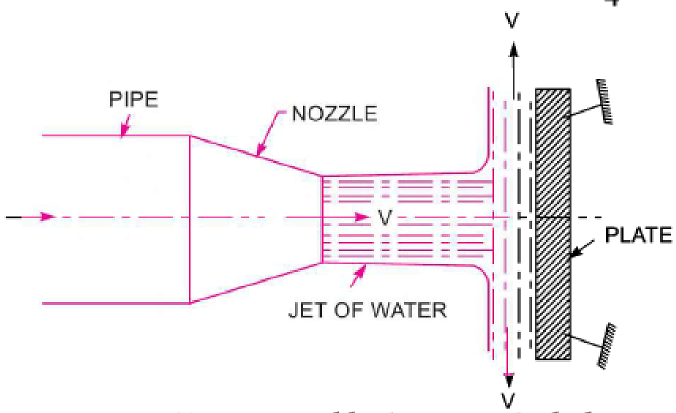


Fig. 17.1 *Force exerted by jet on vertical plate.*

The jet after striking the plate, will move along the plate. But the plate is at right angles to the jet. Hence the jet after striking, will get deflected through 90°. Hence the component of the velocity of jet, in the direction of jet, after striking will be zero.

The force exerted by the jet on the plate in the direction of jet,

$$F_x = \text{Rate of change of momentum in the direction of force}$$

$$= \frac{\text{Initial momentum} - \text{Final momentum}}{\text{Time}}$$

$$= \frac{(\text{Mass} \times \text{Initial velocity} - \text{Mass} \times \text{Final velocity})}{\text{Time}}$$

$$= \frac{\text{Mass}}{\text{Time}} [\text{Initial velocity} - \text{Final velocity}]$$

$$= (\text{Mass/sec}) \times (\text{velocity of jet before striking} - \text{velocity of jet after striking})$$

$$= \rho a V[V - 0] \qquad (\because \text{ mass/sec} = \rho \times a\, V)$$

$$= \rho a V^2 \qquad ...(17.1)$$

5.

Show that for a curved radial vane, the work done per second is given by. $\rho a V_1 [V_{w1} u_1 \pm V_{w2} u_2]$.

17.4.6 Force Exerted on a Series of Radial Curved Vanes. For a radial curved vane, the radius of the vane at inlet and outlet is different and hence the tangential velocities of the radial vane at inlet and outlet will not be equal. Consider a series of radial curved vanes mounted on a wheel as shown in Fig. 17.23. The jet of water strikes the vanes and the wheel starts rotating at a constant angular speed.

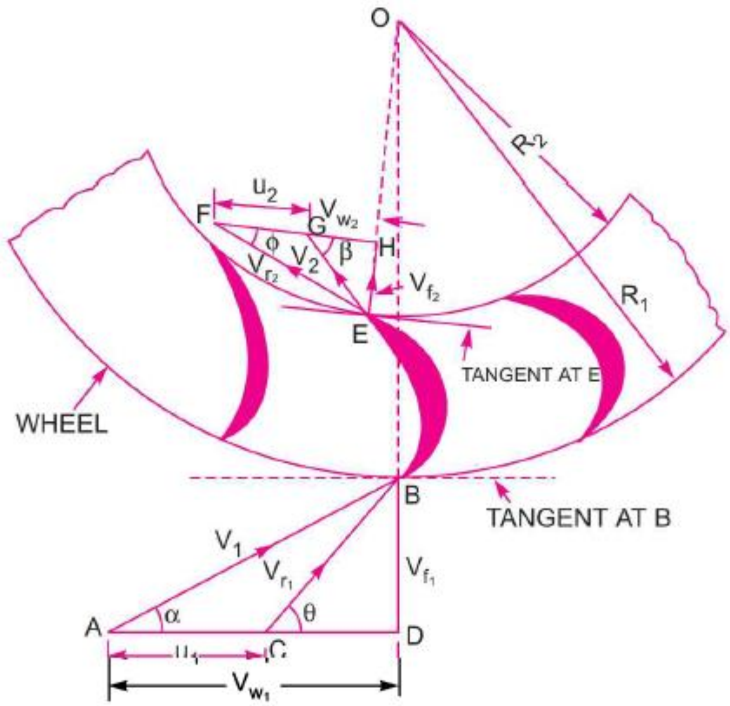


Fig. 17.23 *Series of radial curved vanes mounted on a wheel.*

Let R_1 = Radius of wheel at inlet of the vane,

R_2 = Radius of the wheel at the outlet of the vane,

ω = Angular speed of the wheel.

Then $u_1 = \omega R_1$ and $u_2 = \omega R_2$

The velocity triangles at inlet and outlet are drawn as shown in Fig. 17.23.

The mass of water striking per second for a series of vanes

= Mass of water coming out from nozzle per second

$= \rho a V_1$, where a = Area of jet and V_1 = Velocity of jet.

Momentum of water striking the vanes in the tangential direction per sec at inlet

= Mass of water per second × Component of V_1 in the tangential direction

$= \rho a V_1 \times V_{w_1}$ ($\because$ Component of V_1 in tangential direction $= V_1 \cos \alpha = V_{w_1}$)

Similarly, momentum of water at outlet per sec

$= \rho a V_1 \times$ Component of V_2 in the tangential direction

$= \rho a V_1 \times (-V_2 \cos \beta) = -\rho a V_1 \times V_{w_2}$ ($\because$ $V_2 \cos \beta = V_{w_2}$)

–ve sign is taken as the velocity V_2 at outlet is in opposite direction.

Now, angular momentum per second at inlet

= Momentum at inlet × Radius at inlet

$= \rho a V_1 \times V_{w_1} \times R_1$

Angular momentum per second at outlet

= Momentum of outlet × Radius at outlet

$= -\rho a V_1 \times V_{w_2} \times R_2$

Torque exerted by the water on the wheel,

$$T = \text{Rate of change of angular momentum}$$
$$= [\text{Initial angular momentum per second} - \text{Final angular momentum per second}]$$
$$= \rho a V_1 \times V_{w_1} \times R_1 - (-\rho a V_1 \times V_{w_2} \times R_2) = \rho a V_1 [V_{w_1} \times R_1 + V_{w_2} R_2]$$

Work done per second on the wheel

$$= \text{Torque} \times \text{Angular velocity} = T \times \omega$$
$$= \rho a V_1 [V_{w_1} \times R_1 + V_{w_2} R_2] \times \omega = \rho a V_1 [V_{w_1} \times R_1 \times \omega + V_{w_2} R_2 \times \omega]$$
$$= \rho a V_1 [V_{w_1} u_1 + V_{w_2} \times u_2] \qquad (\because \quad u_1 = \omega R_1 \text{ and } u_2 = \omega R_2)$$

If the angle β in Fig. 17.23 is an obtuse angle then work done per second will be given as

$$= \rho a V_1 [V_{w_1} u_1 - V_{w_2} u_2]$$

$\therefore$ The general expression for the work done per second on the wheel

$$= \rho a V_1 [V_{w_1} u_1 \pm V_{w_2} u_2] \qquad \text{...(17.26)}$$

Problem 18.1 *A Pelton wheel has a mean bucket speed of 10 metres per second with a jet of water flowing at the rate of 700 litres/s under a head of 30 metres. The buckets deflect the jet through an angle of 160°. Calculate the power given by water to the runner and the hydraulic efficiency of the turbine. Assume co-efficient of velocity as 0.98.*

Solution. Given :

Speed of bucket, $u = u_1 = u_2 = 10$ m/s

Discharge, $Q = 700$ litres/s $= 0.7$ m^3/s, Head of water, $H = 30$ m

Angle of deflection $= 160°$

∴ Angle, $\phi = 180° - 160° = 20°$

Co-efficient of velocity, $C_v = 0.98$.

The velocity of jet, $V_1 = C_v\sqrt{2gH} = 0.98\sqrt{2 \times 9.81 \times 30} = 23.77$ m/s

∴ $V_{r_1} = V_1 - u_1 = 23.77 - 10$

$= 13.77$ m/s

$V_{w_1} = V_1 = 23.77$ m/s

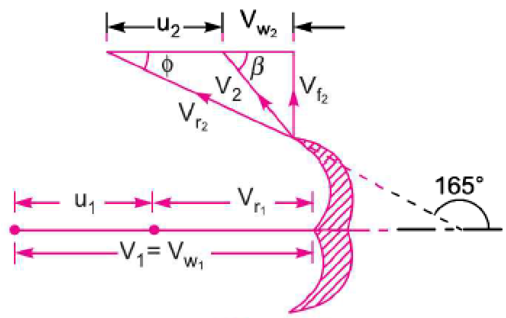


Fig. 18.6

From outlet velocity triangle,

$V_{r_2} = V_{r_1} = 13.77$ m/s

$V_{w_2} = V_{r_2} \cos \phi - u_2$

$= 13.77 \cos 20° - 10.0 = 2.94$ m/s

Work done by the jet per second on the runner is given by equation (18.9) as

$$= \rho a V_1 \left[V_{w_1} + V_{w_2}\right] \times u$$

$$= 1000 \times 0.7 \times [23.77 + 2.94] \times 10 \qquad (\because \ aV_1 = Q = 0.7 \text{ m}^3/\text{s})$$

$$= 186970 \text{ Nm/s}$$

∴ Power given to turbine $= \dfrac{186970}{1000} =$ **186.97 kW. Ans.**

The hydraulic efficiency of the turbine is given by equation (18.12) as

$$\eta_h = \frac{2\left[V_{w_1} + V_{w_2}\right] \times u}{V_1^2} = \frac{2[23.77 + 2.94] \times 10}{23.77 \times 23.77}$$

$=$ **0.9454** or **94.54%. Ans.**

Problem 18.2 *A Pelton wheel is to be designed for the following specifications :*
Shaft power = 11,772 kW ; Head = 380 metres ; Speed = 750 r.p.m. ; Overall efficiency = 86% ; Jet diameter is not to exceed one-sixth of the wheel diameter. Determine :

(i) The wheel diameter, *(ii) The number of jets required, and*
(iii) Diameter of the jet.

Take $K_{v_1} = 0.985$ *and* $K_{u_1} = 0.45$

Solution. Given :

Shaft power, S.P. $= 11,772$ kW

Head, $H = 380$ m

Speed, $N = 750$ r.p.m.

Overall efficiency, $\eta_0 = 86\%$ or 0.86

Ratio of jet dia. to wheel dia. $= \dfrac{d}{D} = \dfrac{1}{6}$

Co-efficient of velocity, $K_{v_1} = C_v = 0.985$

Speed ratio, $K_{u_1} = 0.45$

Velocity of jet, $V_1 = C_v\sqrt{2gH} = 0.985\sqrt{2 \times 9.81 \times 380} = 85.05$ m/s

The velocity of wheel, $u = u_1 = u_2$

$$= \text{Speed ratio} \times \sqrt{2gH} = 0.45 \times \sqrt{2 \times 9.81 \times 380} = 38.85 \text{ m/s}$$

But $$u = \frac{\pi DN}{60} \quad \therefore \quad 38.85 = \frac{\pi DN}{60}$$

or $$D = \frac{60 \times 38.85}{\pi \times N} = \frac{60 \times 38.85}{\pi \times 750} = \mathbf{0.989\ m.\ Ans.}$$

But $$\frac{d}{D} = \frac{1}{6}$$

$\therefore$ Dia. of jet, $$d = \frac{1}{6} \times \text{D} = \frac{0.989}{6} = \mathbf{0.165\ m.\ Ans.}$$

Discharge of one jet, q = Area of jet × Velocity of jet

$$= \frac{\pi}{4} d^2 \times V_1 = \frac{\pi}{4}(.165) \times 85.05 \text{ m}^3\text{/s} = 1.818 \text{ m}^3\text{/s}$$

Now $$\eta_o = \frac{\text{S.P.}}{\text{W.P.}} = \frac{11772}{\dfrac{\rho g \times Q \times H}{1000}}$$

$$0.86 = \frac{11772 \times 1000}{1000 \times 9.81 \times Q \times 380}, \text{ where } Q = \text{Total discharge}$$

$\therefore$ Total discharge, $$Q = \frac{11772 \times 1000}{1000 \times 9.81 \times 380 \times 0.86} = 3.672 \text{ m}^3\text{/s}$$

$\therefore$ Number of jets $$= \frac{\text{Total discharge}}{\text{Discharge of one jet}} = \frac{Q}{q} = \frac{3.672}{1.818} = \mathbf{2\ jets.\ Ans.}$$

Problem 18.3 *The penstock supplies water from a reservoir to the Pelton wheel with a gross head of 500 m. One third of the gross head is lost in friction in the penstock. The rate of flow of water through the nozzle fitted at the end of the penstock is 2.0 m*^{3}*/s. The angle of deflection of the jet is 165°. Determine the power given by the water to the runner and also hydraulic efficiency of the Pelton wheel. Take speed ratio = 0.45 and* $C_v = 1.0$.

Solution. Given :

Gross head, $H_g = 500$ m

Head lost in friction, $h_f = \dfrac{H_g}{3} = \dfrac{500}{3} = 166.7$ m

∴ Net head, $H = H_g - h_f = 500 - 166.7 = 333.30$ m

Discharge, $Q = 2.0\ m^3/s$

Angle of deflection $= 165°$

∴ Angle, $\phi = 180° - 165° = 15°$

Speed ratio $= 0.45$

Co-efficient of velocity, $C_v = 1.0$

Velocity of jet, $V_1 = C_v\sqrt{2gH} = 1.0 \times \sqrt{2 \times 9.81 \times 333.3} = 80.86$ m/s

Velocity of wheel, $u = \text{Speed ratio} \times \sqrt{2gH}$

or $u = u_1 = u_2 = 0.45 \times \sqrt{2 \times 9.81 \times 333.3} = 36.387$ m/s

∴ $V_{r_1} = V_1 - u_1 = 80.86 - 36.387$

$= 44.473$ m/s

Also $V_{w_1} = V_1 = 80.86$ m/s

From outlet velocity triangle, we have

$$V_{r_2} = V_{r_1} = 44.473$$

$$V_{r_2} \cos \phi = u_2 + V_{w_2}$$

or $44.473 \cos 15° = 36.387 + V_{w_2}$

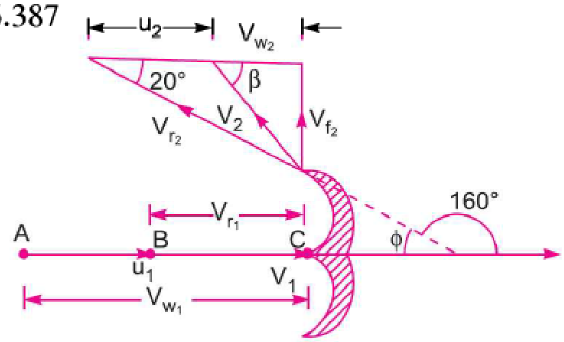


Fig. 18.7

or $V_{w_2} = 44.473 \cos 15° - 36.387 = 6.57$ m/s.

Work done by the jet on the runner per second is given by equation (18.9) as

$$\rho a V_1 [V_{w_1} + V_{w_2}] \times u = \rho Q [V_{w_1} + V_{w_2}] \times u \qquad (\because \quad aV_1 = Q)$$

$$= 1000 \times 2.0 \times [80.86 + 6.57] \times 36.387 = 6362630 \text{ Nm/s}$$

∴ Power given by the water to the runner in kW

$$= \frac{\text{Work done per second}}{1000} = \frac{6362630}{1000} = \mathbf{6362.63\ kW.\ Ans.}$$

Hydraulic efficiency of the turbine is given by equation (18.12) as

$$\eta_h = \frac{2\left[V_{w_1} + V_{w_2}\right] \times u}{V_1^2} = \frac{2[80.86 + 6.57] \times 36.387}{80.86 \times 80.86}$$

$= \mathbf{0.9731}$ or $\mathbf{97.31\%}$. **Ans.**

Problem 18.4 *A Pelton wheel is to be designed for a head of 60 m when running at 200 r.p.m. The Pelton wheel develops 95.6475 kW shaft power. The velocity of the buckets = 0.45 times the velocity of the jet, overall efficiency = 0.85 and co-efficient of the velocity is equal to 0.98.*

Solution. Given :

Head, $H = 60$ m

Speed $N = 200$ r.p.m

Shaft power, S.P. = 95.6475 kW

Velocity of bucket, $u = 0.45 \times$ Velocity of jet

Overall efficiency, $\eta_o = 0.85$

Co-efficient of velocity, $C_v = 0.98$

Design of Pelton wheel means to find diameter of jet (d), diameter of wheel (D), Width and depth of buckets and number of buckets on the wheel.

(*i*) *Velocity of jet*, $V_1 = C_v \times \sqrt{2gH} = 0.98 \times \sqrt{2 \times 9.81 \times 60} = 33.62$ m/s

$\therefore$ Bucket velocity, $u = u_1 = u_2 = 0.45 \times V_1 = 0.45 \times 33.62 = 15.13$ m/s

But $u = \frac{\pi D N}{60}$, where D = Diameter of wheel

$$\therefore \quad 15.13 = \frac{\pi \times D \times 200}{60} \quad \text{or} \quad D = \frac{60 \times 15.13}{\pi \times 200} = \textbf{1.44 m. Ans.}$$

(*ii*) *Diameter of the jet* (d)

Overall efficiency $\eta_o = 0.85$

But
$$\eta_o = \frac{\text{S.P.}}{\text{W.P.}} = \frac{95.6475}{\left(\frac{\text{W.P.}}{1000}\right)} = \frac{95.6475 \times 1000}{\rho \times g \times Q \times H} \qquad (\because \quad \text{W.P.} = \rho g Q H)$$

$$= \frac{95.6475 \times 1000}{1000 \times 9.81 \times Q \times 60}$$

$$\therefore \quad Q = \frac{95.6475 \times 1000}{\eta_o \times 1000 \times 9.81 \times 60} = \frac{95.6475 \times 1000}{0.85 \times 1000 \times 9.81 \times 60} = 0.1912 \text{ m}^3/\text{s}.$$

But the discharge, Q = Area of jet $\times$ Velocity of jet

$$\therefore \quad 0.1912 = \frac{\pi}{4} d^2 \times V_1 = \frac{\pi}{4} d^2 \times 33.62$$

$$\therefore \quad d = \sqrt{\frac{4 \times 0.1912}{\pi \times 33.62}} = 0.085 \text{ m} = \textbf{85 mm. Ans.}$$

(*iii*) *Size of buckets*

Width of buckets $= 5 \times d = 5 \times 85 = 425$ mm

Depth of buckets $= 1.2 \times d = 1.2 \times 85 =$ **102 mm. Ans.**

(*iv*) *Number of buckets on the wheel* is given by equation (18.17) as

$$Z = 15 + \frac{D}{2d} = 15 + \frac{1.44}{2 \times .085} = 15 + 8.5 = \textbf{23.5 say 24. Ans.}$$

Problem 18.16 *A reaction turbine works at 450 r.p.m. under a head of 120 metres. Its diameter at inlet is 120 cm and the flow area is 0.4 m². The angles made by absolute and relative velocities at inlet are 20° and 60° respectively with the tangential velocity. Determine :*

(a) The volume flow rate, (b) The power developed, and

(c) Hydraulic efficiency.

Assume whirl at outlet to be zero.

Solution. Given :

Speed of turbine, $N = 450$ r.p.m.

Head, $H = 120$ m

Diameter at inlet, $D_1 = 120$ cm $= 1.2$ m

Flow area, $\pi D_1 \times B_1 = 0.4 \text{ m}^2$

Angle made by absolute velocity at inlet, $\alpha = 20°$

Angle made by the relative velocity at inlet, $\theta = 60°$

Whirl at outlet, $V_{w_2} = 0$

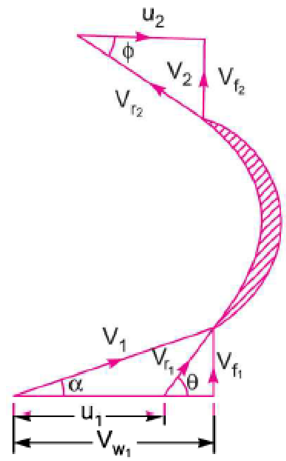


Fig. 18.13

Tangential velocity of the turbine at inlet,

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 1.2 \times 450}{60} = 28.27 \text{ m/s}$$

From inlet velocity triangle,

$$\tan \alpha = \frac{V_{f_1}}{V_{w_1}} \text{ or } \tan 20° = \frac{V_{f_1}}{V_{w_1}} \text{ or } \frac{V_{f_1}}{V_{w_1}} = \tan 20° = 0.364$$

$\therefore$ $$V_{f_1} = 0.364 \, V_{w_1}$$

Also $$\tan \theta = \frac{V_{f_1}}{V_{w_1} - u_1} = \frac{0.364 \, V_{w_1}}{V_{w_1} - 28.27} \qquad (\because \; V_{f_1} = 0.364 \, V_{w_1})$$

or $$\frac{0.364 \, V_{w_1}}{V_{w_1} - 28.27} = \tan \theta = \tan 60° = 1.732$$

$\therefore$ $$0.364 V_{w_1} = 1.732(V_{w_1} - 28.27) = 1.732 V_{w_1} - 48.96$$

or $$(1.732 - 0.364) \, V_{w_1} = 48.96$$

$\therefore$ $$V_{w_1} = \frac{48.96}{(1.732 - 0.364)} = 35.789 = 35.79 \text{ m/s.}$$

From equation (*i*), $$V_{f_1} = 0.364 \times V_{w_1} = 0.364 \times 35.79 = 13.027 \text{ m/s.}$$

(*a*) Volume flow rate is given by equation (18.21) as $Q = \pi D_1 B_1 \times V_{f_1}$

But $$\pi D_1 \times B_1 = 0.4 \text{ m}^2 \qquad \text{(given)}$$

$$Q = 0.4 \times 13.027 = \mathbf{5.211 \; m^3/s.} \; \textbf{Ans.}$$

(*b*) Work done per sec on the turbine is given by equation (18.18),

$$= \rho Q \, [V_{w_1} u_1] \qquad \left(\because \; V_{w_2} = 2\right)$$

$$= 1000 \times 5.211 \, [35.79 \times 28.27] = 5272402 \text{ Nm/s}$$

$\therefore$ $$\text{Power developed in kW} = \frac{\text{Work done per second}}{1000} = \frac{5272402}{1000} = \mathbf{5272.402 \; kW.} \; \textbf{Ans.}$$

(*c*) The hydraulic efficiency is given by equation (18.20*B*) as

$$\eta_h = \frac{V_{w_1} u_1}{gH} = \frac{35.79 \times 28.27}{9.81 \times 120} = 0.8595 = \mathbf{85.95\%.} \; \textbf{Ans.}$$

Problem 18.17 *As inward flow reaction turbine has external and internal diameters as 1.0 m and 0.6 m respectively. The hydraulic efficiency of the turbine is 90% when the head on the turbine is 36 m. The velocity of flow at outlet is 2.5 m/s and discharge at outlet is radial. If the vane angle at outlet is 15° and width of the wheel is 100 mm at inlet and outlet, determine : (i) the guide blade angle, (ii) speed of the turbine, (iii) vane angle of the runner at inlet, (iv) volume flow rate of turbine and (v) power developed.*

Solution. Given :

External diameter, $D_1 = 1.0$ m

Internal diameter, $D_2 = 0.6$ m

Hydraulic efficiency, $\eta_h = 90\% = 0.90$

Head, $H = 36$ m

Velocity of flow at outlet, $V_{f_2} = 2.5$ m/s

Discharge is radial, $V_{w_2} = 0$

Vane angle at outlet, $\phi = 15°$

Width of wheel, $B_1 = B_2 = 100$ mm $= 0.1$ m

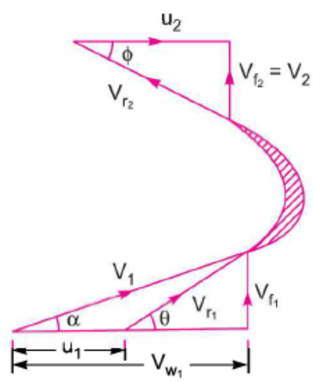


Fig. 18.14

Using equation (18.20 *B*) for hydraulic efficiency as

$$\eta_h = \frac{V_{w_1} u_1}{gH} \text{ or } 0.90 = \frac{V_{w_1} . u_1}{9.81 \times 36}$$

$\therefore$ $$V_{w_1} u_1 = 0.90 \times 9.81 \times 36 = 317.85 \quad ...(i)$$

From outlet velocity triangle, $\tan \phi = \frac{V_{f_2}}{u_2} = \frac{2.5}{u_2}$

$\therefore$ $$u_2 = \frac{2.5}{\tan \phi} = \frac{2.5}{\tan 15°} = 9.33$$

But $$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.6 \times N}{60}$$

$\therefore$ $$9.33 = \frac{\pi \times 0.6 \times N}{60} \text{ or } N = \frac{60 \times 9.33}{\pi \times 0.6} = \mathbf{296.98.\ Ans.}$$

$\therefore$ $$u_1 = \frac{\pi \times D_1 \times N}{60} = \frac{\pi \times 1.0 \times 296.98}{60} = 15.55 \text{ m/s.}$$

Substituting this value of 'u_1' in equation (*i*),

$$V_{w_1} \times 15.55 = 317.85$$

$\therefore$ $$V_{w_1} = \frac{317.85}{15.55} = 20.44 \text{ m/s}$$

Using equation (18.21), $\pi D_1 B_1 V_{f_1} = \pi D_2 B_2 V_{f_2}$ or $D_1 V_{f_1} = D_2 V_{f_2}$ $(\because B_1 = B_2)$

$\therefore$ $$V_{f_1} = \frac{D_2 \times V_{f_2}}{D_1} = \frac{0.6 \times 2.5}{1.0} = 1.5 \text{ m/s.}$$

(*i*) *Guide blade angle* (α).

From inlet velocity triangle, $\tan \alpha = \frac{V_{f_1}}{V_{w_1}} = \frac{1.5}{20.44} = 0.07338$

$\therefore$ $$\alpha = \tan^{-1} 0.07338 = \mathbf{4.19°} \text{ or } \mathbf{4° \ 11.8'.\ Ans.}$$

(*ii*) *Speed of the turbine,* $N = \mathbf{296.98}$ **r.p.m. Ans.**

(*iii*) *Same angle of runner at inlet* (θ)

$$\tan\theta = \frac{V_{f_1}}{V_{w_1} - u_1} = \frac{1.5}{(20.44 - 15.55)} = 0.3067$$

$\therefore$ $\theta = \tan^{-1} .3067 = 17.05°$ or **17° 3′. Ans.**

(*iv*) *Volume flow rate of turbine* is given by equation (18.21) as

$$= \pi D_1 B_1 V_{f_1} = \pi \times 1.0 \times 0.1 \times 1.5 = \mathbf{0.4712\ m^3/s.\ Ans.}$$

(*v*) *Power developed* (in kW)

$$= \frac{\text{Work done per second}}{1000} = \frac{\rho Q\left[V_{w_1} u_1\right]}{1000}$$

[Using equation (18.18) and $V_{w_2} = 0$]

$$= 1000 \times \frac{0.4712 \times 20.44 \times 15.55}{1000} = \mathbf{149.76\ kW.\ Ans.}$$

Problem 18.23 *A Francis turbine with an overall efficiency of 75% is required to produce 148.25 kW power. It is working under a head of 7.62 m. The peripheral velocity = 0.26 $\sqrt{2gH}$ and the radial velocity of flow at inlet is 0.96$\sqrt{2gH}$. The wheel runs at 150 r.p.m. and the hydraulic losses in the turbine are 22% of the available energy. Assuming radial discharge, determine :*

(i) The guide blade angle, *(ii) The wheel vane angle at inlet,*

(iii) Diameter of the wheel at inlet, and *(iv) Width of the wheel at inlet.*

Solution. Given :

Overall efficiency $\eta_o = 75\% = 0.75$

Power produced, S.P. = 148.25 kW

Head, $H = 7.62$ m

Peripheral velocity, $u_1 = 0.26\sqrt{2gH} = 0.26 \times \sqrt{2 \times 9.81 \times 7.62} = 3.179$ m/s

Velocity of flow at inlet, $V_{f_1} = 0.96\sqrt{2gH} = 0.96 \times \sqrt{2 \times 9.81 \times 7.62} = 11.738$ m/s.

Speed, $N = 150$ r.p.m.

Hydraulic losses = 22% of available energy

Discharge at outlet = Radial

$$V_{w_2} = 0 \text{ and } V_{f_2} = V_2$$

Hydraulic efficiency is given as

$$\eta_h = \frac{\text{Total head at inlet} - \text{Hydraulic loss}}{\text{Head at inlet}}$$

$$= \frac{H - .22\,H}{H} = \frac{0.78\,H}{H} = 0.78$$

But $$\eta_h = \frac{V_{w_1} u_1}{gH}$$

$$\therefore \quad \frac{V_{w_1} u_1}{gH} = 0.78$$

$$\therefore \quad V_{w_1} = \frac{0.78 \times g \times H}{u_1}$$

$$= \frac{0.78 \times 9.81 \times 7.62}{3.179} = 18.34 \text{ m/s.}$$

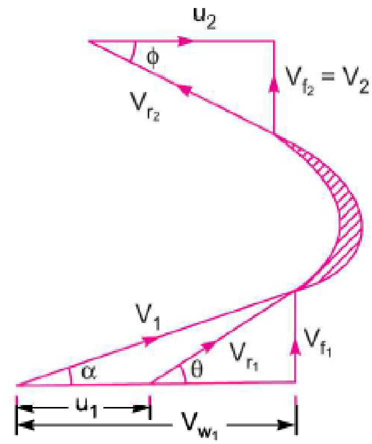


Fig. 18.21

(*i*) *The guide blade angle*, *i.e.*, α. From inlet velocity triangle,

$$\tan \alpha = \frac{V_{f_1}}{V_{w_1}} = \frac{11.738}{18.34} = 0.64$$

$$\therefore \quad \alpha = \tan^{-1} 0.64 = \mathbf{32.619°} \text{ or } \mathbf{32° \; 37'. \; Ans.}$$

(*ii*) *The wheel vane angle at inlet*, *i.e.*, θ

$$\tan \theta = \frac{V_{f_1}}{V_{w_1} - u_1} = \frac{11.738}{18.34 - 3.179} = 0.774$$

$$\therefore \quad \theta = \tan^{-1} .774 = \mathbf{37.74} \text{ or } \mathbf{37° \; 44.4'. \; Ans.}$$

(*iii*) *Diameter of wheel at inlet* (D_1).

Using the relation, $$u_1 = \frac{\pi D_1 N}{60}$$

$$D_1 = \frac{60 \times u_1}{\pi \times N} = \frac{60 \times 3.179}{\pi \times 50} = \mathbf{0.4047 \; m. \; Ans.}$$

(*iv*) *Width of the wheel at inlet* (B_1)

$$\eta_o = \frac{\text{S.P.}}{\text{W.P.}} = \frac{148.25}{\text{W.P.}}$$

But $$\text{W.P.} = \frac{\text{WH}}{1000} = \frac{\rho \times g \times Q \times H}{1000} = \frac{1000 \times 9.81 \times Q \times 7.62}{1000}$$

$$\therefore \quad \eta_o = \frac{148.25}{\frac{1000 \times 9.81 \times Q \times 7.62}{1000}} = \frac{148.25 \times 1000}{1000 \times 9.81 \times Q \times 7.62}$$

or $$Q = \frac{148.25 \times 1000}{1000 \times 9.81 \times 7.62 \times \eta_o} = \frac{148.25 \times 1000}{1000 \times 9.81 \times 7.62 \times 0.75} = 2.644 \text{ m}^3\text{/s}$$

Using equation (18.21), $$Q = \pi D_1 \times B_1 \times V_{f_1}$$

$$\therefore \quad 2.644 = \pi \times .4047 \times B_1 \times 11.738$$

$$\therefore \quad B_1 = \frac{2.644}{\pi \times .4047 \times 11.738} = \mathbf{0.177 \; m. \; Ans.}$$

Problem 18.24 *The following data is given for a Francis Turbine. Net head $H = 60$ m ; Speed $N = 700$ r.p.m.; shaft power = 294.3 kW ; $\eta_o = 84\%$; $\eta_h = 93\%$; flow ratio = 0.20 ; breadth ratio $n = 0.1$; Outer diameter of the runner = 2 × inner diameter of runner. The thickness of vanes occupy 5% of circumferential area of the runner, velocity of flow is constant at inlet and outlet and discharge is radial at outlet. Determine :*

(i) Guide blade angle, *(ii) Runner vane angles at inlet and outlet,*
(iii) Diameters of runner at inlet and outlet, and *(iv) Width of wheel at inlet.*

Solution. Given :

Net head, $H = 60$ m

Speed, $N = 700$ r.p.m.

Shaft power = 294.3 kW

Overall efficiency, $\eta_o = 84\% = 0.84$

Hydraulic efficiency, $\eta_h = 93\% = 0.93$

Flow ratio, $\dfrac{V_{f_1}}{\sqrt{2gH}} = 0.20$

$\therefore$ $V_{f_1} = 0.20 \times \sqrt{2gH}$

$= 0.20 \times \sqrt{2 \times 9.81 \times 60} = 6.862$ m/s

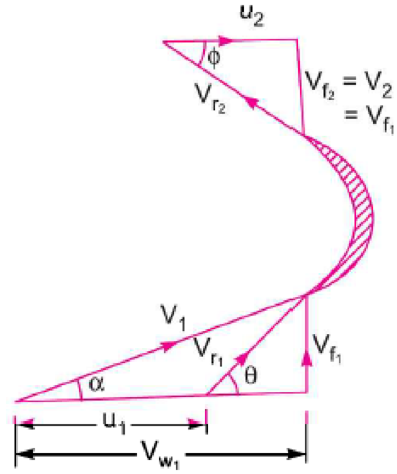


Fig. 18.22

Breadth ratio, $\dfrac{B_1}{D_1} = 0.1$

Outer diameter, $D_1 = 2 \times \text{Inner diameter} = 2 \times D_2$

Velocity of flow, $V_{f_1} = V_{f_2} = 6.862$ m/s.

Thickness of vanes = 5% of circumferential area of runner

$\therefore$ Actual area of flow $= 0.95\ \pi D_1 \times B_1$

Discharge at outlet = Radial

$\therefore$ $V_{w_2} = 0$ and $V_{f_2} = V_2$

Using relation, $\eta_o = \dfrac{\text{S.P.}}{\text{W.P.}}$

$$0.84 = \frac{294.3}{\text{W.P.}}$$

$\therefore$ $\text{W.P.} = \dfrac{294.3}{0.84} = 350.357$ kW.

But $\text{W.P.} = \dfrac{WH}{1000} = \dfrac{\rho \times g \times Q \times H}{1000} = \dfrac{1000 \times 9.81 \times Q \times 60}{1000}$

$\therefore$ $\dfrac{1000 \times 9.81 \times Q \times 60}{1000} = 350.357$

$\therefore$ $Q = \dfrac{350.357 \times 1000}{60 \times 1000 \times 9.81} = 0.5952\ \text{m}^3/\text{s}.$

Using equation (18.21), Q = Actual area of flow × Velocity of flow

$$= 0.95\ \pi D_1 \times B_1 \times V_{f_1}$$

$$= 0.95 \times \pi \times D_1 \times 0.1 D_1 \times V_{f_2} \qquad (\because \quad B_1 = 0.1\ D_1)$$

or

$$0.5952 = 0.95 \times \pi \times D_1 \times 0.1 \times D_1 \times 6.862 = 2.048\ D_1^2$$

$$\therefore \qquad D_1 = \sqrt{\frac{0.5952}{2.048}} = 0.54 \text{ m}$$

But

$$\frac{B_1}{D_1} = 0.1$$

$$\therefore \qquad B_1 = 0.1 \times D_1 = 0.1 \times .54 = .054 \text{ m} = 54 \text{ mm}$$

Tangential speed of the runner at inlet,

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.54 \times 700}{60} = 19.79 \text{ m/s.}$$

Using relation for hydraulic efficiency,

$$\eta_h = \frac{V_{w_1} u_1}{gH} \text{ or } 0.93 = \frac{V_{w_1} \times 19.79}{9.81 \times 60}$$

$$\therefore \qquad V_{w_1} = \frac{0.93 \times 9.81 \times 60}{19.79} = 27.66 \text{ m/s.}$$

(*i*) *Guide blade angle* (α)

From inlet velocity triangle, $\tan \alpha = \frac{V_{f_1}}{V_{w_1}} = \frac{6.862}{27.66} = 0.248$

$$\therefore \qquad \alpha = \tan^{-1} 0.248 = 13.928° \text{ or } \mathbf{13° \ 55.7'.\ Ans.}$$

(*ii*) *Runner vane angles at inlet and outlet* (θ *and* ϕ)

$$\tan \theta = \frac{V_{f_1}}{V_{w_1} - u_1} = \frac{6.862}{27.66 - 19.79} = 0.872$$

$$\therefore \qquad \theta = \tan^{-1} 0.872 = 41.09° \text{ or } \mathbf{41° \ 5.4'.\ Ans.}$$

From outlet velocity triangle, $\tan \phi = \frac{V_{f_2}}{u_2} = \frac{V_{f_1}}{u_2} = \frac{6.862}{u_2}$ ...(*i*)

But

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times D_1}{2} \times \frac{N}{60} \qquad \left(\because \quad D_2 = \frac{D_1}{2} \text{ given}\right)$$

$$= \pi \times \frac{.54}{2} \times \frac{700}{60} = 9.896 \text{ m/s.}$$

Substituting the value of u_2 in equation (*i*),

$$\tan \phi = \frac{6.862}{9.896} = 0.6934$$

$$\therefore \qquad \phi = \tan^{-1} .6934° = 34.74 \text{ or } \mathbf{34° \ 44.4'.\ Ans.}$$

(*iii*) *Diameters of runner at inlet and outlet*

$$D_1 = 0.54 \text{ m}, D_2 = \mathbf{0.27 \text{ m. Ans.}}$$

(*iv*) *Width of wheel at inlet*

$$B_1 = \mathbf{54 \text{ mm. Ans.}}$$

Problem 18.29 *A Kaplan turbine runner is to be designed to develop 9100 kW. The net available head is 5.6 m. If the speed ratio = 2.09, flow ratio = 0.68, overall efficiency = 86% and the diameter of the boss is 1/3 the diameter of the runner. Find the diameter of the runner, its speed and the specific speed of the turbine.*

Solution. Given :

Power, $P = 9100$ kW

Net head, $H = 5.6$ m

Speed ratio $= 2.09$

Flow ratio $= 0.68$

Overall efficiency, $\eta_o = 86\% = 0.86$

Diameter of boss $= \frac{1}{3}$ of diameter of runner

or $D_b = \frac{1}{3} D_o$

Now, speed ratio $= \frac{u_1}{\sqrt{2gH}}$

$\therefore$ $u_1 = 2.09 \times \sqrt{2 \times 9.81 \times 5.6} = 21.95$ m/s

Flow ratio $= \frac{V_{f_1}}{\sqrt{2gH}}$

$\therefore$ $V_{f_1} = 0.68 \times \sqrt{2 \times 9.81 \times 5.6} = 7.12$ m/s

The overall efficiency is given by, $\eta_o = \frac{P}{\left(\frac{\rho \times g.Q.H}{1000}\right)}$

or
$$Q = \frac{P \times 1000}{\rho \times g \times H \times \eta_o} = \frac{9100 \times 1000}{1000 \times 9.81 \times 5.6 \times 0.86} \qquad (\because \rho g = 1000 \times 9.81 \text{ N/m}^3)$$

$= 192.5$ m^3/s.

The discharge through a Kaplan turbine is given by

$$Q = \frac{\pi}{4}[D_o^2 - D_b^2] \times V_{f_1}$$

or
$$192.5 = \frac{\pi}{4}\left[D_o^2 - \left(\frac{D_o}{3}\right)^2\right] \times 7.12 \qquad \left(\because D_b = \frac{D_o}{3}\right)$$

$$= \frac{\pi}{4}\left[1 - \frac{1}{9}\right] D_o^2 \times 7.12$$

$\therefore$
$$D_o = \sqrt{\frac{4 \times 192.5 \times 9}{\pi \times 8 \times 7.12}} = \textbf{6.21 m. Ans.}$$

The speed of turbine is given by, $u_1 = \frac{\pi DN}{60}$

$\therefore$
$$N = \frac{60 \times u_1}{\pi \times D} = \frac{60 \times 21.95}{\pi \times 6.21} = \textbf{67.5 r.p.m. Ans.}$$

The specific speed is given by, $N_s = \frac{N\sqrt{P}}{H^{5/4}} = \frac{67.5 \times \sqrt{9100}}{5.6^{5/4}} = $ **746. Ans.**

Problem 18.39 *A Pelton wheel develops 8000 kW under a net head of 130 m at a speed of 200 r.p.m. Assuming the co-efficient of velocity for the nozzle 0.98, hydraulic efficiency 87%, speed ratio 0.46 and jet diameter to wheel diameter ratio* $\frac{1}{9}$*, determine :*

(i) the discharge required, *(ii) the diameter of the wheel,*
(iii) the diameter and number of jets required, and *(iv) the specific speed.*
Mechanical efficiency is 75%.

Solution. Given :

Power developed, $P = 8000$ kW

Net head, $H = 130$ m

Speed, $N = 200$ r.p.m.

Co-efficient of velocity, $C_v = 0.98$

Hydraulic efficiency, $\eta_h = 87\% = 0.87$

Speed ratio, $\frac{u_1}{\sqrt{2gH}} = 0.46$

$$\therefore \quad u_1 = 0.46 \times \sqrt{2gH} = 0.46 \times \sqrt{2 \times 9.81 \times 130} = 23.23 \text{ m/s.}$$

Jet diameter to wheel diameter $= \frac{d}{D} = \frac{1}{9}$

Mechanical efficiency, $\eta_m = 75\% = 0.75$

Overall efficiency is given by equation (18.6) as

$$\eta_o = \eta_h \times \eta_m = 0.87 \times 0.75 = 0.6525$$

Also $$\eta_o = \frac{\text{Power developed}}{\text{Water power}} = \frac{8000}{\text{W.P. in kW}} \quad \text{or} \quad 0.6525 = \frac{8000}{\text{W. P. in kW}}$$

$$\therefore \quad \text{W.P. in kW} = \frac{8000}{0.6525} = 12260.536 \text{ kW}$$

But W.P. in kW $$= \frac{\rho \times g \times Q \times H}{1000}$$

$$= \frac{1000 \times 9.81 \times Q \times H}{1000} \qquad (\because \rho g \text{ in S.I.} = 1000 \times 9.81)$$

$$= Q \times H \times 9.81 = Q \times 130 \times 9.81$$

$$\therefore \quad 12260.536 = Q \times 130 \times 9.81$$

$$\therefore \quad Q = \frac{12260.536}{130 \times 9.81} = \mathbf{9.614 \text{ m}^3\text{/s. Ans.}}$$

(*i*) *Discharge required*

$$Q = \mathbf{9.614 \text{ m}^3\text{/s. Ans.}}$$

(*ii*) *Diameter of wheel (D)*

Using the relation, $$u_1 = \frac{\pi D N}{60}$$

$$\therefore \quad D = \frac{60 \times u_1}{\pi \times N} = \frac{60 \times 23.23}{\pi \times 200} = \mathbf{2.218 \text{ m. Ans.}}$$

(iii) Diameter of jet (d) and number of jets required

$$\frac{d}{D} = \frac{1}{9}$$

$\therefore$ $$d = \frac{D}{9} = \frac{2.218}{9} = 0.2464 \text{ m} = \mathbf{246.4 \text{ mm. Ans.}}$$

$\therefore$ Area of jet, $$a = \frac{\pi}{4} d^2 = \frac{\pi}{4}(.2464)^2 = .04768 \text{ m}^2.$$

Velocity of jet is given by, $V_1 = C_v\sqrt{2gH} = 0.98\sqrt{2 \times 9.81 \times 130} = 49.49$ m/s

$\therefore$ Discharge through one jet $= \text{Area of jet} \times \text{Velocity of jet} = a \times V_1$

$$= .04768 \times 49.49 = 2.359 \text{ m}^3/\text{s}$$

$\therefore$ Number of jets $$= \frac{\text{Total discharge}}{\text{Discharge through one jet}}$$

$$= \frac{Q}{2.359} = \frac{9.614}{2.359} = 4.07 \text{ say } \mathbf{4.0. \text{ Ans.}}$$

(*iv*) *Specific speed* is given by equation (18.28) as

N_s (S.I. units) $$= \frac{N\sqrt{P}}{H^{5/4}} = \frac{200 \times \sqrt{8000}}{130^{5/4}} = \frac{17888.54}{438.96} = \mathbf{40.75. \text{ Ans.}}$$

$\therefore$ Force exerted in the direction of motion,

F_x = Mass of water striking the vane per second $\times$ (Initial velocity with which the jet strikes in the *direction of motion* – final velocity)

$$= \rho a V_{r1}[V_{r1}\cos\theta - (-V_{r2}\cos\phi)]$$

But, $V_{r1}\cos\theta = (V_{w1} - u_1)$, and $V_{r2}\cos\phi = (u_2 + V_{w2})$ (Refer to Fig. 1.17)

$\therefore$ $$F_x = \rho a V_{r1}[(V_{w1} - u_1) - \{-(u_2 + V_{w2})\}]$$

$$= \rho a V_{r1}[V_{w1} - u_1 + u_2 + V_{w2}]$$

or, $$F_x = \rho a V_{r1}(V_{w1} + V_{w2}) \quad (\because u_1 = u_2) \quad ...(i)$$

The eqn. (*i*) is *true only when* β *is an actue angle* (See Fig. 1.17), When β = 90°, $V_{w2} = 0$ the eqn. (*i*) redues to

$$F_x = \rho a V_{r1}(V_{w1})$$

If β is an *obtuse angle,* the expression for F_x will become

$$F_x = \rho a V_{r1}(V_{w1} - V_{w2})$$

Thus, in general F_x is written as:

$$F_x = \rho a V_{r1}(V_{w1} \pm V_{w2}) \quad ...(1.27)$$

2.3.2. Work done and Efficiency of a Pelton Wheel

Fig. 2.5 shows the velocity triangles.

Let, N = Speed of wheel in r.p.m.,

D = Diameter of the wheel,

d = Diameter of the jet,

u = Peripheral (or circumferential) velocity of runner. It will be same at inlet and outlet of the runners at the mean pitch. (*i.e.* $u = u_1 = u_2$)

$$= \frac{\pi DN}{60},$$

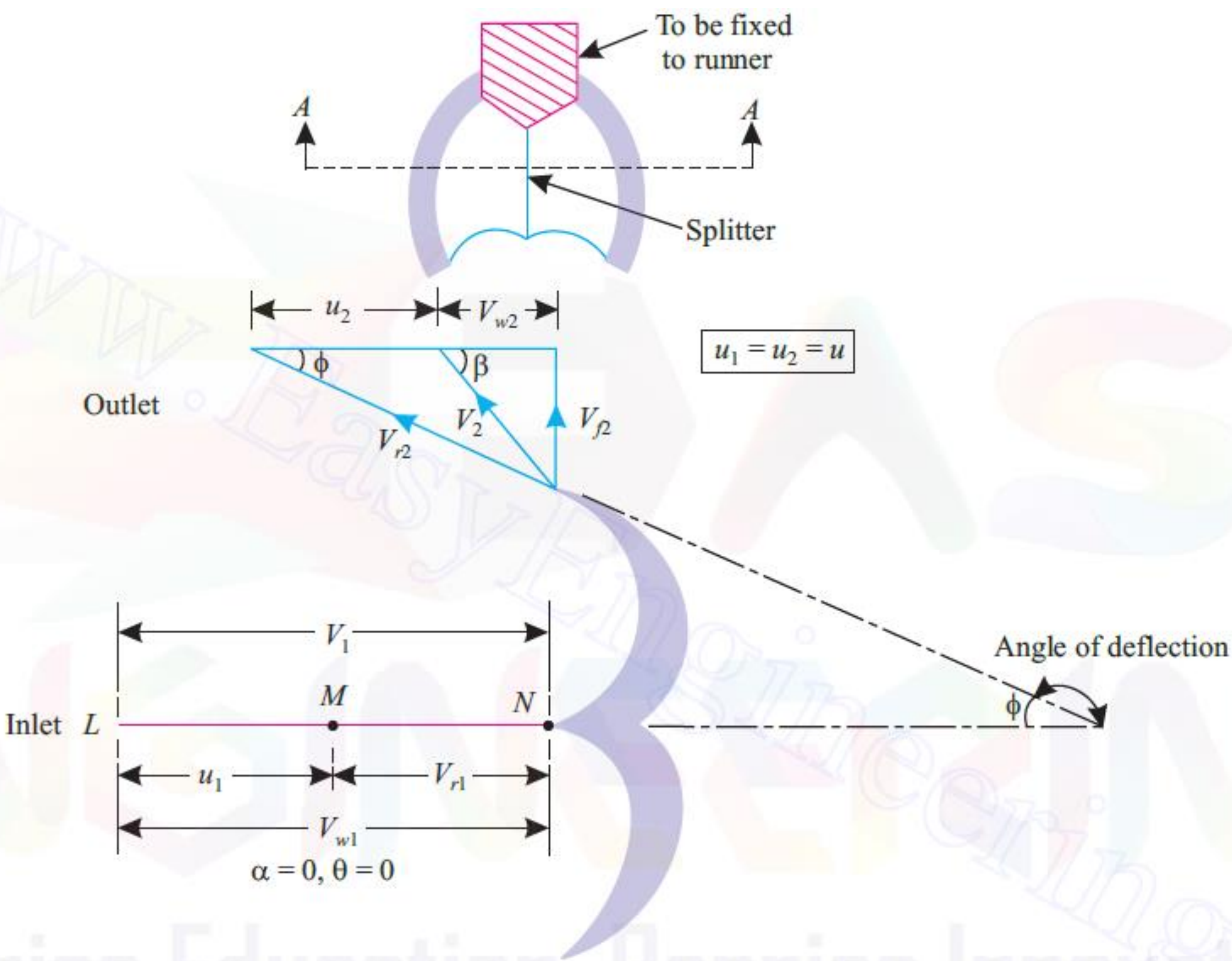


Fig. 2.5. Velocity triangles.

V_1 = Absolute velocity of water at inlet,

V_{r1} = Jet velocity relative to vane/bucket at inlet,

α = Angle between the direction of the jet and direction of motion of the vane/ bucket (also called *guide angle*),

θ = Angle made by the relative velocity (V_{r1}) with the direction of motion at inlet (also called *vane angle at inlet*),

V_{w1} and V_{f1} = The components of the velocity of the jet V_1, in direction of motion and perpendicular to the direction of motion of the vane respectively;

V_{w1} is also known as *velocity of whirl* at inlet,

V_{f1} is also known as *velocity of flow* at inlet,

V_2 = Velocity of jet, leaving the vane or velocity of jet at outlet of the vane,

V_{r2} = Relative velocity of the jet with respect to the vane at outlet,

ϕ = Angle made by the relative velocity V_{r2} with the direction of motion of the vane at outlet and also called *vane angle at outlet*,

β = Angle made by the velocity V_2 with the direction of motion of the vane at outlet, and

V_{w2} and V_{f2} = Components of the velocity V_2, in the direction of motion of vane and perpendicular to the direction of motion of vane at outlet;

V_{w2} is also called the *velocity of whirl at outlet*, and

V_{f2} is also called the *velocity of flow at outlet*.

Inlet. The velocity triangle at *inlet* will be a *straight line* where

$$V_{r1} = V_1 - u_1 = V_1 - u, \; V_{w1} = V_1 \qquad (\because u_1 = u_2 = u)$$

$$\alpha = 0 \text{ and } \theta = 0$$

Outlet : From velocity triangle at outlet, we have

$$V_{r2} = KV_{r1},$$

[where, K = blade friction co-efficient, *slightly less than unity*. Idealy when bucket surfaces are *perfectly smooth* and energy losses due to impact at splitter are *neglected*, $K = 1$]

and, $$V_{w2} = V_{r2} \cos\phi - u_2 = V_{r2} \cos\phi - u \qquad (\because u_1 = u_2 = u) \text{ (When } \beta < 90°)$$

The energy supplied to the jet at inlet is in the form of K.E. and is equal to $\frac{1}{2} mV_1^2$.

$\therefore$ Kinetic energy (K.E.) of jet per second = $\frac{1}{2}(\rho a V_1) \times V_1^2$

$$\therefore \quad \text{Hydraulic efficiency, } \eta_h = \frac{\text{Work done per second}}{\text{K.E. of jet per second}} = \frac{\rho a V_1 (V_{w1} + V_{w2}) \times u}{\frac{1}{2}(\rho a V_1) \times V_1^2}$$

or, $$\eta_h = \frac{2(V_{w1} + V_{w2}) \times u}{V_1^2}$$

The force exerted by the jet of water in the direction of motion is given as:

$$F = \rho a V_1 (V_{w1} + V_{w2})$$

$$\left[\rho \text{ and } a \text{ are the mass density and area of jet } \left(a = \frac{\pi}{4} d^2\right) \text{ respecitvely.}\right]$$

Now work done by the jet on runner per second

$$= F \times u = \rho a V_1 (V_{w1} + V_{w2}) \times u$$

Work done per second per unit weight of water striking

$$= \frac{\rho a V_1 (V_{w1} + V_{w2}) \times u}{\text{Weight of water striking}} = \frac{\rho a V_1 (V_{w1} + V_{w2}) \times u}{\rho a V_1 \times g}$$

$$= \frac{1}{g} (V_{w1} + V_{w2})\, u$$

Co-efficient of velocity, C_v

Speed ratio, K_u

Velocity of jet, $V_1 = C_v \sqrt{2gH}$

Velocity of wheel, $u = K_u \sqrt{2gH}$

(Assume $C_v = 1.0$ if not given)

Work done by the jet on the runner per second

$$= \rho Q (V_{w1} + V_{w2}) \times u$$

Mean diameter of Pelton wheel, D

Speed of wheel, N

Tangential velocity of the wheel, $u = \frac{\pi D N}{60}$

Power available at the nozzle : $= wQH$ **kW**

Gross head, H_g

Head lost in friction, $h_f = \frac{H_g}{3}$

Net head, $H = H_g - h_f$

Hydraulic efficiency of the Pelton wheel, η_h :

$$\eta_h = \frac{2 (V_{w1} + V_{w2}) \times u}{V_1^2}$$

Alternatively :

$$\eta_h = \frac{(V_{w1} + V_{w2})\, u}{gH}$$

Hydraulic efficiency, η_h :

$$\eta_h = \frac{2 (V_1 - u)(1 + K \cos \phi)\, u}{V_1^2}$$

18.6.2 Points to be Remembered for Pelton Wheel

(*i*) The velocity of the jet at inlet is given by $V_1 = C_v\sqrt{2gH}$

where C_v = Co-efficient of velocity = 0.98 or 0.99

H = Net head on turbine

(*ii*) The velocity of wheel (u) is given by $u = \phi\sqrt{2gH}$

where ϕ = Speed ratio. The value of speed ratio varies from 0.43 to 0.48.

(*iii*) The angle of deflection of the jet through buckets is taken at 165° if no angle of deflection is given.

(*iv*) The mean diameter or the pitch diameter D of the Pelton wheel is given by

$$u = \frac{\pi DN}{60} \text{ or } D = \frac{60u}{\pi N}.$$

(*v*) **Jet Ratio.** It is defined as the ratio of the pitch diameter (D) of the Pelton wheel to the diameter of the jet (d). It is denoted by 'm' and is given as

$$m = \frac{D}{d} \text{ (= 12 for most cases)} \quad \text{...(18.16)}$$

(*vi*) Number of buckets on a runner is given by

$$Z = 15 + \frac{D}{2d} = 15 + 0.5 \text{ m} \quad \text{...(18.17)}$$

where m = Jet ratio

18.6.1 Velocity Triangles and Work done for Pelton Wheel. Fig. 18.5 shows the shape of the vanes or buckets of the Pelton wheel. The jet of water from the nozzle strikes the bucket at the splitter, which splits up the jet into two parts. These parts of the jet, glides over the inner surfaces and comes out at the outer edge. Fig. 18.5 (*b*) shows the section of the bucket at *Z-Z*. The splitter is the inlet tip and outer edge of the bucket is the outlet tip of the bucket. The inlet velocity triangle is drawn at the splitter and outlet velocity triangle is drawn at the outer edge of the bucket, by the same method as explained in Chapter 17.

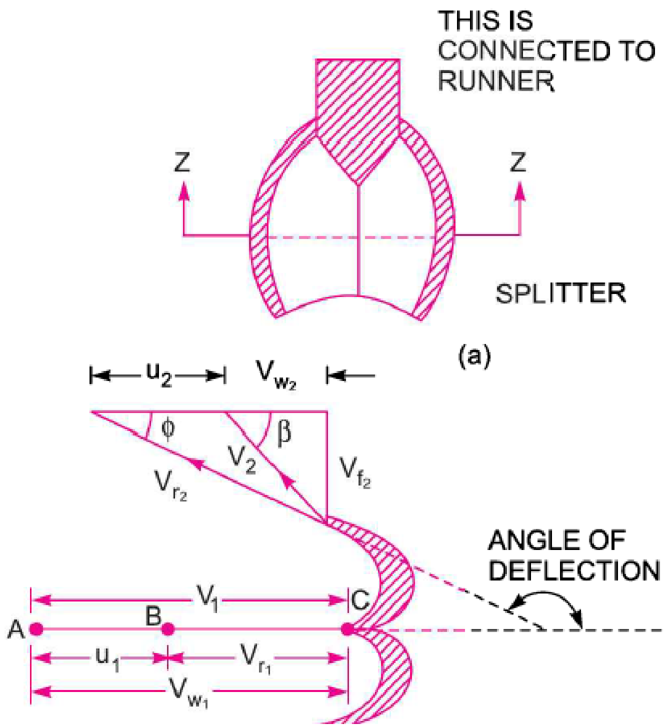


Fig. 18.5 *Shape of bucket.*

Let H = Net head acting on the Pelton wheel

$$= H_g - h_f$$

where H_g = Gross head and $h_f = \dfrac{4fLV^2}{D^* \times 2g}$

where D^* = Dia. of Penstock, N = Speed of the wheel in r.p.m.,
D = Diameter of the wheel, d = Diameter of the jet.

Then

$$V_1 = \text{Velocity of jet at inlet} = \sqrt{2gH} \quad \text{...(18.7)}$$

$$u = u_1 = u_2 = \frac{\pi DN}{60}.$$

The velocity triangle at inlet will be a straight line where

$$V_{r_1} = V_1 - u_1 = V_1 - u$$

$$V_{w_1} = V_1$$

$$\alpha = 0° \text{ and } \theta = 0°$$

From the velocity triangle at outlet, we have

$$V_{r_2} = V_{r_1} \text{ and } V_{w_2} = V_{r_2} \cos \phi - u_2.$$

The force exerted by the jet of water in the direction of motion is given by equation (17.19) as

$$F_x = \rho a V_1 \left[V_{w_1} + V_{w_2}\right] \qquad \text{...(18.8)}$$

As the angle β is an acute angle, +ve sign should be taken. Also this is the case of series of vanes, the mass of water striking is $\rho a V_1$ and not $\rho a V_{r_1}$. In equation (18.8), 'a' is the area of the jet which is given as

$$a = \text{Area of jet} = \frac{\pi}{4} d^2.$$

Now work done by the jet on the runner per second

$$= F_x \times u = \rho a V_1 \left[V_{w_1} + V_{w_2}\right] \times u \text{ Nm/s} \qquad \text{...(18.9)}$$

Power given to the runner by the jet

$$= \frac{\rho a V_1 \left[V_{w_1} + V_{w_2}\right] \times u}{1000} \text{ kW} \qquad \text{...(18.10)}$$

Work done/s per unit weight of water striking/s

$$= \frac{\rho a V_1 \left[V_{w_1} + V_{w_2}\right] \times u}{\text{Weight of water striking/s}}$$

$$= \frac{\rho a V_1 \left[V_{w_1} + V_{w_2}\right] \times u}{\rho a V_1 \times g} = \frac{1}{g}\left[V_{w_1} + V_{w_2}\right] \times u \qquad \text{...(18.11)}$$

The energy supplied to the jet at inlet is in the form of kinetic energy and is equal to $\frac{1}{2} m V^2$

$\therefore$ K.E. of jet per second $= \frac{1}{2} (\rho a V_1) \times V_1^2$

$\therefore$ **Hydraulic efficiency**, $\eta_h = \dfrac{\text{Work done per second}}{\text{K.E. of jet per second}}$

$$= \frac{\rho a V_1 \left[V_{w_1} + V_{w_2}\right] \times u}{\frac{1}{2} (\rho a V_1) \times V_1^2} = \frac{2\left[V_{w_1} + V_{w_2}\right] \times u}{V_1^2} \qquad \text{...(18.12)}$$

Problem 1 *The internal and external diameters of the impeller of a centrifugal pump are 200 mm and 400 mm respectively. The pump is running at 1200 r.p.m. The vane angles of the impeller at inlet and outlet are 20° and 30° respectively. The water enters the impeller radially and velocity of flow is constant. Determine the work done by the impeller per unit weight of water.*

Solution. Given :

Internal diameter of impeller, $D_1 = 200$ mm $= 0.20$ m

External diameter of impeller, $D_2 = 400$ mm $= 0.40$ m

Speed, $N = 1200$ r.p.m.

Vane angle at inlet, $\theta = 20°$

Vane angle at outlet, $\phi = 30°$

Water enters radially* means, $\alpha = 90°$ and $V_{w_1} = 0$

Velocity of flow, $V_{f_1} = V_{f_2}$

Tangential velocity of impeller at inlet and outlet are,

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.20 \times 1200}{60} = 12.56 \text{ m/s}$$

and $$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.4 \times 1200}{60} = 25.13 \text{ m/s.}$$

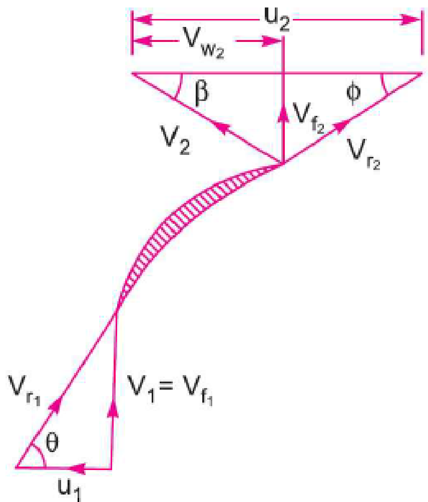


Fig. 19.4

From inlet velocity triangle, $\tan\theta = \frac{V_{f_1}}{u_1} = \frac{V_{f_1}}{12.56}$

$\therefore$ $$V_{f_1} = 12.56 \tan\theta = 12.56 \times \tan 20° = 4.57 \text{ m/s}$$

$\therefore$ $$V_{f_2} = V_{f_1} = 4.57 \text{ m/s.}$$

From outlet velocity triangle, $\tan\phi = \frac{V_{f_2}}{u_2 - V_{w_2}} = \frac{4.57}{25.13 - V_{w_2}}$

or $$25.13 - V_{w_2} = \frac{4.57}{\tan\phi} = \frac{4.57}{\tan 30°} = 7.915$$

$\therefore$ $$V_{w_2} = 25.13 - 7.915 = 17.215 \text{ m/s.}$$

The work done by impeller per kg of water per second is given by equation (19.1) as

$$= \frac{1}{g} V_{w_2} u_2 = \frac{17.215 \times 25.13}{9.81} = \textbf{44.1 Nm/N. Ans.}$$

Problem 2 *A centrifugal pump delivers water against a net head of 14.5 metres and a design speed of 1000 r.p.m. The vanes are curved back to an angle of 30° with the periphery. The impeller diameter is 300 mm and outlet width is 50 mm. Determine the discharge of the pump if manometric efficiency is 95%.*

Solution. Given :

Net head, $H_m = 14.5$ m

Speed, $N = 1000$ r.p.m.

Vane angle at outlet, $\phi = 30°$

Impeller diameter means the diameter of the impeller at outlet

$\therefore$ Diameter, $D_2 = 300$ mm = 0.30 m

Outlet width, $B_2 = 50$ mm = 0.05 m

Manometric efficiency, $\eta_{man} = 95\% = 0.95$

Tangential velocity of impeller at outlet,

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.30 \times 1000}{60} = 15.70 \text{ m/s.}$$

Now using equation (19.8), $\eta_{man} = \dfrac{gH_m}{V_{w_2} \times u_2}$

$$\therefore \qquad 0.95 = \frac{9.81 \times 14.5}{V_{w_2} \times 15.70}$$

$$\therefore \qquad V_{w_2} = \frac{0.95 \times 14.5}{0.95 \times 15.70} = 9.54 \text{ m/s.}$$

Refer to Fig. 19.5. From outlet velocity triangle, we have

$$\tan \phi = \frac{V_{f_2}}{(u_2 - V_{w_2})} \text{ or } \tan 30° = \frac{V_{f_2}}{(15.70 - 9.54)} = \frac{V_{f_2}}{6.16}$$

$$\therefore \qquad V_{f_2} = 6.16 \times \tan 30° = 3.556 \text{ m/s.}$$

$\therefore$ Discharge, $Q = \pi D_2 B_2 \times V_{f_2}$

$= \pi \times 0.30 \times 0.05 \times 3.556$ m^3/s = **0.1675 m^3/s. Ans.**

Problem 3 *A centrifugal pump having outer diameter equal to two times the inner diameter and running at 1000 r.p.m. works against a total head of 40 m. The velocity of flow through the impeller is constant and equal to 2.5 m/s. The vanes are set back at an angle of 40° at outlet. If the outer diameter of the impeller is 500 mm and width at outlet is 50 mm, determine* :

(*i*) *Vane angle at inlet,* (*ii*) *Work done by impeller on water per second, and*

(*iii*) *Manometric efficiency.*

Solution. Given :

Speed, $N = 1000$ r.p.m.

Head, $H_m = 40$ m

Velocity of flow, $V_{f_1} = V_{f_2} = 2.5$ m/s

Vane angle at outlet, $\phi = 40°$

Outer dia. of impeller, $D_2 = 500$ mm = 0.50 m

Inner dia. of impeller, $D_1 = \dfrac{D_2}{2} = \dfrac{0.50}{2} = 0.25$ m

Width at outlet, $B_2 = 50$ mm = 0.05 m

Tangential velocity of impeller at inlet and outlet are

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.25 \times 1000}{60} = 13.09 \text{ m/s}$$

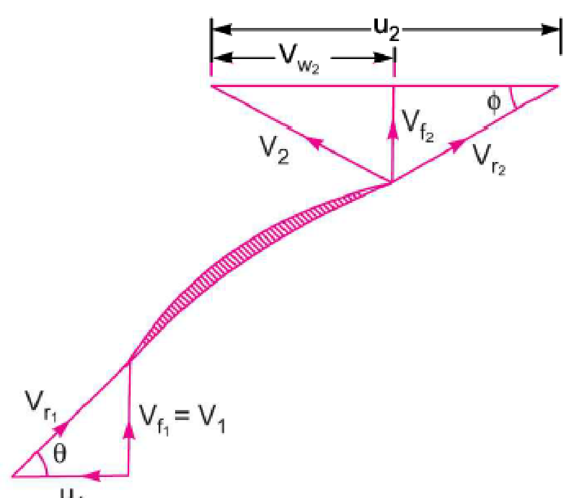


Fig.

and $u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.50 \times 1000}{60} = 26.18$ m/s.

Discharge is given by, $Q = \pi D_2 B_2 \times V_{f_2} = \pi \times 0.50 \times .05 \times 2.5 = 0.1963 \text{ m}^3/\text{s}.$

(*i*) **Vane angle at inlet (θ).**

From inlet velocity triangle $\tan \theta = \frac{V_{f_1}}{u_1} = \frac{2.5}{13.09} = 0.191$

$$\therefore \quad \theta = \tan^{-1} .191 = 10.81° \text{ or } \mathbf{10° \ 48'}. \quad \textbf{Ans.}$$

(*ii*) **Work done by impeller on water per second** is

$$= \frac{W}{g} \times V_{w_2} u_2 = \frac{\rho \times g \times Q}{g} \times V_{w_2} \times u_2$$

$$= \frac{1000 \times 9.81 \times 0.1963}{9.81} \times V_{w_2} \times 26.18 \qquad ...(i)$$

But from outlet velocity triangle, we have

$$\tan \phi = \frac{V_{f_2}}{u_2 - V_{w_2}} = \frac{2.5}{(26.18 - V_{w_2})}$$

$$\therefore \quad 26.18 - V_{w_2} = \frac{2.5}{\tan \phi} = \frac{2.5}{\tan 40°} = 2.979$$

$$\therefore \quad V_{w_2} = 26.18 - 2.979 = 23.2 \text{ m/s.}$$

Substituting this value of V_{w_2} in equation (*i*), we get the work done by impeller as

$$= \frac{1000 \times 9.81 \times 0.1963}{9.81} \times 23.2 \times 26.18$$

$$= \mathbf{119227.9 \ Nm/s.} \quad \textbf{Ans.}$$

(*iii*) **Manometric efficiency (η_{man}).** Using equation (19.8), we have

$$\eta_{man} = \frac{gH_m}{V_{w_2} u_2} = \frac{9.81 \times 40}{23.2 \times 26.18} = 0.646 = \mathbf{64.4\%}. \ \textbf{Ans.}$$

Problem 4 *A centrifugal pump discharges 0.15 m³/s of water against a head of 12.5 m, the speed of the impeller being 600 r.p.m. The outer and inner diameters of impeller are 500 mm and 250 mm respectively and the vanes are bent back at 35° to the tangent at exit. If the area of flow remains 0.07 m² from inlet to outlet, calculate :*

(i) Manometric efficiency of pump, (ii) Vane angle at inlet, and

(iii) Loss of head at inlet to impeller when the discharge is reduced by 40% without changing the speed.

Solution. Given :

Discharge, $Q = 0.15 \text{ m}^3/\text{s}$

Head, $H_m = 12.5 \text{ m}$

Speed, $N = 600$ r.p.m.

Outer dia., $D_2 = 500 \text{ mm} = 0.50 \text{ m}$

Inner dia., $D_1 = 250 \text{ mm} = 0.25 \text{ m}$

Vane angle at outlet, $\phi = 35°$

Area of flow, $= 0.07 \text{ m}^2$

As area of flow is constant from inlet to outlet, then velocity of flow will be constant from inlet to outlet.

Discharge = Area of flow × Velocity of flow

or 0.15 = 0.07 × Velocity of flow

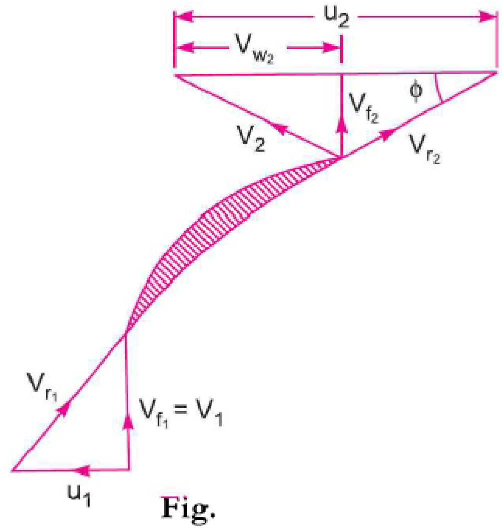


Fig.

$$\therefore \quad \text{Velocity of flow} = \frac{0.15}{0.07} = 2.14 \text{ m/s.}$$

$$\therefore \quad V_{f_1} = V_{f_2} = 2.14 \text{ m/s.}$$

Tangential velocity of impeller at inlet and outlet are

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.25 \times 600}{60} = 7.85 \text{ m/s}$$

and

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.50 \times 600}{60} = 15.70 \text{ m/s}$$

From outlet velocity triangle, $V_{w_2} = u_2 - \frac{V_{f_2}}{\tan \phi} = 15.70 - \frac{2.14}{\tan 35°} = 12.64 \text{ m/s}$

(*i*) *Manometric efficiency of the pump*

Using equation (19.8), we have $\eta_{man} = \frac{g \times H_m}{V_{w_2} \times u_2} = \frac{9.81 \times 12.5}{12.64 \times 15.7} = 0.618$ or **61.8%. Ans.**

(*ii*) *Vane angle at inlet* (θ)

From inlet velocity triangle, $\tan \theta = \frac{V_{f_1}}{u_1} = \frac{2.14}{7.85} = 0.272$

$$\therefore \quad \theta = \tan^{-1} 0.272 = \mathbf{15° \; 12'}. \textbf{ Ans.}$$

(*iii*) *Loss of head at inlet to impeller when discharge is reduced by 40% without changing the speed.*

When there is an increase or decrease in the discharge from the normal discharge, a loss of head occurs at entry due to shock. In this case, discharge is reduced by 40%. Hence the new discharge is given by,

$$Q^* = 0.6 \times Q$$

where $Q = 0.15\ \text{m}^3/\text{s}$

As area of flow is constant, hence new velocity of flow $(V_{f_1}{}^*)$ will be given by,

$$V_{f_1}{}^* = \frac{Q^*}{\text{Area of flow}}$$

$$= \frac{0.6 \times Q}{0.07} = \frac{0.6 \times 0.15}{0.07} = 1.284\ \text{m/s.}$$

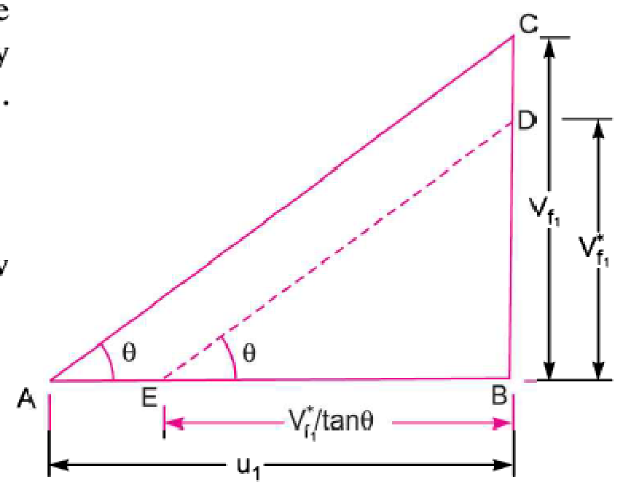


Fig. 19.6 (*b*)

Fig. 19.6 (*b*) shows the velocity triangle at inlet corresponding to normal discharge and reduced discharge. *ABC* is the velocity triangle due to normal discharge. Triangle *BDE* is corresponding to reduced discharge *BD* = 1.284 m/s and *DE* is parallel to *AC*.

The blade angle θ at inlet cannot change and hence *DE* will be parallel to *AC*.

There will be a sudden change in the tangential velocity from *AB* to *BE*. Hence due to this shock, there will be a loss of head at inlet.

$$\therefore \quad \text{Head lost at inlet} = \frac{(\text{change in tangential velocity at inlet})^2}{2g}$$

$$= \frac{(AB - BE)^2}{2g} = \frac{\left(u_1 - \dfrac{V_{f_1}^*}{\tan\theta}\right)^2}{2g} = \frac{\left(7.85 - \dfrac{1.284}{\tan 15.2°}\right)^2}{2 \times 9.81} = \mathbf{0.5\ m.\ Ans.}$$

Problem 5 *A centrifugal pump has the following dimensions : inlet radius = 80 mm ; outlet radius = 160 mm ; width of impeller at the inlet = 50 mm ;* $\beta_1 = 0.45$ *radians ;* $\beta_2 = 0.25$ *radians ; width of impeller at outlet = 50 mm.*

Assuming shockless entry determine the discharge and the head developed by the pump when the impeller rotates at 90 radians/second.

Solution. Given :

Inlet radius,	$R_1 = 80\ \text{mm} = 0.08\ \text{m}$
Outlet radius	$R_2 = 160\ \text{mm} = 0.16\ \text{m}$
Width at inlet,	$B_1 = 50\ \text{mm} = 0.05\ \text{m}$
Width at outlet,	$B_2 = 50\ \text{mm} = 0.05\ \text{m}$
Angles,	$\beta_1 = 0.45$ radians and $\beta_2 = 0.25$ radians.

Here β_1 is the vane angle at inlet and β_2 is the vane angle at outlet.

∴ Vane angle at inlet, $\theta = \beta_1 = 0.45$ radians

Vane angle at outlet, $\phi = \beta_2 = 0.25$ radians.

Angular velocity, $\omega = 90$ rad/s

Find :

(*i*) discharge, and (*ii*) head developed.

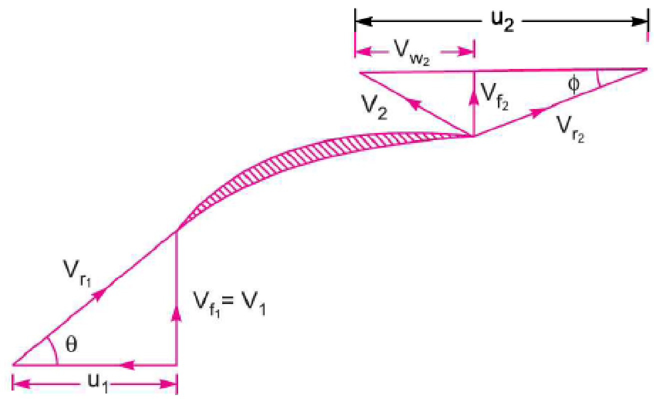


Fig.

Now tangential velocity of impeller at inlet and outlet are,

$$u_1 = \frac{\pi D_1 \times N}{60} = \frac{2\pi N}{60} \times \frac{D_1}{2} = \omega \times R_1 = 90 \times 0.08 = 7.2 \text{ m/s}$$

and $$u_2 = \omega \times R_2 = 90 \times 0.16 = 14.4 \text{ m/s}$$

From inlet velocity triangle, $\tan\theta = \dfrac{V_{f_1}}{u_1}$

∴ $$V_{f_1} = u_1 \times \tan\theta = 7.2 \times \tan(0.45 \text{ rad}) = 7.2 \times 0.483 = 3.478 \text{ m/s}$$

(*i*) *Discharge* (*Q*)

Discharge is given by, $$Q = \pi D_1 \times B_1 \times V_{f_1} = \pi \times (2R_1) \times B_1 \times V_{f_1}$$
$$= \pi \times 2 \times 0.08 \times 0.05 \times 3.478 \text{ m}^3/\text{s} = \mathbf{0.0874\ m^3/s.\ Ans.}$$

(*ii*) *Head developed* (H_m)

For the shockless entry, the losses of the pump will be zero. Hence, the head developed (H_m) will be given by equation (19.5).

∴ $$H_m = \frac{V_{w_2} \times u_2}{g} \qquad \text{...}(i)$$

where from outlet velocity triangle, $V_{w_2} = u_2 - V_{f_2} \times \cot\phi$

The value of V_{f_2} is obtained from $Q = \pi D_2 \times B_2 \times V_{f_2}$

or $$0.0874 = \pi \times (2R_2) \times B_2 \times V_{f_2}$$
$$= \pi \times (2 \times 0.16) \times 0.05 \times V_{f_2}$$

$$V_{f_2} = \frac{0.0874}{\pi \times 2 \times 0.16 \times 0.05} = 1.7387 \text{ m/s}$$

$$V_{w_2} = u_2 - V_{f_2} \times \cot\phi$$

$$= 14.4 - 1.7387 \times \cot(0.25 \text{ radians})$$
$$= 14.4 - 1.7387 \times 3.9163 = 14.4 - 6.809 = 7.591 \text{ m/s}$$

Substituting this value in equation (*i*) above, we get

$$H_m = \frac{V_{w_2} \times u_2}{g} = \frac{7.591 \times 14.4}{9.81} = \mathbf{11.142\ m.\ Ans.}$$

6. What is a reciprocating pump ? Describe the principle and working of a reciprocating pump with a neat sketch. Why is a reciprocating pump not coupled directly to the motor ? Discuss the reason in detail.

MAIN PARTS OF A RECIPROCATING PUMP

The following are the main parts of a reciprocating pump as shown in Fig.

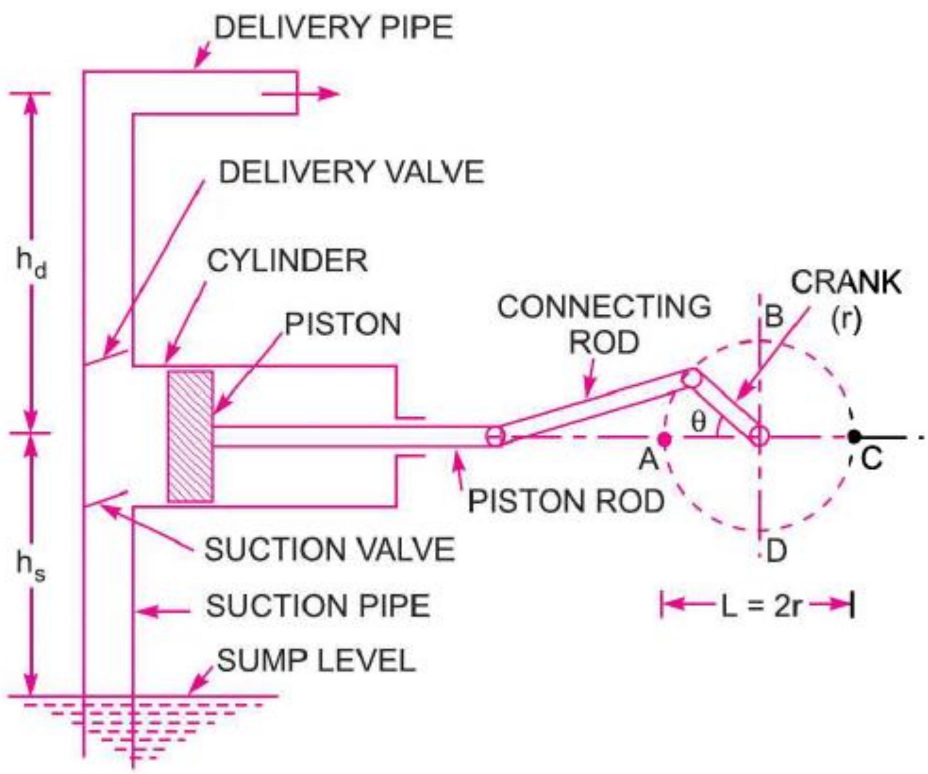


Fig. *Main parts of a reciprocating pump.*

1. A cylinder with a piston, piston rod, connecting rod and a crank,
2. Suction pipe,
3. Delivery pipe,
4. Suction valve, and
5. Delivery valve.

WORKING OF A RECIPROCATING PUMP

Fig. shows a single acting reciprocating pump, which consists of a piston which moves forwards and backwards in a close fitting cylinder. The movement of the piston is obtained by connecting the piston rod to crank by means of a connecting rod. The crank is rotated by means of an electric motor. Suction and delivery pipes with suction valve and delivery valve are connected to the cylinder. The suction and delivery valves are one way valves or non-return valves, which allow the water to flow in one direction only. Suction valve allows water from suction pipe to the cylinder which delivery valve allows water from cylinder to delivery pipe only.

When crank starts rotating, the piston moves to and fro in the cylinder. When crank is at *A*., the piston is at the extreme left position in the cylinder. As the crank is rotating from *A* to *C*, (*i.e.*, from $\theta = 0°$ to $\theta = 180°$), the piston is moving towards right in the cylinder. The movement of the piston towards right creates a partial vacuum in the cylinder. But on the surface of the liquid in the sump atmospheric pressure is acting, which is more than the pressure inside the cylinder. Thus, the liquid is forced in the suction pipe from the sump. This liquid opens the suction valve and enters the cylinder.

When crank is rotating from *C* to *A* (*i.e.*, from $\theta = 180°$ to $\theta = 360°$), the piston from its extreme right position starts moving towards left in the cylinder. The movement of the piston towards left increases the pressure of the liquid inside the cylinder more than atmospheric pressure. Hence suction valve closes and delivery valve opens. The liquid is forced into the delivery pipe and is raised to a required height.

Problem 7 *A single-acting reciprocating pump, running at 50 r.p.m., delivers 0.01 m³/s of water. The diameter of the piston is 200 mm and stroke length 400 mm. Determine :*

(i) The theoretical discharge of the pump, (ii) Co-efficient of discharge, and (iii) Slip and the percentage slip of the pump.

Solution. Given :

Speed of the pump, $N = 50$ r.p.m.

Actual discharge, $Q_{act} = .01 \text{ m}^3/\text{s}$

Dia. of piston, $D = 200 \text{ mm} = .20 \text{ m}$

∴ Area, $A = \frac{\pi}{4}(.2)^2 = .031416 \text{ m}^2$

Stroke, $L = 400 \text{ mm} = 0.40 \text{ m}$.

(*i*) Theoretical discharge for single-acting reciprocating pump is given by equation (20.1) as

$$Q_{th} = \frac{A \times L \times N}{60} = \frac{.031416 \times .40 \times 50}{60} = \mathbf{0.01047 \text{ m}^3/\text{s. Ans.}}$$

(*ii*) Co-efficient of discharge is given by

$$C_d = \frac{Q_{act}}{Q_{th}} = \frac{0.01}{.01047} = \mathbf{0.955. \text{ Ans.}}$$

(*iii*) Using equation (20.8), we get

$$\text{Slip} = Q_{th} - Q_{act} = .01047 - .01 = \mathbf{0.00047 \text{ m}^3/\text{s. Ans.}}$$

And percentage slip

$$= \frac{(Q_{th} - Q_{act})}{Q_{th}} \times 100 = \frac{(.01047 - .01)}{.01047} \times 100$$

$$= \frac{.00047}{.01047} \times 100 = \mathbf{4.489\%. \text{ Ans.}}$$

Problem 8 *A double-acting reciprocating pump, running at 40 r.p.m., is discharging 1.0 m^3 of water per minute. The pump has a stroke of 400 mm. The diameter of the piston is 200 mm. The delivery and suction head are 20 m and 5 m respectively. Find the slip of the pump and power required to drive the pump.*

Solution. Given:

Speed of pump, $N = 40$ r.p.m.

Actual discharge, $Q_{act} = 1.0\ m^3/min = \frac{1.0}{60}\ m^3/s = 0.01666\ m^3/s$

Stroke, $L = 400$ mm $= 0.40$ m

Diameter of piston, $D = 200$ mm $= 0.20$ m

$\therefore$ Area, $A = \frac{\pi}{4} D^2 = \frac{\pi}{4} (.2)^2 = 0.031416\ m^2$

Suction head, $h_s = 5$ m

Delivery head, $h_d = 20$ m.

Theoretical discharge for double-acting pump is given by equation (20.5) as,

$$Q_{th} = \frac{2ALN}{60} = \frac{2 \times .031416 \times 0.4 \times 40}{60} = .01675\ m^3/s.$$

Using equation (20.8), Slip $= Q_{th} - Q_{act} = .01675 - .01666 =$ **.00009 m^3/s. Ans.**

Power required to drive the double-acting pump is given by equation (20.7) as,

$$P = \frac{2 \times \rho g \times ALN \times (h_s + h_d)}{60{,}000} = \frac{2 \times 1000 \times 9.81 \times .031416 \times .4 \times 40 \times (5 + 20)}{60{,}000}$$

$=$ **4.109 kW. Ans.**

9 What is negative slip in a reciprocating pump ? Explain with neat sketches the function of air vessels in a reciprocating pump.

SLIP OF RECIPROCATING PUMP

Slip of a pump is defined as the difference between the theoretical discharge and actual discharge of the pump. The discharge of a single-acting pump given by equation (20.1) and of a double-acting pump given by equation (20.5) are theoretical discharge. The actual discharge of a pump is less than the theoretical discharge due to leakage. The difference of the theoretical discharge and actual discharge is known as slip of the pump. Hence, mathematically.

$$\text{Slip} = Q_{th} - Q_{act}$$

But slip is mostly expressed as percentage slip which is given by,

$$\text{Percentage slip} = \frac{Q_{th} - Q_{act}}{Q_{th}} \times 100 = \left(1 - \frac{Q_{act}}{Q_{th}}\right) \times 100$$

$$= (1 - C_d) \times 100 \qquad \left(\because \frac{Q_{act}}{Q_{th}} = C_d\right)$$

where C_d = Co-efficient of discharge.

Negative Slip of the Reciprocating Pump. Slip is equal to the difference of theoretical discharge and actual discharge. If actual discharge is more than the theoretical discharge, the slip of the pump will become –ve. In that case, the slip of the pump is known as negative slip.

Negative slip occurs when delivery pipe is short, suction pipe is long and pump is running at high speed.

AIR VESSELS

An air vessel is a closed chamber containing compressed air in the top portion and liquid (or water) at the bottom of the chamber. At the base of the chamber there is an opening through which the liquid (or water) may flow into the vessel or out from the vessel. When the liquid enters the air vessel, the air gets compressed further and when the liquid flows out the vessel, the air will expand in the chamber.

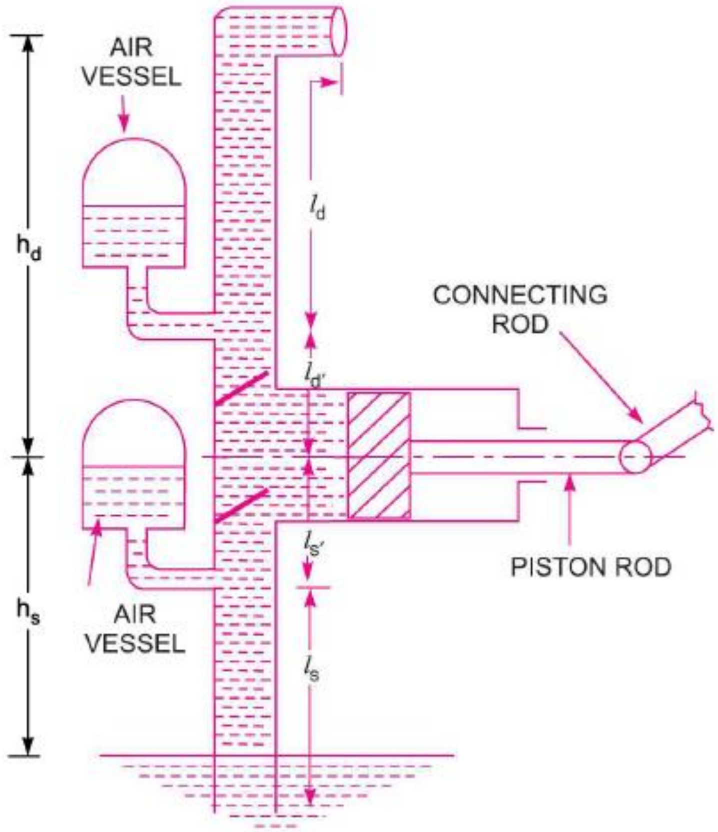
An air vessel is fitted to the suction pipe and to the delivery pipe at a point close to the cylinder of a single-acting reciprocating pump :

(*i*) to obtain a continuous supply of liquid at a uniform rate,

(*ii*) to save a considerable amount of work in overcoming the frictional resistance in the suction and delivery pipes, and

(*iii*) to run the pump at a high speed without separation.

Fig. shows the single-acting reciprocating pump to which air vessels are fitted to the suction and delivery pipes. The air vessels act like an intermediate reservoir. During the first half of the suction stroke, the piston moves with acceleration, which means the velocity of water in the suction pipe is more than the mean velocity and hence the discharge of water entering the cylinder will be more than the mean discharge. This excess quantity of water will be supplied from the air vessel to the cylinder in such a way that the velocity in the suction pipe below the air vessel is equal to mean velocity of flow. During the second half of the suction stroke, the piston moves with retardation and hence velocity of flow in the suction pipe is less than the mean velocity of flow. Thus, the discharge entering the cylinder will be less than the mean discharge. The velocity of water in the suction pipe due to air vessel is equal to mean velocity of flow and discharge required in cylinder is less than the mean discharge. Thus, the excess water flowing in suction pipe will be stored into air vessel, which will be supplied during the first half of the next suction stroke.



Air vessels fitted to reciprocating pump.

When the air vessel is fitted to the delivery pipe, during the first half of delivery stroke, the piston moves with acceleration and forces the water into the delivery pipe with a velocity more than the mean velocity. The quantity of water in excess of the mean discharge will flow into the air vessel. This will compress the air inside the vessel. During the second half of the delivery stroke, the piston moves with retardation and the velocity of water in the delivery pipe will be less than the mean velocity. The water already stored into the air vessel will start flowing into the delivery pipe and the velocity of flow in the delivery pipe beyond the point to which air vessel is fitted will become equal to the mean velocity. Hence, the rate of flow of water in the delivery pipe will be uniform.

Problen 10 *A single-acting reciprocating pump has a stroke length of 15 cm. The suction pipe is 7 metre long and the ratio of the suction diameter to the plunger diameter is 3/4. The water level in the sump is 2.5 metres below the axis of the pump cylinder, and the pipe connecting the sump and pump cylinder is 7.5 cm diameter. If the crank is running at 75 r.p.m., determine the pressure head on the piston :*

(*i*) *in the beginning of the suction stroke,* (*ii*) *in the end of the suction stroke, and*

(*iii*) *in the middle of the suction stroke.*

Take co-efficient of friction as 0.01.

Solution. Given :

Stroke length, $L = 15 \text{ cm} = 0.15 \text{ m}$

$\therefore$ Crank radius, $r = \frac{L}{2} = \frac{0.15}{2} = 0.075 \text{ m}$

Length of suction pipe, $l_s = 7.0 \text{ m}$

$$\frac{\text{Suction pipe diameter}}{\text{Plunger diameter}} = \frac{d_s}{D} = \frac{3}{4}$$

$$\therefore \quad \frac{\text{Area of suction pipe}}{\text{Area of plunger}} = \frac{a_s}{A} = \left(\frac{3}{4}\right)^2 = \frac{9}{16}$$

Suction head, $h_s = 2.5$

Diameter of suction pipe, $d_s = 7.5 \text{ cm} = 0.075 \text{ m}$

Crank speed, $N = 75 \text{ r.p.m.}$

$\therefore$ Angular speed, $\omega = \frac{2\pi N}{60} = \frac{2 \times \pi \times 75}{60} = 2.5\ \pi \text{ rad/s.}$

Friction co-efficient, $f = 0.01$

The pressure head due to acceleration in suction pipe is given by equation (20.14), as

$$h_{as} = \frac{l_s}{g} \times \frac{A}{a_s} \times \omega^2 \times r \cos\theta$$

$$= \frac{7.0}{9.81} \times \frac{16}{9} \times (2.5\ \pi)^2 \times 0.075 \cos\theta \qquad \left(\because \frac{A}{a_s} = \frac{16}{9}\right)$$

$$= 5.87 \cos\theta$$

The loss of head due to friction in suction pipe is given by equation (20.18)as

$$h_{fs} = \frac{4 f l_s}{d_s \times 2g} \times \left(\frac{A}{a_s} \omega r \sin\theta\right)^2$$

$$= \frac{4 \times 0.01 \times 7.0}{0.075 \times 2 \times 9.81} \times \left(\frac{16}{9} \times 2.5\pi \times 0.075 \times \sin\theta\right)^2 = 0.208 \sin^2\theta.$$

(*i*) *Pressure head on the piston in the beginning of suction stroke :*

(Refer to Fig. 20.8). At the beginning of suction stroke, $\theta = 0^\circ$ and pressure head

$= (h_s + h_{as})$ m below atmospheric pressure head

$= 2.5 + 5.87 =$ **8.37 m vacuum. Ans.**

(*ii*) *Pressure head on the piston at the end of suction stroke :*

At the end of the suction stroke, $\theta = 180^\circ$ and hence $\cos\theta$

$= -1$ and $\sin\theta = 0$

The pressure head $= (h_s - h_{as})$ m below atmospheric pressure head

$= H_{atm} - (h_s - h_{as})$ m abs. $= H_{atm} - (2.5 - 5.87)$ m abs.

$= H_{atm} - (-3.37)$ m abs. $= H_{atm} + 3.37$ m abs. = **3.37 m (gauge). Ans.**

(*iii*) *Pressure head on the piston in the middle of suction stroke :*

In the middle of suction stroke, $\theta = 90^\circ$ and hence $\cos\theta = 0$ and $\sin\theta = 1$. The pressure head

$= (h_s + h_{fs})$ m below atmospheric pressure head

$= (2.5 + 0.208)$ m vacuum = **2.708 m vacuum. Ans.**