SREENIVASA INSTITUTE OF TECHNOLOGY AND MANAGEMENT STUDIES, CHITTOOR – 517127
(AUTONOMOUS)
Affiliated to JNTUA, Anantapuramu, NBA & NAAC Accredited

DEPARTMENT
OF
ELECTRONICS & COMMUNICATION ENGINEERING

ANALOG AND DIGITAL COMMUNICATIONS
(23ECE241T)

LECTURE NOTES

Prepared by
Mrs. Shaik Reshma
Mrs. B.R.Usharani

INSTITUTE VISION AND MISSION

➢ VISION

- To emerge as a centre of excellence for learning and research in the domains of engineering, computing and management.

➢ MISSION

- Provide congenial academic ambience with state-art of resources for learning and research.
- Ignite the students to acquire self-reliance in the latest technologies.
- Unleash and encourage the innate potential and creativity of students.
- Inculcate confidence to face and experience new challenges.
- Foster enterprising spirit among students.
- Work collaboratively with technical Institutes/Universities/Industries of National and International repute.

DEPARTMENT VISION AND MISSION

➢ VISION

- To become a centre of excellence in Electronics and Communication Engineering and provide necessary skills to the students to meet the challenges of industry and society.

➢ MISSION

- Provide congenial academic ambience with necessary infrastructure and learning resources
- Inculcate confidence to face and experience new challenges from industry and society.
- Ignite the students to acquire self-reliance in State-of-the-Art Technologies
- Foster Enterprising spirit among students

PROGRAMME EDUCATIONAL OBJECTIVES (PEO's)

PEO1: Have Professional competency through the application of knowledge gained from subjects like Mathematics, Physics, Chemistry, Inter-Disciplinary and core subjects like Signal Processing, VLSI, Embedded Systems, Communication and Automation. **(Professional Competency)**

PEO2: E*xcel* in one's career by critical thinking towards successful services and growth of the organization or as an entrepreneur or through higher studies. **(Successful Career Goals)**

PEO3: Enhance knowledge by updating advanced technological concepts for facing the rapidly changing world and contribute to society through innovation and creativit*y*. **(Continuing Education and Contribution to Society)**

PROGRAM SPECIFIC OUTCOMES (PSO'S)

PSO1: Apply the knowledge obtained in core areas for the analysis and design of components in Signal Processing, VLSI, Embedded Systems, Communication and Automation.

PSO2: Adapt Innovation, Creativity and design to develop products which meet industrial and societal needs.

PROGRAMME OUTCOMES (PO's)

PO1. **Engineering knowledge**: Apply the knowledge of mathematics, science, engineering fundamentals, and an engineering specialization to the solution of complex engineering problems.

PO2. **Problem analysis**: Identify, formulate, research literature, and analyze complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences.

PO3. **Design/development of solutions**: Design solutions for complex engineering problems and design system components or processes that meet the specified needs with appropriate consideration for the public health and safety, and the cultural, societal, and environmental considerations.

PO4. **Conduct investigations of complex problems**: Use research-based knowledge and research methods including design of experiments, analysis and interpretation of data, and synthesis of the information to provide valid conclusions.

PO5. **Modern tool usage**: Create, select, and apply appropriate techniques, resources, and modern engineering and IT tools including prediction and modeling to complex engineering activities with an understanding of

the limitations.

PO6. **The engineer and society**: Apply reasoning informed by the contextual knowledge to assess societal, health, safety, legal and cultural issues and the consequent responsibilities relevant to the professional engineering practice.

PO7. **Environment and sustainability**: Understand the impact of the professional engineering solutions in societal and environmental contexts, and demonstrate the knowledge of, and need for sustainable development.

PO8. **Ethics**: Apply ethical principles and commit to professional ethics and responsibilities and norms of the engineering practice.

PO9. **Individual and team work**: Function effectively as an individual, and as a member or leader in diverse teams, and in multidisciplinary settings.

PO10. **Communication**: Communicate effectively on complex engineering activities with the engineering community and with society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentations, and give and receive clear instructions.

PO11. **Project management and finance**: Demonstrate knowledge and understanding of the engineering and management principles and apply these to one's own work, as a member and leader in a team, to manage projects and in multidisciplinary environments.

PO12. **Life-long learning:** Recognize the need for, and have the preparation and ability to engage in independent and life-long learning in the broadest context of technological change.

SYLLABUS

UNIT – I : Continuous Wave Modulation

Introduction: The communication Process, Communication Channels, Baseband and Passband Signals, Analog vs Digital Communications, Need for the modulation.

Amplitude Modulation (AM): AM and its modifications – DSB, SSB, VSB. Frequency Translation, Frequency Division Multiplexing (FDM).

Angle Modulation: Frequency Modulation (FM), Phase Modulation, PLL, Nonlinear Effects in FM, Super heterodyne Receivers.

UNIT – II: Noise and Pulse Modulation

Introduction to Noise: Types of Noise, Receiver Model, Noise in AM, DSB, SSB, and FM Receivers, Pre-Emphasis and De-emphasis in FM.

Introduction to Pulse Modulation: The Sampling Process, PAM, TDM, Bandwidth-Noise Trade off, Quantization process, PCM, Noise considerations in PCM systems, Delta Modulation, DPCM, Coding speech at low bit rates.

UNIT – III: Baseband Pulse Transmission

Introduction, Matched Filter, Properties of Matched Filter, Error rate due to noise, Inter Symbol Interference (ISI), Nyquist Criterion for distortion less baseband binary transmission, Correlative level coding, Baseband M-ary PAM transmission, QAM, MAP and ML decoding, Equalization, Eye pattern.

UNIT – IV: Digital Passband Transmission

Introduction, Passband Transmission Model, Gram-Schmidt Orthogonalization Procedure, Geometric Interpretation of Signals, Response of bank of correlators in noise, Correlation receiver, Probability of Error, Detection of Signals with unknown phase.

UNIT – V: Digital Modulation Schemes

Coherent Digital Modulation Schemes – ASK, BPSK, BFSK, QPSK, Non-coherent BFSK, and DPSK. M- ary Modulation Techniques, Power Spectra, Bandwidth Efficiency, Timing and Frequency synchronization.

Information theory: Entropy, Mutual Information and Channel capacity theorem.

Textbooks:

1. Simon Haykin, "Communication Systems", JohnWiley& Sons, 4th Edition, 2004.

2. B. P. Lathi, Zhi Ding " Modern Digital and Analog Communication Systems", Oxford press, 2011.

References:

1. Sam Shanmugam, "Digital and Analog Communication Systems",JohnWiley& Sons, 1999.

2. Bernard Sklar, F. J. harris"Digial Communications: Fundamentals andApplications", Pearson Publications, 2020.

3. Taub and Schilling, " Principles of Communication Systems", Tata McGraw Hill, 2007.

UNIT-I
CONTINUOUS WAVE MODULATION

INTRODUCTION

Communication is the process of establishing connection or link between two points for information exchange. Communication is simply the process of conveying message at a distance or communication is the basic process of exchanging information. Generally communication can be classified into two types.

(1) Communication within line of sight

(2) Communication beyond line of sight between point to point.

COMMINCATION PROCESS AND ITS COMPONENTS
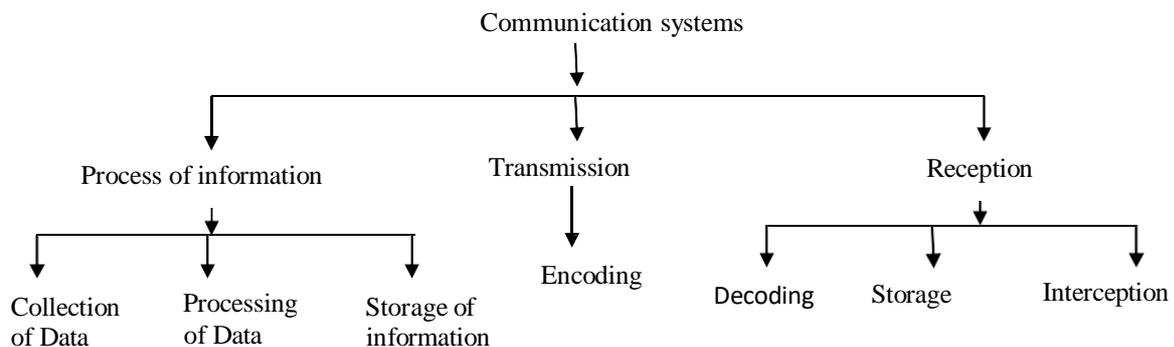
In Communication engineering, the physical message such as sounds, words, pictures etc. are converted into equivalent electrical values called signals.

This electrical signal is conveyed to a distance place, through a communication media, and at receiving end, this electrical signal is reconverted back into the original message through same media.

NOTE: Different electronic equipment's which are used for communication purpose, are called communication equipment's. Different communication equipment's when assembled together form a communication system.

ELEMENTS OF A COMMUNICATION SYSTEMS

In general, communication involves the transmission of information from one point to another through a succession of process i.e.,

(1) The generation of a thought pattern or image in the mind of an originator.

(2) The description of that image.

(3) The encoding of these symbols.

(4) The transmission of the encoded symbols to destination.

(5) Decoding and reproduction of the original symbols.

(6) The recreation of the original through pattern or image.

The purpose of communication system is to transmit an information bearing signal, from a source located at one point to a user or destination, located at another point some distance away.

The block diagram of general communication system (or) elements of communication system.
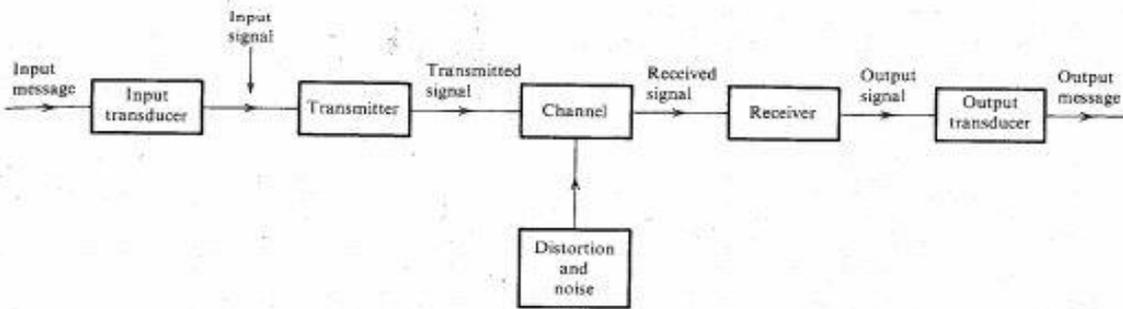

Fig: Block diagram of Communication System

INFORMATION SOURCE:

A communication system serves to communicate a message or information. This message or information originates in the information source. The function of information source is to produce required message which has to be transmitted.

INPUT TRANSDUCER

A transducer is a device which converts one form of energy into another form. The message from which the information source may or may not be electrical in nature. In case when the message produced by the information source is not electrical in nature, an input transducer is used to convert it into a time varying electrical signal.

TRANSMITTER

The function of the transmitter is to process the electrical signal from different aspects. In long distance radio communication or broadcast, signal amplification is necessary before modulation. Modulation is the main function of the transmitter. In modulation, the message signal is super imposed upon the high frequency carrier signal. Simply, inside the transmitter, signal processing such as restriction of range of frequencies, amplification and modulation are achieved.

THE CHANNEL AND THE NOISE

The term channel means the medium through which the message travels from the transmitter to receiver. The function of the channel is to provide a physical connection between the transmitter and the receiver. There are two types of channels, namely point to point channel and broadcast channels. Examples of point to point channels are wire lines, microwave links and optical fibers.

During the process of transmission and reception the signal gets distorted due to noise introduced in the system. Noise is an unwanted signal which tend to interfere with the required signal. Noise signal is always random in character. Noise has its greatest effect on the signal in the channel.
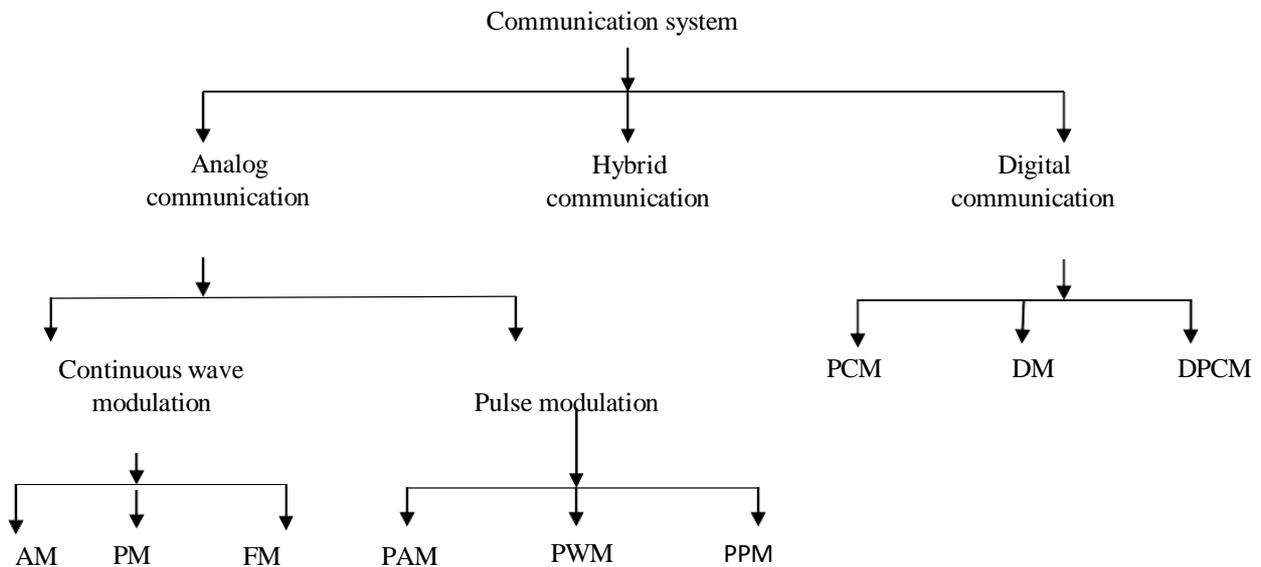
RECIEVER

The main function of the receiver is to reproduce the message signal in electrical form from the distorted received signal. The reproduction of the original signal is accomplished by a process known as demodulation or detection. Demodulation is the reverse process of modulation carried out at the transmitter.

DESTINATION

Destination is the final stage which is used to convert an electrical message signal into original form.

TYPES OF COMMUNICATION SYSTEMS

Communication system
```
                          Communication system
                                  |
         ┌────────────────────────┼────────────────────────┐
         ↓                        ↓                         ↓
      Analog                   Hybrid                   Digital
   communication            communication            communication
         |                                                 |
    ┌────┴────┐                                    ┌────────┼────────┐
    ↓         ↓                                    ↓        ↓        ↓
Continuous wave   Pulse modulation               PCM       DM      DPCM
  modulation
    |              |
 ┌──┼──┐        ┌──┼──┐
 ↓  ↓  ↓        ↓  ↓  ↓
AM  PM  FM     PAM PWM PPM
```

ANALOG COMMUNICATION SYSTEM

      It is designed to transmit analog information using analog modulation schemes. Such as Amplitude modulation and Angle modulation.

DIGITAL COMMUNICATION SYSTEM

      It is designed to transmit digital information using digital modulation schemes. Such as PCM, DM, DPCM etc.

HYBRID COMMUINCATION SYSTEM

It is designed to use digital modulation schemes for transmitting sampled and quantized value of analog signals.

MESSAGES

      Messages are digital or analog. Digital messages are constructed with a finite no of symbols. For example printed language consists of 26 letters, 10 numbers, a space and a several punctuation marks. Thus a text is a digital message constructed from about 50 symbols. Human speech is digital message, because it is made up from a finite vocabulary in a language. A digital message constructed with M-symbols is called as M-ary message.

Analog messages are characterized by data whose values vary over a continuous range. For example temperature or the atmospheric pressure of a certain location can vary over a continuous rage and can assume an infinite no of possible values.

SIGNALS

Digital messages are transmitted by using a finite set of electrical waveforms. For example in the Marse code, a mark can be transmitted by an electrical pulse of amplitude $+A/2$ and a space can be transmitted by a pulse of amplitude $-A/2$. In an M-ary case, M-distinct electrical pulses are used. Each of M-pulses represents one of the M possible symbols. The task of the receiver is to extract a message from a distorted and noisy signal at the channel output.

CLASSIFICATION OF COMMUNICATION

Communication engineering is further divided into two categories depending on the transmission media (or) channel used, such as

(1) Line communication   (2) Radio communication

LINE COMMUNICATION

In line communication, the medium of transmission is a pair of conductors called transmission line. This is also called as line channel. This means that in line communication, the transmitter and receiver are connected through a wire or lines.

The main problems associated with the line communication are

(1) Installation and maintenance of transmission line is costly and it overcrowds open space

(2) Transmission capability is limited.

WIRLESS OR RADIO COMMUNICATION

In wireless and radio communication, a message is transmitted through open space by electromagnetic waves called a radio wave. Radio waves are radiated from the transmitter in open space through a device called antenna. A receiving antenna intercepts the radio waves at the receiver. The advantages of wireless communication are cost effectiveness, possible long distance communication and simplicity.

The radio frequency spectrum is classified into seven service bands and its uses are,

Type of signal	Frequency Range	Applications
Very low frequency(VHL)	3 to 3KHz	Long distance, point-to-point communication.
Low frequency (LF)	30KHz to 300 KHz	Radio Navigation
Medium frequency (MF)	300KHz to 3 MHz	Broadcasting, Marine applications
High frequency (HF)	3 MHz to 30MHz	Radio Telephony
Very high frequency(VHF)	30MHz to 300MHz	FM broadcasting, TV, Mobile radio, radio navigation
Ultra high frequency (UHF)	300MHz to 3000 MHz	FM broadcasting, TV, Mobile radio, radio navigation
Extremely high frequency(EHF)	30GHz to 300GHz	Multichannel telephony links, Radar, satellite communication

MODULATION

Modulation may be defined as the process by which the characteristics of a signal called carrier are varied in accordance with the instantaneous value of another signal called Modulating signal. The modulating signal consists of information and this information bearing signal is also called as Baseband signal. The frequency of carrier signal is greater than modulating frequency. The signal resulting from the process of modulation is called modulated signal.

The receiver recreates the original message signal from a degraded version of the transmitted signal after propagation through the channel is called demodulation.

MODULATION METHODS (OR) TYPES OF MODULATION

Modulation is basically of two types

(i) Continuous wave modulation: When the carrier is continuous in nature, the modulation process is known as continuous wave modulation or analog modulation.

(ii) Pulse modulation: When the carrier wave is pulse type waveform, the modulation process is known as pulse modulation. In pulse modulation, the carrier consists of a periodic sequence of rectangular pulses. Pulse modulation can be of analog or digital.
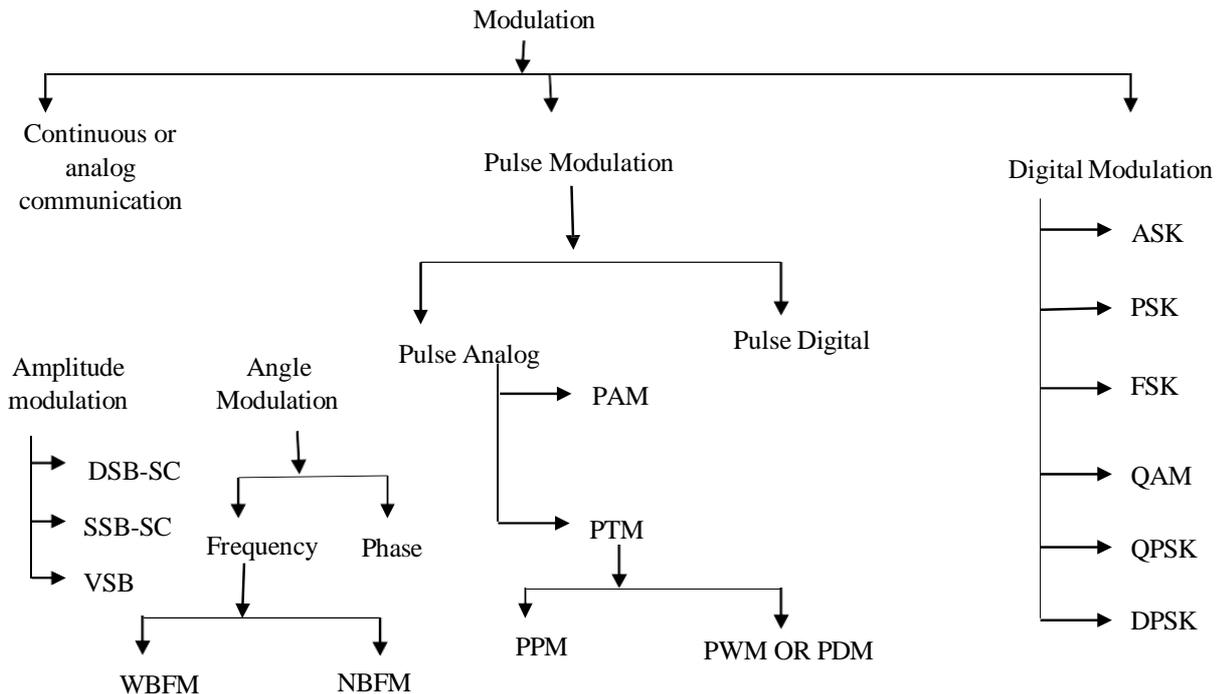
NEED OF MODULATION (OR) BENEFITS OF MODULATION

The various aspects why we should modulate the message signal are

(a) For easy transmission (or) To reduce antenna height:

If the communication medium is the free space, then antenna are needed to radiate and receive the signal. The antenna radiates effectively when its height is of the order of wave length of the signal to be transmitted.

For example: For a frequency of 1 KHz, the height of the antenna required for effective radiation would be half the wavelength,

i.e.,   Antenna height $\dfrac{\lambda}{2} = \dfrac{C}{2f} = \dfrac{3 \times 10^8}{2 \times 1 \times 10^3} = 150 km$

Where λ = Wavelength of the signal to be transmitted

C = Velocity of light f = Frequency of the signal to be transmitted.

But it is highly impractical to construct and install such an antenna. So, the height of the antenna can be reduced by modulation technique and it achieves effective radiation.

The process of modulation provides frequency shifting or frequency translation i.e., audio frequency (AF) signals are translated into Radio frequency (RF). These RF signals act as carrier signal and AF signals act as message signals. Hence height of the antenna very much reduced. Here, the 1 KHz baseband signal is translated into a high frequency signal of 1MHZ

Hence the height of antenna $\dfrac{\lambda}{2} = \dfrac{C}{2f} = \dfrac{3 \times 10^8}{2 \times 1 \times 10^6} = 150m$

This height of the antenna is practically achievable.

(b) Avoids mixing of signals

All sound signals are concentrated within the range of 20Hz to 20 KHz. The transmission of baseband signals from various sources causes the mixing of signal and then it is difficult to separate at the receiver end.

In order to separate the various signals, it is necessary to translate them to different portions of the electromagnetic spectrum (channel0, each must be given its own bandwidth commonly known as channel bandwidth. This can be achieved by taking different carrier frequencies for different signal source. Once the signals have been transmitted, a tuned circuit at the receiver end selects the portion of the electromagnetic spectrum it is tuned for. Therefore modulating different signal sources by different carrier frequencies avoid mixing of signals.

(c) Increases the range of communication

At low frequencies radiation is poor and signal gets highly attenuated. Therefore baseband signals cannot be transmitted directly over long distance. Modulation effectively increases the frequency of the signal to be radiated and thus increases the distance over which signals can be transmitted faithfully.

(d) Allows multiplexing of signals

The modulation permits multiplexing. Multiplexing means transmission of two or more signals simultaneously over the same channel. The common examples of multiplexing are the number of TV channels operating simultaneously or number of radio stations broadcasting the signal in MW and SW band simultaneously.

(e) Improves quality of reception

The signal communication using modulation techniques such as frequency modulation, pulse code modulation reduce the effect of noise to great extent. Reduction in noise improves the quality of reception.

APPLICATIONS OF MODULATION

1. FM Broadcasting

2. Radio navigation

3. Telephone communication

4. Mobile communication

5. TV communication

6. Radar communication

7. Satellite communication

8. Frequency division multiplexing

BASEBAND AND CARRIER COMMUNICATION:

In baseband communication, baseband signals are transmitted without modulation, i.e., without any shift in the range of frequencies of the signal. Communication that uses modulation to shit frequency spectrum of a signal is known as carrier communication.

In carrier communication, one of the basic parameters (i.e. amplitude, frequency or phase) of a sinusoidal carrier of high frequency are varied in proportion to the baseband signal.

AMPLITUDE MODULATION

Amplitude modulation is the process of changing the amplitude of the carrier signal in accordance with the amplitude of a modulating signal. Frequency and phase of the carrier signal are not altered during this process.

Let the modulating signal and carrier signal can be written as

$$V_m(t) = V_m \sin \omega_m t \qquad (1)$$

$$V_c(t) = V_c \sin \omega_c t \qquad (2)$$

According to the definition, the amplitude of the carrier signal is changed after modulation.

$$V_{AM} = V_c + V_m(t) = V_c + V_m \sin \omega_m t$$

$$= V_c \left[1 + \frac{V_m}{V_c} \sin \omega_m t \right] = V_c (1 + m_a \sin \omega_m t)$$

$$m_a = \frac{V_m}{V_c} = \quad \text{Modulation index (or) depth of modulation}$$

The shape of the modulating signal is called as AM envelope, because it contains all frequencies that make up the AM signal and it is used to communicate the information through the system.

The instantaneous amplitude of modulated signal or AM envelope can be written as,

$$V_{AM}(t) = V_{AM} \sin \omega_c t$$

Substitute the value of V_{AM} in above equation

$$V_{AM}(t) = V_c (1 + m_a \sin \omega_m t) \sin \omega_c t$$

$$= V_c \sin \omega_c t + m_a V_c \sin \omega_m t . \sin \omega_c t$$

Since $\sin \omega_m t . \sin \omega_c t = \frac{1}{2} [\cos(\omega_c - \omega_m)t - \cos(\omega_c + \omega_m)t]$

$$V_{AM}(t) = V_c \sin\omega_c t + \frac{m_a V_c}{2}\left[\cos(\omega_c - \omega_m)t - \cos(\omega_c + \omega_m)t\right]$$

The graphical representation of amplitude modulation wave is
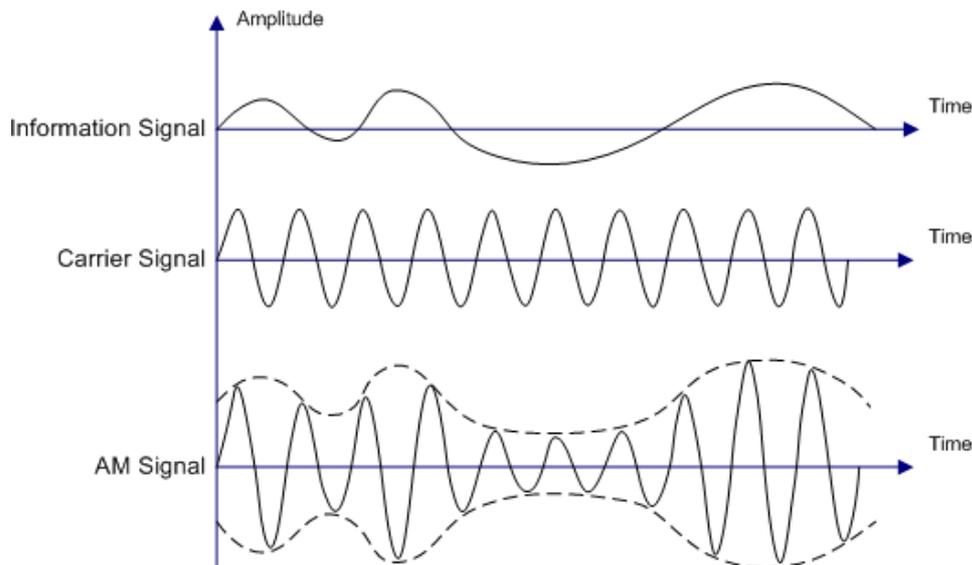
It is important to note that,

(i) If message signal is absent the output is simply the carrier signal

(ii) The shape of the envelope is identical to shape of modulating signal.

The equation of amplitude modulated signal consists of three terms, the first terms of RHS represents the carrier wave. The second and third terms are identical which are called as lower sideband (LSB) and upper sideband (USB).
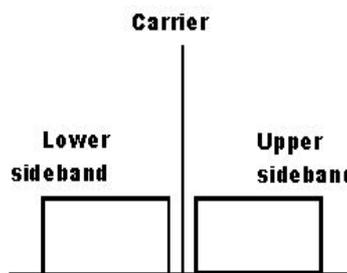
Fig. Frequency spectrum of AM with carrier

From above fig, the range of frequency between $(\omega_c - \omega_m)$ is known as LSB and $(\omega_c + \omega_m)$ is known as USB. The spacing between these two bands w.r.t carrier is ω_m. The band width of AM can be determined by using these sidebands. Hence "BW" is twice the frequency of modulating signal.

COEFFICIENT OF MODULATION (OR) PERCENT MODULATION (OR) MODULATION INDEX:

The modulation index used to describe the amount of amplitude change occurred in AM envelopes.
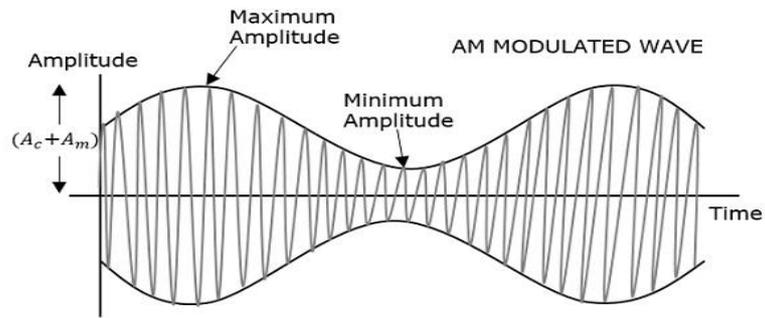
From above fig,

$$2V_{(\mathrm{mod}\ ulating\ )\max} = V_{\max} - V_{\min}$$

$$V_m = V_{(\mathrm{mod}\ ulating\ )\max} = \frac{V_{\max} - V_{\min}}{2}$$

And

$$V_{(carrier\ )\max} = V_{\max} - V_m$$

$$= V_{\max} - \left(\frac{V_{\max} - V_{\min}}{2}\right)$$

$$= \frac{V_{\max} + V_{\min}}{2}$$

$$\therefore m_a = \frac{V_m}{V_c} = \frac{V_{\max} - V_{\min}}{V_{\max} + V_{\min}}$$

$$\% m_a = \frac{V_{\max} - V_{\min}}{V_{\max} + V_{\min}} \times 100$$

There are three degrees of modulation depending upon the amplitude of the message signal relative to carrier amplitude

(i) Under modulation   (ii) critical modulation   (iii) Over modulation

UNDER MODULATION

In this case modulation index $m_a < 1, V_m < V_c$. From the figure the envelope of AM signal does not reach the zero amplitude axis. This is known as "under modulation".

CRITICAL MODULATION

In this case modulation index $m_a = 1, V_m = V_c$. From the figure the envelope of AM signal just reaches the zero amplitude axis. This is known as "critical modulation".

OVER MODULATION

In this case modulation index $m_a > 1, V_m > V_c$. From the figure the envelope of AM signal is greater than carrier amplitude. Therefore the portion of envelope of modulating signal crosses zero axis. This is known as "over modulation". Due to this envelope detector provides distorted message signal.

POWER DISTRIBUTION

The modulated wave contains three terms such as carrier, LSB and USB. Therefore the modulated wave contains more power than the carrier had before modulation.

The total power in modulated wave will be,

$$P_t = P_c + P_{LSB} + P_{USB}$$

$$P_t = \frac{V_{carrier}^2}{R} + \frac{V_{LSB}^2}{R} + \frac{V_{USB}^2}{R}$$

Where $V_{carrier}$ = RMS value of carrier voltage

$V_{LSB} = V_{USB}$ = RMS value of upper and lower side band voltages

R = Resistance

$$P_{carrier} = \frac{V_{carrier}^2}{R} = \frac{\left(\frac{V_c}{\sqrt{2}}\right)^2}{R} = \frac{V_c^2}{2R}$$

$$^{ly} /// \quad P_{LSB} = P_{USB} = \frac{V_{SB}^2}{R} = \frac{\left[\frac{m_a V_c}{\sqrt{2}}\right]^2}{R} = \frac{m_a^2 V_c^2}{8R}$$

V_c = maximum carrier wave amplitude

$V_{SB} = \dfrac{m_a V_c}{2}$ = maximum amplitude of sidebands

Wkt, $$P_t = P_c + P_{LSB} + P_{USB}$$

$$= \frac{V_c^2}{2R} + \frac{m_a^2 V_c^2}{8R} + \frac{m_a^2 V_c^2}{8R}$$

$$V^2 \quad V^2 m^2$$

8R

$$\therefore P_t = \frac{c}{2R} + \frac{c \quad a}{4R} = \frac{c}{2R}\left[1 + \frac{a^2}{2}\right]$$

$$P_t = P_c\left(1 + \frac{m_a^2}{2}\right)$$

$$\therefore \frac{P_t}{P_c} = \left[1 + \frac{m_a^2}{2}\right]$$

If $m_a = 1$, for 100% modulation, $P_t = 1.5 P_c$

CURRRENT RELATION AND EFFICIENCY

Wkt, $\quad P_t = P_c\left(1 + \frac{m_a^2}{2}\right)$

$$P_t = I_t^2 R \quad \text{and} \quad P_c = I_c^2 R$$

Hence $I_t^2 = I_c^2\left[1 + \frac{m_a^2}{2}\right]$ or $I_t = I_c\sqrt{1 + \frac{m_a^2}{2}}$

Where I_t = total or modulated current, I_c = carrier current

% Efficiency: It can be defined as the ratio of power in sidebands to total power, because sidebands only contain useful information

$$\%\eta = \frac{power \quad in \quad sideband}{totalpower} \times 100$$

$$\%\eta = \frac{P_{LSB} + P_{USB}}{P_{total}} \times 100 = \frac{\dfrac{m_a^2 V_c^2}{8R} + \dfrac{m_a^2 V_c^2}{8R}}{\dfrac{V_c^2}{2R}\left[1 + \dfrac{m_a^2}{2}\right]} \times 100$$

$$\%\eta = \frac{\dfrac{m_a^2 P_c}{2}}{P_c\left[1 + \dfrac{m_a^2}{2}\right]} \times 100 = \frac{m_a^2}{2 + m_a^2} \times 100$$

If $m_a = 1$ then $\%\eta = \frac{1}{3} \times 100 = 33.3\%$

From this we conclude that only 33.3% of energy is used and remaining power is wasted by the carrier transmission along with sidebands.

GENERATION OF AM SIGNALS:

The device which is used to generate an amplitude modulated wave is known as Amplitude Modulator.

SQUARE LAW DIODE MODULATION:

Definition: square law diode modulation circuit make use of nonlinear current voltage characteristics of diode. This method is suited at low voltage levels because of the fact that current voltage characteristic of a diode is highly nonlinear particularly in the low voltage region.

(a) Square law modulator
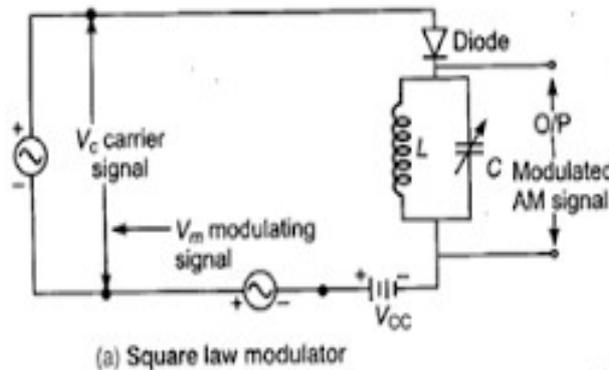
From the fig, carrier and modulating signals are applied across the diode. When two different frequencies are passed through a nonlinear device, the process of amplitude modulation takes place. Different frequency terms appear at the output of the diode. These different frequency terms are applied across the tuned circuit which is tuned to the carrier frequency and has a narrow bandwidth just to pass two sidebands along with the carrier and reject other frequencies.
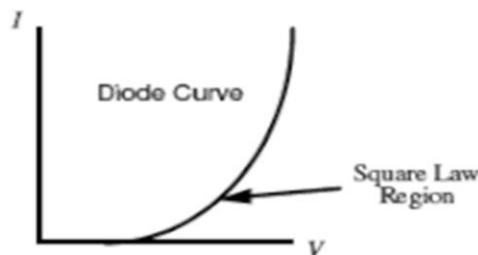
Fig: V-I characteristics of diode.

Carrier voltage is expressed as

$$V_c = V_c \cos\omega_c t$$

Modulating voltage $V_m = V_m \cos\omega_m t$

Where ω_c = carrier frequency $\qquad$ ω_m = modulating frequency

The total ac voltage across the diode is given by

$$V_s = V_c + V_m$$
$$V_s = V_c \cos\omega_c t + V_m \cos\omega_m t \qquad (1)$$

The non linear relationship between voltage and current for a diode is expressed as

$$i = a + bV_s$$

Where a,b,c are constants

i = current through the diode V_s = voltage across the diode.

$$i = a + bV_s + cV_s^2 = a + b(V_c \cos\omega_c t + V_m \cos\omega_m t) + c(V_c \cos\omega_c t + V_m \cos\omega_m t)^2$$

$$i = a + bV_c \cos\omega_c t + bV_m \cos\omega_m t + cV_c^2 \cos^2\omega_c t + cV_m^2 \cos^2\omega_m t + 2cV_c V_m \cos\omega_c t \cos\omega_m t$$

$$i = \left(a + \frac{1}{2}cV_c^2 + \frac{1}{2}cV_m^2\right) + bV_c \cos\omega_c t + bV_m \cos\omega_m t + \left(\frac{1}{2}cV_c^2 \cos2\omega_c t + \frac{1}{2}cV_m^2 \cos2\omega_m t\right) +$$

$$cV_c V_m \cos(\omega_c + \omega_m)t + cV_c V_m \cos(\omega_c - \omega_m)t \qquad (2)$$

Substitute the value of V_s from equation (1) in equation (2) we get,

Equation (3) consists of six terms, term(1) is dc term, (2) is the carrier signal , (3) is the modulating signal, (4) consists of harmonics of carrier and modulating signals, (5) is the upper sideband, (6) is the lower sideband.

In the diode modulation circuit, the load impedance is a tuned circuit which is tuned to carrier frequency. Therefore, this tuned circuit responds to a narrow band of frequencies centered about the carrier frequency. Thus the frequency components which are actually developed in the output are terms of frequency ω_c, $(\omega_c + \omega_m)$ and $(\omega_c - \omega_m)$. The rest of the frequency components are rejected by the tuned circuit.

The requires expression of output current will be

$$i_0 = bV_c \cos\omega_c t + cV_c V_m \cos(\omega_c + \omega_m)t + cV_c V_m \cos(\omega_c - \omega_m)t$$

$$i_0 = bV_c \cos\omega_c t + 2cV_c V_m \cos\omega_c t \cos\omega_m t$$

$$i_0 = bV_c \left[1 + \frac{2cV_m}{b}\cos\omega_m t\right]\cos\omega_c t$$

$$i_0 = bV_c (1 + m_a \cos\omega_m t)\cos\omega_c t \qquad (3)$$

Where $m_a = \dfrac{2cV_m}{}$, is the modulation index and equation (4) is the required expression for AM current.

DEMODULATION OF AM SIGNALS

The process of extracting a modulating or baseband signal from the modulated signal is called demodulation or detection. (OR) demodulation or detection is the process by which the message is recovered from the modulated signal at receiver.

The detectors are categorized as,

(i) Square law detectors

(ii) Envelope detectors

SQUARE LAW DECTECTOR/RECTIFIER DETECTOR:

The square law detector circuit is used for detecting modulated signal of small magnitude (i.e. below 1 volt). The circuit is very similar to square law modulator. The only difference lies in the filter circuit. In a square law modulator, the filter used is a band pass filter where as in a square law detector a LPF is used.

From the circuit, the operation is limited to the non-linear region of the diode characteristics, the lower half portion of the modulated waveform is compressed. This produces envelope applied distortion. Due to this, the average value of the diode current is no longer constant, rather it varies with time.
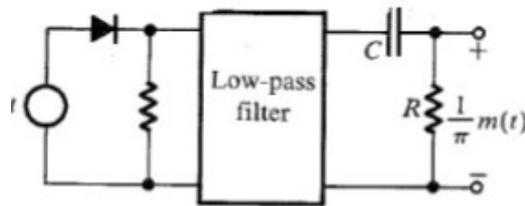
Fig: Square law detector
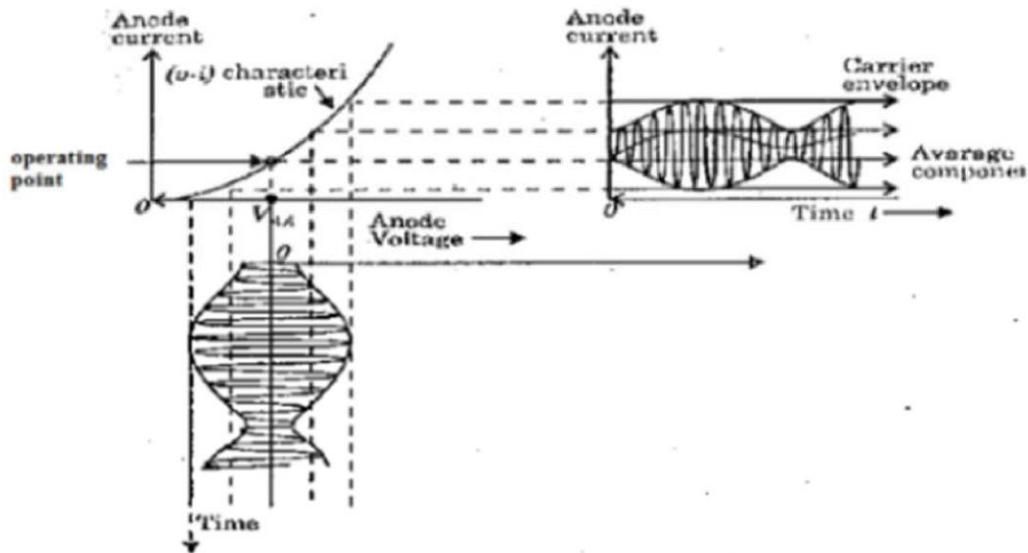
The distorted output diode current is expressed by the non liner V-I relationship (i.e., square law) as

$$i = aV + bV^2 \quad \text{where V = input modulated voltage}$$

The AM wave is $\quad V = A\left(1 + m_a \cos\omega_m t\right)\cos\omega_c t$

By substituting 'v' in above equation

$$i = a\left[A\left(1 + m_a \cos\omega_c t\right)\cos\omega_c t\right] + b\left[A\left(1 + m_a \cos\omega_m t\right)\cos\omega_m t\right]^2$$

If the above expression is expanded, we get frequencies like $2\omega_c, 2\left(\omega_c \pm \omega_m\right), \omega_m$ and $2\omega_m$ besides input

frequency terms. Hence, the diode current 'i' containing all these frequency terms is passed through a LPF which allows to pass the frequencies below or up to ω_m and reject the other frequency components. Therefore, the modulating or baseband signal with frequency ω_m is recovered from the input modulated signal.

LINEAR DIODE OR ENVELOPE DETECTOR:

A diode operating in a linear region of its V-I characteristics can extract the envelope of an AM wave. This type of detector is known as envelope detector or linear detector

From the fig, in the input portion of the circuit, the tuned transformer provides perfect tuning at the desired carrier frequency. RC network is the time constant network. If the magnitude of the modulated signal at the input of the detector is '1' volt or more, the operation takes place in the linear portion of the V-I characteristic of diode
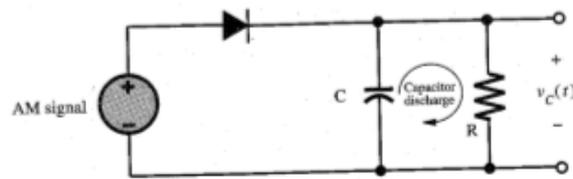
Fig: Linear diode detector.
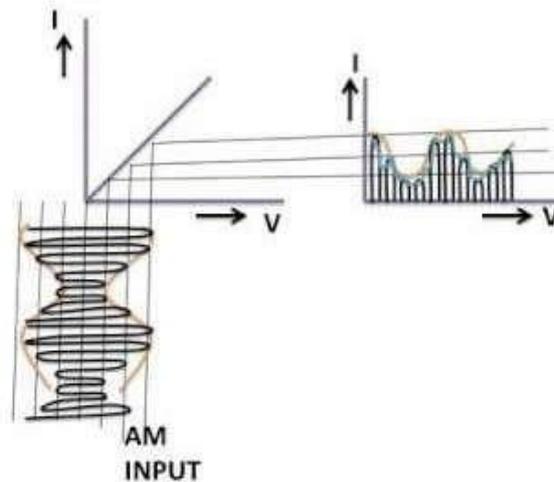
Fig: Characteristics of linear diode detector.

OPERATING PRINCIPLE:

Let us assume that the capacitor is absent, in this case the detector current will work as a half-wave rectifier. Therefore, the output waveform would be a half rectified modulated signal. Let us consider that the capacitor is introduced in the circuit. For the positive half cycle, the diode conducts and the capacitor is charged to peak value of the carrier voltage. So, for a negative half cycle the diode is reverse biased and doesn't conduct. This means that the input carrier voltage is disconnected from the R-C circuit. Therefore the capacitor starts discharging through the resistance 'R' with a time constant $\tau = RC$.

If the time constant is suitably chosen, the voltage across the capacitor 'C' will not fall during the small period of negative half cycle and by that time the next positive cycle appears. This positive cycle again charges the capacitor 'C' to the peak value of the carrier voltage and thus this process repeats again and again. Hence, the output voltage across the capacitor 'C' is the spiky modulating or baseband signal. We can reduce these spikes to a negligible amount by keeping the time constant RC large so that the capacitor 'C' discharges negligible small. Therefore, the time constant is an important consideration for envelope detector.

OTHER TYPES OF AM:

The amplitude modulation is also called as the "Double sideband full carrier system (DSB-FC). There are some other types of AM are
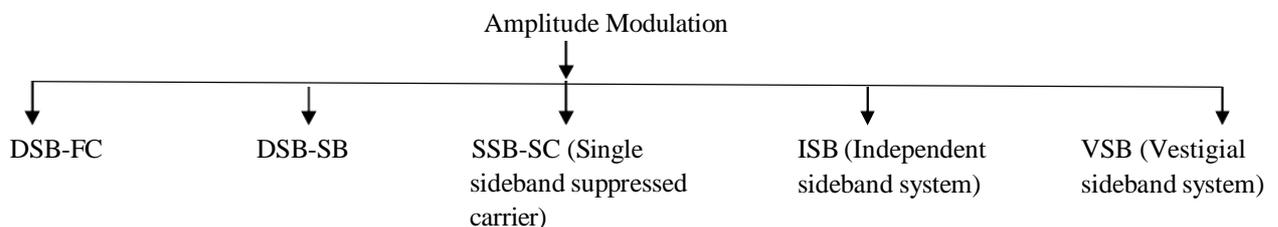
DISADVANTAGES OF DSB-FC (STANDARD AM):

(i) Power wastage in DSB-FC transmission:

The carrier signal in the DSB-FC system does not convey any information. The information is contained in the two sidebands only. But, the sidebands are images of each other and hence both of them contains same information. Thus all the information can be conveyed by only one sideband.

Total power transmitted by an AM wave,

$$P_t = P_c + P_{USB} + P_{LSB} = P_C + \frac{m_a^2}{4}P_c + \frac{m_a^2}{4}P_c$$

The carrier component does not contain any information and one sideband is redundant. Hence, out of the total power $P_t = P_c\left(1+\frac{m_a^2}{2}\right)$, the wasted power is

$$\text{Power wastage} = P_c + \frac{m_a^2}{4}P_c = \left[1+\frac{m_a^2}{4}\right]P_c$$

(ii) Bandwidth Requirement of DSB-FC

The bandwidth of DSB_FC system is $2f_m$. This is due to simultaneous transmission of both the sidebands, out of which only one is sufficient to convey all the information. So, the bandwidth of DSB-FC is double than actually required. Therefore, DSB-FC is a bandwidth inefficient system.

(iii) Effect of Noise:

When the AM wave travels from the transmitter to receiver over a communication channel, noise gets added to it. The noise will change the amplitude of the envelope of AM in a random manner. So, the noise will contaminate the information contents in the AM. Hence, the performance of AM is very poor in presence of noise.

APPLICATION OF AM:

(i) Radio broadcasting

(ii) Picture transmission in a TV system.

DOUBLE SIDE BAND SUPPRESSED CARRIER AM (DSB-SC-AM):

Two important parameters of a communication system are transmitting power and band width. Hence saving of power and band width are highly desirable in a communication system.

In AM with carrier scheme, there is wastage in both transmitted power and band width. In order to save the power in amplitude modulation the carrier is suppressed, because it does not contain any useful information. This scheme is called as double side band suppressed carrier amplitude modulation(DSB-SC-AM).It contains only LSB& USB terms and BW=2fm.

Let the modulating signal $v_m(t) = v_m \cos\omega_m t$

and carrier signal $v_c(t) = v_c \cos\omega_c t$

When multiplying both the carrier and message signal, the resultant signal is the DSB-SC-AM signal.

$$v(t)_{DSBSC} = v(t)_m v_c(t)$$

$$\therefore v(t)_{DSBCSC} = v_m \cos\omega_m t . v_c \cos\omega_c t$$

$$v(t)_{DSB-SC} = \frac{v_m v_c}{2}\left[\cos(\omega_m + \omega_c)t + \cos(\omega_m - \omega_c)t\right] \text{----------(1)}$$

Here the product of $v_c(t)$ and $v_m(t)$ produces the DSB-SC –AM signal. Thus we require product modulator to generate DSB SC signal.

We know that,

$$v(t)_{AM} = v_c \cos\omega_c t + \frac{m_a v_c}{2}\left[\cos(\omega_c + \omega_m)t + \cos(\omega_c - \omega_m)t\right]$$

When the above equation is compared with equation of DSB-SC, the unmodulated term $v_c \cos\omega_c t$ is missing and only two side bands are present. Hence the eq (1) is called DSB-SC-AM.
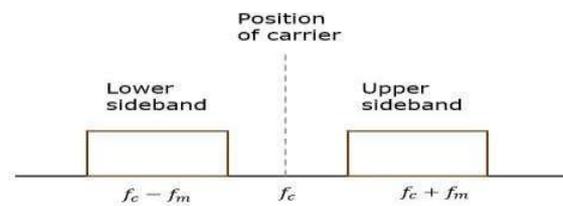
Fig: Frequency spectrum of DSB-SC-AM
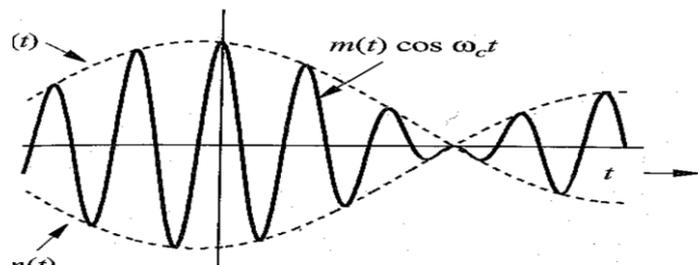
Fig: DSB-SC modulated signal

POWER CALCULATION:

We know that, the total power transmitted in AM is

$$p_t = p_{carrier} + p_{LSB} + p_{USB}$$

$$= \frac{v_c^2}{2R} + \frac{m_a^2 v_c^2}{8R} + \frac{m_a^2 v_c^2}{8R} = \frac{v_c^2}{2R}\left[1 + \frac{m_c^2}{2}\right]$$

$$p_t = p_c\left(1 + \frac{m_a^2}{2}\right)$$

Where $p_c = \frac{v_c^2}{2R}$

Hence the carrier is suppressed, then the total power transmitted in DSB-SC-AM is,

$$p_t{}^1 = p_{LSB} + p_{USB}$$

We know that,

$$p_{LSB} = p_{USB} = \frac{m_c^2 v_c^2}{8R}$$

$$\therefore p_t^1 = \frac{m_a^2 v_c^2}{8R} + \frac{m_a^2 v_c^2}{8R} = \frac{m_a^2}{2}\left[\frac{v_c^2}{2R}\right] = \frac{m_a^2}{2} p_c$$

Power saving = $\dfrac{p_t - p_t^1}{p_t}$

$$= \frac{\left[1 + \dfrac{m_a^2}{2}\right]p_c - \left[\dfrac{1}{2} m_a^2 p_c\right]}{\left[1 + \dfrac{m_a^2}{2}\right]p_c} = \frac{p}{\left(1 + \dfrac{m_a^2}{2}\right)p_c}$$

$$\therefore Powersaving = \frac{1}{\left(1 + \dfrac{m_a^2}{2}\right)} *100 = \frac{2}{2 + m_a^2} *100$$

If $m_a = 1$ then power saving $= \dfrac{2}{3} *100 = 66.7\%$

i.e:66.7% of power is saved.

Due to suppression of the carrier wave, the power saving is increasing from 33.3% to 66.7%.

GENERATION OF DSB-SC:

The expression for DSB-SC signal is given as,

$$s(t) = x(t)\cos\omega_c t$$

Where $x(t)$=Base band signal & $\cos\omega_c t$ =carrier signal

From this expression, a DSB-SC signal is basically the product of the modulating or base band signal and carrier signal.

- A circuit to achieve the generation of DSB-SC signal is called a product modulator.
- We have two types of product modulators namely, the balanced modulator and the ring modulator.

BALANCED MODULATOR USING TWO AM MODULATORS:
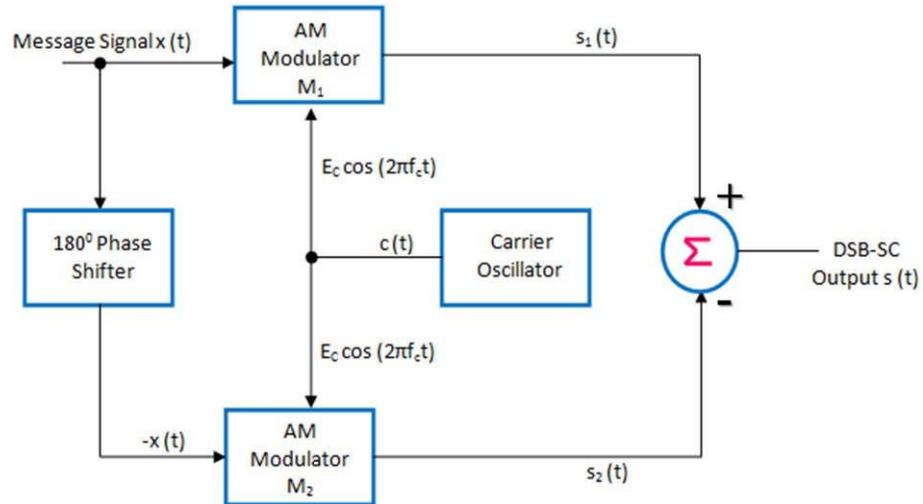
The above figure shows the balanced modulator circuit. It consists of two standard amplitude modulators arranged in a balanced configuration so as to suppress the carrier wave. Here, the two modulators are identical, except for the sign reversal of the modulating wave applied to the input of one of them. Therefore, the outputs of two modulators can be given as,

$$s_1(t) = A_c \left[1 + k_a m(t)\right] \cos 2\Pi f_c t$$

$$s_2(t) = A_c \left[1 - k_a m(t)\right] \cos 2\Pi f_c t$$

Subtracting $s_2(t)$ from $s_1(t)$,we obtain

$$s(t) = s_1(t) - s_2(t$$

$$= 2k_a A_c \cos(2\Pi f_c t) m(t)$$

Hence, except for the scaling factor $2k_a$, the balanced modulator output is equal to the product of the m (t) and carrier

RING MODULATOR OR CHOPPER MODULATOR:

Below figure shows the circuit diagram of a diode ring modulator. It consists of four diodes, an audio frequency transformer T1 and RF transformer T2. The carrier signal is assumed to be a square wave with frequency 'f_c' and it is connected between center taps of the two transformers. The DSB-SC output is obtained at the secondary of RF transformer T2.
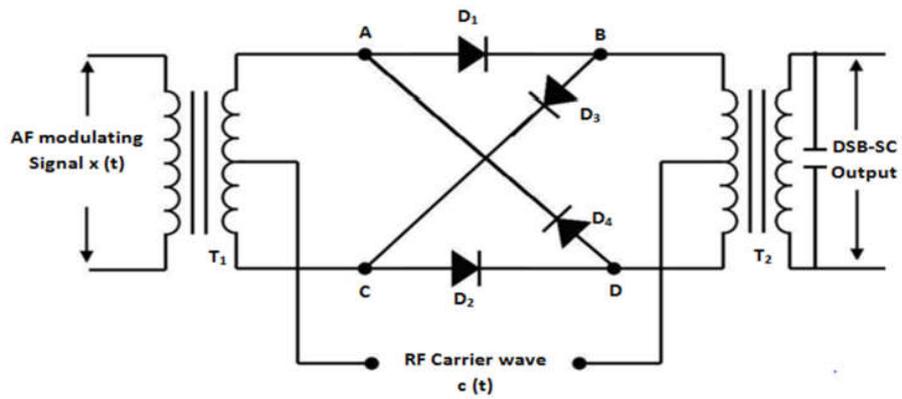
Fig: A diode ring modulator

WORKING OPERATION:

The operation is explained with the assumptions that the diodes act as perfect switches and that they are switched ON and OFF by the RF carrier signal. This is because the amplitude and frequency of the carrier is higher than that of the modulating signal. The operation can be divided into different modes without the modulating signal and with modulating signal follows.

(1) Mode1: Operation in absence of modulating signal

To understand how carrier suppression takes place, let us assume that the modulating signal is absent and only the carrier signal is applied. Hence x (t)=0

OPERATION IN THE POSITIVE HALF CYCLE OF CARRIER:

In this mode, let us assume that the modulating signal is zero, and only the carrier signal is applied

The equivalent circuit for this mode of operation is as shown in figure. The diodes D1 and D2 are forward biased. Diodes D3 and D4 are reverse biased.
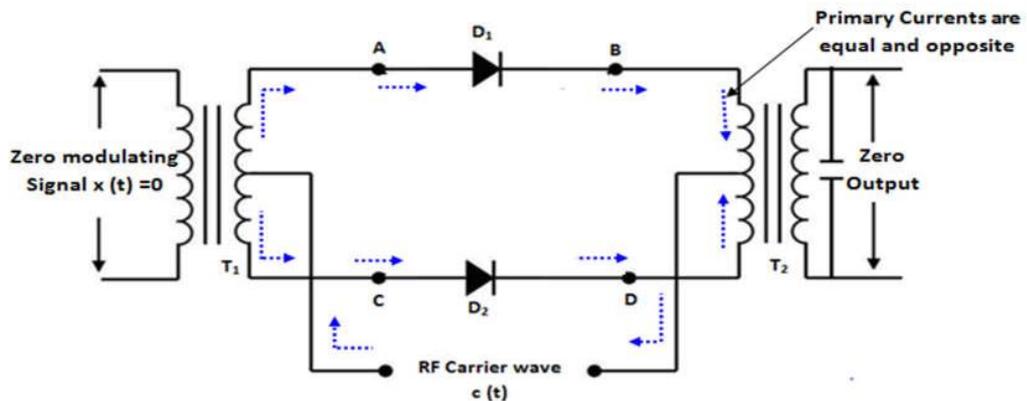
Fig: Equivalent circuit in mode 1

Let us observe the directions of currents flowing through the primary windings of output transformer T2, they are equal and opposite to each other. Therefore, the magnetic fields produced by the currents are equal and opposite and cancel each other. Hence induced voltage in the secondary winding is zero. Thus, the carrier is supposed in the positive half cycle.

OPERATION IN NEGATIVE HALF CYCLE OF THE CARRIER:

In this mode also the modulating signal is zero. In the negative half cycle of the carrier, the diodes D3 and D4 are forward biased and diodes D1 and D2 are reverse biased.

In figure the currents flowing in the upper and lower halves of primary winding of T2 are again equal and opposite directions. This is going to cancel the magnetic fields as explained in mode 1. Thus the output voltage in this mode is also zero. Thus the carrier is suppressed in the negative half cycle as well. The perfect cancellation of carrier will take place if and only if the characteristics of diodes are perfectly matched.
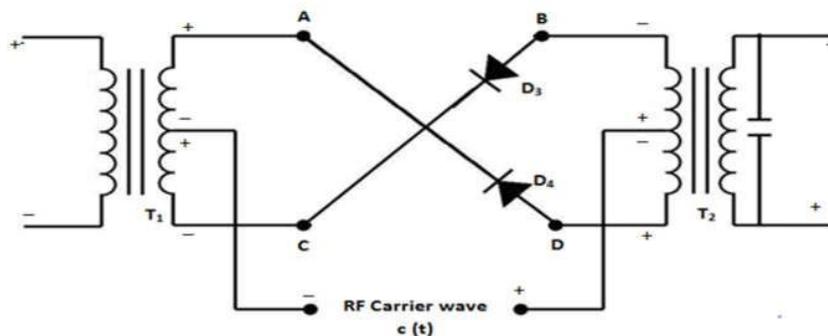
Fig: Equivalent circuit in mode 2

(2). Mode 2: Operation in presence of modulating signal

When RF carrier and the modulating signal both are applied

(A) OPERATION IN THE POSITIVE HALF CYCLE OF MODULATING SIGNAL:

(a) As we apply the low frequency modulating signal through the input audio transformer T1, there are many cycles of the carrier signal, in the positive half cycle of the modulating signal.

(b) In the positive half cycle of the carrier, D1 and D2 are ON and secondary of T1 is applied as it is across the primary of T2. Hence during the positive half cycle of the carrier, the output of T2 is positive as shown in fig (a).

(c) In the negative half cycle of the carrier, D3 and D4 are turned ON and the secondary of T1 is applied in a reversed manner across the primary of T2 as shown in equivalent circuit of fig (b). Thus, the primary voltage of T2 is negative and output voltage becomes negative.
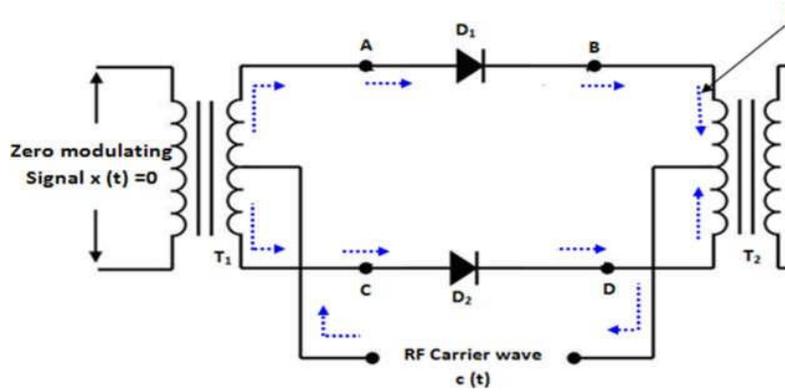
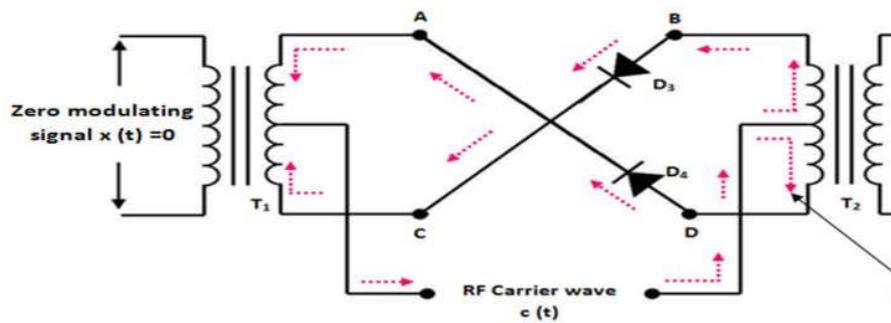**Fig: Equivalent circuit in the half cycle of m (t) carrier positive**

Fig: Equivalent circuit in the half cycle of m (t) carrier negative

(B) OPERATION IN THE NEGATIVE HALF CYCLE OF MODULATING SIGNAL

When modulating signal reverses the polarities, the operation of the circuit is same as that in the positive half cycle. Now, the only difference is that the diode pair D3D4 will produce a positive output voltage. Whereas D1D2 will produce a negative output voltage as shown in figure.
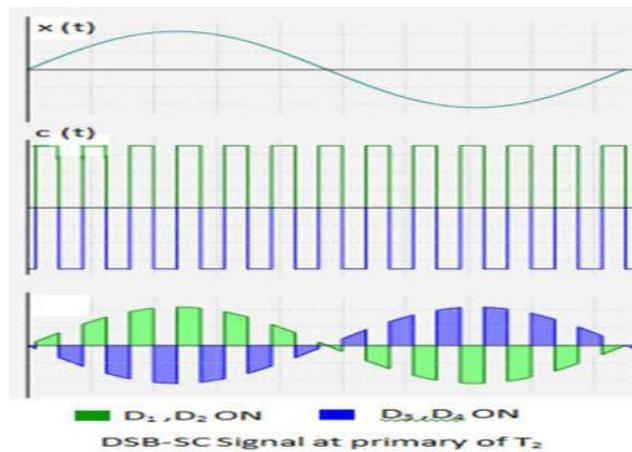
DEMODULATION OF DSB-SC SIGNAL:

COHERENT DETECTION OF DSB-SC MODULATED WAVES:

The modulating signal m (t) is recovered from a DSB-SC wave s(t) by first multiplying s(t) with a locally generated sinusoidal wave and then the low pass filtering the product.

For faithful recovery of modulating signal m(t), the local oscillator output should be exactly coherent or synchronized, in both frequency and phase with the carrier wave c(t) used in the product modulator to generate s(t). This method of demodulation is called coherent detection or synchronous detection.
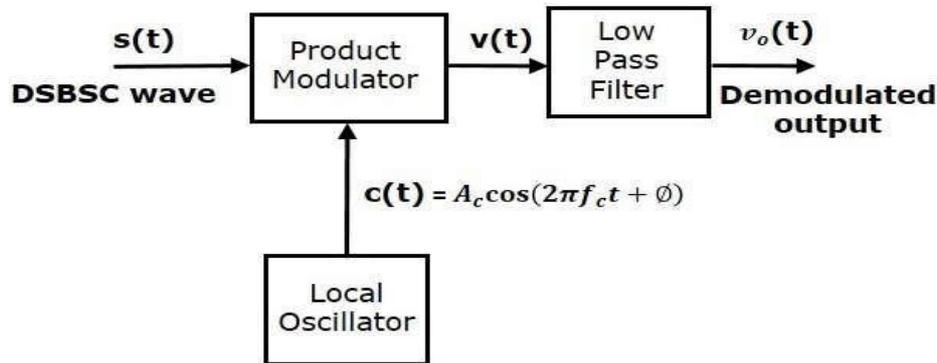
Fig: Coherent detection DSBSC modulated wave

From figure, the product modulator output is,

$$v(t) = \cos(2\Pi f_c t + \Phi)s(t)$$

$$= A_c \cos(2\Pi f_c t)\cos(2\Pi f_c t)m(t)$$

$$v(t) = \frac{1}{2}A_c \cos\Phi m(t) + \frac{1}{2}A_c \cos(4\Pi f_c t + \Phi)m(t)$$

The output consists of two terms;

Scaled version of message signal and unwanted term. The unwanted term is removed with the help of low pass filter. The overall output v $_0$(t) is therefore given as,

$$v_0(t) = \frac{1}{2}A_c \cos\Phi m(t)$$

Where phase error 'Φ' is constant, the demodulated signal v $_0$(t) is proportional to m (t). It is maximum when Φ=0 and minimum, when Φ=±Π/2. The zero demodulated signal, which occurs for Φ=±Π/2, represents the "quadrature null effect" of the coherent detector.

Because of phase error Φ, the detector output is attenuated by factor cosΦ. As long as phase error (cosΦ) is constant, the detector output provides an undistorted modulating signal m (t). Generally cosΦ is not constant, it varies randomly. Therefore, it is necessary to provide additional circuitry to maintain the local oscillator in perfect synchronism, in both frequency and phase with the carrier wave used to generate the DSB- SC modulated wave in the transmitter.

Note: Coherent detector is a combination of "AM DSB-SC modulator+filter".

It is also called synchronous or homodyne detector.

PHASOR DIAGRAMS:

AM (DSBFC):

$$V_{AM}(t) = A_c \cos\omega_c t + \frac{m_a v_c}{2} \left[\cos(\omega_c - \omega_m)t + \cos(\omega_c + \omega_m)t\right]$$
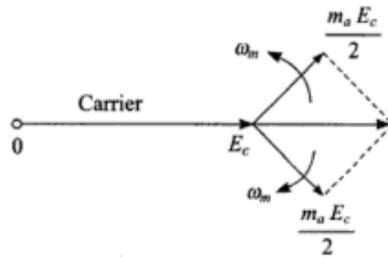

Fig: The phase representation of AM with carrier

Here v_c is the carrier wave phasor, taken as reference phasor. The two sidebands having a frequency of $(\omega_c+\omega_m)$ and $(\omega_c-\omega_m)$ are represented by two phasors rotating in opposite directions with angular frequency of 'ω_m'. The net or resultant phasor is s(t). It depends on the position of the sideband phasor and carrier wave phasor.

DSB-SC-AM:

$$s(t)_{DSB-SC} = \frac{m_a v_c}{2} \left[\cos(\omega_c + \omega_m)t + \cos(\omega_c - \omega_m)t\right]$$
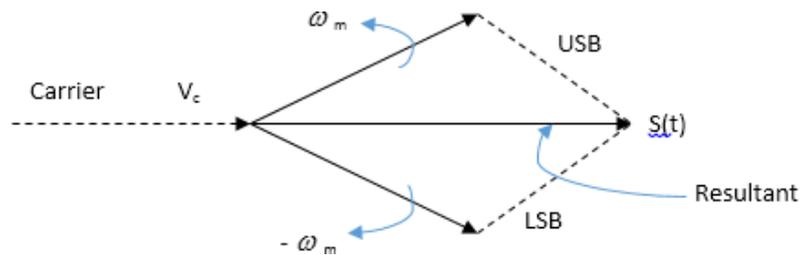

Fig: Phasor Diagram of DSB-SC-AM

Let, the carrier phasor is the reference phasor and oriented in horizontal direction as shown in figure, by the dotted line (because it is suppressed after modulation).

The USB term $m_a v_{c/2}\cos(\omega_c+\omega_m)t$ rotates at an angular frequency 'ω_m' in anti-clockwise direction and the LSB term $m_a v_{c/2}\cos(\omega_c-\omega_m)t$ rotates at an angular frequency of 'ω_m' in clockwise direction. Hence the resultant amplitude of modulated wave at any point is the vector sum of the side bands.

FREQUENCY MIXER OR CONVERTER:

A frequency mixer or frequency converter, used to change the carrier frequency of a modulated signal $m(t)\cos\omega_c t$ by $2\cos\omega_{mix}t$. Where $\omega_{mix} = \omega_c + \omega_I$ or $\omega_c - \omega_I$, and then band pass filtering the product.

The product x (t) is

$$x(t) = 2m(t)\cos\omega_c t \cos\omega_{mix} t$$

$$= m(t)\left[\cos(\omega_c - \omega_{mix})t + \cos(\omega_c + \omega_{mix})t\right]$$

If we select $\omega_{mix} = \omega_c - \omega_I$

$$x(t) = m(t)\left[\cos\omega_I t + \cos(2\omega_c - \omega_I)t\right]$$

If we select $\omega_{mix} = \omega_c + \omega_I$

$$x(t) = m(t)\left[\cos\omega_I t + \cos(2\omega_c + \omega_I)t\right]$$

In either case, a band pass filter at the output, turned to ω_I, will pass the term $m(t)\cos\omega_I t$. Thus the carrier frequency has been translated to ω_I from ω_c.

The operation of frequency mixing or frequency conversion is identical to the operation of modulation with a modulating carrier frequency (the mixer oscillator frequency ω_{mix}) that differs from the incoming carrier frequency by ω_I.

When we select the carrier frequency $\omega_{mix} = \omega_c + \omega_I$, the operation is called up-conversion, and when we select $\omega_{mix} = \omega_c - \omega_I$, the operation is called down-conversion.
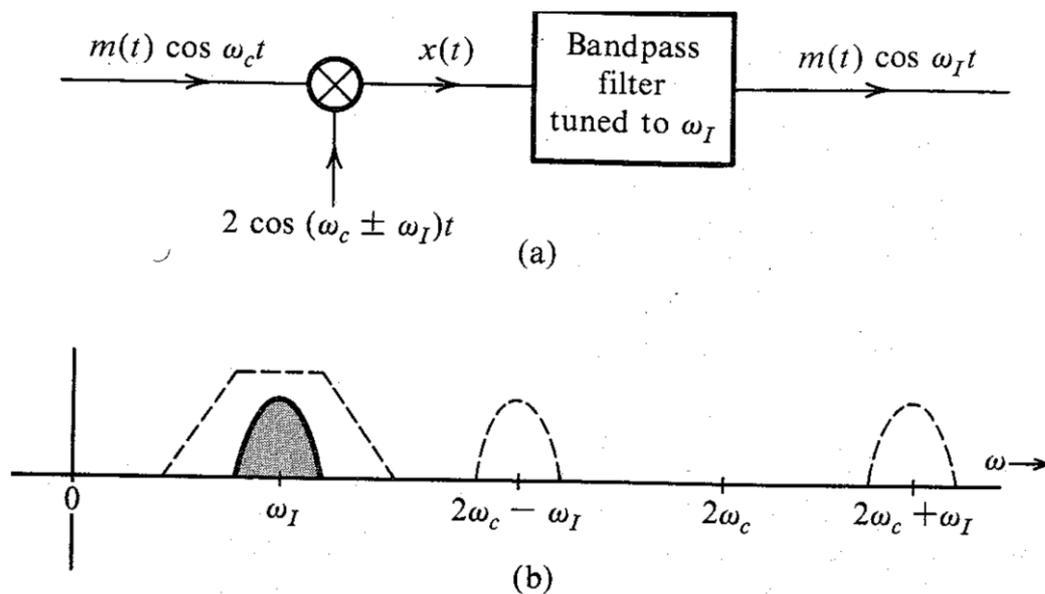

Fig: Frequency mixer or converter

SINGLE SIDE BAND SUPPRESSED CARRIER AM (SSB-SC-AM):

In AM with carrier both the transmitting power and bandwidth are wasted. Hence the DSB-SC scheme has been introduced in which power is saved by suppressing the carrier component but band width remains the same i.e; BW= $2\omega_m$.

To increase the saving of power is possible by eliminating one side band in addition to carrier component, because the USB and LSB are uniquely related by symmetry about the carrier frequency. So either one side band is enough for transmitting as well as recovering the useful message.

In addition to that, transmission bandwidth can be cut into half, one side band is suppressed along the carrier. This scheme is known as "SSB-SC-AM'

FREQUENCY SPECTRUM:

From the definition AM, the AM signal,

$$V_{Am}(t) = A_c \cos \omega_c t + \frac{A_c m_a}{2}\left[\cos(\omega_c - \omega_m)t + \cos(\omega_c + \omega_m)t\right]$$

By the definition of SSB-SC, carrier signal and one side band is suppressed. From the frequency spectrum of SSB-SC,

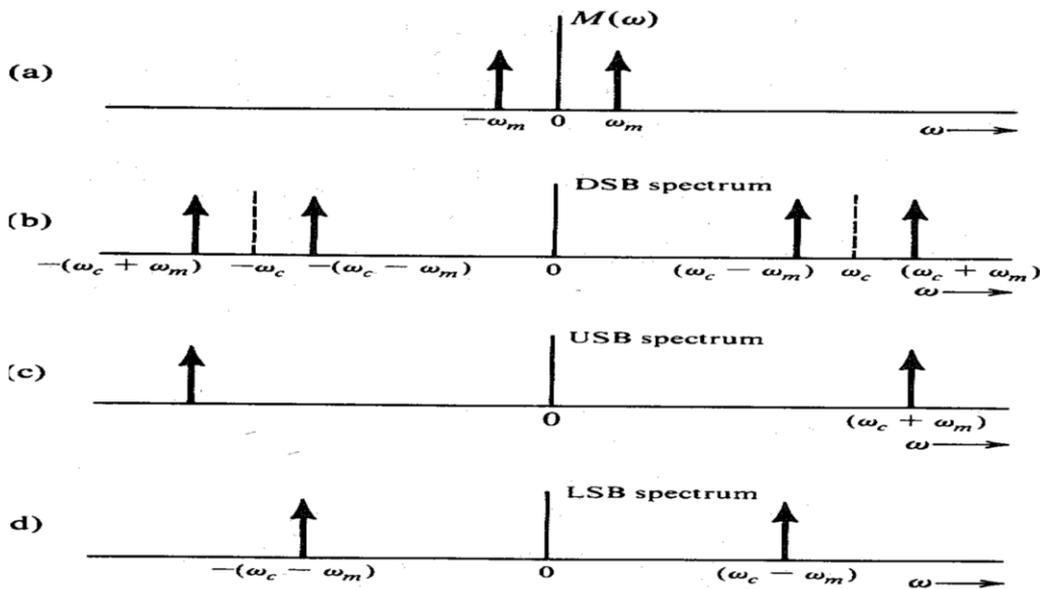$$V_{SSB-SC}(t) = \frac{m_a A_c}{2}\cos(\omega_c \pm \omega_m)t$$

Fig (a): Spectrum of base band signal   (b): Spectrum of DSB-SC

(c): Spectrum of SSB-SC wave with upper side band transmitted

(d) Spectrum of DSB-SC wave with lower side band transmitted

POWER CALCULATION:

Power in SSB-SC-AM is

$$p_t{}^{II} = p_{SB} = \frac{m_a{}^2}{4} p_c$$

Power saving with respect to AM with carrier

$$powersaving = \frac{p_t - p_t{}^{II}}{p_t}$$

Where P_t=total power transmitted

$$powersaving = \frac{\left[1 + \frac{m_a{}^2}{2}\right] p_c - \left[\frac{m_a{}^2}{4} p_c\right]}{\left[1 + \frac{m_a{}^2}{2}\right] p_c} = \left[\frac{1 + \frac{m_a{}^2}{4}}{1 + \frac{m_a{}^2}{2}}\right] * 100$$

Therefore, for 100% modulation i.e., ma=1 then,

% Power saving=1.25/1.5=83.3%

Saving of power in SSB-SC-AM w.r.t DSB-SC-AM

$$= \frac{P_t{}^{1} - P_t{}^{II}}{p_t{}^{1}} = \frac{\frac{1}{2} m_a{}^2 P_c - \frac{1}{4} m_a{}^2 P_c}{\frac{1}{2} m_a{}^2 p_c} = \frac{1}{2} * 100$$

Therefore %power saving=50%.

Frequency spectrum of SSB-SC-AM is as shown in figure.
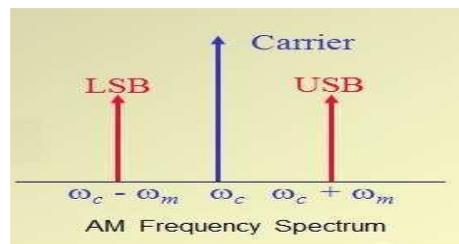
It shows that only one side band signal is present, the carrier and other (upper) side band signal are suppressed. Thus the bandwidth required reduces from $2\omega_m$ to ω_{ms} i.e; bandwidth requirement is reduced to half compared to AM and DSB-SC signals.

$$bandwidth = \omega_c - (\omega_c - \omega_m) = \omega_m$$

Graphical representation of SSB-SC-AM is
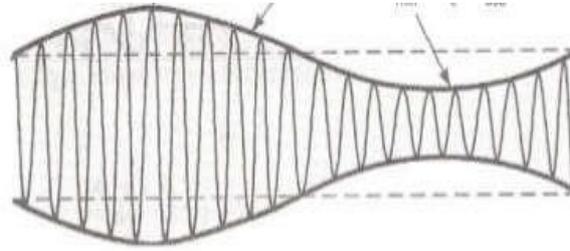
TIME DOMAIN DESCRIPTION OF THE SSB-SC WAVE:

Let us consider a single tone modulating signal is

$$x(t) = \cos\omega_m t$$

The frequency spectrum of this modulating signal consists of two pulses located at $\omega = \pm\omega_m$ as shown in figure (a). If this modulating signal, modulates a carrier signal $\cos\omega_c t$, then resulting spectrum of DSB-SC signal is shown in fig (b).

To get SSB-SC wave form, we have to eliminate one of two side bands. From fig (c), the spectrum of SSB-SC wave with lower side bands, corresponds to a time domain signal $\cos\omega_c$ t

Because the frequency spectrum of cosine function contains two impulses in its frequency domain.i.e; a SSB-SC wave with lower side band may be expressed as,

$$\cos(\omega_c + \omega_m)t = \cos\omega_m t \cos\omega_c t + \sin\omega_m t \sin\omega_c t ----> (1)$$

Similarly, when upper side band may be expressed as,

$$\cos(\omega_c + \omega_m)t = \cos\omega_m t \cos\omega_c t - \sin\omega_m t \sin\omega_c t ------> (2)$$

Since,

$$\sin\omega_c t = \cos\left(\omega_c t - \frac{\Pi}{2}\right)$$

$$\sin\omega_m t = \cos\left(\omega_m t - \frac{\Pi}{2}\right)$$

i.e; the sine terms may be obtained using the corresponding cosine terms simply by giving phase shift of (-Π/2).

Combining eq (1) and (2) we get,

$$s(t)_{SSB} = \cos\omega_m t \cos\omega_c t \pm \sin\omega_m t \sin\omega_c t$$

x (t) may be expressed as a continuous sum of sinusoidal signals.

Hence,

$$s(t)_{SSB} = x(t)\cos\omega_c t \pm x_n(t)\sin\omega_c t$$

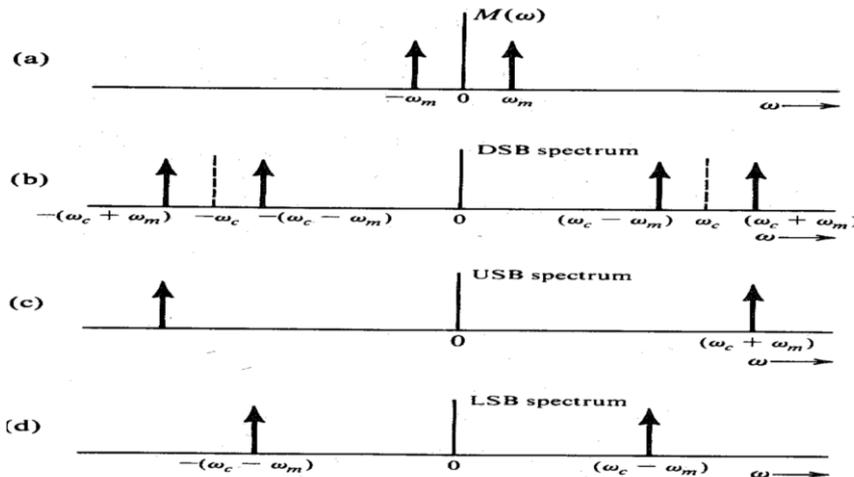Where x (t)=signal obtained by shifting the of every component present in x(t) and (-Π/2).

Fig: (a) Frequency spectrum of modulating signal

   (b) Frequency spectrum of DSB-SC wave

   (c) Spectrum of SSB with USB

   (d) Spectrum of SSB with LSB

GENERATION OF SSB-SC SIGNALS:

SSB-SC signals may be generated by two methods,

(a) Frequency discrimination method or filter method
(b) Phase discrimination method or phase shift method

(A)FREQUENCY DISCRIMINATION METHOD:

In a frequency discrimination method, firstly a DSB-SC signal is generated simply by using an ordinary product modulator. After this, form the DSB-SC, signal one of the two side bands is filtered out by a suitable band pass filter (BPF).
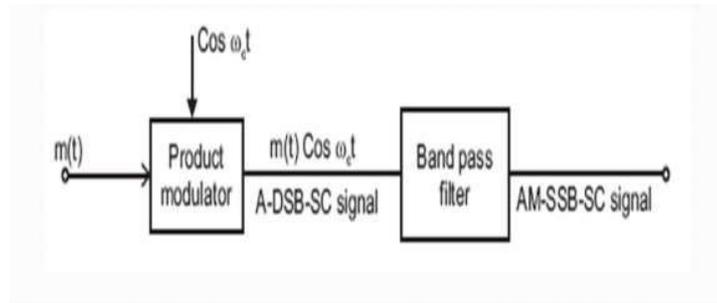
Fig: Frequency Discrimination Method

The design of band pass filter is quite critical and puts same limitations on the modulating or baseband and carrier frequencies.

The limitations for frequency discrimination method,

(a) The frequency discrimination method is useful only if the baseband signal is restricted at its lower edge due to which the upper and lower side bands are non-overlapping.

(b) Another restriction of the frequency discrimination method is that the baseband signal must be appropriately related to the carrier frequency. The design of the BPF becomes difficult if the carrier frequency is quite higher than the bandwidth of the band pass signal.

PHASE SHIFT METHOD FOR SSB-SC GENERATION:
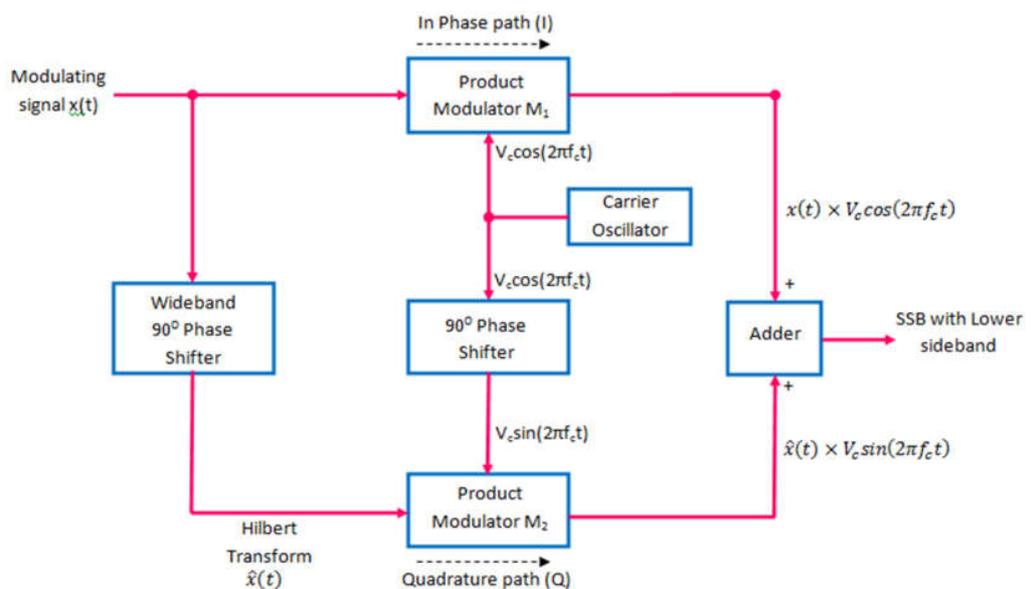
The above system is used for the suppression of upper side band. This system uses two balanced modulators M_1 and M_2. The output of the wideband 90^0 phase shifter is x^(t)

The output of the carrier oscillator is applied to modulator M_1. Whereas it is passed through a 90^0 phase shifter and applied to the modulator M_2.

Output of M_1 is,

$$x(t)v_c \cos(2\Pi f_c t)$$

Output of M_2 is,

$$\hat{x}(t)v_c \sin(2\Pi f_c t)$$

The outputs of M_1 and M_2 are applied to an adder. The negative sign for the quadrature path. And the adder output is,

$$= x(t)v_c \cos(2\Pi f_c t) + \hat{x}(t)v_c \sin(2\Pi f_c t)$$
$$= v_c \left[x(t)\cos(2\Pi f_c t) + \hat{x}(t) \sin(2\Pi f_c t) \right]$$

This represents the SSB signal with only LSB i.e., it rejects USB.

DEMODULATION OF SSB-SC SIGNALS:
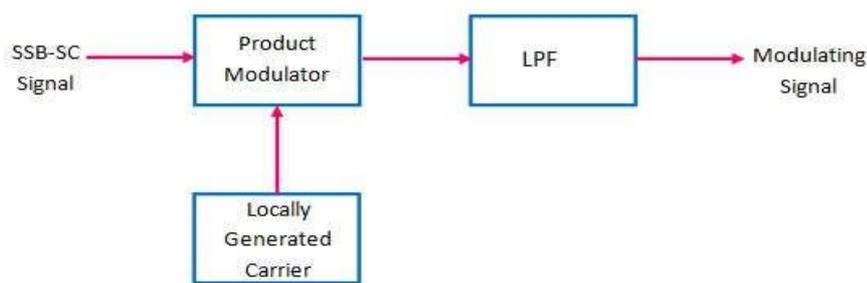
COHERENT DETECTION OF SSB-SC WITHOUT CARRIER:

Fig: Coherent detector

The SSB-SC signals can be coherently demodulated.

$$V_{SSB}(t) = m(t)\cos\omega_c t \mp \hat{m}(t)\sin\omega_c t$$

$$V_{SSB}(t)\cos\omega_c t = \frac{1}{2}m(t)\left[1 + \cos 2\omega_c t\right] \mp \frac{1}{2}\hat{m}(t)\sin 2\omega_c t$$

$$V_{SSB}(t)\cos\omega_c t = \frac{1}{2}m(t) + \frac{1}{2}\left[m(t)\cos 2\omega_c t \mp \hat{m}(t)\sin 2\omega_c t\right]$$

Thus, the product $v_{SSB}(t)\cos\omega_c t$ yields the baseband signal and another SSB signal with a carrier $2\omega_c$. A low pass filter will suppress the unwanted SSB terms, giving the desired baseband signal m (t)/2.

Hence the demodulator is identical to the synchronous demodulator used for the DSB-SC. Thus any one of the synchronous DSB-SC demodulators can be used to demodulate an SSB-SC signal.

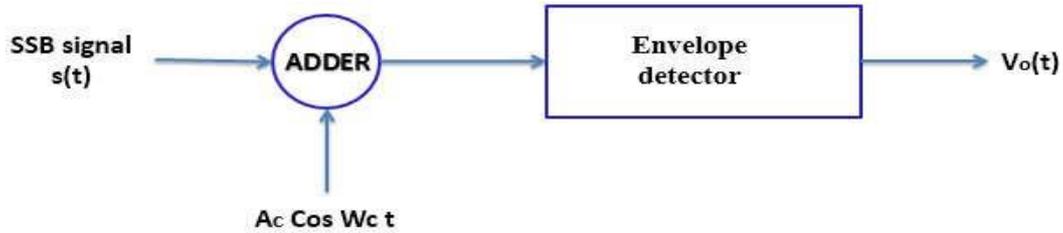## *ENVELOPE DETECTION OF SSB SIGNALS WITH A CARRIER:*

Fig: Envelope detection

Let us consider SSB signals with an additional carrier (SSB+C). Such a signal is expressed as,

$$V_{SSB+C}(t) = A\cos\omega_c t + [m(t)\cos\omega_c t + \hat{m}(t)\sin\omega_c t]$$

Although m(t) can be recovered by synchronous detection (multiplying V_{SSB+c} by $\cos\omega_c t$) of A, the carrier amplitude is enough, m(t) can also be recovered from V_{SSB+C} by envelope or rectifier detection, this can be shown by rewriting V_{SSB+C} as,

$$V_{SSB+C} = [A + m(t)]\cos\omega_c t + \hat{m}(t)\sin\omega_c t = E(t)\cos(\omega_c t - \theta)$$

Where E (t), the envelope of the V_{SSB+C} is given by,

$$E(t) = \left\{[A + m(t)]^2 + \hat{m}(t)\right\}^{1/2}$$

$$= A\left[1 + \frac{2m(t)}{A} + \frac{m^2(t)}{A^2} + \frac{\hat{m}^2(t)}{A^2}\right]^{1/2}$$

If A>>m (t), then in general, A>>|m^(t)| and the terms $m^2(t)/2$ and $m^{\wedge 2}(t)/A^2$ can be ignored. Thus,

$$E(t) \cong A\left[1 + \frac{2m(t)}{A}\right]^{1/2}$$

Using binomial expansion and discarding higher order terms (because m (t)/A<<1), we get

$$E(t) \cong A\left[1 + \frac{m(t)}{A}\right]$$

$$= A + m(t)$$

It is evident that for a large carrier, the SSB+C can be demodulated by an envelope detector.

SSB-SC PHASOR DIAGRAM:

The SSB-SC signal with lower side band,

$$s(t)_{SSB} = \frac{1}{2} A_c A_m \cos(\omega_c - \omega_m)t$$

The signal represents a single sinusoid with an amplitude $1/2 A_c A_m$ and a frequency $(\omega_c - \omega_m)$. Therefore the phasor diagram consists of a single phasor and there will be no amplitude fluctuation in the modulated wave.
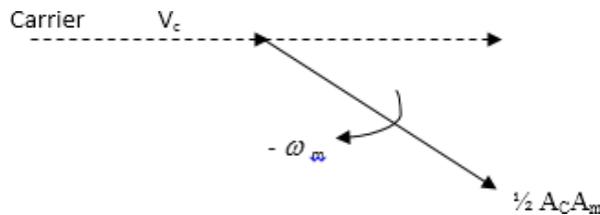

Fig: Phasor diagram for SSB-SC

VSB TRANSMISSION (VESTIGIAL SIDEBAND TRANSMISSION):

The stringent frequency response requirements on the side band filter in SSB-SC system can be relaxed by allowing a part of the unwanted sideband (called as vestige) to appear in the output of the modulator. Due to this, the design of the sideband filter is simplified to a great extent. But the bandwidth of the system is increased slightly.

For the generation of VSB signal, we have to first generate a DSB-SC signal and then pass it through a sideband filter. This filter will pass the wanted sideband as it is along with a part of unwanted sideband.

FREQUENCY DOMAIN DESCRIPTION:

Frequency spectrum: The spectrum of VSB is as shown in figure. The spectrum of message signal x(t) has also shown. In the frequency spectrum it is assumed that the upper sideband is transmitted as it is and the lower sideband is modified into vestigial sideband.
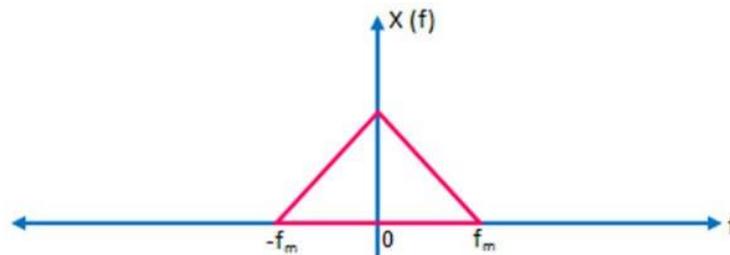

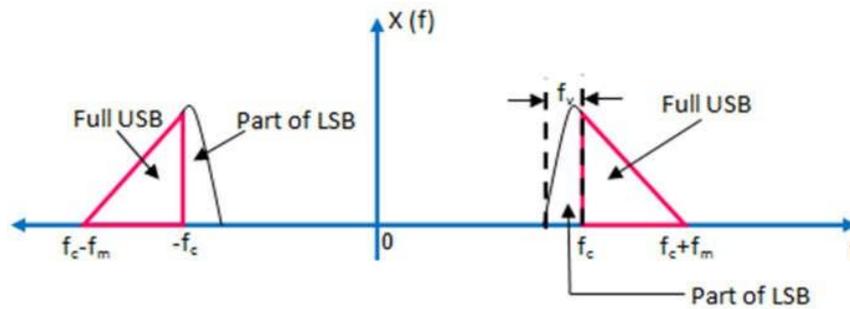**Fig (a): Spectrum of message signal**

Fig (b): Spectrum of VSB signal

From figure the transmission bandwidth of VSB modulated wave is given by,

$$B = (f_m + f_v) \text{ Hz}$$

Where f_m=message signal bandwidth & f_v=width of VSB

(1) The main advantage of VSB modulation is the reduction in bandwidth. It is almost as efficient as the SSB.

(2) Due to allowance of transmitting a part of LSB, the constraint on the filters have been relaxed. So, practically easy to design filters can be used.

GENERATION OF VSB MODULATED WAVE:
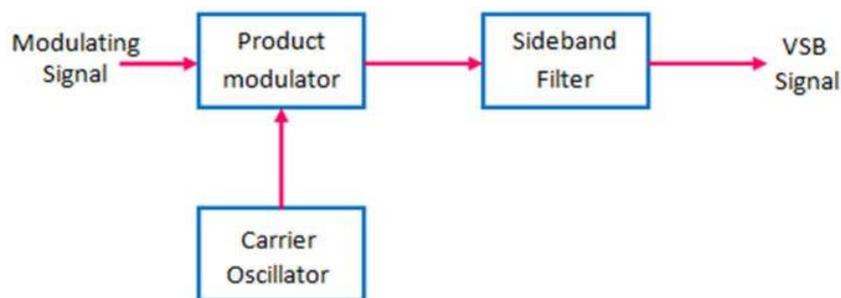
Fig: Generation of VSB signal

The modulating signal x (t) is applied to a product modulator. The output of carrier oscillator is also applied to the other input of the product modulator is given by,

$$m(t) = x(t) * c(t) = x(t) * v_c \cos(2\Pi f_c t)$$

This DSB-SC signal is then applied to a sideband shaping filter. The design of this filter depends on the desired spectrum of the VSB modulated signal. This filter will pass the wanted sideband as it is and the vestige of the unwanted sideband.

DEMODULATION OR DETECTION OF VSB WAVE:

The VSB modulated wave is passed through a product modulator where it is multiplied with the locally generated synchronous carrier. Hence the output of the product modulator is given by,

$$m(t) = s(t) * c(t) = s(t)\cos(2\Pi f_c t)$$

Fig: Synchronous detection of VSB
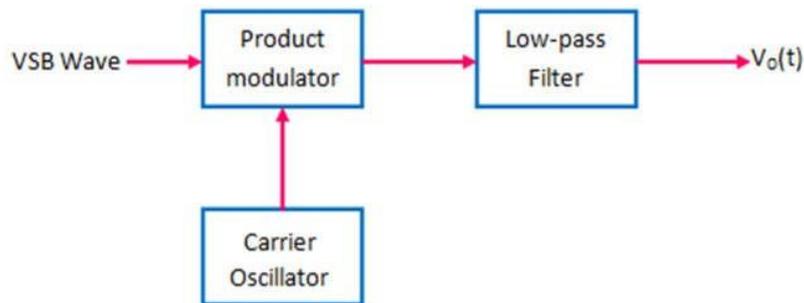
If we want to obtain the undistorted message signal x (t) at the output of the demodulator, then V_0 should be scaled version of x(t) .

THE EFFECT OF FREQUENCY AND PHASE ERROR IN DEMODULATION OF DSB-SC WAVE SYNCHRONOUS DETECTOR:

<u>FREQUENCY ERROR:</u>

In the demodulation of DSB-SC signal using synchronous detector, the phase and frequency of local oscillator should be similar to the phase and frequencies of the carrier used in DSB-SC signal generation. The error which occurs due to the same phase and different frequency of local oscillator with respect to carrier is known as frequency error.

Let the carrier signal used in the generation of DSB-SC signal is '$A_c \cos 2\pi f_c t$' and local oscillator signal be "$A_c' \cos 2\pi \left(f_c + \Delta f \right) t$". Here Δf represent the frequency shift between the local oscillator signal and the carrier. Then, the output of product modulator becomes as

$$
\begin{aligned}
S_c(t) &= S(t) \times A_c' \cos 2\pi \left(f_c + \Delta f \right) t \\
&= \left[A_c m(t)\cos 2\pi f_c t \right] A_c' \cos 2\pi \left(f_c + \Delta f \right) t \\
&= \frac{A_c A_c' m(t)}{2} \left[2\cos 2\pi \left(f_c + \Delta f \right) t \cos 2\pi f_c t \right] \\
&= \frac{A_c A_c^1 m(t)}{2} \left[\cos 2\pi \left(f_c + \Delta f \right) t + \cos 2\pi \Delta f t \right] \\
&= \frac{A_c A_c' m(t)}{2} \cos \left[2\pi \left(2 f_c + \Delta f \right) t \right] + \frac{A_c A_c' m(t)}{2} \cos 2\pi \Delta f t
\end{aligned}
$$

The signal obtained by passing $S_c(t)$ through LPF of cutoff frequency f_c is

$$S_o(t) = \frac{A_c A_c' m(t)}{2} [\cos 2\pi \Delta f t]$$

If $\Delta f \geq \omega$ then, the resultant signal cannot represent the required message signal. However, Δf is a small variation in f_c. Such that $f_c \gg \omega$, which means Δf comparable with ω.

After demodulation process, the resultant signal does not contain actual frequency components present in the message signal. This lead to distortion in the message signal.

PHASE ERROR:

The error which occurs due to the same frequency and different phase of local oscillator with respect to the carrier (at the transmitter) is known as phase error.

Let the carrier signal used in the generation of DSB-SC signal be '$A_c \cos 2\pi f_c t$' and local oscillator signal be $A_c' \cos(2\pi f_c t + \phi)$. Here ϕ represents the phase difference between the local oscillator signal and the carrier. Then the output of product modulator is

$$S_c(t) = S(t) \times A_c' \cos(2\pi f_c t + \phi)$$
$$= [A_c m(t) \cos 2\pi f_c t] A_c' \cos(2\pi f_c t + \phi)$$
$$= \frac{A_c A_c' m(t)}{2} [2\cos(2\pi f_c t + \phi)\cos 2\pi f_c t]$$
$$= \frac{A_c A_c' m(t)}{2} [\cos(4\pi f_c t + \phi) + \cos\phi]$$
$$= \frac{A_c A_c' m(t)}{2} [\cos(4\pi f_c t + \phi)] + \frac{A_c A_c' m(t)}{2} \cos\phi$$

The signal obtained by passing the signal $S_c(t)$ through a LPF of cutoff frequency f_c is

$$S_o(t) = \frac{A_c A_c' m(t)}{2} \cos\phi$$

From the above equation it is evident that output of demodulator is proportional to message signal for constant phase error ϕ

For $\phi = 0$, $S_o(t) = \frac{A_c A_c' m(t)}{2}$

i.e., Maximum amplitude of the demodulated signal is obtained

For $\phi = \pm\frac{\pi}{2}$, $S_o(t) = 0$

i.e., Minimum amplitude of the demodulated signal is obtained. This zero output is referred as *"Quadrature Null Effect "* of synchronous detector.

Thus the demodulator output completely depends on the factor '$\cos\phi$'. The output is a distorted version of actual message signal till the phase error ϕ is made constant.

FREQUENCY DIVISION MULTIPLEXING:

        Freq. division multiplexing (FDM) is a technique for transmitting multiple messages simultaneously over a wideband channel by first modulating the message signals on to a several sub carriers and forming a composite baseband signal that consists of the sum of these modulated sub carriers.

        This composite signal may then be modulated on to the main carrier. Any type of modulation such as AM, DSB, SSB, PM, FM and so on can be used. The types of modulation used on the sub carriers, as well as the type used on the main carrier, may be different from figure (b), the composite signal spectrum must consists of modulated signals that do not have over lapping spectra otherwise crosstalk will occur between the message signal at the receiver output. The composite baseband signal then modulates a main transmitter to produce the FDM signal that is transmitted over the wideband channel.

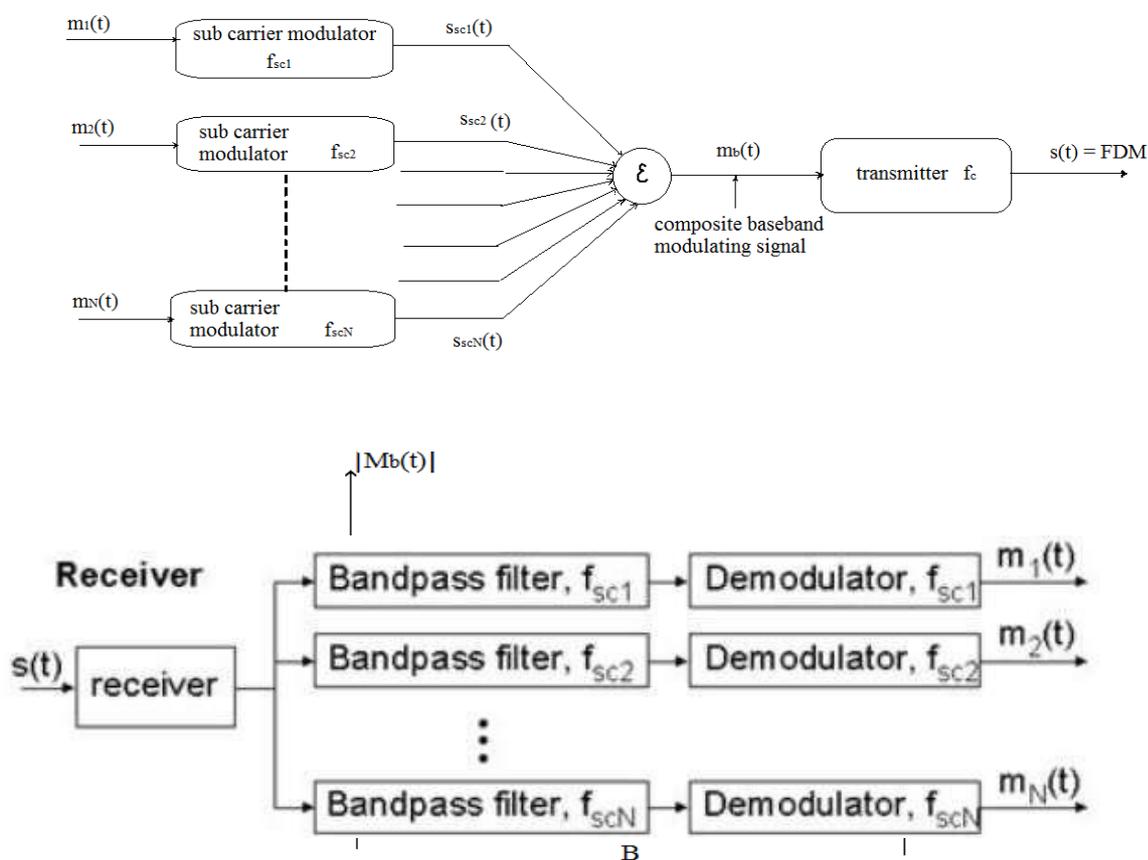

fig : the spectrum component of baseband signal

The receiver FDM signal is fast demodulated to reproduce the composite baseband signal that is passed through filters to separate the individual modulated sub carriers then the sub carriers are demodulated to produce the message signals m1(t), m2(t),... .

ANGLE MODULATION &DEMODULATION

Angle Modulation is the process by which the angle (i.e., Frequency or Phase) of the carrier signal is changed in accordance with the instantaneous amplitude of modulating or message signal. It is also known as "Exponential modulation" but the amplitude of the carrier remains constant.

This is an advantage over AM since, all natural internal and external noises consist of electrical amplitude variations. The receiver cannot distinguish between amplitude variations that represent noise and those that represent designed signal.

CONCEPT OF INSTANTANEOUS FREQUENCY:

The time dependent angular frequency ω_i of the phasor φ provides a time varying instantaneous frequency fi of the carrier wave φ (t). This implies that frequency of the carrier wave changes from one cycle to another. The instantaneous frequency fi may vary with some function of time. Figure (a) shows a variation in the frequency of a carrier. The variation is linear with time from a value ω_0 and $2\omega_0$. Figure (b) shows the corresponding waveform of the carrier with varying frequencies. The frequency is maximum at $t=t_1$ and t_3 and minimum at 0 and t_2.
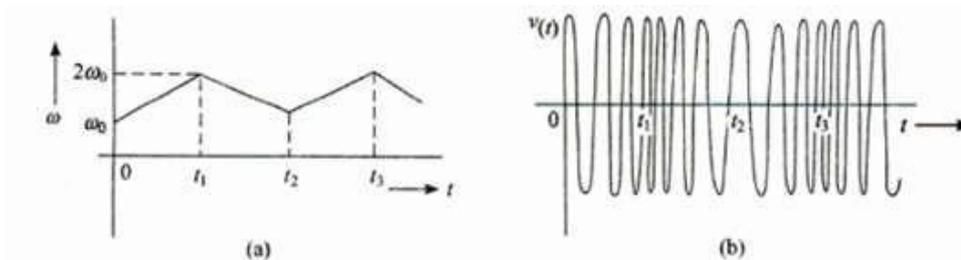

Fig: Concept of Instantaneous Frequency: (a) variation of ω with Time, (b) waveform of a carrier wave with varying frequency.

GENERALIZED CONCEPT OF ANGLE MODULATION:

Angle modulation is a method in which either the phase or frequency of the carrier wave is varied according to the message signal. In this method of modulation the amplitude of the carrier wave is maintained constant.

"Angle Modulation is a method of modulation in which either Frequency or Phase of the carrier wave is varied according to the message signal."

The Angle modulation can be broadly classified into two types such as

1. Frequency Modulation
2. Phase Modulation.

Let V_{FM} (t) $=V_c \cos(\omega c$ t$+ \theta$ (t)) $= V_c \cos \varnothing$ (t)

Where V_{FM} (t) = Angle modulated wave

V_c = Maximum amplitude of carrier signal

ω_c = Angular frequency of carrier signal= $2\pi f_c$

θ (t) = Instantaneous phase deviation

$\emptyset$ (t) = [ω_c t+ θ (t)] = phase angular of carrier

In angle modulation, the phase angle of carrier signal is varied in accordance with the instantaneous amplitude of message signal.

> We know that $\emptyset$ (t) =ω_ct+ θ (t) , whenever the frequency of carrier is varied, and vice versa thus FM&PM both occur even if either of the terms are varied. As a result direct FM modulation cause indirect PM& vice versa.

Instantaneous phase (Ø (t)):

It can be defined as the phase of the carrier at any instant of time.

Instantaneous phase deviation θ (t):

It is defined as the change in phase of the carrier at any instant of time with respect to its reference phase.

Instantaneous frequency (ω_i):

It is the frequency of the carrier at any instant of time i.e.

ω_i (t)= d/dt $\emptyset$ (t) = d/dt [ω_ct+ θ (t)]

$=\omega_c$t+ θ^1(t)

Instantaneous frequency deviation θ^1(t):

It is the change in frequency of the carrier. It can be defined as the first time derivation of instantaneous phase deviation.

Deviation Sensitivity:

The deviation sensitivity provides relationship between o/p parameter changes in respect to input parameters, for FM, the output frequency is varied in accordance with the amplitude of the modulating signal.

i.e., $K_f = \dfrac{\Delta\omega}{V_{in}} = \dfrac{\text{Change in output frequenCy}}{\text{Change in input voltage}}$

Similarly for PM, the o/p phase is varied w.r.t. amplitude of modulating signal.

i.e., $K_p = \dfrac{\Delta\theta}{V_{in}}$ or $\Delta\theta= K_p * V_{ik}$.

FREQUENCY MODULATION:

Frequency Modulation can be defined as the process by which the frequency of the carrier wave is altered in accordance with the instantaneous amplitude of modulating or message signals. The mathematical representation of frequency modulation is obtained as follows,

Let the message signal $V_m(t) = V_m \cos\omega_m t$ ……………(1) and

The carrier signal $V_c(t) = V_c \cos(\omega_c t + \theta)$............... (2)

V_m = Maximum Amplitude of message or modulating signal.

V_c = Maximum Amplitude of carrier signal.

ω_m = angular frequency of modulating signal.

ω_c = angular frequency of carrier signal.

$\emptyset$ = total instantaneous phase angle of carrier.

$\emptyset = (\omega_c t + \theta)$

$V_c(t) = V_c \cos\emptyset = V_c \sin(\omega_c t + \theta)$ ....(3)

To find angular velocity, differentiate the eq. (3) w.r.t

i.e., $\dfrac{d\emptyset}{dt} = \omega_c = \theta^1(t)$.

The frequency of the carrier after modulation is,

$$\omega_i = \omega_c + K_f Vm(t) = \omega_c + KV_m \cos\omega_m t .... (4)$$

Where k= constant of proportionality.

The instantaneous phase angle of modulated signal integrate eq. (4)

$\emptyset_i = \int \omega_i \, dt = \int(\omega_c + KV_m \cos\omega_m t)dt$

$$= \omega_c t + \dfrac{KV_m}{\omega m} \sin\omega_m t + \theta 1$$

$\theta 1$ = Integration constant, it is neglected because it plays no role in modulation process.

The instantaneous amplitude of the modulating signal is given by,

$V(t)_{FM} = V_c \cos\emptyset i$

$$= V_c \cos(\omega_c t + \frac{KVm}{\omega_m} \sin\omega_m t) \ldots\ldots\ldots (5)$$

$$V(t)_{FM} = v_c \cos(\omega_c t + m_f \sin\omega_m t) \ldots\ldots\ldots\ldots (6)$$

Where $m_f = \dfrac{KfVm}{\omega_m}$ = modulation index of FM.

General Expression for FM is given by

$S_{FM}(t) = v_c \cos(\omega_c t + m_f \sin\omega_m t)\ldots\ldots\ldots\ldots$ in terms of modulation index

$S_{FM}(t) = v_c \cos(\omega_c t + K_f \int v_m(t)\, dt)\ldots\ldots\ldots\ldots$ in terms of deviation sensitivity

From eq. (4) the instantaneous angular frequency of FM signal is ,

$\omega_i = \omega_c + KV_m \cos\omega_m t$

The maximum and minimum value of cosine term is ±1. Hence the maximum value of angular frequency is given by $\omega_{max} = \omega_c + KV_m$.

The minimum value of angular frequency is $\omega_{min} = \omega_c - KV_m$.

Then frequency deviation is

$$\boldsymbol{\omega_d = \omega_{max} - \omega_c = \omega_c - \omega_{min} = KV_m}$$

Modulating sine-wave signal
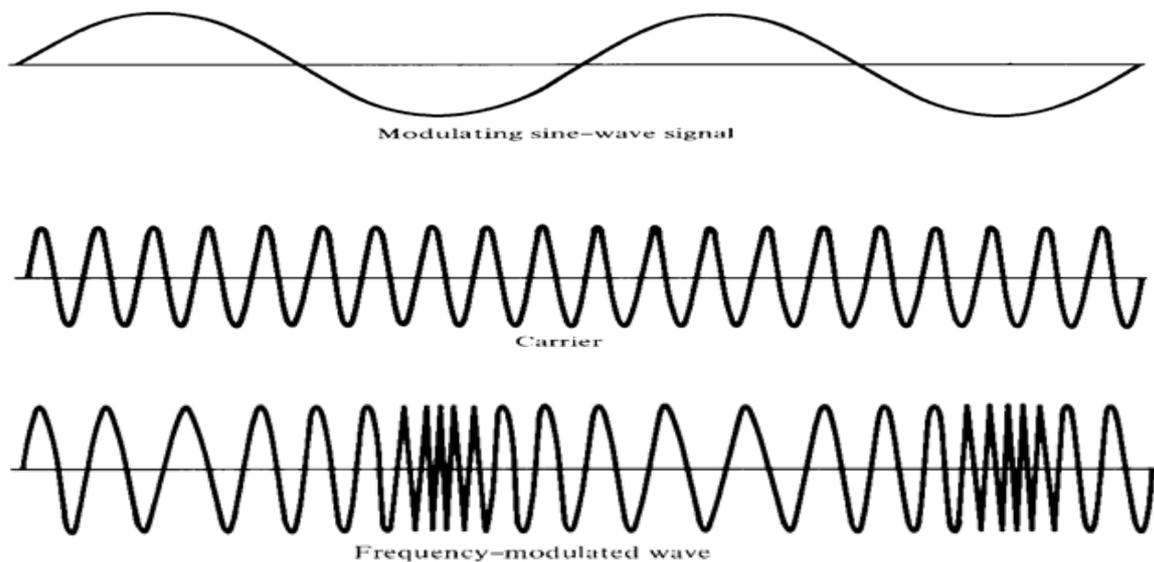
Carrier

Frequency-modulated wave

Fig: Graphical representation of FM wave

The modulation index of FM system can be defined as the ratio of max. Frequency deviation to the modulating frequency.

i.e., $m_f = \omega_d / \omega_m = K_p V_m / \omega_m$

$\omega_d = K_p V_m$ = Maximum frequency deviation

FREQUENCY DEVIATION:

It is defined as the change in frequency of the carrier w.r.t. amplitude of modulating signal, it can be written as $\Delta\omega = K_f V_m$ where K_f =Deviation sensitivity in terms of modulation index,

$m_f = \Delta\omega / \omega_m = \Delta f/f_m$ or $\Delta f = m_f f_m$.

SINGLE TONE FREQUENCY MODULATION:

The general expression for FM wave is given as,

$S(t) = Vc \cos [\omega_c t + K_f \int V_m(t) dt]$

In general expression of FM wave, a modulating or baseband signal $V_m(t)$ which may consist of any no. of frequency components.

A single tone frequency modulation is that type of frequency modulation (FM) in which the modulating or baseband signal contains a single frequency.

MATHEMATICAL EXPRESSION:

Let the expression for carrier signal is $V_C(t) = V_C \cos\omega_c t$.

Let the modulating signal is $V_m(t) = V_m \cos\omega_m t$.

Let the expression for FM wave is $S(t) = V_C \cos \emptyset_i$.

Where $\emptyset_i$ = Instantaneous phase angle of modulated wave

Instantaneous frequency of modulated signal is $\omega_i = \omega_C + K_f V_m(t)$

$$= \omega_C + K_f V_m \cos\omega_m t.$$

But frequency deviation, $\Delta\omega = K_f V_m$

$$\therefore \omega_i = \omega_c + \Delta\omega \cos\omega_m t.$$

The total phase angle $\emptyset_i$ of the modulated wave is given as,

$$\emptyset_i = \int \omega_i dt = \int [\omega_c + \Delta\omega\cos\omega_m t] dt$$

$$= \omega_c t + \Delta\omega / \omega_m \sin\omega_m t.$$

Since modulation index, $m_f = \Delta\omega / \omega_m$.

$$\therefore \emptyset_i = \omega_c t + m_f \sin\omega_m t.$$

The expression for single tone frequency FM wave as,

$$S(t) = V_C \cos \emptyset_i.$$

$$S(t) = V_C \cos[\omega_c t + m_f \sin\omega_m t].$$

MULTI TONE MODULATION:

Modulation done with more than one message signal is called **"Multi Tone Modulation".**

Let us consider the message signal as

$$V_m(t) = V_{m1} \cos\omega_{m1}t + V_{m2} \cos\omega_{m2}t.$$

Let the carrier signal be

$$V_C(t) = Vc \sin(\omega_c t + \theta) = Vc \sin\emptyset$$

Let $(\omega_c t + \theta) = \emptyset$ and angular velocity $\omega_c = d\emptyset/dt$

The instantaneous frequency is

$$\omega_i = \omega_c + K_f V_m(t)$$

$$= \omega_c + K_f [V_{m1} \cos\omega_{m1}t + V_{m2} \cos\omega_{m2}t]$$

$$= \omega_c + K_f V_{m1} \cos\omega_{m1}t + K_f V_{m2} \cos\omega_{m2}t$$

The frequency duration will be maximum when $\cos w_{m1} = \pm1$, $\cos w_{m2} = \pm1$. The frequency deviation is Proportional to amplitude of modulating signal.

$$\omega_i = \omega c + K_f V_{m1} \cos\omega_{m1}t + K_f V_{m2} \cos\omega_{m2}t$$

$$\omega_i = \omega_c + \Delta\omega_1 \cos\omega_{m1}t + \Delta\omega_2 \cos\omega_{m2}t$$

The instantaneous phase in $\emptyset_i = \int \omega_i dt$

$$\emptyset_i = \int (\omega_c + \Delta\omega_1 \cos\omega_{m1}t + \Delta\omega_2 \cos\omega_{m2}t) dt$$

$$= \omega_c t + \Delta\omega_1/\omega_{m1} (\sin\omega_{m1}t) + \Delta\omega_2/\omega_{m2} (\sin\omega_{m2}t)$$

$$\emptyset_i = \omega_c t + \Delta f_1/f_{m1} (\sin\omega_{m1}t) + \Delta f_2/f_{m2} (\sin\omega_{m2}t)$$

After frequency modulation,

$$V_{FM}(t) = V_C \sin\emptyset_i$$

$$V_{FM}(t) = V_C \sin[\omega_c t + \Delta f_1/f_{m1} \sin\omega_{m1}t + \Delta f_2/f_{m2} \sin\omega_{m2}t]$$

$$V_{FM}(t) = V_C \sin[\omega_c t + mf_1 \sin\omega_{m1}t + mf_2 \sin\omega_{m2}t]$$

If $\alpha 1 = mf_1 \sin\omega_{m1}t$, $\alpha 2 = mf_2 \sin\omega_{m2}t$,

Then $V_{FM}(t) = V_C \sin[\omega_c t + (\alpha 1 + \alpha 2)]$

$$= V_C [\sin\omega_c t \cos(\alpha 1 + \alpha 2) + \cos w_C t \sin(\alpha 1 + \alpha 2)]$$

$$= V_C \{ \sin\omega_c t [\cos\alpha 1 \cos\alpha 2 - \sin\alpha 1 \sin\alpha 2] + \cos\omega_c t [\sin\alpha 1 \cos\alpha 2 + \cos\alpha 1 \sin\alpha 2]\}$$

In order to simplify the above eq. the Bessel function can be used hence the resultant equation:

$$V_{FM}(t) = Vc \sum_{n=-\infty}^{n=\infty} J_n(\alpha 1) J_m(\alpha 2) (\cos\omega_c t \pm n\omega_1 t \pm n\omega_2 t)$$

From the above eq., it has four frequency terms.

(i) Carrier frequency component with an amplitude $[J_0(\omega 1). J_0(\omega 2)]$
(ii) A set of side bands corresponding to first tone w1.the side bands have amplitude $J_n(\alpha 1). J_0(\alpha 2)$ and frequency $(\omega 1 + n\omega 2)$ where $n = 1,2,3,\ldots\ldots$

TRANSMISSION BANDWIDTH OF FM:

The bandwidth of FM signal depends upon the value of modulation index m_f. With the increase of modulation index m_f, more and more no of sidebands acquire significant amplitudes and thus bandwidth is increased.

➢ The separation between the two extreme significant side frequencies on the two sides of the carrier is known as effective bandwidth of FM.
➢ For large value of 'β', the bandwidth of FM is slightly greater than the total frequency excursion $(2\Delta f)$ where as for small value of 'β'. The bandwidth approaches to '$2f_m$'.

Hence, the bandwidth of FM signals can be

$$B_T = 2\Delta f + 2f_m = 2(\Delta f + f_m)$$

$$= 2\Delta f(1 + \frac{f_m}{\Delta f})$$

CARSON'S RULE:

The bandwidth of a signal tune wide band FM can be determined by using Carson's rule according to the Carson's rule the bandwidth of FM signal is

$$BW = 2(\Delta\omega + \omega_m) = 2\Delta\omega(1 + \frac{\omega_m}{\Delta\omega})$$

$$BW = 2(\Delta\omega + \omega_m) = 2\Delta\omega \left(1 + \frac{1}{\beta}\right) \quad \text{Radians/sec}$$

$$BW = 2\Delta f \left(1 + \frac{1}{\beta}\right) Hz \qquad \text{(Or)}$$

$$BW = 2\omega_m[1 + \beta] \quad \text{Rad/sec}$$

(i)   If $\Delta\omega \ll \omega_m$ i.e., m_f(or)$\beta \ll 1$ as in the case for narrowband FM,
      then $BW = 2\omega_m$.

(ii) If $\Delta\omega \gg \omega_m$ i.e., $\beta \gg 1$ as in the case for wideband FM, then BW =$2\beta\omega_m$
             But $\beta\omega_m = \Delta\omega$
      Therefore $B = 2\Delta\omega$

CARRIER SWING:

The total variation in frequency from the lowest to highest point is called **"Carrier Swing".**

The carrier swing = 2*frequency deviation =$2*\Delta\omega$

PERCENT MODULATION:

It is used in reference to FM refers to the ratio of actual frequency deviation to the maximum allowable frequency deviation.

Percent modulation, M= $\frac{\Delta faCtual}{\Delta f_{max}}$ *100

BANDWIDTH OF ANGLE MODULATED WAVES:

TYPES OF FREQUENCY MODULATION:

Depending on modulation index 'β', the FM can be classified into two types. If the value of the modulation index 'β' is very small compared to one radian, it is called **"Narrow-Band Frequency Modulation (NBFM)"**.

If the modulation index values very large compared to one radian, it is called **"Wide-Band Frequency Modulation (WBFM)"**.

NOTE:

 For NBFM

> ➢ Modulation index, $\beta \ll 1$
> ➢ Narrow B.W which is equal to twice the message bandwidth

For WBFM

> ➢ Modulation index, $\beta \gg 1$
> ➢ Large B.W which is ideally infinite

NARROW BAND FREQUENCY MODULATION (NBFM):

Let the message signal, $V_m(t) = V_m Cos(\omega_m t)$

Carrier signal, $V_C(t) = V_C \cos(\omega_C t)$ then

FM signal is $S(t) = V_C \cos[\omega_C t + \beta \sin(\omega_m t)]$

W.K.T $\cos(a + b) = \cos a \cos b - \sin a \sin b$ then

$$S(t) = V_C[\cos\omega_C t + \cos(\beta \sin t \omega_m) - \sin \omega_C t \sin(\beta \sin \omega_m t)]$$

Since the value of β is very small for NBFM,

Therefore, $\cos(\beta \sin(\omega_m t)) = 1$ [where $\cos \theta = 1$ if θ is small]

$\sin(\beta \sin(\omega_m t)) = \beta \sin(\omega_m t)$ [Where $\sin \theta = \theta$ if θ is small]

Substitute the above approximations,

$$S(t) = V_C[\cos \omega_C t - \sin \omega_C t(\beta \sin \omega_m t)]$$

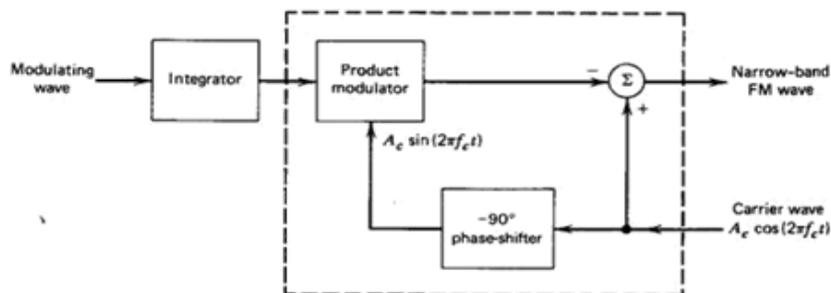With the help of above equation, we can set an arrangement for generating NBFM signal

Fig: Block Diagram for Generation of NBFM

The message signal Vm (t) is fed into the integrator and it follows to one input of the product modulator. The other input to the product modulator is obtained from the 90° phase shifter circuit; the input to the 90° phase shifter circuit is the carrier signal $V_C \cos \omega_C t$. The output of the product modulator is inverted and it is given to one input of the adder, the other for the adder is carrier signal. The output of the adder is nothing but the Narrow Band FM signal.

$\therefore$ Input signal, $V_m(t) = kV_m(\cos(\omega_m t))$

After integration, $\int V_m(t) = k\dfrac{V_m}{\omega_m} \sin(\omega_m t)$

This integration output is multiplied with the 90° phase shifting of carrier signal

Product modulator output $= k\dfrac{V_m}{\omega_m} \sin(\omega_m t)*V_C \sin(\omega_C t)$
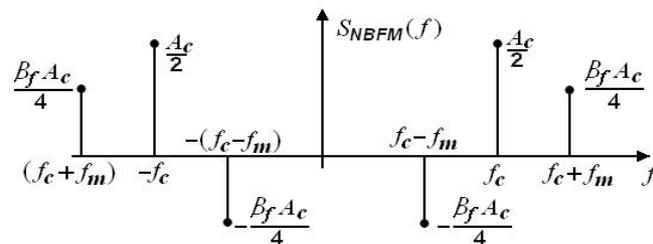
$= V_C \sin \omega_C t(\beta \sin \omega_m t)$

Product modulator output is subtracted from carrier signal,

$$S(t)_{NBFM} = V_C[\cos \omega_C t - \sin \omega_C t(\beta \sin \omega_m t)]$$

This is the resultant NBFM signal

FREQUENCY DOMAIN REPRESENTATION OF NBFM:

$$S(t)_{NBFM} = \frac{V_C}{2}[\delta(f - f_C) + \delta(f + f_C)] + \frac{\beta V_C}{2} FT\{[\cos 2\pi(f_C + f_m)t - \frac{\beta V_C}{2}\cos 2\pi(f_C - f_m)t]\}$$

$$= \frac{V_C}{2}[\delta(f - f_C) + \delta(f + f_C)] + \frac{\beta V_C}{4}[\delta(f - f_C - f_m) + \delta(f + f_C + f_m)]$$

$$- \frac{\beta V_C}{4}[\delta(f - f_C + f_m) + \delta(f + f_C - f_m)]$$

$$= \frac{V_C}{2}[\delta(f - f_C) + \delta(f + f_C)] + \frac{\beta V_C}{4}[\delta(f - f_C + f_m) + \delta(f + f_C + f_m)]$$

Bandwidth of NBFM = 2fm

This narrow band FM is somewhat similar to corresponding one defining an AM wave i.e.

$$S(t)_{AM} = V_C \cos 2\pi f_C t + \frac{\mu V_C}{2}\cos 2\pi(f_C + f_m)t + \frac{\mu V_C}{2}\cos 2\pi(f_C - f_m)t$$

➢ If we compare NBFM with single tone AM, the basic difference between an AM wave and NBFM is that the sign of the lower side frequency in the narrow band FM is reversed.
➢ A narrow band FM wave requires essentially the same transmission bandwidth (i.e., 2 fm) as the AM wave.
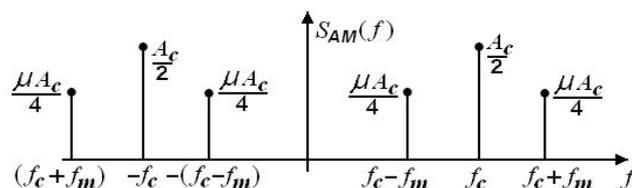
PHASOR DIAGRAM OF NFBM AND AM:

$$S(t)_{NBFM} = V_C \cos 2\pi f_C t + \frac{\beta V_C}{2}\cos(2\pi(f_C + f_m)t) - \frac{\beta V_C}{2}\cos 2\pi(f_C - f_m)t$$

$$S(t)_{AM} = V_C \cos 2\pi f_C t + \frac{\mu V_C}{2} \cos 2\pi (f_c + f_m)t + \frac{\mu V_C}{2} \cos 2\pi (f_c - f_m)t$$
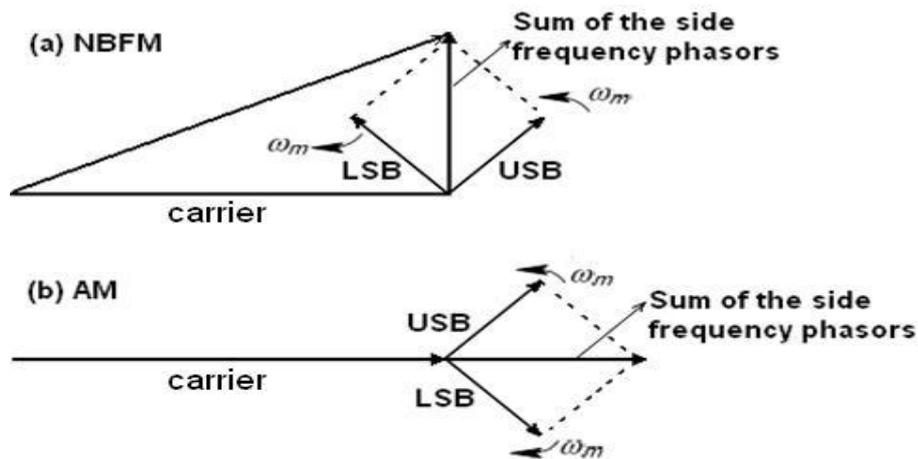

Fig: Phasor Diagram of NFBM and AM

WIDE BAND FREQUENCY MODULATION (WBFM):

If the value of the modulation index (β) is very large, then the resulting FM modulation is wide band FM. The bandwidth of WBFM signal is ideally infinite.

The FM signal is given by,

$$S(t) = V_C \cos(2\pi f_C t + \beta \sin(2\pi f_m t))$$

$$S(t) = V_C \, Re \, [e^{j(2\pi f_C t + \beta \sin(2\pi f_m t))}]$$

$$= V_C \, Re \, [e^{j2\pi f_C t} e^{j\beta \sin 2\pi f_m t}]\dots \dots \dots \dots \dots 1$$

The envelope of the FM signal S (t) is

$$\widetilde{S}(t) = V_C[e^{j\beta \sin(2\pi f_m t)}]\dots \dots \dots \dots \dots \dots \dots \dots \dots 2$$

Since the envelop is complex in nature substitute equation 2 equation 1

$$S(t) = Re \, [\widetilde{S}(t) \, e^{j2\pi f_C t}]\dots \dots \dots \dots \dots \dots \dots 3$$

The complex envelop $\widetilde{S}(t)$ is a periodic function of time with a fundamental frequency fm. Since it is periodic function it can be represented with the help of the complex Fourier series i.e.

$$\widetilde{S}(t) = \sum_{n=-\infty}^{\infty} c_n e^{j2\pi f_m n t}\dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots 4$$

Where C_n is the complex Fourier coefficient. The value of C_n

$$C_n = f_m \left[\int_{-\frac{1}{2}f_m}^{\frac{1}{2}f_m} \widetilde{S(t)} \ e^{-j2\pi nf_m t} dt \right] \ldots\ldots\ldots\ldots\ldots\ldots 5$$

Substitute equation 2 in 5

$$C_n = V_c \ f_m \int_{-\frac{1}{2}f_m}^{\frac{1}{2}f_m} e^{j(\beta \sin 2\pi f_m t - 2\pi nf_m t)} dt \ldots\ldots\ldots 6$$

The Bessel function of nth order is given by

$$J_n(\beta) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j(\beta \sin x - nx)} \ dx \ldots\ldots\ldots\ldots\ldots\ldots 7$$

From equation 6 the complex Fourier coefficient C_n is

$$C_n = V_c \left[f_m \int_{-\frac{1}{2}f_m}^{\frac{1}{2}f_m} e^{j(\beta \sin 2\pi f_m t - 2\pi nf_m t)} dt \right]$$

Take $u = 2\pi f_m t \implies du = 2\pi f_m dt$ and

Limits: if $t = -1/2f_m \implies u = -\pi$

if $t = 1/2f_m \implies u = \pi$

Substitute the new variables 'u' in equation, we get

$$C_n = V_c \left[\frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j(\beta \sin u - nu)} \ du \right] \ldots\ldots\ldots\ldots\ldots 8$$

Substitute $J_n(\beta)$ in equation 8

$$C_n = V_C \ J_n(\beta) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 9$$

Substitute the equation 9 in equation 4

$$\widetilde{S(t)} = \sum_{n=-\infty}^{\infty} V_C J_n(\beta) e^{j2\pi nf_m t} \ldots\ldots\ldots\ldots\ldots 10$$

Substitute equation 10 in equation 3

$$S(t) = V_C \ R_e \left[\sum_{n=-\infty}^{\infty} J_n(\beta) e^{j2\pi nf_m t} \ e^{j2\pi nf_m t} \right]$$

$$= V_C \ R_e \left[\sum_{n=-\infty}^{\infty} J_n(\beta) e^{j2\pi (f_C + nf_m)t} \right] \ldots\ldots\ldots\ldots 11$$

$$= V_C \sum_{n=-\infty}^{\infty} J_n(\beta) \ \cos[2\pi(f_C + nf_m)t] \ldots\ldots 12$$

Equation 12 gives the Fourier series representation of FM signal S (t)

Expanding equation 12 we get

$$S(t) = V_C J_0(\beta) \cos 2\pi f_C t + V_C J_1(\beta) \cos(2\pi(f_C + f_m)t) + \ldots + V_C J_{-1}(\beta) \cos(2\pi(f_C - f_m)t) + V_C J_{-2}(\beta) \cos(2\pi(f_C - 2f_m)t) \ldots\ldots\ldots\ldots\ldots\ldots 13$$

Properties of Bessel functions

For even values of 'n' $J_n(\beta) = J_{-n}(\beta)$

For odd values of 'n' $J_n(\beta) = (-1)^n J_{-n}(\beta)$

By using the properties of Bessel function equation 13 can be written as

$S(t) = V_C J_0(\beta) \cos(2\pi f_C t) - V_C J_1(\beta)[\cos(2\pi(f_C - f_m)t) - \cos(2\pi(f_C + f_m)t)] + V_C J_2(\beta)[\cos(2\pi(f_C - 2f_m)t) + \cos(2\pi(f_C + 2f_m)t] + \ldots\ldots$

The variation of the Bessel function $J_n(\beta)$ which determines the amplitude of various side band frequency component of WBFM has been plotted against the modulation index β for different values of 'n'.

Fig: Plots of Bessel Functions
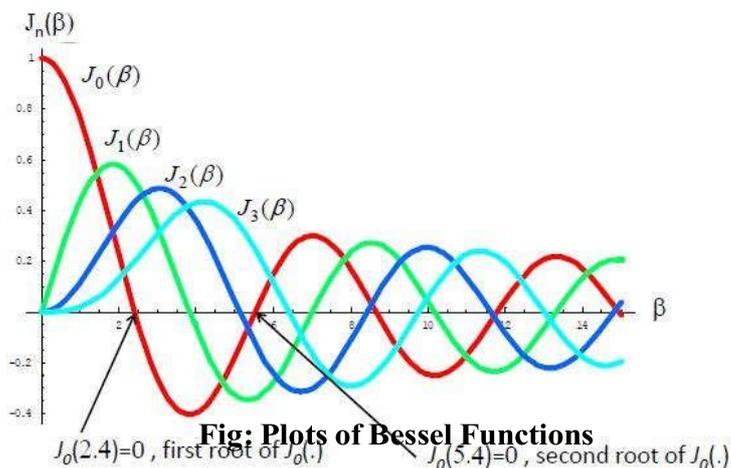
For small values of β, $J_0(\beta) = 1$, $J_1(\beta) = \frac{\beta}{2}$, $J_N(\beta) = 0$ for $n > 2$

Substituting, so that the FM signal is effectively composed of a carrier and a single side band frequencies at $f_C \pm f_m$ for small values of 'β'.

PHASE MODULATION:

Phase modulation can be defined as the process by which changing the phase of the carrier signal in accordance with the instantaneous amplitude of the message signal. The amplitude and frequency remains constant even after the modulation process.

Let the message signal $V_m(t) = V_m \cos\omega_m t$ ………….(1) and

The carrier signal $V_c(t) = V_c \sin(\omega_c t + \theta)$…............(2)

Where θ = Phase angle of carrier signal. It changed in accordance with amplitude of the message signal $V_m(t)$

$\theta = K_p V_m(t) = K_p V_m \cos\omega_m t$

Where K_p = Phase deviation sensitivity

After phase modulation the instantaneous voltage is

$V_{PM}(t) = Vc \sin(\omega_c t + \theta)$

$\qquad = Vc \sin(\omega_c t + K_p Vm \cos\omega_m t)$

$\qquad \therefore \quad V_{PM}(t) = Vc \sin(\omega_c t + m_p \cos\omega_m t)$

Where $m_p = K_p Vm$, modulation index of phase modulation.

PHASE DEVIATION AND MODULATION INDEX:

W.K.T $\qquad V_{PM}(t) = Vc \sin(\omega_c t + m_p \cos\omega_m t)$

$\qquad\qquad\qquad = Vc \sin(\omega_c t + \theta(t))$

$\qquad\qquad$ Where $\theta(t)$= instantaneous phase deviation

$\qquad\qquad\qquad = m_p \cos\omega_m t$

If the modulating signal is single tone sinusoid, then the phase angle of carrier varies from its unmodulated signal is known as phase deviation.

The modulation index of FM system can be defined as the ratio of Maximum frequency deviation to the modulating frequency.

$\qquad$ i.e., $m_f = \omega_d / \omega_m = K_p V_m / \omega_m \qquad\qquad \omega_d = K_p V_m = $ Maximum frequency deviation

For PM :

The modulation index depends on modulating signal, i.e., $m_p = K_p V_m$

Where K_p =Deviation sensitivity.

BANDWIDTH OF PM:

The PM bandwidth as per Carson's rule $(BW)_{PM} = 2\Delta\omega = 2K_p V_m \omega_m$

Thus the B.W of PM signal varies tremendously with a change in modulating frequency ω_m .

RELATIONSHIP BETWEEN PHASE MODULATION (PM) AND FREQUENCY MODULATION (FM) :

The Angle Modulation Wave Is $S(T) = V \cos\emptyset_i$

Where V = Amplitude, $\emptyset_i$ = Instantaneous total phase angle

The expression for PM wave is $S(t) = V \cos[\omega_c t + Kp V_m(t)]$

FM wave is $S(t) = Vc \cos[\omega_c t + K_f \int V_m(t) \, dt]$

From the above equations that Phase Modulation (PM) and Frequency Modulation (FM) are closely related to each other because in both the cases there is a variation in the total phase angle.

- ➤ In **PM** the phase angle varies linearly with baseband signal Vm(t).where as in case of frequency modulation , the phase angle varies linearly with the integral of baseband signal $V_m(t)$.
- ➤ This means that **FM** wave may be obtained by using PM and vice versa.

CONVERSION OF PM TO FM OR FREQUENCY MODULATION:

Frequency modulation wave can be obtained from **PM** This is done by integrating the modulating signal before applying it to the phase modulator it is shown in fig .
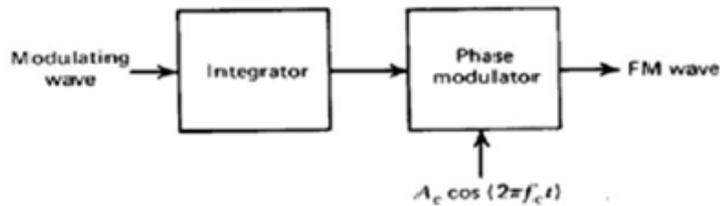

Fig: Conversion of PM to FM (or) Frequency Modulator

Let $V_m(t) = V_m \cos\omega_m t$.

After integration, $\int V_m(t) = \int V_m \cos\omega_m t.dt = V_m/\omega_m \sin\omega_m t$

After phase modulation, $\theta \propto \int V_m(t)$

$$\theta = K \int V_m(t) = K V_m/\omega_m \sin\omega_m t$$

The instantaneous value of modulated voltage in given by

$V_{FM}(t) = Vc \sin(\omega_c t + \theta)$

$= Vc \sin(\omega_c t + K V_m/\omega_m \sin\omega_m t)$

$= Vc \sin(\omega_c t + m_f \sin\omega_m t)$

Where $m_f = \Delta f/f_m = k V_m/w_m$

$\therefore V_{FM}(t) = Vc \sin[\omega_c t + m_f \sin w_m t]$

This is expression for FM wave.

CONVERSION OF FM TO PM (OR) PHASE MODULATOR:

The PM wave can be obtained from FM by differentiating the modulating signal before applying it to the frequency modulator is shown in fig.
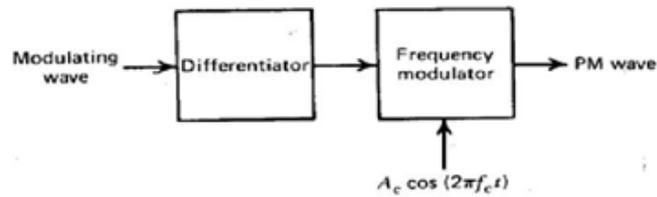
Fig: Conversion of FM to PM (or) Phase Modulator

We know that, $V_m(t) = V_m \cos\omega_m t$.

After differentiation, $d/dt \, V_m(t) = -\omega_m V_m \sin\omega_m t$.

After frequency modulation, $\omega_i = \omega_c + K \, d/dt \, V_m(t)$

$$= \omega_c + K(-\omega_m V_m \sin\omega_m t)$$

We know that the instantaneous phase angle of FM signal is

$$\emptyset_i = \int \omega_i \, dt = \int (\omega_c - \omega_m V_m \sin\omega_m t) dt$$

$$= \omega_c t + \frac{K \, \omega m V m}{\omega m} \cos\omega_m t$$

$$\emptyset_i = \omega_c t + K V_m \cos\omega_m t$$

The instantaneous voltage after modulation, $V_{PM}(t) = Vc \sin \emptyset_i$

$$V_{PM}(t) = Vc \sin(\omega_c t + K Vm \cos\omega_m t)$$

$$V_{PM}(t) = Vc \sin(\omega_c t + m_p \cos\omega_m t)$$

This is expression for PM wave.

FEATURES OF ANGLE MODULATION:

The important features of angle modulation are:

(i)   The process of angle modulation is non-linear in nature.
(ii) The spectral components of angle modulated wave are completely different from that of message spectrum.
(iii) The bandwidth of angle modulated wave is higher (usually double) than that of message wave.
(iv) The improved signal-to-noise ratio at the o/p of the receiver compensates the complexity and bandwidth of an angle modulator system.
(v)   This type of modulation is immune to noise and effectively- rejects the interference of undesired signals.
(vi) It is also immune to the propagation induced selective fading.

(vii)   Angle modulated permits the efficient use of transmitters and greater dynamic range of modulating signal.

(viii)   Due to the non-linear property of angle modulation, it supports the exchange of signal power for transmission bandwidth.

GENERATION OF FM SIGNALS:

Frequency modulated signals can be generated by two methods,

    1. Direct method   2. Indirect method
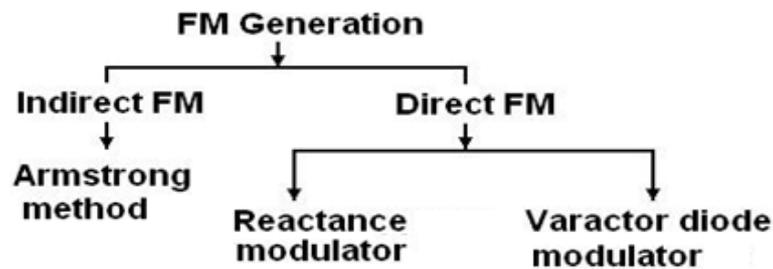
INDIRECT METHOD (ARMSTRONG METHOD) OF FM GENERATION:

In the direct methods of generation of FM, LC oscillators are to be used. The crystal oscillators cannot be used. The LC oscillators are not suitable enough for the communication or broadcast purpose. Thus, the direct methods cannot be used for the broadcast applications. The alternative method is to use the indirect method called as Armstrong method of FM generation. In this method, the FM is obtained through phase modulation. A crystal oscillator can be used hence the frequency stability is very high. The Armstrong method uses the phase modulator to generate a frequency modulated wave.
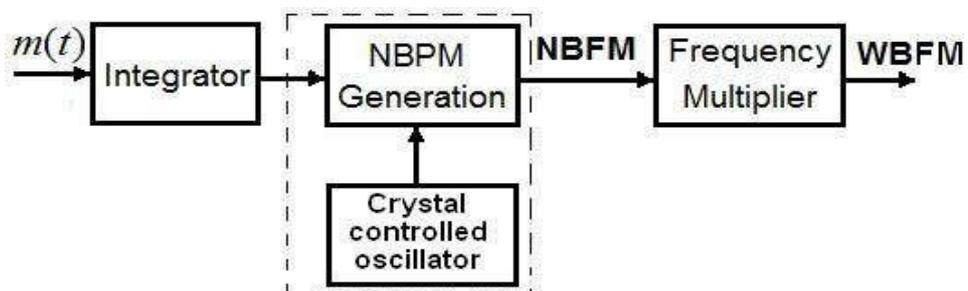

Fig: Block Diagram of Armstrong Method for Generating WBFM Signal

In this method of FM generation the modulating signal is integrated and it is phase modulated by crystal oscillator to get narrow band FM signal which later passed to a frequency multiplier to get a wideband FM signal.

The crystal oscillator provides frequency stability. The value of $'\beta'$ is kept small to produce narrow band FM signal. The frequency multiplier consists of a memory less nonlinear device followed by a band pass filter.
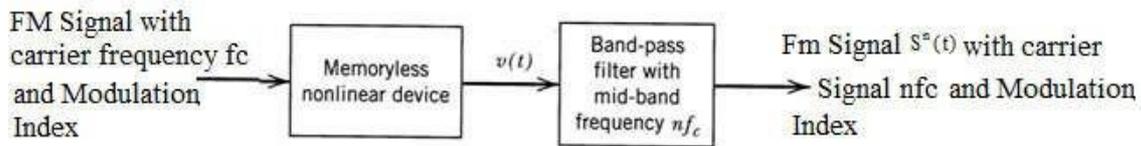

Fig: Block diagram frequency multiplier

If FM input signal is S(t), then output $V_0(t)$ of memory less nonlinear device is given.

$$V_0(t) = a_1 S(t) + a_2 S^2(t) + \dots\dots\dots\dots a_n S^n(t)$$

Where $a_1, a_2, \dots\dots\dots\dots\dots, a_n$ are the coefficients determined by the operating point of the device and 'n' is the higher order of non linearity.

The input $S(t) = V_C \cos[\omega_C t + k_f \int V_m(t)dt]$

The instantaneous frequency, $\omega(t) = \omega_C + k_f V_m(t)$

Consider the maximum non linearity term then the instantaneous frequency $nf_i(t)$ is

$$nf_i(t) = nf_C + nk_f V_m(t)$$

Therefore new wideband FM is given by

$$S'(t) = V_C' \cos[n\omega_C t + nk_f \int V_m(t)dt]$$

Where $V_C' = nV_C$

From this it is clear that the frequency of FM signal can be varied by varying the value of 'n'.

DIRECT METHOD OF FM GENERATION:

In the direct method of FM, the carrier frequency is directly varied in accordance with the input baseband signal. In direct method of FM generation, the message is first integrated and their phase modulated to get a frequency modulated signal. The basic requirement of a frequency modulated system is a variable output frequency with the variation proportion to the instantaneous amplitude of the message signal.

The FM signal can be generated by a no. of ways. One way of FM generation is by varying the capacitance or inductance of an LC oscillator circuit or tank circuit, frequency modulation of some form will be generated. If the capacitance or inductance of the tank circuit is varied proportionately to the voltage of the modulating signal, then FM signal is generated.

Several devices are existing which provides a variation of capacitance according to change in voltage. If such a voltage variable reactance circuit is connected across the tank circuit and the tank circuit is tuned to the carrier frequency signal. If the reactance of the voltage variable reactance circuit is varied according to the amplitude of the modulating signal, the frequency of the tank circuit is changed. The change in frequency of the tank

circuit is proportional to the amplitude of the modulating signal. The most common voltage variable reactance circuits are reactance modulators.

REACTANCE MODULATOR:

In the direct FM generation, the instantaneous frequency of the carrier is changed directly in proportion with the message signal. For this advice called voltage controlled oscillator (VCO) is used. A VCO can be implemented by using a sinusoidal oscillator with a tuned circuit having a high value of 'Q'. The frequency of this oscillator is changed by incremented variation in the reactive components involved in the tuned circuit. If the 'L' or 'C' of a tuned circuit of an oscillator is changed in accordance with the amplitude of modulating signed then FM can be obtained across the tuned circuit.
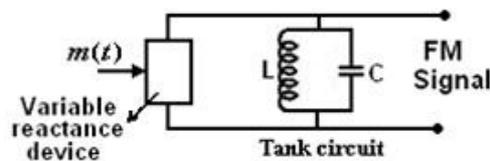

Fig: Principle of Reactance Modulator

A two or three terminal device is placed across the tuned circuit. The reactance of the device is varied proportional to the modulating signal voltage. This will vary the frequency of the oscillator to produce FM. The devices used are FET, transistor or varactor diode. The direct FM is shown in the figure which uses a Hartley oscillator along with varactor diode.

The varactor diode is reverse biased. Its capacitance is dependent on the reverse voltage applied across it

Frequency of oscillation of Hartley oscillator is

$$f_i = \frac{1}{2\sqrt{(L_1 + L_2)C(t)}}$$

Where C (t)=C+$C_{varaCtor}$. This means that C(t) is the effective capacitance of the fixed tuned circuit capacitance C and the varactor diode capacitance $C_{varaCtor}$

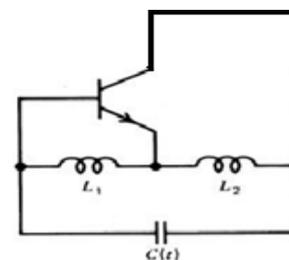Let the relation between the modulating voltage x(t)=0 and the capacitance C(t) is represented as

$$C(t) = C - k_C x(t)$$

Where C= total capacitance when x (t) =0 and k_C is the sensitivity of the varactor capacitance to change in voltage.

Substituting expression for C (t) in $f_i(t)$

$$f_i(t) = \frac{1}{2\pi\sqrt{(L_1 + L_2)(c - k_C x(t))}}$$

$$f_i(t) = \frac{1}{2\sqrt{(L_1+L_2)C - (L_1+L_2)k_C\ x(t)}}$$

But, let $\dfrac{1}{2\ \sqrt{(L_1+L_2)C}} = f_0$, which is the oscillator frequency in absence of the modulating signal[x (t) = 0]

Therefore $f_i(t) = f_0[1 - \dfrac{k_C}{C} x(t)]^{\frac{-1}{2}}$

If the maximum change in the capacitance corresponding to the modulating wave,

$$f_i(t) = f_0 \left[1 + \dfrac{k_C}{2C} x(t)\right] = f_0 + \dfrac{f_0 k_C}{2C}\ x(t)$$

Let $\dfrac{f_0 k_C}{2C} = k_f \Rightarrow f_i(t) = f_0 + k_f x(t)$

Where k_f = frequency sensitivity

DEMODULATION OF FM WAVES:

The demodulation process of FM wave is exactly opposite to that of the frequency modulation. After demodulation, we get the original modulating signal at the demodulation output. The input to the demodulator is the FM wave. The FM demodulator (detector) operates on an altogether different principle then AM detector. The AM detector is basically a rectifier. But FM demodulator is basically a frequency to amplitude converter. It is used to convert the frequency variations in FM wave at its input into amplitude variations at its output to recover the original modulating signal.

Classification of FM Demodulators
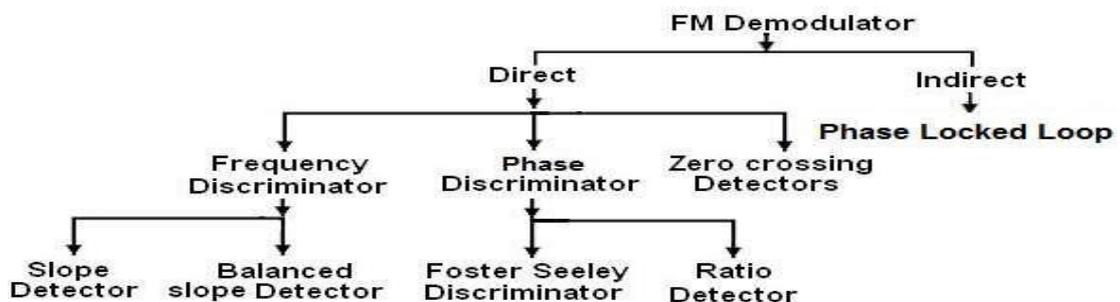
FREQUENCY DISCRIMINATOR:

SIMPLE SLOPE DETECTOR:

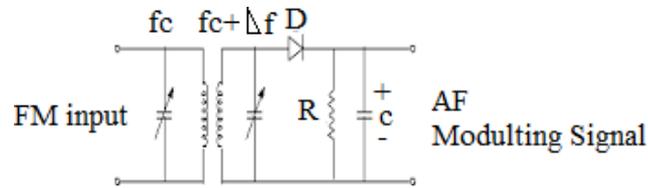The circuit diagram of a simple slope detector is

Fig: Simple Slope Detector

The output voltage of the tank circuit is then applied to simple diode detector with an RC load with proper time constant. The detector is identical to the AM diode detector. Even though the slope detector circuit is simple it has the following drawbacks.

(i) It is inefficient.

(ii) It is linear only over a limited frequency range.

An advantage of the basic slope detector circuit is its simplicity .To overcome the drawback of the simple slope detector, a balanced slope detector is used.
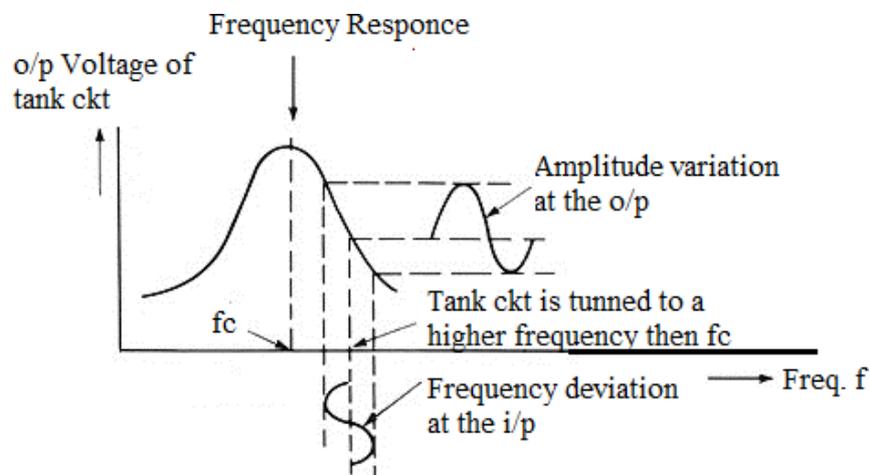

Fig: Characteristics of Slope Detector

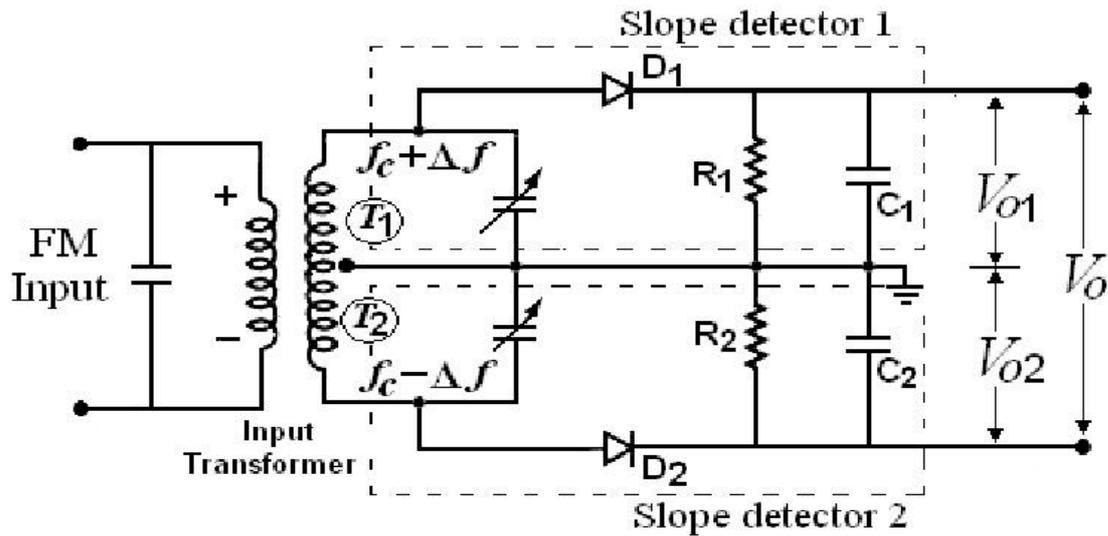## BALANCED SLOPE DETECTOR (BALANCED FREQUENCY DETECTOR):

Fig: Balanced Slope Detector

The circuit diagram shows that the balanced slope detector consists of two slope detector circuits. The input transformer has a center tapped secondary. Hence, the input voltages to two slope detectors are 180° out of phase. There are three tuned circuits. The primary tuned to I_f i.e.f_C.

The upper tuned circuit of the secondary (T_1) is tuned above $'f_C'$ by Δf i.e., its resonant frequency is ($f_C + \Delta f$). The lower tuned circuit of the secondary is tuned below $'f_C'$ by Δf i.e., at ($f_C - \Delta f$). R_1C_1 and R_2C_2 are the filters used to bypass the RF ripple. V_{O1} and V_{O2} are the output voltages of the two slope detectors. The final output voltage V_O is obtained by taking the subtraction of the individual output voltages V_{O1} and V_{O2} i.e. $V_O = V_{O1} - V_{O2}$

WORKING OPERATION OF THE CIRCUIT:

The circuit operation by dividing the input frequency into three ranges as follows.

(i) $f_{in} = f_C$: When input frequency is instantaneously equal to f_c , the induced voltage in the 'T_1' winding of secondary is exactly equal to that induced in the winding T_2.Thus the input voltage is to both the diodes D_1 and D_2 will be the same. Therefore, their DC output voltage V_{O1} and V_{O2} will also identical but they have opposite polarities. Hence net voltage output $V_O = 0$

(ii) **$f_c < f_{in} < (f_c + \Delta f)$:** In this range of input frequency, the induced voltage in the winding T_1 is higher than that induced in T_2. Therefore, the input to the D_1 is higher than D_2. Hence, the +ve output 'V_{O1}' of 'D_1' is higher than –ve output V_{O2} of D_2.Therefore the output voltage is V_O is +ve. As the input frequency increases towards the +ve output voltage increases as shown in the fig.

If the output frequency goes outside the range of (fc - Δf) to (fc + Δ f), the output voltage will fall due to reduction in the tuned circuit response.

Fig. Characteristics of Balanced Slope Detector

ADVANTAGES:

(i) This circuit is more efficient than simple slope detector.

(ii) It has better linearity than the simple slope detector.

LIMITATIONS:

(i) Even though linearity is good, it is not good enough.

(ii) This circuit is difficult to tune since the three tuned circuits are to be

tuned at different frequencies, fc , (fc - Δf) and (fc+ Δ f)

Amplitude limiting is not provided.

PLL-FM DEMODULATOR:

A phase locked loop (PLL) is primarily used in tracking the phase and frequency of the carrier component of an incoming FM signal. PLL is also useful for synchronous demodulation of AM-SC (i.e., Amplitude Modulation with suppressed carrier) signals (or) waves with few cycles of pilot carrier. PLL is also useful for demodulating FM signals in presence of large noise and low signal power. A phase locked loop (PLL) is basically a negative feedback system. It contains of three major components.
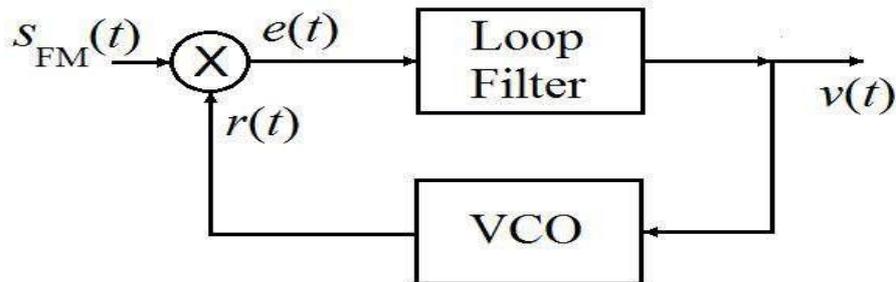

Fig: Block diagram of Phase Locked Loop

These components are multiplier, a loop filter and a voltage controlled oscillator (VCO) connected in the form of a feedback loop. A VCO is a sine wave generator whose frequency is determined by the voltage applied to it from an external source.

WORKING OPERATION:

The operation of a PLL is similar to any other feedback system. In any feedback system the feedback signal tends to follow the input signal. If the signal feedback is not equal to the input signal the error signal will change the value of the feedback signal until it is equal to the input signal. The difference signal between s(t) and b(t) is called an error signal.

A PLL operates on a similar principle except for the fact that the quantity feedback is not the amplitude, but a generalized phase $\Phi(t)$. The error signal or difference signal e(t) is utilized to adjust the VCO frequency. In such a way that the instantaneous. Phase angle comes close to the angle of the incoming signal s(t). At this point, the two signals s(t) and b(t)are in synchronism and the PLL is locked to the incoming signal s(t).
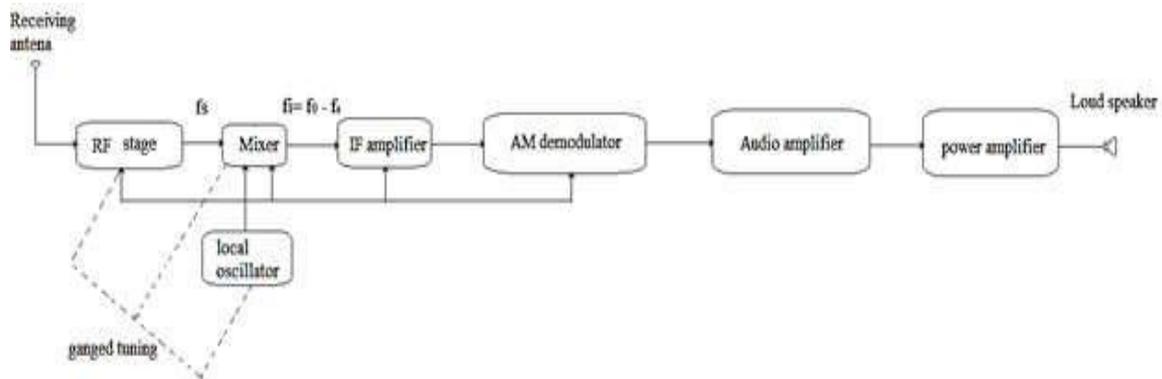
SUPER HETERODYNE AM RECEIVER:

Fig: Super Heterodyne AM Receiver

In super heterodyne receiver, the incoming RF signal freq. is combined with the local oscillator signal freq. through a mixer and is converted into a signal of lower freed freq. the lower fixed freq. is known as intermediate freq. However, the intermediate freq. signal is same modulation as the original signal. The intermediate freq. signal is now amplified and demodulated to reproduce the original signal.

The word heterodyne stands for mixing. Here we have mixed the incoming signal freq. with a local oscillator freq. therefore this receiver is called super heterodyne receiver.

In a super heterodyne receiver, a constant freq. difference is maintained between the local oscillator signal freq. and incoming RF signals freq. through the capacitance tuning in which the capacitance are ganged together and operated by a common control knob. The intermediate freq. (IF) amplifier generally contains a number of transformers is consisting of a pair of mutually coupled tuned circuits. The IF amplifier determines the sensitivity and selectivity of the super heterodyne receiver. The IF amplifier works at a fixed IF freq., the design of this system is quite easy to provide high gain and constant bandwidth. Because of its narrow band width, the IF amplifier rejects all over the frequencies except intermediate freq. (IF).

After the IF amplifier, the signal is applied to the input of demodulated which extracts the original modulating signal. The audio signal is amplified by the audio amplifier to get a particular voltage level. This amplifier freq. audio signal is further amplified by a power amplifier to get a specified power level so that it may activate the loud speaker. The loud speaker is a transducer which converts their audio signal into audio sound signal and thus the original signal is reproduced ie., the original transmission is received.

The advantages of the super heterodyne receiver moves it the most suitable for the majority of radio receiver applications like M, FM, communications, single sideband, television and even the radar receiver, all use super heterodyne principle.

SUPER HETERODYNE FM RECEIVER:

The FM receiver is used to receive the FM signal from an FM transmitter and then convert it into the original modulating signal.
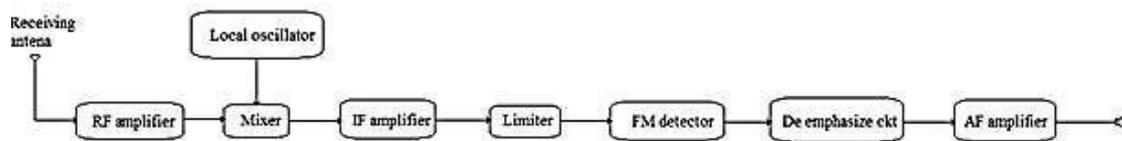

Fig: Block diagram of Super Heterodyne FM Receiver

RF AMPLIFIER:

RF amplifiers are used in FM receiver to minimize the noise figure and to match the input impedance of the receiver with the impedance of antenna. This amplifier increases the strength of the signal to a satisfactory level and feeds the amplified output to the receiver.

LOCAL OSCILLATOR:

The local oscillator used in FM receiver generates carrier waves if frequencies lower than the input signal frequency.

MIXER:

A mixer is used to combine the RF amplified output with the output of the local oscillator to produce a high intermediate frequency (IF). This high IF helps in attaining effective image rejection capability of the receiver.

IF AMPLIFIER:

IF amplifiers are used for amplifying intermediate frequency. These amplifiers provide high gain and larger band width of the order of 150 KHz.

LIMITER:

Limiter is a form of clipping device, in which the output remains constant irrespective of the variations in the input signal. FM receiver uses an amplitude limiter to clip off the amplitude variation present in the signal. As a result, noise gets reduce without affecting the information content of the signal. The constant frequency modulated carrier is than applied to a discriminator circuit.

DISCRIMINATOR:

Discriminator or an FM detector, applied next to the limiter circuit, extracts the original audio frequency from the frequency modulated receiver.

DE-EMPHASIS NETWORK:

A De-emphasis network is employed to reduce the high audio frequencies which are directly proportional to the frequency of the transmitter. These circuits also help in reduce in the frequency modulated noise which enters the front end of the receiver.

AF AND POWER AMPLIFIER: The audio frequency power amplifier accepts input from the de-emphasis network, amplifiers the audio signal to a desired level. This amplified output is then fed to the loud speaker at the receiver end.