UNIT-2

NOISE AND PULSE MODULATION

✓ In electrical terms, noise may be defined as an unwanted from of energy which tend to interfere with the proper reception and reproduction of transmitted signals.

✓ Although there are several ways of classifying the noise, but conveniently noise may be classified in two broad groups as under
(i) External Noise (ii) Internal Noise

✓ External noise may be defined as that type of noise which is generated external to a communication system i.e., whose source are external to the communication system.

✓ External Noise may be classified as under
(i) Atmospheric Noise (ii) Extraterrestrials Noise (iii) Industrial Noise

✓ Atmospheric Noise, which is also called Static, is produced by lightening discharges in thunder storms and other natural, electrical disturbances which occur in the atmosphere.

✓ There are several types of extraterrestrial noise or space noise depending upon their sources. Extraterrestrial noise may be divided into following two sub groups as under,
(i) Solar Noise (ii) Cosmic Noise

✓ Solar noise is the electrical nose emitting from the sun.

✓ Distant stars can also be considered suns. These distant stars have high temperature and therefore radiate noise in the same manner as the sun. The noise received from these distant stars is thermal noise and is distributed almost uniformly are the entire sky.

✓ The industrial noise or man-made noise is that type of noise which is produced by such sources as automobiles and aircraft ignition, electrical motors, switch gears and leakage from high voltage transmission lines and several other heavy electrical equipment's.

✓ Internal noise in that type of noise which is generated internally or within the communication system or receiver

✓ Shot noise arises in active devices due to random behavior of charge carries. In electron tubes, Shot noise is generated due to the random emission electrons from cathodes, where as in semiconductor devices, shot noises is generated due to random diffusion of minority carries or simply random generation and recombination of electron-hole pairs.

✓ The transit time of an electron in a diode depends upon anode voltage 'V' and may be expressed as $\tau = 3.36 \times \frac{d}{\sqrt{V}}$ Here d=the spacing between anode and cathode.

✓ Partition noise is generated in a circuit when a current has to divide between two or more paths. This means that the partition noise results from the random fluctuations in the division.

✓ At low frequencies (below few kHz), a particular type of noise appears. The power spectral density of this noise increase as the frequency decreases. This noise in called *Flicker Noise* or (1/f) noise. In case of vacuum tubes, the main causes of flicker noise are slow changes which take place in the oxide structure of oxide coated cathodes and migration of impurity ions.

✓ In semiconductor devices, flicker noise is generated from the flue ration in the carrier density and creates more problems in semiconductor amplifying device than in vacuum tubes at low frequencies.

✓ It is generally observed in semiconductor devices, when the transit-time of change carriers crossing a junction is comparable with the time period of the signal, some charge carries diffuse back to the source oremitters.

Thermal Noise:

The Thermal Noise or Johnson noise is the random noise which is generated in a resistor or the resistive component of a complex impedance due to rapid and random motion of molecules, atoms and electrons. According to the kinetic theory of thermodynamics, the temperature of a particle denotes its internal kinetic energy. This means that the temperature of a body expresses the rms value of the velocity of motion of the particles in body. As per this kinetic theory, the kinetic energy of these particles becomes approximately zero (i.e., zero velocity) at absolute zero.

Therefore, the noise power produced in a resistor is proportional to its absolute temperature. Also the noise power is proportional to the bandwidth are which the noise in measured.
Therefore the expression for maximum noise power output of a resistor,

$$Pk \propto TB$$
$$Pk = KTB$$

Where K= Boltzmann's constant = 1.38*10-23 J/
T= absolute temperature B= bandwidth of interest in Hz

An equivalent circuit of a resistor as a noise voltage generator is shown in fig. This equivalent circuit is also called as Voltage model of Noisy resistor.

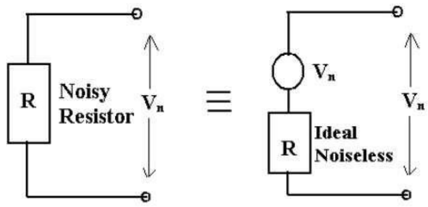


FIGURE Equivalent Circuit of Thermal Noise Voltage

From this equivalent circuit, we can compute the resistors equivalent rms noise voltage 'V_n'.

According to maximum power transfer theorem, for maximum transfer of power from noise voltage source V_n to load resistor R_L, R_L= R

Then the maximum noise power so transfer will be given as,

$P_n = \frac{V^2}{R_L}$ but $R_L = R$

$P_n = \frac{V^2}{R}$

Applying voltage divider rule in fig., $V = \frac{V_n}{2}$

$P_n = \frac{V^2}{R_L} = \frac{(\frac{V_n}{2})^2}{R_L} = \frac{V_n^2}{4R}$

$V_n^2 = 4RKTB \Rightarrow V_n = \sqrt{4RKTB}$

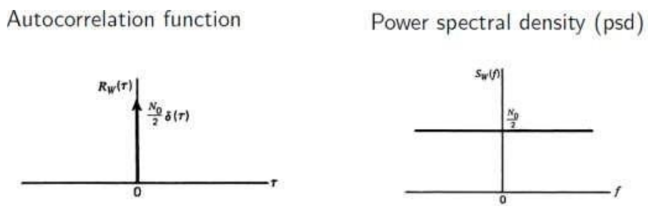
White Noise:

Noise in an idealized form is known as White Noise. This means that in a communication system, the noise analysis is based on idealized form of noise i.e., White noise.

- ✓ White light contains equal amount of all frequencies with in the visible band of electron-genetic radiation, in the same manner white noise contains all frequencies in equal amount.
- ✓ The power density spectrum of white noise is dependent of frequency.
- ✓ If the probability of occurrence of a white noise is specified by a Gaussian distribution function, it is called al White Gaussian Noise.
- ✓ The power spectral density of white noise is $S_w(f) = \frac{N_0}{2}$.
- ✓ The dimensions of N_o and in watts per Hertz.

  $N_o = KT_e$

Where K= Boltzmann'sconstant, T_e= equivalent noise temperature of Rx



The auto correlation function of PSD of white noise is $R_w(\tau) = \frac{N_0}{2}\delta(\tau)$

That is auto correlation of white noise consist of a delta function weighted by the factor $\frac{N_0}{2}$ occurring at $\tau = 0$.

$$R_w(\tau) = 0 \text{ for } \tau \neq 0$$

Ideal Lowpass Filter White Noise:

Let Gaussian noise of zero mean and psd $\frac{N_0}{2}$ is applied to an ideal low pass filter of bandwidth 'B' and pass band amplitude response of one.

The psd of the noise $k(t)$appearing at the filter output,

$$S_N(f) = \begin{cases} \frac{N_0}{2} & -B < f < B \\ 0 & |f| > B \end{cases}$$

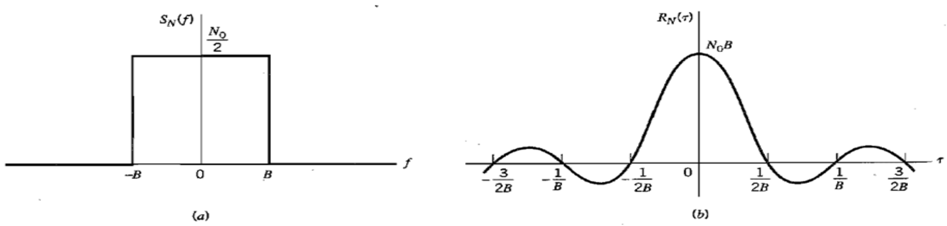


FIGURE Characteristics of low-pass filtered white noise. (*a*) Power spectral density. (*b*) Autocorrelation function.

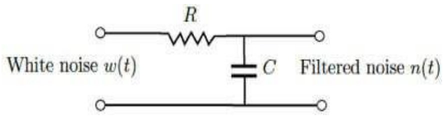
The auto correlation function of n(t) is the inverse Fourier Transform of the psd,

$$R_N(\tau) = \int_{-B}^{B} \frac{N_o}{2} e^{j2\Pi f t}\, df$$

$$= N_o B \,\text{sinc}(2B\tau)$$

RC LOWPASS Filter White noise:

A White Gaussian noise w (t) of zero mean and psd $\frac{N_o}{2}$ applied to a LPF RC filter. The transfer function is H (*f*)= $\frac{1}{1+j2hRC}$



The psd of the noise $k(t)$ appearing at the low pass RC filter output is $S_N\ (f) = \frac{N_o/2}{1+(2hRC)^2}$

Autocorrelation function of the filtered noise $k(t)$ is

$$R_N(\tau) = \frac{N_o}{4RC} e^{\left(-\frac{|\tau|}{RC}\right)}$$

Narrowband Noise:

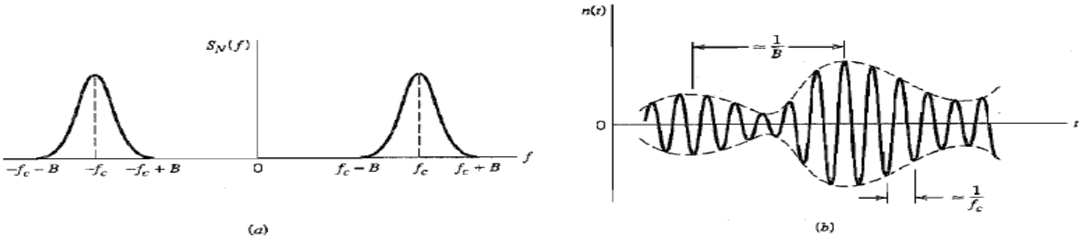
The front end of the receivers of practical communication system using carrier modulation consists of frequency selective filters. These filter process the desired signal and the noise. The filters are designed to have a bandwidth large enough to pass the signal without distortion but not to admit noise through the receiver. This filter is usually narrowband in the sense that its bandwidth is small compared to the middle band frequency. The noise appearing at the output of such a filter is called "Narrowband Noise".

Representation of Narrow band noise in terms of In-Phase and Quadrature Components:

Consider a narrowband noise $k(t)$ of bandwidth 2B centered on frequency 'f_c' as show in fig. In light of the theory of band pass signals and system, $k(t)$ in the standard form,

$$k(t) = k_I(t)\cos w_c t - k_Q(t)\sin w_c t$$

Where $k_I(t)$ is called the in-phase component of $k(t)$ and $k_Q(t)$ is called Quadrature component of $k(t)$. Both $k_I(t)$ and $k_Q(t)$ are low pass signals.



(a) Power spectral density of narrowband noise. (b) Sample function of narrow- band noise.

The in-phase and quadrature components of a narrow band noise have important properties

1. The in-phase component $k_I(t)$ and quadrature component $k_Q(t)$of narrow band noise $k(t)$ have zero mean.
2. If the narrow band noise $k(t)$is Gaussian, then its in-phase component $k_I(t)$ and quadrature component $k_Q(t)$ are jointly stationary.
3. Both the in-phase $k_I(t)$ and quadrature component $k_Q(t)$ have the same power spectral density.
4. The in-phase component $k_I(t)$and quadrature component $k_Q(t)$have the same variance as the narrow band noise $k(t)$.

Representation of Narrow band noise in terms of Envelope and Phase Quadrature Components:

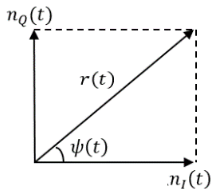
Narrow band noise $k(t)$ in terms of its envelope its envelope and phase components as,

$$k(t) = r(t)\cos(2\pi f_c t + \quad (t))$$

Where,

$$r(t) = (k_I^2(t) + k_Q^2(t))^2$$

$$(t) = \tan^{-1}\left(\frac{k_Q(t)}{k_I(t)}\right)$$



The function $r(t)$ is called the envelope of $k(t)$ and the function $\quad (t)$ is called the phase of $k(t)$.Theenvelope $r(t)$ and phase $\quad (t)$ are both sample functions of low pass random processes.

Sine wave plus Narrow band Noise:

Let the sine wave signal A$\cos(2\pi f_c t)$is added to the Gaussian noise$k(t)$, where 'A' and f_c are both constants. Assume that the frequency of the sinusoidal wave is same as the nominal carrier frequency for Narrow band noise. A sample function of the sinusoidal wave plus noise is,

$$\text{x(t)} = \text{A}\cos(2\pi f_c t) + k(t)$$

Let us represent the narrow band noise $k(t)$ interms of its in-phase and quadrature components,

$$x(t) = \text{A}\cos(2\pi f_c t) + k_I(t)\cos \text{w}_c t - k_Q(t)\sin \text{w}_c t$$

$$= (A + k_I(t))\cos 2\pi f_c t - k_Q(t)\sin 2\pi f_c t$$

$$= k_I'(t)\cos(2\pi f_c t) - k_Q(t)\sin 2\pi f_c t$$

Where

$$k_I'(t) = A + \, + k_I(t)$$

We assume that $k(t)$ is Gaussian with zero mean and variance σ^2. Accordingly

1. Both $k_I(t)$ and $k_Q(t)$ are Gaussian and statistically independent.
2. The mean of $k_I(t)$ is 'A' and that of $k_Q(t)$ is Zero.

3. The variance of both $k_I(t)$ and $k_Q(t)$ is σ^2.

Signal to Noise Ratio:

Signal to Noise Ratio is defined as the ratio of the signal power to the noise power at the same point in the system. Signal to Noise ratio is denoted by S/N

$$\frac{S}{N} = \frac{P_s}{P_n} = \frac{V_s^2}{V_n^2}$$

The ratio is dimension less and is often expressed in decides

$$(S/N)_{o/p}\ ik\ dB = 10\ \log_{10}\left(S_o/N_o\right)$$

The power spectrum density in power per unit bandwidth, hence

$$\frac{S}{N} = \frac{S_s(\omega)}{S_n(\omega)} = \frac{power\ spectrum\ deksity\ of\ sigkal\ voltage}{power\ spectrum\ deksity\ of\ koise\ voltage}$$

Noise Figure:

The noise figure 'F' is defined as the ratio of the signal-to-noise power ratio supplied to the input terminals of a receiver or amplifier to the Signal to noise power ratio supplied to the output or load resistor.

$$\text{Noise figure, } F = \frac{input\ SNR}{output\ SNR} = \frac{(SNR)_i}{(SNR)_o}$$

Noise figure sometimes called as Noise factor.

Figure of Merit:

To compare different CW modulation systems, we have to normalize the receiver performance by dividing the output signal to noise ratio $(SNR)_0$ by the channel signal to noise ratio$(SNR)_c$.

Hence the figure of merit for a receiver is

$$\text{Figure of Merit} = \frac{(SNR_o)}{(SNR_c)} = \frac{SNR_o}{SNR_i}$$

Figure of merit can be less than, greater than or equal to 1 depending of the type of modulation.

✓ The figure of merit must be as high as possible because high value of figure of merit indicates better noise performance of the receiver.

Power Spectral density of a Random process:

- ✓ By definition, the Power spectral density $S_X(t)$ and autocorrelation function $R_X(\tau)$ of an ergodic random process X(t) form a Fourier Transform pair with *'T'* and *'f'* as the variables of interest.

$$S_X(f) = \int_{-\infty}^{\infty} R_X(\tau)e^{-j2hf\tau}\,d\tau \quad ; \quad R_X(\tau) = \int_{-\infty}^{\infty} S_X(f)e^{j2hf\tau}\,df$$

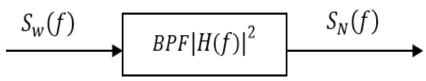
- ✓ The power of an ergodic random process X(t) is equal to area under the graph of *psd*.,

$$E[x^2(t)] = \sigma^2 = \int_{-\infty}^{\infty} S_X(f)\,df$$

The *psd* is that characteristic of a random process which is easy to measure and which is used to characterize noise.

Note: The power of White Noise is infinite. Only the band limited white noise has a finite power.

Power Spectral Density (PSD) of Band pass filtered Noise:



$$S_N(f) = |H(f)|^2 S_w(f) = |H(f)|^2 \frac{N_0}{2}$$

Where $H(f)$the frequency response of channel is filter and$S_w(f)$ denotes the psd of white noise. The average noise power may be calculated from the psd.

The average power 'N' of filtered Gaussian white noise is

$$N = \sigma^2 = 2\frac{N_0}{2}B_T = N_0 B_T$$

$$i.e., \quad {}^2 = N = \int_{-\infty}^{\infty} S_w(f)\,df = \int_0^B \frac{N}{2}\,df = BN_0\,watts = meak\ square\ value$$

Narrow Band Noise:

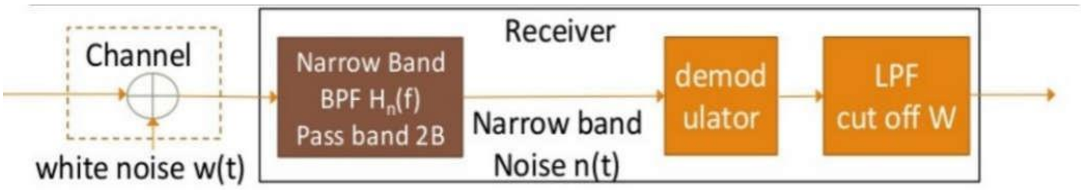
The front end of the receiver of practical communication system consists of frequency selective filter. These filter process the desired signal and noise. I.e., Preprocessing of received signals done by a Narrow band filter. Narrow band filter is used to pass the modulated signal. Noise is also pass through this filter.

This filter is usually narrow band in the sense that its bandwidth is small compared to the middle band frequency. The Noise appearing at the output of such a filter is called "Narrow band Noise".

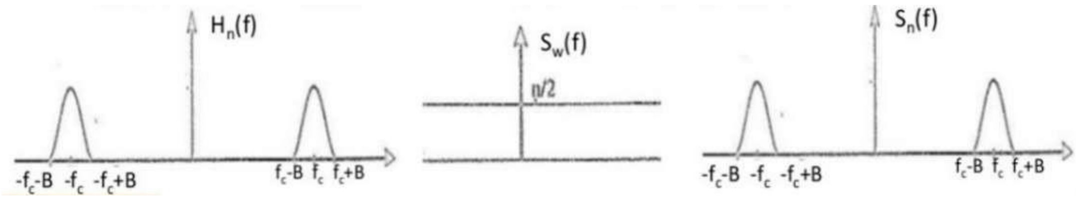
There are two specific representations of narrow band noise. (Depending on application)

1. Narrow Band Noise is defined in terms of a pair of components called the in-phase and quadrature components.
2. Narrow Band Noise is defined in terms of two other components called Envelope and phase.

Representation of Narrowband Noise in terms of In-phase and Quadrature components:



- ✓ First stage at the Receiver end is BPF of pass band $2B$ to limit the noise.
- ✓ Spectral density of White NoiseS_w (f)at the input of BPF is constant ($\frac{N_0}{2}$).
- ✓ The power spectral density of Noise $S_N(f)$ at the output of BPF is Narrow band.



Time domain representation of Narrow band Noise:

From the narrow band spectral density function of noise, it is evident that narrow band noise $k(t)$ is approximately a sinusoidal function of frequency 'f_c' with amplitude and phase varying randomly.

$$k(t) = A_n(t)\cos[2\pi f_c t + \emptyset_n(t)]$$
$$= A_n(t)\cos[\emptyset_n(t)]\cos[2\pi f_c t] - A_n(t)\sin[\emptyset_n(t)]\sin[2\pi f_c t]$$
$$= k_I(t)\cos[2\pi f_c t] - k_Q(t)\sin[2\pi f_c t]$$

Where In-phase component of narrowband noise, $k_I(t) = A_n(t)\cos[\emptyset_n(t)]$and
Quadrature phase component of narrow band noise, $k_Q(t) = A_n(t)\sin[\emptyset_n(t)]$.
Thus narrow band noise can be viewed as sum of in-phase component of noise modulated on carrier signal and quadrature phase component of noise modulated with quadrature shifted version of carrier signal.

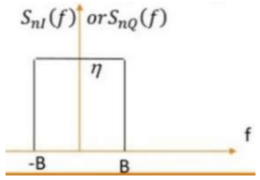
Properties of Narrow band noise:

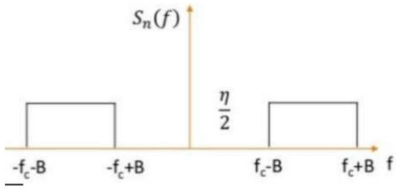
1. If $k(t)$ has zero mean then $k_I(t)$ and $k_Q(t)$ has zero mean.
2. $S_n(f) = \frac{N_0}{2}$ for $f_c - B \leq |f| \leq f_c + B$

3. Both $k_I(t)$ and $k_Q(t)$ has same psd which is related to spectral density of narrowband noise, $S_n(f)$ as

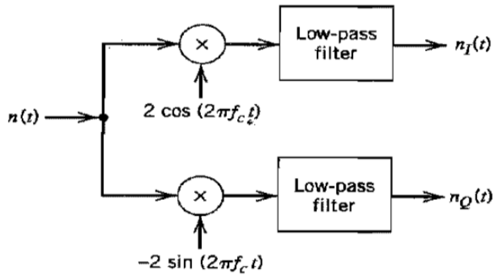
$$S_n(f) = S_Q(f) = S_n(f - f_c) - S_n(f + f_c) = N \; for \; |f| \leq B$$

Thus $\overline{k^2(t)} = \overline{k_I^2(t)} = \overline{k_Q^2(t)}$



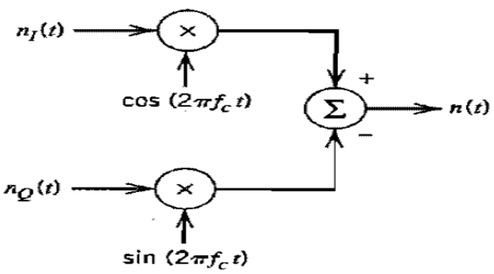


Extraction of $n_I(t)$ and $n_Q(t)$ from $n(t)$:



Each LPF have bandwidth 'B'.
This is known as
"*Narrowbakd Noise Akalyzer*".

Generation of $n(t)$ from $n_I(t)$ and $n_Q(t)$:



This is known as
"*Narrowbakd koise sykthesizer*".

Note: If $k(t)$ is Gaussian and stationary then $k_I(t)$ and $k_Q(t)$ are jointly Gaussian and Stationary.

Noise Temperature:

The available noise power is directly proportional to temperature and is independent of resistor value.

The power specified in terms of temperature is called as "Noise Temperature".

$$T_n = \frac{P_n}{kB} \quad where\ P_n = max.koise\ power$$

Noise figure in terms of noise temperature. i.e., $F = 1 + \frac{T_e}{T_0}$

Where T_e = Effective noise temperature = $(F-1)T_0$

Noisy Receiver Model:

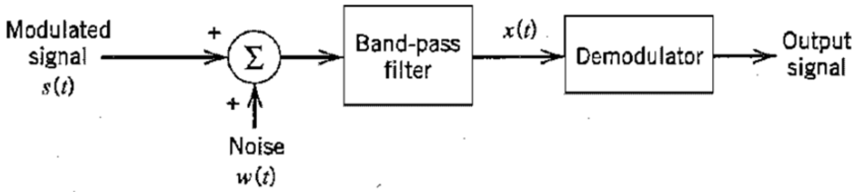


FIGURE Receiver model

Note: Noise $k(t)$ is band pass filtered version of $\omega(t)$

- ➔ Where the receiver noise is included in N_0 given by$N_0 = kT_e$. The band width and center frequency of ideal band pass channel filter are identical to the transmission Bandwidth B_T and the center frequency of modulated waved form, respectively.
- ➔ The filtered noisy received signal $x(t)$ available for demodulation is defined by,

$$x(t) = s(t) + k(t)$$

- ➔ Signal to Noise Ratio (SNR) is a measure of the degree to which a signal is contaminated with additive noise.
- ➔ Assume that band pass filter is ideal, having a band width equal to the transmission B.W. B_T of the modulated signal s(t) and a mid-band frequency equal to the carrier frequency fc $f_c \gg B_T$.
- ➔ The filtered noise $k(t)$ may be treated as a narrow band noise represented in the canonical form, $k(t) = k_I(t)\cos 2\pi f_c\, t - k_Q(t)\sin 2\pi f_c\, t$
- ➔ The average noise power at the demodulator input is equal to the total area under the curve of the $psd\ S_N(f)$.

$$P_{avg-noise} = 2 \times B_T \times \frac{N_0}{2} = B_T N_0$$

Figure of Merit of CW modulation schemes:

Goal: Compare the performance of different continuous wave modulation schemes.

- ➔ Assume that the only source of degradation in message signal quality is the additive noise$\omega(t)$.
- ➔ $(SNR)_0$is well defined only if the recovered message signal and noise appear additively at demodulator output.

The condition is,

- ➔ Always valid for coherent demodulators.
- ➔ But is valid for non-coherent demodulators only if the input signal to noise ratio$(SNR)_I$is high enough.
- ➔ $(SNR)_0 depekds\ ok$: -Modulation scheme and type of demodulator.

Conditions of comparison:

To get a fair comparison of continuous wave modulation schemes and receiver configuration, it must be made on as equal basis.

- ➔ Modulated signal $s(t)$transmitted by each modulation scheme has the same average power.
- ➔ Channel and receiver noise $\omega(t)$has the same average power measured in the message bandwidth 'ω'.

Note: The power P_x(average power) of a Random process $X(t)$ is its means square value $\overline{X^2}$

$$\therefore\ P_X = \overline{X^2} = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_X(\omega)\,d\omega$$

$$= 2\int_0^{\infty} S_X(\omega)df, \quad f = \frac{\omega}{2\pi}$$

- ➔ The mean square value (power) of a random process $x(t) = A\cos(\omega_c t + \theta)$ is $\frac{A^2}{2}$.
- ➔ The noise powers of narrowband noise signal, and its in-phase quadrature components are equal i.e., $\overline{k}_I^2 = \overline{k}_Q^2 = \overline{k}^2 = 2N_0\omega$
- ➔ The psd's of $S_{n_I}(\omega) = S_{n_Q}(\omega) = N_0$; $S_n(\omega) = \frac{N_0}{2}$
- ➔ $S(t) = A\cos x - B\sin x$; $R(t) = \sqrt{A^2 + B^2}$; $\emptyset = \tan^{-1}\frac{B}{A}$ $\quad cakakical\ form$
- ➔ $E = [x^2(t)] = \int_{-\infty}^{\infty} S_X(\omega)d\omega = meak\ square\ value$

  $= total\ area\ ukder\ graph\ the\ of\ psd$
- ➔ $E[XY] = E[X]E[Y]$

The Noise Performance of DSB-SC Scheme:

The model of a DSB-SC receiver using a coherent detector

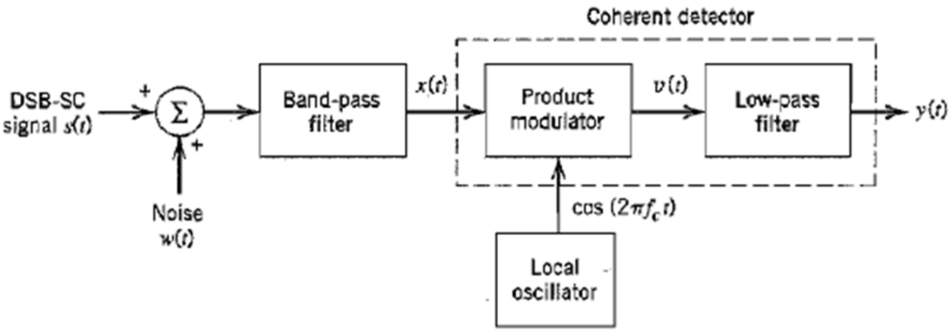


FIGURE Model of DSB-SC receiver using coherent detection.

DSB-SC uses coherent detection scheme at the receiver. The performance of the DSB-SC system can be estimated by calculating the Figure of Merit (γ) as,

$$Figure\ of\ Merit\ (\gamma) = \frac{output\ SNR}{input\ SNR} = \frac{s_0/n_0}{s_i/n_i}$$

Where s_0 = output signal power $\quad$ s_i = input signal power

k_0= output noise power $\quad$ k_i = input noise power

(i) Input signal power :(s_i)

The modulated signal at the input of detector is $S(t) = A_c\cos 2\pi f_c t.m(t)$

Input signal power, $S_i = A_c\cos 2\pi f_c t.A_c\cos 2\pi f_m t = \left[\frac{A_c}{\sqrt{2}}\right]^2\left[\frac{A_m}{\sqrt{2}}\right]^2$

$$= \frac{A_c^2}{2}.\frac{A_m^2}{2} = \frac{A_c^2}{2}P \qquad \because \frac{A_m^2}{2} = P$$

(ii) Noise power at the input :(k_i)

The noise at the input of the receiver is white Gaussian Noise. After passing through BPF, the noise is converted into Band pass noise.

Noise power at the output of filter, $k_i = \int S_\omega(f)\,df$

$$k_i = \int_{-\omega}^{\omega} \frac{N_0}{2}\, df = \omega N_0 \, watts \; = meak\ square\ vale$$

(iii) Output Signal Power :(s_o)

Input to the BPF is signal with noise having amplitude $\frac{N^0}{2}$. The input to the product modulator is (DSB + noise). In synchronous detection method, the incoming DSB-SC signal at detector input is multiplied by the synchronous carrier $'\cos \omega_c t'$.

i.e., Input to the Product Modulator is

$$A_c \cos 2\pi f_c t . m(t) + k_I(t) \cos 2\pi f_c t - k_Q(t) \sin 2\pi f_c t$$

Output of product modulator is,

$$= [A_c \cos 2\pi f_c t . m(t) + k_I(t) \cos 2\pi f_c t - k_Q(t) \sin 2\pi f_c t] \cos(2\pi f_c t)$$

$$= A_c m(t) \cos^2 2\pi f_c t + k_I(t) \cos^2 2\pi f_c t - k_Q(t) \sin 2\pi f_c t \cos(2\pi f_c t)$$

$$= A_c m(t) \left[\frac{1 + \cos 4\pi f_c t}{2}\right] + k_I(t) \left[\frac{1 + \cos 4\pi f_c t}{2}\right] - \frac{k_Q(t)}{2} \sin(4\pi f_c t)$$

After passing through LPF, LPF output is, i.e., $\frac{A_c m(t)}{2} + \frac{n_I(t)}{2}$ i.e., $S_0(t) + k_0(t)$

$$S_0 : S_0(t) = \frac{A_c A_m}{2} \cos 2\pi f_m t$$

Output signal power, $S_0 = \frac{\left[\frac{A_c A_m}{2}\right]^2}{2} = \frac{A_c^2}{4} P$

(iv) Output noise power (n_o):

Here $k_o(t) = \frac{n_I(t)}{2}$

$$k_o = \text{Means square value} = \text{E}[k_o^2] = \frac{1}{4} \overline{k_7^2}(t)$$

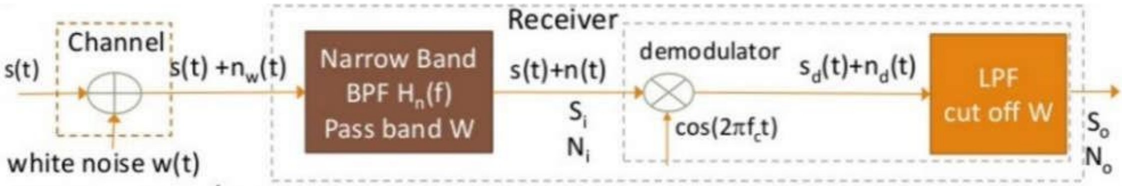
The average power output can be calculated by, $N_o \times B_T$

Where B_T= Transmission B.W of respective modulation system

$$\therefore k_o = \frac{1}{4} \times N_o \times 2\omega = \frac{N_o \omega}{2} \ watts$$

$$\therefore Figure\ of\ merit|_{DSB-SC} = \frac{(S/N)_0}{(S/N)_I} = \frac{s_o}{k_o} . \frac{k_i}{s_i} = \frac{A_c^2 P/4}{N_o \omega/2} \times \frac{N_o \omega}{A_c^2 P/2} = 1 = \gamma$$

Calculation of Figure of Merit of SSB-SC communication system:



Input signal power: The incoming signal to the SSB-SC receiver is,

$$S(t) = \frac{A_c}{2} \cos 2\pi f_c t . m(t) \pm \frac{A_c}{2} \sin(2\pi f_c t) \hat{m}(t)$$

$$= \frac{1}{2} A_c A_m \cos 2\pi f_c t \cos 2\pi f_m t \pm \frac{1}{2} A_c A_m \sin 2\pi f_c t \sin 2\pi f_m t$$

$$= \frac{1}{2} A_c A_m \cos[2\pi (f_c \pm f_m) t]$$

$$(\tfrac{1}{2} A_c A_m)^2 \quad A^2 A^2 \quad \underline{A^2}$$

$$ikput\ sigkal\ power, S_i = \frac{2}{2} = \frac{c}{4}\frac{m}{2} = \frac{c}{4}P$$

Input noise power:

Noise power at the output of filter, $k_i = \int S_\omega(f)df$

$$k_i = \int_{-\omega}^{\omega} \frac{N_o}{2}\, df = \omega N_o\, watts$$

Output signal power:

Input to the product modulator is SSB-SC + Noise

i.e., $\frac{1}{2} A_c A_m \cos[2\pi(f_c \pm f_m)t] + k_I(t)\cos 2\pi f_c t - k_Q(t)\cos 2\pi f_c t$

Output of product modulator is, Product Modulator multiplied by $\cos 2\pi f_c t$.

➔ In SSB be will transmit, receive only one sideband i.e., either LSB or USB,

$$= \{[\frac{A_c}{2}\cos(2\pi f_c t)m(t) \pm \frac{A_c}{2}\sin(2\pi f_c t)\hat{m}(t)] + k_I(t)\cos 2\pi f_c t - k_Q(t)\sin 2\pi f_c t\}\cos 2\pi f_c t$$

$$= \frac{A_c}{2} m(t)\cos^2 2\pi f_c t \pm \frac{A_c}{2}\sin(2\pi f_c t)\hat{m}(t)\cos 2\pi f_c t + k_I(t)\cos^2 2\pi f_c t$$

$$+ k_Q(t)\sin 2\pi f_c t \cos 2\pi f_c t$$

$$= \frac{A_c}{2} m(t)\left[\frac{1+\cos 4\pi f_c t}{2}\right] + \frac{A_c}{2}\hat{m}(t)\frac{\sin 4\pi f_c t}{2} + k_I(t)\left[\frac{1+\cos 4\pi f_c t}{2}\right] + k_Q(t)\frac{\sin 4\pi f_c t}{2}$$

➔ The output of LPF is $\frac{A_c}{4} m(t) + \frac{n_I(t)}{2}$

$$\therefore S_o(t) = \frac{A_c}{4} m(t) = \frac{A_c}{4} A_m \cos 2\pi f_m t$$

$$S_o = \frac{[A_c A_m/4]^2}{2} = \frac{A_c^2}{16} \times \frac{A_m^2}{2} = \frac{A_c^2}{16} P$$

Output Noise power:

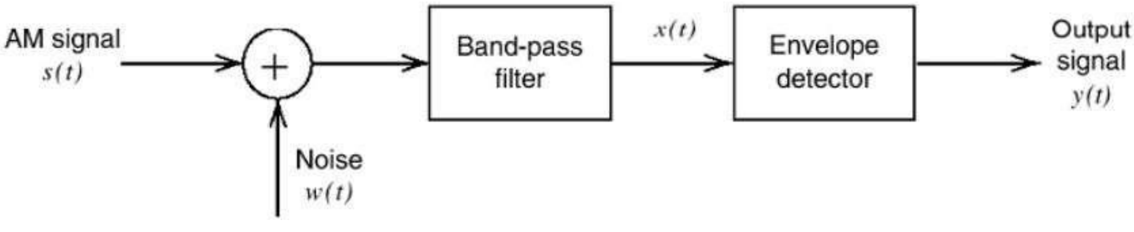
$$k_o(t) = \frac{k_I(t)}{2}$$

$$k_o = E[k_o(t)] = \frac{1}{4} k_I(t) = \frac{1}{4} \times N_o \times B_T \qquad ; where\ B_T = \omega\ for\ ssb - sc$$

$$= \frac{1}{4} \times N_o \times \omega = \frac{N_o \omega}{4}$$

$$\therefore Figure\ of\ merit|_{SSB-SC}, \gamma = \frac{(S/N)_0}{(S/N)_I} = \frac{s_o}{k_o} \cdot \frac{k_i}{s_i} = \frac{A_c^2 P/16}{N_o \omega/4} \times \frac{N_o \omega}{A_c^2 P/4} = 1$$

Figure of Merit for AM System:



Input Signal Power:

The standard equation of AM signal

$$S(t) = A_c[1 + K_a m(t)]\cos 2\pi f_c t$$

$$= A_c \cos 2\pi f_c t + A_c K_a m(t) \cos 2\pi f_c t$$

Input signal power, $S_i = \frac{A_c^2}{2} + \frac{A_c^2 K_a^2 A_m^2}{4} = \frac{A_c^2}{2}[1 + K_a^2 P]$

Note: $A_c[1 + K_a m(t)]$ is reference Phasor $k_I(t)$ and $k_Q(t)$ are in-phase and quadrature phasors, each 90° phase difference. So, total three phasors.

Input Noise Power:

Noise power at the output of the filter, $k_i = \int S_\omega(f)df$

$$k_i = \int_{-\omega}^{\omega} \frac{N_o}{2}\, df = \omega N_o \,watts$$

Output Signal Power:

The input signal to the envelope detector is AM + Noise

$$= A_c \cos 2\pi f_c t + A_c K_a m(t) \cos 2\pi f_c t + k_I(t) \cos 2\pi f_c t - k_Q(t) \sin 2\pi f_c t$$

$$= [A_c + A_c K_a m(t) + k_I(t)] \cos 2\pi f_c t - k_Q(t) \sin 2\pi f_c t$$

To find the envelope of above system, write above equation, in polar-form

i.e., $R(t) \cos[\omega_t + \emptyset(t)]$

Where envelope, $R(t) = \sqrt{[A_c + A_c K_a m(t) + k_I(t)]^2 + k_Q^2(t)}$

The envelope detector output is $R(t)$.

Here, if $[A_c + A_c K_a m(t) + k_I(t)] >> k_Q(t)$, So, $k_Q(t)$ is neglected.

$$\left\{ \begin{array}{c} \because \text{Since if desired signal strength is more than noise i.e.}, A_c + A_c K_a m(t) + k_I(t) \gg \text{n(t)} \\ \therefore\ A_c + A_c K_a m(t) + k_I(t) \gg k_Q(t) \end{array} \right\}$$

$$\therefore R(t) = A_c + A_c K_a m(t) + k_I(t)$$

The dc component 'A_c' of envelope detector output $R(t)$ is blocked by a capacitor (because it does not contain any information), yielding $m(t)$ as the useful signal and $k_I(t)$ as the noise.

$$\therefore R(t) = A_c K_a m(t) + k_I(t)$$

$$= S_o(t) + k_o(t)$$

Output Signal Power for $S_o(t) = A_c K_a m(t)$

Output Signal Power, $S_o = \frac{[A_c K_a A_m]^2}{2} = A_c^2 K_a^2 P$

Output Noise Power:

$$k_o(t) = k_I(t)$$

$$k_o = E[k_o^2(t)] = \overline{k_I^2(t)} = N_o \times B_T$$

$$k_o = 2N_o\omega$$

$$\therefore Figure\ of\ merit|_{AM}, \gamma = \frac{(S/N)_0}{(S/N)_I} = \frac{s_o}{k_o} \cdot \frac{k_i}{s_i}$$

$$= \frac{A_c^2 K_a^2 P}{2N_o\omega} \cdot \frac{N_o\omega}{\frac{A_c^2}{2}[1 + K_a^2 P]} = \frac{K_a^2 P}{1 + K_a^2 P}$$

Here $P = \frac{A_m^2}{2}$; $\mu = K_a A_m$

$$Figure\ of\ merit, \gamma = \frac{K_a^2 A_m^2/2}{1 + K_a^2 A_m^2/2} = \frac{\mu^2}{\ } \quad (\because \mu = K_a A_m,)$$

$$1 + K_a^2 A_m^2/2 \qquad 2 + \mu^2 \left\{ 1 + \frac{\mu^2}{2} = \frac{2 + \mu^2}{2} \right\}$$

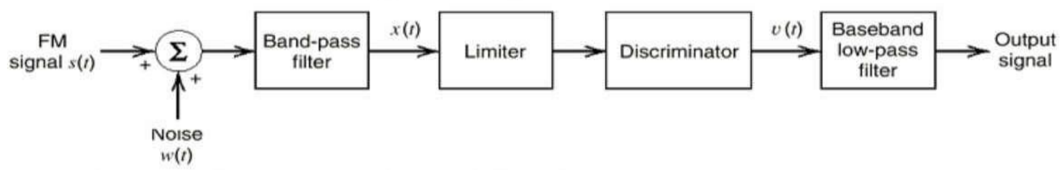
When envelope detector is used maximum value of μ is 1.

$$Figure\ of\ merit, \gamma_{AM} = \frac{1}{3}$$

This mean that, other factors being equal, an AM system (using envelope detection) must transmit three times as much average power as a suppressed carrier system (using coherent detection) in order to achieve the same quality of noise performance.

Noise in FM Receiver:

The Receiver model is given by,



- ➔ The noise $\omega(t)$ is modeled as white Gaussian noise of zero mean and power spectral density $N_o/2$.
- ➔ The received FM signal $s(t)$ has a carrier frequency 'f_c' and transmission bandwidth $'B'_T$ of such that only a negligible amount of power lies outside the frequency band $f_c \pm \frac{B_T}{2}$ for positive frequencies.
- ➔ The BPF has a mid-band frequency 'f_c' and Bandwidth $'B'_T$ and therefore passes the FM signal essentially without distortion.
- ➔ Ordinary, $'B_T'$ is small compared with the mid-band frequency 'f_c' so that we may use the narrow band representation for $k(t)$, the filtered version of receiver noise $\omega(t)$, in terms of its in-phase and quadrature components.
- ➔ In an FM system, the message information is transmitted by variations of the instantaneous frequency of a sinusoidal carrier wave, and its amplitude is maintained constant.
- ➔ Any variations of the carrier amplitude at the receiver input must result from noise or interference.
- ➔ The limiter is used to remove amplitude variations by clipping the modulated wave at the filter output almost to the zero axis.
  - (i) The resultant rectangular wave is rounded off by another BPF that is an integral part of the limiter, there by suppressing harmonics of the carrier frequency.
  - (ii) The filter output is again sinusoidal, with an amplitude that is practically independent of the carrier amplitude at the receiver input.
- ➔ The discriminator consists of two components,
  - (i) A slope network or differentiator with a purely imaginary transfer function that varies linearly with frequency. It produces a hybrid modulated wave in both amplitude and frequency vary in accordance with the message signal.
  - (ii) An envelope detector that recovers the amplitude variation and thus reproduces the message signal.

(iii) The slope network and envelope detector are usually implemented as integral parts of a single physical unit.

- ➔ The post detection filter, labeled "Baseband low pass filter" has a Bandwidth that is just large enough to accommodate the highest frequency component of the message signal.
- ➔ This filter remove the out of band components of the noise at the discriminator output and there by keeps the effect of the output noise to a minimum.

✓ The filtered noise at the BPF output is defined as

$$k(t) = k_I(t)\cos 2\pi f_c t - k_Q(t)\sin 2\pi f_c t$$

An equivalent representation in envelope and phase is

$$k(t) = r(t)\cos[2\pi f_c t + \psi(t)] \text{ --------------------(1)}$$

Where envelope is $r(t) = [k_I^2(t) + k_Q^2(t)]^{\frac{1}{2}}$ ---------------- (2)

and phase is $\psi(t) = \tan^{-1}\left[\frac{n_Q(t)}{n_I(t)}\right]$ ------------------ (3)

Here the envelope is Rayleigh distributed and the phase is uniformly distributed. The FM signal is, $S(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(t)\, dt\right]$ ------------------ (4)

$\phi(t) = 2\pi k_f \int_0^t m(t)\, dt$ ------------------ (5)

$\therefore \quad S(t) = A_c\cos[2\pi f_c t + \phi(t)]$ ------------------ (6)

The noisy signal at the BPF output is,

$x(t) = S(t) + k(t)$

$= A_c\cos[2\pi f_c t + \phi(t)] + r(t)\cos[2\pi f_c t + \psi(t)]$------------------- (7)

The resulting phase $\theta(t)$representing $x(t)$. Here we have used signal term as reference.

$\because [x(t) = \mathrm{A}\cos 2\pi f_c t + \theta(t)]$

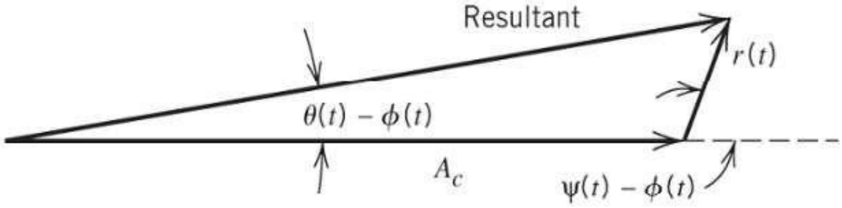


fig: phasor diagram of FM

$$\theta(t) - \phi(t) = \tan^{-1}\left\{\frac{r(t)\sin[\psi(t) - \phi(t)]}{A_c + r(t)\,\mathrm{Cos}[\psi(t) - \phi(t)]}\right\} \text{ ------------------(8)}$$

The envelope of $x(t)$ is of no interest to us, because any envelope variations at the band pass output are removed by limiter.

- ➢ Our motivation is to determine the error in the instantaneous frequency of the carrier wave caused by the presence of the filtered noise $k(t)$.

$$\theta(t) = \phi(t) + \tan^{-1}\left\{\frac{r(t)\sin[\psi(t) - \phi(t)]}{A_c + r(t)\,\mathrm{Cos}[\psi(t) - \phi(t)]}\right\} \text{-------------------(9)}$$

For small noise analysis, $A_c \gg r(t)$and if $x \ll 1 \Rightarrow \tan^{-1}[x] = x$

$$\theta(t) \cong \phi(t) + \frac{r(t)}{A_c}\sin[\psi(t) - \phi(t)] \text{ -----------------------(10)}$$

From eq.5; $\theta(t) = 2\pi k_f \int_0^t m(t)dt + \frac{r(t)}{A_c}\sin[\ \ (t) - \phi(t)]$ -------------------(11)

- With the discriminator assumed ideal, its output is proportional to $\frac{\theta^{F}(t)}{2h}$ where $\theta'(t)$ is the derivative of $\theta(t)$ with respect to time.
- The discriminator output is $v(t) = \frac{1}{2h}\frac{d}{dt}\theta(t)$

$$v(t) = k_f m(t) + k_d(t) \text{ --------------------(12)}$$

And is proportional to the message. Where noise is

$$k_d(t) = \frac{1}{2hA_c}\frac{d}{dt}\{r(t)\sin[\ \ (t) - \phi(t)]\} \text{ ------------------ (13)}$$

Since the phase $\ (t)$ is uniformly distributed over [0, 2π], we may assume $\ (t) - \phi(t)$ is also uniformly distributed over [0, 2π]and thus the noise $k_d(t)$at the discriminator output is independent of the modulating signal.

$$k_d(t) = \frac{1}{2hA_c}\frac{d}{dt}\{r(t)\sin[\ \ (t)]\} \text{ ------------------ (14)}$$

Here $k_Q(t) = r(t)\sin[\ \ (t)]$

$$\therefore\ k_d(t) = \frac{1}{2hA_c}\frac{d}{dt}\{k_Q(t)\} \text{ ------------------(15)}$$

This means that the addition noise $k_d(t)$ appearing at the discriminator output is determined effectively by the carrier amplitude 'A_c' and quadrature component $k_Q(t)$of the narrowband noise $k(t)$.

- From eq.12 the message component in the discriminator output and therefore the LPF output is $k_f m(t)$.
- The average output signal power is equal to $k_f^2 p$, where 'p' is the average power of the message signal $m(t)$.
- To determine the average output noise power, we note that the noise $k_d(t)$ at the discriminator output is proportional to the time derivative of the quadrature noise component $k_Q(t)$.
- The differentiation of a function respect to time corresponds to multiplication of its F.T. byj$2\pi f$. We may obtain the noise $k_d(t)$by passing $k_Q(t)$ through a linear filter with a transfer function,

$$\frac{j2hf}{2hA_c} = \frac{jf}{A_c} \text{ - - - - - - - - - - - - (16)}$$

The power spectral density $S_{n_d}(f)$of $k_d(f)$is related to psd of $k_Q(t)S_{n_Q}(f)$

$$S_{n_d}(f) = \frac{f^2}{A_c^2}S_{n_Q}(f) \text{ ------------------(17)}$$

Since $S_{n_Q}(f)$is from LP filtered AWGN, we have

$$S_{n_d}(f) = \begin{cases} \frac{N_o f^2}{A_c^2}, & |f| \le \frac{B_T}{2} \\ 0, & otherwise \end{cases} \text{ --------------------------------(18)}$$

For wideband FM, $'\omega'$ is usually less than ^{B_T}

$$S_{n_0}(f) = \begin{cases} \frac{N_o f^2}{A_c^2}, & |f| \le \omega \\ 0 & , otherwise \end{cases} \text{-------------------(19)}$$

In FM system, increasing the carrier power has a noise – quieting effect.

Average power of output noise,

$$k_0 = \int_{-\omega}^{\omega} \frac{N_0}{A_c^2} f^2 \, df = \frac{2N_0\omega^3}{3A_c^2} \text{-------------------(20)}$$

Output SNR is $(SNR)_0 = \frac{S_o}{N_o} = \frac{PK_f^2}{2N_o\omega^3/3A_c^2} = \frac{3PK_f^2A_c^2}{2N_o\omega^3}$ -------------------------------- (21)

➔The average power in modulated signal $s(t)$ is $\frac{A^2}{2}$, and the average noise power in the message Bandwidth is

$$k_i = \int_{-\omega}^{\omega} \frac{N_0}{2} \, df = N_0 \omega \text{------------------- (22)}$$

∴The input Signal to Noise ratio, $(SNR)_I = \frac{A_c^2}{2N_0\omega}$ ------------------------------- (23)

$$\therefore Figure\ of\ merit|_{FM},\ \gamma = \frac{(S/N)_0}{(S/N)_I} = \frac{3K_f^2P}{\omega^2} \text{-------------------(24)}$$

For single tone modulation, $S(t) = A_c \cos[2\pi f_c t + 2\pi k_f \int_0^t m(t)\, dt]$

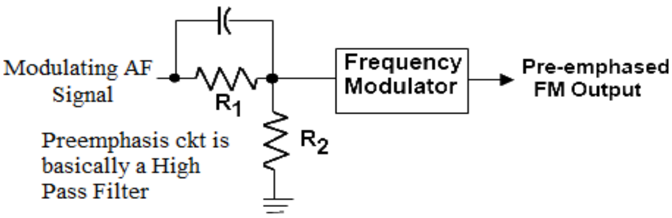
$$S(t) = A_c \cos[2\pi f_c t + [\frac{K_f A_m}{f_m}] \sin 2\pi f_m t]$$

$$S(t) = A_c \cos[2\pi f_c t + \beta \sin 2\pi f_m t]$$

From eq.24; $\gamma = \frac{3K_f^2A_m^2}{2\omega^2}$ $\qquad [\because P = \frac{A_m^2}{2}\ ;\ K_f A_m = \Delta f = \Delta\omega]$

PRE-EMPHASIS:

In FM the noise has a greater effect on the higher modulating frequencies. This effect can be reduced by increasing the value of modulation (m_f) for higher modulating frequencies (f_m).



$$\therefore \quad \gamma_{FM} = \frac{3}{2}\beta^2$$

Fig (a): Typical Pre-Emphasis Circuit

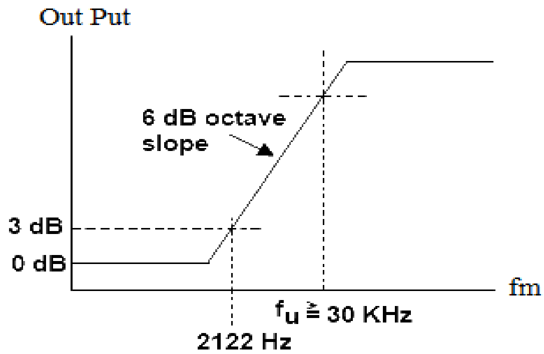


Fig (b): Pre-Emphasis Characteristics

This can be done by increasing the deviation Δf and Δf can be increased by increasing the amplitude of modulating signal at higher modulating frequencies.

Thus if we boost the amplitude of higher frequency modulating signals artificially then it will be possible to improve the noise immunity at higher modulating frequencies. The artificial boosting of higher modulating frequencies is called as pre-emphasis. Boosting of higher frequency modulating signal is achieved by using the pre-emphasis circuit of fig (a). The modulating AF signal is passed through a high pass RC filter, before applying it to the FM modulator.

As f_m increases, reactance of ‘c’ decreases and modulating voltage applied to FM modulator goes on increasing. The frequency response characteristic of the RC high pass network is shown in fig (b). The pre-emphasis circuit is basically a high pass filter. The pre-emphasis is carried out at the transmitter.

i.e., $f = \frac{1}{2\ \ RC} = \frac{1}{2\ \ \times 75\times 10^{-6}}$ =2.122 Hz.

The demodulated FM is applied to the De-emphasis circuit with increase in f_m the reactance of ‘c’ goes on increasing and the o/p of de-emphasis circuit will also reduce.

Sampling Process:

It is a process of converting continues time signal into discrete time signal and recover back when we sample the signal is at Nyquist rate.
Let us consider a continuous signal g(t) is shown in below fig(a).

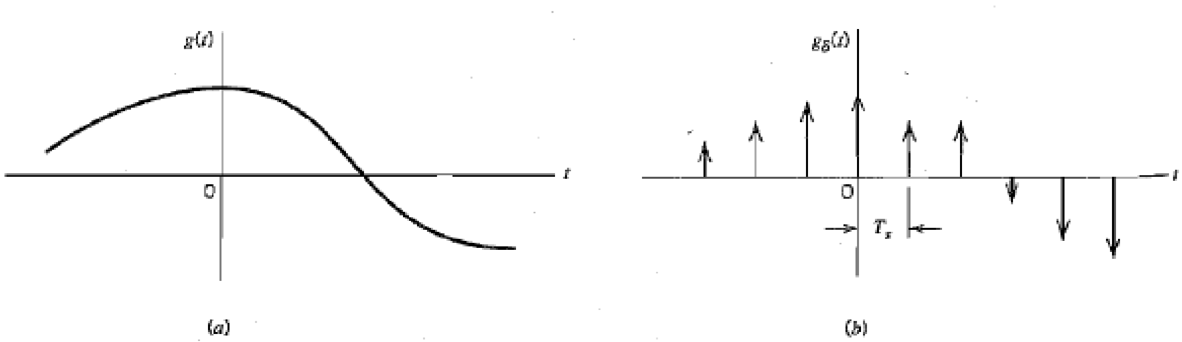


FIGURE The sampling process. (*a*) Analog signal. (*b*) Instantaneously sampled version of the analog signal.

This Continuous signal is converted into discrete time signal by taking the samples at regular interval of times. The space between two successive samples is called as Sampling period(T_s). Reciprocal of sampling period gives the sampling Frequency(f_s).

Let $g_\delta(t)$ is the sampled version of analog signal. It is expressed as

$$g_\delta(t) = \sum_{n=-\infty}^{\infty} g(kT_s)\eth(t-kT_s) \qquad ------1$$

Using Fourier transform pairs, we may write

$$g_\delta(t) \rightleftarrows f_s \sum_{m=-\infty}^{\infty} G(f-mf_s) \qquad ------2$$

Where G(f) is the Fourier transform of original signal g(t)

f_s is the sampling rate.

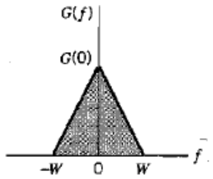
Apply Fourier transform to both sides of equation 1, we get ------3

$$G_\delta(f) = \sum_{n=-\infty}^{\infty} g(kT_s)e^{-j2\pi fnT_s} -$$

From equation 2 ,

$$G_\delta(f) = f_s G(f) + f_s \sum_{\substack{m=-\infty \\ m \neq 0}}^{\infty} G(f - mf_s) \qquad ----4$$

Let us consider the input signal g(t) is strictly band limited to W hertz, i.e whose Fourier transform G(f) of a signal g(t) has a property that G(f)=0 for |f|≥W. It is shown in below figure.



Spectrum of band limited signal

i) f_s =2W then $T_s = \frac{1}{2W}$ (Perfect sampling)

Substitute in Equation 3 we get,

$$G_\delta(f) = \sum_{n=-\infty}^{\infty} g\left(\frac{k}{2W}\right) e^{\frac{-j2\pi nf}{W}} \qquad ------5$$

For a band limited signal equation 4 can be modified as,

$$G_\delta(f) = f_s G(f) \qquad -W < f < W$$

$$G(f) = \frac{1}{f_s} G_\delta(f)$$

$$G(f) = \frac{1}{2W} \sum_{n=-\infty}^{\infty} g\left(\frac{k}{2W}\right) e^{\frac{-j2\pi nf}{W}} ------6 \qquad (From\ eqk\ 5)$$

The above equation gives the fourier transform of signal g(t) with a sampling rate f_s=2W.

In order to reconstruct the g(t) from G(f) we need to apply inverse Fourier transform

$$g(t) = \int_{-\infty}^{\infty} G(f)\, e^{j2\pi ft}\, df$$

$$g(t) = \int_{-W}^{W} \frac{1}{2W} \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) e^{\frac{-j2\pi nf}{W}} e^{j2\pi ft}\, df \qquad \text{(from equation 6)}$$

Interchanging the order of summation and integration, we get

$$g(t) = \sum^{\infty} g\left(\frac{k}{2W}\right) \frac{1}{2W}\int_{-W}^{W} e^{j2hf\left(t-\frac{n}{2W}\right)}\, df$$

$$g(t) = \sum_{n=-\infty}^{\infty} g\left(\frac{k}{2W}\right) \frac{\sin(2\pi Wt - k\pi)}{(2\pi Wt - k\pi)}$$

$$g(t) = \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right)\ sikc(2Wt - k)\ ,\ -\infty < t < \infty \text{-----------}7$$

The above equation provides interpolation formula for re constructing the original signal g(t). In this case the better reproduction occurs.

The corresponding spectrum of sampled signal is shown in below figure,

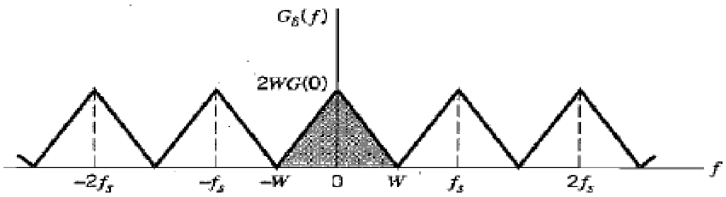
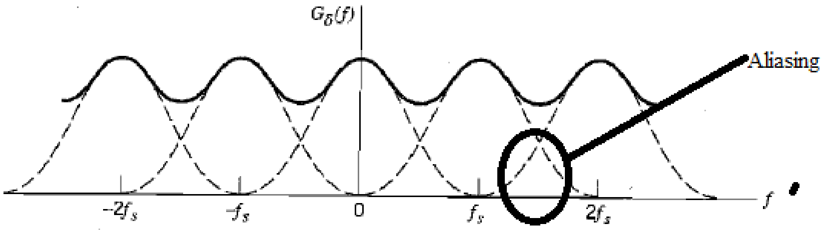


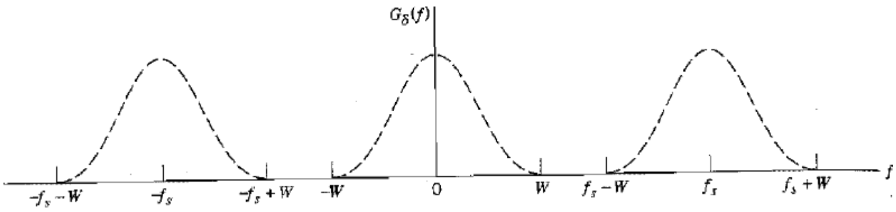
Fig : Spectrum of sampled version of g(t) for a sampling rate fs=2W

ii) f_s< 2W, when we sample the signal lower than the Nyquist rate then the successive samples of the spectrum are overlapped . This effect is called **aliasing effect**. The corresponding spectrum is shown in below fig.



It causes there is no better reproduction. This one can be eliminated by using the following cases.
1. Prior to sampling, a low pass anti-aliasing filters are used to eliminate unnecessary high frequency components.
2. Sampling frequency slightly higher than the Nyquist rate.

iii) f_s > 2W, When we sample the signal higher than the Nyquist rate there is no overlap between the successive cycles of the spectrum. The better reproduction is possible but the transition band extending from W to $f_s - W$.



<u>PULSE AMPLITUDE MODULATION (PAM):-</u>

<u>Generation of PAM:-</u>

- It is very easy to generate and demodulate PAM.

- The signal to be converted to PAM is fed through switch which is controlled by a pulse train.
- When pulse is present i.e signal is at high level switch is closed. When pulse is absent i.e signal is at low level switch is open.
- With this control action of switch we get pulse amplitude modulated wave form at the output terminal of the switch.
- This pulse amplitude modulated signal is passed through a pulse shaping network,which gives them flat tops.
- These output pulses can be used to frequency modulate the carrier to form PAM-FM system.

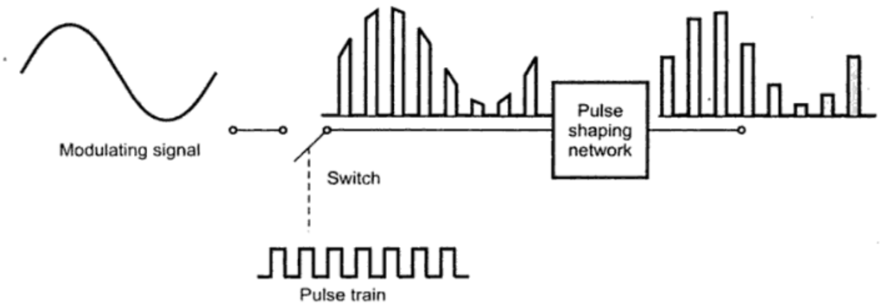
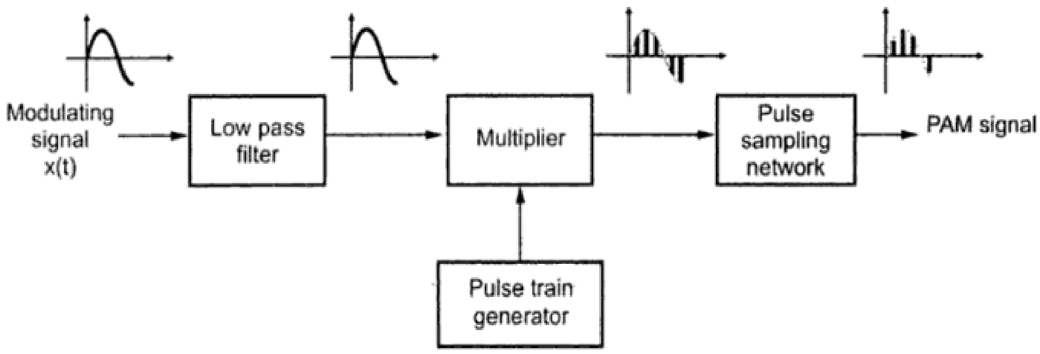
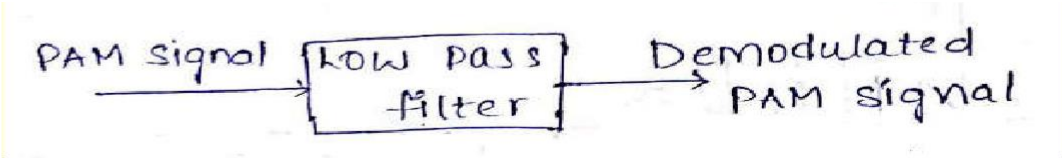


Fig. 2.4 Generation of pulse amplitude modulated wave



- The fig shows the block schematic of PAM generator
- It consists of a low pass filter (LPF), a multiplier and a pulse train generator.
- Initially, the modulating signal X(t) is passed through the LPF.
- The LPF removes all the frequency components which are higher than frequency fm. This is known as band limiting.
- The band limiting is necessary to avoid the aliasing effect in the sampling process.
- The pulse train generator generates a pulse train at a frequency fs such that $fs>=2f_m$. thus the Nyquist criteria are satisfied.
- The pulse sampling network does the shaping work to give flat tops.
- <u>Reconstruction of original signal X(t) (or) Detection of PAM:-</u>
-
- The original modulating signal can be detected from the natural PAM by passing naturally modulated PAM signal through a low pass filter .the low pass filter cut off frequency equal to fm removes the high frequency ripple and recovers original modulating signal .this is illustrated in below fig.



- In case of flat top PAM to reduce aperture effect, an equalizer is used as shown in fig, the Receiver consists of LPF reconstruction filter with cut off frequency stability higher than the maximum frequency in message signal .The equalizer compensates for the aperture effect It also compensates for the attenuation by a low pass construction filter.

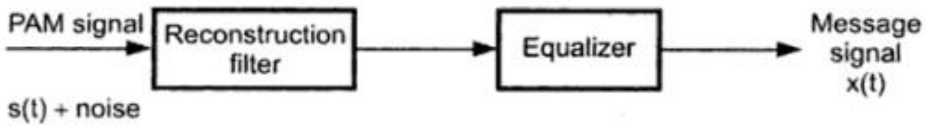
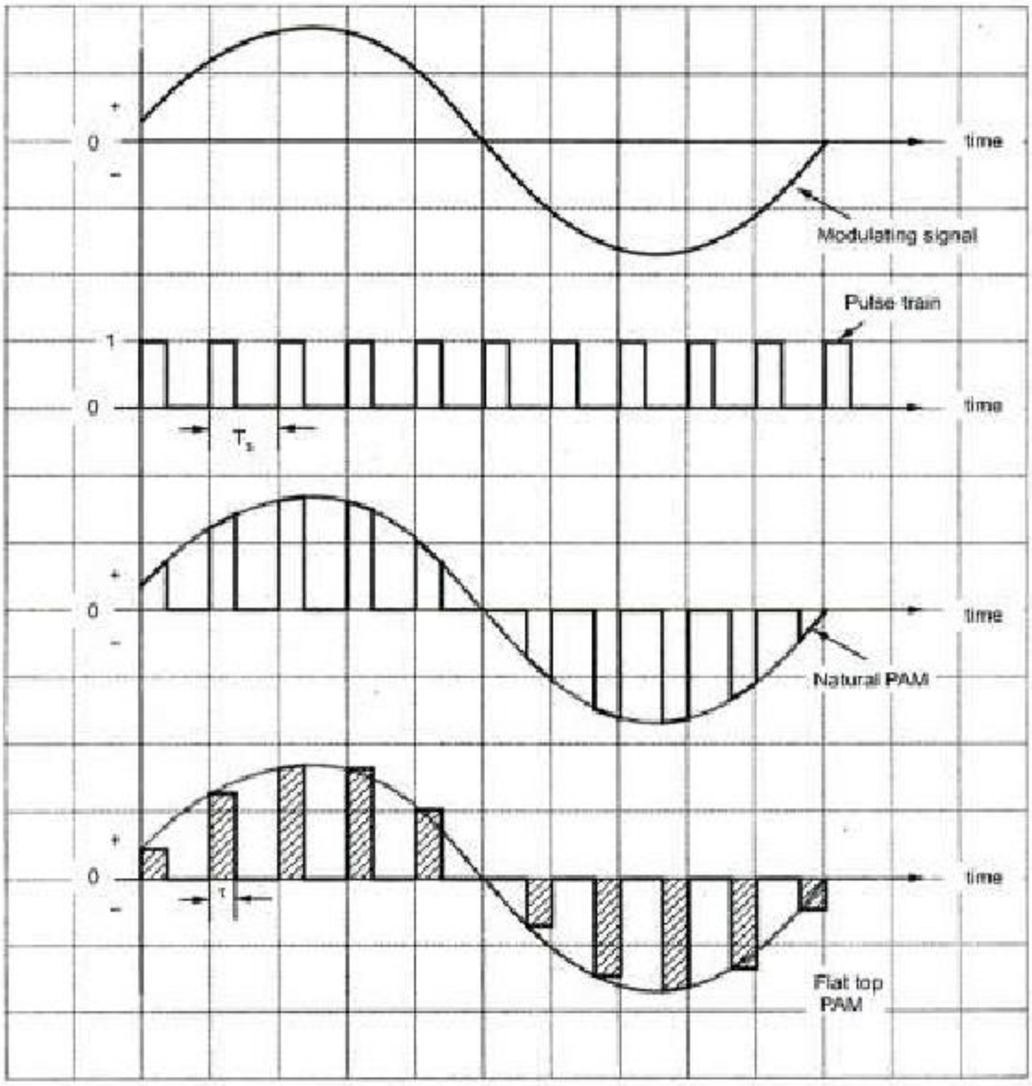


Fig. 2.17 Recovering x(t)



- **Fig: Waves of PAM**

<u>Quantization Process :</u>It is used to discretizing the amplitude of the sampled signal.

The `L' level quantizer compares sampled signal($m(kT_s)$) with the fixed digital levels in the quantizer and assigns any one of the **nearest** digital level to sampled signal($m(kT_s)$) which results some distortion or error. This error is called quantization error. The output of quantizer is denoted as $m_q(kT_s)$.

Quantization error = $m_q(kT_s)$- $m(kT_s)$

Quantization process can be classified into 2 types.

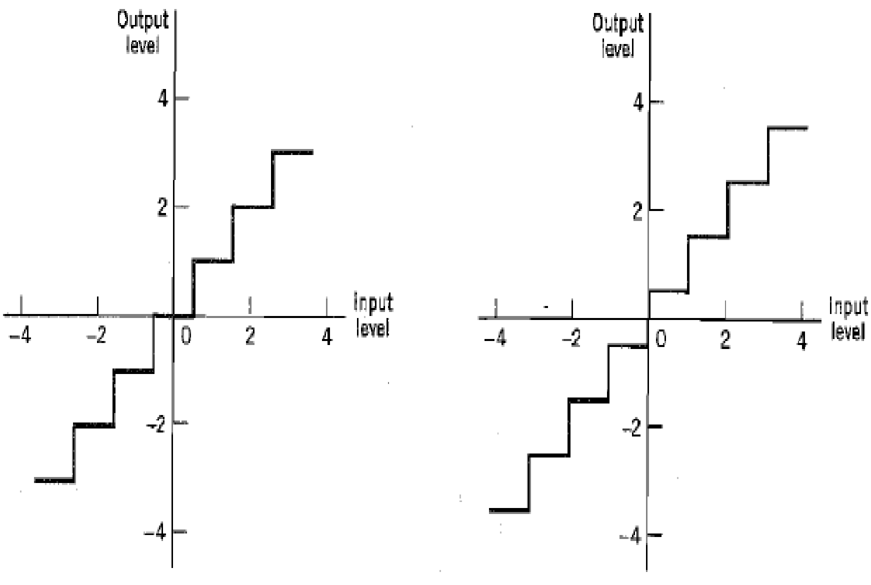
1. Uniform quantization or Linear quantization (Symmetric type)
2. Non uniform quantization or Nonlinear quantization(Non symmetric type)

<u>Uniform quantization</u>: In this type of quantization step size is same for throughout the input message. It is classified into two types

i) Mid tread type uniform quantization

ii) Mid rise type uniform quantization

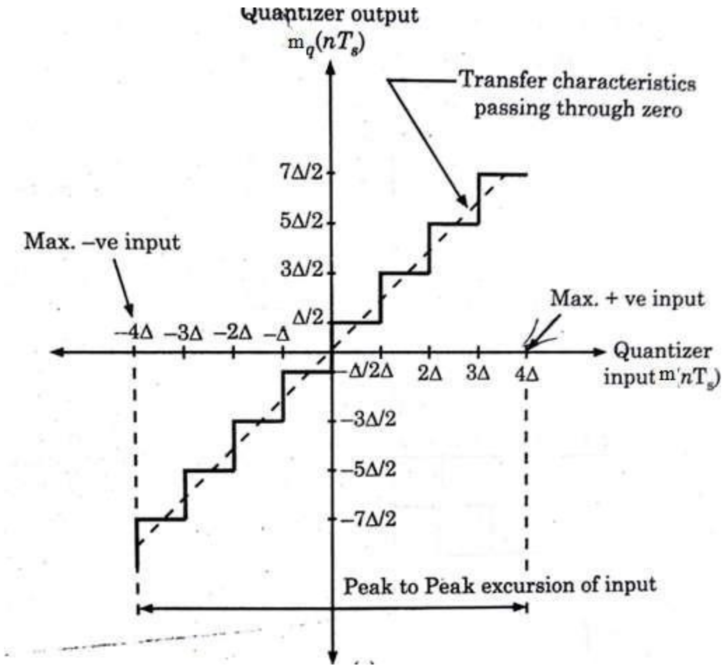
In fig(a), the origin lies in the middle of tread of the staircase like graph.
In fig(b), the origin lies in the middle of rising part of the staircase like graph.



(a) Mid Tread (b) Mid Rise

<u>Working principle of Quantizer:</u>

Let us consider Mid rise type uniform quantization with step size Δ and whose amplitude is varies from -4Δ to +4Δ. It is shown in below figure.



Form the fig,

$m(kT_s)$ = 4Δ then $m_q(kT_s) = \frac{7\Delta}{2}$. The quantization error is given by

Quantization error q = $m_q(kT_s)$- $m(kT_s)$

$$= \frac{7\Delta}{2} - 4\Delta = -\frac{\Delta}{2}$$

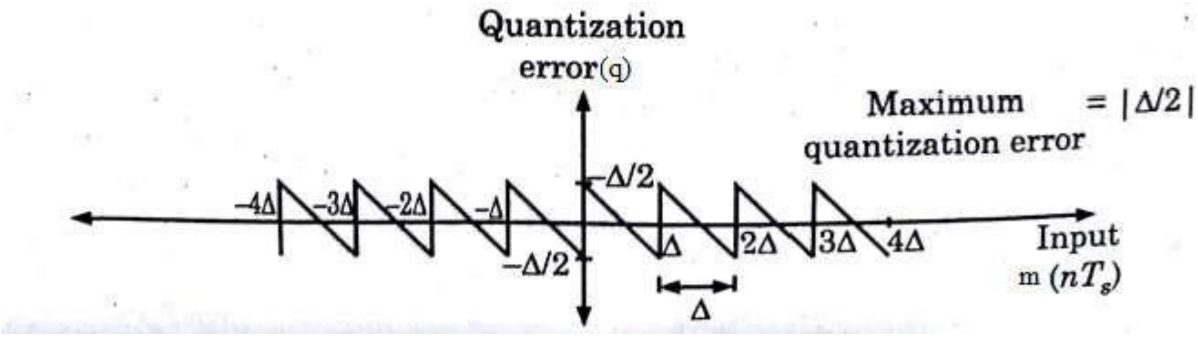
Form the fig,

$m(kT_s)$ = -4Δ then $m_q(kT_s) = -\frac{7\Delta}{2}$. The quantization error is given by

Quantization error q = $m_q(kT_s)$- $m(kT_s)$

$$= -\frac{7\Delta}{2} + 4\Delta = +\frac{\Delta}{2}$$

Therefore, the maximum possible quantization error is equal to $\pm\frac{\Delta}{2}$. The spectrum for maximum quantization is shown in below figure.



Non uniform Quantization:

The step size variable with respect to the input message. It is also known as nonlinear quantization. It is varied based on the amplitude of the input signal.

Generally Non uniform quantization is used to improve the signal to noise ratio of the weak signals.

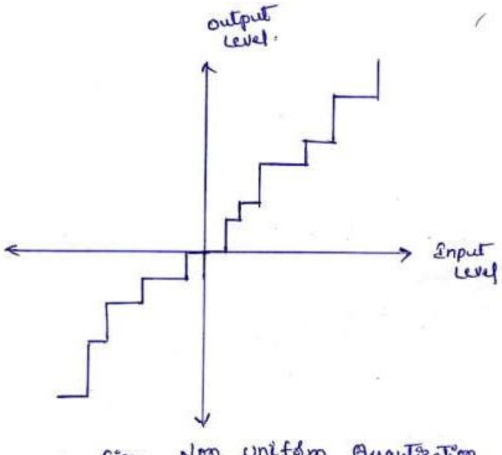


Fig:- Non uniform Quantization

Non uniform quantization is practically achieved through Companding technique.

Non Uniform quantization has an advantage over the uniform quantization:

In uniform quantization , the step size fixed. For weak signals the difference between the quantized signal to sampled signal (Quantization error) increases. So that it decreases to overall signal to noise ration of the system.

In Non-uniform quantization , the step size variable. For weak signals the difference between the quantized signal to sampled signal (Quantization error) decreases. So that it increases to overall signal to noise ration of the system.

From the above discussion we can conclude that Non uniform quantization has an advantage over the uniform quantization.

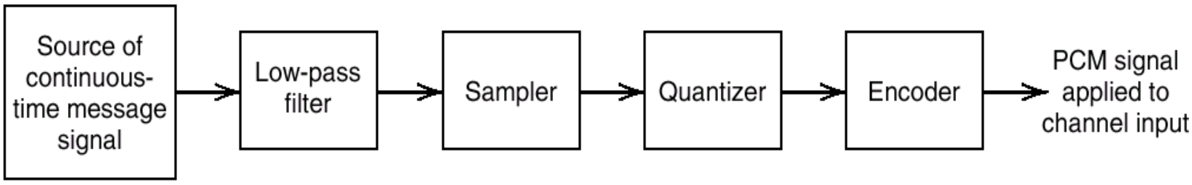
Pulse code Modulation:

- PCM is known as digital pulse modulation technique.
- PCM is quite complex as compared to analog pulse modulation techniques because the message signal is subjected to more no of iterations.

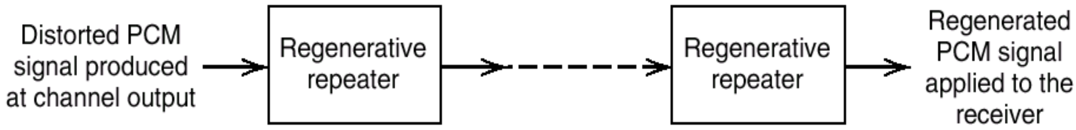
The basic block diagram of PCM is shown in the below figure.

The basic block diagram consisting of three parts. They are transmitter, Transmission path and receiver.

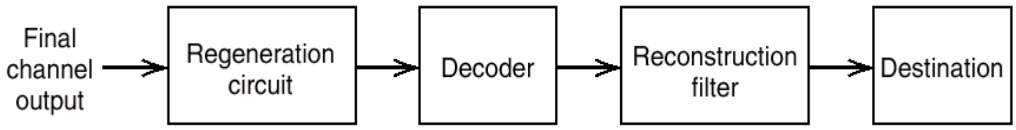
<u>PCM Transmitter</u> :



(*a*) Transmitter



(*b*) Transmission path



(*c*) Receiver

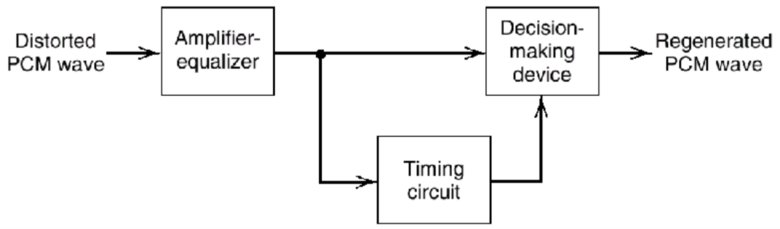
- The continues time signal m(t) is passed through the low pass filter whose cut off frequency is equals to max frequency of message signal ($f_c = f_m$). So the message signal is band limited to the cutoff frequency of low pass filter .
- The output of LPF is given to sampler. It is a process of converting continuous time signal into discrete time signal. In order to get the better reproduction we should sample at Nyquist rate or more than Nyquist rate ($f_s \geq 2f_m$).
- This sampled signal is given to quantizer. The `L' level quantizer compares sampled signal $m(kT_s)$ with the fixed digital levels and assign any one of the **nearest** digital level to $m(kT_s)$ which results some distortion or error. This error is called quantization error. The output of quantizer is denoted as $m_q(kT_s)$.

Quantization error = $m_q(kT_s)$- $m(kT_s)$

- The output of the quantizer is given to encoder. It will generates R bits per sample. This encoder is also known as digitizer. The output of the encoder is given parallel to serial converter.

<u>PCM transmission path (Regenerative repeater):</u>

The PCM transmission contains regenerative repeaters. It will generate clean PCM signals from the distorted PCM signals. The Block Diagram of Regenerative repeater is shown in fig.



- The amplitude equalizer shapes the distorted PCM wave so as to compensate the effects of amplitude and phase distortion produced by the non-ideal nature of the channel.
- The timing circuit provides a periodic pulse train , derived from received pulses , used for sampling the equalized pulses at the instant time where the signal to noise ratio is maximum.
- The decision making device having a predefined threshold value. The decision making device makes a decision based on whether the received signal exceeds the threshold or not. If it is exceeded it indicates as binary `1' else binary `0'.

In practice , the regenerated signal deviates from the actual signal for the following two reasons
1. The unavailable presence of noise and interference causes the regenerative repeater make some occasional decision errors.
2. If the spacing between received pulses deviates from its assigned value a jitter is introduced into the regenerated pulse position it causes distortion

PCM Receiver:

- First the distorted PCM signals is given to regenerative repeater, it gives clean PCM signal.
- The clean PCM signals are given to serial to parallel converter. These binary words are given to the DAC blocks , it decodes and converts to discrete time signal.
- This output is given to S&H circuit then it gives the continuous signal.
- It passes through the low pass filter to get the appropriate message signal denoted as m(t).

Transmission Bandwidth in PCM system:

Let assume that quantizer having L number of quantization levels. PCM system uses R bits per sample. Then the signaling rate (number of bits per second) expressed as

Signaling rate **r** (bits/sec) = R (bits/sample) . f_s (samples/sec)

$$r = R.f_s \text{ --------------------- } 1$$

In case of PCM transmission the required transmission bandwidth must be greater than or equal to half of the signaling rate.

$$BW \geq \frac{1}{2}(\text{ signaling rate})$$
$$\geq \frac{1}{2} R.f_s$$
$$\geq \frac{1}{2} R.2f_m$$
$$BW \geq R\,f_m$$

Advantages of PCM:

- Robustness to channel noise and interference.
- Large amount of noise is tolerated by using regenerative repeaters.
- It improves the signal to noise ratio performance by using Companding technique.

- Secure communication takes place by using encryption and decryption.
- We can store the PCM signal due to digital nature.

Disadvantages of PCM:

- The encoding, decoding and quantizing circuitry of PCM is complex.
- PCM requires large transmission bandwidth.

Applications of PCM:

- With the advances in fiber optics, PCM is used in telephone system.
- It is used in space communication.

Encoding:

By using sampling and quantization the continuous signal is converted into discrete signal but this is not suitable to transmit over the channel. To use the advantage of sampling and quantization for the purpose of making the transmitted signal more robust to noise and interference we require to use encoding process.

Encoding is used to translate the discrete set of samples to a particular arrangement of discrete events called as code. One of the discrete event is called code element or symbol. Suppose a binary code, it is having two discrete symbols they are binary 1 and binary 0.

For example below table shows the representation levels and level number expressed as sum of powers of 2 for 4 bits/sample

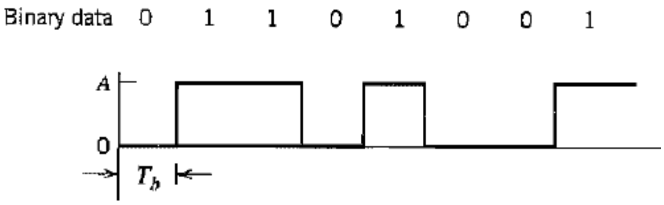
Ordinal Number of Representation Level	Level Number Expressed as Sum of Powers of 2	Binary Number
0		0000
1	2^0	0001
2	2^1	0010
3	$2^1 + 2^0$	0011
4	2^2	0100
5	$2^2 + 2^0$	0101
6	$2^2 + 2^1$	0110
7	$2^2 + 2^1 + 2^0$	0111
8	2^3	1000
9	$2^3 + 2^0$	1001
10	$2^3 + 2^1$	1010
11	$2^3 + 2^1 + 2^0$	1011
12	$2^3 + 2^2$	1100
13	$2^3 + 2^2 + 2^0$	1101
14	$2^3 + 2^2 + 2^1$	1110
15	$2^3 + 2^2 + 2^1 + 2^0$	1111

Line codes: These are used for the electrical representation of binary data stream. There are different types of line codes.

Unipolar Non Return to Zero(NRZ) signaling

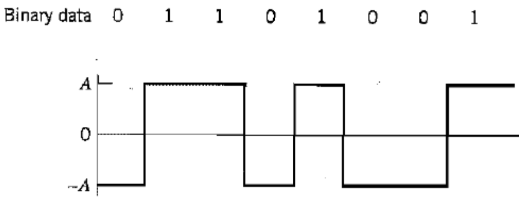
Here the Symbol 1 is transmitted by a pulse of A amplitude for complete symbol duration and Symbol 0 is transmitted by no pulse. It is also called as on-off signaling.

For ex:



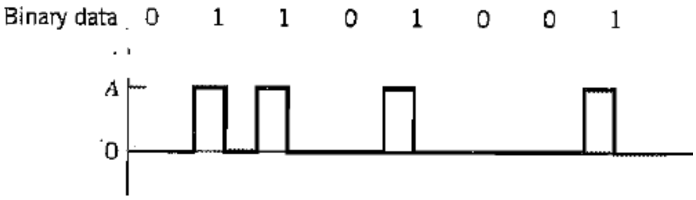
Polar Non Return to Zero(NRZ) signaling:

Symbol 1 & 0 are transmitted by pulse of +A and –A amplitude for complete symbol duration respectively.For example :



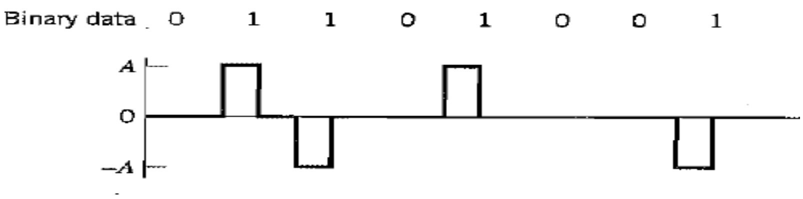
Unipolar Return to Zero(RZ) signaling

Here the Symbol 1 is transmitted by a pulse of A amplitude for half symbol duration and Symbol 0 is transmitted by no pulse. For example,



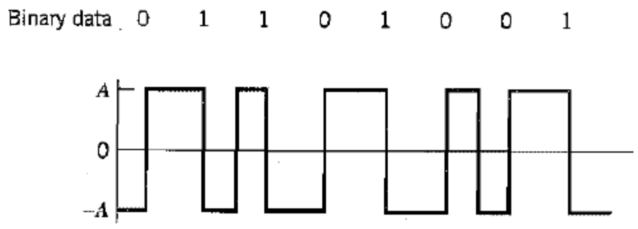
The disadvantage of this line code is it requires 3dB more power than polar return to zero signaling for the same probability of error.

Bipolar RZ signaling: Binary 1 is transmitted by +A and –A amplitude pulse alternatively for half symbol duration . Binary 0 is transmitted by no pulse. For example,



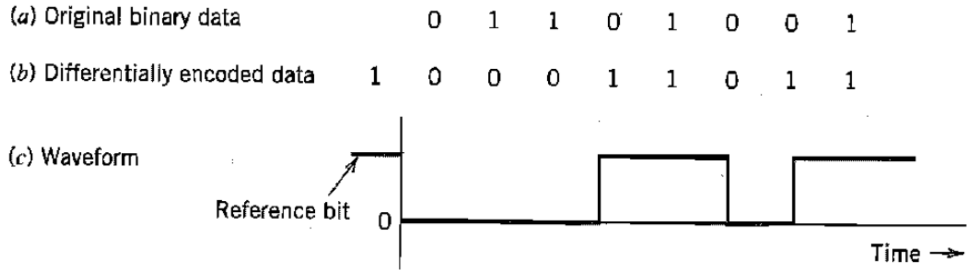
- It is also called alternative mark inversion(AMI).

Split-phase signaling (Manchester code signaling): Binary 1 is transmitted by +A pulse followed by –A pulse , both pulses having half symbol duration. For binary 0 the polarities are reversed. For example,



Differential Encoding:

This encoding process is completely based on signal transition. Here the transition is encoded by binary 0 and no transition is encoded by binary 1. Before initiating the encoding process we should consider one reference bit. It is either '1' or '0'.For example



The original binary information is recovered simply by comparing the polarity of adjacent binary symbols to establish whether transition occur or not.

Decoding and Filtering:

The first operation of the receiver is to generate the received pulses. These clean pulses are then regrouped into codewords and decoded into a quantized PAM signal. The decoding process involves generating a pulse whose amplitude is the linear sum of all bits in the codeword with each bit being weighted by its place value.

The final operation in the receiver is to recover the message signal by passing decoder output through a low pass reconstruction filter whose cut off frequency is equals to the message bandwidth f_m. Assuming that the channel is error free and the received signal includes no noise except initial distortion introduced by quantizer.

Noise considerations in PCM system: The performance of PCM system is influenced by two major sources of noise.

1. Channel noise
2. Quantization noise

1. Channel noise is introduced anywhere in between the transmitter output and receiver input. Channel noise is always present once the equipment is switched ON.

- The main effect of channel noise is to introduce bit errors into the received signal. It means instead binary '1' there is a binary '0' (or) vice versa in the received signal. It can be measured by using average probability of symbol error or bit error rate. To optimize this effect we need to minimize average probability of symbol error. The effect of channel noise is practically negligible by using sufficient signal energy to noise density ratio by providing short enough spacing between regenerative repeaters.

2. Quantization noise is introduced in the transmitter and it is carried all the way along to the receiver output. It is signal dependent it means when the signal is switched off it will be disappears.

- The quantization effect completely under designer control. This effect can be minimized by using Companding technique.

Time division multiplexing:

According to sampling theorem message signal m(t) converted into sampled signal and sampled at a rate of more than or equal to Nyquist rate. Form this discussion sample present for the shorter duration and the gap between two successive samples doesn't carry any information. For example if we take PAM,PWM and PPM the pulse present for shorter duration of time and most of the time between two pulses no signal is present. This free space between two pulses is occupied by the another signal from another source. This is know as TDM. It improves the utilization of channel. The TDM block diagram is shown in below fig,

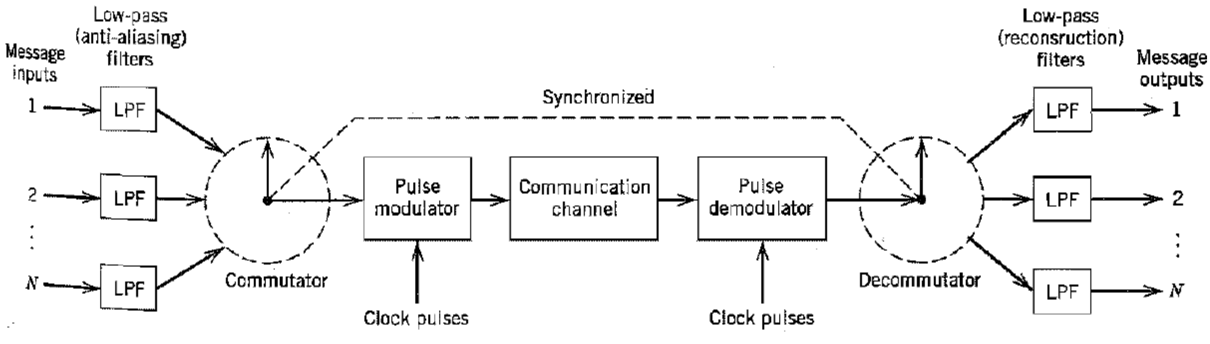


FIGURE : Block diagram of TDM system.

First the message signal are restricted to cutoff frequency of the low pass anti aliasing filters. The output of these filters are applied to commutator.

Commutator selects narrow samples of each of N input messages at a rate of $f_s \geq 2f_m$. It sequentially interleave N samples inside the sampling interval T_s. This multiplexed signal is given to pulse modulator.

Pulse modulator transform these multiplexed signals into the another form which is suitable to transmit over the communication channel. It increases the BW by a factor N.

In the receiver end, the received signal is applied to the pulse demodulator which performs the reverse operation of modulator. The output of Pulse demodulator given to decommutator. It distributes to appropriate destinations through reconstruction filter.

Decommutator must be time synchronized with the commutator for satisfactory operation of the system.

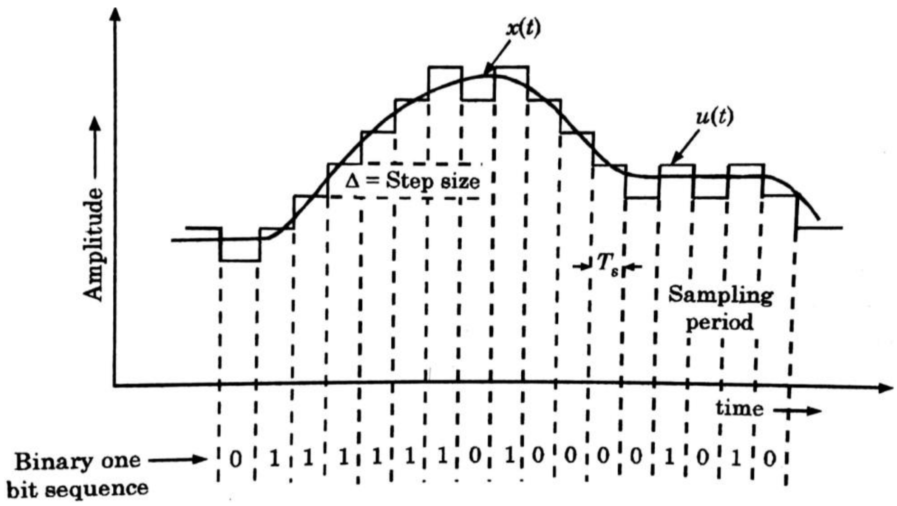
Delta Modulation:

It transmits only one bit per sample. That is the present sample value is compared with the previous sample value and checks whether amplitude increases or decreases.

Here the input signal m(t) is approximated by delta modulator. Within the delta modulator we have staircase approximated signal. The step size is fixed. The difference between m(t) and staircase approximated signal represented by two levels +Δ and –Δ.

i) If the Difference is positive , the approximated signal is increased by one step size ,+ Δ that is binary '1' is Transmitted..
ii) If the Difference is negative , the approximated signal is decreased by one step size -Δ i.e binary '0' is Transmitted.

The below fig shows approximation of signal m(t) by delta modulator.



It is mathematically expressed as,

The error between sample value of m(t) and last approximated sample is given as

$$e(nT_s) = m(nT_s) - \hat{m}(nT_s)$$

where $m(nT_s)$ is the sampled signal of m(t)

$\hat{m}(nT_s)$ is the last sample approximation of staircase waveform

let $b(nT_s)$ is defined as, $b(nT_s) = \Delta \operatorname{sgn}(e(nT_s))$.

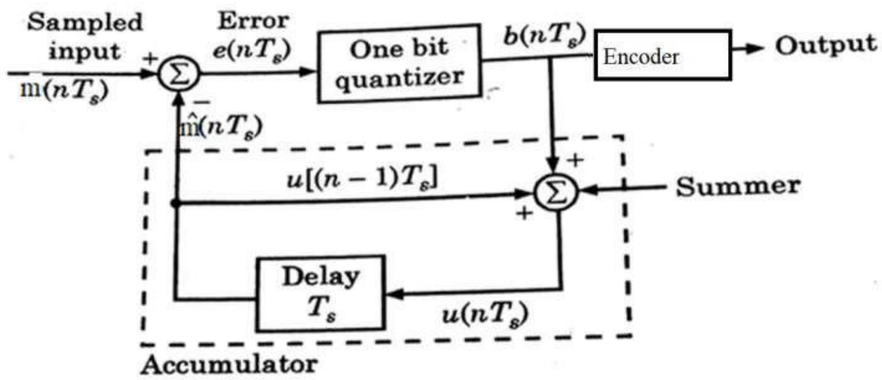
$b(nT_s) = +\Delta$ if $m(nT_s) > \hat{m}(nT_s)$; binary 1 is transmitted

$b(nT_s) = -\Delta$ if $m(nT_s) < \hat{m}(nTs))$; binary 0 is transmitted

let $u(nT_s)$ represent the present sample approximation of staircase output.

$u((n-1)T_s)$ represent the last sample approximation of staircase output.

<u>DM Transmitter:</u>



The summer of accumulator adds quantizer output with the previous sample approximation. It gives the present sample approximation

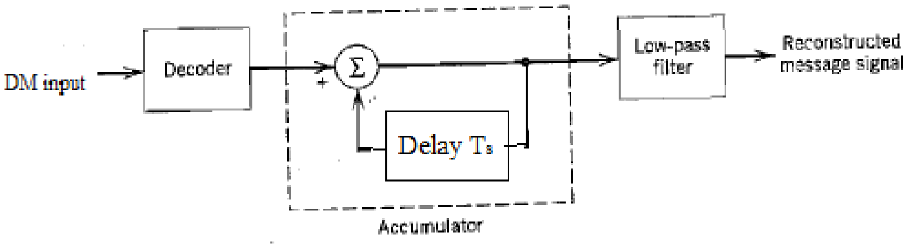
$$u(nT_s) = u((n\text{-}1)T_s) \pm b((nT_s)$$
$$u(nT_s) = u((n\text{-}1)T_s) \pm \Delta$$

The input signal and stair case approximate signal are subtracted to get the error signal. Depending on sign of $e(nT_s)$,one bit quantizer produce + Δ or – Δ.

If it is + Δ , Encoder generates '1' so binary 1 is transmitted

If it is – Δ, Encoder generates '0' so binary 0 is transmitted.

DM Receiver:



It contains accumulator and reconstruction filter. Accumulator generates approximated staircase signal delayed by Ts.

If input is binary 1, it adds +Δ to previous output. If it is binary 0 it subtracts Δ from the delayed signal. The filter smoothers the staircase signal to reconstruct m(t).

In DM the step size is fixed so this is also known as linear delta Modulation.

Advantages:

- It transmits only one bit per sample so it requires less transmission bandwidth.
- Transmitter and receiver implementation is simple.

Disadvantages of DM:

There are two drawbacks

1. Slope over load distortion
2. Granular noise or idle noise

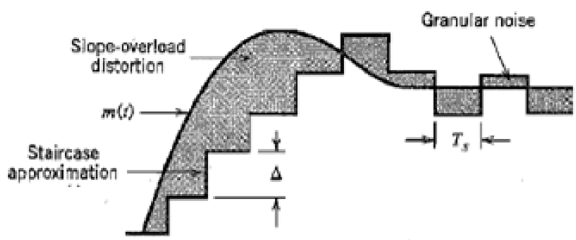


Illustration of the two different forms of quantization error in delta modulation.

1) Slope over load distortion – The rate of raise of input signal is very high, then the stair case signal can't approximate it. It means the difference between staircase approximated signal and original input signal is very large. This error is called as slope overload distortion. It is reduced by increasing the step size when the slope of the signal m(t) is high.
2) Granular noise –The variations in input signal is small , then the staircase approximate signal keeps on oscillating between +Δ and –Δ. This error is called granular noise. It is reduced by decreasing the step size when the variations in the input signal are small.

Condition for occurrence of slope over load distortion

Consider a sine wave with frequency f_m and amplitude A_m, it is applied to delta modulator of step size Δ

$$m(t) = A_m \text{Sin}(2\pi f_m t) \text{-------}1$$

Slope over load distortion occurs at slope of input signal is more than the slope of the delta modulator signal

Maximum slope of delta modulator is = $\frac{step\ size}{samplind\ period} = \frac{\Delta}{T_s}$

$$\text{i.e , max} \left| \frac{d}{dt}(m(t)) \right| > \frac{\Delta}{T_s}$$

$$\text{max} \left| \frac{d}{dt}(\text{Am Sin}(2\pi f_m t)) \right| > \frac{\Delta}{T_s}$$

$$\text{Am } 2\pi\, f_m \text{ max}|\text{Cos}(2\pi\, f_m t)| > \frac{\Delta}{T_s}$$

$$\text{Am } 2\pi\, f_m > \frac{\Delta}{T_s}$$

$$A_m > \frac{\Delta}{2hf_m T_s}$$

This is the condition for the occurrence of slope overload distortion in delta modulation.

Differential Pulse Code Modulation:

The successive samples of the signal are highly correlated with each other. This is because any signal does not change fast. It means the value of present sample to next sample does not differ by a large amount. Whenever these samples are encoded by PCM, the resultant encoded data contains redundant information. It is shown in below fig,

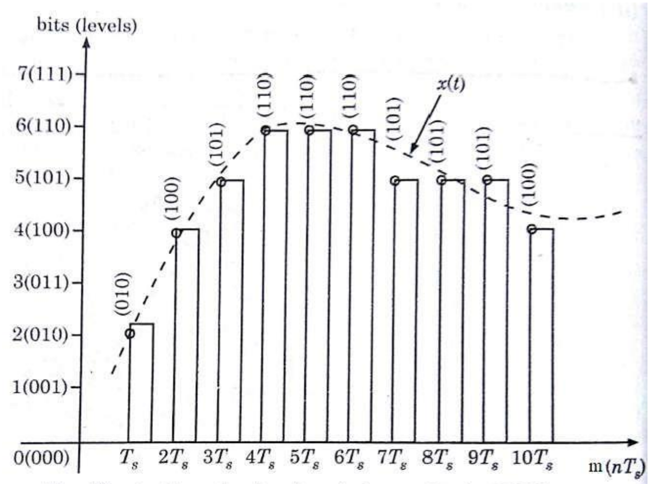
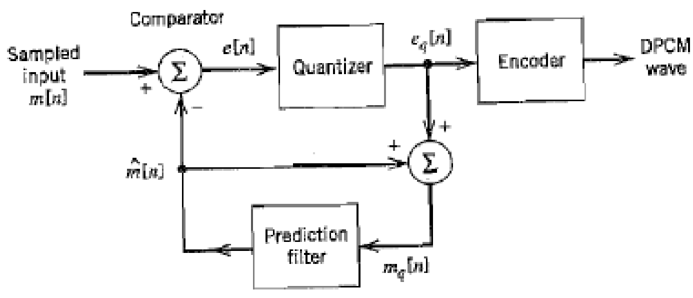


Fig. *Illustration of redundant information in PCM.*

In the above fig samples at $4T_s$, $5T_s$&$6T_s$ carries the same information and these are encoded by 101, these three carries the same information. It is referred as redundancy. This redundancy can be reduced by using DPCM.

If this redundancy is reduced, then the overall bit rate will decreases and number of bits required to transmit one sample will also be reduced. This type of digital pulse modulation scheme is called differential pulse code modulation.

DPCM Transmitter:



DPCM works on prediction principle. The value of present sample is predicted from the past samples. The prediction may or may not be exact but it is very close to the actual sample value.

The error signal e(kT_s) is obtained by taking the difference between the input sampled signal and the predicted signal.

$$e(kT_s) = m(kT_s) - \hat{m}(kT_s) \text{ ----------1}$$

The predicted value is produced using a prediction filter. The quantizer output signal $e_q[kT_s]$ and previous prediction is added and given to the input of the prediction filter. It is denoted by expressed as $m_q[kT_s]$. This makes prediction is more closer to actual sampled signal. The $e_q[kT_s]$ is very small and encoded by using small number of bits.

The quantizer output is expressed as,

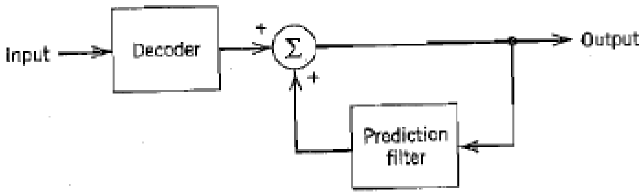
$$e_q[kT_s]=e[kT_s]+q[kT_s]$$

From fig,

$$m_q[kT_s]=\hat{m}(kT_s)+e_q[kT_s]$$
$$=\hat{m}(kT_s)+e[kT_s]+q[kT_s]$$
$$=\hat{m}(kT_s)+e[kT_s]+q[kT_s]$$
$$=\hat{m}(kT_s)+m(kT_s)-\hat{m}(kT_s)+q[kT_s]$$
$$=m[kT_s]+q[kT_s]\text{--------}2$$

The quantized version of the signal is the sum of original sample value and quantization error. The quantization error is positive or negative. Equation 2 doesn't depends on prediction filter.

DPCM Receiver :



The decoder first reconstructs the quantized error signal from incoming binary signal. The prediction filter output and quantized error signal are summed up to give the quantized version of the original signal. Thus the signal at the receiver differs from the actual signal by quantization error q(nTs), which is introduced permanently in the reconstructed signal.