Unit - III SITAMS(R20)

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Structures For Realization of FIR Systems :-

In general a FIR System is described by the diff. equ.,

$$y(n) = \sum_{k=0}^{N-1} b_k x(n-k)$$

On taking Z- transform

$$Y(Z) = \sum_{k=0}^{N-1} b_k Z^{-k} X(Z)$$

$$\therefore \frac{Y(Z)}{X(Z)} = H(Z) = \sum_{k=0}^{N-1} b_k Z^{-k} = b_0 + b_1 Z^{-1} + b_2 Z^{-2} + \dots + b_{N-1} Z^{-(N-1)} \quad \text{--- (1)}$$

where, $H(Z)$ is the transfer fun. of FIR System

Also we know that,

$H(Z) = Z\{h(n)\}$, where $h(n)$ - impulse response of FIR sys.

Let us replace the index ~~n~~ n by k.

$$\therefore H(Z) = Z\{h(k)\} = \sum_{k=0}^{N-1} h(k) Z^{-k} = h(0) + h(1)Z^{-1} + h(2)Z^{-2} + \dots + h(N-1)Z^{-(N-1)} \quad \text{--- (2)}$$

Comparing equ (1) & (2),

$$b_k = h(k), \text{ for } k = 0, 1, 2, \dots (N-1)$$

* different types of structures,

(1) Direct form realization.

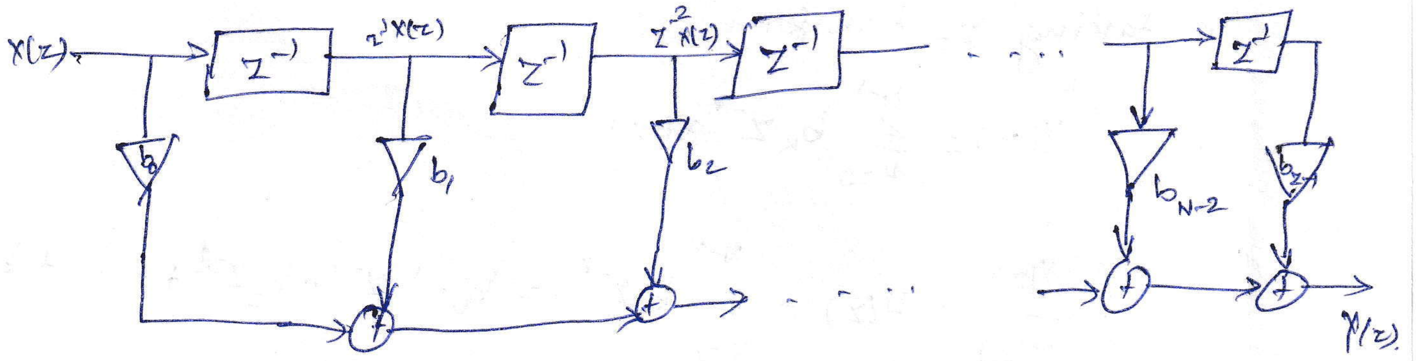
(2) Cascade Realization.

(3) Linear Phase Realization.

(1) Direct Form Realization:-

$$\therefore Y(z) = \sum_{k=0}^{N-1} b_k z^{-k} X(z)$$

$$= b_0 X(z) + b_1 z^{-1} X(z) + b_2 z^{-2} X(z) + \dots\dots + b_{N-1} z^{-(N-1)} X(z).$$



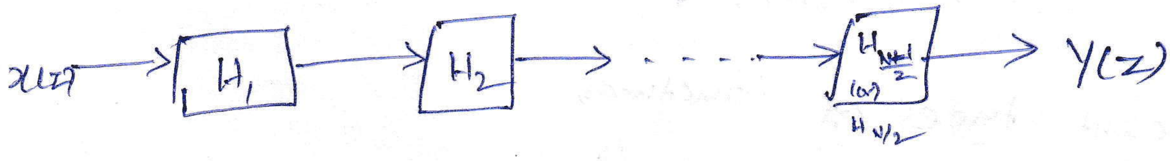
(2) Cascade Realization of FIR System:-

when N is odd, $H(z) = \sum_{k=0}^{N-1} b_k z^{-k} = \prod_{i=1}^{(N-1)/2} (C_{0i} + C_{1i} z^{-1} + C_{2i} z^{-2}) = H_1 \cdot H_2 \cdots H_{\frac{N-1}{2}}$

when, N is odd, H(z) is writting into second order form.

When N is even, $H(z) = \sum_{k=0}^{N-1} b_k z^{-k} = (C_{01} + C_{11} z^{-1}) \prod_{i=2}^{N/2} (C_{0i} + C_{1i} z^{-1} + C_{2i} z^{-2})$

$= H_1 \, H_2 \dots H_{N/2}$



When N is even, H(z) — 1^{st} writting to 1^{st} order form.
— 2^{nd} onward writting into 2^{nd} order form.

(2)

Liner phase Realizations:-

In FIR systems for linear phase response the impulse response should be symmetrical,

i; $h(k) = h(N-1-k)$ ——— (1)

* In symmetrical, the no. of multipliers are reduced in realization tech.

* when N is even,

$$H(z) = \sum_{k=0}^{N-1} h(k) z^{-k} = \sum_{k=0}^{\frac{N}{2}-1} h(k) z^{-k} + \sum_{k=\frac{N}{2}}^{N-1} h(k) z^{-k} \quad \text{——— (2)}$$

Let, $m = N-1-k$, $\therefore k = N-1-m$ ——— (3)

when $k = \frac{N}{2}$, $m = N-1-\frac{N}{2} = \frac{N}{2} - 1$

when $k = N-1$, $m = N-1-(N-1) = 0$ ——— (4)

using equ (3) to (5), ~~in~~ in equ. (2) can be written as,

$$H(z) = \sum_{k=0}^{\frac{N}{2}-1} h(k) z^{-k} + \sum_{m=0}^{\frac{N}{2}-1} h(N-1-m) z^{-(N-1-m)}$$

on replacing m by k in equ,

$$H(z) = \sum_{k=0}^{\frac{N}{2}-1} h(k) z^{-k} + \sum_{k=0}^{\frac{N}{2}-1} h(N-1-k) z^{-(N-1-k)}$$

$$= \sum_{k=0}^{N/2-1} h(k) z^{-k} + \sum_{k=0}^{\frac{N}{2}-1} h(k) z^{-(N-1-k)} \quad \left[\because h(k) = h(N-1-k)\right.$$

$$= \sum_{k=0}^{N/2-1} h(k) \left[z^{-k} + z^{-(N-1-k)} \right]$$

Let,

$$H(z) = \frac{Y(z)}{X(z)} = \sum_{k=0}^{\frac{N}{2}-1} h(k)\left[z^{-k} + z^{-(N-1-k)}\right]$$

$$\therefore Y(z) = \sum_{k=0}^{\frac{N}{2}-1} h(k)\left[z^{-k} + z^{-(N-1-k)}\right]X(z)$$

$$= \sum_{k=0}^{N/2-1} h(k)\left[z^{-k}X(z) + z^{-(N-1-k)}X(z)\right]$$

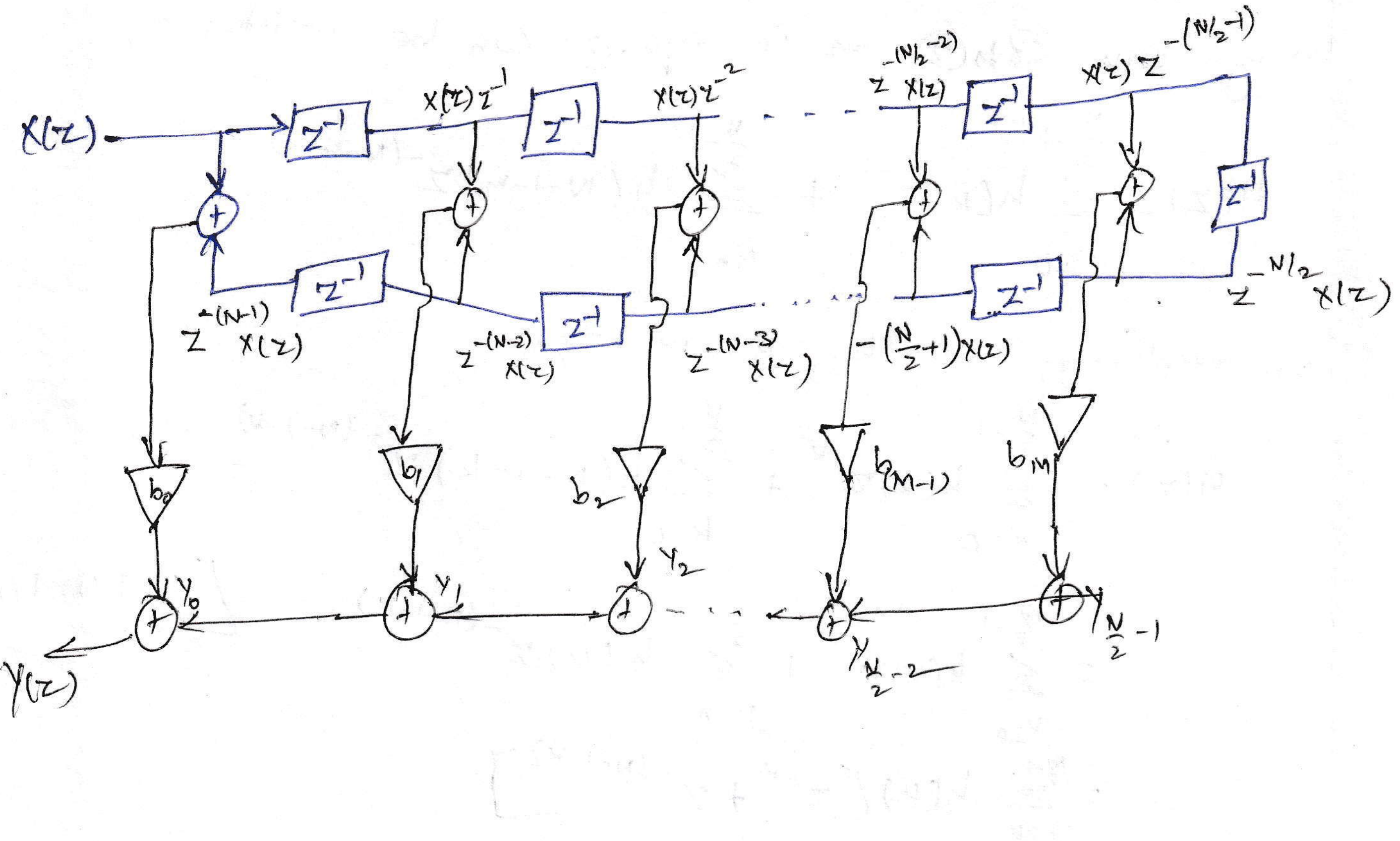
$$= h(0)\left[X(z) + z^{-(N-1)}X(z)\right] + h(1)\left[z^{-1}X(z) + z^{-(N-2)}X(z)\right] + \cdots$$

$$\cdots + h\left(\tfrac{N}{2}-2\right)\left[z^{-\left(\frac{N}{2}-2\right)}X(z) + z^{-\left(\frac{N}{2}+1\right)}X(z)\right] + h\left(\tfrac{N}{2}-1\right)\left[z^{-\left(\frac{N}{2}-1\right)}X(z) + z^{-\frac{N}{2}}X(z)\right]$$

we know that, $h(k) = b_k$,

$$\therefore Y(z) = b_0\left[X(z) + z^{-(N-1)}X(z)\right] + b_1\left[z^{-1}X(z) + z^{-(N-2)}X(z)\right] + \cdots + b_{\frac{N}{2}-2}\left[z^{-\left(\frac{N}{2}-2\right)}X(z)\right.$$

$$\left. + z^{-\left(\frac{N}{2}+1\right)}X(z)\right] + b_{\frac{N}{2}-1}\left[z^{-\left(\frac{N}{2}-1\right)}X(z) + z^{-N/2}X(z)\right],$$



③ When,
N - is odd,

$$H(Z) = \sum_{k=0}^{N-1} h(k) Z^{-k} = \sum_{k=0}^{\frac{N-3}{2}} h(k) Z^{-k} + h\left(\frac{N-1}{2}\right) Z^{-\left(\frac{N-1}{2}\right)} + \sum_{k=\frac{N+1}{2}}^{N-1} h(k) Z^{-k} \quad \text{--- } ①$$

Let, $m = N-1-k$, $\therefore$ $k = N-1-m$ ——— ②

when, $k = \frac{N+1}{2}$; $m = N-1-\frac{N+1}{2} = \frac{N-3}{2}$ ——— ③

when, $k = N-1$; $m = N-1-(N-1) = 0$ ——— ④

Sub ②&④, ~~in~~ the eqn. ① can be written as,

$$H(z) = \sum_{k=0}^{\frac{N-3}{2}} h(k) Z^{-k} + h\left(\frac{N-1}{2}\right) Z^{-\left(\frac{N-1}{2}\right)} + \sum_{m=0}^{\frac{N-3}{2}} h(N-1-m) Z^{-(N-1-m)} \quad \text{--- } ⑤$$

On replacing the index m by k in equ ⑤, we get.

$$H(Z) = \sum_{k=0}^{\frac{N-3}{2}} h(k) Z^{-k} + h\left(\frac{N-1}{2}\right) Z^{-\left(\frac{N-1}{2}\right)} + \sum_{k=0}^{\frac{N-3}{2}} h(N-1-k) Z^{-(N-1-k)}$$

$$= \sum_{k=0}^{\frac{N-3}{2}} h(k) Z^{-k} + h\left(\frac{N-1}{2}\right) Z^{-\left(\frac{N-1}{2}\right)} + \sum_{k=0}^{\frac{N-3}{2}} h(k) Z^{-(N-1-k)}$$ [Symmetric $h(k) = h(N-1-k)$]

$$= h\left(\frac{N-1}{2}\right) Z^{-\left(\frac{N-1}{2}\right)} + \sum_{k=0}^{\frac{N-3}{2}} h(k) \left[Z^{-k} + Z^{-(N-1-k)}\right]$$

Let,
$$H(Z) = \frac{Y(Z)}{X(Z)} = h\left(\frac{N-1}{2}\right) Z^{-\left(\frac{N-1}{2}\right)} + \sum_{k=0}^{\frac{N-3}{2}} h(k) \left[Z^{-k} + Z^{-(N-1-k)}\right]$$

$$Y(z) = h\left(\frac{N-1}{2}\right) z^{-\left(\frac{N-1}{2}\right)} X(z) + \sum_{k=0}^{\frac{N-3}{2}} h(k)\left[z^{-k} X(z) + z^{-(N-1-k)} X(z)\right]$$

$$= h\left(\frac{N-1}{2}\right) z^{-\left(\frac{N-1}{2}\right)} X(z) + h(0)\left[X(z) + z^{-(N-1)} X(z)\right] + h(1)\left[z^{-1} X(z) + z^{-(N-2)} X(z)\right]$$

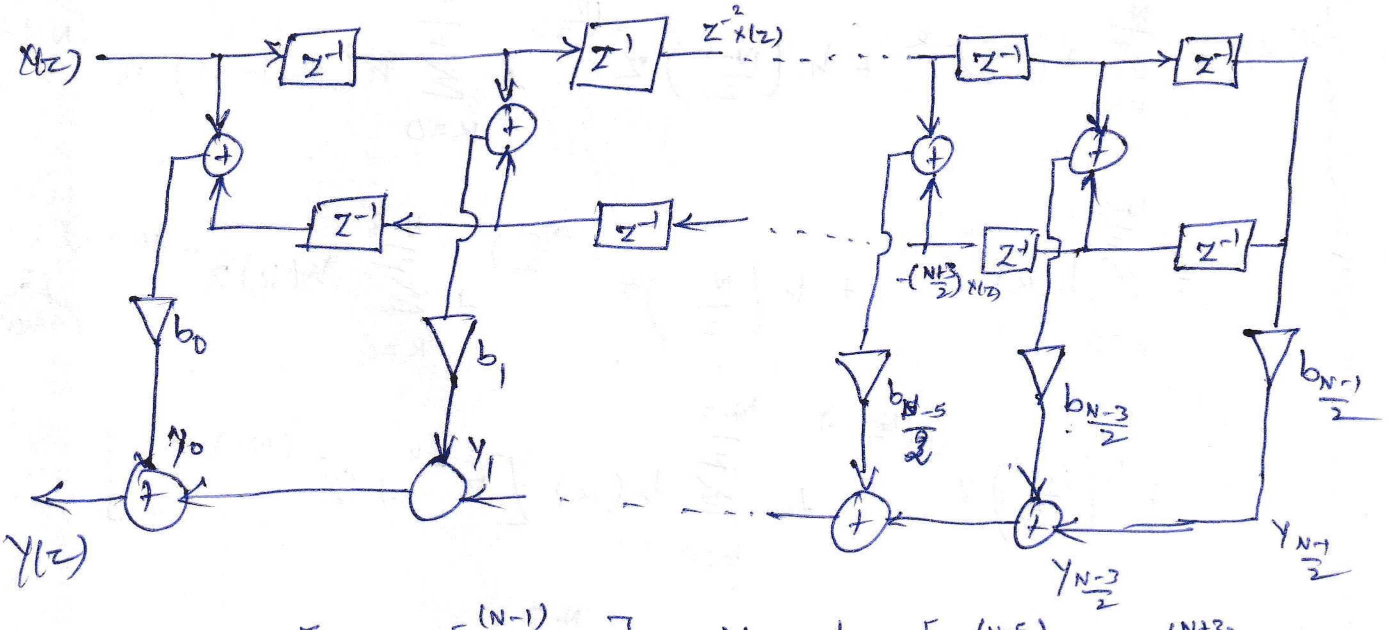
$$+ \ldots\ldots + h\left(\frac{N-5}{2}\right)\left[z^{-\left(\frac{N-5}{2}\right)} X(z) + z^{-\left(\frac{N+3}{2}\right)} X(z)\right] \;\&$$

$$h\left(\frac{N-3}{2}\right)\left[z^{-\left(\frac{N-3}{2}\right)} X(z) + z^{-\left(\frac{N+1}{2}\right)} X(z)\right]$$

we know that $h(k) = b_k$

$$\therefore Y(z) = b_{\frac{N-1}{2}} z^{-\left(\frac{N-1}{2}\right)} X(z) + b_0\left[X(z) + z^{-(N-1)} X(z)\right] + b_1\left[z^{-1} X(z) + z^{-(N-2)} X(z)\right] +$$

$$\ldots + b_{\frac{N-5}{2}}\left[z^{-\left(\frac{N-5}{2}\right)} X(z) + z^{-\left(\frac{N+3}{2}\right)} X(z)\right] + b_{\frac{N-3}{2}}\left[z^{-\left(\frac{N-3}{2}\right)} X(z) + z^{-\left(\frac{N+1}{2}\right)} X(z)\right]$$



$$y_0 = b_0\left[X(z) + z^{-(N-1)} X(z)\right]; \quad Y_{\frac{N-5}{2}} = b_{\frac{N-5}{2}}\left[z^{-\left(\frac{N-5}{2}\right)} X(z) + z^{-\left(\frac{N+3}{2}\right)} X(z)\right]; \quad Y_{\frac{N-1}{2}} = b_{\frac{N-1}{2}} z^{-\left(\frac{N-1}{2}\right)} X(z)$$

$$y_1 = b_1\left[z^{-1} X(z) + z^{-(N-2)} X(z)\right]; \quad Y_{\frac{N-3}{2}} = b_{\frac{N-3}{2}}\left[z^{-\left(\frac{N-3}{2}\right)} X(z) + z^{-\left(\frac{N+1}{2}\right)} X(z)\right]$$

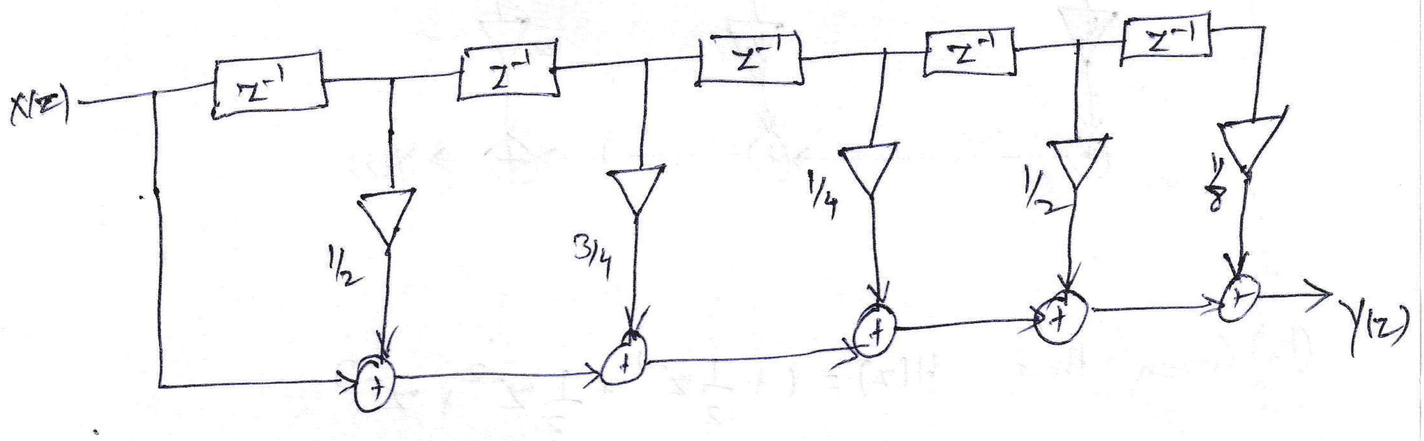
→ Draw the direct form structure of the FIR system described by the transfer function.

$$H(z) = 1 + \frac{1}{2}z^{-1} + \frac{3}{4}z^{-2} + \frac{1}{4}z^{-3} + \frac{1}{2}z^{-4} + \frac{1}{8}z^{-5}$$

Sol:-

$$H(z) = \frac{Y(z)}{X(z)} = 1 + \frac{1}{2}z^{-1} + \frac{3}{4}z^{-2} + \frac{1}{4}z^{-3} + \frac{1}{2}z^{-4} + \frac{1}{8}z^{-5}$$

$$Y(z) = X(z) + \frac{1}{2}z^{-1}X(z) + \frac{3}{4}z^{-2}X(z) + \frac{1}{4}z^{-3}X(z) + \frac{1}{2}z^{-4}X(z) + \frac{1}{8}z^{-5}X(z)$$



→ Realize the following system with minimum no. of Multipliers.

(a) $H(z) = \frac{1}{4} + \frac{1}{2}z^{-1} + \frac{3}{4}z^{-2} + \frac{1}{2}z^{-3} + \frac{1}{4}z^{-4}$

Sol

$$H(z) = \sum_{n=0}^{\infty} h(n)z^{-n} = h(0) + h(1)z^{-1} + h(2)z^{-2} + h(3)z^{-3} + \dots$$

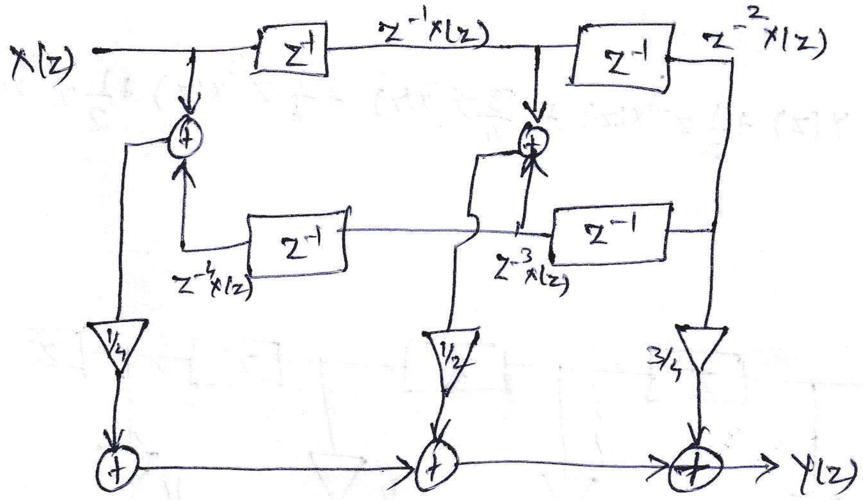
Comparing with given values,

$\therefore$ Impulse response $h(n) = \left\{\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \frac{1}{2}, \frac{1}{4}\right\}$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{4} + \frac{1}{2}z^{-1} + \frac{3}{4}z^{-2} + \frac{1}{2}z^{-3} + \frac{1}{4}z^{-4}$$

$$\therefore \quad Y(z) = \frac{1}{4}X(z) + \frac{1}{2}z^{-1}X(z) + \frac{3}{4}z^{-2}X(z) + \frac{1}{2}z^{-3}X(z) + \frac{1}{4}z^{-4}X(z).$$

$$= \frac{1}{4}\left[X(z) + z^{-4}X(z)\right] + \frac{1}{2}\left[z^{-1}X(z) + x^{-3}X(z)\right] + \frac{3}{4}z^{-2}X(z)$$

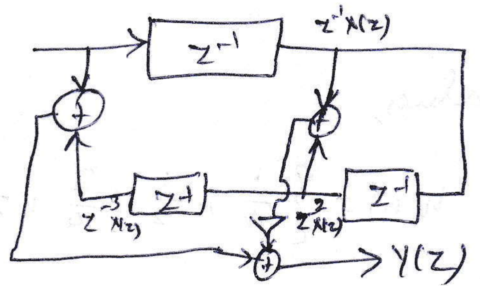


(b) Given that, $H(z) = 1 + \frac{1}{2}z^{-1} + \frac{1}{2}z^{-2} + z^{-3}$

Let $H(z) = \frac{Y(z)}{X(z)} = 1 + \frac{1}{2}z^{-1} + \frac{1}{2}z^{-2} + z^{-3}$

$$Y(z) = X(z) + \frac{1}{2}z^{-1}X(z) + \frac{1}{2}z^{-2}X(z) + z^{-3}X(z)$$

$$Y(z) = \left[X(z) + z^{-3}X(z)\right] + \frac{1}{2}\left[z^{-1}X(z) + z^{-2}X(z)\right]$$



$\rightarrow \quad H(z) = \left(1 + \frac{1}{2}z^{-1} + z^{-2}\right)\left(1 + \frac{1}{4}z^{-1} + z^{-2}\right)$

Sol:

$$H(z) = H_1(z)\, H_2(z)$$

$$H_1(z) = 1 + \frac{1}{2}z^{-1} + z^{-2}$$

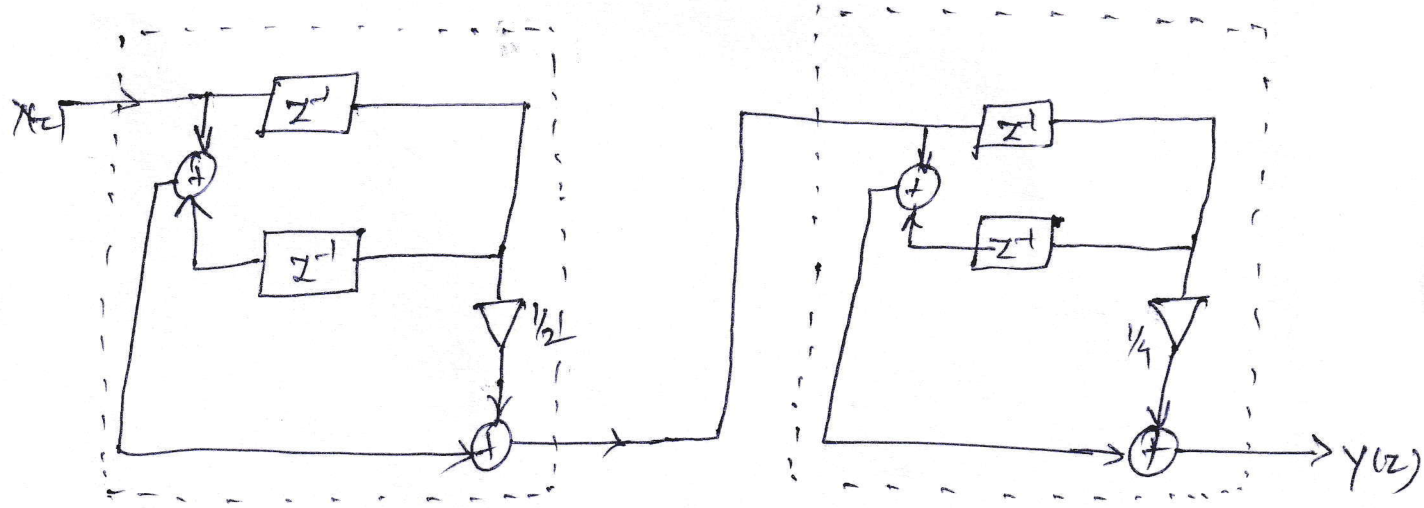
$$H_1(z) = \frac{Y_1(z)}{X(z)} =$$

$$Y_1(z) = X(z) + \frac{1}{2}z^{-1}X(z) + z^{-2}X(z) = \left[X(z) + z^{-2}X(z)\right] + \frac{1}{2}z^{-1}X(z)$$

Similiarly.

$$H_2(z) = \frac{Y_2(z)}{X_2(z)} = 1 + \frac{1}{4}z^{-1} + z^{-2}$$

$$Y_2(z) = X_2(z) + \frac{1}{4}z^{-1}X_2(z) + z^{-2}X_2(z) = \left[X_2(z) + z^{-2}X_2(z)\right] + \frac{1}{4}z^{-1}X_2(z)$$



Design Procedure for Digital FIR Filter by using Fourier Series Method.

Step-1 :
Choose the desired freq. range $H_d(e^{j\omega})$

Step-2 :-
Determine the desired Impulse response $h_d(n)$ by taking Inverse Fourier transform.

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})\, e^{j\omega n}\, d\omega$$

Step-3 :-
Calculate N- Samples of $h_d(n)$ for $n = -\left(\frac{N-1}{2}\right)$ to $\left(\frac{N-1}{2}\right)$ and for the impulse response $h(n)$ of FIR filters.

$\therefore$ Impulse response, $h(n) = h_d(n)\Big|_{n=-\frac{(N-1)}{2} \text{ to } \frac{(N-1)}{2}}$

The Impulse response is symmetric with $n=0$

so, $h(-n) = h(n)$

Step-4
Take Z-transform of the Impulse response to get non-causal transfer fun. of FIR filter $H_N(z)$

$$H_N(z) = Z\{h(n)\} = \sum_{n=-\left(\frac{N-1}{2}\right)}^{\frac{N-1}{2}} h(n) z^{-n}$$

Alternatively,

$$H_N(z) = h(0) + \sum_{n=1}^{\frac{N-1}{2}} h(n)\,(z^{n} + z^{-n})$$

Step-5

Convert the non-causal transfer function, $H_n(z)$ to causal transfer fun. $H(z)$ by Multiplying $H_N(z)$ by $z^{-\frac{N-1}{2}}$

$$H(z) = z^{-\left(\frac{N-1}{2}\right)}\left[h(0) + \sum_{n=1}^{\frac{N-1}{2}} h(n)\,(z^{n} + z^{-n})\right]$$

Step-6

Realize the Linear phase Structure of $H(z)$

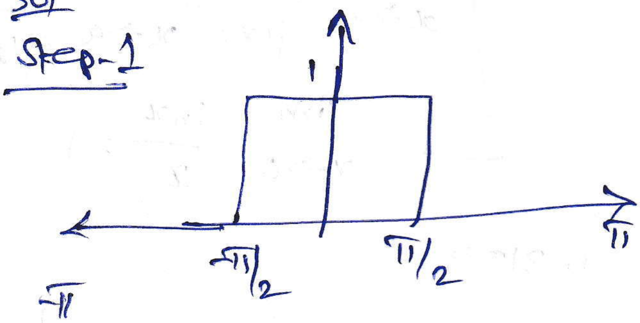
→ Design an Ideal LPF using fourier series Method with a freq. response of

$$H(e^{j\omega}) = \begin{cases} 1 & \text{for } \frac{-\pi}{2} \le \omega \le \frac{\pi}{2} \\ 0 & \text{for } \frac{\pi}{2} \le |\omega| \le \pi \end{cases}$$

find the value of h(n). for N=11. and also find H(Z) ~~for~~ & plot the magnitude response.

Sol

Step-1



Step-2

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H(e^{j\omega})\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi/2}^{\pi/2} 1 \cdot e^{j\omega n}\, d\omega = \frac{1}{2\pi}\left[\frac{e^{j\omega n}}{jn}\right]_{-\pi/2}^{\pi/2}$$

$$= \frac{1}{2\pi j n}\left[e^{j\frac{\pi}{2}n} - e^{-j\frac{\pi}{2}n}\right]$$

$$h_d(n) = \frac{\sin\frac{\pi}{2}n}{\pi n}$$

$$\left[\begin{array}{l} e^{j\theta} = \cos\theta + j\sin\theta \\ e^{-j\theta} = \cos\theta - j\sin\theta \\ e^{j\theta} - e^{-j\theta} = 2j\sin\theta \end{array}\right.$$

Step 3

$$h(n) = h_d(n)\Big|_{n=\frac{-(N-1)}{2} \text{ to } \left(\frac{N-1}{2}\right)}$$

$$h(n) = h_d(n)\Big|_{n=-5 \text{ to } 5}$$

$$h(-n) = h(n)$$

$$h(0) = \frac{1}{2\pi}\int_{-\pi/2}^{+\pi/2} e^{0} \cdot d\omega = \frac{1}{2\pi}[\omega]_{-\pi/2}^{\pi/2} = \frac{1}{2\pi}\left[\frac{\pi}{2}+\frac{\pi}{2}\right] = \frac{\pi}{2\pi} = \frac{1}{2}$$

$$e^{0}=1$$

$$h(0) = \frac{1}{2}\lim_{\theta \to 0} \frac{\sin\frac{\pi}{2}n}{\frac{\pi}{2}n}$$

$$= \frac{1}{2}(1) = \frac{1}{2}$$

L- Hospital rule

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$$

$$\lim_{x\to 0}\frac{\sin x}{x} = 1$$

$$\Downarrow$$

$$h(0) = \frac{1}{2}\lim_{\theta\to 0}\frac{\sin \pi n}{\sin \pi n}$$

$$= \frac{1}{2}(1)$$

$$h(1) = h(-1) = \frac{\sin\frac{\pi}{2}}{\pi} = 0.3183$$

$$h(2) = h(-2) = \frac{\sin\pi}{\pi} = 0$$

$$h(3) = h(-3) = \frac{\sin\frac{3\pi}{2}}{3\pi} = -0.106$$

$$h(4) = h(-4) = \frac{\sin 2\pi}{4\pi} = 0$$

$$h(5) = h(-5) = \frac{\sin\left(\frac{5}{2}\pi\right)}{5\pi} = 0.0636$$

~~Step 5:~~

~~$H(z) = z^{-\left(\frac{N-1}{2}\right)} H_N(z)$~~

Step-4

$$H_N(z) = h(0) + \sum_{n=1}^{5} h(n)\left(z^n + z^{-n}\right)$$

$$= \frac{1}{2} + h(1)\left(z + z^{-1}\right) + h(2)\left(z^2 + z^{-2}\right) + h(3)\left(z^3 + z^{-3}\right)$$

$$+ h(4)\left(z^4 + z^{-4}\right) + h(5)\left(z^5 + z^{-5}\right)$$

$$= 0.5 + 0.3183\left(z + z^{-1}\right) - 0.106\left(z^3 + z^{-3}\right)$$
$$+ 0.0636\left[z^5 + z^{-5}\right]$$

Step-5

$$H(z) = z^{-\left(\frac{N-1}{2}\right)} H_N(z)$$

$$= z^{-5}\left[0.5 + 0.3183\left(z + z^{-1}\right) - 0.106\left(z^3 + z^{-3}\right) + 0.0636\left[z^5 + z^{-5}\right]\right]$$

$$= z^{-5}\left[0.5 + 0.3183z + 0.3183z^{-1} - 0.106z^3 - 0.106z^{-3} + 0.0636z^5\right.$$
$$\left. + 0.0636z^{-5}\right]$$

$$= 0.5z^{-5} + 0.0318z^{-4} + 0.03183z^{-6} - 0.106z^{-2} - 0.106z^{-8}$$
$$+ 0.0636\cancel{z} + 0.0636z^{-10}$$

$$= 0.0636\cancel{z} - 0.106z^{-2} + 0.0318z^{-4} + 0.5z^{-5} + 0.0318z^{-6}$$
$$- 0.106z^{-8} + 0.0636z^{-10}$$

~~$= 0.0636 - 0.106[z$~~

$$= 0.0636\left[1 + z^{-10}\right] - 0.106\left[z^{-2} + z^{-8}\right] + 0.0318\left[z^{-4} + z^{-6}\right] + 0.5z^{-5}$$

→ Design FIR LPF with Cut-off freq of 1 KHz & Sampling freq of 4 KHz with 11-samples using Fourier Series method.

Sol $f_c = 1\ KHz.$ $f_s = 4KHz.$

$N = 11$

$$\omega_c = \frac{2\pi f_c}{f_s} = \frac{2\pi \times 1 \times 10^3}{4 \times 10^3} = 0.5\pi \text{ rad/sec.}$$

Step-1

$$H(e^{j\omega}) = \begin{cases} 0, & -\pi \le \omega \le -\omega_c \\ 1, & -\omega_c \le \omega \le \omega_c \\ 0, & \omega_c \le \omega \le \pi \end{cases}$$

Step-2

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H(e^{j\omega}) e^{j\omega n} d\omega = \frac{1}{2\pi}\left[\int_{-\pi}^{-\omega_c} 0 + \int_{-\omega_c}^{\omega_c} 1.e^{j\omega n} d\omega + \int_{\omega_c}^{\pi} 0.d\omega\right]$$

$$= \frac{1}{2\pi}\left[\frac{e^{j\omega n}}{jn}\right]_{-\omega_c}^{\omega_c} = \frac{1}{2\pi j n}\left[e^{j\omega_c n} - e^{-j\omega_c n}\right]$$

$$h_d(n) = \frac{\sin \omega_c n}{\pi n}$$

Step-3

$\cancel{H_N(z)}$ $h(n) = h_d(n)\Big|_{n = -\left(\frac{N-1}{2}\right) \text{ to } \left(\frac{N+1}{2}\right)}$

$\therefore n = -5 \text{ to } 5$

$$h(0) = \frac{\omega_c}{\pi} \lim_{n\to 0} \frac{\sin n}{n} = \frac{\omega_c}{\pi}(1) = \frac{\omega_c}{\pi}\frac{\sin n}{n} = \frac{\omega_c}{\pi}(1) = \frac{\omega_c}{\pi} = \frac{0.5\pi}{\pi}$$

$= 0.5$

$h(1) = 0.3183$

$$h(2) = \frac{\sin 0.5\pi \times 2}{2\pi} = 0$$

$$h(3) = \frac{\sin 0.5\pi \times 3}{3\pi} = -0.1061$$

$$h(5) = \frac{\sin 0.5\pi \times 5}{5\pi} = 0.0637$$

$$h(4) = \frac{\sin 0.5\pi \times 4}{4\pi} = 0$$

Step-4

$$H_N(z) = h(0) + \sum_{n=1}^{5} h(n)(z^n + z^{-n})$$

$$= h(0) + h(1)(z^1 + z^{-1}) + h(2)(z^2 + z^{-2}) + h(3)(z^3 + z^{-3})$$

$$+ h(4)(z^4 + z^{-4}) + h(5)(z^5 + z^{-5})$$

$$= 0.5 + 0.3183z^1 + 0.3183z^{-1} + 0 - 0.1061z^3 - 0.1061z^{-3}$$

$$+ 0 + 0.0637z^5 + 0.0637z^{-5}$$

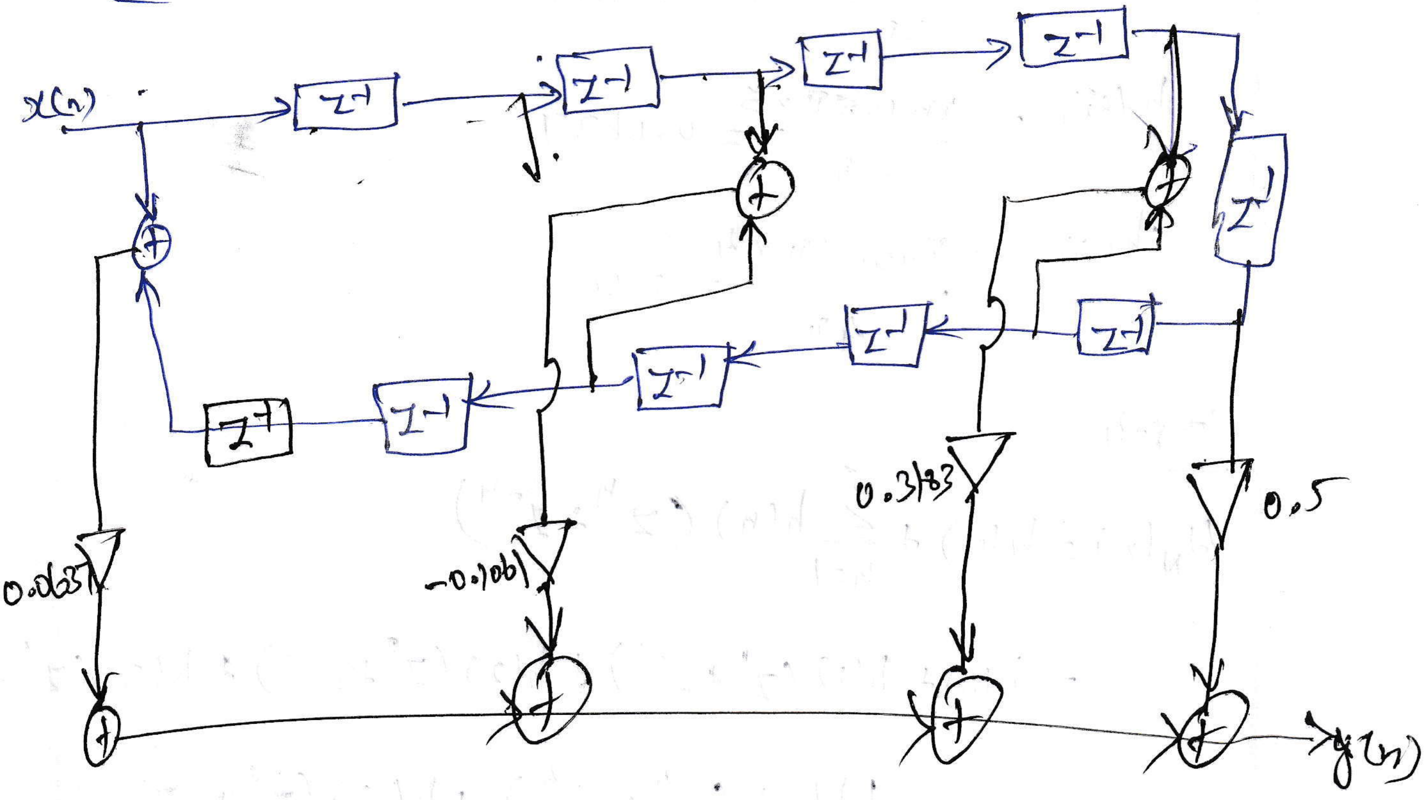
Step-5

$$H(z) = z^{-\left(\frac{N-1}{2}\right)} \times H_N(z)$$

$$= z^{-5}\left[0.5 + 0.3183z + 0.3183z^{-1} - 0.1061z^{3} - 0.1061z^{-3} + 0.0637z^{5} + 0.0637z^{-5}\right]$$

$$= 0.0637 - 0.1061z^{-2} + 0.3183z^{-4} + 0.5z^{-5} + 0.318z^{-6} - 0.1061z^{-8} + 0.0637z^{-10}$$

$$= 0.0637\left[1+z^{-10}\right] - 0.1061\left[z^{-2}+z^{-8}\right] + 0.3183\left[z^{-4}+z^{-6}\right] + 0.5z^{-5}$$



FIR Filters:-

* It is defined Finite Impulse Response.

* Represent in terms of $H_d(\omega)$ ie; given in terms of ideal freq. response.

* When $H_d(\omega)$ is taken Inverse Fourier transform then we will get finite no. of sample of impulse response $h_d(n)$.

Adv:-

(1) The Linear Phase FIR filters can be easily designed

(2) Efficient realization of FIR filters exist. as both recursive and non-recursive structure.

(3) FIR filters realized non-recursively are always stable

(4) The round-off noise can be made small in non-recursive realization of FIR filters.

Dis:-

(1) The duration of impulse response should be large to realize sharp cut-off filters.

(2) The non-Integral delay can lead to problems in some signal processing application.

FIR Filter Design - using Windows Tech:-

* In this method, given desired freq. response specification is $H_d(\omega) \xrightarrow{T}$ get ~~get~~ corresponding unit sample response $h_d(n)$.

* $h_d(n)$ is Inverse Fourier transform of $H_d(\omega)$.
  - They have Infinite Sequence
  - They want to truncate at some point say at $n=N-1$ to yield an FIR filter ~~of~~ length N.

* Truncation is obtained by multiplying $h_d(n)$ by a window sequence $w(n)$. [i.e; $h(n) = h_d(n) * w(n)$].

* ~~Freq.~~ $h(n)$ → write in freq response of filter is denoted by $H(\omega)$.

* Then taking z-transform of $H(z)$.
* Draw the Linear phase magnitude function

* Desirable char. of the freq. response of window function are the following:-

(1) The width of the mainlobe should be small and it should contain as much of the total energy as possible.

(2) The sidelobes should decrease in energy rapidly as ω tend to π.

There have many widows, they are Rectangular, Hanning, Hamming, Blackman and Kaiser windows.

FIR Filter Design using Windows:- (steps for Calculation)

(1) Choose the desired freq. response of the filter $H_d(\omega)$.

(2) Take Inverse Fourier transform of $H_d(\omega)$ to obtain the desired Impulse response $h_d(n)$. By definition of Inverse fourier transform,

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(\omega)\, e^{j\omega n}\, d\omega.$$

(3) Choose a window seq. $w(n)$ and determine the product of $h_d(n)$ and $w(n)$. Let this product be $h(n)$.

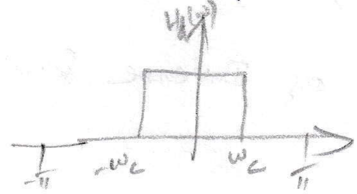
$$\therefore\ h(n) = h_d(n)\, w(n).$$

(4) The ~~transferfun~~ transfer fun. $H(z)$ of the filter is obtained by taking Z-transform of $h(n)$. Realize the filter by a suitable structure.

(5) Choose a linear phase mag. fun., $|H(\omega)|$. using $h(n)$ obtain an eqn. for $|H(\omega)|$. Calculate $|H(\omega)|$ for various values of ω in the range $0 \le \omega \le \pi$ and sketch the graph b/w $|H(\omega)|$ and ω, which is the ~~res~~ freq. response of the filter.

6.23

Type of filter.	Ideal (desired) freq. range	Ideal (desired) Impulse response.
Lowpass filter.	$H_d(\omega) = \begin{cases} e^{-j\omega\alpha} ; & -\omega_c \le \omega \le \omega_c \\ 0 ; & -\pi \le \omega \le -\omega_c \\ 0 ; & \omega_c < \omega \le \pi \end{cases}$	$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(\omega) e^{j\omega n} d\omega$

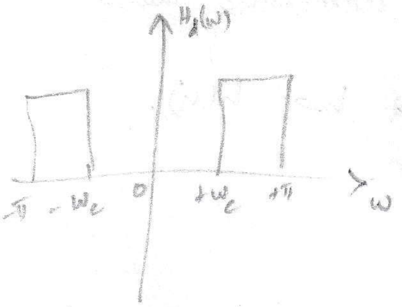


$$= \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} e^{-j\omega\alpha} e^{j\omega n} d\omega$$

$\because H_d(\omega) = 0$ in the range $-\pi \le \omega \le -\omega_c$ & $\omega_c < \omega \le \pi$

Highpass filter

$$H_d(\omega) = \begin{cases} e^{-j\omega\alpha} ; & -\pi \le \omega \le -\omega_c \\ e^{-j\omega\alpha} ; & \omega_c \le \omega \le \pi \\ 0 ; & -\omega_c < \omega < \omega_c \end{cases}$$

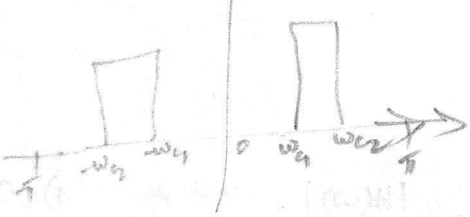


$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(\omega) e^{j\omega n} d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi}^{-\omega_c} e^{-j\omega\alpha} e^{j\omega n} d\omega + \frac{1}{2\pi}\int_{\omega_c}^{\pi} e^{-j\omega\alpha} e^{j\omega n} d\omega$$

Bandpass filter

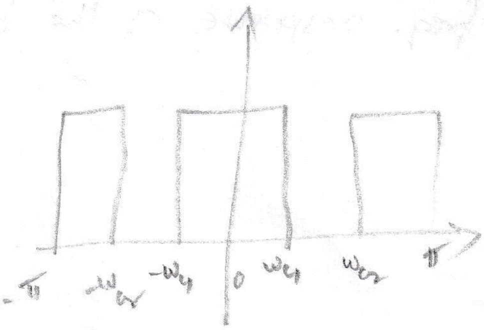
$$H_d(\omega) = \begin{cases} e^{-j\omega\alpha} ; & -\omega_{c_2} \le \omega \le -\omega_{c_1} \\ e^{-j\omega\alpha} ; & \omega_{c_1} \le \omega \le \omega_{c_2} \\ 0 ; & -\pi \le \omega \le -\omega_{c_2} \\ 0 ; & -\omega_{c_1} < \omega < \omega_{c_1} \\ 0 ; & \omega_{c_2} < \omega \le \pi \end{cases}$$



$$h_d(n) = \frac{1}{2\pi}\int_{-\omega_{c_2}}^{-\omega_{c_1}} e^{-j\omega\alpha} e^{j\omega n} d\omega + \frac{1}{2\pi}\int_{\omega_{c_1}}^{\omega_{c_2}} e^{-j\omega\alpha} e^{j\omega n} d\omega$$

Bandstop filter.

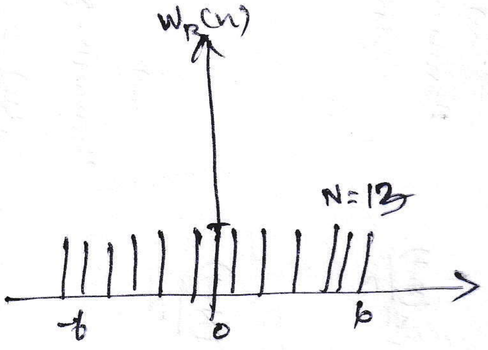
$$H_d(\omega) = \begin{cases} e^{-j\omega\alpha} ; & -\pi \le \omega \le -\omega_{c_2} \\ e^{-j\omega\alpha} ; & -\omega_{c_1} \le \omega \le \omega_{c_1} \\ e^{-j\omega\alpha} ; & \omega_{c_2} \le \omega \le \pi \\ 0 ; & -\omega_{c_2} < \omega < -\omega_{c_1} \\ 0 ; & \omega_{c_1} < \omega < \omega_{c_2} \end{cases}$$



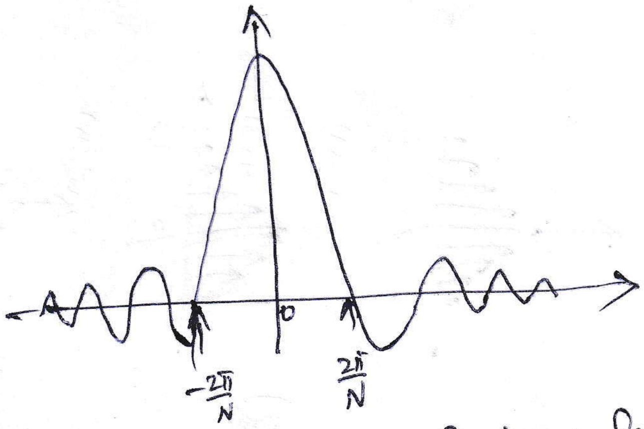
(1) <u>Rectangular window:-</u>

* N-point rectangular window has a weighting function.

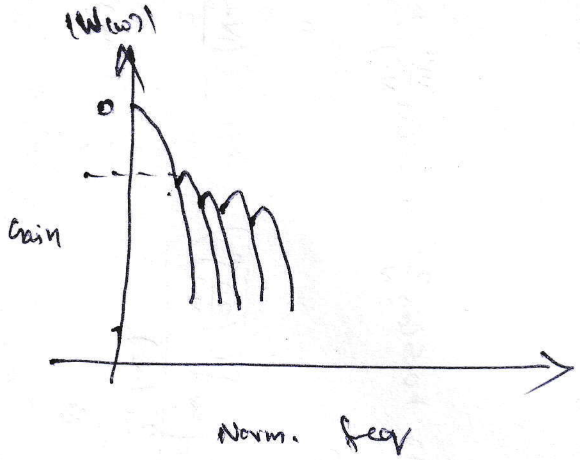
$$w_R(n) = \begin{cases} 1 ; & -\left(\frac{N-1}{2}\right) \le n \le \left(\frac{N-1}{2}\right) \\ 0 ; & \text{Elsewhere.} \end{cases} \quad \text{(or)} = w_R(n) = \begin{cases} 1 ; & 0 \le n \le N-1 \\ 0 ; & \text{otherwise} \end{cases}$$



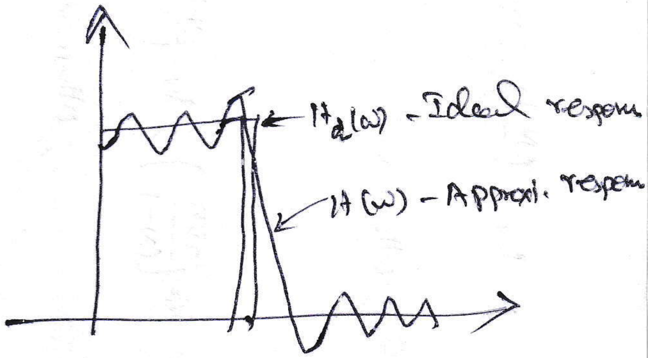
(a) Rectangular window seq.



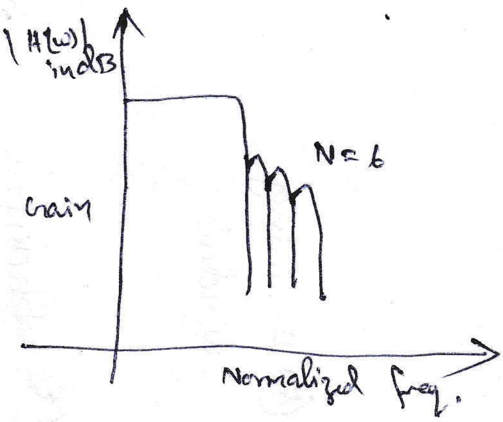
(b) Mag. response of Rectangular window.



(C) Log-mag. response of rectangular window



(d) Mag. response of lowpass filter approximated using rectangular window.



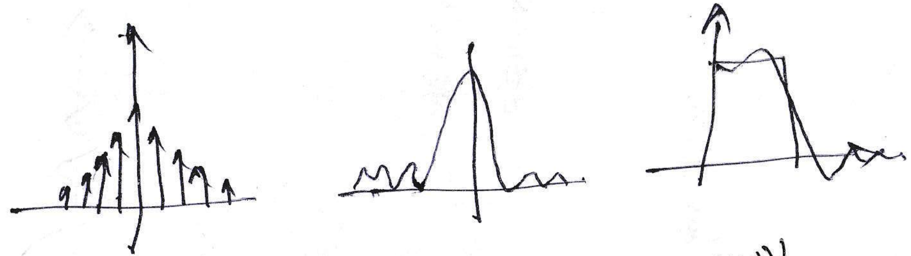
(e) Log-Mag. response of FIR lowpass filter designed using rectangular window.

6.40

6.53

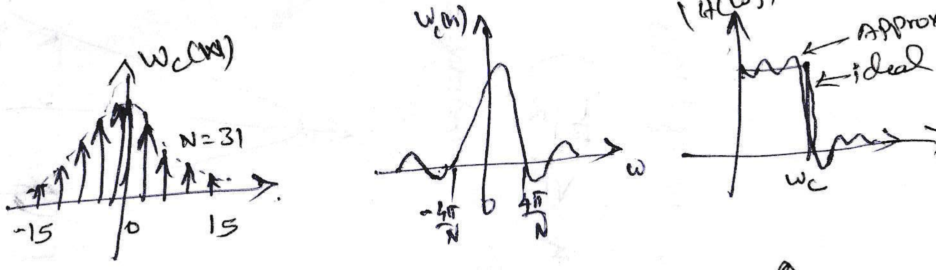
② Triangular window

$$w_T(n) = \begin{cases} 1 - \dfrac{2|n|}{N-1} & ; \text{ for } -\left(\dfrac{N-1}{2}\right) \le n \le \left(\dfrac{N-1}{2}\right) \\ 0 & ; \text{ Otherwise.} \end{cases}$$



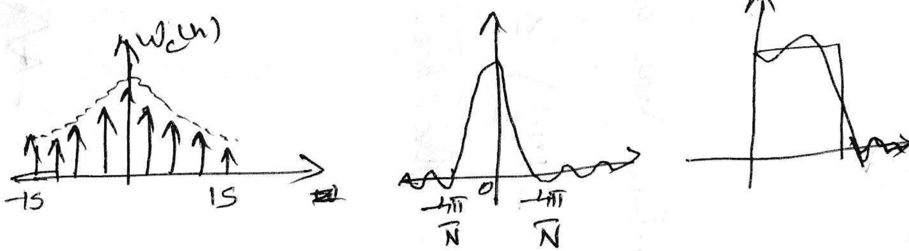
③ Hanning window

$$w_c(n) = \begin{cases} 0.5 + 0.5\cos\left(\dfrac{2\pi n}{(N-1)}\right) & ; \text{ for } -\left(\dfrac{N-1}{2}\right) \le n \le \left(\dfrac{N-1}{2}\right) \\ 0 & ; \text{ Otherwise.} \end{cases}$$



④ Hamming window

$$w_H(n) = \begin{cases} 0.54 + 0.46\cos\left(\dfrac{2\pi n}{N-1}\right) & ; \text{ for } -\left(\dfrac{N-1}{2}\right) \le n \le \left(\dfrac{N-1}{2}\right) \\ 0 & ; \text{ Otherwise.} \end{cases}$$



⑤ Blackman window

$$w_B(n) = \begin{cases} 0.42 + 0.5\cos\dfrac{2n\pi}{N-1} + 0.08\cos\dfrac{4n\pi}{N-1} & , -\dfrac{(N-1)}{2} \le n \le \dfrac{(N-1)}{2} \\ 0 \end{cases}$$

⑥ Kaiser

$$w_K(n) = \begin{cases} \dfrac{I_0\left(\alpha\sqrt{1-\left(\frac{2n}{N-1}\right)^2}\right)}{I_0(\alpha)} & ; -\dfrac{(N-1)}{2} \le n \le \dfrac{(N-1)}{2} \\ 0 & ; \text{ Otherwise.} \end{cases}$$

(or) Hanning,

$$w_c(n) = \begin{cases} 0.5 - 0.5\cos\dfrac{2n\pi}{N-1} & ; 0 \le n \le N-1 \\ 0 & ; \text{ Otherwise} \end{cases}$$

(or) Hamming,

$$w_H(n) = \begin{cases} 0.54 - 0.46\cos\dfrac{2n\pi}{N-1} & ; 0 \le n \le N-1 \\ 0 & ; \text{ Otherwise.} \end{cases}$$

→ Design a lowpass Filter using Rectangular window by taking 9- samples of $w(n)$ and with a cut-off freq. of 1.2 rad/sec.

Sol:

$\omega_c = 1.2$ rad/sec & $N=9$

[where α - Constant phase delay in sample

For a lowpass filter the desired freq. response is,

$$H_d(\omega) = \begin{cases} e^{-j\omega\alpha} ; & -\omega_c \le \omega \le \omega_c \\ 0 ; & \text{Otherwise} \end{cases}$$

$$* \quad h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(\omega) e^{j\omega n} d\omega = \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} e^{-j\omega\alpha} e^{j\omega n} d\omega$$

$$= \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} e^{j\omega(n-\alpha)} d\omega = \frac{1}{2\pi}\left[\frac{e^{j\omega(n-\alpha)}}{j(n-\alpha)}\right]_{-\omega_c}^{\omega_c}$$

$$= \frac{1}{2\pi}\left[\frac{e^{j\omega_c(n-\alpha)}}{j(n-\alpha)} - \frac{e^{-j\omega_c(n-\alpha)}}{j(n-\alpha)}\right] = \frac{1}{\pi(n-\alpha)}\left[\frac{e^{j\omega_c(n-\alpha)} - e^{-j\omega_c(n-\alpha)}}{2j}\right]$$

$$= \frac{1}{\pi(n-\alpha)} \sin\omega_c(n-\alpha)$$

when $n=\alpha$, the term $\frac{\sin\omega_c(n-\alpha)}{(n-\alpha)}$ will become 0/0 which is indeterminate.

Hence, $h_d(n) = \frac{\sin\omega_c(n-\alpha)}{\pi(n-\alpha)}$; for $n \neq \alpha$

For $n=\alpha$, $h_d(n)$ can be evaluated using L'Hospital rule.

$\boxed{\displaystyle \underset{\theta\to 0}{Lt} \frac{\sin A\theta}{\theta} = A}$ $\therefore$ $n=\alpha$; $h_d(n) = \underset{n\to\alpha}{Lt} \frac{\sin\omega_c(n-\alpha)}{\pi(n-\alpha)} = \frac{1}{\pi}\underset{n\to\alpha}{Lt}\frac{\sin\omega_c(n-\alpha)}{(n-\alpha)} = \frac{1}{\pi}\omega_c = \frac{\omega_c}{\pi}$

$\therefore$ Impulse Response, $h(n) = h_d(n)\, w_R(n)$

where, $w_R(n) = \begin{cases} 1 ; & \text{for } n=0 \text{ to } (N-1) \\ 0 ; & \text{Otherwise} \end{cases}$

$\therefore h(n) = h_d(n)$; for $n=0$ to $(N-1)$

Here, $N=9$; $\omega_c = 1.2$ rad/sec ; $\alpha = \frac{N-1}{2} = \frac{9-1}{2} = 4$

when $n=0$; $h(0) = h_d(0) = \frac{\sin(1.2 \times (0-4))}{\pi(0-4)} = -0.0793$

$n=1$; $h(1) = h_d(1) = \frac{\sin(1.2 \times (1-4))}{\pi(1-4)} = -0.0470$

$n=2$; $h(2) = \frac{\sin(1.2 \times (2-4))}{\pi(2-4)} = 0.1075$

$n=3$; $h(3) = \frac{\sin(1.2 \times (3-4))}{\pi(3-4)} = 0.2967$

$n=4$; $h(4) = \frac{1.2}{\pi} = 0.3820$

when $n=5$; $h(5) = \frac{\sin(1.2 \times 1)}{\pi} = 0.2967$

$h(6) = 0.1075$

$h(7) = -0.047$

$h(8) = -0.0793$

The Impulse response is satisfying the Symmetry Condition

i.e. $h(N-1-n) = h(n)$

when $n=5$; $h(5) = h(8-5) \Rightarrow h(5) = h(3)$

$n=6$; $h(6) = h(8-6) \Rightarrow h(6) = h(2)$

$n=7$; $h(7) = h(8-7) \Rightarrow h(7) = h(1)$

$n=8$; $h(8) = h(8-8) \Rightarrow h(8) = h(0)$

The mag. fun. of FIR filter when the impulse response is Symmetric and N is odd is given below.

$$|H(w)| = h\left(\frac{N-1}{2}\right) + \sum_{n=1}^{\frac{N-1}{2}} 2h\left(\frac{N-1}{2}-n\right)\cos nw$$

$$= h(4) + 2h(3)\cos w + 2h(2)\cos 2w + 2h(1)\cos 3w + 2h(0)\cos 4w$$

$$= 0.382 + 2\times 0.2967\times \cos w + 2\times 0.1075\times \cos 2w + 2\times(-0.047)\cos 3w + 2\times(-0.0793)\cos 4w.$$

$$= 0.382 + 0.5934\cos w + 0.215\cos 2w - 0.094\cos 3w - 0.1586\cos 4w$$

The Transfer fun. of the filter is given by,

$$H(z) = \sum_{n=0}^{N-1} h(n) z^{-n} = \sum_{n=0}^{8} h(n) z^{-n}$$

$$= \sum_{n=0}^{3} h(n) z^{-n} + h(4) z^{-4} + \sum_{n=5}^{8} h(n) z^{-n}$$

$$= \sum_{n=0}^{3} h(n) z^{-n} + h(4) z^{-4} + \sum_{n=0}^{3} h(8-n) z^{-(8-n)} \quad [\because h(N-1-k) = h(k)$$

$$= \sum_{n=0}^{3} h(n) z^{-n} + h(4) z^{-n} + \sum_{n=0}^{3} h(n) [z^{-(8-n)}]$$

$$= h(4) z^{-n} + \sum_{n=0}^{3} h(n)\left[z^{-n} + z^{-(8-n)}\right]$$

$$\therefore H(z) = \frac{Y(z)}{X(z)} = \sum_{n=0}^{3} h(n)\left[z^{-n} + z^{-(8-n)}\right] + h(4) z^{-4}$$

$$= h(0)\left[1 + z^{-8}\right] + h(1)\left[z^{-1} + z^{-7}\right] + h(2)(z^{-2} + z^{-6}) + h(3)(z^{-3} + z^{-5}) + h(4) z^{-4}$$

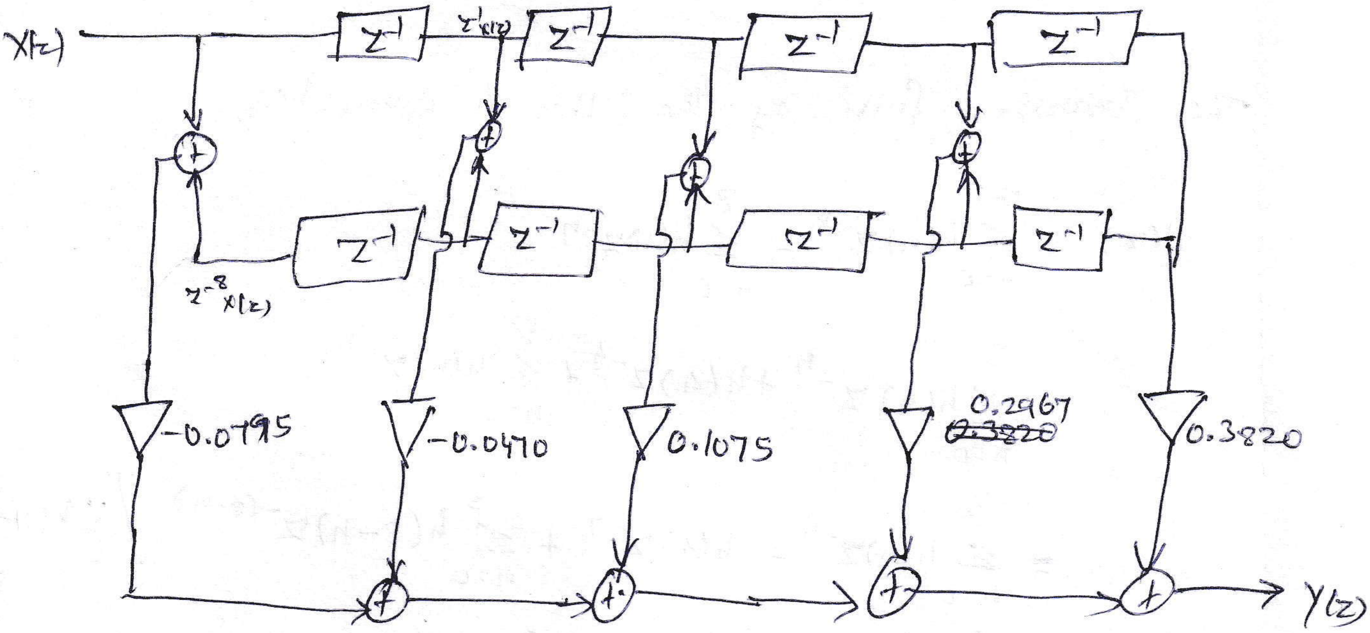
$$Y(z) = h(0)\left[X(z) + z^{-8}X(z)\right] + h(1)\left[z^{-1}X(z) + z^{-7}X(z)\right] + h(2)\left[z^{-2}X(z) + z^{-6}X(z)\right]$$

$$+ h(3)\left[z^{-3}X(z) + z^{-5}X(z)\right] + h(4)\, z^{-4}X(z).$$

$$= -0.0795\left[X(z) + z^{-8}X(z)\right] + 0.0470\left[z^{-1}X(z) + z^{-7}X(z)\right]$$

$$+ 0.1075\left[z^{-2}X(z) + z^{-6}X(z)\right] + 0.2967\left[z^{-3}X(z) + z^{-5}X(z)\right].$$

$$+ 0.3820\, z^{-4}X(z)$$



<u>Note:-</u>

In <u>Linear Phase</u> FIR Filters,

<u>Case-I</u>: Impulse Response is symmetric & N-is odd

$$|H(\omega)| = h\left(\frac{N-1}{2}\right) + \sum_{n=1}^{\frac{N-1}{2}} 2h\left(\frac{N-1}{2} - n\right)\cos\omega n$$

<u>Case-II</u>: Impulse response is symmetric and N-is even

$$|H(\omega)| = \sum_{n=1}^{N/2} 2h\left(\frac{N}{2} - n\right)\cos\left(\omega\left(n - \frac{1}{2}\right)\right)$$

→ Design a highpass filter using hamming window, with a cut-off freq. of 1.2 radians/sec and $N=9$.

Sol

High pass filter, $H_d(\omega) = \begin{cases} e^{-j\omega\alpha} ; & -\pi \le \omega \le -\omega_c \ \& \ \omega_c \le \omega \le \pi \\ 0 ; & \text{otherwise.} \end{cases}$

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(\omega) e^{-j\omega n} d\omega = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{-j\omega\alpha} e^{-j\omega n} d\omega = \frac{1}{2\pi}\int_{-\pi}^{-\omega_c} e^{-j\omega\alpha} e^{j\omega n} d\omega + \frac{1}{2\pi}\int_{\omega_c}^{\pi} e^{-j\omega\alpha} e^{j\omega n} d\omega$$

$$= \frac{1}{2\pi}\left[\frac{e^{j\omega(n-\alpha)}}{j(n-\alpha)}\right]_{-\pi}^{-\omega_c} + \frac{1}{2\pi}\left[\frac{e^{j\omega(n-\alpha)}}{j(n-\alpha)}\right]_{\omega_c}^{\pi}$$

$$= \frac{1}{2\pi}\left[\frac{e^{-j\omega_c(n-\alpha)} - e^{-j\pi(n-\alpha)} + e^{j\pi(n-\alpha)} - e^{j\omega_c(n-\alpha)}}{j(n-\alpha)}\right]$$

$$= \frac{1}{\pi(n-\alpha)}\left[\frac{e^{j\pi(n-\alpha)} - e^{-j\pi(n-\alpha)}}{2j} - \frac{e^{j\omega_c(n-\alpha)} - e^{-j\omega_c(n-\alpha)}}{2j}\right]$$

$$= \frac{1}{\pi(n-\alpha)}\left[\sin(n-\alpha)\pi - \sin\omega_c(n-\alpha)\right]$$

When $n=\alpha$, the terms $\frac{\sin(n-\alpha)\pi}{(n-\alpha)}$ and $\frac{\sin\omega_c(n-\alpha)}{(n-\alpha)}$ become $0/0$ which is indeterminate

Hence, $h_d(n) = \dfrac{\sin(n-\alpha)\pi - \sin w_c(n-\alpha)}{\pi(n-\alpha)}$; for $n \neq \alpha$

For $n = \alpha$, $h_d(n)$ can be evaluated using L' Hospital Rule

$$h_d(n) = \lim_{n\to\alpha} \frac{\sin(n-\alpha)\pi - \sin w_c(n-\alpha)}{\pi(n-\alpha)} \qquad \left[\lim_{\theta\to 0} \frac{\sin A\theta}{\theta} = A\right]$$

$$= \frac{1}{\pi}\left[\lim_{n\to\alpha} \frac{\sin\pi(n-\alpha)}{(n-\alpha)} - \lim_{n\to\alpha} \frac{\sin w_c(n-\alpha)}{n-\alpha}\right] = \frac{1}{\pi}\left[\pi - w_c\right] = 1 - \frac{w_c}{\pi}$$

The window sequences for hamming window is,

$$W_H(n) = 0.54 - 0.46\cos\left(\frac{2\pi n}{N-1}\right) \; ; \text{ for } n = 0 \text{ to } (N-1)$$

$$\therefore h(n) = h_d(n)\, W_H(n) = \frac{1}{\pi(n-\alpha)}\left[\sin\pi(n-\alpha) - \sin(n-\alpha)w_c\right]$$

$$\left[0.54 - 0.46\cos\left(\frac{2\pi n}{N-1}\right)\right] \; ; \text{ for } n \neq \alpha$$

$$h(n) = \left(1 - \frac{w_c}{\pi}\right)\left(0.54 - 0.46\cos\left(\frac{2\pi n}{N-1}\right)\right) \; ; \text{ for } n = \alpha$$

* $N = 9$, $w_c = 1.2$ rad/sec, $\therefore \alpha = \frac{N-1}{2} = \frac{9-1}{2} = 4$

n & α are Integers, $\therefore (n-\alpha)$ also Integer, $(n-\alpha)\pi$ - is Integral multiple of π

SO, $\sin(n-\alpha)\pi = 0$

Also $(N-1) = 8$

$n=0 \Rightarrow h(0) = \dfrac{1}{\pi(-4)} \sin(-4 \times 1.2)\left[0.54 - 0.46\cos 0\right]$

$= 0.0063$

$n=1$ $h(1) = \dfrac{\sin(-3\times 1.2)}{\pi(-3)}\left[0.54 - 0.46\cos\frac{\pi}{4}\right] = 0.0101.$

4

Similiarly.

$h(2) = -0.0581$

$h(3) = -0.2567$

$$h(4) = \left(1 - \frac{1.2}{\pi}\right)\left(0.54 - 0.46 \cos\frac{4\pi}{4}\right) = 0.6180$$

$h(5) = -0.2567$

$h(6) = -0.0581$

$h(7) = 0.0101$

$h(8) = 0.0063$

$$H(z) = \sum_{n=0}^{N-1} h(n) z^{-n} = \sum_{n=0}^{8} h(n) z^{-n} = \sum_{n=0}^{3} h(n) z^{-n} + h(4) z^{-4} + \sum_{n=5}^{8} h(n) z^{-n}$$

$$= \sum_{n=0}^{3} h(n) z^{-n} + h(4) z^{-4} + \sum_{n=0}^{3} h(N-1-n) z^{-(N-1-n)}$$

$$= \sum_{n=0}^{3} h(n) z^{-n} + h(4) z^{-4} + \sum_{n=0}^{3} h(8-n) z^{-(8-n)}$$

$$H(z) = \frac{Y(z)}{X(z)} = \sum_{n=0}^{3} h(n)\left[z^{-n} + z^{-(8-n)}\right] + h(4) z^{-4}$$

$$Y(z) = \sum_{n=0}^{3} h(n)\left[z^{-n} X(z) + z^{-(8-n)} X(z)\right] + h(4) z^{-4} X(z)$$

$$= h(0)\left(X(z) + z^{-8} X(z)\right) + h(1)\left(z^{-1} X(z) + z^{-7} X(z)\right)$$
$$+ h(2)\left(z^{-2} X(z) + z^{-6} X(z)\right) + h(3)\left(z^{-3} X(z) + z^{-5} X(z)\right) + h(4) z^{-4} X(z)$$

UNIT- 3

Finite word Length Effect

Introduction:-

Decimal to Binary

① Integer:-

$$
\begin{array}{r|l}
2 & 24 \\
2 & 12 - 0 \\
2 & 6 - 0 \\
2 & 3 - 0 \\
 & 1 - 1
\end{array}
$$

$(24)_{10} = (11000)_2$

② floating

0.256

$0.256 \times 2 = 0.512$

$0.512 \times 2 = 1.024$

$0.024 \times 2 = 0.048$

$0.048 \times 2 = 0.096$

$(0.256)_{10} = (0.0100)_2$

$(24.256)_{10} = (11000.0100)_2$

(3) Binary to Decimal

11000.0100

11000

$$\begin{array}{ccccc} 1 & 1 & 0 & 0 & 0 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 2^4 & 2^3 & 2^2 & 2^1 & 2^0 \end{array}$$

$1\times 2^4 + 1\times 2^3 + 0 + 0 + 0 =$

$16 + 8 = 24$

0.0100

$$\begin{array}{cccc} 0 & 1 & 0 & 0 \\ \downarrow & \downarrow & \downarrow & \downarrow \\ 2^{-1} & 2^{-2} & 2^{-3} & 2^{-4} \end{array}$$

$0 + 0.256 + 0 + 0 = 0.256$

$$(11000.0100)_2 = (24.256)_{10}$$