

24/2/14

Unit - II

BEAMS

⇒ Introduction :-

A Structural Member Subjected to Transverse Load (Load perpendicular to the longitudinal axis) is called a Beam. When a beam is provided in buildings they are called with specific terminology known as "Joists". When a large beam supporting a no. of Joists is called a "Girder". Beams which carry roof loads in trusses are called "Purlins" and the beam which supports from the masonry is called "Lintels". Beams are more critical members in any structure. Their design must be economical & safe. The main guidelines for the design of beams are as follows:

$$K = \frac{P}{\Delta} \xrightarrow{\text{Lateral}}$$

1. Beams should be proportioned for strength in Bending, keeping in view the Lateral and Local stability of the compression flange.
2. Beams should be proportioned for stiffness (K) keeping in mind their deflections and deformations under certain service conditions.
3. They should be proportioned for economy paying attention to the size & grades of steel to produce most economical design.

It is difficult task for the designer to select a beam size for a given span and load which will satisfy all the above

⇒ Rolled steel beam sections :-

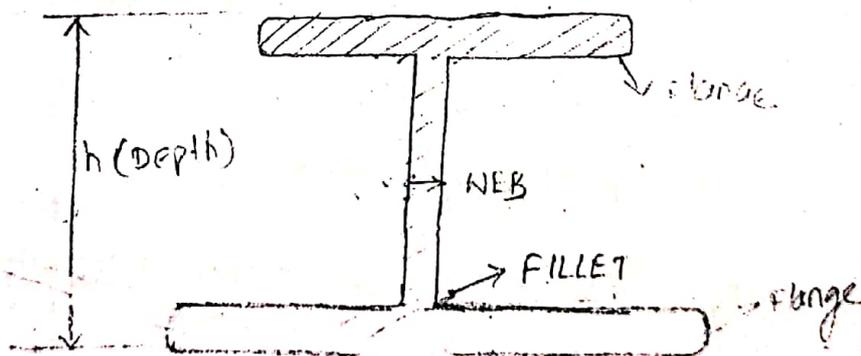
The Rolled steel beam sections are classified into 4 types

1. Indian standard Junior Beams (ISJB)
2. Indian standard Light Beams (ISLB)
3. Indian standard medium weight Beams (ISMb)
4. Indian standard wide flange Beams (ISWB)

sometimes rolled steel columns are heavy weight which may be classified into two types :-

1. Indian standard heavy weight Beams (ISHB)
2. Indian standard column sections (ISCS)

The c/s of the Rolled steel beam has been shown in following figure :-



The beam section consist of a web and two flanges both at top and bottom portion. The Junction between the web & flange is known as "fillet." The Rolled steel beams are mainly employed to resist bending. The Rolled steel beams are

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Principles of Limit state Design :-

The Aim of Limit state Design is to see that the structure which is constructed or built must be safe. A structure may become unfit when it collapses. So the requirements of deflection, shear, Bending moment, Bearing stress etc, must be satisfied in the design. Anyway the design life of permanent structures should not exceed the limiting conditions. The design is based on the load applied and the strength of materials.

Types of Beams :-

Based on steel Design :-

As per code IS: 800 - 2007 is classified three categories.

1. Laterally supported Beams (or) Laterally Restrained beam
2. Laterally unsupported Beams
3. Built-up Beams

1. Laterally supported Beams (or) Laterally Restrained

The beam which is provided with lateral support on one end (or) other end is known as laterally supported beam. Generally Bending is produced due to the load acting on the beam in which tensions & compressions are developed at the

flanges. The compression takes place at the top portion of the flange and tension takes place at the bottom portion of the flange. These compression flanges acts as a 'column' which tends to buckle (bending) in the beam to avoid such buckling, we provide lateral supports.

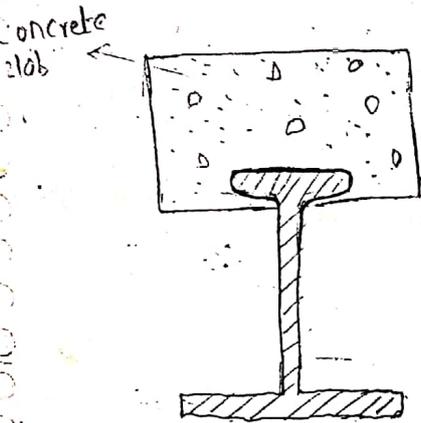
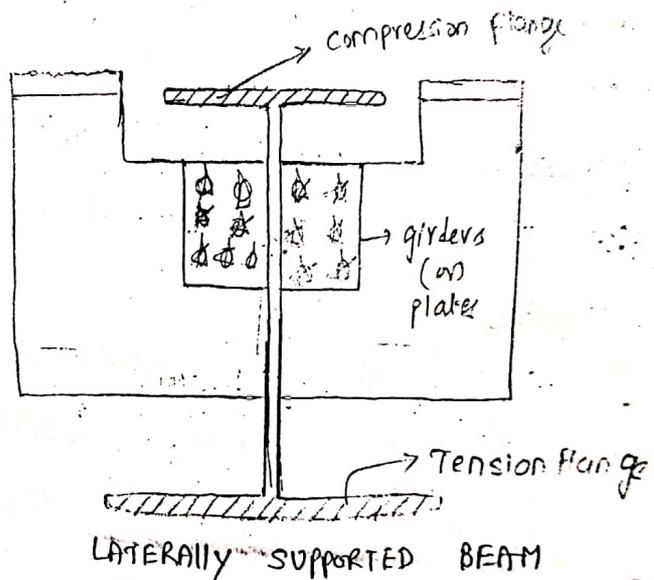


fig (11)



→ Design steps for laterally supported beam :-

Step 1 :- calculation of B.M

IF the load is acting in the form of UDL

then B.M $M = \frac{Wl^2}{8}$

IF the load is acting in the form of concentrated load then B.M this is $M = \frac{Wl}{4}$

IF necessary assume self weight of the beam

step 2 :- calculation of shear force (S.F)

IF the load is U.D.L max s.f $V \text{ (or) } F = \frac{Wl}{2}$

Step 3:- calculation of section modulus (Z) required for
section Modulus (Z) required for the beam is

$$Z_{req} = \frac{M}{\sigma_b}$$

where σ_b = permissible bending stress
 $= 0.66 f_y$
 $= 165 \text{ kN/mm}^2$

$$\frac{I}{y} = Z$$

Step 4:- Selection of suitable beam section

We have to select a beam section from steel tables.

From Beam sections like
I_{STB}, I_{SLB}, I_{SMB}, I_{SWB} etc., from those sections determine (or) list out all the properties like depth of the web, thickness of web (t_w), thickness of flange (t_f) etc.,

Step 5:- check for section Modulus

(1) for checking the section Modulus we have to satisfy the following conditions with reference to properties of steel section.

(a) $\frac{D}{t_f}$

(b) $\frac{t_f}{t_w} < 9.0$

(c) $\frac{d_1}{t_w} < 85$

where,

t_f = thickness of flange

t_w = thickness of web

h (or) D = depth of the section beam

$$d_1 = h_1 - 2h_2$$

$$d_1 = h_1 - 2h_2$$

step 6 :- Determination of slenderness Ratio (λ) (or) K

$$\lambda = \frac{l_{eff}}{r_{yy}} \quad \left(\begin{array}{l} l_{eff} = 0.2l \\ r_{yy} \text{ (radius of gyration)} \end{array} \right)$$

step 7 :- check for shear force stress

Due to shear force there will be forming shear stresses

shear stress required = $\tau = \frac{F \text{ (or) } V}{h \times t_w}$ V = shear force

shear stress calculated should be less than permissible shear stress

step 8 :- check for deflection

$$\sigma_E = 0.4 f_y = 100 \text{ N/mm}^2$$

Deflection is based on type of loading when

(a) A simply supported beam (SSB) is subjected to

$$\text{U.D.L} \quad y_{max} = \frac{5}{384} \frac{wL^4}{EI}$$

(b) If the (SS) beam is subjected to point load

$$y_{max} = \frac{wL^3}{48EI}$$

maximum deflection calculated must be less than allowable deflection.

Allowable deflection $\frac{1}{Allow} = \frac{\text{span length}}{325}$

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Problems on Laterally supported beams (or) Laterally restrained (or) The beam is not free to rotate at the bearing

~~Q~~ Design a simply supported beam to carry an u.d.l. of 44 kN/m. The effective span of the beam is 8m. The effective length of the compression flange is also 8m. The beam is laterally supported. The ends of the beam are not free to rotate at the bearings.

Sol :- step 1 :- calculation of B.M :
 Given u.d.l acting on the beam $w = 44 \text{ kN/m}$
 Assume self weight = 1.0 kN/m to the beam

\therefore Total load = $44 + 1 = 45 \text{ kN/m}$

The max. B.M for u.d.l $M = \frac{wL^2}{8}$
 $= \frac{45 \times (8)^2}{8}$

$M = 360 \text{ kN-m}$

$M = 360 \times 10^6 \text{ N-mm}$

step 2 :- calculation of S.F :

The max. s.f $V = \frac{wL}{2}$

$V = \frac{45 \times 8}{2}$

$F \text{ (or) } V = 180 \text{ kN}$

$\therefore R_A + R_B = wL$

$R_A = \frac{wL}{2}$

$R_B = \frac{wL}{2}$

step 3 :- Section Modulus Required (Z_{req} required)

Section modulus required $Z_{req} = \frac{M}{\sigma_{bc}}$

(To take sec)

Permissible compressive stress $\sigma_{bc} = 0.66 f_y$

Assume $f_y = 250 \text{ N/mm}^2$

$$\therefore \sigma_{bc} = 0.66 \times 250$$

$$\sigma_{bc} = 165 \text{ N/mm}^2$$

$$\therefore Z_{req} = \frac{360 \times 10^6}{165} \frac{\text{N} \cdot \text{mm}}{\text{N/mm}^2} (\text{mm}^3)$$

$$Z_{req} = 2.181 \times 10^6 \text{ mm}^3$$

Trial Section Modulus = $1.5 \times Z$
(Factor)

~~Factor~~ $Z_{req} = 1.5 \times 2.181 \times 10^6$

(Calculated) $Z_{req} = 3.272 \times 10^6 \text{ mm}^3$

$$Z_{req} = \sqrt{3.272 \times 10^8} \text{ mm}^3$$

Step 4: Selection of suitable steel sections.

The steel section is selected based on calculated Section Modulus (Z_{req}) which should be more than $Z_{table} > (Z_{req} \text{ calculated})$
 Z_{req} from steel tables

This means Z_{xx} from steel tables must be in the range of $3.00 \times 10^6 \text{ mm}^3$ to $3.900 \times 10^6 \text{ mm}^3$

\therefore Select a section of ISWB 600 @ $\overset{\text{depth of section}}{\uparrow} \underset{\text{self weight}}{199.7 \text{ Kg}}$

\Rightarrow properties of section :-

Thickness of web $t_w = 11.2 \text{ mm}$

Thickness of flange $t_f = 21.3 \text{ mm}$

Depth of the section $(\text{mm}) h = 600 \text{ mm}$

Moment of Inertia $I_{xx} = 106198.5 \text{ cm}^4$

$$I_{xx} = 106198.5 \times 10^4 \text{ mm}^4$$

section Modulus $Z_{xx} = 3540.0 \text{ cm}^3$

$$h_2 = 42.90 \text{ mm}$$

Step 5 :- check for section Modulus

In order to check section Modulus, it may be classified into 3 categories

$$(a) \frac{D}{t_f} \text{ (or) } \frac{h}{t_f} = \frac{600}{21.3} = 28.169$$

$$(b) \frac{t_f}{t_w} < 2.0$$

$$\Rightarrow \frac{21.3}{11.2} = 1.902 < 2.0$$

$$(c) \frac{d_1}{t_w} < 85 \quad \text{(or)} \quad \frac{h_1}{t_w} < 85$$

$$\text{As } d_1 = h - 2h_2$$
$$= 600 - 2(42.9)$$

$$d_1 = 514.2$$

$$\therefore \frac{d_1}{t_w} = \frac{514.2}{11.2} = 45.91 < 85$$

We have satisfied above three requirements with properties of steel section. Section Modulus to be checked

$$Z_{\text{check}} = \frac{M}{\sigma_{bc}}$$

Here σ_{bc} should be calculated from code book

IS : 800 - 2007

from IS : 800 - 2007 which should be < 165

Slenderness ratio $k(\text{or}) \lambda = \frac{\lambda_{\text{eff}}}{r_{yy}}$

(r_{yy} from properties of steel tables)

$$k = \frac{8000}{58.5}$$

$$k = 152.38$$

\times $f_y = 250 \text{ N/mm}^2$

The 'k' value is in b/w 150 + 160

(from code book page NO: 40)

$$\frac{150}{160} \rightarrow 68.9$$

$$\rightarrow 61.4$$

σ_{bc} value is obtained by interpolation technique

$$\sigma_{bc} = 68.9 - \frac{(68.9 - 61.4)}{160 - 150} \times (152.38 - 150)$$

$$\sigma_{bc} = 67.11 \text{ N/mm}^2 < 165 \text{ N/mm}^2$$

$$Z_{\text{check}} = \frac{M}{\sigma_{bc}} = \frac{360 \times 10^6}{67.11}$$

$$Z_{\text{check}} = 5.363 \times 10^6 \text{ mm}^3$$

where " λ_{eff} " is the effective length of the beam based on support.

When the beam is laterally supported (or) fully restrained then effective length

$$\lambda_{\text{eff}} = 0.7 l$$

$$\lambda_{\text{eff}} = 0.7 \times 8000$$

$$\therefore \lambda_{\text{eff}} = 5600$$

$$k = \frac{5600}{58.5}$$

$$k = 106.64$$

Here σ_{bc} value is calculated based on the values of slenderness ratio 'k' and yield stress

(From code book page no: 40)

'k' lies (around) b/w 100 & 110

$$k \rightarrow 100 \rightarrow 132$$

$$k \rightarrow 110 \rightarrow 115$$

$\therefore \sigma_{bc}$ value is obtained by Interpolation technique

$$\sigma_{bc} = 132 - \frac{(132 - 115)}{110 - 100} \times (106.67 - 100)$$

$$\therefore \sigma_{bc} = 120.67 \text{ N/mm}^2 < 165 \text{ N/mm}^2$$

$$Z_{\text{check}} = \frac{M}{\sigma_{bc}} = \frac{360 \times 10^6}{120.67} \quad \left[\begin{array}{l} \text{To check whether it lies in} \\ \text{the limit (or not)} \end{array} \right]$$

$$Z_{\text{check}} = 2.983 \times 10^6 \text{ mm}^3$$

$$\therefore Z_{\text{check}} = 2983 \times 10^3 \text{ mm}^3$$

\therefore The section which we are selected is safe

$$\therefore \boxed{Z_{\text{check}} < Z_{xx}} \text{ from (Tables)}$$
$$2983 \times 10^3 < 3540 \times 10^3 \text{ mm}^3$$

step 7 :- check for shear stress

$$\text{Max. shear stress } \tau_v = \frac{V}{h \times t_w}$$
$$= \frac{180 \times 10^3}{600 \times 11.2}$$

$$\therefore \tau_v = 26.78 \text{ N/mm}^2$$

The max. shear stress calculated must be less than permissible shear stress

$$\begin{aligned} \text{permissible shear stress} &= 0.4 f_y \\ &= 0.4 \times 250 \\ &= 100 \text{ N/mm}^2 \end{aligned}$$

$$26.78 \text{ N/mm}^2 < 100 \text{ N/mm}^2$$

step 8 :- check for deflection (y_{\max})

Max. deflection for a simply supported beam due to

$$\text{U.D.L is } y_{\max} = \frac{5}{384} \frac{wL^4}{EI}$$

Take E for steel is i.e. $E = 2 \times 10^5 \text{ N/mm}^2$

Here ' I ' value is nothing but I_{xx} value from properties of the section

$$y_{\max} = \frac{5}{384} \times \frac{45 \times (8000)^4}{2 \times 10^5 \times 106198.5 \times 10^4}$$

$$y_{\max} = 11.29 \text{ mm}$$

The max. deflection must be less than allowable deflection

$$\begin{aligned} \text{Allowable deflection } y_{\text{allow}} &= \frac{\text{span length}}{325} \\ &= \frac{8000}{325} \\ &= 24.61 \text{ mm} \end{aligned}$$

$$\therefore y_{\max} < y_{\text{allow}}$$

Note :-

From steel tables the moment of inertia about x-x axis (I_{xx}) value is taken, because max. bending takes place about the x-axis and also higher value is

— taken (i.e. I_{xx} value is more than I_{yy})
 & In the case of Radius of gyration (r_{yy}). Least value is taken

$\therefore r_{yy}$ is preferred than r_{xx}

Prbl
 Exam
 8/1/14

Q) A simply supported beam having a effective span of 8 m which carries an U.D.L of 40 kN/m. Take $f_y = 250$ N/mm² and $E = 2 \times 10^5$ N/mm². Design the beam if it is laterally supported. Draw the details of the beam.

Sol :- Step 1 :- Calculation of B.M (M) :-

Given : U.D.L $W = 40$ kN/m

Assume self weight = 1.0 kN/m if necessary

\therefore total U.D.L $W = 40 + 1 = 41$ kN/m

The max. B.M for U.D.L $M = \frac{wl^2}{8}$

$$= \frac{41 \times (8)^2}{8}$$

$$= 328 \text{ kN-m}$$

$$\therefore M = 328 \times 10^6 \text{ N-mm}$$

Step 2 :- calculation of s.f (V) :-

The max. s.f for a U.D.L $V = \frac{wl}{2}$

$$= \frac{41 \times 8}{2}$$

$$V = 164 \text{ kN}$$

$$\therefore V = 164 \times 10^3 \text{ N}$$

Step 1:- Section Modulus required (Z_{req})

$$\text{Section Modulus required } Z_{req} = \frac{M}{\sigma_{bc}}$$

Here σ_{bc} is permissible Bending stress

$$\sigma_{bc} = 0.66 f_y \quad (\text{Assume } f_y = 250 \text{ N/mm}^2)$$
$$= 0.66 \times 250$$

$$\therefore \sigma_{bc} = 165 \text{ N/mm}^2$$

$$\therefore Z_{req} = \frac{328 \times 10^6}{165}$$

$$= 1.987 \times 10^6 \text{ mm}^3$$

$$\therefore Z_{req} = 1987 \times 10^3 \text{ mm}^3$$

Trial section Modulus $= 1.5 \times Z_{req}$

$$\therefore \text{Finally required section modulus } Z_{req} = 1.5 \times 1987 \times 10^3$$
$$= 2981 \times 10^3 \text{ mm}^3$$

Step 4:- selection of suitable steel section

Here the beam section is selected which is based on Z_{req} value which should be more than

Z_{req}
 \therefore select ISMB 600 @ 122.6 kg for the complete

design.

\therefore properties of the beam section are as follows:

Thickness of the web $t_w = 12.0 \text{ mm}$

Thickness of the flange $t_f = 20.8 \text{ mm}$

Depth of the section $h = 600 \text{ mm}$

Moment of inertia $I_{yy} = 4.12 \text{ cm}^4$
Radius of gyration $r_{yy} = 41.2 \text{ mm}$

section modulus $Z_{xx} = 3060.4 \times 10^3 \text{ mm}^3$
 $h_i = 509.7 \text{ mm}$
 $h_e = 45.15 \text{ mm}$

Step 5 :- check for section Modulus (Z_{check})

As we know that $Z_{check} = \frac{M}{\sigma_{bc}}$ we have to

In order to check the section Modulus we have to satisfy the following categories

(a) $\frac{h}{t_f} = \frac{600}{20.8} = 28.84$

(b) $\frac{t_f}{t_w} < 2.0$

$\Rightarrow \frac{20.8}{12.0} = 1.73 \text{ mm} < 2.0$

(c) $\frac{h_i}{t_w} < 85$

$\frac{509.7}{12.0} = 42.475 < 85$

From above eq σ_{bc} value is calculated by Interpolated
technique done b/w steel slenderness ratio k vs

The σ_{bc} value obtained must be less than permissible stress which is nothing but 165 N/mm^2 .

(From code book Pg No: 40)

Slenderness ratio $k = \frac{l_{eff}}{r_{yy}}$

when the support is fully restrained then

$$\begin{aligned} l_{eff} &= 0.7 \times l \\ &= 0.7 \times 8000 \\ &= 5,600 \end{aligned}$$

$$\therefore k = \frac{5,600}{41.2} = 135.9$$

$$\therefore k = 135.9$$

k lies b/w 130 & 140

for $f_y = 250 \text{ N/mm}^2$

$$k = 130 \rightarrow 88.8$$

$$k = 140 \rightarrow 77.8$$

$$\therefore \sigma_{bc} = 88.8 - \frac{(88.8 - 77.8)}{140 - 130} \times (135.9 - 130)$$

$$\sigma_{bc} = 82.10 < 165 \text{ N/mm}^2$$

$$\therefore Z_{check} = \frac{M}{\sigma_{bc}} = \frac{328 \times 10^6}{82.10}$$

$$Z_{check} = 3.994 \times 10^6$$

$$Z_{check} = 3994 \times 10^3 \text{ mm}^3$$

$\therefore Z_{check} > Z$ from tables

$$3994 \times 10^3 > 3060.4 \times 10^3$$

\therefore The section is not safe then again redesign

with another section

Try with another section \rightarrow Take ISWB 600 @ 133.7 kg

∴ properties of the beam section are

Thickness of web: $t_w = 11.2 \text{ mm}$

$$t_f = 21.3 \text{ mm}$$

$$Z_{xx} = 3540.0 \times 10^3 \text{ mm}^3$$

$$Y_{yy} = 5.25 \times 10^6 \text{ mm}^3$$

$$r_{yy} = 52.5 \text{ mm}$$

$$h_1 = 514.2 \text{ mm}$$

$$h_2 = 42.9 \text{ mm}$$

step 6:- check for section Modulus (Z_{class}):

In order to check the section modulus we have to satisfy the following categories:

$$(a) \quad \frac{h}{t_f} = \frac{600}{21.3} = 28.16$$

$$(b) \quad \frac{t_f}{t_w} < 2.0$$

$$\Rightarrow \frac{21.3}{11.2} = 1.90 < 2.0$$

$$(c) \quad \frac{h_1}{t_w} < 85$$

$$\frac{514.2}{11.2} = 45.91 < 85$$

From the above eq. i.e. $Z_{class} = \frac{M}{\sigma_{bc}}$, σ_{bc} value
calculated by interpolation technique done. A/w

slenderness ratio $k \leq f_y$

The σ_{bc} value obtained must be less than permissible stress which is nothing but 165 MPa

(from code book if $n = 41$)

slenderness ratio $K = \frac{x_{\text{eff}}}{r_{yy}}$

when the support is fully restrained then

$$l_{\text{eff}} = 0.7 \times l$$
$$= 0.7 \times 8000$$

$$l_{\text{eff}} = 5600$$

$$\therefore K = \frac{5600}{52.5}$$

$$\therefore K = 106.67$$

K lies b/w 100 & 110

for $f_y = 250 \text{ N/mm}^2$ (from code book) \therefore

$$K = 100 \rightarrow 132$$

$$K = 110 \rightarrow 115$$

$$\therefore \sigma_{bc} = 132 - \frac{(132 - 115)}{110 - 100} \times (106.67 - 100)$$

$$\sigma_{bc} = 120.66 \text{ N/mm}^2 < 165 \text{ N/mm}^2$$

$$\therefore Z_{\text{check}} = \frac{M}{\sigma_{bc}} = \frac{328 \times 10^6}{120.66}$$

$$\therefore Z_{\text{check}} = 2.718 \times 10^6 \text{ mm}^3$$

$$\therefore Z_{\text{check}} = 2.718 \times 10^6 \text{ mm}^3$$

$$Z_{\text{check}} < Z_{\text{tables}}$$

$$2.718 \times 10^6 \text{ mm}^3 < 3540.0 \times 10^6 \text{ mm}^3$$

Step 7 :- check for shear stress (τ_v):

$$\text{max. shear stress } \tau_v = \frac{V}{h \times t_w}$$
$$= \frac{164 \times 10^3}{600 \times 11.2}$$

$\therefore \tau_v = 24.40 \text{ N/mm}^2$
 The max. shear stress calculated must be less than the permissible shear stress

$$\text{permissible shear stress} = 0.4 f_y \\ = 100 \text{ N/mm}^2$$

$$\therefore \tau_v < 100 \text{ N/mm}^2$$

$$24.40 < 100 \text{ N/mm}^2$$

Hence safe

step 8 :- check for deflection (y_{\max}) :-

max. deflection for a SSB due to U.D.L is

$$y_{\max} = \frac{5}{384} \frac{wL^4}{EI}$$

$$\therefore E = 2 \times 10^5 \text{ N/mm}^2$$

Here 'I' value is nothing but I_{xx} value from properties of the section

$$y_{\max} = \frac{5}{384} \times \frac{41 \times (8000)^4}{2 \times 10^5 \times 106198.5 \times 10^4}$$

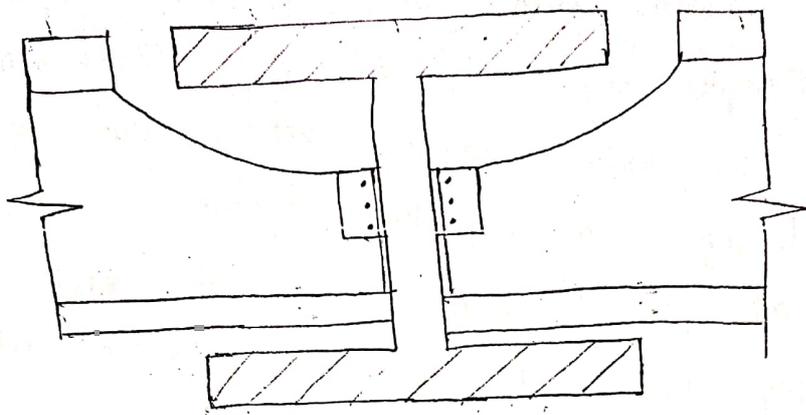
$$y_{\max} = 10.29 \text{ mm}$$

\therefore The max def should be less than allowable deflection

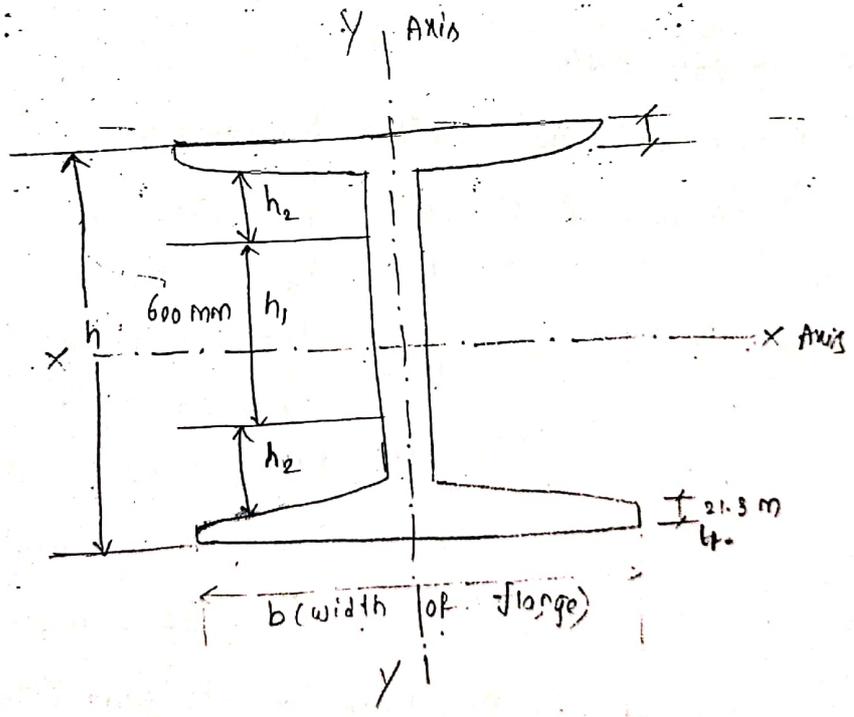
$$\text{Allowable deflection} = \frac{\text{span length}}{325} = \frac{8000}{325} = 24.61$$

$$\therefore 10.29 < 24.61 \text{ mm}$$

\therefore We provide a section of ISWB 600 for complete section because it satisfies all the requirements of deflection, stress, section modulus etc.,



LATERALLY SUPPORTED BEAMS ISWB 600



5) The effective length of compression flange of a supported beam with beam section of ISMB 500 0.869 (self weight) KN/m is of 8 m. Determine the safe U.D.L per meter length which can be placed over beam having an effective span of 8 m. Adopt permissible stresses as per IS: 800 - 2007. The end of the beam is fully restrained against rotation at the beam.

Sol :- step 1 :- properties of the beam section
In above problem the given section is ISMB 500

@ 0.869 KN/m

Depth of the section $h = 500 \text{ mm}$

Thickness of flange $t_f = 17.2 \text{ mm}$

$t_w = 10.2 \text{ mm}$

$h_1 = 424.1 \text{ mm}$

$h_2 = 37.95 \text{ mm}$

$I_{xx} = 45218.3 \times 10^4 \text{ mm}^4$

$Z_{xx} = 1808.7 \times 10^3 \text{ mm}^3$

$r_{yy} = 35.2 \text{ mm}$

step 2 :- Moment of Resistance of the beam (M_R)

As we know that $z = \frac{M_R}{\sigma_{bc}} \Rightarrow$

$$\therefore M_R = z \times \sigma_{bc}$$

In above equation 'z' is nothing but Z_{xx} value

steel tables

Here σ_{bc} value is calculated by interpolation

Technique based on the values of slenderness ratio and yield stress

$$K = \frac{L_{eff}}{r_{yy}}$$

$$K = \frac{0.7 \times 8000}{85.2}$$

$$K = 159.09 \approx 160$$

For $f_y = 250 \text{ N/mm}^2$ and for $K = 160$

From code book $\sigma_{bc} = 61.4 \text{ N/mm}^2$

(Pg. No: 40)

Substitute the value of σ_{bc} in above eq, we get

$$\begin{aligned} \text{Moment of resistance} - M_R &= Z \times \sigma_{bc} \\ &= 1808.7 \times 10^3 \times 61.4 \end{aligned}$$

$$\therefore M_R = 111.054 \times 10^6 \text{ N-mm}$$

Step 3 :- Load supported over the beam (W)

Let 'W' be the U.D.L per meter length

$$\therefore \text{max. B.M (M)} = \frac{Wl^2}{8}$$

$$\text{As } M = M_R$$

$$111.054 \times 10^6 = \frac{W \times (8000)^2}{8}$$

$$W = 13.88 \text{ kN/m}$$

$$\therefore W = 13.88 \text{ kN/m}$$

\therefore safe U.D.L per meter length = W - self weight of the beam

$$\text{self weight of the beam} = 0.869 \text{ kN/m}$$

$$= 13.88 - 0.869$$

safe U.D.L per meter length = 13.011 kN/m

⇒ Note :-

$$\text{Kg} \rightarrow \text{KN} \quad \frac{1 \text{ Kg}}{\text{conversion}} = 9.81 \text{ N (or)} 10 \text{ N}$$

$$86.9 \text{ Kg/m}$$

$$= 86.9 \times 10 \text{ N/m}$$

$$= 869 \text{ N/m}$$

$$= 0.869 \text{ KN/m}$$

$$1 \text{ kg} = 10 \text{ N}$$

$$1 \text{ kg} = 10^3 \text{ kg}$$

$$1 \text{ kg} = 10 \text{ N}$$

4) A laterally supported beam having an effective span of 6m consist of ISMB 550 @ 103.7 kg/m with cover plate of 2500 x 16 mm connected to each flange. Determine the safe u.d.l of the beam in addition to its own weight. Take $f_y = 250 \text{ N/mm}^2$

5) A laterally supported beam having a span of 6m. which carries an u.d.l of 20 kN/m. Take $f_y = 250 \text{ N/mm}^2$. $E = 2 \times 10^5 \text{ N/mm}^2$. The beam is free to rotate at the bearings.

4A) Step 1:- Properties of the Beam section

In the above problem the given section is ISMB 550 @ 103.7 kg/m i.e @ 1.037 KN/m

properties of the Beam section :-

$$h = 550 \text{ mm}$$

$$b_f = 19.3 \text{ mm}$$

$$t_w = 11.2 \text{ mm}$$

$$h_1 = 467.5 \text{ mm}$$

$$h_2 = 41.25 \text{ mm}$$

$$I_{xx} = 64893.6 \times 10^4 \text{ mm}^4$$

$$Z_{xx} = 2559.8 \times 10^3 \text{ mm}^3$$

ISMB 550

103.7 kg/m

1.037 KN/m

1.037

$$r_{yy} = 37.3 \times 10 \text{ mm}$$

$$\therefore r_{yy} = 37.3 \text{ mm}$$

step 2 :- Moment of Resistance of the beam (M_R)

$$\text{As we know that } z = \frac{M_R}{\sigma_{bc}}$$

$$\therefore M_R = z \times \sigma_{bc}$$

In the above equation 'z' is nothing but z_{xx} value from steel tables

Here σ_{bc} value is calculated by Interpolation technique based on the values of slenderness ratio and yield stress

$$K = \frac{left}{r_{yy}}$$

$$\therefore K = \frac{0.7 \times 6000}{37.3}$$

$$\times \left\{ \begin{array}{l} \therefore K = 150 \text{ or } 150 \text{ or } 112.6 \\ \text{for } f_y = 250 \text{ N/mm}^2 \text{ and for } K = 150 \text{ } K = 112.6 \end{array} \right.$$

(from code page NO : 40)

$$\sigma_{bc} = 68.9 \text{ N/mm}^2$$

Moment of Resistance

$$M_R = z \times \sigma_{bc}$$

$$= 2359.8 \times 10^3 \times 68.9$$

$$M_R = 162.59 \times 10^6 \text{ N-mm}$$

step 3 :- Load supported over the beam (W)

let 'W' be the u.d.l per meter length

$$\therefore \text{max. B.M. } M = \frac{wl^2}{8}$$

$$K = \frac{0.7 \times 6000}{87.3}$$

$$\therefore K = 112.60$$

K lies b/w 110 & 120

for $f_y = 250 \text{ N/mm}^2$ (from code book)

$$K \quad 110 \longrightarrow 115$$

$$120 \longrightarrow 101$$

$$\therefore \sigma_{bc} = 115 - \frac{(115-101)}{120-110} \times (112.60 - 110)$$

$$\sigma_{bc} = 111.36 \text{ N/mm}^2$$

Substitute the value of σ_{bc} in eq we get

$$\begin{aligned} \text{Moment of Resistance } M_R &= Z \times \sigma_{bc} \\ &= 2359.8 \times 10^3 \times 111.36 \end{aligned}$$

$$\therefore M_R = 262.787 \times 10^6 \text{ N-mm}$$

Step 3 :- Load supported over the beam (W)

let 'w' be the U.D.L per meter length

$$\therefore \text{max. B.M } M = \frac{wl^2}{8}$$

As $M = M_R$

$$262.787 \times 10^6 = \frac{W \times (6000)^2}{8}$$

$$\therefore W = 58.39 \text{ kN/mm}$$

$$\therefore W = 58.39 \text{ kN/m}$$

\therefore safe U.D.L per meter length = W - self weight of the beam

$$\text{self weight of the beam} = 1.037 \text{ kN/m}$$

$$= 58.39 - 1.037$$

$$\text{safe U.D.L} = 57.353 \text{ kN/m}$$

5A) Step 1:- calculation of B.M

Given U.D.L acting on the beam $W = 20 \text{ kN/m}$

Assume self weight = 1.0 kN/m to the beam

$$\therefore \text{Total Load} = 20 + 1 = 21 \text{ kN/m}$$

$$\begin{aligned} \text{The max. B.M for U.D.L } M &= \frac{wL^2}{8} \\ &= \frac{21 \times (6)^2}{8} \end{aligned}$$

$$\therefore M = 94.5 \text{ kN-m}$$

$$M = 94.5 \times 10^6 \text{ N-mm}$$

Step 2:- calculation of S.F :-

$$\begin{aligned} \text{The max. S.F } V &= \frac{wL}{2} \\ &= \frac{21 \times 6}{2} \\ V &= 63 \text{ kN} \end{aligned}$$

Step 3:- Section Modulus Required (Z_{req}):

$$\text{Section modulus required } Z_{req} = \frac{M}{\sigma_{bc}}$$

σ_{bc} is permissible bending stress

$$\begin{aligned} \sigma_{bc} &= 0.66 f_y \\ &= 0.66 \times 250 \end{aligned}$$

$$\sigma_{bc} = 165 \text{ N/mm}^2$$

$$Z_{req} = \frac{94.5 \times 10^6}{165}$$

$$\therefore Z_{req} = 572.727 \times 10^3 \text{ mm}^3$$

$$\text{trial section modulus} = 1.5 \times Z_{\text{req}}$$

$$\therefore \text{finally required section modulus } Z_{\text{req}} = 1.5 \times 572 \times 10^3$$

$$\therefore Z_{\text{req}} = 858 \times 10^3 \text{ mm}^3$$

Laterally unsupported beam :-

When the compression flange of the beam is laterally unsupported, the compression flange may buckle sideways, leading to failure at the elastic limit.

Laterally unsupported beams are also known as laterally unrestrained beams that means the ends of the beam is free to rotate at the bearings. In symmetrical beams even if loaded in the plain containing principle axis may bend out of plain of load under certain conditions the shape of the beams are more economical in the manner. Such shapes have less resistance to bending and torsion in the direction of xy axis. The bending of beams is accommodated by twisting. This phenomenon is lateral buckling (or) later torsion.

The buckling tendency of compression flange increases as the stress increases on the flange. When the strain attains a max. critical stress maximum. In order to avoid lateral buckling due to a load the beam should have enough torsional stiffness to resist torsional forces and the beam should be laterally supported in the direction perpendicular to the weak principle axis.

⇒ Note :-

The laterally unsupported means laterally the beam is free to rotate at the bearing. Sometimes the beam is laterally restrained against torsion.

Design steps of laterally unsupported beams: —

step 1: calculation of B.M (M)

step 2: calculation of s.f (V)

step 3: section modulus required (Z_{req})

step 4: selection of suitable beam section

step 5: check for section modulus (Z_{check})

Before calculating section modulus we have to satisfy the following categories

(a) $\frac{b_f}{t_f}$

(b) $\frac{t_f}{t_w} < 2.0$

(c) $\frac{h_i}{t_w} < 85$

$$Z_{check} = \frac{M}{\sigma_{bc}}$$

From above equation σ_{bc} is calculated by making interpolation b/w elastic critical stress (f_{cb}) vs f_y

from code (pg: 55)

calculation of critical stress f_{cb}

Generally f_{cb} is calculated by double interpolation b/w slenderness ratio (K) vs (h/t_f) from code book

pg: 57

step 6: check for shear stress

step 7: check for deflection.

PROBLEM

1. Design a rolled steel I-section for a simply supported beam with a clear span of 6m. It carries an UDL of 50 kN/m exclusive of self wt. of the girder. The beam is laterally unsupported.

Sol.:- calculation of B.M

Given clear span = 6m

Assume 0.3m eff. cover at both ends

$$\therefore \text{eff. span length} = 6 + \frac{0.3}{2} + \frac{0.3}{2} = 6.3 \text{ m}$$

Given U.D.L. = 50 kN/m

Assume self weight = 1.0 kN/m if necessary

$$\therefore \text{Total U.D.L.} = 50 + 1 = 51 \text{ kN/m}$$

$$\begin{aligned} \therefore \text{max. B.M for U.D.L} &= \frac{wL^2}{8} = \frac{51 \times 6.3^2}{8} \\ &= 253.023 \text{ kN-m} \\ &= 253.02 \times 10^6 \text{ N-mm} \end{aligned}$$

Step 2:- calculation of S.F (V)

$$\begin{aligned} \text{max. S.F for UDL} \quad V &= \frac{wL}{2} \\ &= \frac{51 \times 6}{2} \end{aligned}$$

$$\therefore V = 160.65 \times 10^3 \text{ N}$$

Step 3:- $Z_{req} = \frac{M}{\sigma_{bc}}$

$$\sigma_{bc} = 0.66 f_y \quad (f_y = 250 \text{ N/mm}^2)$$

$$\sigma_{bc} = 0.66 \times 250$$

$$\therefore \sigma_{bc} = 165 \text{ N/mm}^2$$

$$\therefore Z_{req} = \frac{M}{\sigma_{bc}}$$

$$= \frac{253.02 \times 10^6}{165}$$

$$= 1.533 \times 10^6$$

$$Z_{req} = 1553.45 \times 10^3 \text{ mm}^3$$

$$\begin{aligned} \text{Total section Modulus} &= 1.5 \times Z \\ &= 1.5 \times 1553.45 \times 10^3 \\ &= 2300.75 \times 10^3 \text{ mm}^3 \end{aligned}$$

∴ select ISMB 550 @ 103.7 Kg/m

step 4 :- properties of section

$$h = 550 \text{ mm}$$

$$t_f = 19.3 \text{ mm}$$

$$t_w = 11.2 \text{ mm}$$

$$r_{yy} = 3.73 \text{ m} = 37.3 \text{ mm}$$

$$I_{xx} = 64893.6 \times 10^4 \text{ mm}^4$$

$$Z_{xx} = 2359.8 \times 10^3 \text{ mm}^3$$

$$h_1 = 467.5 \text{ mm}$$

$$h_2 = 41.25 \text{ mm}$$

step 5 :- check for section Modulus

$$Z_{check} = \frac{M}{\sigma_{bc}}$$

$$(a) \frac{h}{t_f} = \frac{550}{19.3} = 28.49$$

$$(b) \frac{t_f}{t_w} = \frac{19.3}{11.2} = 1.72 < 2.0$$

$$(c) \frac{h_1}{t_w} = \frac{467.5}{11.2} = 41.75 < 85$$

from above eq. σ_{bc} is calculated by making interpolation b/w critical stress (f_{cb})

calculation of critical stress :-

To calculate f_{cb} interpolation is done

$(h/t_f) \text{ Vs } (K)$

$$K = \frac{l_{eff}}{r_{yy}}$$

when support is laterally unrestrained (Dr) laterally unsupported

$$l_{eff} = \text{span length} = 6.3$$

$$= \frac{6300}{37.3}$$

$$K l_{eff} = 168.9 \text{ mm}$$

K lies b/w 160 & 170, h/t_f lies b/w 25 & 30

$$K \rightarrow 160 \text{ for } 25 = 148.2$$

$$K \rightarrow 170 \text{ for } 30 = 132.0$$

$$X = 148.2 - \frac{(148.2 - 132)}{(30 - 25)} \times (28.49 - 25)$$

$$X = 136.89$$

for $170 \rightarrow K$

25	30	170	$K \rightarrow 25 = 136.7$
136.7	121.3		30 = 121.3

$$Y = 136.7 - \frac{(136.7 - 121.3)}{30 - 25} \times (28.49 - 25)$$

$$Y = 125.95$$

Again interpolating the values x & y we get
critical stress ' f_{cb} '

$$x = 136.89$$

$$y = 125.95$$

$$f_{cb} = 136.89 - \frac{(136.89 - 125.95)}{170 - 160} \times (168.9 - 160)$$

$$\therefore f_{cb} = 127.153 \text{ N/mm}^2$$

f_{cb} should be increased by 20% for safe design.

$$f_{cb} \text{ increases} = 127.15 + 127.15 \times \frac{20}{100}$$
$$= 152.58 \text{ N/mm}^2$$

$$\therefore f_{cb} = 152.58 \text{ N/mm}^2$$

Now interpolation done b/w f_{cb} vs f_y
 f_{cb} lies b/w 150 & 200 pg. 55 (from code book)

$$f_{cb} \text{ @ } 150 = 106.8$$

$$f_{cb} \text{ @ } 200 = 134.1$$

$$\sigma_{bc} = 134.1 - \frac{(134.1 - 106.8)}{200 - 150} \times (152.58 - 150)$$

$$= 132.6 < 165 \text{ N/mm}^2$$

$$Z_{check} = \frac{M}{\sigma_{bc}}$$

$$= \frac{255.02 \times 10^6}{132.6}$$

$$\therefore Z_{\text{check}} = 1908.14 \times 10^3 \text{ mm}^3$$

$$Z_{\text{check}} < Z_{\text{tables}}$$

$$1908.14 \times 10^3 < 2359 \times 10^3 \text{ mm}^3$$

step 6:- check for shear stress

$$\text{max. shear stress } \tau_b = \frac{V}{h \times t_w}$$

$$= \frac{160.65 \times 10^3}{550 \times 11.2}$$

$$= 26.07 \text{ N/mm}^2$$

check for deflection:-

$$\text{Max. deflection for ss beam } Y_{\text{max}} = \frac{5}{384} \frac{wl^4}{EI}$$

$$E = 2 \times 10^5 \text{ N/mm}^2$$

I value is I_{xx} from properties of section

$$I_{xx} = 64893.6 \times 10^4 \text{ mm}^4$$

$$Y_{\text{max}} = \frac{5}{384} \frac{wl^4}{EI}$$

$$= \frac{5}{384} \times \frac{51 \times (6000)^4}{2 \times 10^5 \times 64893.6 \times 10^4}$$

$$Y_{\text{max}} = 6.631 \text{ mm}$$

$$Y_{\text{allow}} = \frac{\text{span length}}{325} = \frac{6000}{325}$$

$$Y_{\text{allow}} = 18.48 \text{ mm}$$

$$Y_{\text{max}} < Y_{\text{allow}}$$

$$6.631 < 18.48$$

10) Design a rolled steel I-section for a simply supported beam with clear span of 8m it carries a u.d.l of 45 kN/m exclusive of self weight. The beam is free to rotate at the bearings. Draw the

exclusive - we have to apply
inclusive - self weight is added in the load

step 1 :- calculation of B.M

Given : clear span = 8m.

Assume 0.3 m effective cover at both ends of the beam

$$\therefore \text{Effective span} = 8 + 0.3 \left(\frac{0.3}{2} + \frac{0.3}{2} \right)$$

$$= 8.3 \text{ m}$$

Given u.d.l $W = 45 \text{ kN/m}$, Assume self weight = 1.0 kN/m

$$\therefore \text{max. B.M } M = \frac{Wl^2}{8} = \frac{46 \times (8.3)^2}{8}$$

$$\therefore M = 396.11 \times 10^6 \text{ N-mm}$$

step 2 :- calculation of s.f

$$\text{max. s.f for u.d.l } V = \frac{Wl}{2}$$

$$= \frac{46 \times 8.3}{2}$$

$$\therefore V = 190.97 \times 10^3 \text{ N}$$

step 3 :- Section Modulus required

$$\text{Section Modulus required} = Z_{\text{req}} = \frac{M}{\sigma_{bc}}$$

$$\sigma_{bc} = 0.66 \text{ fy}$$

$$\sigma_{bc} = 165 \text{ N/mm}^2$$

$$Z_{req} = 2400.667 \times 10^3 \text{ mm}^3$$

Trial section Modulus $= 1.5 \times Z_{req}$
 $= 1.5 \times 2400.667 \times 10^3$
 $\therefore Z_{req} = 3600.99 \times 10^3 \text{ mm}^3$

Step 4 :- selection of suitable steel section

select ISWB 600 @ 145.1 kg/m³

properties of the section :-

$$h = 600 \text{ mm}$$

$$\text{Thickness of flange } t_f = 23.6 \text{ mm}$$

$$t_w = 11.8 \text{ mm}$$

$$I_{xx} = 115626.6 \times 10^4 \text{ mm}^4$$

$$r_{yy} = 53.5 \text{ mm}$$

$$Z_{xx} = 3854.3 \times 10^3 \text{ mm}^3$$

$$h_1 = 507.9 \text{ mm}$$

$$h_2 = 46.05 \text{ mm}$$

check for section Modulus :-

Before checking the section Modulus we have to satisfy the following requirements.

$$(a) \frac{h}{t_f} = \frac{600}{23.6} = 25.42$$

$$(b) \frac{t_f}{t_w} \leq 2.0 \Rightarrow \frac{23.6}{11.8} = 2$$

$$(c) \frac{h_1}{t_w} < 85 \Rightarrow \frac{507.9}{11.8} = 43.04$$

$$Z_{check} = \frac{M}{\sigma_{bc}}$$

Here the bending stress σ_{bc} may be calculated

by doing : Interpolation b/w f_{cb} (elastic critical stress)
 V_s f_y)
 In order to calculate f_{cb} (critical stress) we have to
 make double Interpolation b/w slenderness ratio 'K'

K Vs $\frac{h}{t_f}$

From code book page no : 57

slenderness Ratio $K = \frac{\text{Effective length}}{r_{yy}}$

In the case of laterally unsupported (or) laterally unrestrained
 $L_{eff} = 8.3 \text{ m}$ (span length)

$\therefore K = \frac{8.3 \times 1000}{58.5}$

$K = 155.14$

K lies b/w 150 & 160 and $\frac{h}{t_f}$ lies b/w 25 & 30

calculation of critical stress (f_{cb})

K	$\frac{h}{t_f}$		[code book page NO :- 57]
	25	30	
150 →	161.5	144.8	

Interpolation (variable) → $X = 161.5 - \frac{(161.5 - 144.8)}{30 - 25} \times (25.42 - 25)$

$\therefore X = 160.16$

K → 160 →	$\frac{h}{t_f}$	
	25	30
	148.2	132.0

$$y = 148.2 - \frac{(148.2 - 132) \cdot x}{30 - 25} \quad (25.42 - 25)$$

$$y = 146.90$$

Now, again interpolate the values of x & y directly

get

f_{cb}

$$x = 160.16 \rightarrow 150$$

$$y = 146.90 \rightarrow 160$$

$$f_{cb} = 160.16 - \frac{(160.16 - 146.90)}{160 - 150} \times (155.14 - 150)$$

$$f_{cb} = 153.80 \text{ N/mm}^2$$

f_{cb} should be increased by 20% for safe design

$$f_{cb} = 153.80 + 153.80 \times \frac{20}{100}$$

$$\therefore f_{cb} = 183.96 \text{ N/mm}^2$$

From code book pg no :- 55 f_{cb} should be around 150 & 200

By interpolating f_{cb} vs f_y we get directly

$$\sigma_{bc} < 165 \text{ N/mm}^2$$

$$f_{cb} \rightarrow 150 \rightarrow 106.8$$

$$200 \rightarrow 134.1$$

$$\sigma_{bc} = 134.1 - \frac{(134.1 - 106.8)}{200 - 150} \times (183.96 - 150)$$

$$\therefore \sigma_{bc} = 115.5 < 165 \text{ N/mm}^2$$

$$\text{check} = \frac{V}{\sigma_b C} = \frac{270.11 \times 10^3}{115.5}$$

$$Z_{\text{check}} = 3429.52 \times 10^3$$

$$Z_{\text{check}} < Z_{\text{tables}}$$

$$3429.18 \times 10^3 < 3854.1 \times 10^3 \text{ mm}^3$$

∴ provide ISWB 600 @ 145.1 for the given beam

check for shear stress (τ_v):

$$\begin{aligned} \text{max. s/r } \tau_v &= \frac{V}{h \times t_w} = \frac{190.9 \times 10^3}{600 \times 11.8} \\ &= 26.96 \text{ N/mm}^2 \end{aligned}$$

$$\begin{aligned} \text{permissible shear stress} &= 0.4 f_y \\ &= 100 \text{ N/mm}^2 \end{aligned}$$

$$\tau_v < 100 \text{ N/mm}^2$$

check for Deflection:

$$\begin{aligned} y_{\text{max}} &= \frac{5}{384} \frac{w l^4}{EI} \\ &= \frac{5}{384} \times \frac{46 \times (8300)^4}{2 \times 10^5 \times 115626.6 \times 10^4} \end{aligned}$$

$$\therefore y_{\text{max}} = 12.29 \text{ mm}$$

$$y_{\text{allow}} = \frac{\text{span length}}{325} = \frac{8300}{325}$$

$$y_{\text{allow}} = 25.5 \text{ mm}$$

$$\therefore y_{\text{max}} < y_{\text{allow}}$$

3) A beam section having ISMB : 550 @ 1.037 KN/m
 a simply supported beam with span length of 4m.
 The ends of the beam are restrained against torsion.
 The beam is laterally unsupported. The ends of the
 beam is free to rotate at the bearings. Deflection
 the safe U.D.L. per meter length which the beam
 carry

sol:- step 1 :- Properties of the section:

Given section : ISMB : 550 @ 1.037 KN/m

$$h = 550 \text{ mm}$$

$$t_f = 19.5 \text{ mm}$$

$$t_w = 11.2 \text{ mm}$$

$$I_{xx} = 64893.6 \times 10^4 \text{ mm}^4$$

$$Z_{xx} = 2359.8 \times 10^3 \text{ mm}^3$$

$$r_{yy} = 37.3 \text{ mm}$$

$$h_1 = 467.5 \text{ mm}$$

$$h_2 = 41.25 \text{ mm}$$

Step 2 :- Moment of Resistance of the beam (M_R)

We have know that $z = \frac{M}{\sigma_{bc}}$

$$\therefore M_R = Z_{xx} \times \sigma_{bc}$$

From above eq σ_{bc} is calculated by making
 Interpolation b/w f_{cb} vs f_y

calculation of critical stress (f_{cb})

To calculate \dot{t}_{cb} make double interpolation b/w

K vs h/t_f

$$\therefore K = \frac{I_{eff}}{r_{yy}} = \frac{4000}{37.3}$$

$$K = 107.23$$

$$h/t_f = \frac{550}{19.3} = 28.49$$

K lies b/w 100 & 110 [Pg:-57]

h/t_f lies b/w 25 & 30

Interpolation

h/t_f

25 30

$K \rightarrow 100 \rightarrow 291.4 \quad 270.9$

$$X = 291.4 - \frac{(291.4 - 270.9)}{30 - 25} \times \frac{(28.49 - 25)}{28.49}$$

$$X = 277.09$$

Interpolation

h/t_f

25 30

$K \rightarrow 110 \rightarrow 251.8 \quad 232.1$

$$Y = 251.8 - \frac{(251.8 - 232.1)}{30 - 25} \times (28.49 - 25)$$

$$\therefore Y = 238.06$$

Now Interpolating x & y we get f_{cb}

$$f_{cb} = 277.09 - \frac{(277.09 - 238.06)}{110 - 100} \times (107.23 - 100)$$

$$\therefore f_{cb} = 248.87 \text{ N/mm}^2$$

* f_{cb} should be increased by 20% for safe design

$$f_{cb} = 248.87 + 248.87 \times \frac{20}{100}$$

Actual $f_{cb} = 298.64 \text{ N/mm}^2$

from pg no: 55 f_{cb} should be around 250 & 300

\therefore By interpolating f_{cb} vs f_y we get directly σ_{bc}

$$\sigma_{bc} < 165 \text{ N/mm}^2$$

$$f_{cb} \rightarrow \begin{matrix} 250 & \rightarrow & 152.3 \\ 300 & \rightarrow & 163.6 \end{matrix}$$

$$\sigma_{bc} = 163.6 \frac{(163.6 - 152.3)}{300 - 250} \times (298.64 - 250)$$

$$\therefore \sigma_{bc} = 152.60 < 165 \text{ N/mm}^2$$

$$M_R = 2359.8 \times 10^3 \times 152.60$$

$$\therefore M_R = 360.10 \times 10^6 \text{ N-mm}$$

Step 3 :- Load supported for the beam

Let w be the u.d.l acting

$$\therefore \text{max B.M } M = \frac{wL^2}{8} \quad (\text{As } M = M_R)$$

$$\therefore M_R = \frac{wL^2}{8}$$

$$3.60 \cdot 133 \times 10^6 = \frac{W \times (4000)^2}{8}$$

$$\therefore W = 180.06 \text{ N/mm}^2$$

$$W = 180.06 \text{ KN/m}$$

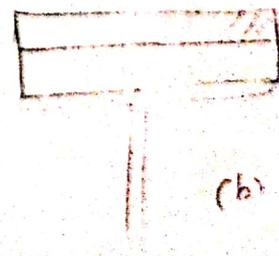
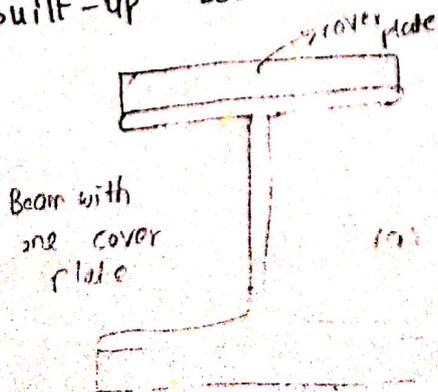
$$\begin{aligned} \text{safe u.d.l} &= W - \text{self weight} \\ &= 180.06 - 1.037 \end{aligned}$$

$$W = 179.02 \text{ KN/m}$$

→ Built-up Beams (or) Compound Beams :-

When span lengths are more, a very large B.M's are generated due to the applications of high loads. Also the depth of the beam may be limited or restricted. Rolled steel beams of largest size cannot resist for that purpose. built-up-beams are used.

The built-up-beams are also termed as Compound beams (or) compound girders. The built-up beams are used when the rolled steel beam section are in adequate in the case of the strength of the beam section. We have to attach cover plates both at top & bottom flanges of the beam. The following are the built-up beams shown diagrammatically as follows.



⇒ Design steps for built-up beams :-

step 1 :- calculation of B.M

step 2 :- calculation of s.f

step 3 :- section Modulus required

step 4 :- Selection of suitable steel section :-

Here in the case of built-up beams, beam depth is limited. so, we have to take the section based on the depth of the beam. since section Modulus from tables will be less than

Z_{req} . Here plates are attached both at top

← bottom flanges

Step 5 :- Dimensions of the cover plate

Here Area of the cover plate must be calculated

∴ Area of cover plate $A_p = \frac{Z_{req} - Z_{xx}}{h}$

Width of the plate $= b = \frac{\text{Area } A_p}{\text{thickness of cover plate}}$

Here thickness of cover plate may be selected by trial & error Method which ranges from 10 mm

12 mm, 14, 16, 18, 20, 22, 25, 30 - - - -

step 6 :- calculation of Bending stress in compression

Tension

Here Bending stress in compression

$$\sigma_{b \text{ Comp}} = \frac{\text{Max. B.M.}}{\text{Gross M.I}} \times y$$

$$\text{Gross Moment of Inertia } I_{zz} = I_{xx(\text{table})} + 2A_p \left(\frac{h'}{2}\right)^2$$

Area of plate

y = depth of Neutral axis including thickness of plates provided both at top and bottom.

h' = distance b/w centroid at top & bottom flange

$$\therefore \frac{h'}{2} = \frac{h}{2} + \frac{\text{thickness of one cover plate}}{2}$$

Step 7 :- Bending stress in Tension σ_{b_t}

$$\sigma_{b_t} = \sigma_{b \text{ Comp}} \times \frac{\text{Gross area of flange}}{\text{Net area of flange}}$$

$$\text{Gross area of flange} = (b_f \times t_f) + A_p$$

width of flange



$$\text{Net area of flange} = \text{Gross area} - 23.5 \times (t_f + t_w)$$

where 23.5 = Gross diameter of the rivet

The calculated Bending stress both in compression and Tension must be $< 165 \text{ N/mm}^2$



Step 8 :- It's check for shear stress $\tau = \frac{V}{h \times t_w}$

Step 9 :- check for deflection

$$y_{\text{max}} = \frac{5}{384} \frac{W L^4}{E I_{zz}}$$

1) Design a simply supported beam which carries an UDL of 50 kN/m inclusive. The effective span of the beam is 10 m. Depth of the beam is limited to 500 mm. The beam is laterally restrained at both ends.

Step 1 :- calculation of B.M. :-

Given U.D.L. $w = 50 \text{ kN/m}$

Effective span = 10 m

$$\text{max. B.M. } M = \frac{wL^2}{8} = \frac{50 \times (10)^2}{8}$$

$$\therefore M = 625 \times 10^6 \text{ N-mm}$$

Step 2 :- calculation of S.F (V)

$$\text{max. S.F. } V = \frac{wL}{2} = \frac{50 \times 10}{2}$$

$$\therefore V = 250 \times 10^3 \text{ N}$$

Step 3 :- section Modulus required (Z_{req})

$$Z_{req} = \frac{M}{\sigma_{bc}}$$

$$\sigma_{bc} = 0.66 f_y$$

$$\therefore \sigma_{bc} = 165 \text{ N/mm}^2$$

$$\therefore Z_{req} = \frac{625 \times 10^6}{165} = 3787.87 \times 10^3 \text{ mm}^3$$

$$\therefore Z_{req} = 3787.87 \times 10^3 \text{ mm}^3$$

Trial section Modulus = $1.5 \times Z$

$$= 1.5 \times 3787.87 \times 10^3$$

$$Z_{req} = 5681.81 \times 10^3 \text{ mm}^3$$

$$Z_{req} = 5681.81 \times 10^3 \text{ mm}^3$$

steps:- selection of suitable steel section:-

Here steel section is selected based on beam depth, Here depth of beam is limited to 500.

Select ISWB: 450 @ 79.4

properties of section:-

$$h = 450 \text{ mm}$$

$$t_f = 15.4 \text{ mm}$$

$$t_w = 9.2 \text{ mm}$$

$$I_{xx} = 35057.6 \times 10^4 \text{ mm}^4$$

$$r_{yy} = 41.1 \text{ mm}$$

$$Z_{xx} = 1558.1 \times 10^3 \text{ mm}^3$$

Dimensions of cover plate:-

The dimensions of cover plate nothing but area of plate

$$A_p = \frac{Z_{req} - Z_{xx}}{h}$$

$$\Rightarrow \frac{5681.81 \times 10^3 - 1558.1 \times 10^3}{450}$$

$$A_p = 9168.81 \text{ mm}^2$$

$$\text{width of the plate} = \frac{A_p}{\text{thickness of cover plate}}$$

let us Assume thickness of plate = 25 mm

$$= \frac{9163.8}{25}$$

$$= 366.55 \text{ mm}$$

$$\approx 366 \text{ mm}$$

∴ Provide the dimensions of cover plate is $366 \times 25 \text{ mm}$

step :- calculation of Bending stress in compression and Tension

Bending stress in compression $\sigma_{b \text{ comp}} = \frac{\text{max B.M}}{\text{Gross M.I}} \times y$

$$y = \frac{500}{2} = 250 \text{ mm}$$

(Including plates)

$$\text{Gross M.I} = I_{xx(\text{table})} + 2A_p \left(\frac{h'}{2}\right)^2$$

$$h' = \frac{450}{2} + \frac{25}{2} = 237.5 \text{ mm}$$

$$\text{Gross M.I} = 35057.6 \times 10^4 + 2 \times 9163.81 \times (237.5)^2$$

$$\text{Gross M.I} = 1.384 \times 10^9 \text{ mm}^4$$

$$\sigma_{b \text{ comp}} = \frac{\text{max. B.M}}{\text{Gross M.I}} \times y$$

$$= \frac{625 \times 10^6}{1.384 \times 10^9} \times 250$$

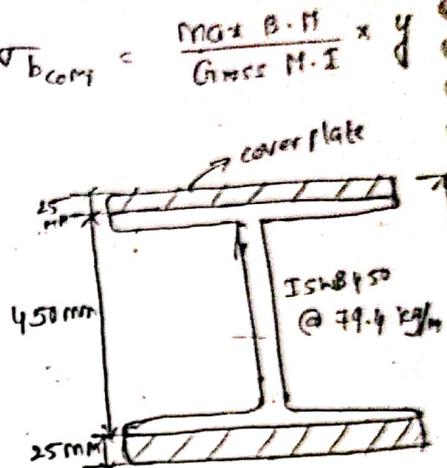
$$\sigma_{b \text{ comp}} = 11286 \text{ N/mm}^2 < 165 \text{ N/mm}^2$$

Bending stress in Tension $\sigma_{b \text{ t}} = \sigma_{b \text{ c}} \times \frac{\text{Gross area}}{\text{Net area}}$

$$\text{Gross area} = (b_f \times t_f) + A_f$$

$$= (200 \times 15.4) + 9163.81$$

$$\text{Gross Area} = 12243.81 \text{ mm}^2$$



$$\text{Net area} = \text{Gross area} - 23.5 (t_f + t_w)$$

$$= 12243.81 - 23.5 (15.4 + 9.2)$$

$$\text{Net area} = 11665.71 \text{ mm}^2$$

$$\sigma_{bt} = \frac{112.86 \times 12243.81}{11665.71}$$

$$\therefore \sigma_{bt} = 118.456 \text{ N/mm}^2 < 165 \text{ N/mm}^2$$

check for shear

$$I_{max} = \frac{5}{384} \cdot \frac{wl^4}{E I_{22} (I_{max})}$$

$$= \frac{5}{384} \cdot \frac{50 \times (10000)^4}{2 \times 10^5 \times 1.384 \times 10^9 \text{ mm}^4}$$

$$= 23.52 \text{ mm}$$

$$\tau_v = \frac{V}{b \times t_w}$$

$$= \frac{250 \times 10^3}{450 \times 1.2}$$

$$\tau_v = 60.38 \text{ N/mm}^2$$

$$< 100 \text{ N/mm}^2$$

$$y_{allow} = \frac{\text{span length}}{325} = \frac{10000}{325} = 30.76 \text{ mm}$$

$$y_{max} < y_{allow}$$

Q:- Design a beam with 6.5 m effective span carrying an u.d.l. 40 kN/m inclusive of self weight over the entire span. The overall depth of the beam is restricted to 850 mm. The compression flange of the beam is laterally supported. Take $f_y = 250 \text{ N/mm}^2$

Step 1 :- calculation of B.M

Given Load ^{u.d.l} $w = 40 \text{ kN/m}$

$$l = 6.5 \text{ m}$$

$$M = \frac{40 \times (6.5)^2}{8} = 211.25 \times 10^6 \text{ N-mm}$$

Step 2 :- calculation of S.F :

$$\text{max S.F } V = \frac{wl}{2} = \frac{40 \times 6.5}{2} = 130 \times 10^3 \text{ N}$$

$$\therefore V = 130 \times 10^3 \text{ N}$$

step 3:- Section Modulus required (Z_{req}):

$$\begin{aligned} Z_{req} &= \frac{M}{\sigma_{bc}} \\ &= \frac{211.25 \times 10^6}{165} \\ &= 1.280 \times 10^3 \text{ mm}^3 \end{aligned}$$

$$\text{Trial} = 1.5 \times Z$$

$$Z_{req} = 1920 \times 10^3 \text{ mm}^3$$

step 4:- selection of suitable steel section:

As the given beam depth is limited to 350 mm.

Therefore the section is selected based on depth of the beam.

select IS MB 300 @ 44.2 kg/m

properties of the section:-

$$h = 300 \text{ mm}, \quad b_f = 140 \text{ mm}$$

$$t_f = 12.4 \text{ mm}$$

$$t_w = 7.5 \text{ mm}$$

$$I_{xx} = 8603.6 \times 10^4 \text{ mm}^4$$

$$Z_{xx} = 573.6 \times 10^3 \text{ mm}^3$$

step 5:- Dimensions of the cover plate

In order to increase the strength of the section we have to attach cover plates both at top & bottom using rivets

Area of the plate $A_f = \frac{Z_{req} - Z_{xy}}{h}$

$$= \frac{1920 \times 10^5 - 573.6 \times 10^5}{300}$$

$$\therefore A_f = 4489.5 \text{ mm}^2$$

width of the plate = $\frac{A_f}{\text{thickness of cover plate}}$

let us assume 25 mm thickness of cover plate

$$\therefore \text{width} = \frac{4489.5}{25} = 179.58$$

$$\approx 180 \text{ mm}$$

\therefore provide dimensions of cover plate 180 mm x 25 mm

Step 6:- calculation of Bending stress in compression and

Tension.

Bending stress in compression $\sigma_{b_{comp}} = \frac{\text{max B.M}}{\text{Gross M.I}}$

$$y = \frac{350}{2} = 175 \text{ mm}$$

(Including plates)

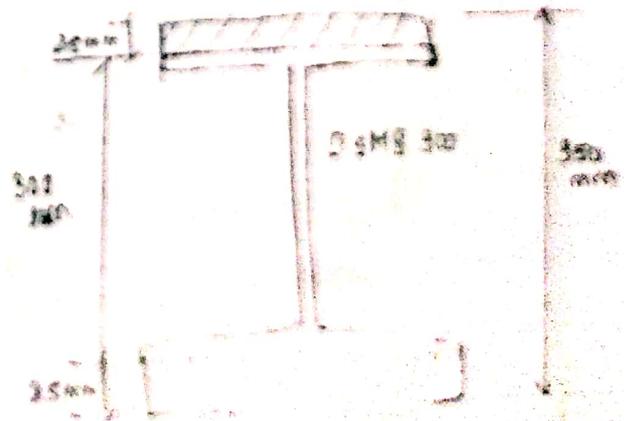
$$\text{Gross M.I} = I_{xx(\text{table})} + 2 A_f \left(\frac{h'}{2}\right)^2$$

$$\frac{h'}{2} = \frac{300}{2} + \frac{25}{2} = 162.5 \text{ mm}$$

$$\frac{h'}{2} = 162.5 \text{ mm}$$

$$\therefore I_{22} = 8603.6 \times 10^4 + 2 \times 4489.5 \times (162.5)^2$$

$$I_{22} = 323.16 \times 10^6 \text{ mm}^4$$



$$\sigma_{b \text{ comp}} = \frac{211.25 \times 10^6}{323.16 \times 10^6} \times 175$$

$$\sigma_{b \text{ comp}} = 114.40 \text{ N/mm}^2 < 165 \text{ N/mm}^2$$

Bending stress in tension $\sigma_{b t} = \sigma_{b \text{ comp}} \times \frac{\text{Gross area}}{\text{Net area}}$

$$\begin{aligned} \text{Gross area} &= (b_f \times t_f) + A_p \\ &= (140 \times 12.4) + 489.5 \\ &= 6225.5 \text{ mm}^2 \end{aligned}$$

$$\begin{aligned} \text{Net area} &= \text{Gross area} - 23.5 (t_f + t_w) \\ &= 6225.5 - 23.5 (12.4 + 9.5) \\ &= 5757.85 \text{ mm}^2 \end{aligned}$$

$$\therefore \sigma_{b t} = 114.40 \times \frac{6225.5}{5757.85}$$

$$\sigma_{b t} = 123.69 \text{ N/mm}^2 < 165 \text{ N/mm}^2$$

step 7:-
check for shear stress

$$\begin{aligned} \tau_v &= \frac{V}{h \times t_w} = \frac{130 \times 10^3}{300 \times 9.5} \\ &= 57.78 \text{ N/mm}^2 < 100 \text{ N/mm}^2 \end{aligned}$$

step 8:-
check for deflection :-

$$\begin{aligned} y_{\text{max}} &= \frac{5}{384} \frac{w l^4}{E I_{22}} \\ &= \frac{5}{384} \times \frac{40 \times (6500)^4}{2 \times 10^5 \times 323.16 \times 10^6} \end{aligned}$$

$$y_{\text{max}} = 14.384 \text{ mm}$$

$$y_{\text{allow}} = \frac{\text{span length}}{325} = \frac{6500}{325} = 20$$

$$y_{\text{max}} < y_{\text{allow}}$$