ENGINEERING THERMODYNAMICS

PURPOSE

This course provides the basic knowledge about thermodynamic laws, relations and their application to various processes.

INSTRUCTIONAL OBJECTIVES

Expected to

1. Understand the thermodynamic laws and their applications.
2. Know the concept of entropy and availability.
3. Know the thermodynamic relations.
4. Have clear idea about the properties of steam and the use of steam tables and Mollier chart.

UNIT – I

Introduction: Basic Concepts: System, Control Volume, Surrounding, Boundaries, Universe, Types of Systems, Macroscopic and Microscopic viewpoints, Concept of Continuum, Thermodynamic Equilibrium, State, Property, Process, Cycle – Reversibility, Quasi – static Process, Irreversible Process, Causes of Irreversibility – Energy in State and in Transition - Types, Work and Heat, Point and Path function.

DEFINITION:

- **Thermodynamics-** The field of science which deals with the energies possessed by gases & vapours. (It also includes the conversion of these energies in terms of heat & mechanical work & their relationship with properties of the system.)

- **Heat Engine-** A machine which converts heat into mechanical work & vice-versa.

UNITS:

Fundamental Units- The measurement of physical quantities.
Eg: Length (l), mass (m), time (t), temperature (T) etc.,
Derived Units- Units expressed in terms of other units & derived from fundamental units.
Eg: mass density (ρ), force (F), weight (W), pressure (p), power (P) etc.,
Systems of units- CGS (cm-gram-sec), FPS (foot-pound-sec,), MKS (m-kg-sec) & SI (international standard)

- **Newton's laws of motion:**

  I-law—It states, Every body continuous in its state of rest or in uniform motion in a straight line, unless it is acted upon by some external forces. (Law of Inertia)

  II-law—It states, The rate of change of momentum is directionally proportional to the impressed force & takes place in the same direction in which the force acts.

  III-law—It states, To every action, there is always an equal & opposite reaction.

**ISO- International Standard Organisation, ISI- Indian Standards Institution,
BIS- Bureau of Indian Standards.**

- **Mass:** It is the amount of matter contained in a given body & does not vary with the change in its position on the earth surface. (Unit-kg)
- **Weight**: It is the amount of matter contained in a given body will also vary with the change in its position on the earth surface. (Unit-N)

$$\boxed{W = mg \quad \text{(or)} \quad m = W/g \quad \text{where: } W\ (N),\ m\ (kg)\ \&\ g\ (m/s^2)}$$

- **Force:** According to Newton's II law of motion, the applied force is directly proportional to the rate of change of momentum.

$F \propto ma$ $F \propto m\,[(v-u)/t]$ where: m = mass, a = acceleration =change of momentum / unit time
(Momentum= mass x velocity) In S.I units: $1\ N = 1\ kg\text{-}m/s^2$
$1\ kg\text{-}f = 9.81 kg\text{-}m/s^2 = 9.81\ N$ Mass, m =100 kg $F = ma = mg = 100 \times 9.81 = 981\ N$

- **Thermodynamic Systems:** It is definite area or space where some thermodynamic process is takes place.
- **Classification of Thermodynamic systems:** i. Closed, ii. Open & iii. Isolated

(i) ***Closed System:*** Fixed mass and identity whose boundaries are determined by the space of the matter occupied in it. Eg:- piston-cylinder.

A closed system does not permit any mass transfer across its boundary, but it permits transfer of energy (heat & work).

(ii) ***Open System (Control Volume :)*** An open system permits both mass & energy (heat & work) transfer across its boundary and mass within the system may not constant.
Eg:- Air compressor (low Pressure air into high pressure air)

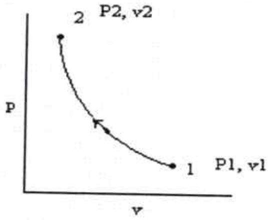
(iii) ***Isolated System (Universe):*** An isolated system does not have transfer of either mass & energy with the surroundings. Eg:- Rare.

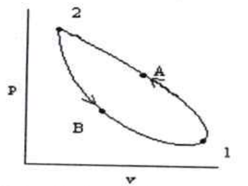
- **Properties of System:** All the observable quantities (p, t, v, ρ etc.), which identify the state of a system are called properties.

(i) **Extensive Properties:** The properties of the system, whose value for the entire system is **equal** to the sum of their value of the individual parts of the system.
Eg:- Total Volume, Total Mass, Total Energy etc.

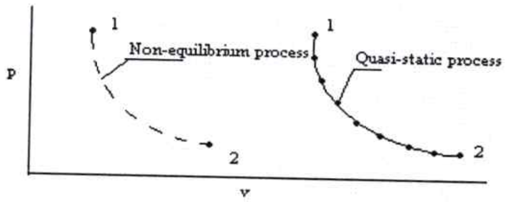
(ii) **Intensive Properties:** The properties of the system, whose value for the entire system is **not equal** to the sum of their value of the individual parts of the system.
Eg:- Pressure, Temperature, Density etc.

- **State of a System:** When the system is in **thermodynamic equilibrium** the condition of the system at any particular moment which can be identified by the statement of its properties such as p, t, v etc.
In Fig. p1, v1, T1.. & p2, v2, T2..





(1-A-2) & (2-B-1): Thermodynamic Processes
(1-A-2-B-1): Thermodynamic Cycle or Cyclic Process



Path of Change of State: When the system is in equilibrium states during a change of state.

Path of the Process: It known as when the path is completely specified.

Thermodynamic Process: In fig.(1-A-2) & (2-B-1)

- **Thermodynamic Cycle:** In fig. (1-A-2-B-1)

(i) **Reversible Cycle**: The initial conditions (heat & work) are restored at the end of the cycle. In this process there should not loss of heat due to friction, radiation or conduction.

(ii) **Irreversible Cycle:** The initial conditions (heat & work) are not restored at the end of the cycle. (It is also known as actual or real process). In this process there is a loss of heat due to friction, radiation or conduction.

- **Quasi-Static Process:** When the process is carried out in such a way that at every instant, the system deviation from the thermodynamic equilibrium is infinitesimal, then the process is known as Quasi-static or Quasi-equilibrium process and each state in the process may be considered as an equilibrium state. (It also known as reversible process).

- **Thermodynamic Equilibrium:**

(i) Mechanical Equilibrium: When there is no unbalanced forces acting on any part of the system.

(ii) Thermal Equilibrium: When there is no temperature difference between the parts of the system or between the system & the surroundings.

(i) Open System : ←

(ii) closed system : ←

(iii) Isolated system : ←

(iv) Adiabatic system: An adiabatic system is one which is thermally insulated from its surroundings. It can, however, exchange work with its surroundings. If it does not, it becomes an isolated system.

(v) Homogeneous system: A system which consists of a single phase is termed as homogeneous system.
Ex: Mixture of air & water vapour, water plus nitric acid..

(vi) Heterogeneous system: A system which consists of two or more phases is called a heterogeneous system.
Ex: Water plus steam, Ice plus water...

(iii) Chemical Equilibrium: When there is no chemical reaction within the system & also there is no movement of any chemical constituent from one part of the system to the other.

- **Temperature:** It is an intensive property, which determines the degree of hotness or the level of heat intensity of a body.

Relation between Celsius scale and Fahrenheit scale: (C/100) = (F-32) / 180 (or) (C/5) = (F-32) / 9

- **Pressure:** The normal force per unit area.

$P_{abs} = P_{atm} + P_g \quad P_{abs} = P_{atm} - P_{vac}$

1 bar = 1×10^5 N/m^2, 1 Pa = 1 N/m^2
1mm of Hg = 133.3 N/m^2

Normal Temperature and Pressure (N.T.P): (0^0c & 760mm of Hg)

Standard Temperature and Pressure (S.T.P): The temp. & pressure of any gas, under standard atmospheric conditions. (15^0c & 760mm of Hg)

- **Law of Conservation of Energy:** It states, Energy can neither be created nor be destroyed, through it can be transformed from one form to any other form, in which the energy can exit.
- **Energy:** Capacity to do work. Types: 1. Stored energy, 2. Transit energy.

1. Stored energy--- The energy possessed by the system with in its boundaries. Eg: Internal, kinetic and potential energy.
2. Transit energy--- The energy possessed by the system which is capable of crossing the boundaries. Eg: Heat, Work, etc..

Potential Energy (P.E): The energy possessed by a body or a system for doing work by virtue of its position above the ground level. P.E = mgZ (N-m)

Kinetic Energy (K.E): The energy possessed by a body or a system for doing work by virtue of its mass and velocity of motion. K.E = $mV^2/2$ (N-m)

Internal Energy (U): The energy possessed by a body or a system due to its molecular arrangement and motion of the molecules. **1N =1kg-m/s^2**

Total Energy: E = P.E + K.E + U

- **Heat (Q):** The energy transformed, without transfer of mass, across the boundary of a system because of a temperature difference between the system and the surroundings.

The heat can transferred in three distinct ways.

a. Conduction: In solids, **b. Convection:** In fluids, &

c. Radiation: It is an electromagnetic wave phenomenon in which energy can be transported through transparent substances & even through a vacuum.

- **Specific Heat (c):** The amount of heat required to raise the temperature of a unit mass of any substance through one degree. (kJ / kg K)

Heat required = m c $(T_2 - T_1)$ kJ

In solids & liquids only one 'c' value because of no change of volume while heating but in gases two 'c' values are there depends upon process. (c_p and c_v). Where always $c_p > c_v$

- **Thermal capacity (or) Heat capacity:** The amount of heat required to raise the temperature of whole mass of a substance through one degree. (kJ). Thermal capacity = m c (kJ)
- **Water equivalent:** The quantity of water which requires the same quantity of heat as the substance to raise its temperature through one degree. (kg). Water equivalent = m c (kg)

OPTIMAL PIN FIN HEAT EXCHANGER SURFACE

ABSTRACT:

- This research presents the results of numerical study of heat transfer and pressure drop in a heat exchanger that is designed with different shape pin fins. The heat exchanger used for this research consists of a rectangular duct fitted with different shape pin fins, and is heated from the lower plate. The pin shape and the compact heat exchanger (CHE) configuration were numerically studied to **maximize the heat transfer and minimize the pressure drop** across the heat exchanger.
- A three dimensional finite volume based numerical model by using analyzing software. The simulation applied to estimate the heat transfer coefficient and pressure drop for a wide range of Reynolds numbers with different pin fins.
- Circular pin configuration variations included changes in pin spacing, axial pitch and pin height ratio.
- Rectangular and drop-shaped pin variations also included changes in length and aspect ratio.
- The data and conclusions of this study can be applied to the optimization of different heat exchangers which are used in industry, especially oil cooler in power transformers which are currently working with low cooling efficiency. It can also be used in the design of electronic components, turbine blade cooling or in other high heat flux dissipation applications.

t =

A fluid at a pr. of 3 bar & with specific volume of 0.18 m^3/kg, contained in a cylinder behind a piston expands reversibly to a pr. of 0.6 bar. Acc. to a law $p = \frac{c}{v^2}$, where 'c' is a const.. Calculate the weight by the fluid on the piston.

Soln: $W = \int_{v_1}^{v_2} p.dv = \int_{v_1}^{v_2} \frac{c}{v^2} dv = c\left[\frac{v^{-2+1}}{-2+1}\right]_{v_1}^{v_2} = c\left[\frac{1}{v_1} - \frac{1}{v_2}\right]$

$p = \frac{c}{v^2} \Rightarrow pv^2 = c \Rightarrow P_1 V_1^2 = 3 \times 0.18^2 = 0.0972 \ (m^3/kg)^2$

$\text{III}^{ly} \ c = P_2 V_2^2 = V_2 = \sqrt{\frac{c}{P_2}} = \sqrt{\frac{0.0972}{0.6}} = 0.402 \ m^3/kg$

$\therefore W = 0.0972 \times 10^5 \left[\frac{1}{0.18} - \frac{1}{0.402}\right] = 29840 \ N\text{-}m/kg$

Relationship between C_p & C_v :-

Consider a perfect gas being heated at const. pr. from T_1 to T_2.

Acc. to Non-flow equation: $Q = W + \Delta U$

also for a perfect gas, $\Delta U = m.C_v(T_2 - T_1)$

$\therefore Q = W + m.C_v(T_2 - T_1)$

In a const. pr. process, the workdone by the fluid, $W = P.dv$
$= P(V_2 - V_1)$

$\Rightarrow W = m.R(T_2 - T_1)$ $[\because PV = mRT]$

$\therefore Q = m.R(T_2 - T_1) + m.C_v(T_2 - T_1) = m(R + C_v)(T_2 - T_1) \rightarrow ①$

But for a const. pr. process; $Q = m.C_p(T_2 - T_1) \rightarrow ②$

Equating ① & ②; $R + C_v = C_p \Rightarrow \boxed{C_p - C_v = R}$

$\left\{\because \gamma = \text{Ratio of sp. heats. } \gamma = \frac{C_p}{C_v}\right\}$

$\div$ by C_v; $\frac{C_p}{C_v} - 1 = \frac{R}{C_v} \Rightarrow \gamma - 1 = \frac{R}{C_v} \Rightarrow \boxed{C_v = \frac{R}{\gamma - 1}}$

$\div$ by C_p; $1 - \frac{C_v}{C_p} = \frac{R}{C_p} \Rightarrow 1 - \frac{1}{\gamma} = \frac{R}{C_p} \Rightarrow \boxed{C_p = \frac{\gamma . R}{\gamma - 1}}$

LAWS OF THERMODYNAMICS:

Zeroth Law:- [Basis of temperature measurement.]

It states that when two bodies are seperately in thermal equilibrium with a 3rd body, then they are in thermal equilibrium with each other.

Thermometer	Thermometric Property
> Const. Volume gas	Pressure (P)
> Const. Pressure gas	Volume (V)
> Alcohol (or) Hg in glass	Length (L)
> Electric Resistance	Resistance (R)
> Thermocouple	EMF (E)
> Radiation [Pyrometer]	Intensity of Radiation (I (or) J)

← Six different kinds of thermometers, and the names of the corresponding thermometric properties.

The thermometers are classified as follows:-

(i) Expansion thermometers:
  (a) Liquid in glass thermometers.
  (b) Bimetallic thermometers.
(ii) Pressure thermometers:
  (a) Vapour Pressure thermometers.
  (b) Liquid-filled thermometers.
  (c) Gas-filled thermometers.
(iii) Thermocouple thermometers,
(iv) Resistance thermometers.
(v) Radiation Pyrometers &
(vi) Optical Pyrometers.

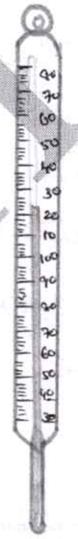
Note: Instruments for measuring ordinary temp. are known as : Thermometers.
" " " high temp. " : Pyrometers.

1. Expansion Thermometer's:

It make use of the differential expansion of two different substances. Thus in liquid in glass thermometers, it is the difference in expansion of liquid & the containing glass. In Bi-metallic thermometers, the indication is due to the difference in expansion of the two solids.

(a) Liquid-in-glass Thermometer's:

This is a very familiar type of thermometer. The mercury (or) other liquid fills the glass bulb & extends into the bore of the glass stem. Mercury is most suitable liquid & is used from -38.9°c to about 600°c



Liquid	Boiling point	Freezing point
Pentane	36°c	-130°c
Ethyl Alcohol	78°c	-100°c
Toluene	110°c	-92°c

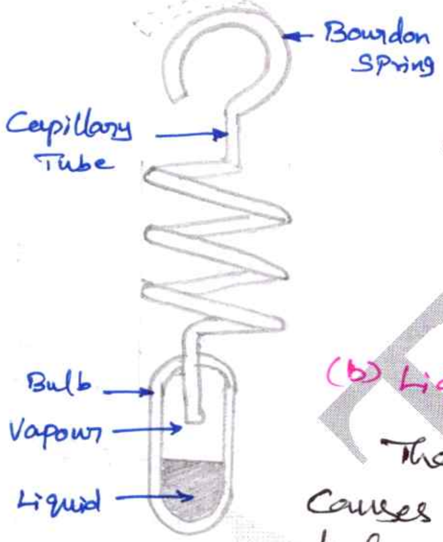
→ These liquids are normally colourless. A dye is added to facilitate reading.

(b) Bi-Metallic Thermometers:

In a bi-metallic thermometers differential expansion of bimetallic strips is used to indicate the temperature. It has the advantage over the liquid-in-glass thermometer's, that it is less fragile & is easier to read. In this type, two flat strips of different metals are placed side by side & are welded together. Generally one is a low expanding metal & the other is high expanding metal. The bimetal strip is coiled in the form of a sprial (or) helix. Due to rise in temperature, the curvature of the strip changes. The differential expansion of a strip causes the pointer to move on the dial of the thermometer.

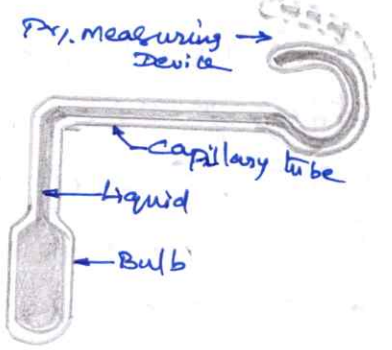
2. Pressure thermometers: In pressure thermometers liquids, gases & vapours can all be used. The principle on which they work is quite simple. The fluid is confined in a closed system (V=C). In this case the pressure is a function of the temperature, so that when the fluid is heated, the pressure will rise & the temperature can be indicated by Bourdon type pressure gauge. In general, the thermometer consists of a bulb which contains the fluid (Hg). The bulb is placed in the region whose temperature is required. A capillary tube connects the bulb to a Bourdon tube, which is graduated with a temperature scale.

(a). Vapour pressure thermometers:



Some of the fluid is vapourizes & increases the vapour pressure. This change of pressure is indicated on the Bourdon tube. The relation between temperature & vapour pressure of a volatile liquid is of the exponential form. Therefore, the scale of a pressure thermometer will not be linear.

(b) Liquid-filled thermometers:

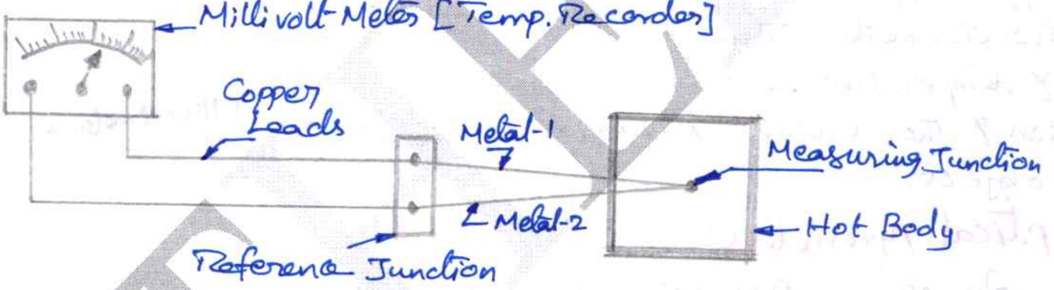


The expansion of the liquid causes the pointer to move in the dial. Therefore liquids having high co-efficient of expansion should be used. Hg, Alcohal, Toulene & Glycerine have been successfully used in practice. The operating pressure varies from about 3 to 100 bar. These type of thermometers could be used for a temperature upto 650°C in which Hg could be used as the liquid.

To minimize the effect of variation in temperature to which the capillary tube is subjected, the volume of the bulb is made as large as possible as compared with the volume of the capillary.

(c) Gas-Filled Thermometers: It is similar to liquid-filled thermometers. The Nitrogen & helium gases used as general. Both these gases are chemically inert, have good values for their co-efficient of expansion & low specific heats. For good performance the bulb volume should be made at least 8 times than that of the rest of the system. It's generally used for pressure below 35 bar.

3. Thermocouple Thermometers: It's suitable for higher range of temperature (above 650°c). It consists of two dissimilar metals (or) alloys which develop e.m.f. when the reference & measuring junctions are at different temperatures. The reference junction (or) cold junction is usually maintained at some constant temperature 0°c.

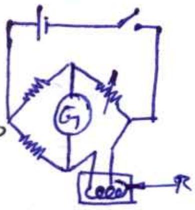


In many industrial installations the instruments are equipped with automatic compensating devices for temperature changes of the reference junction, thus eliminating the necessity of maintaining this junction at constant temperature.

4. Resistance Thermometers: (Below 150°c)

It consists of a resistance element (or) bulb, electrical leads & a resistance measuring instrument. The resistance element (temp. sensitive element) is usually supplied by the manufacturers with its protecting tube & is ready for electrical connections. The resistance element should be reproducible at any given temperature. The resistance element The resistance is reproducible if the composition (or) physical properties of the metal do not change with temperature. The fine resistance wire is wound in a spiral form on a mica frame.

The delicate coil is then enclosed in a porcelain (or) quartz tube. The change of resistance of this unit can be measured by instruments such as wheatstone bridge, potentiometer (or) galvanometer.

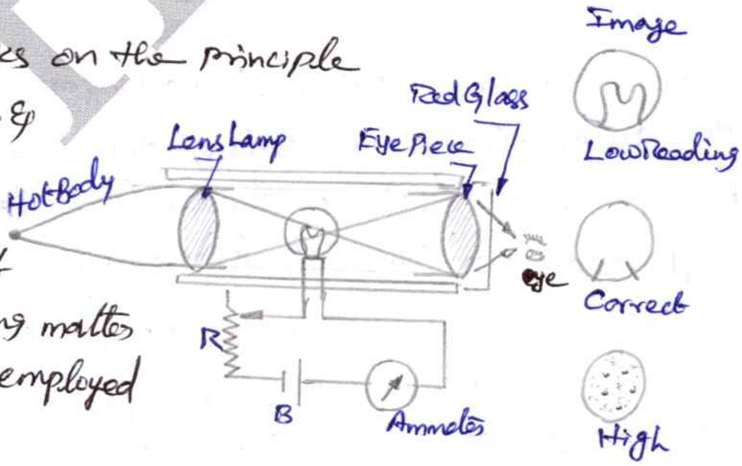


5. Radiation pyrometers:

A device which measures the total intensity of radiation emitted from a body is called radiation pyrometer. It collects the radiation from an object (hot body) whose temperature is required. A mirror is used to focus this radiation on a thermocouple. This energy which is concentrated on the thermocouple raises its temperature & in turn generates an e.m.f. This e.m.f. is then measured either by the galvanometer (or) potentiometer method. Thus rise of temperature is a function of the amount of radiation emitted from the object.

6. Optical pyrometers:

An optical pyrometer works on the principle that matters glow above 480°C & the colour of visible radiation is proportional to the temperature of the glowing matter. The amount of light radiated from the glowing matter (solid or liquid) is measured & employed to determine the temperature.



Operation: > The optical pyrometer is sighted at the hot body & focused.

> In the beginning filament will appear dark as compared to the background which is bright (being hot).

> By varying the resistance (R) in the filament circuit more & more current is fed into it, till filament becomes equally bright as the background & hence disappears.

> The current flowing in the filament at this stage is measured with the help of an ammeter which is calibrated directly in terms of temperature.

> If the filament current is further increased, the filament appears brighter as compared to the background which then looks dark.

> An optical pyrometer can measure temperature ranging from 700°c to 4000°c.

Methods in Use after 1954!

Since 1954 only one fixed point has been in use, viz. the triple point of water, the state at which ice, liquid water & water vapour co-exist in equilibrium. The temperature at which this state exists is arbitrarily assigned the value of 273.16 K. Designating the triple point of water by θ_t & with X_t being the value of the thermometric property when the body, whose temperature θ is to be measured, is placed in contact with water at its triple point, it follows that,

$$\theta_t = aX_t \Rightarrow a = \frac{\theta_t}{X_t} = \frac{273.16}{X_t}$$

$$\therefore \theta = a.X = \frac{273.16X}{X_t}$$

The temperature of the triple point of water, which is an easily reproducible state, is now the standard fixed point of thermometry.

Comparison of Thermometers:

1. Const. Volume Gas thermometer, $\theta(P) = 273.16\frac{P}{P_t}$
2. Const. pressure Gas thermometer, $\theta(V) = 273.16\frac{V}{V_t}$
3. Electric Resistance thermometer, $\theta(R) = 273.16\frac{R}{R_t}$
4. Thermocouple, $\theta(\varepsilon) = 273.16\frac{\varepsilon}{\varepsilon_t}$
5. Liquid in glass thermometer, $\theta(L) = 273.16\frac{L}{L_t}$

Temperature scale:

Based on the available method of measurement, the whole temp. scale may be divided into 4 ranges. The equations for interpolation for each range are as follows:

(1) From 0 to 660°C: A platinum resistance thermometer with a platinum wire whose diameter must lie between 0.05 & 0.20 mm is used & the temperature is given by the eqn:

$$R = R_0(1+At+Bt^2)$$

where, the constants R_0, A & B are computed by measurements at the ice point, steam point & sulphur point.

(2) From -190 to 0°C: The same platinum resistance thermometer is used & the temperature is given by,

$$R = R_0(1+At+Bt^2+C(t-100)t^3)$$

where, R_0, A & B are the same as before & C is determined from a measurement at the oxygen point.

(3) From 660 to 1063°C: A thermocouple, one wire of which is made of platinum & the other of an alloy of 90% plantinum & 10% rhodium, is used with one junction at 0°C. The temperature is given by the formula:

$$E = a+bt+ct^2$$

where, a, b & c are computed from measurements at the antimony point, silver point & gold point. The dia. of each wire of the thermocouple must lie between 0.35 to 0.65 mm.

(4) Above 1063°C: It is based on the intensity of radiation J_T @ temperature T emitted by a black body at a wavelength λ in the visible spectrum & by comparing this to the intensity of radiation J_{Au} at the same wavelength emitted by a black body at the gold point.

The temperature is calculated from planck's eqn. for black body radiation:

$$\frac{J_T}{J_{Au}} = \frac{\exp\left(\frac{C_2}{\lambda T_{Au}}\right)-1}{\exp\left(\frac{C_2}{\lambda T}\right)-1}$$

where; $C_2 = 0.01438$ °c

λ = wavelength (m).

A temperature scale of certain thermometer is given by the relation: $t = a \ln p + b$, where a & b are Constants & p is the thermometric property of the fluid in the thermometer. If at the ice point & steam point the thermometric properties are found to be 1.5 & 7.5 resp/. what will be the temp/. corresponding to the thermometric property of 3.5 on Celsius scale.

Soln: $t = a \ln p + b$

on celsius scale: Ice point = 0°c & steampoint = 100°c

∴ From the given conditions, we have;

$0 = a \ln 1.5 + b \quad ; \quad 100 = a \ln 7.5 + b$

$\Rightarrow 0 = a \times 0.4054 + b \qquad 100 = a \times 2.015 + b$

solving the eqn; ① & ②; $a = 62.112$

$\therefore b = -25.18$

∴ when, $p = 3.5$ the value of temp/. is given by;

$t = 62.112 \ln 3.5 - 25.18$

$t = 52.63°c$

IIIles problem: $t = a \ln k + b$, 0° & 100°c : Ice & steam point resp/.

K : 1.83 & 6.78

Determine the temp. corresponding to a reading of K equal to 2.42

Ans: $t = 21.346°c$

Find the temperature which has the same value on both the Celsius & Fahrenheit scales.

Soln: x = Temperature which has the same value on both Celsius & Fahrenheit scale.

WKT; $\frac{C}{5} = \frac{F-32}{9} \Rightarrow 9C = 5(F-32)$

$9x = 5(x-32)$

$\therefore \Rightarrow x = -40$

Hence −40° on the celsius scale is equal to −40° on the Fahrenheit scale.

Absolute Temperature:

The absolute temperature in celsius scale is called (°K) $(K = {}^\circ C + 273)$

IIIly; The absolute temperature in Fahrenheit scale is called °Rankine (°R) $({}^\circ R = {}^\circ F + 460)$

The properties of a closed system change following the relation between pr. & volume as PV = 3, where; P is in bar, V is in m^3? Calculate the workdone when the pressure increases from 1.5 bar to 7.5 bar.

Soln:

$$PV = 3 \Rightarrow V_1 = \frac{3}{P_1} = \frac{3}{1.5} = 2\,m^3$$

$$\therefore V_2 = \frac{3}{P_2} = \frac{3}{7.5} = 0.4\,m^3$$

$$W = \int_{V_1}^{V_2} P\,dV = \int_{V_1}^{V_2} \frac{PV}{V} \cdot dV = 3\times10^5 \left[\log_e V\right]_{2}^{0.4}$$

$$W = 3\times10^5 \left[\log_e 0.4 - \log_e 2\right]$$

$$W = -4.83\times10^5\,J = -483\,kJ$$

A barometer reads 76cm of Hg. What would be the absolute pressure if;

(i) a pressure gauge connected to turbine inlet reads 28 bar &

(ii) a vaccum gauge connected in the exhaust line of the same turbine reads 70 cm of Hg.

Soln: 1 mm of Hg $= 133.3\ N/m^2$

Barometer reads $= 76$ cm of Hg

$= 760 \times 133.3 = 0.102 \times 10^6\ N/m^2$

(i) $P_{abs} = P_{atm} + P_g = (0.102 \times 10^6) + 28 \times 10^5$

$= 29.02 \times 10^5\ N/m^2$ (Pa)

$= 29.02$ bar.

(ii) $P_{abs} = P_{atm} - P_v = (0.102 \times 10^6) - (700 \times 133.3)$

$= 7998\ N/m^2 = 0.08$ bar

Determine the workdone by the air which enters into an evacuated vessel from atmosphere when the valve is opened. The atm. Pr, is 1.013 bar & $1.5 m^3$ of air at atm. condition enters into the vessel.

Soln: $dW = p.dv \Rightarrow W = \int_{v_1}^{v_2} P.dv = P\int_{1.5}^{0} dv$

$= 1.013 \times 10^5 (0 - 1.5)$

$= -1.5195 \times 10^5 J = -1.5195 kJ$

Since the free air boundary is contracting, the workdone by the system is -ve & the surroundings do +ve work upon the system.

A mass of 1.5kg of air is compressed in a quasi-static process from 0.1 MPa to 0.7 MPa for which PV = constant. The initial density of air is $1.16 kg/m^3$. Find the workdone by the piston to compress the air!

Soln: $m = 1.5$ kg, $P_1 = 0.1$ MPa $= 0.1 \times 10^6$ Pa $= 0.1 \times 10^6\ N/m^2$

$\rho_1 = 1.16 kg/m^3$, $P_2 = 0.7$ MPa $= 0.7 \times 10^6\ N/m^2$

WKT;

$Pv = c$; $W_{1-2} = 2.3 P_1 V_1 \log \frac{V_2}{V_1}$

where; $P_1 V_1 = P_2 V_2$

$\because \rho_1 = \frac{m}{V_1}$

$\Rightarrow V_1 = \frac{m}{\rho_1} = \frac{1.5}{1.16}$

$0.1 \times 1.29 = 0.7 V_2$

$\Rightarrow V_2 = \underline{\underline{0.185 \text{ m}^3}}$

$V_1 = \underline{\underline{1.29 \text{ m}^3}}$

$\therefore W_{1-2} = 2.3 \times (0.1 \times 10^6 \times 1.29) \log \left(\frac{0.185}{1.29}\right)$

$= \underline{\underline{-0.2504 \text{ J}}}$ (or) $\underline{\underline{-250.4}}$ kJ = 250.4 kJ

_ve sign indicates compression.

A fluid at a pressure of 3 bar & with specific volume of 0.18 m³/kg, contained in a cylinder behind a piston expands reversibly to a pressure of 0.6 bar acc. to law $p = \frac{c}{V^2}$, where c is a constant. Calculate the workdone by the fluid on the piston.

<u>Soln:</u> $P_1 = 3 \text{ bar} = 3 \times 10^5 \text{ N/m}^2$, sp. volume; $= 0.18 \text{ m}^3/\text{kg}$,

$P_2 = 0.6 \text{ bar} = 0.6 \times 10^5 \text{ N/m}^2$, $P = \frac{c}{V^2} \Rightarrow PV^2 = c$; $\therefore n = 2$.

$PV^n = c \rightarrow$ polytropic process.

$W_{1-2} = \frac{P_2 V_2 - P_1 V_1}{n-1}$

$V_1 = 0.18 \text{ m}^3$ (per unit mass)

WKT; $\frac{V_1}{V_2} = \left(\frac{P_2}{P_1}\right)^{\frac{1}{n}}$

$\Rightarrow V_2 = \frac{0.18}{(0.6/3)^{1/2}} = \underline{0.402 \text{ m}^3}$

$\therefore W = \frac{(0.6 \times 10^5 \times 0.402) - (3 \times 10^5 \times 0.18)}{2-1}$

$= -29880 \text{ J}$ (or) $\underline{-29.9 \text{ kJ}}$

First law of TD: It states that, "When a system undogoes a thermodynamic cycle then the net heat supplied to the system from the surroundings is equal to net workdone by the system on its surroundings."

(TD-II Law)

(i.e) $\oint dQ = \oint dW$.

(or)

Heat & work are mutually convertible but since energy can neither be created nor destroyed, the total energy associated with an energy conversion remains constant.

$Q = W + \Delta U$. (Possible)

Limitations of First law:

→ It does not specify the direction of flow of heat & work. (i.e) whether the heat flow from hot body to cold body (or) vice-versa.

→ The heat & work are mutually convertible. But actually Q - cannot be converted fully into work. This violates the foresaid statement.

A machine which violates the I law of TD is known as PMM-I [Perpetual Motion Machine] of the first kind which is impossible.

> A closed system receives an I/p heat of 450 kJ & increases the internal energy of the system for 325 kJ. Determine the workdone by the system?

G.D: $Q = 450$ kJ ($+^{ve}$), $\Delta U = 325$ kJ.

soln: By First law of TD; $Q = W + \Delta U$

$\therefore W = Q - \Delta U = 450 - 325 = 125$ kJ

Workdone by the system, $W = 125$ kJ.

> During the compression stroke of reciprocating compressor, the workdone to the air in the cylinder is 95 kJ/kg & 43 kJ/kg of heat is rejected to the surroundings. Determine the change in internal energy?

G.D: Workdone to the system (cylinder) (∴ −ve sign.)

$\Rightarrow W = -95$ kJ/kg.

Heat is rejected from the system (or) to the surrounding (∴ −ve sirgn.)

$\Rightarrow Q = -43$ kJ/kg.

Soln: $Q = W + \Delta U \Rightarrow \Delta U = Q - W$

$= -43 - (-95) = 52$ kJ/kg.

> Calculate the distance moved by a locomotive from consuming 2 tonne of coal if 10% of the heat generated by the coal is converted into coal gas then into work. The tractive effort reqd. is 30 N/tonne of dead mass of the locomotive where the dead mass is 2500 tonnes. Assume 1 kg of coal liberates 35000 kJ of heat on burning.

G.D: mass of coal, m_c = 2 tonne = 2000 kg.

W = 10% Q

Tractive effort = 30 N/tonne

Mass of the locomotive = 2500 tonnes.

CV = 35000 kJ/kg.

Soln:

W = Force × Distance.

$Q = m_c \times CV$

$= 2000 \times 35000 = 70 \times 10^6$ kJ

$\therefore W = \frac{10}{100} \times 70 \times 10^6 = 70 \times 10^5$ kJ

$W = \underline{\underline{70 \times 10^8}}$ J

⇒ Force, F :-

F = tractive effort × mass of locomotive

$= 30 \times 2500 = \underline{\underline{75000}}$ N

$\Rightarrow W = F \times D$

$\Rightarrow 70 \times 10^8 = 75000 \times$ Distance

$\Rightarrow$ Distance $= 93.333 \times 10^3$ m $= \underline{\underline{93.3}}$ km

Law's of Perfect Gases:

The physical properties of a gas are controlled by P, V & t. variable's.

The behaviour of a perfect gas, undergoing any change in the above variables, is governed by the following laws.

(i) Boyle's law: The absolute pr. of a given mass of a perfect gas varies inversely as its volume, when the temp. remains constant. (i.e) $P \propto \frac{1}{V}$ (or) $PV = \text{const.}$

$$\Rightarrow P_1 V_1 = P_2 V_2 = \dots\dots = \text{const.}$$

Robert Boyle (1662)

(ii) Charle's law; The vol. of a given mass of a perfect gas varies directly as its abs. temp., when the abs. pr. remains constant. (i.e) $V \propto T$ (or) $\frac{V}{T} = \text{const.}$

$$\Rightarrow \frac{V_1}{T_1} = \frac{V_2}{T_2} = \dots\dots = \text{const.}$$

Jacques A.C. Charles 1787

(iii) Gay-Lussac law; The abs. pr. of a given mass of a perfect gas varies directly as its abs. temp, when the vol. remains constant. (i.e) $P \propto T$ (or) $\frac{P}{T} = \text{Const.}$

$$\Rightarrow \frac{P_1}{T_1} = \frac{P_2}{T_2} = \dots\dots = \text{Const.}$$

(iv) General Gas Equation. The above gas laws, which gives us the relation btw. the two variables when the third variable remains const. But in actual practice, all the 3 variables are change simultaneously.

In order to deal with all practical cases, the Boyles law & charles law are combined together, which give us a "General Gas Equation".

Acc. to Boyle's law; & Acc. to Charles law;

$P \propto \frac{1}{V}$ (or) $V \propto \frac{1}{P}$ $\qquad V \propto T$

$$\therefore \Rightarrow V \propto \frac{T}{P} \Rightarrow PV \propto T \Rightarrow PV = C \cdot T$$

$$\Rightarrow \frac{PV}{T} = \text{const.}$$

$$\Rightarrow \frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} = \dots\dots = \text{Constant.}$$

(v) Joule's law:

It states that, 'the change of internal energy of a perfect gas is directly proportional to the change of temp/.

(i.e) $du \propto dT$ $\quad (\because du \text{ (or) } dE$ (or ΔU))

$$du = m.c.dT$$

$$du = m.c(T_2 - T_1)$$

where; c is const. of proportionality, known as specific heat.

(vi) Characteristic Equation of Gas:

It is modified form of general gas equation.

If the volume (V) taken as sp. vol. (V_s) & constant (c) taken as of characteristic gas const. (R)

; for any mass 'm' kg of a gas, the eqn. becomes;

$$m.p.v_s = m.R.T \quad [\text{where; } R = 287 \text{ J/kg.k.}]$$

$$\Rightarrow \boxed{p.V = mRT}$$

Enthalpy of a Gas:

The sum of internal energy & the product of pr. & vol., is known as enthalpy (H).

(ie) $\boxed{H = U + PV}$

{Enthalpy is made up entirely of properties U, P & V. ∴ H is also a property}

For unit mass,

sp. enthalpy, $\boxed{h = u + pv_s}$

WKT;

$$Q_{1-2} = W_{1-2} + dU$$

$$= p.dv + dU.$$

$$= p(v_2 - v_1) + (U_2 - U_1)$$

$$\boxed{Q_{1-2} = (U_2 + Pv_2) - (U_1 + Pv_1) = H_2 - H_1}$$

For unit mass; $\boxed{q_{1-2} = h_2 - h_1}$

FIRST LAW of Thermodynamics:

It states that:

"Heat & Work are mutually convertible but since energy can neither be created nor destroyed, the total energy associated with an energy conversion remains constant."

This law is based on "Joules experiment."

$$Q = \Delta E + W \longrightarrow \int^{ing};\quad Q_{1-2} = E_2 - E_1 + W_{1-2}$$

$$(\delta Q = \Delta E + \delta W \ (kJ))$$

$$Q_{1-2} - W_{1-2} = E_2 - E_1$$

for Unit mass; $q_{1-2} - w_{1-2} = e_2 - e_1$

where; $E_1 = PE_1 + KE_1 + U_1$

$$= mgz_1 + \frac{mv_1^2}{2} + U_1$$

|||ly $E_2 = mgz_2 + \frac{mv_2^2}{2} + U_2$

$$\therefore Q_{1-2} - W_{1-2} = mg(z_2 - z_1) + \frac{m}{2}(v_2^2 - v_1^2) + (U_2 - U_1)$$

for Unit mass; $q_{1-2} - w_{1-2} = g(z_2 - z_1) + \frac{1}{2}(v_2^2 - v_1^2) + (u_2 - u_1)$

If, There is no change in Potential Energy (PE) [From datum same height]

$$Q_{1-2} - W_{1-2} = \frac{m}{2}(v_2^2 - v_1^2) + (U_2 - U_1)$$

If; there is no change of PE & also there is no flow; (∴ PE = 0, KE = 0)

$$Q_{1-2} - W_{1-2} = (U_2 - U_1) = dU$$ → Non-Flow Energy Equation.

for an Isolated system; (Q = 0, W = 0)

$\therefore E_2 = E_1$ → It show's I Law of Thermodynamics, is the law of conservation of energy.

——·——

Perpetual Motion Machine of the First kind:-

PMM-I: There can be no machine which would continously supply mechanical work without some other form of energy disappearing simultaneously. Such a fictitious machine is called PMM-I. A PMM-I is thus impossible.

A stationary mass of gas is compressed without friction from an initial state of 0.3 m^3 & 0.105 MPa to a final state of 0.15 m^3 & 0.105 MPa. The pressure remains const. during the process. There is a transfer of 37.6 kJ of heat from the gas during the process. How much does the internal energy of the gas change?

Soln:- Acc. to I law; $Q_{1-2} = W_{1-2} + \Delta U$

$$\Rightarrow W_{1-2} = \int_1^2 P \cdot dv = P(V_2 - V_1) = 0.105(0.15 - 0.3) = -15.75 \text{ kJ}$$

$$\therefore -37.6 = -15.75 + \Delta U$$

$$\Rightarrow \Delta U = (U_2 - U_1) = -21.85 \text{ kJ}$$

The internal energy of gas decreases by 21.85 kJ in the process.

A mass of 0.25 kg of air in a closed system from 2 bar, 60°c to 1 bar, 40°c while receiving 1.005 kJ of heat from a reservoir at 100°c. The surrounding atm. is at 0.95 bar & 27°c. Determine the max. work. How much of this work would be done on the atm.?

Soln:-

$m = 0.25 \text{ kg}$

$P_1 = 2 \text{ bar} = 2 \times 10^5 \text{ N/m}^2$

$T_1 = 60°c = 333 \text{ k}$

$P_2 = 1 \text{ bar} = 1 \times 10^5 \text{ N/m}^2$

$T_2 = 40°c = 313 \text{ k}$

$Q = 1.005 \text{ kJ}$

$T_R = 100°c = 373 \text{ k}$

$P = 0.95 \text{ bar} = 0.95 \times 10^5 \text{ N/m}^2$

$T = 27°c = 303 \text{ k}$

Workdone on the atm.:

$$W_1 = \int p\,dv = P(V_2 - V_1)$$

where; $P_1 V_1 = m R T_1$ $(\because R = 287 \text{ J/kg.k})$

$$\Rightarrow V_1 = \frac{0.25 \times 287 \times 333}{2 \times 10^5} = 0.119 \text{ m}^3$$

IIIly;

$$V_2 = \frac{0.25 \times 287 \times 313}{1 \times 10^5} = 0.224 \text{ m}^3$$

∴ $W_1 = 0.95\times10^5(0.224-0.119) = 9.975\times10^3\ J = 9.975\ kJ$

We know that; $\Delta U = m.C_v(T_2-T_1)$

$= 0.25\times0.71(313-333)$

$= -3.56\ kJ$

$\left\{\begin{array}{l}\text{for air; } C_p = 1 \\ C_v = 0.71\end{array}\right\}$

−ve sign indicates decrease a internal energy.

Net workdone; $W_2 = Q - dU$ (∵ $Q = W + dU$)

$= 1.005 - (-3.56) = 4.565\ kJ$

∴ Max. workdone; $W = W_1 + W_2 = 9.975 + 4.565 = 14.54\ kJ$

A fluid is confined in a cylinder by a spring loaded frictionless piston so that the pressure in the fluid is a linear function of the volume ($P = a + bV$). The internal energy of the fluid is given by the following eqn: $U = 33.5 + 3PV$. where 'U' is in kJ, 'P' in kN/m² & 'V' in m³. If the fluid changes from an initial state of 1.7 bar, 0.03 m³ to a final state of 4 bar, 0.06 m³, with no work other than that done on the piston. Find the direction & magnitude of the work & heat transfer.

Soln:

$P = a + bV$

$U = 33.5 + 3PV$

$P_1 = 1.7$ bar $= 170\ kN/m^2$

$V_1 = 0.03\ m^3$

$P_2 = 4$ bar $= 400\ kN/m^2$

$V_2 = 0.06\ m^3$

WKT; $dU = U_2 - U_1 = (33.5 + 3P_2V_2) - (33.5 + 3P_1V_1)$

$= 3(P_2V_2 - P_1V_1)$

$= 3[(400\times0.06) - (170\times0.03)] = 56.7\ kJ$

We also know that; $P = a + bV$

for initial state; $P_1 = a + bV_1 \Rightarrow 170 = a + 0.03b \rightarrow ①$

for final state; $P_2 = a + bV_2 \Rightarrow 400 = a + 0.06b \rightarrow ②$

from eqn. ① & ②; $b = 7667$; $a = -60$

∴ $W_{1-2} = \int_{V_1}^{V_2} P.dV = \int_{V_1}^{V_2} (a + bV)\,dV = \left[aV + \frac{bV^2}{2}\right]_{V_1}^{V_2}$

$\Rightarrow a(V_2 - V_1) + \frac{b}{2}(V_2^2 - V_1^2) = -60(0.06 - 0.03) + \frac{7667}{2}(0.06^2 - 0.03^2) = 8.55\ kJ$

∴ Heat transfer, $Q_{1-2} = W_{1-2} + dU = 8.55 + 56.7 = 65.25\ kJ$. Since both Q & W are +ve.

∴ Workdone is by the fluid & heat is supplied to the fluid.

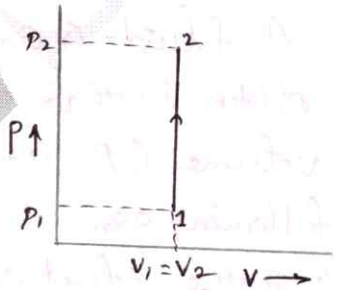
Heating & Expansion of gases in Non-flow Processes:

1. Reversible non-flow processes.
   - a. Const. Volume Process (Isochoric)
   - b. Const. pressure Process (Isobaric)
   - c. Const. Temperature Process (Isothermal)
   - d. Hyperbolic Process
   - e. Adiabatic process (Isentropic) &
   - f. Polytropic process.
2. Irreversible non-flow processes.

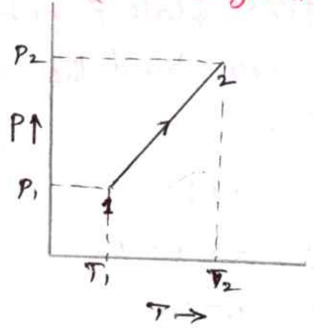
Cooling & compression is regarded as negative heating & negative expansion.

a. Constant Volume Process:

This Process is governed by Gay-Lussac Law.



(P-V)-diagram



(P-T)-diagram

(i) P-V-T relationship:

WKT; general gas eqn: $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$ $(\because V_1 = V_2)$

$\therefore \frac{P_1}{T_1} = \frac{P_2}{T_2}$ (or) $\frac{P}{T}$ = Constant.

(ii) Workdone by the gas:

$\delta W = P.dV$

$\int_1^2$ing

$W_{1-2} = P(V_2 - V_1) = 0$ $(\because V_1 = V_2)$

(iii) change in Internal energy: $dU = m.C_v.dT$

$\int_1^2$ing

$U_2 - U_1 = m.C_v(T_2 - T_1)$

(iv) Heat supplied (or) heat transfer:

$\int_1^2$ing

$\delta Q = \delta W + dU$

$Q_{1-2} = W_{1-2} + (U_2 - U_1)$

$Q_{1-2} = m.C_v(T_2 - T_1)$

(v) Change in enthalpy: $dH = dU + d(PV)$

$\int_1^2$ing

$H_2 - H_1 = (U_2 - U_1) + (P_2V_2 - P_1V_1)$

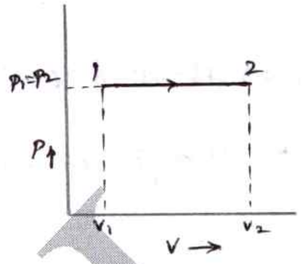
$= m.C_v(T_2 - T_1) + mR(T_2 - T_1)$

$H_2 - H_1 = m.C_p(T_2 - T_1)$ $(\because C_p - C_v = R)$

b. Constant Pressure Process :

This process is governed by Charle's Law.

(i) P-V-T relationship:

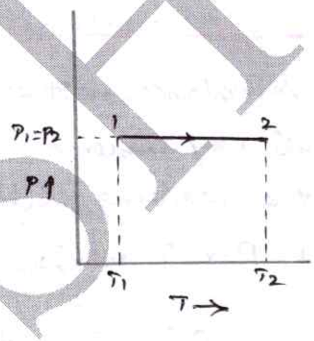


$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \qquad (\because P_1=P_2)$$

$$\therefore \frac{V_1}{T_1} = \frac{V_2}{T_2} \text{ (or) } \frac{V}{T} = \text{constant.}$$

(ii) Workdone by the gas : $\delta W = p.dv$

$$W_{1-2} = P(V_2-V_1) = mR(T_2-T_1)$$



(iii) change in internal Energy : $dU = U_2-U_1 = m.C_v(T_2-T_1)$

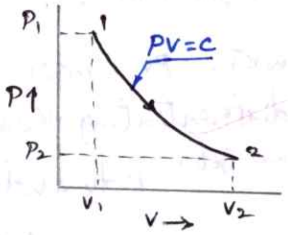
(iv) Heat Supplied ; $\delta Q = \delta w + dU$

$$Q_{1-2} = mR(T_2-T_1) + m.C_v(T_2-T_1)$$

$$Q_{1-2} = m.C_p(T_2-T_1)$$

(v) change in enthalpy : $dH = H_2-H_1 = m.C_p(T_2-T_1)$

c. Hyperbolic process :

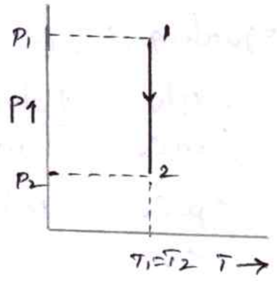


This process is the product of its pressure & volume (P.V) remains constant, is called a hyperbolic process.

This process is governed by Boyle's Law. (PV=c)

It is merely a theoretical case.

Its Practical application is isothermal process.



d. Constant Temperature Process :

(i) P-V-T relationship: $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \Rightarrow PV = \text{Constant.}$

(ii) Workdone by the gas : $\delta W = P.dv \qquad (\because PV = P_1V_1 \Rightarrow P = \frac{P_1V_1}{V})$

$$W_{1-2} \int \frac{P_1V_1}{V} dv$$

$$W_{1-2} = P_1V_1 \left[\log_e V\right]_{V_1}^{V_2} = P_1V_1 \log_e\left(\frac{V_2}{V_1}\right) \text{ (or) } P_1V_1 \log_e r$$

where; $r = \frac{V_2}{V_1}$ is known as expansion ratio.

(or)

$$W_{1-2} = 2.3\, P_1V_1 \log_{10} r$$

$$= 2.3\, mRT \log r$$

This eqn. also written as;

$$W_{1-2} = 2.3\, P_1V_1 \log\left(\frac{P_1}{P_2}\right)$$

(iii) Change in Internal energy: $dU = U_2 - U_1 = m.C_v(T_2 - T_1) = 0$

$\therefore U_1 = U_2$

(iv) Heat supplied; $\delta Q = \delta w + dU \Rightarrow Q_{1-2} = W_{1-2}$

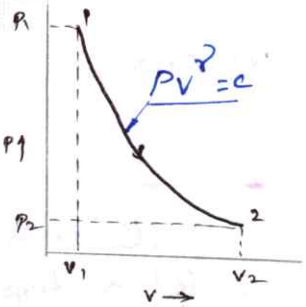
(v) change in enthalpy; $dH = H_2 - H_1 = m.C_p(T_2 - T_1) = 0$

$\therefore H_1 = H_2$

e. Adiabatic Process:(Isentropic)

A process, in which the working substance neither receives nor gives out heat to its surroundings, during its expansion(or) compression, is called an adiabatic process. This will happen the working substance remains thermally insulated.

(i) P-V-T relationship:



$$\delta Q = \delta w + dU \quad (\text{No heat transfer; } \delta Q = 0)$$

$$\delta w + dU = 0$$

$$P\,dv + m\,C_v\,dT = 0 \Rightarrow dT = \frac{-P.dv}{m.C_v} \rightarrow ①$$

WKT; $PV = mRT$

differentiating this expression, we get;

$$p\,dv + v\,dp = mR.dT$$

$$\Rightarrow dT = \frac{p\,dv + v\,dp}{m.R} \rightarrow ②$$

equating ① & ②;

$$\frac{-P\,dv}{m.C_v} = \frac{P\,dv + v\,dp}{m.R}$$

$$\frac{C_p - C_v}{C_v} = \frac{p\,dv + v\,dp}{-p\,dv}$$

$$\gamma - 1 = -1 - \left(\frac{v}{dv} \cdot \frac{dp}{p}\right)$$

$$\therefore \gamma = -\left(\frac{v}{dv} \cdot \frac{dp}{p}\right)$$

$$\gamma \frac{dv}{v} + \frac{dp}{p} = 0$$

∫ing;

$$\gamma \log_e v + \log_e P = c$$

$$\log_e PV^\gamma = \log_e c$$

$$\therefore PV^\gamma = c$$

$$P_1 V_1^\gamma = P_2 V_2^\gamma = c$$

$$\frac{P_1}{P_2} = \left(\frac{V_2}{V_1}\right)^\gamma \rightarrow ③$$

From the general gas eqn. WKT;

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \Rightarrow \frac{P_1}{P_2} = \frac{V_2}{V_1} \cdot \frac{T_1}{T_2} \rightarrow ④$$

④ in ③;

$$\frac{V_2}{V_1} \cdot \frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^\gamma$$

$$\Rightarrow \frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma - 1}$$

(or) $$\frac{V_1}{V_2} = \left(\frac{P_2}{P_1}\right)^{\frac{1}{\gamma}}$$

(or) $$\frac{T_1}{T_2} = \left(\frac{P_1}{P_2}\right)^{\frac{\gamma - 1}{\gamma}}$$

(ii) Workdone ; $\delta W = P.dv$

$$W_{1-2} = \int_1^2 P.dv \qquad PV^\gamma = P_1V_1^\gamma \Rightarrow P = \frac{P_1V_1^\gamma}{V^\gamma}$$

$$= P_1V_1^\gamma \int_1^2 V^{-\gamma} dv$$

$$= P_1V_1^\gamma \left[\frac{V^{-\gamma+1}}{-\gamma+1}\right]_1^2$$

$$= \frac{P_1V_1^\gamma}{1-\gamma}\left(V_2^{1-\gamma} - V_1^{1-\gamma}\right)$$

$$= \frac{P_1V_1^\gamma V_2^{1-\gamma} - P_1V_1}{1-\gamma} \qquad (\because P_1V_1^\gamma = P_2V_2^\gamma)$$

$$= \frac{P_2V_2^\gamma . V_2^{1-\gamma} - P_1V_1}{1-\gamma} = \frac{P_2V_2 - P_1V_1}{1-\gamma} \text{ (or) } \frac{P_1V_1 - P_2V_2}{\gamma-1}$$

$$\text{(or) } W_{1-2} = \frac{mR(T_1-T_2)}{\gamma-1}$$

(iii) Change in Internal Energy; $dU = U_2 - U_1 = m.C_v(T_2-T_1)$

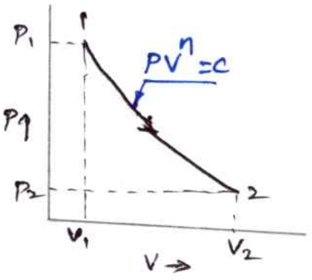
(iv) Heat Supplied, $Q_{1-2} = 0$

(v) Change in Enthalpy; $dH = H_2 - H_1 = m.C_p(T_2-T_1)$

f. Polytropic Process :

$$PV^n = \text{constant.}$$

where; n = Polytropic Index.



(i) P-V-T-relationship: $\parallel^{ler}$ to adiabatic process.

$$P_1V_1^n = P_2V_2^n = C$$

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{n-1} ; \quad \frac{V_1}{V_2} = \left(\frac{P_2}{P_1}\right)^{\frac{1}{n}} ; \quad \frac{T_1}{T_2} = \left(\frac{P_1}{P_2}\right)^{\frac{n-1}{n}}$$

(ii) Workdone : $W_{1-2} = \dfrac{P_1V_1 - P_2V_2}{n-1} = \dfrac{mR(T_1-T_2)}{n-1}$

(iii) Change in Internal Energy; $dU = U_2 - U_1 = m.C_v(T_2-T_1)$

(iv) Heat Supplied: $Q_{1-2} = W_{1-2} + dU$

$$\Rightarrow Q_{1-2} = \frac{m.R(T_1-T_2)}{n-1} + m.C_v(T_2-T_1) \qquad \left(\because C_v = \frac{R}{\gamma-1}\right)$$

$$= \frac{mR(T_1-T_2)}{n-1} + m.\frac{R}{\gamma-1}(T_2-T_1)$$

$$= mR(T_1-T_2)\left[\frac{1}{n-1} - \frac{1}{\gamma-1}\right]$$

$$= mR(T_1-T_2)\left[\frac{\gamma-1-n+1}{(n-1)(\gamma-1)}\right]$$

$$Q_{1-2} = \frac{\gamma-n}{\gamma-1}.\frac{m.R(T_1-T_2)}{n-1} = \frac{\gamma-n}{\gamma-1} \times \text{Workdone.}$$

(v) Change in Enthalpy: $dH = H_2 - H_1 = m.C_p(T_2-T_1)$

A Certain gas occupies a space of $0.3 m^3$ @ a pressure of 2 bar & a temp of 77°c. It is heated at a Consty. Volume until the pressure is 7 bar. Determine: (1) temp, at the end of the process, (2). Mass of the gas, (3) change in internal energy, & (4) change in enthalpy during the process.

Assume: $C_p = 1.005$ kJ/kg.K, $C_v = 0.712$ kJ/kg.K & $R = 287$ J/kg.K

Soln: (i) Temp. at the end of the process: (T_2):

WKT; $\frac{P_1}{T_1} = \frac{P_2}{T_2} \Rightarrow T_2 = \frac{7\times10^5}{2\times10^5}\times 350 = 1225 K = \underline{\underline{952^\circ c}}$

(ii) Mass of the gas (m): $\boxed{P_1V_1 = m.R.T_1}$ (∵ R in J/kg.K)

$\Rightarrow m = \frac{2\times10^5 \times 0.3}{287\times 350} = \underline{\underline{0.597}}$ kg

(iii) change in Internal Energy: $dU = U_2 - U_1 = m.C_v(T_2 - T_1)$

$\Rightarrow dU = 0.597 \times 0.712 (1225 - 350)$

$= \underline{\underline{372}}$ kJ

(iv) change in Enthalpy; $dH = H_2 - H_1 = m.C_p(T_2 - T_1)$

$= 0.597 \times 1.005 (1225 - 350)$

$= \underline{\underline{525}}$ kJ

The values of sp. heats at Consty. pr. and at Consty. vol. for an ideal gas are 0.984 kJ/kg.K & 0.728 kJ/kg.K. Find the values of characteristic gas consty. (R) & ratio of sp. heats (γ) for the gas. If 1 kg of this gas is heated at Consty. Pressure from 25°c to 200°c. Estimate the heat added, ideal workdone & change in internal energy. Also calculate the pr. & final volume, if the initial volume was $2 m^3$.

Soln: (i) char. gas Constant (R): $= C_p - C_v$

$= 0.984 - 0.728$

$= 0.256$ kJ/kg.K

(ii) Ratio of sp. heats γ $= C_p/C_v = \frac{0.984}{0.728} = 1.35$

(iii) Heat Added; $Q_{1-2} = m.C_p(T_2 - T_1)$

$= 1 \times 0.984 [473 - 298] = 172.2$ kJ

(iv) Workdone; $W_{1-2} = m.R(T_2-T_1)$

$= 1\times 0.256(473-298) = 44.8$ kJ

(v) change in Internal Energy: $dU = U_2-U_1 = m.C_v(T_2-T_1)$

$\Rightarrow dU = 1\times 0.728(473-298)$

$= 127.4$ kJ

(vi) Pressure & Final volume:

$P_1 = P_2$ WKT; $\therefore \frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$

$\Rightarrow \frac{V_1}{T_1} = \frac{V_2}{T_2} \Rightarrow V_2 = \frac{473}{298}\times 2$

$V_2 = 3.17\ m^3$

$\therefore \boxed{P_1V_1 = m.R.T_1}$

$\Rightarrow P = \frac{1\times 256\times 298}{2} = 38140\ N/m^2$ (∵ R' in J/kg.k)

$P = 0.3814$ bar

A quantity of air has a volume of 0.4 m³ at a pressure of 5 bar & a temp. of 80°C. It is expanded in a cylinder at a Const. temperature to a pr. of 1 bar. Determine the amount of workdone by the air during expansion & heat supplied.

Soln: WKT; $P_1V_1 = P_2V_2 \Rightarrow V_2 = \frac{P_1V_1}{P_2} = \frac{5\times10^5\times 0.4}{1\times105} = 2\ m^3$

Expansion ratio; $r = \frac{V_2}{V_1} = \frac{2}{0.4} = 5$

Workdone; $W_{1-2} = 2.3\, P_1V_1 \log r = 2.3\times 5\times10^5\times 0.4 \log 5$

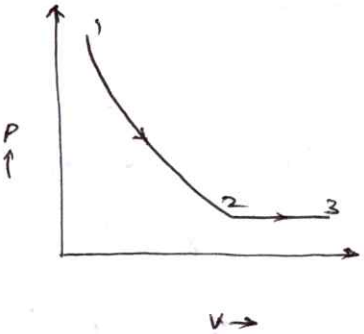
$= 321540\ J = 321.54$ kJ

Heat supplied, $Q_{1-2} = W_{1-2} + dU$

(∵ $dU = 0$)

$\therefore Q_{1-2} = 321.54$ kJ

A System contains 0.15 m^3 of a gas at a pressure of 3.8 bar & 150°C. It is expanded adiabatically till the pressure falls to 1 bar. The gas is then heated at a constt. pressure till is enthalpy increases by 70 KJ. Determine the total workdone. Take $C_p = 1\,KJ/kg.k$ & $C_v = 0.714\,KJ/kg.k$



Soln! $V_1 = 0.15 m^3$, $P_1 = 3.8$ bar $= 3.8\times10^5 N/m^2$
$T_1 = 150°c = 423K$, $P_2 = 1$ bar $= 1\times10^5 N/m^2$
$dH = 70KJ$, $C_p = 1KJ/kg.k$, $C_v = 0.714 KJ/kg.k$

Workdone during Adiabatic process; (1-2)

$$W_{1-2} = \frac{P_1V_1 - P_2V_2}{\gamma - 1}$$

$$\Rightarrow \gamma = C_p/C_v = \frac{1}{0.714} = 1.4$$

WKT; $\frac{V_1}{V_2} = \left(\frac{P_2}{P_1}\right)^{1/\gamma} = \left(\frac{1}{3.8}\right)^{1/1.4} = 0.385$

$\therefore V_2 = 0.15/0.385 = 0.39 m^3$

$$\frac{T_1}{T_2} = \left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\Rightarrow T_2 = \frac{423}{\left(\frac{3.8}{1}\right)^{\frac{1.4-1}{1.4}}}$$

$T_2 = 288.7K$

$$\therefore W_{1-2} = \frac{(3.8\times10^5 \times 0.15) - (1\times10^5\times 0.39)}{1.4-1}$$

$= 45000$ ~~k~~J $= \underline{45 KJ}$

Workdone during constt. pressure process (2-3);

$W_{2-3} = \underline{\underline{P_3V_3 - P_2V_2}}$ (or) $mR(T_3 - T_2)$ where; $R = C_p - C_v$
$W_{2-3} = P_2[V_3 - V_2]$ ($\because P_2 = P_3$)

$= 1 - 0.714$
$= 0.286 KJ/kg.k$
$R = 286 J/kg.k$

$\Rightarrow P_1V_1 = mRT_1 \Rightarrow m = \frac{3.8\times10^5\times0.15}{286\times423} = 0.47kg$

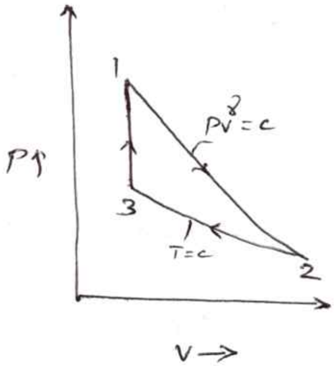
WKT;
$dH = 70 = mC_p(T_3 - T_2)$
$\Rightarrow 70 = 0.47\times1[T_3 - 288.7] \Rightarrow T_3 = 437.6k$

$\therefore W_{2-3} = 0.47\times286[437.6 - 288.7] = 20015.138 J$
$= \underline{20.02 KJ}$

$\therefore$ Total workdone, $W = W_{1-2} + W_{2-3}$
$W = 65.02 KJ$

0.336 m^3 of gas at 10 bar & 150°C expands adiabatically, until its pressure is 4 bar. It is then compressed, isothermally, to its original volume. Find the final temperature & pressure of the gas. Also determine the change in internal energy. Take $C_p = 0.996$ kJ/kg.K & $C_v = 0.703$ kJ/kg.K

Soln:- $V_1 = 0.336 m^3$, $P_1 = 10$ bar $= 10 \times 10^5 N/m^2$,
$T_1 = 150°C = 423 K$, $P_2 = 4$ bar $= 4 \times 10^5 N/m^2$
$V_1 = V_3$; $C_p = 0.996$ kJ/kg.k, $C_v = 0.703$ kJ/kg.k



(1-2): Adiabatic Process:-

$$\gamma = C_p/C_v = \frac{0.996}{0.703} = 1.417$$

$$\frac{T_1}{T_2} = \left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}} \Rightarrow T_2 = \frac{423}{\left(\frac{10\times10^5}{4\times10^5}\right)^{\frac{1.417-1}{1.417}}}$$

$$T_2 = 323 K = 50°C$$

For Isothermal Compression; $T_2 = T_3 = 323 K = 50°C$

$$\therefore \frac{P_1}{T_1} = \frac{P_3}{T_3} \quad [(3-1) \text{ Const. volume process}]$$

$$\Rightarrow P_3 = \frac{P_1 . T_3}{T_1} = \frac{10\times10^5 \times 323}{423} = 7.6\times10^5 N/m^2$$

$$P_3 = 7.6 \text{ bar}$$

change in Internal energy; $dU = U_3 - U_1 = m.C_v(T_3 - T_1)$

where; $P_1 V_1 = mRT_1 \Rightarrow m = \frac{10\times10^5 \times 0.336}{293\times423} = 2.7 kg$

$(\because R = C_p - C_v = 293 J/kg.k)$

$$\therefore dU = 2.7\times0.703[323-423]$$

$$dU = -189.8 kJ$$

The -ve sign indicates that there is a decrease in internal Energy.

0.44 kg of air at 180°c expands adiabatically to three times its original volume & during the process, there is a fall in temperature to 15°c. The workdone during the process is 52.5 KJ. Calculate C_p & C_v.

Soln: $m = 0.44\,kg$, $T_1 = 180^\circ c = 453K$, $V_2 = 3V_1$, $T_2 = 15^\circ c = 288k$

$W_{1-2} = 52.5\,KJ$

For adiabatic process; $\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} \Rightarrow \frac{288}{453} = \left(\frac{1}{3}\right)^{\gamma-1}$

$\Rightarrow 0.6357 = 0.333^{\gamma-1}$

Taking log on both side's; $\log 0.6357 = (\gamma-1)\log 0.333$

$\Rightarrow \gamma = 1.41 = C_p/C_v$

$\therefore W_{1-2} = \frac{mR(T_1-T_2)}{\gamma-1}$ $\Rightarrow C_p = 1.41\,C_v$

$52.5 = \frac{0.44 \times R(453-288)}{1.41-1}$ $\Rightarrow R = 0.296\,KJ/kg.k$

WKT; $R = C_p - C_v$

$0.296 = 1.41C_v - C_v$

$\Rightarrow C_v = 0.722\,KJ/kg.k$

$\therefore C_p = 1.018\,KJ/kg.k$

1 kg of ethane gas is compressed from 1.1 bar, 27°c acc. to law $pv^{1.3} = c$, until the pr. is 6.6 bar. Calculate the heat flow to or from the cylinder walls. Take molecular wt. of ethane = 30, $C_p = 1.75\,KJ/kg.k$

Soln:- Heat flow; $Q = \frac{\gamma-n}{\gamma-1} \times \frac{mR(T_1-T_2)}{n-1}$ (or) $\frac{\gamma-n}{\gamma-1} \times W_{1-2}$

where; $n = 1.3$; $R = \frac{R_0}{M} = \frac{\text{Universal gas constant}}{\text{molecular wt.}}$

$R = \frac{8314}{30} = 277.31$ ~~K~~J/kg.k

(or) $= 0.27731\,KJ/kg.k$

WKT; $R = C_p - C_v$

$\therefore C_v = C_p - R = 1.75 - 0.27731 = 1.473\,KJ/kg.k$

$\therefore \gamma = C_p/C_v = 1.75/1.473 = 1.188$

In Polytropic process; $\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} \Rightarrow T_2 = 453.6k$

$$\therefore \quad Q = \left(\frac{1.188 - 1.3}{1.188 - 1}\right) \times \frac{1 \times 0.27731(300 - 453.6)}{1.3 - 1}$$

$$Q = \underline{\underline{84.5 \text{ kJ}}}$$

Determination of polytropic Index:

We know that for a polytropic process;

$$P_1 V_1^n = P_2 V_2^n$$

Take log on both side's;

$$\log P_1 + n \log V_1 = \log P_2 + n \log V_2$$

$$n \log\left(\frac{V_1}{V_2}\right) = \log\left(\frac{P_2}{P_1}\right)$$

$$\therefore \boxed{n = \log\left(\frac{P_2}{P_1}\right) \Big/ \log\left(\frac{V_1}{V_2}\right)}$$

IIIly for Adiabatic Index;

$$\boxed{\gamma = \log(P_2/P_1) \Big/ \log(V_1/V_2)}$$

A gas initially at 603 K expands until its volume is 5.2 times the initial volume, acc. to $PV^n = C$. If the initial & final pr. are observed to be 8.5 bar & 1 bar. Determine: the Index of expansion, workdone per kg of gas & Heat exchange per kg of gas. Assume; $C_v = 0.712$ kJ/kg.K & $\gamma = 1.4$

Soln: WKT; (i) $n = \log\left(\frac{P_2}{P_1}\right) \Big/ \log\left(\frac{V_1}{V_2}\right)$ $\quad (\because V_2 = 5.2V_1)$

$$= \log\left(\frac{1 \times 10^5}{8.5 \times 10^5}\right) \Big/ \log\left(\frac{V_1}{5.2V_1}\right)$$

$$n = 1.3$$

(ii) $W_{1-2} = \dfrac{P_1V_1 - P_2V_2}{n-1}$ where; $P_1V_1 = mRT_1$

$$\Rightarrow V_1 = \frac{1 \times 284.8 \times 603}{8.5 \times 10^5}$$

$$V_1 = 0.202 \text{ m}^3$$

$$\therefore V_2 = 5.2V_1$$

$$V_2 = 1.05 \text{ m}^3$$

$$= \frac{(8.5 \times 10^5 \times 0.202) - (1 \times 10^5 \times 1.05)}{1.3 - 1}$$

$$= 0.222 \times 10^6 \text{ J}$$

$$W_{1-2} = 222 \text{ kJ}$$

$\because \gamma = C_p/C_v$

$\Rightarrow C_p = 1.4 \times 0.712$

$C_p = 0.997$ kJ/kg.K

$R = C_p - C_v$

$R = 0.997 - 0.712$

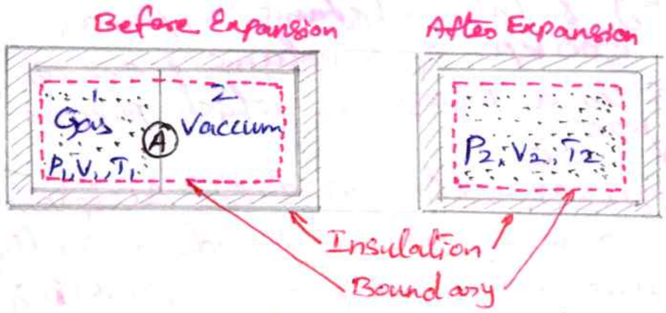
$R = 0.2848$ kJ/kg.K

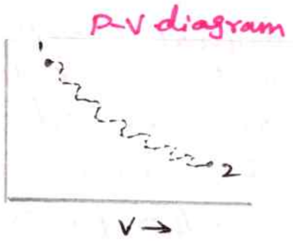
$R = 284.8$ J/kg.K

(iii)

$$Q_{1-2} = \frac{\gamma - n}{\gamma - 1} \times W_{1-2} = \frac{1.4 - 1.3}{1.4 - 1} \times 222 = 55.5 \text{ kJ}$$

Free Expansion [Unresisted Expansion] Process





Consider two vessels 1 & 2 interconnected by a short pipe with a valve 'A' [Partition] & perfectly thermally insulated. Initially let the vessel 1 be filled with a fluid at a certain pressure & let 2 be completely evacuated (Vaccum). When the valve 'A' is opened the fluid in 1 will expand rapidly to fill both vessels 1 & 2. The pressure finally will be lower than the initial pressure in vessel 1. This is known as "Free expansion". The process is highly irreversible. Since the fluid is eddying continously during the process (No workdone & No heat flows).

Now Applying I law of Thermodynamics;

$$Q = W + dU$$

$$\Rightarrow Q_{1-2} = 0 \,;\, W_{1-2} = 0 \,;\, dU = 0$$

$$\Rightarrow U_2 - U_1 = 0 \Rightarrow U_1 = U_2$$

In a free expansion; therefore the internal energy initially is equals to finally.

→ for a perfect gas; $u = C_v T$

∴ for a free Expansion of a perfect gas;

$$C_v T_1 = C_v T_2 \quad \Rightarrow \quad T_1 = T_2$$

That is, for a perfect gas undergoing a free expansion, the initial temperature is equal to the final temperature.

NOTE:

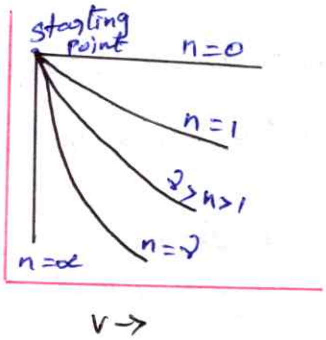
$n = 0$ → Const. Pressure

$n = 1$ → Isothermal

$\gamma > n > 1$ → Polytropic

$\gamma = n$ → Adiabatic

$n = \infty$ → Const. Volume



Curves for various values of 'n'.

The following equation gives the internal energy of a certain Substance $u = 3.64PV + 90$, where: 'u' is in kJ/kg, 'P' in kPa & 'V' is in m^3/kg.

A system composed of 3.5 kg of this Substance expands from an initial pressure of 500 kpa & a volume of $0.25 m^3$ to a final pressure of 100 kpa in a process in which pressure & volume are related by $PV^{1.25} = C$.

(i) If the expansion is quasi-static, find Q, ΔU & W.

(ii) In another process, the same system expands according to the same pressure-volume relationship as in part (i) & from the same initial state to the same final state as in Part (i), but the heat transfer in this case is 32 kJ. Find the work transfer for this process.

(iii) Explain the difference in work transfer in part (i) & (ii).

Soln: $u = 3.64\,PV + 90$; $V_1 = 0.25 m^3$, $P_1 = 500 kpa$, $P_2 = 100 kpa$

$PV^{1.25} = C$.

(i) Now; $u = 3.64\,PV + 90$

$$\Rightarrow \Delta u = u_2 - u_1 = (3.64 P_2V_2 + 90) - (3.64 P_1V_1 + 90)$$

$$\Delta U = 3.64(P_2V_2 - P_1V_1) \quad (\because \underline{\text{for } 3.5 \text{ kg}})$$

Now; $P_1V_1^{1.25} = P_2V_2^{1.25} \Rightarrow V_2 = V_1\left(\frac{P_1}{P_2}\right)^{1/1.25}$

$$= 0.25\left[\frac{500}{100}\right]^{\frac{1}{1.25}}$$

$$V_2 = 0.906 m^3$$

$$\therefore \Delta U = 3.64[(100\times10^3 \times 0.906) - (500\times10^3\times0.25)]$$

$$\Delta U = -125.2 kJ$$

For a quasi-static process; $W = \int p\,dV = \frac{P_1V_1 - P_2V_2}{n-1}$

$$\Rightarrow W = \frac{(500\times10^3 \times 0.25) - (100\times10^3\times0.906)}{1.25 - 1}$$

$$W_{(i)} = 137.6 kJ$$

WKT; $Q = W + \Delta U = 137.6 - 125.2 = \underline{12.4} kJ$

(ii) Here; $Q = 32 kJ$

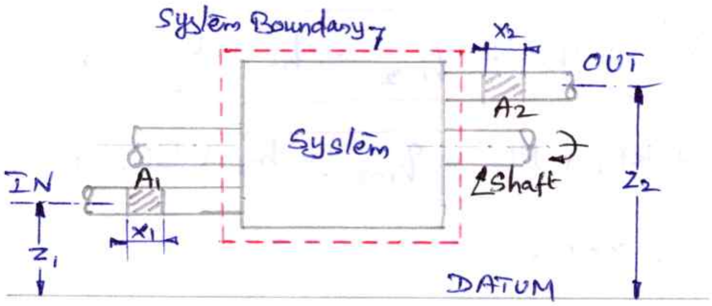
Since the end states are the same, ΔU would remain the same.

$$Q = W + \Delta U \Rightarrow W = Q - \Delta U = 32 + 125.2$$

$$W_{(ii)} = 157.2 kJ$$

(iii) $W_{(ii)}$ is not equal to $W_{(i)}$. Since the process is "<u>not-quasi-static</u>".

Application of First law of Thermodynamics to a Steady Flow Process:



Consider an open system thro' which the working substance flows at a steady state as shown in fig. The working substance enters the system at sec. ①-① & leaves the system at sec. ②-②.

Let; P_1 = pr. at entering (N/m²)
v_{s_1} = sp. volume at entry (m³/kg)
V_1 = Velocity at entry (m/s)
u_1 = sp. internal energy at entry (J/kg)
Z_1 = height above datum at entry (m)

||ly P_2, v_{s_2}, V_2, u_2 & Z_2 = Corresponding values at leaving the system.
q_{1-2} = heat supplied (J/kg) &
W_{1-2} = work delivered by the system (J/kg).

Consider 1 kg of mass of the working substance.

WKT; Total energy entering the system/kg;

e_1 = Internal energy + Flow energy + kinetic energy + Potential energy + Heat supplied.

$$e_1 = u_1 + P_1 v_{s_1} + \frac{V_1^2}{2} + gz_1 + q_{1-2} \text{ (J/kg)}$$

||ly Total energy leaving the system/kg

$$e_2 = u_2 + P_2 v_{s_2} + \frac{V_2^2}{2} + gZ_2 + W_{1-2} \text{ (J/kg)}$$

Assuming, no loss of energy during flow, then acc. to Ist law of Thermodynamics (i.e) $e_1 = e_2$

$$u_1 + P_1 v_{s_1} + \frac{V_1^2}{2} + gz_1 + q_{1-2} = u_2 + P_2 v_{s_2} + \frac{V_2^2}{2} + gZ_2 + W_{1-2}$$

WKT; $h_1 = P_1 v_{s_1} + u_1$ = Enthalpy entering the system (J/kg)

$h_2 = P_2 v_{s_2} + u_2$ = Enthalpy leaving the system (J/kg)

$$\therefore \boxed{h_1 + \frac{V_1^2}{2} + gz_1 + q_{1-2} = h_2 + \frac{V_2^2}{2} + gz_2 + w_{1-2}}$$ (or)

$$\Rightarrow \boxed{h_1 + KE_1 + PE_1 + q_{1-2} = h_2 + KE_2 + PE_2 + w_{1-2}} \text{(J/kg)}$$

Above eqn. represents the energy flow / unit mass.

$$\boxed{m\left(h_1 + \frac{V_1^2}{2} + gz_1 + q_{1-2}\right) = m\left(h_2 + \frac{V_2^2}{2} + gz_2 + w_{1-2}\right)} \text{(J/s)}$$

A steady flow thermodynamic system receives fluid at the rate of 6 kg/min with an initial pressure of 2 bar, velocity 150 m/s, internal energy 800 kJ/kg and density 27 kg/m³. The fluid leaves the system with a final pressure of 8 bar, velocity 200 m/s, internal energy 800 kJ/kg & density 5 kg/m³. If fluid receives 80 kJ/kg of heat during passing through the system & rises thro' 60 m. Determine the workdone during the process.

Soln: $m = 6\,\text{kg/min} = 0.1\,\text{kg/s}$, $P_1 = 2\times10^5\,\text{N/m}^2$, $V_1 = 150\,\text{m/s}$, $u_1 = 800\,\text{kJ/kg}$,
$\rho_1 = 27\,\text{kg/m}^3$, $P_2 = 8\times10^5\,\text{N/m}^2$, $V_2 = 200\,\text{m/s}$, $u_2 = 800\,\text{kJ/kg}$,
$\rho_2 = 5\,\text{kg/m}^3$, $q_{1-2} = 80\,\text{kJ/kg}$, $z_2 = 60\,\text{m}$

$$h_1 + KE_1 + PE_1 + q_{1-2} = h_2 + KE_2 + PE_2 + w_{1-2} \longrightarrow ①$$

where; $h_1 = u_1 + P_1 v_{s_1} = 800\times10^3 + \left(2\times10^5 \times \frac{1}{27}\right)$ $\quad \left(\because \rho = \frac{m}{V}\right)$

$= 807.4\times10^3\,\text{J/kg}$

$h_1 = 807.4\,\text{kJ/kg}$ $\quad \Rightarrow V = \frac{m}{\rho}$

$h_2 = u_2 + P_2 v_{s_2} = 800\times10^3 \left(8\times10^5 \times \frac{1}{5}\right)$

$= 960\times10^3\,\text{J/kg}$

$h_2 = 960\,\text{kJ/kg}$

$KE_1 = \frac{V_1^2}{2} = \frac{150^2}{2} = 11.25\,\text{kJ/kg}$

$KE_2 = \frac{V_2^2}{2} = \frac{200^2}{2} = 20\,\text{kJ/kg}$

$PE_1 = gz_1 = 0$

$PE_2 = gz_2 = 9.81\times60 = 588.6\,\text{J/kg}$

$PE_2 = 0.5886\,\text{kJ/kg}$

Substitute all the values in Eqn ①;

$w_{1-2} = -81.932\,\text{kJ/kg}$

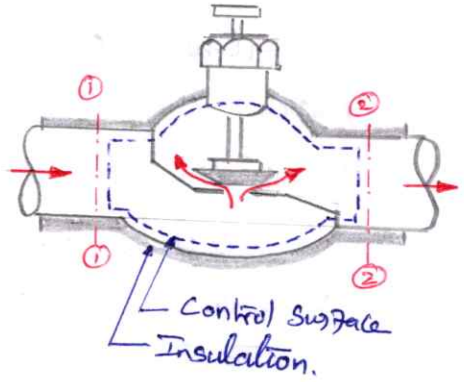
(or) $\quad (\because m = 0.1\,\text{kg/s})$

$w_{1-2} = -8.932\,\text{kJ/s}$

(–ve) sign indicates that the work is done on the system.

(b)

Throttling process:



When a fluid flows through a contricted passage like a partially opened valve, an orifice, there is an appreciable drop in pressure & the flow is said to be throttled. Fluid flow in an insulated pipe, since no heat is supplied (or) rejected & also no work is done.

$$Q_{1-2} = 0 \quad \& \quad W_{1-2} = 0$$

In the steady flow energy equation for unit mass flow is;

$$h_1 + \frac{V_1^2}{2} + gz_1 + q_{1-2} = h_2 + \frac{V_2^2}{2} + gz_2 + w_{1-2}$$

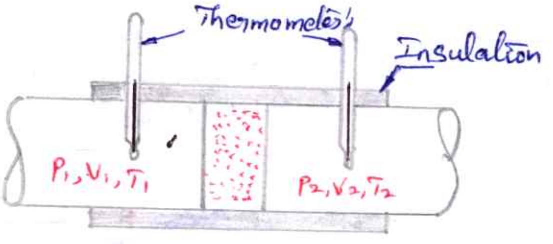
Since there is no considerable change in velocity & the inlet & outlet are at the same level, therefore; $V_1 = V_2$ & $Z_1 = Z_2$.

$$\therefore \; h_1 + \frac{V_1^2}{2} + gz_1 + 0 = h_2 + \frac{V_1^2}{2} + gz_1 + 0$$

$$\Rightarrow \quad \boxed{h_1 = h_2}$$

Thus the throttling process is a "constant enthalpy" process

Joule Thompson porous plug experiment:



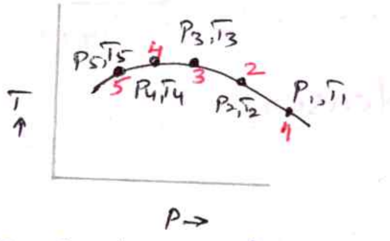


Fig: Constt. Enthalpy curve.

The throttling process was investigated by Joule & Thompson during an experiment known as Joule Thompson porous plug experiment.

In this experiment, a stream of gas at pressure P_1 & temperature T_1 is forced continuously thro' one side of the

Porous plug as shown in fig. The gas comes out from the othes side of the porous plug at P_2 & T_2. The whole appoartments is completely insulated so that no heat transfer takes place. similarly the enthalpy of gas before & aftes the process remains constant $(h_1 = h_2)$.

If the pressure & temperature is kept constant on the downstream of the porous plug is varied to P_1, P_2, P_3.....etc & T_1, T_2, T_3.....etc will be a curve of constant enthalpy $h_1 = h_2 = h_3 = \dots$ etc.

The slope of a constant enthalpy line is called Joule Thompson co-efficient and is denoted by 'μ'.

$$\mu = \left(\frac{\partial T}{\partial P}\right)_h$$

'μ' at a particular state may be $+^{ve}$, 0 (or) $-{ve}$.

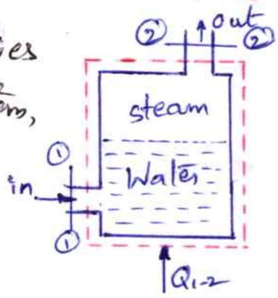
For a perfect gas, the value of μ is zero.

Application of steady flow energy equation to Engineering systems:

(i) Boiler:

A boiler is a device which supplies heat to water & generates steam. In this system, there is no change in KE & PE's

Also there is no workdone by the system.



(ie) $\frac{V_2^2}{2} - \frac{V_1^2}{2} = 0$; $gz_2 - gz_1 = 0$ & $w_{1-2} = 0$

$\therefore \quad h_1 + q_{1-2} = h_2 + 0$

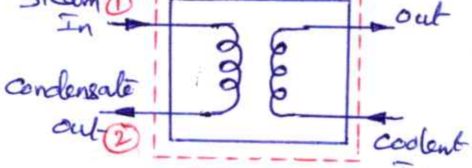
$\Rightarrow \boxed{q_{1-2} = h_2 - h_1}$

This shows that the heat supplied to the system in a boiler increases the enthalpy of the system.

In boiler: heat supplied.

(ii) Condenser:

A condenser is a device used to condense steam in case of steam power plants using water as the cooling medium. For such a system, there is no change in KE & PE's. Also there is no workdone by the system.



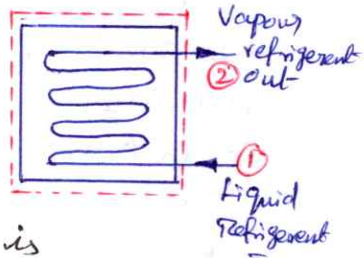
$h_1 + q_{1-2} = h_2$ $\quad [\because \frac{V_2^2}{2} - \frac{V_1^2}{2} = 0 ; gz_2 - gz_1 = 0$ & $w_{1-2} = 0]$

$\therefore q_{1-2} = h_2 - h_1$

$\Rightarrow -q_{1-2} = h_2 - h_1 \Rightarrow \boxed{q_{1-2} = h_1 - h_2}$

In condenser: heat lost.

(iii) Evaporator :



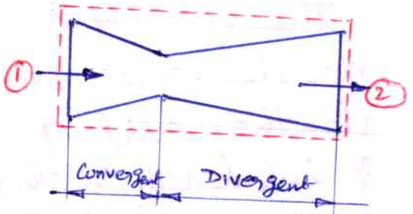
The evaporator is a device used in refrigeration systems in which the liquid refrigerant passes, receives heat & leaves as vapour refrigerent. For such a system, the change in KE & PE's is negligible. Also there is no workdone by the system.

$$KE_2 - KE_1 = 0 \; ; \; PE_2 - PE_1 = 0 \; \& \; w_{1-2} = 0$$

$$\therefore \quad h_1 + q_{1-2} = h_2 \quad \Rightarrow \boxed{q_{1-2} = h_2 - h_1}$$

In evaporator : heat added (supplied)

(iv) Nozzle :



A nozzle is a device which increases the velocity (or) KE of the working substance at the expense of its pressure drop. The nozzle is insulated so that no heat enters or leaves the system (i.e) nozzle considered adiabatic. The system $(Q_{1-2}=0)$ does not deliver any work (i.e) $w_{1-2}=0$ & no PE changes.

$$h_1 + \frac{V_1^2}{2} + 0 + 0 = h_2 + \frac{V_2^2}{2} + 0 + 0$$

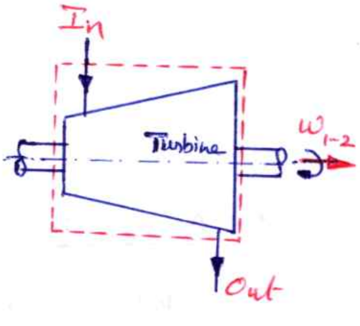
$$\Rightarrow \boxed{\frac{V_2^2 - V_1^2}{2} = h_1 - h_2}$$

$$\Rightarrow V_2 = \sqrt{2(h_1-h_2) + V_1^2}$$

$$\therefore \boxed{V_2 = \sqrt{2(h_1-h_2)}}$$

{V_1 is very small as compared to V_2 $\therefore V_1$ may be neglected}

(v) Turbine :



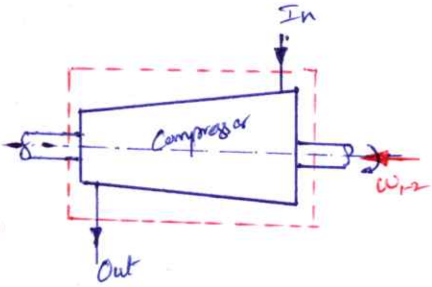
A turbine is a device which converts energy of the working substance (gas/steam) into work in the turbine. The turbine is insulated so that there is no transfer of heat $(q_{1-2}=0)$.

The changes in KE & PE's are negligible.

$$\therefore \quad h_1 = h_2 + W_{1-2}$$

$$\Rightarrow \boxed{W_{1-2} = h_1 - h_2}$$

(vi) Rotary Compressor: A rotary compressor is a device which compresses air & supplies the same at moderate pressure & in large quantities. The rotary compressor is insulated so that no heat transfer takes place ($\because q_{1-2}=0$)



In case the changes in KE & PE's are negligible.

$$h_1 = W_{1-2} + h_2$$

$$\Rightarrow W_{1-2} = h_1 - h_2$$

$$-W_{1-2} = h_1 - h_2 \Rightarrow \boxed{W_{1-2} = h_2 - h_1}$$

The -ve sign indicated the work is done ~~to~~ on the system.

The velocity & enthalpy of fluid at the inlet of a certain nozzle are 50 m/s & 2800 kJ/kg respectively. The enthalpy at the exit of nozzle is 2600 kJ/kg. The nozzle is horizontal & insulated so that no heat transfer takes place from it. Find: (1) Velocity of the fluid at exit of the nozzle, (2) mass flowrate, if the area at inlet of nozzle is 0.09 m² & the sp. volume is 0.185 m³/kg. & (3) exit area of the nozzle, if the sp. volume at the exit is 0.495 m³/kg.

Soln: $V_1 = 50\,m/s$, $h_1 = 2800\,kJ/kg = 28\times10^5\,J/kg$,
$h_2 = 2600\,kJ/kg = 26\times10^5\,J/kg$, $Z_1 = Z_2$, $q_{1-2}=0$,
$A_1 = 0.09\,m^2$, $v_{s_1} = 0.185\,m^3/kg$ & $v_{s_2} = 0.495\,m^3/kg$

(i) Velocity of the fluid at exit (V_2)

$$h_1 + \frac{V_1^2}{2} + \cancelto{0}{gZ_1} + \cancelto{0}{q_{1-2}} = h_2 + \frac{V_2^2}{2} + \cancelto{0}{gZ_2} + \cancelto{0}{W_{1-2}}$$ [No work in nozzle]

$$\therefore\ h_1 - h_2 = \frac{V_2^2}{2} - \frac{V_1^2}{2}$$

$$\Rightarrow V_2 = \sqrt{2(h_1-h_2) + V_1^2}$$

$$= \sqrt{2\left[28\times10^5 - 26\times10^5\right] + 50^2}$$

$$V_2 = \underline{634.4\ m/s}$$

(ii) Mass flow rate: (m)

WKT, $$m = \frac{A_1V_1}{v_{s_1}} = \frac{0.09\times50}{0.185} = \underline{24.3\ kg/s}$$

(iii) Exit area of the nozzle: (A_2)

WKT, $$m = \frac{A_2V_2}{v_{s_2}} \Rightarrow A_2 = \frac{24.3\times0.495}{634.4} = \underline{0.019\ m^2}$$

(d)

Air at a temperature of 15°C passes through a heat exchanger at a velocity of 30 m/s where its temperature is raised to 800°C. It then enters a turbine with the same velocity of 30 m/s & expands until the temp. falls to 650°C. On leaving the turbine, the air is taken at a velocity of 60 m/s to a nozzle where it expands until the temp. has fallen to 500°C. If the air flow rate is 2 kg/s. Calculate: (i) the rate of heat transfer to the air in the heat exchanger, (ii) the power output from the turbine assuming no heat loss & (iii) the velocity at exit from the nozzle, assuming no heat loss. Take the enthalpy of air as $h = C_p.t$, where; $C_p = 1.005$ kJ/kg.K & t is the temperature.

G.D:

$m = 2$ kg/s

$h = C_p.t$

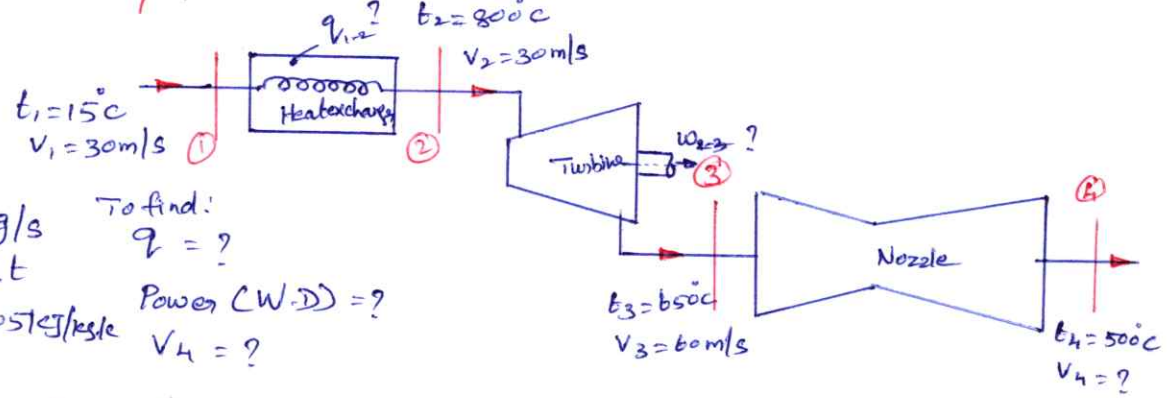
$C_p = 1.005$ kJ/kg.K

To find:

$q = ?$

Power (W.D) = ?

$V_4 = ?$



Soln:

(1-2) Heat exchanger:

by energy Eqn.:
$$m\left[h_1+\cancelto{0}{\frac{V_1^2}{2}}+\cancelto{0}{gZ_1}\right]+Q_{1-2}=m\left[h_2+\cancelto{0}{\frac{V_2^2}{2}}+\cancelto{0}{gZ_2}\right]+\cancelto{0}{W_{1-2}}$$

$$\therefore Q_{1-2} = m.h_2 - mh_1 = m[h_2-h_1]$$
$$= mC_p[t_2-t_1]$$
$$= 2\times1.005[800-15]$$
$$Q_{1-2} = 1580 \text{ kJ/s}$$

(2-3) Turbine:

$$m\left[h_2+\frac{V_2^2}{2}+\cancelto{0}{gZ_2}\right]+\cancelto{0}{Q_{2-3}}=m\left[h_3+\frac{V_3^2}{2}+\cancelto{0}{gZ_3}\right]+W_T$$

$$m.h_2+m\frac{V_2^2}{2} = mh_3+m\frac{V_3^2}{2}+W_T$$

$$\Rightarrow W_T = m(h_2-h_3)+\frac{m}{2}(V_2^2-V_3^2)$$
$$= m.C_p(t_2-t_3)+\frac{m}{2}(V_2^2-V_3^2)$$
$$= 2\times1.005(800-650)+\frac{2}{2}(30^2-60^2)$$
$$W_T = 298.8 \text{ kJ/s} = 298.8 \text{ kW}$$

(3-4) : Nozzle :

$$m\left[h_3+\frac{V_3^2}{2}+\cancel{gZ_3}^{\,0}\right]+\cancel{Q_{3-4}}^{\,0}=m\left[h_4+\frac{V_4^2}{2}+\cancel{gZ_4}^{\,0}\right]+\cancel{W_{3-4}}^{\,0}$$

$$m\left[h_3+\frac{V_3^2}{2}\right]=m\left[h_4+\frac{V_4^2}{2}\right]$$

$$\Rightarrow \quad \frac{V_4}{2}=\sqrt{(h_3-h_4)+\frac{V_3^2}{2}}$$

$$V_4=2\sqrt{Cp(t_3-t_4)+\frac{V_3^2}{2}}$$

$$=2\sqrt{1\cdot005(650-500)+\frac{60^2}{2}}$$

$$V_4=554\ m/s$$

LIMITATIONS of FIRST LAW of THERMODYNAMICS:

(1) "When a closed system undergoes a thermodynamic cycle, the net heat transfer is equal to the net work transfer."

This statement does not specify the direction of flow of heat & work (i.e.) whether the heat flows from a hot body to a cold body (or) vice versa). It also does not given any condition under which these transfers takes place.

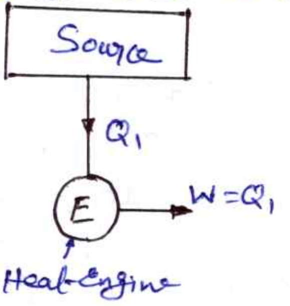
(2) "The heat energy & Mechanical work are Mutually convertible."

Though the mechanical work can be fully converted into heat energy, but only a part of heat energy can be converted into mechanical work. This means that the heat energy & mechanical work are not fully mutually convertible. In other words, there is a limitation on the conversion of one form of energy into another form.

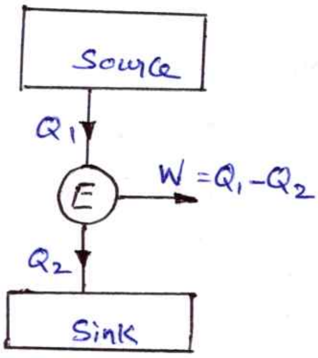
A machine which violates the first law of thermodynamics is known as perpetual Motion machine of the first kind (PMM-I).

Second Law of Thermodynamics:-

(1) Kelvin-planck statement: It is impossible to construct an engine working on a cyclic process, whose sole purpose is to convert all the heat energy supplied to it into work.



(a) PMM-II (Impossible)



(b) Heat Engine.

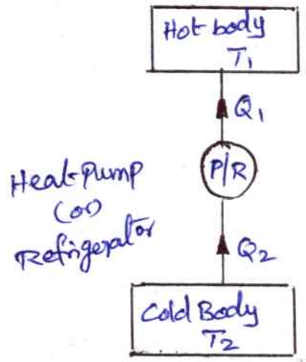
$$\eta_{max} = \frac{\text{Max. work obtained}}{\text{Total heat supplied}}$$

$$= \frac{Q_1 - Q_2}{Q_1}$$

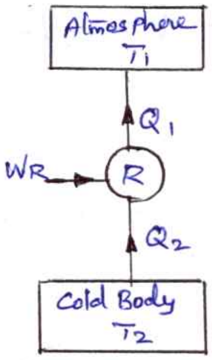
* For a reversible engine, $\frac{Q_1}{T_1} = \frac{Q_2}{T_2}$

$$\eta_{max} = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1}$$

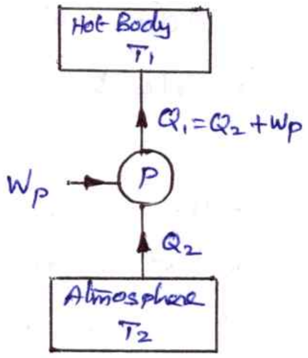
(2) Clausius statement: It is impossible for the heat to flow from a cold body to a hot body without aid of an external work.



(a) PMM-II



(b) Refrigerator



(c) Heat pump.

The Performance of refrigerator & heat pump is measured in terms of C.O.P. [Co-efficient of Performance].

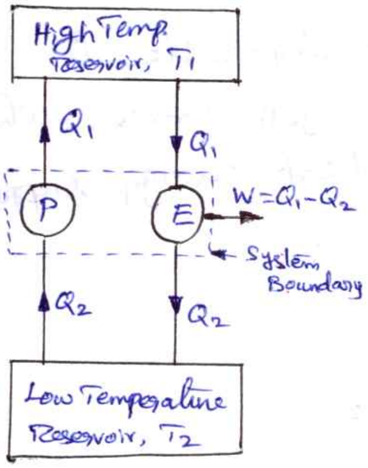
$$(C.O.P)_R = \frac{Q_2}{W_R} = \frac{Q_2}{Q_1 - Q_2} = \frac{T_2}{T_1 - T_2}$$

$$(COP)_P = \frac{Q_1}{W_P} = \frac{Q_1}{Q_1 - Q_2} = \frac{T_1}{T_1 - T_2} = \frac{T_2}{T_1 - T_2} + 1$$

(i.e) $(COP)_P = (COP)_R + 1$

COP of a heat pump is greater than COP of a refrigerator by unity.

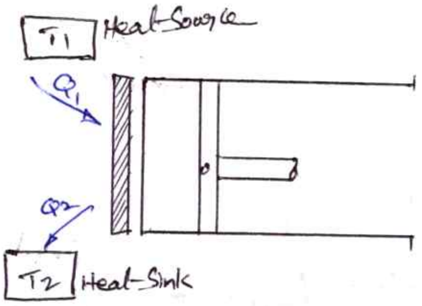
Equivalence of Kelvin-Planck & Clasius statements:-

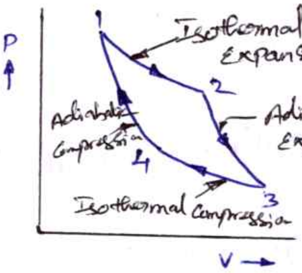


Consider a high temp. reservoir T_1 & low temp. reservoir T_2. Fig. shows a heat pump which requires no work & transfers an amount of Q_2 from a low temperature to a high temp. reservoir (in violation of the clasius statement). Let an amount of heat Q_1 ($Q_1 > Q_2$) be transferred from high temperature reservoir to heat engine which develops a net work, $W = Q_1 - Q_2$ & rejects Q_2 to the low temp. reservoir. Since there is no heat interaction with the low temp. it can be eliminated. The combined system of the heat engine & heat pump acts then like a heat engine exchanging heat with a single reservoir, which is the violation of the Kelvin-planck statement.

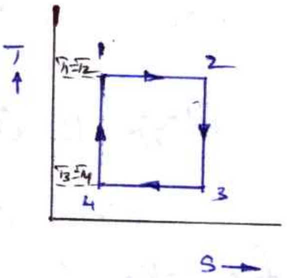
CARNOT Cycle: French Engineer Sadi Carnot in 1824 which works on reversible cycle & is known as "Carnot Cycle".

Any fluid may be used to operate the Carnot cycle which is performed in an engine cylinder the head of which is supposed alternatively to be perfect Conductor or a perfect insulator of a heat. Heat is caused to flow into the cylinder by the application of high temp. energy source to the cylinder head during expansion & to flow from the cylinder by the application of a lower temperature energy source to the head during compression.





(P-V) Diagram



(T-S) Diagram

The assumptions made for describing the working of the Carnot engine are as follows:

(i) The piston moving in a cylinder does not develop any friction during motion.

(ii) The walls of piston & cylinder are considered as perfect insulators of heat.

(iii) The cylinder head is so arranged that it can be a perfect heat conductor or perfect heat insulator.

(iv) The transfer of heat does not affect the temp. of source or sink.

(v) Working medium is a perfect gas & has const. sp. heat.

(vi) Compression & expansion are reversible.

The following 4 stages of Carnot cycle:

Stage-I (1-2) : [Isothermal Expansion]

Hot energy source is applied, heat Q_1 is taken in whilst the fluid expands isothermally & reversibly at const. high temp. 'T_1'.

Heat Supplied ; $Q_{1-2} = dU + W_{1-2}$

$$= W_{1-2} = 2.3\, p_1 V_1 \log \frac{V_2}{V_1}$$

[($\because r = \frac{V_2}{V_1}$) Expansion Ratio]

$$Q_{1-2} = 2.3\, mRT_1 \log . r$$

Stage-II (2-3) : [Reversible Adiabatic Process] Expansion.

The cylinder becomes a perfect insulater, so that no heat flow takes place. The fluid expands adiabatically & reversibily whilst temp. falls from T_1 to T_2.

$$Q_{2-3} = dU + W_{2-3} \qquad (\because Q = 0)$$

$$\Rightarrow \quad -dU = W_{2-3}$$

Decrease in Internal energy $= \frac{P_2V_2 - P_3V_3}{\gamma - 1} = \frac{mR(T_2 - T_3)}{\gamma - 1}$

Stage-III (3-4): [Isothermal Compression] $= \frac{mR(T_1 - T_3)}{\gamma - 1}$

Cold energy source is applied. heat Q_2 flows from the fluid whilst it is compressed isothermally & reversibly at const. lower temp. T_2.

Heat rejected, $Q_{3-4} = 2.3\, mRT_3 \log . r$ [($r = \frac{V_3}{V_4}$) Compression Ratio.]

Stage-IV (4-1) : [Reversible adiabatic compression]

Cylinder head becomes a perfect insulator, so that no heat flow occurs. The compression is continued adiabatically & reversibly during which temp. is raised from T_4 to T_1

$$Q_{4-1} = dU + W_{4-1} \qquad (\because Q = 0)$$

$\therefore$ Increase in Internal energy, $dU = W_{4-1}$

$$= \frac{P_1V_1 - P_4V_4}{\gamma - 1}$$

$$= \frac{mR(T_1 - T_4)}{\gamma - 1} = \frac{mR(T_1 - T_3)}{\gamma - 1}$$

From the above, the decrease in Internal energy during reversible adiabatic expansion (2-3) is equal to the increase in internal energy during reversible adiabatic compression (4-1). Hence the net effect during the whole cycle is zero.

∴ Workdone, W = Heat Supplied − Heat rejected.

$$W = Q_1 - Q_2$$

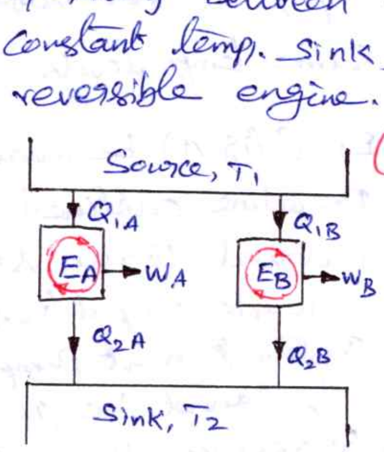
also, thermal efficiency, $\eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$

$$\eta = 1 - \frac{T_2}{T_1}$$

The Carnot cycle cannot be performed in Practice, because of the following reason's.

- It is impossible to perform a frictionless process.
- It is impossible to transfer the heat without temp, potential.
- Isothermal process can be acheived only if the piston moves very slowly to allow heat transfer so that the temp. remains constant. Adiabatic process can be acheived only if the piston moves as fast as possible so that the heat transfer is negligible due to very short time available. Both processes takes place during the same stroke. The variation of motion of the piston during the same stroke is not possible.

Carnot's Theorem: It states that all heat engine operating between a given constant temp. source & a given constant temp. sink, none has a higher efficiency than a reversible engine.

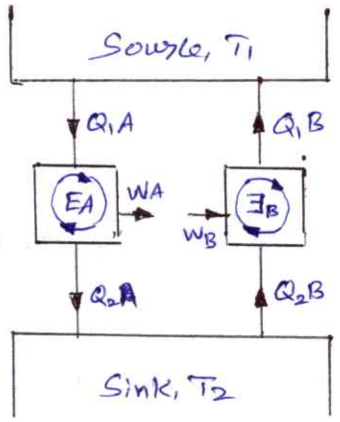


(Fig: A) Two cyclic heat engines E_A & E_B operating between the same source & sink, of which E_B is reversible.

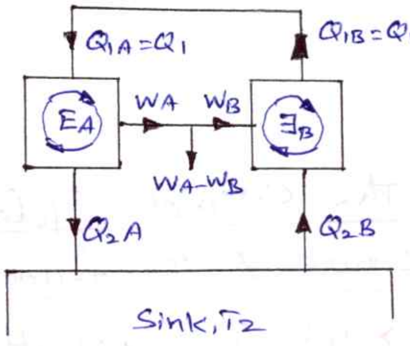
Let E_A be any heat engine & E_B be any reversible heat engine. We have to prove that the η of E_B is more than that of E_A. Let us assume that this is not true & $\eta_A > \eta_B$. Let the rates of working of the engines be such that,

$Q_{1A} = Q_{1B} = Q_1$ since, $\eta_A > \eta_B \Rightarrow \frac{W_A}{Q_{1A}} > \frac{W_B}{Q_{2B}} \Rightarrow W_A > W_B$

Now, Let E_B be reversed. Since E_B is a reversible heat engine, the magnitudes of heat & work transfer quantities will remains the same, but their directions will be reversed as shown in Fig. 1. Since $W_A > W_B$.



(Fig:1) E_B is reversed.



(Fig:2) E_A & $\exists_B$ together violate the kelvin-planck statement.

Some part of W_A (equal to W_B) may be fed to drive the reversed heat engine $\exists_B$.

Since $Q_{1A} = Q_{1B} = Q_1$, the heat discharged by $\exists_B$ may be supplied to E_A. The source may, therefore, be eliminated (Fig. 2). The net result is that E_A & $\exists_B$ together constitute a heat engine which, operating in a cycle, produces net work $W_A - W_B$, while exchanging heat with a single reservoir at T_2. This violates the kolvin-planck statement of the II law. Hence the assumption that $\eta_A > \eta_B$ is wrong.

$$\therefore \eta_B \geq \eta_A$$

Corollary of Carnot's theorem: The η of all reversible heat engines operating between the same temp. levels is the same.

Let both the heat engines E_A & E_B (fig: A) be reversible. Let us assume $\eta_A > \eta_B$. Similar to the procedure outlined in the proceding Carnot theorem, if E_B is reversed to run, say as a heat pump using some part of the work output (W_A) of engine E_A, we see that the combined system of heat pump E_B & engine E_A, becomes a PMM-II. So η_A cannot be greater than η_B. Similarly, if we assume $\eta_B > \eta_A$ & reverse the engine E_A, we observe that η_B cannot be greater than η_A.

$\therefore \eta_A = \eta_B$ Since the η's of all reversible heat engines operating between the same heat reservoirs are the same, "the η of a reversible engine is independent of the nature or amount of the working substance undergoing the cycle..

A carnot engine, working between 650K & 310K, produces 150 KJ of work. Find thermal efficiency & heat added during the process.

<u>Soln</u>: $T_1 = 650K$, $T_2 = 310K$, $Q = ?$ & η ?

$W = 150 KJ$

WKT,
$$\eta = \frac{T_1 - T_2}{T_1} = \frac{650-310}{650} = 0.523 \text{ (or) } 52.3\%$$

$$\eta = \frac{W}{Q_1} \Rightarrow Q_1 = \frac{150}{0.523} = 286.8 \text{ KJ.}$$

An engine operating on a carnot cycle works with in temperature limits of 600K & 300K. If the engine receives 2000KJ of heat. Evaluate the workdone & thermal efficiency of the engine.

<u>Soln</u>: $T_1 = 600K$, $T_2 = 300K$, $Q = 2000 KJ$, W? & η ?

W.K.T;
$$\eta = \frac{T_1 - T_2}{T_1} = \frac{600-300}{600} = 0.5 \text{ (or) } 50\%.$$

$$\eta = \frac{W}{Q} \Rightarrow W = \eta \times Q = 0.5 \times 2000 = 1000 KJ$$

A carnot cycle operates between source & sink temp. of 250°C & −15°C resp.. If the system receives 90KJ from the source. Find: (i) η of the system, (ii) The net work transfer & (iii) Heat rejected to sink.

<u>Soln</u>: $T_1 = 250°C = 523K$, $T_2 = -15°C = 258K$, $Q_1 = 90 KJ$

(i)
$$\eta_{carnot} = \frac{T_1 - T_2}{T_1} = \frac{523-258}{523} = 0.506 \text{ (or) } 50.6\%.$$

(ii) Net work transfer, $W = \eta_{car} \times Q_1 = 0.506 \times 90 = 45.54 KJ$

(iii) Heat rejected to sink, $Q_2 = Q_1 - W$ $[\because W = Q_1 - Q_2]$

$$= 90 - 45.54$$

$$Q_2 = 44.46 KJ$$

A cyclic heat engine operates a source temp. of 1000°c & a sink temp. of 40°c. Find the least rate of heat rejection per kW net output of the engine.

Soln: $T_1 = 1000°c = 1273k$, $T_2 = 40°c = 313K$

For a reversible heat engine, the rate of heat rejection will be minimum.

$$\eta_{max} = \eta_{rev} = \frac{T_1 - T_2}{T_1} = \frac{1273-313}{1273}$$

$$= 0.754 \text{ (or) } 75.4\%$$

Now, $\eta = \frac{W_{net}}{Q_1}$

$$\Rightarrow 0.754 = \frac{1\,kW}{Q_1} \Rightarrow Q_1 = 1.326\,kW$$

Now; $Q_1 - Q_2 = W \Rightarrow Q_2 = Q_1 - W$

$$= 1.326 - 1 = \underline{\underline{0.326\,kW}}$$

Entropy: First it introduced by clausius. It is an important thermodynamic property of a working substance, which increase with the addition of heat & decreases with the removal of heat.

Entropy is a function of a quantity of heat which shows the possibility of conversion of that heat into work.

$$\boxed{ds = \frac{\delta Q}{T}} \Rightarrow \delta Q = T.ds \rightarrow ① \longrightarrow (A)$$

Acc. to First law of thermodynamics;

$$\delta Q = du + \delta w \longrightarrow (B)$$

$$\delta Q = du + p.dv \rightarrow ②$$

From eqn: ① & ②;

$$\Rightarrow \boxed{T.ds = du + pdv} \longrightarrow (C)$$

In Eqn: (A) & (B); δQ & δw are path functions.

∴ these eqn's are true only for "Reversible processes".

But; In eqn: (C); ds, du & dv are point functions.

{ ∴ It is true for Reversible as well as Irreversible processes. }

Importance of Entropy: The maximum possible η obtained by any engine working on a reversible Carnot cycle is given by: $\eta = \frac{T_1 - T_2}{T_1} = 1 - \frac{T_2}{T_1}$

where; T_1 → Highest Absolute Temp. &
T_2 → Lowest Absolute Temp.

In general eqn; $\eta = \frac{\delta w}{\delta Q} \Rightarrow \delta w = \delta Q \times \eta = \delta Q \left(\frac{T_1 - T_2}{T_1}\right)$

For 1° temp. drop, the above expression may be written as } $\Rightarrow \boxed{\delta w = \frac{\delta Q}{T} = ds}$

From this expression, it can be easily understood that;

(1) The change in entropy represents the maximum amount of work obtainable per degree drop in temp.

(2) The change in entropy may be regarded as a measure of the rate of the availability or unavailability of heat for transformation into work.

(3) The increase in entropy is obtained from a given quantity of heat at a low temperature..

Available & Unavailable Heat Energy:

Available Heat Energy: It is that part of the heat energy (or heat supplied) which can be converted into mechanical work by ideal processes which reduces the system in a state of equilibrium.

Unavailable Heat Energy: It is that part of heat energy which can ~~be~~ not be converted into mechanical work even by ideal processes which reduces the system in a state of equilibrium.

Acc. to II Law of T.D;

$$\left.\begin{array}{l}\text{Total heat energy (or}\\ \text{heat supplied to the system}\end{array}\right\}, \delta Q = AHE + UHE$$

$$= \text{workdone} + \text{heat rejected}.$$

WKT; max. possible efficiency; $\eta = 1 - \frac{T_2}{T_1} = \frac{\delta W}{\delta Q}$

$$\delta W = \delta Q - \delta Q \frac{T_2}{T_1}$$

$(\because T_2 = T_0$ (Practical Temp.)$)$
where; T_0 = Temp. of Surrounding

$$\therefore \delta W = \delta Q - \delta Q \frac{T_0}{T_1} \rightarrow ①$$

we also know that,

$$\delta W = \delta Q - UHE \rightarrow ②$$

$$\therefore \delta Q - \delta Q \frac{T_0}{T_1} = \delta Q - UHE$$

$$\therefore UHE = \frac{\delta Q \times T_0}{T_1} = T_0 \times ds$$

$$\Rightarrow \boxed{UHE = T_0 \, ds}$$

UHE (or) heat rejected is the product of the lowest temp. of heat rejection & the change of Entropy of the system during the process of supplying heat.

(Units of Entropy is KJ/K)

Clausius Inequality:- It states that "whenever a closed system undergoes a cyclic process, the cyclic integral of $\delta Q/T$ is less than zero (or) negative.

(i.e) $\oint \frac{\delta Q}{T} < 0$, for an irreversible cyclic process.

$\oint \frac{\delta Q}{T} = 0$, for a reversible cyclic process.

∴ clausius inequality is written as;

$$\boxed{\oint \frac{\delta Q}{T} \leq 0}$$

($\oint \frac{\delta Q}{T} > 0$, the cycle is impossible, since it violates the second law.)

(OR) alitey

Let us consider, two heat engines one is reversible & the other irreversible, operating between the same two constant temp. reservoirs T_1 & T_2. For the supply of the same quantity of heat Q_1 to both the engines, & since the 'η' of the irreversible engine (η_I) is less than that of the reversible engine (η_R).

(i.e) $\eta_I < \eta_R$

$$\frac{W_I}{Q_1} < \frac{W_R}{Q_1} \Rightarrow W_I < W_R \Rightarrow Q_1 - Q_2' < Q_1 - Q_2$$

$$\therefore Q_2' < Q_2$$

where; Q_2' is the heat rejection from the irreversible engine & Q_2 is the heat rejection from the reversible engine.

$$\therefore \oint \frac{\delta Q}{T} = \frac{Q_1}{T_1} - \frac{Q_2}{T_2} = 0 \quad (\text{for Reversible Engine})$$

$$\oint \frac{\delta Q}{T} = \frac{Q_1}{T_1} - \frac{Q_2'}{T_2} < 0 \quad (\text{for Irreversible Engine})$$

So, In general, $\boxed{\oint \frac{\delta Q}{T} \leq 0}$

(Clausius Inequality is also valid for referigerators & heat pumps.)

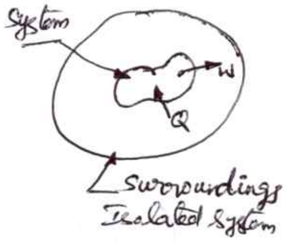
Principle of Increase of Entropy:- (Change of Entropy)

The eqn. for clausius inequality is;

$$\oint \frac{\delta Q}{T} \leq 0 \rightarrow ①$$

WKT; the change in entropy; $ds = \frac{\delta Q}{T}$

Since the entropy is a thermodynamic property & the cyclic integral of a thermodynamic property is zero.



$$\therefore \oint \frac{\delta Q}{T} \leq \oint ds \Rightarrow \frac{\delta Q}{T} \leq ds \quad \&$$

$$\Rightarrow ds \geq \frac{\delta Q}{T} \rightarrow ②$$

when the process is reversible, then;

$$ds = \frac{\delta Q}{T}$$

when the process is Irreversible, then;

$$ds > \frac{\delta Q}{T}$$

If we apply the eqn. ② to an isolated system like universe, for which $\delta Q = 0$, then the eqn. ② may be written as;

$$(ds)_{Univ.} \geq 0 \Rightarrow (ds)_{systems} + (ds)_{surroundings} \geq 0$$

For a reversible cyclic process; $ds = 0$ (or) $S = \text{constant}$.

For a irreversible cyclic process; $ds > 0$

Since in practice all processes are irreversible, therefore the entropy is increasing continually.

This is known as the Principle of Increase of Entropy.

Rudolf clausius summarised the First & Second laws of TD's in the following words.

(1) The energy of the world (Universe) is constant.
(2) The entropy of the world tends towards a maximum.

Third Law of Thermodynamics:

It states that "when a system is at zero absolute temperature, the entropy of system is zero".

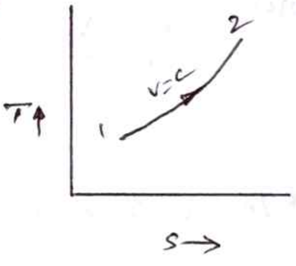
From the above law; that the absolute value of entropy corresponding to a given state of the system could be determined by integrating $\left(\frac{\delta Q}{T}\right)_R$ between the state @ absolute zero & the given state. Zero entropy, however, means the absence of all molecular, atomic, electronic & nuclear disorders.

As it is not practicable to get data at zero absolute temp.

In practice we can determine the change in entropy & not the absolute value of entropy.

Change of Entropy of a perfect Gas during Various TD-Processes:

(1) Constant volume process : $\left[v=c \text{ (or) } \frac{P}{T}=c\right]$



Consider a certain quantity of gas being heated @ const. volume;

Let, m = mass of the gas;
P_1 = Initial pr. of gas,
T_1 = Initial Temp. of gas.
|||ly P_2 & T_2 = Corresponding values of the final conditions.

wkt, for a small change of temp. (dT), the heat supplied;

$$\delta Q = m.C_v.dT$$

$\div T$; $\Rightarrow \frac{\delta Q}{T} = m.c_v.\frac{dT}{T} \Rightarrow ds = m.c_v.\frac{dT}{T}$

by $\int$ing; $$\int_{S_1}^{S_2} ds = m.C_v \int_{T_1}^{T_2} \frac{dT}{T}$$

$$\boxed{S_2 - S_1 = m.C_v \log_e\left(\frac{T_2}{T_1}\right) = 2.3\, mC_v \log\left(\frac{T_2}{T_1}\right)} \rightarrow ①$$

WKT; $\frac{P_1}{T_1} = \frac{P_2}{T_2} \Rightarrow \frac{T_2}{T_1} = \frac{P_2}{P_1}$

$$\therefore \boxed{S_2 - S_1 = 2.3\, mC_v \log\left(\frac{P_2}{P_1}\right)} \rightarrow ②$$

Eqn's ① & ② are valid for both reversible & irreversible process.

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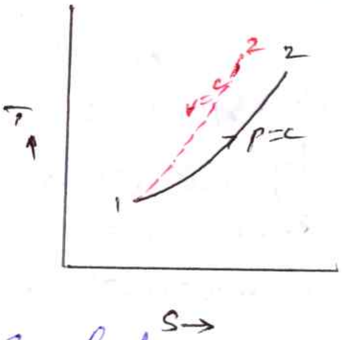
(2) Constant Pressure Process : $[P=C$ (or) $\frac{V}{T}=C]$



Consider a certain quantity of a perfect gas heated at constant pressure.

Let: $m = \dots$
$V_1 = \dots$
$T_1 = \dots$

WKT, for a small change of temp. (dT), the heat supplied;

$$\delta Q = m.C_p.dT$$

$\div T$; $$\frac{\delta Q}{T} = m.C_p.\frac{dT}{T} \Rightarrow ds = m.C_p.\frac{dT}{T}$$

by $\int$ing; $$\int_{S_1}^{S_2} dS = m.C_p \int_{T_1}^{T_2} \frac{dT}{T}$$

$$\Rightarrow \boxed{S_2 - S_1 = m.C_p \log_e\left(\frac{T_2}{T_1}\right) = 2.3\, mC_p \log\left(\frac{T_2}{T_1}\right)} \rightarrow ①$$

WKT; $\frac{V_1}{T_1} = \frac{V_2}{T_2} \Rightarrow \frac{T_2}{T_1} = \frac{V_2}{V_1}$

$$\therefore \boxed{S_2 - S_1 = 2.3\, mC_p \log\left(\frac{V_2}{V_1}\right)} \rightarrow ②$$

Eqn's ① & ② are valid for both reversible & irreversible process.

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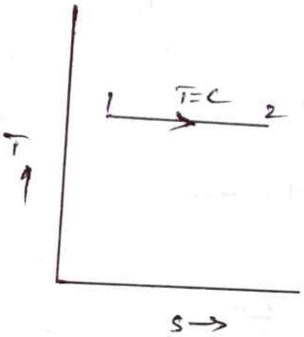
(3) Constant Temperature Process : $[T=C$ (or) $PV=C]$



Consider a certain quantity of a perfect gas being heated at constant Temperature.

Let; $m = \dots$
$P_1 = \dots$
$V_1 = \dots$

WKT; Heat Supplied, $Q_{1-2} = W_{1-2} = 2.3\, mRT \log\left(\frac{V_2}{V_1}\right)$

WKT; $ds = \frac{\text{Heat Supplied}}{\text{Absolute Temp.}} \Rightarrow S_2 - S_1 = \frac{2.3\, mRT \log\left(\frac{V_2}{V_1}\right)}{T}$

$$\therefore \boxed{S_2 - S_1 = 2.3\, mR \log\left(\frac{V_2}{V_1}\right)} \rightarrow ①$$

W.K.T; $P_1V_1 = P_2V_2 \Rightarrow \frac{V_2}{V_1} = \frac{P_1}{P_2}$

$$\therefore \boxed{S_2 - S_1 = 2.3\, mR \log\left(\frac{P_1}{P_2}\right)} \rightarrow ②$$

Eqn's ① & ② are valid for both reversible & irreversible process.

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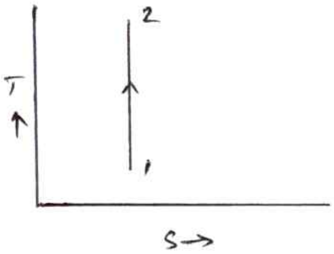
(4) Adiabatic Process: $[PV^\gamma = C]$



During adiabatic process as heat is neither supplied nor rejected

$$dQ = 0 \Rightarrow \frac{dQ}{T} = 0$$

$$\therefore \boxed{ds = 0}$$

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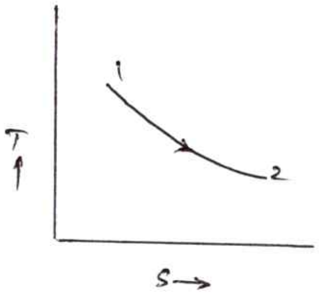
(5) Polytropic Process: $[PV^n = C]$



Consider a certain quantity of a perfect gas being heated by polytropic process.

Let; $m = \ldots$
$P_1 = \ldots$
$V_1 = \ldots$
$T_1 = \ldots$

W.K.T; $dQ = \frac{\gamma - n}{\gamma - 1} \times$ Workdone

÷ by T; $\frac{dQ}{T} = \frac{\gamma - n}{\gamma - 1} \cdot \frac{pdv}{T}$ $(\because PV = mRT)$

$$\therefore ds = \frac{\gamma - n}{\gamma - 1} mR \frac{dv}{V}$$

by $\int$ing; $\int_{S_1}^{S_2} ds = \frac{\gamma - n}{\gamma - 1} \cdot mR \int_{V_1}^{V_2} \frac{dv}{V}$

$$\Rightarrow \boxed{S_2 - S_1 = \frac{\gamma - n}{\gamma - 1} mR \log_e\left(\frac{V_2}{V_1}\right) = 2.3\, m \frac{\gamma - n}{\gamma - 1} R \log\left(\frac{V_2}{V_1}\right)} \rightarrow ①$$

Eqn: ① as valid for both reversible & irreversible process.

W.K.T; $\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{n-1}$ (or) $\frac{V_2}{V_1} = \left(\frac{T_1}{T_2}\right)^{\frac{1}{n-1}}$

In eqn ①;

$$\therefore S_2 - S_1 = 2.3 \frac{\gamma - n}{\gamma - 1} mR \log\left(\frac{T_1}{T_2}\right)^{\frac{1}{n-1}}$$

$$= 2.3\,\frac{\gamma-n}{\gamma-1}\cdot m\cdot R\cdot\frac{1}{n-1}\log\left(\frac{T_1}{T_2}\right)$$

$$= 2.3\,\frac{\gamma-n}{\gamma-1}\cdot m\,C_v(\gamma-1)\,\frac{1}{n-1}\log\left(\frac{T_1}{T_2}\right) \qquad (\because R = C_v(\gamma-1))$$

$$\therefore \boxed{S_2-S_1 = 2.3\cdot m\cdot C_v\,\frac{\gamma-n}{n-1}\log\left(\frac{T_1}{T_2}\right)}$$

WKT; $\frac{V_1}{V_2} = \left(\frac{P_2}{P_1}\right)^{\frac{1}{n}}$ (or) $\frac{V_2}{V_1} = \left(\frac{P_1}{P_2}\right)^{\frac{1}{n}}$

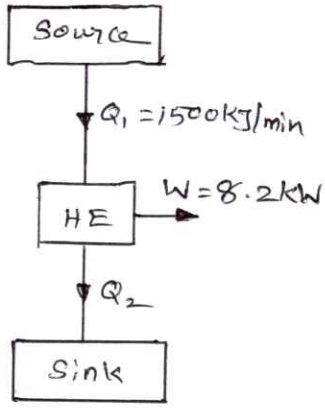
In eqn ①;

$$\therefore S_2-S_1 = 2.3\cdot\frac{\gamma-n}{\gamma-1}\cdot m.R\log\left(\frac{P_1}{P_2}\right)^{\frac{1}{n}}$$

$$= 2.3\,\frac{\gamma-n}{\gamma-1}\cdot m\times C_v(\gamma-1)\cdot\frac{1}{n}\log\left(\frac{P_1}{P_2}\right)$$

$$\therefore \boxed{S_2-S_1 = 2.3\,m.C_v\cdot\frac{\gamma-n}{n}\cdot\log\left(\frac{P_1}{P_2}\right)}$$

A heat engine receives heat at the rate of 1500KJ/min & gives an output of 8.2KW. Determine: (i) $\eta_{thermal}$ & (ii) The rate of heat rejection.



Soln: Heat received by the heat engine.

$Q_1 = 1500 KJ/min = 25 KJ/s$

Work output, $W = 8.2 KW \in (KJ/s)$

(i) $\eta_{thermal} = \frac{W}{Q_1} = \frac{8.2KW}{25KW} = 0.328$

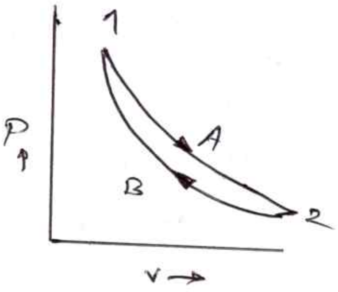
$\eta_{th} = 32.8\%$

(ii) Rate of heat rejection; $Q_2 = Q_1 - W$ $[\because W = Q_1 - Q_2]$

$= 25 - 8.2$

$Q_2 = 16.8 KJ/s$

During a process a system receives 30KJ of heat from a reservoir & does 60KJ of work. Is it possible to reach initial state by an adiabatic process?



Soln: Process (1-2):

Acc. to I law of TD;

$Q_{1-2} = dU + W_{1-2}$

$Q_{1-2} = (U_2 - U_1) + W_{1-2}$

$30KJ = (U_2 - U_1) + 60KJ$

$\Rightarrow (U_2 - U_1) = -30 KJ \longrightarrow ①$

Process (2-1):

Again; $Q_{2-1} = dU + W_{2-1}$ $(\because (U_1 - U_2) = 30KJ)$

$0 = 30 + W_{2-1}$

$\therefore W_{2-1} = -30 KJ$

Thus 30 KJ work has to be done on the system to restore it to original state, by adiabatic process.

A house requires 2×10^5 kJ/h for heating in winter. Heat pump is used to absorb heat from cold air outside in winter & send heat to the house. Work required to operate the heat pump is 3×10^4 kJ/h. Determine: (i) Heat abstracted from outside &(ii) Co-eff. of performance.

Soln: (i) Heat reqd. of the house, $Q_1 = 2\times10^5$ kJ/hr

Work reqd. to operate the heat pump $\Big\}$, $W = 3\times10^4$ kJ/hr

WKT; $W = Q_1 + Q_2$ => Now; $Q_1 = W + Q_2$

∴ Heat abstracted from outside, $Q_2 = Q_1 - W$

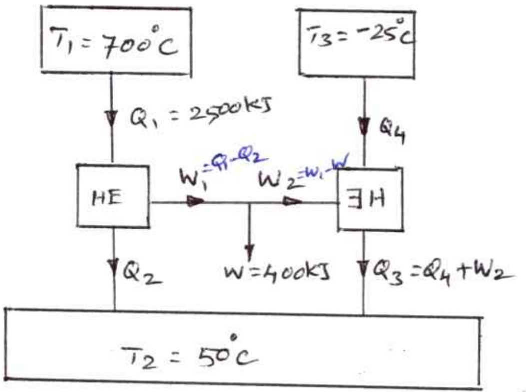
$= 2\times10^5 - 3\times10^4 = \underline{17\times10^4}$ kJ/hr.

(ii) $(COP)_{HP} = \frac{Q_1}{Q_1 - Q_2} = \frac{2\times10^5}{2\times10^5 - 17\times10^4} = \underline{6.66}$

A reversible heat engine operates between two reservoirs at temp.'s 700°C & 50°C. The engine drives a reversible refrigerator which operates between reservoirs at temp.'s of 50°C & −25°C. The heat transfer to the engine is 2500 kJ & the net work output of the combined engine refrigerator plant is 400 kJ.

(i) Determine the heat transfer to the refrigerant & the net heat transfer to the reservoir at 50°C

(ii) Reconsider (i) given that the efficiency of the heat engine & the COP of the refrigerator are each 45% of their maximum possible values.



Soln: $T_1 = 973$K, $T_2 = 323$K,
$T_3 = 248$K, $Q_1 = 2500$kJ

Net work o/p of the combined engine refrigerator plant, $W = W_1 - W_2 = 400$ kJ

(i) $\eta_{max} = 1 - \frac{T_2}{T_1} = 1 - \frac{323}{973} = 0.668$
$= 66.8\%$

$\Rightarrow \eta = \frac{W_1}{Q_1} \Rightarrow W_1 = 0.668 \times 2500 = 1670$ kJ

$(COP)_{max} = \frac{T_3}{T_2 - T_3} = \frac{248}{323 - 248} = 3.306$

$$COP = \frac{Q_4}{W_2} \Rightarrow 3.306 = \frac{Q_4}{W_2}$$

$[\because W = W_1 - W_2]$

$\Rightarrow W_2 = 1670 - 400$

$W_2 = 1270 kJ$

$\therefore Q_4 = 3.306 \times 1270$

$Q_4 = 4198.6 kJ$

Heat rejection to the 50°c reservoir;

$= Q_2 + Q_3$

$= 830 + 5468.6$

$= \underline{\underline{6298.6 kJ}}$

WKT;

$Q_3 = Q_4 + W_2$

$= 4198.6 + 1270$

$Q_3 = 5468.6 kJ$

$Q_2 = Q_1 - W_1$

$= 2500 - 1670$

$Q_2 = 830 kJ$

(ii) Efficiency of actual heat engine cycle;

$\eta = 0.45 \eta_{max}$

$\eta = 0.45 \times 0.668 = 0.3$

$\eta = \frac{W_1}{Q_1} \Rightarrow W_1 = 0.3 \times 2500 = 750 kJ$

$W_2 = W_1 - W = 750 - 400 = 350 kJ$

C.O.P. of the actual refrigerator cycle; $COP = \frac{Q_4}{W_2}$

$\Rightarrow COP = 0.45 \times (COP)_{max}$

$= 0.45 \times 3.306$

$COP = 1.48$

$\therefore Q_4 = 1.48 \times 350$

$Q_4 = 518 kJ$

$Q_3 = Q_4 + W_2 = 518 + 350 = 868 kJ$

$Q_2 = Q_1 - W_1 = 2500 - 750 = 1750 kJ$

Heat rejected to 50°c reservoir; $= Q_2 + Q_3$

$= 1750 + 868$

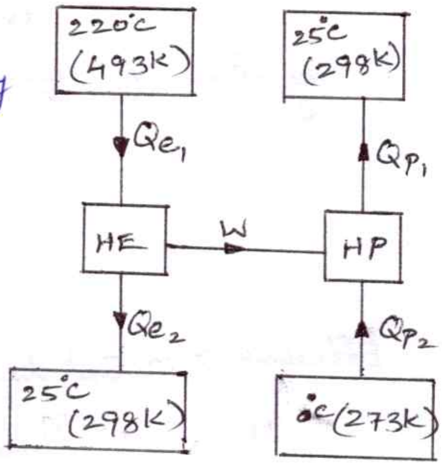
$= \underline{\underline{2618 kJ}}$

An ice plant working on a reversed carnot cycle heat pump produces 15 Tonnes of ice per day. The ice is formed from water at 0°c & the formed ice is maintained at 0°c. The heat is rejected to the atmosphere at 25°c. The heat pump used to run the ice plant is coupled to a carnot engine which absorbs heat from a source which is maintained at 220°c by burning liquid fuel of 44500 kJ/kg calorific value & rejects the heat to the atmosphere.

→

Determine: (i) Power developed by the engine,
(ii) Fuel consumed / hr.
Take enthalpy of fusion of ice = 334.5 kJ/kg.

Soln:



(i) Amount of Ice Produced = 15 Tonnes per day

∴ The amount of heat removed by the heat pump;

$$Q_{P_2} = \frac{15\times1000\times334.5}{24\times60}$$

$$Q_{P_2} = 3484.4 \text{ kJ/min.}$$

$$(COP)_{HP} = \frac{Q_{P_2}}{W} = \frac{273}{298-273} \Rightarrow W = 319.08 \text{ kJ/min.}$$

$$W = 5.3 \text{ kJ/s (kW)}$$

Thus power developed by the engine = 5.3 kW

(ii)

$$\eta_{Carnot} = \frac{W}{Q_{e_1}} = 1-\frac{298}{493} = 0.396 \text{ (or) } 39.6\%$$

$$\therefore Q_{e_1} = \frac{5.3}{0.396} = 13.38 \text{ kJ/s}$$

$$\Rightarrow Q_{e_1}/_{hr.} = 13.38\times(60\times60) = 48168 \text{ kJ}$$

∴ Quantity of fuel consumed per hour $= \frac{48168}{44500} = 1.082$ kg/hr.

Entropy:- A vessel of 2.5 m^3 capacity contains 1 kg-mole of N_2 at 100°c. If the gas is cooled to 30°c. Calculate the change in specific entropy. The ratio of sp. heats is 1.4 & 1 kg-mole N_2 is 28 kg.

Soln: Const. volume process:

$$S_2 - S_1 = 2.3\ m.\ C_v \log\left(\frac{T_2}{T_1}\right)$$

$$R = \frac{R_a}{M} = \frac{8.314 \text{ kJ/kg.k}}{28} = 0.297 \text{ kJ/kg.k}$$

WKT,

$R = C_p - C_v \Rightarrow C_v = C_p - R$ $\quad \gamma = \frac{C_p}{C_v} = 1.4 \Rightarrow C_p = 1.4C_v$

$C_v = 1.4C_v - R$

$\Rightarrow C_v = 0.74$ kJ/kg.k.

$$\therefore S_2 - S_1 = 2.3\times1\times0.74 \log\left(\frac{303}{373}\right) = -0.1536 \text{ kJ/kg.k}$$

The -ve sign indicates decrease in entropy.

Cold air from atm. is circulated through an air heating system, where the temp. of air is increased from 7.2°C to 21.2°C without any pressure loss. If the hourly consumption of warm air at 21.2°C is 850 m^3. Calculate how much heat per hour must be imparted to the air?

What will be the change in entropy of air circulated per hour? Assume proper values of the sp. heat of air.

Soln:- Const. Pressure Process:

$$Q = m \cdot C_p (T_2 - T_1)$$

Given:
$P_1 = P_2 =$ atm. Pr.
$= 1.013$ bar
$= 1.013 \times 10^5$ N/m^2

Assume:
$R = 287$ J/kg.K
$C_p = 1.005$ kJ/kg.K.

where; $m = \dfrac{P_2 V_2}{R T_2}$

$$m = \frac{1.013 \times 10^5 \times 850}{287 \times 294.2} = 1020 \text{ kg/hr.}$$

(i)

$$\therefore Q = 1020 \times 1.005 [294.2 - 280.2]$$

$$Q = \underline{\underline{14351.4}} \text{ kJ/hr}$$

(ii) Change in entropy; $S_2 - S_1 = 2.3\, m \cdot C_p \log(T_2/T_1)$

$$= 2.3 \times 1020 \times 1.005 \times \log\left(\frac{294.2}{280.2}\right)$$

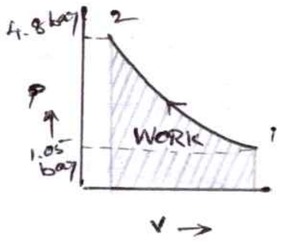
$$S_2 - S_1 = 50 \text{ kJ/K/hr.}$$

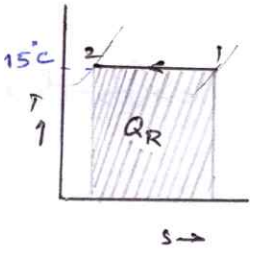
0.04 m^3 of N_2 contained in a cylinder behind a piston is initially at 1.05 bar & 15°C. The gas is compressed isothermally & reversibly until the pressure is 4.8 bar. Calculate; (i) The change of entropy, (ii) The heat flow & (iii) The workdone. Sketch the process on a (P-V) & (T-S) diagram. Assume N_2 to act as a perfect gas. Molecular weight of $N_2 = 28$.

Soln:- Const. Temperature process:

$$R = \frac{R_a}{M} = \frac{8314}{28}$$

$$= 297 \text{ J/kg.K}$$





$$m = \frac{P_1 V_1}{R T_1} = \frac{1.05 \times 10^5 \times 0.04}{297 \times 288}$$

$$m = 0.0491 \text{ kg}$$

(i) $S_2 - S_1 = m.R.\log_e\left(\frac{P_1}{P_2}\right)$

$= 2.3 \times 0.0491 \times 0.297 \log\left(\frac{1.05}{4.8}\right)$

$S_2 - S_1 = -0.0222$ KJ/k

The (-ve) sign indicates decrease in entropy.

(ii) Heat rejected = shaded area in (T-S) diagram.

$= T(S_1 - S_2)$

$Q = 288(0.0222) = 6.382$ kJ

(iii) Net workdone, $W = Q = 6.382$ KJ.

A fluid undergoes a reversible adiabatic compression from 4 bar, $0.3\,m^3$ to $0.08\,m^3$. According to the law, $pV^{1.25} = C$. Determine; (i) dH, (ii) dU, (iii) dS, (iv) Q_{1-2} & (v) W_{1-2}.

<u>Soln:</u> $P_1 = 4 \times 10^5\,N/m^2$, $V_1 = 0.3\,m^3$, $V_2 = 0.08\,m^3$, $n = 1.25$.

$P_1V_1^n = P_2V_2^n \Rightarrow P_2 = P_1 \times \left(\frac{V_1}{V_2}\right)^n = 4\times10^5\left[\frac{0.3}{0.08}\right]^{1.25}$

$\Rightarrow P_2 = 20.87$ bar

(i) dH: w.k.t; $\int_{H_1}^{H_2} dH = \int_{P_1}^{P_2} V.dp$

$P_1V_1^n = P_2V_2^n$

$$= \int_{P_1}^{P_2} \left(\frac{P_1V_1^n}{P}\right)^{\frac{1}{n}} . dp \qquad \left\{ \begin{array}{l} \because P_1V_1^n = PV^n \\ \Rightarrow V = \left[\frac{P_1V_1^n}{P}\right]^{\frac{1}{n}} \end{array} \right.$$

$$= (P_1V_1^n)^{\frac{1}{n}} \left[\frac{p^{-\frac{1}{n}+1}}{-\frac{1}{n}+1}\right]_{P_1}^{P_2}$$

$$= (P_1V_1^n)^{\frac{1}{n}} . \frac{n}{n-1}\left[(P_2)^{\left(1-\frac{1}{n}\right)} - (P_1)^{\left(1-\frac{1}{n}\right)}\right]$$

$$= \frac{n}{n-1}(P_2V_2 - P_1V_1)$$

$$\Rightarrow (H_2 - H_1) = \frac{1.25}{0.25}\left[20.87\times10^5(0.08) - 4\times10^5(0.3)\right]$$

$= 234.8 \times 10^3$ J $= \underline{\underline{234.8}}$ KJ

WKT;

(ii) <u>**du:**</u> $H = U + PV$

$\therefore H_2 - H_1 = (U_2 + P_2V_2) - (U_1 + P_1V_1)$

$\therefore (U_2 - U_1) = (H_2 - H_1) - (P_2V_2 - P_1V_1)$

$= 234.8 - [20.87 \times 10^5 (0.08) - 4 \times 10^5 (0.3)]$

$= 187.84 \times 10^3 \text{ J} = \underline{\underline{187.84}} \text{ kJ}$

(iii) <u>**ds:**</u>

$S_2 - S_1 = 0$

(iv) <u>**Heat Transfer;**</u> $Q_{1-2} = 0$

(v) <u>**Work Transfer;**</u> $W_{1-2} = Q_{1-2} - du \quad (\because Q = du + W)$

$= 0 - (U_2 - U_1)$

Hence; $W_{1-2} = -187.84 \text{ kJ}$

Workdone on the fluid = 187.84 kJ.

Properties of Pure Substances.

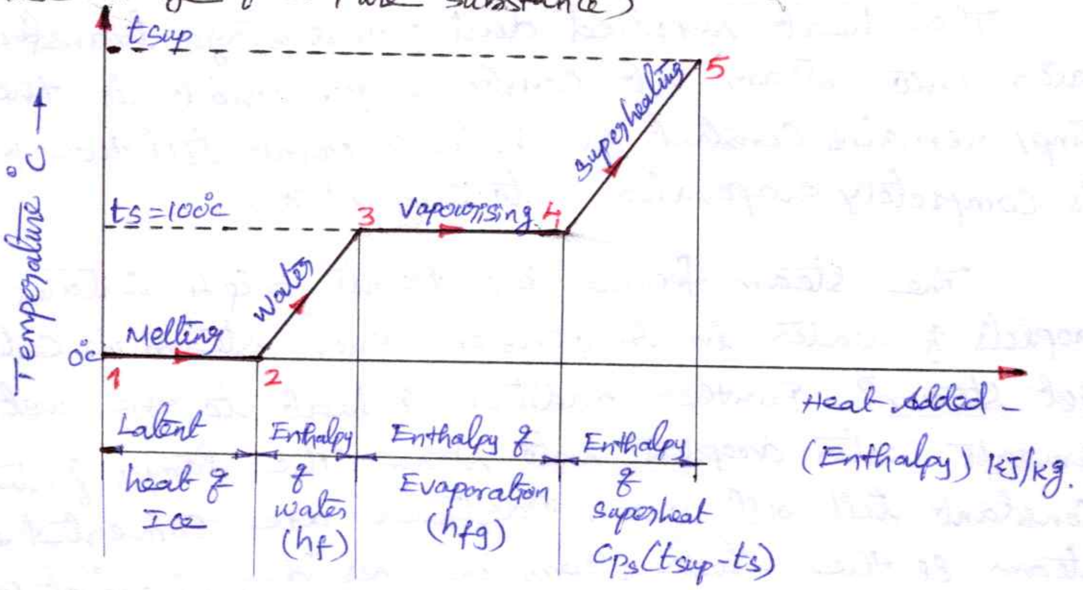
Pure Substances:

A Pure Substance is a substance of constant chemical composition throughout its mass. It is a one-component system. It may exist in one (or) more phases.

Water as the representative of a pure substance.

Formation of steam: [Under Patm = 1.01325 bar]

(phase change of a pure substance)



Consider the formation of steam from 1kg of ice cube, at 0°c, under atm. Pr. Now let heat be added to it. The change that take place is shown in fig. Heat (enthalpy) added in x-axis & temp. on y-axis.

Melting of Ice (1-2):

The ice begins to melt by the heat supplied to it. The temp. remains constant at 0°c. till the ice is completely melts into water.

The amount of heat supplied to melt 1kg of ice completely into water at 0°c is called as "Latent heat of fusion (or) Latent heat of ice".

Liquid stage (2-3): Further addition of heat rises the temperature of water, till the water begins to boil. The temp. at which the water begins to boil is known as saturation temperature.

The boiling point of water under atm. pr. is 100°c. The heat supplied during this stage is known as "sensible heat (or) enthalpy of liquid (water) (hf).

Vapourising stage (3-4):

The heat supplied during this stage transforms water into steam at const. temp. That is, the temp. remains constant at boiling point till the water is completely evaporated into dry steam.

The steam formed btw. points 3 & 4 contains droplets of water in suspension. Such steam is called "Wet steam". Further addition of heat to the wet steam converts water droplets into steam. The temp. of steam constant till all water particles are converted into steam & the whole steam becomes "dry steam" at point 4.

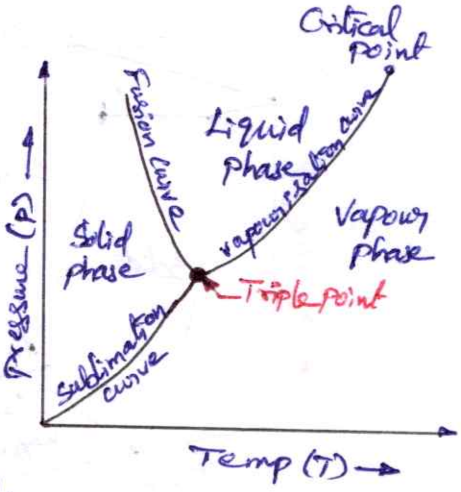
The dry vapour is known as dry steam (or) dry saturated steam. The temp. of dry saturated steam is same as boiling temp. (ts). The heat supplied during this stage is known as "Latent heat of evaporation (or) Enthalpy of evaporation" (hfg).

Superheating stage (4-5):

Further addition of heat to dry steam rises the temp. again. The process of heating dry steam is known as "Superheating" & the steam thus obtained is known as "Superheated steam". Heat added during this stage is known as "Superheated enthalpy".

The difference between 'tsup' & 'ts' is known as "Degree of superheat" (= tsup - ts).

P-T Diagram for a Pure Substance:



If the vapoure pressure of a solid is measured at various temp/. until the triple point is reached &

the liquid is measured until the critical point is reached, the result when plotted on a P-T diagram.

If the substance at the triple point is compressed until there is no vapour left & the pressure on the resulting mixture of liquid & solid is increased, the temp/. will have to be changed for equilibrium to exist b/w. the solid & the liquid.

The points representing the coexistance of

(i) Solid & Vapour lie on the "Sublimation curve",

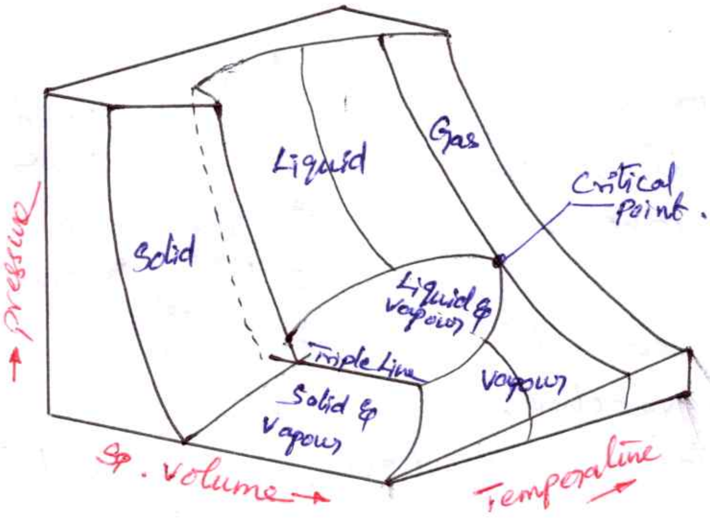
(ii) Liquid & Vapour lie on the "Vapourisation curve" &

(iii) Liquid & solid lie on the "fusion curve".

- → Sublimation curve is called : Frost Line.
- → Vapourisation curve is called : Steam Line. &
- → Fusion curve is called : Ice Line.

Triple point:- It is merely the point of intersection of which Solid, liquid & vapour exist in equilibrium (One point only).

P-V-T [Pressure-Volume-Temperature] Surface:



The relationships between pressure, sp. volume & temp. can be clearly understood with the aid of three-dimensional p-v-T surface as shown in fig.

All the equilibrium states lie on the surface of the model. states represented by the space above or below the surface are not possible. It may be seen that the triple point appears as a line in this representation.

At the critical point the temp. & pressure are called the critical temp. & critical pressure respectively & when the temperature of a substance is above the critical value, it is called a gas. It is not possible to cause a phase change in a gas unless the temp. is lowered to a value less than the critical temp..

Critical point: The limit of distinction between a liquid & vapour.

T-V Diagram [Temp. & Sp. volume]:

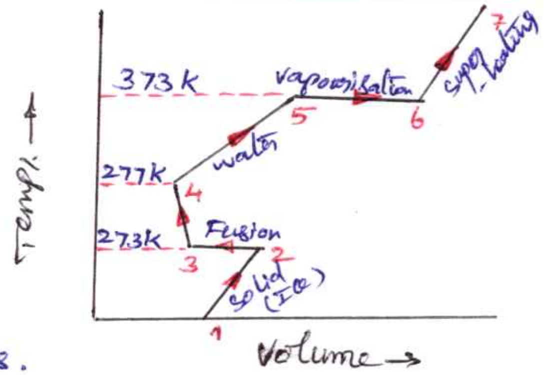


Fig. shows the formation of steam under atm. pressure on temp. (T) - volume (V) diagram.

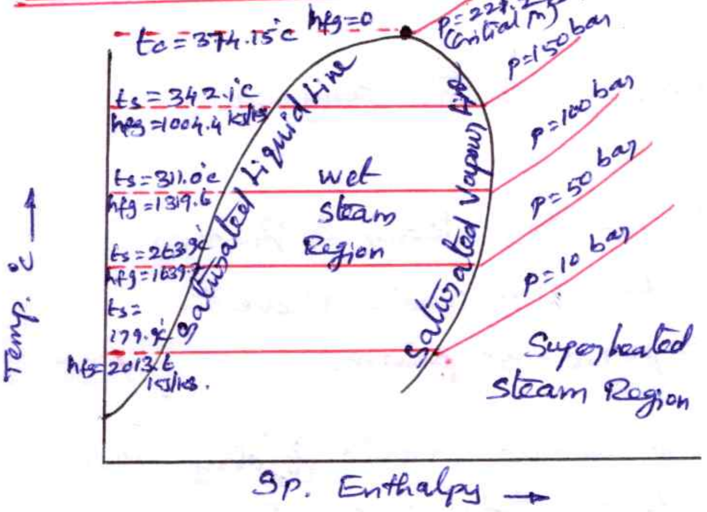
During solid stage (1-2), both the temp. & volume increases.

During melting stage (2-3), temperature remains constant & volume decreases. The volume decreases from 3 to 4 till the temp/. reaches 4°c [277K]. This significance is an exception only in the case of water.

During liquid stage (4-5), both the temp/. & volume increases. The volume increases during vaporisation stage (5-6) & the temp/. remains constant.

The volume & temp/. increases during superheating stage (6-7).

T-h Diagram. [Temp/. & Enthalpy].



The temp/.-Enthalpy (T-h) diagram in the formation of steam @ different pressures is shown in fig.

As the pressure is increased, latent heat of vaporisation (h_{fg}) decreases. Finally, when the pressure reaches 222 bar, latent heat becomes zero. This point is known as "critical point."

Critical point is the point at which the water is directly converted into dry steam, without taking latent heat.

Important Terms Relating to steam Formation:

1. Sensible heat of water (h_f): It is defined as the amount of heat required to raise the temp/. of 1kg of water from 0°c to the saturation temp/. (boiling point) under constant pressure.

This is denoted by the symbol (h_f) & is expressed in KJ/kg.

$$h_f = C_{pw}[t_s - 0°c]$$

$$h_f = 4.186 \times t_s \text{ KJ/kg. (where } t_s \text{ in } °c)$$

(The above equation is approximate only. For accurate values, steam tables are used.)

2. Enthalpy of Evaporation (h_{fg}) (or) Latent heat:

It is defined as the amount of heat required to completely convert 1kg of water at saturation temp. into dry steam.

3. Wet steam: If steam contains water particles is suspension.

4. Dry steam: Dry steam does not contain any water particles.

5. Superheated steam: If the dry steam is further heated at const. pressure, the temp. rises above the boiling point, is called superheated steam.

6. Dryness fraction (x): It is the ratio of the mass of dry saturated steam to the total mass of wet steam containing it. It's also called as "quality of steam".

$$x = \frac{m_s}{m_t} = \frac{m_s}{m_s + m_w} \quad (\%)$$

Dryness fraction of dry steam is "unity".

7. Wetness fraction (y): It is the ratio of mass of water particles to the total mass of wet steam containing it. It's also known as "priming".

$$y = \frac{m_w}{m_t} = \frac{m_w}{m_s + m_w} \quad (\%)$$

$$[x + y = 1]$$

8. Enthalpy of wet steam (h_{wet}) (or Total heat of wet steam:

Specific enthalpy of wet steam is the quantity of heat required to convert 1 kg of water at 0°c at Constt. pressure into wet steam.

$$h_{wet} = h_f + x \cdot h_{fg} \quad (kJ/kg)$$

9. Enthalpy of Dry steam (h_g): It is the quantity of heat required to convert 1 kg of water at 0°c into dry steam at Constt. Pr/.

$$h_g = h_f + x \cdot h_{fg} \qquad (\because x = 1)$$

$$\therefore \; h_g = h_f + h_{fg} \; (kJ/kg)$$

where; h_f = Sensible heat of water
h_{fg} = Latent heat of evaporation
x = Dryness fraction.

10. Enthalpy of Superheated steam (h_{sup}):

It is the amount of heat required to Convert 1 kg of water at 0°c into superheated steam at constt. pressure.

$$h_{sup} = h_f + h_{fg} + C_{ps}(t_{sup} - t_s) \; kJ/kg$$

[$\because C_{ps}(t_{sup} - t_s)$ is superheat Enthalpy]

11. Specific volume of wet steam (V_{wet}):

$$V_{wet} = x \cdot V_g \quad (m^3/kg)$$

12. Specific volume of Dry steam (V_g):

$$V_g \quad (m^3/kg)$$

13. Specific volume of Superheated steam (V_{sup}):

$$V_{sup} = V_g \frac{T_{sup}}{T_s} \quad (m^3/kg)$$

14. Density of steam (ρ):

$$\rho_{wet} = \frac{1}{V_{wet}} \; (kg/m^3),$$

$$\rho_g = \frac{1}{V_g} \quad \& \quad \rho_{sup} = \frac{1}{V_{sup}}.$$

15. External work of Evaporation (W):

$$W = \text{pressure} \times \text{change in volume}$$

$$W = P(V_g - V_f)$$

at lower pressure, V_f is very small & can be neglected.

$\therefore W = P.V_g$ (kJ/kg) → For Dry steam.

$W = P.x.V_g$ → For wet steam.

$W = P.V_{sup}$ → For Superheated steam.

16. Internal Latent heat (IL):

Enthalpy of evaporation (h_{fg})

= Internal Latent heat (IL) + External work of evaporation (W)

$\therefore IL = h_{fg} - W$

$IL = h_{fg} - P.V_g$ (kJ/kg) → For Dry steam.

$= (h_{fg} - P.V_g)x$ (kJ/kg) → For wet steam.

17. Internal Energy of steam (U):

$$h = U + W$$

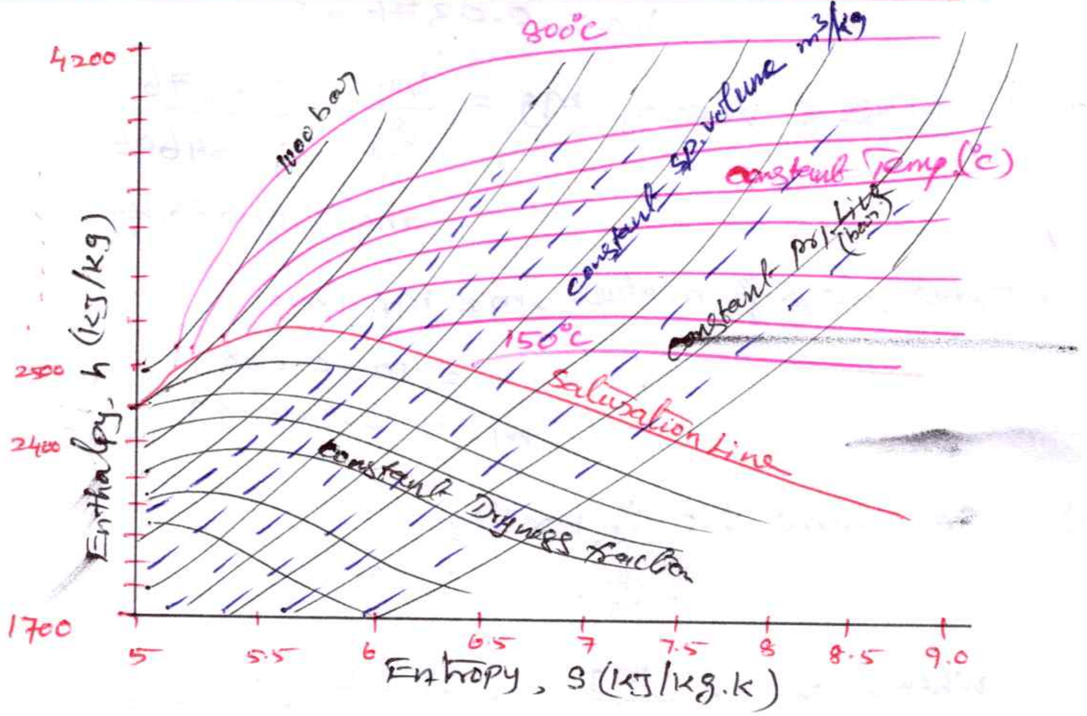
$\therefore U = (h - W)$ kJ/kg.

$U_g = (h_g - W_g) = h_g - P.V_g$ → For Dry steam

$U_{wet} = (h_{wet} - W_{wet}) = [h_f + x h_{fg}] - x P.V_g$ → For wet steam

$U_{sup} = (h_{sup} - W_{sup}) = [h_g + C_{ps}(t_{sup} - t_s)] - P.V_{sup}$ → For Sup. steam.

Enthalpy - Entropy (h-s) chart [MOLLIER DIAGRAM].



Dr. Mollier, in 1904, conceived the idea of plotting total heat against entropy, & his diagram is more widdly used than any other entropy diagram.

A vessel having a capacity of 0.05 m³ contains a mixture of saturated water & saturated steam at a temperature of 245°c. The mass of the liquid present is 10 kg. Find the following:
(i) The pressure, (ii) the mass, (iii) the sp. volume, (iv) the sp. enthalpy, (v) the sp. entropy & (vi) the sp. internal energy.

Soln: From steam tables, corresponding to 245°c;

$P_{sat} = 36.523$ bar, $v_f = 0.001240$ m³/kg, $v_g = 0.054606$ m³/kg
$h_f = 1061.6$ kJ/kg, $h_{fg} = 1740.0$ kJ/kg, $S_f = 2.748$ kJ/kg.k,
$S_{fg} = 3.358$ kJ/kg.k

(i) The pressure, $P = 36.523$ bar $= 3.65$ MPa

(ii) The mass, $m = m_f + m_g$

$\Rightarrow m_f = 10$ kg (given) ; $m_g = \frac{V_g}{v_g} =$

Volume of liquid, $V_f = m_f \cdot v_f = 10 \times 0.001240$
$V_f = \underline{\underline{0.0124\ m^3}}$

Volume of Vapour, $V_g = 0.05 - 0.0124$

$V_g = 0.0376 \text{ m}^3$

$\therefore$ Mass of Vapour, $m_g = \frac{V_g}{v_g} = \frac{0.0376}{0.054606}$

$m_g = 0.688 \text{ kg.}$

$\therefore$ Total mass of mixture, $m = m_f + m_g$

$= 10 + 0.688$

$m = 10.688 \text{ kg}$

(iii) Sp. volume, $v = v_f + x v_{fg}$

$= v_f + x(v_g - v_f)$

where; $x = \frac{m_g}{m_g + m_f} = \frac{0.688}{0.688 + 10} = \underline{\underline{0.064}}$

$\therefore v = 0.001240 + 0.064[0.054606 - 0.001240]$

$v = \underline{\underline{0.004654}} \text{ m}^3/\text{kg}$

(iv) Sp. Enthalpy, $h = h_f + x.h_{fg}$

$= 1061.6 + (0.064 \times 1740.0)$

$h = \underline{\underline{1172.77}} \text{ kJ/kg}$

(v) Sp. Internal energy, $u = h - pv$

$= 1172.77 - \left(\frac{36.523 \times 10^5 \times 0.004654}{1000}\right)$

$u = 1155.78 \text{ kJ/kg.}$

Dry saturated steam at a particular pressure has a total heat of 2750 kJ/kg. If it is superheated until its total heat at that pressure is 3780 kJ/kg. Find the rise in temp. of steam. Assume sp. heat of superheated steam as 2.1227 kJ/kg.k.

Given: $h_g = 2750 \text{ kJ/kg.}$

$h_{sup} = 3780 \text{ kJ/kg.}$

$C_{ps} = 2.1227 \text{ kJ/kg.k}$

$\therefore h_{sup} = h_g + C_{ps}(t_{sup} - t_s)$

$\therefore$ Rise in temp., $(t_{sup} - t_s) = \frac{3780 - 2750}{2.1227}$

$= \underline{\underline{485.23^\circ C}}$

Find the dryness fraction, sp. volume & internal energy of steam at 7 bar & enthalpy 2550 kJ/kg.

Soln: $P = 7$ bar, $h = 2550$ kJ/kg.

From steam tables corresponding to 7 bar pressure;

$h_f = 697.1$ kJ/kg, $h_{fg} = 2064.9$ kJ/kg, $v_g = 0.27268$ m³/kg.

$u_f = 696$ kJ/kg, $u_g = 2573$ kJ/kg

(i) Dryness fraction, x

@ 7 bar, $h_g = 2762$ kJ/kg (from steam table)

hence since the actual enthalpy is given as 2550 kJ/kg, the steam must be in the wet vapour state.

$$\therefore h = h_f + x h_{fg}$$

$$2550 = 697.1 + x \cdot 2064.9 \Rightarrow x = \underline{\underline{0.897}}$$

(ii) Sp. volume of wet steam, $v = x \cdot v_g$

$$= 0.897 \times 0.27268$$

$$v = \underline{\underline{0.2449}} \text{ m}^3/\text{kg}.$$

(iii) Sp. Internal energy of wet steam } $u = (1-x) u_f + x \cdot u_g$

$$= (1-0.897) 696 + (0.897 \times 2573)$$

$$u = \underline{\underline{2379.67}} \text{ kJ/kg}.$$

Steam initially at 0.3 MPa, 250°C is cooled at Const. volume. (a) At what temp. will the steam become saturated vapour? (b) what is the quality at 80°C? what is the heat transferred per kg of steam in cooling from 250°C to 80°C?

Soln: @ 0.3 MPa (3 bar) → $t_s = 133.5$°C

Since; t (250°C) > t_s (133.5°C), the state would be in the superheated region.

From super heated table; @ 3 bar, 250°C;

$h_1 = 2967.9$ kJ/kg, $v_1 = 0.7964$ m³/kg

$\Rightarrow v_1 = v_2 = v_3 = 0.7964$ m³/kg

In steam table;

when; $v_g = 0.84045\ m^3/kg$, $t_s = 122°c$

$v_g = 0.79283\ m^3/kg$, $t_s = 124°c$

∴ when; $v_g = 0.7964$, t_{sat}, by interpolation, would be 123.9°c. steam would become saturated vapour @ $t = 123.9°c$.

At 80°c, $v_f = 0.001029\ m^3/kg$, $v_g = 3.4091\ m^3/kg$,

$h_f = 334.9\ kJ/kg$, $h_{fg} = 2308.9\ kJ/kg$,

$P_s = 0.47360$ bar (47.36 kPa).

$$v_1 = v_2 = 0.7964\ m^3/kg = v_{f\,80°c} + x_2 v_{fg\,80°c}$$

$$0.7964 = 0.001029 + x_2(v_g - v_f)$$

$$0.7964 = 0.001029 + x_2[3.4091 - 0.001029]$$

$$\therefore x_2 = 0.234$$

$$h_2 = 334.9 + 0.234(2308.9) \quad (\because h_2 = h_f + x h_{fg})$$

$$h_2 = 875.9\ kJ/kg.$$

From the First law of thermodynamics;

$$\delta Q = du + \cancel{\delta w}^{\,0} \quad (v = const)$$

$$Q_{1-2} = (u_2 - u_1)$$

$$= (h_2 - P_2 v_2) - (h_1 - P_1 v_1)$$

$$= (h_2 - h_1) + v(P_1 - P_2)$$

$$= (875.9 - 2967.9) + 0.7964(300 - 47.36)$$

$$Q_{1-2} = -1890.2\ kJ/kg$$

A large insulated vessel is divided into two chambers, one containing 5 kg of dry saturated steam at 0.2 MPa & other 10 kg of steam, 0.8 quality at 0.5 MPa. If the partition between the chambers is removed & the steam is mixed throughly & allowed to settle. Find the final pressure, steam quality & entropy change in the process.

Solution:-

From steam table; @ 0.2 MPa (2 bar);

$m_1 = 5\,kg$	$m_2 = 10\,kg$
$x_1 = 1$	$x_2 = 0.8$
$P_1 = 0.2\,MPa$	$P_2 = 0.5\,MPa$

$$\left.\begin{aligned} v_{g_1} &= v_1 = 0.88540\ m^3/kg \\ h_{g_1} &= h_1 = 2706.3\ kJ/kg \\ s_{g_1} &= s_1 = 7.127\ kJ/kg.k \end{aligned}\right\} \left[\because (x_1 = 1)\ \text{dry steam}\right]$$

From steam table; @ 0.5 MPa (5 bar); [$x_2 = 0.8$ (wet steam)]

$v_{f_2} = 0.001093\ m^3/kg$, $h_{f_2} = 640.1\ kJ/kg$, $S_{f_2} = 1.86\ kJ/kg.k$

$v_{g_2} = 0.37466\ m^3/kg$ $h_{fg_2} = 2107.4\ kJ/kg$, $S_{fg_2} = 4.959\ kJ/kg.k$.

Total volume; $V_m = V_1 + V_2$

$= m_1 v_1 + m_2 v_2$

$$\left\{\begin{aligned} v_1 &= 0.8854\ m^3/kg \\ v_2 &= v_{f_2} + x_2 v_{fg_2} \\ &= v_{f_2} + x_2 (v_{g_2} - v_{f_2}) \\ v_2 &= 0.30101\ m^3/kg \end{aligned}\right.$$

$\therefore V_m = 5(0.8854) + 10(0.30101)$

$V_3 = V_m = \underline{\underline{7.4386\ m^3}}$ (Mixture)

Total mass of mixture, $m_m = m_1 + m_2 = 15\,kg. = m_3$

$\therefore$ sp. volume of mixture, $v_m = \dfrac{V_m}{m_m} = \dfrac{7.4386}{15} = \underline{\underline{0.496\ m^3/kg}}$.

enthalpy of mixture, h_m (h_3):-

by energy balance; $m_1 u_1 + m_2 u_2 = m_m \cdot u_m$

$\Rightarrow u_1 = h_1 - P_1 v_1 \simeq 2706.3\ kJ/kg = h_1$

$u_2 = h_2 - P_2 v_2 \simeq 2327.03\ kJ/kg = h_2$

$$\left\{\begin{aligned} h_2 &= h_{f_2} + x_2 h_{fg_2} \\ &= 640.1 + 0.8(2107.4) \\ &= \underline{2327.03\ kJ/kg}. \end{aligned}\right\}$$

$|||^{ly}$ $\therefore u_m = h_m = \dfrac{m_1 u_1 + m_2 u_2}{m_m}$

$= \dfrac{(5 \times 2706.3) + (10 \times 2327.03)}{15}$

$u_m = h_m = \underline{\underline{2453.6}}\ kJ/kg$

$(u_3 = h_3)$

From the Mollier Diagram ; $h_m = 2453.6$ kJ/kg,
$v_m = 0.496$ m³/kg.

∴ $x_3 = 0.87$, $S_3 = 6.29$ kJ/kg.k,
$P_3 = 3.5$ bar,

∴ Entropy change during the process = (Final − Initial)

$$= m_3 S_3 - (m_1 S_1 + m_2 S_2) = m_3 \cdot S_3 - m_1 S_1 - m_2 S_2$$

$$= 15(6.29) - 5(7.127) - 10(5.8292)$$

$$= \underline{\underline{0.43 \text{ kJ/K}}}$$

$$\left\{ \begin{aligned} \because S_2 &= S_{f_2} + x_2 S_{fg_2} \\ &= 1.86 + 0.8(4.959) \\ &= 5.8292 \text{ kJ/kg.k} \end{aligned} \right\}$$

Find the internal energy of 1kg of steam at 20bar, when; (i) It is superheated, its temp. being 400°c, (ii) It is wet, its dryness fraction being 0.9. Assume superheated steam to behave as a perfect gas from the commencement of superheating & thus obeys charle's law. Sp. heat ~~for~~ steam = 2.3 kJ/kg.k.

Soln:- (i) Internal energy of 1kg of superheated steam:

$$u = h_{sup} - P.v_{sup}.$$

from steam table;
@ 20 bar;
$t_s = 212.4$°c,
$v_g = 0.09955$ m³/kg,
$h_f = 908.5$ kJ/kg.
$h_{fg} = 1888.7$ kJ/kg.

$$\Rightarrow h_{sup} = h_f + h_{fg} + C_{ps}(t_{sup} - t_s)$$

$$= 908.5 + 1888.7 + 2.3[400 - 212.4]$$

$$h_{sup} = \underline{3228.68} \text{ kJ/kg.}$$

by charle's law; $\left[\because \frac{v}{T} = c\right]$

$$\frac{v_g}{T_g} = \frac{v_{sup}}{T_{sup}} \Rightarrow v_{sup} = \frac{v_g \cdot T_{sup}}{T_g} = \frac{0.09955 \times (400+273)}{(212.4+273)}$$

$$v_{sup} = \underline{0.1379} \text{ m}^3/\text{kg.}$$

$$\therefore u = 3228.68 - \left[\frac{20\times10^5 (0.1379)}{1000}\right]$$

$$u_{sup} = \underline{\underline{2952.88}} \text{ kJ/kg.}$$

(ii) Internal energy of 1kg of wet steam:-

$$u = h_{wet} - P v_{wet}$$

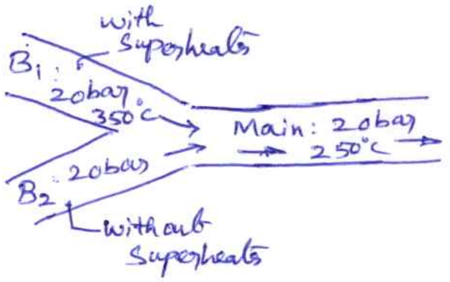
$$\Rightarrow h_{wet} = h_f + x h_{fg} = 908.5 + 0.9(1888.7)$$
$$= 2608.34 \text{ kJ/kg}$$

$$v_{wet} = x . v_g = 0.9 \times 0.09955$$
$$= 0.089595 \text{ m}^3\text{/kg}.$$

$$\therefore u = 2608.34 - \left[\frac{20\times10^5 \times 0.089595}{1000}\right]$$

$$u_{wet} = 2429.24 \text{ kJ/kg}.$$

Two boilers one with Superheater & other without Superheater are delivering equal quantities of steam into a Common main. The pressure in the boilers & main is 20bar. The temp. of steam from a boiler with a superheater is 350°C & temp. of the steam in the main is 250°C. Determine the quality of steam supplied by the other boiler. Take; $C_{ps} = 2.25$ kJ/kg.K



Soln:- Boiler 1 : (B_1) :

20 bar & 350°C;

$$\left\{\begin{array}{l} h_{g_1} = 2797.2 \text{ kJ/kg}, \\ t_s = 212.4°C \end{array}\right\} \text{From steam table;}$$

$$\therefore h_1 = h_{g_1} + C_{ps}(t_{sup} - t_s) = 2797.2 + 2.25(350 - 212.4)$$
$$h_1 = 3106.8 \text{ kJ/kg.} \longrightarrow ①$$

Boiler 2 : (B_2) : 20 bar;

$$h_2 = h_{f_2} + x_2 . h_{fg_2}$$
$$h_2 = 908.5 + x_2(1888.7) \longrightarrow ②$$

$$\left\{\begin{array}{l} \text{From steam table;} \\ 20 \text{ bar;} \\ h_f = 908.5 \text{ kJ/kg.} \\ h_{fg} = 1888.7 \text{ kJ/kg} \end{array}\right\}$$

Main : 20bar & 250°C :

Total heat of 2kg of steam in main :

$$h_{main} = 2[h_g + C_{ps}(t_{sup} - t_s)]$$
$$= 2[2797.2 + 2.25(250 - 212.4)]$$
$$h_{main} = \underline{5763.6} \text{ kJ} \longrightarrow ③$$

equating ①, ② & ③, we get;

$$h_{main} = h_1 + h_2$$

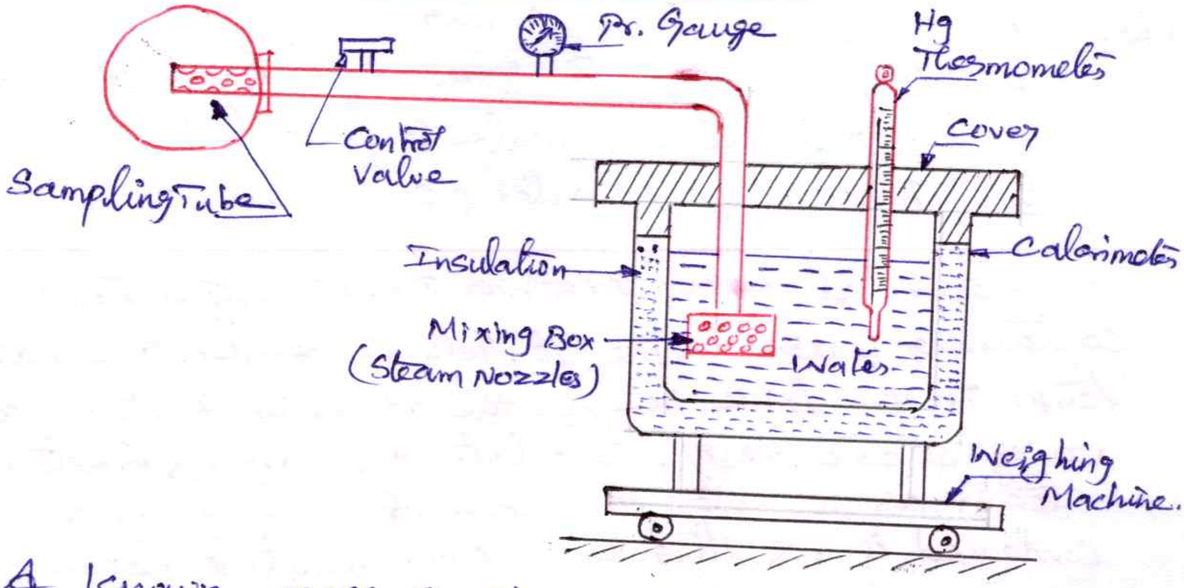
$$5763.6 = 3106.8 + [908.5 + x_2(1888.7)]$$

$$\Rightarrow x_2 = 0.925 \text{ (or) } 92.5\%$$

Measurement of Dryness fraction of steam:

The dryness fraction of steam can be measured by using the following calorimeters.

(i) Barrel / Tank (or) Bucket calorimeter.



A known mass of steam is passed through a known mass of water & steam is completely condensed. The heat lost by steam is equated to heat gained by the water.

The steam is passed thro' the sampling tube into the bucket calorimeter containing a known mass of water.

The weights of calorimeter with water before mixing with steam & after mixing the steam are obtained by weighing. The temp. of water before & after mixing the steam are measured by Hg thermometer & with the help of pr. gauge the pressure of steam passed through the sampling tube are measured.

Acc. to law of conservation, the heat lost by steam is equal to the heat gained by water & calorimeter, provided there is no addition (or) loss of heat.

Heat lost by steam $= m_s[x \cdot h_{fg} + C_w(t - t_2)] \rightarrow ①$

Heat gained by water & Calorimeter $= (m_w \cdot C_w + m_c \cdot C_c)(t_2 - t_1) \rightarrow ②$

equate ① & ② ;

$$m_s[x.h_{fg} + C_w(t-t_2)] = (m_w \cdot C_w + m_c \cdot C_c)(t_2-t_1)$$

From this expression, the dryness fraction of steam (x) may be determined.

where; p = pr. of the steam in bar.
t = temp. of steam @ Pressure 'p' from steam table.
t_1 = Initial temp. of water & calorimeter.
t_2 = Final temp. of water & calorimeter.

Steam at a pr. of 5 bar passes into a tank containing water where it gets condensed. The mass & temp. in the tank before the admission of steam are 50 kg & 20°c resp. Calculate the dryness fraction of steam as it enters the tank if 3 kg of steam gets condensed & resulting temp. of the mixture becomes 40°c. Take water equivalent of tank as 1.5 kg.

G.D :- $p = 5$ bar, $m_e = m_c \cdot C_c = 1.5$ kg, $m_{tank} = 50$ kg,
$t_1 = 20°c$, $m_s = 3$ kg, $t_2 = 40°c$.

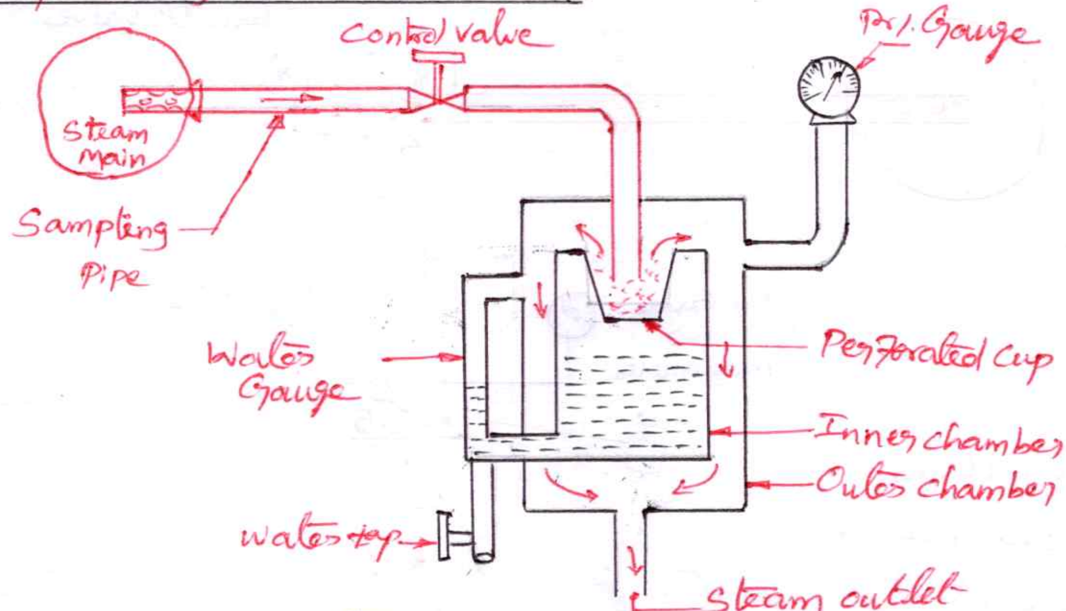
Soln :- From steam table; @ 5 bar;
$t = 151.8°c$, $h_{fg} = 2107.4$ kJ/kg.

$$m_s[x.h_{fg} + C_w(t-t_2)] = [m_w \cdot C_w + m_c \cdot C_c](t_2-t_1)$$

$$3[x.(2107.4) + 4.186(151.8-40)] = [(50 \times 4.186) + 1.5](40-20)$$

$$\Rightarrow x = \underline{\underline{0.45}}$$

(ii) Separating Calorimeter :-



The separating calorimeter is used to determine the dryness fraction of steam by mechanically seperating the water particles from the wet steam.

The wet steam enters at the top of the calorimeter thro' a control valve as shown in fig. It strikes the perforated cup & thereby undergoes a quick reversal of directions of motion. The water particle due to their greater moment of inertia, tend to move on, & consequently get seperated from the mixture. The seperated water collects at the bottom of the inner chamber, & its amount may be determined by the graduated water gauge. The amount of dry steam leaving the outer chamber of the calorimeter may be measured by condensing it in a weighed quantity of cold water (or) alternatively by reading the pr. gauge which is calibrated in terms of rate of steam flow for this purpose.

Let; m = mass of water collected in a certain time.

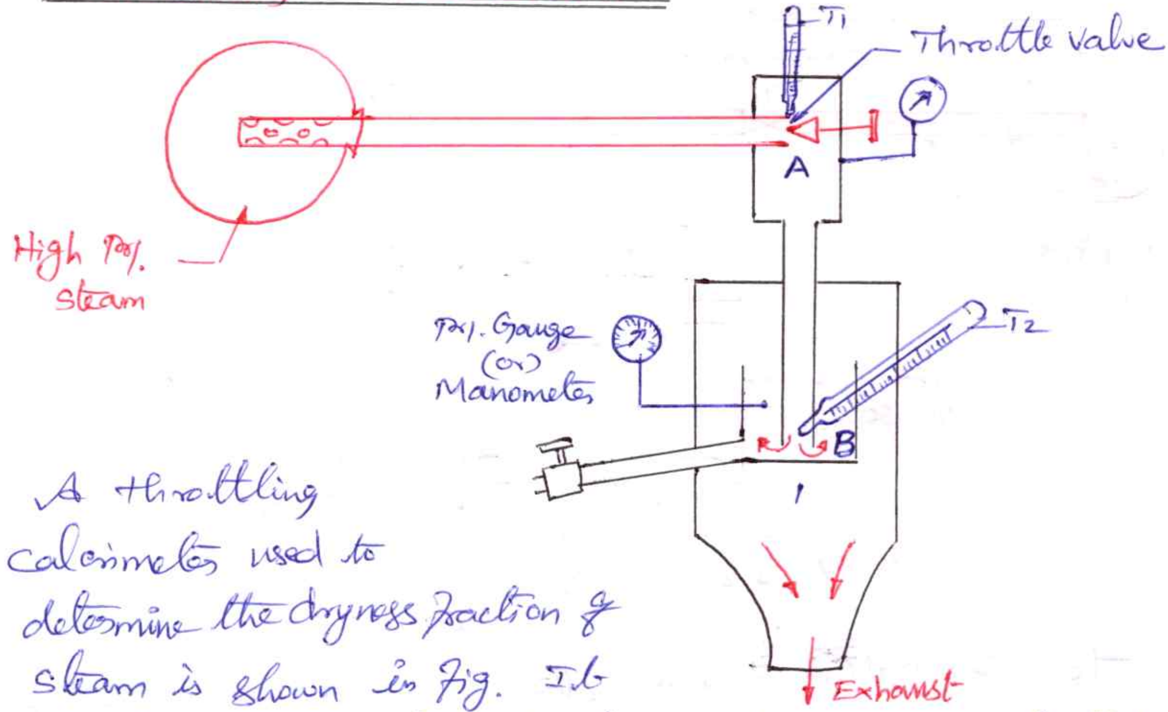
M = mass of dry steam passing in the same time &

x = Dryness fraction of wet steam.

WKT;

$$x = \frac{M}{M+m}$$

(iii) Throttling Calorimeter :-



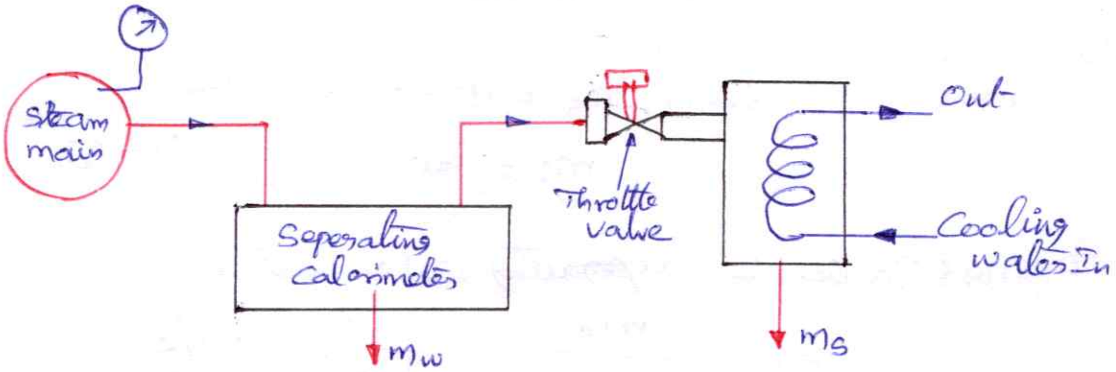
A throttling calorimeter used to determine the dryness fraction of steam is shown in fig. It consists of a separator ('A') into which steam is admitted thro' a control valve from the steam main. The prs. & temp. are measured by the prs. gauge & the thermometer (T_1) provided in this section. It may be noted that temp. recorded by T_1 is same as the saturation temp. corresponding to the prs. of steam in calorimeter (B). This steam is then throttled thro' a narrow aperture of restricted valve opening, its total heat remaining constant. The steam is in the superheated state after throttling at a lower prs. than previous. The temp. & prs. of steam leaving the calorimeter (B) is noted by the thermometer T_2 & manometer respectively.

Since the steam has undergone a throttling process,

∴ Total heat before throttling = Total heat after throttling

$$h_{f_1} + x \cdot h_{fg_1} = h_{g_2} + C_{ps}(t_{sup} - t_2)$$

(iv) Combined Seperating & Throttling Calorimeter:-



In this calorimeter, the wet steam is first collected in a perforated collecting pipe & then passed through a seperating calorimeter. A part of water is removed by the seperating calorimeter. The resulting semi-dry steam is throttled into a throttling calorimeter. This method ensures that the steam will be superheated after throttling. This instrument is well insulated to prevent any loss of heat.

Let; x_1 = Dryness fraction of steam considering seperating calorimeter &
x_2 = Dryness fraction of steam entering the throttling calorimeter.

∴ The actual Dryness fraction of steam in the steam main; $x = x_1 \cdot x_2$

In a laboratory experiment, the following observations were recorded to find the dryness fraction of steam by combined seperating & throttling calorimeter:

Total quantity of steam passed = 36 kg
water drained from seperator = 1.8 kg
steam pr. before throttling = 12 bar
steam temp. after throttling = 110°c
pr. after throttling = 1.013 bar
sp. heat of steam = 2.1 kJ/kg.k

Given: $m_s + m_w = 36\,kg$, $m_w = 1.8\,kg$, $P_1 = 12$ bar, $t_{sup} = 110°c$, $P_2 = 1.013$ bar, $C_p = 2.1\,kJ/kg.k$

Soln:

mass of dry steam, $m_s = (m_s + m_w) - m_w$

$$m_s = 36 - 1.8 = 34.2\,kg$$

∴ Dryness fraction for seperating calorimeter;

$$x_1 = \frac{m_s}{m_s + m_w} = \frac{34.2}{36} = \underline{\underline{0.95}}$$

From steam tables, corresponding @ <u>12 bar pressure (P₁)</u>

$h_{f_1} = 798.4\,kJ/kg$, $h_{fg_1} = 1984.3\,kJ/kg$

& <u>$P_2 = 1.013$ bar</u>; $h_{g_2} = 2676\,kJ/kg$, $t_2 = 100°c$

∴ Dryness fraction for throttling calorimeter;

$$h_{f_1} + x_2 h_{fg_1} = h_{g_2} + C_p(t_{sup} - t_2)$$

$$798.4 + x_2(1984.3) = 2676 + 2.1(110 - 100)$$

$$\Rightarrow x_2 = \underline{\underline{0.957}}$$

∴ The actual dryness fraction of the steam entering the combined seperating & throttling calorimeter;

$$x = x_1 . x_2$$

$$x = 0.95 \times 0.957 = \underline{\underline{0.909}}$$

Thermodynamic Relations:

Some Mathematical Theorems: [Exact Differential]

Let three variables x, y & z, any one variable may be considered as a function of other two, thus;

$$f(x,y,z) = 0 \quad ; \quad x = x(y,z)$$

$$y = y(x,z)$$

$$z = z(x,y)$$

Let x is a function of two independent variable 'y' & 'z'.

$$x = x(y,z)$$

Then the; differential of the dependent variable x is given by;

$$\boxed{dx = \left(\frac{\partial x}{\partial y}\right)_z \cdot dy + \left(\frac{\partial x}{\partial z}\right)_y \cdot dz} \longrightarrow ①$$

where 'dx' is called an "Exact Differential".

If; $\left(\frac{\partial x}{\partial y}\right)_z = M$ & $\left(\frac{\partial x}{\partial z}\right)_y = N$

Then; $dx = M\,dy + N\,dz$

Partial differentiation of M & N with respect to 'z' & 'y' resp. then; -gives;

$$\left(\frac{\partial M}{\partial z}\right)_y = \frac{\partial^2 x}{\partial y . \partial z} \quad \& \quad \left(\frac{\partial N}{\partial y}\right)_z = \frac{\partial^2 x}{\partial z . \partial y}$$

(or) $\boxed{\left(\frac{\partial M}{\partial z}\right)_y = \left(\frac{\partial N}{\partial y}\right)_z} \longleftarrow$ This is the condition of Exact (Perfect) differential.

Similarly if; $y = y(x,z)$ $\qquad$ $z = z(x,y)$

$$\boxed{dy = \left(\frac{\partial y}{\partial x}\right)_z . dx + \left(\frac{\partial y}{\partial z}\right)_x . dz} \to ② \qquad \boxed{dz = \left(\frac{\partial z}{\partial x}\right)_y . dx + \left(\frac{\partial z}{\partial y}\right)_x . dy} \to ③$$

$$\Rightarrow dy = \left(\frac{\partial y}{\partial x}\right)_z . dx + \left(\frac{\partial y}{\partial z}\right)_x \left[\left(\frac{\partial z}{\partial x}\right)_y . dx + \left(\frac{\partial z}{\partial y}\right)_x . dy\right]$$

$$= \left[\left(\frac{\partial y}{\partial x}\right)_z + \left(\frac{\partial y}{\partial z}\right)_x \cdot \left(\frac{\partial z}{\partial x}\right)_y\right] dx + \left(\frac{\partial y}{\partial z}\right)_x \cdot \left(\frac{\partial z}{\partial y}\right)_x . dy$$

$$= \left[\left(\frac{\partial y}{\partial x}\right)_z + \left(\frac{\partial y}{\partial z}\right)_x \cdot \left(\frac{\partial z}{\partial x}\right)_y\right] dx + dy$$

$$\left(\frac{\partial y}{\partial x}\right)_z + \left(\frac{\partial y}{\partial z}\right)_x \cdot \left(\frac{\partial z}{\partial x}\right)_y = 0$$

$$\Rightarrow \left(\frac{\partial y}{\partial z}\right)_x \cdot \left(\frac{\partial z}{\partial x}\right)_y = -\left(\frac{\partial y}{\partial x}\right)_z$$

$$\Rightarrow \boxed{\left(\frac{\partial x}{\partial y}\right)_z \cdot \left(\frac{\partial z}{\partial x}\right)_y \left(\frac{\partial y}{\partial z}\right)_x = -1}$$

Among the Thermodynamic variables P, V & T, the following relation holds good.

$$\boxed{\left(\frac{\partial P}{\partial V}\right)_T \cdot \left(\frac{\partial T}{\partial P}\right)_V \cdot \left(\frac{\partial V}{\partial T}\right)_P = -1}$$

Maxwell's Equations:

A Pure Substance existing in a single phase has only two independent variables. The eight quantities P, V, T, S, U, H, F (Helmholtz function) & G (Gibbs function) any one may be expressed as a function of any two others.

Acc. to I law applied to a closed system;

$$dQ = dU + dW = du + p.dv$$

Acc. to II law; $ds = \frac{dQ}{T} \Rightarrow \therefore Tds = dU + p.dv$

$$\Rightarrow dU = Tds - p.dv \longrightarrow ①$$

$$\Rightarrow dH = dU + pdV + vdp$$

$$dH = Tds + vdp \longrightarrow ②$$

Helmholtz free energy function; $\Rightarrow dF = dU - Tds - sdT$

$$dF = -pdv - sdT \longrightarrow ③$$

Gibb's free energy function; $\Rightarrow dG = dH - Tds - sdT$

$$dG = vdp - sdT \longrightarrow ④$$

Since U, H, F & G are thermodynamic properties & exact differentials of the type;

$$dx = Mdy + Ndz\text{, then}$$

$$\left(\frac{\partial M}{\partial z}\right)_y = \left(\frac{\partial N}{\partial y}\right)_z$$

Applying this to the four equation's:

Eqn: ① $\Rightarrow$ $dU = T\,ds - p.dv$

$$\Rightarrow \boxed{\left[\frac{\partial T}{\partial V}\right]_S = -\left[\frac{\partial p}{\partial s}\right]_V}$$

Eqn: ② $\Rightarrow$ $dH = T\,ds + v\,dp \Rightarrow \left[\frac{\partial T}{\partial P}\right]_S = \left[\frac{\partial V}{\partial s}\right]_P$

Eqn: ③ $\Rightarrow$ $dF = -p\,dv - s\,dT \Rightarrow \left[\frac{\partial P}{\partial T}\right]_V = \left[\frac{\partial S}{\partial V}\right]_T$

Eqn: ④ $\Rightarrow$ $dG = v\,dP - s\,dT \Rightarrow \left[\frac{\partial V}{\partial T}\right]_P = -\left[\frac{\partial S}{\partial P}\right]_T$

These Eqn's are known as "Maxwell's Equations"

Tds Equations [Entropy Eqn's]

Since entropy may be expressed as a function of T & V. Then;

$$S = S(T, V)$$

$$ds = \left(\frac{\partial s}{\partial T}\right)_V . dT + \left(\frac{\partial s}{\partial V}\right)_T . dv$$

$$T\,ds = T\left(\frac{\partial s}{\partial T}\right)_V . dT + T\left(\frac{\partial s}{\partial V}\right)_T . dv$$

For a reversible constant volume change;

$$dQ = C_V (dT)_V = T\,ds \qquad \left[\because ds = \frac{dQ}{T}\right]$$

$$\therefore \Rightarrow C_V = T\left(\frac{\partial s}{\partial T}\right)_V$$

$$\Rightarrow T.ds = C_V.dT + T\left(\frac{\partial s}{\partial V}\right)_T . dv$$

$$\left\{\text{Maxwell's 3}^{\text{rd}}\text{ eqn;} \quad \left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{\partial s}{\partial V}\right)_T\right\}$$

$$\boxed{T.ds = C_V.dT + T\left(\frac{\partial P}{\partial T}\right)_V . dv} \leftarrow \text{First Tds eqn,}$$

(or) $\boxed{T\,ds = C_V\,dT + \frac{\beta}{K} T.dv}$

lll^{ly}; $S = S(T, P)$

$$ds = \left(\frac{\partial s}{\partial T}\right)_P . dT + \left(\frac{\partial s}{\partial P}\right)_T . dp$$

$$T\,ds = T.\left(\frac{\partial s}{\partial T}\right)_P . dT + T\left(\frac{\partial s}{\partial P}\right)_T . dp$$

$$\left\{dQ = C_P\,dT_P = T\,ds \qquad C_P = T\left(\frac{\partial s}{\partial T}\right)_P\right\}$$

$$\Rightarrow T\,ds = C_P.dT + T.\left(\frac{\partial s}{\partial P}\right)_T . dp$$

$$\left\{\text{Maxwell's 4}^{\text{th}}\text{ eqn;} \quad \left(\frac{\partial V}{\partial T}\right)_P = -\left(\frac{\partial s}{\partial P}\right)_T\right\}$$

$$\boxed{T\,ds = C_P.dT - T\left(\frac{\partial V}{\partial T}\right)_P . dp} \leftarrow \text{Second Tds eqn,}$$

(or) $\boxed{T\,ds = C_P.dT - \beta V.T.dp}$

Tds → 3rd eqn. →

Difference in Heat Capacities [Specific Heats].

First Tds eqn: $Tds = C_v \cdot dT + T\left(\frac{\partial P}{\partial T}\right)_v \cdot dv \longrightarrow ①$

Second Tds eqn: $Tds = C_p dT - T\left(\frac{\partial v}{\partial T}\right)_p \cdot dP \longrightarrow ②$

Equating ① & ②;

$$C_p dT - T\left(\frac{\partial v}{\partial T}\right)_p \cdot dP = C_v dT + T\left(\frac{\partial P}{\partial T}\right)_v \cdot dv$$

$$(C_p - C_v)\, dT = T\left(\frac{\partial P}{\partial T}\right)_v \cdot dv + T\left(\frac{\partial v}{\partial T}\right)_p \cdot dP$$

{WKT; $T = T(P,V)$;

$\therefore\ dT = \left(\frac{\partial T}{\partial V}\right)_P dv + \left(\frac{\partial T}{\partial P}\right)_v \cdot dP$}

$$dT = \frac{T\left(\frac{\partial P}{\partial T}\right)_v \cdot dv}{C_p - C_v} + \frac{T\left(\frac{\partial V}{\partial T}\right)_p \cdot dP}{C_p - C_v}$$

$$\therefore \frac{T\left(\frac{\partial P}{\partial T}\right)_v}{C_p - C_v} = \left(\frac{\partial T}{\partial V}\right)_P \quad \& \quad \frac{T\left(\frac{\partial V}{\partial T}\right)_P}{C_p - C_v} = \left(\frac{\partial T}{\partial P}\right)_V .$$

Both these eqn's give;

$$\therefore C_p - C_v = T\left(\frac{\partial P}{\partial T}\right)_v \cdot \left(\frac{\partial V}{\partial T}\right)_p \longrightarrow ③$$

But; In exact differential; $\left(\frac{\partial P}{\partial T}\right)_v \cdot \left(\frac{\partial T}{\partial V}\right)_p \cdot \left(\frac{\partial V}{\partial P}\right)_T = -1 \longrightarrow ④$

WKT; Volume expansivity $(\beta) = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p \Rightarrow \left(\frac{\partial V}{\partial T}\right)_p = \beta V$

Isothermal compressibility $(K_T) = -\frac{1}{v}\left(\frac{\partial v}{\partial P}\right)_T \Rightarrow \left(\frac{\partial v}{\partial P}\right)_T = -K_T V$

From eqn ④;

$$\left(\frac{\partial P}{\partial T}\right)_v = \frac{-1}{\left(\frac{\partial T}{\partial V}\right)_p \cdot \left(\frac{\partial V}{\partial P}\right)_T} = -\left(\frac{\partial V}{\partial T}\right)_p \left(\frac{\partial P}{\partial V}\right)_T = \frac{-\beta V}{-K_T V} = \frac{\beta}{K_T}$$

$\therefore$ The above value's sub. in Eqn. ③:

$$C_p - C_v = T \cdot \frac{\beta}{K} \cdot \beta v = \frac{TV\beta^2}{K_T} \Rightarrow \boxed{C_p - C_v = \frac{TV\beta^2}{K_T}}$$

* Note: $(C_p - C_v)$ is always positive. $\therefore$ C_p is always greater than C_v

Helmholtz Function : (A)

Helmholtz function (A) is a property of a system & is given by substracting the product of absolute temperature (T) and entropy (S) from the internal energy (U).

Mathematically;

Helmholtz Function, $\boxed{A = U - T.S}$

Since (U-TS) is made up entirely of properties, therefore Helmholtz function (A) is also a property.

For per unit mass, specific Helmholtz function;

$a = u - Ts$ where; $u \rightarrow$ sp. internal energy & $s \rightarrow$ sp. entropy.

The work done in a non-flow reversible system;

The general gas energy eqn. is;

$$\delta Q = \delta W + dU \Rightarrow W = Q - (u_0 - u_1)$$
$$= T ds - (u_0 - u_1)$$
$$= T(s_0 - s_1) - (u_0 - u_1)$$
$$\boxed{W = (u_1 - Ts_1) - (u_0 - Ts_0) = a_1 - a_0}$$

The term (u-TS) is known as Helmoltz function.

Gibbs Function : (G) [Thermodynamic Potential]

The Gibbs function is also a property of a system & is denoted by G.

Mathematically; Gibbs Function, $G = U - TS + PV$

$$\boxed{G = H - TS}$$

Since (H-TS) is made up entirely of properties, therefore Gibbs function (G) is also a property.

For per unit mass, specific Gibbs function;

$g = h - Ts$ where; $h \rightarrow$ sp. enthalpy & $s \rightarrow$ sp. entropy.

The term (u-TS) [Helmoltz Function], this gives maximum possible output when the heat 'Q' is transferred at const. temperature & is the case with a very large source.

If work against atmosphere is equal to $P_0(V_0 - V_1)$, then the maximum work available;

$$
\begin{aligned}
W_{max} &= W - \text{work against atmosphere.} \\
&= W - P_0(V_0 - V_1) \\
&= (U_1 - TS_1) - (U_0 - TS_0) - P_0(V_0 - V_1) \\
&= (U_1 + P_0V_1 - TS_1) - (U_0 + P_0V_0 - TS_0) \\
&= (h_1 - TS_1) - (h_0 - TS_0)
\end{aligned}
$$

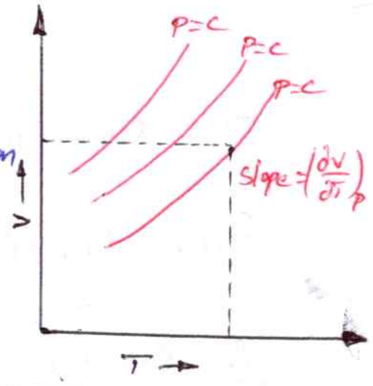
$$\boxed{W_{max} = g_1 - g_0}$$

where; $g = (h - TS)$ is known as Gibb's function (or) Free energy function.

Co-efficient of volume Expansion: (β)

The co-efficient of volume expansion is defined as the change in volume with change in temperature per unit volume, keeping the pressure constant.

$$\boxed{\beta = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P}$$



For specific volume, the co-eff. of volume expansion is written as, $\beta = \frac{1}{V_s}\left(\frac{\partial V_s}{\partial T}\right)_P$

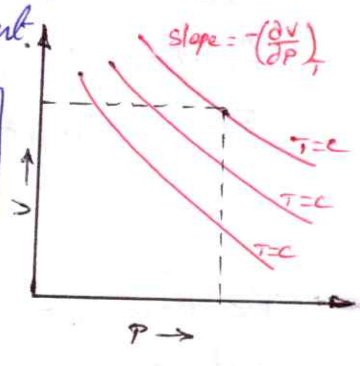
where; V_s = sp. volume in m^3/kg.

Isothermal Compressibility: (K)

The isothermal compressibility of a substance is defined as the change in volume with the change in pressure per unit volume, keeping the temperature constant.

$$K = \frac{-1}{V}\left(\frac{\partial V}{\partial P}\right)_T$$

[$\because$ -ve sign indicates that the slope is decreasing]



For specific volume, the isothermal compressibility is written as;

$$K = \frac{-1}{V_s}\left(\frac{\partial V_s}{\partial P}\right)_T$$

where; V_s = sp. volume in m^3/kg.

Variable specific heat:

Molar specific heats of a gas: [Volumetric sp. heats]

It is defined as the amount of heat required to raise the temp. of unit mole of gas through one degree.

It is denoted by C_m (KJ/Kg-mol.K)

$\boxed{C_m = M \times C}$ where; M = Molar mass of the gas &
c = Sp. heat of the gas.

III^{ly} way;

∴ Molar sp. heat at constant volume;

$$\boxed{C_{mv} = M.C_v}$$

Molar sp. heat at constant pressure;

$$\boxed{C_{mp} = M.C_p}$$

Variation of specific heat with Temperature:

The sp. heat of any gas increases with increase in temperature. The sp. heat increases because of increase in energy of vibration of molecules at high temperature. The vibration are caused because of collisions among molecules which are significant at higher temperature. Therefore, the energy of vibration of a polyatomic gas will undergo a considerable change at ordinary temp., thus increasing the value of sp. heat.

The sp. heat is a linear function of temperature and may be expressed as: $C_p = a + KT$ & $C_v = b + KT$

where; a, b & K are constant.

WKT; characteristic gas constant: $R = C_p - C_v$

$$= (a + KT) - (b + KT)$$

$$\therefore R = a - b$$

Tds - Third eqn: $U = U(P,V) \Rightarrow du = \left(\frac{\partial U}{\partial P}\right)_v dP + \left(\frac{\partial U}{\partial v}\right)_p . dv$

WKT; $\left(\frac{\partial U}{\partial P}\right)_v = \left(\frac{K}{\beta} \cdot C_v\right)$ WKT; $Tds = dU + dw \Rightarrow Tds = du + Pdv$

$\left(\frac{\partial U}{\partial v}\right)_p = \left(\frac{C_p}{\beta v} - P\right)$

$$\therefore Tds = \frac{K}{\beta} \times C_v . dp + \left(\frac{C_p}{\beta . v} - p\right) dv + pdv$$

$$\boxed{Tds = \frac{KC_v}{\beta} . dp + \frac{C_p . dv}{\beta . V}}$$

Isentropic Expansion with Variable specific Heat:

Isentropic expansion is a reversible adiabatic process, in which no heat transfer of gas. Thus;

$\delta Q = 0$ WKT; $ds = \frac{\delta Q}{T}$ Since; $\delta Q = 0$, $\therefore ds = 0$

The expression may be expressed in the following ways:

(a) In terms of V & T;

Acc. to I law of TD; $\delta Q = dU + \delta W$

$$\delta Q = mC_v dT + p.dv$$

÷ by T; $\frac{\delta Q}{T} = m.C_v \frac{dT}{T} + \frac{P}{T}.dv$

Since; $PV = mRT \Rightarrow \frac{P}{T} = \frac{mR}{V}$ & $\frac{\delta Q}{T} = ds = 0.$

$$\therefore m.C_v \frac{dT}{T} + mR.\frac{dV}{V} = 0 \qquad \left[\begin{array}{l}\because C_v = b + kT \\ R = a - b\end{array}\right]$$

(or) $m(b+kT)\frac{dT}{T} + m(a-b)\frac{dV}{V} = 0$

$$mb.\frac{dT}{T} + mk\,dT + m(a-b).\frac{dV}{V} = 0$$

On $\int$ing; $mb\log_e T + mkT + m(a-b)\log_e V = \text{Constant}.$

÷ by mb; $\log_e T + \frac{kT}{b} + \left(\frac{a-b}{b}\right)\log_e V = c$

(or) $\log_e T + \log_e e^{\frac{kT}{b}} + \log_e V^{\left(\frac{a}{b}-1\right)} = c$

(or) $T.e^{\frac{kT}{b}}.V^{\left(\frac{a}{b}-1\right)} = c$ (or) $\boxed{\frac{T}{V}.e^{\frac{kT}{b}}.V^{\frac{a}{b}} = c} \longrightarrow$ Ⓐ

(or) $\boxed{T^{b}.e^{kT}.V^{a-b} = c}$

The more useful form of above eqn. is;

$$\boxed{\frac{T_1}{V_1}.e^{\frac{kT_1}{b}}.V_1^{\frac{a}{b}} = \frac{T_2}{V_2}.e^{\frac{kT_2}{b}}.V_2^{\frac{a}{b}} = \ldots\ldots = \text{Constant}}$$

(b) In terms of P & V;

WKT; General gas eqn; is;

$$PV = mRT \Rightarrow \frac{T}{V} = \frac{P}{mR} = \frac{P}{m(a-b)}$$

$\left(\frac{T}{V}\right)$ value is Substitute in eqn. Ⓐ; we get:

$$\frac{P}{m(a-b)}.e^{\frac{kT}{b}}.V^{\frac{a}{b}} = c \Rightarrow P.e^{\frac{kT}{b}}.V^{\frac{a}{b}} = c \qquad \left[\begin{array}{l}\because m(a-b) \text{ is const.} \\ \text{for a particular gas}\end{array}\right]$$

(or) $\boxed{P^{b}.e^{kT}.V^{a} = c}$

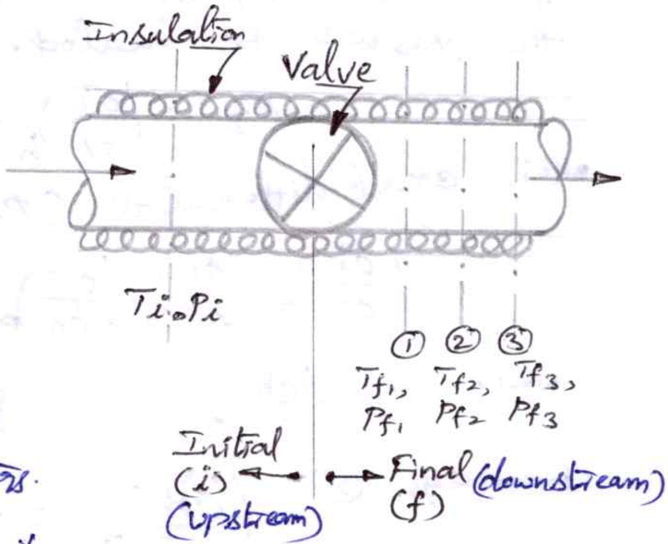
The more useful form of above eqn. is;

$$\boxed{P_1.e^{\frac{kT_1}{b}}.V_1^{\frac{a}{b}} = P_2.e^{\frac{kT_2}{b}}.V_2^{\frac{a}{b}} = \ldots\ldots = \text{Constant}.}$$

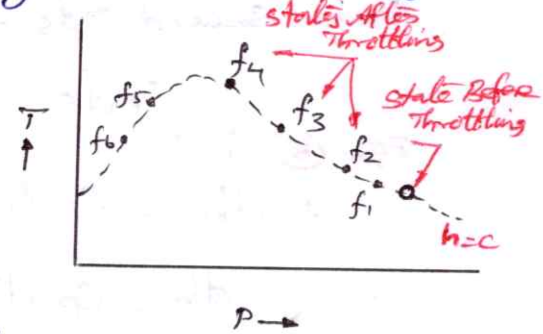
Where; Suffixes 1, 2, 3..... refer to different sets of Conditions.

Joule-Thomson Co-efficient (or) Joule-Kelvin Effect:

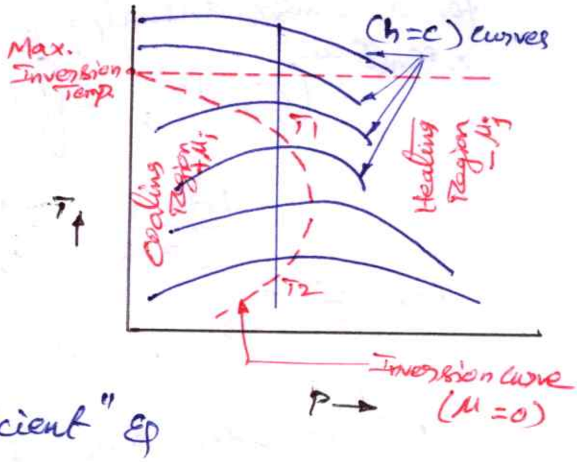
A gas is made to undergo continuous throttling process by a valve, as shown in fig. The pressures & temperatures of the gas in the insulated pipe upstream & downstream of the valve are measured with suitable manometers & thermometers.



Let P_i & T_i be the arbitrarily chosen pr. & Temp. before throttling & let them be kept constant. By operating the valve manually, the gas is throttled successively to different pr.'s & temp's $P_{f_1}, T_{f_1}; P_{f_2}, T_{f_2}; P_{f_3}, T_{f_3}$ & so on. These are then plotted on the P-T diagram as shown in fig.



The initial pr. & temp. of the gas (before Throttling) are then set to new values & by throttling to different states, a family of isenthalpes is obtained for the gas, as shown in fig. The curve passing through the maxima of these isenthalpes is called the "Inversion Curve."



The Numerical value of the slope of an isenthalpe on a P-T diagram at any point is called the "Joule-Kelvin co-efficient" & is denoted by μ_J.

Thus the locus of all points at which μ_J is zero is the inversion curve.

- The region inside the inversion curve is cooling region (+ve).
- The region outside the inversion curve is heating region (μ_J is -ve).

If a gas is throttled in the cooling region, it will get cooled. If it is throttled in the heating region, the gas will be heated. So,

$$\mu_J = \left(\frac{\partial T}{\partial P}\right)_h \rightarrow \text{①}$$

WKT; exact differential (P, T & h);

$$\left(\frac{\partial h}{\partial P}\right)_T \cdot \left(\frac{\partial P}{\partial T}\right)_h \cdot \left(\frac{\partial T}{\partial h}\right)_P = -1 \rightarrow \text{②}$$

WKT; $C_p = \left(\frac{\partial h}{\partial T}\right)_P$ $\quad [dH = C_p \cdot dT_P]$

From eqn: ②; $\left(\frac{\partial H}{\partial P}\right)_T = \frac{-1}{\left(\frac{\partial P}{\partial T}\right)_h \cdot \left(\frac{\partial T}{\partial h}\right)_P} = -\left(\frac{\partial T}{\partial P}\right)_h \left(\frac{\partial h}{\partial T}\right)_P$

[where; $\left(\frac{\partial h}{\partial P}\right)_T$ → is known as Const. Temp. co-efficient] $\Rightarrow \left(\frac{\partial h}{\partial P}\right)_T = -\mu_J C_p \rightarrow \text{③}$

The property relation for dh is $dh = Tds + vdP \rightarrow \text{④}$

From second Tds eqn. we have;

$$Tds = C_p dT - T\left(\frac{\partial v}{\partial T}\right)_P \cdot dP \rightarrow \text{⑤}$$

Eqn: ⑤ in ④;

$$\therefore dh = C_p dT - T\left(\frac{\partial v}{\partial T}\right)_P \cdot dP + vdP$$

$$dh = C_p \cdot dT - \left[T\left(\frac{\partial v}{\partial T}\right)_P - v\right] dP \rightarrow \text{⑥}$$

for a const. enthalpy process (h = c) ⇒ dh = 0.

∴ eqn ⑥ is;

$$\therefore (C_p dT)_h = \left[\left(T\left(\frac{\partial v}{\partial T}\right)_P - v\right) \cdot dP\right]_h$$

$$\left(\frac{\partial T}{\partial P}\right)_h = \frac{1}{C_p}\left[T\left(\frac{\partial v}{\partial T}\right)_P - v\right] \rightarrow \text{⑦}$$

eqn: ⑦ in ①;

$$\therefore \mu_J = \frac{1}{C_p}\left[T\left(\frac{\partial v}{\partial T}\right)_P - v\right] \rightarrow \text{⑧}$$

For an ideal gas; $Pv = RT \Rightarrow v = \frac{RT}{P}$

$$\Rightarrow \left(\frac{\partial v}{\partial T}\right)_P = \frac{R}{P} = \frac{v}{T} \rightarrow \text{⑨}$$

Sub. ⑨ in ⑧;

$$\therefore \mu_J = \frac{1}{C_p}\left[T \times \frac{v}{T} - v\right] = 0$$

There is no change in temperature when an ideal gas is made to undergo a Joule-Kelvin expansion (throttling).

Change in Thermodynamic properties with Variable sp. heat:

We know that already, when a gas is heated, its specific heat increases. Since the thermodynamic properties like U, H & S depends upon sp. heat also, thus there is a considerable change in these properties with change in temp. Consider 'm' kg of gas being heated from the initial state 1 to the final state 2.

Let; m = mass of the gas,

P_1, V_1 & T_1 = Pr., volume & Temp. at the state 1 (Initial)

P_2, V_2 & T_2 = Corresponding values at the final state 2.

$C_p = a + kT$ & $C_v = b + kT$

Now, let us drive the following relations for the change in thermodynamic properties with variable sp. heat.

(i) change in Internal Energy:

$$dU = m.C_v.dT = m(b+kT)dT = mb\,dT + mkT.dT$$

$\int$ing;

$$\int_1^2 dU = mb\int_1^2 dT + mk\int_1^2 T\,dT$$

$$U_2 - U_1 = mb(T_2-T_1) + mk\frac{(T_2^2-T_1^2)}{2}$$

$$= m(T_2-T_1)\left[b + k\frac{(T_2+T_1)}{2}\right]$$

$$= m(T_2-T_1)(b+kT_m)$$

$$\boxed{U_2 - U_1 = mC_{vm}(T_2-T_1)}$$

(ii) change in Enthalpy:

$$dH = m.C_p\,dT = m(a+kT)dT = ma\,dT + mkT\,dT$$

$\int$ing;

$$\int_1^2 dH = ma\int_1^2 dT + mk\int_1^2 T.dT$$

$$H_2 - H_1 = ma(T_2-T_1) + mk\frac{(T_2^2-T_1^2)}{2}$$

$$= m(T_2-T_1)\left[a + k\frac{(T_2+T_1)}{2}\right]$$

$$= m(T_2-T_1)[a + kT_m]$$

$$\boxed{H_2 - H_1 = mC_{pm}(T_2-T_1)}$$

(iii) change in entropy: wkt; $ds = \frac{\delta Q}{T}$

Now the relation for the change of entropy for variable sp. heat can be expressed in the following three ways.

→ (a) In Terms of V & T:

Acc. to I law of T.D; $\delta Q = dU + \delta W$

$\delta Q = mC_v dT + Pdv$

÷ by T; $\frac{\delta Q}{T} = mC_v \cdot \frac{dT}{T} + \frac{P}{T} dv$

$$\because \left(PV = mRT \quad \frac{P}{T} = \frac{mR}{V} = \frac{m(a-b)}{V} \right)$$

$$\therefore ds = m.c_v \frac{dT}{T} + m(a-b) \cdot \frac{dv}{v}$$

$$= m(b+kT)\frac{dT}{T} + m(a-b)\frac{dv}{v}$$

$$= m.b\frac{dT}{T} + mk\,dT + m(a-b)\frac{dv}{v}$$

$\int$ing; $\int_1^2 ds = mb\int_1^2 \frac{dT}{T} + mk\int_1^2 dT + m(a-b)\int_1^2 \frac{dv}{v}$

$S_2-S_1 = mb\log_e\left(\frac{T_2}{T_1}\right) + mk(T_2-T_1) + m(a-b)\log_e\left(\frac{V_2}{V_1}\right)$ → Ⓐ

$$\Rightarrow \boxed{S_2-S_1 = 2.3\,mb\log\left(\frac{T_2}{T_1}\right) + mk(T_2-T_1) + 2.3\,m(a-b)\log\left(\frac{V_2}{V_1}\right)}$$

→ (b) In Terms of P & T: wkt; $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \Rightarrow \frac{V_2}{V_1} = \frac{P_1}{P_2}\cdot\frac{T_2}{T_1}$

$\Rightarrow \log_e\left(\frac{V_2}{V_1}\right) = \log_e\left(\frac{P_1}{P_2}\right) + \log_e\left(\frac{T_2}{T_1}\right)$

∴ In eqn: Ⓐ;

$$S_2-S_1 = m.b\log_e\left(\frac{T_2}{T_1}\right) + mk(T_2-T_1) + m(a-b)\left[\log_e\left(\frac{P_1}{P_2}\right) + \log_e\left(\frac{T_2}{T_1}\right)\right]$$

$$S_2-S_1 = m.\log_e\left(\frac{T_2}{T_1}\right)[b+a-b] + mk(T_2-T_1) + m(a-b).\log_e\left(\frac{P_1}{P_2}\right)$$

$$\boxed{S_2-S_1 = 2.3\,m.a\log\left(\frac{T_2}{T_1}\right) + mk(T_2-T_1) + 2.3\,m(a-b)\log\left(\frac{P_1}{P_2}\right)}$$

→ (c) In Terms of P & V: wkt; $\frac{T_2}{T_1} = \frac{P_2}{P_1}\cdot\frac{V_2}{V_1} \Rightarrow \log_e\left(\frac{T_2}{T_1}\right) = \log_e\left(\frac{P_2}{P_1}\right) + \log_e\left(\frac{V_2}{V_1}\right)$

∴ In eqn Ⓐ: $S_2-S_1 = m.b\left[\log_e\left(\frac{P_2}{P_1}\right) + \log_e\left(\frac{V_2}{V_1}\right)\right] + mk(T_2-T_1) + m(a-b)\log_e\left(\frac{V_2}{V_1}\right)$

$$= mb\log_e\left(\frac{P_2}{P_1}\right) + m\log_e\left(\frac{V_2}{V_1}\right)[b+a-b] + mk(T_2-T_1)$$

$$= m.b\log_e\left(\frac{P_2}{P_1}\right) + mk(T_2-T_1) + m.a.\log_e\left(\frac{V_2}{V_1}\right)$$

$$\boxed{\therefore S_2-S_1 = 2.3\,mb\log\left(\frac{P_2}{P_1}\right) + mk(T_2-T_1) + 2.3\,ma\log\left(\frac{V_2}{V_1}\right)}$$

A Certain quantity of gas having a volume of $5 m^3$ at 1 bar & 27°c is heated at Constt. Pr. to 55°c. Determine:
1. mass of the gas, 2. heat transfer during the process &
3. change of entropy.

Take: $C_p = 0.946 + 0.184 \times 10^{-3} . T$ kJ/kg.k &
$C_v = 0.653 + 0.184 \times 10^{-3} T$ kJ/kg.k

G.D: $V_1 = 5 m^3$, $P_1 = P_2 = 1 bar = 1 \times 10^5 N/m^2$, $T_1 = 27°C = 300k$,
$T_2 = 55°c = 328 k$, $a = 0.946$ kJ/kg.k, $b = 0.653$ kJ/kg.k
$k = 0.184 \times 10^{-3}$ kJ/kg.k

Soln: (i) Mass of the gas: (m):

WKT; $R = C_p - C_v = a - b = 0.946 - 0.653 = 0.293$ kJ/kg.k

WKT; $P_1 V_1 = m.R.T_1 \Rightarrow m = \frac{100 \times 5}{0.293 \times 300} = \underline{5.7 kg}$ ($\because P = 100 kN/m^2$)

(ii) Heat transfer during the process: (Q_{1-2}):

Mean sp. heat @ Constt. Pressure; $C_{p_m} = a + k.T_m$

where; T_m (Mean Temp.) $= \frac{T_1 + T_2}{2} = \frac{300 + 328}{2} = \underline{314 K}$

$\therefore C_{p_m} = 0.946 + 0.184 \times 10^{-3} \times 314 = \underline{1} kJ/kg.k$

$\therefore Q_{1-2} = m . C_{p_m} (T_2 - T_1)$

$= 5.7 \times 1 (328 - 300) = \underline{159.3} kJ$

(iii) Change of Entropy $(S_2 - S_1)$:

$$S_2 - S_1 = 2.3\, m.a \log\left(\frac{T_2}{T_1}\right) + mk(T_2 - T_1) + 2.3\, m(a-b) \log\left(\frac{P_1}{P_2}\right)$$

$$= 2.3 \times 5.7 \times 0.946 \log\left(\frac{328}{300}\right) + 5.7 \times 0.184 \times 10^{-3} (328 - 300)$$

$S_2 - S_1 = \underline{0.509\ kJ/k}$

A mass of 1.5 kg & volume of $0.14 m^3$ of certain gas @ 40 bar is expanded isentropically such that temp. falls to 500k.
Determine: 1. Initial temp. of the gas, 2. Workdone during the process, & 3. pressure of the end of expansion.
Take: $C_p = 0.946 + 0.00184 T$ kJ/kg.k, $C_v = 0.653 + 0.000184 T$ kJ/kg.k

G.D: $m = 1.5\,kg$, $V_1 = 0.14\,m^3$, $P_1 = 40\,bar = 40\times10^5\,N/m^2 = 4000\,kN/m^2$

$T_2 = 500K$, $a = 0.946\,kJ/kg.k$, $b = 0.653\,kJ/kg.k$,

$k = 0.184\times10^{-3}\,kJ/kg.k$

(i) Initial Temperature (T_1):

WKT; $P_1V_1 = m.R.T_1 \Rightarrow T_1 = \frac{P_1V_1}{m.R}$ $\because R = C_p - C_v$

$R = a - b = 0.946 - 0.653$

$\therefore T_1 = \frac{4000\times0.14}{1.5\times0.293} = 1274.2\,k$ $R = 0.293\,kJ/kg.k$

(ii) Work done (W_{1-2}): $\delta Q = du + \delta w$

($\because \delta Q = 0$)

$\therefore \delta w = -du$

$\Rightarrow W_{1-2} = -(U_2 - U_1)$ where; $du = (U_2 - U_1)$

$= m.Cv_m(T_2 - T_1)$

WKT; $Cv_m = b + k.T_m$

$Cv_m = 0.653 + 0.184\times10^{-3}\left(\frac{T_1+T_2}{2}\right)$ $\therefore du = 1.5\times0.816(500-1274.2)$

$Cv_m = 0.816\,kJ/kg.k$ $du = -947.6\,kJ$

$\therefore W_{1-2} = 947.6\,kJ$

(iii) Pressure @ the end of Expansion (P_2):

WKT; $T_1 . e^{\frac{kT_1}{b}} . V_1^{\left(\frac{a}{b}-1\right)} = T_2 . e^{\frac{kT_2}{b}} . V_2^{\left(\frac{a}{b}-1\right)}$

$\therefore 1274.2\times e^{\frac{0.184\times10^{-3}\times1274.2}{0.653}} \times 0.14^{\left(\frac{0.946}{0.653}-1\right)} = 500\times e^{\frac{0.184\times10^{-3}\times500}{0.653}} \times V_2^{\left(\frac{0.946}{0.653}-1\right)}$

$\Rightarrow V_2 = 0.99\,m^3$

$\therefore P_2V_2 = m.R.T_2$

$\Rightarrow P_2 = \frac{1.5\times0.293\times500}{0.99} = 221.97\,kN/m^2$

$P_2 = 2.22\,bar$

Equations of state:

Practically there is no real gas which obeys the gas laws perfectly. The deviation is small @ ordinary temp/. & pressures. But it is large at high pressure & low temp/s. In deriving the perfect gas laws on the basis of kinetic theory of gases, the following assumptions were made which do not hold true for real gases:

→ 1. The molecules of gas are mere mass-points occupying no space &

→ 2. There is no attraction (or) repulsion b/w. the molecules.

In actual practice, the molecules of all actual (or) real gases do occupy some space & do attract each other.

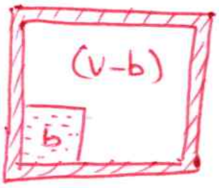
Some of the equations used to correct the perfect gas equation are defined below.

(a) Vander waal's Equation's:

The perfect gas equation $PV = RT$.

The Dutch scientist J.D. Vander waal's was the first scientist to correct this equation by applying corrections for the above two factor's (assumption's).

(i) Correction for the size of molecules:



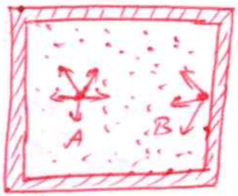
Consider some quantity of a gas contained in a vessel. We know that the molecules of a gas have a finite size.

∴ the space available for the molecules to move about is less than the volume of the gas.

Let; v = volume of the gas &
b = volume occupied by all the molecules.

∴ space actually available (or) the effective volume of the gas $\Big\} = (v-b)$

(ii) Correction for the mutual attraction of molecules:



Consider a gas molecules 'A' well within the body of the gas inside the vessel. It is attracted by other molecules in all directions with the same force & the net force on it is zero. But when it strikes the wall of the vessel, it is pulled back by other molecules. It's velocity & momentum with which it strikes the wall would be less than the momentum with which it will strike in the absence of the force of attraction. It is obvious that when the pressure of the gas drops, the momentum of the molecules also decreases. It is due to this fact, that the decrease in pressure is proportional to:

→ 1. The no. of attracting molecules per unit volume &

→ 2. the no. of attracting molecules striking a unit area of the walls of the containing vessel per unit time.

Each of these factors are proportional to the number of molecules per m^3 (or) the density of the gas.

$\therefore$ Decrease in pressure, $p' \propto (\text{Density of the gas})^2$

$$\Rightarrow p' \propto \frac{1}{V^2}$$

$$\therefore \quad p' = \frac{a}{V^2}$$ (where; a is constant of proportionality)

$\therefore$ Corrected pressure $= P + p'$

$$= P + \frac{a}{V^2}$$

Now with these corrections, the perfect gas equation ($PV = RT$) for the real (or) actual gas becomes;

$$\boxed{\left(P + \frac{a}{V_s^2}\right)(V_s - b) = RT}$$

This is known as Van der Waal's equation for a real gas.

(b) Clausius equation: It is a modification of Vander waal's equation's.

$$P(V_s - b) = RT$$

(c) Beattie Bridgeman equation:

$$P = \frac{RT(1-\varepsilon)}{V_s}(V_s + B) - \frac{A}{V_s}$$

where; $A = A_0\left(1 - \frac{a}{V_s}\right)$, $B = B_0\left(1 - \frac{b}{V_s}\right)$ & $\varepsilon = \frac{c}{V_s T^3}$

(d) Bertholet equation:

$$\left(P + \frac{a}{T V_s^2}\right)(V_s - b) = RT$$

(e) Dieterici equation:

$$P = \frac{RT}{V_s - b} \cdot e^{-a/R.T.V_s}$$

(f) Redlich-Kwong equation:

$$\left(P + \frac{a}{\sqrt{T} \cdot V_s(V_s + b)}\right)(V_s - b) = RT$$

(g) Viral equation:

The viral equation of state was first introduced by the Dutch physicist Kammerlingh Onnes, & is only applicable to gases of low & medium densities. Basically the viral equation of state represents an expansion of the $P.V_s$ product in infinite series form. It is given as;

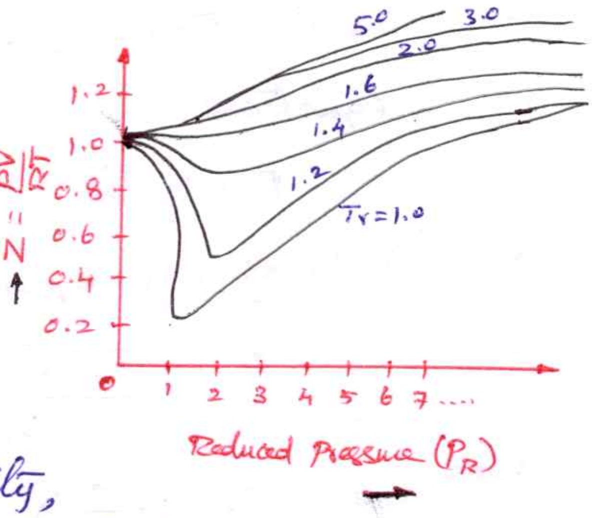
$$P.V_s = RT\left(1 + \frac{B}{V_s} + \frac{C}{V_s^2} + \frac{D}{V_s^3} + \ldots\right)$$

(or) $$P.V_s = RT\left(1 + B'P + C'P^2 + D'P^3 + \ldots\right)$$

where; B, B', C, C', D, D' etc... are called viral co-efficient's.

Note: $\left(\frac{P.V_s}{RT}\right)$ is known as Compressibility Factor & it's denoted by 'Z'.

Compressibility chart: (Z)



The compressibility factor (Z) of any gas is a function of 'P' & 'T'.

So that; $Z = f(T_r, P_r)$

except near the critical point. The value of Z for any real gas may be less (or) more than unity, depending on P & T conditions of the gas.

The general compressibility chart is plotted with (Z) Vs (P_r) for various values of T_r. This is constructed by plotting the known data of one or more gases & can be used for any gas. This chart gives best results for the regions well removed from the critical state for all gases. Fig. shows "Generalised compressibility chart."

Clausius-clapeyron Equation:

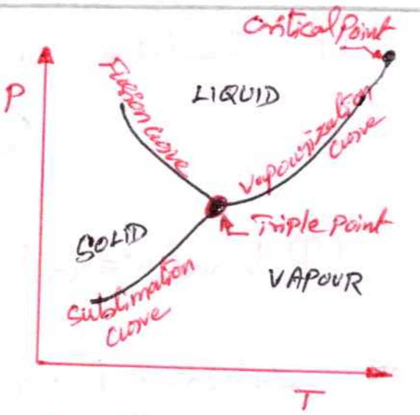


Fig: (P-T) Diagram.

It is a relation between the saturation pressure, temp., the enthalpy of evaporation & the specific volume of the two phases involved. This equation provides a basis for calculations of properties in a two-phase region. It gives the slope of a curve separating the two phases in the (P-T) diagram.

The clausius-clapeyron equation can be derived in different ways. The method given below involves the use of the maxwell 3rd relation.

$$\left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{\partial s}{\partial V}\right)_T$$

Let us consider the change of state from saturated liquid to saturated vapour of a pure substance which takes place at constt. temp. During the evaporation, the pressure & tempi. are independent of volume.

$$\therefore \boxed{\left(\frac{dp}{dT}\right) = \frac{S_g - S_f}{V_g - V_f}} \longrightarrow Ⓐ$$

where; S_g = Sp. entropy of saturated vapour.

S_f = Sp. entropy of saturated liquid.

V_g = Sp. volume of saturated vapour &

V_f = Sp. volume of saturated liquid.

Also, $S_g - S_f = S_{fg} = \frac{h_{fg}}{T}$

& $V_g - V_f = V_{fg}$

where; S_{fg} = Increase in sp. entropy.

V_{fg} = Increase in sp. volume &

h_{fg} = Latent heat added during evaporation at saturation temp. T.

$$\therefore \boxed{\left(\frac{dp}{dT}\right) = \frac{S_{fg}}{V_{fg}}} \longrightarrow Ⓑ \quad \text{(or)} \quad \boxed{\left(\frac{dp}{dT}\right) = \frac{h_{fg}}{T \cdot V_{fg}}} \longrightarrow Ⓒ$$

This is known as [Ⓐ, Ⓑ & Ⓒ] clausius-claperyon eqn. for evaporation of liquids.

PSYCHROMETRICS.

Definition's: It's branch of engineering science, which deals with the study of the properties of moist air [Air-water vapour mixtures].

Psychrometry: The art of measuring the moisture content of air.

Psychrometric Properties:

1. Dry air:- It is a mixture of N_2 & O_2 without presence of water vapour. Other gases present in small percentage are neglected. The composition of N_2 & O_2 by volume is 78.03 % & 20.99% and by mass is 75.47% & 23.19% respectively.

2. Moist air: The amount of water vapour present in the air, depends upon the absolute pr. & temp of the mixture.

It increases with decrease in temp. & decreases with increase in temperature.

3. Saturated air: The moist air is said to be saturated when its condition is such that it can coexist in natural equilibrium with an associated condensed moisture phase presenting a flat surface to it. The temp. of this air is known as saturation temp.

4. Dry Bulb Temperature (DBT): The temp. of air measured by ordinary thermometer is known as DBT.

5. Wet Bulb Temperature (WBT): The temp. measured by a thermometer whose bulb is covered with a wet cloth.

$$WBT < DBT.$$

6. Wet Bulb Depression:

$$WBD = DBT - WBT$$

WBD becomes zero when the air is fully saturated.

7. Dew Point Temperature (DPT):

It is the temp. at which the water vapour in the air just begins to condense, when the air is continuously cooled. At this point water droplets just appear.

DPT is equal to the steam tables saturation temp. corresponding to partial pressure of water vapour in the air.

8. Dew Point Depression:

$$DPD = DBT - DPT$$

9. Humidity:

It is the mass of water vapour present in 1 kg of dry air & is generally expressed in terms of grams per kg of dry air. It is also called "Specific humidity (or) humidity ratio."

$$W = \frac{\text{mass of water vapour in grams}}{\text{mass of dry air in kg.}}$$

10. Absolute Humidity:-

It is the actual weight of water vapour in grams present in the air per cubic meter (m^3) of a mixture of air & water vapour.

$$\text{Absolute Humidity} = \frac{\text{Mass of water vapour in gms}}{\text{Volume of air in } m^3}$$

11. Relative Humidity (RH) (or) (ϕ):

$$(\phi)\ RH = \frac{m_1}{m_2} = \frac{\text{Mass of water vapour in a given volume}}{\text{Mass of water vapour in same volume,}}$$

↳ if the air is saturated at same temperature.

12. Degree of Saturation:

$$\text{Degree of saturation } (\mu) = \frac{\text{Mass of water vapour associated with unit mass of dry air}}{\text{Mass of water vapour associated with saturated unit mass of dry saturated air}}$$

13. Sensible Heat: It is the heat that changes the temp. of a substance when added to (or) abstracted from it.

14. Latent Heat: It is the heat that does not affect the temp., but changes the state of substance when added to (or) abstracted from it.

15. Enthalpy (h): It is the combination energy which represents the sum of internal & flow energy in a steady flow process.

Dalton's Law of Partial Pressures:-

It states, "The total pressure exerted by the mixture of air & water vapour is equal to the sum of the pressures, which each constituent would exert, if it occupied the same space by itself."

Barometric Pressure; $P_b = P_a + P_v$

where; P_a = Partial pr. of dry air &
P_v = partial pr. of water vapour

Psychrometric Relations:

(1) Humidity / Sp. Humidity / Absolute Humidy / Humidity Ratio: (W):

$$\text{Sp. Humidity}, W = \frac{\text{Mass of water vapour}}{\text{mass of dry air}} = \frac{m_v}{m_a}$$

WKT; $PV = mRT \Rightarrow m_a = \frac{P_a \cdot V}{R_a \cdot T}$; $m_v = \frac{P_v \cdot V}{R_v \cdot T}$

where; P_a = Partial pr. of dry air,
P_v = partial pr. of water vapour
V = vol. of mixture.
R_a = charl. gas constant for dry air &
R_v = charl. gas constant for water vapour.

$$\therefore W = \frac{P_v \cdot V}{R_v \cdot T} \times \frac{R_a \cdot T}{P_a \cdot V} = \frac{R_a}{R_v} \times \frac{P_v}{P_a}$$

where; $R_a = \frac{R_0}{M_a} = \frac{8.3143}{28.97} = 0.287\ kJ/kg.K$

$R_v = \frac{R_0}{M_v} = \frac{8.3143}{18} = 0.462\ kJ/kg.K$

Std. value's:
M_a = Molecular wt. of air: 28.97
M_v = Molecular wt. of water vapour: 18

$$\therefore W = \frac{0.287}{0.462} \times \frac{P_v}{P_a} = 0.622 \cdot \frac{P_v}{P_a}$$

Acc. to Dalton's Law; $P_b = P_a + P_v \Rightarrow P_a = P_b - P_v$

$$\therefore \boxed{W = 0.622 \frac{P_v}{P_b - P_v}}$$

Sp. Humidity in terms of sp. volume are;

WKT; $v = \frac{V}{m} \Rightarrow m_a = \frac{V}{v_a}$; IIIly; $m_v = \frac{V}{v_v}$

$$\therefore W = \frac{V}{v_v} \cdot \frac{v_a}{V} = \frac{v_a}{v_v} = \frac{\text{Sp. volume of dry air}}{\text{Sp. volume of water vapour.}}$$

(2) Degree of Saturation (or) Percentage Humidity: (μ)

$$\text{Degree of Saturation; } \mu = \frac{\text{mass of water vapour associated with unit mass of dry air}}{\text{mass of water vapour associated with saturated unit mass of dry saturated air}}$$

(or)

$$\mu = \frac{\text{Actual Sp. Humidity}}{\text{Sp. Humidity of saturated air}}$$

$$\text{(ie) } \mu = \frac{W}{W_s} = \frac{0.622 \times \frac{P_v}{P_b - P_v}}{0.622 \times \frac{P_{vs}}{P_b - P_{vs}}} = \frac{P_v}{P_{vs}} \cdot \frac{P_b - P_{vs}}{P_b - P_v}$$

$$\mu = \frac{P_v}{P_s} \cdot \frac{P_b - P_s}{P_b - P_v}$$

where; P_{vs} : Partial pr. of water vapour when air is fully saturated $= P_s$ (Value's taken from steam table) corresponding to dry Bulb temp. (DBT)

$$\boxed{\mu = \frac{P_v}{P_s}\left[\frac{1 - \frac{P_s}{P_b}}{1 - \frac{P_v}{P_b}}\right]}$$

(3) Relative Humidity :- (ϕ)

$$\phi = \frac{\text{Mass of water vapour in a given volume}}{\text{mass of water vapour in the same volume if air is saturated at the same temperature}}$$

(or)

$$\phi = \frac{\text{Actual partial pr. of water vapour in moist air at a given DBT}}{\text{partial pr. of water vapour in saturated air at the same temperature}}$$

$$\boxed{\phi = \frac{m_v}{m_s} \text{ (or) } \frac{P_v}{P_s}}$$

The value of 'ϕ' substitute in degree of saturation (μ); we get;

$$\mu = \phi\left[\frac{1 - P_s/P_b}{1 - P_v/P_b}\right] = \phi\left[\frac{1 - \frac{P_s}{P_b}}{1 - \phi\frac{P_s}{P_b}}\right] = \phi\left[\frac{P_b - P_s}{P_b - \phi P_s}\right]$$

$$\Rightarrow \mu(P_b - \phi P_s) = \phi(P_b - P_s)$$

$$\Rightarrow \boxed{\phi = \frac{\mu}{1 - (1-\mu)\cdot\frac{P_s}{P_b}}}$$

(4) Enthalpy [Total Heat] of Moist Air:

It is the sum of enthalpy of dry air & enthalpy of water vapour associated with dry air.

$h = h_{air} + W \cdot h_{water\ vapour}$.

$$\boxed{h = C_{p_a} \cdot t_{db} + W \cdot h_{vapour}}$$

where;

C_{p_a} = Sp. heat of dry air = 1.005 kJ/kg.K

t_{db} = DBT

t_{dp} = DPT

$h_{vapour} = h_g + C_{p_s}(t_{db} - t_{dp})$

$\Rightarrow$ h_g = Enthalpy of saturated steam @ dew point temp. (DPT).

C_{p_s} = 1.88 kJ/kg.K = Sp. heat of superheated water vapour

$(t_{db} - t_{dp})$ = Degree of superheat of water vapour.

$$\therefore h = C_{p_a} \cdot t_{db} + W\left[h_g + C_{p_s}(t_{db} - t_{dp})\right]$$

$$= (C_{p_a} + C_{p_s} W) t_{db} + W(h_g - C_{p_s} t_{dp})$$

$$\boxed{h = C_{p_m} \cdot t_{db} + W(h_g - C_{p_s} \cdot t_{dp})}$$

where; $C_{p_m} = (C_{p_a} + C_{p_s} W)$ is the Sp. heat of humid air (or) humid Sp. heat.

The value of C_{p_m} is taken as 1.021 kJ/kg dry air per K.

It is the heat capacity of (1+W) kg of moisture per kg of dry air.

$h_{vapour} \simeq h_g$ @ DBT. So;

$$h = C_{p_a} \cdot t_{db} + W h_g$$

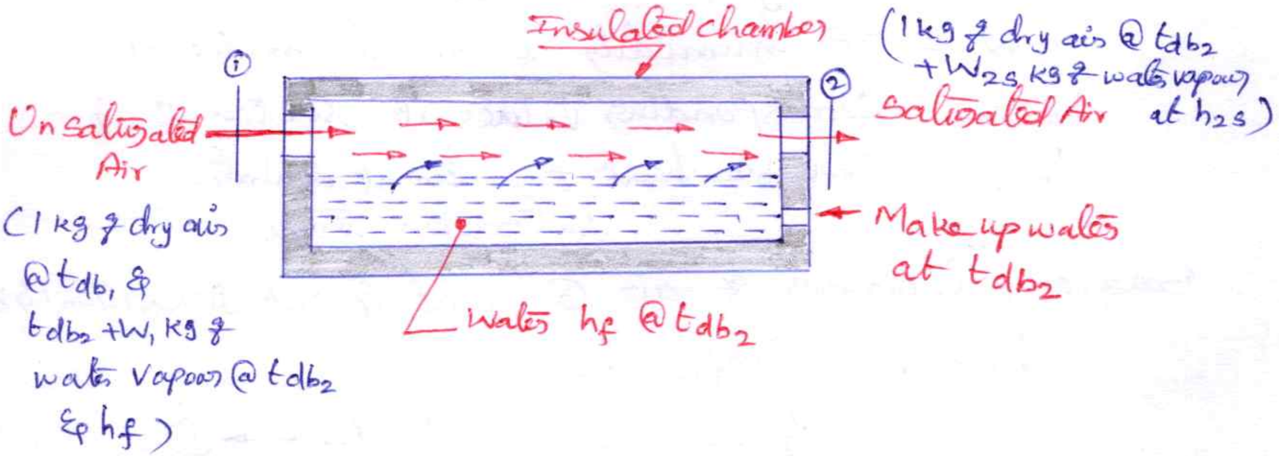
However, a better approximation is given by the following relationship; $h_{vapour} = 2500 + 1.88\, t_{db}$ kJ/kg of water vapour.

where; t_{db} is DBT in °c & the datum state is liquid water @ °c.

$$\therefore \boxed{h = 1.005\, t_{db} + W(2500 + 1.88\, t_{db})} \text{ kJ/kg of dry air}$$

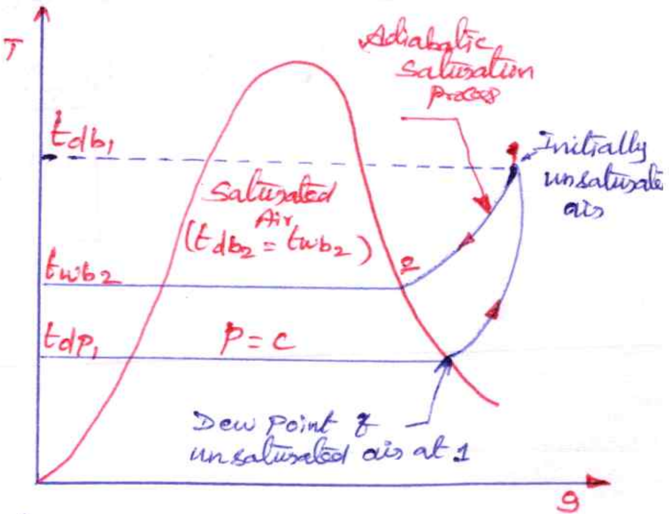
(5) Adiabatic Saturation Temperature (or)

[Thermodynamic wet bulb temperature]



In an insulated chamber when unsaturated air flows over a long sheet of water, the water evaporates, & the sp. humidity of the air increases. As the evaporation takes place both the air & water cooled. The process continues until the energy transferred from the air to the water is equal to the energy required to vapourise the water. When this point is reached thermal equilibrium exists with respect to water, air & water vapour & consequently the air is saturated. The equilibrium temperature is called "Adiabatic saturation temperature (or) the thermodynamic wet bulb temperature". The make up water is introduced at this temp. to make up the water level constant.

The adiabatic cooling process as shown in Ts diagram for the vapour in the air-vapour mixture. Although the total pr. of the mixture is constant, the partial pr. of the vapour increases & in the saturated state corresponding to the adiabatic saturation temp.. The vapour is initially @ DBT t_{db_1} & is cooled adiabatically to DBT, t_{db_2} which is equal to the adiabatic saturation t_{wb_2}.



The adiabatic saturation temp. & WBT are taken to be equal for all practical purposes. The WBT lies between the DBT & DPT.

Let ; h_1 = enthalpy of unsaturated air @ section-1
W_1 = sp. humidity of air @ section-1.
h_2, W_2 = Corresponding values at section 2. &
h_{fw} = Sensible heat of water at adiabatic -saturation temp.

Balancing the enthalpies of air @ inlet & out (i.e. section 1 & 2).

$$h_1 + (W_2 - W_1) h_{fw} = h_2$$

$$h_1 - W_1 h_{fw} = h_2 - W_2 \cdot h_{fw} \longrightarrow ①$$

The term $(h_2 - W_2 \cdot h_{fw})$ is known as sigma heat & remains constant during the adiabatic process.

WKT; $h_1 = ha_1 + W_1 \cdot hs_1$

& $h_2 = ha_2 + W_2 \cdot hs_2$

where; ha_1 = enthalpy of 1 kg of dry air @ DBT, t_{db_1}
hs_1 = enthalpy of superheated vapour at t_{db_1}/kg of -vapour.

ha_2 = enthalpy of 1 kg of air @ WBT, t_w &
hs_2 = enthalpy of saturated vapour @ WBT, t_w/kg of vapour.

Now the equation (1) may be written as;

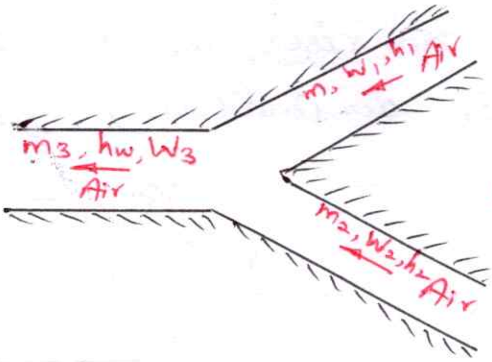
$$ha_1 + W_1 \cdot hs_1 - W_1 h_{fw} = ha_2 + W_2 \cdot hs_2 - W_2 \cdot h_{fw}$$

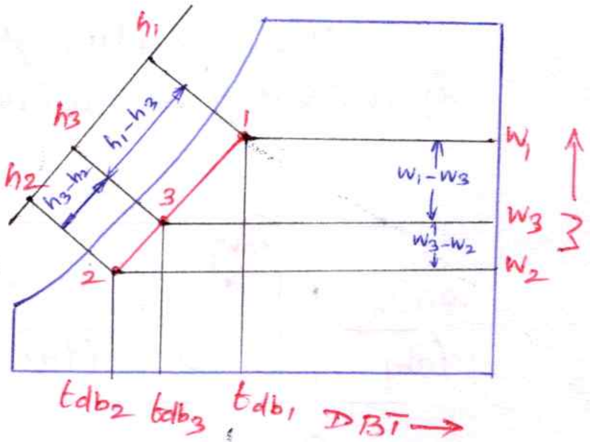
$$W_1 (hs_1 - h_{fw}) = W_2 (hs_2 - h_{fw}) + ha_2 - ha_1$$

$$\boxed{\therefore W_1 = \frac{W_2 (hs_2 - h_{fw}) + ha_2 - ha_1}{(hs_1 - h_{fw})}}$$

Psychrometric Processes: The processes affecting the psychrometric properties of air are called psychrometric processes.

(a) Mixing of air Streams:-





Mixing of several air streams is the process which is very frequently used in air conditioning. This mixing normally takes place without the addition (or) rejection of either heat (or) moisture.

(i.e) Adiabatically & @ constant total moisture content.

$$m_1 + m_2 = m_3 \longrightarrow ①$$

$$m_1 W_1 + m_2 W_2 = m_3 W_3 \longrightarrow ②$$

$$m_1 h_1 + m_2 h_2 = m_3 h_3 \longrightarrow ③$$

eqn: ② $\Rightarrow m_1 W_1 + m_2 W_2 = (m_1+m_2) W_3$

$$\Rightarrow m_1 (W_1 - W_3) = m_2 (W_3 - W_2)$$

$$\Rightarrow \frac{m_1}{m_2} = \frac{W_3 - W_2}{W_1 - W_3}$$

|||^{ly} eqn: ③ $\Rightarrow \boxed{\dfrac{m_1}{m_2} = \dfrac{h_3 - h_2}{h_1 - h_3}}$

(b) Sensible Heating:

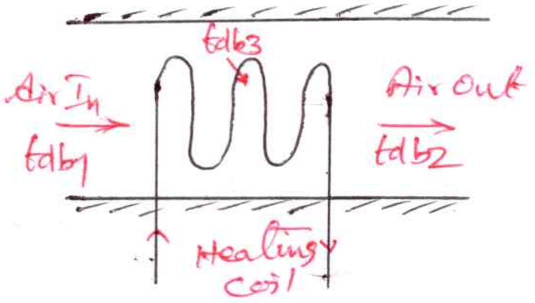
The heating of air without addition of moisture content is known as sensible heating.

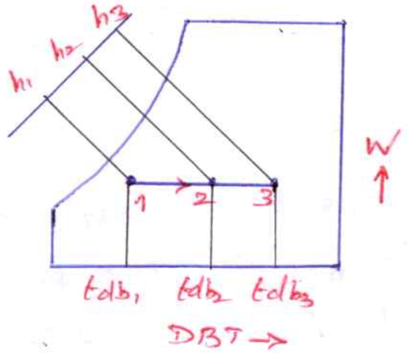
In other words heating air without change in specific humidity.

The heating can be done either by passing the air over electrical heating coils (or) by steam coils.

Thus hot air is obtained. The process is represented on psychrometric chart by a line 1-2 & 2-3 parallel to DBT line as shown in fig.

The heating of air rises, the DBT of air. Heating is necessary is "winter Air conditioning" system.





During ~~a~~ such a process, the sp. humidity (w) remains constant but DBT rises & approaches that of the surface. The extent to which it approaches the mean effective surface temp. of the coil is conveniently expressed in terms of the equivalent by-pass factor.

The by-pass factor (BF) for the process is defined as the ratio of the difference b/w. the ~~s~~ mean surface temp. of the coil & leaving air temp. to the difference b/w. the mean surface temp. & the entering air temp.

$$\boxed{BF = \frac{t_{db_3} - t_{db_2}}{t_{db_3} - t_{db_1}}}$$

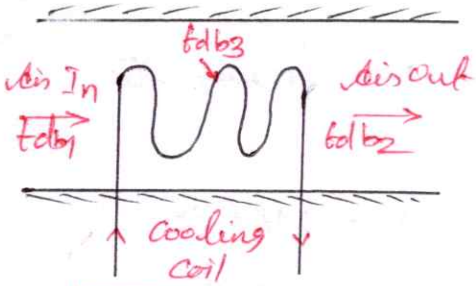
(c) Sensible Cooling: The cooling of air without subtracting of moisture content is known as sensible cooling.

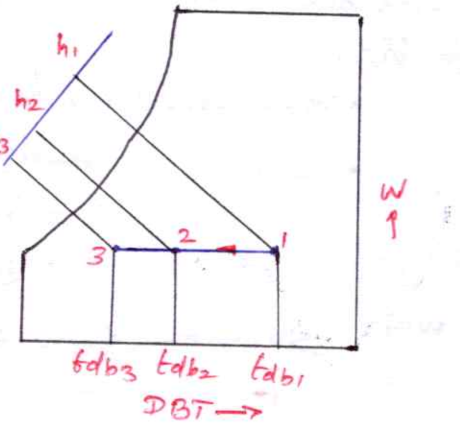
In other words cooling of air without change in sp. humidity.

The cooling is done by passing the air over a cooling coil such as evaporative coil of refrigerating cycle. During the process, the DBT gets lowered.

The sensible cooling process is represented on psychrometric chart by a line 1-2 & 2-3 parallel to DBT line as shown in fig. During the process 'W' remains constant.

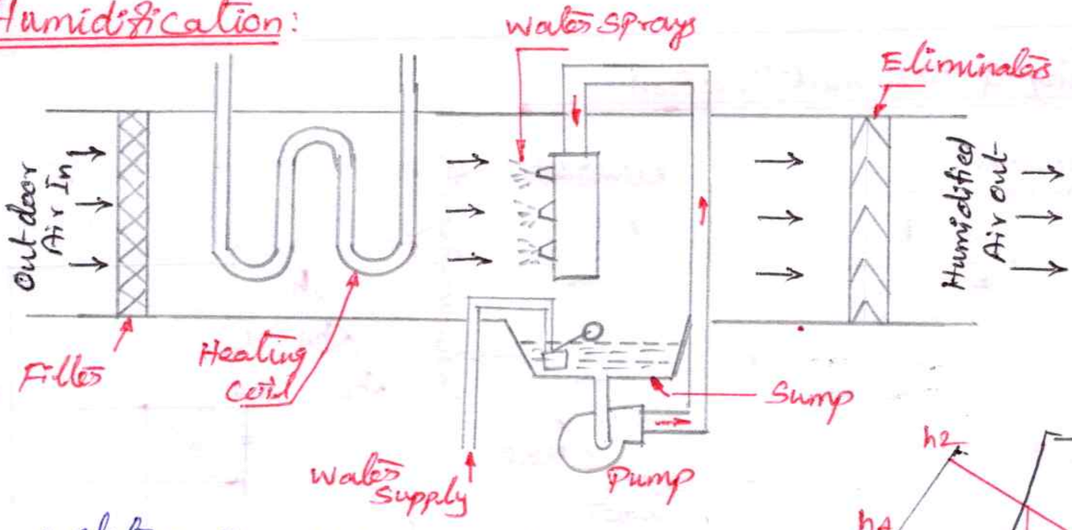
Cooling is necessary in "Summer Air Conditioning".



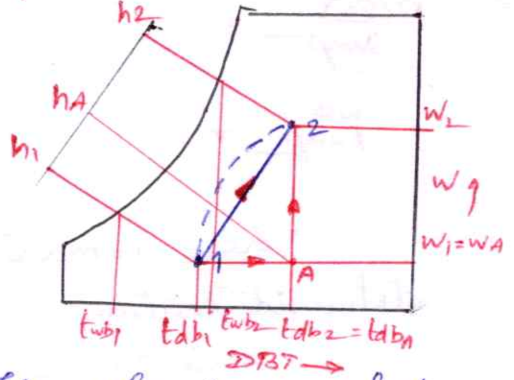


$$\boxed{BF = \frac{t_{db_2} - t_{db_3}}{t_{db_1} - t_{db_3}}}$$

(d) Heating & Humidification:



Addition of moisture (water vapour) to the air is known as humidification.



It is necessary for winter air conditioning system to warm & humidify the air.

A small pump supplies water to the spraying nozzles at high pressure. This water is converted into water vapour by absorbing heat from the air to be humidified. The un-evaporated water is collected in water sump & recirculated. The eliminator provided after the sprays removes water droplets from the air.

Actually, the heating & humidification process follows the path as shown by dotted curve. But for the calculation of psychrometric properties, only the end points are important. Thus the heating & humidification process shown by a line 1-2 on the psychrometric chart may be assumed to have followed the path 1-A (i.e. sensible heating) & A-2 (i.e. humidification) as shown in fig.

∴ The total heat added to the air during the heating & humidification is;

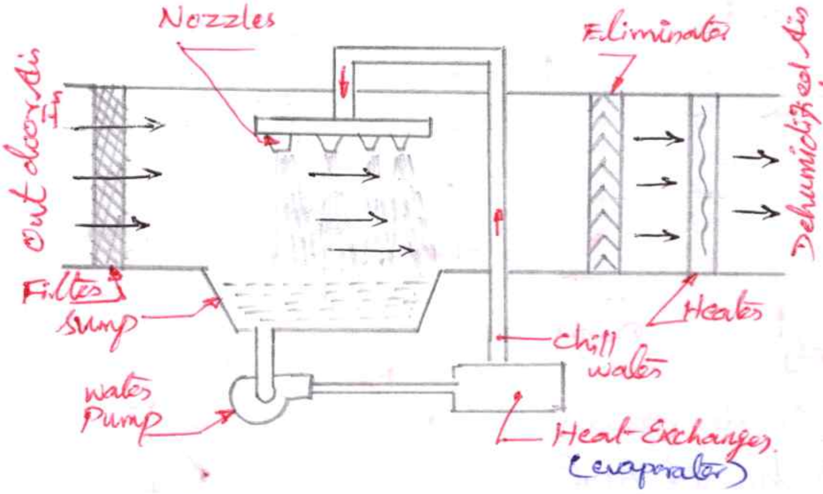
$$h = h_2 - h_1$$

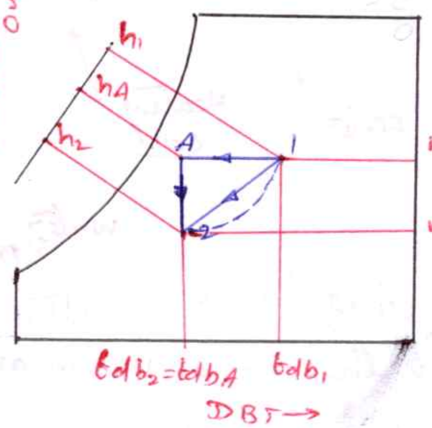
$$h = (h_2 - h_A) + (h_A - h_1) = LH + SH$$

wkt; Sensible heat factor, $SHF = \frac{\text{Sensible Heat}}{\text{Total heat}}$

$$\Rightarrow \boxed{SHF = \frac{SH}{SH+LE} = \frac{h_A - h_1}{h_2 - h_1}}$$

(e) Cooling & Dehumidification:





The removal of moisture from air is known as dehumidification.

This process is generally used in summer Air Conditioning to cool & dehumidify the air.

A small pump circulates water around cooling coil (evaporator) of a refrigerating system. The circulating water is cooled below dew point temp. (DPT).

This very cold water is then sprayed thro' nozzles. The out door air passes & comes in contact with the cold water. The water vapour in the air condenses & falls in the sump. The DBT of air leaving the air washer is very much reduced. Hence a heating element is provided to heat air. The eliminator removes the water droplets in the air.

The total heat removed from the air during the cooling & Dehumidification is;

$$h = h_1 - h_2$$

$$= (h_1 - h_A) + (h_A - h_2)$$

$$h = SH + LH$$

WKT;

$$\boxed{SHF = \frac{SH}{\text{Total heat}} = \frac{h_1 - h_A}{h_1 - h_2}}$$

Psychrometric chart :

A diagram which shows graphically the inter relation of all the psychrometric properties is known as psychrometric chart.

0.004 kg of water vapour / kg of atm. air is removed & temp. of air after removing the water vapour becomes 20°C. Determine; (i) Relative humidity, (ii) Dew point temp. Assume that condition of atm. air is 30°C & 55% RH & pr. is 1.0132 bar.

Soln: From steam tables, the corresponding @ 30°C;

$$p_{vs} = (0.0425 \text{ bar})$$

$$RH, \phi = \frac{p_v}{p_{vs}} \Rightarrow 0.55 = \frac{p_v}{0.0425} \Rightarrow p_v = 0.02337 \text{ bar}$$

Also the sp. humidity,

$$W = 0.622 \frac{p_v}{p_b - p_v}$$

$$= 0.622 \frac{0.02337}{(1.0132 - 0.02337)}$$

$$= 0.01468 \text{ kg/kg of dry air.}$$

The sp. humidity after removing 0.004 kg of water vapour.

It becomes; $0.01468 - 0.004 = 0.01068$ kg/kg of dry air.

$$\therefore W = 0.01068 = 0.622 \frac{p_v}{1.0132 - p_v}$$

$$\Rightarrow p_v = \underline{0.0171} \text{ bar}$$

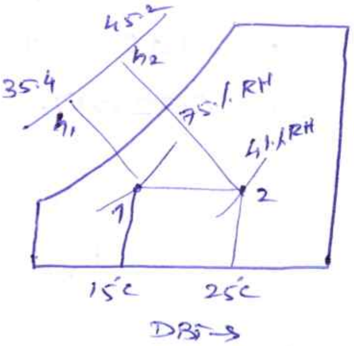
From steam table; @ 20°C; $p_{vs} = 0.0234$ bar

$$\therefore (RH), \phi = \frac{0.0171}{0.0234} = 0.73 \text{ (or) } 73\%$$

From steam table; @ $p_v = 0.0171$ bar; t_{dp} [DPT] = 15°C
(by interpolation)

200 m^3 of air/min @ 15°C DBT & 75% RH is heated until its temp. is 25°C. Find: (i) R.H. of heated air, (ii) WBT of heated air, (iii) Heat added to air/min.

Soln:-



- Locate point 1 on the psy. chart on intersection of DBT: 15°C & RH: 75%
- Thro' point 1 draw a horizontal line to cut DBT: 25°C & get point 2.

where; $h_1 = 35.4$ kJ/kg,
$h_2 = 45.2$ kJ/kg,
$v_{s_1} = 0.8267$ m^3/kg

(i) R.H of heated air = 41%

(ii) WBT of heated air = 16.1°C

(iii) Heat added to air/min. = $m_a(h_2 - h_1)$

$= 241.9[45.2 - 35.4]$

$= \underline{2370.6 \text{ kJ}}$

$$v = \frac{V}{m}$$

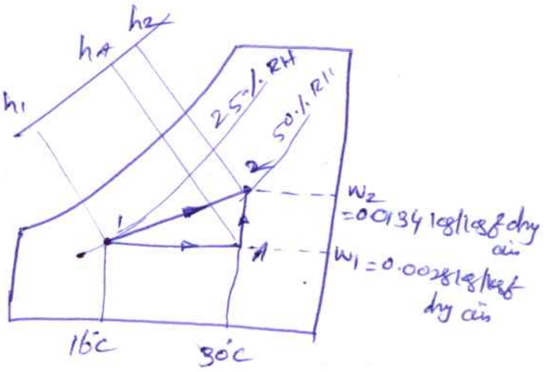
$$\Rightarrow m = \frac{V}{v}$$

$$= \frac{200}{0.8267}$$

$$\underline{m = 241.9 \text{ kg}}$$

Atm. air at a DBT of 16°C & 25% RH passes thro' a furnace & then thro' a humidifier, in such a way that the final DBT is 30°C & 50% RH. Find the heat & moisture added to the air. Also determine the SHF of the process.

Soln: "Heating & Humidification"



$t_{d_1} = 16°C$, $\phi_1 = 25\%$
$t_{d_2} = 30°C$, $\phi_2 = 50\%$

From psy. chart;
$h_1 = 23$ kJ/kg, $h_2 = 64.5$ kJ/kg,
$h_A = 37.5$ kJ/kg.

Heat added to the air; $= h_2 - h_1$
$= 64.5 - 23 = 41.5$ kJ/kg of dry air.

Moisture added to the air $= W_2 - W_1 = 0.0134 - 0.0028$
$= 0.0106$ kg/kg of dry air

Sensible Heat Factor, $SHF = \frac{h_A - h_1}{h_2 - h_1}$

$$= \frac{37.5 - 23}{64.5 - 23}$$

$$SHF = \underline{\underline{0.35}}$$

Atm. air @ 30°C DBT & 45% RH is to be conditioned to 17°C DBT & 15°C WBT. Find the amount of heat rejected by the air. Also find the sensible heat factor of the process. "Cooling & Dehumidification"

Soln: From Psy. chart @ Point 1; & Point 2;

DBT : 30°C & RH : 45% / DBT : 17°C & WBT : 15°C

$\Rightarrow h_1 = 60$ kJ/kg,

$h_2 = 41$ kJ/kg,

$h_A = 49$ kJ/kg.

∴ Amount of heat rejected by the air $= h_1 - h_2$

$= 60 - 41 = 19$ kJ/kg of dry air.

$$SHF = \frac{h_1 - h_A}{h_1 - h_2} = \frac{60 - 49}{60 - 41} = \underline{0.58}$$

One kg of air at 40°C DBT & 50% RH is mixed with 2 kg of air at 20°C DBT & 20°C DPT. Calculate the temp. & Sp. humidity (W) of the mixture.

Soln: "Adiabatic Mixing of two air streams"

$m_1 = 1$ kg, $t_{d1} = 40°C$, $\phi_1 = 50\%$,

$m_2 = 2$ kg, $t_{d2} = 20°C$, $t_{dp} = 20°C$

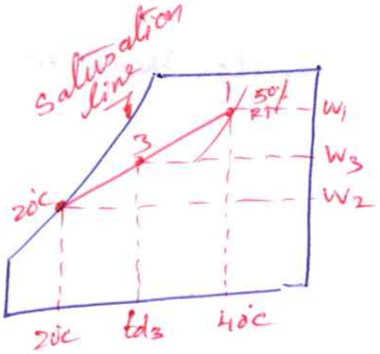
Let; Sp. humidity of mixture (W_3).

From Psy. chart;

@ $t_{d_1} = 40°C$ & RH: 50%.

locate Point 1

$\therefore W_1 = 0.0238$ kg/kg of dry air



& $t_{d_2} = 20°C$ & DPT. $t_{dp} = 20°C$ (lies on saturation line) locate point: 2.

$\therefore W_2 = 0.0148$ kg/kg of dry air.

WKT;
$$\frac{m_1}{m_2} = \frac{W_3 - W_2}{W_1 - W_3} \quad (\text{or}) \quad \frac{h_3 - h_2}{h_1 - h_3}$$

$$\Rightarrow \frac{1}{2} = \frac{W_3 - 0.0148}{0.0238 - W_3} \Rightarrow W_3 = \underline{\underline{0.0178}} \text{ kg/kg of dry air.}$$

Temp. of the mixture (t_{d_3}) @ corresponding value of W_3 & where is lies on point 3 in the line 1-2 ; t_{d_3} (from chart). = $\underline{\underline{26.8°C}}$

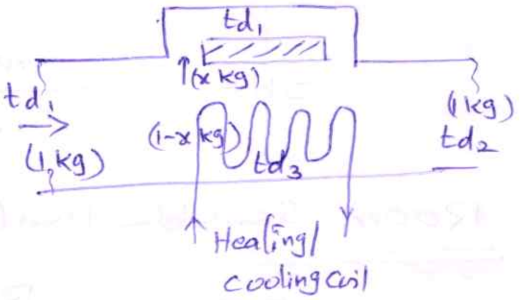
Carrier's equation:

Calculating partial Pr. of water Vapour Carrier's eqn. is most widely used.

$$P_v = P_w - \frac{(P_b - P_w)(t_d - t_w)}{1544 - 1.44 t_w}$$

where;
t_d = DBT,
t_w = WBT,
P_w = Saturation Pr. Corresponding to WBT
P_b = Barometric Pr.

> By-Pass Factor (BPF):



Under Ideal Conditions, the DBT of air leaving (t_{d_2}) should be equal to that of the coil temp. (t_{d_3}). But it not so, bz of the in-efficiency of the coil. This phenomenon is known as "BPF."

Balancing the enthalpies, we get:

$$x \cdot C_{pm}\, t_{d_1} + (1-x) C_{pm} \cdot t_{d_3} = 1 \times C_{pm}\, t_{d_2} \quad (\because C_{pm} = \text{Sp. humid heat})$$

$$x\,(t_{d_1} - t_{d_3}) = t_{d_2} - t_{d_3}$$

$$\Rightarrow \quad x = \frac{t_{d_2} - t_{d_3}}{t_{d_1} - t_{d_3}} = \text{BPF} \; \text{[for Cooling coil.]}$$

|||ly;

$$\text{BPF} = \frac{t_{d_3} - t_{d_2}}{t_{d_3} - t_{d_1}} \; \text{[for heating coil]}.$$

> Contact Factor (or) Efficiency of Coil: $[\eta_H]$ (or) $[\eta_c]$

For Heating Coil:

$$\eta_H = (1 - \text{BPF}) = 1 - \frac{t_{d_3} - t_{d_2}}{t_{d_3} - t_{d_1}} = \frac{t_{d_2} - t_{d_1}}{t_{d_3} - t_{d_1}}$$

For Cooling coil:

$$\eta_c = 1 - \frac{t_{d_2} - t_{d_3}}{t_{d_1} - t_{d_3}} = \frac{t_{d_1} - t_{d_2}}{t_{d_1} - t_{d_3}}$$

> Average Dew Point [ADP]:

The effective surface temp. of the coil is known as ADP.

Sensible Heat Factor (SHF) (or) Sensible Heat Ratio (SHR):

$$SHF = \frac{\text{Sensible Heat}}{\text{Total Heat}} = \frac{SH}{SH+LH}$$

Room Sensible Heat Factor (RSHF):

$$RSHF = \frac{RSH}{RTH} = \frac{RSH}{RSH+RLH}$$

Grand Sensible Heat Factor (GSHF):

$$GSHF = \frac{TSH}{GTH} = \frac{TSH}{TSH+TLH} = \frac{(RSH+OASH)}{(RSH+OASH)+(RLH+OALH)}$$

where; TSH = Total Sensible Heat. = (RSH + OASH)

TLH = Total Latent Heat = (RLH + OALH)

GTH = Grand Total Heat.

OASH = Outside Air Sensible Heat.

$= 0.02044\, V_1 [t_{d_1} - t_{d_2}]$ kW

OALH = Outside Air Latent Heat.

$= 50\, V_1 [W_1 - W_2]$ kW

OATH = Outside Air Total Heat. = (OASH + OALH)

$= 0.02\, V_1 [h_1 - h_2]$ kW

where; V_1, t_{d_1}, W_1 & h_1 = Vol., DBT, sp. humidity & enthalpy of outside Air.

$|||^{ly}$ V_2, t_{d_2}, W_2 & h_2 = ...Room Air.

Effective Room Sensible Heat Factor: [ERSHF].

$$ERSHF = \frac{ERSH}{ERTH} = \frac{ERSH}{ERSH + ERLH}$$

⇒ Effective Room Sensible Heat, $ERSH = RSH + [OASH \times BPF]$

(i.e) $ERSH = RSH + 0.0204 V_1 (td_1 - td_2) \cdot BPF$

⇒ Effective Room Latent Heat, $ERLH = RLH + [OALH \times BPF]$

(i.e) $ERLH = RLH + 50 V_1 (W_1 - W_2) \cdot BPF$

Classification of Air-Conditioning Systems:

> Acc. to the Purpose;
  - Comfort & Industrial A/c system.

> Acc. to season of the year;
  - Winter, Summer & year round A/c system.

> Acc. to the arrangement of equipments;
  - Unitary & Central A/c system.

Equipment Used in an Air-Conditioning System:

- Circulating fan,
- Air Conditioning unit,
- Supply Duct,
- Supply outlets,
- Return outlets,
- Filters.

Factors affecting Comfort Air-Conditioning:

- Temperature of air,
- Humidity of air,
- Purity of air, &
- Motion of air.

Effective Temperature: The degree of warmth/cold felt by a human body depends mainly on the following three factors:

1. DBT ↓ (20°c to 24°c), 2. Relative humidity (ϕ) ↓ (40% to 60%) & 3. Air Velocity. ↓ (5 to 8 m/min)

Factors affecting optimum Effective Temperature:

- climate & seasonal differences.
- clothing
- Age & sex.
- Duration of stay
- Kind of activity
- Desity of occupants.

Factors affecting Human comfort:

- Effective Temperature.
- Heat Production & Regulation in human body.
- Heat & moisture losses from the human body.
- Moisture content of air.
- Quality & Quantity of air.
- Air motion.
- Hot & Cold Surfaces. &
- Air stratification.

DUCTS.

classification of Ducts:

- Supply air duct.
- Return air duct.
- Fresh air duct.
- Low pr. duct (static pr. of < 50 mm of H_2O)
- Medium pr. duct (P is upto 150 mm of H_2O)
- High pr. duct (P is 150 to 250 mm of H_2O)
- Low velocity duct. (Air velocity upto 600 m/min)
- High velocity duct. (Air velocity > 600 m/min).

Gas Power Cycle's. (or Thermodynamic Air Cycle's.

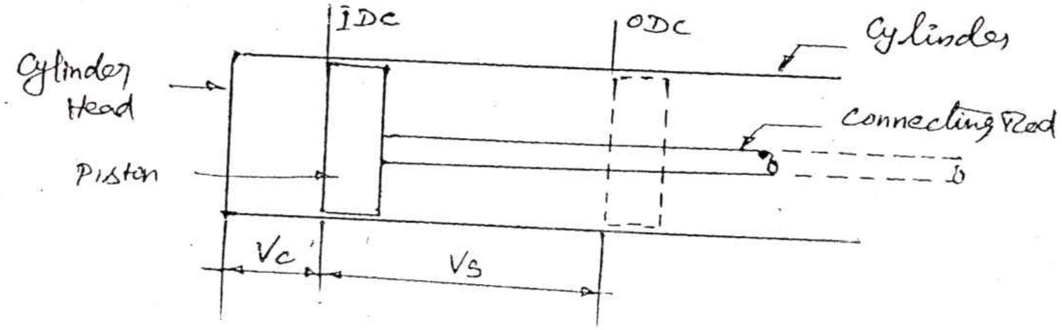
Introduction: A thermodynamic cycle is a series of thermodynamic processes during which a working substance undergoes a change in properties (P, V & T etc) & finally the working substance returns to its initial state.

These processes can be plotted on a P-V & T-S diagram.

Heat Engine: Any machine designed to carryout a thermodynamic cycle to convert heat energy supplied to it, into mechanical energy is called a heat engine.

The cycle on which a heat engine works is known as a heat engine cycle. A cycle requiring two complete revolutions of the crank [or 4 strokes of the piston] is known as 4-stroke cycle. A cycle requiring one complete revolution of the crank [or 2 strokes of the piston] is known as 2-stroke cycle.

Terms used in Thermodynamic cycle:



→ **Cylinder Bore:** This is the diameter of the cylinder in which the piston moves.

→ **Stroke Length:** The piston moves in the cylinder due to the rotation of the crank. (L)

It means : distance btw. (IDC to ODC) (or (TDC to BDC)).

→ Stroke (or) Swept volume (V_s):

The volume swept by the piston when it moves b/w. the two extreme positions (TDC & BDC (or) IDC & ODC) is known as stroke volume.

V_s = piston area × Stroke Length

$$\therefore V_s = A.L = \frac{\pi}{4} d^2 . L$$

→ Clearance volume (V_c):

This is the volume occupied by the working fluid, when the piston reaches TDC (or) IDC & is represented by V_c.

→ Total volume (or) Cylinder volume:

It is the sum of V_s & V_c.

→ Compression Ratio: It's the ratio of total volume to the clearance volume. It denoted by a letter 'r'.

$$r = \frac{V_s + V_c}{V_c} = 1 + \frac{V_s}{V_c}$$

The Compression ratio is also called expansion ratio.

→ Mean Effective Pressure (mep): (or P_m:-

The pr. of the working fluid in the cylinder keeps on changing for different positions of the piston.

It is defined as the average pr. acting on the piston during the working stroke.

$$P_m = mep = \frac{\text{Workdone / cycle (kN-m)}}{\text{Swept volume } (m^3)}$$

$$P_m = \frac{W}{V_s} \ (kN/m^2)$$

→ Diagram Factor:

$$\text{Diagram factor} = \frac{\text{Area of actual Indicator diagram}}{\text{Area of theoretical Indicator diagram}}$$

Efficiency of a cycle-(η) (or) Air Standard Efficiency (ASE)

$$\eta \text{ (or) ASE} = \frac{\text{Workdone per cycle}}{\text{Heat Supplied}} = \frac{W}{Q_S}$$

where; W = heat supplied - heat rejected.

$$\therefore \eta = \frac{Q_S - Q_R}{Q_S}$$

$$\text{Relative efficiency} = \frac{\text{Actual Thermal efficiency}}{\text{Theoretical Thermal efficiency}}$$

Classification of Thermodynamic Cycles:

(a) Reversible cycle & (b) Irreversible cycle.

(a) Reversible cycle:

A thermodynamic process which can be operated in a reversed direction is known as reversible process.

A thermodynamic cycle which consists of reversible process only is known as reversible cycle. At the end of the cycle, the initial conditions are restored.

An engine working on reversible cycle is most efficient one. Carnot cycle is reversible & hence most efficient. But it is impossible to construct an engine working on carnot cycle.

(b) Irreversible cycle:-

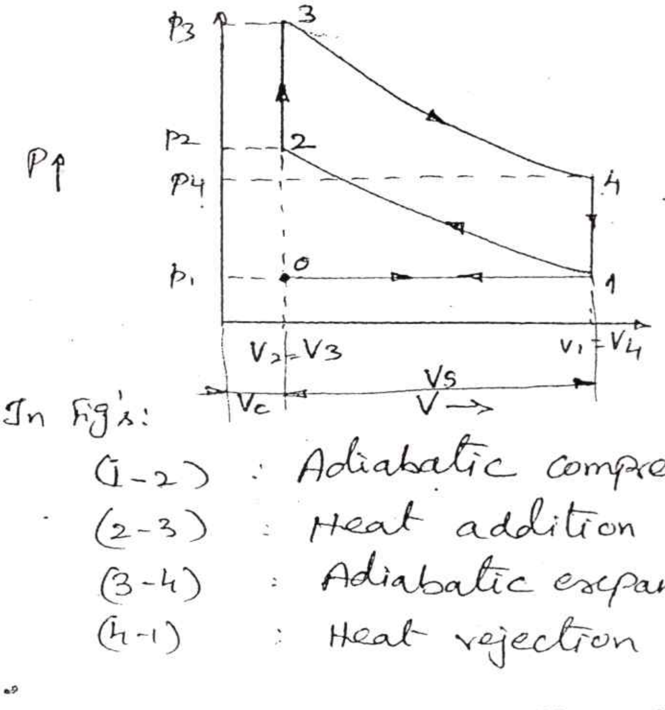
A thermodynamic process which cannot be operated in a reversed direction is known as irreversible process.

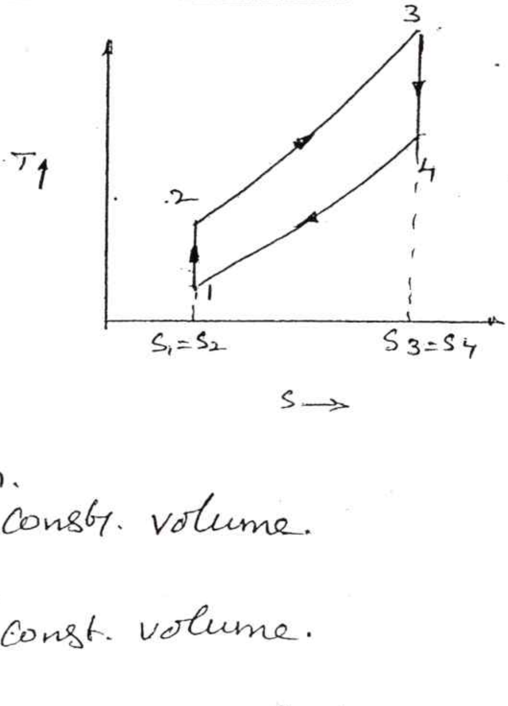
In actual practice, all processes are irreversible. A thermodynamic cycle is said to be irreversible cycle, if any of the process in the cycle is irreversible. At the end of the cycle, the initial conditions are not restored.

Types of Thermodynamic Cycles :-

1. Carnot Cycle (1824) Nicolas Leonard Sadi Carnot - (French)
2. Otto cycle. [Const. volume Cycle] (1876)
3. Brayton Cycle [Const. pressure cycle] (or) Joule cycle.
4. Diesel cycle. (1892) Dr. Rudolph Diesel
5. Dual Combustion cycle.
6. Stirling cycle (1827) Robert Stirling
7. Ericsson Cycle & ~~of~~. Joule cycle.
   (1850)

Otto cycle [Nikolaus A. Otto (German engineer)] :





In Fig's:

(1-2) : Adiabatic compression.
(2-3) : Heat addition at const. volume.
(3-4) : Adiabatic expansion.
(4-1) : Heat rejection at const. volume.

This is the theoretical cycle on which the petrol & gas engines work.

The cycle consists of two adiabatic & two constant volume process as shown in P-V & T-S diagrams. This is also known as "Const. volume cycle" as heat is received at constant volume.

Working of otto cycle:

1. First stage (Adiabatic compression (1-2)):

The air is compressed adiabatically from temperature T_1 to T_2. In this process no heat is absorbed (or) rejected by the air.

2. Second stage (Const. volume of heat addition (2-3)):

The heat is supplied at const. volume. This increases the pr. P_2 to P_3 & the temp. T_2 to T_3.

$$Q_S = m \cdot C_V (T_3 - T_2)$$

3. Third stage (Adiabatic Expansion (3-4)):-

At the end of heat addition, the air is expanded adiabatically by moving the piston from temp. T_3 to T_4. & P_3 to P_4. The volume increases from V_3 to V_4. No heat is absorbed (or) rejected by the air.

4. Fourth stage (Const. volume of heat rejection (4-1)):-

At the end of expansion, the heat is rejected from air at const. volume until the air is brought to its original condition P_1, V_1 & T_1.

$$Q_R = m \cdot C_V (T_4 - T_1)$$

The cycle is thus repeated.

Air standard Efficiency of otto cycle:

$$\eta_{otto} = \frac{\text{Workdone}}{\text{Heat Supplied}} = \frac{Q_S - Q_R}{Q_S}$$

$$= \frac{m C_V (T_3 - T_2) - m \cdot C_V (T_4 - T_1)}{m \cdot C_V (T_3 - T_2)}$$

$$\eta = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)} \longrightarrow Ⓐ$$

The above expression can be simplified by substituting the values of T_4 in terms of T_3 & T_1 in terms of T_2

From adiabatic expansion (3-4);

$$\boxed{\frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{\gamma-1}} \rightarrow ①$$

From adiabatic compression (1-2);

$$\boxed{\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}} \rightarrow ②$$

WKT; $V_1 = V_4$; $V_2 = V_3$

$$\therefore \frac{T_2}{T_1} = \frac{T_3}{T_4} \Rightarrow \frac{T_4}{T_1} = \frac{T_3}{T_2} \Rightarrow \frac{T_4}{T_1} - 1 = \frac{T_3}{T_2} - 1$$

$$\Rightarrow \frac{T_4 - T_1}{T_1} = \frac{T_3 - T_2}{T_2}$$

$$\Rightarrow \frac{T_4 - T_1}{T_3 - T_2} = \frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1} \longrightarrow Ⓑ$$

Sub Ⓑ in Ⓐ;
we get;

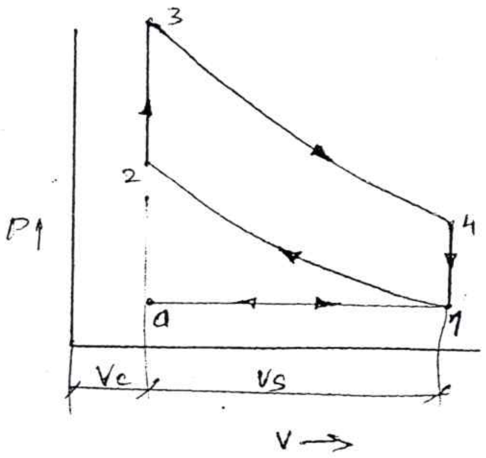
$$\eta_{otto} = 1 - \left(\frac{V_2}{V_1}\right)^{\gamma-1}$$

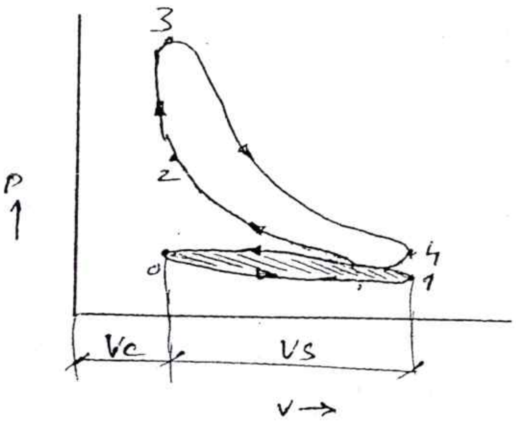
where; Compression ratio; $r = \frac{V_1}{V_2} = \frac{\text{vol. at the beginning of compression}}{\text{vol. at the end of compression}}$

$$\boxed{\therefore \eta_{otto} = 1 - \frac{1}{r^{\gamma-1}}}$$

"The efficiency of the air standard otto cycle is thus a function of the compression ratio only".

Ideal & Actual p-v diagrams of otto cycle:



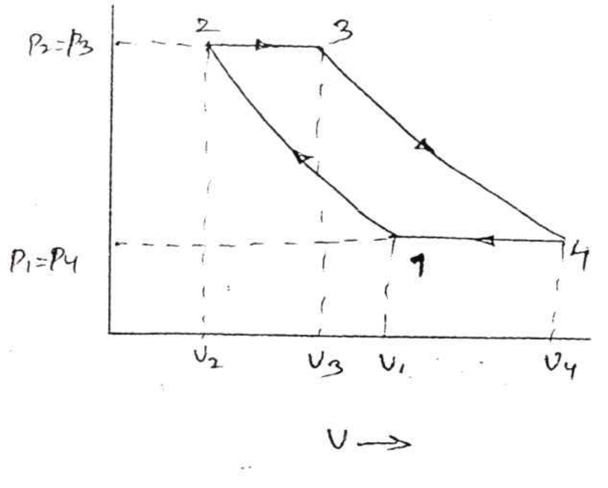


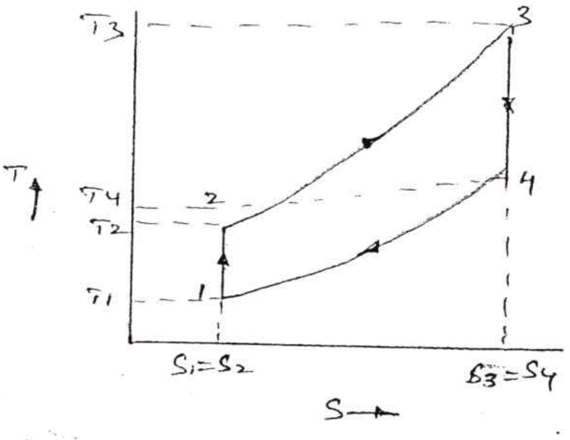
- In ideal cycle, the corner's of p-v diagram are sharp. But in actual p-v diagram, there is "rounding off" of the corner's, because the closing & opening of the valves are not instantaneous but takes some time.
- Time lag between the time of ignition & combustion.
- There is no heat transfer during the adiabatic compression & expansion. But these are not possible in practice because the heat transfer to the metal surface's of cylinder & piston.

The total effect of all the above differences is that the thermal efficiency of the actual cycle is less than ideal (or) Air standard efficiency.

It can be seen from the actual p-v diagram that the area 1-2-3-4 is positive (Actual work developed) & area 4-0-1 is negative (work used for discharging gases to atmosphere) & is called negative power (or) pumping power.

Brayton Cycle (or) Joule's cycle [Constant Pressure Cycle].





This is an ideal cycle for gas turbine plants. The cycle consists of two adiabatic processes & two constant pressure processes.

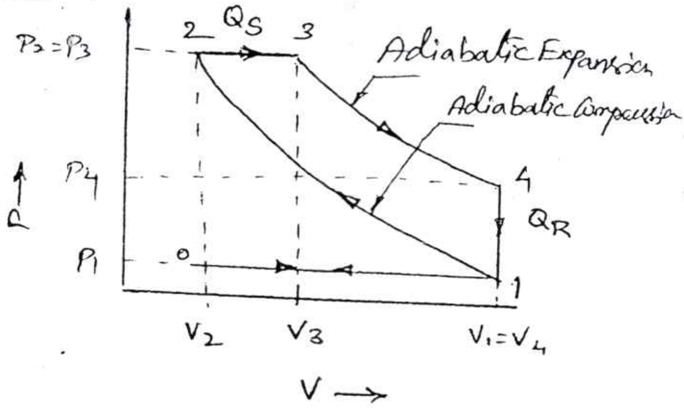
(1-2) : Adiabatic Compression.
(2-3) : Heat addition @ Constt. pressure : $Q_S = m.C_p(T_3-T_2)$
(3-4) : Adiabatic expansion.
(4-1) : Heat rejection @ Constt. Pressure : $Q_R = m.C_p(T_4-T_1)$

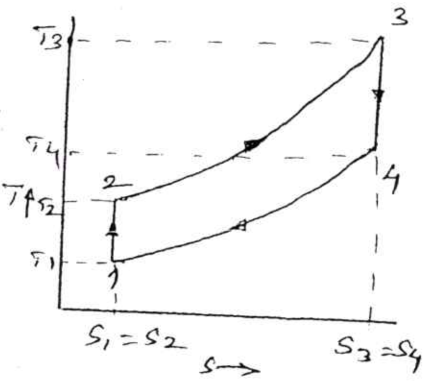
$$\therefore \eta = \frac{W}{Q_S} = \frac{m.C_p(T_3-T_2) - m.C_p(T_4-T_1)}{m.C_p(T_3-T_2)}$$

$$\eta_{Bray} = 1 - \frac{T_4-T_1}{T_3-T_2} = 1 - \frac{1}{r^{\gamma-1}}$$

The efficiency of otto cycle is equal to efficiency of Brayton cycle due to same compression ratio.

Diesel Cycle:-





This is an important cycle in practice on which all diesel engines work. The diesel engine works with high compression ratio & gives higher thermal efficiency than petrol engine working on otto cycle.

The cycle ~~f~~ consists of two adiabatic process, one constant pressure process & one constant volume process.

Working :-

Stage - 1 : [Adiabatic compression] (1-2) :-

At point 1, the cylinder is full of air with volume V_1, pr. P_1 & temp. T_1. At the end of stroke the condition of air is represented by the point 2 on p-v diagram, V_2, P_2 & T_2. No heat is absorbed/Rejected by the air during this process.

Stage-2 : [Heat Addition at Const. Pressure (2-3)] :-

At the point 2, heat is supplied to air at P=c. The piston moves outwards / downwards. At point 3, the supply of heat is stopped. This point is known as "point of cut-off".

$$Q_S = m \cdot C_p (T_3 - T_2)$$

Stage 3 : [Adiabatic Expansion (3-4) :-

From point 3, the air is expanded adiabatically by moving the piston outwards. The work is done on the piston. The pr. & temp. drops from P_3 to P_4 & T_3 to T_4. The volume increases from V_3 to V_4. No heat is absorbed / rejected by the air.

Stage-4 : [Heat Rejection at Constant Volume (4-1)] :-

At point 4, the heat is abstracted from air at $V=C$, until the air brought to its original condition P_1, V_1 & T_1.

$$Q_R = m \cdot C_v (T_4 - T_1)$$

The cycle is thus repeated.

$$\underline{\text{Air standard efficiency}} = \frac{\text{Workdone}}{\text{Heat supplied}}$$

$$\eta_{diesel} = \frac{m \cdot C_p (T_3 - T_2) - m \cdot C_v (T_4 - T_1)}{m \cdot C_p (T_3 - T_2)}$$

$$\eta_{diesel} = 1 - \frac{C_v}{C_p} \frac{(T_4 - T_1)}{(T_3 - T_2)}$$

$$\boxed{\eta_{diesel} = 1 - \frac{1}{\gamma} \frac{(T_4 - T_1)}{(T_3 - T_2)}} \longrightarrow ①$$

The above expression (eqn) can be simplified by substituting all temp's in terms of T_3.

Let Compression ratio, $r = \frac{V_1}{V_2}$, ∴ Expansion ratio $= \frac{V_4}{V_3}$

Cut-off ratio, $\rho = \frac{V_3}{V_2}$

$$= \frac{V_4}{V_2} \cdot \frac{V_2}{V_3}$$

$$= \frac{V_1}{V_2} \cdot \frac{V_2}{V_3} \quad (\because V_4 = V_1)$$

$$\text{Exp. ratio} = \frac{r}{\rho} \longrightarrow ②$$

process (2-3) [Q_S @ $p=C$];

$$\frac{p_2 V_2}{T_2} = \frac{p_3 V_3}{T_3} \quad (\because p_2 = p_3)$$

$$\therefore \frac{V_2}{T_2} = \frac{V_3}{T_3} \Rightarrow \frac{T_3}{T_2} = \frac{V_3}{V_2} = \rho$$

$$\therefore \boxed{T_2 = \frac{T_3}{\rho}} \longrightarrow ③$$

process (1-2) [Adiabatic Compression]:

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = r^{\gamma-1} \quad \therefore \boxed{T_1 = \frac{T_2}{r^{\gamma-1}}} \longrightarrow ④$$

③ in ④; we get; $\therefore \boxed{T_1 = \frac{T_3}{\rho . r^{\gamma-1}}} \longrightarrow ⑤$

process (3-4) [Adiabatic Expansion]:

$$\frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{\gamma-1} = \left(\frac{r}{\rho}\right)^{\gamma-1} \quad \therefore \boxed{T_4 = T_3\left(\frac{\rho}{r}\right)^{\gamma-1}} \longrightarrow ⑥$$

Substituting the values of T_2, T_1 & T_4 in eqn ①, we get;

$$\eta_{diesel} = 1 - \frac{1}{\gamma} \times \frac{\left[T_3\left(\frac{\rho}{r}\right)^{\gamma-1} - \frac{T_3}{\rho . r^{\gamma-1}}\right]}{\left[T_3 - \frac{T_3}{\rho}\right]}$$

$$= 1 - \frac{1}{\gamma} \times \frac{1}{r^{\gamma-1}} \left[\frac{\rho^{\gamma-1} - \frac{1}{\rho}}{1 - \frac{1}{\rho}}\right]$$

$$\boxed{\eta_{diesel} = 1 - \frac{1}{\gamma . r^{\gamma-1}} \left[\frac{\rho^{\gamma} - 1}{\rho - 1}\right]}$$

Comparision of OTTO & DIESEL Cycle:

	Otto Cycle	Diesel Cycle
1.	It is used in petrol engines	- It is used in diesel Engines
2.	The Cycle consists of two adiabatic & two Constt. volume processes.	- It consists of two adiabatic, one Constt. Pr. & one Constt. volume process.
3.	Heat added at Constt. volume.	- Heat added at Constt. Pr.
4.	Compression ratio & expansion ratio are same.	- Not same. Expansion ratio is less than compression ratio.
5.	η of the Cycle depends on Compression ratio. η increases with increase in Compression ratio.	- η of the Cycle depends on compression ratio & cut-off ratio. η increases with increase in compression ratio & with decrease in cut-off ratio.
6.	The Compression ratio is limited from 6 to 8	- 12 to 18
7.	For the Same Compression ratio & Same heat input, the η_{otto} is more than that of diesel cycle.	- η_{diesel} is less than that of otto cycle for the same compression ratio & same heat input.
8.	$\eta_{otto} = 1 - \frac{1}{r^{\gamma-1}}$	- $\eta_{diesel} = 1 - \frac{1}{\gamma . r^{\gamma-1}}\left(\frac{\rho^{\gamma}-1}{\rho-1}\right)$

Comparison of efficiencies of otto & diesel Cycle:

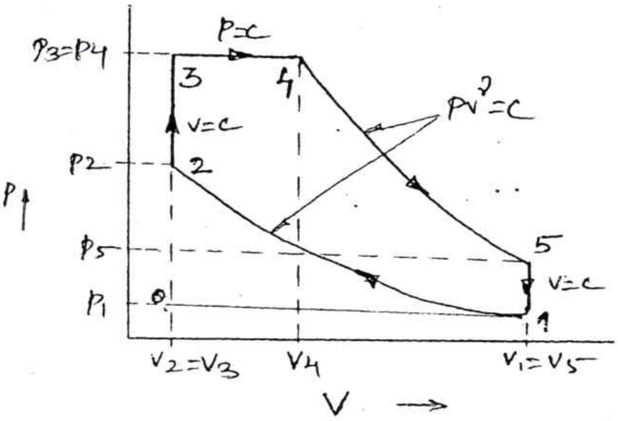
The air standard efficiency of ;

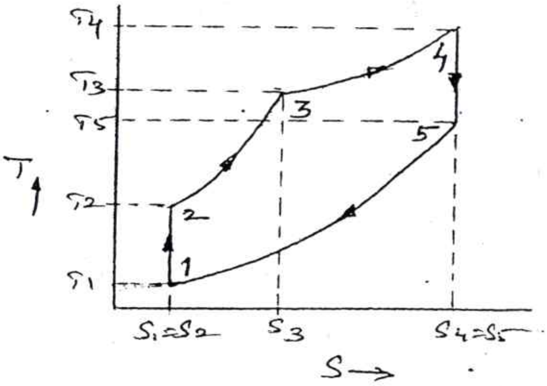
otto cycle, $\eta = 1 - \frac{1}{r^{\gamma-1}}$

Diesel Cycle, $\eta = 1 - \frac{1}{\gamma . r^{\gamma-1}}\left(\frac{\rho^{\gamma}-1}{\rho-1}\right) = 1 - \frac{1}{r^{\gamma-1}}\left[\frac{\rho^{\gamma}-1}{\gamma(\rho-1)}\right]$

In the efficiency eqn. of diesel cycle, the value of the factor $\left[\frac{\rho^{\gamma}-1}{\gamma(\rho-1)}\right]$ depends on the value of cut off ratio (ρ), which is always more than unity. Hence the value of this factor is greater than unity. Therefore for the same compression ratio, η_{diesel} is less than η_{otto}. $[\because \eta_{diesel} < \eta_{otto}]$. That is otto cycle is more efficient than diesel cycle.

Dual Combustion cycle [Limited Prs. Cycle / Mixed cycle]:





This cycle is a combination of otto & Diesel cycles, in a way that heat is added partly at const. volume & partly at const. prs..

This cycle consists of two adiabatic processes, two const. volume processes & one const. prs. process.

I-stage (1-2): Adiabatic compression:

II-stage (2-3): Heat Addition at const. volume.

$$Q_S = m \cdot C_v (T_3 - T_2)$$

III-stage (3-4): Heat Addition at const. pressure.

$$Q_S = m \cdot C_p (T_4 - T_3)$$

IV stage (4-5) : Adiabatic Expansion.

V - stage : (5-1) Heat rejection at constant volume.

$$Q_R = m.C_v(T_5 - T_1)$$

Air standard efficiency, $\eta_{Dual} = \frac{W}{Q_S} = \frac{Q_S - Q_R}{Q_S}$

$$\eta_{Dual} = \frac{[m.C_v(T_3-T_2)+m.C_p(T_4-T_3)] - m.C_v(T_5-T_1)}{[m.C_v(T_3-T_2)+m.C_p(T_4-T_3)]}$$

$$= 1 - \frac{C_v(T_5-T_1)}{C_v(T_3-T_2)+C_p(T_4-T_3)}$$

$$\boxed{\eta_{Dual} = 1 - \frac{(T_5-T_1)}{(T_3-T_2)+\gamma(T_4-T_3)}} \longrightarrow ① \quad \left\{ \because \frac{C_p}{C_v} = \gamma \Rightarrow C_p = \gamma.C_v \right\}$$

The above eqn. can be expressed in terms of the following ratio's & all temp's, in term's of T_4.

Compression ratio, $r = \frac{V_1}{V_2}$

expansion ratio, $r_e = \frac{V_5}{V_4} = \frac{V_1}{V_4}$ $[\because V_1 = V_5]$

Cut-off ratio, $\rho = \frac{V_4}{V_3} = \frac{V_4}{V_2}$ $[\because V_3 = V_2]$.

Pressure ratio at Const. volume, $\beta = \frac{P_3}{P_2}$
((or) Explosion Ratio)

$\therefore$ Compression Ratio, $r = \frac{V_1}{V_2} = \frac{V_1}{V_4} \cdot \frac{V_4}{V_2} = r_e \cdot \rho$

(or) Exp. ratio, $\boxed{r_e = \frac{r}{\rho}}$

Process : (3-4) :-

$$\frac{P_3V_3}{T_3} = \frac{P_4V_4}{T_4} \qquad P = C \quad (\because P_3 = P_4)$$

$$\therefore \frac{V_3}{T_3} = \frac{V_4}{T_4} \Rightarrow \boxed{\begin{array}{l} T_4 = T_3 . \frac{V_4}{V_3} = T_3 . \rho \\ (or) \quad T_3 = \frac{T_4}{\rho} \end{array}} \longrightarrow ②$$

Process (2 - 3) : $\frac{P_2 V_2}{T_2} = \frac{P_3 V_3}{T_3}$ $(\because V_2 = V_3)$

$$\Rightarrow \frac{P_2}{P_3} = \frac{T_2}{T_3} \Rightarrow T_2 = T_3 \cdot \frac{P_2}{P_3} = \frac{T_3}{\beta} \longrightarrow ③$$

eqn. ② in ③;

$$\therefore \boxed{T_2 = \frac{T_4}{\rho . \beta}} \longrightarrow ④$$

Process (1-2) :

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1} = \frac{1}{r^{\gamma-1}}$$

$$\Rightarrow \boxed{T_1 = \frac{T_2}{r^{\gamma-1}} = \frac{T_4}{\rho . \beta . r^{\gamma-1}}} \longrightarrow ⑤$$

Process (4-5) :

$$\frac{T_5}{T_4} = \left(\frac{V_4}{V_5}\right)^{\gamma-1} = \frac{1}{r_e^{\gamma-1}}$$

$$\Rightarrow \boxed{T_5 = \frac{T_4}{r_e^{\gamma-1}} = \frac{T_4 . \rho^{\gamma-1}}{r^{\gamma-1}}} \longrightarrow ⑥$$

Substitute T_3, T_2, T_1 & T_5 value's in eqn. ①; we get;

$$\eta_{Dual} = 1 - \frac{\frac{T_4 . \rho^{\gamma-1}}{r^{\gamma-1}} - \frac{T_4}{\rho . \beta . r^{\gamma-1}}}{\left(\frac{T_4}{\rho} - \frac{T_4}{\rho . \beta}\right) + \gamma\left(T_4 - \frac{T_4}{\rho}\right)}$$

$$\boxed{\eta_{Dual} = 1 - \frac{1}{r^{\gamma-1}} \times \frac{\rho^{\gamma} . \beta - 1}{(\beta - 1) + \gamma \beta (\rho - 1)}}$$

The comparison of otto, Diesel & Dual cycles for the

→ Same compression ratio & heat rejection;

$$\eta_{otto} > \eta_{Dual} > \eta_{Diesel}.$$

→ Same maximum pressure & temperature & heat rejection also;

$$\eta_{Diesel} > \eta_{Dual} > \eta_{otto}$$

The efficiency of an otto cycle is 60%. & $\gamma=1.5$. What is the compression ratio?

Soln:- $\eta = 60\% = 0.6$, $\gamma = 1.5$

$$\eta_{otto} = 1 - \frac{1}{r^{\gamma-1}} \Rightarrow 0.6 = 1 - \frac{1}{r^{1.5-1}}$$

$$\Rightarrow r = 6.25$$

An engine working on the otto cycle is supplied with air at 0.1 MPa, 35°c. The compression ratio is 8. Heat supplied is 2100 kJ/kg. Calculate the max. pressure & temp. of the cycle, the cycle efficiency & the Mean effective pr. (For air, $C_p = 1.005$ kJ/kg k, $C_v = 0.718$ kJ/kg.k & $R = 0.287$ kJ/kg k)

Soln: $T_1 = 35°c = 308k$, $P_1 = 0.1 MPa = 100 kN/m^2$, $Q_s = 2100 kJ/kg$;

$r = 8$, $\gamma = 1.4$.

$$\eta_{otto} = 1 - \frac{1}{r^{\gamma-1}} = 1 - \frac{1}{8^{1.4-1}} = 0.565 \text{ (or) } \underline{56.5\%}$$

Compression Ratio, $r = \frac{V_1}{V_2} \Rightarrow 8 = \frac{V_1}{V_2}$

$\because P_1V_1 = RT_1$ (for unit mass)

$\Rightarrow V_1 = \frac{RT_1}{P_1} = \frac{0.287 \times 308}{100}$

$V_1 = \underline{0.884 \, m^3/kg}$

$\Rightarrow V_2 = \frac{V_1}{8} = \frac{0.884}{8}$

$\therefore V_2 = \underline{0.11 \, m^3/kg}$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = 8^{1.4-1} = 2.3$$

$\Rightarrow T_2 = 2.3 \times T_1 = 2.3 \times 308 = \underline{708.4 K}$

Heat Supplied;

$Q_s = m C_v (T_3 - T_2)$

If for unit mass;

$Q_s = C_v (T_3 - T_2)$

$2100 = 0.718 (T_3 - 708.4)$

$\Rightarrow T_3 = T_{max} = \underline{3633 K}$

$$\frac{P_3V_3}{T_3} = \frac{P_2V_2}{T_2} \quad (\because V_2 = V_3)$$

$$\Rightarrow \frac{P_3}{P_2} = \frac{T_3}{T_2}$$

$$\Rightarrow P_3 = P_{max} = P_2 \times \frac{T_3}{T_2}$$

(1-2); $\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{\gamma} = 8^{1.4} = 18.37$

$\therefore P_2 = P_1 \times 18.37 = 100 \times 18.37 = 1837\ kN/m^2$

$P_2 = \underline{\underline{1.837\ MPa}}$

$\therefore P_3 = P_{max} = 1.837 \times \frac{3633.4}{708.4} = \underline{\underline{9.422\ MPa}}$

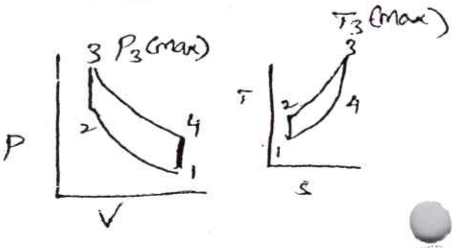
WKT,

$\eta = \frac{W_{net}}{Q_s} \Rightarrow W_{net} = \eta . Q_s = 0.565 \times 2100 = \underline{\underline{1186.5\ KJ/kg.}}$

WKT, $W = p.dv$

$W = P_m (V_1 - V_2)$

$\Rightarrow$ Mean Effective prs., $P_m = \frac{W}{(V_1 - V_2)}$



$\Rightarrow P_m, [m.e.p] = \frac{1186.5}{0.884 - 0.11} = 1533\ kPa$

$= \underline{\underline{1.533\ MPa}}$

The minimum prs. & temp. in an otto cycle are 100kpa & 27°c. The amount of heat added to the air per cycle is 1500 kJ/kg. (i) Determine the prs. & temp. at all points of the air standard otto cycle, (ii) Also calculate the Spl. work & thermal efficiency of the cycle for a compression ratio 8 : 1. Take for air; $C_v = 0.72\ kJ/kg.k$ & $\gamma = 1.4$.

Soln:- $P_1 = 100kPa = 1\ bar$, $T_1 = 27°c = 300k$, $\gamma = 1.4$

$Q_S = 1500\ kJ/kg$. $r = 8:1 = \frac{8}{1} = 8$, $C_v = 0.72\ kJ/kg.k$.

(Consider Unit mass):

(i)

(1-2): ($\because PV^{\gamma} = C$) process.

$\left(\frac{T_2}{T_1}\right) = \left(\frac{V_1}{V_2}\right)^{\gamma - 1} = 8^{1.4-1} = 2.3$

$\Rightarrow T_2 = T_1 \times 2.3 = 300 \times 2.3 = \underline{\underline{689.2\ K}}$

$\left(\frac{P_2}{P_1}\right) = \left(\frac{V_1}{V_2}\right)^{\gamma} = 8^{1.4} = 18.38$

$\Rightarrow P_2 = P_1 \times 18.38 = \underline{\underline{18.38\ bar}}$

(2-3):- ($\because V = C$)

$Q_S = C_v (T_3 - T_2)$

$1500 = 0.72 (T_3 - 689.2) \Rightarrow T_3 = \underline{\underline{2772.5\ K}}$

(@ v=c) $P_2 = T_2 \Rightarrow P_3 = P_2 \times T_3 \quad 18.38 \times 2772.5$

Also; P_3

(3-4) :- $[PV^{\gamma}=C]$ $\quad \frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{\gamma-1} = r^{\gamma-1} = \underline{\underline{2.3}}$

(process)

$$\therefore T_4 = \frac{T_3}{2.3} = \frac{2772.5}{2.3}$$

$$T_4 = \underline{\underline{1205.43\,k}}$$

Also;

$$\frac{P_3}{P_4} = \left(\frac{V_4}{V_3}\right)^{\gamma} = 8^{1.4} = 18.38$$

$$\therefore P_4 = \frac{P_3}{18.38} = \frac{73.94}{18.38} = \underline{\underline{4.023\,bar}}$$

(ii)

$$\eta_{otto} = \eta_{thermal} = 1 - \frac{1}{r^{\gamma-1}} = 1 - \frac{1}{8^{0.4}} = 0.565$$

$$\eta_{the} = \underline{\underline{56.5\%}}$$

(iii) Specific Work, $W = Q_S - Q_R$

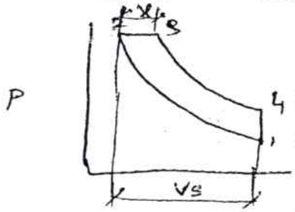
$$= C_v(T_3-T_2) - C_v(T_4-T_1)$$

$$= 0.72[T_3 - T_2 - T_4 + T_1]$$

$$= 0.72[2772.5 - 689.2 - 1205.43 + 300]$$

$$W = \underline{\underline{848.1\,kJ/kg}}$$

A diesel engine has a compression ratio of 14 & cut-off takes place at 6% of the stroke. Find the air standard efficiency?



~~Cut off Ra~~ $\left[\frac{V_3-V_2}{V_1-V_2} = \text{\% of stroke}\right]$

Soln:- $r = \frac{V_1}{V_2} = 14 \Rightarrow V_1 = 14V_2$

$$\frac{V_3-V_2}{V_1-V_2} = 6\% = 0.06$$

$$V_3 - V_2 = 0.06(V_1 - V_2)$$

$$= 0.06 \times 13V_2$$

$$\Rightarrow V_3 = 1.78V_2$$

Cut-off ratio, $\rho = \frac{V_3}{V_2} = \underline{\underline{1.78}}$

$$\therefore \eta_{Diesel} = 1 - \frac{1}{\gamma \cdot r^{\gamma-1}}\left[\frac{\rho^{\gamma}-1}{\rho-1}\right]$$

$$= 1 - \frac{1}{1.4 \times 14^{0.4}}\left[\frac{1.78^{1.4}-1}{1.78-1}\right]$$

$$\eta_{Diesel} = 0.605 = \underline{\underline{60.5\%}}$$

In an air standard diesel cycle, the compression ratio is 16 & @ the beginning of isentropic compression, the temp. is 15°C & the pressure is 0.1 MPa. Heat is added until the temp. at the end of the const. pr. process is 1480°C. Calculate (a) cut-off ratio, (b) the heat supplied per kg of air, (c) the cycle efficiency & (d) the M.E.P.

Soln: $r = \frac{V_1}{V_2} = 16$, $T_1 = 15°C = 288K$, $P_1 = 0.1 MPa$, $T_3 = 1480°C$,
$P_1 = 100 kN/m^2$, $T_3 = 1753 K$

(1-2):- $\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = 16^{0.4} = 3.03$.

$\therefore T_2 = 3.03 \times 288 = \underline{\underline{873 K}}$

(2-3):- $\frac{P_2 V_2}{T_2} = \frac{P_3 V_3}{T_3}$ $(\because V_2 = V_3)$

(a)

$\Rightarrow$ Cut-off Ratio, $\rho = \frac{V_3}{V_2} = \frac{T_3}{T_2} = \frac{1753}{873} = \underline{\underline{2.01}}$

(b) Heat Supplied, $Q_s = C_p(T_3 - T_2)$

$= 1.005(1753 - 873)$

$Q_s = \underline{\underline{884.4 \ kJ/kg}}$

(c) Cycle Efficiency $= 1 - \frac{Q_R}{Q_s}$ $\left[\eta = \frac{W}{Q_s} = \frac{Q_s - Q_R}{Q_s}\right]$

$\Rightarrow Q_R = \cancel{m}.C_v(T_4 - T_1)$

$\therefore Q_R = 0.718(766 - 288)$

$Q_R = \underline{\underline{343.2 \ kJ/kg}}$

$\therefore \eta = 1 - \frac{343.2}{884.4} = 0.612$
$= \underline{\underline{61.2\%}}$

$\Big[\because$ (3-4) process:-

$\frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{\gamma-1} = \left(\frac{r}{\rho}\right)^{\gamma-1} = \left(\frac{16}{2.01}\right)^{0.4}$

$\frac{T_3}{T_4} = 2.29$

$\therefore T_4 = \frac{1753}{2.29} = \underline{766 K}\Big]$

(d) $\underline{\underline{\text{M.E.P}, P_m}} \Rightarrow W_{net} = Q_s - Q_R$

$= 884.4 - 343.2 = \underline{\underline{541.2 \ kJ/kg}}$

WKT, $W = P_m(V_1 - V_2)$

$\Rightarrow P_m = \frac{541.2}{0.825 - 0.0512}$

$P_m = \underline{\underline{699.8 \ kPa}}$

$V_1 = \frac{RT_1}{P_1} = \frac{0.287 \times 288}{100}$
$= 0.8252 \ m^3/kg$.

$V_2 = \frac{V_1}{16} = \frac{0.8252}{16} = 0.0512 \ m^3/kg$

An air standard dual cycle has a compression ratio of 16, & compression begins at 1 bar, 50°c. The max. pr. is 70 bar. The heat transferred to air at const. pr. is equal to that at const. volume. Estimate (a) the pr's & temp's at the cardinal points of the cycle. (b) the cycle efficiency &(c) M.E.P of the cycle. $C_v = 0.718$ kJ/kg.k, $C_p = 1.005$ kJ/kg.k.

Soln: $T_1 = 50°C = 323K$ ✓ $r = 16$, $P_1 = 1$ bar ✓

(1-2): ($pV^{\gamma} = c$) $\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} \Rightarrow T_2 = 323 \times 16^{0.4} = \underline{\underline{979.15K}}$ ✓

$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{\gamma} \Rightarrow P_2 = 1 \times 16^{1.4} = \underline{\underline{48.5 \text{ bar}}}$ ✓

(2-3): $(V=C)$; $\frac{P_2}{P_3} = \frac{T_2}{T_3} \Rightarrow T_3 = T_2 \cdot \frac{P_3}{P_2}$ $\qquad \frac{P_2 \cancel{V_2}}{T_2} = \frac{P_3 \cancel{V_3}}{T_3}$ $(\because V_2 = V_3)$

given: $P_3 = 70$ bar ✓ $\qquad T_3 = 979.15 \times \frac{70}{48.5} = \underline{\underline{1413\,K}}$ ✓

$Q_{2-3} = C_v(T_3 - T_2)$

$= 0.718[1413 - 979.15] = \underline{\underline{312 \text{ kJ/kg}}}$

$Q_{3-4} = Q_{2-3} = 312$ kJ/kg.

(3-4): ($p=c$) $\therefore Q_{3-4} = \cancel{m}.C_p(T_4 - T_3)$

$312 = 1.005[T_4 - 1413] \Rightarrow T_4 = \underline{\underline{1723\,K}}$ ✓

$\frac{V_4}{V_3} = \frac{T_4}{T_3} = \frac{1723}{1413} = 1.22$

(5-1): ($V=C$): $\Rightarrow$ $\frac{V_5}{V_4} = \frac{V_1}{V_4}$ $(\because V_1 = V_5)$

$= \frac{V_1}{V_2} \cdot \frac{V_2}{V_4}$ $(\because V_2 = V_3)$

$\frac{V_5}{V_4} = \frac{V_1}{V_2} \cdot \frac{V_3}{V_4} = \left(16 \times \frac{1}{1.22}\right) = 13.11$

(4-5): $pV^{\gamma} = c$ $\qquad \frac{T_5}{T_4} = \left(\frac{V_4}{V_5}\right)^{\gamma-1} \Rightarrow T_5 = T_4 \times \left(\frac{V_4}{V_5}\right)^{\gamma-1} = 1723 \times \frac{1}{(13.11)^{0.4}}$

$T_5 = \underline{\underline{615.4K}}$ ✓

(5-1): (V=C) $\frac{P_5}{P_1} = \frac{T_5}{T_1} \Rightarrow P_5 = P_1 \cdot \frac{T_5}{T_1}$

$$P_5 = 1 \times \frac{615.4}{323} = \underline{\underline{1.91\ \text{bar}}} \checkmark$$

(4-5): ($PV^\gamma = C$)

$$\left(\frac{P_4}{P_5}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{T_4}{T_5}\right) \Rightarrow \frac{P_4}{P_5} = \left(\frac{T_4}{T_5}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{1723}{615.4}\right)^{\frac{1.4}{0.4}}$$

(or)

$P_3 = P_4 = 70\ \text{bar} \checkmark$

$$\frac{P_4}{P_5} = 36.724$$

$$\therefore P_4 = P_5 \times 36.724 = 1.91 \times 36.724$$

$$P_4 = \underline{\underline{70.14\ \text{bar}}} \checkmark$$

(b) Cycle efficiency, $\eta_{Dual} = 1 - \frac{C_v(T_5 - T_1)}{C_v(T_3 - T_2) + C_p(T_4 - T_3)}$

$$\eta_{Dual} = 1 - \frac{0.718(615.4 - 323)}{312 + 312}$$

$$= 0.6635 = \underline{\underline{66.35\ \%}} \checkmark$$

(c) **M.E.P; P_m:**

$$W_{net} = Q_s \times \eta \qquad \left[\because \eta = \frac{W}{Q_s}\right]$$

$$W_{net} = 624 \times 0.6635 = \underline{\underline{414.1\ \text{kJ/kg}}}$$

$P_1V_1 = RT_1 \Rightarrow \cancel{V_1 = \frac{P_1}{RT_1}}$

$$V_1 = \frac{RT_1}{P_1} = \frac{0.287 \times 323}{100} = \underline{\underline{0.927\ m^3/kg}}$$

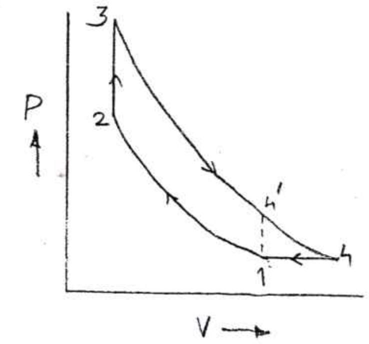
$$\frac{V_1}{V_2} = 16 \Rightarrow V_2 = \frac{0.927}{16} = \underline{\underline{0.058\ m^3/kg}}$$

WKT, $W = P_m(V_1 - V_2) \Rightarrow P_m = \frac{414.1}{0.927 - 0.058} = 476.5\ kN/m^2$

$$\therefore P_m = \underline{\underline{4.76\ \text{bar}}} \checkmark$$

ATKINSON CYCLE:

This cycle consists of two adiabatic process, One constant volume & one constant pressure process.



(1 - 2): Adiabatic Compression.
- No transfer of heat.

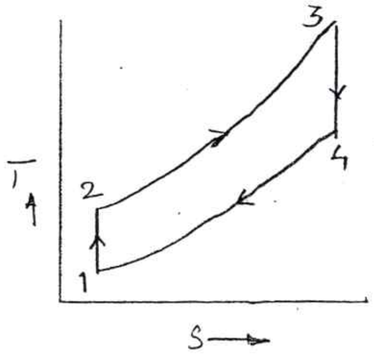
(2 - 3): Heat addition @ V = C.

$$Q_S = m.C_v(T_3 - T_2)$$

(3 - 4): Adiabatic Expansion.
- No transfer of heat. &

(4 - 1): Heat rejection @ P = C.

$$Q_R = m.C_p(T_4 - T_1)$$



$$\therefore \eta_{Air\text{-}std} = \frac{W.D}{Q_S} = \frac{Q_S - Q_R}{Q_S}$$

$$= \frac{m.C_v(T_3 - T_2) - m.C_p(T_4 - T_1)}{m.C_v(T_3 - T_2)}$$

$$\eta_{Air\text{-}std} = 1 - \gamma\left(\frac{T_4 - T_1}{T_3 - T_2}\right) \longrightarrow ①$$

Compression Ratio, $r_c = \frac{V_1}{V_2}$

Expansion Ratio, $r_e = \frac{V_4}{V_3} = \frac{V_4}{V_2}$ $(\because V_2 = V_3)$

<u>Process: (3-4):</u> $(PV^\gamma = C)$

$$\therefore \frac{T_4}{T_3} = \left(\frac{V_3}{V_4}\right)^{\gamma-1} \Rightarrow T_4 = T_3 \cdot \left(\frac{1}{r_e}\right)^{\gamma-1} = \frac{T_3}{r_e^{\gamma-1}}$$

<u>Process: (4-1):</u> $(P = C)$ $\frac{V_4}{T_4} = \frac{V_1}{T_1}$

$$\therefore \frac{T_1}{T_4} = \frac{V_1}{V_4}$$

$$\Rightarrow T_1 = T_4 \times \frac{V_1}{V_2} \times \frac{V_2}{V_4} = T_4 \times r_c \cdot \frac{1}{r_e} = T_4 \times \frac{r_c}{r_e} = \frac{T_3 \cdot r_c}{r_e^\gamma}$$

<u>Process: (1-2):</u> $PV^\gamma = C$

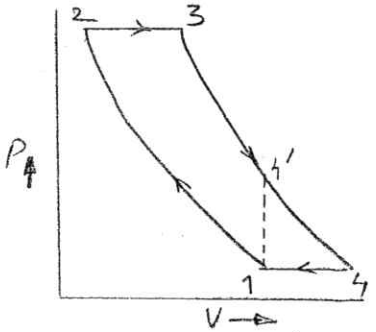
$$\therefore \frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} \Rightarrow T_2 = T_1 \cdot r_c^{\gamma-1} = T_4 \cdot \frac{r_c^\gamma}{r_e} = \frac{T_3 \cdot r_c^\gamma}{r_e^\gamma}$$

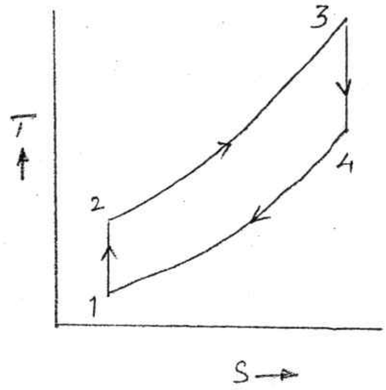
$$\therefore \eta_{Air\text{-}std} = 1 - \gamma \left[\frac{\frac{T_3}{r_e^{\gamma-1}} - \frac{T_3 \cdot r_c}{r_e^{\gamma}}}{T_3 - T_3 \frac{r_c^{\gamma}}{r_e^{\gamma}}} \right]$$

$$\boxed{\eta_{Air\text{-}std} = 1 - \gamma \left[\frac{r_e - r_c}{r_e^{\gamma} - r_c^{\gamma}} \right]}$$

BRAYTON CYCLE : [Joule Cycle].

[Similar to the Diesel cycle for gas turbine's]





(1-2) : Adiabatic compression.

(2-3) : Heat supplied @ P=c.

$Q_S = m \cdot C_p (T_3 - T_2)$

(3-4) : Adiabatic Expansion.

(4-1) : Heat rejection @ P=c.

$Q_R = m \cdot C_p (T_4 - T_1)$

$$\therefore \eta_{Air\text{-}std} = \frac{Q_S - Q_R}{Q_S} = 1 - \frac{T_4 - T_1}{T_3 - T_2}$$

Compression Ratio, $r = \frac{V_1}{V_2}$

Pressure Ratio, $r_p = \frac{P_2}{P_1} = \frac{P_3}{P_4}$

Process : (3-4) : $PV^{\gamma} = c$;

$$\frac{T_3}{T_4} = \left(\frac{P_3}{P_4}\right)^{\frac{\gamma-1}{\gamma}} \Rightarrow T_4 = \frac{T_3}{(r_p)^{\frac{\gamma-1}{\gamma}}}$$

(or)

$$\frac{T_3}{T_4} = \left(\frac{P_3}{P_4}\right)^{\frac{\gamma-1}{\gamma}} = \cancel{V_3}\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = r^{\gamma-1}$$

$$\therefore T_4 = \frac{T_3}{r^{\gamma-1}}$$

Process : (1-2) :

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1} = T_1 = \frac{T_2}{r^{\gamma-1}}$$

$$\therefore \eta_{Air\text{-}std} = 1 - \frac{\frac{T_3}{r^{\gamma-1}} - \frac{T_2}{r^{\gamma-1}}}{T_3 - T_2} = \boxed{1 - \frac{1}{r^{\gamma-1}}}$$

WKT;

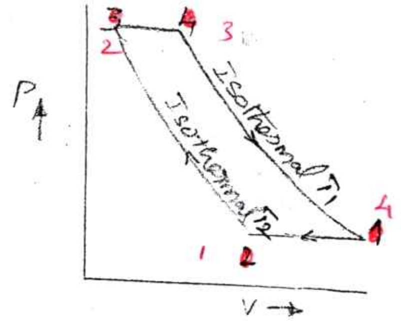
$$r = \frac{V_1}{V_2} = \left(\frac{P_2}{P_1}\right)^{\frac{1}{\gamma}}$$

$$r = r_p^{\frac{1}{\gamma}}$$

$$\boxed{\therefore \eta = 1 - \frac{1}{r_p^{\frac{\gamma-1}{\gamma}}}}$$

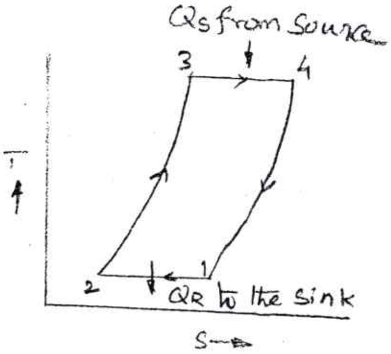
The 'η' depends only on the pressure ratio & the ratio of sp.heat(γ

ERICSSON CYCLE :



It Consists of two isothermal & two constant pressure processes.

The heat addition & rejection takes place @ constant pressure as well as isothermal processes.



Consider 1 kg of air:

volume ratio, $r = \frac{V_1}{V_4} = \frac{V_2}{V_3}$

Heat supplied to air from an external Source = Heat supplied during the isothermal expansion (3-4).

$$Q_S = RT_1 \log_e r.$$

Heat rejected by air to an external sink

$$Q_R = R.T_2 \log_e r.$$

$$\therefore \eta = \frac{W.D}{Q_S} = \frac{RT_1 \log_e r - RT_2 \log_e r}{R.T_1 \log_e r} = \frac{T_1 - T_2}{T_1} = 1 - \frac{T_2}{T_1}$$

Which is the same as "Carnot cycle."

This cycle does not find practical application in piston engines, but is approached by a gas turbine employing a large number of stages with heat exchanger's, insulators and re-heaters.