SREENIVASA INSTITUTE OF TECHNOLOGY AND MANAGEMENT STUDIES (AUTONOMOUS)

Department of Civil Engineering

Year / Semester: II B.Tech IV Semester
Regulation: R23

HAND WRITTEN NOTES

OF

HYDRAULICS AND HYDRAULIC MACHINERY (23ESC242T)

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SYLLABUS

UNIT–I

Laminar & Turbulent flow in pipes: Laminar flow – Laminar flow through circular pipes, annulus and parallel plates. Stoke's law, Measurement of viscosity. Reynolds experiment.
Transition from laminar to turbulent flow.Resistance to flow of fluid in smooth and rough pipes – Moody's diagram.Introduction to boundary layer theory.

UNIT–II

Uniform flow in Open Channels: Open Channel Flow – Comparison between open channel flow and pipe flow,Geometrical parameters of a channel, classification of open channels,classification of open channel flow.Velocity distribution of channel section.Hydraulically efficient channel sections: Rectangular, trapezoidal and triangular channels.Energy and Momentum correction factors.

UNIT–III

Non-Uniform flow in Open Channels: Specific energy, critical flow, discharge curve, specific force, specific depth and critical depth. Measurement of discharge and velocity.Gradually Varied Flow – Dynamic equation of gradually varied flow. Hydraulic Jump and classification –Elements and characteristics. Energy dissipation.

UNIT–IV

Impact of Jets: Hydrodynamic force of jets on stationary and moving flat, inclined and curved vanes.Velocity triangles at inlet and outlet.Work done and efficiency.

Hydraulic Turbines: Classification of turbines, Pelton wheel and its design.Francis turbine and its design – efficiency.Draft tube – theory.Characteristic curves of hydraulic turbines.Cavitation: causes and effects.

UNIT–V

Pumps: Working principles of a centrifugal pump, work done by impeller, heads, losses and efficiencies.Minimum starting speed, priming, specific speed.Limitation of suction lift, Net Positive Suction Head (NPSH).Performance and characteristic curves.Cavitation effects.Multistage centrifugal pumps.Troubles and remedies.

Unit-5 Pumps

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① A Centrifugal Pump rotating at 1000 r.p.m. delivers 160 l/s of water against a head of 30 m. The pump is installed at a place where atmospheric pressure is 1×10^5 pa (abs). and vapour pressure of water is 3 Kpa (abs). The head loss in suction pipe is equivalent to 0.2 m of water. Calculate: ① Minimum NPSH, ② Maximum allowable height of the pump from free surface of water in the sump.
(7QB)

This Problem is based on NPSH (Net Positive Suction Head). it is very commonly used in the pump Industry, it is defined as the absolute pressure head at the inlet to the pump minus the vapour pressure head plus the velocity head.

Solution Given data $N = 1000$ r.p.m, $Q = 160\ l/s = 0.16\ m^3/s$; $H_m = 30\ m$

$\Rightarrow P_a = 1 \times 10^5\ Pa = 1 \times 10^5\ N/m^2 \Rightarrow P_v = 3\ Kpa = 3 \times 10^3\ N/m^2$

$\Rightarrow h_{f_s} = 0.2\ m.$

① Minimum NPSH, we get $\sigma = \dfrac{NPSH}{H_m}$

It is clear that NPSH is directly proportional to Thoma's cavitation factor. (σ) NPSH will be minimum where σ is minimum. But the value of (σ) for no cavitation is σ_c. Hence where $\sigma = \sigma_c$ then NPSH will be minimum.

$$\sigma_c = \frac{(NPSH)_{min}}{H_m}$$

$$(NPSH)_{min} = H_m \times \sigma_c \quad -(1) \text{ Critical value of } \sigma_c$$

$$\sigma_c = 1.03 \times 10^{-3} \times N_s^{4/3} \quad -(2) \text{ eq}$$

Where N_s = Specific Speed of Pump $= \dfrac{N\sqrt{Q}}{H_m^{3/4}}$

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$= 1000 \times \frac{\sqrt{0.16}}{30^{3/4}}$ $\left(\because N = 1000 \text{ r.p.m}, Q = 0.16 m^3/s \text{ \& } H_m = 30m\right)$

Substituting the Value of N_s in eq. (2), We get

$$\sigma_c = 1.03 \times 10^{-3} \times \left[\frac{1000 \times \sqrt{0.16}}{30^{3/4}}\right]^{4/3}$$

$$\sigma_c = 1.03 \times 10^{-3} \times \left[\frac{1000^{4/3} \times (0.16)^{2/3}}{30}\right] = \frac{1.03 \times 10^{-3} \times 10^4 \times 0.2947}{30}$$

$$\sigma_c = 0.1012$$

Substituting the Value of σ_c in Equation (1) We get.

$$(NPSH)_{Min} = H_m \times 0.1012 = 3.036m.$$

(2) Maximum allowable height of the pump from free Surface of water in the Sump (i.e h_s).

Let $(h_s)_{Max}$ = Max. allowable height of Pump from free Surface of water.

$$NPSH = H_a - H_v - h_s - h_{f_s} \quad -(1)$$

from above equation $\left(H_a = \frac{P_a}{\rho g}\right)$, $\left(H_v = \frac{P_v}{\rho g}\right)$, (h_{f_s}), (h_s) will be Maximum.

then NPSH is Minimum.

$$(NPSH)_{Min} = H_a - H_v - (h_s)_{Max} - h_{f_s} \quad -(2)$$

$$(h_s)_{Max} = H_a - H_v - h_{f_s} - (NPSH)_{Min}$$

$$H_a = \frac{P_a}{\rho g} = \frac{1 \times 10^5}{1000 \times 9.81} = 10.193 m \text{ of water}$$

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$H_v = \frac{P_v}{\rho g} = \frac{3\times10^3}{1000\times9.81} = 0.305$ M of water.

$h_{f_s} = 0.2$ M & $(NPSH)_{min} = 3.036$ M.

Now Substituting the values of H_a, H_v, h_{f_s} & $(NPSH)_{min}$ in eq (3) we get: $(h_s)_{max} = 10.193 - 0.305 - 0.2 - 3.036$

$\Rightarrow (h_s)_{max} = 6.652$ M.

(2). A One-fifth Scale Model of a pump was tested in a laboratory at 1000 r.p.M the head developed and the Power input at the best efficieny Point were found to be 8M & 30 kw respectively. if the Prototype pump has to work against a head of 25M, determine its working Speed, the power required to drive it and the ratio of the flow rates handeled by two pumps.

(5Q(B))

Solution: One-fifth Scale Model means that the ratio of linear dimensions of a Model and its prototype is equal to 1/5

Speed of Model, $N_M = 1000$ r.p.M. | Let N_P = Speed of Prototype.

Head of Model $H_M = 8$ M.

Power of Model $P_M = 30$ KW. | P_P = Power of Prototype.

Head of Prototype, $H_P = 25$ M. | Q_P - Flow rate of prototype

Q_M = Flow rate of Model.

(1) Speed of prototype:- We get.

$$\left(\frac{\sqrt{H}}{DN}\right)_M = \left(\frac{\sqrt{H}}{DN}\right)_P \text{ (or) } \frac{\sqrt{H_M}}{D_M N_M} = \frac{\sqrt{H_P}}{D_P N_P}$$

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$$N_p = \frac{\sqrt{H_p}}{\sqrt{H_M}} \times \frac{D_M}{D_p} \times N_M = \frac{\sqrt{25}}{\sqrt{8}} \times \frac{1}{5} \times 1000$$

$N_p = 353.5$ r.p.M.

(2) Power developed by Prototype.

$$\left(\frac{P}{D^5 N^3}\right)_M = \left(\frac{P}{D^5 N^3}\right)_P \text{ (or) } \frac{P_M}{D_M^5 N_M^3} = \frac{P_P}{D_P^5 N_P^3}$$

(or) $P_P = P_M \times \left(\frac{D_P}{D_M}\right)^5 \times \left(\frac{N_P}{N_M}\right)^3 = 30 \times 5^5 \times \left(\frac{353.5}{1000}\right)^3$

$= 30 \times 3125 \times 0.04419 = 4143$ kW.

(3). Ratio of the flow rates of two pumps. (i.e Model & Prototype).

$$\left(\frac{Q}{D^3 N}\right)_M = \left(\frac{Q}{D^3 N}\right)_P \text{ (or) } \frac{Q_M}{D_M^3 N_M} = \frac{Q_P}{D_P^3 N_P}$$

$$\frac{Q_P}{Q_M} = \frac{D_P^3 N_P}{D_M^3 N_M} = \left(\frac{D_P}{D_M}\right)^3 \times \frac{N_P}{N_M} = 5^3 \times \frac{353.5}{1000}$$

$\because \left(\frac{D_P}{D_M} = \frac{5}{1}\right)$

$\Rightarrow$ 44.1875 Answer.

(3) (4 QB) A Three stage Centrifugal Pump has impeller 40Cm in diameter and 2CM wide at outlet. The Vanes are Curved back at the outlet at 45° & reduce the Circumferential area by 10%. the Manometric efficiency is 90% & the Overall efficiency is 80%. Determine the head generated

by the Pump when running at 1000 r.p.M delivering 50 litres/second & The Shaft horse power.

Solution: Given data is

Number of Stages, $n = 3$.

Diameter of impeller at outlet $D_2 = 40CM = 0.40M$.

Width at Outlet, $B_2 = 2CM = 0.02M$.

Vane angle at outlet, $\phi = 45°$

Reduction in area at outlet $= 10\% = 0.1$

$\therefore$ Area of flow at outlet $= 0.9 \times \pi D_2 \times B_2$

$= 0.9 \times \pi \times 0.4 \times 0.2 = 0.02262 M^2$.

Manometric efficiency, $\eta_{man} = 90\% = 0.90$.

Overall efficiency, $\eta_o = 80\% = 0.80$.

Speed $N = 1000$ r.p.M.

Discharge, $Q = 50$ litres/sec $= 0.05 M^3/S$.

Determine ① Head generated by the pump.

② Shaft Power.

Velocity of flow at outlet, $V_{f_2} = \frac{\text{Discharge}}{\text{Area of flow}} = \frac{0.05}{0.02262} = 2.21$ m/s

Tangentially velocity of impeller at outlet

$$U_2 = \frac{\pi D_2 N}{60} = 20.94 M/s.$$

From velocity triangle at outlet.

$$\tan\phi = \frac{V_{f_2}}{U_2 - V_{w_2}}$$

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$\therefore U_2 - V_{w_2} = \frac{V_{f_2}}{\tan\phi} = \frac{2.21}{\tan 45^\circ} = 2.21 m/s$

$V_{w_2} = U_2 - 2.21 = 20.94 - 2.21 = 18.73 m/s$

$\eta_{man} = \frac{gH_m}{V_{w_2} U_2}, \quad 0.90 = \frac{9.81 \times H_m}{18.73 \times 20.94}$

$H_m = \frac{0.90 \times 18.73 \times 20.94}{9.81} = 35.98 m$

Total head Generated by pump.

$\Rightarrow n \times H_m = 3 \times 35.98 = 107.94 M$

Power Output of the pump = $\frac{\text{weight of water lifted} \times \text{Tot}}{1000}$

$= \frac{\rho g \times Q \times 107.94}{1000} = \frac{1000 \times 9.81 \times 0.05 \times 107.94}{1000}$

$= 52.94 KW$

We have

$\eta_o = \frac{\text{Power Output of Pump}}{\text{Power Input of pump}} = \frac{52.94}{S.P}$

$\therefore$ shaft power $= \frac{52.94}{\eta_o} = \frac{52.94}{0.80}$

$\therefore$ Shaft power $= 66.175 KW$ Answer

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$\therefore U_2 - V_{w_2} = \frac{V_{f_2}}{\tan\phi} = \frac{2.21}{\tan 45^\circ} = 2.21 m/s$

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$\eta_{man} = \frac{gH_m}{V_{w_2} U_2}, \; 0.90 = \frac{9.81 \times H_m}{18.73 \times 20.94}$

$H_m = \frac{0.90 \times 18.73 \times 20.94}{9.81} = 35.98 m$

Total head Generated by pump.

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Power Output of the pump $= \frac{\text{weight of water lifted} \times \text{Total H}}{1000}$

$= \frac{\rho g \times Q \times 107.94}{1000} = \frac{1000 \times 9.81 \times 0.05 \times 107.94}{1000}$

$= 52.94 \text{ Kw}$

We have

$\eta_o = \frac{\text{Power Output of Pump}}{\text{Power Input of pump}} = \frac{52.94}{S.P}$

$\therefore$ Shaft power $= \frac{52.94}{\eta_o} = \frac{52.94}{0.80}$

$\therefore$ Shaft power $= 66.175$ Kw Answer

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(4) (3aQB) A Centrifugal Pump with 1.2m diameter runs at 200 r.p.M and pumps 1880 litres/sec, the average lift being 6M. The angle which the Vanes Makes at exit with the tangent to the impeller is 26° and the radial velocity of Flow is 2.5 m/s. Determine the Manometric effeciency and the least Speed to start pumping against a head of 6M, the inner diameter of the impeller being 0.6M.

Solution: Given

Dia. of outlet, $D_2 = 1.2$ M; Speed $N = 200$ r.p.M.

Dischange, $Q = 1880$ liters/sec $= 1.88\ m^3/s$.

Manometric head, $H_m = 6$ M

Angle of Vane at outlet, $\phi = 26°$

Velocity of flow at outlet, $V_{f_2} = 2.5$ m/s

Dia. at inlet, $D_1 = 0.6$ M.

(1) Manometric effeciency $(\eta_{man}) = \dfrac{gH_m}{V_{w_2} \times U_2}$ — (1)

But $U_2 = \dfrac{\pi D_2 N}{60} = 12.56$ m/s. | $\tan\phi = \dfrac{V_{f_2}}{U_2 - V_{w_2}}$

$\Rightarrow U_2 - V_{w_2} = \dfrac{V_{f_2}}{\tan\phi} = \dfrac{2.5}{\tan 26°} = 5.13$

$V_{w_2} = U_2 - 5.13 = 7.43$ m/s. Now substitute the Values in eq (1), We get. $\eta_{man} = 0.63 = 63\%$ Ans

(2) Least Speed to start the pump:

Least speed to start the pump is given by equation

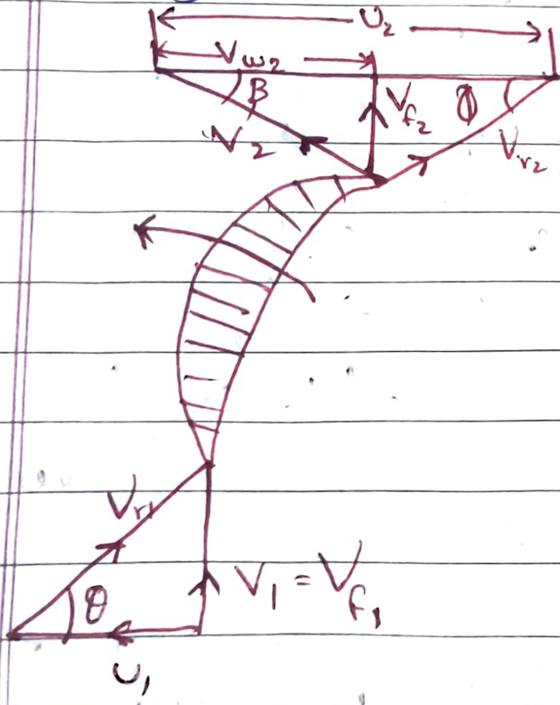
$$\frac{U_2^2}{2g} - \frac{U_1^2}{2g} = H_m \quad \text{- (2) eq.}$$

where U_2 & U_1 are the tangential Velocities of the Vane at the Outlet and inlet respectively, Corresponding to least Speed of Pump But $U_2 = \omega \times r_2$ and $U_1 = \omega \times r_1$

Substituting these Values in eq (2) we get.

$$\Rightarrow \frac{(\omega \times r_2)^2}{2g} - \frac{(\omega \times r_1)^2}{2g} = H_m = 6.0.$$

$$\Rightarrow \frac{\omega^2}{2g}\left[r_2^2 - r_1^2\right] = 6.0 \Rightarrow \left(r_2 = \frac{D_2}{2} = \frac{1.2}{2} = 0.6\,m \text{ \& } r_1 = \frac{D_1}{2} = \frac{0.6}{2} = 0.3\,m \right)$$



$$\omega^2 = \frac{6.0 \times 2.0 \times 9.81}{0.36 - 0.9} = 436$$

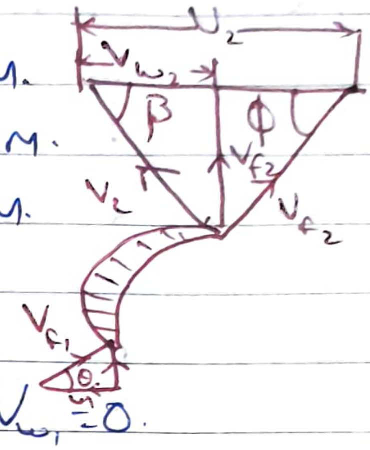
$$\omega = \sqrt{436} = 20.88 = \frac{2\pi N}{60}$$

$$N = \frac{60 \times 20.88}{2 \times \pi} = 200\text{ rpm}$$

(5) (1b QB) The Internal and External diameter of the impeller of a Centrifugal Pump are 200 mm and 400 mm. The pump is running at 1200 r.p.m. The Vane angles of the impeller at inlet & Outlet are 20° & 30°. The water enters the impeller radially & velocity of flow is Constant. Determine the Work done by the impeller.

Per Unit weight of water.

Solution: Given



Internal diameter of impeller $D_1 = 200mm = 0.20M$.

External diameter of impeller $D_2 = 400mm = 0.40M$.

Speed $N = 1200$ r.p.m.

Vane angle at inlet, $\theta = 20°$

Vane angle at outlet $\phi = 30°$

Water enters radially means $\Rightarrow \alpha = 90°$ & $V_{w_1} = 0$.

Velocity of Flow, $V_{f_1} = V_{f_2}$

Tangential velocity of impeller at inlet & Outlet are.

$U_1 = \frac{\pi D_1 N}{60} = 12.56 M/s$, $U_2 = \frac{\pi D_2 N}{60} = 25.13 m/s$.

$\therefore$ From inlet Velocity Triangle, $\tan\theta = \frac{V_{f_1}}{U_1} = \frac{V_{f_1}}{12.56}$

$V_{f_1} = 12.56 \tan\theta = 4.57 m/s.$, $V_{f_2} = V_{f_1} = 4.57 m/s$.

$\therefore$ From outlet Velocity Triangle, $\tan\phi = \frac{V_{f_2}}{U_2 - V_{w_2}} = \frac{4.57}{25.13 - V_{w_2}}$

$\Rightarrow 25.13 - V_{w_2} = \frac{4.57}{\tan\phi} \Rightarrow 7.915$.

$V_{w_2} = 25.13 - 7.915 = 17.215 M/s.$

$\therefore$ The work done by impeller per kg of water per second is given by equation.

$\Rightarrow \frac{1}{g} V_{w_2} U_2 = 44.1 Nm/$ Answer

(6) (1a) QB A Centrifugal Pump has the following characteristics: Outlet diameter of impeller = 800MM; width of impeller vanes at outlet = 100MM; angle of impeller vanes at outlet = 4[illegible]. The impeller runs at 550 r.p.M and delivers 0.98 Cubic meters of water per second water an effective head of 35M. A 500kw Motor is Used to drive the pump. Determine the Manometric, Mechanical and Overall efficiencies of the pump. Assume water enters the impeller vanes radially at inlet.

Solution: Given data.

$Q = 0.98 M^3/s$; $D_1 = 800MM = 0.8M$; $N = 550 r.p.M.$

$B_1 = 100MM = 0.1M$; $\phi = 40°$; $H_M = 35M.$

$P = 500KW$ $U_1 = \frac{\pi D_1 N}{60} = 23.04 M/s$ U_1 = Tangential velocity of impeller at inlet.

$V_{f_1} = \frac{Q}{\pi D_1 B_1} = 3.90 M/s$ V_f = Velocity of flow of liquid.

V_w = Velocity of whirl of the liquid.

$\cot\theta = \frac{1}{\tan\theta}$

$V_{w_1} = (U_1 - V_{f_1} \cot\phi) \Rightarrow 18.39 M/s$ (V_r = relative Velocity)

Manometric efficiency is given by

$$\eta_{man} = \frac{gH_M}{V_{w_1} \times U_1} = 0.81 \text{ (or) } 81\%$$

Overall efficiency is given by $\eta_o = \frac{WQH_M}{P} = 0.67 \text{ (or) } 67\%$

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Mechanical effeciency is given by

$$\eta_{mech} = \frac{\eta_o}{\eta_{mano}} = \frac{0.67}{0.81} = 0.83 \text{ (or) } 83\%$$

(7) (2QB) A Centrifugal pump Operates against a Manometric head of 30M with a manometric efficiency of 75%. The pressure rise through the impeller is 65% of the total Head developed by the pump. The radial velocity of Flow which is Constant is 3 m/s. The Outer diameter of the impeller is 400MM and the width at outlet is 15MM. The blades at inlet are Curved backwards at 60° to the wheel tangent. Calculate (1) the discharge in litres per Minute, (2) speed, (3) blade angle at outlet, (4) diameter of impeller at inlet.

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Solution: Given data.

(1) The discharge 'Q' delivered by the pump is given by

$$Q = K \pi B_1 D_1 V_{f_1}$$

Given $B_1 = 15MM = 0.015M$, $D_1 = 400MM = 0.400M$.

$V_{f_1} = 3M/s$., Assuming $K = 1$, we get.

$$Q = 1 \times \pi \times 0.0015 \times 0.400 \times 3. = 0.0565 M^3/s.$$

$$Q = 3390 \text{ litres/minutes.}$$

(2) Manometric efficiency is given by

$$\eta_{mano} = \frac{gH_M}{V_{w_1} U_1} \Rightarrow \frac{V_{w_1} U_1}{g} = \frac{H_M}{\eta_{mano}}$$

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$H_m = 30M$; $\eta_{mano} = 75\% = 0.75$

$$\therefore \frac{V_{w_1} U_1}{g} = \frac{30}{0.75} = 40M \quad (1)$$

total head developed by the pump = 40M.
The pressure rise through the impeller $\left(\frac{P_1 - P}{w}\right)$ is 65% of the total head developed by the pump.
Thus $\left(\frac{P_1 - P}{w}\right) = (0.65 \times 40) = 26M$.

Applying Bernoulli's Equation between the inlet & Outlet tips of the impeller and neglecting the head loss in the impeller, we have

$$\frac{P}{w} + \frac{V^2}{2g} = \frac{P_1}{w} + \frac{V_1^2}{2g} - \frac{V_{w_1} U_1}{g}$$

$$\left(\frac{P_1 - P}{w}\right) = \frac{V^2}{2g} + \frac{V_{w_1} U_1}{g} - \frac{V_1^2}{2g}$$

$V = V_f = 3m/s$.

Then by substitution, we get.

$$26 = \frac{3^2}{2 \times 9.81} + 40 - \frac{V_1^2}{2g}$$

$V_1 = 16.84 m/s \Rightarrow V_1 = \sqrt{V_{w_1}^2 + V_{f_1}^2}$

$V_{w_1} = \sqrt{V_1^2 - V_{f_1}^2} \Rightarrow V_f = 3m/s$.

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Thus by substitution, We get.

$$V_{w_1} = \sqrt{(16.84)^2 - (3)^2} = 16.57 m/s.$$

By Substituting in eq (1), We get.

$$\frac{16.57 \times U_1}{9.81} = 40$$

$$U_1 = \frac{40 \times 9.81}{16.57} = 23.68 m/s.$$

$$U_1 = \frac{\pi D_1 N}{60}$$

$$23.68 = \frac{\pi \times 0.400 \times N}{60}$$

$$N = \frac{23.68 \times 60}{\pi \times 0.400} = 1131 r.p.M.$$

i.e, Speed of the pump = 1131 r.p.M

(3) From outlet Velocity triangle, We have

$$\tan\phi = \frac{V_{f_1}}{U_1 - V_{w_1}} = 0.4219.$$

$$\phi = 22°52'$$

i.e blade angle at outlet = 22°52'.

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(4) From inlet Velocity triangle:

$$\tan\phi = \frac{V_f}{U}$$

$$U = V_f \cot\theta \Rightarrow \theta = 60^\circ$$

$$U = 3\cot 60^\circ = 1.732\,m/s.$$

$$U = \frac{\pi D N}{60}$$

$$\Rightarrow 1.732 = \frac{\pi \times D \times 1131}{60}$$

$$D = \frac{1.732 \times 60}{\pi \times 1131} \Rightarrow 0.029\,m = 29\,mm.$$

The diameter of the impeller at inlet = 29 mm.

Unit-4 Impact of Jets

QB-(1)(a) pg-1000 pn modi

(1) A Jet of water 75mm diameter having a velocity of 20m/s. strikes normally a flat smooth plate. Determine the thrust on the plate (a) if the plate is at rest, (b) if the plate is moving in the same direction as the jet with a velocity of 5m/s. Also find the work done per second on the plate in each case and the efficiency of the jet when the plate is moving.

Solution: Given data

(a) The Normal thrust on a stationary flat plate is given as $F = \left(\frac{w a V^2}{g}\right) \Rightarrow a = \text{area}$.

Area of Jet $a = \frac{\pi}{4}\left(\frac{75}{1000}\right)^2 = 0.00442 m^2$.

Velocity of Jet $V = 20 m/s$.

$$F = \frac{9810 \times (0.00442) \times (20)^2}{9.81} = 1768 N$$

Since in this case plate is at rest, $U = 0$.

Workdone $= (F \times U) = 0$,

(b) The Normal thrust on a Moving plate is given as

$$F = \frac{w a (V-U)^2}{g} = 994.5 N. \quad \therefore U = 5 m/s.$$

Work done per second by the Jet on the plate.

$= (F \times U) = 4972.5 Nm.$

Thus in this case, Output.

$= 4972.5 \, N.m/s.$

Input = Kinetic Energy of the issuing Jet.

$$= \frac{1}{2}\left(\frac{waV}{g}\right)V^2$$

$$= \frac{1}{2}\left(\frac{9810 \times 0.00462}{9.81}\right)(20)^3 = 17680 \, N.m/s$$

$$\eta \text{ of Jet} = \frac{4972.5}{17680} \times 100 = 28.13\% \left(\frac{\text{workdone}}{\text{Input}}\right)$$

DB-(1) DB pg-1005 pn mode

(2) A Jet of water Moving at 20 m/s impinges on a Symmetrical Curved vane shaped to deflect the Jet through 120° (that is the vane angles θ and ϕ are each equal to 30°). If the Vane is Moving at 5 m/s, find the angle of the Jet so that there is no shock at inlet. Also enough determine the absolute Velocity of exist in Magnitude and direction and the workdone.

Solution: From the Velocity triangle at inlet, applying the Sine rule we have

$$\frac{V}{\sin(180° - 30°)} = \frac{U}{\sin(30° - \alpha)} = \frac{V_r}{\sin\alpha}.$$

Also $V = 20 \, m/s$, $U = 5 \, m/s$.

Thus Considering the First two terms of the above Equation, we have.

$$\frac{20}{\sin 150^\circ} = \frac{5}{\sin(30^\circ - \alpha)} \Rightarrow \alpha = 22^\circ 49'$$

Now Considering the first & third terms of the equation, we have.

$$\frac{20}{\sin 150^\circ} = \frac{V_r}{\sin(22^\circ 49')} \Rightarrow V_r = 15.51 m/s.$$

It is assumed that the relative velocity of flow remains Constant, that is

$$V_{r_1} = V_r = 15.51 m/s.$$

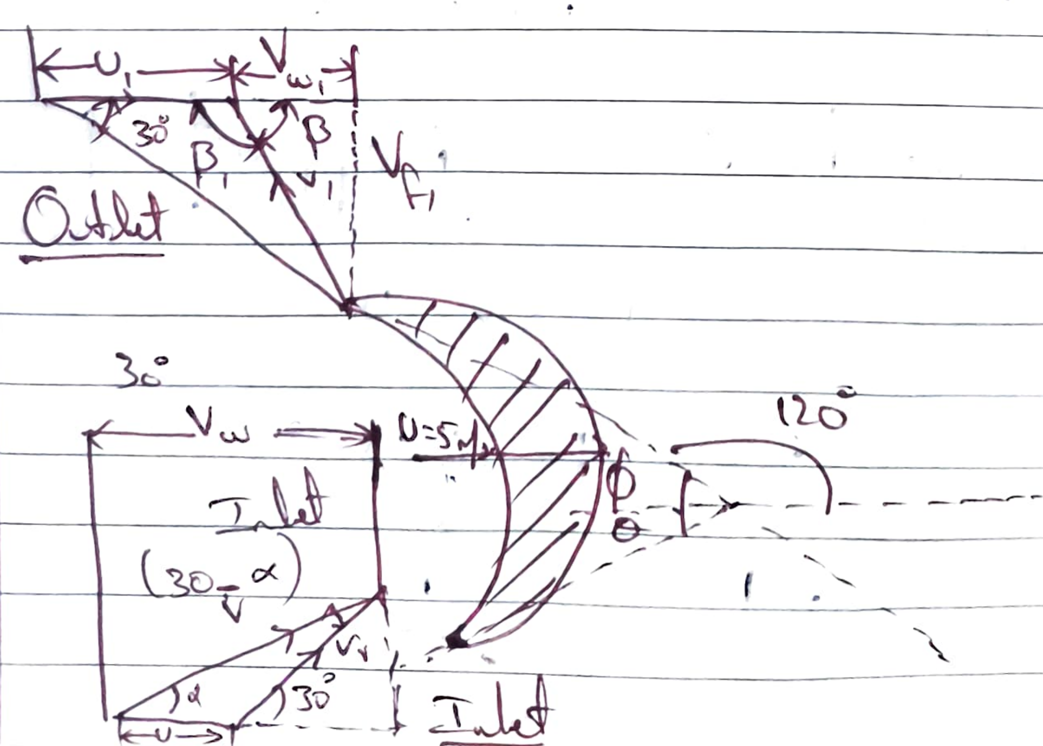
From the outlet velocity triangle applying the Cosine rule.

$$V_1^2 = U_1^2 + V_{r_1}^2 - 2U_1 V_{r_1} (\cos 30^\circ)$$

$$U_1 = U = 5 m/s.$$

$$V_1^2 = (5)^2 + (15.51)^2 - 2(5 \times 15.51)(0.866)$$

$$V_1 = 11.46 m/s.$$



Again from the outlet velocity Triangle.

$$\tan\beta = \frac{V_{r_1}(\sin 30^\circ)}{V_{r_1}(\cos 30^\circ) - u_1}$$

$$\tan\beta = 0.9197$$

$$\beta = 42^\circ 36' \ \& \ \beta' = (180^\circ - \beta) = 137^\circ 24'$$

Workdone per unit weight of fluid.

$$= \frac{1}{g}\left[V_w u \pm V_{w_1} u_1\right].$$

$$V_w = V(\cos\alpha) = 20(\cos 22^\circ 49') = 18.44\, m/s$$

$$V_{w_1} = V_1(\cos\beta) = 8.43\, m/s$$

Since the direction of V_{w_1} is opposite to that of the motion of vane and $u = u_1 = 5\, m/s$. Thus work done.

$$= \frac{1}{g}\left[V_w + V_w\right] u$$

$$= \frac{5}{9.81}\left[18.44 + 8.43\right]$$

$$= 13.70\, N.m/N.$$

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(3) A Jet discharge $0.15 m^3/s$ of water with velocity of 70m/s impinges without shock on a Series of Curved Vanes which move in the Same direction as the Jet. The Shape of each Vane is Such that it would deflect the Jet through an angle of 150; Surface friction reduces the relative Velocity by 8 percent as the water Passes across the vanes and there is a further windage loss equivalent to $\left(\frac{0.5U^2}{2g}\right)$ N.M per N of water, u being the Vane velocity Find.

Q13-2 pcge-1012 pn modi

(1) The velocity of the Vanes Corresponding to Max effic

(2) The Value of this efficiency

(3) The Corresponding Force on the Vanes in, and at right angles to the direction of their motion;

(4) The power of this arrangement.

Solution

Let 'u' be the required Velocity of the vanes,

$$V_r = (V-U) = (70-U)$$

$$V_{r_1} = 0.92V_r = 0.92\,(V-U)$$

$$V_w = V = 70\,m/s. \quad V_{w_1} = \left[U - V_{r_1}(\cos 30^\circ)\right]$$

$$= \left[U - 0.92(V-U)\left(\sqrt{3}/2\right)\right]$$

$\therefore$ Work done by water per Second.

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$$= \frac{W}{g}\left[V_w u - V_{w_1} u_1\right]$$

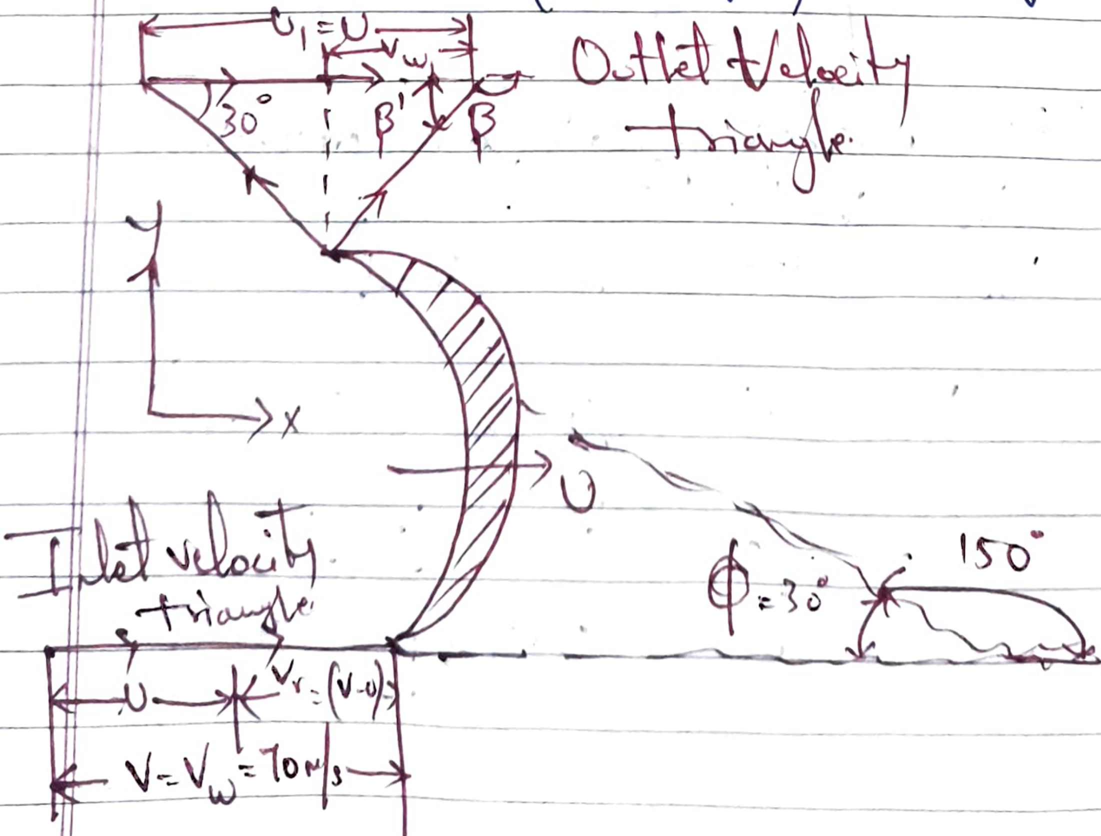
$$= \frac{W}{g}\left[1.797(V-u)u\right] - (0.5)\frac{Wu^2}{2g}$$

Kinetic energy of Jet $= \frac{WV^2}{2g}$

$\therefore$ Efficiency $\eta = \frac{\text{Useful work done}}{\text{Kinetic energy of Jet}}$

$$= \frac{2\left[1.797(V-u)u\right] - 0.5u^2}{V^2}$$

$$= 3.594\left(\frac{u}{V} - \frac{u^2}{V^2}\right) - 0.5\frac{u^2}{V^2}$$



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$\frac{u}{V} = n$, then

$\eta = n(3.594 - 4.094n)$

for Maximum efficiency

$\frac{d\eta}{dn} = 0$.

Thus $3.594 - 2(4.094)n = 0$.

$n = \frac{u}{V} = 0.439$.

For Maximum efficiency

$u = (0.439 \times 70) = 30.73\, m/s$.

Maximum $\eta_{max} = 0.439\{3.594 - (4.094)(0.439)\}$

$= 78.9\%$

Force on the vane at right angle to the direction.

$F_y = \frac{w}{g}(V_{f_1} - V_f) = \frac{w}{g} V_{f_1}$, Since $V_f = 0$)

$F_y = 2709.6\,N$.

The power of the arrangement = Useful work done.

$= \frac{w}{g}[1.797(V-u)u] - 0.5\frac{wu^2}{2g}$

$= 289.872\,kw$.

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4. A Jet of water having a Velocity of 15 m/s, strikes a Curved Vane which is Moving with a velocity of 5 m/s in the Same direction as that of the Jet at inlet. The vane is So shaped that the Jets is deflected through 135°, the diameter of Jet is 100 mm. Assuming the vane to be Smooth, find: (1) Force exerted by the Jet on the Vane in the direction of Motion, (2) Power exerted on the Vane (3) Efficiency of the Vane.

(13-(3) pag NO:- 832 ok

Solution:- Given data.

Velocity of Jet, $V_1 = 15$ m/s.

Velocity of Vane, $U = U_1 = U_2 = 5$ m/s.

At inlet Jet and Vane are in the Same direction. hence $\alpha = 0$.

Diameter of Jet, $d = 100$ mm $= 0.1$ m.

$\therefore$ Area $a = \frac{\pi}{4}(0.1)^2 = 0.007854\ m^2$.

Angle of deflection of Jet $= 135° = 180° - \phi$

$\phi = 180° - 135° = 45°$.

As vane is given Smooth hence $V_{r_1} = V_{r_2}$

From the inlet velocity triangle, which is a straight line in this Case, we have.

$V_{r_1} = V_1 - U_1 = 15 - 5 = 10$ m/s.

$V_{w_1} = V_1 = 15$ m/s.

From the Outlet Velocity triangle DEG, we have

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$V_{r_2} = V_{r_1} = 10\, m/s$

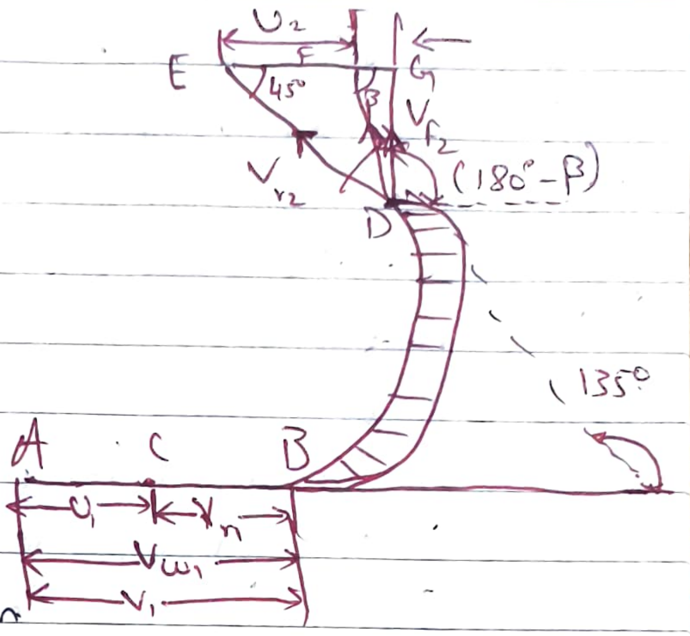
$U_2 = U_1 = U = 5\, m/s$

$V_{r_2} \cos\phi = U_2 + V_{w_2}$

$10 \cos 45^\circ = 5 + V_{w_2}$

$V_{w_2} = 10 \cos 45^\circ - 5$

$V_{w_2} = 7.07 - 5 = 2.07\, m/s$



∴ ① Force exerted by the Jet on the Vane in the direction of Motion is given by equation.

$F_x = \rho a V_{r_1} [V_{w_1} + V_{w_2}]$ (+ve sign is taken as β is an acute angle).

$= 1000 \times 0.007854 \times 10 [15 + 2.07] = 1340.6\, N$ Answer.

② Power of the Vane is given as

$= F_x \times U\ Nm/s = 1340.6 \times 5 = 6703\, W = 6.703\, kW.$

③ Efficiency of the Vane $= \dfrac{\text{Work done per Second on Vane}}{\text{Kinetic energy Supplied by Jet per Second}}$

$= \dfrac{F_x \times U}{\frac{1}{2} \times (\text{Mass per Second}) \times V^2} = \dfrac{F_x \times U}{\frac{1}{2} (\rho a V_1) \times V_1^2}$

$= \dfrac{1340.6 \times 50}{\frac{1}{2} \times (1000 \times 0.007854 \times 15) \times 15^2} = 0.505 = 50.5\%$

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5. A Jet of water having a velocity of 20m/s strikes a Curved Vane, which is Moving with a velocity of 10m/s. The Jet Makes an angle is 20° with the direction of motion of vane at inlet and leaves at an angle of 130° to the direction of Motion of Vane an outlet Calculate:

① Vane angles, So that the water enters and leave the Vane without Shock.

② work done per Second per unit weight of water Striking the vane per Second.

QB - 4
Pg no - 826 - ex

Solution: Given data.

Velocity of Jet, $V_1 = 20m/s$.

Velocity of Vane $U_1 = 10m/s$.

Angle made by Jet at inlet, with direction of Motion of vane, $\alpha = 20°$.

Angle Made by the leaving Jet, with direction of Motion $= 130°$

$\beta = 180° - 130° = 50°$;

In this problem, $U_1 = U_2 = 10m/s$, $V_{r_1} = V_{r_2}$

① Vane angle Means angle made by the relative: Velocities at inlet & Outlet. Such that θ & ϕ.

In ΔABD, We have $\tan\theta = \frac{BD}{CD}$

$$= \frac{V_{f_1}}{AD - AC} = \frac{V_{f_1}}{V_{w_1} - U_1} \quad -①$$

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Where $V_{f_1} = V_1 \sin\alpha = 20 \times \sin 20° = 6.84\, m/s.$

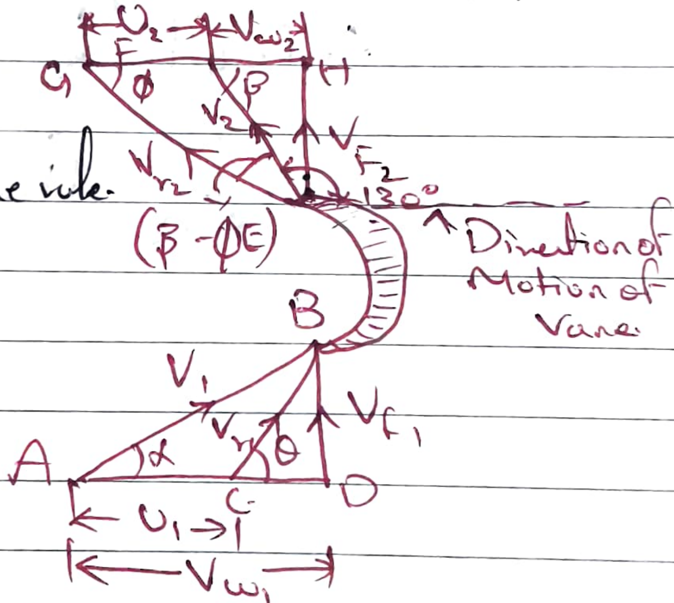
$V_{w_1} = V_1 \cos\alpha = 20 \times \cos 20° = 18.794\, m/s.$

$U_1 = 10\, m/s.$

$$\tan\theta = \frac{6.84}{18.794 - 10} = 0.7778 \text{ or } \theta = 37.875°$$

From, ΔABC, $\sin\theta = \frac{V_{f_1}}{V_{r_1}}$ (or) $V_{r_1} = \frac{V_{f_1}}{\sin\theta} = \frac{6.84}{\sin 37.875°}$

$V_{r_1} = 11.14\, m/s.$



From, $AEFG$, Applying the Sine rule.

$$\frac{V_{r_2}}{\sin(180° - \beta)} = \frac{U_2}{\sin(\beta - \phi)}$$

$$\frac{11.14}{\sin\beta} = \frac{10}{\sin[\beta - \phi]}$$

$$\sin(50° - \phi) = \frac{10 \times \sin 50°}{11.14} \;(\beta = 50°) = 0.6876 \Rightarrow \sin 43.44°$$

$50 - \phi = 43.44°$ (or) $\phi = 50° - 43.46° = 6.56°$

$\phi = 6°\, 33.6'$

(2) Work done per Second per Unit weight of the water striking the vane per second is given by equation.

$= \frac{1}{g}\left[V_{w_1} + V_{w_2}\right] \times u$ (+ve Sign is taken as β is an acute angle).

Where $V_{w_1} = 18.794$ m/s,

$V_{w_2} = GH - GF = V_{r_2}\cos\phi - u_2$

$V_{w_2} = 11.14 \times \cos 6.56^\circ - 10 = 1.067$ m/s.

$u = u_1 = u_2 = 10$ m/s.

$\therefore$ Work done per unit Weight of water

$$= \frac{1}{9.81}\left[18.794 + 1.067\right] \times 10 = 20.24 \text{ Nm/N}$$

UNIT-1

Laminar flow & Turbulent in pipes

Pg: 641, 13.3

① Problem on Circular pipe.

Given data.

Dynamic Viscosity $= (\mu) = 1.766$ pa.s
diameter pipe $= 0.3$m (flow)
Maximum Velocity $= 3$ m/s.

$\tau_0 = ? \Rightarrow$ Pipe wall?
At 50mm for pipe wall.

13.3 For a laminar flow of an oil having dynamic viscosity $\mu =$ 1.766 pa.s in a 0.3m diameter pipe, the velocity distribution is parabolic with a maximum point velocity of 3 m/s at the centre of pipe calculate the shearing stress at the pipe wall and within the fluid 50mm from the pipe wall.

Sol Given,

We have,

$$V = \frac{1}{2} V_{max} = \frac{3}{2} \; 1.5 \text{ m/s}$$

We have
discharge (circular pipe laminar flow)

$$\left(-\frac{\partial p}{\partial x}\right) = \frac{P_1 - P_2}{L} = \frac{32 \mu V}{D^2}$$

$$\left(-\frac{\partial p}{\partial x}\right) = \frac{32 \times 1.761 \times 1.5}{(0.3)^2} = 941.87 \text{ pa/m}$$

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13.3 For a laminar flow of an oil having dynamic viscosity μ = 1.766 pa.s in a 0.3m diameter pipe, the velocity distribution is parabolic with a maximum point velocity of 3 m/s at the centre of pipe. Calculate the shearing stress at the pipe wall and within the fluid 50 mm from the pipe wall.

Sol:

Given,

We have,

$$V = \frac{1}{2} V_{max} = \frac{3}{2} = 1.5 \text{ m/s}$$

We have

discharge (circular pipe laminar flow)

$$\left(-\frac{\partial p}{\partial x}\right) = \frac{P_1 - P_2}{L} = \frac{32 \mu V}{D^2}$$

$$\left(-\frac{\partial p}{\partial x}\right) = \frac{32 \times 1.766 \times 1.5}{(0.3)^2} = 941.87 \text{ pa/m}$$

The shear stress at the pipe wall is

$$\tau_0 = \left(-\frac{\partial p}{\partial x}\right)\frac{R}{2}$$

$$= \left(\frac{941.87 \times 0.3}{2 \times 2}\right) = 70.64 \text{ pa}$$

derivation 60% 13%

The shear stress at 50mm from the pipe wall is

$$\tau_0 = \left(-\frac{\partial p}{\partial x}\right)\frac{R}{2}$$

50mm = 0.05m

$$= 941.87 \times \left(\frac{0.15 - 0.05}{2}\right) = 47.09 \text{ pa}$$

(13.4)/643 Question bank 5(b) :-
Two parallel plates kept 0.1m apart have laminar flow of oil b/w them with a maximum velocity of 1.5m/s Calculate the discharge per metre width, the shear stress at the plates, the difference in pressure in pascals b/w two points 20m apart the velocity gradient at the plate and velocity at 0.02m from the plate. Take viscosity of oil to be 2.453 N/m²

Given
In this case the mean velocity of flow v is Equal to two-thirds of the maximum velocity.

$$v = \frac{2}{3}(1.5) = 1.0 \, m/s$$

the discharge (Q) per metre width the plate is given by

$$Q = VB = (1.0 \times 0.1) = 0.1 \, m^3/s$$

derivation for 612 13.6

we have

$$v = \frac{B^2}{12\mu}\left(-\frac{\partial p}{\partial x}\right)$$

$$\left(-\frac{\partial p}{\partial x}\right) = \frac{12 \mu V}{B^2}$$

$$= \frac{12 \times 2.453 \times 1.0}{(0.1)^2} = 2943.6 \text{ N/m}^2 \text{ per m}$$

The shear stress at plates is given by

$$\tau_0 = \left(-\frac{\partial p}{\partial x}\right)\frac{B}{2} = \frac{2943.6 \times 0.1}{2} = 147.18 \text{ N/m}^2$$

The pressure difference b/w two point given by

$$(p_1 - p_2) = \frac{12 \mu V L}{B^2}$$

$$= \frac{12 \times 2.453 \times 1.0 \times 20}{(0.1)^2}$$

$$= 58872 \text{ N/m}^2 = 58.872 \text{ kN/m}^2$$

The shear stress at plates is also given by

$$\tau_0 = \mu \left(\frac{\partial v}{\partial y}\right)_{y=0}$$

As such the velocity gradient at the plates is given by

$$\left(\frac{\partial v}{\partial y}\right)_{y=0} = \frac{\tau_0}{\mu} = \frac{147.18}{2.453} = 60 s^{-1}$$

The velocity v at a distance of 0.02 m from the plate is given by

$$v = \frac{1}{2\mu}\left(-\frac{\partial p}{\partial x}\right)(By - y^2)$$

$$= \frac{1}{2 \times 2.453} \times (2943.6) \times \left[(0.1 \times 0.02) - (0.02)^2\right]$$

$$= 0.96 \text{ m/s}$$

14.5/683

A Smooth brass pipeline 75mm in diameter and 900m long carries water at the rate of 7 litres persecond. If the kinematic viscosity of water is 0.0195 stokes, Calculate the Loss of head, wall Shearing Stress, Centre line velocity, Shear Stress and velocity at 25mm from the centre line and the thickness of the laminar sublayer, Take $\rho = 1000 kg/m^3$ ($10^{-4} m^2$)

Sol mean velocity, $V = \frac{Q}{A} = \frac{7\times10^{-3}}{\frac{\pi}{4}(0.075)^2} = 1.584 m/s$

Reynolds number $\frac{VD}{\nu} = \frac{1.584\times0.075}{0.0195\times10^{-4}} = 6.09\times10^4$

$$f = \frac{0.316}{(Re)^{1/4}} = \frac{0.316}{(6.09\times10^4)^{1/4}} = 0.0201$$

from darcy-weishbach

$$h_f = \frac{fLV^2}{2gD} = \frac{0.0201\times900\times(1.584)^2}{2\times9.81\times0.075}$$

$= 31m$

wall shearing stress :

$$\tau_0 = \frac{\rho V^2 f}{8} = \frac{1000 \times (1.584)^2 \times 0.0201}{8} = 6.304\ N/m^2$$

Shear velocity $V_* = V\sqrt{\frac{f}{8}} = 1.584\sqrt{\frac{0.0201}{8}} = 0.0794\ m/s$

for turbulent flow in smooth pipe Eq. 14.31

$$\frac{u}{V_*} = 5.75 \log_{10}\left(\frac{V_* y}{\nu}\right) + 5.5$$

$$y = \frac{D}{2} = \frac{0.075}{2} = 0.0375m, \quad u = u_{max}$$

Substituting we get

$$\frac{u_{max}}{0.0794} = 5.75 \log_{10}\left(\frac{0.0794 \times 0.0375}{0.0195 \times 10^{-4}}\right) + 5.5$$

$$u_{max} = 1.89\ m/s$$

Eq. 14.17 we have

$$\tau = \tau_0 \frac{r}{R}$$

The shear stress at $r = 0.025m$ is obtained

$$\tau = 6.304\left(\frac{0.025}{0.0375}\right) = 4.203\ N/m^2$$

Again from Eq. 14.31

$$y = (0.0375 - 0.025) = 0.0125m$$

$$\frac{v}{0.0794} = 5.75 \log_{10}\left(\frac{0.0794 \times 0.0125}{0.0195 \times 10^{-4}}\right) + 5.5$$

$$v = 1.672\ m/s$$

Thickness of laminar sublayer

$$\delta' = \frac{11.6\nu}{V_*} = \frac{11.6 \times 0.0195 \times 10^{-4}}{0.0794}$$

$$= 2.85\ 10^{-4}\ m$$

Question bank 5 (a) 13

Water flows at a steady mean velocity of 1.5 m/s through a 50mm diameter pipe slopping upwards at 45° to the horizontal. At a section some distance downstream of the inlet the pressure is 700kpa and at a section 30m further along the pipe the pressure is 462 k.pa. determine the average shear stress at the wall of the pipe and at a radius of 10mm.

Sol

Given

Assuming datum to be passing through the lower point

$$(h_1 - h_2) = \left(\frac{p_1}{w} + 0\right) - \left(\frac{p_2}{w} + z_2\right)$$

$$= \left(\frac{700 \times 10^3}{9810}\right) - \left(\frac{462 \times 10^3}{9810} + \frac{30}{\sqrt{2}}\right)$$

$$= (71.356 - 68.308)$$

$$= 3.048 m$$

Average shear stress at the wall of pipe is

$$\tau_0 = w\left(-\frac{\partial h}{\partial x}\right)\frac{R}{2}$$

$$= \frac{9810 \times 3.048}{30} \times \frac{0.05}{2 \times 2}$$

$$= 12.459 N/m^2$$

$$= 12.459 pa$$

Average shear stress at the radius of 10mm

$$\tau = \left(-\frac{\partial h}{\partial x}\right)\frac{r}{2}$$

$$= \frac{9810 \times 3.048}{30} \times \frac{0.01}{2}$$

$$= 4.983 N/m^2$$

$$= 4.983 pa$$

7.44-15.

A most Efficient trapezoidal Section is required to give a maximum discharge of $21.5 \, m^3/s$ of water. The slope of the Channel bottom is 1 in 2500. Taking $C = 70 \, m/s$ in chezy's Equation determine the dimensions of the Channel. Also determine the value of manning's n, taking the value of velocity of flow as obtained for the Channel by Chezy's Equation.

Sol: Given,

for most efficient trapezoidal Section slope $z = (1/\sqrt{3})$

we get

$$\frac{B}{y} = \frac{2}{\sqrt{3}} = 1.1547$$

$$B = 1.1547y$$

Using chezy's Equation

$$V = C\sqrt{RS}$$

$$Q = CA\sqrt{RS}$$

$$Q = 21.5 \, m^3/s, \quad C = 70, \; S = \frac{1}{2500}$$

$$A = (B + zy)y$$

$$= \left(1.1547y + \frac{1}{\sqrt{3}}y\right)y = 1.7321y^2$$

$$R = \frac{y}{2} = 0.5y$$

Substituting we get.

$$21.5 = 70 \times 1.7321y^2 \times \sqrt{0.5y \times \frac{1}{2500}}$$

$$y^{3/2} = 12.539$$

$$y = 2.75m$$

$$B = 1.1547 \times 2.75 = 3.175m$$

The wetted perimeter p is given by

$$P = B + 2y\sqrt{1+z^2}$$

$$= 3.175 + 2 \times 2.75 \times \sqrt{1 + \left(\frac{1}{\sqrt{3}}\right)^2}$$

$$= 9.526m$$

As a check it may be show if side slope z is taken either or less than $(1/\sqrt{3})$ the wetted perimeter p is more than the one obtained above for side slope $z = (1/\sqrt{3}) = 0.5774$.

Thus taking side slope, $z = 1$, we have

$$\frac{B+2zy}{2} = y\sqrt{1+z^2}$$

$$B = 0.828y$$

Using Chezy's Condition Equation

$$V = C\sqrt{RS}$$

$$Q = CA\sqrt{R}$$

$$Q = 21.5\, m^3/s, \quad C = 70, \quad S = \frac{1}{2500}$$

$$A = (B+zy)y$$

$$= (0.828y + 1\times y)y = 1.828y^2$$

$$R = \frac{y}{2} = 0.5y$$

Substitution we get

$$21.5 = 70\times 1.828y^2 \times \sqrt{0.5y\times\frac{1}{2500}}$$

$$y^{3/2} = 11.881$$

$$y = 2.692\,m$$

$$B = 2.229\,m$$

wetted perimeter

$$p = B + 2y\sqrt{1+z^2}$$

$$= 2.229 + 2\times 2.692\times\sqrt{1+(1)^2}$$

$$= 9.843\,m$$

Which is more than one obtained above for side slope $z = (1/\sqrt{3}) = 0.5$

Similarly taking side slope $= 0.4$

$$\frac{B+2zy}{2} = y\sqrt{1+z^2}$$

$$B = 1.354y$$

Using Chezy's Equation

$$V = C\sqrt{RS}$$

$$Q = CA\sqrt{RS}$$

$$C = 70, \quad S = \frac{1}{2500}$$

$A = (B+2y)y$

$= (1.345y + 0.4y)y = 1.754y^2$

$R = \frac{y}{2} = 0.5y$

Substituting we get

$21.5 = 70 \times 1.754y^2 \times \sqrt{0.5y \times \frac{1}{2500}}$

$y^{5/2} = 12.382$

$y = 2.736$

$B = 3.705m$

Wetted perimeter is given by

$P = B + 2y\sqrt{1+z^2}$

$= 3.705 + 2 \times 2.736 \times \sqrt{1+(0.4)^2}$

$= 9.599m$

dimension of trapezoidal section

$B = 3.175m, y = 2.75m \quad z = (1/\sqrt{3}) = 0.577$

We have

$C = \frac{1}{n} R^{1/6}$

$R = \frac{y}{2} = \frac{2.75}{2} = 1.375m$

$C = 70$

$70 = \frac{1}{n} \times (1.375)^{1/6}$

$n = \frac{(1.375)^{1/6}}{70} = 0.015$

mannings.

(7) Boundary layers!

(1) The Velocity distribution in the boundary layer is g as $\frac{u}{V} = \frac{3}{2}\eta - \frac{1}{2}\eta^2$ in which $\eta = (y/\delta)$

Compute (δ^*/δ) & (θ/δ).

(1) We know that

$$\eta = \left(\frac{y}{\delta}\right)$$

$$\Rightarrow \delta^* = \int_0^\infty \left(1 - \frac{u}{V}\right) dy$$

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{V}\right) dy + \int_\delta^\infty \left(1 - \frac{u}{V}\right) dy$$

$\Rightarrow$ But outside Boundary Layer $\frac{u}{V} = 1$.

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{V}\right) dy.$$

$$\boxed{\frac{u}{V} = \frac{3}{2}\eta - \frac{1}{2}\eta^2} \text{ \& } dy = \delta d\eta$$

$$\delta^* = \delta \int_0^1 \left(\underset{(1)}{1} - \underset{(2)}{\frac{3}{2}\eta} + \underset{(3)}{\frac{1}{2}\eta^2}\right) d\eta =$$

$\Rightarrow$ (1) $\int_0^1 1\, d\eta = 1$, (2) $\int_0^1 \eta\, d\eta = \frac{1}{2} \Rightarrow \frac{3}{2} \times \frac{1}{2} = \frac{3}{4}$

All Results

(3) $\int_0^1 \eta^2\, d\eta = \frac{1}{3} \Rightarrow \frac{1}{2} \times \frac{1}{3} = \frac{1}{6} \Rightarrow$ $= 1 - \frac{3}{4} + \frac{1}{6}$

$\int x^2 = \frac{x^3}{3}$

$\frac{\delta^*}{\delta} = 0.41$

$$\frac{\theta}{\delta} = \int_0^{\delta} \frac{u}{V}\left(1 - \frac{u}{V}\right) dy$$

$$\theta = \delta \int_0^1 \underbrace{\overbrace{\left(\frac{3}{2}n - \frac{1}{2}n^2\right)}^{(u/v)}}_{①} \underbrace{\overbrace{\left(1 - \frac{3}{2}n + \frac{1}{2}n^2\right)}^{(u/v)}}_{②\ \frac{5}{12}} dn.$$

$$= \frac{19}{120}\delta \Rightarrow (or)\ \frac{\theta}{\delta} = \frac{19}{120.}$$

$$\int_0^1 n = \frac{1}{2} \times \cancel{\frac{3}{2} = \frac{3}{4}},\ \int n^2 = \cancel{\frac{1}{3}} \times \frac{1}{2} = \frac{1}{6}$$

$$= \cancel{\int_0^1 \delta} = \frac{3}{4}\cancel{n - \frac{1}{6}}n^2 = \Rightarrow \left(\cancel{\frac{3}{4} - \frac{1}{6}}\right) \times \delta$$

$$\Rightarrow 1 - \frac{3}{4} + \frac{1}{6} = \cancel{\frac{5}{12}} \Rightarrow \frac{3}{24} \Rightarrow \cancel{\frac{5}{12}} \times \frac{3}{24}$$

Substitute into the definition

Step-②

$$\theta = \delta \int_0^1 \left(\frac{3}{2}n - \frac{1}{2}n^2\right)\left(1 - \frac{3}{2}n + \frac{1}{2}n^2\right) dn$$

Step-③

$$A = \frac{3}{2}n - \frac{1}{2}n^2,\ B = 1 - \frac{3}{2}n + \frac{1}{2}n^2$$

Now multiply terms

$$A \times B = \frac{3}{2}n - \frac{9}{4}n^3 + \frac{3}{4}n^3 - \frac{1}{2}n^2 + \frac{1}{4}n^3 - \frac{1}{4}n^4$$

Combine like terms

$$= \frac{3}{2}n - \frac{11}{4}n^2 + \frac{3}{2}n^3 - \frac{1}{4}n^4$$

Step 4:- Integrate term by term

$$\theta = \delta \int_0^1 \left(\frac{3}{2}n - \frac{11}{4}n^2 + \frac{3}{2}n^3 - \frac{1}{4}n^4\right) dn$$

first term:-

$$\int_0^1 \frac{3}{2}n\, dn = \frac{3}{2} \times \frac{1}{2} = \frac{3}{4}$$

Second term:-

$$\int_0^1 \frac{11}{4}n^2 dn = \frac{11}{4} \cdot \frac{1}{3} = \frac{11}{12}$$

third term

$$\int_0^1 \frac{3}{2}n^3 dn = \frac{3}{2} \times \frac{1}{4} = \frac{3}{8}$$

fourth term

$$\int_0^1 \frac{1}{4}n^4 dn = \frac{1}{4} \times \frac{1}{5} = \frac{1}{20}$$

Step-⑤ Combine the Results

$$= \frac{3}{4} - \frac{11}{12} + \frac{3}{8} - \frac{1}{20}$$

the LCM = 20

$$\frac{90}{120} - \frac{110}{120} + \frac{45}{120} - \frac{6}{120} = \frac{19}{120}$$

Introduction

- Hydraulics deals with flow & conveyance of liquid [Water].

Uses of Hydraulics:-

- Hydraulic Structures : Dam, wiers, Barrages, Embankment, Bridges.
- Water-Distribution System : Drainage canal, Irrigation, Pipe Networks.
- Design of Hydraulic machines : Pumps, Turbines.
- Navigation System : Canal lock, Development of waterways.
- Design of Wells.
- Water Quality.
- Hydroinformatics.

Open channel flow

Openchannel flow (OCF):-

- Liquid flows through any channel with free surface, subjected to atmospheric pressure.
- Liquid flows from higher energy to lower energy level.

Pipe flow (PF):-

- Liquid flows under pressure through any conduit without having a free surface.
- Pressure have to be created for pipe flow.

Energy Equation:-

Head:- Energy per unit weight of fluid with respect to datum.

Potential Head:- $\frac{\text{Energy}}{\text{Unit wt of fluid}} = \frac{W \cdot z}{W} = z$

* The height of the object from datum surface.

Kinetic Head:- $\frac{v^2}{2g}$; Pressure head:- $\boxed{\frac{P}{w} = h} = y$

Total Head in open channel flow

H = Potential head + Kinetic head + Pressure head

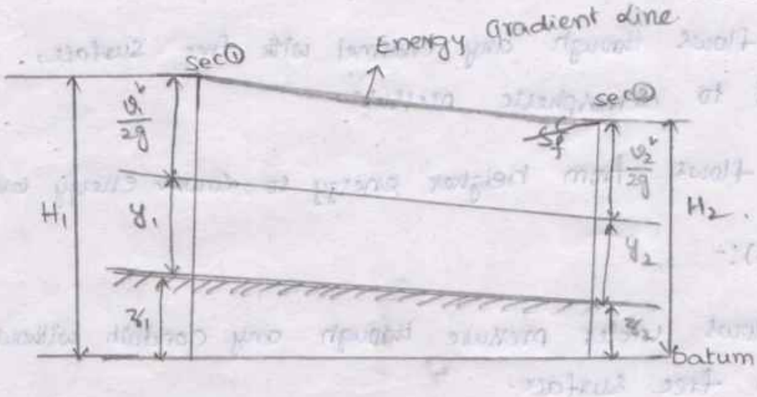
$$H = z + \frac{v^2}{2g} + \frac{P}{w}$$

$$\boxed{H = z + y\cos\theta + \alpha\frac{v^2}{2g}}$$

if θ is small & α is negligible

$$* \boxed{H = z + y + \frac{v^2}{2g}}$$

Energy Gradient Line :-



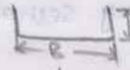
S_f : Energy slope

$$H_1 = \frac{v_1^2}{2g} + y_1 + z_1 \qquad H_2 = \frac{v_2^2}{2g} + y_2 + z_2$$

* Difference b/w Energy Line [Total head line] and piezometric head line represents Velocity head

Types of open channel:-

Based on Shape:-

- Rectangular  

- Trapezoidal 
- Triangular 
- Circular 
- Parabolic 
- Compound channel. 

Based on slope & cross section:-

- **Prismatic** :- Both slope and cross section doesn't change.

$$A_1 = A_2 = A_3 \ldots\ldots$$
$$S_1 = S_2 = S_3 \ldots$$

Ex:- All manmade channels.

- **Non prismatic channel**:-

Have varying c/s and slope.

Ex:- All Natural channels.

Based on boundary characteristics:-

- Rigid boundary channel:- Material on bed & sides of a channel is not movable i.e, Not deformable.

Ex:- Concrete.

- Mobile boundary channel:- Material is easily movable due to flow of water.

Classification of flows:-

1. Temporal : Based on time.

- Steady flow :- flow parameters at any section doesn't varies with time.

$$\frac{\partial V}{\partial t} = 0 \; ; \; \frac{\partial y}{\partial t} = 0 \; ; \; \frac{\partial Q}{\partial t} = 0$$

- Unsteady flow :- varies with time

$$\frac{\partial V}{\partial t} \neq 0 \; ; \; \frac{\partial y}{\partial t} \neq 0 \; ; \; \frac{\partial Q}{\partial t} \neq 0$$

2. Spacial : Based on space :-

- Uniform flow :- flow parameters are constant along the length of the channel.

$$\frac{\partial V}{\partial x} = 0 \; , \; \frac{\partial y}{\partial x} = 0 \; , \; \frac{\partial Q}{\partial x} = 0$$

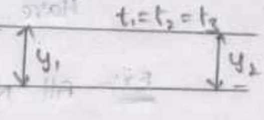
- Non Uniform flow:-

flow parameters changes with space.

$$\frac{\partial V}{\partial x} \neq 0 \; , \; \frac{\partial y}{\partial x} \neq 0 , \; \frac{\partial Q}{\partial x} \neq 0$$

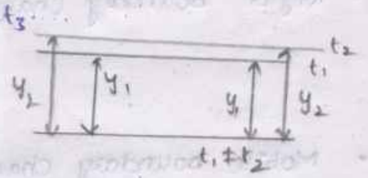
Combination of temporal & spacial flows :-

i) steady uniform :- $y_1 = y_2 = \dots$ $t_1 = t_2 = t_3 = \dots$



ii) Steady Non uniform :- $y_1 \neq y_2 \neq \dots$ $t_1 = t_2 = t_3$

iii) Unsteady uniform flow :- $y_1 = y_2 = \dots$ $t_1 \neq t_2 \neq t_3 \dots$



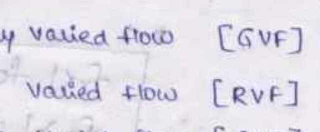
iv) Unsteady Non uniform flow : $y_1 \neq y_2 \neq y_3$
$t_1 \neq t_2 \neq t_3$

Non-Uniform flow:-

- Gradually varied flow [GVF]
- Rapidly varied flow [RVF]
- Spatially varied flow. [SVF]

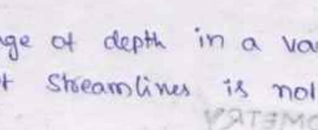
GVF:-

- If the change of depth in a varid flow is gradual so that the curvature of Streamlines is not excessive.



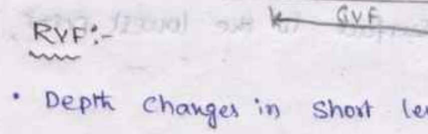
RVF:-

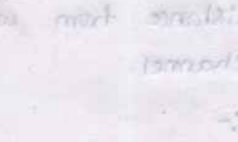
- Depth changes in short length.



SVF:-

- If some flow is added or subtracted results Varied flow, such flow is called as Spatially varied flow.





3. Based on Turbulance:-

- Laminar flow
- Transition
- Turbulent flow

Renold's Number $Re = \frac{\rho V L}{\mu}$ → L: Characteristic Length

$R_e < 2000 \to$ Laminar

$2000 < R_e < 4000 \to$ Transition

$R_e > 4000 \to$ Turbulent

IN OCF:-

$\boxed{R = \frac{D}{4}}$

$R_e < 500 \to$ Laminar

$500 < R_e < 1000 \to$ Transition

$R_e > 1000 \to$ Turbulent.

Hydraulic depth is 4 times of Hydraulic Radius.

HYDRAULIC DEPTH IS 4 TIMES OF HYDRAULIC RADIUS.

4. Based on concept of critical flow:-

froude Number $f_r = \frac{V}{\sqrt{gD}}$

$f_r = 1 \rightarrow$ critical flow.

$f_r > 1 \rightarrow$ super critical flow.

$f_r < 1 \rightarrow$ Sub critical flow.

$f_r < 1$ for Dunes
$f_r > 1$ for Anti Dunes

CHANNEL GEOMETRY

Depth of flow (y):-

- Vertical distance from water surface to the lowest point of the channel.

Top Width (T):-

- Width of the section at water surface.

Bed width (B):-

- Width of the Bed.

Wetted perimeter (P):-

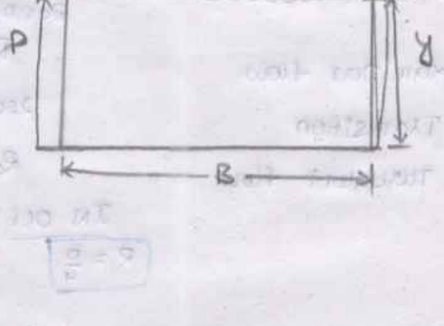
- Length of the line of intersection of the channel wetted surface with the C/S plane normal to the direction of flow.

Hydraulic Radius (R):-

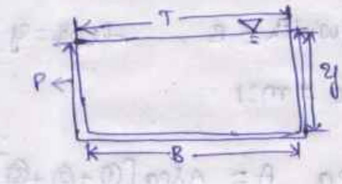
- $\frac{\text{Area}(A)}{\text{Perimeter}(P)}$

Hydraulic Depth (D):-

- $\frac{\text{Area}}{\text{Top width}}$ $\frac{A}{T}$



for Rectangular channel :-



- Bed width $= B$, depth $= y$
- Area $\perp^{ler}$ to direction of flow $= A$

$$* \boxed{A = By}$$

- Perimeter $P = B + y + y$

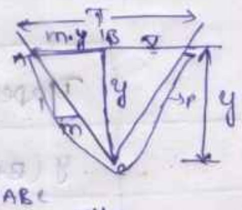
$$* \boxed{P = B + 2y}$$

- Hydraulic Radius $R = \frac{A}{P} = \frac{By}{B+2y}$
- Top width $T = B$
- Hydraulic depth $D = \frac{A}{T} = \frac{By}{B} = y$

for wide rectangular channel $\Rightarrow B >> y$

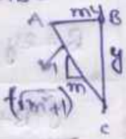
so, $\boxed{R = y}$

for Triangular channel :-



- Slope $= m:1$ depth $= y$
- Area $A = \frac{1}{2}(my)y + \frac{1}{2}(my)(y)$

$$\boxed{A = my^2}$$



- Perimeter $P = \left(y\sqrt{m^2+1}\right) + \left(y\sqrt{m^2+1}\right)$

By Pythagorus

$AC^2 = AB^2 + BC^2$

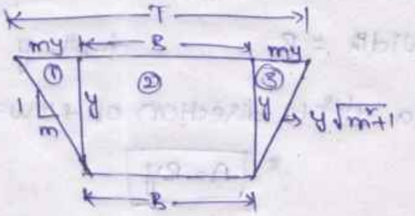
$AC = \sqrt{(my)^2 + y^2}$

$(=) \; y\sqrt{m^2+1}$

$$\boxed{P = 2y\sqrt{m^2+1}}$$

- Hydraulic Radius $R = \frac{A}{P} = \frac{my}{2\sqrt{m^2+1}}$
- Top width $T = my + my = 2my$
- Hydraulic Depth $D = \frac{A}{T} = \frac{my^2}{2my} = \frac{y}{2}$

for trapezoidal channel:-



- Bedwidth = B, depth = y
- Slope = m:1
- Area $A = \text{Area}[①+②+③]$

$$= \frac{1}{2}my^2 + By + \frac{1}{2}my^2$$

$$A = By + my^2$$

$$\boxed{A = y[B+my]}$$

- Perimeter $P = y\sqrt{m^2+1} + B + y\sqrt{m^2+1}$

$$\boxed{P = B + 2y\sqrt{m^2+1}}$$

- Top width $T = my + B + my = B + 2my$
- Hydraulic Radius $R = \dfrac{A}{P} = \dfrac{y(B+my)}{B+2y\sqrt{m^2+1}}$
- Hydraulic Depth $D = \dfrac{A}{T} = \dfrac{y(B+my)}{B+2my}$

Imp
**

	Trapezoidal	Rectangular channel
Area (A)	$y(B+my)$	By
Perimeter (P)	$B+2y\sqrt{m^2+1}$	$B+2y$
Hydraulic Radius (R) = $\frac{A}{P}$	$\dfrac{y(B+my)}{B+2y\sqrt{m^2+1}}$	$\dfrac{By}{B+2y}$
Hydraulic Depth (D) = $\frac{A}{T}$	$\dfrac{y(B+my)}{B+2my}$	y
Top Width (T)	$B+2my$	B

Velocity Distribution

* velocity distribution depends on
  - Shape of the section.
  - Roughness of the channel.
  - Presence of bends.

* Maximum velocity occurs at a depth of $0.05y$ to $0.25y$.

* Energy coefficient = Kinetic Energy correction factor $(\alpha) = \frac{\int v^3 dA}{\bar{v}^3 A}$

* momentum coefficient = momentum correction factor $(\beta) = \frac{\int v^2 dA}{\bar{v}^2 A}$.

*** $\boxed{\alpha > \beta > 1}$ α, β values are always > 1.

* If velocity distribution is uniform $\alpha = \beta = 1$.

UNIFORM FLOW

* A flow is said to be uniform when its flow properties remains constant with respect to distance.

Ex:- Prismatic channel.

Chezy's equation:-

* $$\boxed{V = C\sqrt{R S_0}}$$ where $C = \sqrt{\frac{g}{\rho k}} = \sqrt{\frac{g}{k}}$

S_0 = slope

R = Hydraulic Radius.

* Dimensions for chezy constant $C = L^{1/2} T^{-1}$.

Darcy - weisbach equation:

$$\boxed{h_f = f \frac{L}{D} \frac{v^2}{2g}}.$$

h_f = head loss due to friction

f = friction factor

L = Length

D : Diameter

V : velocity.

for open channels $\boxed{D = 4R}$

$$\boxed{h_f = \frac{fLv^2}{8gR}}$$

$$V = \sqrt{\frac{8g}{f}} \sqrt{R} \sqrt{h_f/L} \rightarrow ①$$

eqn① comparing with Chezy's eqn.

$$* \boxed{C = \sqrt{\frac{8g}{f}}}$$

$$\boxed{C = \frac{87}{1 + \frac{M}{\sqrt{R}}}} \rightarrow \text{Bazin's eqn.}$$

· Manning's formula:-

$$**** \boxed{V = \frac{1}{n} R^{2/3} S^{1/2}}$$

* n = Roughness coefficient $[L^{-1/3} T]$

Comparing with Chezy's eqn,

$$* \boxed{C = \frac{R^{1/6}}{n}}$$

$$* \boxed{Q = VA = \frac{1}{n} A R^{2/3} S^{1/2}}$$

$$\therefore \sqrt{\frac{8g}{f}} = \frac{R^{1/6}}{n}$$

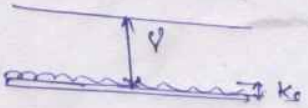
$$* \boxed{f = \frac{n^2}{R^{1/3}} 8g}$$

Bed Shear Stress:-

$$\boxed{\tau = WRS = \rho g R S}$$

$\therefore$ τ = Shear Stress
W = weight of fluid
R = Hydraulic Radius
S = Bed Slope.

$\therefore$ Shear velocity $u_* = \sqrt{\dfrac{\tau_0}{\rho}}$



Sublayer depth

$$\boxed{\delta' = \frac{11.6\,\nu}{u_*}}$$

*

$\dfrac{K_s}{\delta'} \leq 0.25 \rightarrow$ Laminar $\rightarrow$ Smooth Boundary

$0.25 \leq \dfrac{K_s}{\delta'} < 6 \rightarrow$ Transition $\rightarrow$ Rough Surface.

$\dfrac{K_s}{\delta'} \geq 6 \rightarrow$ Rough Surface.

Logarithmic Velocity Distribution equation:-

$$\frac{u}{u_*} = 5.75 \log_{10}\left(\frac{y}{c}\right)$$

- for Smooth Boundary : $c = \dfrac{\nu}{9u_*}$

$$\boxed{\frac{u}{u_*} = 5.75 \log_{10}\frac{u_* y}{\nu} + 5.5}$$

- for Rough Boundary : $c = \dfrac{K_s}{30}$

*
$$\boxed{\frac{u}{u_*} = 5.75 \log_{10}\left(\frac{y}{K_s}\right) + 8.5}$$

Specific Energy:-

$$\text{Total Head } H = z + y\cos\theta + \alpha\frac{v^2}{2g}$$

- If the datum coincides with the channel bed at the section, the resulting expression is known as Specific Energy.

$$* \quad \boxed{E = y\cos\theta + \alpha\frac{v^2}{2g}}$$

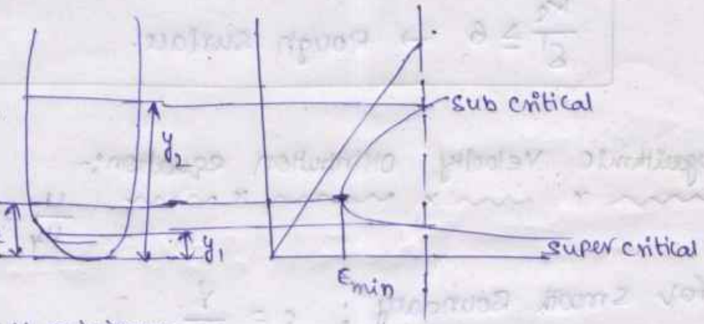
when $\cos\theta = 1$, $\alpha = 1$

$$\boxed{E = y + \frac{v^2}{2g}}$$

Critical & Alternate Depths:-

y_c = critical depth

y_1, y_2 = Alternate depths.



at critical Depth E is minimum,

$$\therefore \frac{dE}{dy} = 0$$

$$E = y + \frac{v^2}{2g} \qquad v = \frac{Q}{A}$$

$$E = y + \frac{Q^2}{2gA^2}$$

$$0 \Rightarrow 1 + \frac{Q^2}{2g}(-2)A^{-3}\frac{dA}{dy} \qquad \left[\because \frac{dA}{dy} = T\right] \rightarrow$$

$$dA = T.dy$$

$$\boxed{T = \frac{dA}{dy}}$$

$$* \quad \boxed{\frac{Q^2 T}{gA^3} = 1}$$

- The 2 depths having same Specific Energy is called as Alternate depths.
- The depth where S.E is min is called as Critical depth.

** Conditions for critical depth :-

1. Specific Energy is minimum for a given Discharge (Q).
2. Q is maximum for SE.
3. Velocity head is $\frac{1}{2}$ hydraulic depth.
4. froud number is unity.
5. spesific force is minimum for a given discharge Q.

$$\therefore \frac{Q^2 T}{gA^3} = 1$$

$$\Rightarrow \frac{v^2 T}{gA} = 1 \Rightarrow \frac{v^2}{g\left(\frac{A}{T}\right)} = 1 \Rightarrow \frac{v^2}{gD} = 1$$

4. $\boxed{\Rightarrow F_r = \frac{v}{\sqrt{gD}} = 1}$

$$\frac{v^2}{g} = D \Rightarrow \frac{v^2}{2g} = \frac{D}{2}$$

3. $\boxed{KE = \frac{1}{2} D}$

* E & Y Relation_ship for critical condition

for Rectangular channel: $E_{min} = \frac{3}{2} y_c$

for Parabolic channel : $E_{min} = \frac{4}{3} y_c$

for Triangular channel : $E_{min} = \frac{5}{4} y_c$

* Imp formulae:-

for Rectangular channel :-

$$\therefore \boxed{y_c^3 = \frac{q^2}{g}} \qquad \boxed{y_c^3 = \frac{2 y_1^2 y_2^2}{y_1 + y_2}}$$

$$\boxed{E = \frac{y_1^2 + y_1 y_2 + y_2^2}{y_1 + y_2}}$$

for triangle

$$y_c = \left[\frac{2Q^2}{g m^2}\right]^{1/5}$$

Specific force:-

$$\boxed{F = A\bar{z} + \frac{Q^2}{gA}}$$

- Specific force is a function of depth (y).

Section factor:-

$\therefore \frac{Q^2 T}{gA^3} = 1$ $\quad \because \boxed{\frac{A}{T} = D}$

$$\frac{Q^2}{g} = A^2\left(\frac{A}{T}\right)$$

$$\frac{Q^2}{g} = A^2 D$$

$$\boxed{\frac{Q}{\sqrt{g}} = A\sqrt{D} = Z}$$

$Z \rightarrow$ section factor.

⇒ for an ideal channel with alluvial bed. the chanel flow sequence will be Ripples - Dunes → Transition - Antidunes.

=) Bed Load = contact load + Saltion load

MOST ECONOMICAL SECTION

→ Also called as Hydraulically Efficient channel.

Rectangle Channel :-

Area $A = By$ $\quad \left[B = A/y\right]$

Perimeter $P = B + 2y$

for 'p' to be minimum $\frac{dP}{dy} = 0$

$$\frac{dP}{dy} = \frac{d}{dy}\left[\frac{A}{y} + 2y\right]$$

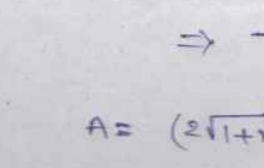
$$0 = \frac{-A}{y^2} + 2$$

$$\boxed{A = 2y^2} \qquad \because A = By \qquad \because R = \frac{A}{P} = \frac{By}{B+2y}$$

$$By = 2y^2 \qquad\qquad = \frac{(2y)y}{2y+2y}$$

$$* \boxed{B = 2y} \quad * \boxed{y = B/2} \ \boxed{R = y/2} \qquad = \frac{2y^2}{4y} = y/2$$

Trapezoidal channel :-



Area $A = (B + my)y$

Perimeter $P = B + 2y\sqrt{1+m^2}$

$$\frac{dP}{dy} = 0 \Rightarrow \frac{d}{dy}\left[\left(\frac{A}{y} - my\right) + 2y\sqrt{1+m^2}\right]$$

$$\Rightarrow -\frac{A}{y^2} - m + 2\sqrt{1+m^2}$$

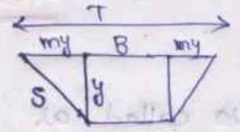
$$A = \left(2\sqrt{1+m^2} - m\right)y^2$$

$$(B + my)y = \left(2\sqrt{1+m^2} - m\right)y^2$$

$$B + 2my = 2y\sqrt{1+m^2} \rightarrow ①$$

* $\boxed{T = 2S}$

∵ Top Width is 2 times of sloping side.



$T = B + 2my$

S = Sloping side

$S = y\sqrt{1+m^2}$ ↳ By pythagorus.

$$R = \frac{A}{y} = \frac{(B+my)\,y}{B + 2y\sqrt{1+m^2}}$$

∵ $B = 2y\sqrt{1+m^2} - 2my$ from eq①.

$$= \frac{\left(2y\sqrt{1+m^2} - 2my + my\right)y}{\left(2y\sqrt{1+m^2} - 2my + 2y\sqrt{1+m^2}\right)}$$

* $\boxed{R = y/2}$

$$\therefore \frac{dP}{dm} = -y + 2y\frac{1}{2\sqrt{1+m^2}} \cdot 2m = 0$$

$$-y + \frac{2my}{\sqrt{1+m^2}} = 0$$

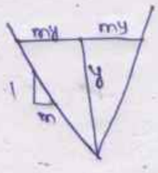
$$3m^2 = 1$$

$\tan\alpha = \frac{1}{m} = \sqrt{3}$

* $\boxed{m = 1/\sqrt{3}}$ $\boxed{\alpha = 60^\circ}$

∵ Hydraulically efficient trapezoidal section is half of a regular hexagon.

Triangular Section:-



Area $A = my^2$ $\quad$ $y = \sqrt{A/m}$

Perimeter $P = 2y\sqrt{1+m^2}$

$$\frac{dP}{dm} = 0 \Rightarrow \frac{d}{dm}\left[2\sqrt{\frac{A}{m}}\left[\sqrt{1+m^2}\right]\right] = 0$$

$$\Rightarrow \frac{d}{dm}\left[2\sqrt{A}\left[\sqrt{\frac{1}{m}+m}\right]\right]$$ By solving this

$$m^2 \Rightarrow 1$$

* $\boxed{m=1}$ $\quad$ $\boxed{\theta = 45^\circ}$ $\quad$ $\boxed{R = \frac{y}{2\sqrt{2}}}$

Circular Section:-

→ Semi circular section is most economical.

Perimeter $P = 2\theta r$

→ for maximum Discharge 'Q' * $\boxed{y = 0.93d}$ $\boxed{R = 0.29\,d}$ $\quad$ d= dia

→ for maximum mean velocity 'v' * $\boxed{y = 0.81d}$ $\boxed{R = 0.30d}$

Simple prblms on Introduction & UNIFORM FLOW

1) A flow of 5 m^3/s is passing at a depth of 1.5m through a rectangular channel of 2.5m width. $\alpha = 1.20$ find Specific Energy!

Sol:-

$$\boxed{E = y + \alpha \frac{v^2}{2g}}$$

$y = 1.5\,m \quad b = 2.5\,m$

$Q = 5\,m^3/s$

$$v = \frac{Q}{A} = \frac{5}{(1.5)(2.5)} = 1.33\,m/s$$

$$E = 1.5 + \left[1.20 \times \frac{1.33^2}{2(9.81)}\right]$$

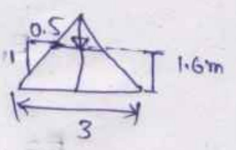
$E = \underline{1.608\,m}$.

2) Water is flowing at a critical depth at a section in a Δ Shaped channel, with side slope of 0.5H : 1V. If critical depth is 1.6m, estimate the discharge in channel and Specific energy at critical depth section. B = 3m.

Sol:-

~~m = 0.5~~ $\quad m = -0.5 \quad B = 3m$

$y_c = 1.6\,m$



$Q = ? \quad E = ?$

if $\vee$ $m = +ve$

it is $\wedge$ $m = -ve = -0.5$

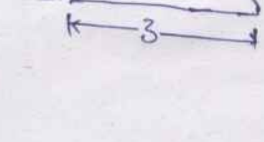
at critical section $\boxed{\frac{Q^2 T}{g A^3} = 1}$

$T = B + 2my = 3 + 2(-0.5)(1.6) = \underline{1.40\,m}$

Area of trapezoidal.

$A = \frac{1}{2}(a+b)h$

$= \frac{1}{2}(3+1.4)(1.6) = \underline{3.52\,m^2}$



$g = 9.81 \qquad \frac{Q^2 (1.4)}{9.81(3.52)^3} = 1$

$$Q^2 = \frac{9.8\,(3.52)^3}{1.4} = 305.2$$

$$Q = \sqrt{305.2} = \underline{17.4\ m^3/s}$$

ii) Specific Energy $E = y + \frac{v^2}{2g}$

$$V = \frac{Q}{A} = \frac{17.4}{3.52} = 4.96\ m/s$$

$$E = 1.60 + \frac{(4.96)^2}{2(9.81)} = \underline{2.85\ m}$$

3) A Trapezoidal channel has a bottom width of 6m and side slopes of 1:1. The depth of flow is 1.5m at a discharge of $15\ m^3/s$. Determine the specific energy and alternate depth?

Sol:- $B = 6m$; $m = 1$; $y_1 = 1.5m$, $Q = 15\ m^3/s$

$$A = (B+my)y$$

$$E = y + \frac{v^2}{2g} \qquad v = Q/A = \frac{15}{(6+1.5)1.5} = \underline{1.33\ m/s}$$

$$= 1.5 + \frac{1.33^2}{2(9.81)}$$

$$E_1 = \underline{1.59\ m}$$

$$E = \frac{y_1^2 + y_1 y_2 + y_2^2}{y_1 + y_2}$$

$$1.59(1.5 + y_2) = 1.5^2 + 1.5\,y_2 + y_2^2$$

$$\Rightarrow y_2^2 + 1.5\,y_2 - 1.59\,y_2 + 1.5^2 - (1.59 \times 1.5) = 0$$

$$\Rightarrow y_2^2 - 0.09\,y_2 - 0.135 = 0$$

$$y_2 = \underline{0.415\ m}.$$

4) In a rectangular channel, the alternate depths are 1m & 2m respectively. The specific energy in m is? (Pr. 2.3) in Subramanya

Sol) $y_1 = 1m \qquad y_2 = 2m$

$$E = \frac{y_1^2 + y_1 y_2 + y_2^2}{y_1 + y_2}$$

$$= \frac{1+2+4}{1+2} = \underline{\underline{2.33m}}.$$

5) A rectangular channel carries a certain flow for which the alternate depths are found to be 3m & 1m. The critical depth in m will be? (Pr: 2.4)

Sol:- $y_1 = 3m, \quad y_2 = 1m$

$$y_c^3 = \frac{2y_1^2 y_2^2}{y_1 + y_2}$$

$$y_c^3 = \frac{2(3)^2(1)^2}{3+1}$$

$$y_c = \left(\frac{18}{4}\right)^{1/3} = \underline{1.65m}$$

6) The froude Number of a flow in a rectangular channel is 0.73. If depth of flow is 1.5m the specific energy in m? (Pr: 2.19)

Sol:- $fr = 0.73 \qquad y = 1.5m$

$$fr = \frac{V}{\sqrt{gD}} = \frac{V}{\sqrt{gy}}$$

$$0.73 = \frac{V}{\sqrt{9.8(1.5)}} \Rightarrow V = 2.8 \text{ m/s}.$$

$$E = y + V^2/2g \Rightarrow 1.5 + \frac{2.8^2}{2(9.81)}$$

$$E = 1.89 \simeq \underline{\underline{1.9m}}$$

7) A trapezoidal channel of bed width of 3.5 m and side slope of 1.5H : 1V carries a flow of 9 m³/s with a depth of 2 m. The froude Number of flow is [P1 2.20]

Sol:- $B = 3.5m$ $m = 1.5$ $Q = 9 m^3/s$ $y = 2m$

$$fr = \frac{V}{\sqrt{gD}}$$

$$V = \frac{Q}{A} = \frac{9}{(B+my)y} = \frac{9}{[3.5+1.5(2)]2} = 0.692 \text{ m/s}$$

$$D = \frac{A}{T} = \frac{(B+my)y}{B+2my} = \frac{13}{9.5} = 1.36$$

$$fr = \frac{V}{\sqrt{gD}} = \frac{0.692}{\sqrt{9.8(1.36)}} = \underline{\underline{0.189}}.$$

8) A Trapezoidal channel is 10m wide and has a side slope of 1.5H : 1V. The bed slope is 0.0003. The channel is lined with smooth concrete of n = 0.012. Compute mean velocity and discharge for a depth of flow of 3m.

Sol:- $y = 3m$, $B = 10m$, $m = 1.5$, $S = 0.0003$, $n = 0.012$

$$V = \frac{1}{n} R^{2/3} S^{1/2}$$

$$R = \frac{A}{P} = \frac{(B+my)y}{B+2y\sqrt{m^2+1}}$$

$$= \frac{[10 + (1.5)(3)]3}{10 + 2(3)\sqrt{1.5^2+1}}$$

$$= \frac{43.5}{20.8} = 2.0m$$

$$V = \frac{1}{0.012}(2.0)^{2/3}(0.0003)^{1/2}$$

$$V = 2.29 \text{ m/s}$$

$$Q = VA = 2.29(43.5) = \underline{99.2} \text{ m}^3/s$$

(9) A rectangular channel of longitudinal slope 0.002 has a width of 0.8 m and carries an oil [rel. density = 0.80] at a depth of 0.4 m in uniform flow mode. The avg shear stress on the channel boundary is - ?

Sol:- $S = 0.002$ $B = 0.8\,m$ $\rho = 0.8$ $y = 0.4\,m$

$\tau = \rho g R S$

$R = \frac{A}{P} = \frac{By}{B+2y}$

$= (0.8)(9.8)(0.2)(0.002)$

$= \frac{(0.8)(0.4)}{0.8 + (0.4)2}$

$= \underline{3.14 \times 10^{-3}}$ Pa

$= 0.2$

(10) A triangular channel with a side slope of 1.5H : 1V is laid on slope of 0.005. The shear stress is N/m² on the boundary for a depth of flow of 1.5 m is - ?

Sol:- $m = 1.5$ $S = 0.005$ $y = 1.5\,m$ $\rho = 1000$ for water

$\tau = \rho g R s$

$R = \frac{A}{P} = \frac{my^2}{2y\sqrt{m^2+1}}$

$= 1000(9.8)(0.624)(0.005)$

$= \underline{30.56\ N/m^2}$.

$= \frac{1.5(1.5)^2}{2(1.5)\sqrt{1.5^2+1}}$

$= \underline{0.624}$

(11) An open channel carries water with a velocity of 0.605 m/s. If avg bed shear stress is 1 N/m², the chezy coefficient 'C' is?

Sol:- $V = 0.605\ m/s$ $\tau = 1\,N/m^2$

$V = C\sqrt{RS}$

$\tau = \rho g R S$

$RS = 1/9.8(1000) = 1.02 \times 10^{-4}$

$0.605 = C\sqrt{1.02 \times 10^{-4}}$

$C = 59.8 \approx \underline{60}$.

(12) In a wide rectangular channel uniform flow is taking place at a depth of 1.2m, the velocity is found to be 1.5 m/s. If a change in discharge causes a uniform flow at a depth of 0.88 m, velocity of flow would be?

Sol:- $y_1 = 1.2$ m $\quad v_1 = 1.5$ m/s

$y_2 = 0.88$ m $\quad v_2 = ?$

Relation b/w v & y

In wide rectangular channel $B >> y$ so $\boxed{R = y}$

$$V = \frac{1}{n} R^{2/3} S^{1/2}$$

$$= \frac{1}{n} y^{2/3} S^{1/2}$$

$$\boxed{V \propto y^{2/3}}$$

$$\frac{V_1}{V_2} = \frac{y_1^{2/3}}{y_2^{2/3}}$$

$$\frac{1.5}{v_2} = \frac{1.2^{2/3}}{0.88^{2/3}}$$

$$V_2 = \frac{1.5\,(0.88^{2/3})}{1.2^{2/3}} = \underline{1.22 \text{ m/s}}.$$

(13) A rectangular channel $B = 4$m, $n = 0.015$ is to carry a uniform discharge at a depth of 1m and $fr = 0.5$. The required bottom slope is?

Sol:- $b = 4$m $\quad n = 0.015 \quad y = 1$m $\quad f = 0.5$

$$fr = \frac{v}{\sqrt{gD}} \Rightarrow 0.5 = \frac{v}{\sqrt{9.8(1)}}$$

$$R = \frac{A}{P} = \frac{By}{B+2y} = 4/6 = 0.66$$

$$v = 1.56$$

$$v = \frac{1}{n} R^{2/3} S^{1/2}$$

$$1.56 = \frac{1}{0.015} (0.66)^{2/3} S^{1/2}$$

$$S = (0.030)^2 = \underline{0.0009}.$$

(14) If the $y_1 = 0.5$, $y_2 = 3m$ for a rectangular channel, $y_c = ?$

Sol:- $$y_c^3 = \frac{2y_1^2 y_2^2}{y_1+y_2} = \left[\frac{2(0.5)^2(3)^2}{3+0.5}\right]^{1/3} = \underline{1.08\ m}$$

(15) for a uniform flow with a depth of 0.6 m and Froude number 2. In a rectangular channel, the specific energy will be,

Sol:- $y = 0.6m$ $\quad f = 2$ $\quad f = \frac{V}{\sqrt{gD}}$ $\quad D = y$ in rect channel

$E = y + \frac{v^2}{2g}$ $\qquad 2 = \frac{V}{\sqrt{9.8(0.6)}}$

$= 0.6 + \frac{4.849^2}{2(9.81)}$ $\qquad V = 4.849\ m/s.$

$= \underline{1.79\ m}$

(16) A trapezoidal channel of bed width 3m and side slopes 2H : 1V is laid on a slope of 0.0025. The avg shear stress in N/m^2, $y = 2m$.

Sol:- $B = 3m$ $\quad m = 2$ $\quad y = 2m$ $\quad S = 0.0025$

$\tau = \rho g R S$ $\qquad R = \frac{A}{P} = \frac{(B+my)y}{B+2y\sqrt{1+m^2}}$

$\tau = 1000(9.8)(1.172)(0.0025)$ $\qquad = \frac{[3+(2)(2)]2}{3+2(2)\sqrt{1+2^2}}$

$= \underline{28.7\ N/m^2}$ $\qquad R = 1.172$

(17) for a hydraulically efficient rectangular channel of bed width 4m the depth of flow is ___.

Sol for hydraulically efficient rec channel $y = B/2$

$y = \frac{4}{2} = \underline{2m}$.

(18) What is the normal depth in a wide rectangular channel carrying 0.5 m³/s discharge at a bed slope of 0.0004 and mannings $n = 0.01$? [IES: 2007]

Sol: $Q = 0.5\ m^3/s$ $S = 0.0004$ $n = 0.01$ $y = ?$

∴ For wide rectangular channel $A = By \approx y$

$$R = \frac{A}{P} \approx y$$

$$Q = \frac{1}{n} A R^{2/3} S^{1/2}$$

$$0.5 = \frac{1}{0.01} (y)(y)^{2/3} (0.0004)^{1/2}$$

$$y^{5/3} = \frac{0.5(0.01)}{(0.0004)^{1/2}} \Rightarrow y = (0.25)^{3/5}$$

$$y = \cancel{0.39m}\ \underline{\underline{0.43\ m}}$$

(19) The critical depth of water flowing through a rectangular channel of width 5m when discharge is 15 m³/s - [IES: 2003]

Sol:- $B = 5m$ $Q = 15\ m^3/s$

$$y_c^3 = \frac{q^2}{g} = \frac{3^2}{9.8} \qquad q = Q/B = 15/5 = 3$$

$$y_c = (0.92)^{1/3}\ m$$

(20) Flow happens at $y_c = 0.5m$ in a rectangular channel of 4m wide. $Q = ?$

Sol:- $$y_c^3 = \frac{(Q/B)^2}{g}$$

$$(0.5)^3 = \frac{Q^2/4^2}{9.8} \qquad Q = \underline{\underline{4.42\ m}}$$

(21) A triangular irrigation lined canal carries a discharge of $25\ m^3/s$ at bed slope $1/6000$. If the side slopes of the canal are 1:1 and $n = 0.018$. the critical depth = ? [GATE: 2005]

Sol:- $Q = 25\ m^3/s$ $\quad S = \frac{1}{6000}$ $\quad n = 0.018$ $\quad m = 1$

$$R = \frac{A}{P} = \frac{my^2}{2y\sqrt{1+m^2}} = \frac{y}{2.82}$$

$$Q = \frac{1}{n} A R^{2/3} S^{1/2}$$

$$25 = \frac{1}{0.018}(my^2)\left(\frac{y}{2.82}\right)^{2/3} S^{1/2}$$

$$25 = y^{2+2/3}(0.359)$$

$$y^{8/3} = 69.5$$

$$\underline{y = 4.91m}$$

(22) The open channel of constant width has a sudden drop in level of 20 cm. The specific energy at up stream of drop is 1.2m. The specific energy at down stream will be?

Sol:- B = constant

Given $\Delta y = y_2 - y_1 = 0.2\ m$

$E_1 = 1.2m$

so $y_1 = 1.2m$

$E = y + \cancel{\frac{v^2}{2g}}$ → Negligible. Both are same because width is constant.

$$\boxed{\begin{array}{l} E_1 = y_1 \\ E_2 = y_2 \end{array}}$$

$$y_2 = 0.2 + 1.2 = 1.4m$$

$$\underline{\underline{E_2 = 1.4m}}$$

(23) In a rectangular channel the depth of flow is 1.6m and specific energy at that section is 2.8m. The flow is ___.

Sol:- $y = 1.6\,m$ $\quad E = 2.8\,m$

$$E = y_1 + \frac{v_1^2}{2g}$$

$$2.8 = 1.6 + \frac{v^2}{2(9.81)}$$

$$v^2 = (1.2)(2)(9.81)$$

$$v = \underline{4.85}\ m/s$$

$$fr = \frac{v}{\sqrt{gD}} = \frac{v}{\sqrt{gy}} = \frac{4.85}{\sqrt{9.8(1.6)}} = 1.2$$

$fr < 1 \rightarrow$ sub critical flow

$fr = 1 \rightarrow$ critical flow

$\boxed{fr > 1 \rightarrow \text{super critical flow}}$ ✓.

24) A rectangular channel of 4m width conveys water at $8\,m^3/s$ under critical condition. The specific Energy = ?

Sol:- $B = 4m$ $\quad Q = 8\,m^3/s$

$$E_c = y_c + \frac{v^2}{2g}$$

$$y_c^3 = \frac{(Q/B)^2}{g}$$

$$y_c = \left[\frac{(8/4)^2}{9.81}\right]^{1/3} = 0.74m$$

$$E_c = 0.74 + \frac{2.69^2}{2(9.81)}$$

$$v = Q/A = 8/_{4(0.7)} = 2.69\ m/s$$

$$= \underline{1.112\ m}$$

(25) Two rectangular channels M & N have same longitudinal slope, roughness coefficient n and depth of flow. The side slope of M channel is 1H:1V, N channel is 2H:1V. The ratio of discharge in channel M and N.

Sol:- B, S, n, y are same, only m is different.

$$Q = \frac{1}{n} A R^{2/3} S^{1/2}$$

$A = my^2$ ①

$$R = \frac{A}{P} = \frac{my^2}{2y\sqrt{1+m^2}} = \boxed{\left[\frac{m}{2\sqrt{1+m^2}}\right]} \rightarrow \text{is different for both channels}$$

②

① → m

② → $\frac{m}{2\sqrt{1+m^2}}$

$$Q_M = m\left[\frac{m}{2\sqrt{1+m^2}}\right]^{2/3} = 0.5$$

$$Q_N = 2\left[\frac{2}{2\sqrt{1+4}}\right]^{2/3} = 1.16$$

$$\frac{Q_M}{Q_N} = \frac{0.5}{1.16} = \underline{0.427}$$

(26) A rigid boundary rectangular channel having a bed slope of 1/800 has its width & depth of flow is 2m & 1m respectively. If the flow is uniform value of chezy's constant is 60, the discharge through channel is — ? [IES:2002]

Sol:- $S = 1/800$ $B = 2m$ $y = 1m$ $C = 60$

$Q = VA$ $V = C\sqrt{RS}$ $R = \frac{A}{P} = \frac{By}{B+2y} = 2/4 = 0.5$

$= 60\sqrt{0.5(1/800)}$

$= 1.5$ m/s

$Q = (1.5)(2) = \underline{3 \text{ m}^3/\text{s}}$

(27) A rectangular channel of 3m wide is laid on a slope of 0.0002. When the depth of flow of channel is 1.5m, what is Boundary Shear stress (nearly)? [IES:2007]

Sol:- $B = 3m \quad S = 0.0002 \quad y = 1.5m$

$\tau = \rho g R S \qquad R = \frac{A}{P} = \frac{By}{B+2y} = \frac{3(1.5)}{3+2(1.5)} = 0.75$

$= 1000(9.8)(0.75)(0.0002)$

$= 1.47 \simeq \underline{1.5 \, N/m^2}$

(28) A rectangular open channel carries a discharge of 15 m³/s when the depth of flow is 1.5m and the bed slope is 1:1440. What will be the discharge through the channel at the same depth if slope is 1:1000? [IES : 1998]

Sol:- $Q_1 = 15 \, m^3/s \quad y_1 = 1.5m \quad S_1 = 1/1440$

$Q_2 = ? \quad y_2 = 1.5m \quad S_2 = 1/1000$

$Q_1 = \underbrace{\left[\frac{1}{n} A R^{2/3}\right]}_{\text{same}} S^{1/2}$

$15 = \left[\frac{1}{n} A R^{2/3}\right]\left(\frac{1}{1440}\right)^{1/2}$

$\frac{1}{n} A R^{2/3} = 569.2$

$Q_2 = (569.2)\left(\frac{1}{1000}\right)^{1/2} = \underline{18 \, m^3/s}$

(29) The froude number of flow in a rectangular channel is 0.8. If the depth of flow is 1.5 m, the critical depth is -? [GATE: 2010].

Sol:- $fr = 0.8$ $\quad y = 1.5$

$$0.8 = \frac{V}{\sqrt{gD}}$$

$$v = 0.8\sqrt{9.8(1.5)} = 3.06 \text{ m/s}$$

$$q = vy = (3.06)(1.5) = 4.6$$

$\because V = Q/A$

$$= \frac{qB}{By}$$

$v = q/y$

$$y_c^3 = \frac{q^2}{g} = \frac{4.6^2}{9.81}$$

$$y_c = \left(\frac{4.6^2}{9.81}\right)^{1/3} = \underline{1.29 \text{ m}}$$

(30) A right-angled triangular channel, symmetrical in section about vertical, carries a discharge of $5 \text{ m}^3/\text{s}$ with a velocity of 1.25 m/s. What is approximate value of the $\underline{fr}$ of flow? [IES: 2007]

Sol:- $Q = 5 \text{ m}^3/\text{s}$ $\quad v = 1.25 \text{ m/s}$

right-angle triangle $\rightarrow \theta = 45^0$

$m = 1$

$$fr = \frac{V}{\sqrt{gD}}$$

$D = \frac{A}{T} = y/2$, $y = ?$

$$v = Q/A$$

$$1.25 = \frac{5}{my^2}$$

$$y^2 = 5/1.25 = 4$$

$$\boxed{y = 2 \text{ m}}$$

$$D = 2/2 = 1$$

$$fr = \frac{1.25}{\sqrt{9.8(1)}}$$

$$= 0.399 \approx \underline{0.4}$$

(31) A trapezoidal channel with $B = 3m$ $m = 1$ carries a discharge of $8\ m^3/s$ with $y = 1.5m$, $fr = ?$ [GATE: 2001]

Sol:- $B = 3m$, $m = 1$, $Q = 8\ m^3/s$, $y = 1.5\ m$

$$fr = \frac{V}{\sqrt{gD}}$$

$$V = Q/A = \frac{8}{(B+my)y} = \frac{8}{(3+1.5)1.5} = 1.01\ m/s$$

$$D = \frac{A}{T} = \frac{(B+my)y}{B+2my} = \frac{(3+1.5)1.5}{3+2(1.5)} = 1.05$$

$$fr = \frac{1.01}{\sqrt{9.81(1.05)}} = \underline{0.314}$$

(32) A rectangular channel has a velocity of $1.5\ m/s$. $\tau = 6\ N/m^2$. $c = ?$

Sol:- $\tau = \rho g R S$

$$6 = 9800\,(RS)$$

$$RS = 6/9800 = 6.12 \times 10^{-4}$$

$$V = C\sqrt{RS} \qquad C = \frac{1.5}{\sqrt{6.12 \times 10^{-4}}} = \underline{60.6}$$

(33) A Rectangular channel $B = 2m$, $y = 1m$, $Q = 4\ m^3/s$, Datum is assumed to be 2 m below channel. Total head & specific energy respectively? $z = 2m$

Sol:- $E = y + v^2/2g$ $\qquad v = Q/A = \frac{4}{2} = 2\ m/s$

$$= 1 + \frac{2^2}{2(9.81)} = 1.2\ m$$

Total head = specific energy + datum head

$$= z + y + \frac{v^2}{2g}$$

$$= 2 + 1 + \frac{2^2}{2(9.81)} = \underline{3.2\ m}.$$

(84) A channel carrying water has an avg height of irregularities, projecting from the surface of the boundary 0.15 mm. The type of boundary is — ?. $\tau = 4.9\,N/m^2$ $\nu = 0.01$ stokes.

Sol:- Equivalent Roughness $K_s = 0.15 \times 10^{-3}\,m$

$$\tau = 4.9\,N/m^2$$

$$\nu = 0.01 \times 10^{-4}\,N/m^2$$

Sublayer depth $\delta' = \dfrac{11.6\,\nu}{u_*}$

Shear velocity $u_* = \sqrt{\dfrac{\tau_0}{\rho}} = \sqrt{\dfrac{4.9}{1000}} = 0.07$

$$\delta' = \frac{11.6\,(0.01 \times 10^{-4})}{0.07} = 1.657 \times 10^{-4}\,m$$

$$\frac{K_s}{\delta'} = \frac{0.15 \times 10^{-3}}{1.65 \times 10^{-4}} = 0.9$$

$\dfrac{K_s}{\delta'} \leq 0.25 \rightarrow$ Smooth Boundary

$\boxed{0.25 \leq \dfrac{K_s}{\delta'} \leq 6 \rightarrow \text{Transition Boundary}}$ ✓.

$\dfrac{K_s}{\delta'} \geq 6 \rightarrow$ Rough Boundary

GRADUALLY VARRIED FLOW (GVF)

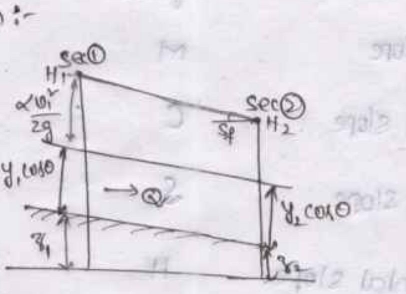
- A steady non-uniform flow in a prismatic channel with gradual changes in its water surface elevation is termed as GVF.

Theory of Gradually varried flow :-

Assumptions:-

- Pressure distribution is assumed to be hydrostatic.
- Head loss is same as that of uniform flow, i.e, Manning's eqn & Chezy's eqn valid.
- The channel is prismatic and $\alpha = 1$.
- Bed slope is small.
- Roughness coefficient $n \rightarrow$ independent of depth.

Derivation of Governing equation :-



Total head $H = z + y\cos\theta + \alpha \frac{v^2}{2g}$

$$\therefore \frac{dH}{dx} = \frac{dz}{dx} + \frac{dy}{dx}\cos\theta + \alpha \frac{d}{dx}\frac{v^2}{2g}$$

* Assumptions $\rightarrow$ $\alpha = 1$, θ = small $\cos\theta = 1$.

- $\frac{dH}{dx}$ represents energy slope. & Energy alway ↓ with direction of flow.

  so, $\frac{dH}{dx} = -S_f$

- $\frac{dy}{dx}$ represents water surface slope relative to bottom of the channel.
- And so, $\frac{dz}{dx} = -S_b$

apply all assumptions.

$$\therefore\ -S_f = -S_b + \frac{dy}{dx} + \frac{d}{dx}\frac{v^2}{2g}$$

$\frac{dA}{dy} = T$

$$S_b - S_f = \frac{dy}{dx}\left[1 - \frac{Q^2T}{gA^3}\right] \qquad \therefore \frac{d}{dx}\left[\frac{v^2}{2g}\right] = -\frac{Q^2T}{gA^3}\frac{dy}{dx}$$

$$\boxed{\frac{dy}{dx} = \frac{S_0 - S_f}{1 - \frac{Q^2T}{gA^3}}}$$ → Dynamic eqn of GVF.

$$\boxed{\frac{dE}{dx} = S_0 - S_f}$$ → Differential energy equation.

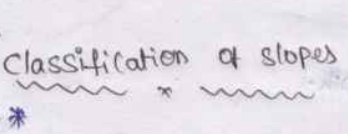
*. Classification of slopes

** y_n = Normal depth, y_c = critical depth

	Symbol	conditions.	
1. Mild slope	M	$y_n > y_c$	$S_b < S_c$
2. Critical slope	C	$y_n = y_c$	$S_b = S_c$
3. Steep slope	S	$y_n < y_c$	$S_b > S_c$
4. Horizontal slope	H	$y_n \to \infty$	$S_b = 0$
5. Adverse slope	A	$S_b < 0$	→ S_b → -ve.

Zones

Zone ①



$y_n > y_c \to$ mild slope.

CDL: Critical depth line

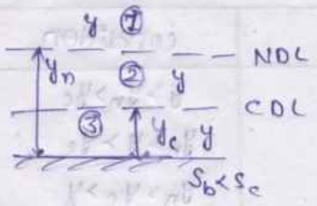
NDL: Normal depth line

Slopes & profiles

1. Mild slope

M1 Profile:- in zone ①

$\boxed{y > y_n > y_c}$



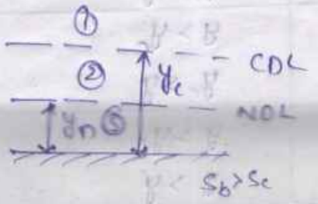
M2 Profile: $y_n > y > y_c$ In zone ②

M3 Profile : In zone ③ $y_n > y_c > y$.

2. Steep slope

S1 Profile :- In zone ①

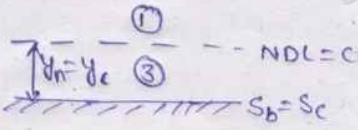
$y > y_c > y_n$



S2 Profile : In zone ② $y_c > y > y_n$

S3 Profile : In zone ③ $y_c > y_n > y$

3. Critical slope

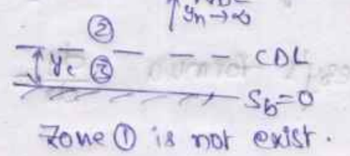


Zone ② is not exist in critical slope.

C1 Profile : $y > y_n$

C3 Profile : $y_n > y$

4. Horizontal slope



Zone ① is not exist.

H_2 Profile: $y > y_c$

H3 Profile: $y_c > y$

5. Adverse slope

A2 Profile : $y > y_c$

A3 Profile : $y_c > y$.

Types & Characteristics of GVF profiles

	Profile	Condition	Characteristic
MILD	M1	$y > y_n > y_c$	Rising profile $\frac{dy}{dx} = +ve$
	M2	$y_n > y > y_c$	falling profile $-ve$
	M3	$y_n > y_c > y$	Rising profile $+ve$
STEEP	S1	$y > y_c > y_n$	Rising
	S2	$y_c > y > y_n$	falling
	S3	$y_c > y_n > y$	Rising
CRITICAL	C1	$y > y_n$ $\quad y_n = y_c$	Rising
	C3	$y_n > y$	Rising
HORIZONTAL	H_2	$y > y_c$	falling
	H_3	$y_c > y$	Rising
ADVERSE	A_2	$y > y_c$	falling
	A_3	$y_c > y$	Rising

*: Total profiles are $= 3+3+2+2+2 = 12$

*. Not existing profiles $\Rightarrow C_2, H_1, A_1 \Rightarrow 3$

* Chezy's formula $\frac{dy}{dx} = S_0 \left[\frac{1-\left(\frac{y_n}{y}\right)^3}{1-\left(\frac{y_c}{y}\right)^3} \right]$

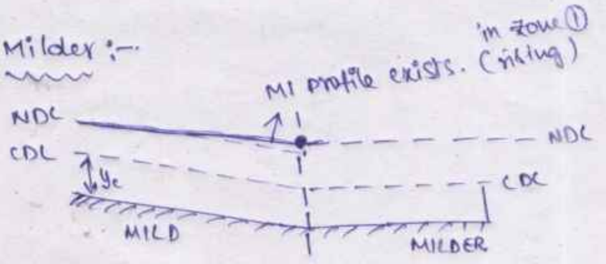
* All ① & ③ profiles are Rising profiles, M1, M3, S1, S3, C1, C3, H3, A3 $\rightarrow$ Rising profiles, $\frac{dy}{dx} = +ve$

* $\frac{dy}{dx} \rightarrow -ve$ for M2, A2, H2, S2 profiles.
↳ falling profiles.

* At transitional depth $\rightarrow$ the slope of GVF profile is horizontal.

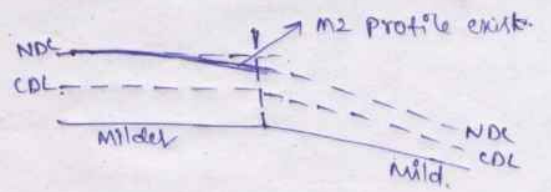
Analysis of flow profile:-

- Mild - Milder:-

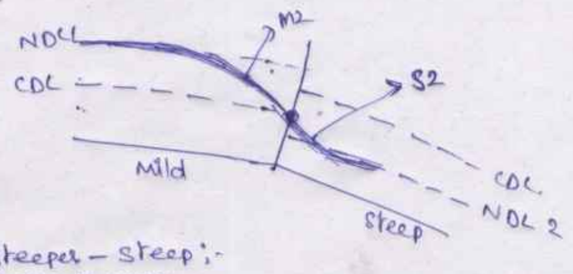


flow have to be in NDL.

- Milder - Mild:-

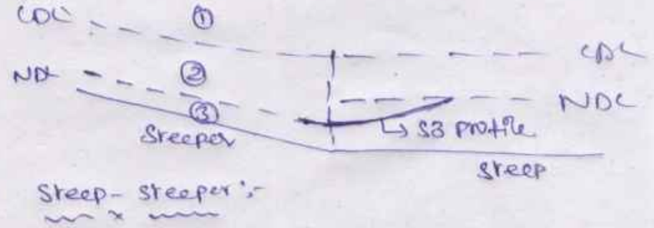


- Mild - Steep:-

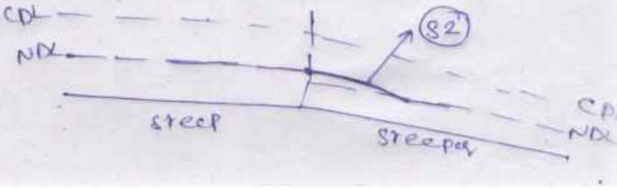


meet as CDL at junction.

- Steeper - Steep:-



- Steep - steeper:-



Simple problems on GVF

① In a 4m wide rectangular channel ($n=0.017$) the bed slope is 0.0006. When the channel is conveying $10\ m^3/s$ of flow, estimate the nature of GVF, where the depth is 2.1m.

Sol:- $B=4m$ $n=0.017$ $S=0.0006$ $Q=10m^3/s$

$y=2.1m$

$$y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{(10/4)^2}{9.8}\right)^{1/3} = 0.86$$

$$\boxed{\begin{matrix} A=By \\ R=y \end{matrix}}$$

$$Q = \frac{1}{n}(B\,y_n)(y_n)^{2/3}(0.0006)^{1/2}$$

$$10 = \frac{1}{0.017}\,4\,y_n^{5/3}(0.0006)^{1/2}$$

$$y_n^{5/3} = 1.73$$

$$y_n = (1.73)^{3/5} = 1.39$$

$y > y_n > y_c \rightarrow$ M1 profile

② In a very long trapezoidal channel with bed width $B=3m$, $m=1.5$, $n=0.016$, $S=0.0004$, $y_n=1.20m$. Determine the type of GVF at i) 0.5m ii) 0.8m & iii) 1.50m ?

Sol:- $B=3m$, $m=1.5$ $n=0.016$, $S=0.0004$ $\boxed{y_n=1.20m}$

$$Q = \frac{1}{n} A R^{2/3} S^{1/2}$$

$$A = (B+my)y = [3+(1.5)(1.2)]1.2$$

$$A = 5.76m^2$$

$$R = \frac{5.76}{B+2y\sqrt{1+m^2}} = \frac{5.76}{3+2(1.5)\sqrt{1+1.5^2}}$$

$$R = 0.6$$

$$Q = \frac{1}{0.016}(5.76)(0.6)^{2/3}(0.0004)^{1/2}$$

$$Q = 5.59\ m^3/s.$$

$$y_c^3 = \frac{q^2}{g} = \frac{(Q/B)^2}{g} = \frac{(5.59/3)^2}{9.8}$$

$$\boxed{y_c = 0.70}$$

i) $y=0.5$ $y_n > y_c > y \rightarrow$ M3 profile

ii) $y=0.8$ $y_n > y > y_c \rightarrow$ M2

iii) $y=1.50$ $y > y_n > y_c \rightarrow$ M1

③ Uniform flow is taking place in a rectangular channel having a longitudinal slope of 0.004 and manning's $n = 0.013$. The discharge per unit width in channel is measured as $1.2\ m^3/s/m$. The slope of channel is classified as!

Sol:- $S = 0.004$ $\quad n = 0.013$ $\quad q = 1.2\ m^3/s/m$

$\hookrightarrow B = 1m$

$Q = 1.2\ m^3/s$

$$y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{1.2^2}{9.8}\right)^{1/3} = \boxed{0.52}$$

$$\boxed{Q = \frac{1}{n} A R^{2/3} S^{1/2}}$$

$$R \approx \frac{y}{1+2y} \approx y$$

$$Q = \frac{1}{n}(By)\left(\frac{y}{1+2y}\right)^{2/3} S^{1/2}$$

$$1.2 = \frac{1}{0.013} y^{5/3} (0.004)^{1/2}$$

$$\underline{y_n = 0.6\,m}$$

$y_c > y_n \rightarrow$ <u>Steep</u>.

④ A 2m wide rectangular channel has normal depth of 1.25m when $Q = 8.75\ m^3/s$. Slope is classified as!

Sol:- $B = 2m$ $\quad \boxed{y_n = 1.25\,m}$ $\quad Q = 8.75\ m^3/s$

$$y_c = \left[\frac{(Q/B)^2}{g}\right]^{1/3} = \left[\frac{(8.75/2)^2}{9.81}\right]^{1/3}$$

$$\boxed{y_c = 1.25}$$

$y_n = y_c \rightarrow$ critical <u>slope</u>.

⑤ In a wide rectangular river the depth of flow is 3m, $S_0 = 1/5000$ and $q = 3\,m^3/s/m$. If chezy formula with $C = 70$ is used, the water surface slope relative to bed at section is?

Sol:- $\boxed{\dfrac{dy}{dx} = \dfrac{S_0 - S_f}{1 - F_r^2}}$

$q = 3\,m^3/s/m \quad y = 3m$

$Q = 3\,m^3/s \quad B = 1m$

$C = 70$

$\boxed{V = C\sqrt{RS_f}}$

$R = y = 3m$

$V = Q/A = \dfrac{3}{1(3)} = 1\,m/s$

$1 = 70\sqrt{3(\cancel{1/5000})S_f}$

② $\boxed{S_f = 6.8 \times 10^{-5}}$

① $S_0 = \dfrac{1}{5000} = 2 \times 10^{-4}$

② $S_f = 6.8 \times 10^{-5}$

$F_r = \dfrac{V}{\sqrt{gD}} = \dfrac{1}{\sqrt{9.8(3)}} = 0.18$

③ $F_r^2 = 0.0324$

$\dfrac{dy}{dx} = \dfrac{2 \times 10^{-4} - 6.8 \times 10^{-5}}{1 - 0.0324}$

$\dfrac{dy}{dx} = \underline{1.37 \times 10^{-4}}$

⑥ A rectangular channel has $B = 20m$, $n = 0.020$, $S_0 = 0.0004$. If the $y_n = 1m$, a depth of 0.8m in a GVF in this channel is a part of?

Sol:- $B = 20m \quad n = 0.020 \quad S = 0.0004 \quad y_n = 1m$

$y = 0.8$

$Q = \dfrac{1}{n} A R^{2/3} S^{1/2}$

$R = \dfrac{By}{B + 2y} = \dfrac{20}{22} = 0.9$

$= \dfrac{1}{0.020}(20)(1)(0.9)^{2/3}(0.0004)^{1/2}$

$Q = 18.6\,m^3/s$

$y_c = \left[\dfrac{(Q/B)^2}{g}\right]^{1/3} = \left[\dfrac{(18.6/20)^2}{9.81}\right]^{1/3} = \underline{\underline{0.44\,m}}$

$y_n > y > y_c \rightarrow$ M2 ✓

(7) In a rectangular channel of width $B = 10m$, $y = 5m$, $Q = 50 m^3/s$. $S_0 = \frac{1}{4000}$, $S_f = ?$ Chezy's constant $c = 55$. S_1 slope at water surface = ?

Sol:- $V = C\sqrt{R S_f}$

$V = Q/A = \frac{50}{50} = 1 m/s$

$R = A/P = \frac{50}{10 + 2(5)} = 50/20 = 2.5$

$1 = 55\sqrt{2.5\, S_f}$

$S_f = \left[\frac{1}{55\sqrt{2.5}}\right]^2 = 1.32 \times 10^{-4}$

$\frac{dy}{dx} = \frac{S_0 - S_f}{1 - F_r^2}$

$F_r = \frac{V}{\sqrt{gD}} = \frac{1}{\sqrt{9.8(5)}} = 0.142$

$= \frac{2.5 \times 10^{-4} - 1.32 \times 10^{-4}}{1 - 0.142^2}$

$\frac{dy}{dx} = \underline{0.00012}$

(8) A Brickwork rectangular channel with $B = 4m$, $Q = 8 m^3/s$, $S = 0.00175$. Is this mild, critical or steep? What type of GVF will we find at local water depth i) 1m ii) 1.5m iii) 2m ($n = 0.015$)

Sol:- $Q = 8 m^3/s$ $B = 4m$ $S = 0.00175$

$y_c = \left[\frac{(Q/B)^2}{g}\right]^{1/3} = \left[\frac{(8/4)^2}{9.81}\right]^{1/3} = 0.74m$

$Q = \frac{1}{n} A R^{2/3} S^{1/2}$

$R \approx y_n$

$A = B y_n$

$8 = \frac{1}{0.015}(4)\, y^{5/3}\, (0.00175)^{1/2}$

$\boxed{y_n = 0.819\, m}$ $y_n > y_c \rightarrow$ Mild slope

for all $y = 1m, 1.5m, 2m$, $y > y_n > y_c \rightarrow$ M1 Profile

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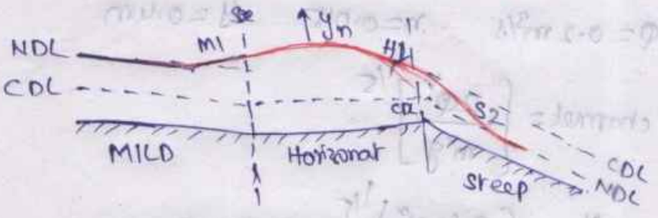
(9) A 2m wide rectangular channel flowing at its normal depth of 1.2m carries a discharge of 6 m³/s. If at a section, the depth of flow is 1.10m, the type of GVF = ?

Sol:- $B = 2m$, $y_n = 1.2m$, $Q = 6\ m^3/s$, $y = 1.10$

$$y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{(6/2)^2}{9.8}\right)^{1/3} = 0.97$$

$y_n > y > y_c \rightarrow$ (M2) ✓

(10) A channel with a mild slope is followed by a horizontal channel and then followed by a steep channel. What GVF will occur. GATE: 2006



Water has to pass Normal Depth line,

$\boxed{M1,\ H2,\ S2}$

(11) MACTHING [IES-2005]

M1 -> Back water profile

H3 -> Hydraulic Jump occurs

S2 -> Hydraulic Jump drops

A2 -> slope upward in the direction of flow.

(12) A discharge of 1 cumec is following in a rectangular channel one meter wide at a depth of 20cm. The Bed slope of channel is GATE: 1992.

Sol:- $Q = 1\ m^3/s$ $B = 1m$ $y_n = 0.2m$

$$y_c^3 = \frac{(Q/B)^2}{g} = \frac{1^2}{9.8}$$

$$y_c = \left(\frac{1}{9.8}\right)^{1/3} = 0.46m$$

$y_c > y_n \rightarrow \underline{\text{steep}}$.

(13) A very wide rectangular channel, $q = 8 m^3/s/m$, $S = 0.004$, $n = 0.015$ GATE:2003
$y = 1m$, what type of GVF it is

Sol:- $q = 8 m^3/s/m$ $S = 0.004$ $n = 0.015$ $y = 1m$

$$y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{8^2}{9.8}\right)^{1/3} = 1.86$$

$$q = \frac{1}{0.015} y^{5/3} (0.004)^{1/2}$$

$$y_n = 1.46 \qquad \boxed{y_c > y_n > y \rightarrow S_3}$$

(14) Water flows in a triangular channel of side slope 1H:1V and $S = 0.001$. $Q = 0.2 m^3/s$; $n = 0.015$, $y = 0.4m$,
find slope of channel & type of profile.

Sol:- $S = 0.001$ $Q = 0.2 m^3/s$ $n = 0.015$ $y = 0.4m$

$$y_c \text{ for } \triangle^{lar} \text{ channel} = \left[\frac{2Q^2}{m^2 g}\right]^{1/5}$$

$$y_c = \left(\frac{2(0.2)^2}{9.8}\right)^{1/5} = 0.382 m$$

$A = my^2$

$$Q = \frac{1}{n} A R^{2/3} S^{1/2} \qquad R = \frac{my}{2\sqrt{1+m^2}} = \frac{y}{2\sqrt{2}}$$

$$0.2 = \frac{1}{0.015} (1)(y^2)\left(\frac{y}{2\sqrt{2}}\right)^{2/3} (0.001)^{1/2}$$

$$(0.2)(0.015) = y^{8/3} \left(\frac{1}{2\sqrt{2}}\right)^{2/3} (0.001)^{1/2}$$

$$y_n = 0.53 m \qquad y = 0.4m$$

$y_n > y_c \rightarrow$ mild slope

$y_n > y > y_c \rightarrow$ M2 profile

Rapidly Varied flow [RVF]

* Sudden change of depth of flow. (or)
* The flow depth changes rapidly in the direction of flow within a short length.

Ex :- → Hydraulic Jump.

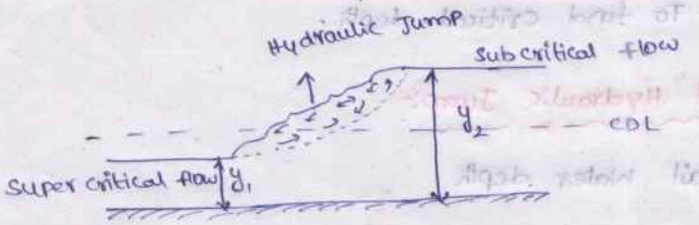
→ flow through channel transition.

→ flow over hump.

Hydraulic Jump :-

- When the flow changes from supercritical to subcritical condition, Hydraulic Jump (or) occurs.
- A supercritical stream meets a subcritical stream of sufficient depth.

* Depth before & after hydraulic Jump - sequent depth.



y_1, y_2 → sequent depths.

* Applications of Hydraulic Jump :-
  - for energy dissipation of downstream of a dam, sluicegate.
    - * Energy of flow decreases with Hydraulic Jump.
  - To increase water depth in irrigation canal.
  - for mixing of chemicals.
  - In aeration of streams.

Sequent depth ratio:-

for horizontal, frictionless and rectangular channel.

** $$\boxed{\frac{y_2}{y_1} = \frac{1}{2}\left[-1 + \sqrt{1+8F_1^2}\right]}$$ → Belanger momentum eqn.

(or)

$$\boxed{\frac{y_1}{y_2} = \frac{1}{2}\left[-1 + \sqrt{1+8F_2^2}\right]}$$

Where $y_1, y_2 \rightarrow$ Sequent depths

$F_1, F_2 \rightarrow$ froud Number $\boxed{F = \frac{V}{\sqrt{gy}}}$

another relation

** $$\boxed{y_1 y_2 (y_1+y_2) = 2\left[\frac{q^2}{g}\right] = 2\,y_c^3}$$ $\boxed{\because \frac{q^2}{g} = y_c^3}$

↳ To find critical depth.

Classification of Hydraulic Jump:-

→ Based on tail water depth

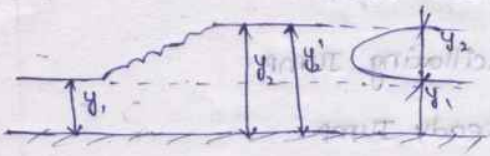
- free Jump
- Repelled Jump
- Submerged Jump

→ Based on froud Number at upstream

- Undular Jump
- Weak Jump
- Oscillating Jump
- Steady Jump
- Strong (or) choppy Jump

Free Jump:-

at y_1, y_2 specific force is same.

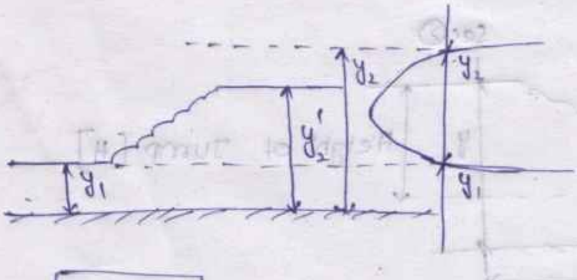


y_1, y_2 → Sequent depths

y_2' → Jump occuring at this depth. [Tail water depth]

$\boxed{y_2' = y_2}$ → free Jump

Repelled Jump:-

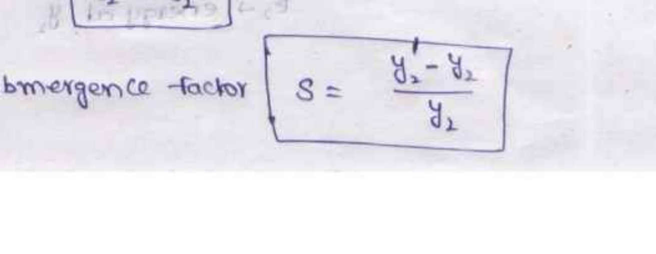


$\boxed{y_2' < y_2}$

Tail water depth (y_2'):-

The depth downstream of a hydraulic struct [sluice gate etc], controlled by downstream channel → Tail water depth.

Submerged Jump:-



$\boxed{y_2' > y_2}$

Submergence factor $\boxed{S = \dfrac{y_2' - y_2}{y_2}}$

***

f_1 = 1 - 1.7 → Undular Jump [Not a Jump]

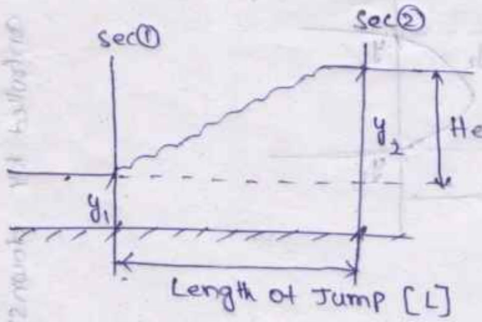
f_1 = 1.7 - 2.5 → Week Jump

f_1 = 2.5 - 4.5 → oscillating Jump

f_1 = 4.5 - 9.0 → steady Jump

f_1 > 9.0 → strong Jump.

Length & Height of Jump:-



$H = y_2 - y_1$

$$\boxed{\frac{L}{H} = 6-7} \quad \text{(or)}$$

* $$\boxed{L = 6.9H = 6.9(y_2 - y_1)}$$

Efficiency:-

$$\boxed{\frac{E_2}{E_1} = 1 - \frac{E_L}{E_1}}$$

E_L → energy loss = $E_1 - E_2$

E_1 → energy at sec ① (or) y_1

E_2 → energy at y_2

Energy Loss :-

** * $$\boxed{E_L = E_1 - E_2 = \frac{(y_2-y_1)^3}{4y_1y_2}}$$ $\Rightarrow$ relative energy loss $$\frac{E_L}{y_1} = \frac{\left(\frac{y_2}{y_1}-1\right)^3}{4(y_2/y_1)}$$

Imp formulae

→ for Rectangular channel

*
$$\frac{2q^2}{g} = y_1y_2(y_1+y_2)$$

$$\Delta E = \frac{(y_2-y_1)^3}{4y_1y_2} = \frac{(V_1-V_2)^3}{2g(V_1+V_2)}$$

$$\frac{y_1}{y_2} = \left[\frac{F_2}{F_1}\right]^{2/3}$$

Energy loss in terms of Fr

$$\frac{E_L}{E_1} = \frac{-3+\sqrt{1+8F_1^2}}{8(2+F_1^2)(-1+\sqrt{1+8F_1^2})}$$

Simple problems on RVF.

① A hydraulic jump occurs in a horizontal rectangular channel with sequent depths of 0.7m and 4.2m. Calculate the rate of flow per unit width, energy loss, and Initial froude Number.

Sol:- $y_1 = 0.7m \qquad y_2 = 4.2m$

$$\frac{2q^2}{g} = y_1 y_2 (y_1 + y_2)$$

$$\frac{2q^2}{9.81} = (0.7)(4.2)[0.7 + 4.2]$$

$$q^2 = \frac{14.4 \times 9.81}{2} = 70.66$$

$$\boxed{q = 8.4 \ m^3/s/m.}$$

* Energy loss $E_L = \dfrac{(y_2 - y_1)^3}{4 y_1 y_2} = \dfrac{(4.2 - 0.7)^3}{4(0.7)(4.2)}$

$$\underline{E_L = 3.6m.}$$

* $\dfrac{y_2}{y_1} = \dfrac{1}{2}\left[-1 + \sqrt{1 + 8f_1^2}\right]$

$$\frac{4.2}{0.7} = \frac{1}{2}\left[-1 + \sqrt{1 + 8f_1^2}\right]$$

$$6(2) = -1 + \sqrt{1 + 8f_1^2}$$

$$(12+1)^2 = 1 + 8f_1^2$$

$$169 - 1 = 8f_1^2$$

$$f_1^2 = \frac{168}{8}$$

$$\underline{f_1 = 4.58}$$

② A hydraulic jump in a rectangular channel has the froude number at the begining of the jump $f_1 = 5$. find the froude number $f_2 = ?$

Sol:-

$$\frac{y_2}{y_1} = \frac{1}{2}\left[\sqrt{1+8f_1^2} - 1\right]$$

$$= \frac{1}{2}\left[\sqrt{1+8(5)^2} - 1\right]$$

$$\frac{y_2}{y_1} = 6.58$$

$$\frac{y_1}{y_2} = 1/6.58 = 0.15$$

$$\frac{y_1}{y_2} = \frac{1}{2}\left[\sqrt{1+8f_2^2} - 1\right]$$

$$\frac{\left[(0.15)2+1\right]^2 - 1}{8} = f_2^2$$

$$f_2 = \underline{0.296}.$$

③ In a Hydraulic jump takes place in a horizontal, rectangular channel a sequent depth ratio of 10. What initial froude number would produce this ratio? What would be the froude number after jump?

Sol:-

$$y_2/y_1 = 10 = \frac{1}{2}\left[-1+\sqrt{1+8f_1^2}\right]$$

$$f_1^2 = \frac{\left[((10)2)+1\right]^2 - 1}{8}$$

$$f_1 = \underline{7.41}.$$

$$\frac{y_1}{y_2} = \frac{1}{10} = \frac{1}{2}\left[-1+\sqrt{1+8f_2^2}\right]$$

$$f_2^2 = \frac{\left[2/10+1\right]^2 - 1}{8}$$

$$f_2 = \underline{0.296}.$$

④ The sequent depth ratio in a hydraulic jump formed in a horizontal rectangular channel is 16.48. The froude number of the super critical stream is! 28

Sol:- Super critical = f_1 Sub critical = f_2

$$\frac{y_2}{y_1} = 16.48 = \frac{1}{2}\left[-1+\sqrt{1+8F_1^2}\right]$$

$$f_1^2 = \frac{\left[(16.48)2+1\right]^2-1}{8}$$

$$f_1 = \underline{12}$$

⑤ The froude number of a subcritical stream at the end of hydraulic jump in a horizontal rectangular channel is 0.22. The sequent depth ratio of this channel is?

Sol:- Sub critical $f_2 = 0.22$

$$\frac{y_1}{y_2} = \frac{1}{2}\left[-1+\sqrt{1+8f_2^2}\right]$$

$$\frac{y_1}{y_2} = \frac{1}{2}\left[-1+\sqrt{1+8(0.22)^2}\right]$$

$$\frac{y_1}{y_2} = 0.088$$

sequent depth ratio $\frac{y_2}{y_1} = \underline{11.25}$.

⑥ The initial depth of a hydraulic jump is 0.2 m, and sequent depth ratio is 10. The length of the jump is

Sol:- $y_1 = 0.2$ m $\frac{y_2}{y_1} = 10$

$y_2 = 10(0.2) = 2$ m

Length of the Jump = $6.9\,(y_2 - y_1)$

$= \underline{12.42\text{ m}}$.

⑦ If a $F_r = 5.5$, then jump is classified as

$F_r = 4.5 - 9.0 \rightarrow$ Steady Jump.

(8) In a hydraulic jump of a rectangular channel of 3m width, the discharge is 7.8 m^3/s and depth before jump is 0.28m. Estimate i) sequent depth ii) Energy loss.

Sol:- $B = 3m \quad y_1 = 0.28m \quad Q = 7.8\ m^3/s$

$$\left[V_1 = Q/A = \frac{7.8}{3(0.28)} = 9.286\ m/s \right]$$

$$F_{r_1} = \frac{V_1}{\sqrt{gD_1}} = \frac{9.286}{\sqrt{9.8(0.28)}} = 5.603$$

$$\frac{y_2}{y_1} = \frac{1}{2}\left[-1 + \sqrt{8+8F_1^2}\right]$$

$$y_2 = \left[\frac{1}{2}\left[-1+\sqrt{1+8(5.603)^2}\right]\right]0.28m$$

i) $y_2 = \underline{2.08m}$.

ii) Energy loss $E_L = \frac{(y_2-y_1)^3}{4y_1y_2} = \frac{(2.08-0.28)^3}{4(2.08)(0.28)}$

$$E_L = \underline{2.503m}$$

(9) A hydraulic jump takes place in a rectangular channel with sequent depths of 0.25m and 1.50m at the begining and end of the jump respectively. Estimate i) discharge per unit width ii) Energy loss.

Sol:- $y_1 = 0.25m \quad y_2 = 1.50m$

$$\frac{y_2}{y_1} = \frac{1}{2}\left[-1+\sqrt{1+8F_1^2}\right]$$

$$\frac{1.50}{0.25} = \frac{1}{2}\left[-1+\sqrt{1+8F_1^2}\right]$$

$F_1 = \underline{4.583m}$

$$F_1 = \frac{V_1}{\sqrt{gy_1}}$$

$4.583(\sqrt{9.81}(0.25)) = V_1$

$V_1 = 7.177\ m/s$.

i) $q = V_1y_1 = 7.177(0.25) = 1.794\ m^3/s/m$.

ii) $E_L = \frac{(y_2-y_1)^3}{4y_1y_2} = \frac{(1.50-0.25)^3}{4(1.50)(0.25)} = \underline{1.802m}$.

10) Matching [IES:2004]

Conjugate depth - Same Specific force

Critical depth - Minimum Specific Energy

Alternate depth - Same Specific Energy

Normal depth - Uniform flow.

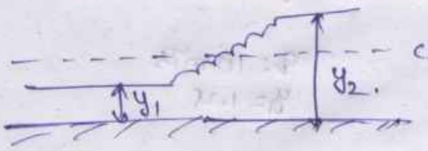
11) A hydraulic Jump occurs at the toe of a spillway. The depth before jump is 0.2m. the sequent depth is 3.2m. What is energy dissipated in m? [IES:2007]

Sol:- $y_1 = 0.2m$ $y_2 = 3.2m$

$$E_L = \frac{(y_2-y_1)^3}{4y_1y_2} = \frac{(3.2-0.2)^3}{4(0.2)(3.2)} = \underline{10.5m}$$

12) The hydraulic Jump always occurs from [GATE:94]

Below critical depth to above critical depth.



13) The conjugate depths at a location in a horizontal rectangular channel, 4m wide are 0.2m and 1m. The discharge in the channel is — m³/sec. [GATE-1991]

Sol:- $B = 4m$ $y_1 = 0.2m$
$y_2 = 1m$

$$\frac{2q^2}{g} = y_1y_2(y_1+y_2)$$

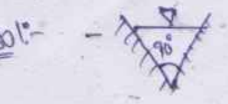
$$=(0.2)(1)(1+0.2)$$

$$q^2 = \frac{(0.2)(1.2)(9.8)}{2}$$

$q = 1.08$ m³/s/m width.

for $B = 4m$ $Q = qB = 1.08(4) = \underline{\underline{4.33}}$ m³/sec

14) A hydraulic jump takes place in a triangular channel of vertex angle 90°. The discharge is $1 m^3/s$ and the pre-depth is 0.5m. What will be the post-jump depth? ($g = 9.8 m/s^2$) GATE:2003.

Sol:- 

$$\frac{Q^2}{gA_1} + A_1\bar{z}_1 = \frac{Q^2}{gA_2} + A_2\bar{z}_2$$

$A_1 = y_1^2$

$A_2 = y_2^2$

$\bar{z}_1 = y_1/3$

$\bar{z}_2 = y_2/3$

$Q = 1 m^3/s$

$y_1 = 0.5 m$

$$\Rightarrow \frac{1^2}{9.8(0.5)^2} + (0.5)^2\left(\frac{0.5}{3}\right) = \frac{1^2}{9.8(y_2^2)} + (y_2^2)\frac{y_2}{3}$$

$$0.4494 = \frac{0.1019}{y_2^2} + \frac{y_2^3}{3}$$

By trail & error method.

$$y_2 = \underline{1.02m}$$

15) Water flows in a wide channel at $q = 10 m^2/s/m$ and $y_1 = 1.25m$ if the flow undergoes a hydraulic jump, the depth of supercritical flow section is!

$q = 10 m^2/s$

$y_1 = 1.25$

Sol:- $$\frac{2q^2}{g} = y_1y_2(y_1+y_2)$$

$$\frac{2(10)^2}{g} = 1.25\,y_2(1.25+y_2)$$

$$20.3 = 1.25y_2^2 + 1.56y_2$$

$$1.25y_2^2 + 1.56y_2 - 20.3 = 0$$

$$y_2 = \underline{3.46m}$$

16) find velocity for above prblm.
→ at subcritical region. (y_2)

$$q = VA$$

$$V = q/A_2 = \frac{10}{3.46} = \underline{2.89 m/s}$$

DESIGN OF CANAL [Alluvial channel].

Canal :-

1. Lined canal [Rigid Boundary]
2. Unlined canal [Mobile Boundary] → Erosion of boundaries.
   - ↳ i. Alluvial channel
   - ii. Non-alluvial channel.

* Alluvial channel design is important topic in this.

Design of alluvial channel :-

1. Kennedy's Theory.
2. Lacey's Theory.
3. Tractive force method.

1. Kennedy's Silt Theory :-

Critial velocity (V_0) :-

$$\boxed{V_0 = 0.55\, D^{0.64}}$$

V_0 = m/sec

D = Depth of flow in m.

$$\boxed{V_K = 0.55\, m\, D^{0.64}}$$

Critical velocity ratio

* $$\boxed{m = V_K / V_0}$$

$m > 1 \rightarrow$ coarser silt

$m < 1 \rightarrow$ finer silt.

Design Procedure:-

1. Assume a trial value 'D' in meters.

2. Calculate $V_K = 0.55\, m\, D^{0.64}$.

3. Calculate c/s $A = Q/V$

4. find B.

5. Calculate mean velocity $V = \frac{1}{n} R^{2/3} S^{1/2}$.

6. If $V \approx V_K$ V is nearly equal to V_K then assumed Depth is correct.

7. If not assume another 'D'.

Lacey's Theory:-

- Lacy developed regime theory

Regime:-

A Stable channel whose width, depth and bed slope have undergone modification by silting to attain equilibrium.

Initial regime:-

- slope & Depth attain equilibrium.

final regime:-

- Slope, Depth & Width also attain equilibrium.

* Lacy's eqns are applicable for a channel which has attained final or true regime.

Equations of Lacey's Theory :-

Particle size $\boxed{d = D_{50}}$

1. Silt factor $f = 1.76\sqrt{d}$
2. Mean velocity $V = \left[\frac{Qf^2}{140}\right]^{1/6}$
3. Wetted perimeter $P = 4.75\sqrt{Q}$ $P \to m$ $Q \to m^3/sec.$
4. velocity $V = \sqrt{\frac{2}{5} fR}$
5. Bed slope $S = \frac{f^{5/3}}{3340\, Q^{1/6}}$
6. Normal scour depth $R = 1.35\left[\frac{q^2}{f}\right]^{1/3}.$

Tractive force method :-

- The force exerted by flowing liquid on the perimeter.
- Drag force (or) unit tractive force :-

$$\boxed{\tau = WRS}$$

- Tractive force ratio $\boxed{K = \frac{\tau_s}{\tau_b} = \cos\theta\sqrt{1 - \frac{\tan^2\theta}{\tan^2\phi}}}$.

① Design a canal by Lacy's theory to convey $40 m^3/s$ of water. The canal is to be cut in an alluvial soil of median size 0.6mm

Sol:- $Q = 40 m^3/s$ $d_m = 0.6mm$

Silt factor $f_s = 1.76\sqrt{d_m}$

$= 1.76\sqrt{0.6} = 1.36$

Longitudinal slope $S_0 = \dfrac{0.0003 \times f_s^{5/3}}{Q^{1/6}}$

$= \dfrac{0.0003 \times (1.36)^{5/3}}{(40)^{1/6}} = 2.72 \times 10^{-4}$

Hydraulic Radius $R = 0.48\left(\frac{Q}{f_s}\right)^{1/3}$

$= 0.48\left[\frac{40}{1.36}\right]^{1/3} = 1.482m.$

Wetted perimeter $P = 4.75\sqrt{Q}$

$= 4.75\sqrt{40} = 30.04m$

⇒ Perimeter $P = B + 2y\sqrt{1+m^2}$ → for trapezoidal,

final slope assume 0.5H : 1V

$m = 0.5$

$30.04 = B + 2y\sqrt{1+0.5^2}$

$30.04 = B + 2.236y$

$\boxed{B = 30.04 - 2.236y}$

$R = A/P$

Area $A = (B+my)y = (30.04 \times 1.481)$ $\therefore \boxed{A = PR}$

$(30.04 - 2.236y + 0.5y)y = 44.52$

$1.736y^2 - 30.04y + 44.52 = 0$

$y = 1.636m$ & $15.66m$

Neglect higher value of y, it is impractical.

$B = 30.04 - 2.236 y_0$

$= 30.04 - 2.236(1.636)$

$B = 26.38m$, $y_0 = 1.636m$, $S_0 = 2.72\times10^{-4}$ Slope = 0.5:1:1

② A regime Lacey channel having a full supply discharge of 30m³/s has a bed material of 0.12mm median size. What would be the Manning's roughness coefficient n for this channel?

Sol:- $Q = 30 m^3/s$ $d = 0.12mm$

$n = \frac{S_0^{1/6}}{10.8}$

$f_s = 1.76\sqrt{d_m}$

$= 1.76\sqrt{0.12}$

$f_s = 0.609$

$S_0 = \frac{0.0003 f_s^{5/3}}{Q^{1/6}}$

$= \frac{0.0003(0.609)^{5/3}}{30^{1/6}}$

$= 7.46\times10^{-5}$

$n = \frac{(7.46\times10^{-5})^{1/6}}{10.8} = \underline{0.019}$.

Unsteady flow associated with WAVE.

Wave:-
Temporal variation in the water surface which propagated through fluid media.

**Types of Waves:-

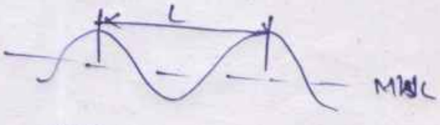
1. Capillary wave → due to surface tension.
2. Elastic wave → due to fluid compressibility.
3. Gravitational wave → due to weight of fluid. [Gravitational force].

⇒* Classification.

- Oscillatory wave → Average mass transport is zero. [NULL].
- Translatory wave → Transport of fluid occurs in the direction of wave propagation.
- Deep Water Wave → $\boxed{\frac{y}{L} > 0.5}$
- shallow Water Wave → $\boxed{\frac{y}{L} < 0.05}$

y = Water depth

L = Wave length.



MWL - mean water level

Wave formation in Nature:-

- Large waves → due to tide in sea.
- Tsunami → due to earth quake.
- Huge waves → due to failure of dam in river.

Celerity :- [c]

- Speed of wave propagation relative to fluid speed.

$$\boxed{C = \sqrt{gy}}$$

$$\boxed{C = \sqrt{\frac{g}{2}\,\frac{y_2}{y_1}(y_1+y_2)}}$$

Height $\boxed{h = y_2 - y_1}$

⇒ Surge moving downstream $\boxed{C_1 = V_w - V_1}$

⇒ Surge moving upstream $\boxed{C_1 = V_w + V_1}$

① A surge travels upstream at a velocity of 3m/s. If the steady state velocity in the channel was 0.6 m/s, and flow depth in channel is 1m. The celerity should be? & height of surge will be!

Sol:- $V = 0.6\ m/s \qquad V_w = 3\ m/s$

$C = V + V_w = 0.6 + 3 = 3.6\ m/s$

$y_1 = 1m$

$$C = \sqrt{\frac{g}{2}\frac{y_2}{y_1}(y_1+y_2)}$$

$$3.6 = \sqrt{\frac{9.8}{2}\, y_2(1+y_2)}$$

$$\frac{(3.6)^2(2)}{9.8} = y_2 + y_2^2$$

$$y_2^2 + y_2 - 2.644 = 0$$

$$y_2 = 1.20$$

$$\text{height} = y_2 - y_1 = 1.20 - 1 = \underline{0.2\,m}$$

② A positive surge of height 0.5m was found to occur in a rectangular channel with a depth of 2m. The celerity of the surge is in m/s.

Sol:- $y_1 = 2m \qquad h = 0.5$

$h = y_2 - y_1 \qquad y_2 = 2 + 0.5 = 2.5m$

$$C = \pm\sqrt{\frac{g}{2}\frac{y_2}{y_1}(y_1+y_2)}$$

$$C = \pm\sqrt{\frac{9.8}{2}\left(\frac{2.5}{2}\right)(2+2.5)}$$

$$C = \pm\underline{5.25}\ m/s.$$

③ A trapezoidal channel width $B = 0.6m$, $m = 1m$, $y = 2m$ has a positive surge of height 0.80m. The celerity will be

sol:- $y_1 = 2m$ $h = y_2 - y_1 = 0.8$

$y_2 = 2.8$

$$C = \sqrt{\frac{g}{2}\frac{y_2}{y_1}(y_1+y_2)}$$

$$C = \sqrt{\frac{9.8}{2}\frac{2.8}{2}(2+2.8)}$$

$C = 5.73$ m/s.

④ A stone thrown into a shallow pound produced a wave of amplitude 2cm and velocity of 1.80 m/s. The depth of pond in m is.

sol:- $v = 1.8$ m/s $a = 2cm = 0.02m$

$$C = v = \sqrt{gy}\boxed{\sqrt{\frac{2y+4a}{2y+a}}}$$

→ as amplitude when compared to y we are neglecting this term in main formu. In this problem we are including this

$$1.8 = \sqrt{9.8y}\sqrt{\frac{2y+4(0.02)}{2y+0.02}}$$

$$(1.8)^2 = \frac{9.8y(2y+0.08)}{2y+0.02}$$

$$(1.8)^2(2y+0.02) = 9.8y(2y+0.08)$$

$$6.48y + 0.0648 = 19.6y^2 + 0.784y$$

$$19.6y^2 = 0.784y - 6.48y - 0.0648 = 0$$

$$19.6y^2 - 5.696y - 0.0648 = 0$$

$$y = \underline{0.301}\ m.$$

⑤ A canal has a velocity of 2.5 m/s and a depth of flow 1.63 m. A negative wave formed due to decrease in the discharge at an upstream control moves at this depth with a celerity of =?

⇒ Surge moving upstream $C_1 = V_w + V_1$

$$C \; V_w = \sqrt{gy} + 2.5$$

$$= \sqrt{9.8(1.63)} + 2.5$$

$$= \underline{6.5 \text{ m/s}}$$

—— × —— × ——

All The Best

Khadir.P

- KHADIR PATAN
- R092502
- CIVIL - 3B [G-8]
- RGUKT RKV

Just believe in smart work give your best, get succeed