LECTURE NOTES

Subject Name: Induction and Synchronous Machines

Year / Branch: II year / EEE

Regulation: R23

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AP/EEE

23EEE242	INDUCTION AND SYNCHRONOUS MACHINES	L	T	P	C
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PRE-REQUISITE: A course on DC Machines and Transformers.

COURSE EDUCATIONAL OBJECTIVES:
1. Understand the construction, principle and operation of single phase and three phase induction motors
2. Understand the construction, principle and operation of synchronous generator and synchronous motor
3. Understand various applications of various alternating machines
4. Apply the above concepts to solve various mathematical and complex problems
5. Analyze the characteristics of induction motor, synchronous motor and synchronous generators

UNIT-I: 3-PHASE INDUCTION MOTORS: 09
Construction of Squirrel cage and Slipring induction motors– production of rotating magnetic field – principle of operation – rotor emf and rotor frequency – rotor current and power factor at standstill and during running conditions– rotor power input, rotor copper loss and mechanical power developed and their inter-relationship –equivalent circuit – phasor diagram, Applications.

UNIT-II: PERFORMANCE OF 3-PHASE INDUCTION MOTORS: 09

Torque equation – expressions for maximum torque and starting torque – torque-slip characteristics – double cage and deep bar rotors – No load, Brake test and Blocked rotor tests – circle diagram for predetermination of performance- methods of starting –starting current and torque calculations - speed control of induction motor with V/f control method, rotor resistance control and rotor emf injection technique –crawling and cogging – induction generator operation.

UNIT-III: SINGLE PHASE MOTORS: 09
Single phase induction motors – constructional features – double revolving field theory, Cross field theory – equivalent circuit- starting methods: capacitor start capacitor run, capacitor start induction run, split phase & shaded pole, AC series motor, Applications.

UNIT-IV: SYNCHRONOUS GENERATOR: 09
Constructional features of non-salient and salient pole type alternators- armature windings – distributed and concentrated windings – distribution & pitch factors – E.M.F equation – armature reaction – voltage regulation by synchronous impedance method – MMF method and Potier triangle method – two reaction analysis of salient pole machines - methods of synchronization- Slip test – Parallel operation of alternators.

UNIT-V: SYNCHRONOUS MOTOR: 09
Synchronous motor principle and theory of operation – Effect of excitation on current and power factor– synchronous condenser –expression for power developed –hunting and its suppression – methods of starting, Applications.

COURSE OUTCOMES:

On successful completion of the course, students will be able to	POs related to COs
CO1 Understand the construction, principle and operation of single phase and three phase induction motors	**PO1,PO2**
CO2 Understand the construction, principle and operation of synchronous generator and synchronous motor	**PO1,PO2**
CO3 Understand various applications of various alternating machines	**PO1,PO2,PO3**
CO4 Apply the above concepts to solve various mathematical and complex problems	**PO1,PO2**
CO5 Analyze the characteristics of induction motor, synchronous motor and synchronous generators	**PO1,PO2,PO3**

Textbooks:
1. Electrical Machinery, Dr. P.S. Bhimbra, Khanna Publishing, 2021, First Edition.
2. Performance and analysis of AC machines by M.G. Say, CBS, 2002.

Reference Books:
1. Electrical machines, D.P. Kothari and I.J. Nagrath, McGraw Hill Education, 2017, Fifth Edition.
2. Theory & Performance of Electrical Machines by J.B.Gupta, S.K.Kataria & Sons,2007.
3. Electric Machinery, A.E.Fitzgerald, Charles kingsley, Stephen D.Umans, McGraw-Hill, 2020, Seventh edition.

REFERENCE WEBSITE:
1. https://nptel.ac.in/courses/108/105/108105131
2. https://nptel.ac.in/courses/108106072

CO-PO MAPPING:

CO-PO	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12
CO.1	3											
CO.2	3											
CO.3	3	2										
CO.4	3											
CO.5	3	2										
CO*	3	2										

Unit-I.
Three Phase Induction Motors.

Introduction:

With the almost universal adoption of AC system of distribution of electric energy, the field of application of AC motors has widened considerably during recent years.

As a result, motor manufactures have tried over a last few decades, various types of AC motors suitable for all ~~das~~ classes of industrial needs.

Of all the AC motors, the polyphase induction motor is the one which is extensively used for various kinds of industrial needs.

It has the following advantages and disadvantages:

Advantages:
1) very simple, extremely rugged and almost unbreakable construction

2) Cost is low and very reliable
3) Sufficiently high efficiency and reasonably good power factor
4) Requires minimum of maintenance
5) Starting of motor is simple.

Disadvantages:
1) Speed cannot be varied without sacrificing some of its efficiency.
2) Speed decreases with increase in load
3) Starting torque is somewhat inferior.

Principle of Operation:

The electro magnetic induction is the basic principle for the operation of induction motor

(The electromotive force induces across the electrical conductor, when it is placed in a rotating magnetic field).

When the three phase supply is given to the stator, the rotating magnetic field

Produced on it.

The conductors of the rotor are stationary. This stationary conductor cut the rotating magnetic field of the stator and because of electromagnetic induction, an emf is induced on the rotor conductor.

The conductors of the rotor are short-circuited either by the end rings or by the external resistance.

As a result, the current is induced in rotor conductor. and thus the flux induces on it.

Now, we have two fluxes, one because of the rotor and another because of the stator.

These fluxes interact with each other. The rotor flux will try to catch up with the stator flux.

Thus, the rotor rotates in the same

direction as that of stator flux, to minimize the relative velocity.

However, the rotor never succeeds in catching up the synchronous speed

This is the basic working principle of induction motor.

Constructional details:

An induction motor consists essentially of two main parts:

a) a stator and
b) a rotor

a) Stator:

The stator of an induction motor is made up of ~~Stampings~~ a number of stampings, which are slotted to receive the windings.

The stator carries a 3-phase winding and is fed from a 3-phase supply.

It is wound for a definite number of poles, the exact number of poles being

determined by the requirement of speed.

The stator windings, when supplied with 3-phase current, produce a magnetic flux, which is of constant magnitude but which revolves (or rotates) at synchronous speed (given by $N_s = \frac{120f}{P}$).

This revolving magnetic flux induces an emf in the rotor by mutual inductance
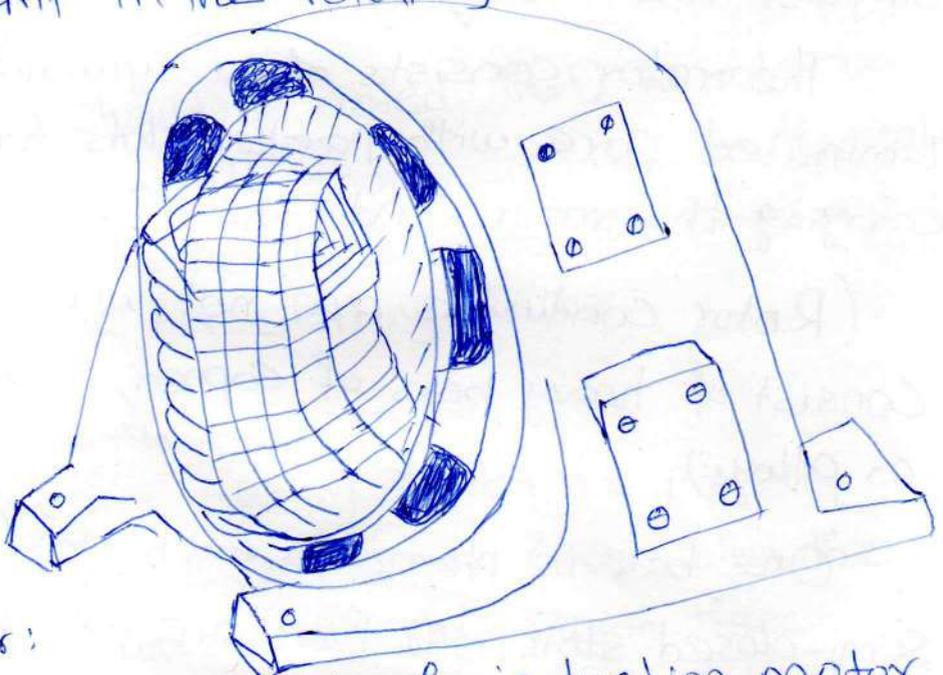
Rotor:

Rotating part of induction motor is rotor.

There are two types of Construction of

rotor of an three phase induction motor.

i) Squirrel-cage rotor

ii) Phase-wound or wound rotor

~~(or) Slip-ring me~~

i) Squirrel-cage Rotor:

Almost 90% of induction motors are squirrel-cage type, because it has the simplest and most rugged construction

The rotor consists of a cylindrical laminated core with parallel slots for carrying the rotor conductors.

(Rotor conductors are not wires but consist of heavy bars of copper, alluminium or alloys).

One bar is placed in each slot (for semi-closed slots, the bars are inserted from the end).

This rotor bars are electrically welded or bolted to two heavy end-rings, which looks like a squirrel-case.
short circuiting
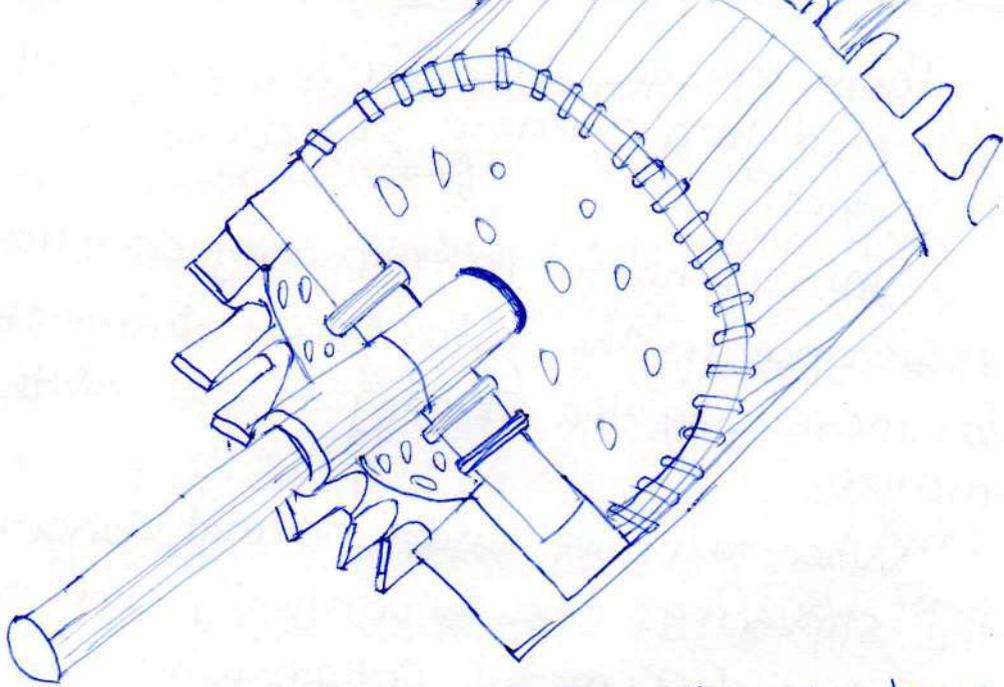
It should be noted that, the rotor bars are permanently short-circuited on themselves, hence, it is not possible to add any external resistance for starting purpose.

ii) Phase-wound rotor:

This type of rotor is properly wound for three-phases (even when the stator is wound two-phase) with three connecting leads brought out through slip rings as shown in fig.

The slip-rings are tapped by brushes.

These brushes are further externally connected to a 3-phase Star connected resistors.

This is helpful in introducing additional resistance in the rotor circuit during starting for increasing the starting torque of the motor.

When running under normal conditions, the slip-rings are automatically short-circuited by means of a metal collar, which is pushed along the shaft and connects all the rings together.

The brushes are automatically lifted from slip-rings to reduce frictional losses

slip-rings
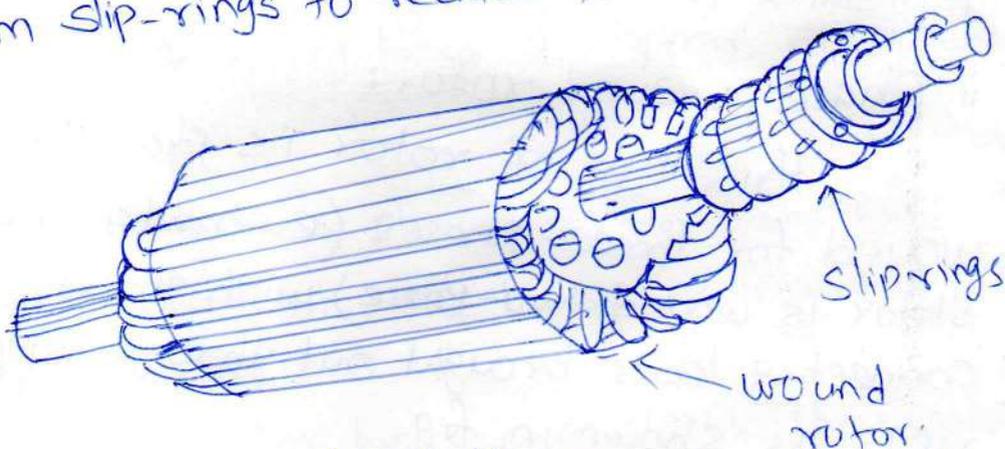
wound rotor.

and wear and tear. Hence, under running condition, the wound rotor is short circuited

On itself just like the squirrel-cage rotor.

Rotating Magnetic field:

When three-phase windings displaced by 120° in space, are fed by three-phase currents displaced by 120° in time, produce a resultant magnetic flux which rotates as if actual magnetic poles were being rotated mechanically.

Assume, a 3-phase, two pole stator having 3 identical winding placed 120° apart as shown in fig. below
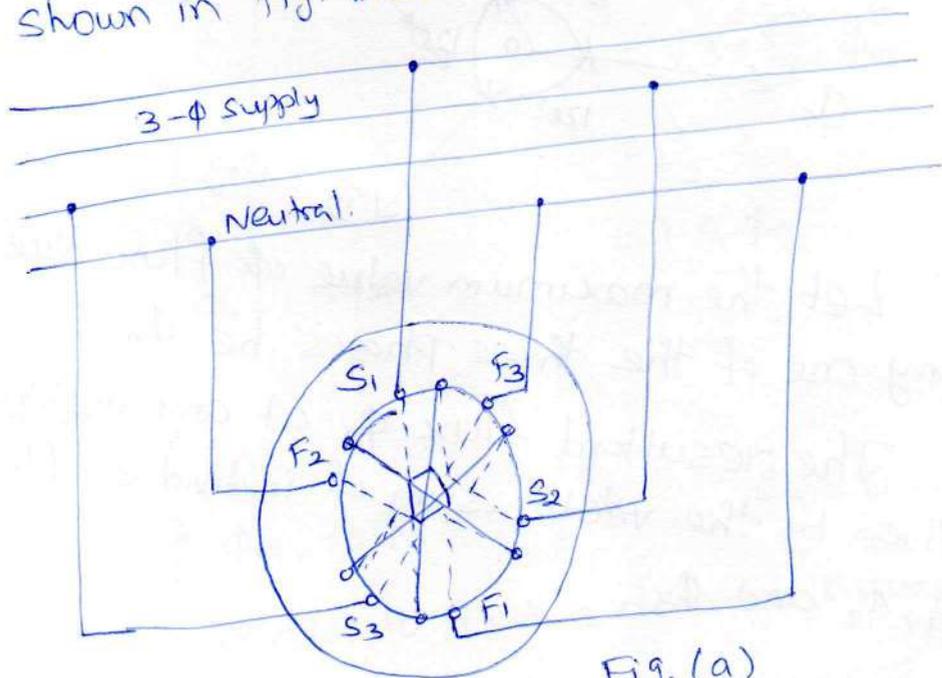
Fig. (a)

The flux (assumed sinusoidal) due to 3-phase
windings is shown in fig. below.
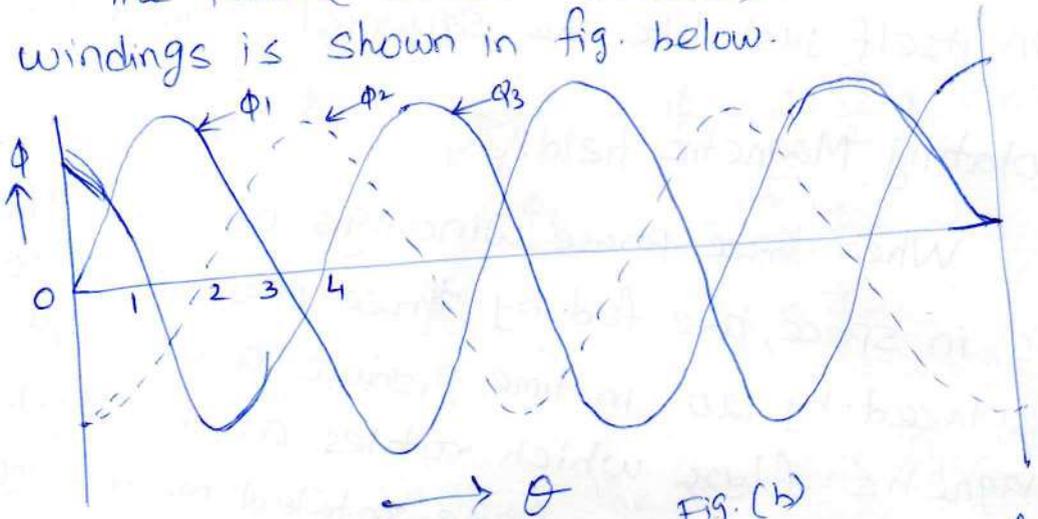
Fig. (b)

The assumed positive directions of
fluxes are shown in fig. below
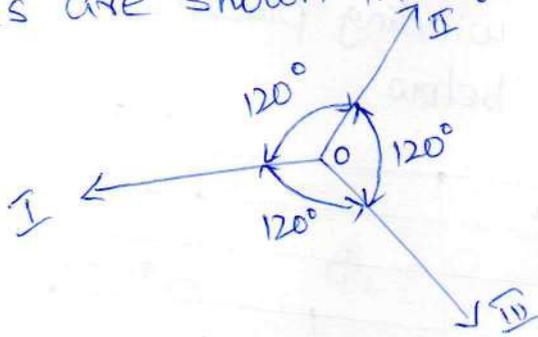
Fig. (c)

Let the maximum value of flux due to
any one of the three phases be ϕ_m.

The resultant flux ϕ_r at any instant is
given by the vector sum of individual fluxes
ϕ_1, ϕ_2 and ϕ_3.

Now, Consider the value of ϕ_r at four instants $\frac{1}{6}$th time-period apart marked as $0,1,2$. and 3 in fig@above.

i) @ At Point 0,

$$\theta = 0°$$

$$\phi_1 = 0, \quad \phi_2 = -\frac{\sqrt{3}}{2}\phi_m, \quad \phi_3 = \frac{\sqrt{3}}{2}\phi_m$$

The vector for ϕ_2 is drawn in a direction opposite to the direction assumed positive in a previous fig. (c)

$$\phi_r = 1.54\phi_m$$
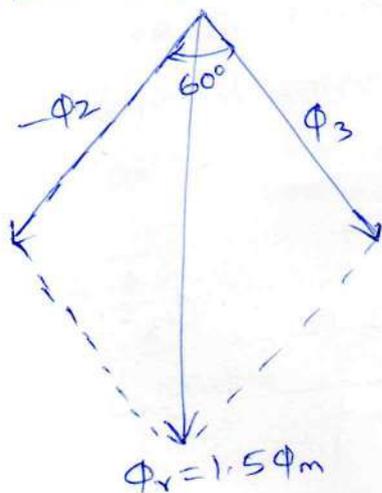
$$\therefore \phi_r = 2 \times \frac{\sqrt{3}}{2}\phi_m$$
$$\cos \frac{60°}{2}$$

$$= \sqrt{3} \times \frac{\sqrt{3}}{2}\phi_m$$

$$= \frac{3}{2}\phi_m$$

$$= 1.5\phi_m.$$

ii) At Point 1,

$$\theta = 60°$$

$$\phi_1 = \frac{\sqrt{3}}{2}\phi_m, \quad \phi_2 = -\frac{\sqrt{3}}{2}\phi_m, \quad \phi_3 = 0$$

ϕ_1 drawn parallel to positive direction assumed,

Φ_2 drawn in opposition to positive
direction assumed.

$$\therefore \Phi_r = 2 \times \frac{\sqrt{3}}{2} \Phi_m \cos 30°$$

$$= \frac{3}{2} \Phi_m$$

$$= 1.5 \Phi_m.$$

$\Phi_r = 1.5 \Phi_m$
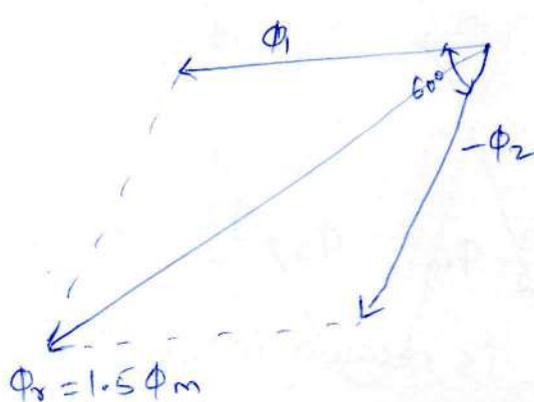
It is found that the resultant flux is
again $\frac{3}{2} \Phi_m$, but rotated clockwise through an
angle of $60°$.

iii) At Point 2,
$$\theta = 120°$$

$$\Phi_1 = \frac{\sqrt{3}}{2} \Phi_m \qquad \Phi_2 = 0, \qquad \Phi_3 = -\frac{\sqrt{3}}{2} \Phi_m$$

$$\therefore \Phi_r = \frac{3}{2} \Phi_m$$

So, the resultant flux is again of the
same value, but has further rotated
clockwise through angle of $60°$ as in fig.
below.

$\phi_r = 1.5\phi_m$

$I - ④$

$-\phi_3$

$60°$

ϕ_1
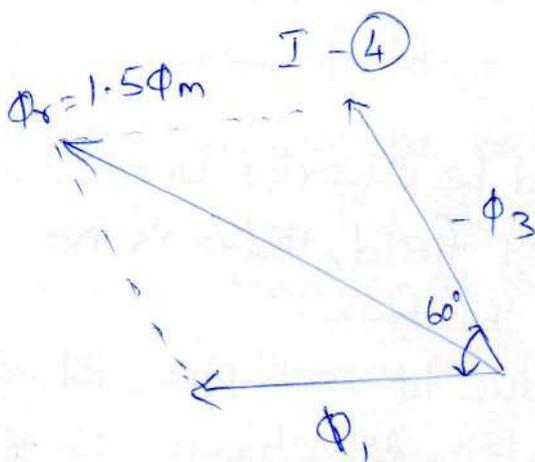
iv) At point 3,

$$\theta = 180°$$

$$\phi_1 = 0 \quad , \quad \phi_2 = \frac{\sqrt{3}}{2}\phi_m, \quad \phi_3 = -\frac{\sqrt{3}}{2}\phi_m$$

The resultant flux is of constant value $\frac{3}{2}\phi_m$ and has rotated clockwise through an additional angle 60°, or through an angle of 180° from the start.

Conclusions:

1) The resultant flux is of constant value $= \frac{3}{2}\phi_m$ (ie. 1.5 times the maximum flux due to any phase).

2) The resultant flux rotates around the stator at synchronous speed given by,

$$N_s = \frac{120 f}{P}$$

However, it should be clearly understood that in this revolving field, there is no actual revolution of the flux.

But, the flux due to each phase changes periodically, according to the changes in the phase current.

Therefore, it is only the resultant flux which keeps on shifting synchronously around the stator.

Slip:

In practice, the rotor never succeeds in "catching up" with stator field.

If it really did so, then there would be no relative speed between the two, hence no rotor emf, no rotor current and so no torque to maintain rotation.

That is why the rotor runs at a speed which is always less than the speed

of the stator field.

The difference between the synchronous speed N_s and the actual speed N of the rotor is known as slip.

Though it may be expressed in so many revolutions/second, yet it is usual to express it as a percentage of the synchronous speed.

$$\% \text{ Slip } S = \frac{N_s - N}{N_s} \times 100$$

Sometimes, $N_s - N$ is called the slip speed.

Obviously, rotor (or) motor speed is

$$N = N_s(1-S)$$

Rotor current frequency:

When the rotor is stationary, the frequency of rotor current is the same as the supply frequency.

But, when the rotor starts revolving, then the frequency depends on the relative

speed or on slip-speed.

Let at any slip speed, the frequency of rotor current be f'.

Then, $N_s - N = \dfrac{120 f'}{P}$ Also, $N_s = \dfrac{120 f}{P}$

Diviading one by the other,

$$\frac{f'}{f} = \frac{N_s - N}{N_s} = S$$

$$\therefore f' = sf$$

Rotor currents have a frequency of $f' = sf$ and when flowing through the individual phases of rotor winding, give rise to rotor magnetic fields.

These individual rotor magnetic fields produce a combined rotating magnetic field, whose speed relative to rotor is

$$= \frac{120 f'}{P} = \frac{120 \, sf}{P} = S N_s$$

However, the rotor itself is running at speed N with respect to space.

$\therefore$ speed of the rotor field in space =

= Speed of rotor magnetic field relative to
rotor + speed of rotor relative to space.

$$= S N_s + N$$
$$= S N_s + N_s (1-s)$$
$$= N_s$$

It means that, no matter what the value
of slip, rotor currents and stator currents
each produce a magnetic field of constant
magnitude and constant speed of N_s.

Problems:

1) A slip-ring induction motor runs at 290 rpm
at full-load, when connected to 50 Hz supply.
Determine the number of Poles and slip.

Soln:

1) The stator of a 3-ϕ induction motor has 3
slots per pole per phase. If supply frequency
is 50 Hz. Calculate i) the number of stator
Poles produced and total number of slots on
the stator, ii) speed of the rotating stator flux
(or) magnetic field.

Soln.:

(i) $P = 2n$

P → number of Poles produced in rotating field

n → number of stator slots/pole/phase

$P = 2 \times 3 = 6$ Poles

Total No. of slots

$= 3$ slots/poles/phase $\times$ 6 pole $\times$ 3 Phase.

$= 54$

(ii) $N_s = \dfrac{120f}{P} = \dfrac{120 \times 50}{6} = 1000$ rpm.

2) A 3-φ induction motor is wound for 4 poles and is supplied from 50-Hz system calculate (i) The synchronous speed, (ii) the rotor speed, when slip is 4% and (iii) rotor frequency when rotor runs at 600 rpm.

Soln.:

(i) Synchronous speed

$N_s = \dfrac{120f}{P} = \dfrac{120 \times 50}{4} = 1500$ rpm

ii) Rotor speed:

$$N = N_s(1-s)$$
$$= 1500(1-0.04)$$
$$= 1440 \, rpm.$$

iii) Rotor current frequency:

When rotor speed is 600 rpm, slip

is, $\quad s = \dfrac{N_s - N}{N_s} = \dfrac{1500 - 600}{1500} = 0.6$

Rotor current frequency

$$f' = sf = 0.6 \times 50 = 30 \, Hz.$$

3) A 12-pole, 3 phase alternator driven at a speed of 500 rpm supplies power to an 8-pole 3 phase induction motor. If the slip of the motor at full-load is 3%, calculate the full-load speed of the motor.

Soln.:

Let $N \to$ actual motor speed

supply frequency $f = \dfrac{PN}{120} = \dfrac{12 \times 500}{120}$

$$= 50 \, Hz.$$

Synchronous Speed

$$N_s = \frac{120f}{P} = \frac{120 \times 50}{8} = 750 \text{ rpm}$$

% Slip $s = \frac{N_s - N}{N_s} \times 100$

$$3 = \frac{750 - N}{750} \times 100$$

$$\Rightarrow N = 727.5 \text{ rpm}$$

Development of torque:

In case of an induction motor, the torque is proportional to the product of flux per stator pole and the rotor current. (Similar to torque in DC motors).

However, there is one more factor that has to be taken into account i.e. Power factor of the rotor.

$$\therefore T \propto \phi I_2 \cos \phi_2$$

$$T = K \phi I_2 \cos \phi_2$$

where, $I_2 \rightarrow$ rotor current at stand still
$\phi_2 \rightarrow$ angle between rotor emf and rotor current

$k \rightarrow$ a Constant

Denoting rotor emf at standstill by E_2, we have that $E_2 \propto \phi$

$$\therefore T \propto E_2 I_2 \cos \phi_2$$

or $T = K_1 E_2 I_2 \cos \phi_2$

where, K_1 is another Constant.

From the above expression for torque, it is clear that as ϕ_2 increases (and hence $\cos \phi_2$ decreases) the torque decreases and vice versa.

Starting Torque:

The torque developed by the motor at the instant of starting is called starting torque.

Let, $E_2 \rightarrow$ rotor emf per phase at standstill

$R_2 \rightarrow$ rotor resistance/Phase.

$X_2 \rightarrow$ rotor reactance/phase at standstill

$$\therefore Z_2 = \sqrt{(R_2^2 + X_2^2)} \rightarrow \text{rotor impedance/phase at standstill}.$$

Then, $I_2 = \dfrac{E_2}{Z_2} = \dfrac{E_2}{\sqrt{R_2^2 + x_2^2}}$

and

$\cos\phi_2 = \dfrac{R_2}{Z_2} = \dfrac{R_2}{\sqrt{R_2^2 + x_2^2}}$

Starting torque $T_{st} = k_1 E_2 I_2 \cos\phi_2$

or $T_{st} = k_1 E_2 \cdot \dfrac{E_2}{\sqrt{R_2^2 + x_2^2}} \times \dfrac{R_2}{\sqrt{R_2^2 + x_2^2}}$

$= \dfrac{k_1 E_2^2 R_2}{R_2^2 + x_2^2}$

Now, $k_1 = \dfrac{3}{2\pi N_s}$

$\therefore T_{st} = \dfrac{3}{2\pi N_s} \cdot \dfrac{E_2^2 R_2}{R_2^2 + x_2^2}$

where, $N_s \rightarrow$ synchronous speed in rps

If supply voltage v is constant, then the flux ϕ and hence E_2 are constant

$\therefore T_{st} = k_2 \cdot \dfrac{R_2}{R_2^2 + x_2^2} = k_2 \cdot \dfrac{R_2}{Z_2^2}$

where, $k_2 \rightarrow$ some other constant.

Condition for maximum starting torque:

It can be proved that starting torque is maximum when rotor resistance equals rotor reactance.

Now, $T_{st} = K_2 \cdot \dfrac{R_2}{R_2^2 + x_2^2}$

$$\dfrac{dT_{st}}{dR_2} = K_2 \cdot \left[\dfrac{1}{R_2^2 + x_2^2} - \dfrac{R_2(2R_2)}{(R_2^2 + x_2^2)^2} \right] = 0$$

$$\Rightarrow R_2^2 + x_2^2 = 2R_2^2$$

$$\therefore \boxed{R_2 = x_2}$$

Problems:

1) A 3-phase, 400V star-connected induction motor has a star-connected rotor with a stator to rotor turn ratio of 6.5. The rotor resistance and standstill reactance per phase to be are 0.05Ω and 0.25Ω respectively. What should be the value of external resistance per phase to be inserted in the rotor circuit to obtain maximum torque at starting and what will be rotor starting current with

this resistance?

Soln:

Here, $k = \frac{1}{6.5}$

because, transformation ratio k is

$$= \frac{\text{rotor turns/phase}}{\text{stator turns/phase}}$$

Stand still rotor emf /phase

$$E_2 = \frac{400}{\sqrt{3}} \times \frac{1}{6.5} = 35.5 \text{ volt}$$

Starting torque is maximum when,

$$R_2 = X_2$$

i.e. when $R_2 = 0.25\,\Omega$ in this case.

∴ Exter resistance /phase required

$$= 0.25 - 0.05 = 0.2\,\Omega$$

Rotor impedance /phase

$$= \sqrt{0.25^2 + 0.25^2} = 0.3535\,\Omega$$

Rotor current /phase,

$$I_2 = \frac{V}{2} = \frac{35.5}{0.3535} \approx 100\,A.$$

Torque under Running Conditions:

$$T \propto E_r I_r \cos \phi_2 \quad \text{(or)} \quad T \propto \phi I_r \cos \phi_2$$

where,

$E_r \to$ rotor emf/phase under running condition.

$I_r \to$ rotor current/phase " " "

Now. $E_r = SE_2$ & $X_r = SX_2$

$$\therefore I_r = \frac{E_r}{Z_r} = \frac{SE_2}{\sqrt{R_2^2 + (SX_2)^2}}$$

$$\cos \phi_2 = \frac{R_2}{\sqrt{R_2^2 + (SX_2)^2}}$$

$$\therefore T \propto \frac{S\phi E_2 R_2}{R_2^2 + (SX_2)^2} = \frac{K\phi \cdot S E_2 R_2}{R_2^2 + (SX_2)^2}$$

Also, $T = \frac{K_1 S E_2^2 R_2}{R_2^2 + (SX_2)^2} \qquad (\because E_2 \propto \phi)$

where, $K_1 \to$ another constant

$$= \frac{3}{2\pi N_s}$$

Hence, expression for torque becomes,

$$T = \frac{3}{2\pi N_s} \cdot \frac{SE_2^2 R_2}{R_2^2 + (SX_2)^2}$$

$$= \frac{3}{2\pi N_s} \cdot \frac{SE_2^2 R_2}{Z_r^2}$$

At standstill, when s=1, obviously,

$$T_{st} = \frac{K_1 E_2^2 R_2}{R_2^2 + X_2^2} \qquad (or)$$

$$= \frac{3}{2\pi N_s} \cdot \frac{E_2^2 R_2}{R_2^2 + X_2^2} \qquad \text{same as previous discussion.}$$

Condition for maximum torque under Running Condition:

The torque of a rotor under running Conditions is

$$T = \frac{K \phi SE_2 R_2}{R_2^2 + (SX_2)^2} = K_1 \cdot \frac{SE_2^2 R_2}{R_2^2 + (SX_2)^2}$$

The condition for maximum torque may be obtained by differentiating the above expression with respect to slip s and then

equating it to zero.

However, it is simpler to put $y = \frac{1}{T}$
and then differentiate it.

$$y = \frac{R_2^2 + (SX_2)^2}{K\phi SE_2 R_2}$$

$$= \frac{R_2}{K\phi SE_2} + \frac{SX_2^2}{K\phi E_2 R_2}$$

$$\frac{dy}{ds} = \frac{-R_2}{K\phi S^2 E_2} + \frac{X_2^2}{K\phi E_2 R_2} = 0$$

$$\frac{R_2}{K\phi S^2 E_2} = \frac{X_2^2}{K\phi E_2 R_2}$$

$$\Rightarrow R_2^2 = S^2 X_2^2 \quad (or) \quad \boxed{R_2 = SX_2}$$

Hence, torque under running condition
is maximum at that value of slip s which
makes rotor reactance per phase equal to
rotor resistance per phase.

Slip corresponding to maximum torque,

$$S = \frac{R_2}{X_2}$$

Problems:

1) A 3-phase, slip-ring induction motor with star-connected rotor has an induced emf of 120V between slip-rings at standstill with normal voltage applied to the stator. The rotor winding has a resistance per phase of 0.3 ohm and standstill leakage reactance per phase of 1.5 ohm. Calculate (i) rotor current/phase when running short-circuited with 4 percent slip and (ii) the slip and rotor current per phase when the rotor is developing maximum torque.

Soln.:

Induced emf/phase under running condition,
$$E_r = sE_2 = 0.04 \times \frac{120}{\sqrt{3}} = 2.77V$$

rotor reactance/phase,
$$X_r = sX_2 = 0.04 \times 1.5 = 0.06\,\Omega$$

Rotor impedance/phase
$$Z_r = \sqrt{0.3^2 + 0.06^2} = 0.306\,\Omega$$

① Rotor current/phase $= \dfrac{E_r}{Z_r} = \dfrac{2.77}{0.306} = 9A$

(ii) For developing maximum torque,

$$R_2 = S X_2$$

$$S = \frac{R_2}{X_2} = \frac{0.3}{1.5} = 0.2$$

$$X_r = S X_2 = 0.2 \times 1.5 = 0.3 \,\Omega$$

$$Z_r = \sqrt{0.3^2 + 0.3^2} = 0.42 \,\Omega$$

∴ Rotor current /phase $= \dfrac{E_r}{Z_r}$

$$E_r = S E_2 = 0.2 \times \frac{120}{\sqrt{3}} = 13.86 \text{ V}$$

Rotor current /phase

$$= \frac{13.86}{0.42} = 33 \text{ A}$$

2) Calculate the torque exerted by an 8-pole, 50 Hz, 3 phase induction motor operating with a 4 percent slip which develops a maximum torque of 150 kg-m at a speed of 660 rpm. The resistance per phase of the rotor is 0.5 Ω.

Soln.:

$$N_S = \frac{120 f}{P} = \frac{120 \times 50}{8} = 750 \text{ rpm.}$$

Speed at maximum torque = 660 rpm

Corresponding slip, $s = \dfrac{750-660}{750} = 0.12$

For maximum torque,

$$R_2 = SX_2$$

$$\therefore X_2 = \dfrac{R_2}{S} = \dfrac{0.5}{0.12} = 4.167\,\Omega$$

we know that,

$$T_{max} = K\phi E_2 \cdot \dfrac{S}{2R_2}$$

$$= K\phi E_2 \cdot \dfrac{0.12}{2 \times 0.5} = 0.12\, K\phi E_2 \quad -①$$

when slip is 4%,

$$Torque,\ T = \dfrac{K\phi E_2 \cdot SR_2}{R_2^2 + (SX_2)^2}$$

$$= K\phi E_2 \cdot \dfrac{0.04 \times 0.5}{0.5^2 + (0.04 \times 4.167)^2}$$

$$= \dfrac{0.02\, K\phi E_2}{0.2778} \quad -②$$

② ÷ ①

$$\therefore \dfrac{T}{T_{max}} = \dfrac{T}{150} = \dfrac{0.02}{0.2778 \times 0.12}$$

$$\therefore T = 90 \text{ kg-m}.$$

Relation between Torque and Slip:

A family of torque/slip curves is shown in fig. below for a range of $s = 0$ to $s = 1$ with R_2 as the parameter.
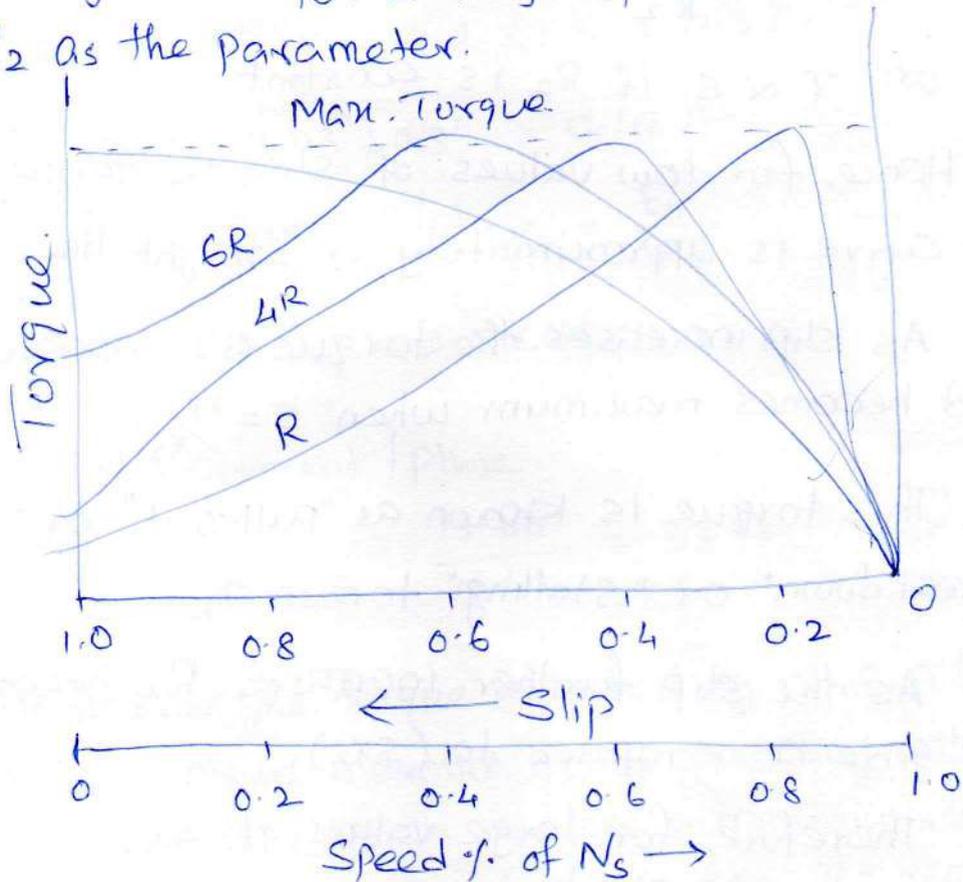
We know that,

$$T = \frac{K\phi s E_2 R_2}{R_2^2 + (sX_2)^2}$$

It is clear that, when $s = 0$, $T = 0$ hence

the curve starts from point O.

At normal speeds, close to synchronism, the term (sx_2) is small and hence negligible

$$\therefore T \alpha \frac{s}{R_2}$$

or $T \alpha s$ if R_2 is constant

Hence, for low values of slip, the torque/ slip curve is approximately a straight line.

As slip increases, the torque also increases and becomes maximum when $s = \frac{R_2}{x_2}$.

This torque is known as "pull-out" or "breakdown" or "stalling" torque T_b.

As the slip further increases, R_2 becomes negligible as compared to (sx_2).

Therefore, for large values of slip,

$$T \alpha \frac{s}{(sx_2)^2} \alpha \frac{1}{s}$$

Hence, the torque/slip curve is a rectangular hyperbola.

So, beyond the point of maximum torque,

any further increase in motor load results in decrease of torque developed by the motor.

The result is that the motor slows down and eventually stops.

Torque-speed characteristics:

The torque developed by a conventional 3-phase induction motor depends on its speed but the relation between the two cannot be represented by a simple equation.

It is easier to show the relationship in the form of a curve.
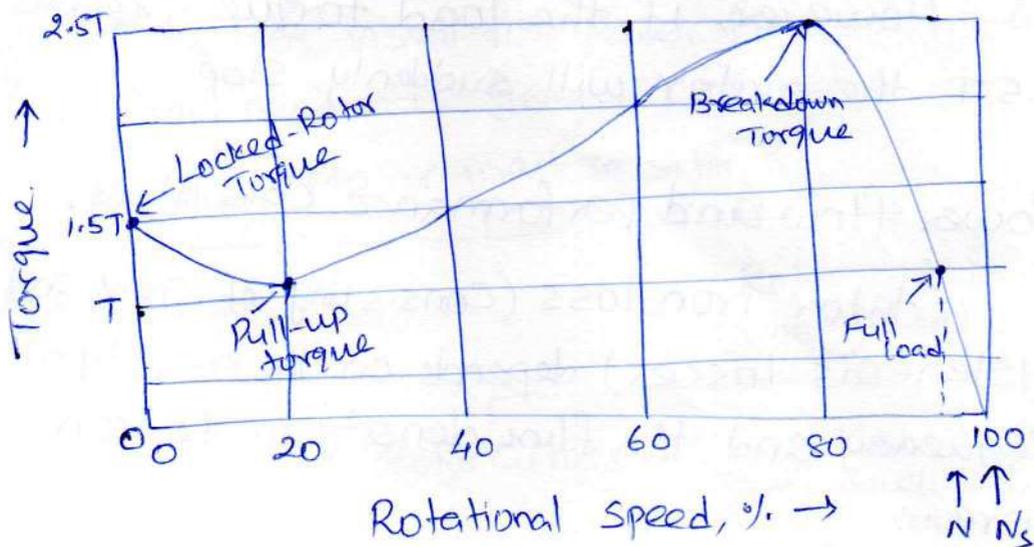
Here, T represents the nominal full-load torque of the motor.

As seen, the starting torque (at $N = 0$) is $1.5T$ and the maximum torque (also called breakdown torque) is $2.5T$

At full-load, the motor runs at a speed N.

When mechanical load increases, motor speed decreases till the motor torque again becomes equal to the load torque.

As long as the two torques are in balance, the motor will run at constant (but lower) speed.

However, if the load torque exceeds $2.5T$, the motor will suddenly stop.

Power flow and performance Calculations:

Stator iron loss (Consisting of eddy and hysteresis losses) depends on the supply frequency and the flux density in the iron core.

It is practically constant.

The iron loss of the rotor is, however negligible because frequency of rotor current under normal running conditions is always small.

$$\text{Total rotor Cu loss} = 3J_2^2 R_2$$

Different stages of power development in an induction motor are as under:
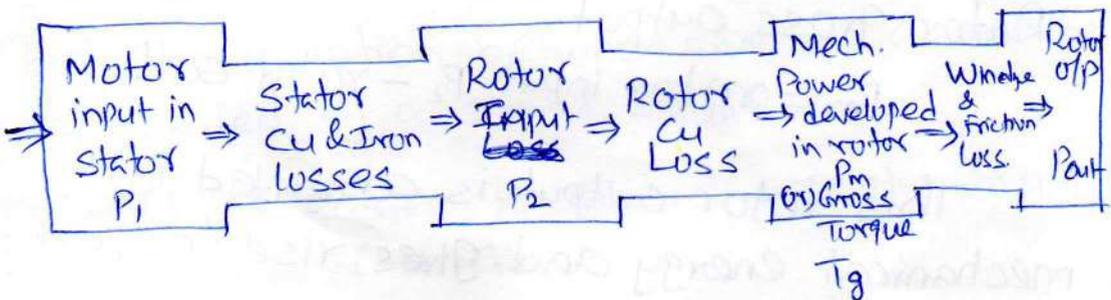
A better visual for power flow, within an induction motor is given in fig. below.
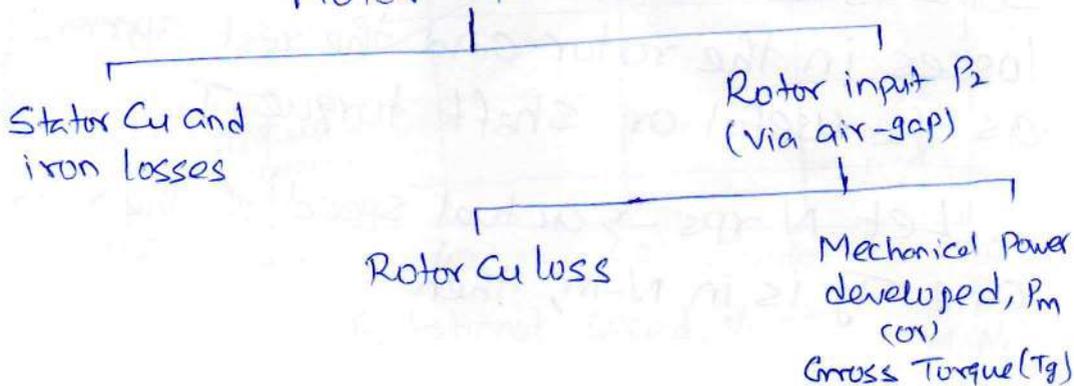
```
                              |
              ┌───────────────┴────────────────┐
         Windage and                      Rotor output
         friction loss                    (or) motor
                                          output (P_out)
```

Stator input P_1 = Stator output + Stator losses.

    The stator output is transferred entirely inductively to the rotor circuit

    Obviously, rotor input

$$P_2 = \text{Stator output}$$

Rotor gross output,

$$P_m = \text{rotor input } P_2 - \text{rotor Cu loss}$$

    This rotor output is converted into mechanical energy and gives rise to gross torque T_g.

    Out of this gross torque developed, some is lost due to windage and friction losses in the rotor and the rest appears as the useful or shaft torque T_{sh}.

    Let N rps → actual speed of the rotor and T_g is in N-m, then

$Tg \times 2\pi N$ = rotor gross output in watts, P_m

$\therefore Tg = \dfrac{\text{rotor gross output in watts, } P_m}{2\pi N}$ N-m

                                 —①

   If there were no Cu losses in the rotor, then rotor output will equal rotor input and the rotor will run at synchronous speed.

$$Tg = \dfrac{\text{rotor input } P_2}{2\pi N_s} \quad —②$$

From ① & ②, we get,

Rotor gross output $P_m = Tg \cdot \omega = Tg \times 2\pi N$ —③

Rotor input $P_2 = Tg \cdot \omega_s = Tg \times 2\pi N_s$ — ④

   The difference of these two equals rotor Cu loss.

$\therefore$ rotor Cu loss = $P_2 - P_m$

               $= Tg \times 2\pi(N_s - N) \quad — ⑤$

From ④ & ⑤,

$$\dfrac{\text{Rotor Cu loss}}{\text{Rotor input}} = \dfrac{N_s - N}{N_s} = S$$

∴ rotor Cu loss = s × rotor input

$$= S P_2$$

Also, rotor input = $\dfrac{\text{rotor cu loss}}{S}$

Rotor gross output,

P_m = input P_2 − rotor cu loss

= input P_2 − (s × rotor input)

= (1 − s) rotor input P_2

⟹ $\dfrac{\text{Rotor gross output}}{\text{rotor input}} = 1 - S = \dfrac{N}{N_s}$

∴ $\dfrac{P_m}{P_2} = \dfrac{N}{N_s}$

∴ Rotor efficiency = $\dfrac{N}{N_s}$

Problems:

1) A 440V, 3−ϕ, 50 Hz, 4-Pole Y-connected induction motor has a full-load speed of 1425 rpm. The rotor has an impedance of (0.4+j4) ohm and rotor/stator turn ratio of 0.8. Calculate (i), full-load torque, (ii) rotor

Current and full-load rotor Cu loss, (iii).
Power output if windage and friction losses
amount to 500W, (iv) maximum torque, (🔸).
and speed at which it occurs, (v). Starting
Current and (vi) starting torque.

Soln.:

$$N_s = \frac{120f}{P} = \frac{120 \times 50}{4} = 1500 \text{ rpm}$$

$$= 25 \text{ rps}$$

$$\text{Slip, } s = \frac{1500 - 1425}{1500} = 0.05$$

$$E_1 = \frac{440}{\sqrt{3}} = 254 \text{ V/Phase}.$$

(i). Full-load torque.

$$T_f = \frac{3}{2\pi N_s} \times \frac{SE_2^2 R_2}{R_2^2 + (SX_2)^2}$$

$$= \frac{3}{2\pi \times 25} \times \frac{0.05 (0.8 \times 254)^2 \times 0.4}{(0.4)^2 + (0.05 \times 4)^2}$$

$$(\because E_2 = kE_1)$$

$$T_f = 78.87 \text{ N-m}.$$

ii) Rotor Current,

$$I_r = \frac{SE_2}{\sqrt{R_2^2 + (Sx_2)^2}} = \frac{SKE_1}{\sqrt{R_2^2 + (Sx_2)^2}}$$

$$= \frac{0.05 \times (0.8 \times 254)}{\sqrt{(0.4)^2 + (0.05 \times 4)^2}} = 22.73 \, A$$

Total cu loss

$$= 3I_r^2 R_2$$

$$= 3 \times 22.73^2 \times 0.4 = 620 \, W$$

(iii)

$$P_m = \frac{2\pi NT}{60}$$

$$= \frac{2\pi \times 1425 \times 78.87}{60} = 11745 \, W$$

$$\therefore P_{out} = P_m - \text{windage and friction loss}$$

$$= 11745 - 500 = 11245 \, W$$

(iv). For maximum torque,

$$S = \frac{R_2}{X_2} = \frac{0.4}{4} = 0.1$$

$$T_{max} = \frac{3}{2\pi N_s} \times \frac{SE_2^2 R_2}{R_2^2 + (Sx_2)^2}$$

$$= \frac{3}{2\pi \times 25} \times \frac{0.1 \times (0.8 \times 254)^2 \times 0.4}{(0.4)^2 + (0.1 \times 4)^2}$$

I-⑪

$$= 98.5 \text{ N-m.}$$

Since, $s = 0.1,$

$$\text{Slip Speed} = S N_s = 0.1 \times 1500$$
$$= 150 \text{ rpm.}$$

∴ Speed for maximum torque

$$= 1500 - 150$$
$$= 1350 \text{ rpm.}$$

(v) Starting Current:

$$= \frac{E_2}{\sqrt{R_2^2 + X_2^2}} = \frac{0.8 \times 254}{\sqrt{0.4^2 + 4^2}} = 50.5 \text{ A}$$

(vi) Starting torque.

At Start, $s = 1.$

$$\therefore T_{st} = \frac{3}{2 \pi N_s} \cdot \frac{E_2^2 R_2}{R_2^2 + X_2^2}$$

$$= \frac{3}{2 \pi \times 25} \times \frac{(0.8 \times 254)^2 \times 0.4}{(0.4)^2 + 4^2}$$

$$= 19.5 \text{ N-m}$$

Note:
   As compared to full-load torque, the

starting torque is much less - almost 25 percent.

Equivalent circuit:

As in the case of a transformer, in induction motor also, the secondary values may be transferred to the primary and vice versa.

It should be remembered that when shifting impedance or resistance from secondary to primary, it should be divided by k^2, whereas current should be multiplied by k.

The equivalent circuit of an induction motor where all values have been referred to primary i.e. stator is shown in fig.
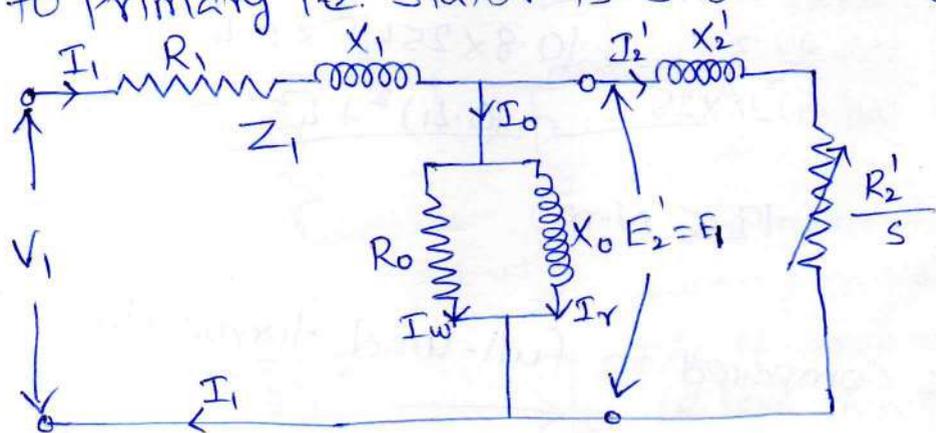
As shown in fig. below, the exciting circuit may be transferred to the left, because inaccuracy involved is negligible but the circuit and hence the calculations are very much simplified.

$$R_{01} = R_1 + R_2' \qquad X_{01} = X_1 + X_2'$$

$G_0 \rightarrow$ exciting Conductance
$B_0 \rightarrow$ exciting Susceptance
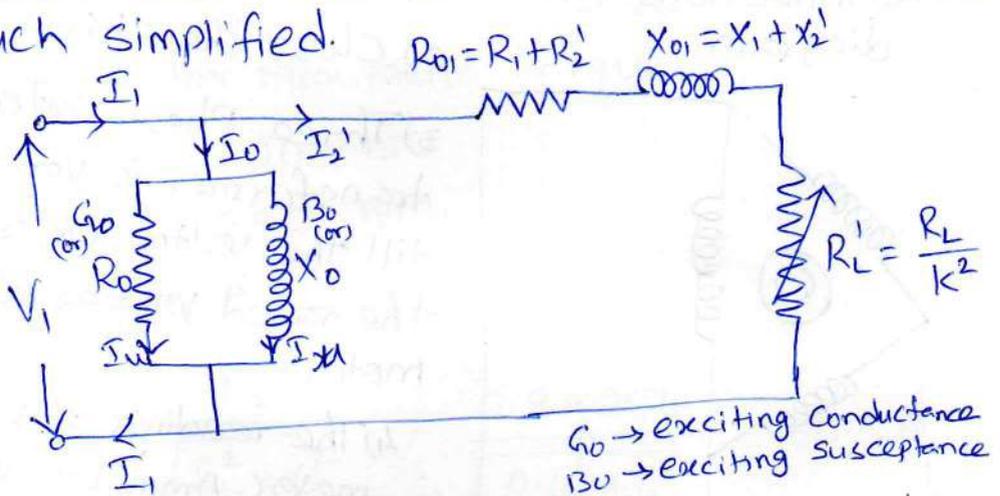
This is known as the approximate equivalent circuit of the induction motor.

Calculation of equivalent circuit parameters:

  1) No-load test

  2) Blocked rotor test

1) No-load test:

    In practice, it is neither necessary nor feasible to run the induction motor synchronously for getting G_0 and B_0.

Instead, the motor is run without any external mechanical load on it.
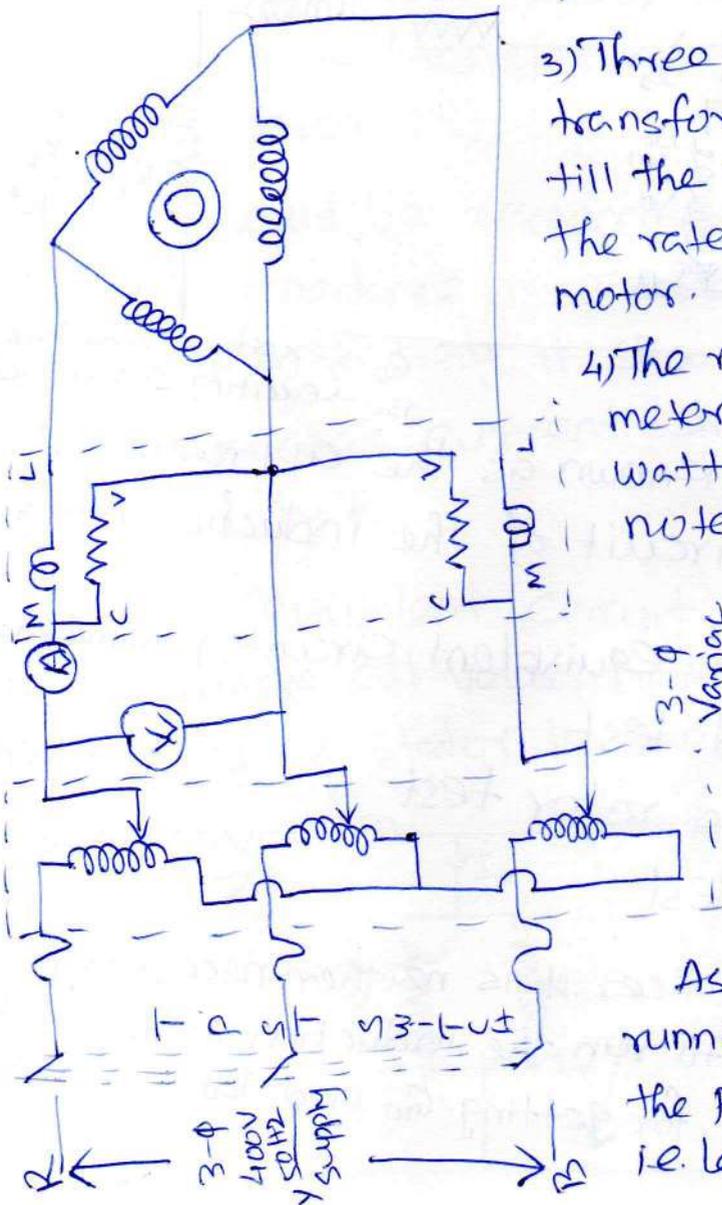
Procedure:
1) Connections are made as per the Circuit diagram.
2) Close the TPST Switch.
3) Three phase auto-transformer is varied till the voltmeter reads the rated voltage of the motor.
4) The readings of volt meter, Ammeter and wattmeter are noted and tabulated.
5) The motor is turned off after bringing back the 3-φ Variac to its initial position.

3-φ Variac

3-φ 400V 50Hz Supply

As motor is running on light load, the pf would be low i.e. less than 0.5.

I-⑫

No-load input power W_0 consists of

i) Stator cu. loss, $3I_0^2 R_1$

ii) Stator Core loss, $3G_0 V^2$

& iii) loss due to friction and windage.

The losses (ii) and (iii) are collectively known as fixed losses. because they are independent of load.

The readings of the total power Calculation of equivalent circuit parameters:

Let, $V \rightarrow$ applied voltage/phase

$I_0 \rightarrow$ motor current/phase.

$W \rightarrow$ Wattmeter reading i.e. input power.

Then, for 3-phase induction motor,

$$W = 3 G_0 V^2$$

(or) $G_0 = \dfrac{W}{3V^2}$

Also, $I_0 = V Y_0$

where, $Y_0 \rightarrow$ exciting admittance of the motor.

$$Y_0 = \dfrac{I_0}{V}$$

$$B_0 = \sqrt{(Y_0^2 - G_0^2)}$$

$$= \sqrt{(I_{0N})^2 - G_0^2}$$

Hence, G_0 and B_0 can be found.

Additionally, ϕ_0 can also be found from the relation,

$$W_0 = \sqrt{3} \, V_L \, I_0 \, \cos \phi_0$$

$$\therefore \cos \phi_0 = \frac{W_0}{\sqrt{3} \, V_L I_0}$$

where, $V_L \to$ line voltage

$W_0 \to$ no-load stator input.

Problems:

1) In a no-load test, an induction motor took 10A and 450 watts with a line voltage of 110V. If stator resistance/phase is 0.05Ω and friction and windage losses amount to 135 watts, calculate the exciting conductance and susceptance/phase.

Soln.:

Stator Cu. loss $= 3 I_0^2 R_1$

$= 3 \times 10^2 \times 0.05 = 15 \, W$

$\therefore$ Stator Core loss $= 450 - 135 - 15$

$$= 300 \text{ W}$$

$$\text{Voltage/phase } V = \frac{110}{\sqrt{3}}$$

$$\text{Core loss} = 3 \, G_0 V^2$$

$$300 = 3 \, G_0 \left(\frac{110}{\sqrt{3}}\right)^2$$

$$\Rightarrow G_0 = 0.025 \text{ siemens/phase.}$$

$$Y_0 = \frac{I_0}{V} = 10 \times \frac{\sqrt{3}}{110} = 0.158 \text{ siemens/phase.}$$

$$B_0 = \sqrt{Y_0^2 - G_0^2}$$

$$= \sqrt{(0.158^2 - 0.025^2)} = 0.156 \text{ siemens/phase.}$$

2)

Blocked Rotor test:

This test is used to find,

i) short-circuit current with normal voltage applied to stator

ii) Power factor on short-circuit

iii) Total leakage reactance X_{01} of the motor as referred to Primary (ie. stator)

iv) Total resistance R_{01} of the motor as referred to Primary. (stator).

Procedure:

1) Connections are made as per the circuit diagram

2) Close the TPST switch.

3) Three phase auto-transformer is slowly varied till the ammeter reads the rated full-load current while blocking the rotor

4) The readings of ammeter, voltmeter and wattmeter are noted and tabulated

5) The motor is turned off after bringing back the 3-ϕ auto-transformer to its initial Position.

Calculations:

If V is normal stator voltage, V_s is the Short-circuit voltage, then Short-circuit or Standstill rotor current with normal voltage to stator is given by,

$$I_{SN} = I_s \left(\frac{V}{V_s} \right)$$

where, $I_{SN} \rightarrow$ Short-circuit current obtainable with normal voltage.

$I_s \Rightarrow$ Short-circuit current with voltage V_s.

Power factor on short-circuit is found from,

$$W_s = \sqrt{3} \, V_{SL} \, I_{SL} \cos \phi_s$$

$$\Rightarrow \cos \phi_s = \frac{W_s}{\sqrt{3} \, V_{SL} \, I_{SL}}$$

where,

$W_s \rightarrow$ total power input on short-circuit

$V_{SL} \rightarrow$ line voltage on short-circuit

$I_{SL} \rightarrow$ line current on short-circuit

Now, the motor input on short circuit consists of

i) mainly stator and rotor Cu losses (Wcл)

ii) Core-loss (negligible) because the applied voltage is only a small percentage of the normal voltage. (Wcл)

∴ Total Cu. loss = $W_s - W_{CL}$

$$3 I_s^2 R_{01} = W_s - W_{CL}$$

$$\Rightarrow R_{01} = \frac{W_s - W_{CL}}{3 I_s^2}$$

The motor leakage reactance per phase X_{01} as referred to the stator may be calculated as follows:

$$Z_{01} = \frac{V_s}{I_s}$$

$$\therefore X_{01} = \sqrt{Z_{01}^2 - R_{01}^2}$$

Usually, X_1 is assumed equal to X_2' where X_1 and X_2 are stator and rotor reactances per phase respectively, as referred to stator.

$$X_1 = X_2' = \frac{X_{01}}{2}$$

If the motor has wound rotor, then stator and rotor resistances are separated by dividing R_{01} in the ratio of Dc resistances of stator and rotor windings.

In case of squirrel-cage rotor, R_1 is determined as usual and the effective rotor resistance (R_2') as referred to stator is given by

$$R_2' = R_{01} - R_1$$

Problem:

1) A 110V, 3-ϕ, star connected induction motor takes 25 A at a line voltage of 30V with rotor locked. With this line voltage, Power input to motor is 440W and Core loss is 40W. The DC resistance between a pair of stator terminals is 0.1Ω. If the ratio of AC to DC resistance is 1.6, find the equivalent leakage reactance/phase of the motor and the Stator and rotor resistance per phase.

Soln.:

Short circuit voltage/phase

$$V_s = \frac{30}{\sqrt{3}} = 17.3 \text{ V}$$

$I_s = 25 A$ per phase.

$$Z_{01} = \frac{17.3}{25} = 0.7\Omega \text{ (approx)} \text{ per phase}$$

Stator and rotor cu. losses

= input - Core losses

= 440 - 40

= 400 W

$$\therefore \ 3 \times 25^2 \times R_{01} = 400$$

$$\Rightarrow \ R_{01} = \frac{400}{3 \times 625} = 0.21 \, \Omega$$

Leakage reactance /phase

$$X_{01} = \sqrt{(0.7^2 - 0.21^2)} = 0.668 \, \Omega$$

DC resistance /phase of stator $= \frac{0.1}{2} = 0.05 \, \Omega$

AC resistance /phase $R_1 = 0.05 \times 1.6$
$$= 0.08 \, \Omega$$

Hence, effective resistance /phase of rotor as referred to stator

$$R_2' = 0.21 - 0.08$$
$$= 0.13 \, \Omega$$

——————×——————

End of Unit - I.

Important 10 Marks questions:

1. Explain the construction and principle of operation of 3-phase induction motor with neat sketch.

2. Elaborately explain how a rotating magnetic field is produced in the air-gap between stator and rotor of a three phase induction motor.

3. Define the term slip and explain its importance in obtaining rotor current frequency

4. Draw the power flow diagram of a 3phase induction motor and explain each stage?

5. Explain the principle of operation of 3-phase induction motor and explain how the rotating magnetic field is produced by 3-phase currents.

6. Derive an expression for the torque of an induction motor and obtain the condition for maximum torque.

Unit - II
Starting And Speed Control Methods

Introduction:

The performance characteristics of an induction motor are derivable from a circular locus.

The data necessary to draw the circle diagram may be found from no-load and blocked-rotor tests.

Predetermination of performance characteristics using circle diagram:

Construction:

Circle diagram of an induction motor can be drawn by using the data obtained from

1. no-load test
2. Blocked rotor test and
3. Stator resistance test as shown below.

Step 1:

From no-load test, I_0 and ϕ_0 can be calculated.

Hence as shown in fig; vector for I_0 can be laid off lagging ϕ_0 behind the applied voltage V.
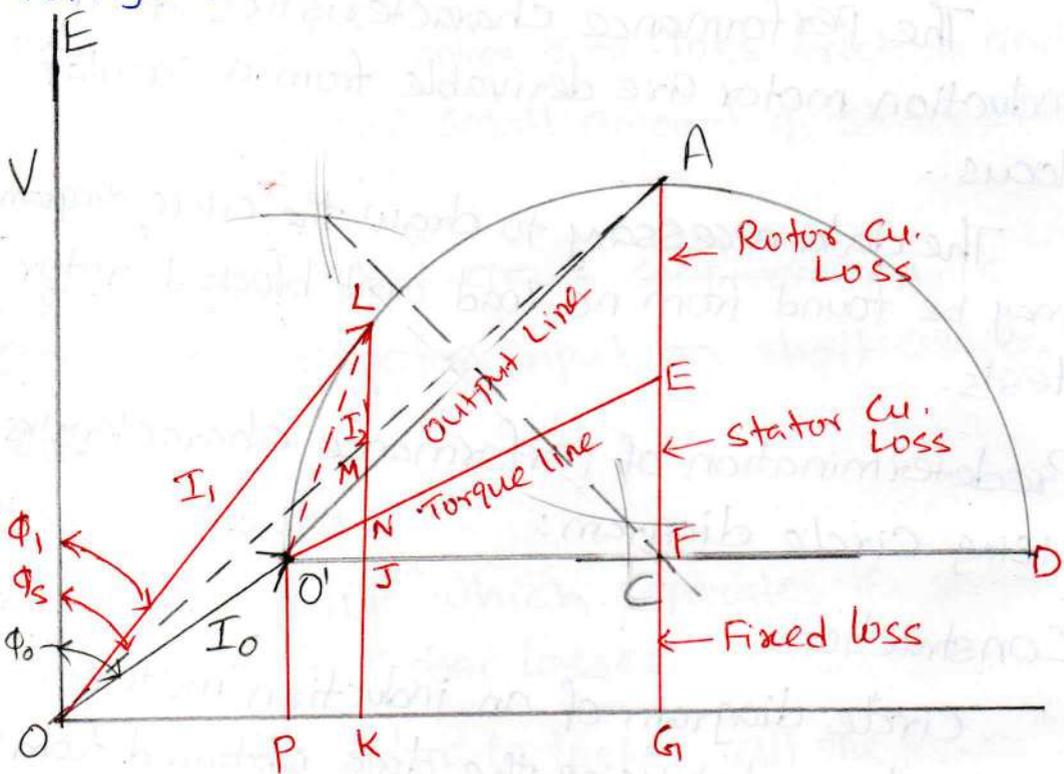
Step 2:

Next, from blocked rotor test, short-circuit current I_{SN} corresponding to normal voltage and ϕ_s are found.

The vector OA represents $I_{SN} = \dfrac{I_s V}{V_s}$ in magnitude and Phase

Vector O'A represents rotor current I_2'

as referred to stator.

Clearly, the two points O'and A lie on the required circle.

For finding the center C of this circle, chord O'A is bisected at right angles - its bisector giving point C.

The diameter O'D is drawn perpendicular to the voltage vector.

As a matter of practical contingency, it is recommended that the scale of current vectors should be so chosen that the diameter is <u>more than 25 cm</u>, in order that the performance data of the motor may be read with reasonable accuracy from the circle diagram.

With center C and radius = CO', the circle can be drawn.

The line O'A is known as <u>output line</u>.

Note:
As the voltage vector is drawn vertically, all vertical distances represent the active or

Power or energy components of the currents.

For example,

1) O'P of no-load current OO' represents no-load input (supplies core-loss, friction and windage loss and small amount of stator I^2R loss).

2) AG of short-circuit Current Component OA is Proportional to motor-input on short circuit.

Step 3:

Torque line:

This is line which separates the stator and the rotor copper losses.

When the rotor is locked, all the power supplied to the motor goes to meet core losses and Cu. losses in stator and rotor windings.

The power input is proportional to AG.

Out of this, FG (=O'P) represents fixed losses.

AF is proportional to sum of stator and rotor Cu. loss.

The Point E is such that,

$$\frac{AE}{EF} = \frac{Rotor\ Cu.loss}{Stator\ Cu.loss.}$$

For Squirrel-cage rotor,

Stator resistance/phase R_1 is found from Stator resistance test,

Now,

Stator Cu-loss = $3I_s^2 R_1$

$\therefore$ rotor Cu-loss = $W_s - 3I_s^2 R_1$,

where, $W_s \rightarrow$ Short-Circuit motor input

$$\therefore \frac{AE}{EF} = \frac{W_s - 3I_s^2 R_1}{3I_s^2 R_1}$$

As said earlier, the line O'E is known as torque line.

Let us assume that the motor is running and taking a current OL, then, the perpendicular

JK → fixed losses   NL → rotor input

JN → Stator Cu. loss   NM → rotor Cu. loss.

ML → rotor output and
Lk → the total motor input.

Hence, it is seen that, at least, theoretically, it is possible to obtain all the characteristics of an induction motor from its circle diagram..

Load test (Brake test):

The performance of a three phase induction motor may be determined by applying mechanical load (Belt Brake) directly on the rotor.

Circuit Diagram:
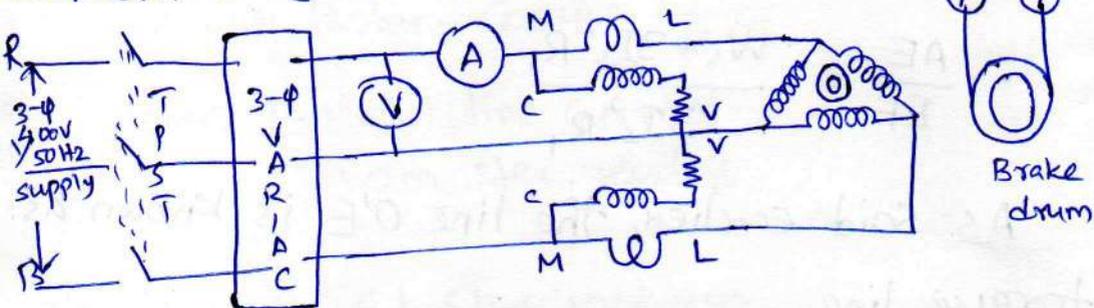
Procedure:

1. Connections are made as per the circuit diagram.

2. After observing the precautions, the TPST switch is closed and the motor is started with

the help of auto-transformer Starter.

3. The readings on voltmeter, ammeter, wattmeter on no-load are noted and tabulated, while S_1 and S_2 are each = 0.

4. The load is applied gradually in steps by tightening the belt brake and the corresponding meter readings, speed and spring balance readings S_1 and S_2 are noted and tabulated.

5. The load is adjusted up to the full-load Current of the motor.

6. After gradually reducing the load, the motor is stoped by bringing back the auto-transformer to its original position.

7. Switch off the motor by TPST switch.

8. The Circumference of the pulley is measured and the radius of the pulley is determined such that $R = Circumference/2\pi$ mts.

Tabulation:

Sl-No.	Input(V) Voltage	Line (A) Current	Input (W) Power	Speed N	S_1	S_2	S $\frac{S}{S_1\sim S_2}$	Torque	o/p Powr	η%

Calculations:

Net force $= S = S_1 - S_2$ kgr.
$$= 9.81 (S_1 - S_2) \, Nw.$$

Torque $T = 9.81 (S). R \, N-m.$

where $R \rightarrow$ radius of brake drum

Output power $P_0 = \dfrac{2 \pi N T}{60}$ Watts

Input power $P_i =$ Wattmeter reading

% efficiency $(\eta) = \dfrac{\text{output power } (P_0)}{\text{input power } (P_i)} \times 100$

Input power $P_i = \sqrt{3} \, V I \cos \phi$

$\therefore$ power factor $= \cos \phi = \dfrac{P_i}{\sqrt{3} \, V I}$

where, $I \rightarrow$ input line current
$\rightarrow$ ammeter reading

~~Starting of Induction motors:~~

Problems:

1) A 3-Ph, 400V induction motor gave the
following test readings;

No load: 400V, 1250 W, 9A

Short-Circuit: 150 V, 38 A, 4 kW. Draw the circle diagram. If the normal rating is 14.9 kW, find from the circle diagram, the full-load value of current, p.f. and slip.

Soln.:

$$\cos \phi_0 = \frac{1250}{\sqrt{3} \times 400 \times 9} = 0.2004$$

$$\phi_0 = 78.5°$$

$$\cos \phi_s = \frac{4000}{\sqrt{3} \times 150 \times 38} = 0.405$$

$$\phi_s = 66.1°$$

Short-circuit current with normal voltage is,
$$I_{SN} = 38 \left(\frac{400}{150}\right) = 101.3 \text{ A}$$

Power taken would be $= 4000 \left(\frac{400}{150}\right)^2$
$$= 28440 \text{ W}$$

In fig, OO' represents $I_0 = 9A$.

If current scale is 1 cm = 5 A, then vector

$OO' = \frac{9}{5} = 1.8$ cm.

and is drawn at an angle of $\phi_0 = 78.5°$ with the vertical OV.

Similarly,

OA represents $I_{SN} = 101.3$ A

It measures $\frac{101.3}{5} = 20.26$ Cm

and is drawn at an angle of 66.1° with the vertical OV.

Line O'D is drawn parallel to OX.

NC is the right-angle bisector of O'A.

The semi circle O'AD is drawn with C as center and CO' as radius.

This semi-circle is the locus of current vector for all load conditions from no-load to Short-circuit.

Now, AF represents 28,440 W and measures 8.1 cm.

Hence, Power scale becomes

$$1 \text{ cm} = \frac{28440}{8.1} = 3510 \text{ W}$$

Now,

full-load motor output = 14.9×10^3
$$= 14,900 \text{ W}.$$

∴ The intercept between the semi-circle and output line O'A should measure

$$= \frac{14,900}{3510} = 4.25 \text{ cm}.$$

For locating full-load point P, BA is extended. to S.

AS is made equal to 4.25 cm and SP is drawn parallel to output line O'A.

PL is perpendicular to OX.

Line current = OP = 6 cm.

$$= 6 \times 5 = 30 \text{ A}$$

Angle $\phi = 30°$ (by measurement)

$\therefore Pf = \cos 30° = 0.886$

(or) $\cos \phi = \dfrac{PL}{OP} = \dfrac{5.2}{6} = 0.\overset{865}{\cancel{886}}$

Now, Slip $= \dfrac{\text{rotor cu. loss}}{\text{rotor input}}$

In fig., EK represents rotor cu. loss and PK represents rotor input

$\therefore$ Slip $= \dfrac{EK}{PK} = \dfrac{0.3}{4.5} = 0.067$ (or) 6.7%.

Starting of Induction Motors:

Induction motors, when direct-switched take five to seven times their full-load current and develop only 1.5 to 2.5 times their full-load torque.

This initial excessive current is objectionable because it will produce large

line-voltage drop that, in turn, will affect the operation of other electrical equipment connected to the same lines.

We know that, the starting torque of an induction motor can be improved by increasing the rotor resistance.

The initial in-rush current can also be controlled by applying a reduced voltage to the stator during starting period.

Full normal voltage being applied when the motor has run up to speed.

Rheostat/reactor starter:

The purpose of this starter is to drop some voltage and hence reduce the voltage applied across the motor terminals.

Hence, the initial current drawn by the motor is reduced.

However, it should be noted that, whereas current varies directly as the

Voltage, the torque varies as square of applied voltage.

By using Primary resistors (or) rheostt or reactors as shown in fig., the applied voltage/phase can be reduced by a fraction 'x'.
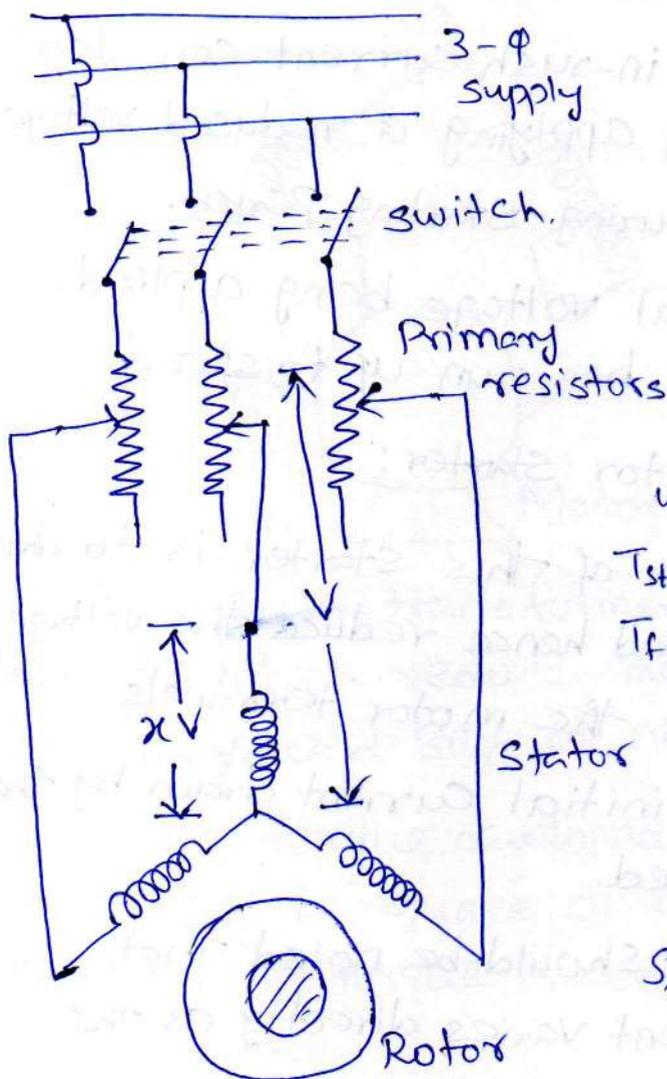
$$I_{st} = x \cdot I_{sc}$$

and

$$T_{st} = x^2 T_{sc}.$$

As we know,

$$\frac{T_{st}}{T_f} = \left(\frac{I_{st}}{I_f}\right)^2 . S_f$$

where,

$T_{st} \rightarrow$ Starting torque

$T_f \rightarrow$ full-load torque

$I_{st} \rightarrow$ Starting current

$I_f \rightarrow$ normal full-load current

$S_f \rightarrow$ full-load slip

3-ϕ Supply

switch.

Primary resistors

Stator

$x \downarrow \uparrow$

Rotor

$$\therefore \frac{T_{st}}{T_f} = \left(\frac{x \cdot I_{sc}}{I_f} \right)^2 \cdot S_f$$

$$= x^2 \left(\frac{I_{sc}}{I_f} \right)^2 \cdot S_f \qquad = x^2 \cdot a^2 \cdot S_f$$

where,

$$a = \frac{I_{sc}}{I_f}$$

$I_{sc} \rightarrow$ short-circuit current

$\therefore$ It is observed that, the ratio of the starting torque to full-load torque is x^2 of that of direct switching.

This method is useful for the smooth starting of small machines only.

Auto-transformer starter :

Fig. shows a 3-phase star-connected auto-transformer used for applying reduced voltage across motor stator during starting period.

3 φ supply — R, Y, B

Auto-transformer

T2

T1

T3
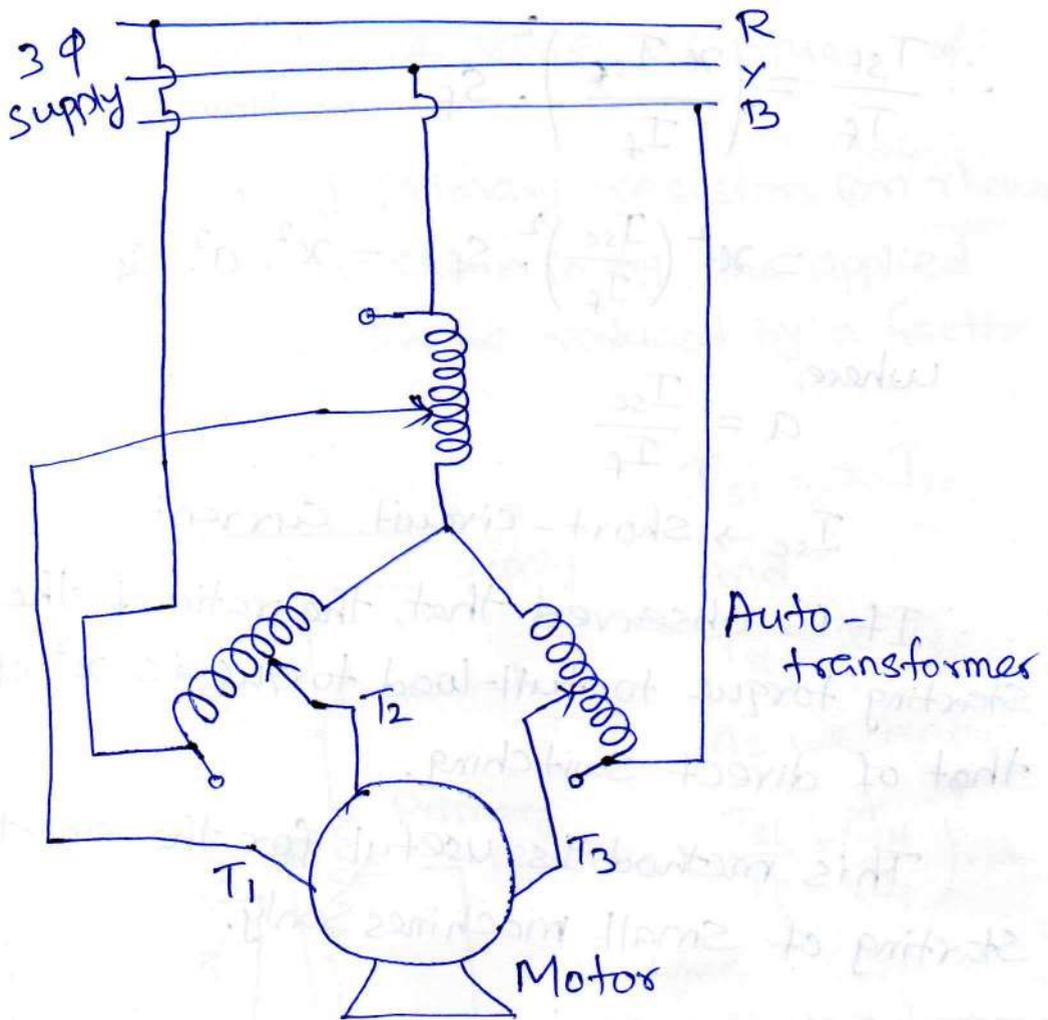
Motor

The auto-transformer usually has tappings which reduce the line voltage to 50%, 65% and 80% of normal value.

Since, torque developed by the motor varies as the square of applied voltage, the starting torque is considerably reduced.

Generally, a double-throw switch is used for connecting the auto-transformer in the circuit for starting purpose.

After the motor has run up to speed, the switch is moved into RUN position which connects the motor directly to the supply.

Star-delta Starter:

In this method, the stator winding of the motor is connected in star for starting and in delta for normal running.

3-φ supply
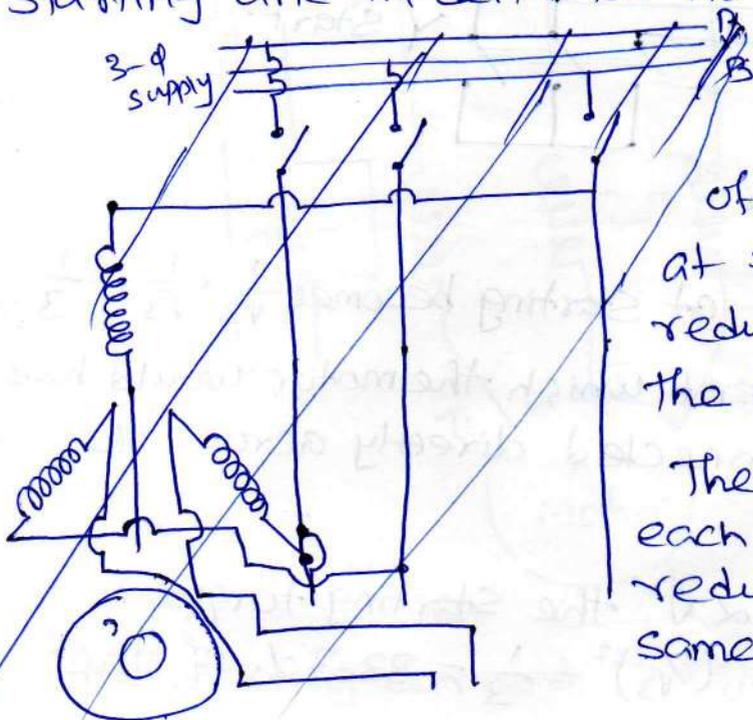
The voltage of each phase at starting is reduced to $\frac{1}{\sqrt{3}}$ of the line voltage.

The current in each phase is also reduced by the same factor so that
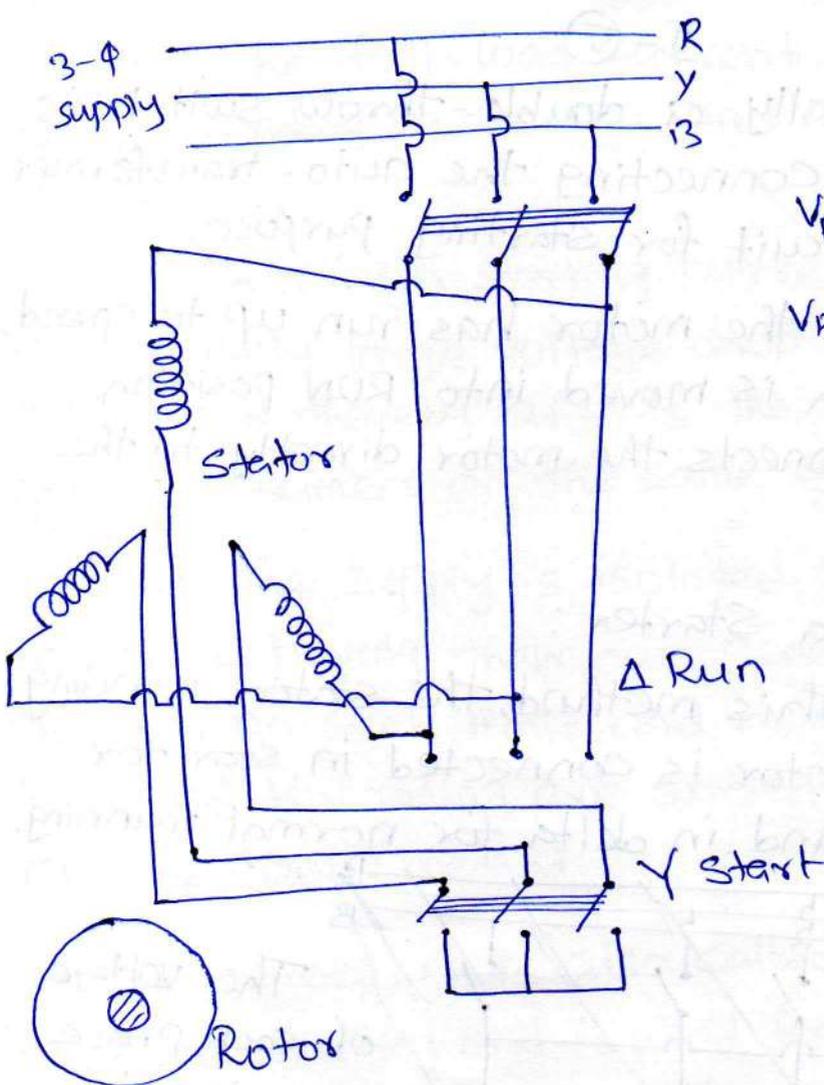
$V_{ph} = \frac{1}{\sqrt{3}} V_L \rightarrow$ Star

$V_{ph} = V_L \rightarrow$ Delta.

3-ϕ supply

R
y
B

Stator

Δ Run

Y start

Rotor

line current at starting becomes $\frac{1}{\sqrt{3}} \times \frac{1}{\sqrt{3}} = \frac{1}{3}$
of the current which the motor would have
taken if connected directly across the
supply.

Since, $T \propto V^2$, the starting torque is
reduced to $(\frac{1}{\sqrt{3}})^2 = \frac{1}{3} = 33.3\%$. of the

normal value.

   The change-over from star-starting to delta-running is made by a double-throw switch with inter-locks to prevent motor starting with the switch in the RUN position.

Direct-on-line (DOL) starter:
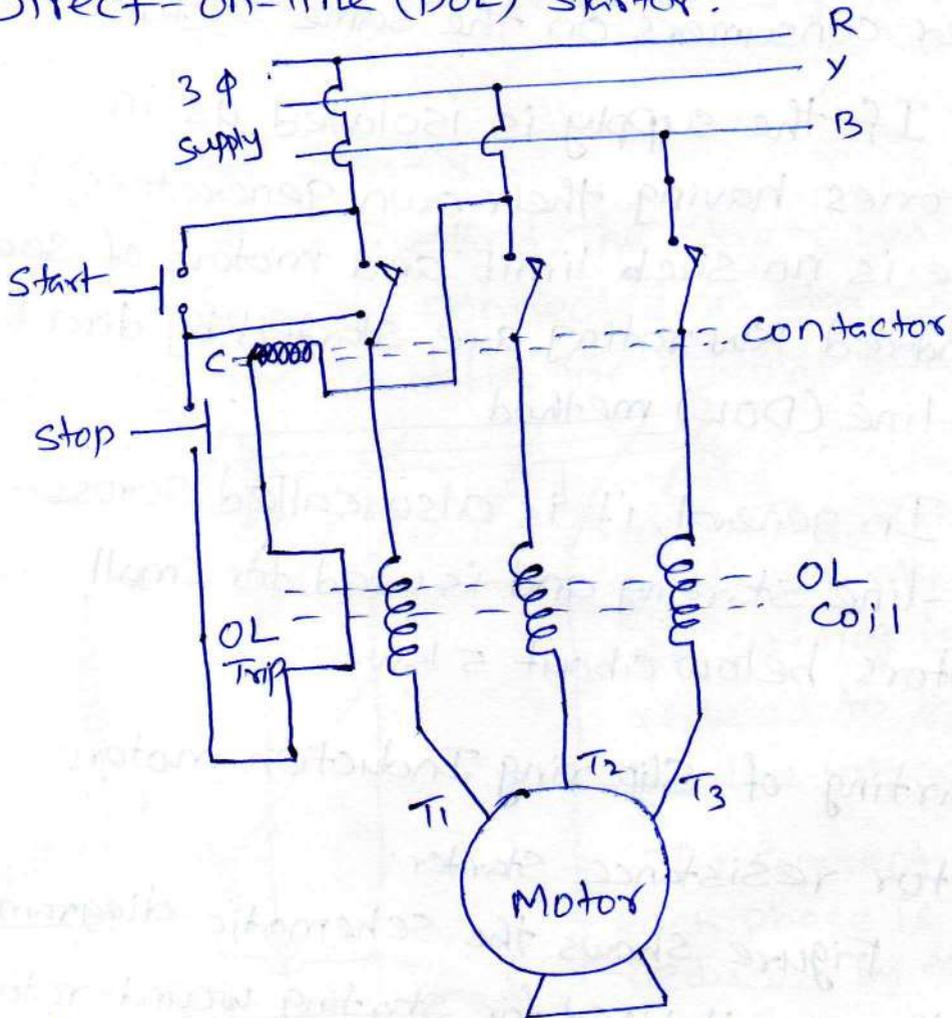
The starting current is high about 4 to

7 times the full-load current, the actual value depending on the size and design of the motor.

Such a high starting current causes a relatively large voltage drop in the cables and thereby affects the supplies to other consumers on the same system.

If the supply is isolated as in factories having their own generators, there is no such limit and motors of several hundred kw rating are started by direct-on-line (DOL) method.

In general, it is also called across-the-line starting and is used for small motors below about 5 kw.

Starting of Slip-ring Induction motors:

Rotor resistance starter:

Figure shows the schematic diagram of the circuit used for starting wound-rotor

Or slip-ring induction motor.

Wye - Connected
speed controller.
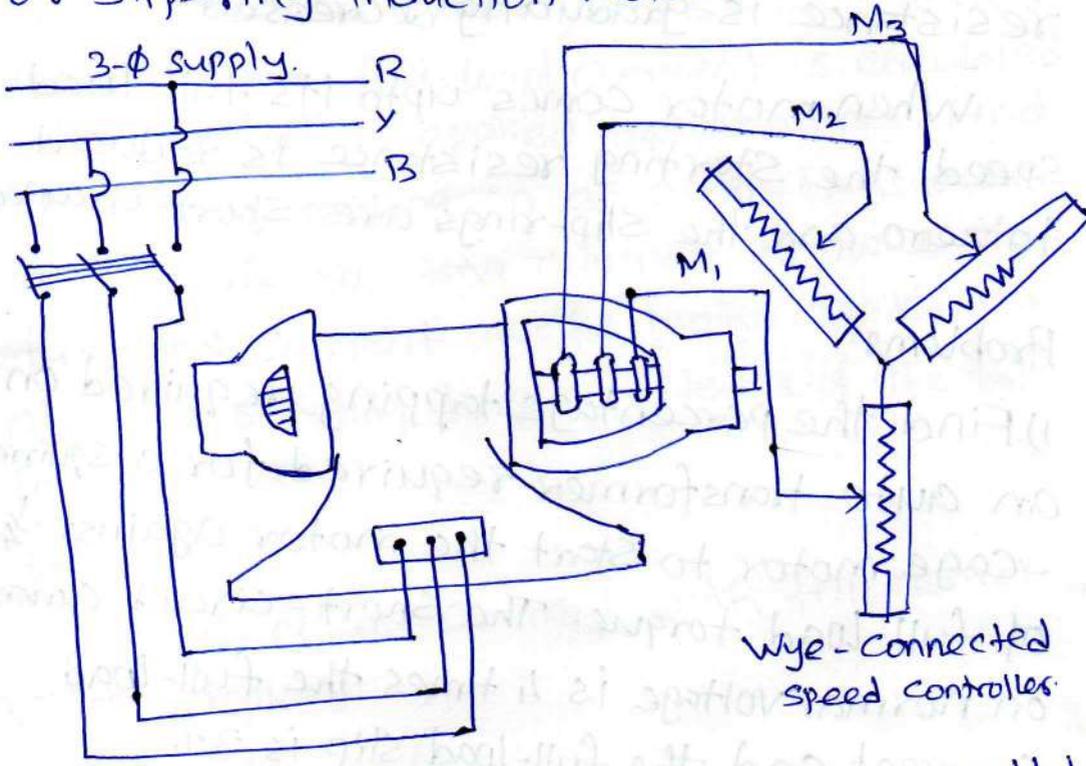
As seen, variable resistance can be added into the rotor circuit, for reducing the starting current and improving the starting torque

The rotor windings are connected to the Y-connected external resistors through slip-rings and brushes.

The motor is started with all the starter resistance circuit in thus giving maximum starting torque.

As motor speed increases, the starter resistance is gradually decreased.

When motor comes upto its full-load speed, the starting resistance is reduced to zero and the slip-rings are short-circuited.

Problems:

1) Find the percentage tapping required on an auto-transformer required for a squirrel-cage motor to start the motor against $\frac{1}{4}$ of full-load torque. The short-circuit current on normal voltage is 4 times the full-load current and the full-load slip is 3%.

Soln.:

$$\frac{T_{st}}{T_f} = \frac{1}{4} \; ; \; \frac{I_{sc}}{I_f} = 4, \; S_f = 0.03$$

∴ Now, $\dfrac{T_{st}}{T_f} = k^2 \left(\dfrac{I_{sc}}{I_f}\right)^2 \cdot S_f$

$$\frac{1}{4} = k^2 \times 4^2 \times 0.03$$

$$\Rightarrow k = 0.722$$

$$(or) \; k = 72.2\%$$

2) The full-load slip of a 400v, 3-phase cage induction motor is 3.5% and with locked rotor, full-load current is circulated when 92 volt is applied between lines. Find necessary tapping on an auto-transformer to limit the starting current to twice the full-load current of the motor. Determine also the starting torque in terms of the full-load torque.

Soln.:

Short-circuit current with full normal voltage applied is.

$$I_{sc} = \left(\frac{400}{92}\right) I_f$$

Supply line current $= I_{st} = 2 I_f$

Now, line current $I_{st} = K^2 I_{sc}$

$$\therefore 2 I_f = K^2 I_{sc}.$$

$$= K^2 \left(\frac{400}{92}\right) I_f$$

$$\Rightarrow K^2 = 0.46, \quad K = 0.678$$
$$= 67.8\%$$

Also, $\dfrac{T_{st}}{T_f} = K^2 \left(\dfrac{I_{sc}}{I_f}\right)^2 \times S_f.$

$$= 0.46 \times \left(\frac{400}{92}\right)^2 \times 0.035$$

$$= 0.304$$

$$\therefore T_{st} = 30.4\% \text{ of full-load torque.}$$

3) A 3-phase, 6-pole, 50-Hz induction motor takes 60 A at full-load speed of 940 rpm and develops a torque of 150 N-m. The starting current at rated voltage is 300 A. What is the starting torque? If a star/delta starter is used, determine the starting torque and starting current.

Soln.:

For direct-switching of induction motor,

$$\frac{T_{st}}{T_f} = \left(\frac{I_{sc}}{I_f}\right)^2 \cdot S_f$$

Here, $I_{st} = I_{sc} = 300$ A (line value)

$$I_f = 60 \text{ A (line value)}$$

$$S_f = \frac{N_s - N}{N_s} = \frac{1000 - 940}{1000} \qquad \left(\because N_s = \frac{120 f}{P} = \frac{120 \times 50}{6} \right)$$

$$= 0.06$$

$$T_f = 150 \text{ N-m.}$$

$$\therefore T_{st} = 150 \left(\frac{300}{60}\right)^2 \times 0.06$$

$$= 225 \text{ N-m}.$$

When star/delta starter is used,

Starting torque $= \frac{225}{3} = 75$ N-m.

Starting current $= \frac{1}{3} \times$ starting current with direct starting

$$= \frac{1}{3} \times 300$$

$$= 100 \text{ A}.$$

4) A squirrel-cage type induction motor when started by means of a star/delta starter takes 180% of full-load line current and develops 35% of full-load torque at starting. Calculate the starting torque and current in terms of full-load values, if an auto-transformer with 75% tapping were employed.

Soln.:

With star-delta starter,

$$\frac{T_{st}}{T_f} = \frac{1}{3} \left(\frac{I_{sc}}{I_f}\right)^2 S_f$$

Line current on line-start

$$I_{sc} = 3 \times 180\% \text{ of } I_f$$
$$= 3 \times 1.8 \, I_f = 5.4 \, I_f$$

Now, $\dfrac{T_{st}}{T_f} = 0.35$ (given)

$$\frac{I_{sc}}{I_f} = 5.4$$

$$\therefore \; 0.35 = \frac{1}{3} \times 5.4^2 \times S_f$$

$$5.4^2 \, S_f = 1.05$$

For auto-transformer starter,

Here, $k = 0.75$

Line starting current

$$= k^2 \, I_{sc} = (0.75)^2 \times 5.4 \, I_f$$
$$= 3.04 \, I_f$$
$$= 304\% \text{ of F.L. current}$$

$$\frac{T_{st}}{T_f} = k^2 \left(\frac{I_{sc}}{I_f} \right)^2 \cdot S_f$$
$$= (0.75)^2 \times 5.4^2 \, S_f$$
$$= (0.75)^2 \times 1.05 = 0.59$$

$$\therefore \; T_{st} = 0.59 \, T_f = 59\% \text{ FL torque.}$$

Crawling and Cogging:

The important characteristics normally shown by a squirrel-cage induction motors are crawling and cogging.

These characteristics are the result of improper functioning of the motor that means either motor is running at very slow speed or it is not taking the load.

Crawling:

It has been observed that squirrel cage type induction motor has a tendency to run at very low speed, compared to its synchronous speed.

This phenomenon is known as Crawling of an induction motor.

The resultant speed is nearly $\frac{1}{7}^{th}$ of its synchronous speed.

This is due to the fact that harmonics fluxes produced in the gap of the stator winding of odd harmonics like 3rd, 5th, 7th etc.

These harmonics create additional torque fields in addition to the synchronous torque.

The torque produced by these harmonics rotates in the forward or backward direction at $N_s/3$, $N_s/5$, $N_s/7$ speed respectively.

3rd harmonics are absent in a balanced 3-phase system.

The total motor torque now consist three components as:
i) the fundamental torque with synchronous speed N_s
ii) 5th harmonic torque with synchronous speed $N_s/5$
iii) 7th harmonic torque with synchronous speed $N_s/7$
(Provided, higher harmonics are neglected).

Now, 5th harmonic currents have a phase difference of $5 \times 120° = 600° = -120°$ in stator windings.

Hence, the revolving speed set up will be

in reverse direction with speed $N_s/5$.

The small amount of 5th harmonic torque produces breaking action and can be neglected.

The 7th harmonic currents will have phase difference of $7 \times 120 = 840° = 2 \times 360° + 120° = +120°$.

Hence, this will set up rotating field in forward direction with synchronous speed equal to $N_s/7$.

If we neglect all the higher harmonics, the resultant torque will be equal to sum of fundamental torque and 7th harmonic torque.
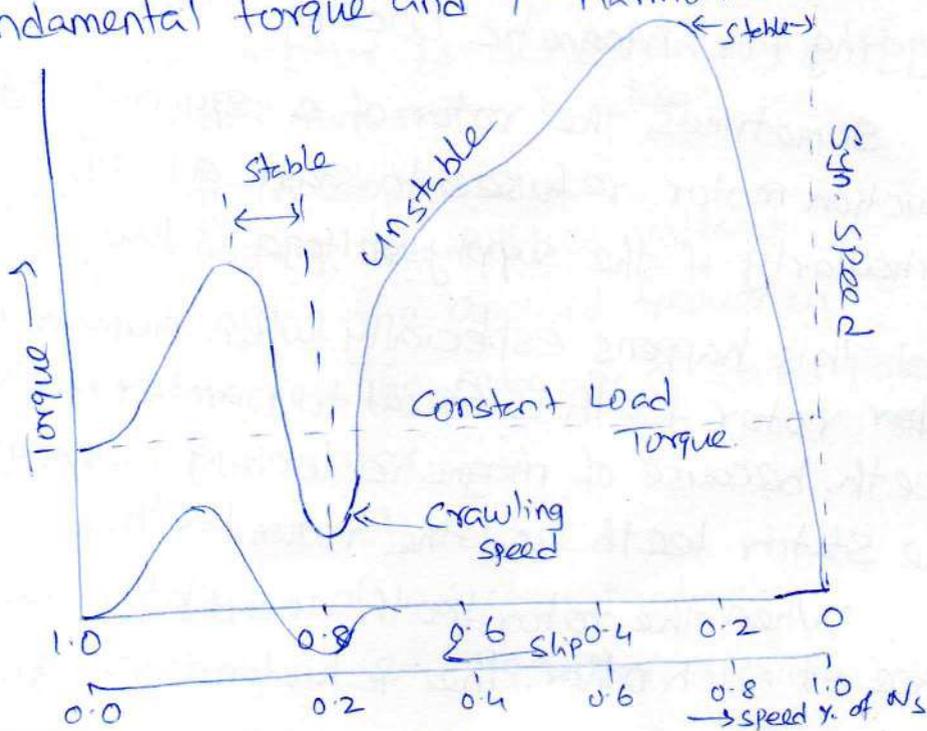
7th harmonic torque reaches its maximum positive value just before $\frac{1}{7}$th of N_s.

If the mechanical load on the shaft involves a constant load torque, the torque developed by the motor may fall below this load torque.

When this happens, the motor will not accelerate upto its normal speed, but will remain running at a speed, which is nearly $\frac{1}{7}$th of its normal speed.

This phenomenon is called as crawling in induction motors.

Cogging (or) Magnetic Locking:

Sometimes, the rotor of a squirrel cage induction motor refuses to start at all, particularly if the supply voltage is low.

This happens especially when number of ~~rotor~~ rotor teeth is equal to number of stator teeth, because of magnetic locking between the stator teeth and the rotor teeth.

When the rotor teeth and stator teeth face each other, the reluctance of the

magnetic path is minimum, that is why the rotor tends to remain fixed.

This Phenomenon is called cogging or magnetic locking of induction motor.

Cogging of squirrel cage motors can be easily overcome by making the number of rotor slots prime to the number of stator slots.

Speed Control of Induction Motors:

A speed regulation of an induction motor is usually less than 5% at full-load.

Different methods by which speed control of induction motor is achieved, may be grouped under two main headings:

1. Control from stator side
   a) by changing the applied voltage
   b) by changing the applied frequency
   c) by changing the number of stator poles

2. Control from rotor side
   a) rotor rheostat control
   b) cascading of induction motors
   c) injecting an emf in rotor circuit.

Voltage Control :

This method, though the cheapest and the easiest, is rarely used because

i) a large change in voltage is required for a relatively small change in speed

ii) a large change in voltage will result in a large change in the flux density thereby disturbing the magnetic conditions of the motor.

Frequency Control :

We have seen that the synchronous speed of an induction motor is given by,

$$N_s = \frac{120 f}{P}$$

∴ the synchronous speed of an induction motor can be changed by changing the supply frequency 'f'.

However, this method could only be used in cases where the induction motor is the only load on the generators, (here the supply frequency can be controlled by controlling the speed of prime movers of the generators).

But, the range over which the motor speed may be varied is limited by the economical speed of the prime movers.

Pole Changing:

From the speed equation, it is clear that, the synchronous speed can also be changed by changing the number of stator poles.

This change of number of poles is achieved by having two or more independent stator windings in the same slots.

Each winding gives a different number of poles and hence different synchronous speed.

Motors with four independent stator winding are also in use and they give four different synchronous speeds.

Of course, one winding is used at a time, the others being entirely disconnected.

This method has been used for elevator motors, traction motors and also for small motors driving machine tools.

Rotor resistance control:

In this method, which is applicable to slip-ring motors alone, the motor speed is reduced by introducing an external resistance in the rotor circuit.
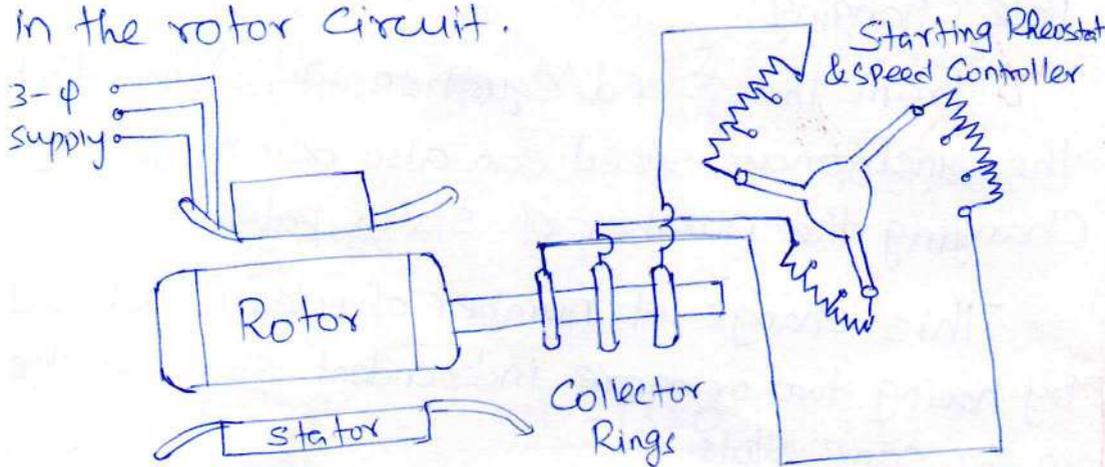
This method is, in fact, similar to the armature control method of DC shunt motors.

It has been known that, near synchronous speed, $T \propto \dfrac{s}{R_2}$

It is obvious that for a given torque, slip can be increased ie. speed can be decreased by increasing the rotor resistance R_2.

One disadvantage of this method is that, with increase in rotor resistance, $I^2 R$ losses

also increase which decrease the operating efficiency of the motor.

Problems:

1) The rotor of a 4-pole, 50-Hz slip ring induction motor has a resistance of 0.30 Ω per phase and runs at 1440 rpm at full-load. Calculate the external resistance per phase which must be added to lower the speed to 1320 rpm, the torque being the same as before.

Soln.:

The motor torque is given by,

$$T = \frac{ks R_2}{R_2^2 + (sx_2)^2}$$

Since, x_2 is not given,

$$T = \frac{ks R_2}{R_2^2} = \frac{ks}{R_2}$$

In the first case,

$$T_1 = \frac{ks_1}{R_2}$$

In second case, $T_2 = \frac{ks_2}{(R_2 + r)}$

where, $r \rightarrow$ external resistance/phase. added

to the rotor circuit.

Since, $T_1 = T_2$

$$\frac{k S_1}{R_2} = \frac{k S_2}{R_2 + r} \implies \frac{R_2 + r}{R_2} = \frac{S_2}{S_1}$$

$$S_1 = \frac{N_s - N_1}{N_s} \times 100 \quad \text{\&} \quad N_s = \frac{120 f}{P} = \frac{120 \times 50}{4}$$

$$= 1500 \, rpm.$$

$$S_1 = \frac{1500 - 1440}{1500} = 0.04$$

$$S_2 = \frac{N_s - N_2}{N_s} = \frac{1500 - 1320}{1500} = 0.12$$

$$\therefore \quad \frac{0.30 + r}{0.30} = \frac{0.12}{0.04}.$$

$$\implies r = 0.6 \, \Omega$$

Cascading Operation:

In this method, two motors are used and are ordinarily mounted on the same shaft, so that both run at the same speed.
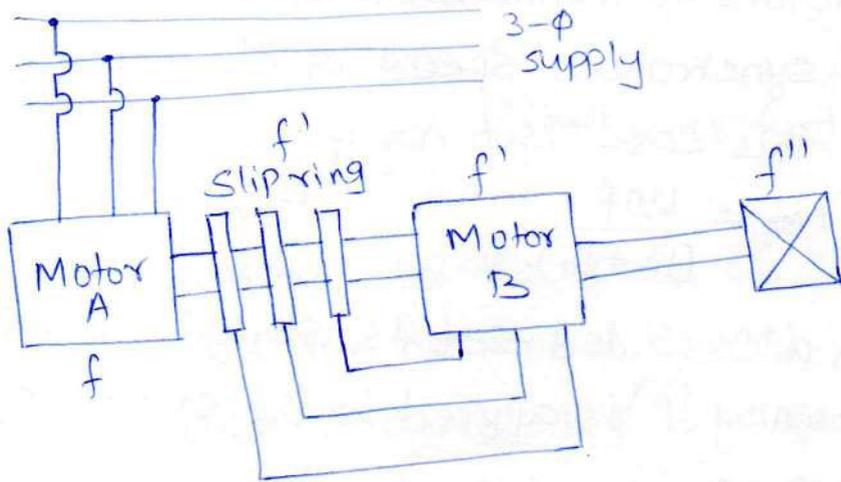
The stator winding of the main motor A is connected to the mains as usual, while that of the auxiliary motor B is fed from the rotor circuit of motor A.

For satisfactory operation, the main motor A should be phase-wound i.e. Slip-ring type with stator to rotor winding ratio of 1:1.

In addition to Concatenation (cascade), each motor Can also be run from supply mains separately.

The combination may be connected in Cumulative Cascade i.e. in such a way that the phase rotation of the stator fields of

both motors is in the same direction.

The synchronous speed of the cascaded set, in this case is

$$N_{sc} = \frac{120f}{(P_a + P_b)}$$

When a cascaded set is started, the voltage at frequency f is applied to the stator winding of machine A.

An induced emf of the same frequency is produced in rotor A which is supplied to auxiliary motor B.

Both motors develop a forward torque.

As the shaft speed rises, the rotor frequency of motor A falls and so does the synchronous speed of motor B.

The set settles down to a stable speed when the shaft speed becomes equal to the speed of rotating field of motor B.

Considering load conditions, the electrical power taken in by stator A is partly used

to meet I^2R and core losses and the rest is given to its rotor.

The power given to rotor is further divided into two parts:

i) Proportional to the speed of set i.e. N (converted into mechanical power)

ii) Proportional to $(N_{sa} - N)$ (developed as electrical power at slip frequency) and is passed on to the auxiliary motor B, which uses it for producing mechanical power and losses.

Hence, approximately, the mechanical outputs of the two motors are in the ratio $N : (N_{sa} - N)$.

where,

$N \rightarrow$ actual speed of concatenated set

$N_{sa} \rightarrow$ Synchronous speed of motor A, it being independent of N.

In fact, it comes to that the mechanical outputs are in the ratio of the number of poles of the motors.

Problem:

1) A cascaded set consists of two motors A and B with 6 poles and 4 poles respectively. The motor A is connected to a 50-Hz supply. Find, (i) the speed of the set, (ii) the electric power transferred to motor B when the input to motor A is 25 kW. Neglect losses.

Soln.: As all losses are neglected, the speed = synchronous speed

(i) Synchronous speed of the set is,

$$N_{sc} = \frac{120 f}{P_a + P_b} = \frac{120 \times 50}{6 + 4} = 600 \text{ rpm}$$

(ii) The output of the two motors are proportional to the number of their poles.

∴ output of 4-Pole motor, B

$$= 25 \times \frac{4}{10} = 10 \text{ kW}$$

Rotor emf injection:

In this method, speed of an induction motor is controlled by injecting a voltage in rotor circuit.

It is necessary that voltage (emf) being injected must have same frequency as of the slip frequency.

However, there is no restriction to the phase of injected emf.

If we inject emf which is in opposite phase with the rotor induced emf, rotor resistance will be increased.

If we inject emf which is in phase with the rotor induced emf, rotor resistance will be decreased.

Thus, by changing the phase of injected emf, speed can be controlled.

The main advantage of this method is a wide range of speed control (above normal as well as below normal) can be achieved.

The emf can be injected by various methods such as Kramer System, Scherbius System etc.

Double-cage rotors:

The main disadvantage of a squirrel-cage motor is its poor starting torque, because of its low rotor resistance.

The starting torque could be increased by having a cage of high resistance, but then the motor will have poor efficiency under normal running conditions. (because of more rotor cu. losses).

The difficulty with a cage motor is that its cage is permanently short-circuited, so no external resistance can be introduced temporarily in its rotor circuit during starting period.

In order to overcome this, issue, Boucheort introduced a motor, which has two independent cages on the same rotor, one inside the other.

A punching for such a double cage rotor is shown in fig.

The outer cage consists of bars of a high-resistance metal, whereas the inner cage

has low-resistance copper bars.

Inner cage — | Outer cage
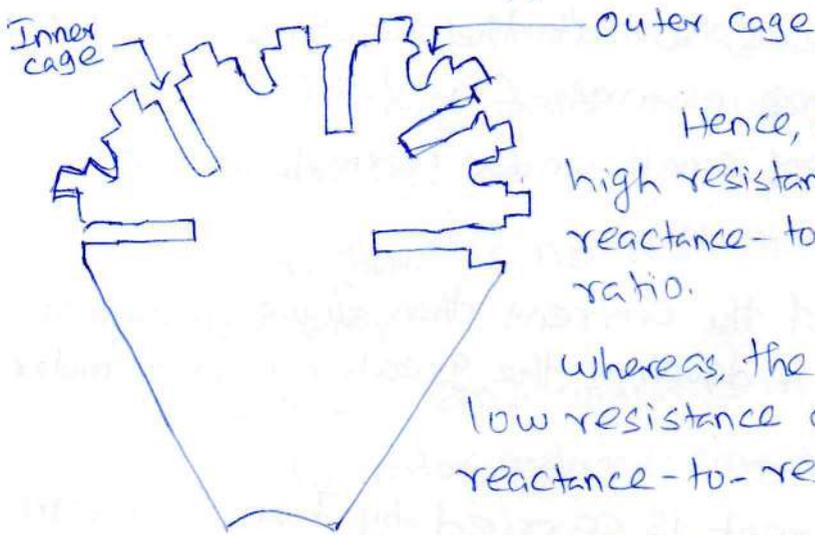
Hence, outer cage has high resistance and low reactance-to-resistance ratio.

Whereas, the inner cage has low resistance and large reactance-to-resistance ratio.

Hence, the outer cage develops maximum torque at starting, while the inner cage does so at about 15% slip.

As we know, at starting and at large slip values, frequency of induced emf in the rotor is high.

So, the reactance of inner cage ($= 2\pi f L$) and therefore, its impedance are high. Hence, very little current flows in it.

Most of the starting current is confined to outer cage, despite its high resistance.

Hence, the motor develops a high starting torque due to high-resistance outer cage.

As the speed increases, the frequency of rotor emf decreases, so that the reactance and hence the impedance of inner cage decreases and becomes very small, under normal running conditions.

Most of the current then flows through it and hence it develops the greater part of motor torque.

The current is carried by two cages in parallel, giving a low combined resistance.

Hence, it has been made possible to construct a single machine, which has a good starting torque with reasonable starting current which maintains high efficiency and good speed regulation under normal operating conditions.

Such motors are particularly useful where frequent starting under heavy loads is required.

Induction generator:

It has been observed that an induction machine is in generating mode when $s < 0$

(negative slip).

An induction generator is asynchronous in nature because of which it is commonly used as windmill generator as a windmill runs at non-fixed speed.

A transmission line connected to an induction generator feeding a local load is shown in fig.
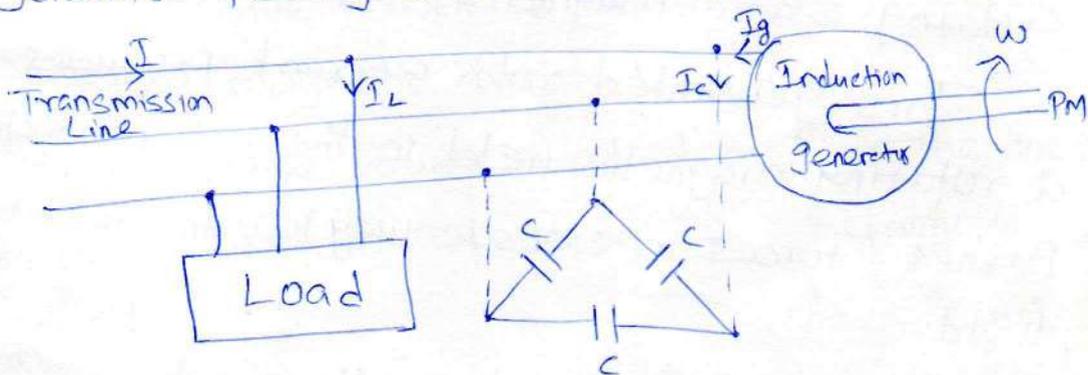
The prime mover must be provided with automatic control to increase the generator speed when it is required to meet increased load.

Principle of operation:

If the rotor is accelerated to the synchronous speed by means of a prime mover, the slip will be zero and hence the

net torque will be zero. The rotor current will become zero.

If the rotor is made to rotate at a speed more than the synchronous speed, the slip becomes negative.

A rotor current is generated in the opposite direction, due to the rotor conductors cutting static magnetic field.

This generated rotor current produces a rotating magnetic field in the rotor which pushes (forces in opposite way) onto the stator field.

This causes a stator voltage which pushes current flowing out of the stator winding against the applied voltage.

Thus, the machine is now working as an induction generator (asynchronous generator).

Induction machine.

Prime mover (eg. Petrol engine)

→P

←Q

3-φ AC supply line
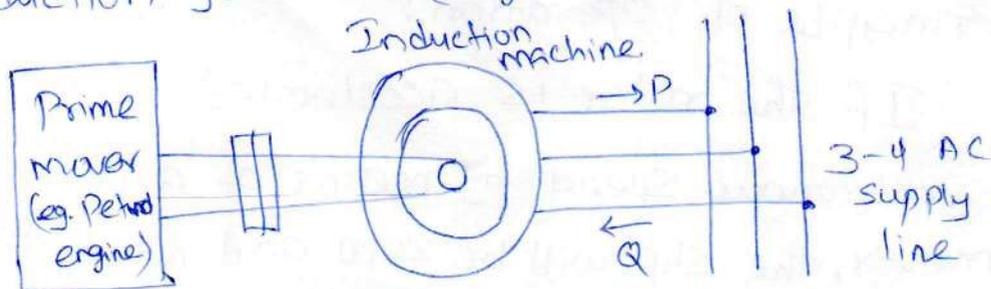
P → active Power

Q → Reactive Power

Induction generator is not a self-excited machine. Therefore, when running as a generator, the machine takes reactive power from the AC power line and supplies active power back into the line.

Reactive power is needed for producing rotating magnetic field.

The active power supplied back in the line is proportional to slip above the synchronous speed.

A capacitor bank can be connected across the stator terminals to supply reactive power to the machine as well as to the load. (Self-excited case).

When the rotor is rotated at enough speed, a small voltage is generated across the stator terminals, due to residual magnetism.

Due to this small voltage, capacitor current is produced which provides further

reactive power for magnetization.

Advantages:
- more rugged
- require no commutator and brush
  arrangement.

Disadvantage:
- they take quite large amount of reactive
Power.

Applications:
- Produce useful power even at varying
rotor speeds, therefore suitable in wind
turbines.

_____ Y _____

End of Unit - II

Important 10 Marks questions:

1. Derive the value of maximum starting and maximum running torque of a three phase induction motor.
2. Illustrate the phenomenon of cogging and crawling in induction motor.
3. Explain the tests required to be performed to obtain the data for the circle diagram
4. Explain in detail the construction of circle diagram of an induction motor
5. Explain the methods of starting of induction motor with neat sketches.
6. Explain the speed control of induction motor from stator side.
7. Why starters are necessary for starting induction motors? What are the various types of starters? Explain star-delta type starter in detail.

Alternators

Introduction:

A.C. generators are usually called as alternators.

Constructional / features:-

It operate on the same fundamental principle of electromagnetic induction as D.C. generators.

They also consist of an armature winding and a magnetic field.

But, there is one important difference between the two. Whereas, in d.c. generators, the armature rotates and field system is stationary,

the arrangement in alternators is just the reverse.

In their case, standard construction consists of armature winding mounted on a stationary element called 'stator' and field windings on the rotating element called 'rotor'.

Constructional features:-

The details of construction are shown in figure.

3 phase Supply

Stator

3-phase winding
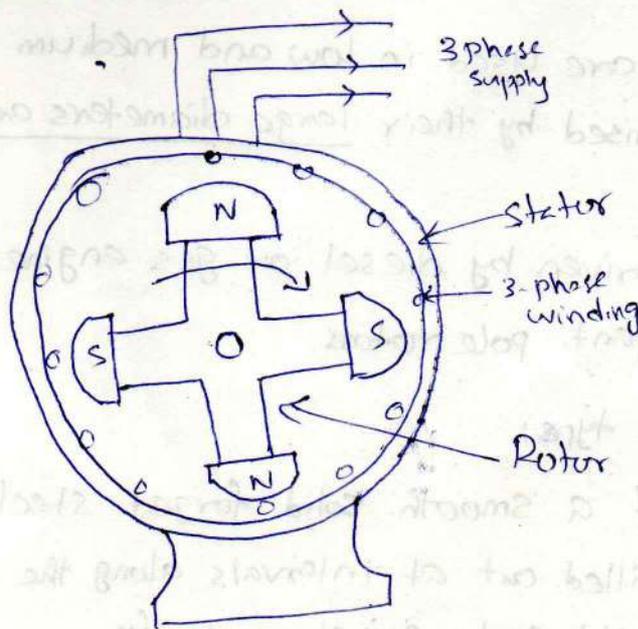
Rotor

a) Stator:

It consists of a cast-iron frame which supports the laminated armature core having slots on its inner periphery for housing the 3-phase winding.

b) Rotor:

These are of two types:

i) Salient pole (or) projecting type

& ii) Smooth cylindrical type.

i) Salient pole type:

It is like a flywheel which has a large number of alternate North and South poles bolted to it as shown in fig.
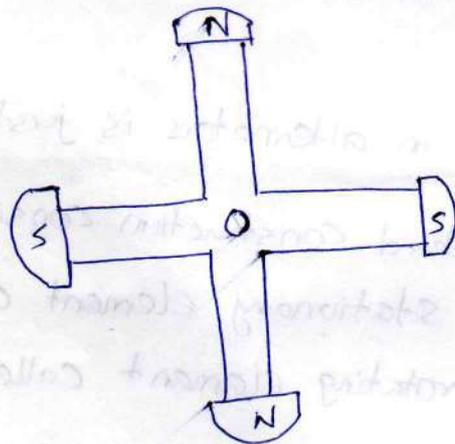
The magnetic wheel is made of cast iron or steel of good magnetic quality.

Such rotors are used in low and medium speed alternators which are characterised by their large diameters and short axial lengths.

Alternators driven by diesel or gas engines and gas turbines have salient pole rotors.

ii) Smooth cylindrical type:

It consist of a smooth solid forged-steel cylinder having a number of slots milled out at intervals along the outer periphery for accommodating field coils as shown in fig.

Polar Area

slots

Non-salient Pole

N

S

S

N

DC supply

N

S
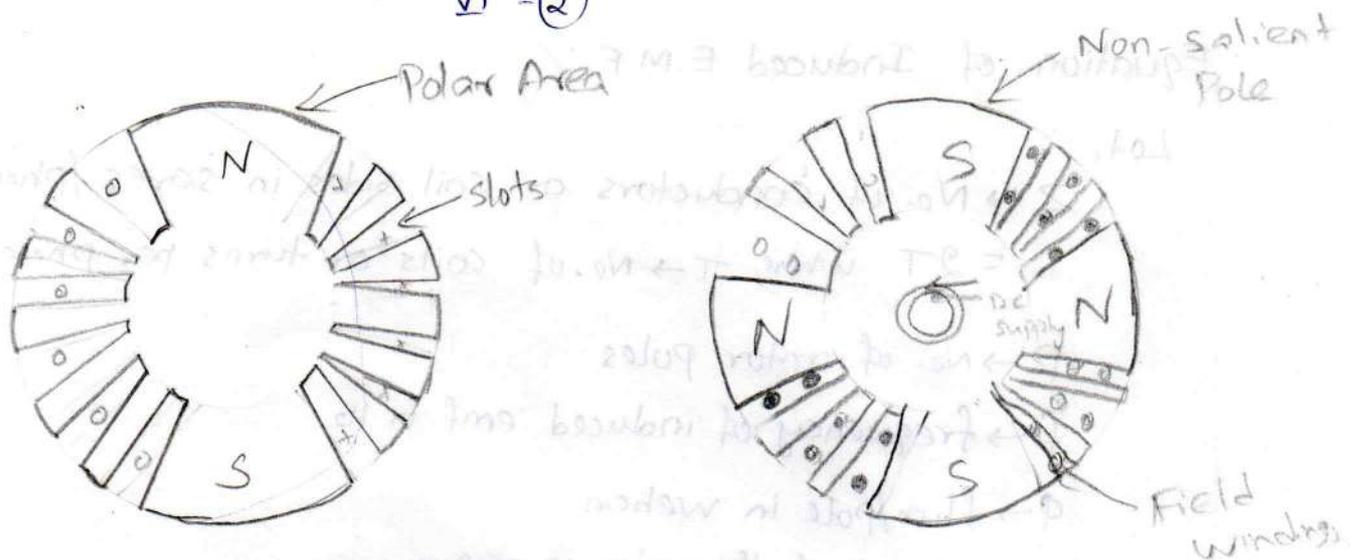
Field windings.

Two or four regions corresponding to the central Polar areas areas left unslotted.

The Central polar areas are surrounded by the field windings placed in slots.

Obviously, in this case, the poles are non-salient i.e. they do not project out from the surface of the rotor.

Such rotors are used in steam turbine-driven alternators i.e. turbo alternators which run at very high speeds and are characterised by their small diameters and very long axial lengths.

Principle of operation:

When the rotor is rotated by the prime mover, the stator winding or conductor are cut by the magnetic flux of the rotor poles.

Hence, an emf is induced in the stator conductors.

Because the rotor poles are alternately N and S, they induce an alternating emf in the stator conductors.

The frequency of this induced emf is given by $f = \frac{PN}{120}$ and its direction can be found by applying Fleming's Right-hand rule.

The emf generated in the stator conductors is taken out from the three leads connected to the stator winding.

Equation of Induced E.M.F. :

Let,

$Z \to$ No. of conductors or coil sides in series /phase.
$= 2T$ where, $T \to$ No. of coils or turns per phase.

$P \to$ No. of rotor poles

$f \to$ frequency of induced emf in Hz.

$\phi \to$ flux/pole in webers.

$N \to$ Speed of the rotor in r.p.m.

In one revolution of rotor (ie. in $\frac{60}{N}$ second), each stator conductor is cut by a flux of ϕP webers.

$$\therefore d\phi = \phi P \quad \text{and} \quad dt = \frac{60}{N}$$

$\therefore$ Average emf induced per conductor

$$= \frac{d\phi}{dt} = \frac{\phi P}{60/N} = \frac{\phi N P}{60} \text{ volt.}$$

Now, we know that $f = \frac{PN}{120} \Rightarrow N = \frac{120 f}{P}$

$\therefore$ Average emf per conductor

$$= \frac{\phi P}{60} \times \frac{120 f}{P} = 2 f \phi \text{ volt.}$$

If there are Z number of conductors in series /phase, then,

$$\text{Average emf per phase} = 2 f Z \phi = 4 f \phi T \text{ volt.}$$

$\therefore$ R.M.S. value of emf /phase $= 1.11 \times 4 f \phi T$

$$= 4.44 f \phi T \text{ volt.} \quad (\text{Same as Transformer})$$

If the alternator is star-connected, then the line voltage is $\sqrt{3}$ times the phase voltage.

Problems:

1) What is the frequency of voltage generated by an alternator having 10-poles and rotating at 720 rpm.?

Soln.:
$$f = \frac{PN}{120} = \frac{10 \times 720}{120} = 60 \text{ Hz}$$

2) A 3 phase, 16-pole alternator has a star-connected winding with 144 slots and 10 conductors per slot. The flux per pole is 30 mwb sinusoidally distributed and the speed is 375 rpm. Find the frequency, the phase and line emf.

Soln.:
$$f = \frac{PN}{120} = \frac{16 \times 375}{120} = 50 \text{ Hz}$$

No. of slots per phase = $\frac{144}{3}$ = 48

No. of conductors/slot = 10

No. of conductors/phase, Z = 48 × 10 = 480

emf per phase = 2.22 f Z φ volts

= 2.22 × 50 × 480 × 30 × 10⁻³

= 1600 V

For a star connection, $V_L = \sqrt{3} \ V_{ph}$

∴ Line voltage $V_L = \sqrt{3} \times 1600$

= 2770 V

Types:

The alternators are classified in various ways as follows:

1) According to number of phases

a) Single phase alternator

b) Three phase alternator

The use of 1-φ machine is limited. Normally 3-φ machines are used in power plants. and industrial sector.

2) According to the rotating Part.

    a) Rotating armature type → armature winding is at rotor

    b) Rotating field type → field winding is at rotor

Rotating armature type is similar to that of a D.C. generator except that the commutator of dc machine is replaced by slip-rings in the alternator.

3) According to type of rotor for rotating field type:

    a) Salient pole type

    b) Non-salient pole type.

Terminology:

1) **Conductor:**

    It is the active length of wire placed in the armature slot, which is responsible for cutting the main magnetic flux and inducing the emf in it.

2) **Turn:**

    A conductor in one slot is connected to a conductor in another slot to form a turn

    ← single turn

    ← conductor 2
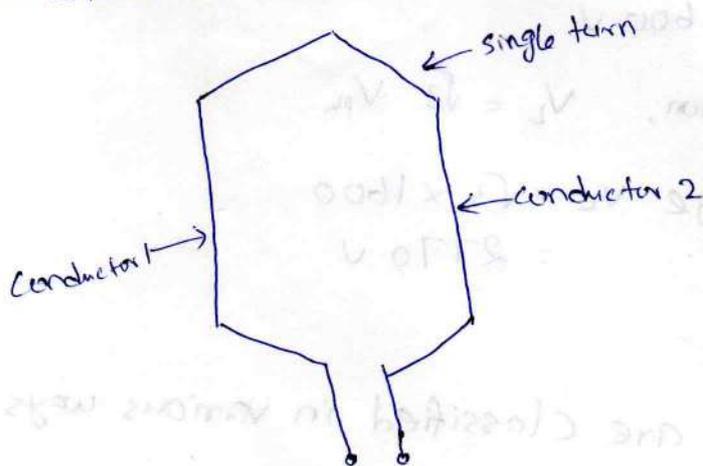
Conductor 1 →

3) **Coil:**

    Number of turns are grouped together to form a coil as shown in fig.

← Coil Side 2
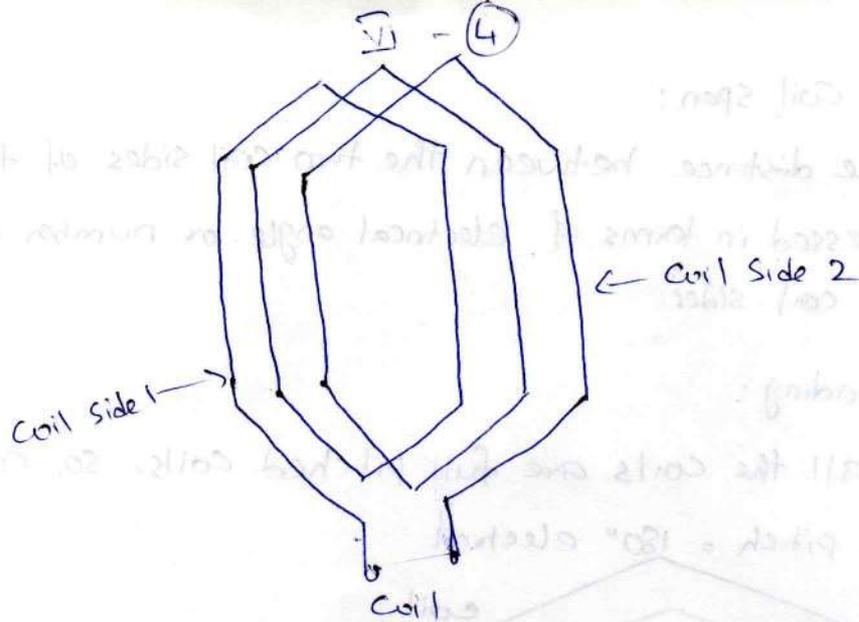
Coil Side 1 →

Coil

4) Coil Side :

It is the group of conductors of a coil in one slot. ie. there are two coil side of a coil placed in two different slots.

5) Pole Pitch:

During one rotation, if the conductor cuts the flux of two N and S poles, emf of $360°$ electrical angle is induced in it.

i.e. One pole flux induces emf of $180°$ electrical.

This centre to centre distance between the two adjacent poles is known as a pole pitch. It is $180°$ electrical.

Generally the pole pitch is expressed in terms of number of slots under one pole.

6) Single layer and double layer windings:

A single layer winding has only one coil side in each slot.

A double layer winding has two coil sides of two different coils in each slot.

Coil side

↓

slot

a) single layer

Top coil side →

Bottom coil side →
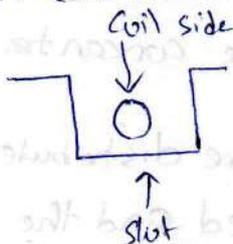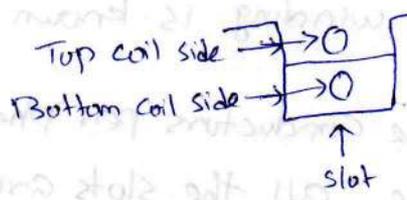
slot

(b) double layer

7) Coil Pitch or Coil span:

It is the distance between the two coil sides of the same coil. It is expressed in terms of electrical angle. or number of slots between the two coil sides.

8) Full Pitch winding:

In this, all the coils are full pitched coils. So, coil Pitch is equal to pole pitch = 180° electrical.

coil.

|← Coil pitch = Pole pitch = n = 180° electrical →|

Slot 1   Slot (n+1)
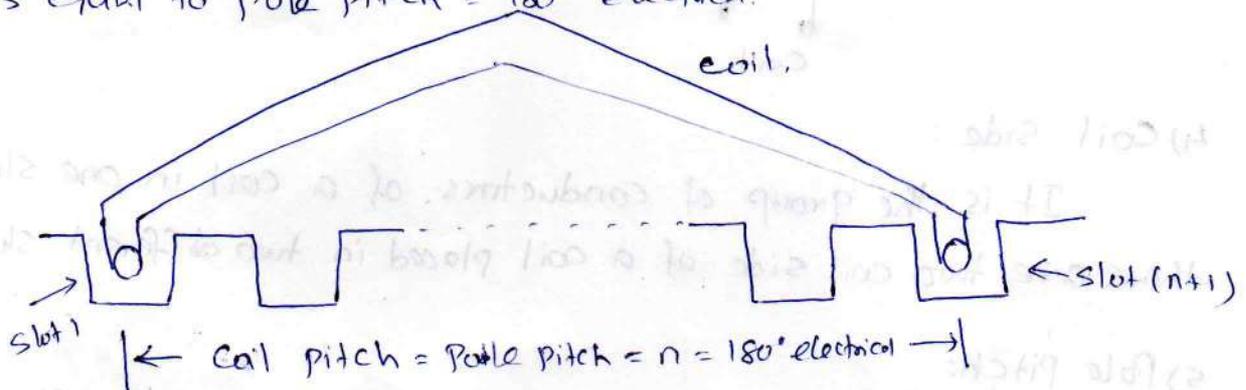
9) Short Pitch winding:

If the coil pitch is less than pole pitch ie. less than 180° electrical, the winding is said to be short pitch winding.

The corresponding coils are known as short-pitched coils.

|← Coil pitch < Pole Pitch →|← Short Pitching →|

Slot 1   Slot (n+1)
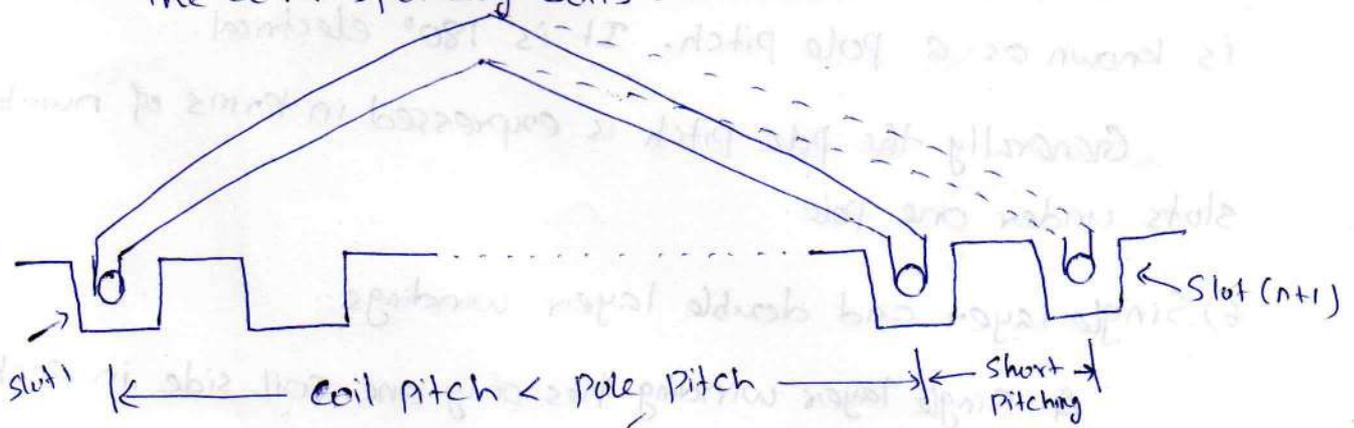
10) Concentrated and Distributed winding:

If all the conductors per phase are placed in only one slot per pole, the winding is known as the concentrated winding.

If all the conductors per phase are distributed uniformly in $n/3$ slots/pole, all the slots are filled and the corresponding

winding is called distributed winding.

11) Pitch factor or Coil span factor (k_p):

Short pitched coils are preferred over full pitched coils due to their outstanding advantages. But then such winding gives reduced emf.

In order to account for this reduction in emf due to short pitching, a factor is deviced in the emf equation of the alternator. It is known as pitch factor (k_p) or coil span factor.

The pitch factor k_p or coil span factor is defined as the factor by which the induced emf gets reduced due to short pitched coils. It is the ratio of resultant emf for short pitch to the resultant emf for full pitch.

$$\therefore k_p = \text{Pitch factor} = \frac{E_r \text{ for short pitch}}{E_r \text{ for full pitch}}$$

The short pitched coil is short pitched by an angle known as "short pitching angle (α)". It is the angle of slots by which the coil is short pitched.
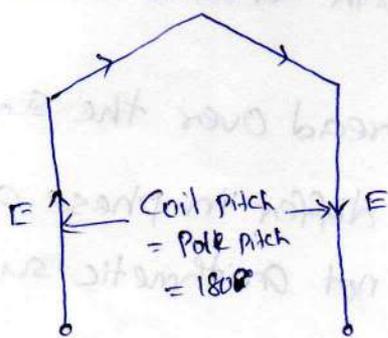
(a) Full pitched coil
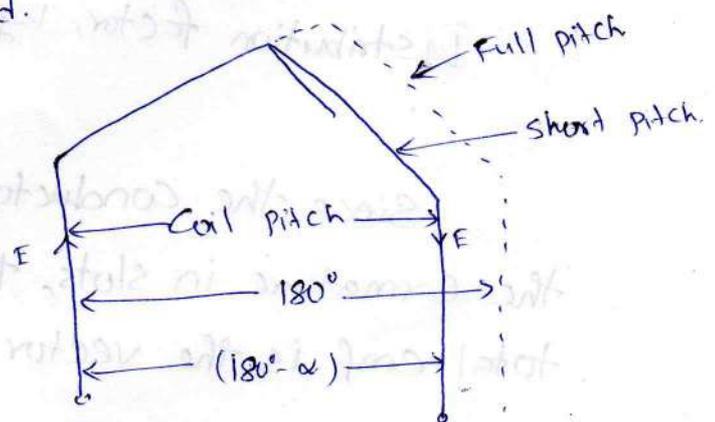
b) Short pitched coil

Figure above, shows the emfs E induced in the two coil sides of a coil for full pitched coil and short pitched coil.
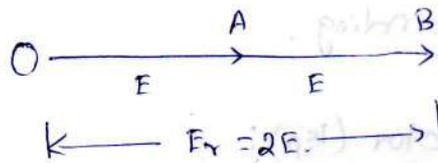
a) for full pitched coil
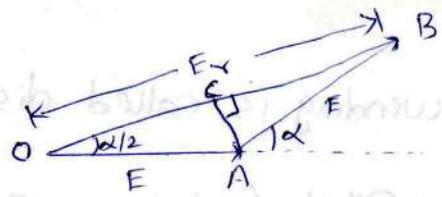
b) for short pitched coil

From above fig(a) the resultant coil emf E_r is the algebraic sum of the coil-side emfs E and E. i.e. $E_r = 2E$ because the two coil side emfs are 180° electrical apart.

From Fig (b), the resultant coil emf E_r is the vector sum of the coil side emfs E and E. These two emfs E and E have a phase difference of α between them.

$$\therefore E_r = OB = OC + CB = 2 OC$$
$$= 2 OA \cos(\alpha/2)$$
$$= 2E \cos(\alpha/2)$$

$$\therefore \text{Pitch factor, } k_p = \frac{2E \cos(\alpha/2)}{2E}$$

$$= \cos \alpha/2$$

where, α → short pitching angle.

2) Distribution factor (k_d):

$$\text{Distribution factor, } k_d = \frac{emf \text{ with distributed winding}}{emf \text{ with concentrated winding}}$$

Since the conductors are spread over the surface of the armature in slots, their emfs differ in phase and the total emf is the vector sum and not arithmetic sum.

Hence, $k_d = 1$ for concentrated winding but it is less than 1 for distributed winding.

Short-pitch Coil : (Chording)

Advantages.

1. They save copper of end connections

2. They improve the wave-form of the generated emf i.e. the generated emf can be made to approximate to a sine wave more easily and the distorting harmonics can be reduced or totally eliminated.

3. Due to elimination of high frequency harmonics, eddy current and hysteresis losses are reduced thereby increasing the efficiency.

Disadvantages.

1. The total voltage around the coil is somewhat reduced.

EMF Equation:

$$E_{rms} = 4.44 \, K_p \, K_d \, f \, \Phi \, T$$

$$= 4 \, K_f \, K_p \, K_d \, f \, \Phi \, T$$

$K_f \rightarrow$ form factor

$K_p \rightarrow$ Pitch factor

$K_d \rightarrow$ distribution factor.

Advantages of Stationary Armature and rotating field system:

1. Ease of Construction:

For a large three-phase synchronous machines, the armature winding is more complex than the field winding

The coil and phase connections including bracing of the windings can be done more easily and securely on a Stationary structure i.e. on the Stator than on the rotor.

2. Number of Slip-rings required:

When armature winding is made rotating, atleast 3 slip-rings are needed to receive the generated power.

For large synchronous machines rated in MVAs, transferring power through brush and slip-ring arrangement may cause some Problems.

It is also difficult to insulate the slip-

rings from the rotating shaft for high
voltage.

.The distance between the slip-rings is
to be kept sufficiently large so that flash-
over does not take place.

With the stationary armature and
rotating field arrangement, none of these
problems occur.

Only two slip-rings of much smaller
size are required to supply excitation
current to the rotating windings, as power
required for excitation is much less and
is supplied at a low voltage.

3. **Better insulation to Armature:**

It is easier to insulate the armature
coils from the core if the windings are
placed on the stator instead of on the
rotor.

It is comparatively easier to insulate

the low voltage DC winding placed on the rotor.

4. ~~Rotor~~ Reduced rotor weight and rotor Inertia :

The weight of the field system placed on the rotor is comparatively much lower than the armature winding placed on the Stator.

This is because, the field winding are made with thinner wires and are required to be insulated for a lower voltage.

The inertia of the rotor is, therefore reduced

With rotating field system, the rotor will take comparatively less time to come up to the rated speed.

5. Improved ventilation arrangement :

Arrangement for forced air-cooling or hydrogen cooling for large machine can

easily be made on a stationary armature by enlarging the stator core and providing radial air-ducts and ventilation holes.

Due to the reasons mentioned, the idea of rotating armature for commercial synchronous machines has been unpopular altogether.

All the large synchronous machines built today have stationary armature and rotating field structure.

Voltage regulation:

The voltage regulation of an alternator is defined as the percentage rise in terminal voltage when full-load is removed.

ie. % regulation = $\dfrac{E - V}{V} \times 100$

where, E → no-load voltage per phase

V → full-load voltage per phase

In case of small machines, the regulation may be found by direct loading. But in large machines the direct loading becomes prohibitive.

In direct loading method, alternator is driven out synchronous speed and the terminal voltage is adjusted to its rated value.

The load is varied untill the voltmeter, ammeter and wattmeter indicate the rated value of desired power factor.

The entire load is thrown off while the speed and field excitation are kept constant. The open circuit voltage or no-load voltage E_0 is read and regulation can be found from.

% regulation = $\dfrac{E_0 - V}{V} \times 100$.

But there are 3 different methods are available for predetermining the regulation of large machines without applying actual load.

They are,

→ Synchronous impedance method (or) emf method
→ Ampere turns method (or) mmf method
→ Zero power factor (zpf) method (or) Potier method.

Predetermination of regulation by Synchronous Impedance Method:

By conducting proper open circuit (o.c.) and short circuit (s.c.) test on alternator, one can predetermine the value of regulation by Synchronous Impedance method.
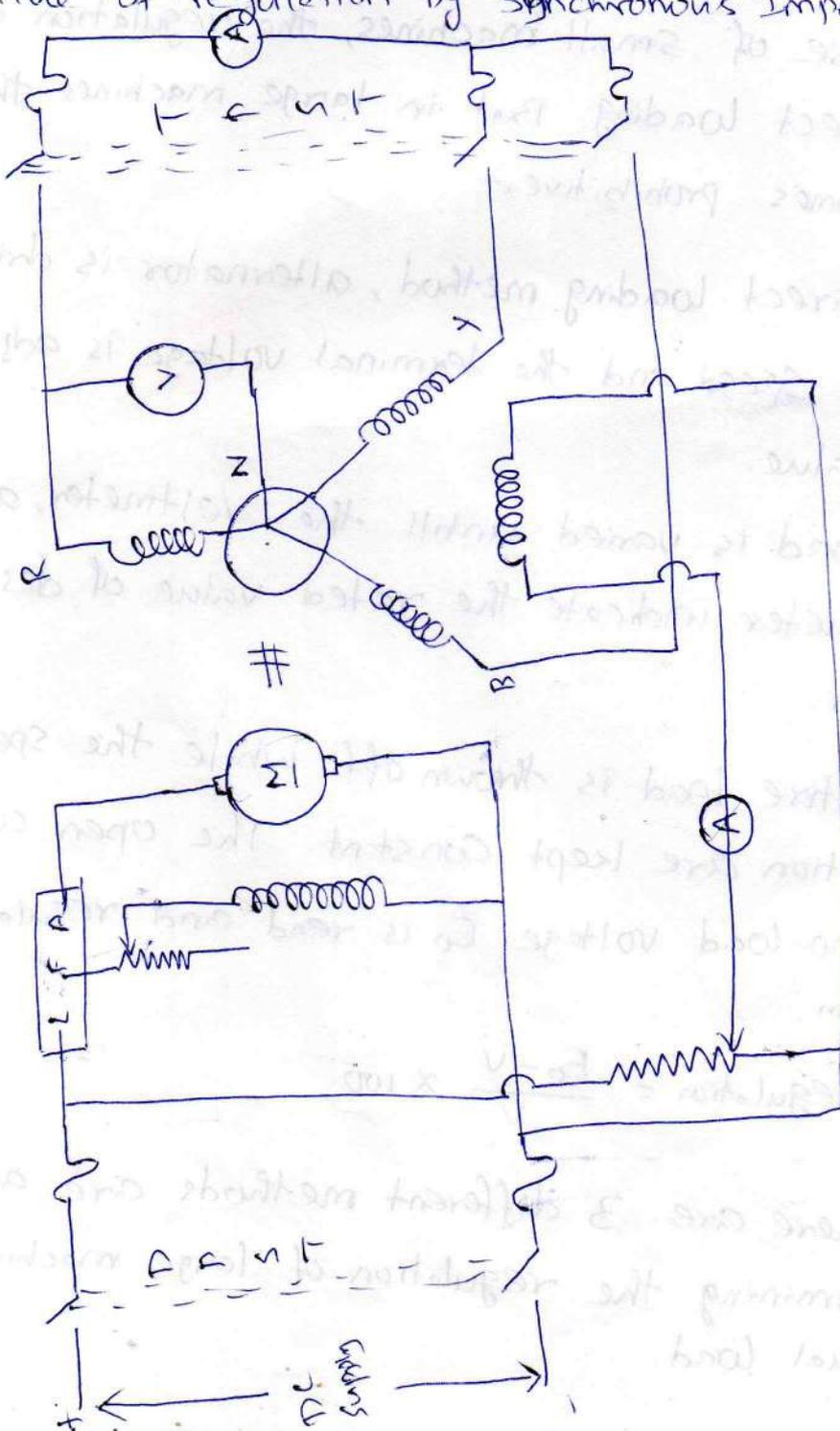
O.C. & S.C test:

→ Connect the circuit as per the circuit diagram

→ Verify the precautions. ie. * DPST & TPST should be in open position.

    * Motor field rheostat is kept at minimum position and generator field rheostat is kept at maximum position.

→ The DPST is closed and the motor is started with the help of 3point starter.

→ By adjusting the motor field rheostat, run the alternator at rated speed.

→ By varying the generator field rheostat, the different meter readings are taken. upto 125% of rated voltage.

→ The generator field rheostat is brought into its initial position, and TPST is closed.

→ the generator field rheostat is so adjusted such that the ammeter reads the rated current value.

→ Bring back the generator field rheostat to its original position and open the TPST and reduce the speed of the alternator by motor field rheostat and open the DPST.

O.C test

Field current I_f (A)	OC voltage. (v)

S.C. test

Field current I_f (A)	Armature current I_a(A)
1.	
2.	

Graph axes: vertical axis labeled $V_{oc}(v)$ & $I_a(A)$, horizontal axis labeled I_f (Amp). Curves labeled O.C.C. and S.C.C. with points C, B, D, E, A, O.
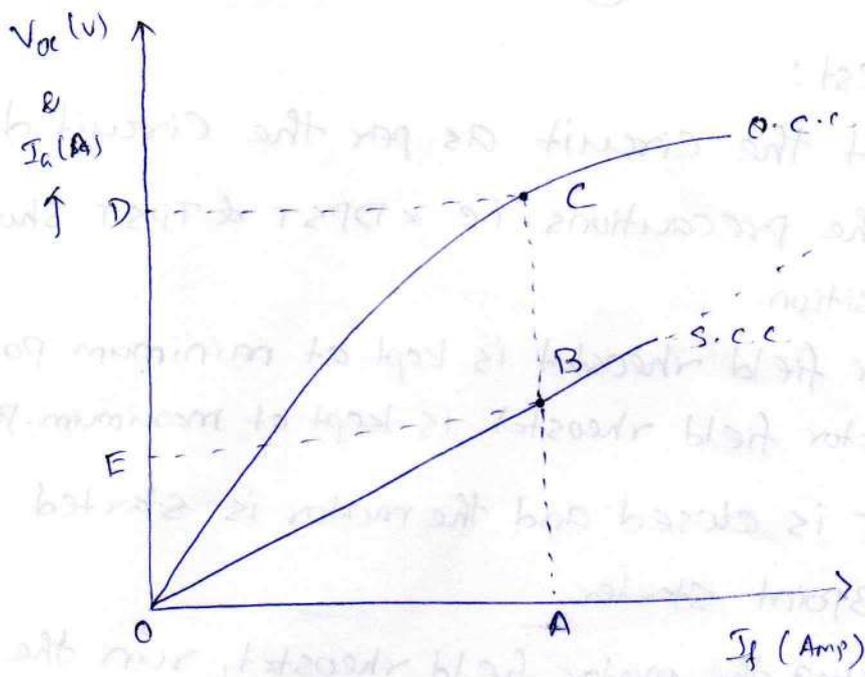
Determination of Z_s:

The synchronous impedance Z_s of the alternator changes as load condition changes.

In s.c test, external load impedance is zero. The short circuit, armature current is circulated against the impedance of the armature winding which is Z_s.
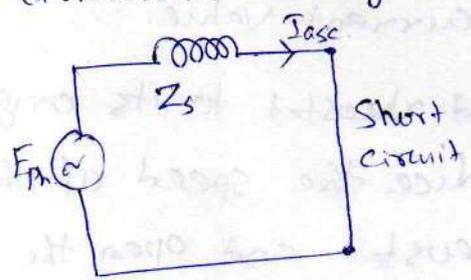
From equivalent circuit, we can write,

$$Z_s = \frac{E_{ph}}{I_{asc}}$$

Value of I_{asc} is known, which can be observed on the ammeter. But induced emf can not be observed under short circuit condition.

To determine Z_s it is necessary to determine the value of E.

Now, induced emf is proportional to the flux ie. field current I_f.

$$E_{ph} \alpha \phi \alpha I_f \qquad (\text{from emf equation})$$

So, if the terminals of the alternator are opened without disturbing I_f which was present at the time of short circuited condition. But now Current will be Zero.
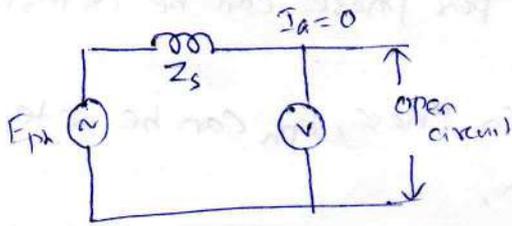
Now, $E_{ph} = V_{oc}$ on open circuit.

$$\therefore Z_s = \frac{V_{oc}}{I_{asc}} \bigg| \text{ for same } I_f.$$

So O.C.C and S.C.C. can be used effectively to calculate Z_s.

From S.C.C. determine I_f required to drive this full load Short Circuit Current (I_a). This is OA from graph.

Now, for this value of I_f, V_{oc} can be obtained from O.C.C. Extend line from point A, till it meets O.C.C. at Point C.

The Corresponding (V_{oc}) value is available at point D.

$$V_{oc} = OD$$
while $I_{asc} = OE$.

$$\therefore Z_s \text{ at full load} = \frac{V_{oc}}{I_{asc}} \bigg| \text{ same } I_f$$

$$= \frac{OD}{OE} \bigg| \text{ same } I_f = OA$$

Regulation Calculation:

From O.C.C. and S.C.C., Z_s can be determined for any load condition.

The Armature resistance per phase (R_a) can be measured by different methods.

Now, $Z_s = \sqrt{(R_a)^2 + (X_s)^2}$

$X_s = \sqrt{Z_s^2 - R_a^2} \quad (\ldots) / ph.$

So, Synchronous reactance per phase can be determined.

No-load induced emf per Phase, E_{ph} can be determined by the mathematical expression,

$$E_{ph} = \sqrt{(V_{ph} \cos\phi + I_a R_a)^2 + (V_{ph} \sin\phi \pm I_a X_s)^2}$$

where,

$V_{ph} \rightarrow$ Phase value of rated voltage

$I_a \rightarrow$ phase value of current depending on the load condition

$\cos\phi \rightarrow$ P.f. of load

+'ve sign for lagging pf while

-'ve sign for leading pf

The regulation then can be determined by using formula,

$$\% \text{ Regulation} = \frac{E_{ph} - V_{ph}}{V_{ph}} \times 100$$

Advantages:

→ the value of Synchronous impedance Z_s for any load condition can be calculated. Hence regulation of the alternator at any load condition and load power factor can be determined.

→ Actual load need not be connected to the alternator and hence method can be used for very high capacity alternators.

Limitations:

→ It gives large values of Synchronous reactance. This leads to high values of percentage regulation than the actual results. Hence this method is called pessimistic method.

Problems:

1) The open circuit and short sh circuit test is conducted on a 3 phase, star connected 866V, 100 kVA alternator.

The O.C. test results are,

I_f Amp	1	2	3	4	5	6	7
V_{oc} line (v)	173	310	485	605	728	790	840

The field current of 1A, produces a short circuit current of 25 A.

The armature resistance per phase is $0.15\ \Omega$. Calculate its full load regulation at 0.8 lagging power factor condition.

Soln. :

$$V_L = 866\ V$$

$$kVA = 100 = \sqrt{3}\ V_L\ I_L \times 10^{-3}$$

$$I_L = \frac{100 \times 10^3}{\sqrt{3} \times 866} = 66.67\ A$$

$$I_{a\ ph.\ (F.L)} = I_L = 66.67\ A \quad (as\ star\ connected\ alternator)$$

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{866}{\sqrt{3}} = 500\ V.$$

O.C.C. & S.C.C
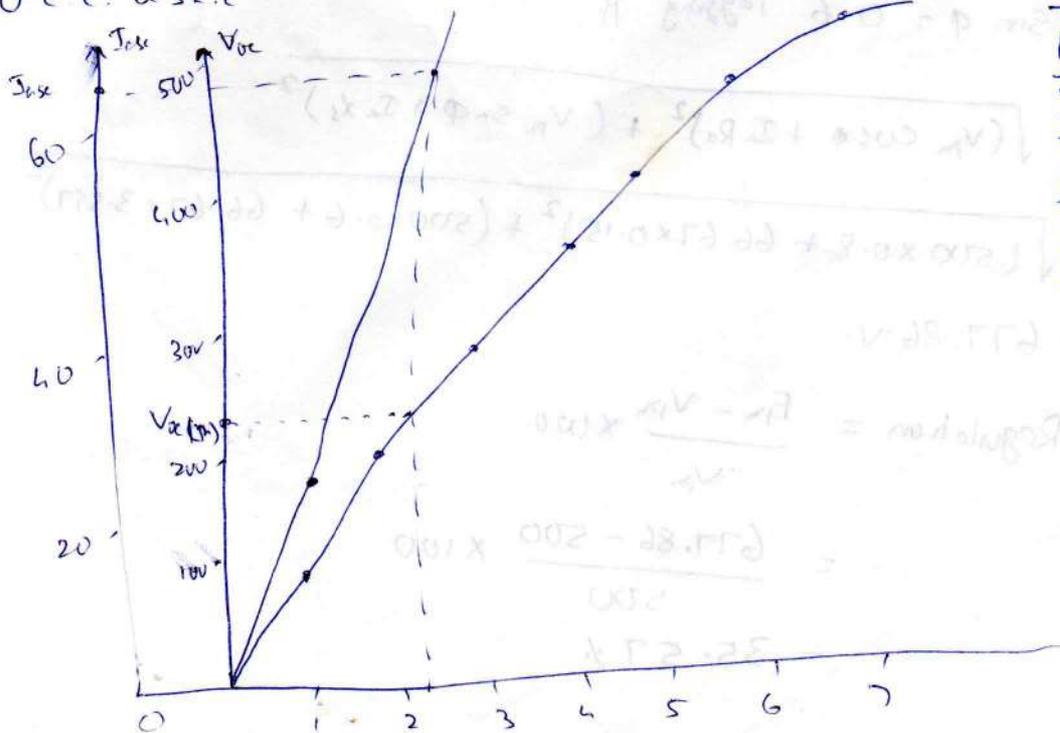
V_L	V_{ph}
173	100
310	180
485	280
605	350
728	420
790	456
840	485

In this problem,

$$I_{asc} = 25 \text{ A for } I_f = 1 \text{ A}$$

we need to calculate Z_s for I_{asc} = its full load value = 66.67A

From, S.C.C.

for $I_{asc} = 66.67$ A, $I_f = 2.4$ A

From O.C.C.

for $I_f = 2.4$ A, $V_{oc\,(ph)} = 240$ V.

From the graph, Z_s for full load is

$$Z_s = \left.\frac{(V_{oc})_{ph}}{I_{asc\,(ph)}}\right| \text{ for same excitation.}$$

$$= \left.\frac{240}{66.67}\right| \text{ for } I_f = 2.4 \text{ A}$$

$$= 3.6 \,\Omega \,/ \text{phase.}$$

$$R_a = 0.15\,\Omega\,/\text{phase.}$$

$$X_s = \sqrt{(Z_s)^2 - (R_a)^2} = 3.597\,\Omega\,/\text{ph.}$$

$$V_{ph.F.L} = 500 \text{ V}$$

$$\cos\varphi = 0.8$$

$$\sin\varphi = 0.6 \text{ lagging } p.f.$$

$$E_{ph} = \sqrt{(V_{ph}\cos\varphi + I_a R_a)^2 + (V_{ph}\sin\varphi + I_a X_s)^2}$$

$$= \sqrt{(500\times0.8 + 66.67\times0.15)^2 + (500\times0.6 + 66.67\times3.597)^2}$$

$$= 677.86 \text{ V.}$$

$$\% \text{ Regulation} = \frac{E_{ph} - V_{ph}}{V_{ph}} \times 100$$

$$= \frac{677.86 - 500}{500} \times 100$$

$$= 35.57 \%$$

EMF method problems.

1) A 500 kVA, three phase, star connected alternator has a rated line-to-line terminal voltage of 3300V. The resistance and synchronous reactance per phase are 0.3 and 4.0Ω respectively. Calculate the voltage regulation at full-load 0.8 power-factor lagging.

Soln.:

Output Power in VA $= \sqrt{3} V_L I_L = 500 \, kVA$,

$$I_L = \frac{500 \times 10^3}{\sqrt{3} \times 3300} = 87.5 \, A$$

For Star connected alternator, line current is equal to phase current.

Therefore, $I_a = 87.5 \, A$

$$\cos \phi = 0.8 \; ; \; \sin \phi = 0.6$$

$$R_a = 0.3 \; ; \; X_s = 4.0$$

$$V_{ph} = \frac{3300}{\sqrt{3}} = 1905 \, V$$

Induced emf,

$$E = \sqrt{(V \cos \phi + I_a R_a)^2 + (V \sin \phi \pm I_a X_s)^2}$$

$$= \sqrt{(1905 \times 0.8 + 87.5 \times 0.3)^2 + (1905 \times 0.6 + 87.5 \times 4)^2}$$

$$= 2152 \text{ V}/\text{Phase}$$

$$\therefore \text{% Regulation} = \frac{E_{ph} - V_{ph}}{V_{ph}} \times 100$$

$$= \frac{2152 - 1905}{1905} \times 100$$

$$= 12.96 \text{%}.$$

2) In a 2000V, single-phase synchronous generator, a full-load current of 100A is produced on a short-circuit by a field excitation of 2.5 A; an emf of 500V is produced on open-circuit by the same excitation. The armature resistance is 0.8Ω. Determine the voltage regulation when the generator is delivering a current of 100 A at (a) unity power factor, (b) 0.71 Pf lagging and (c) 0.8 pf leading.

Soln.:

Synchronous Impedance,

$$Z_s = \frac{OC \; voltage}{SC \; current} = \frac{500}{100} = 5\Omega$$

$$X_s = \sqrt{Z_s^2 - R_a^2} = \sqrt{5^2 - 0.8^2} = 4.935\Omega$$

Induced emf,

$$E = \sqrt{(V\cos\phi + I_a R_a)^2 + (V\sin\phi \pm I_a X_s)^2}$$

(a) At unity pf,

$$E = \sqrt{(V\cos\phi + I_a R_a)^2 + (V\sin\phi + I_a X_s)^2}$$

$$= \sqrt{(2000 \times 1 + 100 \times 0.8)^2 + (2000 \times 0 + 100 \times 4.935)^2}$$

$$= 2138 \; V$$

$$\therefore \% \; Regulation = \frac{E - V}{V} \times 100$$

$$= \frac{2138 - 2000}{2000} \times 100$$

$$= 6.9 \%$$

(b) At 0.71 pf lagging,

$$\cos\phi = 0.71$$

$$\sin\phi = 0.704$$

$$E = \sqrt{(2000 \times 0.71 + 100 \times 0.8)^2 + (2000 \times 0.704 + 100 \times 4.935)^2}$$

$$= 2422 \; V$$

1. Regulation $= \dfrac{E-V}{V} \times 100$

$$= \dfrac{2422-2000}{2000} \times 100$$

$$= 21.1 \%.$$

(c) At 0.8 pf leading,

$$\cos \phi = 0.8, \quad \sin \phi = 0.6$$

Armature Reaction, Phasor diagram:

The resistance of each phase winding of a Synchronous generator is designated as Ra.

Some of the flux lines, produced by the Armature, which do not cross the air-gap are called leakage flux.

The reactance due to these leakage fluxes is called leakage reactance X_L.

When Synchronous generator is loaded, there will be a change in terminal voltage due to a voltage drop in Ra as well as in X_L.

The change in terminal voltage due to armature reaction effect can also be viewed as a reactance voltage drop.

Explanation:

The rotor field flux, ϕ_f produces induced emf E in the armature winding.

When loaded, this emf causes an armature current I_a to flow through the winding and the load.

The armature ampere-turns produces a flux ϕ_a in the air-gap.

This flux ϕ_a produces another emf E_a in the armature windings.

The phase relationship between the

- field flux ϕ_f
- Armature induced emf due to field flux, E
- the armature current I_a
- flux produced by armature current, ϕ_a
- & emf induced in the armature due to ϕ_a
  at different power factor loads, E_a

are shown in fig.

In general, induced emf E will lag the field flux ϕ_f as shown in fig.
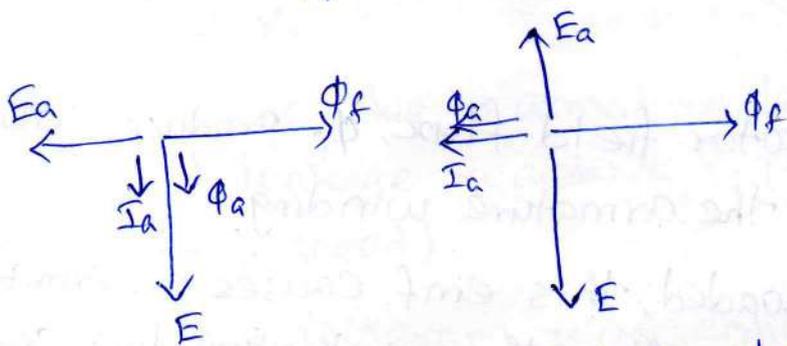
(a) Resistive load
(upf)

(b) Inductive load
(zpf lagging)
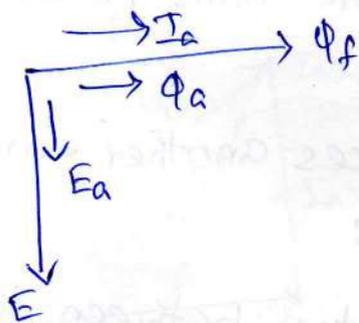
(c) Capacitive load
(zpf leading)

The phase relationship between the induced emf, E and the current flowing through the armature winding, I_a will depend upon the power factor of the load.

At unity pf, I_a will be in phase with E.

At zero lagging pf, I_a will lag E by $90°$

At zero leading pf, I_a will lead E by $90°$

Flux, ϕ_a Produced by armature current I_a will be in time-phase.

Emf induced E_a in the armature windings due to ϕ_a will lag ϕ_a by $90°$

A component of the generated voltage that would be necessary to overcome this armature reaction voltage must act in the opposite direction.

Since, the armature reaction induced voltage always lags the armature current and the flux producing it by $90°$, the component necessary to overcome this generated voltage will always lead the armature current by $90°$.

Thus, the voltage induced due to armature

reaction effect can be considered as a reactance drop in the armature winding.

This reactance due to armature flux, ϕ_a is called X_a.

Reactance due to armature leakage flux, is called leakage reactance X_s. (flux lines rotates at synchronous speed).

Phasor Diagram representing the various quantities for a Synchronous generator on different power-factor loads are shown in fig.
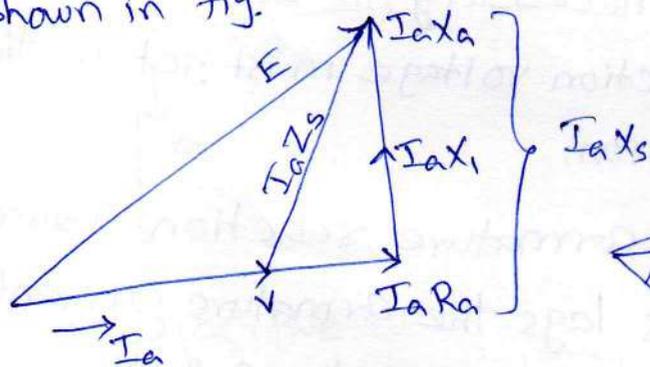
(a) Unity PF load.
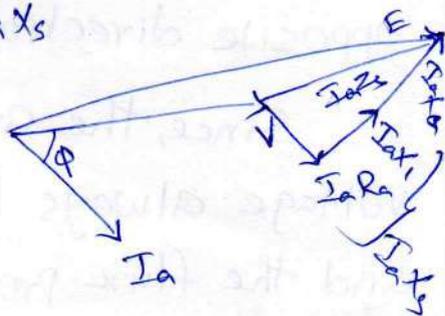
(b) Lagging Pf load.
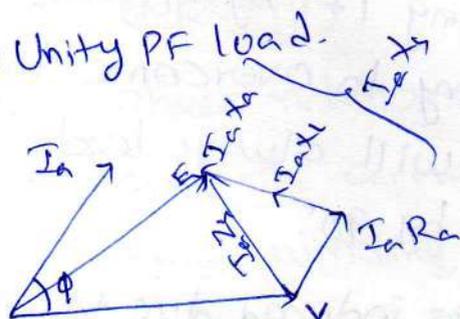
(c) Leading pf load.

MMF method:

This method also utilizes O.C. and S.C. data.

Here, armature leakage reactance is treated as an additional armature reaction.

Now, field A.T. required to produce a voltage of V on full-load is the vector sum of the following:

i) Field A.T. required to produce V on no-load. (found from O.C.C.) and

ii) Field A.T. required to overcome the demagnetising effect of armature reaction on full-load. (found from SC test)

E_o

IX_a → entirely due to armature reaction.

O ϕ V I_aR_a

I
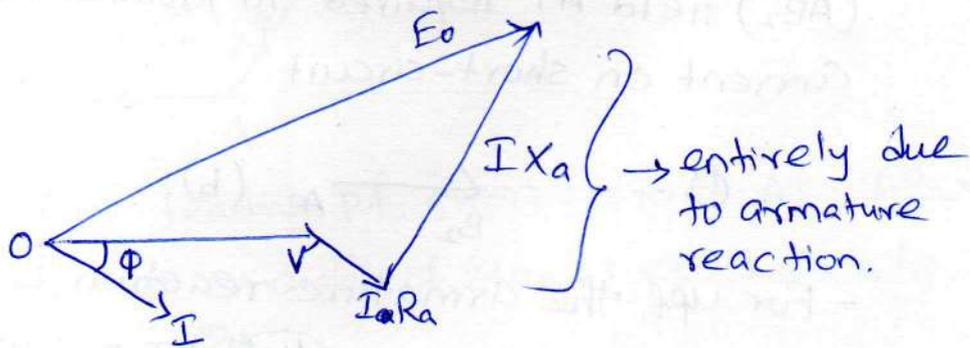
Now, if the alternator, instead of being on short-circuit, is supplying full-load current

at its normal voltage and zero pf lagging, then total field AT required is the vector sum of

i) the field AT (=OA) necessary to produce normal voltage (as from OCC). and

ii) the field AT necessary to neutralize the armature reaction (= AB₁)

- total field AT is represented by OB₁ in fig.(a) and equals the vector sum of OA and AB₁.

$$O \xrightarrow{\hspace{0.5cm} A \hspace{0.5cm}} B_1 \quad (a)$$

- For leading pf, the armature reaction is wholly magnetising. In this case, field AT required is OB₂ which is less than OA by (AB₂) field AT required to produce full-load current on short-circuit.

$$O \cdot ----- \overset{I}{\underset{B_2}{\longleftarrow}} A \quad (b)$$
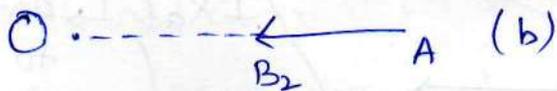
- For upf, the armature reaction is cross-magnetising. In this case, field AT required is OB₃ (vector sum of OA and AB₃ which is drawn at right angles to OA as in fig.(c)).

(c)
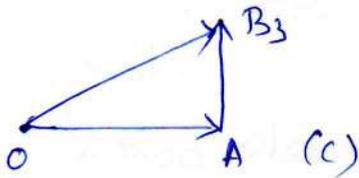
General Case:

Let us consider, the general case, when the pf has any value between zero to unity.

Field AT 'OA' Corresponding to v is laid off horizontally.

Then AB_1 representing full-load short circuit field AT, is drawn at an angle of $(90°+\phi)$ for lagging pf.

The total field AT is given by OB_1 as in fig.
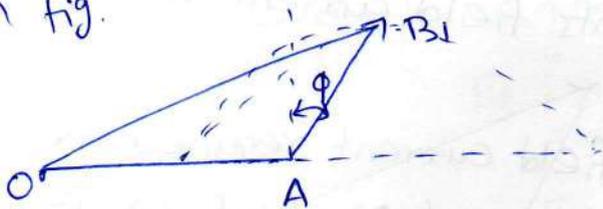
For leading pf, short-circuit $AT = AB_2$ is drawn at an angle of $(90-\phi)$ as shown in fig.
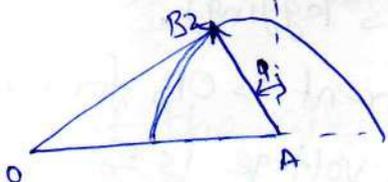
For unity power factor AB_3 is drawn at right angles as shown in fig. below.
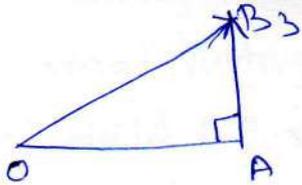
In those cases, where number of turns on the field coils is not known, it is usual to work in terms of field current.
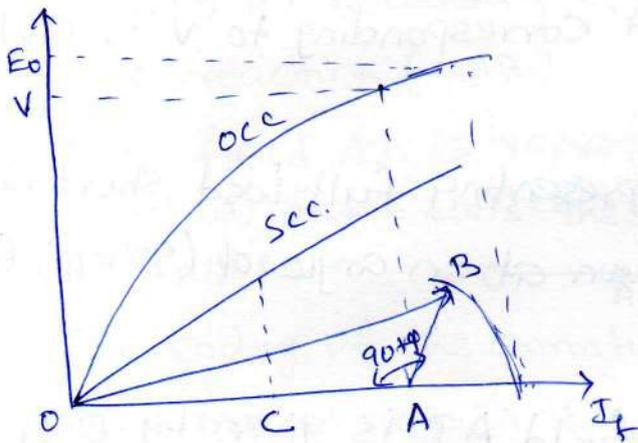
In Fig. OA represents field current for normal voltage V.

OC represents field current required for Producing full-load current on short-circuit.

Vector AB = OC is drawn at an angle of $(90 + \phi)$ to OA (if the pf is lagging).

The total field current is OB for which the corresponding O.C. voltage is E_0.

$$\therefore \% \text{ Regulation} = \frac{E_0 - V}{V} \times 100$$

It should be noted that this method gives results which are less than the actual results, that is why it is sometimes referred to as optimistic method.

Problems:

1) A 3.5 MVA, Y-connected alternator rated at 4160 volts at 50-Hz has an open circuit characteristics given by the following data.

$I_f(A)$	50	100	150	200	250	300	350	400	450
Emf(v)	1620	3150	4160	4750	5130	5370	5550	5650	5750

A field current of 200 A is found necessary to circulate full-load current on short-circuit of the alternator. Calculate by (i) synchronous impedance method and (ii) ampere-turns method the full-load voltage regulation at 0.8 pf lagging. Neglect resistance. Comment on the results obtained.

Soln.:

(i). Synchronous Impedance method:

As seen from given data, a field-current of 200 A produces OC voltage of 4750 and full-load current on short-circuit which is

$$= \frac{3.5 \times 10^6}{\sqrt{3} \times 4160} = 486\, A$$

OC voltage$/_{Ph} = \frac{4750}{\sqrt{3}} = 2740\, V$

$$Z_s = \frac{OC\ voltage}{SC\ current} = \frac{2740}{486} = 5.64\, \Omega\, /\,Phase$$

Since, $R_a = 0$, $X_s = Z_s$

$\therefore I R_a = 0$, $I x_s = I Z_s = 486 \times 5.64$
$$= 2740\, V.$$

F.L. voltage$/ph. = \frac{4160}{\sqrt{3}} = 2400\, V.$

$$Cos\varphi = 0.8, \quad sin\varphi = 0.6$$

$$E_0 = \sqrt{(V\cos\varphi + IR_a)^2 + (V\sin\varphi + I x_s)^2}$$

$$= \sqrt{(2400 \times 0.8 + 0)^2 + (2400 \times 0.6 + 2740)^2}$$

$$= 4600\, V$$

1. Regulation $= \dfrac{E_0 - V}{V} \times 100 = \dfrac{4600 - 2400}{2400} \times 100$

$$= 92.5\%.$$

(ii) Ampere-turns method:

It is seen from given data, for normal voltage of 4160 V, field current needed is 150 A.

Field current necessary for F.L. Short-circuit current is 200 A.
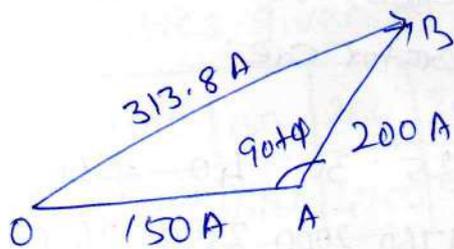
In fig. $OA \rightarrow 150$ A

$AB \rightarrow 200$ A is vectorially added to

OA at $(90 + \phi) = 90° + 36° = 126°$.

Vector OB represents excitation necessary to produce a terminal P.d. of 4160 at 0.8 lagging Pf at full-load.

$$\therefore OB = \sqrt{150^2 + 200^2 + \left(2 \times 150 \times 200 \times \cos(180° - 126°)\right)}$$

$= 313.8$ A

The generated phase emf E_o corresponding to this excitation as found from OCC (if drawn) is 3140 V.

$$\% \text{ Regulation} = \frac{3140 - 2400}{2400} \times 100$$

$$= 30.7 \%.$$

2) The open-and short circuit test readings for a 3-ϕ Star-connected, 1000 kVA, 2000 V, 50 Hz, Synchronous generator are:

Field Amps:	10	20	25	30	40	50
O.C terminal V:	800	1500	1760	2000	2350	2600
SC current A:	—	200	250	300	—	—

The armature effective resistance is 0.2Ω per phase. Draw the characteristic curves and estimate the full-load percentage regulation at (a) 0.8 pf lagging, (b) 0.8 pf leading.

Soln.:

The OCC and SCC are plotted in the graph.

The Phase voltages are:

462   866   1016 1155   1357   1502

Full-load phase voltage : $= \dfrac{2000}{\sqrt{3}} = 1155\,V$

Full-load current $= \dfrac{1000 \times 10^3}{\sqrt{3} \times 2000}$

$$= 288.7\,A$$

Voltage/phase at full-load 0.8 pf

$= V + I Ra \cos\phi$

$= 1155 + (288.7 \times 0.2 \times 0.8) = 1200\,V$
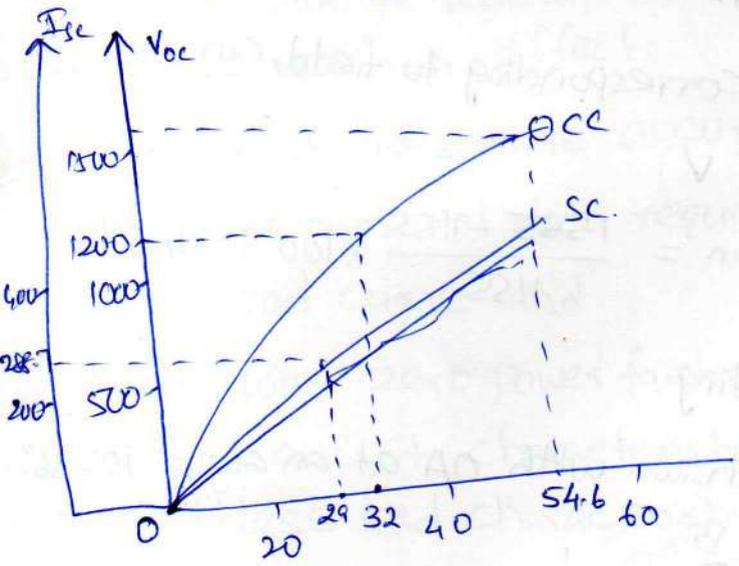
From OC curve, field current necessary to

Produce 1200 V $= 32\,A$

From SC characteristics, field current

necessary to produce full-load current of
288.7 A = 29 A.

(a) 0.8 pf lagging:

$\cos \phi = 0.8$, $\phi = 36°$ (lagging)
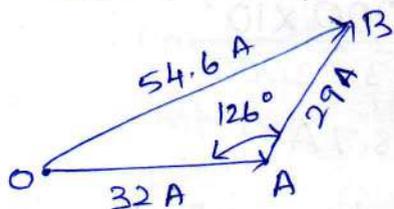
From fig,
$OA = 32$ A
$AB = 29$ A
angle $= 90° + 36° = 126°$ with OA.

$$OB = \sqrt{32^2 + 29^2 + 2 \times 32 \times 29 \times \cos(180 - 126°)}$$

$$= 54.6 \text{ A}$$

O.C. voltage corresponding to field current of

54.6 A $= 1555$ V

$$\text{% Regulation} = \frac{1555 - 1155}{1155} \times 100 = 34.6\%$$

(b) 0.8 pf leading:

AB is drawn with OA at an angle $90° - 36°$

$= 54°$,
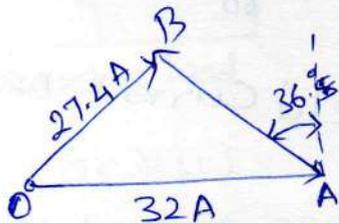
$$OB = \sqrt{32^2 + 29^2 - 2 \times 32 \times 29 \times \cos 54°}$$

$OB = 27.4\,A$

OC voltage Corresponding to $27.4\,A$ of field

excitation $= 1080\,V$

∴ Regulation $= \dfrac{1080 - 1155}{1155} \times 100$

$= -6.4\%.$

Zero power factor Method :

　　(or) Potier Method.

　　This method is based on the separation of armature leakage reactance drop and the armature reaction effects.

　　Hence, it gives more accurate results.

　　The experimental data required is,

i) no-load curve and

ii) full-load zero power factor curve (not the short-circuit characteristics). Called wattless load characteristics.

　　(It is the curve of terminal volts against excitation when armature is delivering F.L. current at zero pf.).

The reduction in voltage due to armature reaction is found from above and

Voltage drop due to armature leakage reactance X_L (also called Potier reactance) is found from both.

By combining these two, E_0 can be calculated.
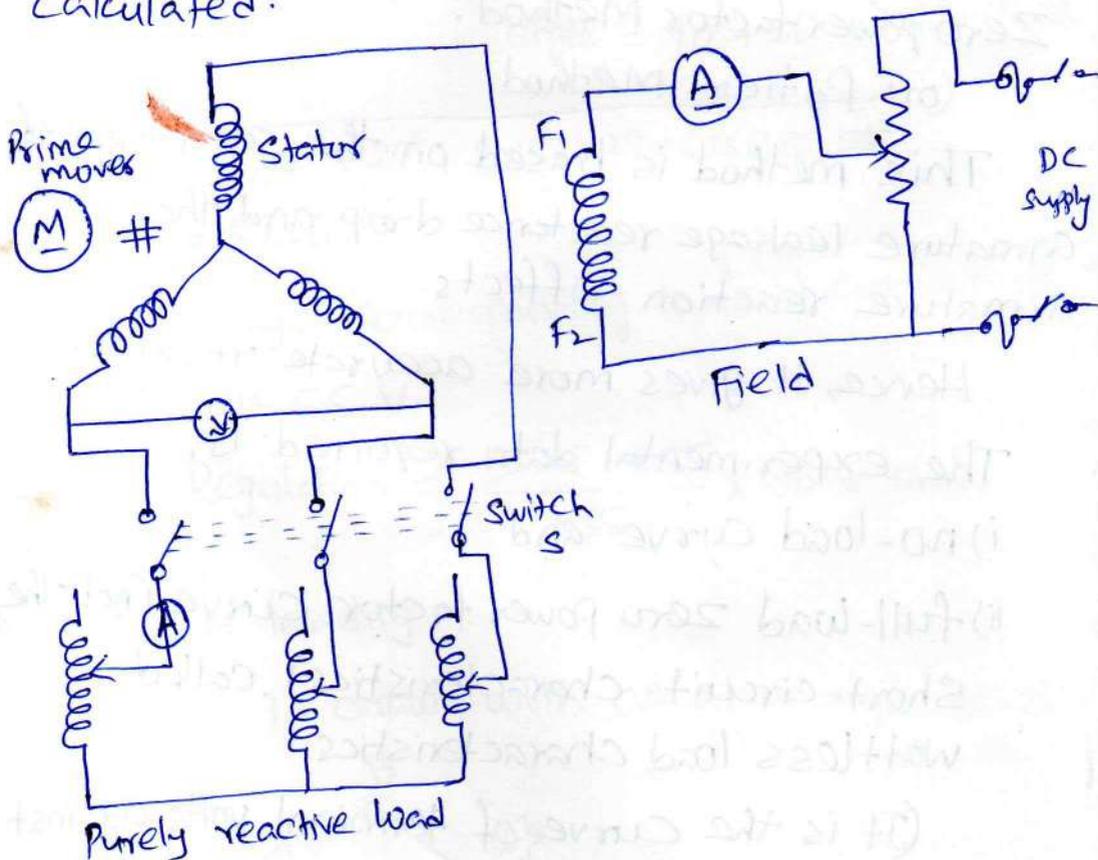
Procedure: (OC test)
1. The switch s is opened.

2. The alternator is made to rotate using Prime mover. at synchronous speed. (Same speed is maintained throughout the test).

3. The excitation value is changed by using a Potential divider, from zero to rated value in steps. (Open circuit voltage is measured with voltmeter).

4. A graph be of I_f and $(V_{oc})_{ph}$ is plotted. This is OCC.

ZPf test:

1. Switch s is kept closed.

2. Purely inductive load is connected to an alternator through an ammeter. (has a Pf of $\cos 90°$ i.e. zero lagging)

3. The machine speed is maintained constant at its synchronous value.

4. Load current is maintained constant at its rated full-load value by varying excitation. and by adjusting variable inductance on the load.

To plot graph:

1. Plot OCC on a graph (terminal voltage against excitation).

2. Plot the excitation corresponding to zero terminal voltage ie short circuit full ZPf armature current. This Point is 'A' in below figure.
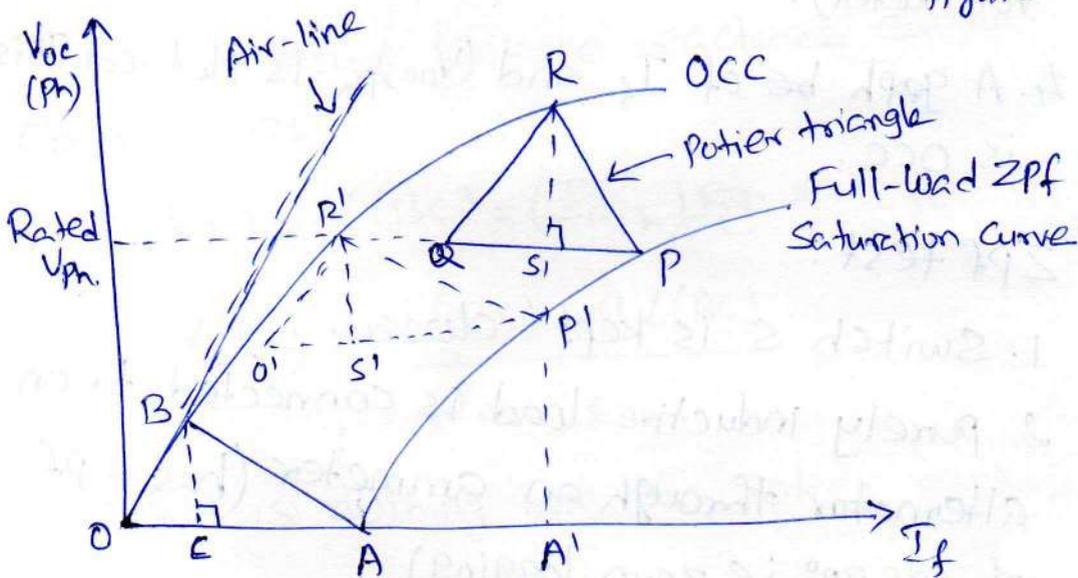
Another point is the rated voltage when the alternator is delivering full current at zero Pf lagging. (Point P)

3. Draw the tangent to OCC through origin which is line OB as shown dotted (called the air line)

4. Draw the horizontal line PQ parallel and equal to OA.

5. From Q, draw a line parallel to the air-line which intersects OCC at point R.

Join RQ and PR. The triangle PQR is called potier triangle.

6. From point R, draw a perpendicular on PQ to meet at point S.

7. The ZPf full-load saturation curve is now be constructed by moving triangle PQR so that R remains always on OCC and line PQ always remains horizontal.

The dotted triangle is in above figure.

The potier triangle once obtained is constant for a given armature current and hence can be transferred as it is.

8. The perpendicular RS gives the voltage drop due to armature leakage reactance ($I X_L$).

9. The length PS gives field current necessary to overcome the demagnetising effect of armature reaction at full-load.

10. The length SQ represents field current required to induce an emf for ~~balancing~~ balancing leakage reactance drop RS.

So, armature leakage reactance can be obtained as,

$$l(RS) = l(BC) = (I_{aph})_{F.L} \times X_{L\,ph}.$$

$$\therefore X_L(ph) = \frac{l(RS) \text{ or } l(BC)}{(I_{aph})_{F.L.}} \, \Omega$$

This is nothing but the potier reactance.

To ~~detter~~ determine Regulation:

Draw the phasor diagram using following procedure.

1. Draw the rated terminal voltage V_{ph} as a reference phasor.

2. Depending on the power factor, draw the

Current phasor I lagging or leading V_{ph} by ⑦
angle ϕ.

3. Draw IR_a drop to V_{ph}. which is in phase
with I

4. Drop IX_L is drawn perpendicular to IR_a
vector, but leading IR_a at the extremity
of V_{ph}.

⑩.
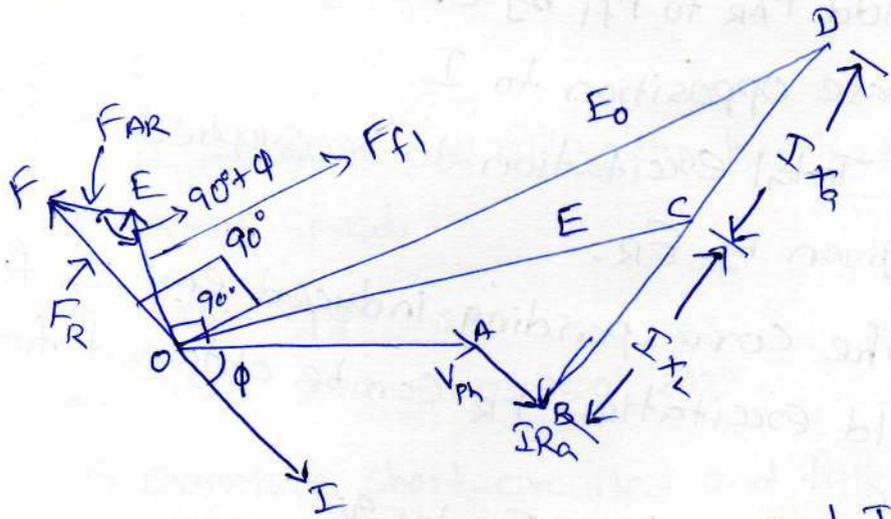
5. Phasor sum of V_{ph} rated, IR_a and IX_L
gives the emf (say E)

$$E = V_{ph} + IR_a + IX_L$$

6. Obtain the excitation corresponding to E
from OCC. (Let this be F_{f1}). (Does not Consider
the effect of armature reaction).

7. The field current required to balance armature reaction can be obtained from Potier triangle (say F_{AR}).

8. The total excitation required is the vector sum of F_{f1} and F_{AR}.

9. Draw vector F_{f1} leading E by $90°$.

10. Add F_{AR} to F_{f1} by drawing vector F_{AR} in phase opposition to I

   Total excitation to be supplied by field is given by F_R.

11. The corresponding induced emf E_0 for field excitation F_R can be obtained from OCC.

   This E_0 lags F_R by $90°$.

12. The length CD (IX_a drop) due to armature reaction is perpendicular to AB. $\cancel{on}/\cancel{current}$

13. Once E_0 is known, the regulation of an alternator can be predicted as,

$$\% \text{ Regulation} = \frac{E_0 - V_{ph}}{V_{ph}} \times 100.$$

The only drawback of this zpf method is that the separate curve for every load condition is necessary to plot if potier triangles for various load conditions are required.

Problems:

1) A 3-phase, ~~6000V~~ 6000V, alternator has the following OCC at normal speed.

Field Amps:	14	18	23	30	43
Terminal volts:	4000	5000	6000	7000	8000

With armature short-circuited and full-load current flowing the field current is 17A and when the machine is supplying full-load of 2000 KVA, at zero power factor, the field current is 42·5 A and the terminal voltage is 6000 V. Determine the field current required when the machine is supplying the full-load at 0·8 pf lagging.

Soln.:

The OCC is drawn with phase voltages

| 2310 | 2828 | 3465 | 4042 | 4620 |
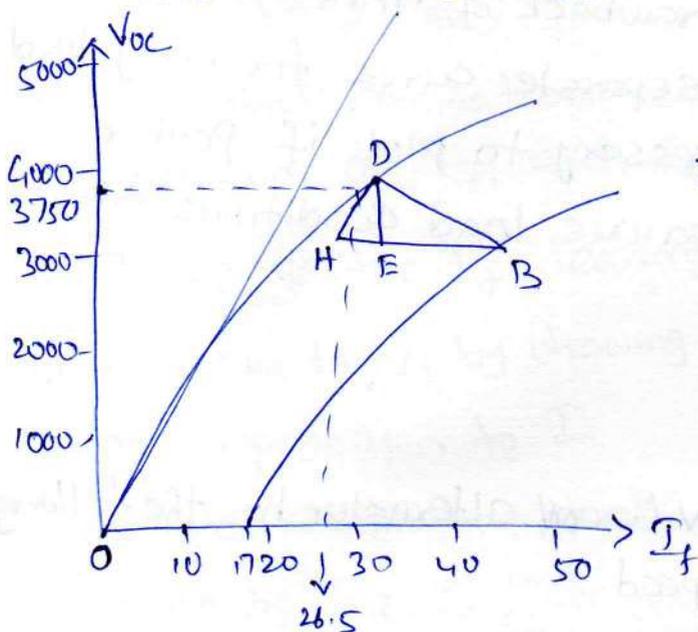
The full-load Zpf curve is drawn with two point given

$(17, 0)$ and
$(42.5, 3465)$

In Potier triangle $\triangle BDH$, line DE represents the leakage reactence drop (IX_L) and equal to 450 V
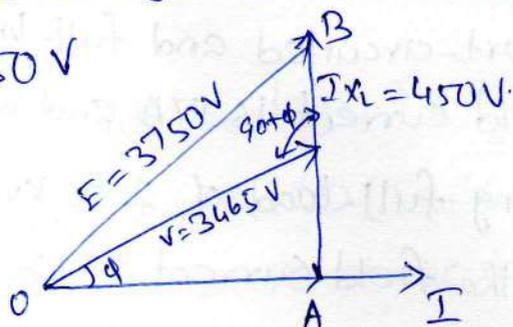
From above fig.

$$E = \sqrt{(V\cos\phi)^2 + (V\sin\phi + IX_L)^2}$$

$$= \sqrt{(3465 \times 0.8)^2 + (3465 \times 0.6 + 450)^2}$$

$$= 3750 \, V.$$

From OCC, it is found that field amperes required for this voltage $= 26.5 \, A$

Field ampere required for balancing armature reaction $= BE = 14.5 \, A$ (from potier triangle measurement).
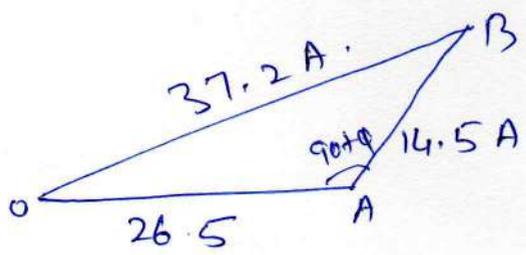
From fig. the field current are added vectorially at an angle of $(90 + \phi) = 126°$

∴ Resultant field current

$$OB = \sqrt{26.5^2 + 14.5^2 + 2 \times 26.5 \times 14.4 \cos 54°}$$

$$= 37.2 \, A$$

Parallel Operation of Synchronous Generator

Introduction:

The operation of connecting an alternator in parallel with another alternator or with common bus-bar is known as synchronizing.

Often the electrical system to which the alternator is connected, has already so many alternators and loads connected to it that, no matter what power is delivered by the incoming alternator, the voltage and frequency of the system remain the same.

In that case, the alternator is said to be connected to infinite bus-bar.

For proper synchronization of alternators, the following three conditions must be satisfied:

1. The terminal voltage (effective) of the incoming alternator must be the same as bus-

bar voltage.

2. The speed of the incoming machine must be such that its frequency $(= \frac{PN}{120})$ equals bus-bar frequency.

3. The phase of the alternator voltage must be identical with the phase of the bus-bar voltage.

(It means that, the switch must be closed at the instant the two voltages have correct phase relationship).

Condition (1) is indicated by a voltmeter, Conditions (2) and (3) are indicated by Synchronizing lamps or a synchronoscope.

Synchronization of Alternators:

(a) Single-phase alternators:

Suppose machine 2 is to be synchronized with or 'put on' the bus-bars to which machine 1 is already connected.

This is done with the help of two lamps L_1 and L_2 (known as synchronizing lamps)

Connected as shown in figure.

Bus-bars

L_1 L_2

→ Dark Lamp method.

E_1

Machine 1

E_2
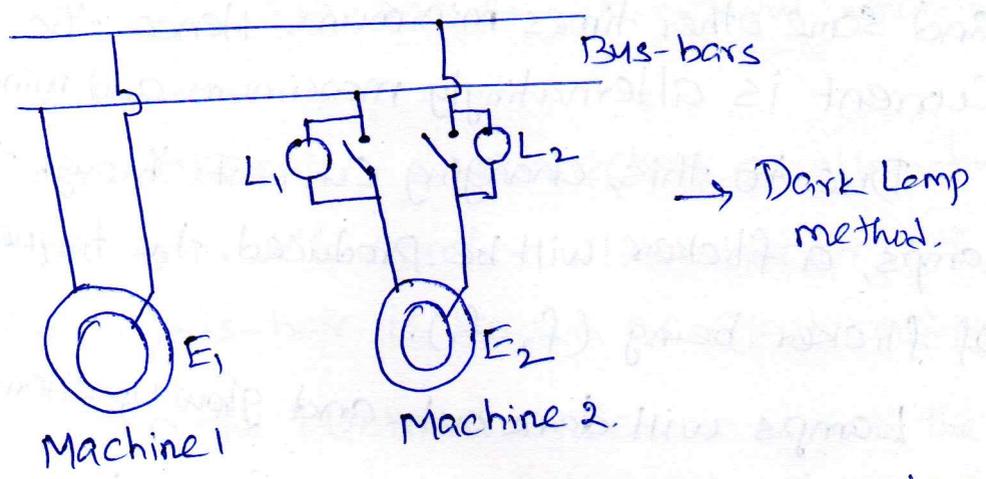
Machine 2.

If the speed of incoming machine 2, is not brought upto machine 1, then its frequency will also be different, hence there will be a phase-difference between their voltages (even at same magnitude).

This phase-difference will be Continuosly changing with the changes in their frequencies.

e_1

e_2

$e_1 + e_2$
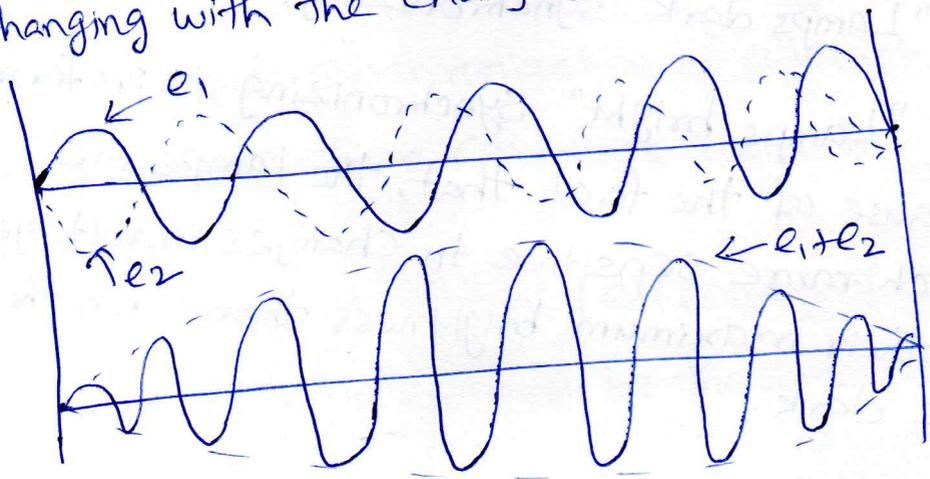
Sometimes, the resultant voltage is maximum and some other times minimum. Hence, the current is alternatingly maximum and minimum.

Due to this, changing current through the lamps, a flicker will be produced, the frequency of flicker being $(f_2 - f_1)$.

Lamps will dark out and glow up alternatively.

Darkness indicates that the two voltages E_1 and E_2 are in exact phase opposition relative to the local circuit and hence there is no resultant current through the lamps.

Synchronizing is done at the middle of the dark period. That is why, it is known as "Lamps dark" synchronizing.

"Lamps bright" synchronizing is preferred, because of the fact that, the lamps are much more sensitive to changes in voltage at their maximum brightness than when they are dark.

Hence, a sharper and more accurate synchronization is obtained. It that case, the lamps are connected as shown in fig. below.

Bus-bars

L_1 L_2

→ Bright lamp method

E_1

Machine 1

E_2
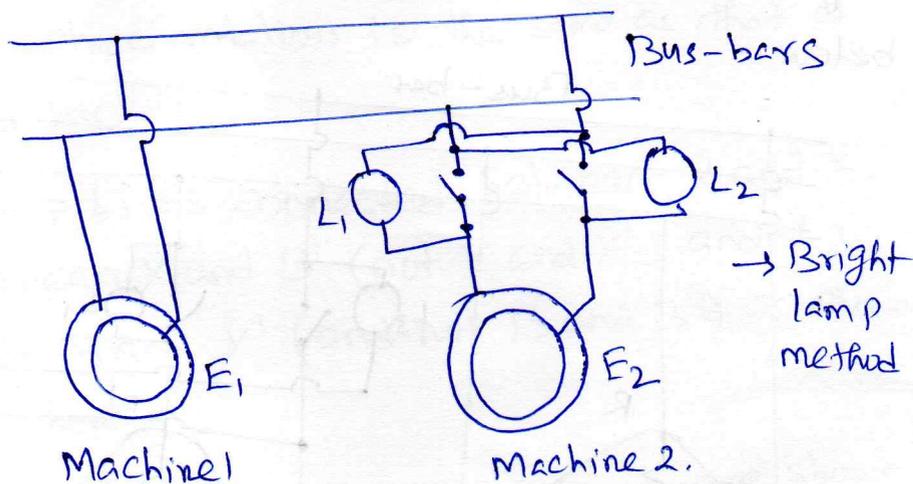
Machine 2.

Now, the lamps will glow brightest when the two voltages are in-phase with the bus-bar voltage, because then voltage across them is twice the voltage of each machine.

(b). Three-Phase Alternators:

In 3-φ alternators, it is necessary to synchronize one phase only, the other two phases will then be synchronized automatically.

It is necessary that the incoming alternator is correctly 'phased out' i.e. the phases are

connected in the proper order of R, Y, B and
not R, B, Y etc.

In this case, three lamps are used, but
they are connected asymmetrically as shown in fig.
below.
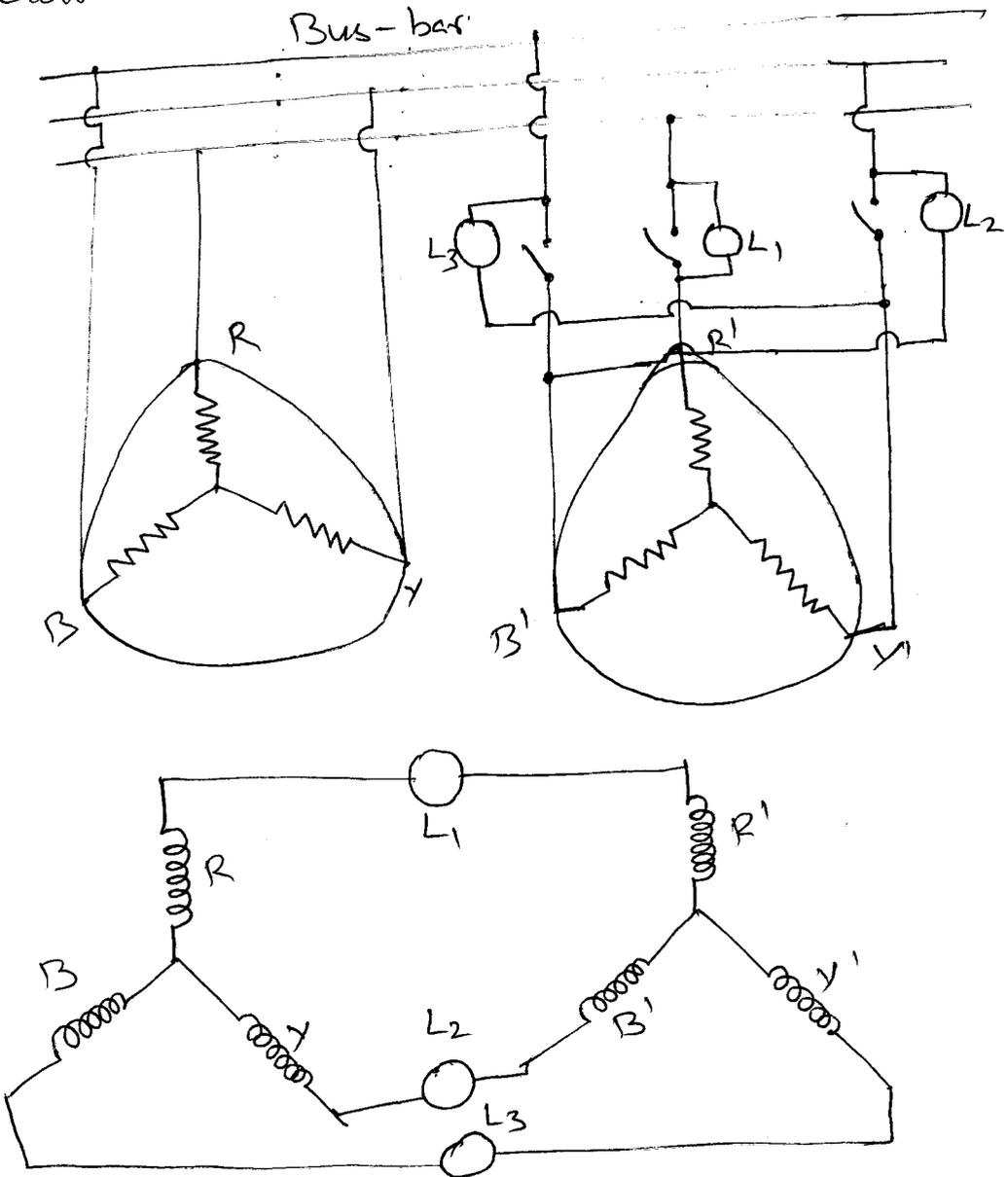
This transposition of two lamps, indicate whether the incoming machine is running too slow. If lamps were connected symmetrically, they would dark out or glow up simultaneously. (If the phase rotation is the same as that of the bus-bar).

Lamp L_1 is connected between R and R', L_2 between Y and B' (not Y and Y') and L_3 between B and Y' (and not B and B') as shown in fig. above.

Voltage stars of two machines are shown superimposed on each other in fig. below.

(a)   (b)
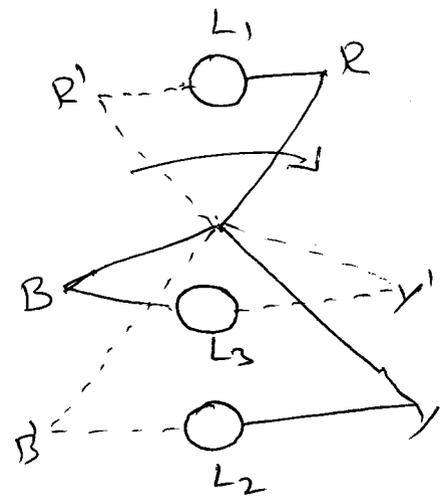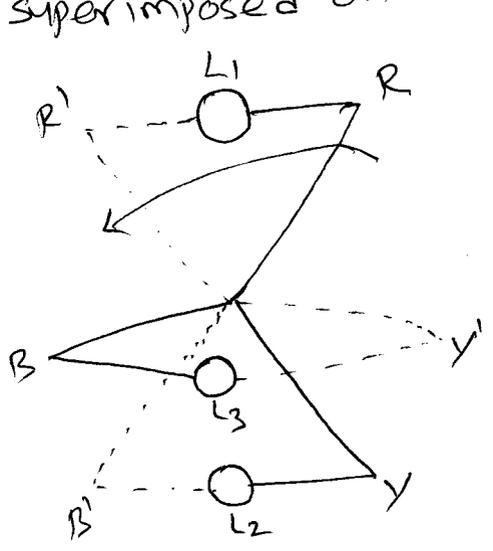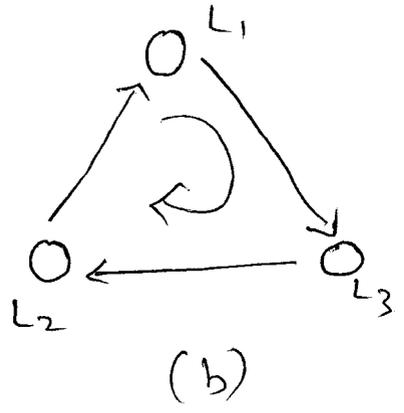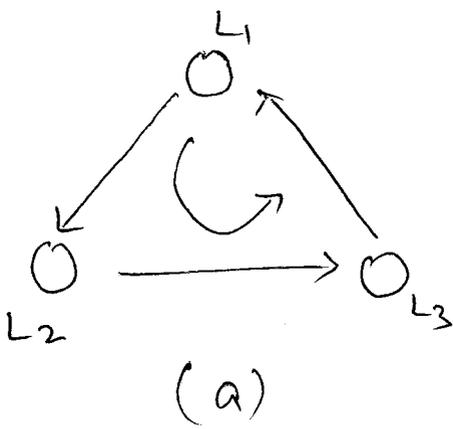
If the incoming alternator is running faster, then voltage star R'Y'B' will appear to rotate anticlockwise with respect to bus-bar voltage star RYB at a speed corresponding to the difference between their frequencies.

With reference to fig. (a), it is seen that the voltage across L_1 is RR' which is increasing from zero,

that across L_2 is YB' which is decreasing, having just passed through its maximum,

that across L_3 is BY' which is increasing and approaching its maximum.

Hence, the lamps will light up one after the other in the order 2, 3, 1; 3, 1, 2 or 1, 2, 3.

Now, suppose that the incoming machine is slightly slower, then the star R'Y'B' will appear to be rotating clockwise relative to voltage star RYB (fig. (b)).

Here, the voltage across L_3 i.e Y'B is decreasing having just passed through its maximum,

that across ~~blue~~ L_2 i.e. YB' is increasing and approaching its maximum.

that across L_1 i.e. RR' is decreasing having passed through its maximum earlier.

Hence, the lamps will light up one after the other in the order 3,2,1; 2,1,3, etc. which is just the reverse of the first order.

Usually, the three lamps are mounted at the three corners of a triangle and the apparent direction of rotation of light indicates whether the incoming alternator is running too fast or too slow.

Synchronization is done at the moment, the uncrossed lamp L_1 is in the middle of the dark period.

It will be noted that when the uncrossed lamp L_1 is dark, the other two 'crossed' lamps L_2 and L_3 are dimly but equally bright.

Hence, this method of synchronizing is also sometimes known as 'two bright and one dark' lamp method.

It should be noted that synchronization by lamps is not quite accurate, because it depends on the sense of correct judgement of the operator.

Hence, to eliminate the element of personal judgement in routine operation of alternators, the machines are synchronized by a more accurate device called a synchronoscope.

Parallel operation of Two-Alternators:

Consider two alternators with identical speed/load characteristics connected in parallel as shown in fig.
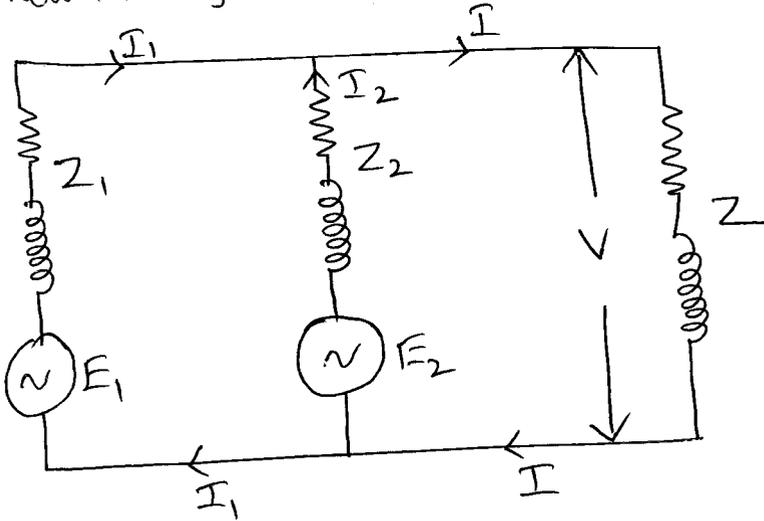
The common terminal voltage V is given by,

$$V = E_1 - I_1 Z_1 = E_2 - I_2 Z_2$$

$$\therefore E_1 - E_2 = I_1 Z_1 - I_2 Z_2$$

$$I = I_1 + I_2 \quad \text{and} \quad V = IZ$$

$$\therefore E_1 = I_1 Z_1 + IZ$$

$$= I_1 Z_1 + (I_1 + I_2) Z = I_1 (Z + Z_1) + I_2 Z$$

$$E_2 = I_2 Z_2 + IZ$$

$$= I_2 (Z + Z_2) + I_1 Z$$

$$\therefore I_1 = \frac{(E_1 - E_2)Z + E_1 Z_2}{Z(Z_1 + Z_2) + Z_1 Z_2}$$

$$I_2 = \frac{(E_2 - E_1)Z + E_2 Z_1}{Z(Z_1 + Z_2) + Z_1 Z_2}$$

$$I = \frac{E_1 Z_2 + E_2 Z_1}{Z(Z_1 + Z_2) + Z_1 Z_2}$$

$$V = IZ = \frac{E_1 Z_2 + E_2 Z_1}{Z_1 + Z_2 + \left(\frac{Z_1 Z_2}{Z}\right)}$$

$$\therefore I_1 = \frac{E_1 - V}{Z_1} \quad ; \quad I_2 = \frac{E_2 - V}{Z_2}$$

The circulating current under no-load condition is,

$$I_c = \frac{E_1 - E_2}{Z_1 + Z_2}$$

Using Admittances,

The terminal voltage may also be expressed in terms of admittances as shown below:

$$V = IZ = (I_1 + I_2)Z$$

$$\therefore I_1 + I_2 = \frac{V}{2} = Vy \quad - \quad ①$$

Also,
$$I_1 = \frac{E_1 - V}{Z_1} = (E_1 - V)y_1 \; ;$$

$$I_2 = \frac{E_2 - V}{Z_2} = (E_2 - V)y_2 \; ;$$

$$\therefore I_1 + I_2 = (E_1 - V)y_1 + (E_2 - V)y_2 \quad - \quad ②$$

From eqns. ① & ②, we get

$$Vy = (E_1 - V)y_1 + (E_2 - V)y_2$$

$$\Rightarrow \quad V = \frac{E_1 y_1 + E_2 y_2}{y_1 + y_2 + y} \; .$$

Synchronizing current :
~~Parallel operation of synchronous generator to~~
~~Transformer Bank~~

    Once synchronized properly, two alternators continue to run in synchronism.

    When in exact synchronism, the two alternators have equal p.d's and are in exact phase opposition, so far as the local circuit is concerned.

    Hence, there is no current circulating round

the local circuit.

As shown in fig. (a), emf of machine 1 (E_1) is in exact phase opposition to the emf of machine 2 (i.e. E_2).

The two emfs are in opposition, so far as their local circuit is concerned, but are in same direction with respect to the external circuit.

Hence, there is no resultant voltage ($E_1 = E_2$ in magnitude) round the local circuit.

(a)
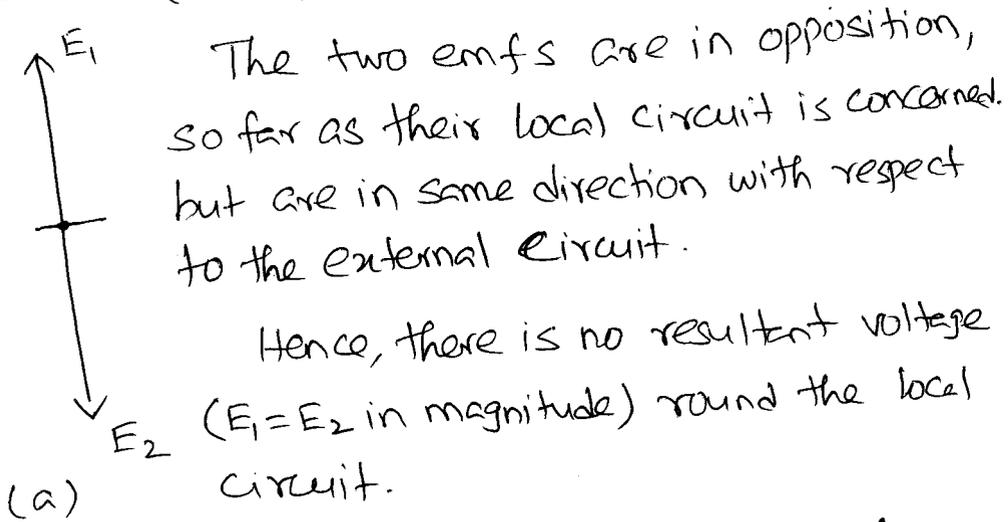
Now, suppose due to change in speed of machine 2, E_2 falls back by an angle α as shown in fig. (b). (Still $E_1 = E_2$ in magnitude)

Now, there exist a resultant voltage E_r, which acting on the local circuit, circulates a current known as "synchronizing current".

The value of this synchronizing current is given by,

(b)
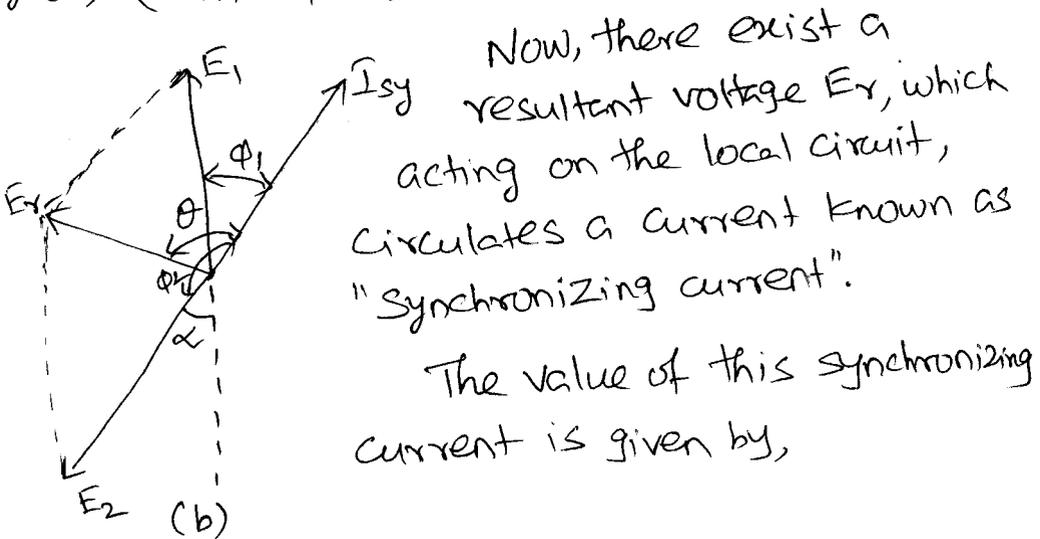
$$I_{sy} = \frac{E_r}{Z_s}$$

where, $Z_s \rightarrow$ the synchronous impedance of the phase windings of both the machines.

The current I_{sy} lags behind E_r by an angle θ given by

$$\tan \theta = \frac{X_s}{R_a}$$

where, $X_s \rightarrow$ Combined synchronous reactance of the two machines

and $R_a \rightarrow$ their armature resistance.

Similarly, if E_2 tends to advance in phase (fig. (c)). then I_{sy} circulates.

(c)
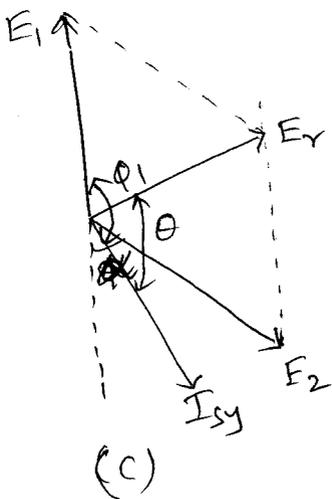
Therefore, any departure from synchronism results in the production of a synchronizing current I_{sy}, which sets up synchronizing torque to re-establish synchronism between the two machines by retarding the leading machine and by accelerating the lagging one.

Consider fig. (b), the synchronizing power

$= E_1 I_{sy} \cos \phi_1$ which is approximately equal to $E_1 I_{sy}$ (ϕ_1 is small).

Since, $\phi_1 = (90 - \theta)$, Synchronizing power

$$= F_1 I_{sy} \cos \phi_1$$
$$= E_1 I_{sy} \cos(90 - \theta) = E_1 I_{sy} \sin \theta$$
$$\simeq E_1 I_{sy} \quad (\because \theta \simeq 90°)$$

This power output from machine 1, goes to supply,

(a). Power input to machine No. 2 (which is motoring) and

(b). the cu losses in the local armature circuit of the two machines.

Power input to machine 2 is $E_2 I_{sy} \cos \phi$ which is approximately equal to $E_2 I_{sy}$.

$$\therefore E_1 I_{sy} = E_2 I_{sy} + Cu. \text{ losses.}$$

Now, let $E_1 = E_2 = E$ (say)

Then, $E_r = 2E \cos \left[(180° - \alpha)/2 \right]$
$$= 2E \cos \left[90° - (\alpha/2) \right]$$
$$= 2E \sin \frac{\alpha}{2}$$
$$= 2E \times \frac{\alpha}{2} = \alpha E \qquad (\because \alpha \text{ is small}).$$

Now, $I_{sy} = \dfrac{E_r}{Z_s} \simeq \dfrac{E_r}{2X_s} = \dfrac{\alpha E}{2X_s}$

(if R_a of both machines is negligible).

Here, $X_s \rightarrow$ Synchronous reactance of one machine and not of both.

Synchronizing Power (by machine 1) is,

$$P_{sy} = E_1 I_{sy} \cos \phi_1$$
$$= E I_{sy} \cos (90° - \theta)$$
$$= E I_{sy} \sin \theta$$
$$\simeq E I_{sy}.$$

Substituting the value of I_{sy} from above,

$$P_{sy} = E \cdot \dfrac{\alpha E}{2X_s} = \dfrac{\alpha E^2}{2X_s} \text{ per phase}$$

Total Synchronizing Power for three phases,

$$= 3 P_{sy}$$
$$= \dfrac{3 \alpha E^2}{2X_s}$$

This is the value of the synchronizing Power when two alternators are connected in

Parallel and are on no-load.

Parallel operation of synchronous Generator to infinite Bus:

Now, consider the case of an alternator which is connected to infinite bus-bars.

The expression for Psy is still applicable but with one important difference i.e. impedance (or) reactance of only that one alternator is considered (and not of two).

Hence, expression for synchronizing power in this case becomes

$$E_r = \alpha E \quad - \quad \text{as before}$$

$$I_{sy} = \frac{E_r}{Z_s} = \frac{E_r}{X_s} = \frac{\alpha E}{X_s} \quad \text{if } R_a \text{ negligible}$$

∴ Synchronizing Power,

$$P_{sy} = E \cdot I_{sy} = E \cdot \frac{\alpha E}{X_s} = \frac{\alpha E^2}{X_s} \quad \text{per phase}$$

Now, $\dfrac{E}{X_s}$ = s.c. current = I_{sc}.

∴ $P_{sy} = \alpha E \cdot \dfrac{E}{X_s} = \alpha E \cdot I_{sc}$. — per phase.

Total synchronizing power for three phases,

$$= 3 \, P_{sy}.$$

Synchronizing Torque T_{sy}:

    Let T_{sy} be the synchronizing torque per phase in (N-m).

(a) when two-alternators in parallel:

$$T_{sy} \times \frac{2\pi N_s}{60} = P_{sy}$$

$$\therefore T_{sy} = \frac{P_{sy}}{(2\pi N_s / 60)} = \frac{\alpha E^2 / 2 X_s}{2\pi N_s / 60} \quad N\text{-}m.$$

Total torque due to three phases,

$$= \frac{3 P_{sy}}{(2\pi N_s / 60)} = \frac{(3\alpha E^2 / 2 X_s)}{(2\pi N_s / 60)} \quad N\text{-}m.$$

(b) Alternator connected to infinite bus-bars.

$$T_{sy} \times \frac{2\pi N_s}{60} = P_{sy}.$$

$$T_{sy} = \frac{P_{sy}}{(2\pi N_s / 60)} = \frac{\alpha E^2 / X_s}{2\pi N_s / 60} \quad N\text{-}m.$$

Torque due to three phases, $= \dfrac{3 P_{sy}}{(2\pi N_s / 60)}$

$$= \frac{3 \alpha E^2 / X_S}{2 \pi N_s / 60} \quad N-m.$$

where,

$N_s \rightarrow$ synchronous speed in rpm $= \frac{120 f}{P}$

Problems:

1) A 3000 kVA, 6-pole alternator runs at 1000 rpm in parallel with other machines on 3300 V bus-bars. The synchronous reactance is 25%. Calculate the synchronizing power for one mechanical degree of displacement and the corresponding synchronizing torque.

Soln.:

It may please be noted that here the alternator is working in parallel with many alternators.

Hence, it may be considered to be connected to infinite bus-bars.

$$\text{Voltage / phase} = \frac{3300}{\sqrt{3}} = 1905 \text{ V}$$

$$\text{F.L. Current, } I = \frac{3000 \times 10^3}{\sqrt{3} \times 3300} = 525 \text{ A}$$

Now, $IX_s = 25\%$ of 1905

$$\therefore X_s = \frac{0.25 \times 1905}{525} = 0.9075\,\Omega$$

Also, $P_{sy} = \frac{3\alpha E^2}{X_s}$

Here, $\alpha = 1°$ (mech.)

$$\alpha \,(\text{elect.}) = 1 \times \frac{6}{2} = 3°$$

(electrical degree
= No. of pole
pairs
= $\frac{\text{no. of poles}}{2}$)

$$\therefore \alpha = 3 \times \frac{\pi}{180} = \frac{\pi}{60} \text{ elect. radian.}$$

$$P_{sy} = \frac{3 \times \pi \times 1905^2}{60 \times 0.9075}$$

$$= 628.4 \text{ kW}$$

$$T_{sy} = \frac{60 \cdot P_{sy}}{2\pi N_s} = 9.55 \cdot \frac{P_{sy}}{N_s}$$

$$= 9.55 \times \frac{628.4 \times 10^3}{1000}$$

$$= 6,000 \text{ N-m.}$$

Sharing of Real and Reactive Powers:

Load sharing is defined as the Proportional division of kW and kVAR total load between multiple generator ~~sets~~ sets in a paralleled System.

Load sharing is essential to avoid overloading and stability problems on the systems' generator sets.

The speed of the generator set determines the proportional sharing of total active power requirements of the system. This can be achieved by increasing or decreasing fuel to the system's engines.

If the fuel to the engine of one generator set is increased, it will not lead to an increase in speed and frequency, but will lead to increase in the proportion of total kw load that it will deliver.

The control system of the generator sets monitors and controls the sharing of total kw load in proportion to the relative rating of the engines.

When generators operate in parallel, the field excitation system of each generator controls the proportional sharing of the total

reactive power requirement of the system.

The KVAR load sharing is achieved by increasing or decreasing the field excitation to the systems' alternators.

If the field excitation of one generator in a group is increased i.e. over excited, it will not lead to an increase in voltage, but it will lead to an increase in the proportion of the total KVAR load it will deliver and a decrease in its power factor.

An undesirable circulating reactive current will flow in the system if the excitation of the alternators is not matched.

The voltage control system of the generator sets monitors and controls the sharing of the total KVAR load in proportion to the relative rating of the alternators on the system.

Capability curve:

The capability curve of a synchronous

generator defines a boundary within which the machine can operate safely.

It is also known as "operating Charts" or "capability Charts".

The permissible region of operation is restricted to the following points given below

- The MVA loading should not exceed the generator rating
- The MW loading should not exceed the rating of the Prime mover.
- The field current should not be allowed to exceed a specified value.
- For steady-state or stable operation, the load angle δ must be less than $90°$

The Capability Curve is based upon the phasor diagram of the synchronous machine.

The phasor diagram of an alternator at lagging power factor is shown in fig.

Assumed, armature resistance to be negligible.

The machine is connected to constant voltage bus-bar.

∴ The length $O'O = V_p$ is fixed

The axes OX and OY are drawn with their origin O at the tip of V_p.

$OB = I_a X_s$; $OA = I_a X_s \sin \phi$ and

$AB = I_a X_s \cos \phi$.

The real power output P is given by,

$$P = 3 V_p I_a \cos \phi$$

The reactive power output Q is given by,

$$Q = 3 V_p I_a \sin \phi$$

A typical capability curve for a synchronous generator is shown below:
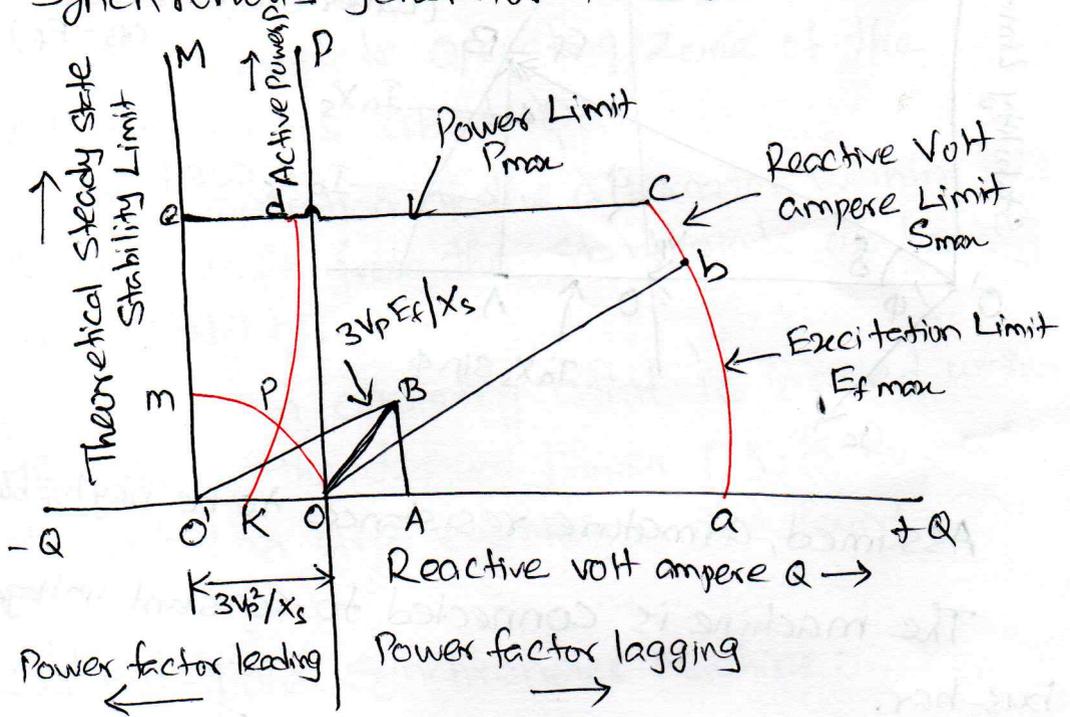
The curve is plotted on the S-plane, where P is the vertical axis and Q is the horizontal axis.

For constant Power, I_a and volt-ampere, the locus is a circle with a center at O and radius $OB (= 3V_p I_a)$

Constant p operation lies on a line parallel to Q axis.

The Constant excitation locus is a circle

with center O' and radius O'B $(=3 V_p E_f / X_s)$.

For excitation E_f equal to zero, the armature current is given as:

$$I_a = \frac{V_p}{X_s} = \text{short circuit current at rated voltage.}$$

$$= OO'$$

The theoretical stability limit is a straight line O'M at right angles to O'O at O'.

Between a and b, the operation of the alternator is limited by the maximum field current, and a circle of radius $(3 V E_f / X_s)$ with centre O'.

Between b and c, the operation is limited by MVA limit. Here I_a is the maximum permissible armature current.

Between c and d, the operation is limited by the power of prime mover.

Between d and e, the operation is limited by the practical stability limit.

The theoretical limit of stability occurs where $\delta = 90°$.

The practical limit is usually taken 10% less than the theoretical stability limit.

The complete operating zone of the alternator is abcdkOa.

The operation of the alternator within this area is safe from the standpoints of heating and stability.

Once an operating point is located within this area, the desired power P, S, Q, current, power factor and excitation are found.

Salient-Pole Synchronous Machine:

A synchronous machine with salient or projecting poles has non-uniform air-gap, due to which its reactance varies with the rotor position.

A cylindrical rotor machine possesses one axis of symmetry (Pole axis or direct axis) whereas salient pole machine possesses two axes of geometric symmetry.

i) field poles axis, called direct axis or

d-axis and

ii) axis passing through the center of the
inter polar space, called the quadrature axis
or q-axis as shown in fig.
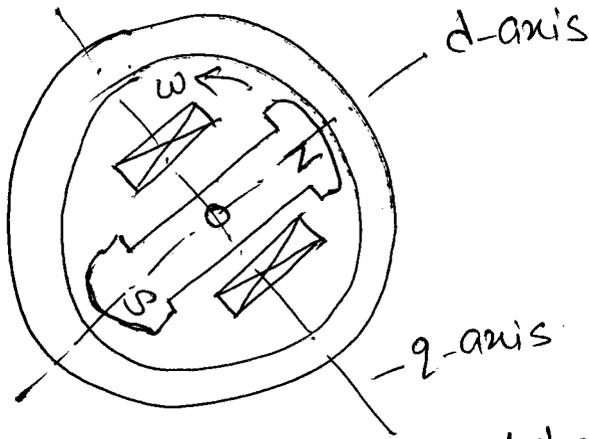
Obviously, two
mmfs act on the d-axis
of a salient-pole
synchronous machine
i) field mmf and
ii) armature mmf
whereas, only one mmf
i.e. armature mmf acts on the q-axis, because
field mmf has no component in q-axis.

Two-Reaction theory:

The magnetic reluctance is low along
the poles and high between the poles.

The above facts form the basis of the two-
reaction theory proposed by Blondel, according
to which,
i) armature current I_a can be resolved into
two components. i.e. I_d perpendicular to E_0 and

I_q along E_0 as shown in fig.
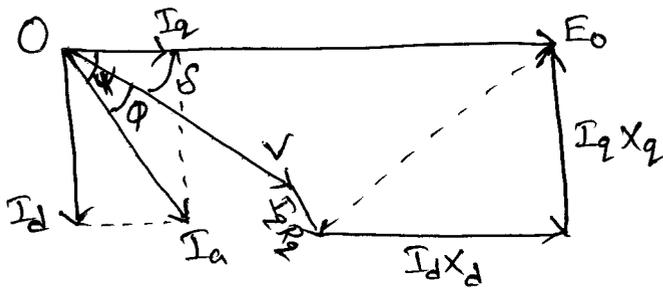
ii) Armature reactance has two components i.e. d-axis armature reactance X_{ad} associated with I_d and q-axis armature reactance X_{aq} linked with I_q.

If we include the armature leakage reactance X_l which is the same on both axes, we get

$$X_d = X_{ad} + X_l \quad \text{and}$$
$$X_q = X_{aq} + X_l$$

Since, reluctance on the q-axis is higher, owing to the larger air-gap, hence,

$$X_{aq} < X_{ad} \quad \text{or} \quad X_q < X_d$$
$$\text{or} \quad X_d > X_q$$

The equivalent circuit of a salient-pole synchronous generator is shown in fig. below.

Angle ψ (b/w E_0 and I_a) → internal power factor angle.

Angle δ (b/w E_0 and V) → power angle
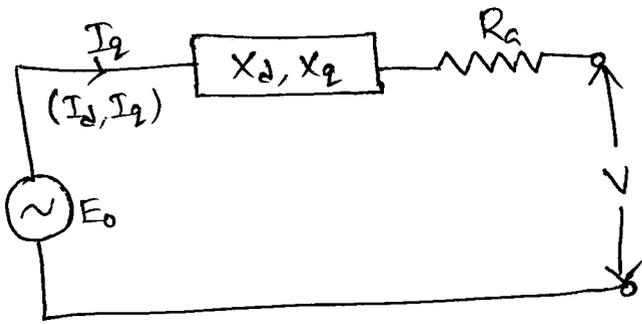
$$E_0 = V + I_a R_a + j I_d X_d + j I_q X_q$$

$$\text{and } I_a = I_d + I_q$$

If R_a is neglected,

$$E_0 = V + j I_d X_d + j I_q X_q.$$

Determination of Direct Axis and Quadrature Axis
Reactances of Salient-Pole Machines:

Direct and quadrature axis reactances of
a salient-pole synchronous machine can be
estimated by means of test known as the
"slip test" on synchronous machine.
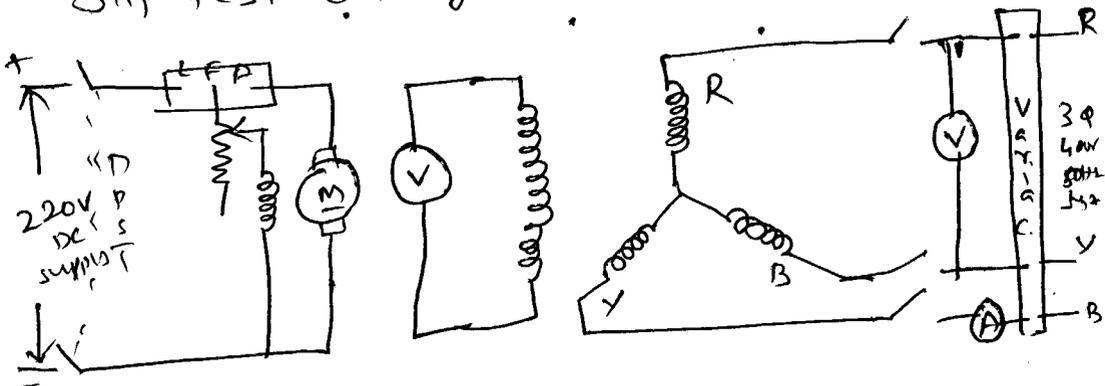
Procedure:

- Connect the circuit as per circuit Diagram.
- Start the DC motor with the help of 3-point starter.
- Adjust the speed of alternator near to synchronous speed.
- A balanced reduced external voltage is applied by varying 3-phase variac.
- The reading of maximum & minimum induced voltage indicated by the voltmeter connected across the field circuit of the alternator are noted and tabulated.
- The balanced applied voltage of the alternator is reduced to zero and TPST is opened.
- The speed is reduced to normal value and the motor is switched off by using DPST switch.

Tabulation:

Maximum Voltage	minimum Voltage	maximum Current	Minimum Current

Calculation:

Direct axis armature reactance is given by

$$X_d = \frac{\text{maximum induced voltage across the field}}{\text{minimum armature current}}$$

Quadrature axis armature reactance is given by,

$$X_q = \frac{\text{minimum induced voltage across the field}}{\text{maximum armature current}}.$$

Power-angle characteristics :

Synchronous and Single Phase Motors.

Introduction:

A synchronous motor is one in which, the rotor normally rotates at the same speed as the revolving ~~speed~~ field in the machine.

The main features of synchronous motor are:

- The speed of the synchronous motor is independent of the load.

- The synchronous motor is not self-Starting. The prime-mover is used for rotating the motor at its synchronous Speed.

- The synchronous motor can operate both for leading and lagging power factor.

Principle of Operation:

Synchronous motors are a doubly excited machine i.e. two electrical inputs are provided to it.

i) Three phase supply to three-phase stator winding.

ii) DC Supply to the rotor winding.

The field current of a Synchronous motor produces a steady state magnetic field. ~~cost thereby~~ B_R ~~Produces set of currents in generator rotating~~

~~Produces set of currents in generator rotating~~
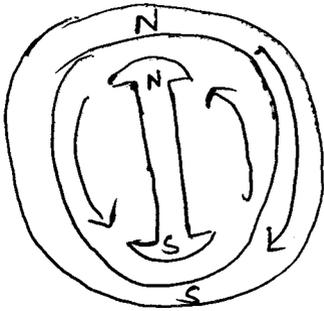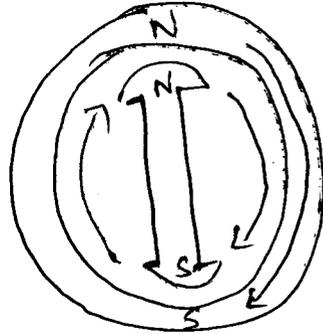
(a)   (b)

The three-phase voltage applied to the stator, produces a three-phase current in the windings, which produces a uniform rotating magnetic field B_s in the air-gap.

The stator rotates at synchronous speed

Therefore, there are two magnetic fields (B_R and B_s) present in the machine, The rotor field will tend to line up with the stator field.

Since the stator magnetic field is rotating, the rotor magnetic field (and the rotor itself) will try to catch up with the rotating magnetic

field of stator.

This is possible when the rotor also rotates at synchronous speed.

The basic principle of operation of synchronous motor is that the rotor chases the stator magnetic field.

If the rotor of the synchronous motor is rotated by some external means at the start, there will be a continuous force of attraction between the stator and the rotor.

This is called "magnetic locking".

Once, this stage is reached, the rotor pole is dragged by the revolving stator field and thus the rotor will continue to rotate.

Starting Methods:

If three-phase supply is given to the stator phases of a stationary synchronous m/c. with the rotor excited, no steady starting torque will be developed.

Instead, a sinusoidally time-varying torque is developed, the average value of which is zero.

That is why a synchronous motor as such is not self-starting.

The starting of a synchronous motor from its stationary condition can be achieved by the following methods:

a) Starting with the help of Damper winding:

To enable the synchronous machine to start independently as a motor, a damper winding is made on pole face slots.
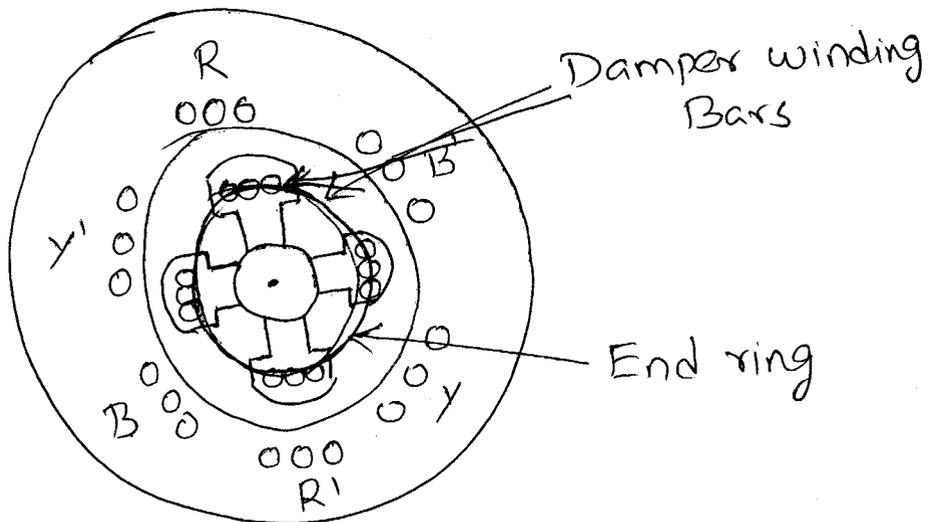
Bars of copper, aluminium, bronze or similar alloys are inserted in slots made on pole shoes as shown in fig.

These bars are short-circuited by end-rings on each side of the poles.

Thus these short circuited bars form a squirrel-cage winding.

On application of three-phase supply to the stator, a synchronous motor with damper winding will start as a three-phase induction motor and rotate at a speed near to synchronous speed.

Now, with the application of DC excitation to the field winding, the rotor will be pulled into synchronous speed since the rotor poles are now rotating at only slip-speed with respect to the stator rotating magnetic field.

To limit the starting current drawn by the motor, reduced voltage can be applied through an auto-transformer or through star-delta starter.

During starting period, before applying DC excitation, the field windings are kept closed

- either through a resistor (where DC is supplied from an independent source) or

- through the armature of the DC exciter (namely the DC generator) on the shaft of synchronous motor.

If this is not done, a high voltage induced in the DC winding during starting period will strain the insulation of the field winding.

Since, starting of the motor is done as an induction motor, the starting torque is rather low and therefore, large capacity motors may not be able to start on full-load.

b) Starting with the help of separate small Induction motor :

In this method, the synchronous motor

is brought to synchronous speed with the help of a separate induction motor.

The number of poles of the induction motor should be less than the number of poles of the synchronous motor to enable it to rotate at the synchronous speed of the synchronous motor.

In this method, however, the motor will have to be synchronised with the bus-bar.

c) Starting by using DC motor coupled to synchronous motor:

The DC motor drives the synchronous motor and brings it to synchronous speed.

The synchronous machine is then synchronised with the bus-bar.

Out of the three methods mentioned, starting by using damper winding is the most

commonly used method since no auxiliary machine is required.

Phasor Diagram:

In a synchronous machine, a back emf E_b is set up in the armature (stator) by the rotor flux which opposes the applied voltage V.

The net voltage in armature (stator) is the vector difference of V and E_b.

(a)
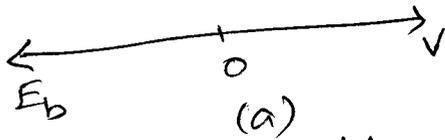
Fig(a) shows the condition when the motor is running on no-load, and has no-losses and is having field excitation which makes $E_b = V$.

It is seen that vector difference of E_b and V is zero and so is the armature current.

If motor is on no-load, but it has losses, then the vector E_b falls back by a certain small angle α. (Shown in fig(b) below), so that a resultant voltage E_R. and hence current I_a is brought into existence, which supplies losses.

5-③
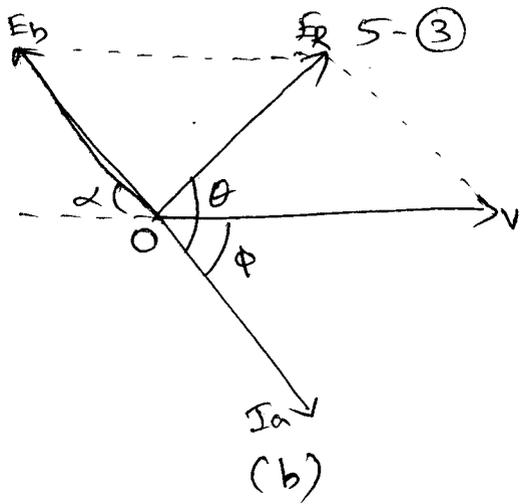
If the motor is loaded, then its rotor will further fall back in phase by a greater value of angle α - called the load angle or coupling angle.

(b)

The resultant voltage E_R is increased and motor draws an increased armature current (shown in fig.(c)) though at a slightly decreased power factor.

(c)
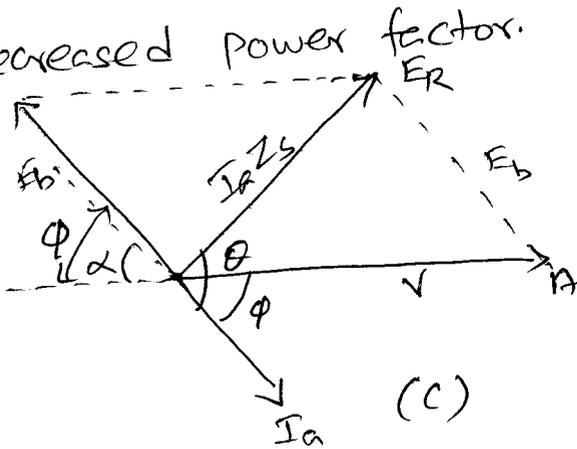
Effect of changing load and changing excitation on machine performance :

A synchronous motor is said to have normal excitation when its $E_b = V$.

If field excitation is such that $E_b < V$, the motor is said to be under-excited.

In both these conditions, it has lagging power factor as shown in fig.(a & b)
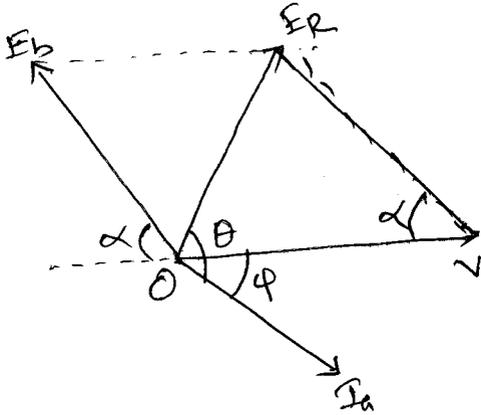
$E_b = V$
lagging Pf. (a)
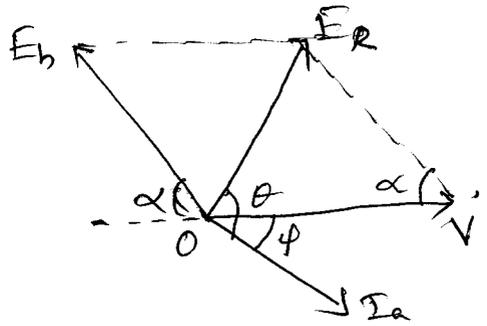
$E_b < V$
lagging Pf
(b)

On the other hand, if the field excitation is such that, $E_b > V$, then motor is said to be over-excited and draws a leading current. as shown in fig.(c).
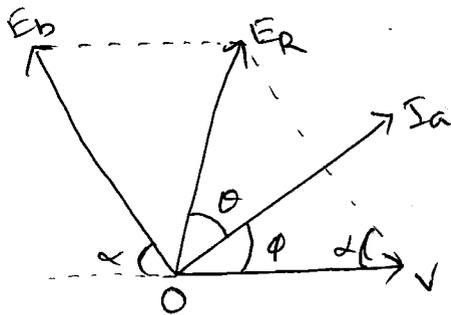
(c)
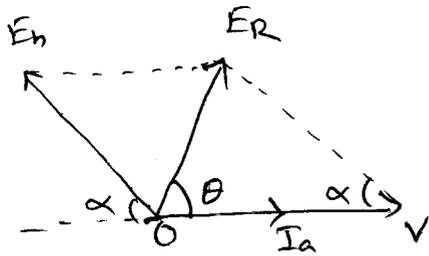
$\phi = 0$
(d)

There will be some value of excitation for which armature current will be in phase with V, so that the power factor will become unity, as shown in fig. (d)

Now, the effect of change in load on a synchronous motor under various conditions (i.e. normal, under and over-excitation) will be discussed.

Whatever the value of excitation, it would be kept "constant".

It is also assumed that, R_a is negligible as compared to X_s, so that phase angle between E_R and I_a i.e. $\theta = 90°$.

i) Normal excitation:

$E_b = V$ (a)

$E_b = V$ (b)
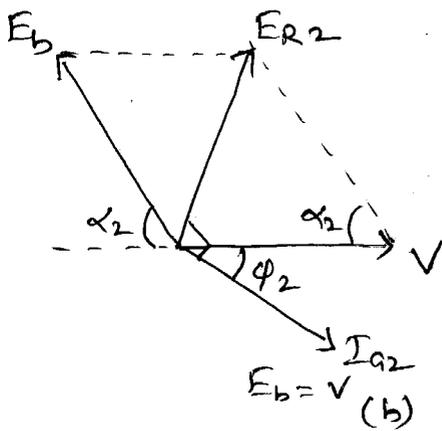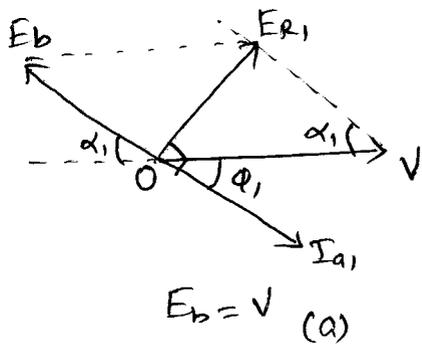
Fig. (a) shows the condition when motor is running with light load so that,

(i) torque angle α_1 is small

ii) so E_{R1} is small   iii) hence I_{a1} is small

and °iv) ϕ_1 is small so that $\cos \phi_1$ is large.

Now, if the load on the motor is increased, as shown in fig. (b).

To meet out this extra load, motor must develop more torque by drawing more armature current.

What actually happens is as under:

- rotor falls back in phase i.e. load angle increases to α_2 (shown in fig. (b)).

- the resultant voltage in armature is increased considerably (E_{R2}).

- As a result, I_{a1} increases to I_{a2}, thereby increasing the torque developed by the motor.

- ϕ_1 increases to ϕ_2, so that power factor decreases from $\cos \phi_1$ to $\cos \phi_2$.

Since, increase in I_a is much greater than the slight decrease in power factor, the torque developed by the motor is increased to meet out the extra load put on the motor.

ii) Under excitation: $(E_b < V)$
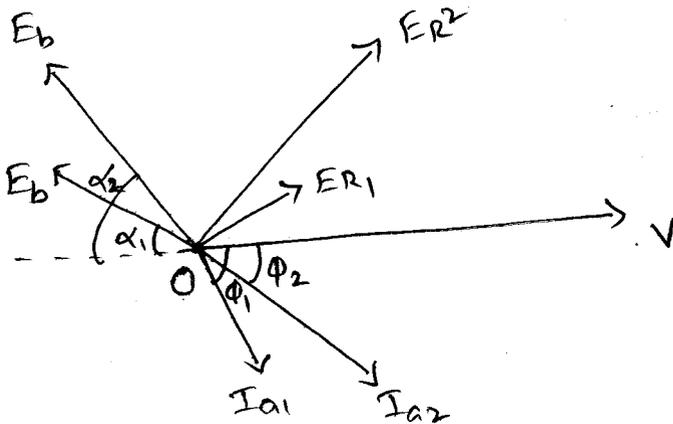
As shown in fig. with a small load and hence, small torque angle α_1, I_{a_1} lags behind V by a large phase angle ϕ_1, which means poor power factor.

Unlike, normal excitation, a much larger armature current must flow for developing the same power because of poor power factor.

As load increases, E_{R_1} increases to E_{R_2}, consequently I_{a_1} increases to I_{a_2} and power

factor angle decreases from ϕ_1 to ϕ_2 or power factor increases from $\cos \phi_1$ to $\cos \phi_2$.

Therefore, power generated by the armature increases to meet the increased load

iii) Over-excitation: $(E_b > V)$:

When running on light load, α_1 is small but I_{a1} is comparatively larger and leads V by a larger angle ϕ_1.

Like the under-excited motor, increase in load, improves the powerfactor and approaches unity.

The armature current also increases thereby producing the necessary increased armature power to meet the increased applied load.
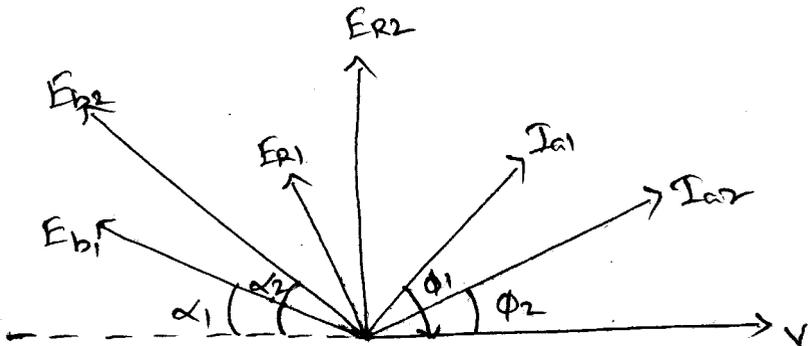
However, in this case, power factor angle ϕ decreases (or power factor increases) at a faster

rate than the armature current thereby producing the necessary increased power to meet the increased load applied to the motor.

'V' and Inverted 'V' curves:

The V-curves of a synchronous motor show how armature current varies with its field current, when motor input is kept constant.

These are obtained by plotting AC armature current against DC field current while motor input is kept constant and are so called because of their shape.

Single-phase induction motors:

Constructionally, this motor is more or less similar to polyphase induction motor, except that

i) its stator is provided with a single-phase winding and

ii) a centrifugal switch is used in some type of motors, in order to cut out a winding used only for starting purpose.

When fed from a single-phase supply, its stator winding produces a flux which is alternating (It is not a revolving or rotating flux as in the case of three-phase stator winding.)

Now, an alternating flux acting on a stationary squirrel-cage rotor cannot produce rotation (only a revolving flux can).

That is why a single-phase motor is not self-starting.

However, if the rotor of such machine is given an initial start by hand or otherwise

(by use of small motors), in either direction, immediately a torque arises and the motor accelerates to its final speed.

This peculiar behaviour of the motor has been explained in two ways:

i) by two-field or double-field revolving theory and ii) by cross-field theory.

Double-field revolving theory :

It is based on the idea that, an alternating uni-axial quantity can be represented by two oppositely-rotating vectors of half magnitude.

Accordingly, an alternating flux can be represented by two revolving fluxes, each equal to half the value of the alternating flux and each rotating synchronously $\left(N_s = \frac{120f}{P}\right)$ in opposite direction.

As shown in fig. (a), let the alternating flux have a maximum value of ϕ_m, with two components A and B which is equal to $\phi_m/2$ revolving in opposite directions.
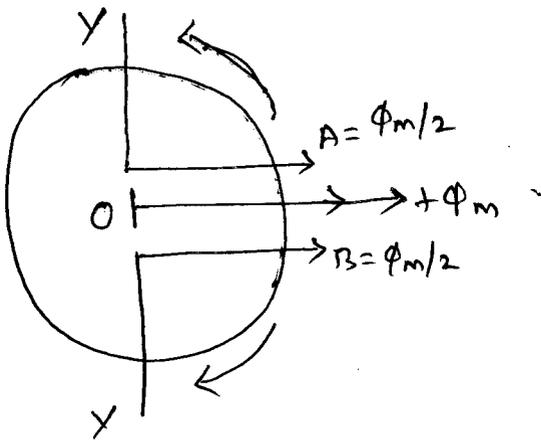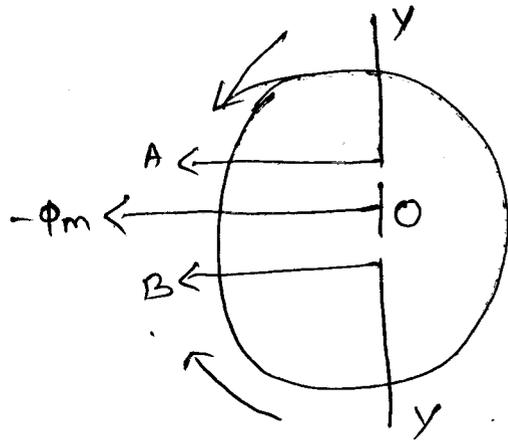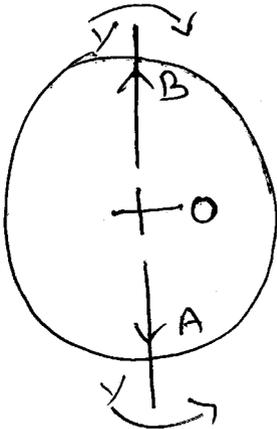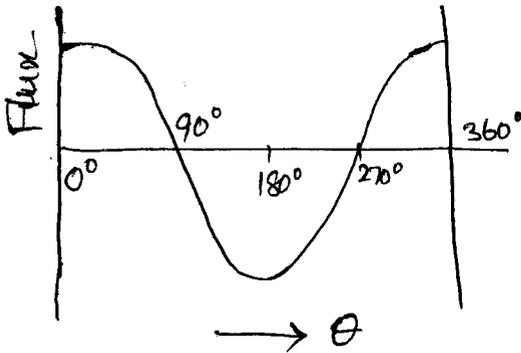
(a)
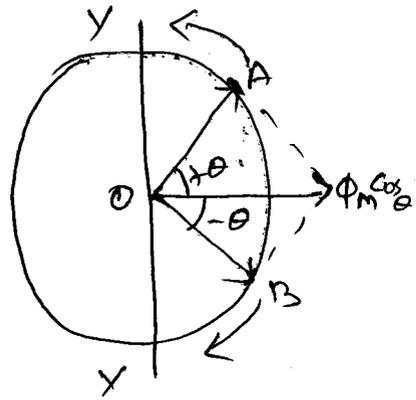
(b)
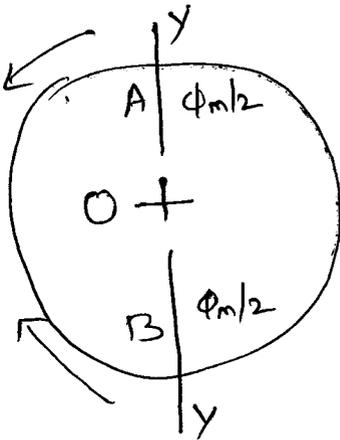
(c)

(d)

(e)

After some definite time, A and B would have rotated through angle $+\theta$ and $-\theta$ as in fig.(b).

The resultant flux would be,

$$= 2 \times \frac{\phi_m}{2} \cos \frac{2\theta}{2}$$

$= \phi_m \cos \theta.$

(f)

After a quarter cycle of rotation, flux A and B will be oppositely-directed as shown in fig. (C). So that the resultant flux would be zero.

After half a cycle, A and B will have resultant of $-2 \times \frac{\phi_m}{2} = -\phi_m$. (fig. (d)).

After three-quarters of a cycle, again the resultant is zero as shown in fig. (e) and so on.

The plot of resultant fluxes against $\theta°$ between 0° to 360°, is similar to the one shown in fig. (f).

That is why, an alternating flux can be looked upon as composed of two revolving fluxes, each of half the value and revolving synchronously in opposite directions.

Note: If the slip of the rotor is 's' with respect

to the forward rotating flux (ie. rotates in same
direction as rotor), then its ~~flux~~ slip with respect
to backward rotating flux is (2-S).

  Each of the two component fluxes, while
revolving round the stator, cuts the rotor conductors,
induces an emf and this produces its own torque.

  Obviously, the forward and backward torques
are oppositely-directed, so that the net or resultant
torque is equa to their difference as shown in.
fig.
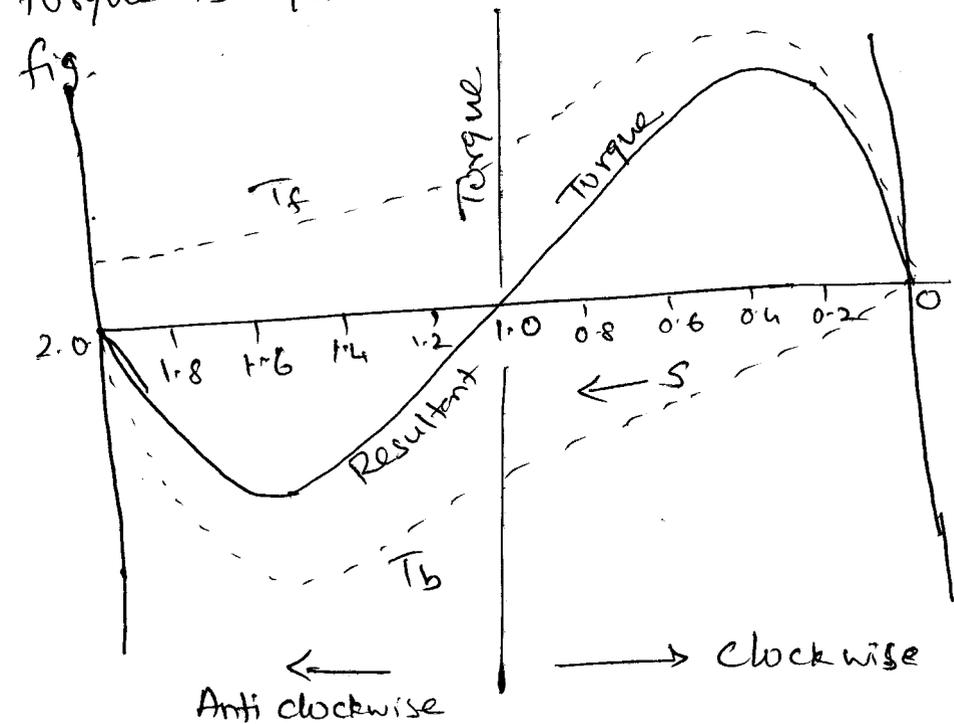
Now, power developed by the rotor is
$$P_g = \left(\frac{1-S}{S}\right) J_2^2 R_2$$

If N is the rotor rps, then torque is given by,

$$T_g = \frac{1}{2\pi N}\left(\frac{1-s}{s}\right)I_2^2 R_2$$

Now, $N = N_s(1-s)$

$$T_g = \frac{1}{2\pi N_s} \cdot \frac{I_2^2 R_2}{s} = k \cdot \frac{I_2^2 R_2}{s}$$

Hence, the forward and backward torque are given by,

$$T_f = k \cdot \frac{I_2^2 R_2}{s} \qquad \text{and} \qquad T_b = -k \cdot \frac{I^2 R_2}{2-s}$$

$$\text{or } T_f = \frac{I^2 R_2}{s} \text{ synch. watt.} \qquad \text{and}$$

$$T_b = -\frac{I_2^2 R_2}{2-s} \text{ synch. watt.}$$

$$\text{Total torque } T = T_f + T_b$$

At standstill, $s=1$ and $(2-s)=1$. Hence, T_f and T_b are numerically equal but oppositely directed, produce no resultant torque.

That explains why there is no starting torque in a single-phase induction motor.

However, if the rotor is started somehow, say in clockwise direction, the clockwise torque starts increasing and at the same time, the anticlockwise torque starts decreasing.

Hence, there is a certain amount of net torque in the clockwise direction which accelerates the motor to full speed.

Principle of operation of split phase motors:

Since, a single-phase induction motor is not self-starting, to make it self-starting, it is temporarily converted into a two-phase motor during starting.

For this purpose, the stator of a single-phase motor is provided with an extra winding called starting or auxiliary winding in addition to the main or running winding.
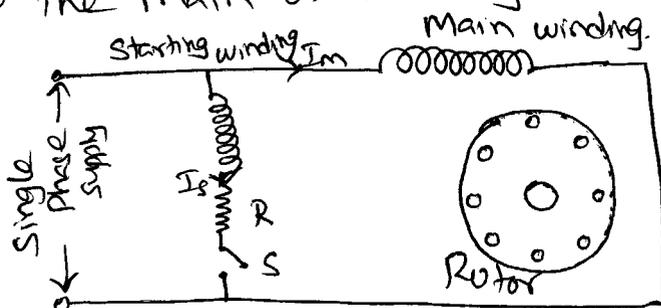
In this split-phase machine, the main winding has low resistance but high reactance whereas starting winding has a high resistance but low reactance.
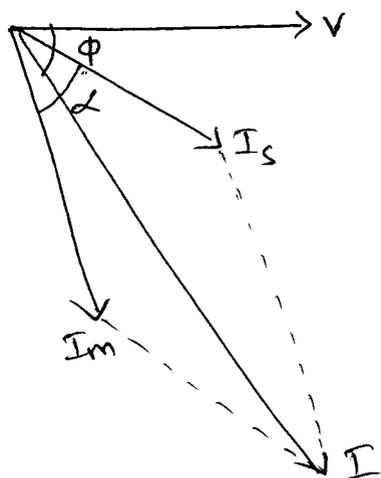
Hence, as shown in fig. the current I_s drawn by starting winding lags behind the applied voltage V by a small angle

whereas, current I_m taken by main winding lags behind V by very large angle.

Phase angle between I_s and I_m is made as large as possible because the starting torque of a split-phase motor is proportional to $\sin \alpha$.

Important 10 Marks questions:

1. Explain briefly about the V and inverted V curves and mention its significant.
2. Explain the effect of changing field current excitation at constant load.
3. Describe the various methods of starting the synchronous motor
4. Derive the expression for power developed in a synchronous motor. Also find the condition for maximum power developed

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