

Analysis of Indeterminate Structures

Introduction:

Anything built by man by using material and machines are called structures.

Ex: Building, bridges, Dam, Tunnel, Highway, ways, Harbour etc.,

Calculation of unknowns is called analysis. There are two types of unknowns:

- Reactive unknowns
- Displacement unknowns

Reactive unknowns are $R_v, R_H, BM, SF, RS, T, \sigma, E, P, \tau$ etc., and

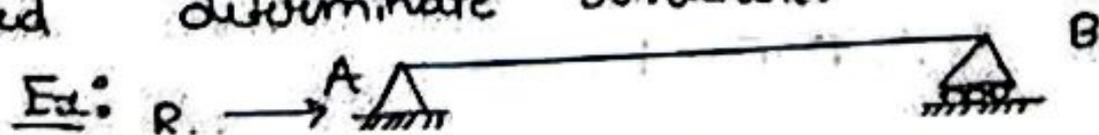
Displacement unknowns are Slope (θ), Displacement (Δ), Settlement, Sway, etc.,

Equilibrium Equations:

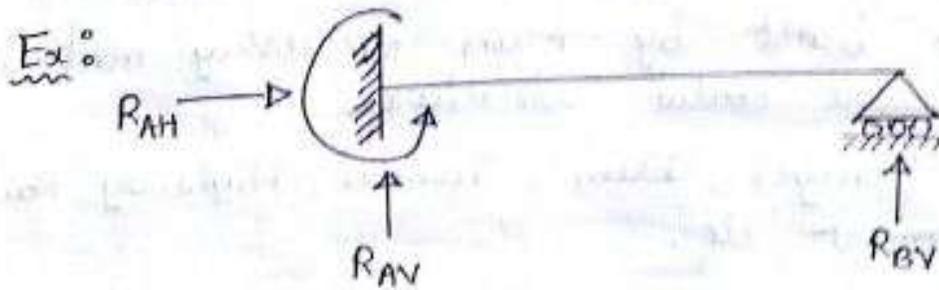
- $\sum M = 0$; Body does not rotate in any direction
- $\sum H = 0$; Body does not move in Horizontal direction
- $\sum V = 0$; Body does not move in vertical direction

Note: All structures are in equilibrium condition.

If the unknowns can be calculated making use of three equilibrium equations - called determinate structure.



If the unknowns cannot be calculated by using the equilibrium equation is called indeterminate structure.
 We need extra equation to calculate unknowns.

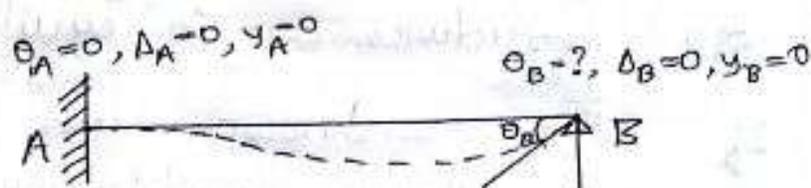


$$\begin{aligned} \text{No. of unknowns} &= 4 \\ \text{Equilibrium eq}^{\text{ns}} &= 3 \\ 4 - 3 &= 1 \end{aligned}$$

Number of extra equations required to calculate unknown reactive components is called static Indeterminacy. It is also called as degree of redundancy (DOR).

Number of extra equations required to calculate unknown displacement components is called Kinematic Indeterminacy. It is also called as Degree of freedom (DOF).

Degree of freedom is defined as 'the possible movement of a structure at the support @ at the joints.'



$$\text{DOF} = \theta_B$$

$$\therefore \text{DOF} = 1$$

There are various methods to analyse statically Indeterminate structures!

1. Slope - Deflection method
2. Moment Distribution method

3. Kani's rotation method
4. Stiffness matrix method
5. Flexibility matrix method

Sign Convention :

*) \uparrow +ve \downarrow -ve

*) \rightarrow +ve \leftarrow -ve

*) \curvearrowright +ve \curvearrowleft -ve

Imp. *

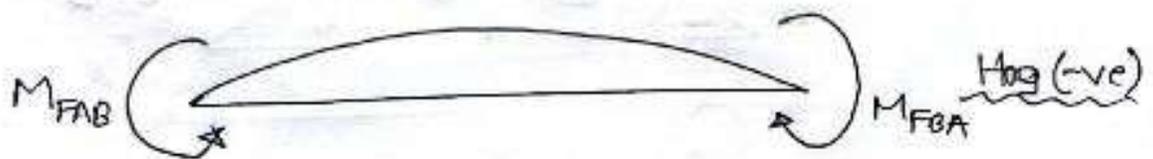
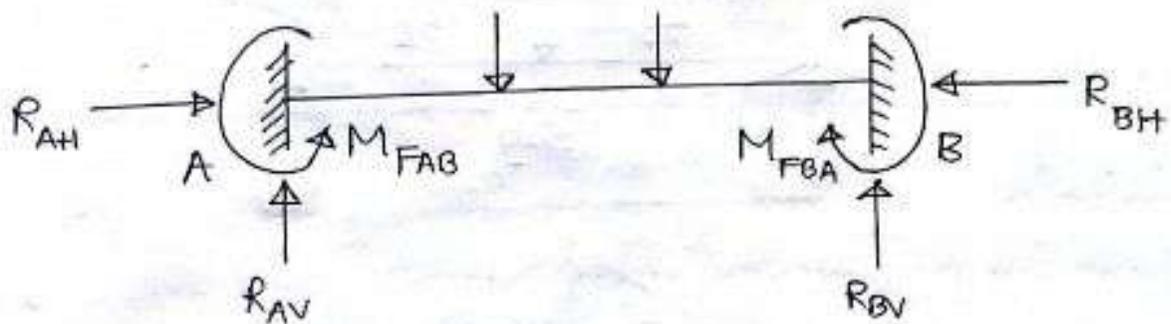
\curvearrowleft Hogging '-ve'

\curvearrowright Sagging '+ve'

The moments which are developed at the support due to fixity end is called fixed end moment (FEM). The effect of fixed end moment is to 'Hog' the beam.

The moment developed at the centre called sagging bending moment @ free bending moment (FBM).

Ex :



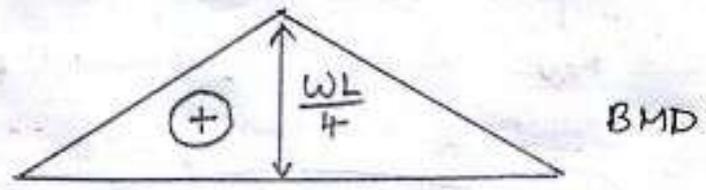
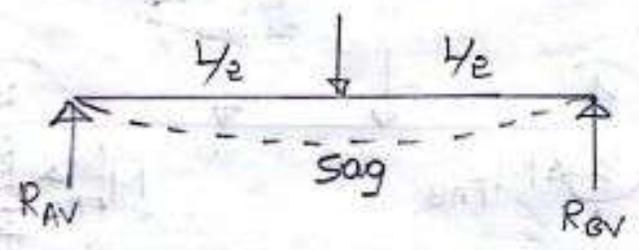
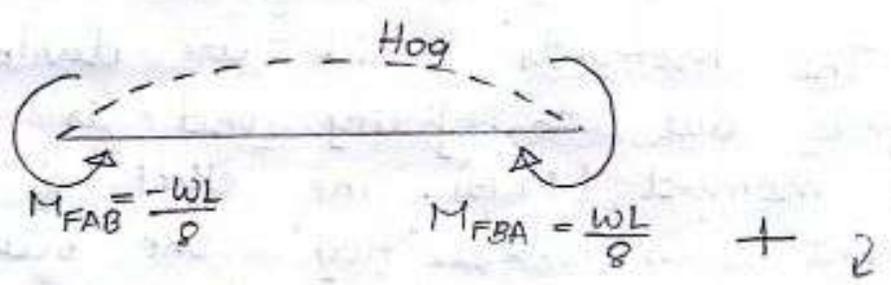
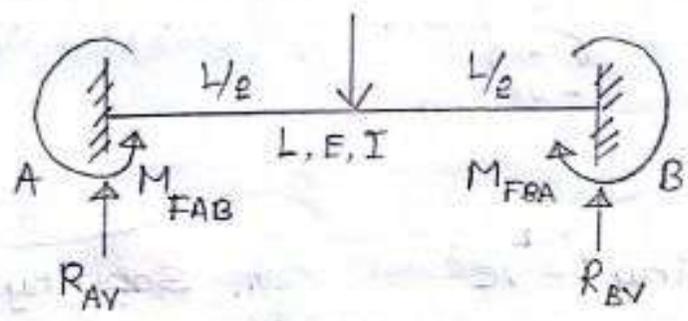
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If the cross-section of the beam is uniform throughout is called "prismatic beam".

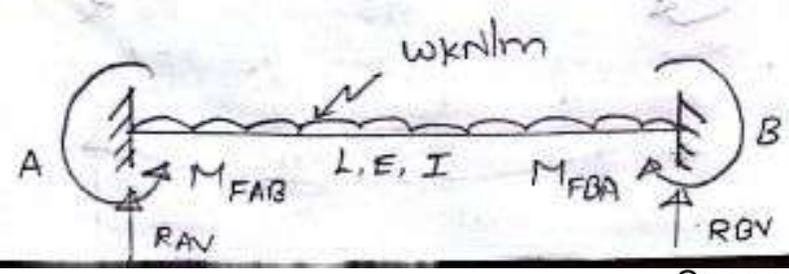
If the beam is made of same material throughout the length called "homogeneous", if elastic properties are same (E, G, K) called "isotropic".

Fixed end moments and free bending moments

1.

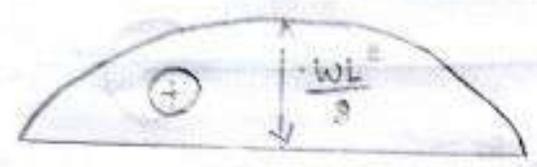


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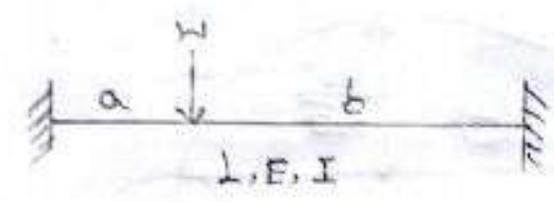


$$M_{FAB} = -\frac{wL^2}{12}$$

$$M_{FBA} = \frac{wL^2}{12}$$

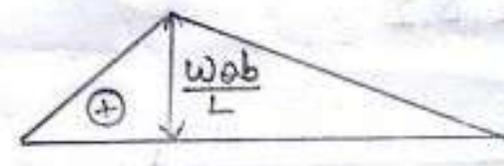


3.

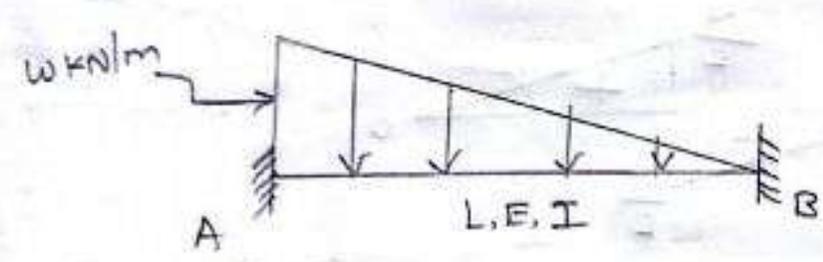


$$M_{FAB} = -\frac{wab^2}{L^2}$$

$$M_{FBA} = \frac{wab^2}{L^2}$$

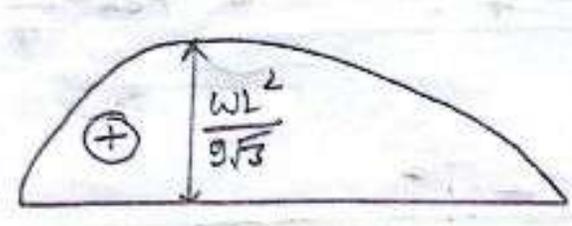


4.

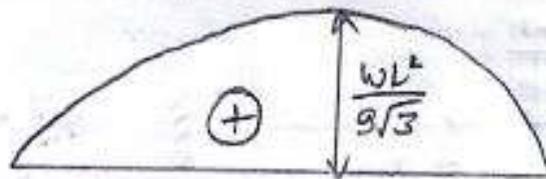
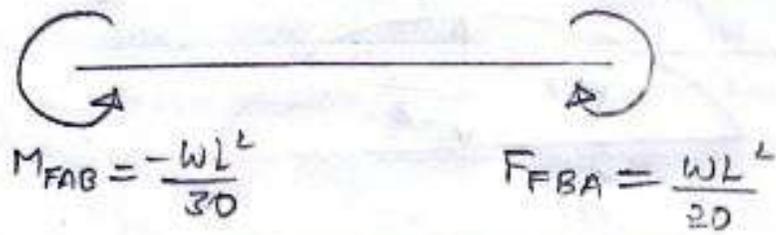
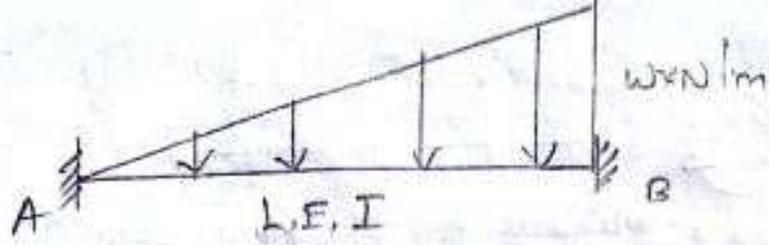


$$M_{FAB} = -\frac{wL^2}{20}$$

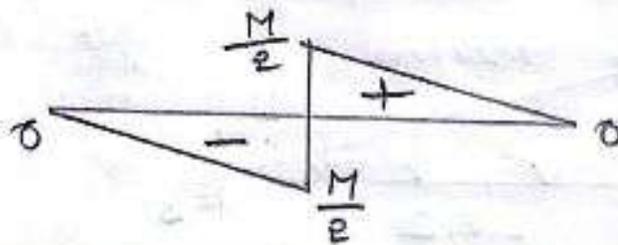
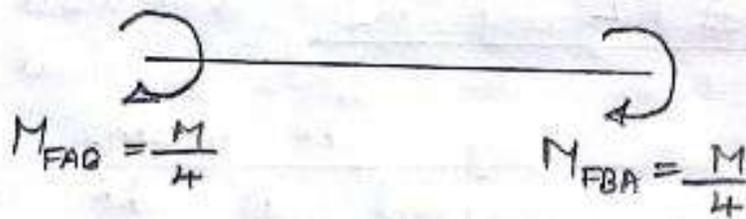
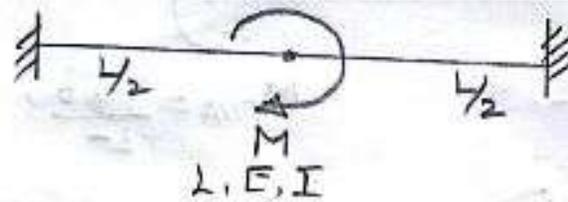
$$M_{FBA} = \frac{wL^2}{30}$$



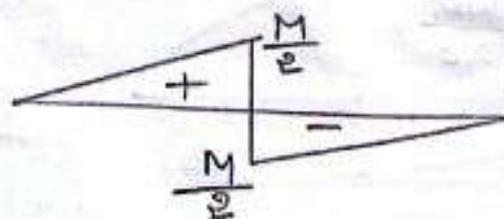
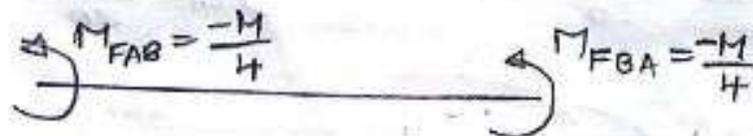
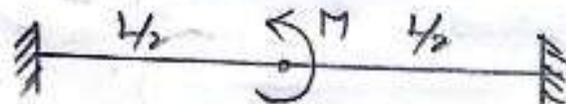
5.



6.

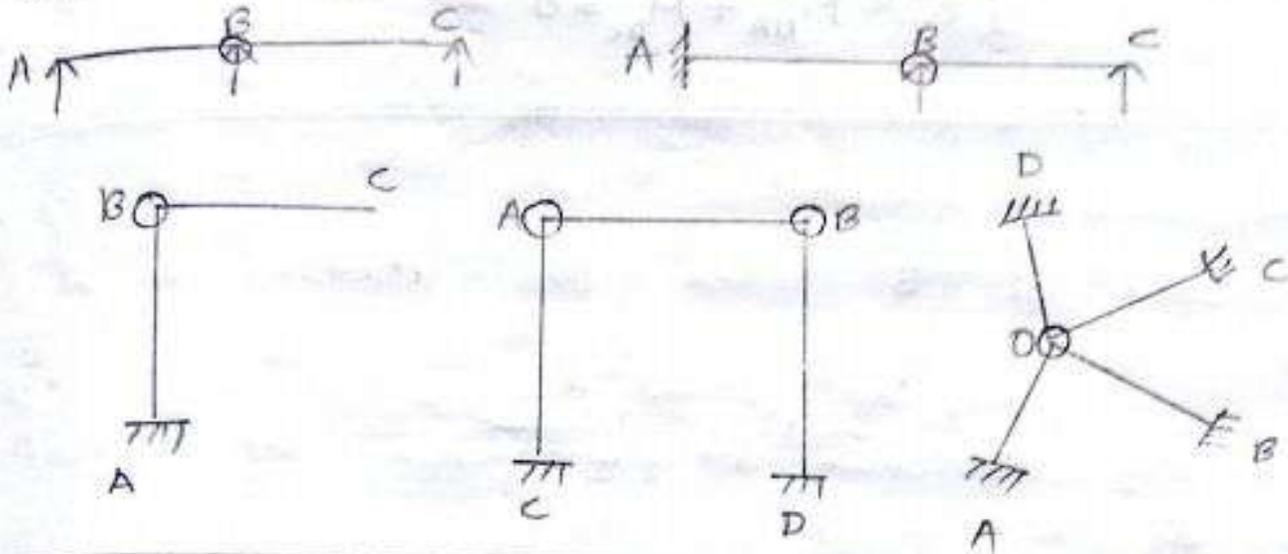


7.



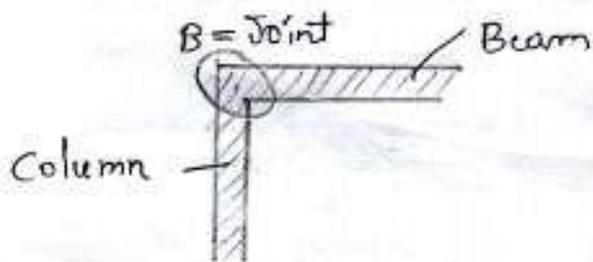
Joint :

Joint is a point where two or more than two members meet.

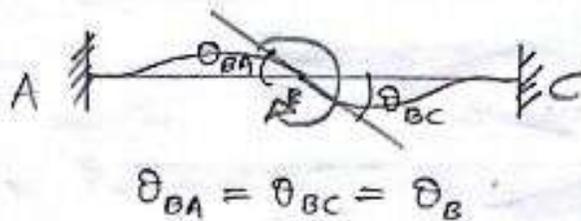


Characteristics of a joint :

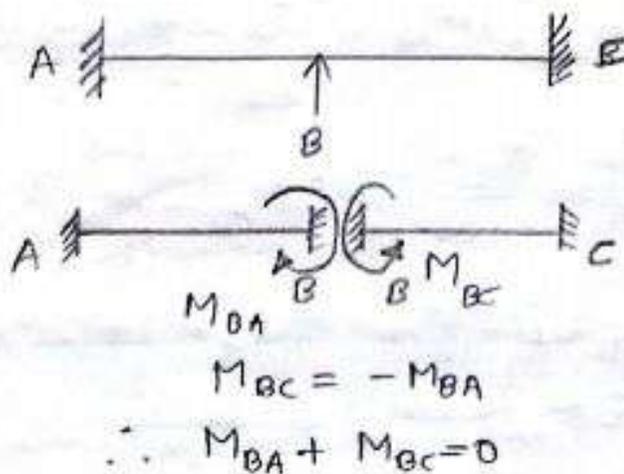
1. Joint is monolithic and rigid.



2. Rotation at the joint is constant.



3. Joints can be replaced by two fixed points.



4. Sum of the moments @ the joint is zero.

$$\sum M_B = 0$$

$$\text{i.e., } M_{BA} + M_{BC} = 0$$
