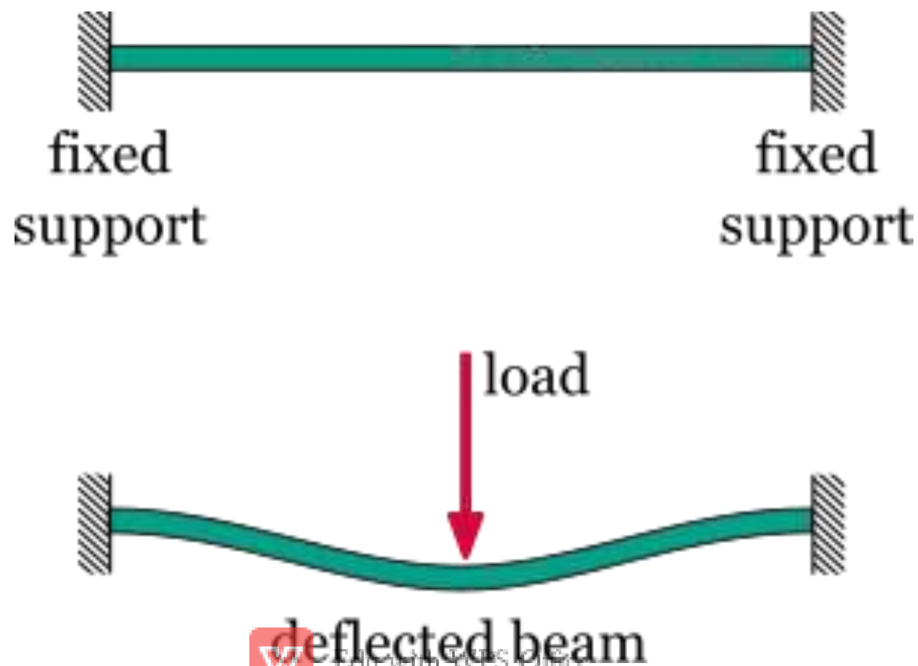
FIXED BEAMS

FIXED BEAMS

- A beam whose both ends are fixed is known as a fixed beam. Fixed beam is also called as built-in or encaster beam.
- Incase of fixed beam both its ends are rigidly fixed and the slope and deflection at the fixed ends are zero



Advantages of fixed beams

(i) For the same loading, the maximum deflection of a fixed beam is less than that of a simply supported beam.

(ii) For the same loading, the fixed beam is subjected to lesser maximum bending moment.

(iii) The slope at both ends of a fixed beam is zero.

(iv) The beam is more stable and stronger.

Disadvantages of a fixed beam

(i) Large stresses are set up by temperature changes.

(ii) Special care has to be taken in aligning supports accurately at the same level.

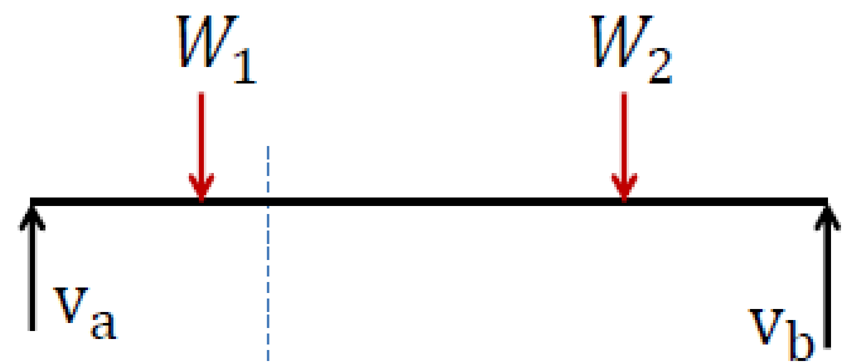
(iii) Large stresses are set if a little sinking of one support takes place.

(iv) Frequent fluctuations in loading render the degree of fixity at the ends very uncertain

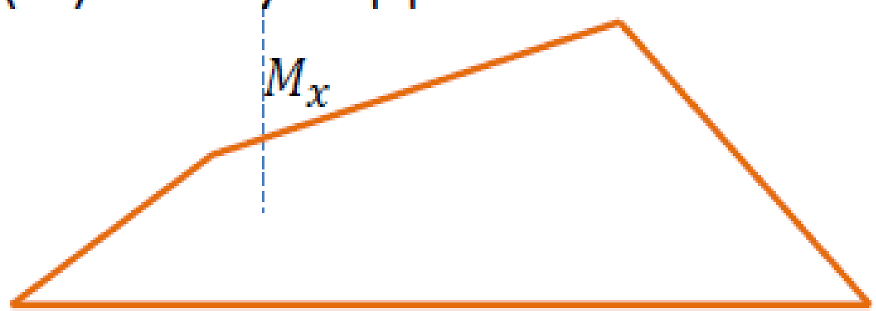
The beam may be analyzed in the following stages.

(i) Let us first consider the beam as Simply supported.

Let v_a and v_b be the vertical reactions at the supports A and B. Figure (ib) shows the bending moment diagram for this condition. At any section the bending moment M_x is a sagging moment.



(ia) Freely supported condition



(ib) Free B.M.D.

- (ii) Now let us consider the effect of end couples M_A and M_B alone.

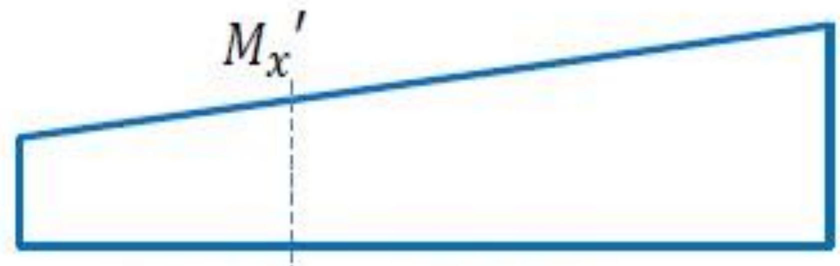
Let v be the reaction at each end due to this condition.

Suppose $M_B > M_A$.

Then $V = \frac{M_B - M_A}{L}$.

If $M_B > M_A$ the reaction V is upwards at B and downwards at A.

(iia) Effect of end couples

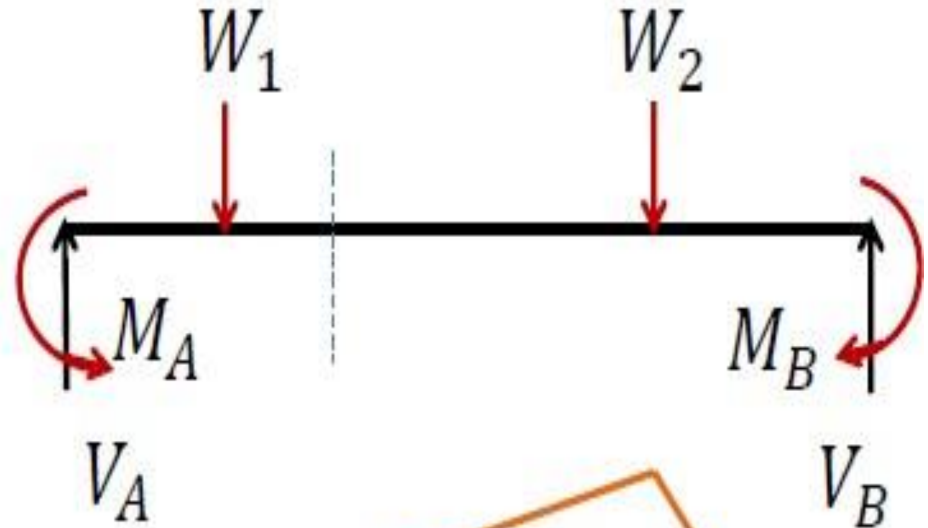


(iib) Fixed B.M.D.

Fig (iib). Shows the bending moment diagram for this condition.

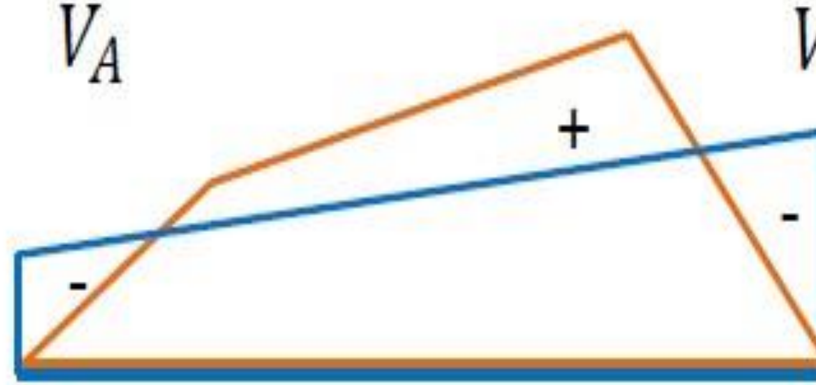
At any section the bending moment $M_{x'}$ is hogging moment.

- Now the final bending moment diagram can be drawn by combining the above two B.M. diagrams as shown in Fig. (iiib)



Now the final reaction $V_A = v_a - v$

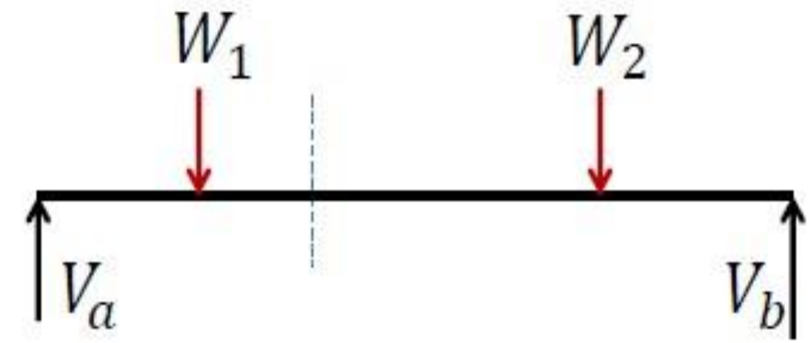
and $V_B = v_b + v$



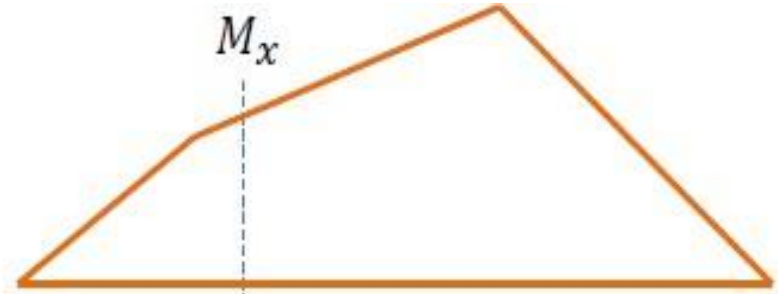
(iiib) Resultant B.M.D.

The actual bending moment at any section X, distance x from the end A is given by,

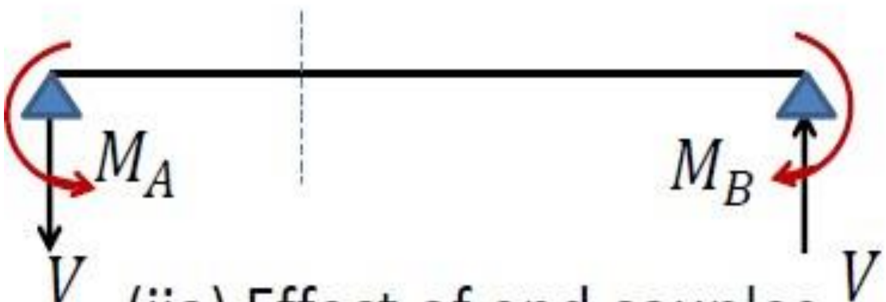
$$EI\,\frac{d^2y}{dx^2} = M_x - M_x'$$



(ia) Freely supported condition



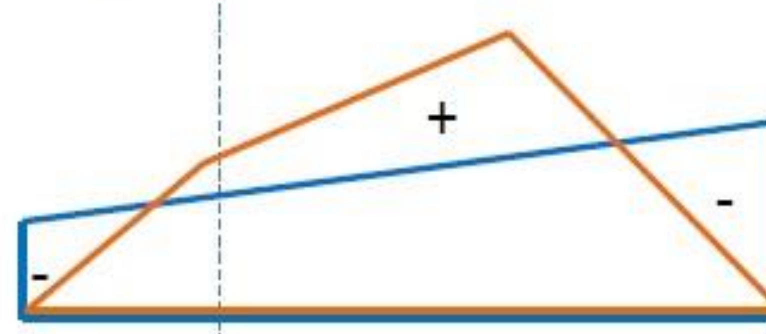
(ib) Free B.M.D.



(iia) Effect of end couples

(iib) Fixed B.M.D.

(iiia) Fixed beam



(iiib) Resultant B.M.D.

$$EI\frac{d^2y}{dx^2} = M_x - M_x'$$

- Integrating, we get,
- $EI\left[\frac{dy}{dx}\right]_0^l = \int_0^l M_x dx - \int_0^l M_x' dx$
- But at x=0, $\frac{dy}{dx} = 0$

 and at $x = l, \frac{dy}{dx} = 0$

Further $\int_0^l M_x dx$ = area of the Free BMD = a

$$\int_0^l M_x' dx = \text{area of the fixed B.M.D} = a'$$

Substituting in the above equation, we get,

$0 = a - a'$

$$\therefore a = a'$$

$$a = a'$$

∴ Area of the free B.M.D. = Area of the fixed B.M.D.

Again consider the relation,

$$EI\,\frac{d^2y}{dx^2} = M_x - M_x'$$

$Multying\ by\ x\ we\ get,$

$$EIx\,\frac{d^2y}{dx^2} = M_x x - M_x' x$$

- Integrating we get,
- $\int_0^l EIx\,\frac{d^2y}{dx^2} = \int_0^l M_x x\,dx\; - \int_0^l M_x' x\,dx$
- $\therefore EI\left[x\frac{dy}{dx} - y\right]_0^l$ = a$\bar{x}$-a'$\bar{x}'$
- Where $\bar{x}$= distance of the centroid of the free B.M.D. from A.

  and $\bar{x}'$= distance of the centroid of the fixed B.M.D. from A.

- Further at x=0, y=0 and $\frac{dy}{dx} = 0$
- and at x=l, y=0 and $\frac{dy}{dx} = 0$.
- Substituting in the above relation, we have

$$0 = a\bar{x}\text{-}a'\bar{x}'$$

$$a\bar{x}=a'\bar{x}'$$

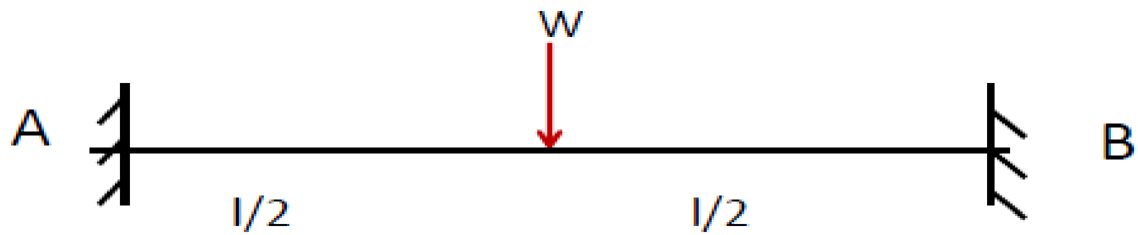
or $$\bar{x} = \bar{x}$$

∴ The distance of the centroid of the free B.M.D . From A= The distance of the centroid of the fixed B.M.D. from A.

$$\therefore a = a'$$

$$\bar{x} = \bar{x}$$

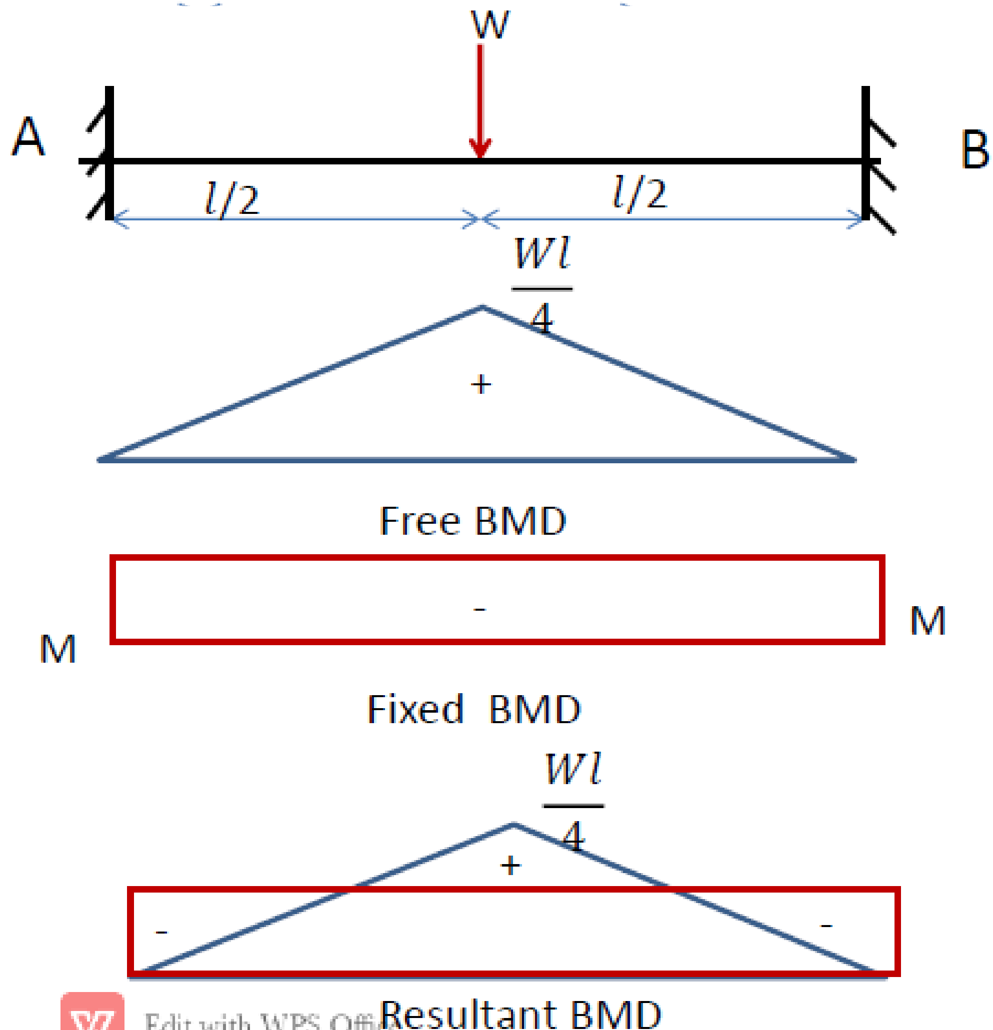
- Find the fixed end moments of a fixed beam subjected to a point load at the center.



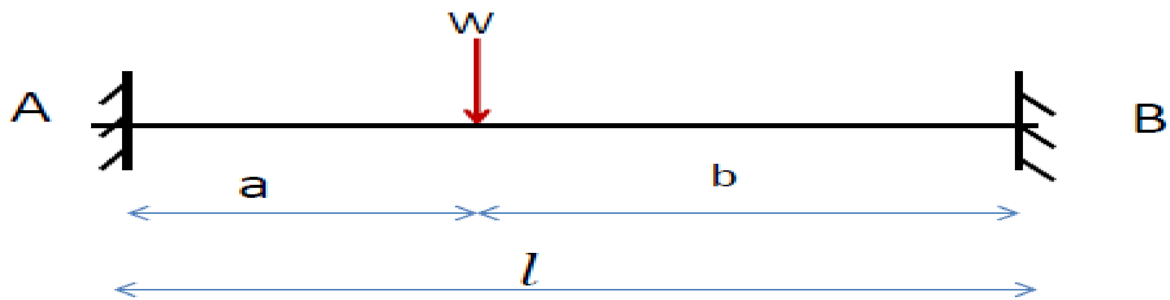
- $A' = A$

$$M \times l = \frac{1}{2} \times l \times \frac{Wl}{4}$$

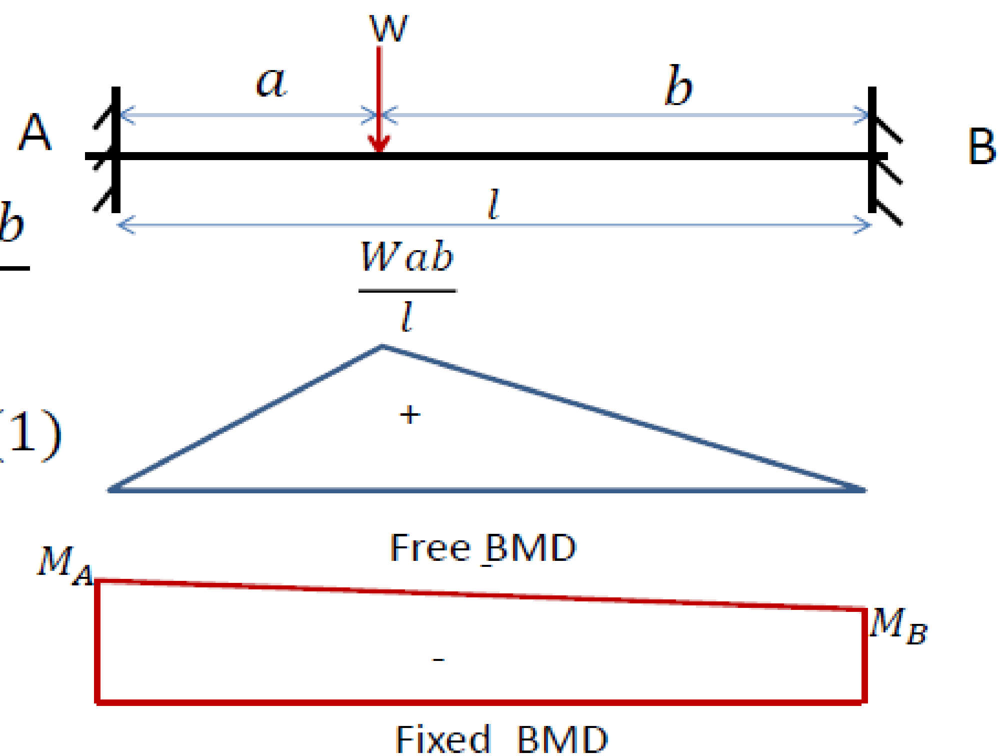
$$M = \frac{Wl}{8} = M_A = M_B$$



- Find the fixed end moments of a fixed beam subjected to a eccentric point load.



- $A' = A$



$$\frac{M_A + M_B}{2} \times l = \frac{1}{2} \times l \times \frac{Wab}{l}$$

$$M_A + M_B = \frac{Wab}{l} ----(1)$$

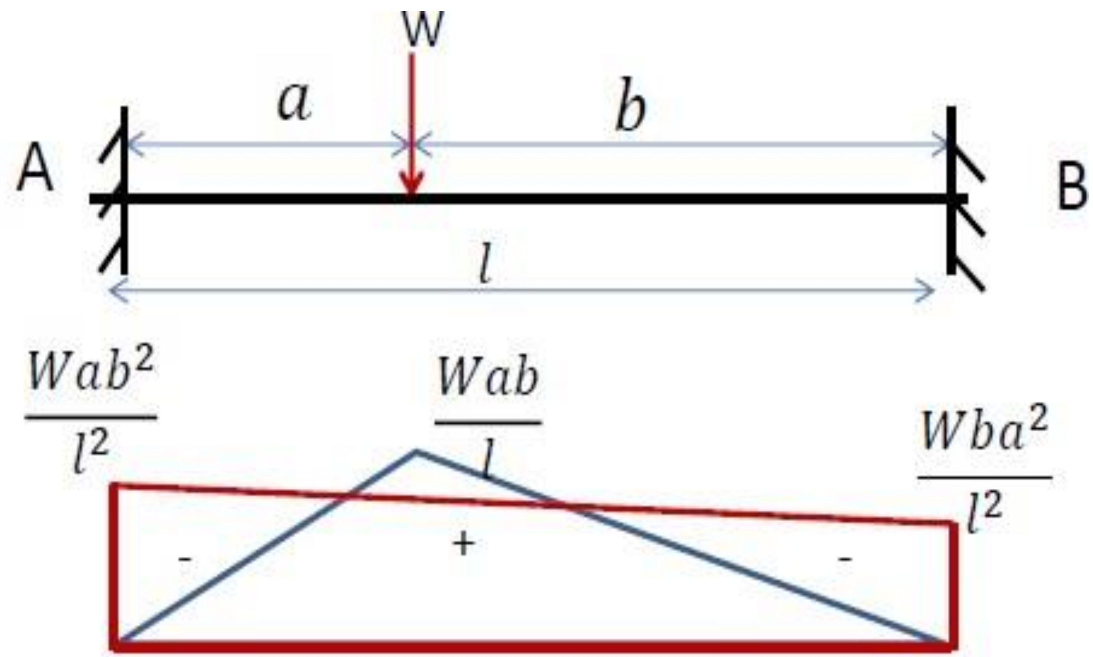
- $x' = x$

$$\frac{M_A + 2M_B}{M_A + M_B} \times \frac{l}{3} = \frac{l + a}{3}$$

$$M_B = M_A \times \frac{a}{l - a}$$

$$M_B = \frac{a}{l} ---(2)$$

$$M_A + M_B = \frac{Wab}{l} - - - - -(1)$$



Resultant BMD

$$M_B = M_A \times \frac{a}{b} - - - -(2)$$

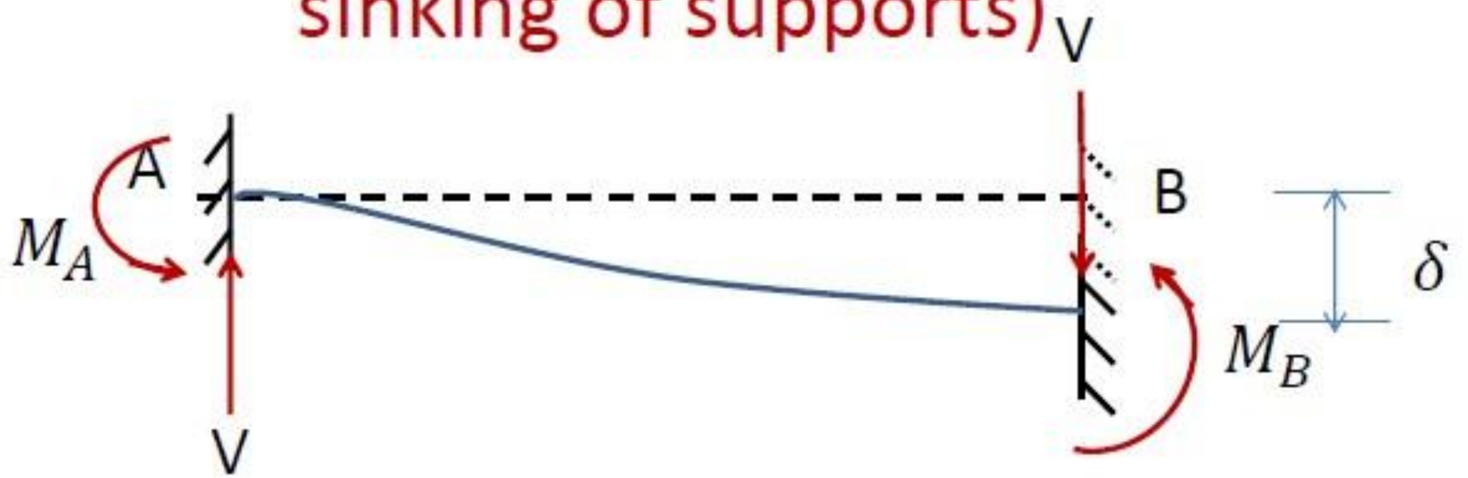
By substituting (2) in (1),

$$M_A = \frac{Wab^2}{l^2}$$

From (2),

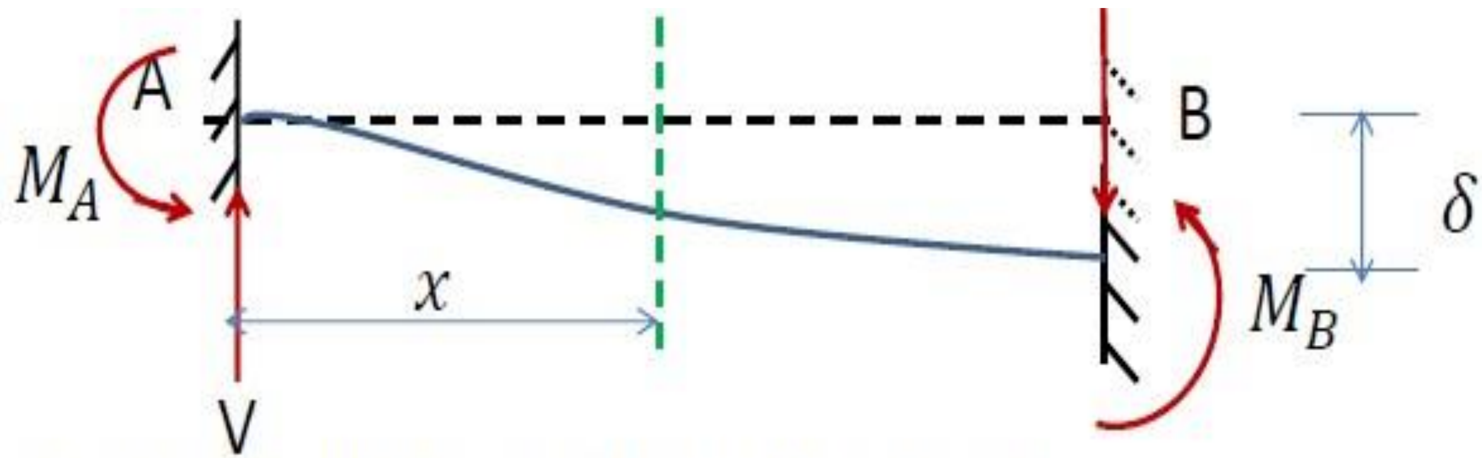
$$M_B = \frac{Wba^2}{l^2}$$

Fixed beam with ends at different levels (Effect of sinking of supports)



M_A is negative (hogging) and M_B is positive (sagging). Numerically M_A and M_B are equal.

Let V be the reaction at each support.



Consider any section distance x from the end A.

Since the rate of loading is zero, we have, with the usual notations

$$EI\frac{d^4y}{dx^4} = 0$$

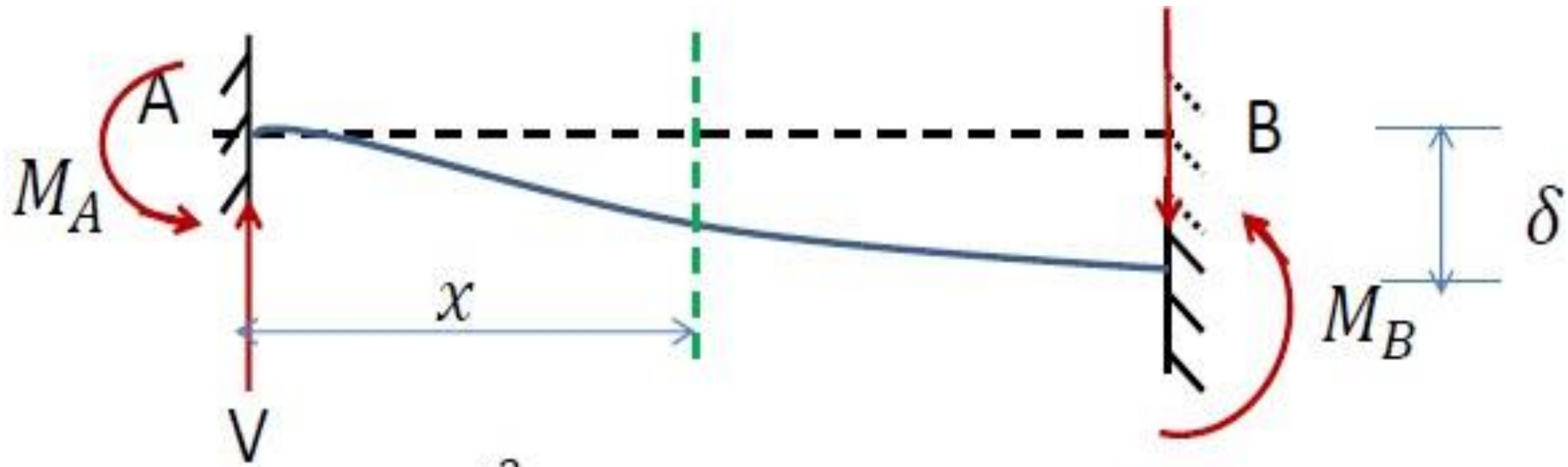
Integrating, we get,

Shear force = $EI\frac{d^3y}{dx^3} = C_1$

Where C_1 is a constant

At $x = 0$, $S.F. = +\mathrm{V}$

$$\therefore C_1 = \mathrm{V}$$



B.M. at any section = $EI\frac{d^2y}{dx^2} = Vx + C_1$

At $x = 0,\ B.M. = -M_A$

$$\therefore\ C_2 = -M_A$$

$$\therefore EI\frac{d^2y}{dx^2} = Vx - M_A$$

Integrating again,

$EI\frac{dy}{dx} = \frac{V}{2}x^2 - M_A x + C_3$ (Slope equation)

But at $x = 0,\ \frac{dy}{dx} = 0 \qquad \therefore C_3 = 0$

Integrating again,

$EI\,y = \frac{Vx^3}{6} - \frac{M_A x^2}{2} + C_4$ ------ (Deflection equation)

But at $x = 0, y = 0$

$$\therefore C_4 = 0$$

At $x = l, y = -\delta$

$$-EI\,\delta = \frac{Vl^3}{6} - \frac{M_A l^2}{2} ---------(\mathrm{i})$$

But we also know that at B, $x = l\ and\ \frac{dy}{dx} = 0$

And substitute in slope Eq. $EI\frac{dy}{dx} = \frac{V}{2}x^2 - M_A x$

$$\therefore 0 = \frac{Vl^2}{2} - M_A l$$

$$\therefore V = \frac{2M_A}{l} ---------(\mathrm{ii})$$

Substituting in deflection Eq.(i) i.e., $-EI\,\delta = \frac{Vl^3}{6} - \frac{M_A l^2}{2}$;we have,

$$-EI\,\delta = \frac{2M_A}{l} \times \frac{l^3}{6} - \frac{M_A l^2}{2}$$

$$EI\,\delta = \frac{M_A l^2}{6}$$

$$\therefore M_A = \frac{6EI\delta}{l^2}$$

Hence the law for the bending moment at any section distant x from A is given by,

$$M = EI\frac{d^2y}{dx^2} = \mathrm{V}x - M_A$$

$$\therefore M = \frac{2M_A}{l}x - \frac{6EI\delta}{l^2}$$

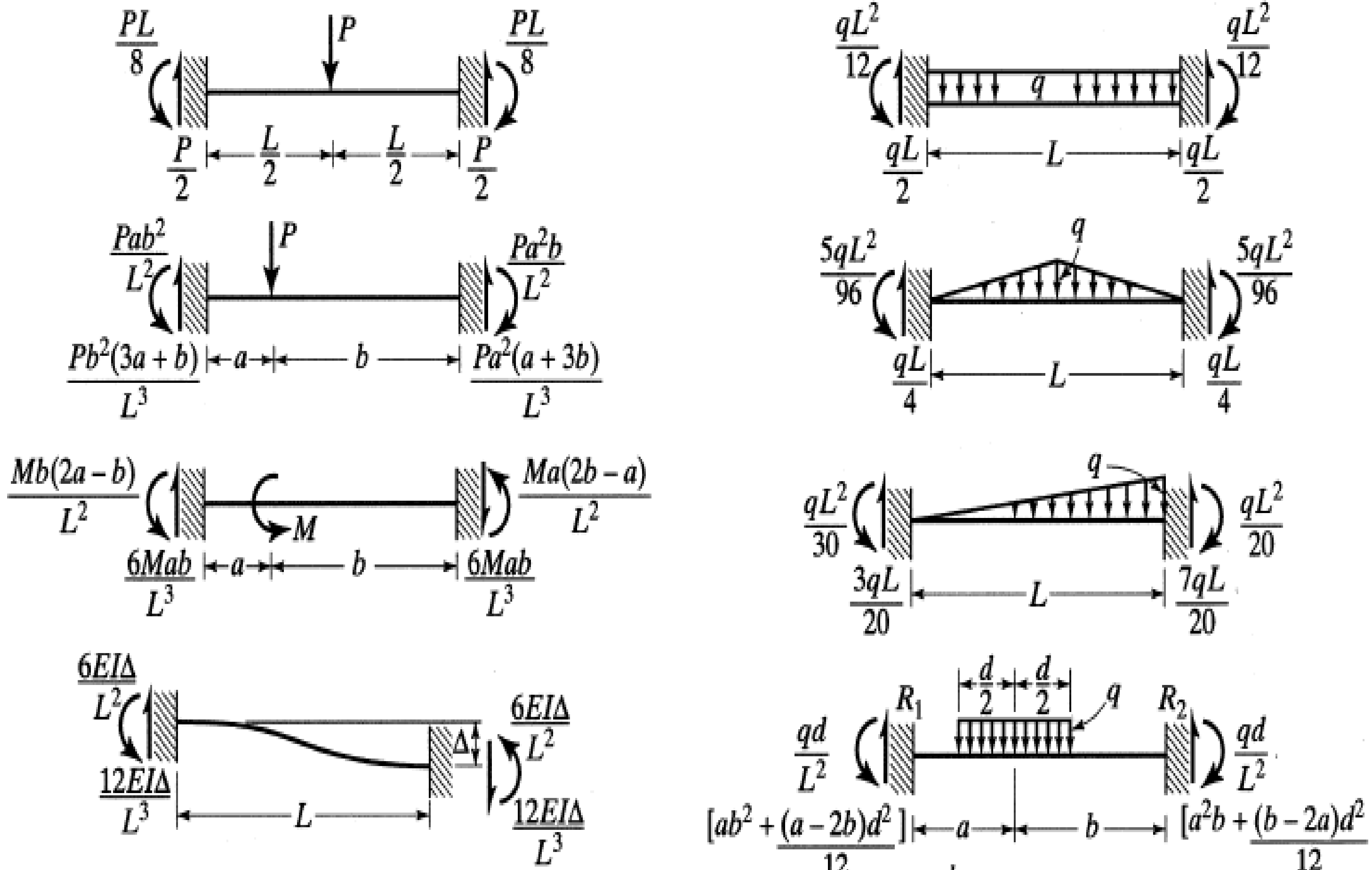
$$\text{But for B. M. at B, put x} = \text{l},$$

$$\therefore M_B = \frac{2M_A}{l} \times l - \frac{6EI\delta}{l^2} = \frac{12EI\delta}{l^2} - \frac{6EI\delta}{l^2} = \frac{6EI\delta}{l^2}$$

Hence when the ends of a fixed beam are at different levels,

The fixing moment at each end = $\frac{6EI\delta}{l^2}$ numerically.

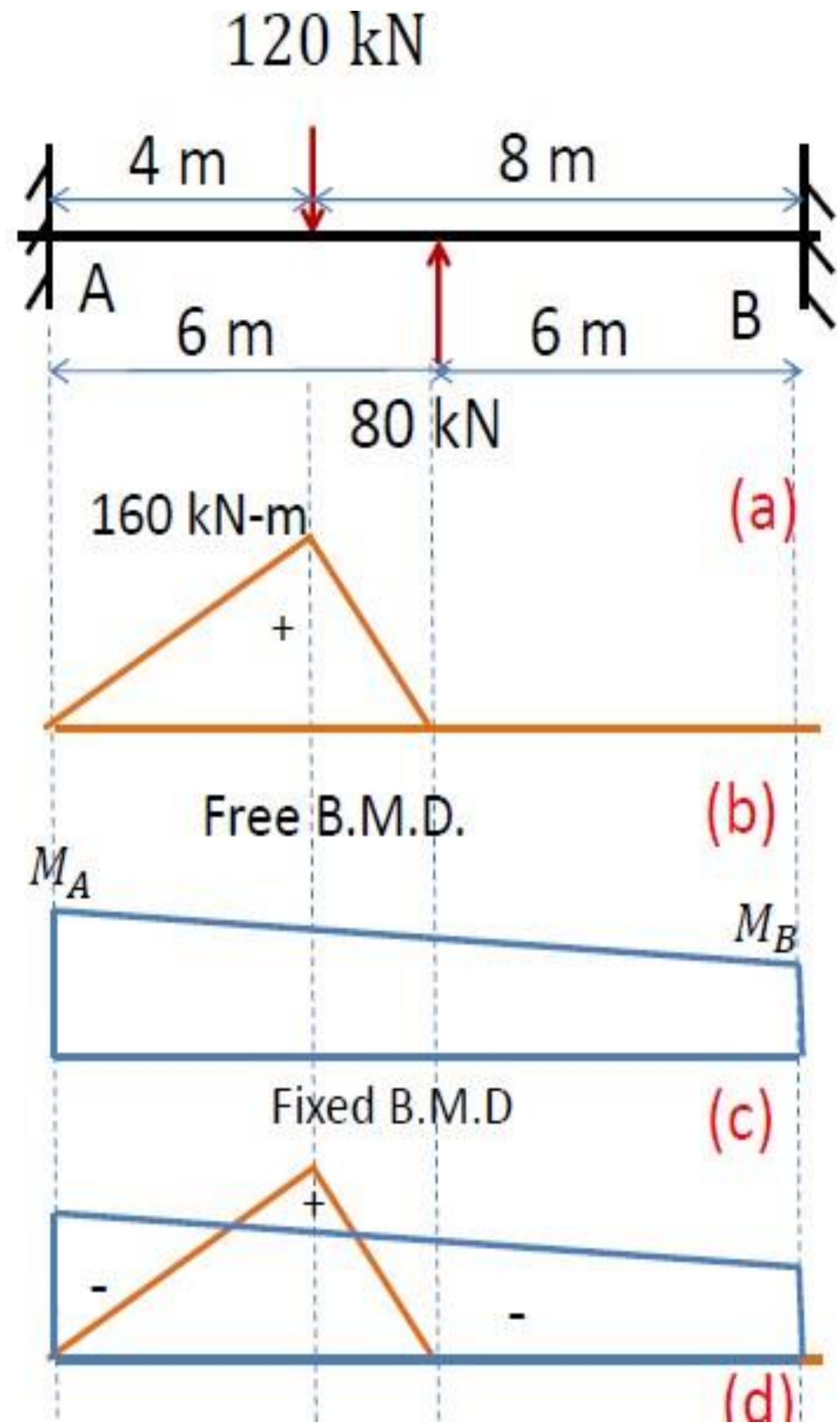
At the higher end this moment is a hogging moment and at the lower end this moment is a sagging moment.



$$R_1 = \frac{qd}{L^3}\left[(2a+L)b^2 + \left(\frac{a-b}{4}\right)d^2\right]$$

$$R_2 = \frac{qd}{L^3}\left[(2b+L)a^2 - \left(\frac{a-b}{4}\right)d^2\right]$$

- Solution:
- The M (Free B.M.) and M' (Fixed B.M.) diagrams have been shown in Fig.(b) and (c) respectively.



For the M-Diagram:

$$A = \frac{1}{2} \times 6 \times 160 = 480\ kNm$$

For the M' diagram:

$$A' = \frac{M_A + M_B}{2} \times 12 = 6(M_A + M_B)$$

- Area of the fixed B.M. D. = Area of the free B.M.D.

A' = A

$$6(M_A + M_B) = 480$$

$$M_A + M_B = 80\text{------------}(1)$$

The distance of the centroid of the free B.M. D. from A = The distance of the centroid of the fixed B.M.D. from A.

i.e., $x = x'$

$$\frac{6+4}{3} = \left(\frac{M_A + 2M_B}{M_A + M_B}\right) \times \frac{12}{3}$$

$$(M_A + 2M_B)12 = (M_A + M_B)10$$

$$12M_A + 24\,M_B - 10\,M_A - 10M_B = 0$$

$$2M_A + 14\,M_B = 0$$

$$M_A = -7M_B \text{ -----}(2)$$

- Substitute $M_A = -7M_B$ in equation (1)

$$-7M_B + M_B = 80$$

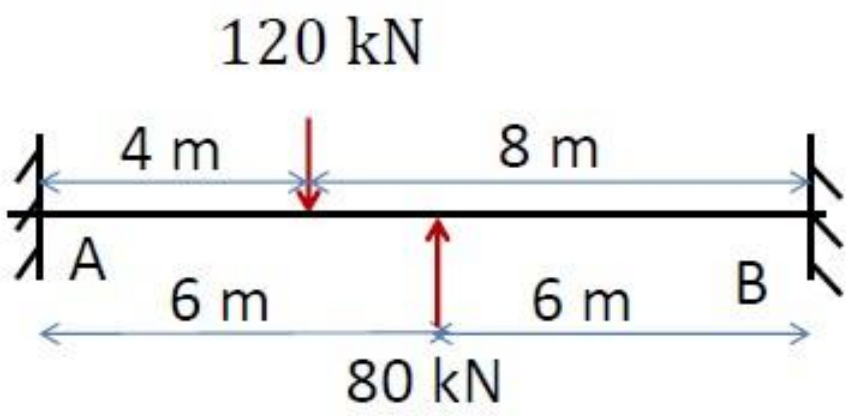
$$\therefore M_B = \frac{-80}{6} = -13.33$$

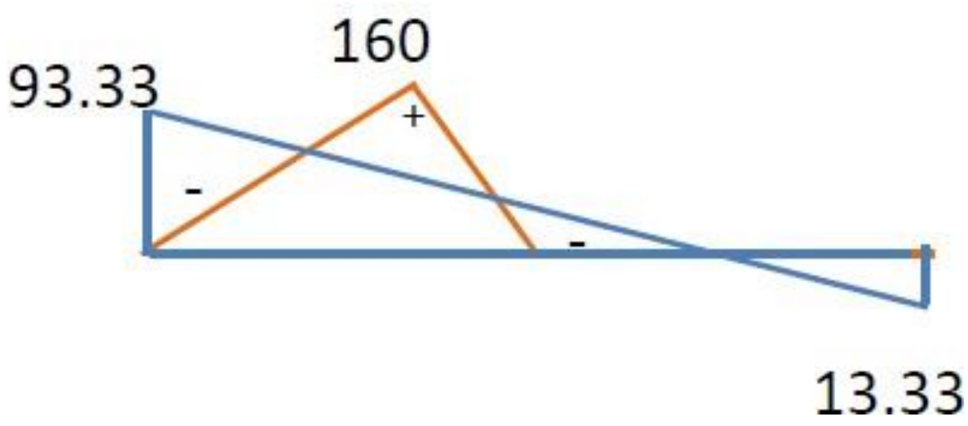
$$M_B = -13.33 \text{ kNm}$$

$$M_A = -7M_B$$

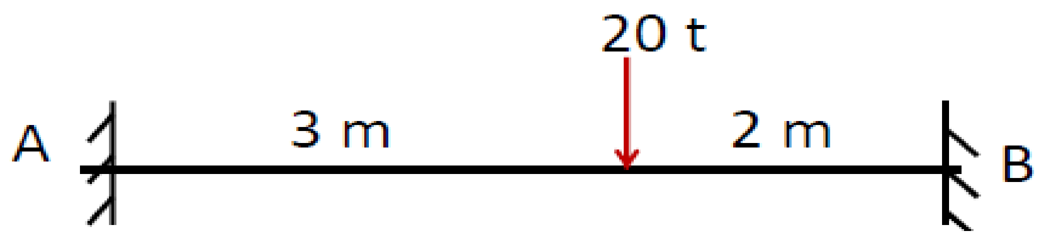
$$= -7(-13.33) = 93.33$$

$$\therefore M_A = 93.33\ kNm$$

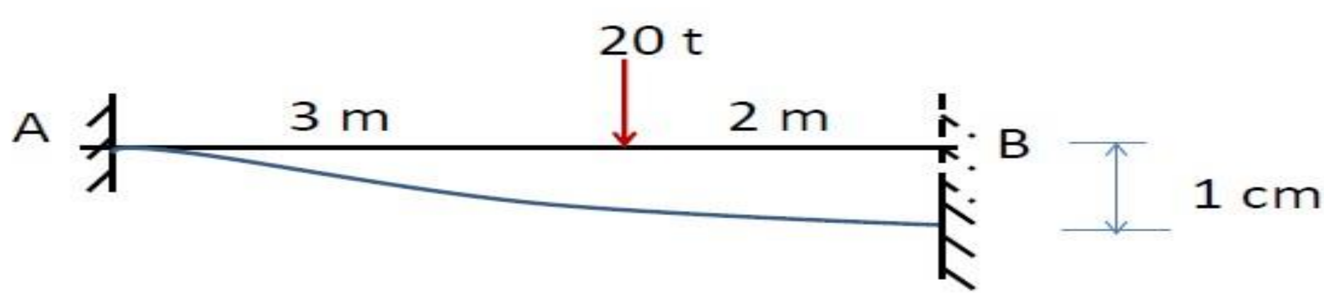




- A fixed beam of span 5 metres carries a concentrated load of 20 t at 3 meters from the left end. If the right end sinks by 1 cm, find the fixing moments at the supports. For the beam section take $I=30{,}000\ cm^4$ and $E=2\times10^3\ t/cm^2$. Find also the reaction at the supports.

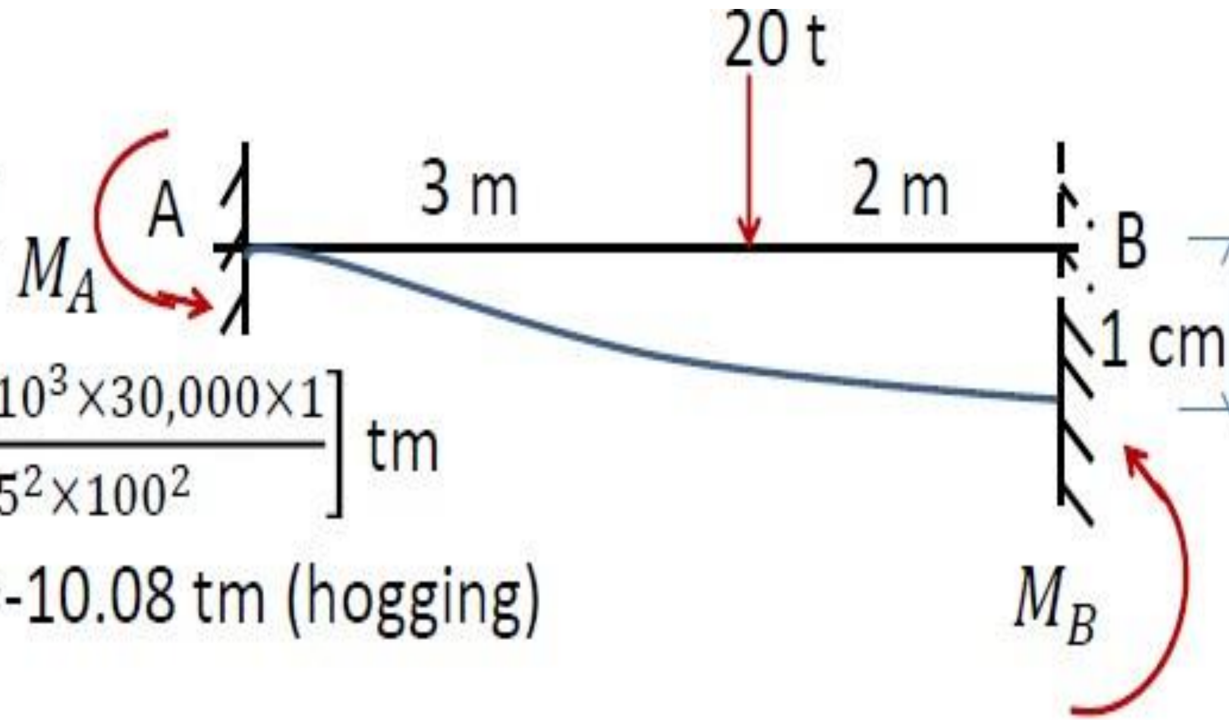


- A fixed beam of span 5 metres carries a concentrated load of 20 t at 3 meters from the left end.



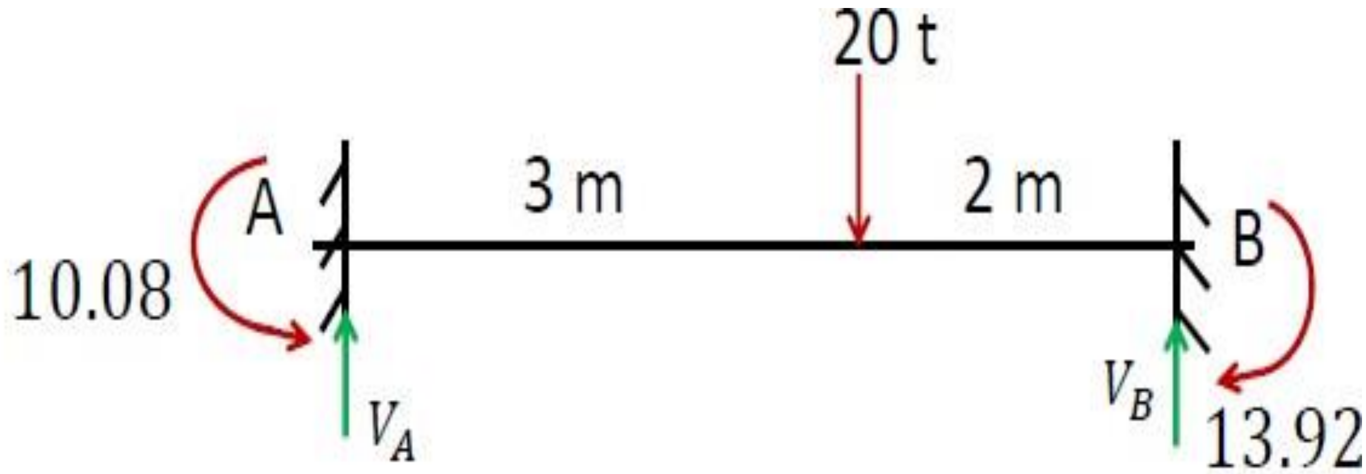
- The right end sinks by 1 cm, find the fixing moments at the supports.

- $M_A = -\frac{Wab^2}{l^2} - \frac{6EI\delta}{l^2}$



- $= -\left[\frac{20 \times 3 \times 2^2}{5^2} + \frac{6 \times 2 \times 10^3 \times 30{,}000 \times 1}{5^2 \times 100^2}\right]$ tm
- $= -[9.6 + 0.48]$ tm=-10.08 tm (hogging)

- $M_B = -\frac{Wba^2}{l^2} + \frac{6EI\delta}{l^2}$
- $= \left[-\frac{20 \times 2 \times 3^2}{5^2} + \frac{6 \times 2 \times 10^3 \times 30{,}000 \times 1}{5^2 \times 100^2}\right]$ tm
- $= [-14.4 + 0.48]$ tm= -13.92 tm (hogging)



- **Reaction at A:**
- $\sum M_B = 0,$
- $V_A \times 5 + 13.92 - 10.08 - (20 \times 2) = 0$
- $\therefore V_A = 7.232$ t
- **Reaction at B:**
- $\therefore V_B = 20 - 7.232 = 12.768$ t.

Continuous Beams

Introduction:

- Beams are made continuous over the supports to increase structural integrity.
- A continuous beam provides an alternate load path in the case of failure at a section.
- In regions with high seismic risk, continuous beams and frames are preferred in buildings and bridges.
- A continuous beam is a statically indeterminate structure.
-

The advantages of a continuous beam as compared to a simply supported beam are as follows

1) For the same span and section, vertical load capacity is more.
2) Mid span deflection is less.
3) The depth at a section can be less than a simply supported beam for the same span. Else, for the same depth the span can be more than a simply supported beam.

   ⇒ The continuous beam is economical in material.
4) There is redundancy in load path.

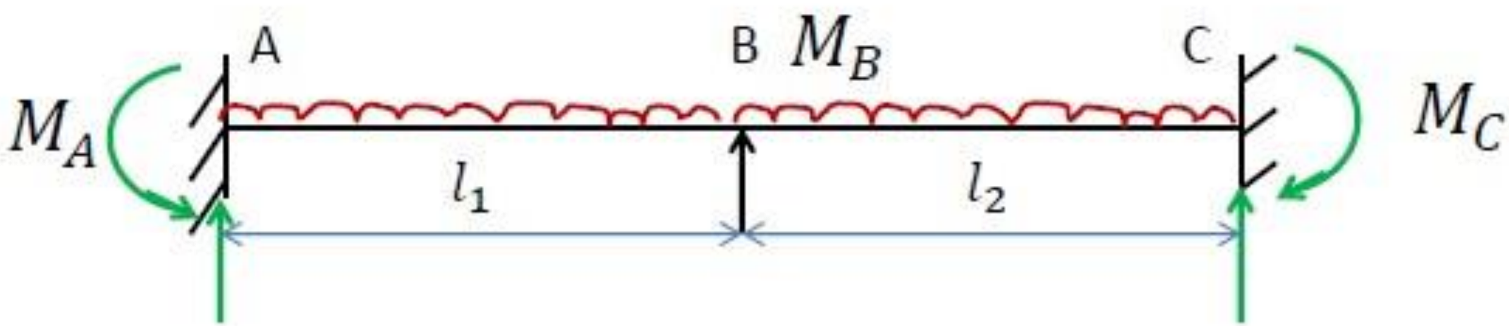
   ⇒ Possibility of formation of hinges in case of an extreme event.
5) Requires less number of anchorages of tendons.
6) For bridges, the number of deck joints and bearings are reduced.

   ⇒ Reduced maintenance

There are of course several disadvantages of a continuous beam as compared to a simply supported beam.

1) Difficult analysis and design procedures.
2) Difficulties in construction, especially for precast members.
3) Increased frictional loss due to changes of curvature in the tendon profile.
4) Increased shortening of beam, leading to lateral force on the supporting columns.
5) Secondary stresses develop due to time dependent effects like creep and shrinkage, settlement of support and variation of temperature.
6) The concurrence of maximum moment and shear near the supports needs proper detailing of reinforcement.
7) Reversal of moments due to seismic force requires proper analysis and design.

Clapeyron's theorem of three moments



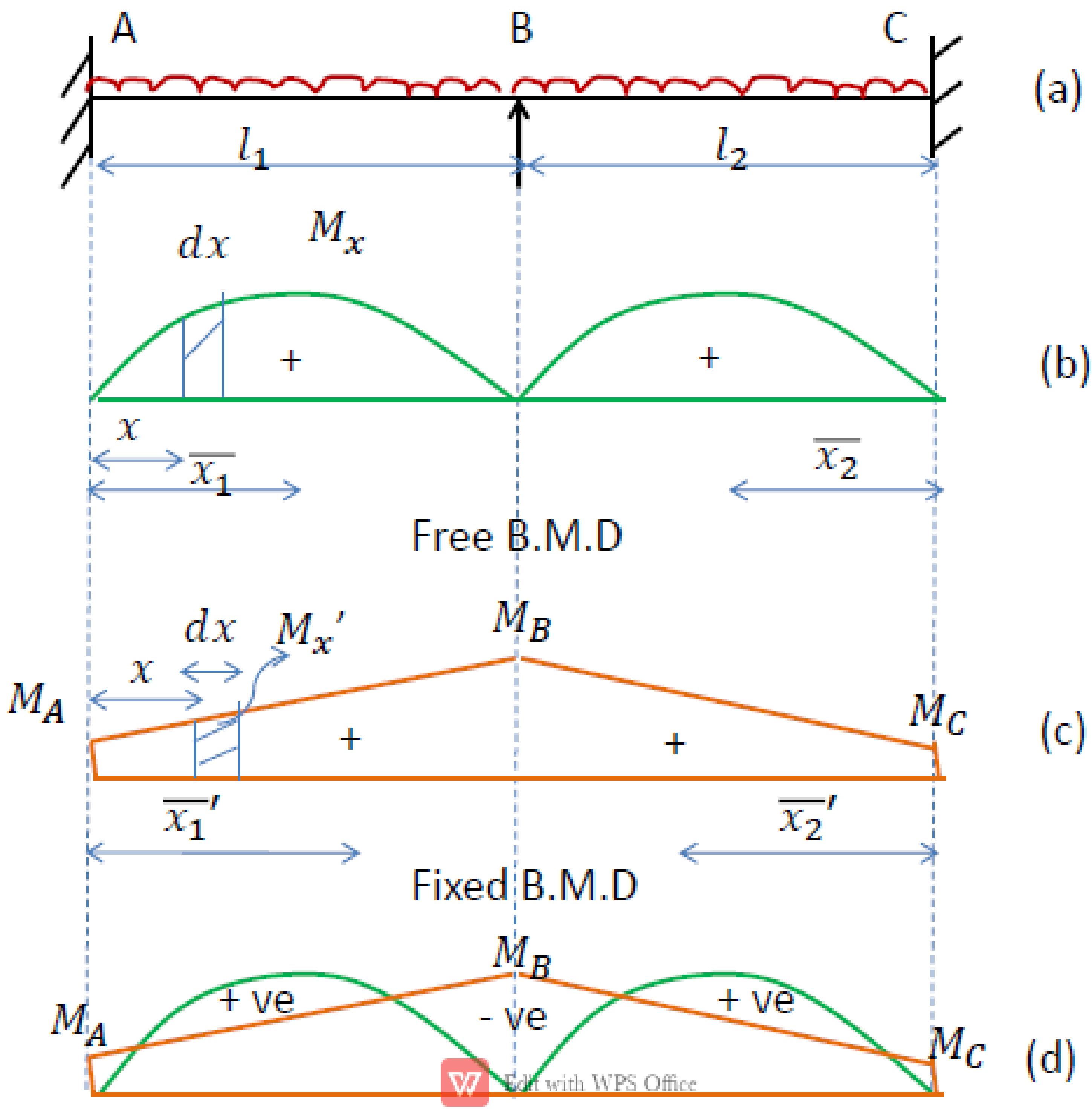
- As shown in above Figure, AB and BC are any two successive spans of a continuous beam subjected to an external loading.

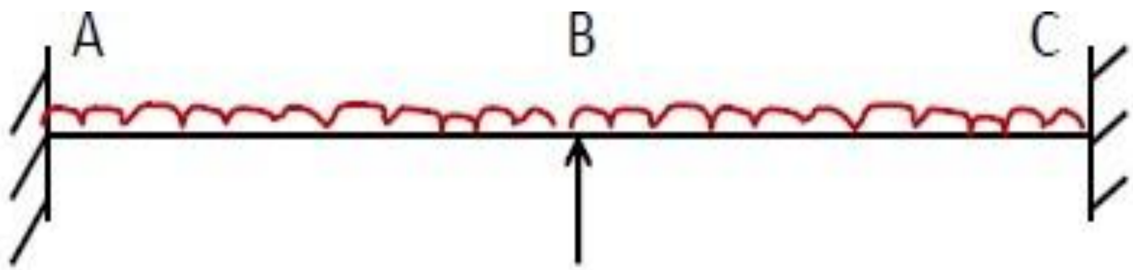
- If the extreme ends A and C fixed supports, the support moments M_A, M_B and M_C at the supports A, B and C are given by the relation,

$$M_A l_1 + 2M_B(l_1 + l_2) + M_C(l_2) = \frac{6a_1\overline{x_1}}{l_1} + \frac{6a_2\overline{x_2}}{l_2}$$

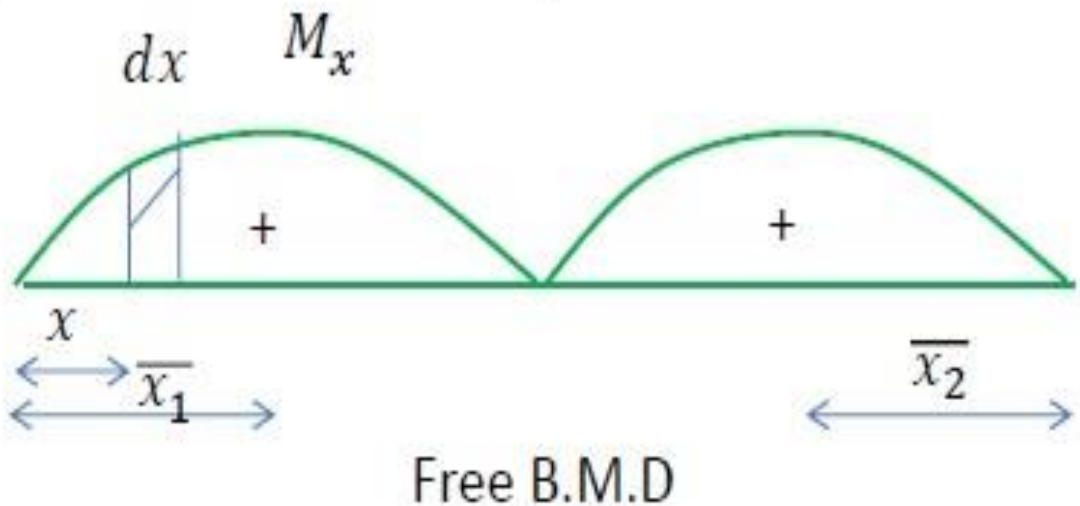
$$M_A l_1 + 2M_B(l_1 + l_2) + M_C(l_2) = \frac{6a_1\overline{x_1}}{l_1} + \frac{6a_2\overline{x_2}}{l_2}$$

- Where,
- a_1 = area of the free B.M. diagram for the span AB.
- a_2 = area of the free B.M. diagram for the span BC.
- $\overline{x_1}$= Centroidal distance of free B.M.D on AB from A.
- $\overline{x_2}$= Centroidal distance of free B.M.D on BC from C.

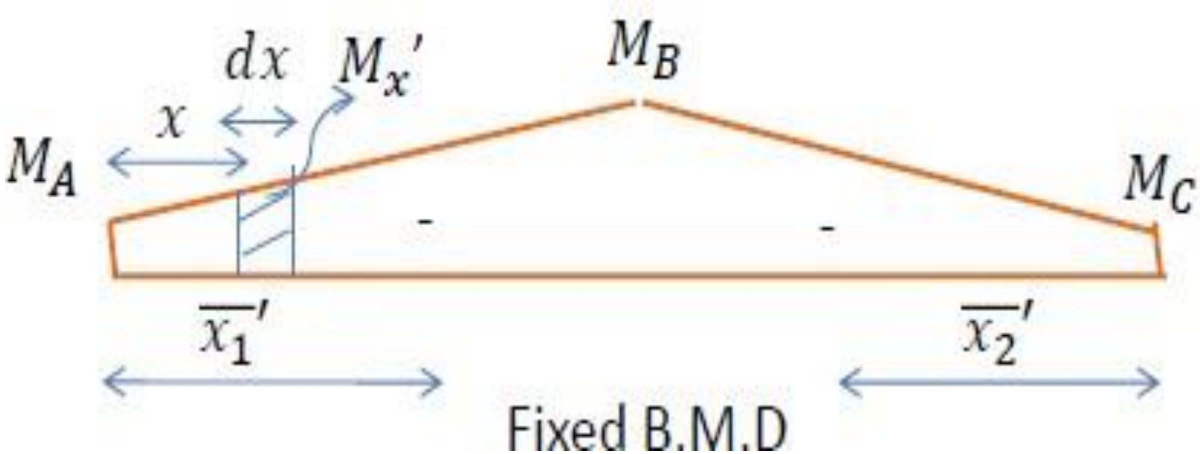

A
B
C
(a)
l_1
l_2
dx
M_x
+
+
(b)
x
$\overline{x_1}$
$\overline{x_2}$
Free B.M.D
dx
M_x'
M_B
x
M_A
M_C
+
+
(c)
$\overline{x_1}'$
$\overline{x_2}'$
Fixed B.M.D
M_B
M_A
+ ve
- ve
+ ve
M_C
(d)



(a)The given beam



(b) Free B.M.D.



(c) Fixed B.M.D.

- <u>Consider the span AB:</u>
- Let at any section in AB distant x from A the free and fixed bending moments be M_x and $M_x{}'$ respectively.
- Hence the net bending moment at the section is given by

$$EI\frac{d^2y}{dx^2} = M_x - M_x{}'$$

- Multiplying by x, we get

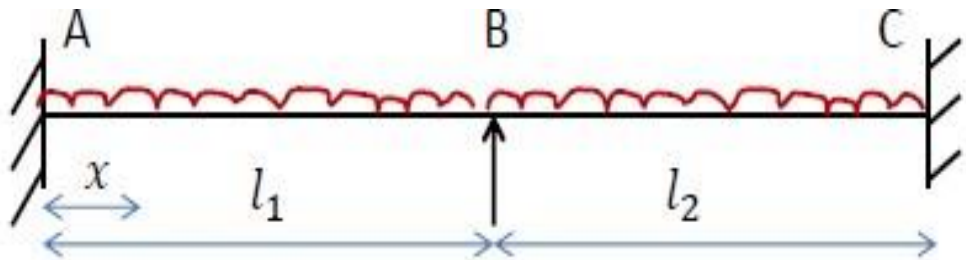
$$EIx\frac{d^2y}{dx^2} = M_x x - M_x{}' x$$

- $EIx\frac{d^2y}{dx^2} = M_x x - M_x' x$

- Integrating from $x = 0\ to\ x = l_1$, we get,

$$EI\int_0^{l_1} x\frac{d^2y}{dx^2} = \int_0^{l_1} M_x x\,dx - \int_0^{l_1} M_x' x\,dx$$

$$EI\left[x.\frac{dy}{dx} - y\right]_0^{l_1} = \int_0^{l_1} M_x x\,dx - \int_0^{l_1} M_x' x\,dx \qquad ---(1)$$



- But it may be such that

  At $x = 0$, deflection $y = 0$

- At $x = l_1, y = 0$; *and slope at B for AB*, $\frac{dy}{dx} = \theta_{BA}$

- $\int_0^{l_1} M_x x\, dx = a_1 \overline{x_1}$ = Moment of the free B. M. D. on AB about A .

- $\int_0^{l_1} M_x' x\, dx = a_1' \, \overline{x_1}'$ = Moment of the fixed B. M. D. on AB about A.

$$EI\left[x.\frac{dy}{dx} - y\right]_0^{l_1} = \int_0^{l_1} M_x x\,dx - \int_0^{l_1} M_x{}' x\,dx \quad --(1)$$

- Therefore the equation (1) is simplified as,

$$EI\,[l_1\theta_{BA} - 0] = a_1\overline{x_1} - a_1'\overline{x_1}'.$$

But a_1' = area of the fixed B.M.D. on AB = $\frac{(M_A+M_B)}{2} l_1$

$\overline{x_1}'$ = Centroid of the fixed B. M. D. from A = $\frac{(M_A+2M_B)}{M_A+M_B}\frac{l_1}{3}$

- Therefore,

$$a_1'\overline{x_1}' = \frac{(M_A + M_B)}{2} l_1 \times \left(\frac{M_A + 2M_B}{M_A + M_B}\right) \frac{l_1}{3} = (M_A + 2M_B) \frac{l_1^2}{6}$$

$$EI\, l_1 \theta_{BA} = a_1 \overline{x_1} - (M_A + 2M_B) \frac{l_1^2}{6}$$

$$6EI\, \theta_{BA} = \frac{6a_1 \overline{x_1}}{l_1} - (M_A + 2M_B) l_1 \quad ----(2)$$

Similarly by considering the span BC and taking C as origin it can be shown that,

$$6EI\, \theta_{BC} = \frac{6a_2 \overline{x_2}}{l_2} - (M_C + 2M_B) l_2 \quad ----(3)$$

θ_{BC} = slope for span CB at B

- But $\theta_{BA} = -\theta_{BC}$ as the direction of x from A for the span AB, and from C for the span CB are in opposite direction.
- And hence, $\theta_{BA} + \theta_{BC} = 0$

$$6EI\,\theta_{BA} = \frac{6a_1\overline{x_1}}{l_1} - (M_A + 2M_B)l_1 \qquad ----(2)$$

$$6EI\,\theta_{BC} = \frac{6a_2\overline{x_2}}{l_2} - (M_C + 2M_B)l_2 \qquad ----(3)$$

- Adding equations (2) and (3), we get

$$EI\,\theta_{BA} + 6EI\,\theta_{BC} = \frac{6a_1\overline{x_1}}{l_1} + \frac{6a_2\overline{x_2}}{l_2} - (M_A + 2M_B)l_1 - (M_C + 2M_B)l_2$$

$$6EI(\theta_{BA} + \theta_{BC}) = \frac{6a_1\overline{x_1}}{l_1} + \frac{6a_2\overline{x_2}}{l_2} - [M_A l_1 + 2M_B(l_1 + l_2) + M_C l_2]$$

$$0 = \frac{6a_1\overline{x_1}}{l_1} + \frac{6a_2\overline{x_2}}{l_2} - [M_A l_1 + 2M_B(l_1 + l_2) + M_C l_2]$$

$$[M_A l_1 + 2M_B(l_1 + l_2) + M_C l_2] = \frac{6a_1\overline{x_1}}{l_1} + \frac{6a_2\overline{x_2}}{l_2}$$

Problem 15.3. *A fixed beam AB of length 6 m carries point loads of 160 kN and 120 kN at a distance of 2 m and 4 m from the left end A. Find the fixed end moments and the reactions at the supports. Draw B.M. and S.F. diagrams.*

Sol. Given :

Length = 6 m

Load at C, $W_C = 160$ kN

Load at D, $W_D = 120$ kN

Distance $AC = 2$ m

Distance $AD = 4$ m

For the sake of convenience, let us first calculate the fixed end moments due to loads at C and D and then add up the moments.

(*i*) Fixed end moments due to load at C.

For the load at C, $a = 2$ m and $b = 4$ m

$$\therefore \quad M_{A_1} = \frac{W_C . a . b^2}{L^2}$$

$$= \frac{160 \times 2 \times 4^2}{6^2} = 142.22 \text{ kNm}$$

$$M_{B_1} = \frac{W_C . a^2 . b}{L^2} = \frac{160 \times 2^2 \times 4}{6^2} = 71.11 \text{ kNm}$$

(*ii*) Fixed end moments due to load at D.

Similarly for the load at D, $a = 4$ m and $b = 2$ m

$$\therefore \quad M_{A_2} = \frac{W_D . a . b^2}{L^2}$$

$$= \frac{120 \times 4 \times 2^2}{6^2} = 53.33 \text{ kNm}$$

and
$$M_{B_2} = \frac{W_D . a^2 . b}{L^2} = \frac{160 \times 4^2 \times 2}{6^2} = 106.66 \text{ kNm}$$

Total fixing moment at A,

$$M_A = M_{A_1} + M_{A_2} = 142.22 + 53.33$$

$$= \mathbf{195.55 \text{ kNm.}} \quad \textbf{Ans.}$$

and total fixing moment at B,

$$M_B = M_{B_1} + M_{B_2} = 71.11 + 106.66$$

$$= \mathbf{177.77 \text{ kNm.}} \quad \textbf{Ans.}$$

B.M. diagram due to vertical loads

Consider the beam AB as simply supported. Let $R_A{}^*$ and $R_B{}^*$ are the reactions at A and B due to simply supported beam. Taking moments about A, we get

$$R_B{}^* \times 6 = 160 \times 2 + 120 \times 4$$

$$= 320 + 480 = 800$$

$$\therefore \quad R_B{}^* = \frac{800}{6} = 133.33 \text{ kN}$$

and
$$R_A{}^* = \text{Total load} - R_B{}^* = (160 + 120) - 133.33$$

$$= 146.67 \text{ kN}$$

B.M. at $A = 0$

B.M. at $C = R_A{}^* \times 2 = 146.67 \times 2 = 293.34$ kNm

B.M. at $D = R_B{}^* \times 2 = 133.33 \times 2 = 266.66$ kNm

B.M. at $B = 0$.

S.F. Diagram

Let R_A = Resultant reaction at A due to fixed end moments and vertical loads

R_B = Resultant reaction at B.

Equating the clockwise moments and anti-clockwise moments about A, we get

$$R_B \times 6 + M_A = 160 \times 2 + 120 \times 4 + M_B$$

$$R_B \times 6 + 195.55 = 320 + 480 + 177.77$$

$$\therefore \quad R_B = \frac{800 + 177.77 - 195.55}{6} = 130.37 \text{ kN}$$

$$R_A = \text{Total load} - R_B$$

$$= (160 + 120) - 130.37 = 149.63 \text{ kN}$$

S.F. at $A = R_A$ = 149.63 kN

S.F. at C = 149.63 – 160 = – 10.37 kN

S.F. at D = – 10.37 – 120 = – 130.37 kN

S.F. at B = – 130.37 kN

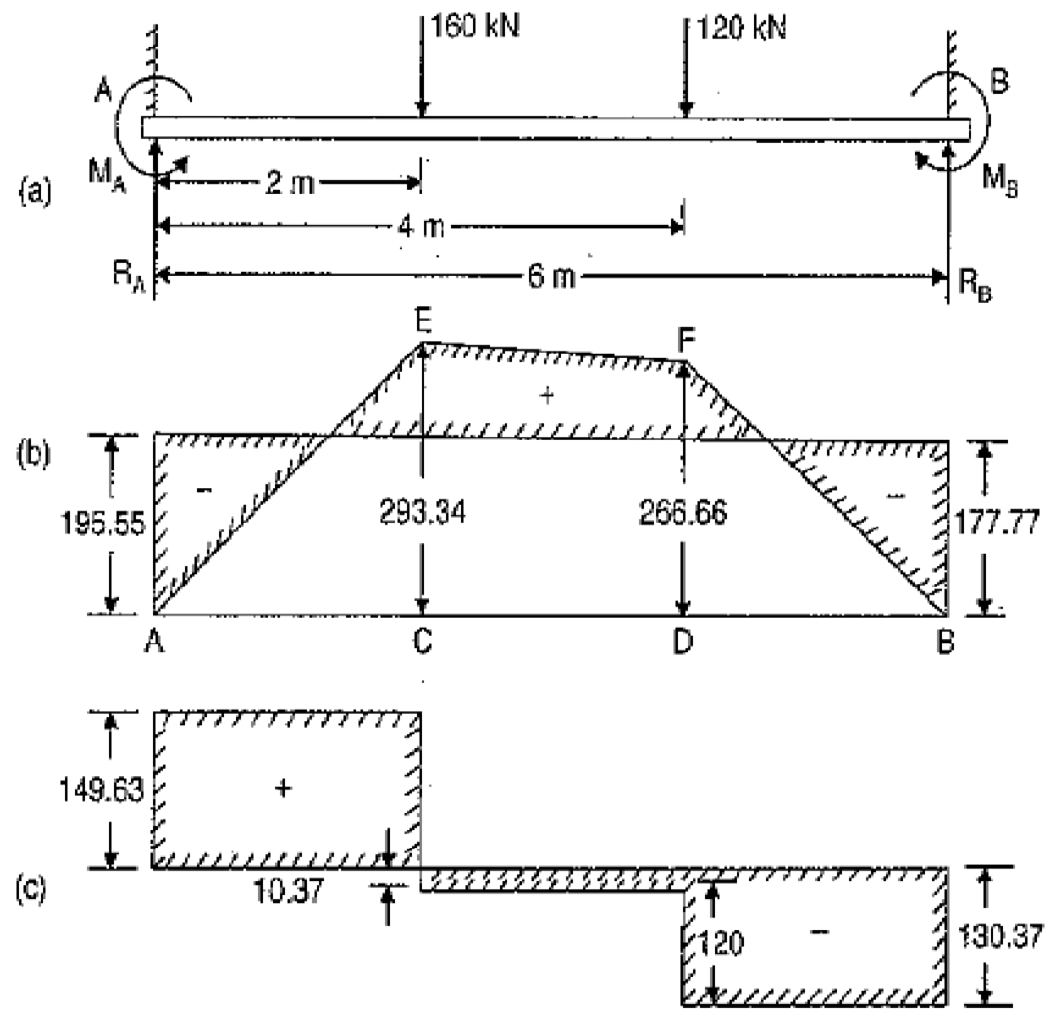


Fig. 15.8

Problem 15.6. *Find the fixing moments and support reactions of a fixed beam AB of length 6 m, carrying a uniformly distributed load of 4 kN/m over the left half of the span.*

Macaulay's method can be used and directly the fixing moments and end reactions can be calculated. This method is used where the areas of B.M. diagrams cannot be determined conveniently.

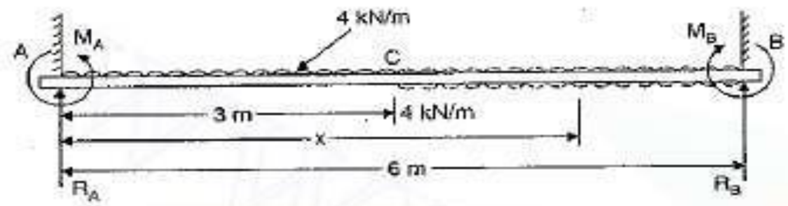


Fig. 15.12

For this method it is necessary that u.d.l. should be extended upto B and then compensated for upward u.d.l. for length BC as shown in Fig. 15.12.

The B.M. at any section at a distance x from A is given by

$$EI\frac{d^2y}{dx^2} = R_A x - M_A - w \times x \times \frac{x}{2} \;\vdots\; + w \times (x-3) \times \frac{(x-3)}{2}$$

$$= R_A x - M_A - \frac{4 \times x^2}{2} \;\vdots\; + \frac{4(x-3)^2}{2}$$

$$= R_A . x - M_A - 2x^2 \;\vdots\; + 2(x-3)^2 \qquad ...(A)$$

Integrating, we get

$$EI\frac{dy}{dx} = R_A . \frac{x^2}{2} - M_A . x - \frac{2x^3}{3} + C_1 \;\vdots\; + \frac{2(x-3)^3}{3} \qquad ...(i)$$

when $x = 0$, $\frac{dy}{dx} = 0$.

Substituting this value in the above equation upto dotted line, we get

$C_1 = 0$.

Therefore equation (*i*) becomes as

$$EI\frac{dy}{dx} = R_A . \frac{x^2}{2} - M_A . x - \frac{2x^3}{3} \;\vdots\; + \frac{2(x-3)^3}{3} \qquad ...(ii)$$

Integrating again, we get

$$EI\, y = \frac{R_A}{2} . \frac{x^3}{3} - \frac{M_A . x^2}{2} - \frac{2}{3}\frac{x^4}{4} + C_2 \;\vdots\; + \frac{2}{3}\frac{(x-3)^4}{4} \qquad ...(iii)$$

when $x = 0$, $y = 0$.

Substituting this value upto dotted line, we get

$C_2 = 0$

Therefore equation (*iii*) becomes as

$$EI\, y = \frac{R_A . x^3}{6} - \frac{M_A . x^2}{2} - \frac{1}{6}x^4 \;\vdots\; + \frac{1}{6}(x-3)^4 \qquad ...(iv)$$

when $x = 6$, $y = 0$.

Substituting this value in equation (*iv*) [Here complete equation is taken], we get

$$0 = \frac{R_A \times 6^3}{6} - \frac{M_A \times 6^2}{2} - \frac{1}{6} \times 6^4 + \frac{1}{6} \times (6-3)^4$$

$$= 36R_A - 18M_A - 216 + 13.5$$

$$202.50 = 36R_A - 18M_A$$

$$101.25 = 18R_A - 9M_A \qquad ...(v)$$

At $x = 6$ m, $\frac{dy}{dx} = 0$.

Substituting these values in the complete equation (*ii*), we get

$$0 = R_A \times \frac{6^2}{2} - M_A \times 6 - \frac{2}{3} \times 6^3 + \frac{2}{3}(6-3)^3$$

$$= 18R_A - M_A \times 6 - 144 + 18$$

$$126 = 18R_A - 6M_A \qquad \text{...}(vi)$$

Substracting equation (*v*) from equation (*vi*), we get

$$126 - 101.25 = 9M_A - 6M_A$$

or $$24.75 = 3M_A$$

$\therefore$ $$M_A = \frac{24.75}{3} = \mathbf{8.25\ kNm.}\quad \textbf{Ans.}$$

Substituting this value in equation (*vi*), we get

$$126 = 18R_A - 6 \times 8.25$$

$\therefore$ $$R_A = \frac{126 + 6 \times 8.25}{18} = \mathbf{9.75\ kN.}\quad \textbf{Ans.}$$

Now $$R_B = \text{Total load} - R_A$$

$$= 4 \times 3 - 9.75 = \mathbf{2.25\ kN.}\quad \textbf{Ans.}$$

To find the value of M_B, we must equate the clockwise moments and anti-clockwise moments about B. Hence

Clockwise moments about B = Anti-clockwise moments about B.

$$M_B + R_A \times 6 = M_A + 4 \times 3 \times (4.5)$$

or $$M_B + 9.75 \times 6 = 8.25 + 54 \qquad (\because\ R_A = 9.75 \text{ and } M_A = 8.25)$$

or $$M_B + 58.50 = 62.25$$

$\therefore$ $$M_B = 62.25 - 58.50 = \mathbf{3.75\ kNm.}\quad \textbf{Ans.}$$

Problem 15.7. *A fixed beam of length 20 m, carries a uniformly distributed load of 8 kN/m on the left hand half together with a 120 kN load at 15 m from the left hand end. Find the end reactions and fixing moments and magnitude and the position of the maximum deflection. Take $E = 2 \times 10^8$ kN/m^2 and $I = 4 \times 10^8$ mm^4.*

Sol. Given :

Length, $L = 20$ m

U.d.l., $w = 8$ kN/m

Point load, $W = 120$ kN

Value of $E = 2 \times 10^8$ kN/m^2

Value of $I = 4 \times 10^8$ mm^4 $= 4 \times 10^{-4}$ m^4

Lengths, $AC = 10$ m, $AD = 15$ m

Fig. 15.13 shows the loading on the fixed beam.

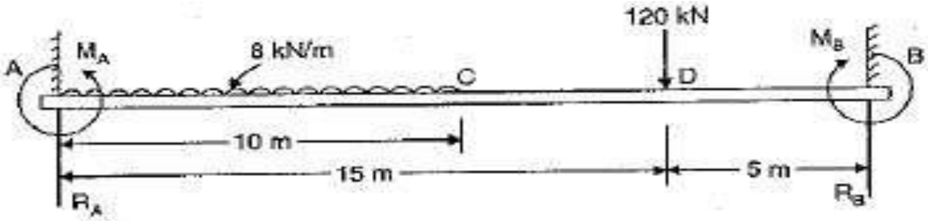


Fig. 15.13

Let R_A and R_B = End reactions at A and B

M_A and M_B = Fixing moments at A and B

Let us apply Macaulay's method for this case. Hence it is necessary that the u.d.l. should be extended upto B and then compensated for upward u.d.l. for length BC as shown in Fig. 15.14.

The B.M. at any section at a distance x from A is given by,

$$EI \frac{d^2y}{dx^2} = R_A x - M_A - w \times x \times \left(\frac{x}{2}\right) \Bigg| - 120(x - 15) \Bigg| + w \times (x - 10) \times \left(\frac{x-10}{2}\right)$$

$$= R_A \times x - M_A - 8 \times \frac{x^2}{2} \Bigg| - 120(x - 15) \Bigg| + \frac{8 \times (x-10)^2}{2}$$

$$= R_A x - M_A - 4x^2 \Big| - 120(x - 15) \Big| + 4(x - 10)^2$$

Integrating the above equation, we get

$$EI \frac{dy}{dx} = R_A \cdot \frac{x^2}{2} - M_A \cdot x - 4 \cdot \frac{x^3}{3} + C_1 \Bigg| - \frac{120(x-15)^2}{2} \Bigg| + \frac{4(x-10)^3}{3} \quad \ldots(i)$$

when $x = 0$, $\frac{dy}{dx} = 0$. Substituting this value in the above equation upto first dotted line, we get $C_1 = 0$. Therefore, equation (*i*) becomes as

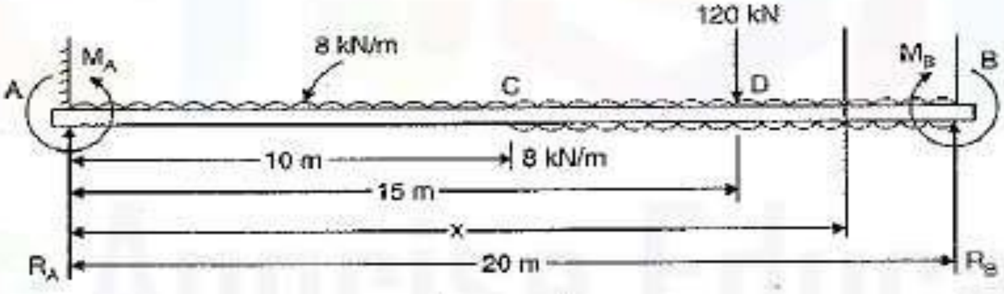


Fig. 15.14

$$EI \frac{dy}{dx} = \frac{R_A}{2} \cdot x^2 - M_A \cdot x - \frac{4}{3} x^3 \Bigg| - 60(x - 15)^2 \Bigg| + \frac{4}{3}(x - 10)^3 \quad \ldots(ii)$$

Integrating again, we get

$$EIy = \frac{R_A.x^3}{6} - \frac{M_A.x^2}{2} - \frac{4x^4}{3 \times 4} + C_2 \;\vdots\; - \frac{60(x-15)^3}{3} \;\vdots\; + \frac{4}{3}\frac{(x-10)^4}{4} \qquad ...(iii)$$

when $x = 0$, $y = 0$. Substituting this value in the above equation upto first dotted line, we get $C_2 = 0$. Therefore equation (*iii*) becomes as

$$EIy = \frac{R_A.x^3}{6} - \frac{M_A.x^2}{2} - \frac{x^4}{3} \;\vdots\; - 20(x-15)^3 \;\vdots\; + \frac{1}{3}(x-10)^4 \qquad ...(iv)$$

when $x = 20$, $y = 0$. Substituting these values in complete equation (*iv*), we get

$$0 = \frac{R_A \times 20^3}{6} - \frac{M_A \times 20^2}{2} - \frac{20^4}{3} - 20(20-15)^3 + \frac{1}{3}(20-10)^4$$

$$= \frac{20}{6}R_A - \frac{M_A}{2} - \frac{20^2}{3} - \frac{125}{20} + \frac{1}{3} \times \frac{10^4}{400} \qquad \text{(Dividing by } 20^2\text{)}$$

$$= \frac{20}{6}R_A - \frac{M_A}{2} - \frac{400}{3} - \frac{12.5}{2} + \frac{25}{3}$$

$$= \frac{20R_A - 3M_A - 800 - 37.5 + 50}{6}$$

or $\quad 20R_A - 3M_A = 800 + 37.5 - 50 = 787.5 \qquad ...(v)$

At $x = 20$, $\frac{dy}{dx} = 0$. Substituting these values in complete equation (*ii*), we get

$$0 = \frac{R_A}{2} \times 20^2 - M_A \times 20 - \frac{4}{3} \times 20^3 - 60(20-15)^2 + \frac{4}{3}(20-10)^3$$

$$= \frac{20}{6}R_A - \frac{M_A}{2} - \frac{20^2}{3} - \frac{125}{20} + \frac{1}{3} \times \frac{10^4}{400} \qquad \text{(Dividing by } 20^2\text{)}$$

$$= \frac{20}{6}R_A - \frac{M_A}{2} - \frac{400}{3} - \frac{12.5}{2} + \frac{25}{3}$$

$$= \frac{20R_A - 3M_A - 800 - 37.5 + 50}{6}$$

or $\quad 20R_A - 3M_A = 800 + 37.5 - 50 = 787.5 \qquad ...(v)$

At $x = 20$, $\frac{dy}{dx} = 0$. Substituting these values in complete equation (*ii*), we get

$$0 = \frac{R_A}{2} \times 20^2 - M_A \times 20 - \frac{4}{3} \times 20^3 - 60(20-15)^2 + \frac{4}{3}(20-10)^3$$

$$= 10R_A - M_A - \frac{4 \times 400}{3} - 3 \times 25 + \frac{4}{3} \times \frac{1000}{20} \qquad \text{(Dividing by 20)}$$

$$= 10R_A - M_A - \frac{1600}{3} - 75 + \frac{200}{3}$$

or $\quad 10R_A - M_A = \frac{1600}{3} + 75 - \frac{200}{3} = \frac{1400}{3} + 75$

or $\quad 10R_A - M_A = 541.66$

or $\quad 20R_A - 2M_A = 1083.32 \qquad$ (Multiplying by 2 both sides) $\quad ...(vi)$

Substracting equation (*v*) from equation (*vi*), we get

$M_A = 1083.32 - 787.50 =$ **295.82 kNm. Ans.**

Substituting this values of M_A in equation (*vi*), we get

$20R_A - 2 \times 295.82 = 1083.32$

$$\therefore \quad R_A = \frac{1083.32 + 2 \times 295.82}{20}$$

= **83.7d8 kN. Ans.**

Now $\quad R_B$ = Total load on beam $- R_A$

$= (10 \times 8 + 120) - 83.748 =$ **116.252 kN. Ans.**

Equating the clockwise moment and anti-clockwise moment about B, we get

$M_B + R_A \times 20 = M_A + 120 \times 5 + 8 \times 10 \times 15$

or $\quad M_B + 83.748 \times 20 = 295.82 + 600 + 1200$

or $\quad M_B = 2095.82 - 83.748 \times 20 =$ **420.86 kNm. Ans.**

Problem 15.8.s A *fixed beam AB of length 3 m is having moment of inertia $I = 3 \times 10^6$ mm^4. The support B sinks down by 3 mm. If $E = 2 \times 10^5$ N/mm^2, find the fixing moments.*

Sol. Given :

Length, $L = 3 \text{ m} = 3000 \text{ mm}$

Value of $I = 3 \times 10^6 \text{ mm}^4$

Value of $E = 2 \times 10^5 \text{ N/mm}^2$

The amount by which the support B sinks down,

$$\delta = 3 \text{ mm}.$$

The fixing moments at the ends is given by,

$$M_A = M_B = \frac{6EI\delta}{L^2}$$

$$= \frac{6 \times 2 \times 10^5 \times 3 \times 10^6 \times 3}{3000^2}$$

$$= 12 \times 10^5 \text{ Nmm} = 12 \times 10^3 \text{ Nm} = \textbf{12 kNm.} \quad \textbf{Ans.}$$

The fixing moment at A will be a hogging moment whereas at B it will be a sagging moment.

EXAMPLE 24.5. *A fixed beam AB of span 6 m is carrying a uniformly distributed load of 4kN/m over the left half of the span. Find the fixing moments and support reactions.*

SOLUTION. Given: Span $(l) = 6$ m ; Uniformly distributed load $(w) = 4$ kN/m and loaded portion $(l_1) = 3$ m.

Fixing moments

Let M_A = Fixing moment at A and,

M_B = Fixing moment at B.

First of all, consider the beam AB on a simply supported. Taking moments about A,

$$R_B \times 6 = 4 \times 3 \times 1.5 = 18$$

$$\therefore \quad R_B = \frac{18}{6} = 3 \text{ kN}$$

and $$R_A = 3 \times 4 - 3 = 9 \text{ kN}$$

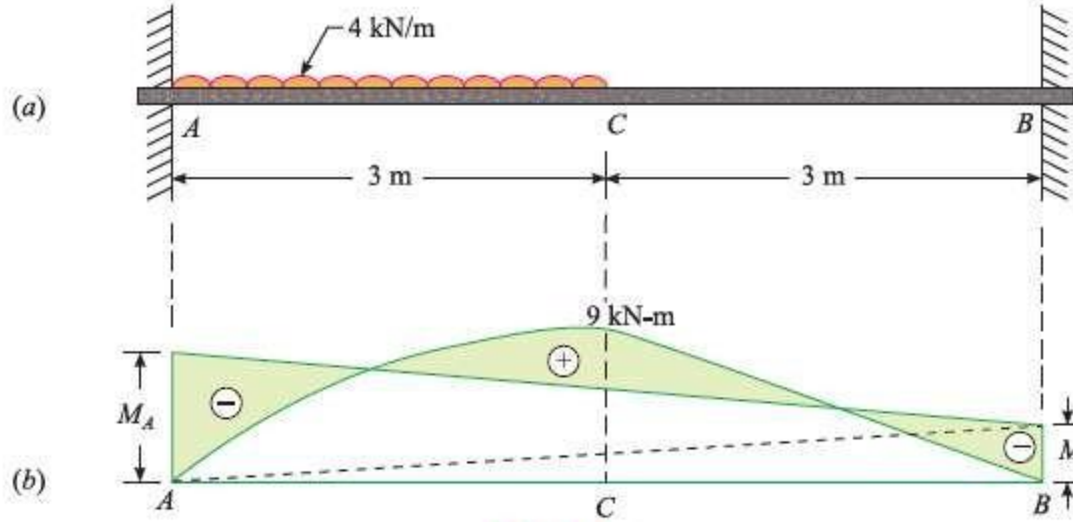


Fig. 24.6

We know that µ-diagram will be parabolic from A to C and triangular from C to B as shown in ig. 24.6 (b). The bending moment at C (treating the beam as a simply supported),

$$M_C = R_B \times 3 = 3 \times 3 = 9 \text{ kN-m}$$

The bending moment at any section X in AC, at a distance x from A (treating the beam as a simply ıpported),

$$M_X = 9x - 4x \cdot \frac{x}{2} = 9x - 2x^2$$

$\therefore$ Area µ-diagram from A to B,

$$a = \int_0^3 (9x - 2x^2)\, dx + \frac{1}{2} \times 9.0 \times 3$$

$$= \left[\frac{9x^2}{2} - \frac{2x^3}{3}\right]_0^3 + 13.5$$

$$= \frac{9 \times (3)^2}{2} - \frac{2 \times (3)^3}{3} + 13.5 = 36$$

nd area of µ′-diagram, $$a' = (M_A + M_B) \times \frac{6}{2} = 3\,(M_A + M_B)$$

We know that $$a' = -a$$

$$\therefore \quad 3\,(M_A + M_B) = -36$$

or $$M_A + M_B = -\frac{36}{3} = -12$$

Moment of μ-diagram area about A (by splitting up the diagram into AC and CB),

$$a\bar{x} = \int_0^3 (9x^2 - 2x^3)\,dx + \frac{1}{2}\times 9\times 3\times 4$$

$$a\bar{x} = \left[\frac{9x^3}{3} - \frac{2x^4}{4}\right]_0^3 + 54$$

$$= \left[\frac{9\times(3)^3}{3} - \frac{2\times(3)^4}{4}\right] + 54 = 94.5$$

and moment of $\bar{x}'$-diagram area about A (by splitting up the trapezium into two triangles) as shown in Fig. 24.6 (a),

$$a'\bar{x}' = \left(M_A\times\frac{6}{2}\times\frac{6}{3}\right) + M_B\times\frac{6}{2}\times\frac{2\times 6}{3}$$

$$= 6M_A + 12M_B = 6(M_A + 2M_B)$$

We know that $a'\bar{x} = -a\bar{x}$

$$6(M_A + 2M_B) = -94.5$$

$$\therefore \quad M_A + 2M_B = -\frac{94.5}{6} = -15.75 \quad \text{...(ii)}$$

Solving equations (i) and (ii),

M_A = **– 8.25 kN-m** **Ans.**

M_B = **– 3.75 kN-m** **Ans.**

Now complete the bending moment diagram as shown in Fig. 24.6 (b).

Support reactions

Let R_A = Reaction at A, and

R_B = Reaction at B.

Equating the clockwise moments and anticlockwise moments about A,

$$R_B\times 6 + 8.25 = (4\times 3\times 1.5) + 3.75 = 21.75$$

$$\therefore \quad R_B = \frac{21.75 - 8.25}{6} = \mathbf{2.25\ kN} \quad \textbf{Ans.}$$

and $R_A = 4\times 3 - 2.25 =$ **9.75 kN** **Ans.**

EXAMPLE 24.6. *A beam AB of uniform section and 6 m span is built-in at the ends. A uniformly distributed load of 3 kN/m runs over the left half of the span and there is in addition a concentrated load of 4 kN at right quarter as shown in Fig. 24.7.*

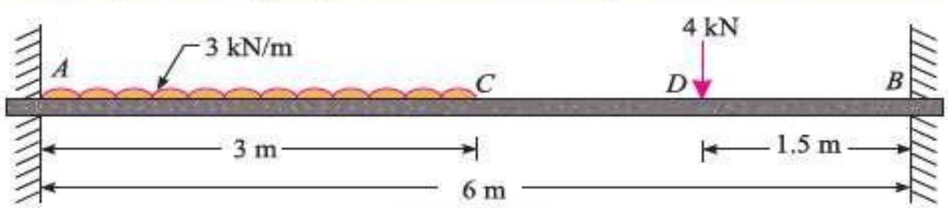


Fig. 24.7

Determine the fixing moments at the ends, and the reactions. Sketch neatly the bending moment and shearing force diagram marking thereon salient values.

SOLUTION: Given: Span (l) = 6 m ; Uniformly distributed load on AC (w) = 3 kN/m ; Loaded portion (l_1) = 3 m and concentrated load at D (W) = 4 kN.

Fixing moments at the ends

Let M_A = Fixing moment at A and

First of all, consider the beam AB as a simply supported. Taking moments about A,

$$R_B \times 6 = (3 \times 3 \times 1.5) + (4 \times 4.5) = 31.5$$

$$\therefore \quad R_B = \frac{31.5}{6} = 5.25 \text{ kN}$$

and

$$R_A = (3 \times 3 + 4) - 5.25 = 7.75 \text{ kN}$$

We know that the μ-diagram will be parabolic from A to C, trapezoidal from C to D and triangular from D to B as shown in Fig. 24.8(b). The bending moment at D (treating the beam at a simply supported),

$$M_D = 5.25 \times 1.5 = 7.875 \text{ kN-m}$$

and

$$M_C = 5.25 \times 3 - 4 \times 1.5 = 9.75 \text{ kN-m}$$

The bending moment at any section X in AC, at a distance x from A (treating the beam as a simply supported),

$$M_X = 7.75x - 3x\,\frac{x}{2} = 7.75x - 1.5x^2$$

∴ Area of μ-diagram from A to B,

$$\therefore \quad a = \int_0^3 (7.75x - 1.5x^2)\,dx + \left(\frac{1}{2}(9.75 + 7.875) \times 1.5\right) + \left(\frac{1}{2} \times 7.875 \times 1.5\right)$$

$$= \left[\frac{7.75x^2}{2} - \frac{1.5x^3}{3}\right]_0^3 + 19.125$$

$$= \frac{7.75 \times (3)^2}{2} - \frac{1.5 \times (3)^3}{3} + 19.125 = 40.5$$

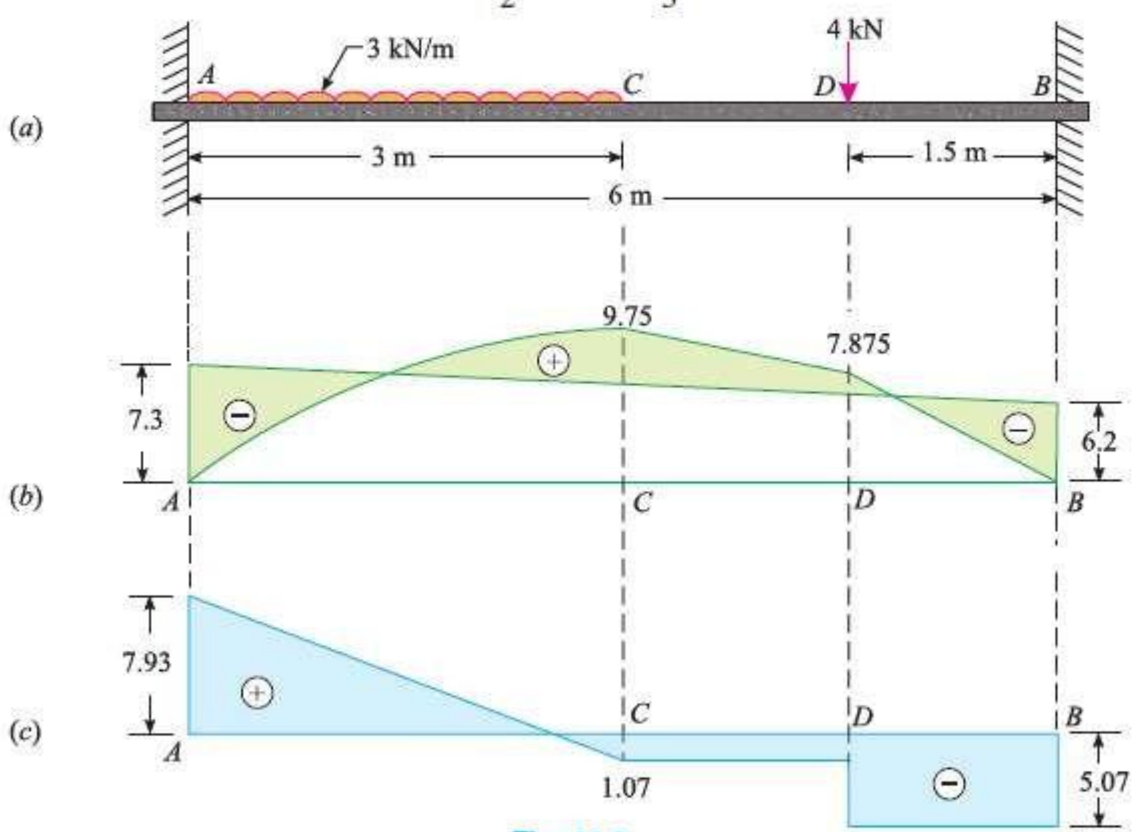


Fig. 24.8

and area of μ-diagram, $a' = (M_A + M_B) \times \frac{6}{2} = 3\,(M_A + M_B)$

We know that $a' = -a$

$$\therefore \quad 3\,(M_A + M_B) = -40.5 \qquad ...(\because a = 40.5)$$

$$\text{or} \quad M_A + M_B = -13.5 \qquad ...(i)$$

Moment of μ-diagram area about *A* (by splitting up the diagram into *AC*, *CD* and *DB*),

$$a\bar{x} = \int_0^3 (7.75x^2 - 1.5x^3)\,dx + \left(\frac{1}{2} \times 9.75 \times 1.5 \times 3.5\right)$$

$$+ \left(\frac{1}{2} \times 7.875 \times 1.5 \times 4\right) + \left(\frac{1}{2} \times 7.875 \times 1.5 \times 5\right)$$

$$= \left[\frac{7.75x^3}{3} - \frac{1.5x^4}{4}\right]_0^3 + 78.75$$

$$= \left[\frac{7.75 \times (3)^3}{3} - \frac{1.5 \times (3)^4}{4}\right] + 78.75 = 118.1$$

and moment of μ′-diagram area about *A* (by splitting up the trapezium into two triangles),

$$a'\bar{x}' = \left(M_A \times \frac{6}{2} \times \frac{6}{3}\right) + \left(M_B \times \frac{6}{2} \times \frac{2 \times 6}{3}\right)$$

$$= 6M_A + 12M_B = 6\,(M_A + 2M_B)$$

We know that $a'\bar{x}' = -a\bar{x}$

$$\therefore \quad 6\,(M_A + 2M_B) = -118.1$$

$$\text{or} \quad M_A + 2M_B = -\frac{118.1}{6} = -19.7 \qquad ...(ii)$$

Solving equations (*i*) and (*ii*), we get

$$M_A = -7.3 \text{ kN-m} \quad \text{and} \quad M_B = -6.2 \text{ kN-m}$$

Now complete the bending moment diagram as shown in Fig. 24.8 (*b*).

Shearing force diagram

Let R_A = Reaction at *A* and

R_B = Reaction at *B*.

Equating the clockwise moments and anticlockwise moments about *A*,

$$R_B \times 6 + 7.3 = (3 \times 3 \times 1.5) + (4 \times 4.5) + 6.2 = 37.7$$

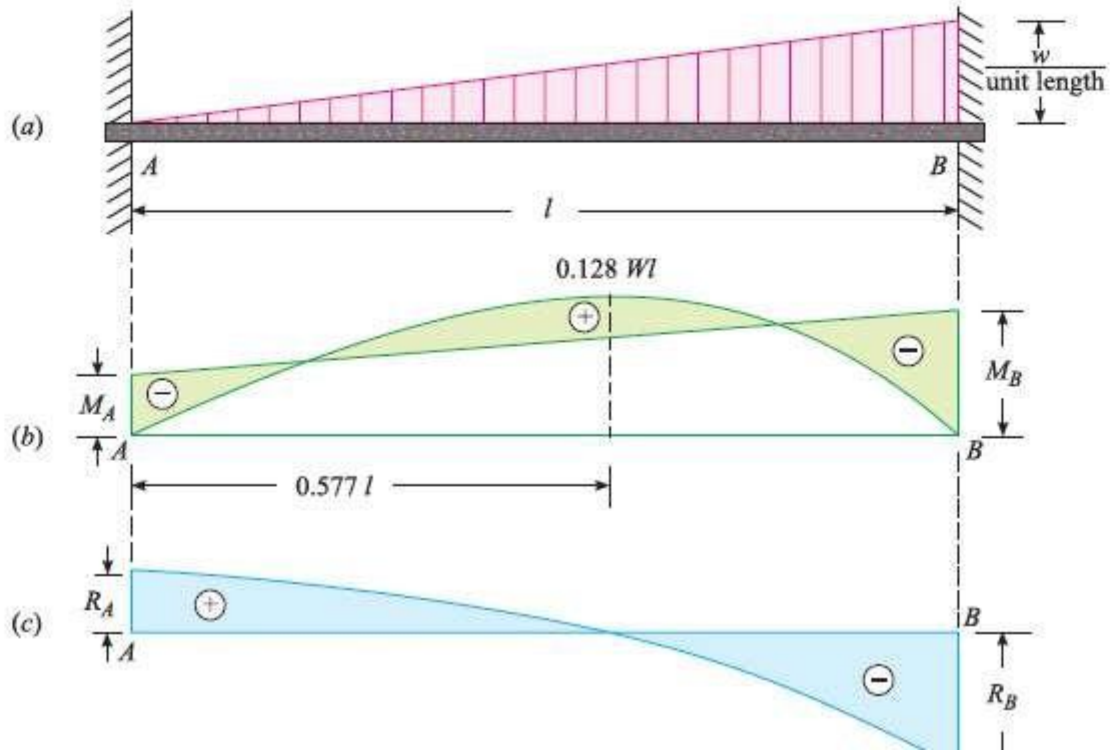
$$\therefore \quad R_B = \frac{37.7 - 7.3}{6} = 5.07 \text{ kN}$$

$$\text{and} \quad R_A = (3 \times 3 + 4) - 5.07 = 7.93 \text{ kN}$$

Now complete the shear force diagram as shown in Fig. 24.8 (*c*).

24.8. Fixing Moments of a Fixed Beam Carrying a Gradually Varying Load from Zero at One End to *w* per unit length at the Other

Consider a beam *AB* fixed at *A* and *B* and carrying a gradually varying load from zero at *A* to *w* per unit length at *B* as shown in Fig. 24.9 (*a*).



Let l = Span of the beam,

M_A = Fixing moment at *A* and

M_B = Fixing moment at *B*.

First of all, consider the beam *AB* as a simply supported and taking moments about *A*,

$$R_B \times l = w \times \frac{l}{2} \times \frac{2l}{3} = \frac{wl^2}{3}$$

$$\therefore \quad R_B = \frac{wl}{3}$$

and
$$R_A = \frac{wl}{2} - \frac{wl}{3} = \frac{wl}{6}$$

We know that the μ-diagram will be parabolic from *A* to *B*. The bending moment at any section *X*, at a distance *x* from *A* (treating the beam as a simply supported),

$$M_X = \frac{wl}{6} \times x - \frac{wx}{l} \times \frac{x}{2} \times \frac{x}{3} = \frac{wlx}{6} - \frac{wx^3}{6l}$$

$\therefore$ Area of μ-diagram,

$$a = \int_0^l \left(\frac{wlx}{6} - \frac{wx^3}{6l} \right) dx$$

$$= \frac{w}{6} \int_0^l \left(lx - \frac{x^3}{l} \right) dx$$

$$= \frac{w}{6} \left[\frac{lx^2}{2} - \frac{x^4}{4l} \right]_0^l$$

$$= \frac{w}{6} \left(\frac{l^3}{2} - \frac{l^3}{4} \right) = \frac{wl^3}{24}$$

and area of μ′-diagram, $$a' = \frac{l}{2}(M_A + M_B)$$

We know that $$a' = -a$$

$$\therefore \quad \frac{l}{2}(M_A + M_B) = -\frac{wl^3}{24}$$

or $$M_A + M_B = -\frac{wl^2}{12}$$

Moment of μ-diagram area about A,

$$a\bar{x} = \int_0^l \left(\frac{wlx^2}{6} - \frac{wx^4}{6l}\right) dx$$

$$= \frac{w}{6}\int_0^l \left(lx^2 - \frac{x^4}{l}\right) dx$$

$$= \frac{w}{6}\left[\frac{lx^3}{3} - \frac{x^5}{5l}\right]_0^l$$

$$= \frac{w}{6}\left(\frac{l^4}{3} - \frac{l^4}{5}\right) = \frac{wl^4}{45}$$

and moment of μ′-diagram about A (by splitting up the trapezium into two triangles),

$$a'x' = M_A \times \frac{l}{2} \times \frac{l}{3} + M_B \times \frac{l}{2} \times \frac{l}{3}$$

$$= \frac{l^2}{6}(M_A + 2M_B)$$

We know that $$a'\bar{x}' = -a\bar{x}$$

$$\therefore \quad \frac{l^2}{6}(M_A + 2M_B) = -\frac{wl^4}{45}$$

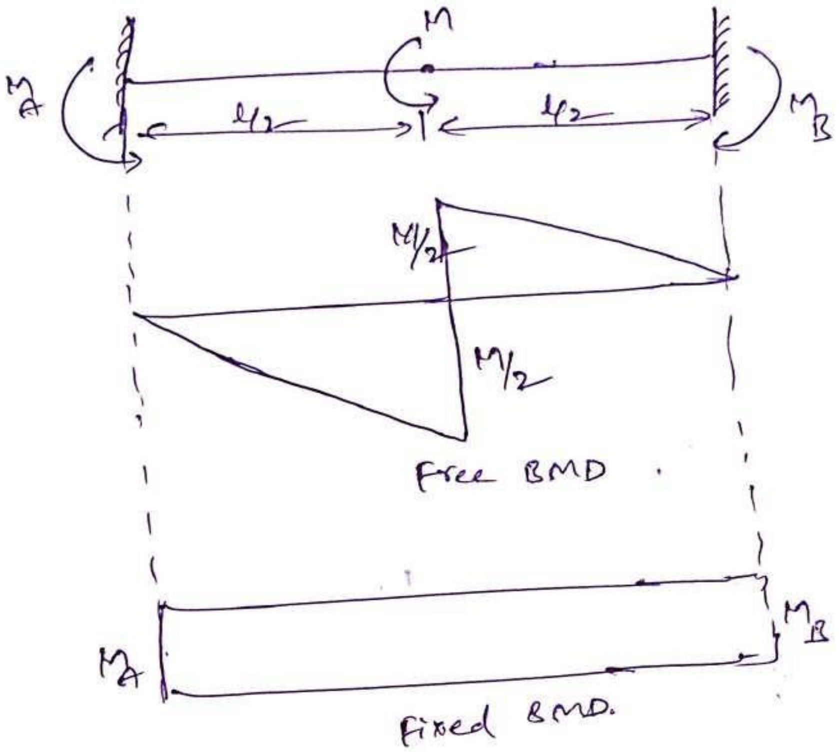
or $$M_A + 2M_B = -\frac{2wl^2}{15} \quad \text{...}(ii)$$

Solving equations (*i*) and (*ii*),

$$M_A = -\frac{wl^2}{30} = -\frac{Wl}{15} \quad \text{...}\left(\because W = \frac{wl}{2}\right)$$

and $$M_B = -\frac{wl^2}{20} = -\frac{Wl}{10} \quad \text{...}\left(\because W = \frac{wl}{2}\right)$$

Fixed Beam subjected to Concentrated Couple at mid span :-



Area of free BMD = $A = \left(\frac{1}{2} \times \frac{l}{2} \times \frac{M}{2}\right) \times 2 = \frac{Ml}{4}$

Area of fixed BMD = $A' = M_A \times l$

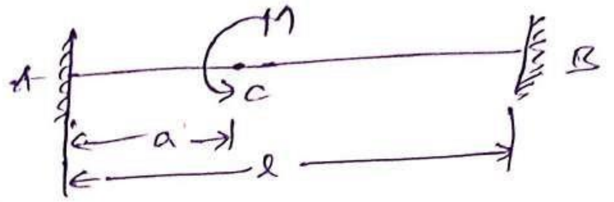
$$A' = A$$

$$M_A l = \frac{Ml}{4}$$

$$\boxed{M_A = M_B = \frac{M}{4}}$$

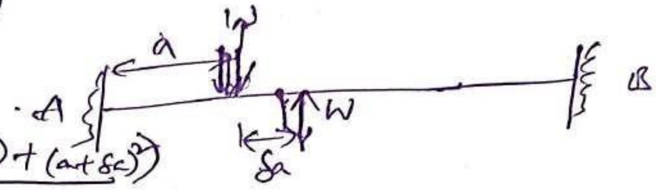
Fixed Beam subjected to a couple applied eccentrically on the span

$M_A = \frac{Wab^2}{l^2}$



$W = W\uparrow + W\downarrow$

$\therefore M_A = \frac{W(a)(l-a)^2}{l^2} - \frac{W(a+\delta a)(l-(a+\delta a))^2}{l^2}$



$= \frac{Wa(l-a)^2 - W(a+\delta a)(l^2 - 2l(a+\delta a) + (a+\delta a)^2)}{l^2}$

as δa is small, $(\delta a)^2$ very small, so neglected.

$\therefore M_A = \frac{Wa(l-a)^2 - W[al^2 + 2al(a+\delta a) + \delta a l^2]}{l^2}$

$= \frac{Wa(l-a)^2 - W(a+\delta a)[(l-a)^2 - 2\delta a(l-a) + (\delta a)^2]}{l^2}$

$= \frac{W\delta a(la)}{l^2}[2a-(l-a)]$

$= \frac{-W\delta a}{l^2}[(l-a)(l-3a)]$

If δa is small moment $M = W \times \delta a$.

$\therefore M_A = \frac{-M(l-a)(l-3a)}{l^2}$

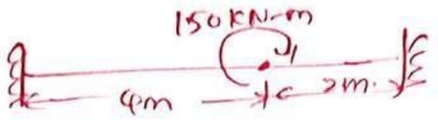
$M_B = \frac{Wa^2b}{l^2} = \frac{Wa^2(l-a)}{l^2} - \frac{W((\delta a+a)^2)(l-(a+\delta a))}{l^2}$

$= \frac{Wa^2(l-a)}{l^2} - \frac{W(a^2+2a\delta a+\delta a^2)(l-a-\delta a)}{l^2}$

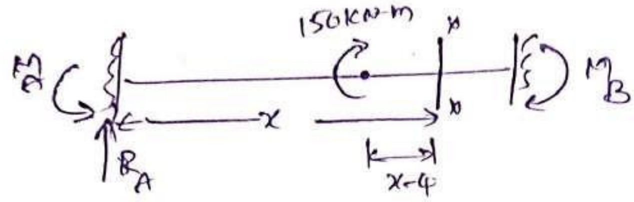
$= \frac{Wa^2(l-a)}{l^2} + \frac{Wl(a+\delta a)^3}{l^3} = \frac{M}{l^2}[a(2l-3a)]$

Prob 6 for the given beam find (a) Moment at fixed ends.

(b) Reactions (c) SFD & BMD



Sol:



Bending moment at section x-x = $M_x = R_A x - M_A + 150(x-4)^0$

but $M = EI\frac{d^2y}{dx^2}$

$$EI\frac{d^2y}{dx^2} = R_A x - M_A + 150(x-4)^0$$

$$EI\frac{dy}{dx} = \frac{R_A x^2}{2} - M_A x + C_1 + 150(x-4)^1$$

At $x=0$, $\frac{dy}{dx}=0 \Rightarrow C_1 = 0$.

$$EI\frac{dy}{dx} = \frac{R_A x^2}{2} - M_A x + 150(x-4) \longrightarrow (1)$$

$$EIy = \frac{R_A x^3}{6} - \frac{M_A x^2}{2} + C_2 + 150\frac{(x-4)^2}{2}$$

At $x=0$, $y=0$, $\Rightarrow C_2 = 0$.

$$EIy = \frac{R_A x^3}{6} - \frac{M_A x^2}{2} + \frac{150(x-4)^2}{2} \longrightarrow (2)$$

At $x=6m$, $\frac{dy}{dx}=0$ from eq (1)

$$0 = \frac{R_A(6)^2}{2} - M_A(6) + 150(6-4)$$

$$0 = \frac{R_A \times 6 \times 6}{2} - 6M_A + 150(2) \Rightarrow 18R_A - 6M_A + 300 = 0$$

$$18R_A - 6M_A = -300 \longrightarrow (3)$$

At $x=6m$, $y=0$, from eq (2).

$$0 = \frac{R_A(6^3)}{6} - \frac{M_A(6)^2}{2} + \frac{150(6-4)^2}{2}$$

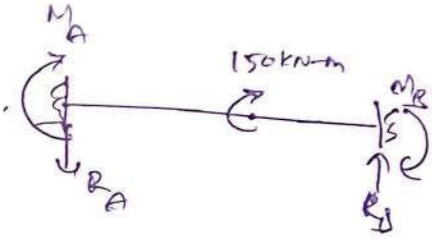
$$0 = 36R_A - 18M_A + 300 \Rightarrow 36R_A - 18M_A = -300 \longrightarrow (4)$$

$R_A = \frac{-100}{3}$ kN, $M_A = -50$ kN-m.

$\Sigma V = 0 \Rightarrow R_A + R_B = 0.$

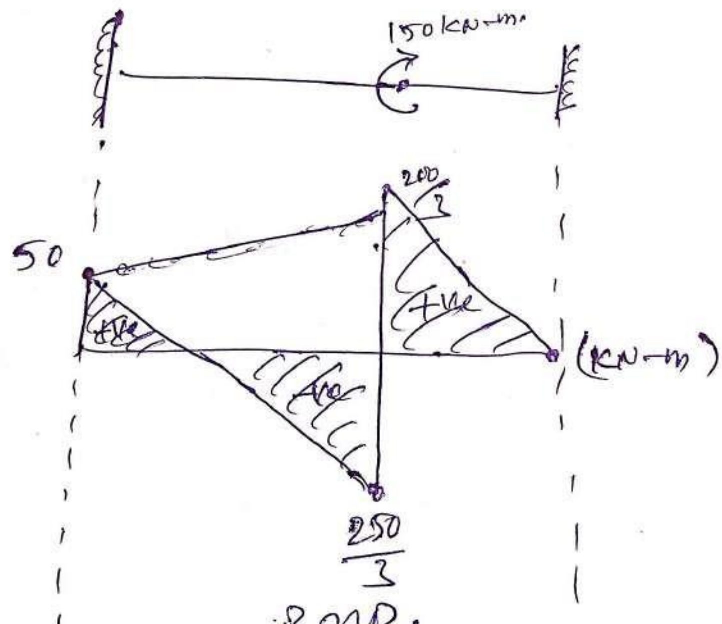
$R_B = \frac{100}{3}$ kN.

$\Sigma M_A = 0 \Rightarrow -R_B \times 6 + M_B + 150 + M_A = 0.$

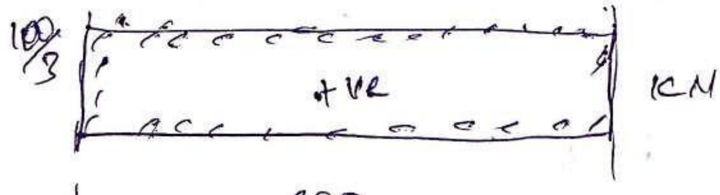


$\frac{-100}{3} \times 6 + M_B + 150 + 50 = 0$

$M_B = 0$

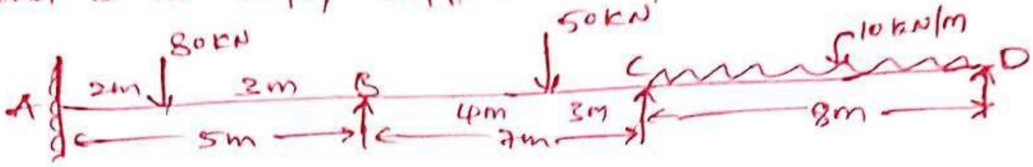


BMD.

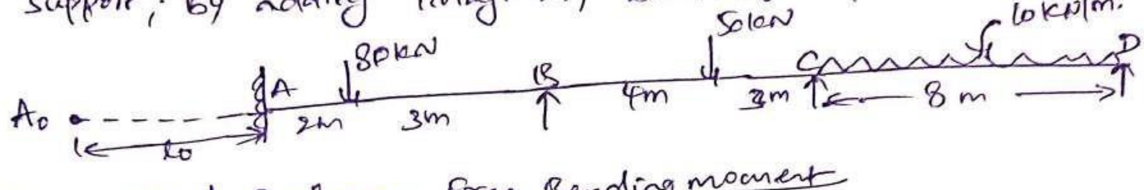


SFD

Prob: Solve the given continuous beam as shown in fig. If the end 'A' is fixed & end 'D' is simply supported.



Sol: To handle fixed end 'A' case at end A, end 'A' is made continuous support, by adding imaginary beam A_0A span l_0.



Step ①: ~~Fixed End mom.~~ Free Bending moment

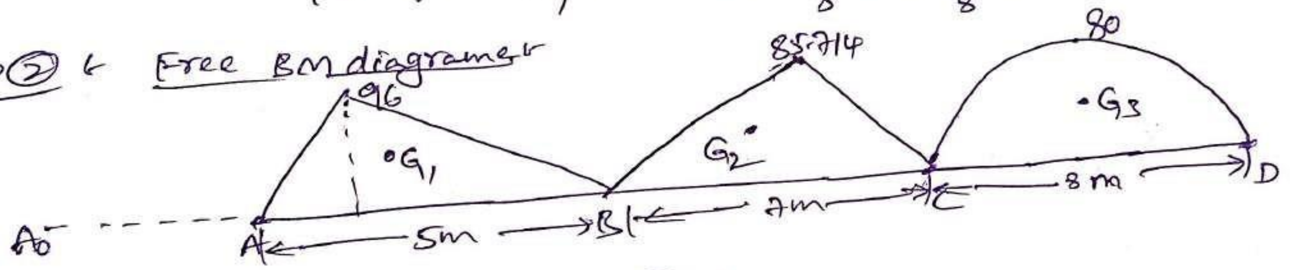
For span $A_0A = 0$,

For span AB, The BM = $\frac{Wab}{l} = \frac{80\times2\times3}{5} = 96$ kN-m.

For span BC, The BM = $\frac{Wab}{l} = \frac{50\times4\times3}{7} = 85.714$ kN-m

For span CD, the BM = $\frac{wl^2}{8} = \frac{10\times8^2}{8} = 80$ kN-m

Step ②: Free BM diagram:



For span A_0A: $A = 0$, $\bar{x} = 0$.

For span AB: $A = \frac{1}{2}\times5\times96 = 240$, $x_l = \frac{a+l}{3} = \frac{2+5}{3} = 2.3$ m.

$x_R = \frac{b+l}{3} = \frac{3+5}{3} = 2.6$ m.

For span BC: $A = \frac{1}{2}\times7\times85.714 = 299.99 \approx 300$

$x_l = \frac{a+l}{3} = \frac{4+7}{3} = 3.66$ m, $x_R = \frac{b+l}{3} = \frac{3+7}{3} = 3.33$ m

For span CD: $A = \frac{2}{3}\times8\times80 = 426.6$,

$x_l = x_R = \frac{l}{2} = \frac{8}{2} = 4$ m.

Step ③: Apply Three moment Theorem:

For span A_0A & AB:

$$M_{A0}l_0 + 2M_A(l_0+l_1) + M_B(l_1) = \frac{-6A_0\bar{x}_0}{l_0} - \frac{6A_1\bar{x}_1}{l_1}$$

$$0 + 2M_A(0+5) + M_B(5) = 0 - \frac{6\times240\times2.66}{5}$$

$$10M_A + 5M_B = -766.08 \Rightarrow 2M_A + M_B = -153.216 \rightarrow ①$$

For span AB & BC :-

$$M_A l_1 + 2M_B(l_1+l_2) + M_C l_2 = -\frac{6A_1\bar{x}_1}{l_1} - \frac{6A_2\bar{x}_2}{l_2}$$

$$M_A(5) + 2M_B(5+7) + M_C(7) = -\frac{6\times 240\times 2.33}{5} - \frac{6\times 300\times 3.33}{7}$$

$$5M_A + 24M_B + 7M_C = -1527.32 \rightarrow (2)$$

For span BC & CD :-

$$M_B l_1 + 2M_C(l_1+l_2) + M_D(l_2) = \frac{-6A_1\bar{x}_1}{l_1} - \frac{6A_2\bar{x}_2}{l_2}$$

$$M_B(7) + 2M_C(7+8) + 0 = -\frac{6\times 300\times 3.66}{7} - \frac{6\times 426.6\times 4}{8}$$

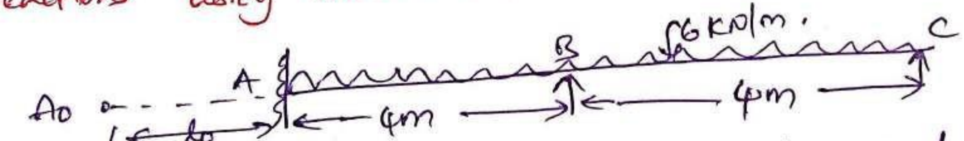
$$7M_B + 30M_C = -2220.94 \rightarrow (3)$$

From eq (1) & eq (2) & eq (3)

$M_A = 61.002$ kN-m.
$M_B = 31.567$ kN-m.
$M_C = 66.701$ kN-m.
$M_D = 0$.

Prob :- A Continous beam ABC of uniform section with AB & BC as 4m each. is fixed at 'A' & simply support at 'B' & 'C'. The beam is carrying a uniformly distributed load of 6kN/m run throughout it's length - find support moments & reactions using three moment theorem. Also draw SFD & BMD.

Sol :-



Considering imaginary point 'Ao' having length lo from support 'A'.

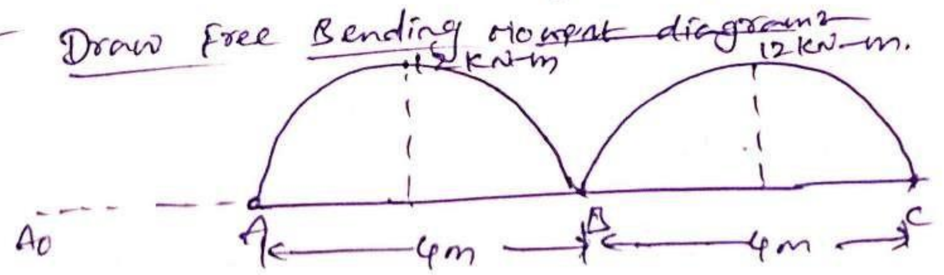
Step (1) :- Free Bending moments

For span AoA = 0.

For span AB $= \frac{wl^2}{8} = \frac{6\times 4^2}{8} = 12$ kN-m.

For span BC $= \frac{wl^2}{8} = \frac{6\times 4^2}{8} = 12$ kN-m

Step (2) :- Draw free Bending moment diagram :-



For span A_0A: $A = 0$, $\bar{x} = 0$.

For span AB: $A = \frac{2}{3} \times 12 \times 4 = 32$, $x_l = x_R = \frac{4}{2} = 2m$.

For span BC: $A = \frac{2}{3} \times 12 \times 4 = 32$, $x_l = x_R = \frac{4}{2} = 2m$.

Step ③: Apply three moment theorem

For span A_0A & AB:

$$M_{A_0} + 2M_A(l_0 + l_1) + M_B(l_1) = \frac{-6A_0\bar{x}_0}{l_0} - \frac{6A_1\bar{x}_1}{l_1}$$

$$0 + 2M_A(0+4) + M_B(4) = 0 - \frac{6 \times 32 \times 2}{4}$$

$$8M_A + 4M_B = -96$$

$$2M_A + M_B = -24 \longrightarrow ①$$

For span AB & BC: support 'C' is simple support, $M_C = 0$.

$$M_A l_1 + 2M_B(l_1 + l_2) + M_C l_2 = \frac{-6A_1\bar{x}_1}{l_1} - \frac{6A_2\bar{x}_2}{l_2}$$

$$4M_A + 2M_B(4+4) + 0 = -96 - 96$$

$$4M_A + 16M_B = -192$$

$$M_A + 4M_B = -48 \longrightarrow ②$$

From eq ① & eq ②

$$2(-48 - 4M_B) + M_B = -24$$

$$-96 - 8M_B + M_B = -24$$

$$-7M_B = 72 \Rightarrow M_B = -10.28 \text{ kN-m}$$

~~$M_A = -89.12$ kN-m~~. $M_A = -6.88$ kN-m.

Final moments $M_A = -6.88$ kN-m, $M_B = -10.28$ kN-m, $M_C = 0$.

Reactions:

For span AB:

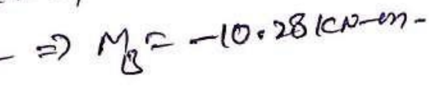
$\Sigma V = 0 \Rightarrow R_A + (R_B)_1 = 6 \times 4 = 24$

$\Sigma M_A = 0 \Rightarrow -(R_B)_1 \times 4 + 6 \times 4 \times \frac{4}{2} - M_A = 0.$

$(R_B)_1 = 10.28$ kN.

$R_A = 13.72$ kN.

For span BC:



$\Sigma V = 0 \Rightarrow (R_B)_2 + R_C = 6 \times 4 = 24$

$\Sigma M_B = 0 \Rightarrow -R_C \times 4 + 6 \times 4 \times \frac{4}{2} - M_B = 0$

$R_C = 9.43$ kN.

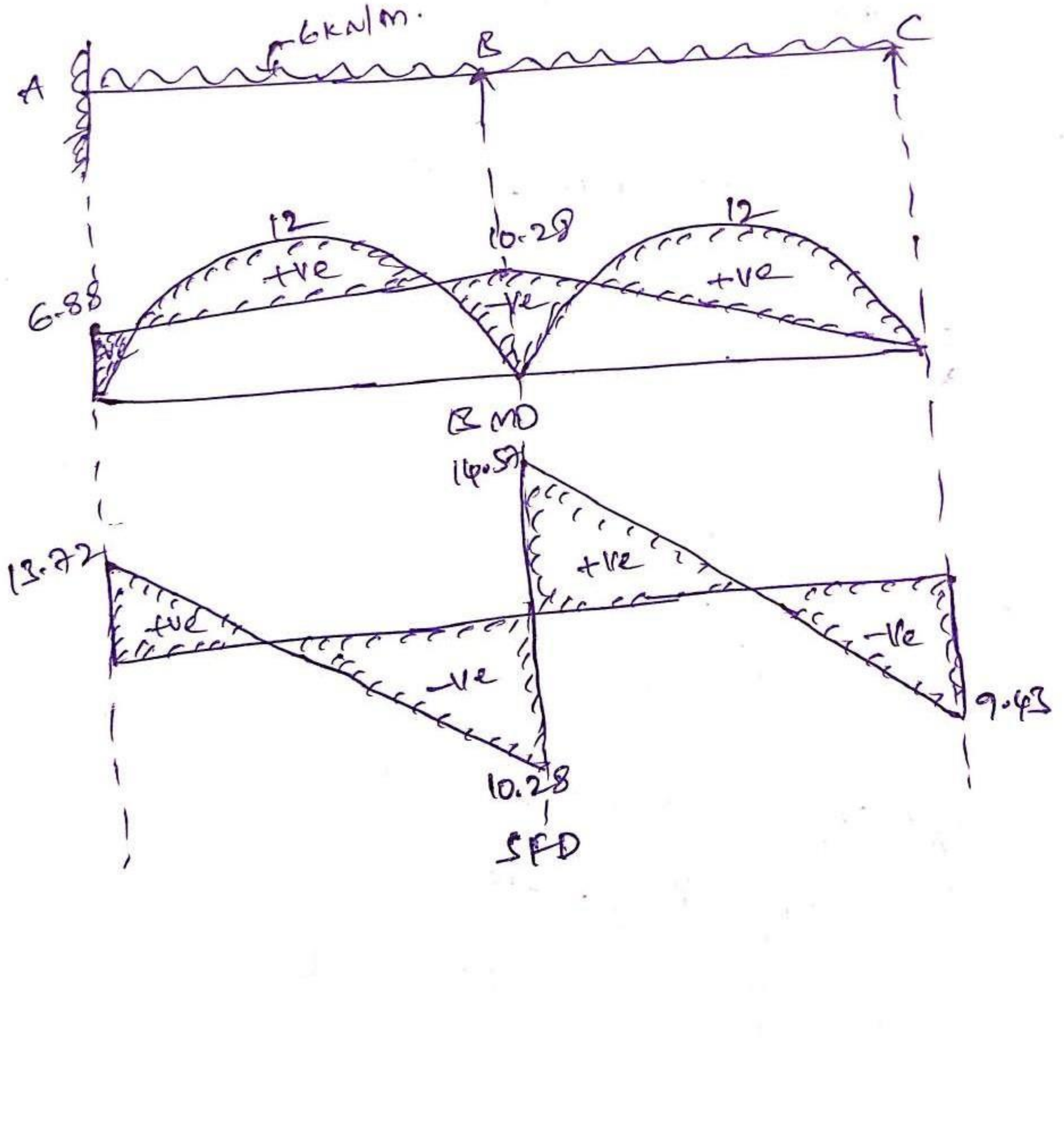
$(R_B)_2 = 14.57$ kN.

Final reactions.

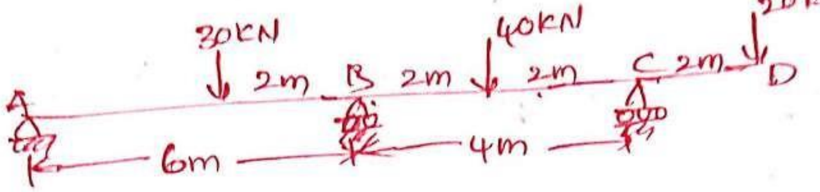
$R_A = 13.72\,kN.$

$R_B = (R_B)_1 + (R_B)_2 = 10.28 + 14.57 = 24.85\,kN.$

$R_C = 9.43\,kN.$



Prob 6: Analyse the continuous beam ABCD as shown in fig. If support 'C' settles down by 5mm. Take E = 15 kN/mm² & MOI is constant throughout $I = 5 \times 10^9\ mm^4$.



Sol:

Step ①: Free Bending moments:

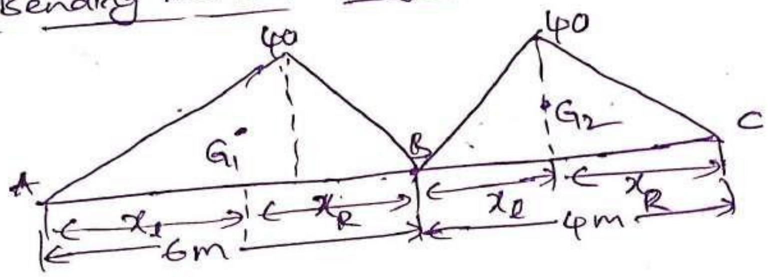
For span AB = $\frac{Wab}{l} = \frac{30 \times 4 \times 2}{6} = 40$ kN-m.

For span BC = $\frac{Wl}{4} = \frac{40 \times 4}{4} = 40$ kN-m.

For span CD = $Wl = 20 \times 2 = 40$ kN-m $= M_C$

$M_A = 0$, since it is simply supported.

Step ②: Free Bending moment diagrams



For span AB: $A_1 = \frac{1}{2} \times 6 \times 40 = 120$

$x_l = \frac{a+l}{3} = \frac{4+6}{3} = \frac{10}{3}$ & $x_R = \frac{b+l}{3} = \frac{2+6}{3} = 8/3$

For span BC: $A_2 = \frac{1}{2} \times 4 \times 40 = 80$

$x_l = x_R = \frac{l}{2} = \frac{4}{2} = 2m.$

$M_C = -40$ kN-m, $\delta_C = -5mm = -0.005m.$

$E = 15\ kN/mm^2 = 15 \times 10^6\ kN/m^2$

$I = 5 \times 10^9\ mm^4 = 5 \times 10^{-3}\ m^4$

Step ③: Apply three moment theorem:

For span AB & BC $M_A l_1 + 2M_B(l_1+l_2) + M_C l_2 = -\frac{6A_1\bar{x}_1}{l_1} - \frac{6A_2\bar{x}_2}{l_2} + \frac{6EI}{l_2}\left(\frac{\delta_1}{l_1} + \frac{\delta_2}{l_2}\right)$

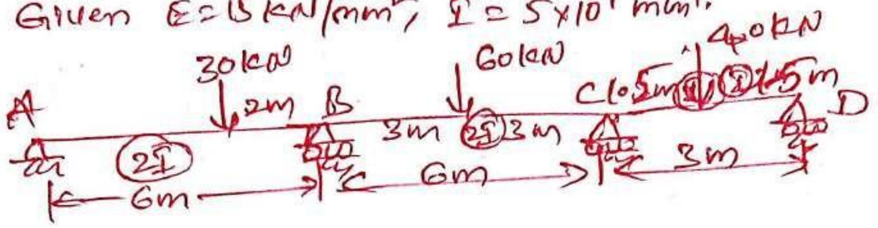
$0 + 2M_B(6+4) + (-40 \times 4) = -\frac{6 \times 120 \times 10/3}{6} - \frac{6 \times 80 \times 2}{4} + \frac{6 \times 15 \times 10^6 \times 5 \times 10^{-3}(-0.005)}{4}$

$2M_B \times 10 = -1042.5$

$M_B = -52.125$ kN-m.

∴ Final moments $M_A = 0$, $M_B = -52.125$ kN-m & $M_C = -40$ kN-m.

Prob: Analyse the Continuous beam ABCD shown in fig. If support 'C' sinks by 5mm, Given E = 15 kN/mm², I = 5×10⁹ mm⁴.



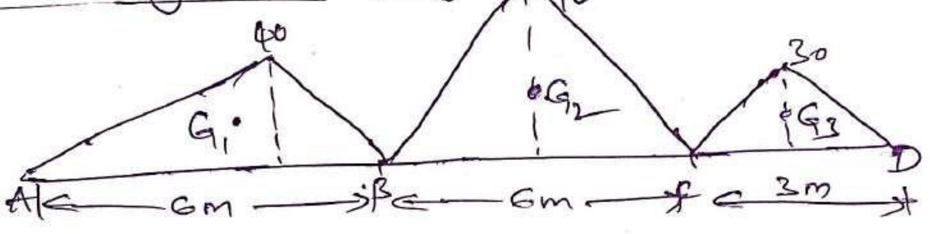
Sol:- Step ①: Free Bending Moment

For span AB, $BM = \frac{Wab}{l} = \frac{30\times4\times2}{6} = 40$ kN-m.

For span BC, $BM = \frac{Wl}{4} = \frac{60\times6}{4} = 90$ kN-m.

For span CD, $BM = \frac{Wl}{4} = \frac{40\times3}{4} = 30$ kN-m.

Step ②: Free Bending moment diagrams



For span AB: $A = \frac{1}{2}\times6\times40 = 120$,

$\bar{x}_l = \frac{a+l}{3} = \frac{4+6}{3} = 10/3$, $\bar{x}_R = \frac{b+l}{3} = \frac{2+6}{3} = 8/3$

For span BC: $A = \frac{1}{2}\times6\times90 = 270$,

$\bar{x}_l = \bar{x}_R = l/2 = \frac{6}{2} = 3$m.

For span CD: $A = \frac{1}{2}\times3\times30 = 45$,

$\bar{x}_l = \bar{x}_R = \frac{3}{2} = 1.5$m.

$\delta_C = 5mm = 0.005$m

~~E = 12 kN/m~~ $E = 12$ kN/mm² $= 12\times10^6$ kN/m²

$I = 5\times10^9$ mm⁴ $= 5\times10^{-3}$ m⁴.

Step ③: Apply theorem of three moments:

For span A-B-C: We know support A & D is simple supports

$M_A = M_D = 0$. $\delta_1 = 0$, $\delta_2 = -0.005$

$$M_A\frac{l_1}{I} + 2M_B\left(\frac{l_1}{I_1}+\frac{l_2}{I_2}\right) + M_C\left(\frac{l_2}{I_2}\right) = -\frac{6A_1\bar{x}_1}{l_1I_1} - \frac{6A_2\bar{x}_2}{l_2I_2} + \frac{6E\delta_1}{l_1I_1} + \frac{6E\delta_2}{l_2I_2}$$

$$0 + 2M_B\left(\frac{6}{2I}+\frac{6}{2I}\right) + M_C\left(\frac{6}{2I}\right) = \frac{-6\times120\times10/3}{6(2I)} - \frac{6\times270\times3}{6(2I)} + 0 + \frac{6\times12\times10^6\times(-0.005)}{6}$$

$$\frac{12M_B}{I} + \frac{3M_C}{I} = -\frac{160}{I} - \frac{145}{I} - \left(\frac{6\times15\times10^6\times(0.005)}{6}\right)$$

$$12M_B + 3M_C = -160 - 145 - \left(\frac{6\times15\times10^6\times0.005\times5\times10^{-3}}{6}\right)$$

$$12M_B + 3M_C = -160 - 405 - 375$$

$$12M_B + 3M_C = -940 \longrightarrow (1)$$

For span B-C-D :-

$\delta_1 = 0.005m$, $\delta_2 = 0.005m$.

$$M_B\left(\frac{l_1}{I_1}\right) + 2M_C\left(\frac{l_1}{I_1} + \frac{l_2}{I_2}\right) + M_D\left(\frac{l_2}{I_2}\right) = -\frac{6A_1\bar{x}_1}{l_1 I_1} - \frac{6A_2\bar{x}_2}{l_2 I_2} + \frac{6EI_1\delta_1}{l_1 I_1} + \frac{6EI\delta}{l_2 I_2}$$

$$M_B\left(\frac{6}{2I}\right) + 2M_C\left(\frac{6}{2I} + \frac{3}{I}\right) + 0 = \frac{-6\times270\times3}{6\times(2I)} - \frac{6\times45\times1.5}{3\times I} + \frac{6\times12\times10^6(0.005)}{6} + \left(\frac{6\times15\times10^6\times0.005}{3}\right)$$

$$\frac{3M_B}{I} + \frac{12M_C}{I} = \frac{-405}{I} - \frac{135}{I} + \left(\frac{6\times12\times10^6(0.005)}{6}\right) + \left(\frac{6\times15\times10^6\times0.005}{3}\right)$$

$$3M_B + 12M_C = -405 - 135 + \left(\frac{6\times12\times10^6\times0.005\times5\times10^{-3}}{6}\right) + \left(\frac{6\times15\times10^6\times0.005\times5\times10^{-3}}{3}\right)$$

$$3M_B + 12M_C = -405 - 135 + 375 + 750$$

$$3M_B + 12M_C = 585 \longrightarrow (2)$$

From eq (1) & eq (2)

$M_B = -96.556$ KN-m,

$M_C = -72.889$ KN-m.

Final moments

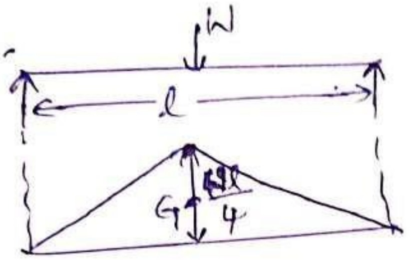
$M_A = 0$,

$M_B = -96.556$ KN-m.

$M_C = -72.889$ KN-m.

$M_D = 0$.

(1)

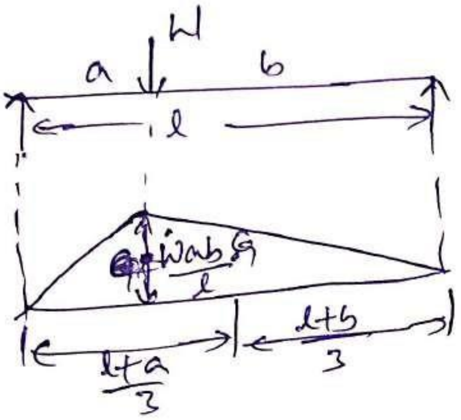


Max BM = $\frac{Wl}{4}$,

Area $A = \frac{1}{2} \times l \times \frac{Wl}{4}$,

CG from left & right = $\frac{l}{2}$

(2)



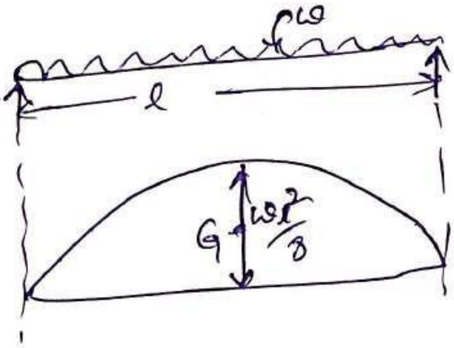
Max BM = $\frac{Wab}{l}$,

Area $A = \frac{1}{2} \times l \times \frac{Wab}{l}$

CG from left support = $\frac{l+a}{3}$

CG from right support = $\frac{l+b}{3}$

(3)



Max BM = $wl^2/8$

Area (A) = $\frac{2}{3} \times l \times \frac{wl^2}{8}$

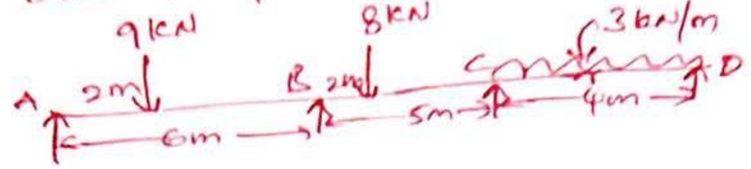
CG from left & right = $\frac{l}{2}$

~~clapeyro.~~

Clapeyron's theorem:

$$M_A l_1 + 2M_B(l_1+l_2) + M_C l_2 = -\left(\frac{6A_1\bar{x}_1}{l_1} + \frac{6A_2\bar{x}_2}{l_2}\right)$$

prob: Analyse the Continuous beam ABCD shown in fig. using theorem of Three moments. Draw SFD & BMD



Sol: Step① Free Bending Moment :-

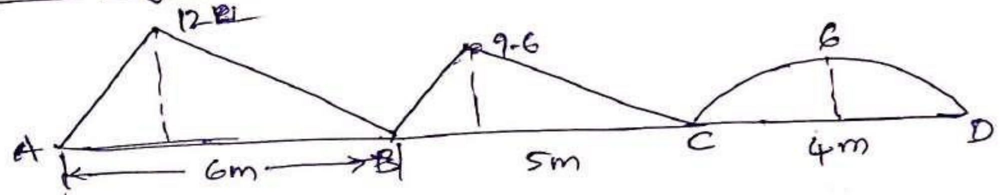
Considering each span Simply supported.

for span AB, $BM = \frac{Wab}{l} = \frac{9\times2\times4}{6} = 12$ KN-m.

for Span BC, $BM = \frac{Wab}{l} = \frac{8\times2\times3}{5} = 9.6$ KN-m.

for span CD, $BM = \frac{wl^2}{8} = \frac{3\times4^2}{8} = 6$ KN-m.

Step②: Free BM diagram:-



For Span AB:- Area $A = \frac{1}{2}\times b\times h = \frac{1}{2}\times12\times6 = 36$.

$\bar{x}_L = \frac{a+l}{3} = \frac{2+6}{3} = 2.67$ m, $\bar{x}_R = \frac{b+l}{3} = \frac{4+6}{3} = 3.33$ m

For Span BC:- Area $(A) = \frac{1}{2}\times b\times h = \frac{1}{2}\times9.6\times5 = 24$

$\bar{x}_L = \frac{a+l}{3} = \frac{2+5}{3} = 2.33$ m $\bar{x}_R = \frac{b+l}{3} = \frac{3+5}{3} = 2.66$ m.

For span CD:- Area $(A) = \frac{2}{3}\times$ base$\times$height $= \frac{2}{3}\times4\times6 = 16$

$\bar{x}_L = \bar{x}_R = \frac{l}{2} = \frac{4}{2} = 2$ m.

Step③: Apply three moment Theorem:-

For Span A-B-C:-

$$M_Al_1 + 2M_B(l_1+l_2) + M_Cl_2 = -\left(\frac{6A_1\bar{x}_1}{l_1} + \frac{6A_2\bar{x}_2}{l_2}\right)$$

$$0 + 2M_B(6+5) + M_C(5) = -\left(\frac{6\times36\times2.67}{6} + \frac{6\times24\times2.66}{5}\right)$$

$22M_B + 5M_C = -172.728 \rightarrow ①$

$2M_B(11) = -(172.728) \Rightarrow 22M_B = -172.728$

For span B-C-D:

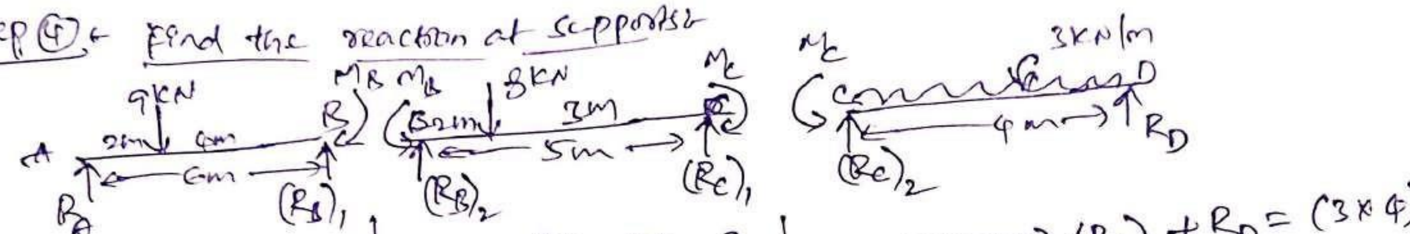
$$M_B l_1 + 2M_C(l_1+l_2) + M_D(l_2) = -\left(\frac{6A_1\bar{x}_1}{l_1} + \frac{6A_2\bar{x}_2}{l_2}\right)$$

$$M_B(5) + 2M_C(5+4) + 0 = -\left(\frac{6\times24\times2.33}{5} + \frac{6\times16\times2}{4}\right)$$

$$5M_B + 18M_C = -115.10 \longrightarrow (2)$$

From eq (1) & eq (2). $M_B = -6.83$ kNm & $M_C = -4.497$ kNm.

Step (4): Find the reaction at supports:



$\Sigma V = 0 \Rightarrow R_A + (R_B)_1 = 9$

$\Sigma M_A = 0 \Rightarrow 9\times2 - (R_B)_1\times6 + 6.83 = 0.$

$(R_B)_1 = 4.138$ KN.

$R_A = 4.862$ KN.

$\Sigma V = 0 \Rightarrow (R_B)_2 + (R_C)_1 = 8$

$\Sigma M_B = 0 \Rightarrow$

$-M_B + 8\times2 - (R_C)_1\times5 + M_C = 0.$

$-6.83 + 16 - (R_C)_1\times5 + 4.497 = 0$

$(R_C)_1 = 2.733$ KN

$(R_B)_2 = 5.267$ KN.

$\Sigma V = 0 \Rightarrow (R_C)_2 + R_D = (3\times4)$

$\Sigma M_C = 0 \Rightarrow$

$-M_C + 3\times4\times2 - R_D\times4 = 0.$

$-4.497 + 24 - R_D\times4 = 0.$

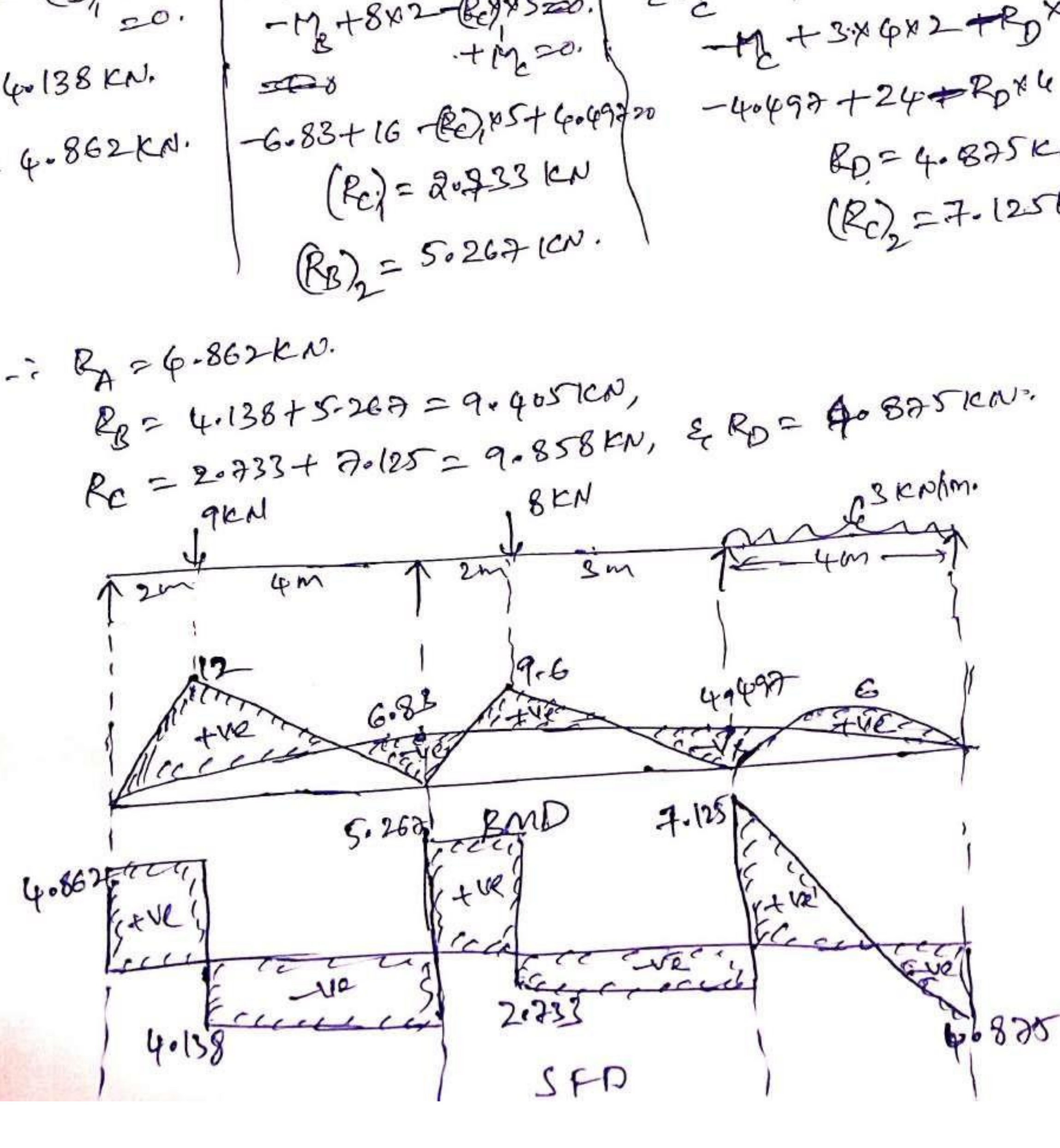
$R_D = 4.875$ KN

$(R_C)_2 = 7.125$ KN.

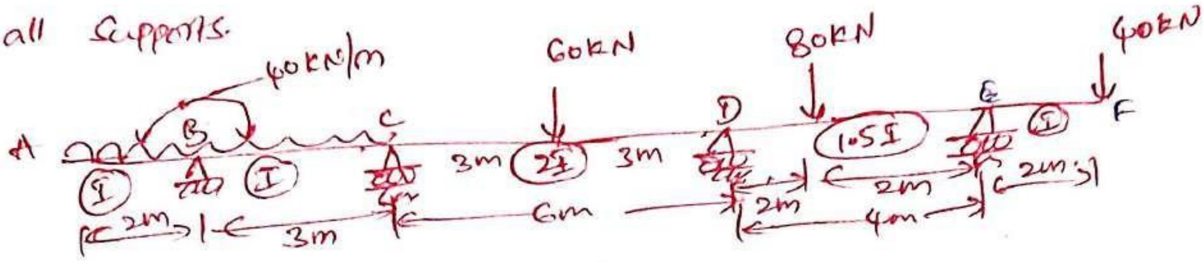
$\therefore R_A = 4.862$ KN.

$R_B = 4.138 + 5.267 = 9.405$ KN, & $R_D = 4.875$ KN.

$R_C = 2.733 + 7.125 = 9.858$ KN,



Prob :- Analyse the Continuous beam as shown in fig. & determine the moment at all supports.



Sol:- Step ① :- Free Moment :-

for span AB = $w \frac{l^2}{2} = 40 \times 2 \times \frac{2}{2} = 80$ kN-m. $= M_B$

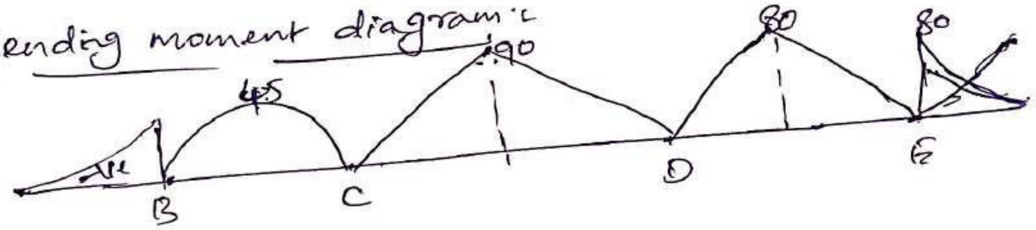
for span BC = $\frac{wl^2}{8} = \frac{40 \times 3^2}{8} = 45$ kN-m.

for span CD = $\frac{Wl}{4} = \frac{60 \times 6}{4} = 90$ kN-m.

for span DE = $\frac{Wl}{4} = \frac{80 \times 4}{4} = 80$ kN-m.

for span EF = $40 \times 2 = 80$ kN-m. $= M_E$

Step ②:- Free Bending moment diagram :-



For span BC :- $(A) = \frac{2}{3} \times 3 \times 45 = 90$, $\bar{x}_l = \bar{x}_R = \frac{l}{2} = \frac{3}{2} = 1.5$m.

For span CD :- $A = \frac{1}{2} \times 6 \times 90 = 270$, $\bar{x}_l = \bar{x}_R = \frac{l}{2} = \frac{6}{2} = 3$m.

For span DE :- $A = \frac{1}{2} \times 4 \times 80 = 160$, $\bar{x}_l = \bar{x}_R = \frac{l}{2} = \frac{4}{2} = 2$m.

Step ③ :- Apply three moment theorem :-

For span B-C-D :-

$$M_B\left(\frac{l_1}{I}\right) + 2M_C\left(\frac{l_1}{I} + \frac{l_2}{2I}\right) + M_D\left(\frac{l_2}{2I}\right) = -\left(\frac{6A_1\bar{x}_1}{l_1 I}\right) - \left(\frac{6A_2\bar{x}_2}{l_2(2I)}\right)$$

$$(-80)\times\frac{3}{I} + 2M_C\left(\frac{3}{I} + \frac{6}{2I}\right) + M_D\left(\frac{6}{2I}\right) = -\left(\frac{6 \times 90 \times 1.5}{3I}\right) - \left(\frac{6 \times 270 \times 3}{(2I) \times 6}\right)$$

$$\frac{-240}{I} + \frac{12M_C}{I} + \frac{3M_D}{I} = \frac{-270}{I} - \frac{405}{I}$$

$$4M_C + M_D = -145 \longrightarrow ①$$

For span C - D - B :

$$M_C\left(\frac{l_1}{2I}\right)+2M_D\left(\frac{l_1}{2I}+\frac{l_2}{1.5I}\right)+M_B\left(\frac{l_2}{1.5I}\right)=-\left(\frac{6A_1\bar{x}_1}{l_1(2I)}\right)-\left(\frac{6A_2\bar{x}_2}{l_2(1.5I)}\right)$$

$$M_C\left(\frac{6}{2I}\right)+2M_D\left(\frac{6}{2I}+\frac{4}{1.5I}\right)+(-80)\left(\frac{4}{1.5I}\right)=-\left(\frac{6\times 270\times 3}{2I\times 6}\right)-\left(\frac{6\times 160\times 2}{1.5I\times 4}\right)$$

$$\frac{3M_C}{I}+\frac{11.33M_D}{I}-\frac{213.33}{I}=-\frac{405}{I}-\frac{320}{I}$$

$$3M_C+11.33M_D-213.33=-405-320$$

$$3M_C+11.33M_D=-511.66 \longrightarrow (2)$$

From eq (1) & eq (2)

$M_C = -26.732$ kN-m.

$M_D = -38.071$ kN-m.

Final moment $M_B = -80$ kN-m.

$M_C = -26.732$ kN-m.

$M_D = -38.071$ kN-m

$M_B = -80$ kN-m.

UNIT-IV

SLOPE DEFLECTION METHOD

Continuous beams and rigid frames (with and without sway) – Symmetry and antisymmetry – Simplification for hinged end – Support displacements

Introduction:

- This method was first proposed by Prof. George A. Maney in 1915.
- It is ideally suited to the analysis of continuous beams and rigid jointed frames.
- Basic unknowns like slopes and deflections of joints are found out.
- Moments at the ends of a member is first written down in terms of unknown slopes and deflections of end joints.
- Considering the joint equilibrium conditions, a set of equations are formed and solutions of these simultaneous equations gives unknown slopes and deflections.
- Then end moments of individual members are determined.
- It involves solutions of simultaneous equations; a problem with more than three unknowns is considered a difficult problem for hand calculations. Hence this method was sidelined by moment distribution method with the help of computers; solutions for any number of simultaneous equations can be obtained early.
- The development of this method in the matrix form is "Stiffness Matrix Method" (it is commonly used for the analysis of large structures with the help of computers.

Assumptions made in slope-deflection method

- All joints are rigid.
- The rotations of joints are treated as unknowns.
- Between each pair of the supports the beam section is constant.
- The joint in structure may rotate or deflect as a whole, but the angles between the members meeting at that joint remain the same.
- Distortions due to axial deformations are neglected.
- Shear deformations are neglected.

Sign Conventions:

Moments:

- Clockwise moments = $(+)^{ive}$
- Anti-clockwise moments = $(-)^{ive}$