08/08/18

Module - 1

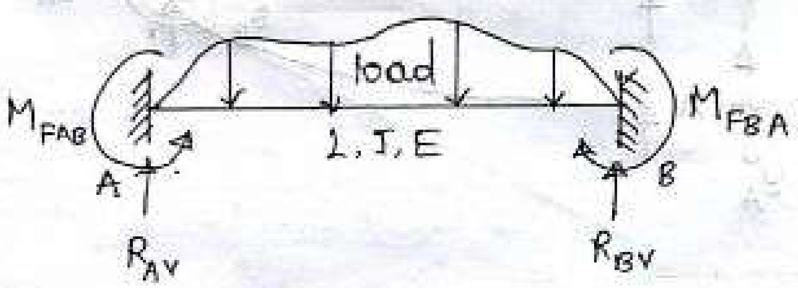
Slope - Deflection Method

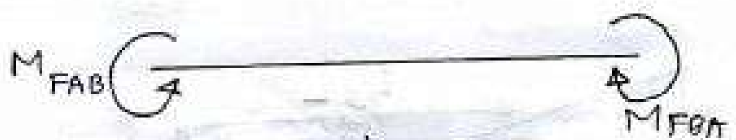
Slope - deflection method is one of the methods to analyse Indeterminate structures.

It is first introduced by an investigator G.A. Maney, hence known as Maney's method.

Slope deflection Equations:

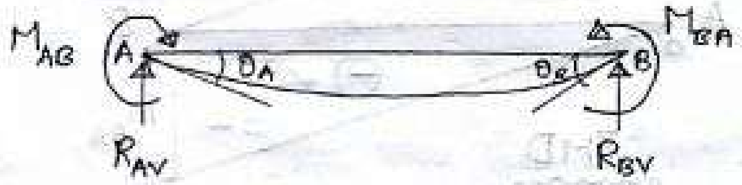
Consider a fixed beam of length 'L', moment of Inertia 'I' and Young's modulus 'E' for general loading condition. as shown in figure.





Moment due to Fixity (M_F)

+



Moment due to rotation (θ)

+

Moment due to settlement (Δ)

$\therefore$ $\left.\begin{matrix}\text{Final Moment} \\ (M)\end{matrix}\right\} = \left\{\begin{matrix}\text{Moment due} \\ \text{to fixity}\end{matrix}\right\} + \left\{\begin{matrix}\text{Moment due} \\ \text{to rotation}\end{matrix}\right\} + \left\{\begin{matrix}\text{moment} \\ \text{to settlement}\end{matrix}\right\}$

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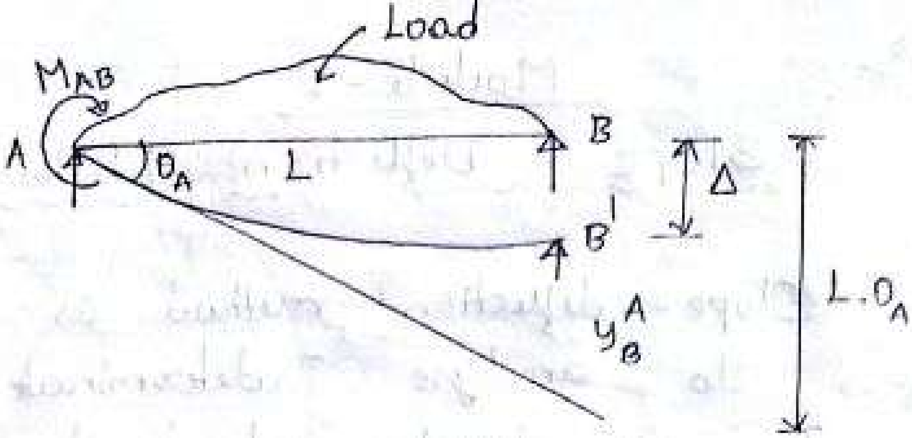


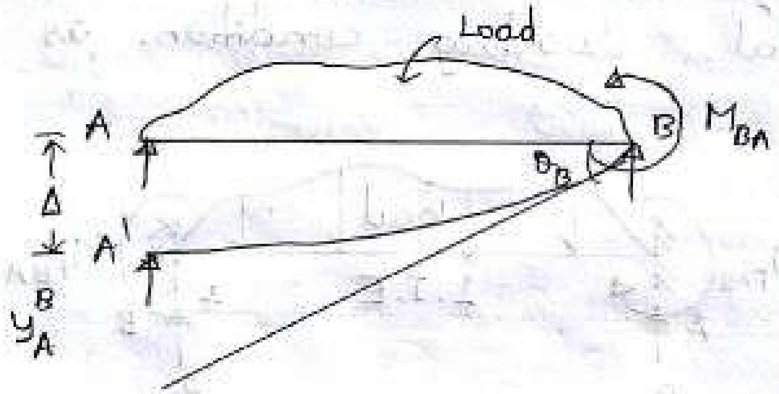
Deviation at 'B' when the moment is at 'A' is i.e.,

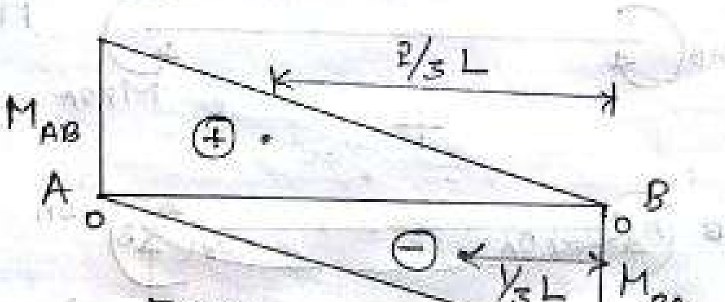
$$Y_B^A = (L.\theta_A - \Delta)$$

Similarly deviation at 'A' when the moment is at 'B' is

$$y_A^B = (L\theta_B - \Delta)$$

i.e.,





BMD

$$Y_B^A = \frac{A\bar{X}}{EI} = \frac{\left(\frac{1}{2} \times m_{AB} \times L\right)\frac{2}{3}L - \left(\frac{1}{2} \times m_{BA} \times L\right)\frac{1}{3}L}{EI}$$

$$Y_B^A = \frac{L^2}{6EI}\left[2m_{AB} - m_{BA}\right]$$

$$\frac{L^2}{6EI}\left[2m_{AB} - m_{BA}\right] = L\theta_A - \Delta$$

$$2m_{AB} - m_{BA} = \frac{6EI}{L^2}(L\theta_A - \Delta)$$

$$2m_{AB} - m_{BA} = \frac{6EI}{L}\left(\theta_A - \frac{\Delta}{L}\right) \longrightarrow (1)$$

Similarly,

$$2m_{BA} - m_{AB} = \frac{6EI}{L}\left(\theta_B - \frac{\Delta}{L}\right) \longrightarrow (2)$$

Solving the equations (1) and (2), we get

$$2m_{AB} - m_{BA} = \frac{6EI}{L}(\theta_A - \Delta/L) \quad \times 2$$

$$2m_{BA} - m_{AB} = \frac{6EI}{L}(\theta_B - \Delta/L)$$

$$3m_{AB} = \frac{12EI}{L}(\theta_A - \Delta/L) + \frac{6EI}{L}(\theta_B - \Delta/L)$$

$$3m_{AB} = \frac{6EI}{L}\left\{\left(2\theta_A - \frac{2\Delta}{L}\right) + \theta_B - \Delta/L\right\}$$

$$m_{AB} = \frac{2EI}{L}\left(2\theta_A + \theta_B - \frac{3\Delta}{L}\right)$$

$$m_{BA} = \frac{2EI}{L}\left(2\theta_B + \theta_A - \frac{3\Delta}{L}\right)$$

$\therefore$ Final moment (M),

$$\boxed{\begin{aligned} M_{AB} &= M_{FAB} + \frac{2EI}{L}\left(2\theta_A + \theta_B - \frac{3\Delta}{L}\right) \rightarrow (3) \\ M_{BA} &= M_{FBA} + \frac{2EI}{L}\left(2\theta_B + \theta_A - \frac{3\Delta}{L}\right) \rightarrow (4) \end{aligned}}$$

Eq^{ns} (3) and (4) are the slope - deflection equations.

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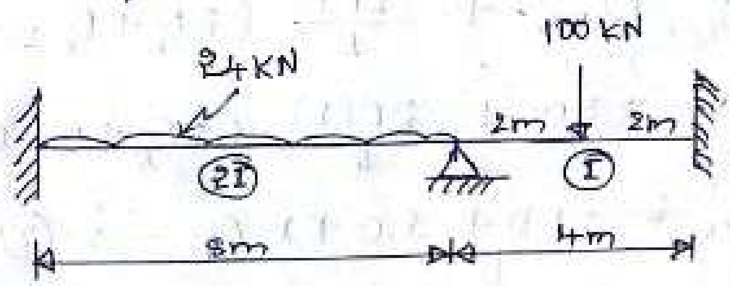
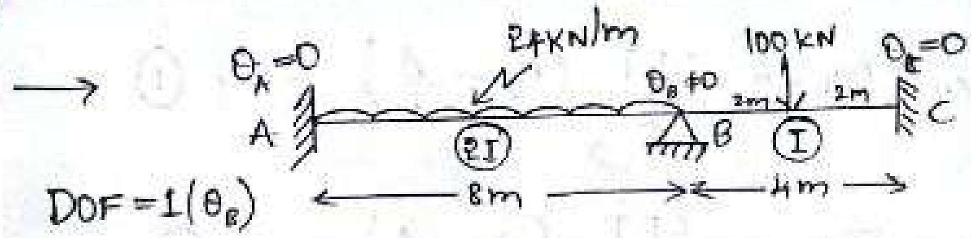
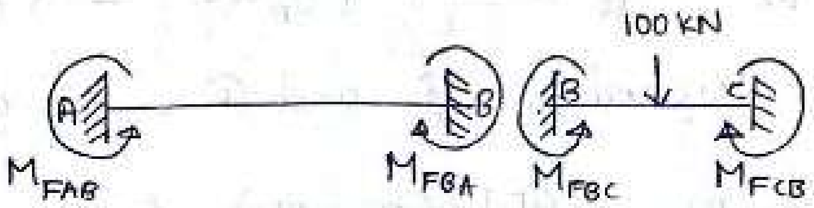
Problems:

1. A continuous beam is loaded as shown below. Draw the BMD and elastic curve by slope - deflection method.





DOF = 1 (θ_B)



Step 1: Fixed End moments:

$$M_{FAB} = \frac{-WL^2}{12} = \frac{-24\times8^2}{12} = -128\ \text{KN-m}$$

$$M_{FBA} = \frac{WL^2}{12} = \frac{24\times8^2}{12} = 128\ \text{KN-m}$$

$$M_{FBC} = \frac{-WL}{8} = \frac{-100\times4}{8} = -50\ \text{KN-m}$$

$$M_{FCB} = \frac{WL}{8} = \frac{100\times4}{8} = 50\ \text{KN-m}$$

Step 2: Slope - deflection Equations:

$$M_{AB} = M_{FAB} + \frac{2EI}{L}\left(2\theta_A^{\,0} + \theta_B - \cancel{\frac{3\Delta}{L}}^{\,0}\right)$$

$$M_{AB} = -128 + \frac{2E(2I)}{8}(\theta_B)$$

$$M_{AB} = -128 + 0.5\ EI\ \theta_B \rightarrow (1)$$

$$M_{BA} = M_{FBA} + \frac{2EI}{L}\left(2\theta_B + \theta_A - \frac{3\Delta}{L}\right)$$

$$= 128 + \frac{2E(2I)}{8}(2\theta_B + 0 - 0)$$

$$M_{BA} = 128 + 1.0\ EI\ \theta_B \rightarrow (2)$$

$$M_{BC} = M_{FBC} + \frac{2EI}{L}\left(2\theta_B + \theta_C - \frac{3\Delta}{L}\right)$$

$$= -50 + \frac{2E(I)}{4}(2\theta_B + 0 - 0)$$

$$M_{BC} = -50 + 1.0\ EI\ \theta_B \rightarrow (3)$$

$$M_{CB} = M_{FCB} + \frac{2EI}{L}\left(2\theta_C + \theta_B - \frac{3\Delta}{L}\right)$$

$$= 50 + \frac{2E(I)}{4}(0 + \theta_B - 0)$$

$$\therefore \quad M_{CB} = 50 + 0.5\ EI\ \theta_B \rightarrow (4)$$

Step 3: Joint - Equilibrium Equations :

$$\Sigma M_B = 0; \quad M_{BA} + M_{BC} = 0$$

$$128 + 1.0\ EI\ \theta_B - 50 + 1.0\ EI\theta_B = 0$$

$$2\ EI\ \theta_B + 78 = 0$$

$$2\ EI\ \theta_B = -78$$

$$\therefore \boxed{\theta_B = \frac{-39}{EI}}$$

Step 4: Final Moments :

(1) $\Rightarrow$

$$M_{AB} = -128 + 0.5\ EI\left(\frac{-39}{EI}\right)$$

$$= -128 - 19.5$$

$$\therefore \quad M_{AB} = -147.5 \text{ kN-m}$$

(2) $\Rightarrow$

$$M_{BA} = 128 + 1.0\ EI\left(\frac{-39}{EI}\right)$$

$$= 128 - 39$$

$$M_{BA} = 89 \text{ kN-m}$$

(3) $\Rightarrow$

$$M_{BC} = -50 + 1.0\ EI\left(\frac{-39}{EI}\right)$$

$$= -50 - 39$$

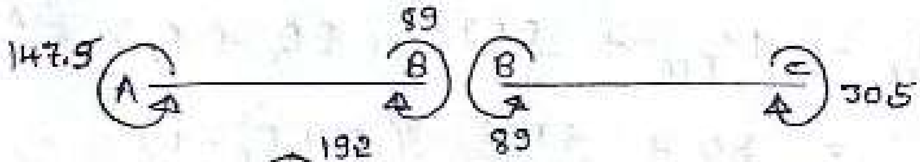
$$M_{BC} = -89 \text{ kN-m}$$

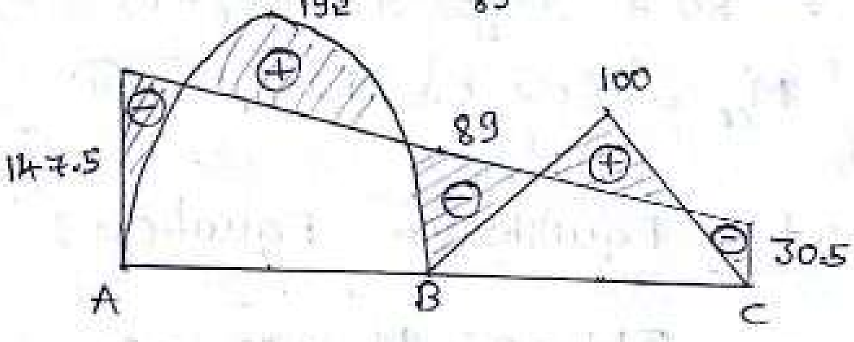
(4) $\Rightarrow$

$$M_{CB} = 50 + 0.5\ EI\left(\frac{-39}{EI}\right)$$

$$= 50 - 19.5$$

$$M_{CB} = 30.5 \text{ kN-m}$$

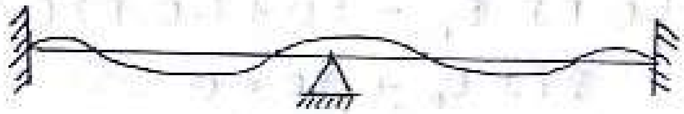




$\frac{wL^2}{8}$; $\frac{wL}{4}$

$=192$; $=100$

BMD



Elastic Curve

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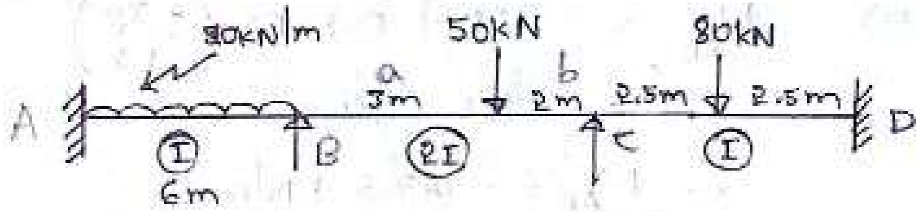
2. Analyse the continuous beam shown in the figure by slope deflection method. Draw BMD, elastic curve and SF diagram.



$\rightarrow$ DOF = 2 (θ_B, θ_C)

Step 1: Fixed end moments :-

$$M_{FAB} = \frac{-wL^2}{12} = -\frac{20\times6^2}{12} = -60 \text{ kNm}$$

$$M_{FBA} = \frac{wL^2}{12} = \frac{20\times6^2}{12} = 60 \text{ kN-m}$$

$$M_{FBC} = \frac{-wab^2}{L^2} = \frac{-50\times3\times2^2}{5^2} = -24 \text{ kN-m}$$

$$M_{FCB} = \frac{wa^2b}{L^2} = \frac{50\times3^2\times2}{5^2} = 36 \text{ kN-m}$$

$$M_{FCD} = \frac{-wL}{8} = \frac{-80\times5}{8} = -50 \text{ kN-m}$$

$$F_{FDC} = \frac{wL}{8} = 50 \text{ kN-m}$$

Step 2 : Slope - Deflection equations :

$$M_{AB} = M_{FAB} + \frac{2EI}{L}\left(2\theta_A + \theta_B - \frac{3\Delta}{L}\right)$$

$$= -60 + \frac{2E(I)}{6}(0 + \theta_B - 0)$$

$$\therefore M_{AB} = -60 + 0.3333\ EI\ \theta_B \rightarrow (1)$$

$$M_{BA} = M_{FBA} + \frac{2EI}{L}\left(2\theta_B + \theta_A - \frac{3\Delta}{L}\right)$$

$$= 60 + \frac{2E(I)}{6}(2\theta_B + 0 - 0)$$

$$\therefore M_{BA} = 60 + 0.6667\ EI\ \theta_B \rightarrow (2)$$

$$M_{BC} = M_{FBC} + \frac{2EI}{L}\left(2\theta_B + \theta_C - \frac{3\Delta}{L}\right)$$

$$= -24 + \frac{2E(2I)}{5}(2\theta_B + \theta_C - 0)$$

$$\therefore M_{BC} = -24 + 1.6\ EI\ \theta_B + 0.8\ EI\ \theta_C \rightarrow (3)$$

$$M_{CB} = M_{FCB} + \frac{2EI}{L}\left(2\theta_C + \theta_B - \frac{3\Delta}{L}\right)$$

$$= +36 + \frac{2E(2I)}{5}(2\theta_C + \theta_B - 0)$$

$$\therefore M_{CB} = 36 + 1.6\ EI\ \theta_C + 0.8\ EI\ \theta_B \rightarrow (4)$$

$$M_{CD} = M_{FCD} + \frac{2EI}{L}\left(2\theta_C + \theta_D - \frac{3\Delta}{L}\right)$$

$$= -50 + \frac{2EI}{5}(2\theta_C + 0 - 0)$$

$$\therefore M_{CD} = -50 + 0.8\ EI\ \theta_C \rightarrow (5)$$

$$M_{DC} = M_{FDC} + \frac{2EI}{L}\left(2\theta_D + \theta_C - \frac{3\Delta}{L}\right)$$

$$= 50 + \frac{2EI}{L}(0 + \theta_C - 0)$$

$$\therefore M_{DC} = 50 + 0.4\ EI\ \theta_C \rightarrow (6)$$

Step 3 : Joint Equilibrium equations :

$$\Sigma M_B = 0 ; \quad M_{BA} + M_{BC} = 0$$

$$60 + 0.6666\ EI\ \theta_B - 24 + 1.6\ EI\ \theta_B + 0.8\ EI\ \theta_C = 0$$

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$$2.2666\ EI\ \theta_B + 0.8\ EI\ \theta_c = -36 \longrightarrow (A)$$

$\Sigma M_c = 0;$ $\quad M_{CB} + M_{CD} = 0$

$$36 + 1.6\ EI\ \theta_c + 0.8\ EI\ \theta_B - 50 + 0.8\ EI\ \theta_c = 0$$

$$0.8\ EI\ \theta_B + 2.4\ EI\ \theta_c = 14 \longrightarrow (B)$$

Now,

$$EI.\ \theta_B = \frac{\begin{vmatrix} -36 & 0.8 \\ 14 & 2.4 \end{vmatrix}}{\begin{array}{cc} \theta_B & \theta_c \end{array} \begin{vmatrix} 2.2666 & 0.8 \\ 0.8 & 2.4 \end{vmatrix}} = \frac{-36 \times 2.4 - 14 \times 0.8}{2.667 \times 2.4 - 0.8 \times 0.8}$$

{Cramer's rule

$$\theta_B = \frac{-20.93}{EI}$$

(or)

Multiplication method:

$$6.798\ EI\ \theta_B + 2.4\ EI\ \theta_c = -108$$

$$0.8\ EI\ \theta_B + 2.4\ EI\ \theta_c = 14$$

$$5.97\ EI\ \theta_B = -122$$

$$\therefore\ \theta_B = \frac{-20.34}{EI}$$

(or) Using calculator (Eqⁿ):

$$\therefore\ \theta_B = -\frac{20.33}{EI},\ \theta_c = \frac{12.61}{EI}$$

Step 4: Final moments:

(1) $\Rightarrow$ $\quad M_{AB} = -60 + 0.3333\ EI\left(\frac{-20.33}{EI}\right)$

$$\therefore\ M_{AB} = -66.8\ KN\text{-}m$$

(2) $\Rightarrow$ $\quad M_{BA} = 46.4\ KN\text{-}m$

(3) $\Rightarrow$ $\quad M_{BC} = -24 + 1.6\ EI\left(\frac{-20.33}{EI}\right) + 0.8\ EI\left(\frac{12.61}{EI}\right)$

$$M_{BC} = -46.4\ kN\text{-}m$$

(4) $\Rightarrow$ $\quad M_{CB} = 39.9\ kN\text{-}m$

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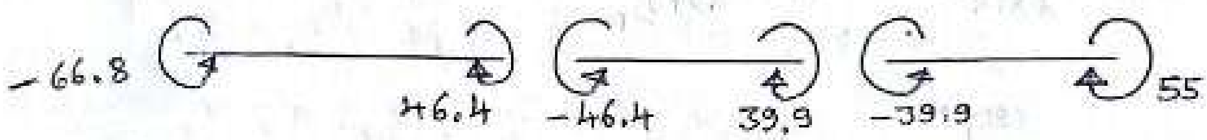
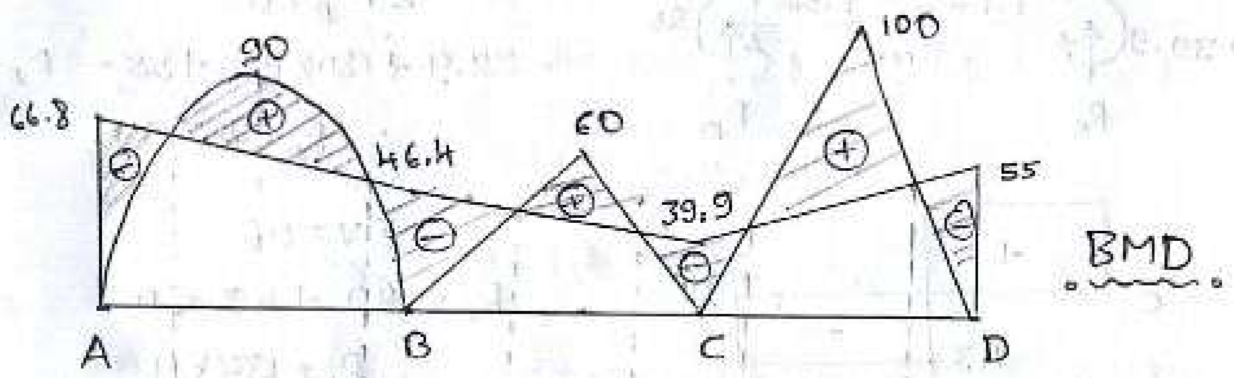
⑤ $\Rightarrow$ $$M_{CD} = -50 + 0.8EI\left(\frac{12.61}{EI}\right)$$

$$M_{CD} = -39.9 \text{ kN-m}$$

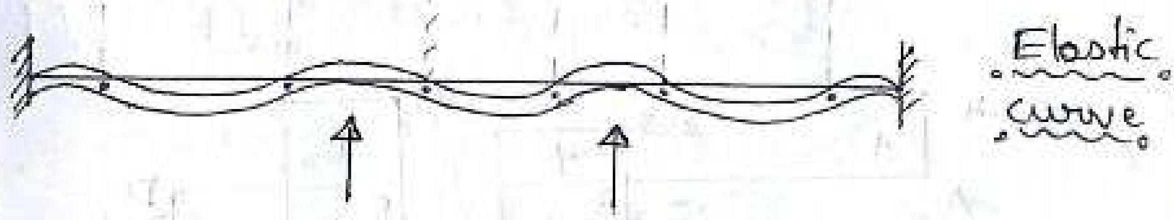
⑥ $\Rightarrow$ $$M_{DC} = 50 + 0.4EI\left(\frac{12.61}{EI}\right)$$

$$\therefore M_{DC} = 55 \text{ kN-m}$$





BMD.

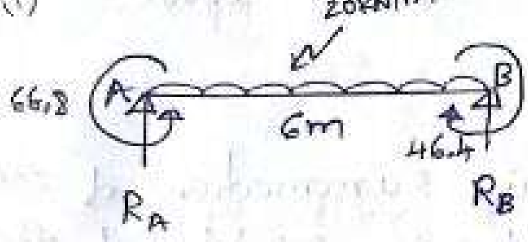


Elastic curve.

Note: The bentform shape of the beam due to external bending is known as 'elastic curve'.

To draw SF diagram:

(i)



$\Sigma M_A = 0$;

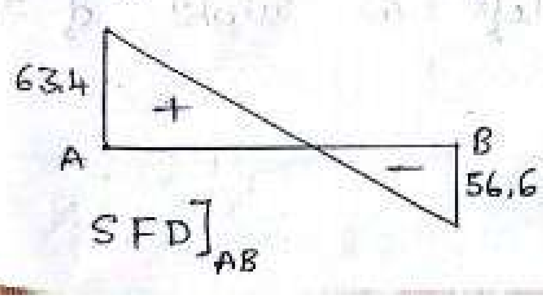
$-66.8 + (20\times6)3 - R_B\times6 + 46.4 = 0$

$\therefore R_B = 56.6$ kN

$(\overset{\uparrow}{+ve})$ $\Sigma V = 0$;

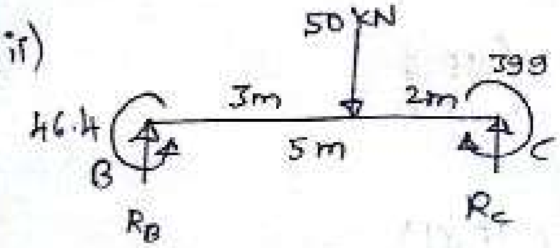
$R_A - (20\times6) + 56.6 = 0$

$\therefore R_A = 63.4$ kN



$SFD]_{AB}$

(ii)



$\Sigma M_B = 0;$

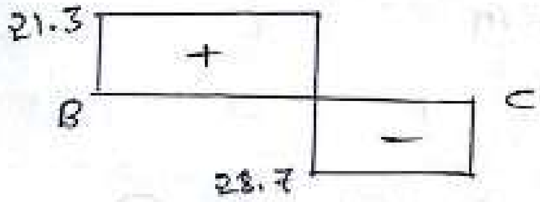
$-46.4 + 50\times3 - R_C\times5 + 39.9 = 0$

$\therefore R_C = 28.7$ KN

$\Sigma V = 0;$

$+R_B - 50 + 28.7 = 0$

$\therefore R_B = 21.3$ KN



SFD]$_{BC}$

(iii)

$\Sigma M_B = 0;$

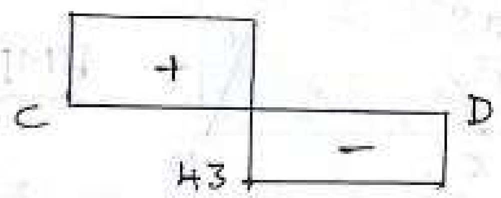
$-39.9 + 80\times2.5 + 55 - R_D\times5 = 0$

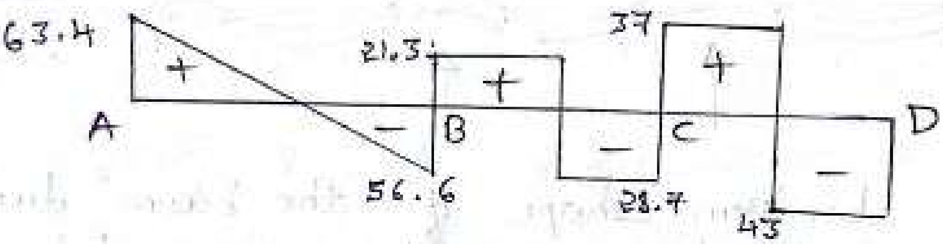
$\therefore R_D = 43$ KN

$\Sigma V = 0;$

$R_C - 80 + 43 = 0$

$\therefore R_C = 37$ KN





SFD

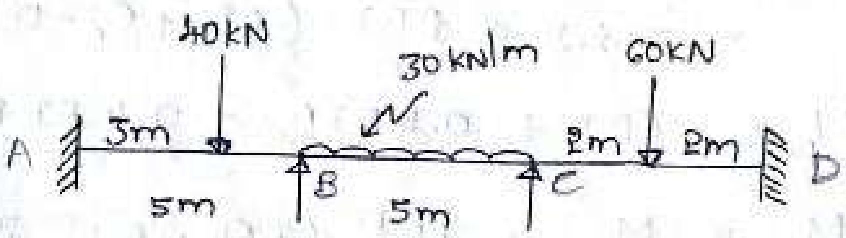
Elastic Curve:

Bekentum shape of the beam due to external load is called elastic curve.

Shear Force:

It is the algebraic summation of the vertical forces either left @ right of the section.

3. Analyse the continuous beam by slope-deflection method. Draw BMD, SFD & elastic curve.



→ Here moment of inertia is not given, hence we assume it as (I) (continuous beam).

DOF = 2 (θ_B, θ_C)

Step 1: Fixed end moments:

$$M_{FAB} = \frac{-wab^2}{L^2} = \frac{-40\times3\times2^2}{2} = -19.2 \text{ kN-m}$$

$$M_{FBA} = \frac{wa^2b}{L^2} = \frac{40\times3^2\times2}{2} = 28.8 \text{ kN-m}$$

$$M_{FBC} = \frac{-wL^2}{12} = \frac{-30\times5^2}{12} = -62.5 \text{ kN-m}$$

$$M_{FCB} = \frac{wL^2}{12} = \frac{30\times5^2}{12} = 62.5 \text{ kN-m}$$

$$M_{FCD} = \frac{-wL}{8} = \frac{-60\times4}{8} = -30 \text{ kN-m}$$

$$M_{FDC} = \frac{wL}{8} = \frac{60\times4}{8} = 30 \text{ kN-m}$$

Step 2: Slope - Deflection Equation:

$$M_{AB} = M_{FAB} + \frac{2EI}{L}\left(2\theta_A + \theta_B - \frac{3\Delta}{L}\right)$$

$$= -19.2 + \frac{2EI}{5}(0 + \theta_B - 0)$$

$$\therefore M_{AB} = -19.2 + 0.4\,EI\,\theta_B \rightarrow (1)$$

$$M_{BA} = M_{FBA} + \frac{2EI}{L}(2\theta_B + \theta_A - 3\Delta/L)$$

$$= 28.8 + \frac{2EI}{5}(2\theta_B + 0 - 0)$$

$$\therefore M_{BA} = 28.8 + 0.8\ EI\ \theta_B \rightarrow (2)$$

Now, $M_{BC} = M_{FBC} + \frac{2EI}{L}\left(2\theta_B + \theta_C - \frac{3\Delta}{L}\right)$

$$= -62.5 + \frac{2EI}{5}\left(2\theta_B + \theta_C - 0\right)$$

$$\therefore M_{BC} = -62.5 + 0.8\ EI\theta_B + 0.4\ EI\ \theta_C \rightarrow (3)$$

Now, $M_{CB} = M_{FCB} + \frac{2EI}{L}\left(2\theta_C + \theta_B - \frac{3\Delta}{L}\right)$

$$= 62.5 + 0.4\ EI\ (2\theta_C + \theta_B - 0)$$

$$\therefore M_{CB} = 62.5 + 0.8\ EI\ \theta_C + 0.4\ EI\ \theta_B \rightarrow (4)$$

Now, $M_{CD} = M_{FCD} + \frac{2EI}{L}\left(2\theta_C + \theta_D - \frac{3\Delta}{L}\right)$

$$= -30 + \frac{2EI}{4}\left(2\theta_C + 0 - 0\right)$$

$$\therefore M_{CD} = -30 + EI.\ \theta_C \rightarrow (5)$$

Now, $M_{DC} = M_{FDC} + \frac{2EI}{L}\left(2\theta_D + \theta_C - 3\Delta/L\right)$

$$\therefore M_{DC} = 30 + 0.5.\ EI.\ \theta_C \rightarrow (6)$$

Step 4: Joint Equilibrium Equations:

$$\Sigma M_B = 0; \quad M_{BA} + M_{BC} = 0$$

$$28.8 + 0.8\ EI\ \theta_B - 62.5 + 0.8\ EI\ \theta_B + 0.4\ EI\ \theta_C = 0$$

$$1.6\ EI\ \theta_B + 0.4\ EI\ \theta_C = 33.7 \rightarrow (A)$$

Now, $\Sigma M_C = 0$; $M_{CB} + M_{CD} = 0$

$$62.5 + 0.8\ EI\ \theta_C + 0.4\ EI\theta_B - 30 + EI.\ \theta_C = 0$$

$$0.4\ EI\ \theta_B + 1.8\ EI\ \theta_C = -32.5 \rightarrow (B)$$

$\therefore$ From eq^{ns} (A) and (B),

$$\theta_B = \frac{27.08}{EI}, \quad \theta_C = \frac{-24.10}{EI}$$

Step 4: Final Moments:

$$\therefore M_{AB} = -19.2 + 0.4\ EI\left(\frac{27.08}{EI}\right) = -8.37\ KN\text{-}m$$

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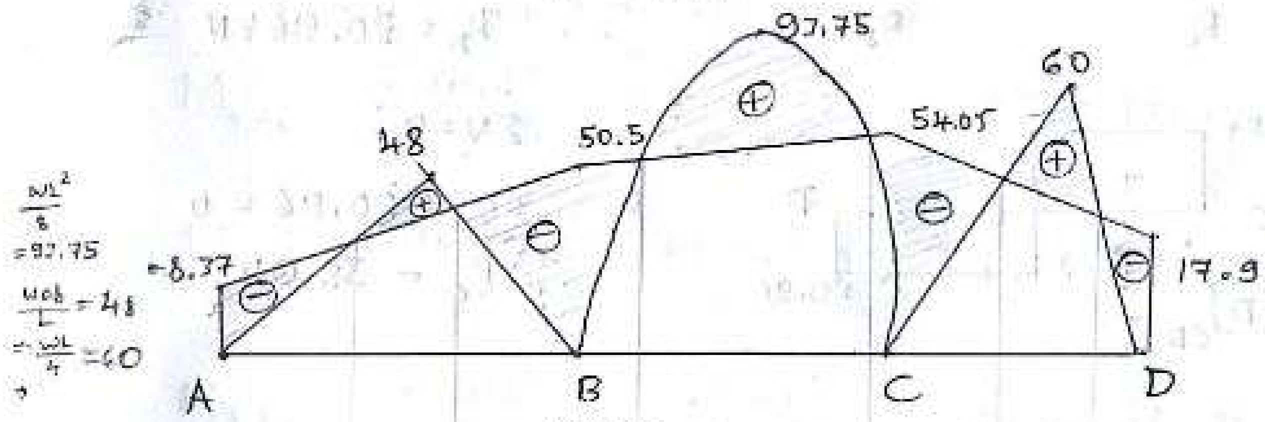
Also, $M_{BA} = 28.8 + 0.8EI\left(\frac{+27.08}{EI}\right) = 50.46$ kN-m

$M_{BC} = -62.5 + 21.664 - 9.64 = -50.47$ KN-m

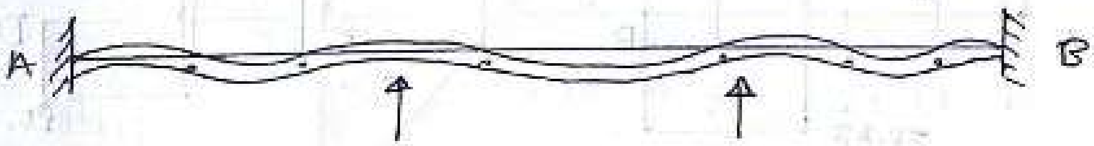
$M_{CB} = 62.5 + (-19.28) + 10.83 = 54.05$ KN-m

Also, $M_{CD} = -30 - 24.10 = -54.1$ kN-m

$M_{DC} = 30 - 12.05 = 17.9$ kN-m



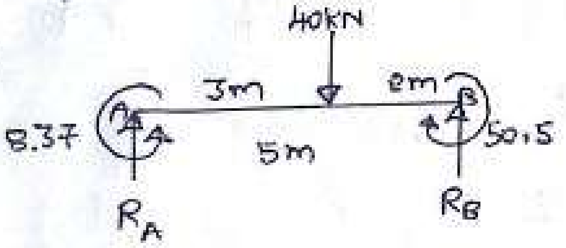
BMD



Elastic curve

Step 5: To draw SF diagram:

(i)



$\Sigma M_A = 0$;

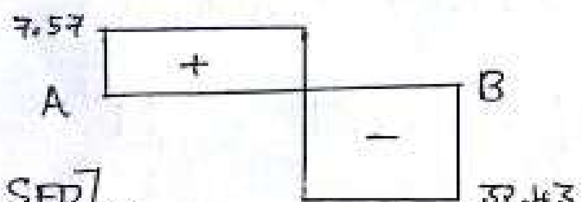
$-8.37 + (40\times3) - (R_B\times5) + 50.5 = 0$

$\therefore R_B = 32.43$ KN

(↑ +ve) $\Sigma V = 0$;

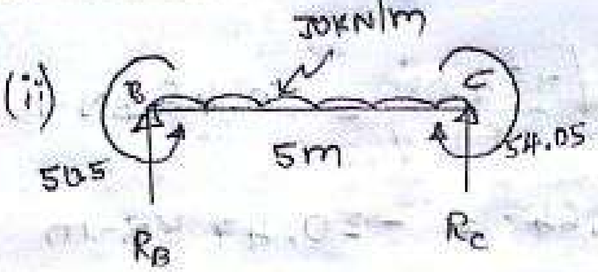
$R_A - 40 + 32.43 = 0$

$\therefore R_A = 7.57$ KN



$SFD]_{AB}$

(ii)



$\Sigma M_A = 0;$

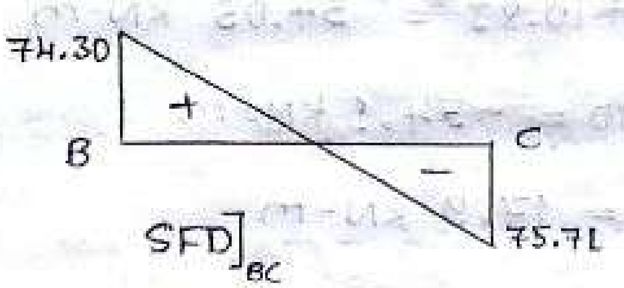
$-50.5 + (30\times5)2.5 + 54.05 - R_c \times 5 = 0$

$\therefore R_c = 75.71$ KN

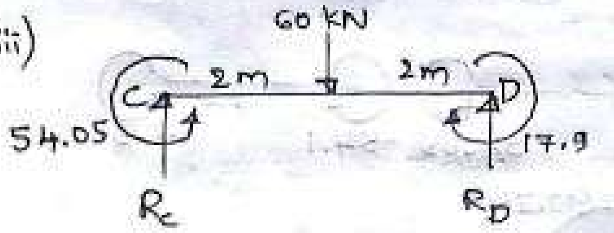
$\Sigma V = 0;$

$R_B - 150 + R_c(75.71) = 0$

$\therefore R_B = 74.30$ KN



(iii)



$\Sigma M_A = 0;$

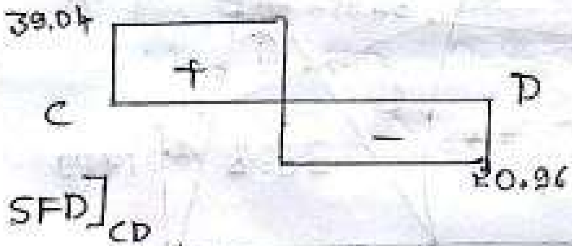
$-54.05 + (60\times2) + 17.9 - R_D \times 4 = 0$

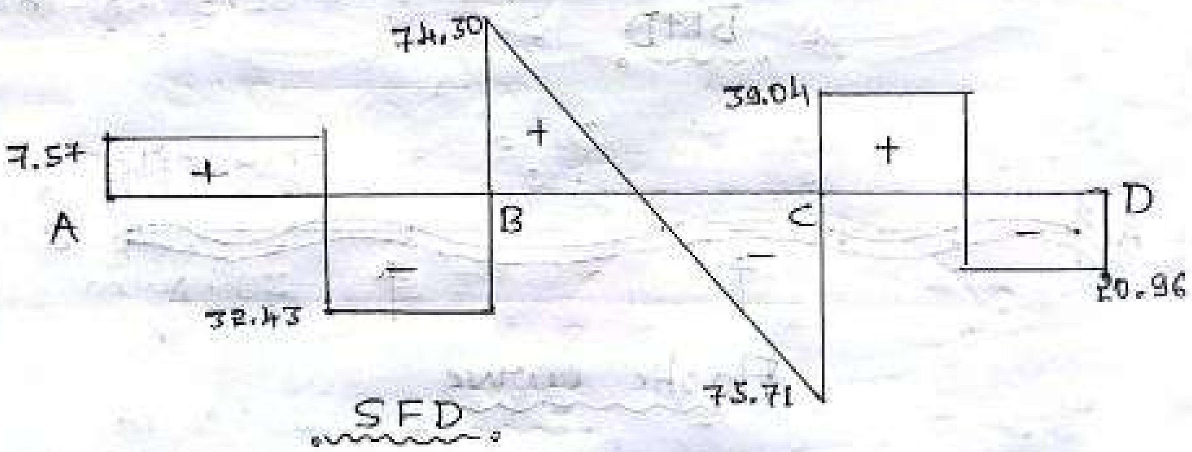
$\therefore R_D = 20.96$ KN

$\Sigma V = 0;$

$R_c - 60 + 20.96 = 0$

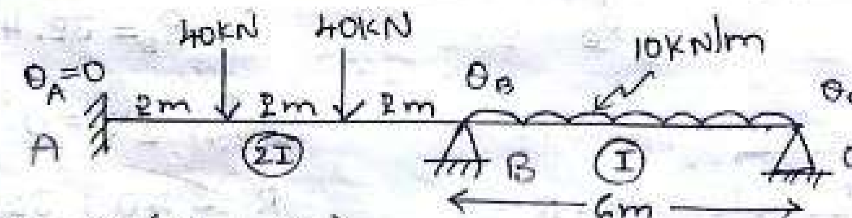
$\therefore R_c = 39.04$





SFD

4. Analyse the continuous beam loaded shown in the figure by slope-deflection method. Draw BMD and SFD.

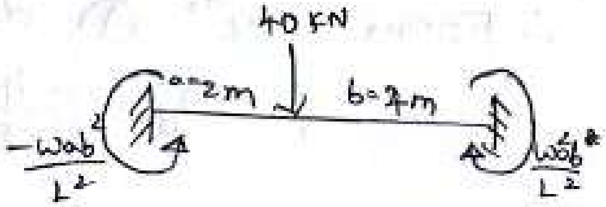


$\longrightarrow$ DOF = 2 (θ_B, θ_C)

20/08/18

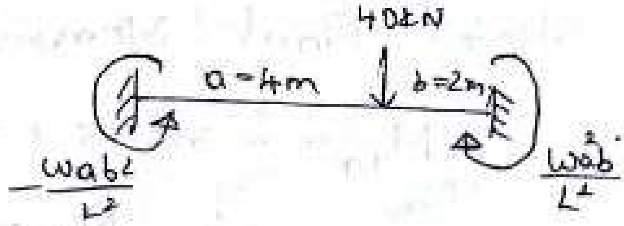
Step 1: Fixed End Moments:

$$M_{FAB} = \frac{-wab^2}{L^2} - \frac{wab^2}{L^2}$$



$$= \frac{-40 \times 2 \times 4^2}{6^2} - \frac{40 \times 4 \times 2^2}{6^2}$$

$$= -53.33 \text{ KNm}$$



$$M_{FBA} = \frac{wa^2b^2}{L^2} + \frac{wa^2b}{L^2}$$

$$= \frac{40 \times 2^2 \times 4}{6^2} + \frac{40 \times 4^2 \times 2}{6^2}$$

$$= 53.33 \text{ KNm}$$

$$M_{FBC} = \frac{-wL^2}{12} = \frac{-10 \times 6^2}{12} = -30 \text{ KNm}$$

$$M_{FCB} = \frac{wL^2}{12} = \frac{10 \times 6^2}{12} = 30 \text{ KNm}$$

Step 2: Slope - Deflection Equations:

$$M_{AB} = -53.33 + \frac{2E(2I)}{6}\left(\cancel{2\theta_A}^{0} + \theta_B - \cancel{\frac{3\Delta}{L}}^{0}\right)$$

$$= -53.33 + 0.667\, EI\, \theta_B \rightarrow (1)$$

$$M_{BA} = 53.33 + \frac{2E(2I)}{6}\left(2\theta_B + \cancel{\theta_A}^{0} - \cancel{\frac{3\Delta}{L}}^{0}\right)$$

$$= 53.33 + 1.333\, EI\, \theta_B \rightarrow (2)$$

$$M_{BC} = -30 + \frac{2E(I)}{6}\left(2\theta_B + \theta_C - \cancel{\frac{3\Delta}{L}}^{0}\right)$$

$$= -30 + 0.667\, EI\, \theta_B + 0.333\, EI\, \theta_C \rightarrow (3)$$

$$M_{CB} = 30 + \frac{2E(I)}{6}\left(2\theta_C + \theta_B - \cancel{\frac{3\Delta}{L}}^{0}\right)$$

$$= 30 + 0.667\, EI\, \theta_C + 0.333\, EI\, \theta_B \rightarrow (4)$$

Step 3: Joint Equilibrium Equations:

$$\Sigma M_B = 0; \quad M_{BA} + M_{BC} = 0$$

$$53.33 + 1.333\, EI\, \theta_B - 30 + 0.667\, EI\, \theta_B + 0.333\, EI\, \theta_C = 0$$

$$2.0\, EI\, \theta_B + 0.333\, EI\, \theta_C = -23.33 \rightarrow (A)$$

$M_{CB} = 0$;

$$0.333\,EI\theta_B + 0.667\,\theta_C = -30 \rightarrow (B)$$

$\therefore$ From eq^{ns} (A) and (B),

$$\theta_B = \frac{-4.55}{EI}, \quad \theta_C = \frac{-42.70}{EI}$$

Step 4: Final Moments:

$$M_{AB} = -53.33 + 0.667\,\cancel{EI}\left(\frac{-4.55}{\cancel{EI}}\right)$$

$$\therefore \quad M_{AB} = -56.4\ \text{KN-m}$$

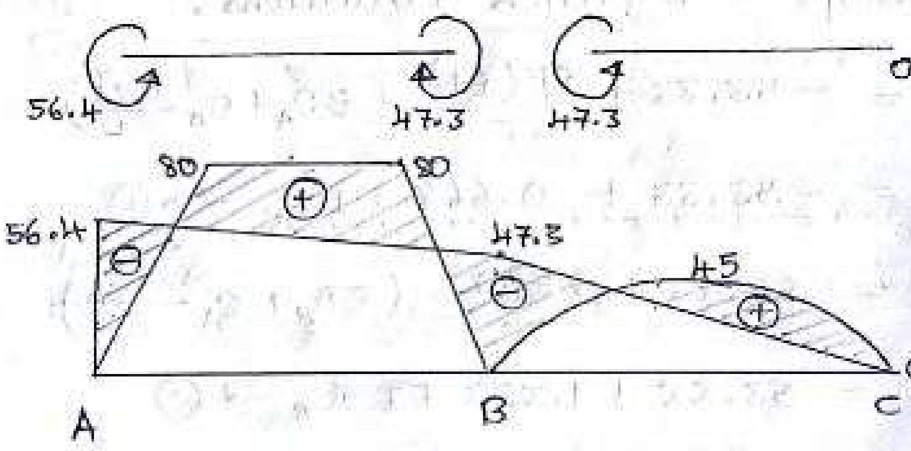
$$M_{BA} = +47.33\ \text{KNm}$$

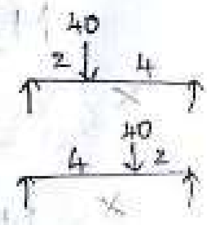
$$M_{BC} = -30 + 0.667\,EI\left(\frac{-4.55}{EI}\right) + 0.333\,EI\left(\frac{-42.70}{EI}\right)$$

$$M_{BC} = -47.3\ \text{KNm}$$

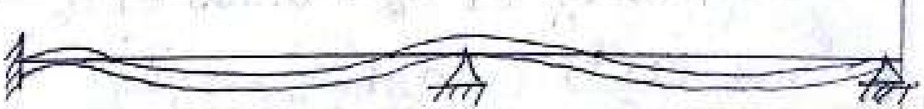
$$M_{CB} = 30 + 0.667\,EI\left(\frac{-42.7}{EI}\right) + 0.333\,EI\left(\frac{-4.55}{EI}\right)$$

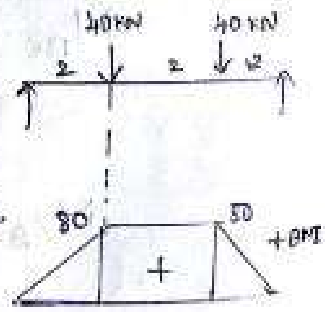
$$M_{CB} = 0\ \text{KN-m}$$





BMD

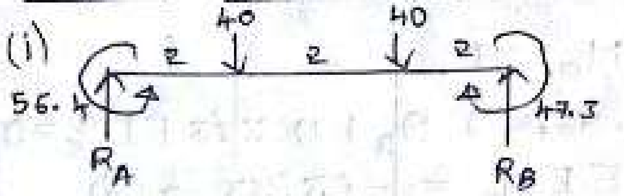




Elastic Curve

Step 5: To draw SFD:

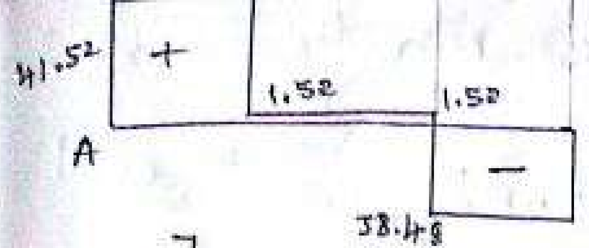
(i)



$\Sigma M_A = 0$;

$$-56.4 + 40\times2 + 40\times4 + 47.3 - R_B\times6 = 0$$

$$\therefore \quad R_B = 38.48\ \text{KN}$$



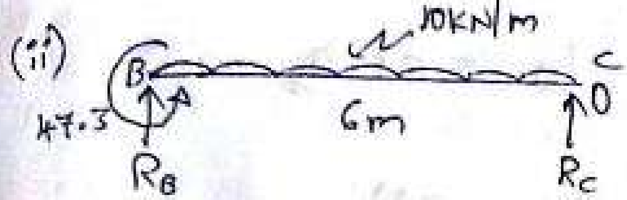
$\Sigma V = 0;$

$R_A - 40 - 40 + 38.48 = 0$

$R_A = 41.52\ kN$

$SFD]_{AB}$

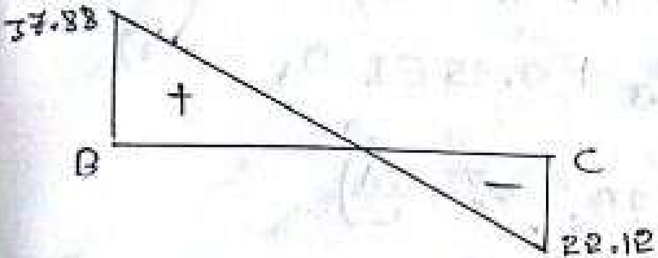
(ii)



$\Sigma M_B = 0;$

$47.3 + 60 \times 3 - R_C \times 6 = 0$

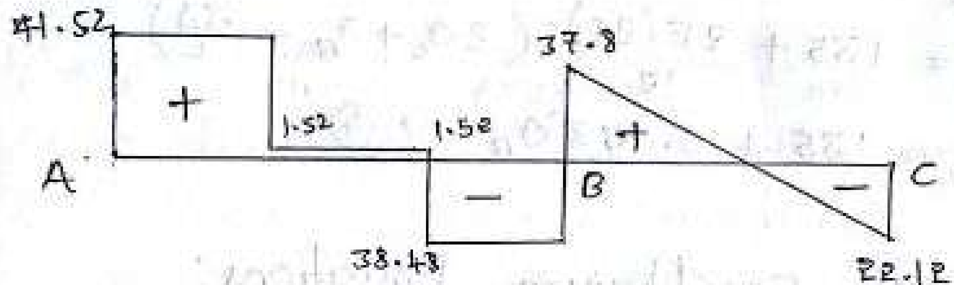
$\therefore R_C = 22.12\ kN$



$\Sigma V = 0;$

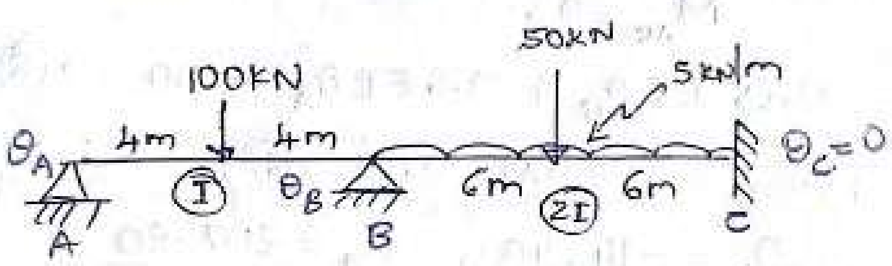
$R_B - 60 + 22.12 = 0$

$R_B = 37.88\ kN$



SFD

5. Analyse the continuous beam by slope deflection method. Draw BMD and SFD.



→ $DOF = 2\ (\theta_A, \theta_B)$

Step I: Fixed End Moments:

$M_{FAB} = \frac{-WL}{8} = -\frac{100 \times 8}{8} = -100\ kN\text{-}m$

$M_{FBA} = \frac{WL}{8} = \frac{100 \times 8}{8} = 100\ kN\text{-}m$

$M_{FBC} = \frac{-WL}{8} - \frac{wL^2}{12} = \frac{-50 \times 12}{8} - \frac{5 \times 12^2}{12} = -135\ kN\text{-}m$

$$M_{FCB} = \frac{WL}{8} + \frac{WL^2}{12} = 135 \text{ KN-m}$$

Step 2: Slope - Deflection Equations:

$$M_{AB} = -100 + \frac{2E(I)}{8}\left(2\theta_A + \theta_B - \cancel{\frac{3\Delta}{L}}^{\,0}\right)$$

$$\therefore M_{AB} = -100 + 0.5\,EI\,\theta_A + 0.25\,EI\,\theta_B \rightarrow (1)$$

$$M_{BA} = 100 + \frac{2E(I)}{8}\left(2\theta_B + \theta_A - \frac{3\Delta}{L}\right)$$

$$\therefore M_{BA} = 100 + 0.5\,EI\,\theta_B + 0.25\,EI\,\theta_A \rightarrow (2)$$

$$M_{BC} = -135 + \frac{2E(2I)}{12}\left(2\theta_B + \cancel{\theta_C}^{\,0} - \cancel{\frac{3\Delta}{L}}^{\,0}\right)$$

$$\therefore M_{BC} = -135 + 0.667\,EI\,\theta_B \rightarrow (3)$$

$$M_{CB} = 135 + \frac{2E(2I)}{12}\left(2\theta_C + \theta_B - \frac{3\Delta}{L}\right)$$

$$\therefore M_{CB} = 135 + 0.333\,EI\,\theta_B \rightarrow (4)$$

Step 3: Joint - Equilibrium Equations;

$$\Sigma M_B = 0; \quad M_{BA} + M_{BC} = 0$$

$$100 + 0.5\,EI\,\theta_B + 0.25\,EI\,\theta_A - 135 + 0.667\,\theta_B = 0$$

$$1.167\,\theta_B + 0.25\,EI\,\theta_A = 35 \rightarrow (A)$$

$$M_{AB} = 0;$$

$$0.25\,EI\,\theta_B + 0.5\,EI\,\theta_A = 100 \rightarrow (B)$$

$\therefore$ From eq^{ns} (A) and (B),

$$\theta_B = \frac{-14.40}{EI}, \quad \theta_A = \frac{207.20}{EI}$$

Step 4: Final moments:

$$M_{AB} = -100 + 0.5\,EI\left(\frac{207.20}{EI}\right) + 0.25\,EI\left(\frac{-14.40}{EI}\right)$$

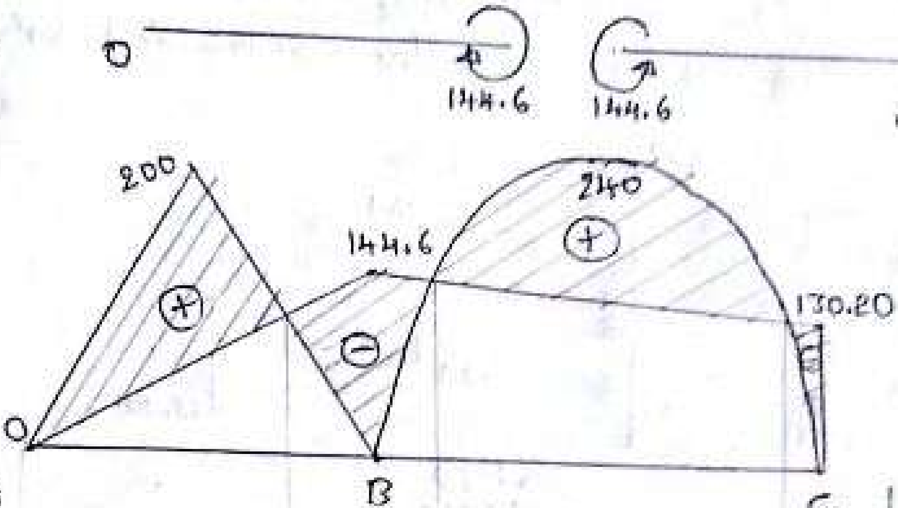
$$\therefore M_{AB} = 0.0 \text{ KN-m}$$

$M_{BA} = 144.6 \text{ KN-m}$

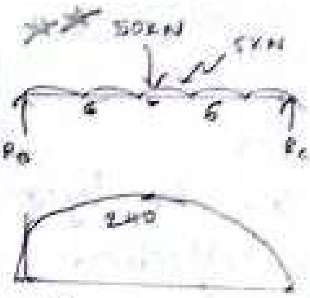
$M_{BC} = -135 + 0.667 (EI)\left(\frac{-14.40}{EI}\right)$

$M_{BC} = -144.6 \text{ KN-m}$

$M_{CB} = 130.2 \text{ KN-m}$



BMD



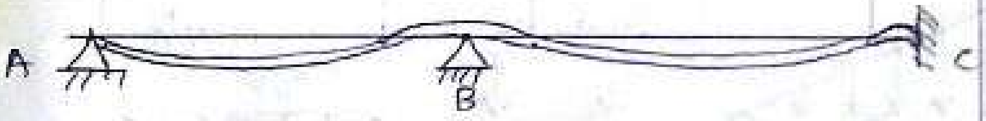
$BM]_{\omega} = 55 \times 6 - (10 \times 6) \times 3$

$\Sigma M_B = 0$

$300 - (60 \times 6) - R_C \times 12 = 0$

$\therefore R_C = 55 KN$

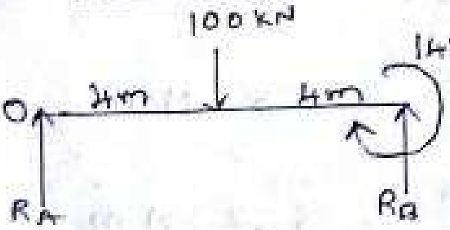
$\Sigma V = 0, \; R_B = 55 KN$



Elastic Curve

Step 5 : To draw SFD :

(1)



$\Sigma M_A = 0;$

$100 \times 4 + 144.6 - R_B \times 8 = 0$

$R_B = 68.10 \text{ kN}$

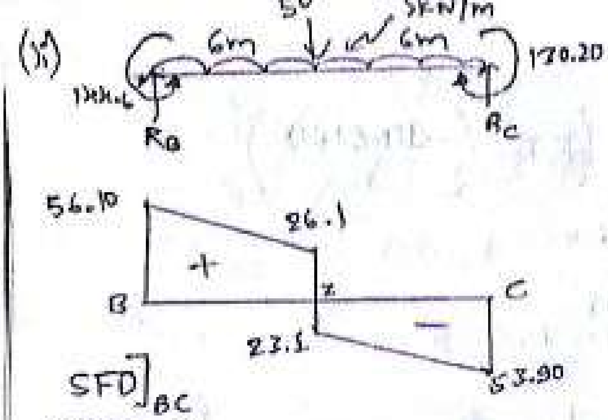
$\Sigma V = 0;$

$R_A - 100 + 68.10 = 0$

$\therefore R_A = 31.9 \text{ KN}$

SFD]$_{AB}$

(xi)



$SFD]_{BC}$

$$\Sigma M_A = 0;$$

$$-144.6 + (60\times 6) + (50\times 6) + 130.20 - R_C \times 12 = 0$$

$$\therefore R_C = 53.90 \text{ kN}$$

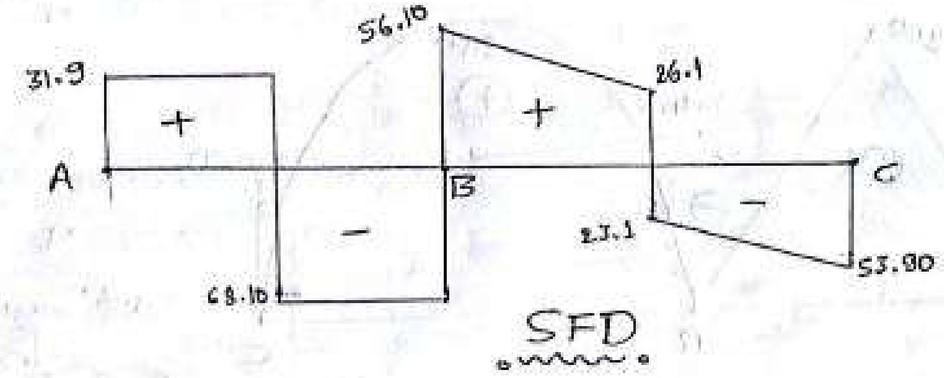
$$\Sigma V = 0;$$

$$R_B - 50 - 60 + 53.90 = 0$$

$$R_B = 56.10 \text{ kN}$$

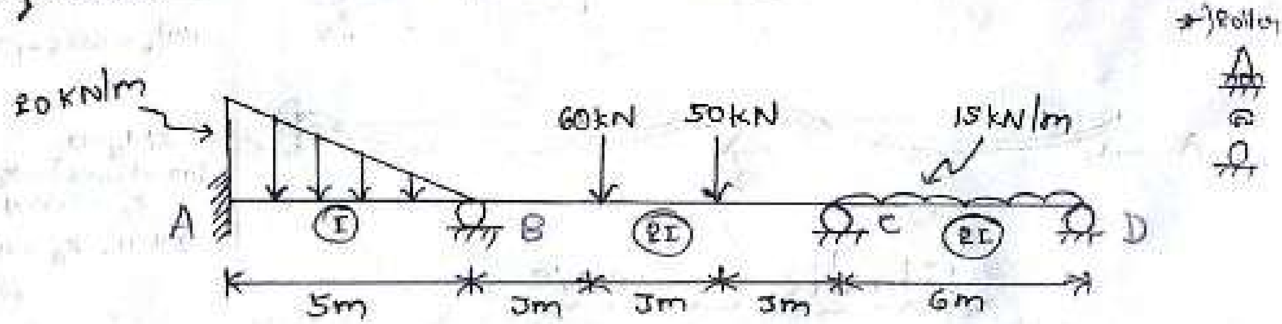
$$\therefore SF]_Z = 56.10 - (5\times 6) = +26.1$$

$$SF]_{PZ} = 56.10 - (5\times 6) - 50 = -23.9$$



SFD

6. Analyse the continuous beam by slope deflection method. Draw BMD and SFD.



→ DOF = 3 $(\theta_B, \theta_C, \theta_D)$

Step 1: Fixed End moments:

$$M_{FAB} = -\frac{wL^2}{20} = \frac{-20\times 5^2}{20} = -25 \text{ kN-m}$$

$$M_{FBA} = \frac{wL^2}{30} = \frac{20\times 5^2}{30} = 16.67 \text{ kN-m}$$

$$M_{FBC} = \frac{-60\times 3\times 6^2}{9^2} - \frac{50\times 6\times 3^2}{9^2} = -113.33 \text{ kN-m}$$

$$M_{FCB} = \frac{60\times 3\times 6^2}{9^2} + \frac{50\times 6^2\times 3}{9^2} = 106.67 \text{ kN-m}$$

$$M_{FCD} = -\frac{15\times 6^2}{12} = -45 \text{ kN-m}$$

$M_{FDC} = 45 \text{ KN-m}$

Step 2: Slope - Deflection Equations:

$$M_{AB} = -25 + \frac{2E(I)}{5}(\theta_B)$$

$$\therefore M_{AB} = -25 + 0.4\, EI\, \theta_B \rightarrow ①$$

Now, $$M_{BA} = 16.67 + \frac{2E(I)}{5}(2\theta_B)$$

$$\therefore M_{BA} = 16.67 + 0.8\, EI\, \theta_B \rightarrow ②$$

$$M_{BC} = -113.33 + \frac{2E(2I)}{9}(2\theta_B + \theta_C)$$

$$\therefore M_{BC} = -113.33 + 0.88\, EI\, \theta_B + 0.44\, EI\, \theta_C \rightarrow ③$$

$$M_{CB} = 106.67 + \frac{2E(2I)}{9}(2\theta_C + \theta_B)$$

$$\therefore M_{CB} = 106.67 + 0.88\, EI\, \theta_C + 0.44\, EI\, \theta_B \rightarrow ④$$

$$M_{CD} = -45 + \frac{2E(2I)}{6}(2\theta_C + \theta_D)$$

$$\therefore M_{CD} = -45 + 1.33 EI\, \theta_C + 0.667\, EI\, \theta_D \rightarrow ⑤$$

$$M_{DC} = 45 + \frac{2E(2I)}{6}(2\theta_D + \theta_C)$$

$$\therefore M_{DC} = 45 + 1.33\, EI\, \theta_D + 0.667\, EI\, \theta_C \rightarrow ⑥$$

Step 3: Joint - Equilibrium Equations:

$$\Sigma M_B = 0; \quad M_{BA} + M_{BC} = 0$$

$$16.67 + 0.8\, EI\, \theta_B - 113.33 + 0.88\, EI\, \theta_B + 0.44\, EI\, \theta_C = 0$$

$$1.68\, EI\, \theta_B + 0.44\, EI\theta_C = 96.66 \rightarrow Ⓐ$$

$$\Sigma M_C = 0; \quad M_{CB} + M_{CD} = 0$$

$$106.67 + 0.88\, EI\theta_C + 0.44\, EI\theta_B - 45 + 1.33 EI\, \theta_C + 0.67 EI\theta_D = 0$$

$$0.44\, EI\theta_B + 2.21\, EI\, \theta_C + 0.667\, EI\theta_D = -61.67 \rightarrow Ⓑ$$

$$\Sigma M_{DC} = 0;$$

$$\cancel{45} * 0.667\, EI\, \theta_C + 1.33\, EI\, \theta_D = -45 \rightarrow Ⓒ$$

$$\therefore \quad \theta_B = \frac{\overset{67.1}{-}}{EI}\,; \quad \theta_C = \frac{\overset{36.58}{-\ \cdot\ 8}}{EI}\,; \quad \theta_D = \frac{\overset{49}{-15.}}{EI}$$

Step 4: Final moments:

$M_{AB} = 1.84$ kN-m

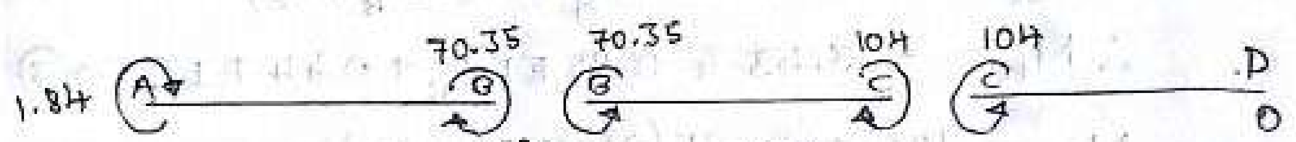
$M_{BA} = 70.35$ kN-m

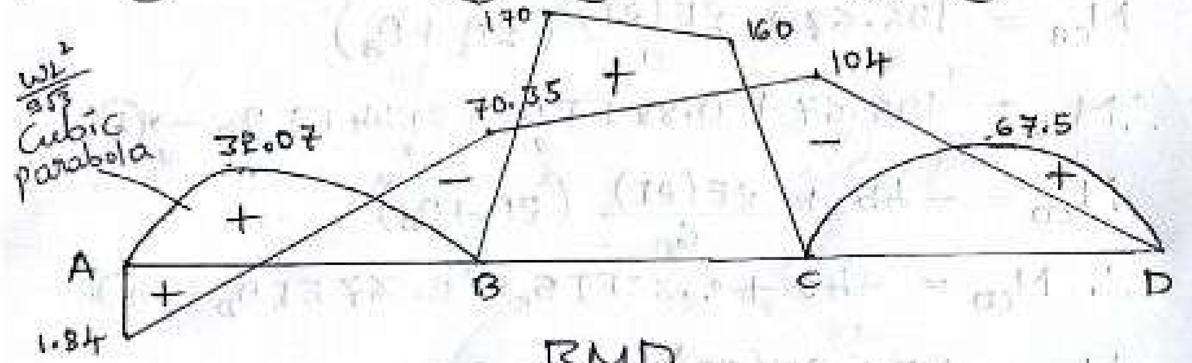
$M_{BC} = -70.35$ kN-m

$M_{CB} = 104$ kN-m

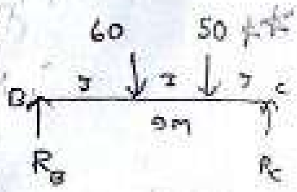
$M_{CD} = -104$ kN-m

$M_{DC} = 0.0$ kN-m





BMD



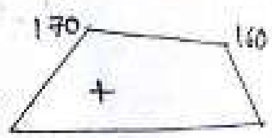
$\Sigma M_B = 0;$

$60\times3 + 50\times3 - R_C\times3$

$R_C = 53.33$

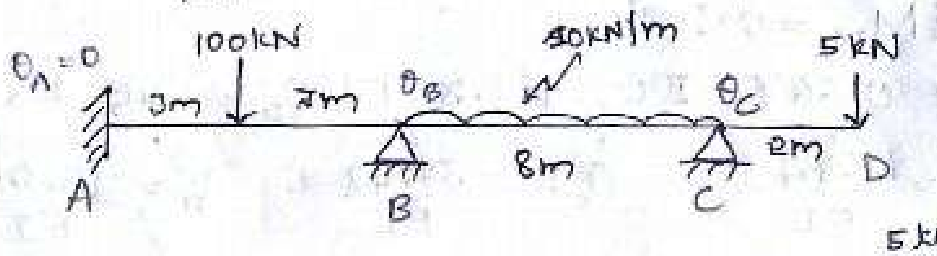
$\Sigma V = 0;$

$R_B = 56.67$

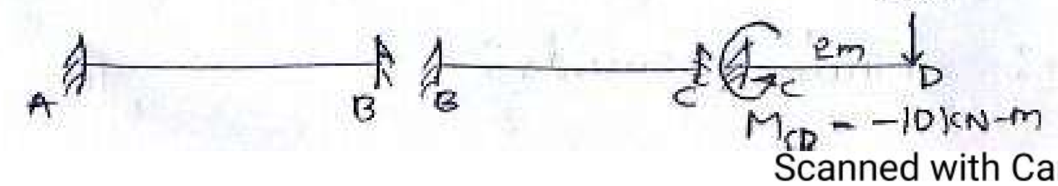


Elastic Curve.

7. Analyse the continuous beam loaded shown in the figure. Draw BMD, SFD & elastic curve.



→



**) Here CD is a cantilever which is a determinate one, hence $M_{CD} = -5 \times 2 = -10\ kN\text{-}m$

No

Step 1: Fixed End moments:

$$M_{FAB} = -\frac{WL}{8} = \frac{-100\times 6}{8} = -75\ kN\text{-}m$$

$$M_{FBA} = \frac{WL}{8} = \frac{100\times 6}{8} = 75\ kN\text{-}m$$

$$M_{FBC} = \frac{-WL^2}{12} = \frac{-10\times 8^2}{12} = -53.33\ kN\text{-}m$$

$$M_{FCB} = \frac{WL^2}{12} = +53.33\ kN\text{-}m$$

Step 2: Slope - Deflection Equation:

$$M_{AB} = -75 + \frac{2E(I)}{6}\left(2\theta_A + \theta_B - \frac{3\Delta}{L}\right)$$

$$\therefore M_{AB} = -75 + 0.33\ EI\ \theta_B \rightarrow (1)$$

$$M_{BA} = 75 + \frac{2E(I)}{6}\left(2\theta_B + \theta_A - \frac{3\Delta}{L}\right)$$

$$\therefore M_{BA} = 75 + 0.667\ EI\ \theta_B \rightarrow (2)$$

$$M_{BC} = -53.33 + \frac{2EI}{8}(2\theta_B + \theta_C)$$

$$\therefore M_{BC} = -53.33 + 0.5\ EI\theta_B + 0.25\ EI\theta_C \rightarrow (3)$$

$$M_{CB} = 53.33 + \frac{2EI}{8}(2\theta_C + \theta_B)$$

$$\therefore M_{CB} = 53.33 + 0.5\ EI\ \theta_C + 0.25\ EI\ \theta_B \rightarrow (4)$$

Step 3: Joint Equilibrium Equations:

$$\Sigma M_B = 0;\ M_{BA} + M_{BC} = 0;$$

$$75 + 0.667\ EI\theta_B - 53.33 + 0.5\ EI\theta_B + 0.25\ EI\theta_C = 0$$

$$1.166\ EI\theta_B + 0.25\ EI\theta_C = -21.67 \rightarrow (A)$$

$$\Sigma M_C = 0;\ M_{CB} + M_{CD} = 0;$$

$$53.33 + 0.5\ EI\theta_C + 0.25\ EI\theta_B - 10 = 0$$

$$0.25\ EI\theta_B + 0.5\ EI\theta_C = -43.33 \rightarrow (B)$$

$\therefore$ From (A) & (B),

$$\theta_C = \frac{-4.88\times 10}{EI},\quad \theta_B = \frac{-86.65}{EI}$$

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Step 4: Final Moments :

$$\therefore M_{AB} = -75 + 0.33\,EI\left(\frac{-4.88\times10^{-3}}{EI}\right)$$

$$\therefore \quad M_{AB} = -75 \text{ kN-m}$$

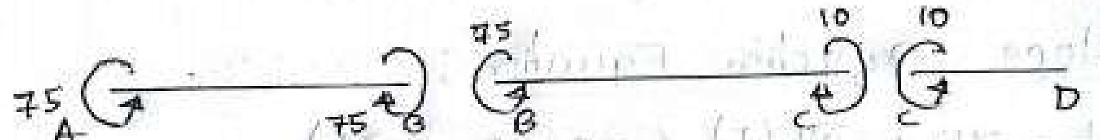
$$M_{BA} = 75 \text{ kN-m}$$

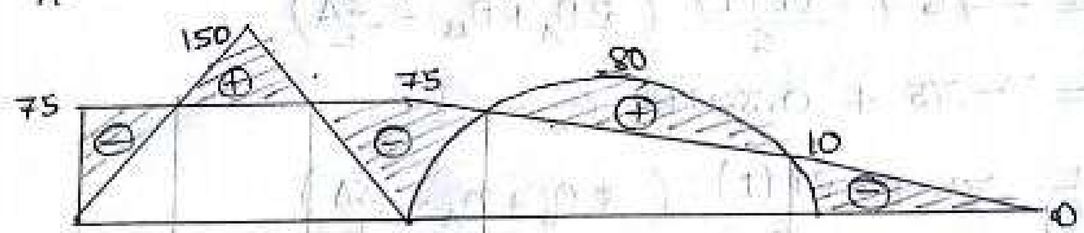
$$M_{BC} = -75 \text{ kN-m}$$

$$M_{CB} = 53.33 + 0.5\,EI\left(\frac{-86.66}{EI}\right)$$

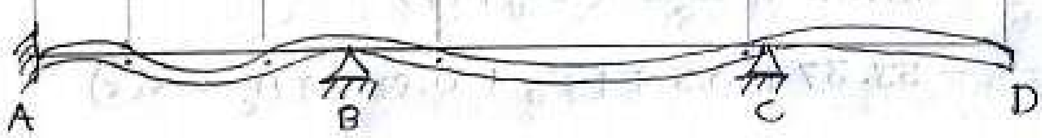
$$\therefore \quad M_{CB} = +10 \text{ kN-m}$$

$$M_{CD} = -10 \text{ kN-m}$$





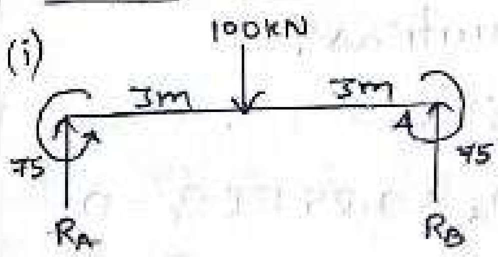
BMD



Elastic Curve

Step 5: To draw SFD :

(i)



$\Sigma M_A = 0$;

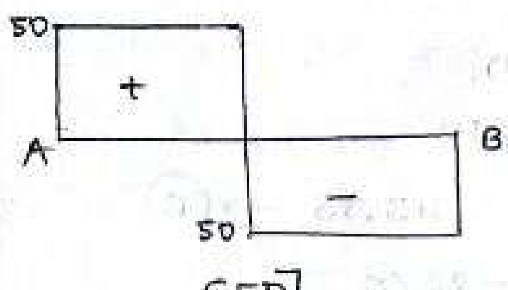
$-75 + 300 + 75 - R_B \times 6 = 0$

$\therefore R_B = 50 \text{ KN}$

$\Sigma V = 0$;

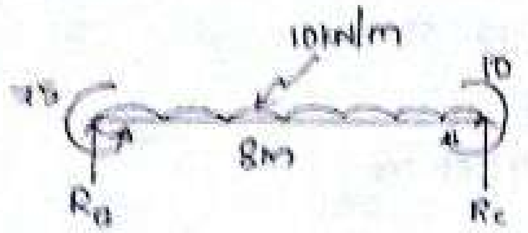
$R_A - 100 + 50 = 0$

$\therefore R_A = 50 \text{ KN}$



SFD$]_{AB}$

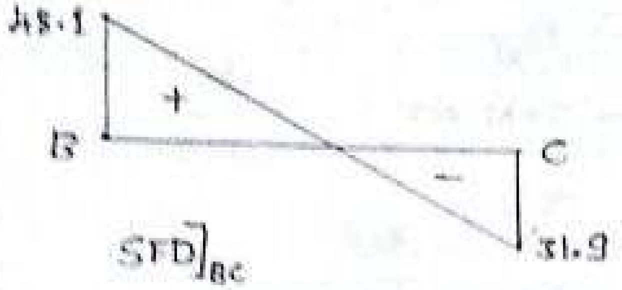
(ii)



$\Sigma M_A = 0$:

$-75 + 80\times4 + 10 - R_c\times 8 = 0$

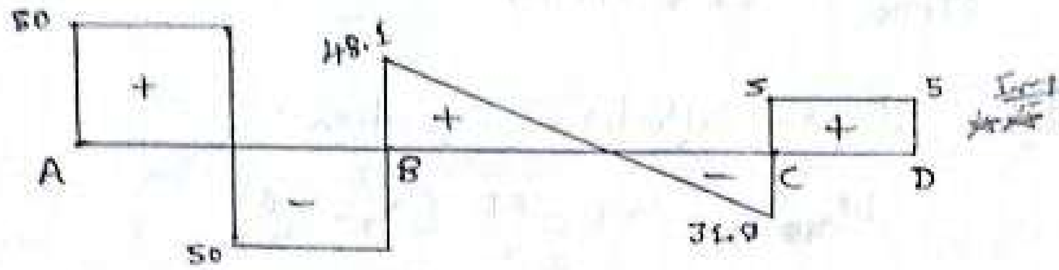
$R_c = 31.90$ KN



$\Sigma V = 0$:

$R_B - 80 + 31.90 = 0$

$\therefore\ R_B = 48.1$ KN



SFD.

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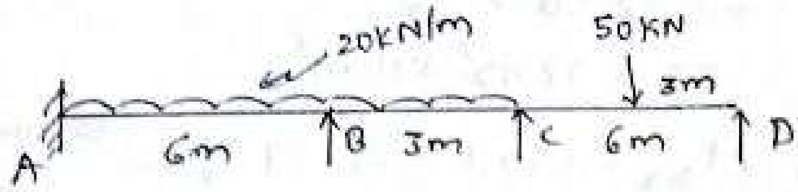
2. Analyse the continuous beam loaded as shown in the figure by slope deflection method. The support 'B' sinks by 10mm. $E = 2\times10^5$ N/mm^2, $I = 16\times10^7$ mm^4. Sketch the BMD.



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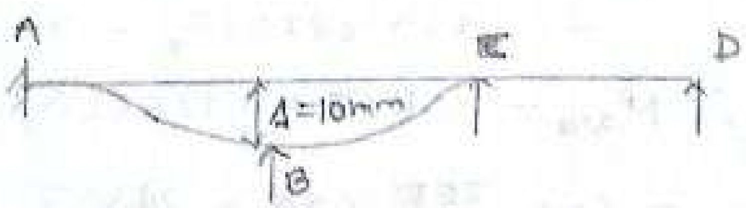

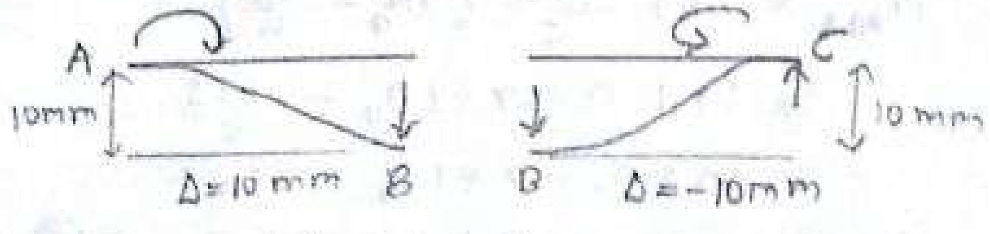

DOF = 3 $(\theta_B, \theta_C, \theta_D)$

Step 1 : Fixed End Moments :

$$M_{FAB} = \frac{-20\times 6^2}{12} = -60\,kN\text{-}m$$

$$M_{FBA} = 60\,kN\text{-}m$$

$$M_{FBC} = -\frac{20\times 3^2}{12} = -15\,kN\text{-}m$$

$$M_{FCB} = +15\,kN\text{-}m$$

$$M_{FCD} = -\frac{W\times L}{8} = -\frac{50\times 6}{8} = -37.5\,kN\text{-}m$$

$$M_{FDC} = 37.5\,kN\text{-}m$$

Step 2 : Slope - Deflection Equation :

$$M_{FAB} = -60 + \frac{2EI}{6}\left(\theta_B - \frac{3\Delta}{L}\right)$$

$$M_{FAB} = -60 + 0.333\,EI\,\theta_B - \frac{EI\cdot\Delta}{6} \rightarrow (4)$$

But , $EI = \left(\frac{1}{1000}\right) kN \left(\frac{1}{1000}\right)^2 m^2 \left\{\begin{array}{l} 1000\,N = 1kN \\ 1\,N = 1/1000\,kN \end{array}\right.$

$$= \frac{1}{10^3}\times\frac{1}{10^6}\,kNm^2$$

$$\therefore\ EI = 1/10^9\ kN\text{-}m^2$$

$$\therefore\ EI = (2\times10^5\times16\times10^7)\times 1/10^9$$

$$= 32\times10^{12}\times\frac{1}{10^9}$$

$$\therefore\ EI = 32\times10^3\ kN\text{-}m^2$$

Now, (4) $\Rightarrow$ $M_{FAB} = -60 + 0.333\,EI\theta_B - \frac{\left(32\times10^3\times\frac{1}{100}\right)}{6}$

$$= -60 + 0.333\,EI\theta_B - 53.33$$

$$\therefore\ M_{FAB} = -113.33 + 0.333\,EI\,\theta_B \rightarrow (1)$$

Now, $M_{FBA} = 60 + \frac{2EI}{6}\left(2\theta_B - \frac{3\Delta}{L}\right)$

$$= 60 + 0.667\,EI\,\theta_B - \frac{EI\Delta}{6}$$

$$= 60 + 0.667\,EI\theta_B - 53.33$$

$$\therefore\ M_{FBA} = 6.67 + 0.667\,EI\,\theta_B \rightarrow (2)$$

Now, $M_{FBC} = -15 + \frac{2EI}{3}\left(2\theta_B + \theta_C - \frac{3\Delta}{3}\right)$

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$$M_{FBC} = -15 + 1.33\,EI\theta_B + 0.667\,EI\theta_C - 0.667\,EI\,\Delta \quad -(1/100)$$

$$\therefore M_{FBC} = -15 + 1.33\,EI\theta_B + 0.667\,EI\theta_C + 213.44$$

$$\therefore M_{FBC} = 198.144 + 1.33\,EI\theta_B + 0.667\,EI\theta_C \rightarrow (3)$$

Now, $M_{FCB} = 15 + \frac{2EI}{3}\left(2\theta_C + \theta_B - \frac{3\Delta}{3}\right)$

$$M_{FCB} = 15 + 1.333\,EI\theta_C + 0.667\,EI\theta_B + 213.44$$

$$\therefore M_{FCB} = 228.44 + 1.333\,EI\theta_C + 0.667\,EI\theta_B \rightarrow (4)$$

Now, $M_{FCD} = -37.5 + \frac{2EI}{6}(2\theta_C + \theta_D)$

$$\therefore M_{CD} = -37.5 + 0.667\,EI\theta_C + 0.333\,EI\theta_D \rightarrow (5)$$

$$M_{DC} = 37.5 + 0.667\,EI\theta_D + 0.333\,EI\theta_C \rightarrow (6)$$

Step 3: Joint Equilibrium Equations:

$\Sigma M_B = 0$; $M_{BA} + M_{BC} = 0$;

$$6.67 + 0.667\,EI\theta_B + 198.144 + 1.33\,EI\theta_B + 0.667\,EI\theta_C = 0$$

$$2.0\,EI\theta_B + 0.667\,EI\theta_C = -205.11 \rightarrow (A)$$

Now, $\Sigma M_C = 0$; $M_{CB} + M_{CD} = 0$;

$$228.44 + 1.333\,EI\theta_C + 0.667\,EI\theta_B - 37.5 + 0.667\,EI\theta_C + 0.333\,EI\theta_D = 0;$$

(B)

$$0.667\,EI\theta_B + 2.0\,EI\theta_C + 0.333\,EI\theta_D = -190.94$$

$M_{DC} = 0$;

$$0.333\,EI\theta_C + 0.667\,EI\theta_D = -37.5 \rightarrow (C)$$

$$\theta_B = \frac{-81.07}{EI} \qquad \theta_C = \frac{-64.43}{EI} \qquad \theta_D = \frac{-24.05}{EI}$$

Step 4: Final moments:

$M_{AB} = -140.33$ KN-m

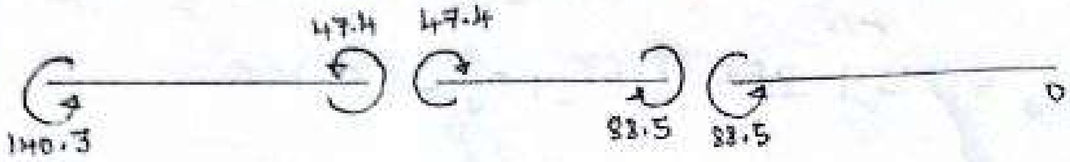
$M_{BA} = -47.40$ KN-m

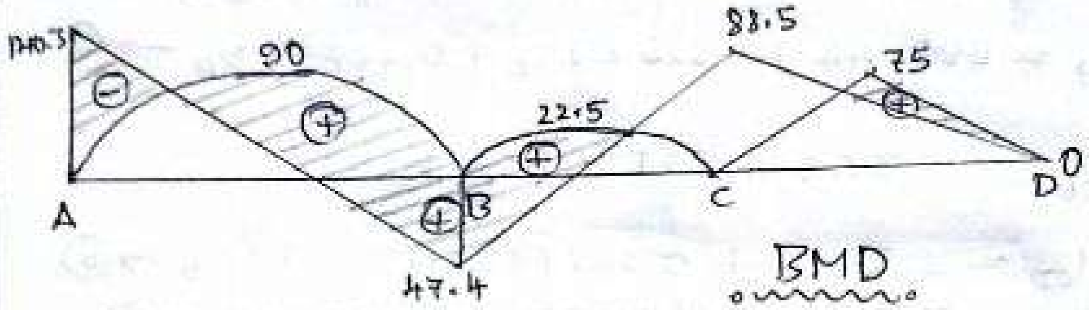
$M_{BC} = 47.40$ KN-m

$M_{CB} = 88.48$ KN-m

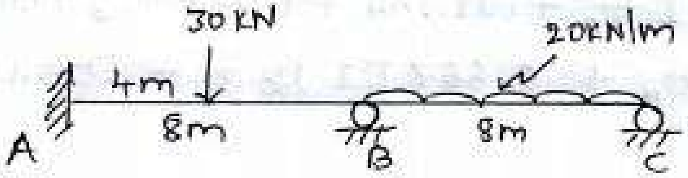
$$M_{CB} = -88.48\ kN\text{-}m$$

$$M_{DC} = 0.0\ kN\text{-}m$$





9. Analyse the continuous beam by slope-deflection method if the support 'B' sinks by 5 mm. Draw BMD. Assume $EI = 4000\ kN\,m^2$



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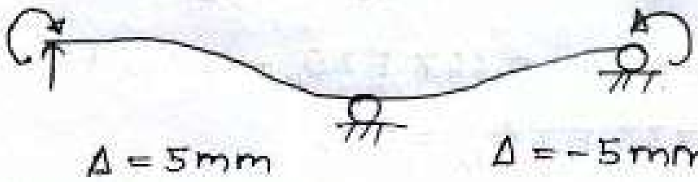


$\Delta = 5\,mm$

$1000\,mm = 1\,m$

$1\,mm = \frac{1}{1000}\,m$

$\therefore 5\,mm = \left(\frac{5}{1000}\right)m$

$DOF = 2\ (\theta_B, \theta_C)$

Step 1: Fixed End Moments:

$$M_{FAB} = \frac{-WL}{8} = -\frac{30\times8}{8} = -30\,kN\text{-}m$$

$$M_{FBA} = \frac{WL}{8} = \frac{30\times8}{8} = 30\,kN\text{-}m$$

$$M_{FBC} = \frac{-WL^2}{12} = -\frac{20\times8^2}{12} = -106.67\,kN\text{-}m$$

$$M_{FCB} = \frac{WL^2}{12} = \frac{20\times8^2}{12} = 106.67\,kN\text{-}m$$

Step 2: Slope-Deflection Equations:

$$M_{AB} = -30 + \frac{2EI}{8}\left(\cancel{2\theta_A}^{\,0} + \theta_B - \frac{3\Delta}{8}\right)$$

$$M_{AB} = -30 + 0.25\,EI\,\theta_B - 0.25 \times 0.375 \times 4000 \times \left(\frac{5}{1000}\right)$$

$$M_{AB} = -31.87 + 0.25\,EI\,\theta_B \rightarrow (1)$$

Now, $$M_{BA} = 30 + \frac{2EI}{8}\left(2\theta_B + \theta_A^{\nearrow 0} - \frac{3\Delta}{L}\right)$$

$$= 30 + 0.5\,EI\,\theta_B - 1.87$$

$$\therefore M_{BA} = 28.13 + 0.5\,EI\,\theta_B \rightarrow (2)$$

Now, $$M_{BC} = -106.67 + \frac{2EI}{8}\left(2\theta_B + \theta_C - \frac{3\Delta}{L}\right)$$

$$= -106.67 + 0.25\,EI\left(2\theta_B + \theta_C - 0.375\left(\frac{5}{1000}\right)\right)$$

$$= -106.67 + 0.50\,EI\,\theta_B + 0.25\,EI\,\theta_C - 1.875$$

$$\therefore M_{BC} = -108.54 + 0.50\,EI\,\theta_B + 0.25\,EI\,\theta_C \rightarrow (3)$$

Now, $$M_{CB} = 106.67 + \frac{2EI}{8}\left(2\theta_C + \theta_B - \frac{3\Delta}{L}\right)$$

$$M_{CB} = 104.67 + 0.25\,EI\,\theta_B + 0.50\,EI\,\theta_C \rightarrow (4)$$

Step 3: Joint Equilibrium Equations:

$$\Sigma M_B = 0; \quad M_{BA} + M_{BC} = 0$$

$$28.13 + 0.5\,EI\,\theta_B - 108.54 + 0.5\,EI\,\theta_B + 0.25\,EI\,\theta_C = 0$$

$$1.0\,EI\,\theta_B + 0.25\,EI\,\theta_C = 80.41 \rightarrow (A)$$

$$\text{Ⅱ}^{ly} \quad M_{CB} = 0;$$

$$0.25\,EI\,\theta_B + 0.50\,EI\,\theta_C = -104.67 \rightarrow (B)$$

$$\therefore \theta_B = \frac{151.71}{EI}; \quad \theta_C = \frac{-285.2}{EI}$$

Step 4: Final Moments:

$$M_{AB} = -31.87 + 0.25\,EI\left(\frac{151.71}{EI}\right)$$

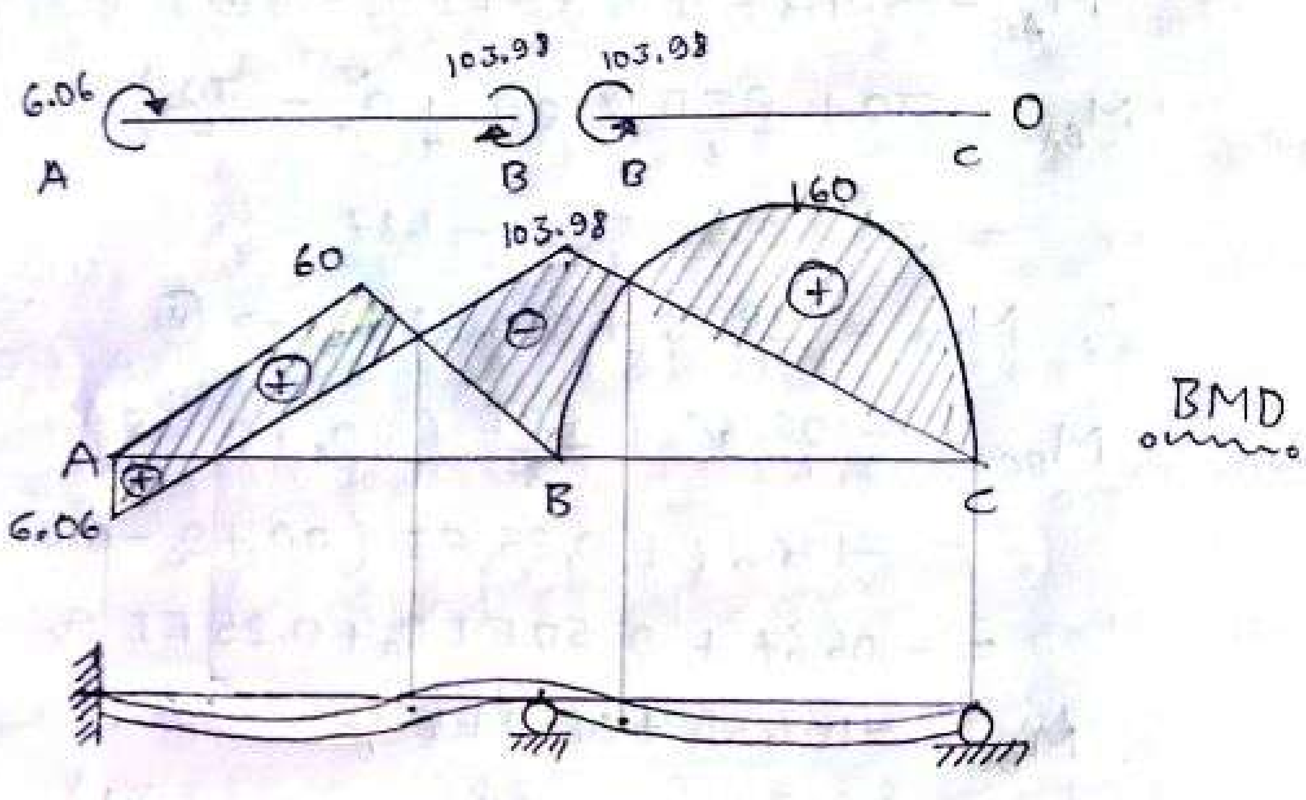
$$\therefore M_{AB} = 6.06 \text{ kN-m};$$

$$M_{BA} = 28.13 + 0.5\,EI\left(\frac{151.71}{EI}\right)$$

$$\therefore M_{BA} = 103.98 \text{ kN-m};$$

$$\text{Ⅱ}^{ly}, \quad M_{BC} = -103.98 \text{ kN-m};$$

III^{ly}. $M_{CB} = 0.0$ kN-m

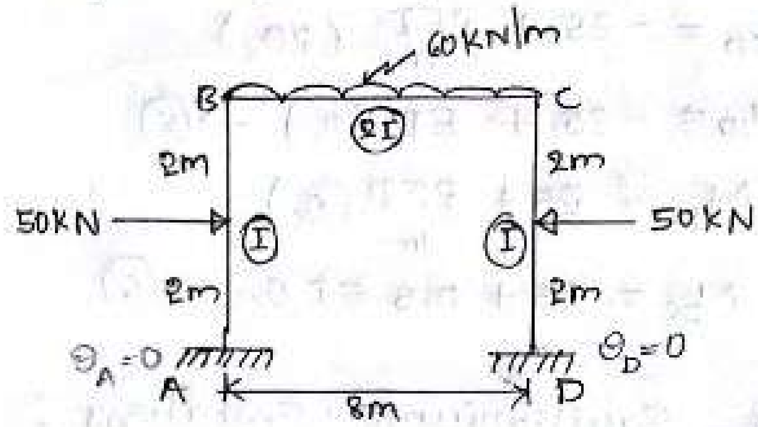


Elastic Curve

25/08/18

Rigid Plane Frames :

1. Analyse the portal frame shown in the figure by Slope - deflection method.



→ Here DOF = 2 (θ_B, θ_C)

Step 1 : Fixed End Moments :

$$M_{FAB} = \frac{-WL}{8} = \frac{-50\times4}{8} = -25 \text{ kNm}$$

$$M_{FBA} = \frac{WL}{8} = 25 \text{ kN m}$$

$$M_{FBC} = -\frac{WL^2}{12} = \frac{-60\times8^2}{12} = -320 \text{ kNm}$$

$$M_{FCB} = \frac{WL^2}{12} = \frac{60\times8^2}{12} = 320 \text{ kNm}$$

$$M_{FCD} = \frac{-WL}{8} = \frac{-50\times4}{8} = -25 \text{ kNm}$$

$$M_{FDC} = \frac{WL}{8} = 25 \text{ kNm}$$

Step 2 : Slope - Deflection Equations :

$$M_{AB} = M_{FAB} + \frac{2EI}{L}\left(2\theta_A + \theta_B - \frac{3\Delta}{L}\right)$$

$$= -25 + \frac{2EI}{4}(\theta_B)$$

$$\therefore M_{AB} = -25 + 0.5\, EI\, \theta_B \rightarrow (1)$$

Now, $M_{BA} = 25 + \frac{2EI}{4}(2\theta_B)$

$$\therefore M_{BA} = 25 + EI(\theta_B) \rightarrow (2)$$

Now, $M_{BC} = -320 + \frac{2E(2I)}{8}(2\theta_B + \theta_C)$

$\therefore \; M_{BC} = -320 + EI\,\theta_B + 0.5\,EI\,\theta_C \rightarrow (3)$

Now, $M_{CB} = 320 + \frac{2E(2I)}{8}(2\theta_C + \theta_B)$

$\therefore \; M_{CB} = 320 + EI(\theta_C) + 0.5\,EI\,\theta_B \rightarrow (4)$

Now, $M_{CD} = -25 + \frac{2EI}{4}(2\theta_C)$

$\therefore \; M_{CD} = -25 + EI(\theta_C) \rightarrow (5)$

Now, $M_{DC} = 25 + \frac{2EI}{4}(\theta_C)$

$\therefore \; M_{DC} = 25 + 0.5\,EI\,\theta_C \rightarrow (6)$

Step 3: Joint Equilibrium Equations:

$\Sigma M_B = 0; \quad M_{BA} + M_{BC} = 0$

$25 + EI\,\theta_B - 320 + EI\,\theta_B + 0.5\,EI\,\theta_C = 0$

$2EI\,\theta_B + 0.5\,EI\,\theta_C = 295 \rightarrow (A)$

Now, $\Sigma M_C = 0; \quad M_{CB} + M_{CD} = 0$

$320 + EI\,\theta_C + 0.5\,EI\,\theta_B - 25 + EI\,\theta_C = 0$

$0.5\,EI\,\theta_B + 2\,EI\,\theta_C = -295 \rightarrow (B)$

$\therefore \; \theta_B = \frac{196.67}{EI} \quad \theta_C = \frac{-196.67}{EI}$

Step 4: Final Moments:

$(1) \rightarrow \quad M_{AB} = -25 + 0.5\,EI\left(\frac{196.7}{EI}\right)$

$= 73.35$ KN-m

$(2) \Rightarrow \quad M_{BA} = 25 + EI\left(\frac{196.7}{EI}\right)$

$= 221.7$ KN-m

$(3) \Rightarrow \quad M_{BC} = -320 + EI\left(\frac{196.7}{EI}\right) + 0.5\,EI\left(-\frac{196.7}{EI}\right)$

$\therefore \; M_{BC} = -221.7$ KN-m

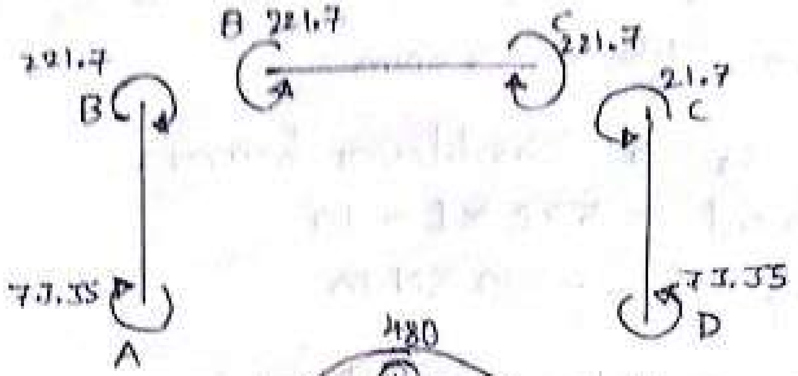
$(4) \Rightarrow \quad M_{CB} = 320 + EI\left(\frac{-196.7}{EI}\right) + 0.5\,EI\left(\frac{196.7}{EI}\right)$

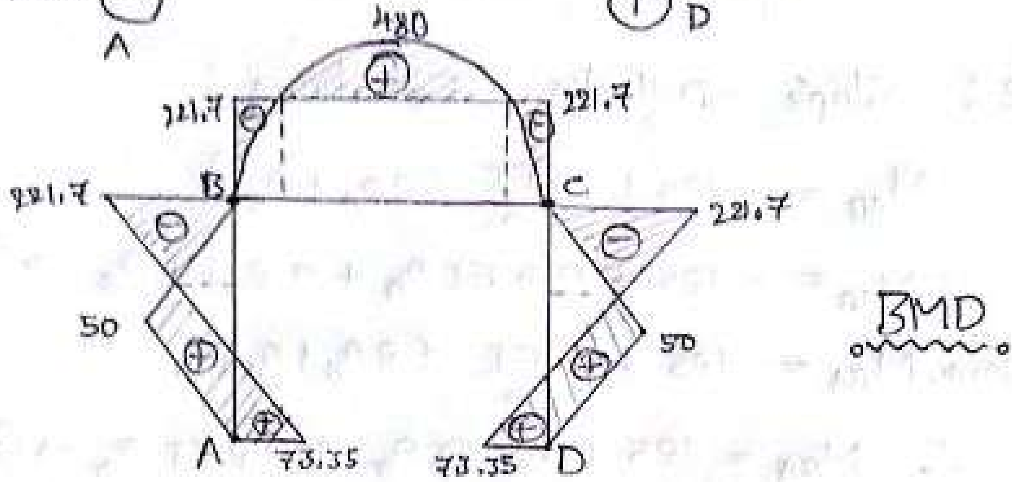
$= 221.7$ KN-m

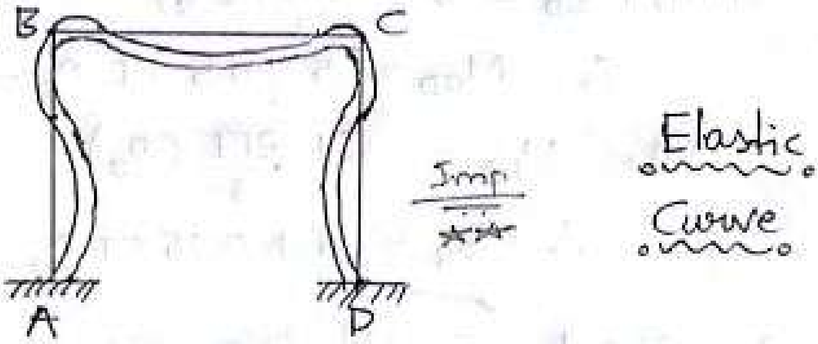
$(5) \Rightarrow \quad M_{CD} = -221.7$ KNm

⑥ ⇒ $M_{DC} = 25 + 0.5\, EI \left(\frac{-196.7}{EI}\right)$

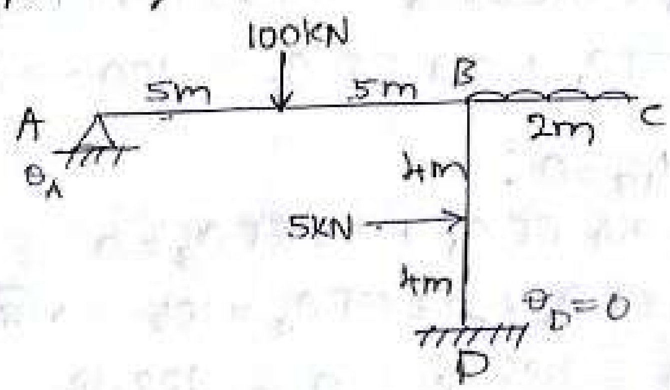
$= -73.35$ KN·m







2. Analyse the frame shown in the figure by slope-deflection method. Draw BMD. **V.Imp

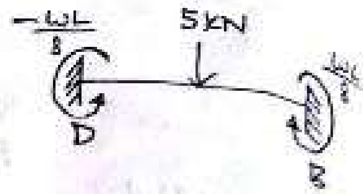


→ Here DOF = 2 (θ_A, θ_B)

Step 1: Fixed End Moments:

$M_{FAB} = -\frac{wl}{8} = \frac{-100 \times 10}{8} = -125$ KNm

$$M_{FBA} = \frac{wL}{8} = 125 \text{ kNm}$$



$$M_{FBD} = \frac{wL}{8} = \frac{5\times8}{8} = 5 \text{ kNm}$$

$$M_{FDB} = \frac{-wL}{8} = -5 \text{ KNm}$$

★ Note : BC is a cantilever beam,
moment $= 5\times2\times1 = 10$
$\therefore \quad M_{BC} = -10 \text{ KN-m}$

Step 2 : Slope - Deflection Equations :

$$M_{AB} = -125 + \frac{2EI}{10}(2\theta_A + \theta_B)$$

$$\therefore M_{AB} = -125 + 0.4\,EI\,\theta_A + 0.2\,EI\,\theta_B \rightarrow (1)$$

Now, $M_{BA} = 125 + \frac{2EI}{10}(2\theta_B + \theta_A)$

$$\therefore \quad M_{BA} = 125 + 0.4\,EI\,\theta_B + 0.2\,EI\,\theta_A \rightarrow (2)$$

Now, $M_{BD} = 5 + \frac{2EI}{8}(2\theta_B)$

$$\therefore \quad M_{BD} = 5 + 0.5\,EI\,\theta_B \rightarrow (3)$$

Now, $M_{DB} = -5 + \frac{2EI}{8}(\theta_B)$

$$\therefore \quad M_{DB} = -5 + 0.25\,EI\,\theta_B \rightarrow (4)$$

Step 3 : Joint Equilibrium Equations :

$$\Sigma M_B = 0; \quad M_{BA} + M_{BC} + M_{BD} = 0$$

$$125 + 0.4\,EI\,\theta_B + 0.2\,EI\,\theta_A - 10 + 5 + 0.5\,EI\,\theta_B = 0$$

$$\therefore \quad 0.2\,EI\theta_A + 0.9\,EI\,\theta_B = -120 \rightarrow (A)$$

$$M_{AB} = 0;$$

$$-125 + 0.4\,EI\,\theta_A + 0.2\,EI\,\theta_B = 0$$

$$0.4\,EI\,\theta_A + 0.2\,EI\,\theta_B = 125 \rightarrow (B)$$

$$\therefore \quad \theta_A = \frac{426.56}{EI}; \quad \theta_B = \frac{-228.12}{EI}$$

Step 4 : Final Moments :

$$M_{AB} = -125 + 0.4\,EI\left(\frac{426.56}{EI}\right) + 0.2\,EI\left(\frac{-228.12}{EI}\right)$$

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$\therefore M_{AB} = 0.0 \text{ kNm}$

Now, $M_{BA} = 125 + 0.4\,EI\left(\frac{-228.12}{EI}\right) + 0.2\,EI\left(\frac{426.56}{EI}\right)$

$\therefore M_{BA} = 119 \text{ kNm}$

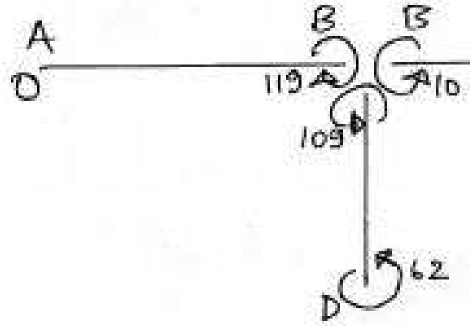
$M_{BC} = -10 \text{ kNm}$

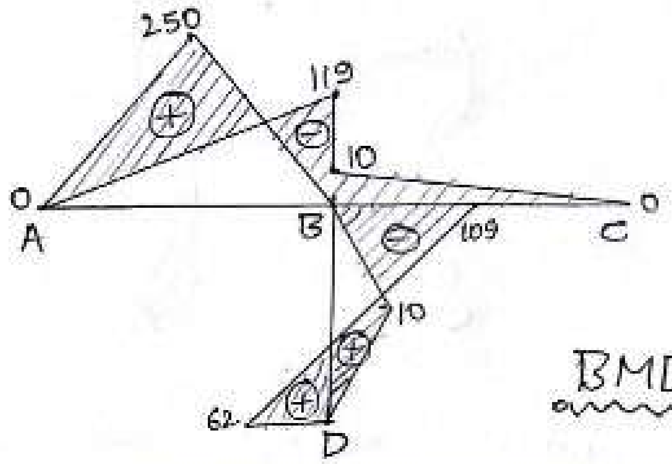
Now, $M_{BD} = 5 + 0.5\,EI\left(\frac{-228.12}{EI}\right)$

$= -109 \text{ kNm}$

Now, $M_{DB} = -5 + 0.25\left(\frac{-228.12}{EI}\right)EI$

$\therefore M_{DB} = -62 \text{ kNm}$





(i) $\frac{\omega L}{4} = \frac{1000 \times 10}{4}$

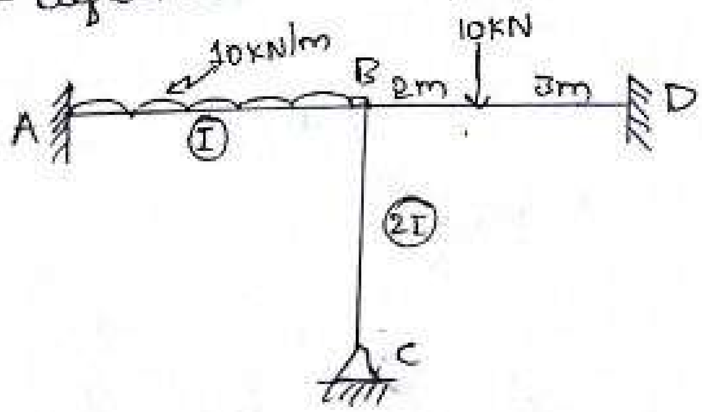
$= 250$

(ii) $\frac{WL}{4} = \frac{5 \times 8}{4}$

$= 10$

BMD

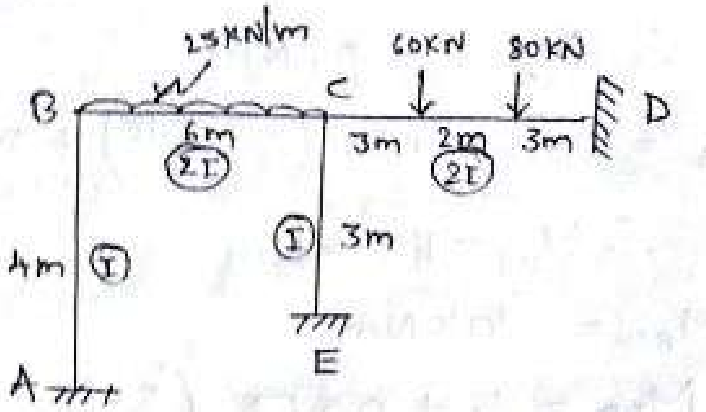
3. Analyse the frame shown in the figure by slope-deflection method.



$\rightarrow$ DOF $= 2$ (θ_B, θ_C)

27/08/18

4.



→ Here DOF = 2 (θ_B, θ_C)

Step 1: Fixed End Moments:

$$M_{FAB} = \frac{-WL}{8} = \frac{-0\times4}{8} = 0 \;;\; M_{FBA} = 0$$

$$M_{FBC} = \frac{-wL^2}{12} = \frac{-25\times6^2}{12} = -75\text{ kNm}$$

$$M_{FCB} = 75\text{ KN-m}$$

$$M_{FCD} = \frac{-Wab^2}{L^2} - \frac{Wab^2}{L^2}$$

$$= \frac{-60\times3\times5^2}{8^2} - \frac{80\times5\times3^2}{8^2}$$

$$\therefore M_{FCD} = -126.56\text{ KNm}$$

$$M_{FDC} = \frac{Wa^2b}{L^2} + \frac{Wa^2b}{L^2} = \frac{60\times3^2\times5}{8^2} + \frac{80\times5^2\times3}{8^2}$$

$$\therefore M_{FDC} = 135.94\text{ KNm}$$

$$M_{FCE} = 0 \;;\; M_{FEC} = 0$$

Step 2: Slope - Deflection Equations:

$$M_{AB} = 0 + \frac{2EI}{4}(\theta_B)$$

$$\therefore M_{AB} = 0.5\,EI\,\theta_B \rightarrow (1)$$

$$M_{BA} = 0 + \frac{2EI}{4}(2\theta_B)$$

$$\therefore M_{BA} = 1.0\,EI\,\theta_B \rightarrow (2)$$

$$M_{BC} = -75 + \frac{2E(2I)}{6}(2\theta_B + \theta_C)$$

$$\therefore M_{BC} = -75 + 1.33\,EI\,\theta_B + 0.67\,EI\,\theta_C \rightarrow (3)$$

Now, $$M_{CB} = 75 + \frac{2E(2I)}{6}(2\theta_C + \theta_B)$$

$$\therefore M_{CB} = 75 + 1.33\,EI\,\theta_C + 0.67\,EI\,\theta_B \rightarrow (4)$$

$$M_{CD} = -126.56 + \frac{2E(2I)}{8}(2\theta_c)$$

$$\therefore M_{CD} = -126.56 + 1.0\,EI\,\theta_c \rightarrow (5)$$

$$M_{DC} = 135.94 + \frac{2E(2I)}{8}(\theta_c)$$

$$\therefore M_{DC} = 135.94 + 0.5\,EI\,\theta_c \rightarrow (6)$$

$$M_{CE} = 0 + \frac{2EI}{3}(2\theta_c)$$

$$M_{CE} = 1.33\,EI\,\theta_c \rightarrow (7)$$

Now, $M_{EC} = 0 + \frac{2EI}{3}(\theta_c)$

$$\therefore M_{EC} = 0.67\,EI\,\theta_c \rightarrow (8)$$

Step 3: Joint Equilibrium Equations:

$$\Sigma M_B = 0; \quad M_{BA} + M_{BC} = 0$$

$$1.0\,EI\,\theta_B + (-75) + 1.33\,EI\theta_B + 0.67\,EI\,\theta_c \rightarrow A$$

$$2.33\,EI\,\theta_B + 0.67\,EI\,\theta_c = 75 \rightarrow (A)$$

$$\Sigma M_c = 0; \quad M_{CB} + M_{CD} + M_{CE} = 0$$

$$75 + 1.33\,EI\theta_c + 0.67\,EI\,\theta_B - 126.56 + 1.0\,EI\,\theta_c + 1.33\,EI\theta_c = 0$$

$$0.67\,EI\theta_B + 3.33\,EI\,\theta_c = 51.56 \rightarrow (B)$$

$$\therefore \quad \theta_B = \frac{29.7}{EI} \quad ; \quad \theta_c = \frac{8.65}{EI}$$

Step 4: Final Moments:

$$M_{AB} = 0.5\,EI\left(\frac{29.7}{EI}\right) = 14.8\text{ KNm}$$

$$M_{BA} = 1.0\,EI\left(\frac{29.7}{EI}\right) = 29.7\text{ KNm}$$

$$M_{BC} = -75 + 1.33\,EI\left(\frac{29.7}{EI}\right) + 0.67\,EI\left(\frac{8.65}{EI}\right) = -29.7\text{ KNm}$$

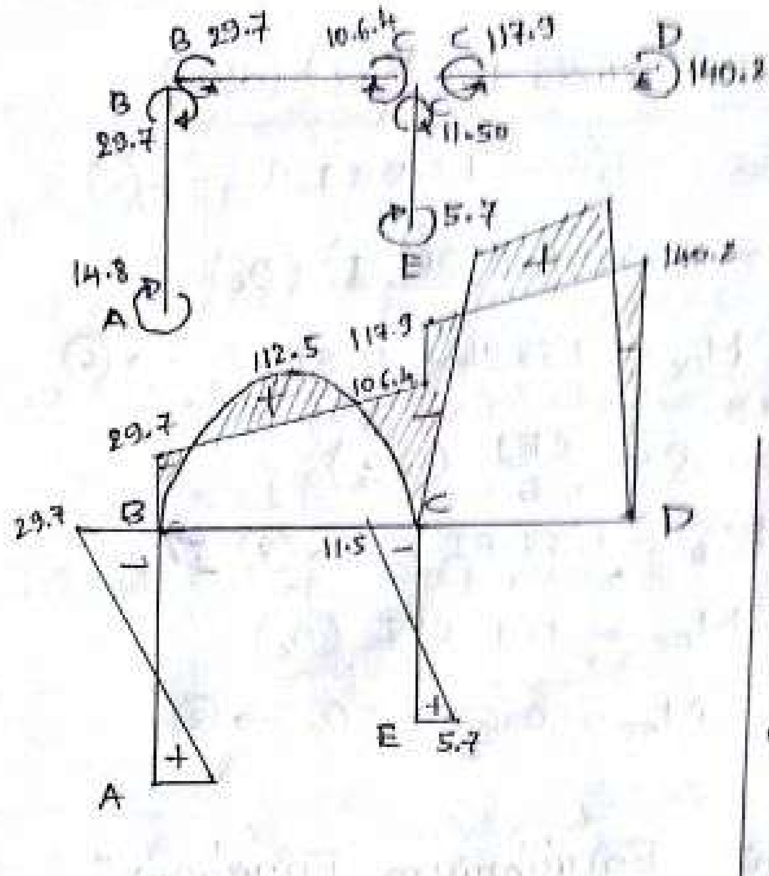
$$M_{CB} = 75 + 1.33\,EI\left(\frac{8.65}{EI}\right) + 0.67\,EI\left(\frac{29.7}{EI}\right) = 106.4\text{ KNm}$$

$$M_{CD} = -117.9\text{ KNm}$$

$$M_{DC} = 140.2\text{ KNm}$$

$$M_{CE} = 11.50\text{ KNm}$$

$$M_{EC} = 5.7\text{ KNm}$$



BMD

$\Sigma M_c = 0;$

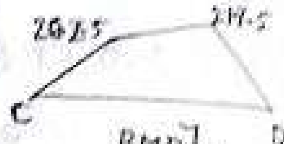
$60 \times 3 + 80 \times 3 - R_D \times 3 = 0$

$R_D = 72.5$ kN

$\Sigma V = 0;$

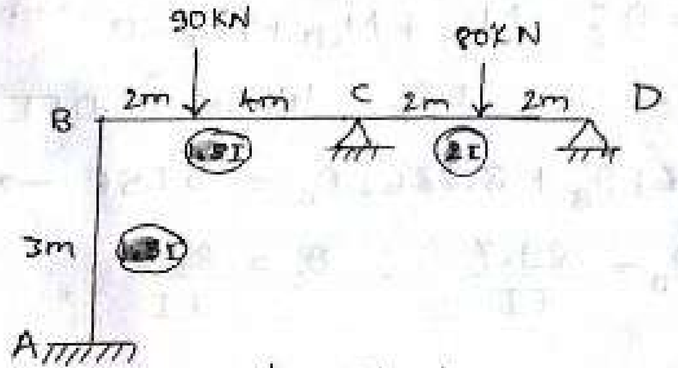
$R_c - 60 - 80 + 72.5 = 0$

$R_c = 67.5$



$BMD]_{CD}$

5.



Draw BMD and Elastic curve.

$\rightarrow$ DOF = 3 $(\theta_B, \theta_C, \theta_D)$

1: $M_{FAB} = 0$; $M_{FBA} = 0$

$$M_{FBC} = \frac{-wab^2}{L^2} = \frac{-90 \times 2 \times 4^2}{6^2} = -80 \text{ KNm}$$

$$M_{FCB} = \frac{+wa^2b}{L^2} = \frac{+90 \times 2^2 \times 4}{6^2} = 40 \text{ KNm}$$

$$M_{FCD} = \frac{-wL}{8} = \frac{-80 \times 4}{8} = -40 \text{ KNm}$$

$$M_{FDC} = \frac{wL}{8} = \frac{8 \times 4}{8} = +40 \text{ KNm}$$

Scanned with CamScanner

Step 2 : Slope - Deflection Equations :

$$M_{AB} = 0 + \frac{2EI}{3}(2\theta_A + \theta_B - 0)$$

$$\therefore M_{AB} = 0.67\, EI\, \theta_A \rightarrow (1)$$

$$M_{BA} = 0 + \frac{2EI}{3}(2\theta_B + 0 - 0)$$

$$\therefore M_{BA} = 1.34\, EI\, \theta_B \rightarrow (2)$$

$$M_{BC} = -80 + \frac{2EI}{6}(2\theta_B + \theta_C - 0)$$

$$M_{BC} = -80 + 0.67\, EI\theta_B + 0.33\, EI\, \theta_C \rightarrow (3)$$

$$M_{CB} = 40 + \frac{2EI}{6}(2\theta_C + \theta_B - 0)$$

$$\therefore M_{CB} = 40 + 0.67\, EI\, \theta_C + 0.33\, EI\, \theta_B \rightarrow (4)$$

$$M_{CD} = -40 + \frac{2EI}{4}(2\theta_C + \theta_D - 0)$$

$$\therefore M_{CD} = -40 + 1.0\, EI\theta_C + 0.5\, EI\, \theta_D \rightarrow (5)$$

$$M_{DC} = 40 + \frac{2EI}{4}(2\theta_D + \theta_C - 0)$$

$$\therefore M_{DC} = 40 + 1.0\, EI\, \theta_D + 0.5\, EI\theta_C \rightarrow (6)$$

Step 3 : Joint Equilibrium Equations :

$$\Sigma M_B = 0 \; ; \quad M_{BA} + M_{BC} = 0$$

$$1.34\, EI\, \theta_B - 80 + 0.67 EI\theta_B + 0.33\, EI\, \theta_C = 0$$

$$2.01\, EI\, \theta_B + 0.33\, EI\, \theta_C = 80 \rightarrow (A)$$

$$\Sigma M_C = 0 ; \quad M_{CB} + M_{CD} = 0$$

$$40 + 0.67\, EI\theta_C + 0.33\, EI\, \theta_B - 40 + 1\, EI\, \theta_C + 0.5\, EI\, \theta_D = 0$$

$$0.33\, EI\theta_B + 1.67\, EI\theta_C + 0.5 EI\theta_D = 0 \rightarrow (B)$$

$$M_{DC} = 0 ;$$

$$\cancel{40}\; 0.5\, EI\, \theta_C + 1.0\, EI\, \theta_D = -40 \rightarrow (C)$$

$$\therefore \theta_B = \frac{38.97}{EI} \; ; \; \theta_C = \frac{5.03}{EI} \; ; \; \theta_D = \frac{-42.5}{EI}$$

Step 4: Final Moments :

$$M_{AB} = 0.67\,EI\left(\frac{38.97}{EI}\right) = 26.11\ KNm$$

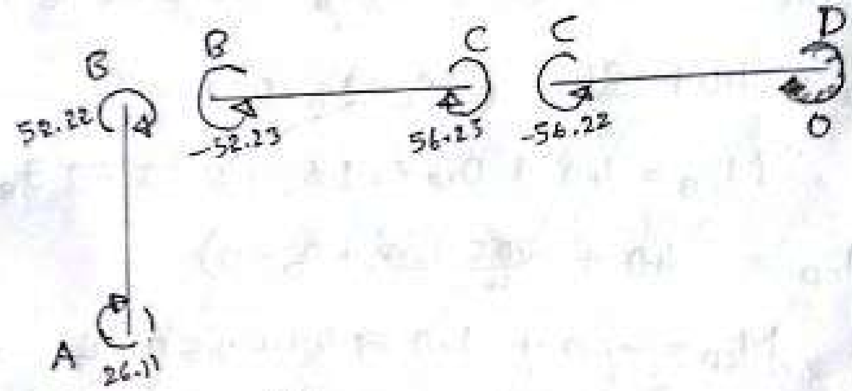
$$M_{BA} = 1.34\,EI\left(\frac{38.97}{EI}\right) = 52.22\ KNm$$

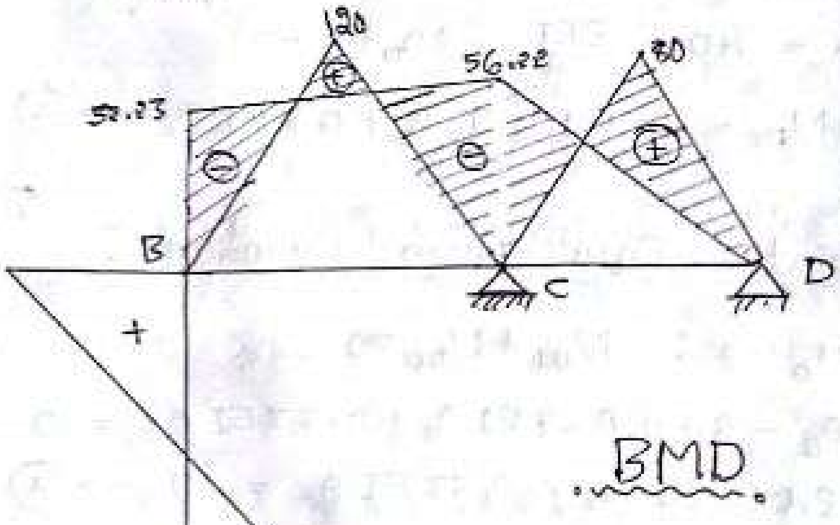
$$M_{BC} = -80 + 0.67\,EI\left(\frac{38.97}{EI}\right) + 0.33\,EI\left(\frac{5.03}{EI}\right) = -52.23\ KNm$$

$$M_{CB} = 40 + 0.67\,EI\left(\frac{5.03}{EI}\right) + 0.33\,EI\left(\frac{38.97}{EI}\right) = 56.23\ KNm$$

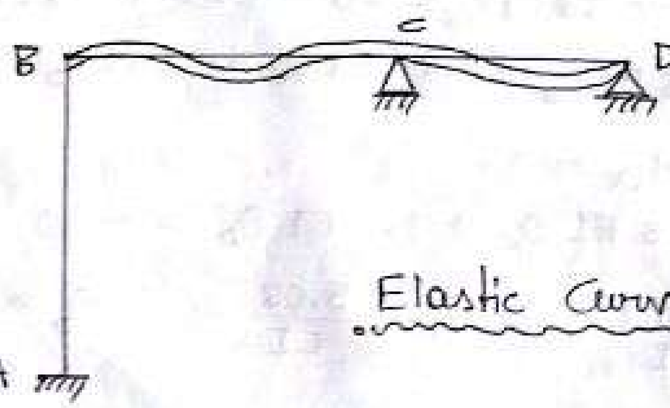
$$M_{CD} = -40 + EI\left(\frac{5.03}{EI}\right) + 0.5\,EI\left(\frac{-42.5}{EI}\right) = -56.22\ KNm$$

$$M_{DC} = 40 + EI\left(\frac{-42.5}{EI}\right) + 0.5\,EI\left(\frac{5.03}{EI}\right) = 0\ KNm$$



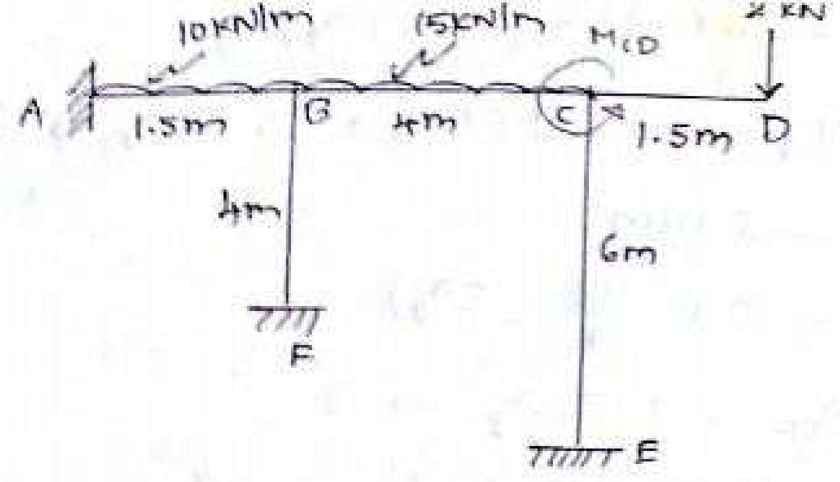


BMD



Elastic Curve

6.



→ DOF = 2 (θ_B, θ_C)

Here $M_{CD} = -2 \times 1.5 = -3$ KNm

Step 1: Fixed End Moments:

$$M_{FAB} = \frac{-wL^2}{12} = \frac{-10 \times (1.5)^2}{12} = -1.875 \text{ KNm}$$

$$M_{FBA} = \frac{wL^2}{12} = \frac{10 \times (1.5)^2}{12} = 1.875 \text{ KNm}$$

$$M_{FBC} = \frac{-wL^2}{12} = \frac{-15 \times 4^2}{12} = -20 \text{ KNm}$$

$$M_{FCB} = \frac{wL^2}{12} = \frac{15 \times 4^2}{12} = 20 \text{ KNm}$$

$M_{FBF} = 0$

$M_{FFB} = 0$

$M_{FCE} = 0$

$M_{FEC} = 0$

Step 2: Slope-deflection Equations:

$$M_{AB} = -1.87 + \frac{2EI}{1.5}(0 + 2\theta_B - 0)$$

$$M_{AB} = -1.87 + 1.33 EI \theta_B \rightarrow (1)$$

$$M_{BA} = 1.87 + \frac{2EI}{1.5}(2\theta_B - 0 - 0)$$

$$M_{BA} = 1.87 + 2.67 \theta_B EI \rightarrow (2)$$

$$M_{BC} = -20 + \frac{2EI}{4}(2\theta_B + \theta_C)$$

$$\therefore M_{BC} = -20 + 1 EI \theta_B + 0.5 EI \theta_C \rightarrow (3)$$

$$M_{CB} = 20 + \frac{2EI}{4}(2\theta_c + \theta_B)$$

$$\therefore M_{CB} = 20 + 1EI\theta_c + 0.5EI\theta_B \rightarrow (4)$$

$$M_{CD} = -3 \text{ kNm}$$

$$M_{BF} = 0 + \frac{2EI}{4}(2\theta_B)$$

$$M_{BF} = 1.EI\theta_B \rightarrow (5)$$

$$M_{FB} = 0 + \frac{2EI}{4}(\theta_B)$$

$$\therefore M_{FB} = 0.5\ EI\theta_B \rightarrow (6)$$

$$M_{CE} = 0 + \frac{2EI}{6}(2\theta_c)$$

$$M_{CE} = 0.67\ EI\theta_c \rightarrow (7)$$

$$|||^{ly},\ M_{EC} = 0 + \frac{2EI}{6}(\theta_c)$$

$$M_{EC} = 0.33\ EI\theta_c \rightarrow (8)$$

Step 3: Joint Equilibrium Equations:

$$\Sigma M_B = 0 \ ; \quad M_{BA} + M_{BC} + M_{BF} = 0$$

$$1.87 + 2.67\ EI\theta_B - 20 + EI\theta_B + 0.5\ EI\theta_c + EI\theta_B = 0$$

$$4.67\ EI\theta_B + 0.5\ EI\theta_c = 18.43 \rightarrow (A)$$

Now, $\Sigma M_C = 0 ; \quad M_{CB} + M_{CD} + M_{CE} = 0$

$$20 + EI\theta_c + 0.5\ EI\theta_B - 3 + 0.67\ EI\theta_c = 0$$

$$0.5\ EI\theta_B + 1.67\ EI\theta_c = -17 \rightarrow (B)$$

$\therefore$ From (A) and (B),

$$\therefore \theta_B = \frac{5.20}{EI} \ ; \ \theta_c = \frac{-11.74}{EI}$$

Step 4: Final Moments:

$$\therefore M_{AB} = -1.87 + 1.33\ EI\left(\frac{5.20}{EI}\right) = 5.05 \text{ KNm}$$

$$M_{BA} = 1.87 + 2.67\ EI\left(\frac{5.20}{EI}\right) = 15.75 \text{ KNm}$$

$$M_{BC} = -20 + EI\left(\frac{5.2}{EI}\right) + 0.5EI\left(\frac{-11.74}{EI}\right) = -20.7 \text{ KN}$$

$$M_{CB} = 20 + EI\left(\frac{-11.74}{EI}\right) + 0.5\,EI\left(\frac{5.2}{EI}\right) = 10.9\ \text{KNm}$$

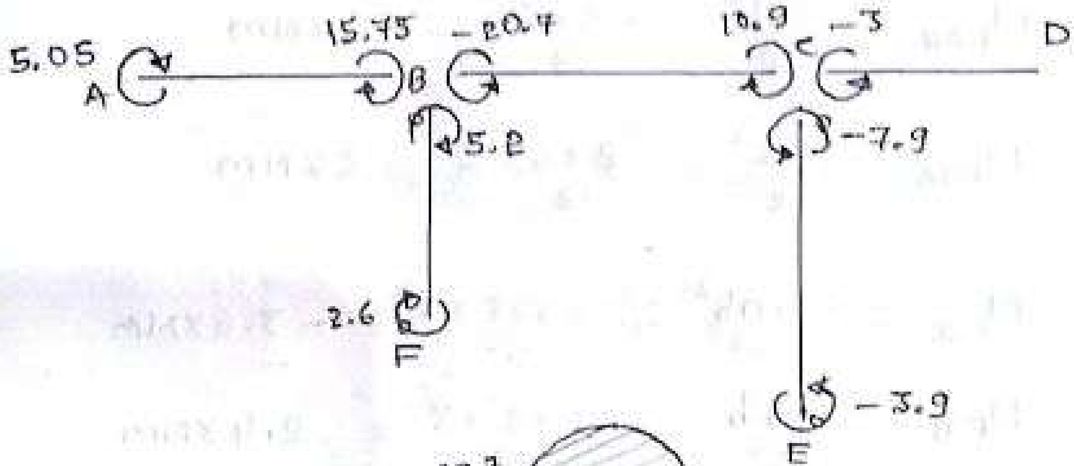
$$M_{CD} = -3\ \text{KNm}$$

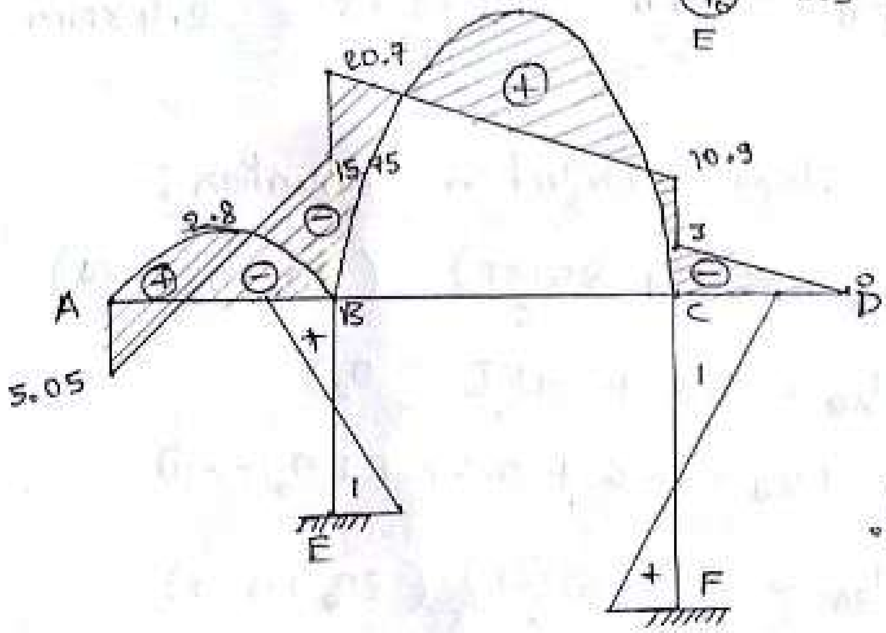
$$M_{BF} = EI\left(\frac{5.20}{EI}\right) = 5.20\ \text{KNm}$$

$$M_{FB} = 0.5\,EI\left(\frac{5.20}{EI}\right) = 2.6\ \text{KNm}$$

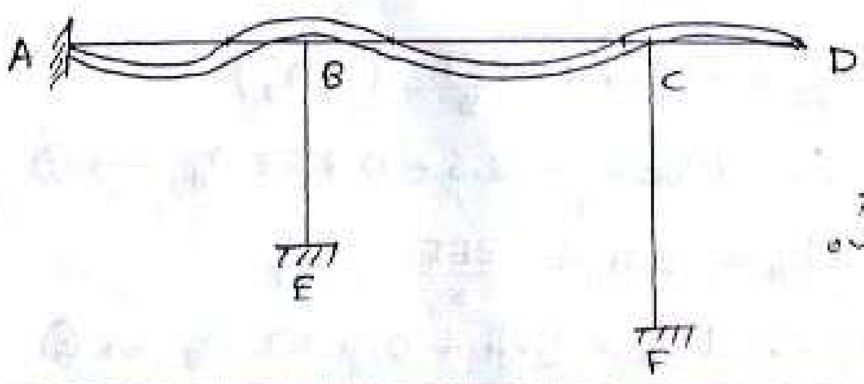
$$M_{CE} = 0.67\,EI\left(\frac{-11.74}{EI}\right) = -7.9\ \text{KNm}$$

$$M_{EC} = 0.33\,EI\left(\frac{-11.74}{EI}\right) = -3.9\ \text{KNm}$$



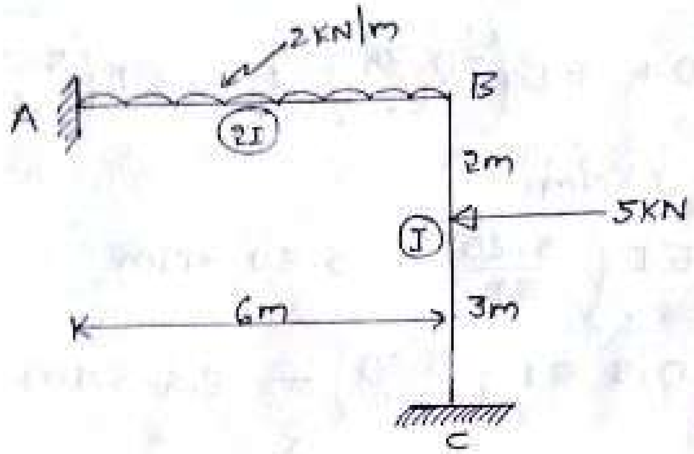


BMD



Elastic Curve

7.



→ DOF = 1 (θ_B)

Step I : Fixed End Moments :

$$M_{FAB} = \frac{-wL^2}{12} = \frac{-2 \times 6^2}{12} = -6\ KNm$$

$$M_{FBA} = \frac{wL^2}{12} = \frac{2 \times 6^2}{12} = 6\ KNm$$

$$M_{FBC} = -\frac{wab^2}{L^2} = \frac{-5 \times 2 \times 3^2}{5^2} = -3.6\ KNm$$

$$M_{FCB} = \frac{wa^2b}{L^2} = \frac{5 \times 2^2 \times 3}{5^2} = 2.4\ KNm$$

Step 2 : Slope - Deflection Equation :

$$M_{AB} = -6 + \frac{2E(2I)}{6}\left(2\theta_A + \theta_B - \frac{3\Delta}{L}\right)$$

$$M_{AB} = -6 + \frac{4EI}{6}(\theta_B)$$

$$\therefore\ M_{AB} = -6 + 0.67\ EI\ \theta_B \rightarrow (1)$$

$$M_{BA} = 6 + \frac{2E(2I)}{6}(2\theta_B + 0 - 0)$$

$$\therefore\ M_{BA} = 6 + 1.34\ EI\ \theta_B \rightarrow (2)$$

$$M_{BC} = -3.6 + \frac{2EI}{5}(2\theta_B)$$

$$\therefore\ M_{BC} = -3.6 + 0.8\ EI\ \theta_B \rightarrow (3)$$

$$M_{CB} = 2.4 + \frac{2EI}{5}(\theta_B)$$

$$\therefore\ M_{CB} = 2.4 + 0.4\ EI\ \theta_B \rightarrow (4)$$

Step 3 : Joint Equilibrium Equation :

$$\Sigma M_D = 0 ; \quad M_{BA} + M_{BC} = 0$$

$$6 + 1.34\ EI\theta_B - 3.6 + 0.8\ EI\ \theta_B = 0$$

$$2.4 + 2.14\ EI\theta_B = 0$$

$$2.14\ EI\ \theta_B = -2.4$$

$$\therefore \quad \theta_B = \frac{-1.12}{EI}$$

Step 4 : Final Moments :

$$M_{AB} = -6 + 0.67\ EI\left(\frac{-1.12}{EI}\right)$$

$$\therefore \quad M_{AB} = -6.7\ kNm$$

$$M_{BA} = 6 + 1.34\ EI\left(\frac{-1.12}{EI}\right)$$

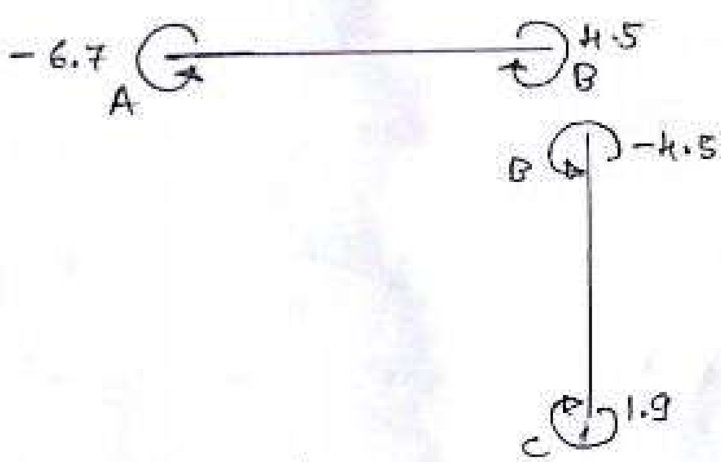
$$\therefore \quad M_{BA} = 4.5\ kNm$$

$$M_{BC} = -3.6 + 0.8\ EI\left(\frac{-1.12}{EI}\right)$$

$$\therefore \quad M_{BC} = -4.5\ kNm$$

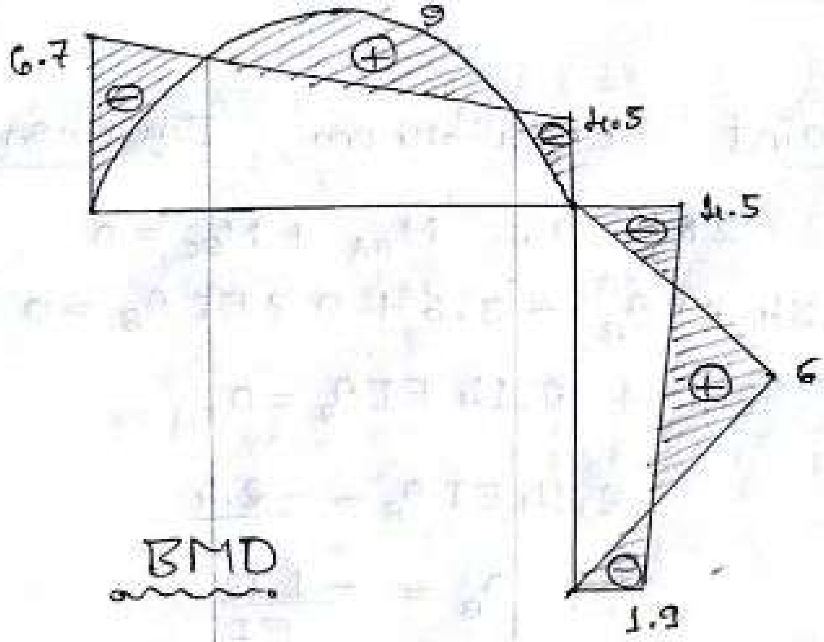
$$M_{CB} = 2.4 + 0.4\ EI\left(\frac{-1.12}{EI}\right)$$

$$\therefore \quad M_{CB} = 1.9\ kNm$$

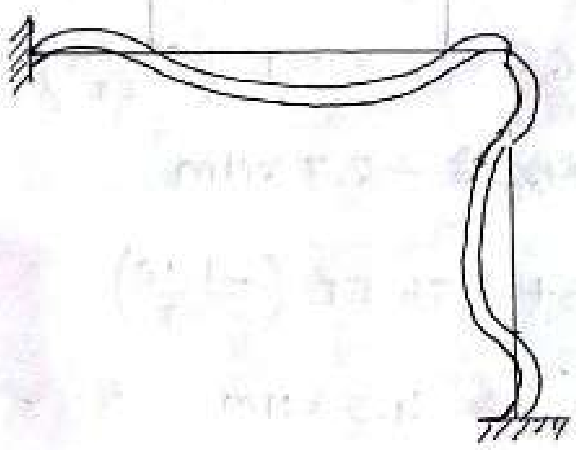


Final Moments

BMD → PTO



BMD



Elastic Curve

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