28/08/18

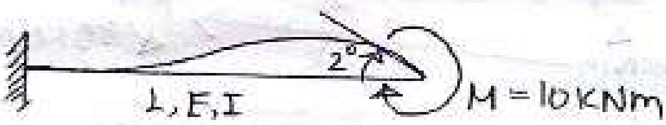
Module - 2 SUJITH. N. S

Moment Distribution Method

It is an another method to analyse statically Indeterminate structure introduced by Prof. Hardy, hence the method is also known as Hardy cross method.

Stiffness : (K) :-

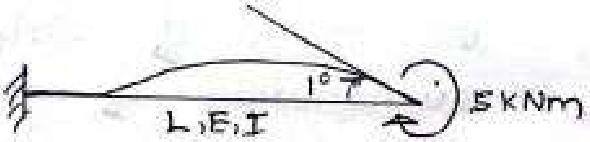
The moment required to produce 1° rotation is called stiffness. Stiffness is also defined as the ratio of moment of Inertia to length.



$2^\circ \longrightarrow 10\,KNm$

For $1^\circ \longrightarrow \frac{10}{2} = 5\,KN\text{-}m$

$\therefore \quad K = \left(\frac{M}{\theta}\right)$

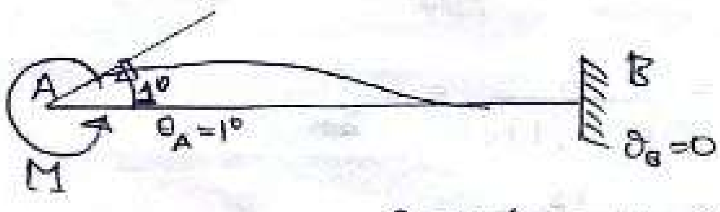


$K = \frac{M}{\theta} = \frac{5}{1^\circ}\,KNm$

OR

$K = \left(I/L\right) \quad \left[\because \begin{matrix} K \propto I \\ K \propto 1/L \end{matrix}\right]$

Consider a beam of length 'L', Young's modulus 'E', MOI 'I' subjected to a moment 'M' to produce 1° rotation as shown in the fig.



Now, $\quad M_{AB} = 0 + \frac{2EI}{L}\left(2\theta_A + 0 - 0\right)$

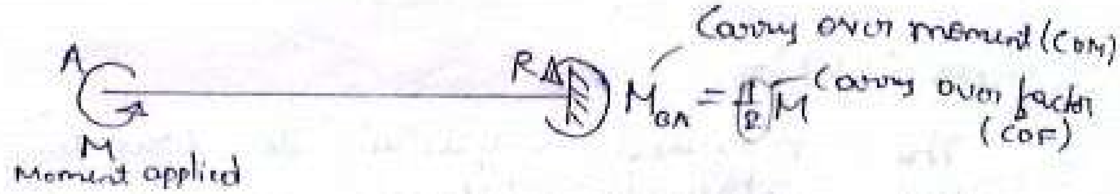
$\therefore \quad M = \frac{4EI}{L}$

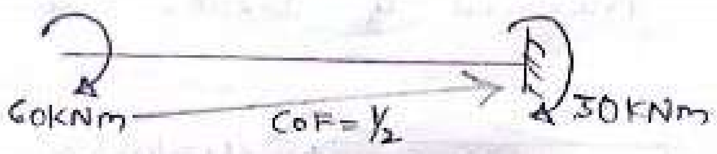
Now, $M_{BA} = 0 + \frac{2EI}{L}(2\times0 + 1 - 0)$

$\therefore M_{BA} = \frac{2EI}{L}$

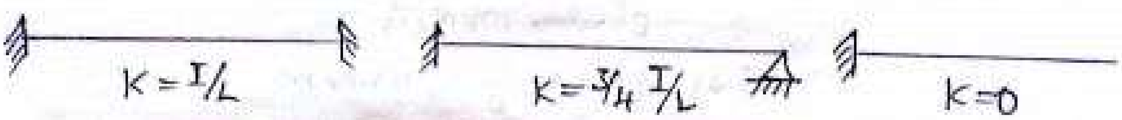
Consider. $\frac{M_{BA}}{M_{AB}} = \frac{M_{BA}}{M} = \frac{2EI/L}{4EI/L}$

$\frac{M_{BA}}{M_{AB}} = \frac{1}{2}$ @ $\boxed{M_{BA} = \frac{1}{2} M}$



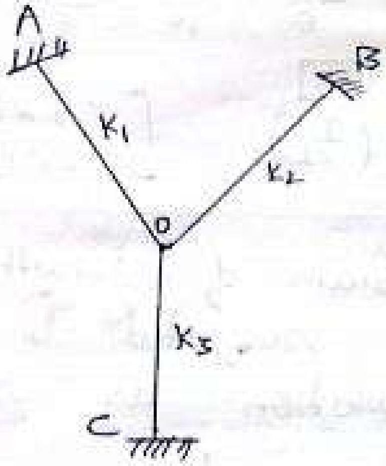


Note :



Distribution factor (DF) @ $\surd$:

It is the ratio of stiffness of one member to the summation of all the members meeting at the joint.



$\surd_{OA} = \frac{K_1}{K_1+K_2+K_3} = \frac{K_1}{\Sigma K}$

$\surd_{OB} = \frac{K_2}{\Sigma K}$

$\surd_{OC} = \frac{K_3}{\Sigma K}$

$$\therefore \; \delta_{OA} + \delta_{OB} + \delta_{OC} = \frac{K_1}{\Sigma K} + \frac{K_2}{\Sigma K} + \frac{K_3}{\Sigma K}$$

$$= \frac{K_1 + K_2 + K_3}{\Sigma K}$$

$$= \frac{\Sigma K}{\Sigma K} = 1$$

$$\boxed{\delta_1 + \delta_2 + \delta_3 = 1}$$

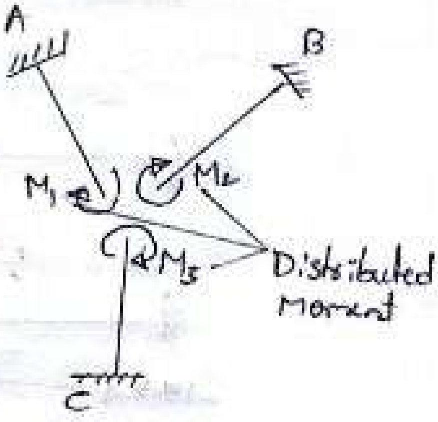
Sum of the distribution factor = 1

If a moment is applied at a joint, mome will be get distributed among the members based on the distribution factor.

$M_1 = \delta_1 M$

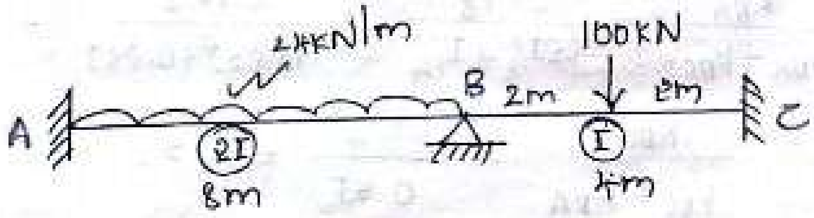
$M_2 = \delta_2 M$

$M_3 = \delta_3 M$



Problems:

1. Analyse the continuous beam by Moment-Distribution method. Draw BMD & SFD.



*Note: Distribution factors are to be calculated only at the joints

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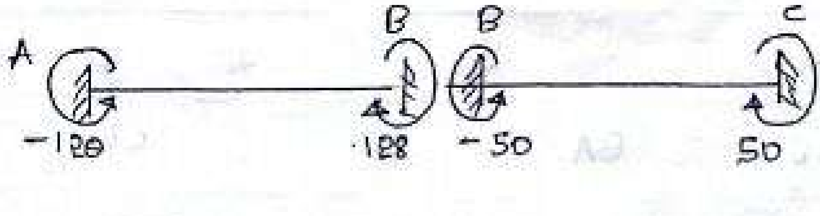


30/08/18

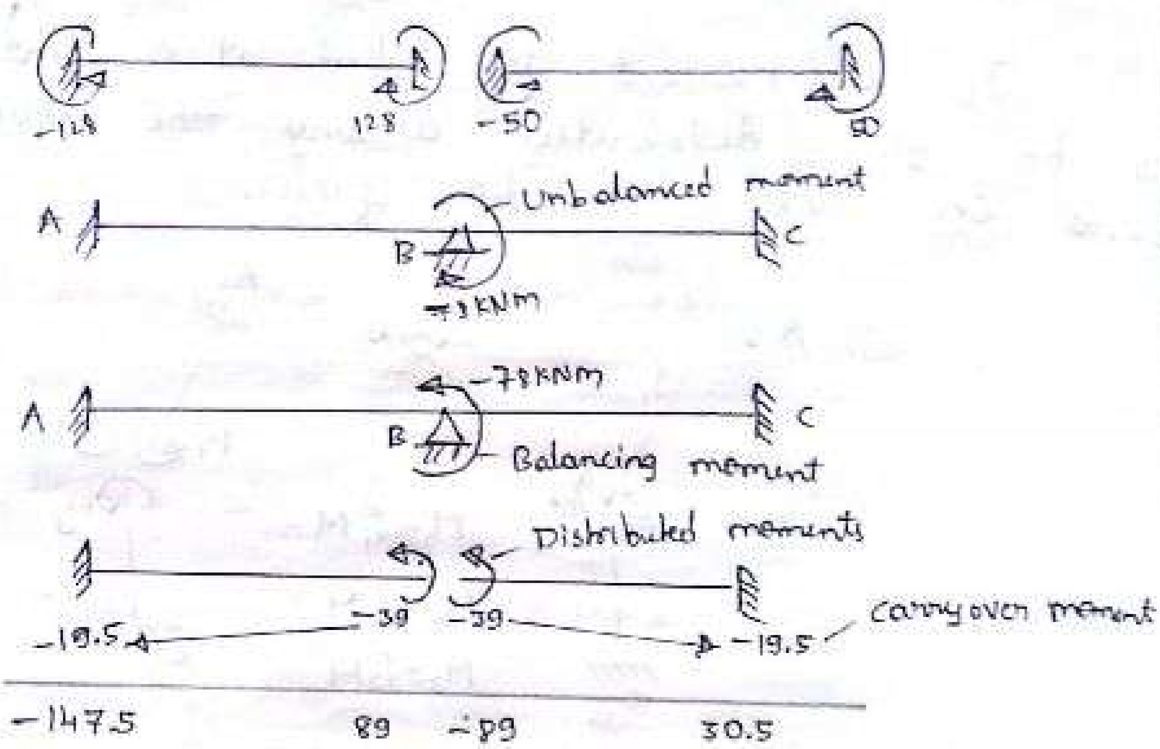
Step 1: Fixed End Moments;

$$M_{FAB} = \frac{-wl^2}{12} = -\frac{24\times8^2}{12} = -128\ KNm$$

$$M_{FBA} = \frac{wL^2}{12} = \frac{24\times8^2}{12} = 128\ KNm$$

$$M_{FBC} = \frac{-100\times4}{8} = -50\ KNm$$

$$M_{FCB} = 50\ KNm$$



Step 2: Distribution Factors;

@ Joint 'B',

$$\delta_{BA} = \frac{K_{BA}}{K_{BA}+K_{BC}} = \frac{2I/8}{2I/8+I/4} = \frac{0.25I}{0.25I+0.25I} = 0.5$$

$$\delta_{BC} = \frac{K_{BC}}{K_{BC}+K_{BA}} = \frac{I/4}{0.5I} = 0.5$$

Step 3: Moment - Distribution table:

Joint	A	B		C
Members	AB	BA	BC	CB
DF	–	0.5	0.5	–

$\delta_{AB} = \frac{K_{AB}}{\Sigma}$

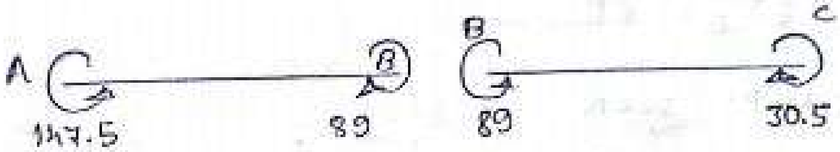
FEM	−128	128	[−78] −50	50
	−19.5	−39	−39	−19.5
Final moments KNm	−147.5	+89	−89	30.5

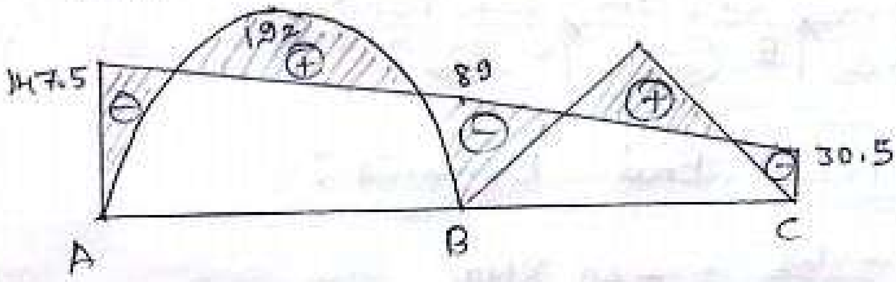
Balancing moment

Distributed moment

Carry over moment

1 cycle

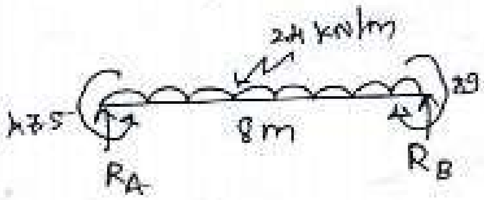




BMD

Elastic curve

To find SF :



$\Sigma M_A = 0$;

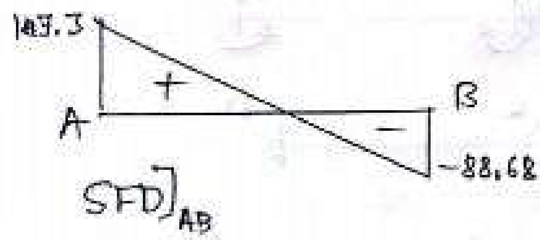
$-147.5 + (24\times8)4 - R_B\times8 + 89 = 0$

$\therefore R_B = 88.68$ KN

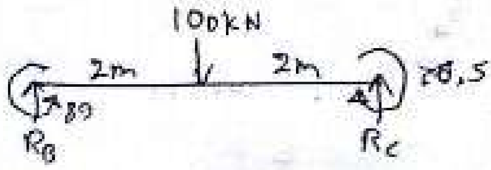
$\Sigma V = 0$;

$R_A - (24\times8) + 88.68 = 0$

$R_A = 103.3$ KN



SFD]$_{AB}$



$\Sigma M_B = 0$;

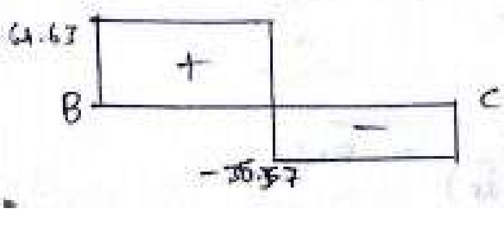
$-89 + 100\times2 + 30.5 - R_c\times4 = 0$

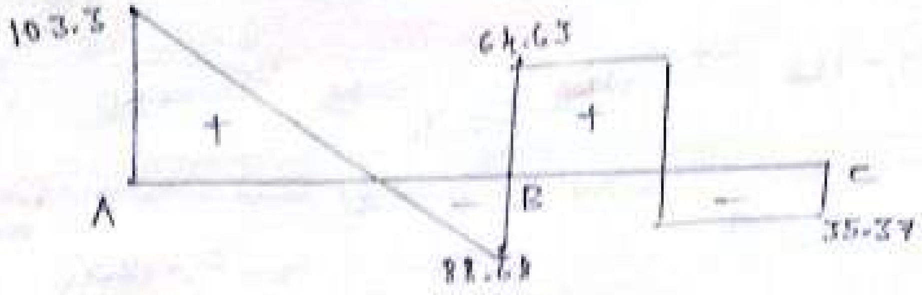
$R_c = 35.37$ KN

$\Sigma V = 0$;

$R_B - 100 + 35.37 = 0$

$R_B = 64.63$ KN

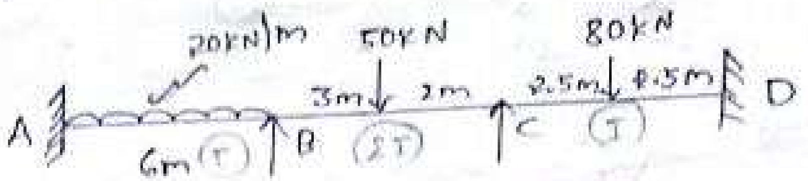




SFD

P. Draw BMD by moment distribution method.

$2I_{AB} = I_{BC} = 2I_{CD} = 2I$



→ Step 1: Fixed End Moments:

$M_{FAB} = \frac{-wL^2}{12} = -60$ KNm

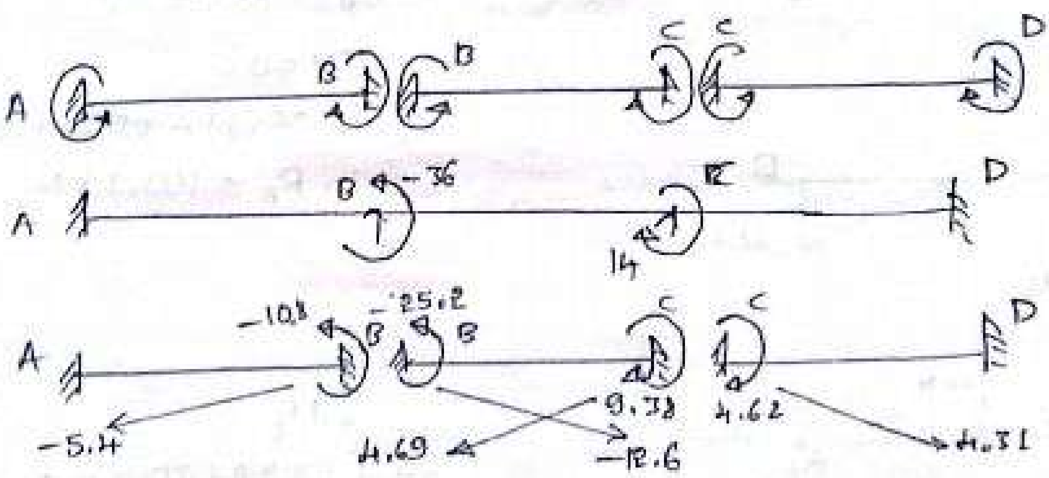
$M_{FBA} = \frac{wL^2}{12} = 60$ KNm

$M_{FBC} = \frac{-wab^2}{L^2} = -24$ KNm

$M_{FCB} = \frac{wa^2b}{L^2} = 36$ KNm

$M_{FCD} = \frac{-WL}{8} = -150$ KNm

$M_{FDC} = \frac{WL}{8} = 50$ KNm



Step 2: DF's:

(a) Joint 'B',

$\delta_{BA} = \frac{K_{BA}}{K_{BA}+K_{BC}} = \frac{I/6}{I/6 + 2I/5} = \frac{(1/6)}{(1/6+2/5)} = 0.3$

Scanned with CamScanner

$$\delta_{BC} = \frac{K_{BC}}{K_{BA}+K_{BC}} = \frac{2I/5}{I/6+2I/5} = 0.7$$

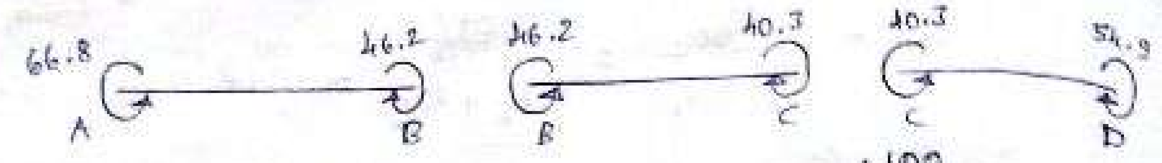
@ Joint 'C';

$$\delta_{CB} = \frac{K_{CB}}{K_{CB}+K_{CD}} = \frac{2I/5}{2I/5+I/5} = 0.67$$

$$\delta_{CD} = \frac{K_{CD}}{K_{CB}+K_{CD}} = \frac{I/5}{2I/5+I/5} = 0.33$$

Step 3 : Moment distribution table :

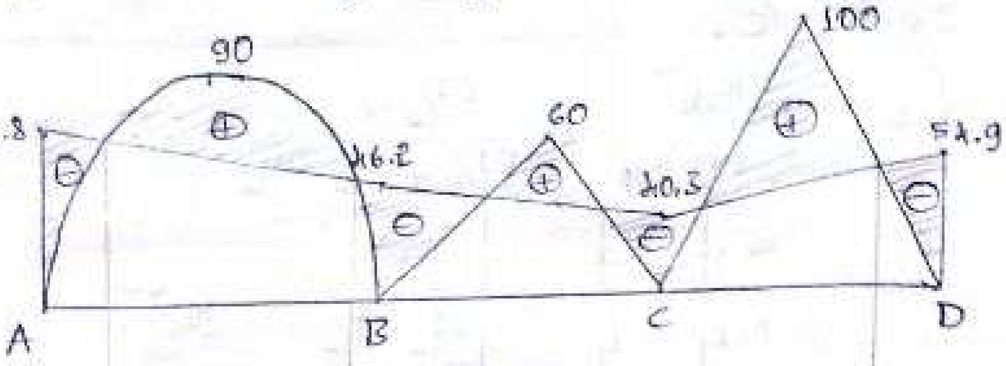
Joint	A	B		C		D	
Members	AB	BA	BC	CB	CD	DC	
DF	—	0.3	0.7	0.67	0.33	—	
FEM	-60	60	-24 [-36]	36	[14] -50	50	
	-5.4	-10.8 [-4.69]	-25.2 4.69	9.38 -12.6	4.62 [12.6]	2.31	1th cycle
	-0.7	-1.407 [-4.22]	-3.283 4.22	8.44 -1.64	4.15 [1.64]	2.07	2th cycle
	-0.63	-1.26 [-0.54]	-2.95 0.54	1.09 -1.47	0.54 [1.47]	0.27	3th cycle
	-0.08	-0.162 [-0.49]	-0.378 0.49	0.985 -0.129	0.485 [0.129]	0.24	4th cycle
	-0.073	-0.147	-0.34 +0.063	0.126 -0.17	0.06	0.03	5th cycle
Final Moments	-66.8	46.2	-46.2	40.3	-40.3	54.94	



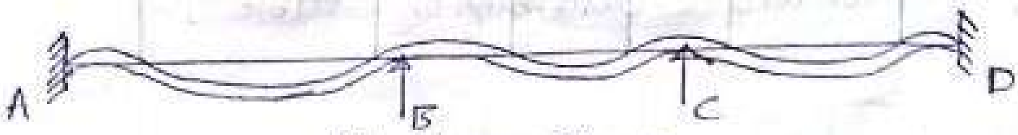
$\frac{WL^2}{8}$

$\frac{Wab}{L}$

$\frac{WL}{4}$



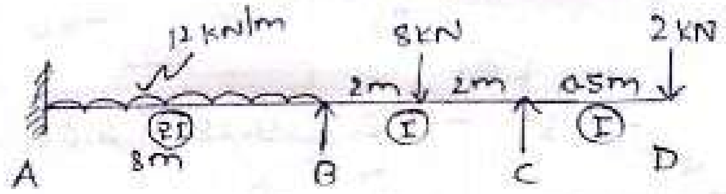
BMD



Elastic Curve

03/09/18

3. Analyse the continuous beam loaded as shown in the figure. Draw BMD and SFD



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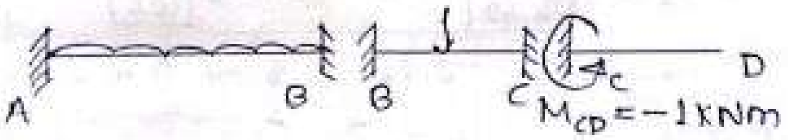


Step 1: Fixed End Moments:

$$M_{FAB} = \frac{-12 \times 8^2}{12} = -64 \text{ kNm}$$

$$M_{FBA} = 64 \text{ kNm}$$

$$M_{FBC} = \frac{-WL}{8} = \frac{-8 \times 4}{8} = -4 \text{ kNm}$$

$$M_{FCB} = 4 \text{ kNm}$$

Step 2: Distribution factor:

@ Joint B, $\delta_{BA} = \frac{K_{BA}}{K_{BA} + K_{BC}} = \frac{2I/8}{2I/8 + I/4} = 0.5$

$$\delta_{BC} = \frac{K_{BC}}{K_{BA} + K_{BC}} = \frac{I/4}{2I/8 + I/4} = 0.5$$

@ Joint C.

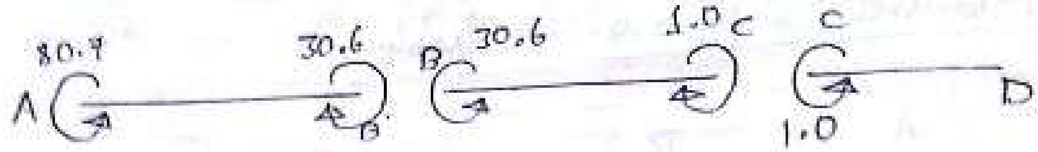
$$DF_{CB} = \frac{K_{CB}}{K_{CB}+K_{CD}} = \frac{I/4}{I/4+0} = 1.0$$

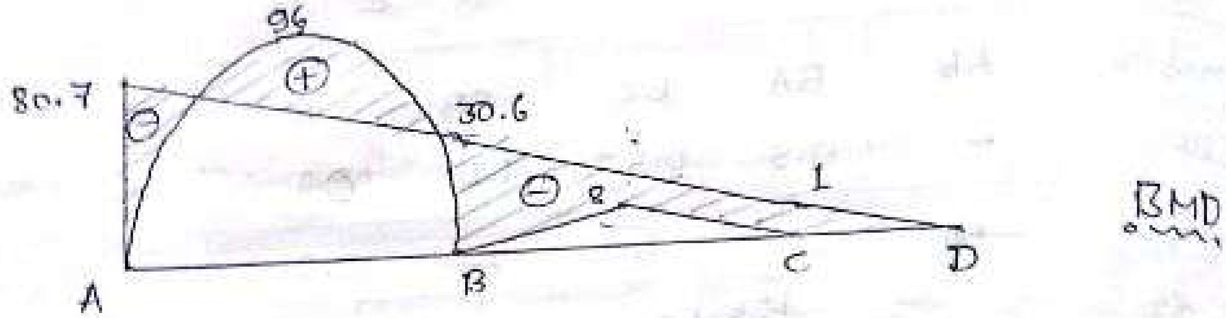
$$DF_{CD} = \frac{K_{CD}}{K_{CB}+K_{CD}} = \frac{0}{K_{CB}+K_{CD}} = 0.0$$

Step 3: Moment distribution table:

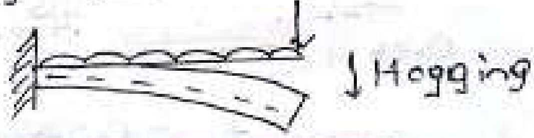
Joint	A	B		C	
Members	AB	BA	BC	CB	CD
DF	–	0.5	0.5	1.0	0.0
FEM	−64	64 [−60]	−4	4 [−3.0]	−1.0
	−30 −15	−30 [1.5]	−30 −1.5	−3.0 −15 [15]	0.0
	0.375	0.75 [−7.5]	0.75 7.5	15.0 0.375 [−0.375]	0.0
	−1.875	−3.15 [0.1875]	−3.75 −0.1875	−0.375 −1.875 [1.875]	0.0
	0.045	0.09 [−0.9]	−0.09 0.9	1.875 0.045 [−0.045]	0.0
	−0.225	−0.45 [0.022]	−0.45 −0.0225	−0.045 −0.225 [0.225]	0.0
	0.005	0.011 [−0.112]	0.011 0.112	0.225 0.005 [−0.005]	0.0 0.0
	−0.0551 −0.022	−0.0551	−0.0551 −0.002	−0.005 −0.022	0.0

Final Moment	-80.7	30.6	-30.6	1.0	-1.0



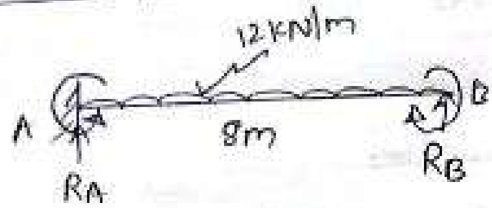


Note: There is no sagging BM in the cantilever. Only Hogging BM occurs.



To draw SFD:

(i)



$\Sigma M_A = 0;$

$-80.7 - (96 \times 4) + 30.6 - R_B \times 8 = 0$

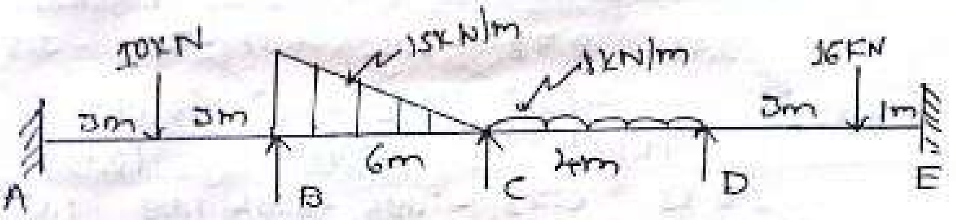
$R_B = -51.26$ kN

$\Sigma M = 0;$

$R_A - 96 - 51.26 = 0$

$\therefore R_A = 147.26$ kN

4.



→ <u>Step 1:</u> <u>Fixed End moments:</u>

$M_{FAB} = -7.5\ KNm$

$M_{FBA} = 7.5\ KNm$

$M_{FBC} = \frac{-15 \times 6^2}{20} = -27\ KNm$

$M_{FCB} = \frac{15 \times 6^2}{30} = 18\ KNm$

$M_{FCD} = \frac{-8 \times 4^2}{12} = -10.67\ KNm$

$M_{FDC} = 10.67\ KNm$

$M_{FDE} = \frac{-16 \times 3 \times 1^2}{4^2} = -3.0\ KNm$

$M_{FED} = \frac{16 \times 3^2 \times 1}{4^2} = 9.0\ KNm$

<u>Step 2:</u> <u>DF:</u> @ Joint 'B'.

$\delta_{BA} = \frac{I/6}{I/6 + I/6} = 0.5 \qquad \delta_{BC} = 0.5$

@ Joint 'C',

$$\delta_{CB} = \frac{I/6}{I/6 + I/4} = 0.4$$

$$\delta_{CD} = \frac{I/4}{I/6 + I/4} = 0.6$$

@ Joint 'D',

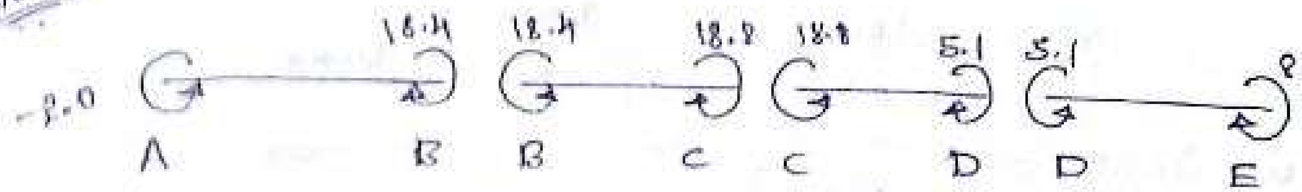
$$\delta_{DC} = \frac{K_{DC}}{K_{DC} + K_{DE}} = \frac{I/4}{I/4 + I/4} = 0.5$$

$$\delta_{DE} = 0.5$$

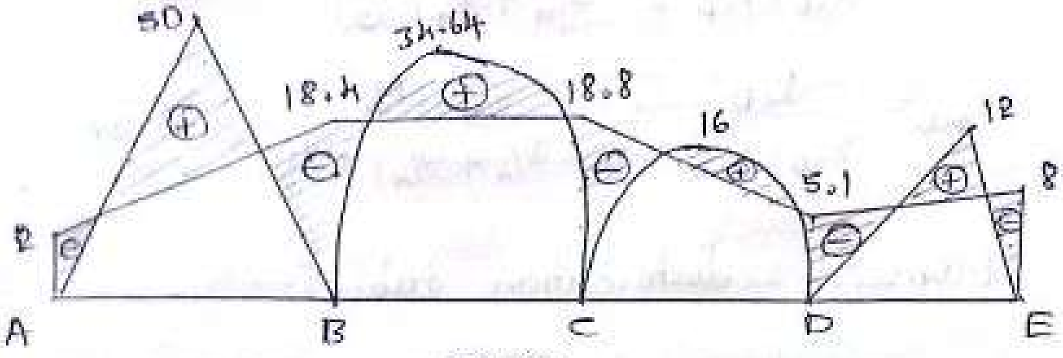
Step 3 : Distribution table :

Joint	A	B		C		D		E
Members	AB	BA	BC	CB	CD	DC	DE	ED
DF	–	0.5	0.5	0.4	0.6	0.5	0.5	–
FEM	-7.5	7.5	-27 [19.5]	18	-10.67 [-7.3]	10.67	-3.0 [-7.63]	9.0
	4.89	9.78	9.78 -1.46 [1.46]	-2.92 4.89	-4.38 -1.9 [-2.99]	-3.8 -2.19	-3.8 [2.19]	-1.9
	0.365	0.73	0.73 -0.598 [0.598]	-1.196 0.365	-1.794 0.545 [-0.91]	1.09 -0.898	1.09 [0.895]	0.545
	0.145	0.29	0.29 -0.18 [0.18]	-0.36 0.145	-0.54 0.223 [-0.368]	0.447 -0.27	0.447 [0.27]	0.223
	0.045	0.09	0.09 -0.07 [0.07]	-0.14 0.045	-0.21 0.067 [-0.112]	0.136 -0.105	0.136 [0.105]	0.067
	0.0175	0.035	0.035 -0.02	-0.04 0.0175	-0.06 -0.025	0.05 -0.02	0.05	0.025
Final Moments	-2.07	18.4	-18.4	18.8	-18.8	5.1	-5.1	7.96≈8

09/09/18



Fixed Moments



BMD

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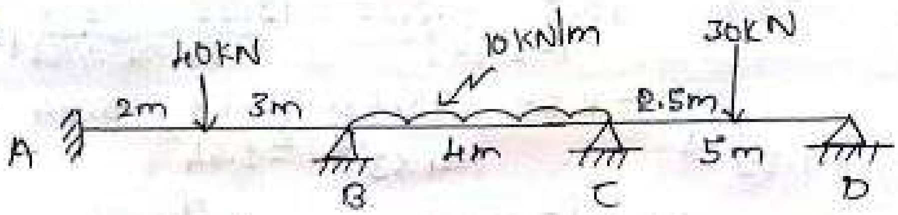
5. Analyse the continuous beam by moment-distribution method. Draw BMD and SFD. EI is constant.



→ Step 1: Fixed End Moments:

$$M_{FAB} = \frac{-40\times 2\times 3^2}{5^2} = -28.8\ KNm$$

$$M_{FBA} = \frac{40\times 2^2\times 3}{5^2} = 19.2\ kNm$$

$$M_{FBC} = \frac{-wL^2}{12} = \frac{-10\times 4^2}{12} = -13.33\ kNm$$

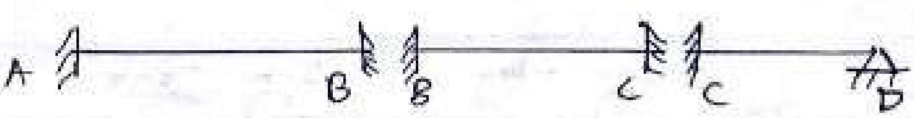
$$M_{FCB} = 13.33\ KNm$$

$$M_{FCD} = \frac{-WL}{8} = \frac{-30\times 5}{8} = -18.7\ KNm$$

$$M_{FDC} = 18.7\ KNm$$

Step 2: Distribution factors:

@ Joint 'B', $\sqrt{}_{BA} = \frac{K_{BA}}{K_{BA}+K_{BC}} = \frac{I/5}{I/5+I/4} = 0.44$



$$\sqrt{}_{BC} = \frac{K_{BC}}{K_{BA} + K_{BC}} = \frac{I/4}{I/5 + I/4} = 0.56$$

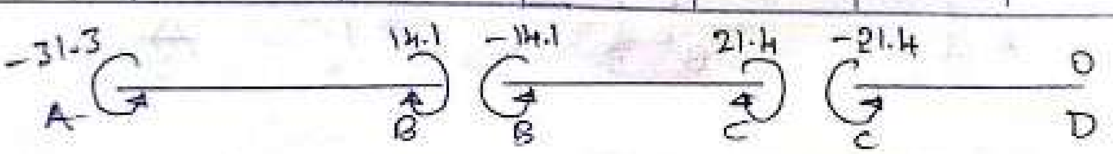
@ Joint 'C':

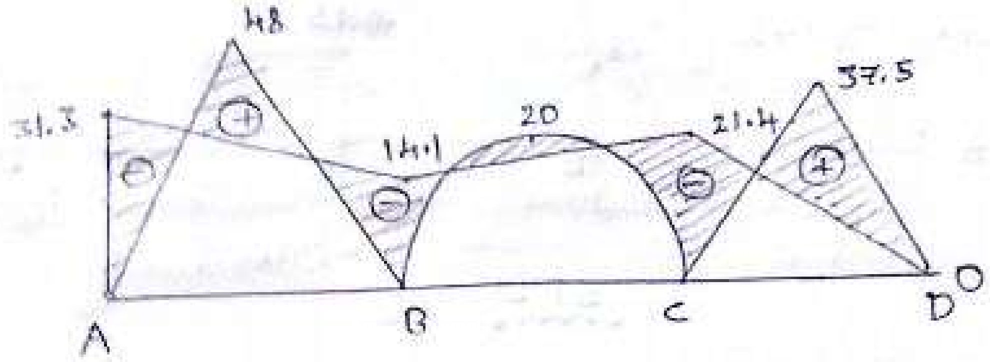
$$\sqrt{}_{CB} = \frac{K_{CB}}{K_{CB} + K_{CD}} = \frac{I/4}{I/4 + 3/4(I/5)} = 0.63$$

$$\sqrt{}_{BCD} = \frac{K_{CD}}{K_{CB} + K_{CD}} = \frac{3/4(I/5)}{I/4 + 3/4(I/5)} = 0.37$$

Step 3: Moment distribution table:

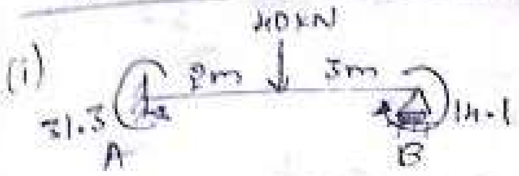
Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
DF	–	0.44	0.56	0.63	0.37	–
FEM	−28.8	19.2	−13.33	13.33	−18.7	18.7
		−5.87		+14.7	−9.35	−18.7 (for V.Im)
		−2.58	−3.28	−9.27	5.44	
	−1.29		4.63	−1.64		
		−4.63		1.64		
		−2.03	−2.59	1.03	0.60	
	−1.02		0.51	−1.29		
		−0.51		1.29		
		−0.22	−0.28	0.81	0.47	
	−0.11		0.405	−0.14		
		−0.405		0.14		
		−0.179	−0.22	0.08	0.051	
	0.08		0.04	−0.11		
		−0.04		0.11		
		−0.017	−0.022	0.067	0.04	
	−0.008		0.035	−0.011		
Final Moments	−31.38	14.1	−14.1	21.40	−21.4	0.0





To draw SFD:

(i)



$\Sigma M_A = 0;$

$-31.3 + 80 + 14.1 - R_B \times 5 = 0$

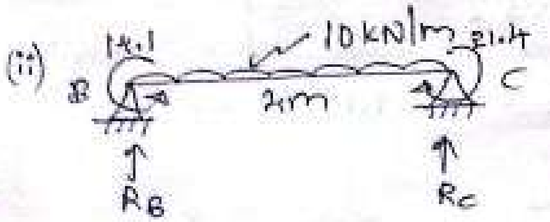
$R_B = 12.56\ kN$

$\Sigma V = 0;$

$R_A - 40 + R_B = 0$

$R_A = 27.44\ kN$

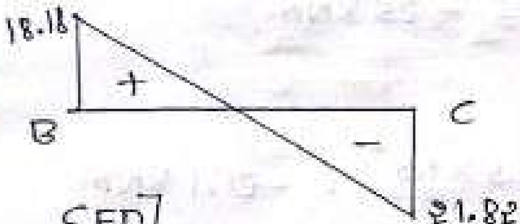
(ii)



$\Sigma M_B = 0;$

$-14.1 + (10 \times 4)2 + 21.4 - R_C \times 4 = 0$

$R_C = 21.82\ kN$



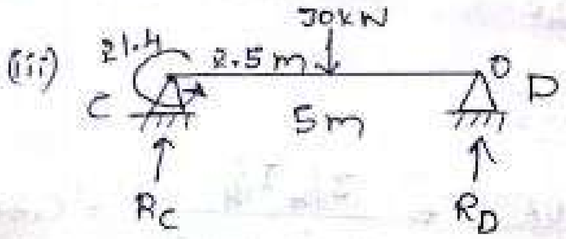
$SFD]_{BC}$

$\Sigma V = 0;$

$R_B - 40 + 21.82 = 0$

$\therefore R_B = 18.18\ kN$

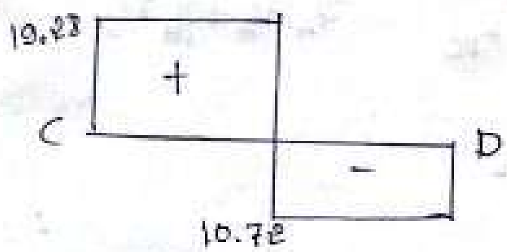
(iii)



$\Sigma M_C = 0;$

$-21.4 + (30 \times 2.5) - R_D \times 5 = 0$

$R_D = 10.72\ kN$



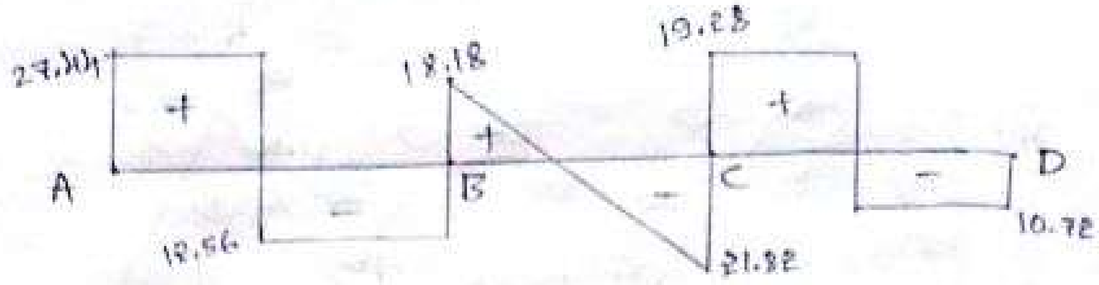
$SFD]_{CD}$

$\Sigma V = 0;$

$R_C - (30) + 10.72 = 0$

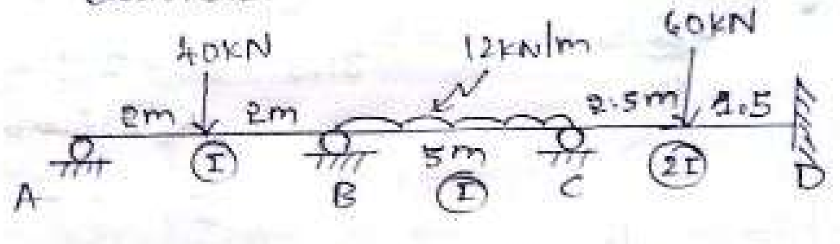
$R_C = 19.28\ kN$

$\therefore$ Shear force is given by,



SFD

6. Analyse the ~~frame~~beam shown in the fig by moment distribution.



→ 

Step I : Fixed End Moments :

$$M_{FAB} = -\frac{wL}{8} = -\frac{40\times4}{8} = -20\,KNm$$

$$M_{FBA} = +20\,KNm$$

$$M_{FBC} = \frac{-wL^2}{12} = \frac{-12\times5^2}{12} = -25\,KNm$$

$$M_{FCB} = 25\,KNm$$

$$M_{FCD} = -\frac{wab^2}{L^2} = \frac{-60\times2.5\times1.5^2}{4^2} = -21.1\,KNm$$

$$M_{FDC} = \frac{wa^2b}{L^2} = \frac{60\times2.5^2\times1.5}{4^2} = 35.15\,KNm$$

Step 2 : Distribution factor :

@ Joint B, $\delta_{BA} = \frac{K_{BA}}{K_{BA}+K_{BC}} = \frac{3/4\ I/4}{3/4\ I/4 + \frac{3}{4}\ I/5} = 0.48$

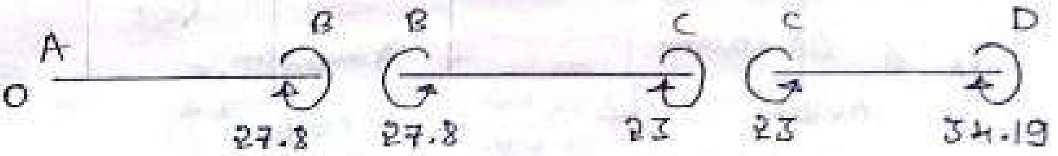
$$\delta_{BC} = \frac{K_{BC}}{K_{BA}+K_{BC}} = 0.52$$

@ C, $\delta_{CB} = \frac{K_{CB}}{K_{CB}+K_{CD}} = 0.3$

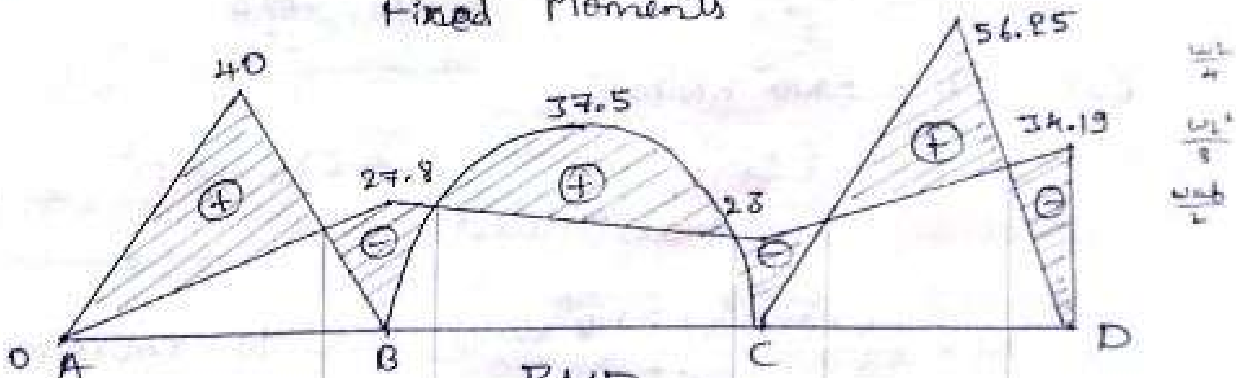
IIIly, $\delta_{CD} = 0.7$

Step 3: Moment Distribution table:

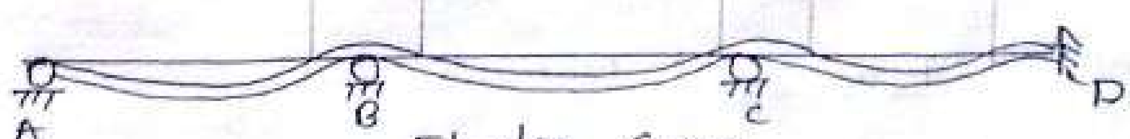
Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
DF	—	0.48	0.52	0.3	0.7	—
FEM	-20	20	-25	25	-21.09	35.15
	+20 →	10	[-5]	[-3.91]		
		-2.4	-2.6	-1.17	-2.73	
			-0.52	-1.3		-1.36
			[0.52]	[1.3]		
		0.28	0.30	0.39	0.91	
			0.195	0.15		0.455
			[-0.195]	[-0.15]		
		-0.094	-0.10	-0.045	-0.105	
			-0.0225	-0.05		-0.052
Final Moments	0.0	27.80	-27.8	23	-23	34.19



Fixed Moments



BMD

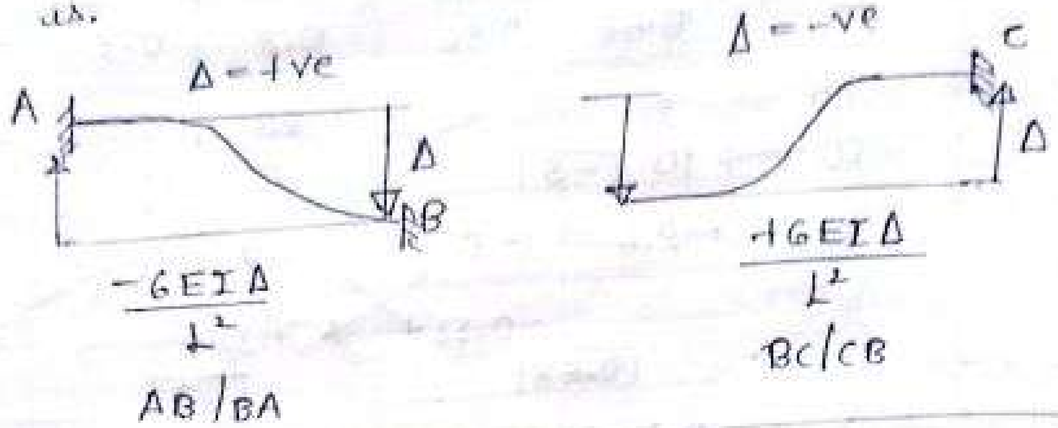


Elastic Curve

05/09/18

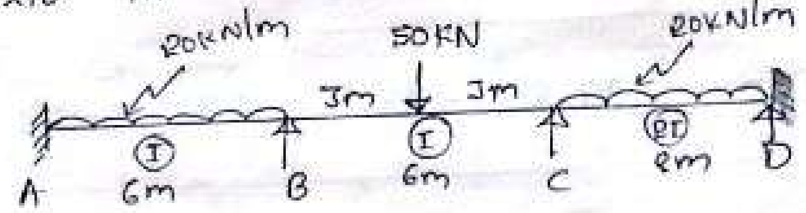
Settelement (Δ):

Settlement 'Δ' is converted to equivalent moment. that moment is added to fixed end moment and adopt distribution process it is.

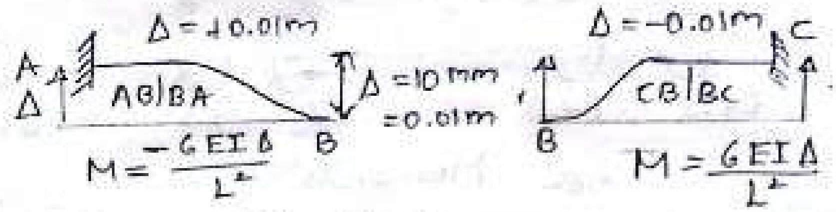


Problems:

1. Analyse the continuous beam loaded shown in the fig. by moment distribution method. E is 200 KN/mm². support 'B' sinks by 10 mm.
$I = 1.2 \times 10^{-4}$ m⁴



→



But $E = 2 \times 10^5$ KN/mm²

$$E = \frac{2 \times 10^5 \left(\frac{1}{1000}\right) KN}{\left(\frac{1}{1000}\right)m \left(\frac{1}{1000}\right)m} = 2 \times 10^8 \, KN/m^2$$

$$\therefore M = \frac{-6 \times 2 \times 10^8 \times 1.2 \times 10^{-4} \times 0.01}{6^2} \qquad \therefore M = +40 KNm$$

$\therefore M = -40$ KNm

Step 1: Fixed End Moments:

$$M_{FAB} = -\frac{wL^2}{12} = -\frac{20 \times 6^2}{12} - 40 = -60 - 40 = -100 \, KNm$$

$$M_{FBA} = \frac{20\times6^2}{12} - 40 = 20\,KNm$$

$$M_{FBC} = \frac{-WL}{8} = \frac{-50\times3}{8} + 40 = 2.5\,KNm$$

$$M_{FCB} = \frac{WL}{8} = \frac{50\times3}{8} + 40 = 77.5\,KNm$$

$$M_{FCD} = \frac{-WL^2}{12} = \frac{-20\times8^2}{12} = -106.67\,KNm$$

$$M_{FDC} = 106.67\,KNm$$

Step 2: Distribution factors:

@ Joint 'B',

$$\delta_{BA} = \frac{K_{BA}}{K_{BA}+K_{BC}} = \frac{I/6}{I/6+I/6} = 0.5$$

$$\delta_{BC} = \frac{I/6}{I/6+I/6} = 0.5$$

@ Joint 'C',

$$\delta_{CB} = \frac{I/6}{\frac{I}{6}+\frac{3}{4}\left(\frac{2I}{8}\right)} = 0.47$$

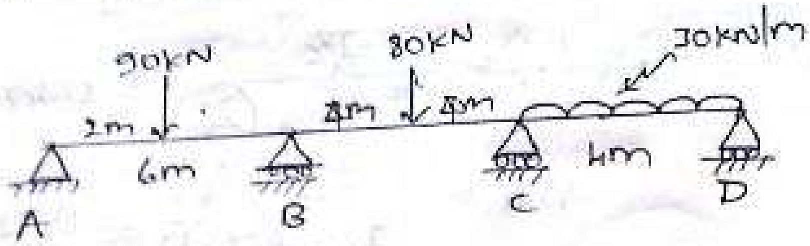
$$\delta_{CD} = \frac{\frac{3}{4}\left(\frac{2I}{8}\right)}{I/6+\frac{3}{4}\left(\frac{2I}{8}\right)} = 0.53$$

Step 3: Moment distribution table:

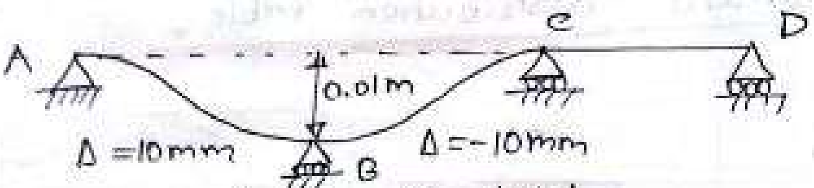
Joint	A	B		C		D
Members	AB	BA	BC	CB	CD	DC
DF	—	0.5	0.5	0.47	0.53	—
FEM	-100	20	2.5	77.5	-106.67	106.67
		[-22.5]		[82.7]	-53.33	-106.67
		-11.25	-11.25	38.77	43.72	
	-5.625		19.38	-5.625		
		[-19.38]		[5.625]		
		-9.69	-9.69	2.64	2.98	
	-4.84		1.32	-4.84		
		[-1.32]		[4.84]		
		-0.66	-0.66	2.27	2.56	
	-0.33		1.135	-0.33		
		[-1.135]		[0.33]		

18/10/18

2. Draw the BMD & SFD for the continuous beam loaded shown in the figure. Support 'B' yields by 10 mm below the level of ACD. $I = 132 \times 10^6\ mm^4$, $E = 200\ GPa$



→ Note: Yielding means settlement



$\Delta = 10mm$ $\qquad$ $\Delta = -10mm$

$M = \frac{-6EI\Delta}{L^2}$ $\qquad$ $M = \frac{6EI\Delta}{L^2}$

AB/BA $\qquad$ BC/CB

Step 1: Fixed End Moments:

$M_{FAB} = \frac{-90 \times 2 \times 4^2}{6^2} - \frac{6EI\Delta}{L^2}$

$\therefore EI = 200 \times 10^6 \times 132 \times 10^{-6}$

$EI = 26400\ kNm^2$

$\therefore M_{FAB} = -80 - \frac{6(26,400)0.01}{6^2}$

$= -80 - 44$

$\therefore M_{FAB} = -124\ kNm$

$M_{FBA} = \frac{90 \times 2^2 \times 4}{6^2} - 44 = -4\ kNm$

$E = 200\ GPa$

$E = 200 \times 10^9\ N/m^2$

$E = 200 \times 10^9 \left(\frac{1}{1000}\right)$

$E = 200 \times 10^6\ kN/m^2$

$I = 132 \times 10^6 \left(\frac{1}{10^3}\right)^4$

$I = 132 \times 10^{-6}\ m^4$

Scanned with CamScanner

$$M_{FBC} = -\frac{80\times8}{8} + 24.75$$

$$= -55.25 \text{ kNm}$$

$$M_{FCB} = \frac{80\times8}{8} + 24.75$$

$$\therefore\ M_{FCB} = 104.75 \text{ kNm}$$

$$M_{FCD} = -\frac{30\times4^2}{12} = -40 \text{kNm}$$

$$M_{FDC} = \frac{30\times4^2}{12} = 40 \text{kNm}$$

Step 2: Distribution factor:

@ Joint 'B'.

$$\delta_{BA} = \frac{K_{BA}}{K_{BA}+K_{BC}} = \frac{3/4\,(I/6)}{3/4\,(I/6)+I/8} = 0.5$$

$$\delta_{BC} = \frac{K_{BC}}{K_{BA}+K_{BC}} = 0.5$$

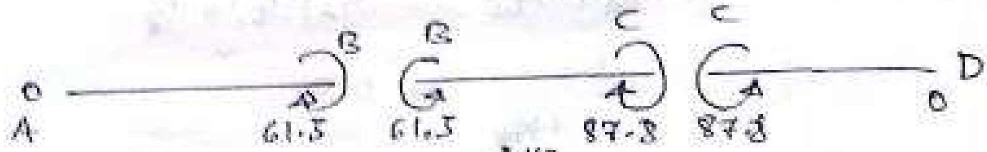
@ Joint 'C', $\delta_{CB} = \dfrac{K_{CB}}{K_{CB}+K_{CD}} = \dfrac{I/8}{I/8+3/4\,(I/4)} = 0.4$

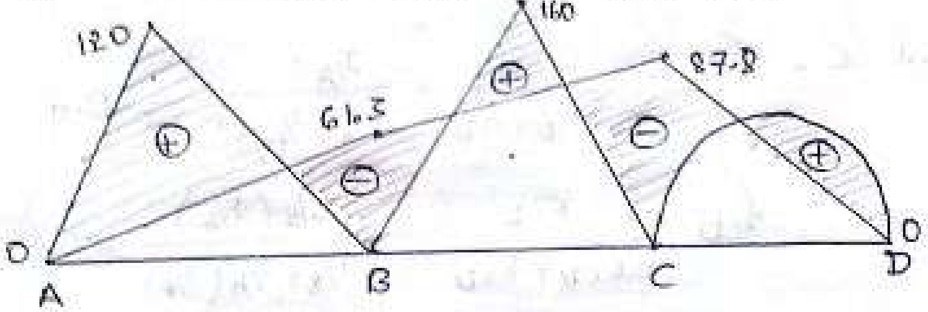
$$\delta_{CD} = \frac{K_{CD}}{K_{CB}+K_{CD}} = \frac{3/4\,(I/4)}{I/8+3/4\,(I/4)} = 0.6$$

Step 3: Moment distribution table:

Joint	A	B		C		D
Members	AB	BA	BC	CB	CD	DC
DF	–	0.5	0.5	0.4	0.6	–
FEM	−124 +124 →	−4 62	−55.25 [−2.75]	104.75 [−44.75]	40 −20 ←	40 −40
		−1.375 [8.95]	−1.375 −8.95	−17.9 −0.68	−26.85 [0.68]	
		4.475 [−0.13]	4.475 0.13	0.27 2.23	0.41 [−2.23]	

	~	-0.065	-0.065	-0.892	-1.338	~
			-0.446	-0.0325		
		[0.446]			[0.0325]	
	~	0.223	0.223	0.013	0.02	~
			0.006	0.1115		
		[-0.006]			[-0.1115]	
	~	-0.003	-0.003	-0.045	-0.07	~
			-0.0225	-0.0015		
Final moments	0.0	61.3	-61.3	87.8	-87.8	0.0

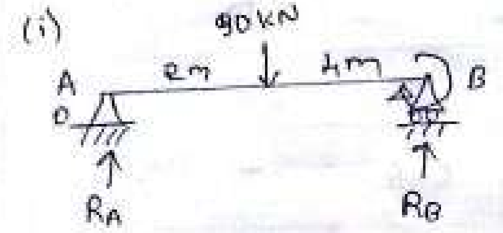




BMD

To find SF :

(i)



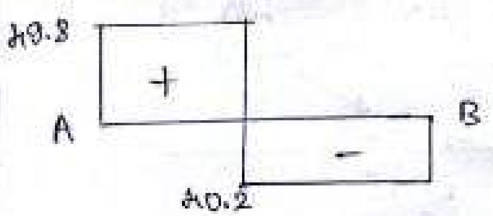
$\Sigma M_A = 0$;

$90 \times 2 - R_B \times 6 + 61.3 = 0$

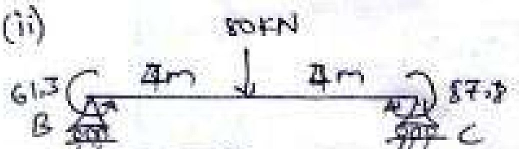
$R_B = 40.2$

$\Sigma V = 0$; $R_A - 90 + 40.2 = 0$

$R_A = 49.8$



(ii)

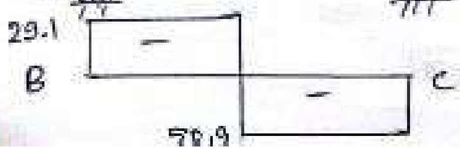


$\Sigma M_B = 0$;

$80 \times 4 + 87.8 - R_c \times 8 - 61.3 = 0$

$\therefore R_c = 50.9$

$\Sigma V = 0$; $R_B = 29.1$



(iii)

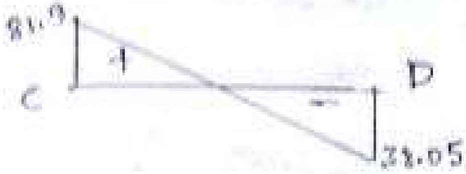
$\Sigma M_c = 0;$

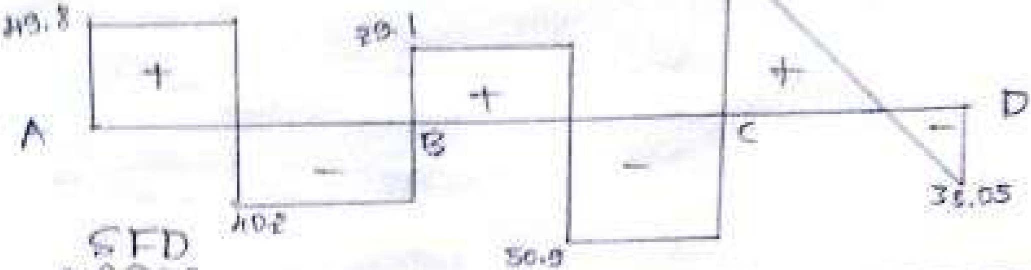
$-87.8 + (30\times4)\times2 - R_D\times4 = 0$

$\therefore R_D = 38.05$ kN

$\Sigma V = 0;\ R_c - (30\times4) + 38.05 = 0$

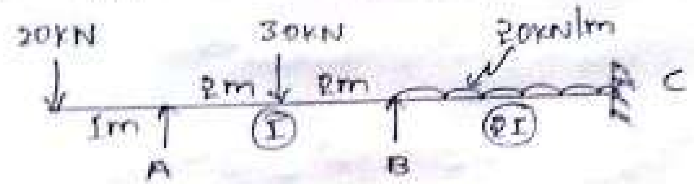
$R_c = 81.9$ kN



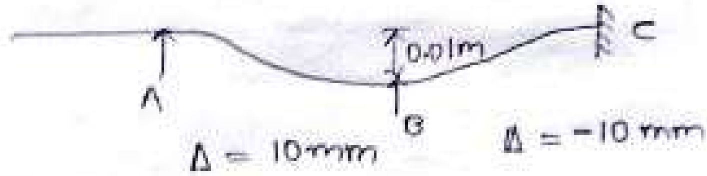


SFD

3. Analyse the continuous beam by moment-distribution method. 'B' sinks by 10mm. EI = 4000 kNm^2. Draw BMD and SFD.



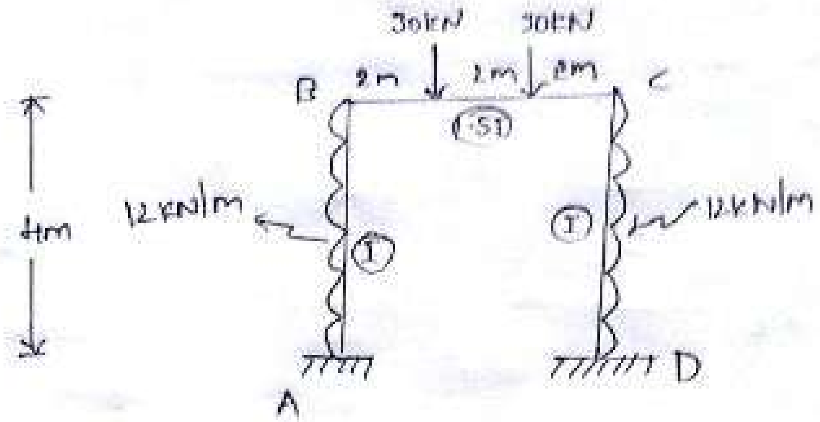
→



07/09/18

Frames :

1. Analyse the portal frame by moment-distribution method. Draw BMD.



→ **Step 1 : Fixed End Moments :**

$$M_{FAB} = \frac{-wL^2}{12} = -16\,KNm$$

$$M_{FBA} = \frac{wL^2}{12} = 16\,KNm$$

$$M_{FBC} = \frac{-30\times2\times4^2}{6^2} - \frac{30\times4\times2^2}{6^2}$$

$$\therefore M_{FBC} = -40\,KNm$$

$$M_{FCB} = \frac{30\times2^2\times4}{6^2} + \frac{30\times4^2\times2}{6^2}$$

$$\therefore M_{FCB} = 40\,KNm$$

$$M_{FCD} = \frac{-wL^2}{12} = -16\,KNm$$

$$M_{FDC} = 16\,KNm$$

Step 2 : DF :

@ Joint 'B', $\delta_{BA} = \frac{K_{BA}}{K_{BA}+K_{BC}} = \frac{I/4}{I/4 + \frac{1.5I}{6}} = 0.5$

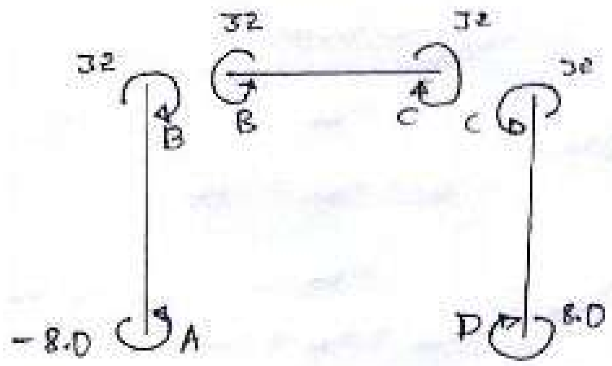
$$\delta_{BC} = \frac{K_{BC}}{K_{BA}+K_{BC}} = 0.5$$

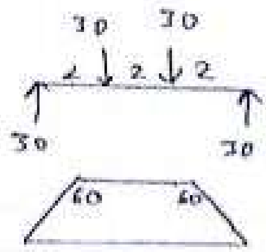
@ Joint 'C', $\delta_{CB} = \frac{K_{CB}}{K_{CB}+K_{CD}} = \frac{1.5I/6}{1.5I/6 + I/4} = 0.5$

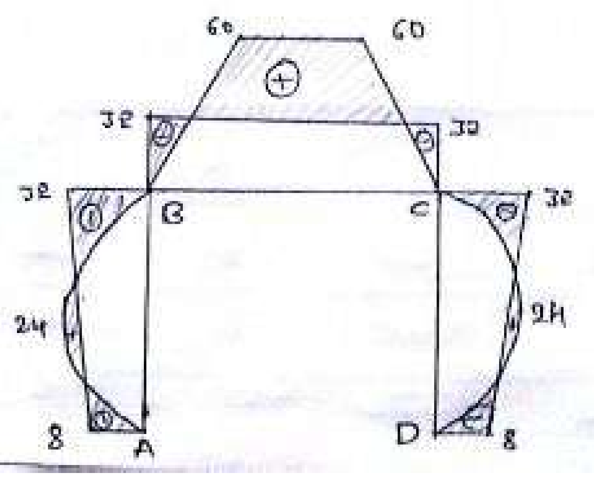
$$\delta_{CD} = \frac{K_{CD}}{K_{CB}+K_{CD}} = 0.5$$

Step 3: Moment Distribution table :

Joint	A	B		C		D
Members	AB	BA	BC	CB	CD	DC
DF	–	0.5	0.5	0.5	0.5	–
FEM	−16.0	16.0	−40	40	−16	16
		[24.0]			[−24]	
	6.0	32.0	12.0 −6.0	−12.0 6.0	12.0	−6.0
		[6.0]			[−6.0]	
	1.5	3.0	3.0 −1.5	−3.0 1.5	−3.0	−1.5
		[1.5]			[−1.5]	
	−0.375	0.75	0.75 −0.375	−0.75 0.375	−0.75	−0.375
		[0.375]			[−0.375]	
	0.09	0.187	0.187 −0.09	−0.187 0.09	−0.187	−0.09
Final Moments	−8.03	32.0	−32.0	32.0	−32.0	8.0

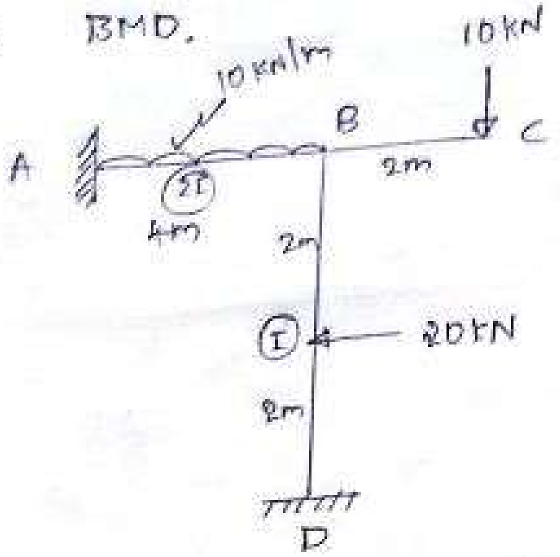






BMD

3. Analyse the frame by moment - distribution -ed. Draw BMD.



→ **Step 1: Fixed End moments:**

$M_{FAB} = \frac{-wL^2}{12} = -13.33$ kN ; $M_{BC} = -20$ kNm

$M_{FBA} = 13.33$ kN

$M_{FBD} = \frac{-WL}{8} = -10$ kNm

$M_{FDB} = 10$ kNm

Step 2: Distribution factor:

@ Joint B, $\delta_{BA} = \frac{K_{BA}}{K_{BA}+K_{BC}+K_{BD}} = \frac{4I/4}{2I/4 + 0 + I/4}$

$\therefore \delta_{BA} = 0.67$

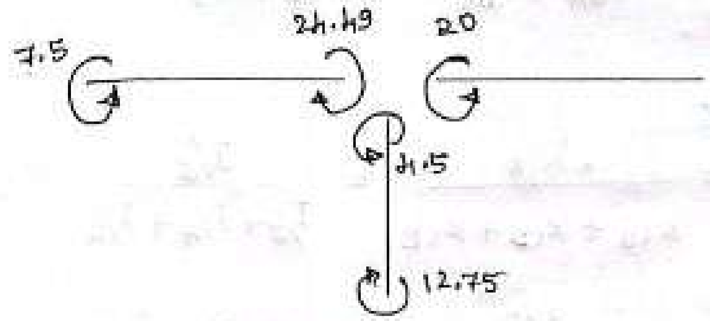
$\delta_{BC} = \frac{K_{BC}}{K_{BA}+K_{BC}+K_{BD}} = \frac{0}{(\ )} = 0$

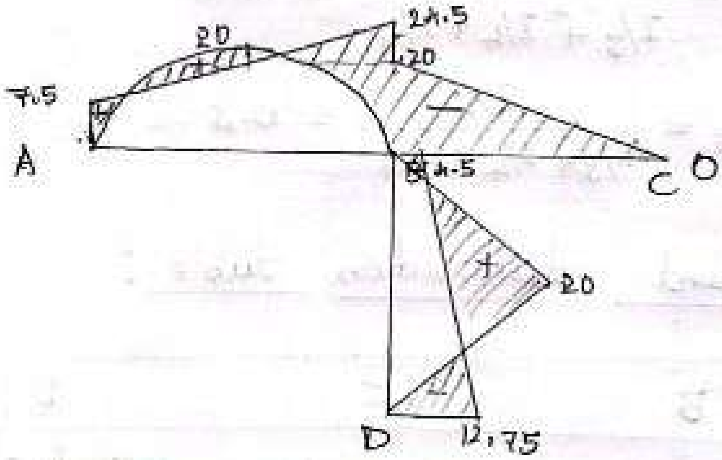
$\delta_{BD} = \frac{K_{BD}}{K_{BA}+K_{BC}+K_{BD}} = 0.33$

Step 3: MDT:

Joint	A	B			D
Members	AB	BA	BC	BD	DB
DF	—	0.67	0	0.33	—
FEM	−13.33	13.33	−20 16.67	−10	10

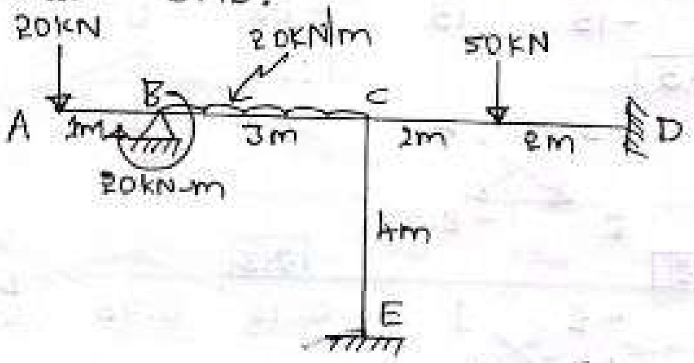
	5.58	11.16	0.0	5.5	2.75
Final moments	−7.5	24.49	−20	−4.5	12.75





BMD

4. Analyse the portal frame by moment distribution method. Draw BMD.



→ Step 1: Fixed End Moments:

$$M_{FBC} = \frac{-WL^2}{12} = \frac{-20 \times 3^2}{12} = -15 \text{ KN-m}$$

$$M_{FCB} = \frac{WL^2}{12} = 15 \text{ KN-m}$$

$$M_{FCD} = \frac{-WL}{8} = \frac{-50 \times 4}{8} = -25 \text{ kN-m}$$

$$M_{FDC} = \frac{WL}{8} = 25 \text{ KN-m}$$

$M_{FCE} = 0$; $M_{FEC} = 0$; $M_{BA} = 20 \text{KN-m}$

12/09/18

Step 2: Distribution factor:

@ Joint 'B',

$$D_{BA} = \frac{K_{BA}}{K_{BA} + K_{BC}} = \frac{0}{0 + I/3} = 0$$

$$D_{BC} = \frac{K_{BC}}{K_{BA} + K_{BC}} = \frac{I/3}{0 + I/3} = 1$$

@ Joint 'C',

$$D_{CB} = \frac{K_{CB}}{K_{CB} + K_{CD} + K_{CE}} = \frac{I/3}{I/3 + I/4 + I/4} = 0.4$$

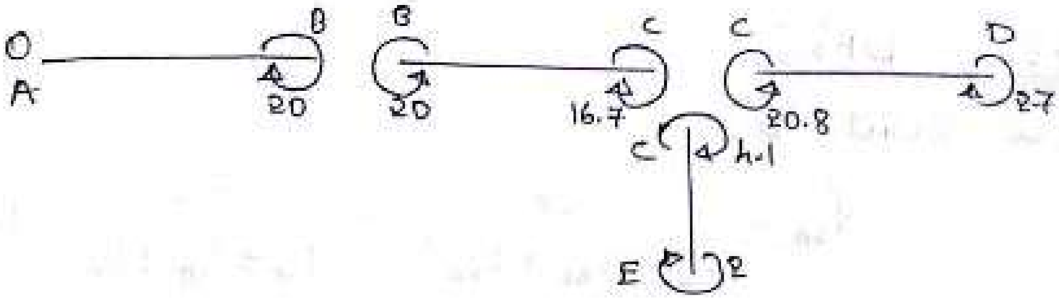
$$D_{CD} = \frac{K_{CD}}{I/3 + I/4 + I/4} = 0.3$$

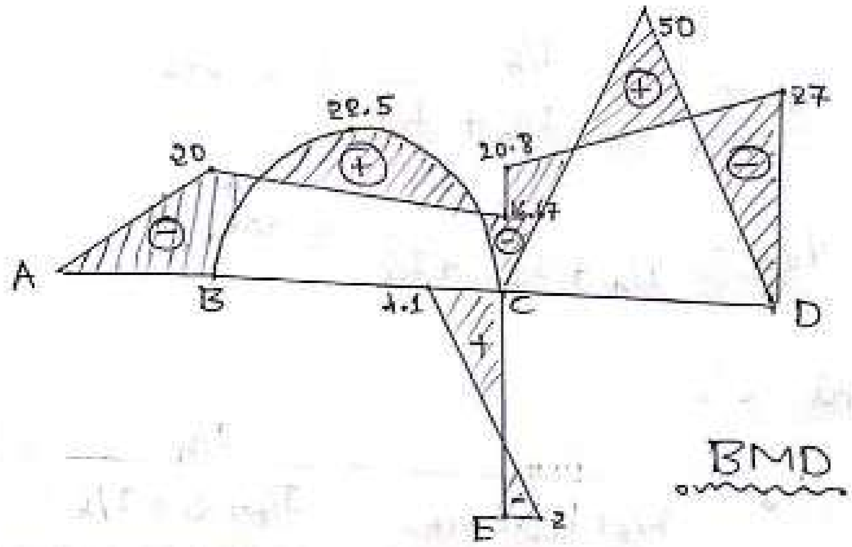
$$D_{CE} = \frac{I/4}{I/3 + I/4 + I/4} = 0.3$$

Step 3: Moment distribution table:

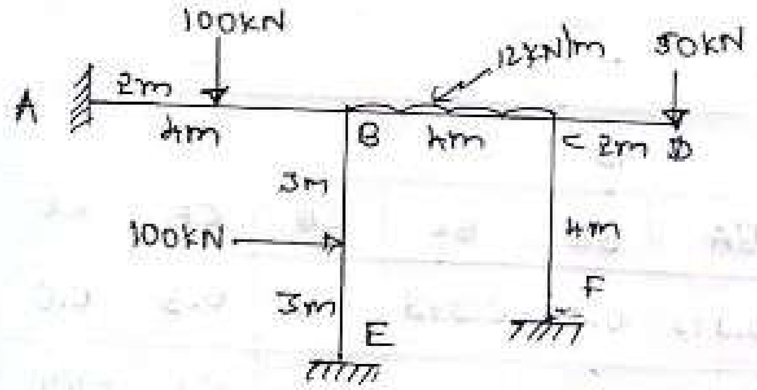
Joint	B		C			D	E
Members	BA	BC	CB	CD	CE	DE	EC
DF	—	1	0.4	0.3	0.3	—	—
FEM	20	-15	15	-25	0	25	0
	[-5]			[10]			
	0	-5	4	3	3	1.5	1.5
		2	-2.5				
	[-2]			[2.5]			
	0	-2	1	0.75	0.75	0.375	0.375
		0.5	-1				
	[-0.5]			[1]			
	0	-0.5	0.4	0.3	0.3	0.15	0.15
		0.2	-0.025				
	[-0.2]			[0.025]			
	0	-0.2	-0.1	0.075	0.075	0.0375	0.0375
		-0.05	-0.1				
	[0.005]			[0.1]			

	0.0	0.05 / 0.02	0.04 / 0.025	0.03	0.03	0.015	0.015
Final Moments	20	−20	16.7	−20.8	4.1	27	2.0





5. Analyse the frame shown in the figure by moment distribution method. Draw BMD.



→ Here $M_{CD} = -50 \times 2 = -100 KNm$

Step 1: $M_{FAB} = \frac{-WL}{8} = -50 KNm$

$M_{FBA} = \frac{WL}{8} = 50 KNm$

$M_{FBC} = \frac{-WL^2}{12} = -16 KNm$

$M_{FCB} = \frac{WL^2}{12} = 16 KNm$

$$M_{FBE} = \frac{100 \times 6}{8} = 75 \text{ KNm}$$

$$M_{FEB} = -75 \text{ kNm}$$

$$M_{FCF} = 0 \;;\; M_{FFC} = 0$$

Step 2: DF's:

@ Joint 'B',

$$\sqrt{}_{BA} = \frac{K_{BA}}{K_{BA} + K_{BC} + K_{BE}} = \frac{I/4}{I/4 + I/4 + I/6} = 0.375$$

$$\sqrt{}_{BC} = \frac{I/4}{I/4 + I/4 + I/6} = 0.375$$

$$\sqrt{}_{BE} = \frac{I/6}{I/4 + I/4 + I/6} = 0.25$$

@ Joint 'C',

$$\sqrt{}_{CB} = \frac{K_{CB}}{K_{CB} + K_{CD} + K_{CF}} = \frac{I/4}{I/4 + 0 + I/4} = 0.5$$

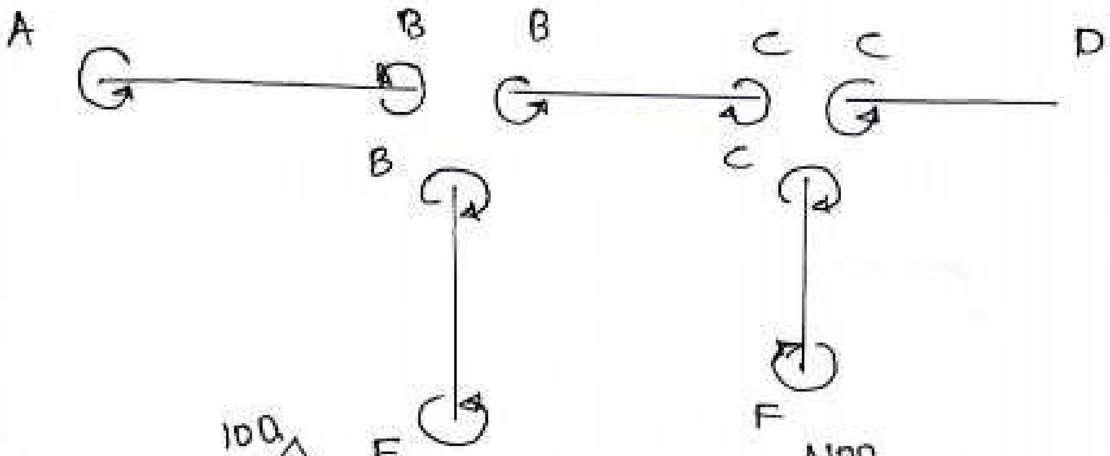
$$\sqrt{}_{CD} = 0$$

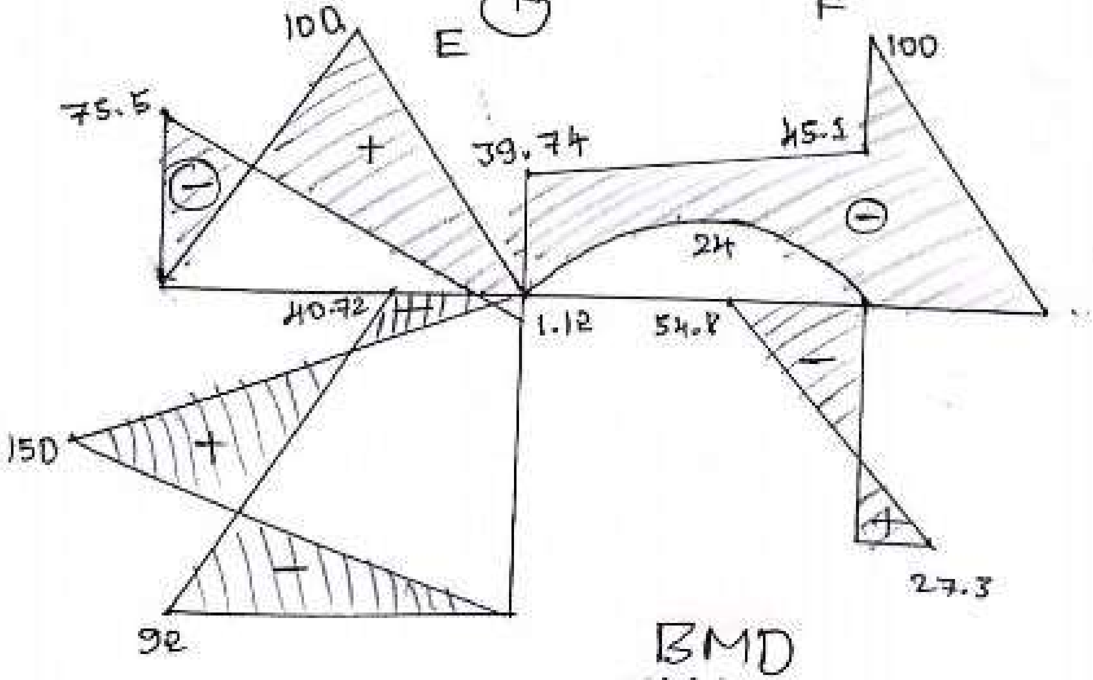
$$\sqrt{}_{CF} = 0.5$$

Step 3: MDT:

Joint	A	B			C			E	F
Members	AB	BA	BE	BC	CB	CF	CD	EB	FC
DF	—	0.375	0.25	0.375	0.5	0.5	0.0	—	—
FEM	-50	50	75	-16	16	0.0	-100	-75	0.0
			[-109]			[84]			
		-40.87	-27.25	-40.87	42	42	0		
	-20.47			21	-20.47			-13.625	21
			[-21]			[21.43]			
		-7.87	-5.25	-7.87	10.21	10.21	0.0		
	-3.93			5.105	-3.93			-2.625	5.105
			[-5.105]			[3.93]			

		-1.914	-1.276	-1.914	1.967	1.967	0.0		
	-0.957			0.983	-0.957			-0.638	0.983
			-0.983			0.957			
		-0.368	-0.245	-0.368	0.48	0.48	0.0		
	-0.184			0.24	-0.184			-0.1225	0.24
			-0.24			0.184			
		-0.09	-0.06	-0.09	0.092	0.09	0.0		
	-0.045			0.046	-0.045			-0.03	0.045
Final Moments	-75.5	-1.112	40.72	-39.73	45.2	54.8	-100	-92.04	27.37





BMD