SREENIVASA INSTITUTE OF TECHNOLOGY AND MANAGEMENT STUDIES

(AUTONOMOUS)

DEPARTMENT OF MECHANICAL ENGINEERING

COURSE MATERIAL

Subject Name	Fluid Mechanics and Hydraulic Machines
Subject Code	23MEC241T
Semester	IV Semester
Academic Year	2025-26
Regulation	R23

Unit-II

Fluid Kinematics, Fluid Dynamics and Closed Conduit Flow

UNIT – II

Fluid Kinematics:

- Stream Line, Path line, streak lines and stream tubes.
- Classification of flows- steady & unsteady, uniform & non-uniform, laminar & turbulent, rotational & irrotational flows.
- Equation of continuity- 1D flow.

Fluid Dynamics:

- Surface & body forces- Euler's and Bernouli's equation for flow along a stream line.
- Momentum equation & its application on force on pipe bend

COURSE OUTLINE

UNIT -2

LECTURE	LECTURE TOPIC	KEY ELEMENTS	Learning objectives
1	Introduction	Stream line, stream tube	Understanding the basics (B2)
2	Classification of flows	Uniform & Non-uniform Laminar & Turbulent	Understanding different types of flows (B2)
3	Equation of Continuity	1 D Flow	• Analyze equation of continuity (B4) • Apply 1D flow equation (B3)
4	Surface & body forces	Euler's Equation	Evaluate Velocity in a flow (B5)
5	Bernouli's Equation of motion	For a stream line	Evaluate velocity & Pressure (B5)
6	Momentum equation	Application on pipe bend	Evaluate force exerted on pipe bend (B5)
7	Example Problems on Bernouli's & Momentum Equations.		

TOPICS TO BE COVERED

- Introduction to Fluid Kinematics
- Stream line
- Path line
- Streak Line
- Stream tube

LECTURE 1

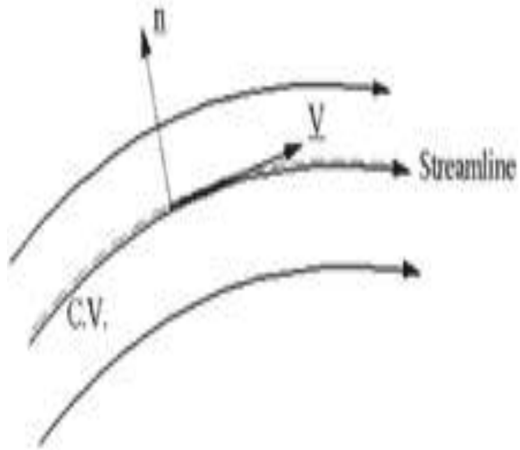
Classification of flows

FLUID KINEMATICS

- Kinematics is defined as a branch of science which deals with motion of particles without considering the forces causing the motion.
- The velocity at any point in a flow field at any time is studied in this.
- Once the velocity is known, then the pressure distribution and hence the forces acting on the fluid can be determined.

STREAM LINE

- A stream line is an imaginary line drawn in a flow field such that the tangent drawn at any point on this line represents the direction of velocity vector.
- From the definition it is clear that there can be no flow across stream line.
- Considering a particle moving along a stream line for a very short distance 'ds' having its components dx , dy and dz, along three mutually perpendicular co-ordinate axes.
- Let the components of velocity vector Vs along x, y and z directions be u, v and w respectively



- The time taken by the fluid particle to move a distance ‘ds’ along the stream line with a velocity Vs is:

$$t = \frac{\mathrm{ds}}{\mathrm{Vs}}$$

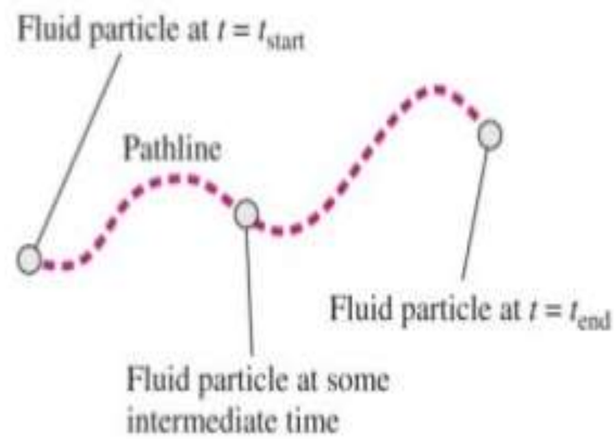
which is same as $t = \frac{\mathrm{dx}}{\mathrm{u}} = \frac{\mathrm{dy}}{\mathrm{v}} = \frac{\mathrm{ds}}{\mathrm{w}}$

- Hence the differential equation of the steam line may be written as:

$$\frac{\mathrm{dx}}{\mathrm{u}} = \frac{\mathrm{dy}}{\mathrm{v}} = \frac{\mathrm{ds}}{\mathrm{w}}$$

PATH LINE

- A path line is locus of a fluid particle as it moves along.
- In other words a path line is a curve traced by a single fluid particle during its motion.
- A stream line at time t_1 indicating the velocity vectors for particles A and B.
- At times t_2 and t_3 the particle A occupies the successive positions.
- The line containing these various positions of A represents its **Path line**

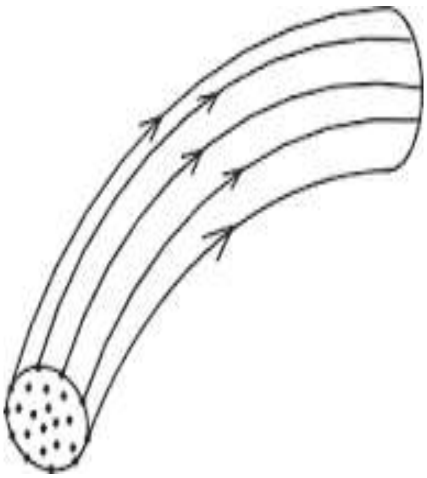


STREAK LINE

- When a dye is injected in a liquid or smoke in a gas, so as to trace the subsequent motion of fluid particles passing a fixed point, the path fallowed by dye or smoke is called the **streak line**.
- Thus the streak line connects all particles passing through a given point.
- In steady flow, the stream line remains fixed with respect to co-ordinate axes.
- Stream lines in steady flow also represent the path lines and streak lines.
- In unsteady flow, a fluid particle will not, in general, remain on the same stream line (except for unsteady uniform flow).
- Hence the stream lines and path lines do not coincide in unsteady non-uniform flow.

STREAM TUBE

- If stream lines are drawn through a closed curve, they form a boundary surface across which fluid cannot penetrate.
- Such a surface bounded by stream lines is known as **Stream tube**.
- From the definition of stream tube, it is evident that no fluid can cross the bounding surface of the stream tube.
- This implies that the quantity of fluid entering the stream tube at one end must be the same as the quantity leaving at the other end.
- The Stream tube is assumed to be a small cross-sectional area, so that the velocity over it could be considered uniform.



TOPICS TO BE COVERED

- Classification of flows
- Steady and unsteady flows
- Uniform & non-uniform flow
- Laminar & turbulent flow
- Compressible & incompressible flow
- Rotational & irrotational flow
- One dimensional flow
- Two & three dinemsional flow
- Rate of flow

LECTURE 2

Classification of flows

CLASSIFICATION OF FLOWS

- The fluid flow is classified as:
  - Steady and unsteady flows.
  - Uniform and Non-uniform flows.
  - Laminar and Turbulent flows.
  - Compressible and incompressible flows.
  - Rotational and Irrotational flows.
  - One, two and three dimensional flows.

STEADY & UNSTEADY FLOW

- Steady flow is defined as the flow in which the fluid characteristics like velocity, pressure, density etc. at a point do not change with time.
- Thus for a steady flow, we have

$$\left(\frac{\partial V}{\partial t}\right)_{x,y,z} = 0, \qquad \left(\frac{\partial p}{\partial t}\right)_{x,y,z} = 0, \qquad \left(\frac{\partial \rho}{\partial t}\right)_{x,y,z} = 0$$

- Un-Steady flow is the flow in which the velocity, pressure, density at a point changes with respect to time.
- Thus for un-steady flow, we have

$$\left(\frac{\partial V}{\partial t}\right)_{x,y,z} \neq 0, \qquad \left(\frac{\partial p}{\partial t}\right)_{x,y,z} \neq 0, \qquad \left(\frac{\partial \rho}{\partial t}\right)_{x,y,z} \neq 0$$

UNIFORM & NON-UNIFORM FLOW

- Uniform flow is defined as the flow in which the velocity at any given time does not change with respect to space. (i.e. the length of direction of flow)
- For uniform flow $\left(\frac{\partial V}{\partial s}\right)_{t=const} = 0$

  Where, ∂V = Change of velocity

  ∂s = Length of flow in the direction of – S
- Non-uniform is the flow in which the velocity at any given time changes with respect to space.
- For Non-uniform flow $\left(\frac{\partial V}{\partial s}\right)_{t=const} \neq 0$

LAMINAR & TURBULENT FLOW

- Laminar flow is defined as the flow in which the fluid particles move along well-defined paths or stream line and all the stream lines are straight and parallel.
- Thus the particles move in laminas or layers gliding smoothly over the adjacent layer. This type of flow is also called streamline flow or viscous flow.
- Turbulent flow is the flow in which the fluid particles move in a zigzag way.
- Due to the movement of fluid particles in a zigzag way, the eddies formation takes place, which are responsible for high energy loss.

LAMINAR & TURBULENT FLOW

- For a pipe flow, the type of flow is determined by a non-dimensional number $\left(\frac{VD}{v}\right)$ called the Reynolds number.

  Where D = Diameter of pipe.

  V = Mean velocity of flow in pipe.

  v = Kinematic viscosity of fluid.

- If the Reynolds number is lessthan2000, the flow is called Laminar flow.
- If the Reynolds number is more than 4000, it is called Turbulent flow.
- If the Reynolds number is between 2000 and 4000 the flow may be Laminar or Turbulent flow.

COMPRESSIBLE & INCOMPRESSIBLE FLOW

- Compressible flow is the flow in which the density of fluid changes from point to point or in other words the density is not constant for the fluid.
- For compressible flow, $\rho \neq$ Constant.
- In compressible flow is the flow in which the density is constant for the fluid flow.
- Liquids are generally incompressible, while the gases are compressible.
- For incompressible flow, $\rho =$ Constant

ROTATIONAL & IRROTATIONAL FLOW

- Rotational flow is a type of flow in which the fluid particles while flowing along stream lines also rotate about their own axis.

- And if the fluid particles, while flowing along stream lines, do not rotate about their own axis, the flow is called Irrotational flow.

ONE DIMENSION FLOW

- **1D flow** is a type of flow in which flow parameter such as velocity is a function of time and one space co-ordinate only, say '*x*'.
- For a steady one- dimensional flow, the velocity is a function of one space co-ordinate only.
- The variation of velocities in other two mutually perpendicular directions is assumed negligible.
- Hence for one dimensional flow **u = f(x), v = 0 and w = 0**

Where u, v and w are velocity components in x, y and z directions respectively.

TWO DIMENSION FLOW

- **2D flow** is the type of flow in which the velocity is a function of time and two space co-ordinates, say x and y.
- For a steady two-dimensional flow the velocity is a function of two space co-ordinates only.
- The variation of velocity in the third direction is negligible.
- Thus for two dimensional flow **$u = f_1 (x, y)$, $v = f_2 (x, y)$ and $w = 0$.**
- **3D flow** is the type of flow in which the velocity is a function of time and three mutually perpendicular directions.
- But for a steady three-dimensional flow, the fluid parameters are functions of three space co-ordinates (x, y, and z) only.
- Thus for **three- dimensional flow $u = f_1 (x, y, z)$, $v = f_2 (x, y, z)$, $z = f_3 (x, y, z)$.**

RATE OF FLOW OR DISCHARGE (Q)

- It is defined as the quantity of a fluid flowing per second through a section of pipe or channel.
- For an incompressible fluid (or liquid) the rate of flow or discharge is expressed as the volume of the liquid flowing cross the section per second. or compressible fluids.
- The rate of flow is usually expressed as the weight of fluid flowing across the section.
- Thus (i) For liquids the unit of Q is m^3/sec or Litres/sec.

  (ii) For gases the unit of Q is Kgf/sec or Newton/sec.
- **The discharge $Q = A \times V$**

  Where, A = Area of cross-section of pipe.

  V= Average velocity of fluid across the section.

TOPICS TO BE COVERED

- Continuity equation
- Continuity equation for 3D flow
- Example Problem

LECTURE 3

Equation of Continuity

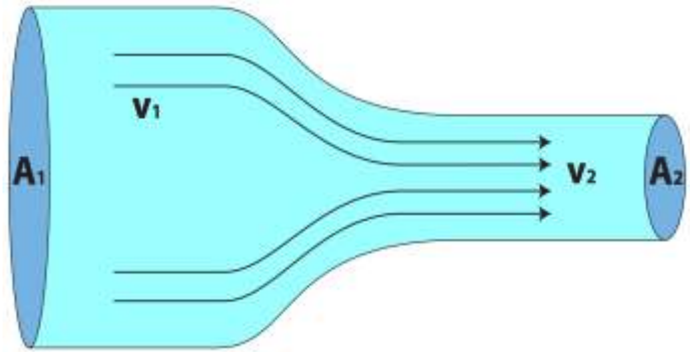
EQUATION OF CONTINUITY

- The equation based on the principle of conservation of mass is called Continuity equation.
- Thus for a fluid flowing through the pipe at all cross- sections, the quantity of fluid per second is constant.
- Consider two cross- sections of a pipe.
- Let V_1 = Average velocity at cross-section 1-1

  ρ_1 = Density of fluid at section 1-1

  A_1 = Area of pipe at section 1-1

  And V_2, ρ_2, A_2 are the corresponding values at section 2-2



- Then the rate flow at section 1-1= $\rho_1 A_1 V_1$
- Rate of flow at section 2-2 = $\rho_2 A_2 V_2$
- According to law of conservation of mass, Rate of flow at section 1-1 = Rate of flow at section 2-2

$$\boldsymbol{\rho_1 A_1 V_1 = \rho_2 A_2 V_2}$$

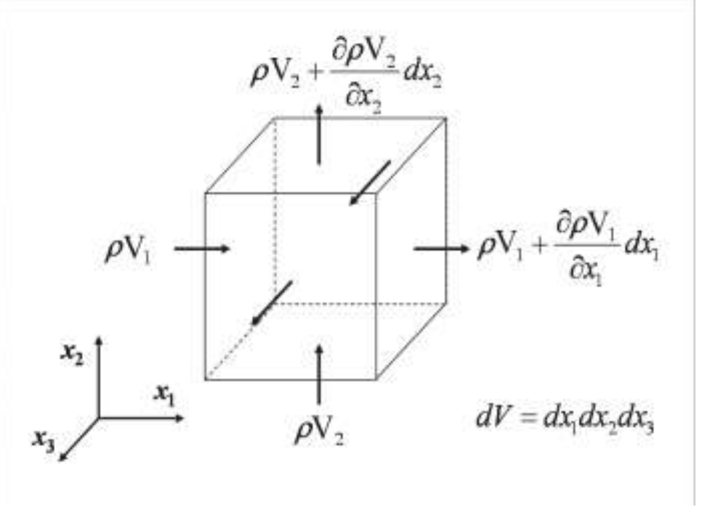
- This equation is applicable to the compressible as well as incompressible fluids and is called **"Continuity equation"**.
- If the fluid is incompressible, then $\rho_1=\rho_2$ and the continuity equation reduces to

$$\boldsymbol{A_1 V_1 = A_2 V_2}$$

EQUATION OF CONTINUITY FOR 3D FLOW

- Consider a fluid element of lengths dx, dy and dz in the direction of x, y and z.
- Let u, v and w are the inlet velocity components in x, y and z directions respectively.
- Mass of fluid entering the face ABCD per second = ρ × velocity in x – direction × Area of ABCD

$$= \rho \times u \times (dy \times dz)$$



- Then the mass of fluid leaving the face EFGH per second

$$= \rho \times u \times (dy \times dz) + \frac{\partial}{\partial x}(\rho\ u\ dy\ dz)dx$$

- Gain of mass in x- direction = Mass through ABCD – Mass through EFGH per second = ρ u dy dz - ρ u dy dz - $\frac{\partial}{\partial x}(\rho\ u\ dy\ dz)dx$

$$= - \frac{\partial}{\partial x}(\rho\ u\ dy\ dz)dx$$

$$= - \frac{\partial}{\partial x}(\rho\ u\ )dxdydz \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_} (1)$$

- Similarly gain of mass in y- direction

$$= -\frac{\partial}{\partial y}(\rho v)\ dx\ dy\ dz \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_} (2)$$

- Similarly gain of mass in z- direction

$$= -\frac{\partial}{\partial z}(\rho w)dx\ dy\ dz \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_} (3)$$

- Net gain of mass = $-\left[\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)+\frac{\partial}{\partial z}(\rho w)\right]$ dxdydz__ (4)
- Since mass is neither created nor destroyed in the fluid element, the net increase of mass per unit time in the fluid element must be equal to the rate of increase of mass of fluid in the element.
- But the mass of fluid in the element is ρ dxdydz and its rate of increase with time is $\frac{\partial}{\partial t}$(ρ. dx. dy. dz.) or $\frac{\partial \rho}{\partial t}$. dx. dy. dz. __(5)
- Equating the two expressions (4) & (5)

$$-\left(\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)+\frac{\partial}{\partial z}(\rho w)\right)dx\,dydz=\frac{\partial \rho}{\partial t}.dx.dy.dz.$$

$$\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)+\frac{\partial}{\partial z}(\rho w)=0 \quad \text{———— (6)}$$

- This equation is applicable to
  - Steady and unsteady flow
  - Uniform and non- uniform flow , and
  - Compressible and incompressible flow.
- For steady flow $\frac{\partial \rho}{\partial t} = 0$ and hence equation (6) becomes

$$\frac{\partial}{\partial x}(\rho\, u) + \frac{\partial}{\partial y}(\rho\, v) + \frac{\partial}{\partial z}(\rho\, w) = 0 \qquad \text{———— (7)}$$

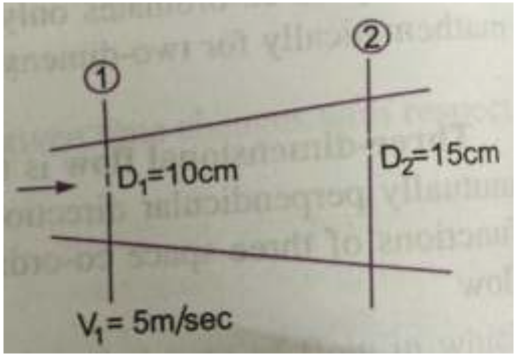
- If the fluid is incompressible, then ρ is constant and the above equation becomes

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \qquad \text{———— (8)}$$

- This is the continuity equation in three - dimensional flow.

PROBLEM 1

- The diameter of a pipe at sections 1 and 2 are 10 cm and 15cms respectively. Find the discharge through pipe, if the velocity of water flowing through the pipe at section 1 is 5m/sec. determine the velocity at section 2.



Sol: Given data

At section 1, D_1 = 10cms = 0.1m

$$A_1 = \frac{\pi}{4}\left(D_1{}^2\right) = \frac{\pi}{4}(0.1)^2 = 0.007254\, m^2$$

V_1 = 5m/sec

At section 2, D_2 = 15cms =0.15

$$A_2 = \frac{\pi}{4}(0.15)^2 = 0.01767\, m^2$$

Discharge through pipe, Q = $A_1 \times V_1$ = 0.007854 ×5 = 0.03927 m^3/sec

We have, $A_1 V_1 = A_2 V_2$

$$V_2 = \frac{A_1 V_1}{A_2} = \frac{0.007854}{0.01767} \times 5.0 = 2.22 m/s$$

TOPICS TO BE COVERED

- Fluid Dynamics
- Surface, Line & Body forces
- Forces acting on fluid in motion
- Euler’s equation of motion

LECTURE 4

Fluid Dynamics- Surface & Body forces

FLUID DYNAMICS

- A fluid in motion is subjected to several forces, which results in the variation of the acceleration and the energies involved in the flow of the fluid.
- The study of the forces and energies that are involved in the fluid flow is known as Dynamics of fluid flow.
- The various forces acting on a fluid mass may be classified as:
  - Body or volume forces
  - Surface forces
  - Line forces.

FORCES

- **Body forces:** The body forces are the forces which are proportional to the volume of the body.

  Examples: Weight, Centrifugal force, magnetic force, Electromotive force etc.

- **Surface forces:** The surface forces are the forces which are proportional to the surface area which may include pressure force, shear or tangential force, force of compressibility and force due to turbulence etc.
- **Line forces:** The line forces are the forces which are proportional to the length.

  Example: surface tension.

- The dynamics of fluid flow is governed by Newton's second law of motion which states that the resultant force on any fluid element must be equal to the product of the mass and acceleration of the element and the acceleration vector has the direction of the resultant vector.
- The fluid is assumed to be in compressible and non- viscous.

$$\sum F_x = M.\, a$$

Where $\sum F$ represents the resultant external force acting on the fluid element of mass **M** and **a** is total acceleration.

- Both the acceleration and the resultant external force must be along same line of action.
- The force and acceleration vectors can be resolved along the three reference directions x, y and z and the corresponding equations may be expressed as ;

$$\sum F_x = M.\ a_x$$

$$\sum F_y = M.\ a_y$$

$$\sum F_z = M.\ a_z$$

Where $\sum F_x$, $\sum F_y$ and $\sum F_z$ are the components of the resultant force in the x, y and z directions respectively, and a_x , a_y and a_z are the components of the total acceleration in x, y and z directions respectively.

FORCES ACTING ON FLUID IN MOTION

- The various forces that influence the motion of fluid are due to gravity, pressure, viscosity, turbulence and compressibility.
- The gravity force ‘Fg’ is due to the weight of the fluid and is equal to Mg . The gravity force per unit volume is equal to “ρ g”.
- The pressure force ‘Fp’ is exerted on the fluid mass, if there exists a pressure gradient between the two points in the direction of the flow.
- The viscous force ‘Fv’ is due to the viscosity of the flowing fluid and thus exists in case of all real fluids.
- The turbulent flow ‘Ft’ is due to the turbulence of the fluid flow.
- The compressibility force ‘Fc’ is due to the elastic property of the fluid and it is important only for compressible fluids.

FORCES ACTING ON FLUID IN MOTION

- If a certain mass of fluid in motion is influenced by all the above forces, then according to Newton's second law of motion
- The net force $F_x = M.\ a_x = (F_g)_x + (F_p)_x + (F_v)_x + (F_t)_x + (F_c)_x$
- If the net force due to compressibility(F_c) is negligible, the resulting net force

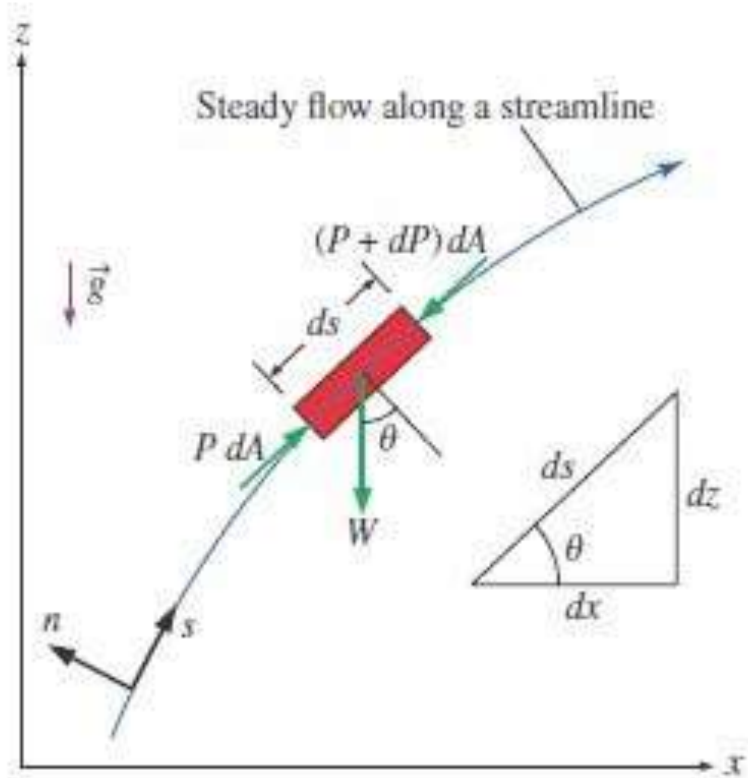
$$F_x = (F_g)_x + (F_p)_x + (F_v)_x + (F_t)_x$$

and the equation of motions are called **Reynolds's equations of motion.**

- For flow where (Ft) is negligible, the resulting equations of motion are known as **Navier – Stokes equation.**
- If the flow is assumed to be ideal, viscous force (F_v) is zero and the equations of motion are known as **Euler's equation of motion.**

EULER'S EQUATION OF MOTION

- In this equation of motion the forces due to gravity and pressure are taken in to consideration.
- This is derived by considering the motion of the fluid element along a stream-line as:
- Consider a stream-line in which flow is taking place in s-direction.
- Consider a cylindrical element of cross-section dA and length ds.



- The forces acting on the cylindrical element are:

➢ Pressure force pdA in the direction of flow.

➢ Pressure force $\left(p + \frac{\partial \mathrm{p}}{\partial \mathrm{s}} \mathrm{ds}\right) dA$

➢ Weight of element ρ g dA.ds

- Let θ is the angle between the direction of flow and the line of action of the weight of the element.

- The resultant force on the fluid element in the direction of S must be equal to the mass of fluid element× acceleration in the direction of s.

$$pdA - \left(p + \frac{\partial \mathrm{p}}{\partial \mathrm{s}} ds\right) dA - \rho g dA ds \cos\theta = \rho dA ds \times a_s \quad \text{\_\_\_\_(1)}$$

Whereas is the acceleration in the direction of s.

- Now, $a_s = \frac{dv}{dt}$ where 'v' is a function of s and t.

$$a_s = \frac{\partial v}{\partial s}\frac{ds}{dt} + \frac{\partial v}{\partial t} = \frac{v\partial v}{\partial s} + \frac{\partial v}{\partial t}$$

- If the flow is steady, then $\frac{\partial v}{\partial t} = 0$. So, $a_s = \frac{v\partial v}{\partial s}$
- Substituting the value of a_s in equation (1) and simplifying, we get

$$-\frac{\partial p}{\partial s}dsdA - \rho g dAds\cos\theta = \rho dAds \times \frac{\partial v}{\partial s}$$

- Dividing by ρ dA.ds, $-\left(\frac{1}{\rho}\right) \times \left(\frac{\partial p}{\partial s}\right) - g\cos\theta = \frac{v\partial v}{\partial s}$

$$\left(\frac{1}{\rho}\right) \times \left(\frac{\partial p}{\partial s}\right) + g\cos\theta + \frac{v\partial v}{\partial s} = 0$$

But we have $\cos\theta = \frac{dz}{ds}$

$$\left(\frac{1}{\rho}\right) \times \left(\frac{\partial p}{\partial s}\right) + g\frac{dz}{ds} + \frac{v\partial v}{\partial s} = 0$$

$$\boxed{\frac{\partial p}{\rho} + gdz + vdv = 0}$$

∴ This equation is known as Euler's equation of motion.

LECTURE 5

Bernoulli's Equation

TOPICS TO BE COVERED

- Bernoulli's equation
- Assumptions of Bernoulli's equation
- Momentum equation

Bernoulli's Equation

BERNOULLI'S EQUATION

- Bernoulli's equation is obtained by integrating the Euler's equation of motion as

$$\int \frac{dp}{\rho} + \int gdz + \int vdv = Constant$$

- If the flow is incompressible, ρ is constant and

$$\frac{\mathrm{p}}{\rho} + \mathrm{gz} + \frac{\mathrm{v}^2}{2} = \mathrm{constant}$$

$$\frac{\mathrm{p}}{\rho \mathrm{g}} + \mathrm{z} + \frac{\mathrm{v}^2}{2\mathrm{g}} = \mathrm{constant}$$

$$\frac{\mathrm{p}}{\rho \mathrm{g}} + \frac{\mathrm{v}^2}{2\mathrm{g}} + \mathrm{z} = \mathrm{constant}$$

- The above equation is Bernoulli's equation in which

$\frac{p}{\rho g}$ = Pressure energy per unit weight of fluid or pressure head.

$\frac{v^2}{2g}$ = Kinetic energy per unit weight of fluid or Kinetic head.

z = Potential energy per unit weight of fluid or Potential head.

ASSUMPTIONS OF BERNOULLI'S EQUATION

- The following are the assumptions made in the derivation of Bernoulli's equation.

➢ The fluid is ideal i.e. Viscosity is zero.

➢ The flow is steady.

➢ The flow is incompressible.

➢ The flow is Irrotational.

MOMENTUM EQUATION

- It is based on the law of conservation of momentum or on the momentum principle, which states that the net force acting on a fluid mass equal to the change in the momentum of the flow per unit time in that direction.
- The force acting on a fluid mass ‘m’ is given by Newton’s second law of motion.

$$F = m \times a$$

- Where a is the acceleration acting in the same direction as force F.

But $a = \frac{dv}{dt}$

$F = m\frac{dv}{dt} = \frac{d(mv)}{dt}$(Since m is a constant and can be taken inside differential)

$$F = \frac{d(mv)}{dt}$$

The above equation is known as the momentum principle.

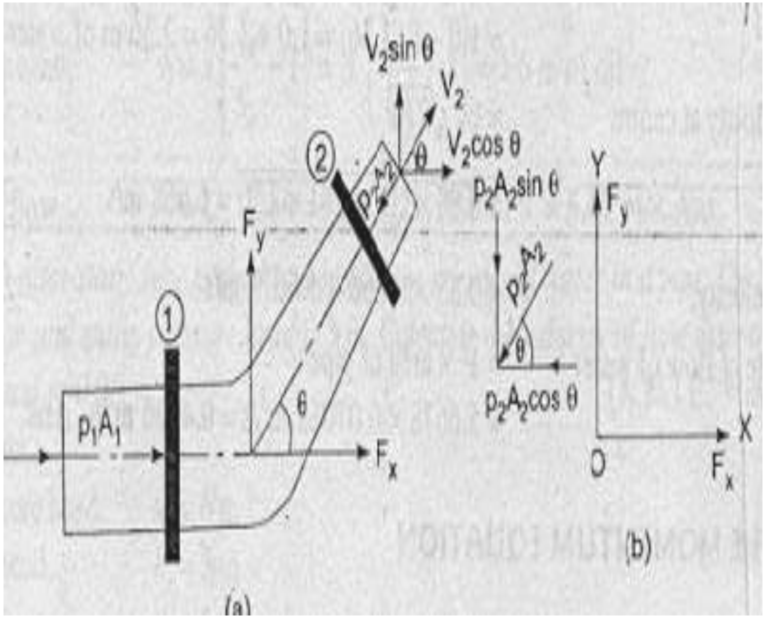
$$F.\ dt = d(mv)$$

The above equation is known as the impulse momentum equation.

- It states that the impulse of a force ‘F’ acting on a fluid mass m in a short interval of time ‘dt’ is equal to the change of momentum ‘d(mv)’ in the direction of force.

FORCE EXERTED BY A FLOWING FLUID ON A PIPE-BEND:

- The impulse momentum equation is used to determine the resultant force exerted by a flowing fluid on a pipe bend.
- Consider two sections (1) and (2) as above
- Let v_1= Velocity of flow at section (1)
- P_1= Pressure intensity at section (1)
- A_1 = Area of cross-section of pipe at section (1)
- And V_2, P_2, A_2 are corresponding values of Velocity, Pressure, Area at section (2)



- Let F_x and F_y be the components of the forces exerted by the flowing fluid on the bend in *x* and *y* directions respectively.
- Then the force exerted by the bend on the fluid in the directions of *x* and *y* will be equal to F_X and F_Y but in the opposite directions.
- Hence the component of the force exerted by the bend on the fluid in the *x* – direction = $-F_x$ and in the direction of *y* = $-F_y$.
- The other external forces acting on the fluid are $p_1 A_1$ and $p_2 A_2$ on the sections (1) and (2) respectively. Then the momentum equation in x-direction is given by
- Net force acting on the fluid in the direction of x = Rate of change of momentum in x –direction

$= p_1 A_1 - p_2 A_2 \cos\theta - F_x$ = (Mass per second) (Change of velocity)

$= \rho Q$ (Final velocity in x-direction – Initial velocity in x-direction)

$= \rho \; Q \; (V_2 \; Cos \; \theta \; - \; V_1)$

$F_x = \rho \; Q \; (V_1 - V_2 \; Cos \; \theta) + p_1 A_1 - p_2 A_2 \; Cos \; \theta$ ________(1)

- Similarly the momentum equation in y-direction gives

$0 - p_2 A_2 \; Sin \; \theta - F_y = \rho \; Q \; (V_2 \; Sin \; \theta - 0)$

$F_y = \rho \; Q \; (-V_2 \; Sin \; \theta) - p_2 A_2 \; Sin \; \theta$ _________________(2)

- Now the resultant force (F_R) acting on the bend

$$F_R = \sqrt{F_x^{\,2} + F_y^{\,2}}$$

- And the angle made by the resultant force with the horizontal direction is given by

$$\boldsymbol{tan\theta = \frac{F_y}{F_x}}$$

TOPICS TO BE COVERED

- Problems on Bernoulli's equation

Bernoulli's Equation

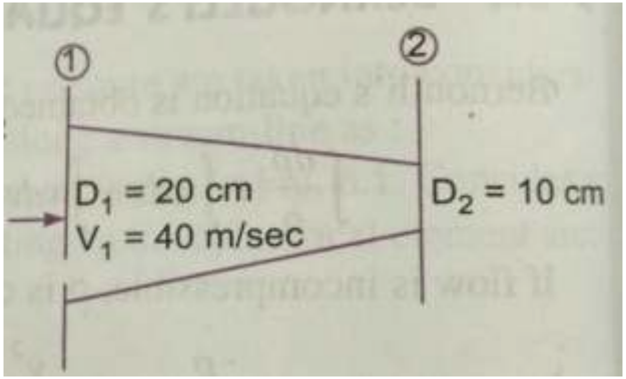
LECTURE 6

Problems

PROBLEM 1

- A pipe through which water is flowing is having diameters 20cms and 10cms at cross-sections 1 and 2 respectively. The velocity of water at section 1 is 4 m/sec. Find the velocity head at section 1 and 2 and also rate of discharge?

Sol: Given data



D_1 = 20cms = 0.2m

$A_1 = \left(\frac{\pi}{4}\right) \times (0.2)^2 = 0.0314 m^2$

V_1 = 4 m/sec

D_2 = 10 cm = 0.1 m

$A_2 = \left(\frac{\pi}{4}\right) \times (0.1)^2 = 0.007854 m^2$

i) Velocity head at section 1

$$\frac{V_1{}^2}{2g} = \frac{4\times4}{2\times9.81} = 0.815m$$

ii) Velocity head at section 2 $\frac{V_2{}^2}{2g}$

To find V_2, apply continuity equation, $A_1 V_1 = A_2 V_2$

$$V_2 = \frac{A_1 V_1}{A_2} = \frac{0.0314\times4}{0.00785} = 16m/sec$$

- Velocity head at section 2
- $$\frac{V_2{}^2}{2g} = \frac{16\times16}{2\times9.81} = 13.047m$$
- **iii)** Rate of discharge

$Q = A_1 V_1 = A_2 V_2$

$= 0.0314 \times 4 = 0.1256\ m^3/sec$

Q = 125.6 Liters/sec

PROBLEM 2

- Water is flowing through a pipe of 5cm dia. Under a pressure of 29.43N/cm^2 and with mean velocity of 2 m/sec. find the total head or total energy per unit weight of water at a cross-section, which is 5m above datum line.

Sol: Given data

Dia. of pipe, d = 5cm = 0.05m

Pressure, P = 29.43N/cm^2 = 29.43 x 10^{4}N/m^2

Velocity, V = 2 m/sec

Datum head, Z = 5m

Total head = Pressure head + Kinetic head + Datum head

$$\text{Pressure head} = \frac{p}{\rho g} = \frac{(29.43 \times 10^4)}{(1000 \times 9.81)} = 30m$$

Kinetic head $= \frac{V^2}{2g} = \frac{2 \times 2}{2 \times 9.81} = 0.204m$

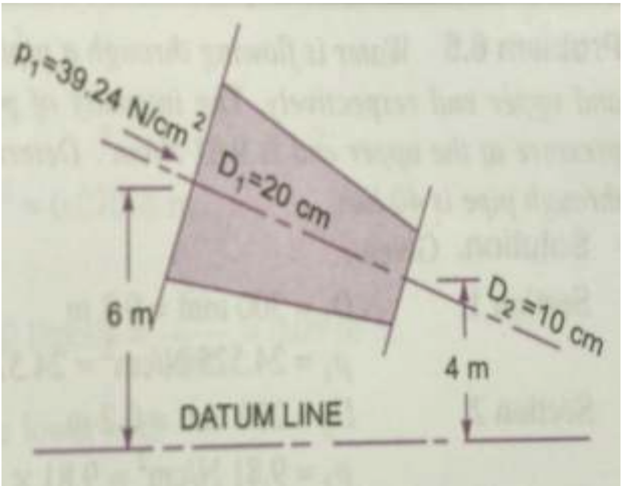
Datum head = Z = 5m

$$\frac{p}{\rho g} + \frac{V^2}{2g} + Z = 30 + 0.204 + 5 = 35.204m$$

Total head = 35.204m

PROBLEM 3

- Water is flowing through a pipe having diameters 20cms and 10cms at sections 1 and 2 respectively. The rate of flow through pipe is 35 liters/sec. The section 1 is 6m above the datum and section 2 is 4m above the datum. If the pressure at section 1 is 39.24n/cm^2. Find the intensity of pressure at section 2?



Sol: Given data

At section 1, D_1 = 20cm =0.2m

$$A_1 = \left(\frac{\pi}{4}\right) \times (0.2)^2 = 0.0314m^2$$

P_1 = 39.24N/cm^2 =39.24 x 10^{4}N/m^2

Z_1 = 6m

At section 2, D_2 =10cm = 0.1m

$$A_2 = \left(\frac{\pi}{4}\right) \times (0.1)^2 = 0.0007854m^2$$

Z_2 = 4m

Rate of flow Q = 35 lt/sec = (35/1000) m^3/sec = 0.035 m^3/sec

Q = $A_1V_1 = A_2V_2$

$$V_1 = \frac{Q}{A_1} = \frac{0.035}{0.0314} = 1.114 m/sec, V_2 = \frac{Q}{A_2} = \frac{0.035}{0.007854} = 4.456 m/sec$$

Applying Bernoulli's equation at sections 1 and 2

$$\frac{P_1}{\rho g} + \frac{{V_1}^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{{V_2}^2}{2g} + z$$

$$=(39.24 \times 10^4)(1000 \times 9.81) + \frac{(1.114)^2}{2\times9.81} + 6 = \frac{P_2}{(1000\times9.81)} + \frac{(4.456)^2}{2\times9.81} + 4$$

$$= 40 + 0.063 + 6 = \frac{P_2}{9810} + 1.102 + 4$$

$$= 46.063 = \frac{P_2}{9810} + 5.102$$

$$= \frac{P_2}{9810} = 46.063 - 5.102 = 41.051$$

Therefore, P_2 = 41.051 ×9810 =402710 N/m^2

P_2 = 40.271N/cm^2

PROBLEM 4

- Water is flowing through a pipe having diameter 300mm and 200mm at the bottom and upper end respectively. The intensity of pressure at the bottom end is 24.525N/cm^2 and the pressure at the upper end is 9.81N/cm^2. Determine the difference in datum head if the rate of flow through is 40lit/sec?

Sol: Given data

Section 1, D_1= 300mm =0.3m

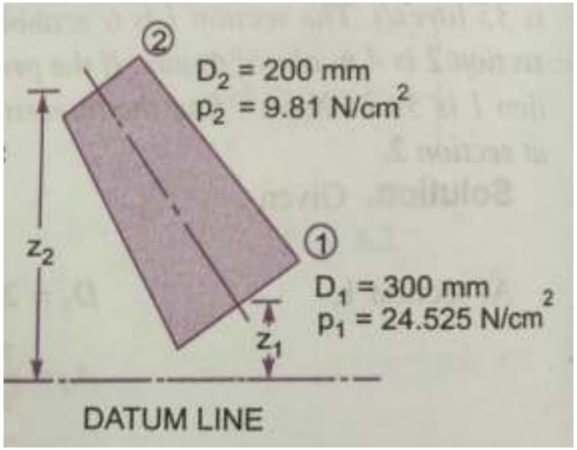
$A_1 = \left(\frac{\pi}{4}\right) \times (0.3)^2 A_1 = 0.07065 \text{ m}^2$

P_1= 24.525N/cm^2 = 24.525 × 10^{4}N/m^2

Section 2, D_2 = 200mm 0.2m

$A_2 = \left(\frac{\pi}{4}\right)) \times (0.2)^2 = 0.0314 \text{ m}^2$

P_2 = 9.81N/cm^2 = 9.81 × 10^4 N/m^2



Rate of flow , Q = 40 lit/Sec =40/1000 = 0.04 m^3/sec

$Q = A_1 V_1 = A_2 V_2$

$$V_1 = \frac{Q}{A_1} = \frac{0.04}{0.07065} = 0.566 \text{m/sec}$$

$$V_2 = \frac{Q}{A_2} = \frac{0.04}{00.0314} = 1.274 \text{m/sec}$$

Applying Bernoulli's equation at sections 1 and 2

$$\frac{P_1}{\rho g} + \frac{{V_1}^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{{V_2}^2}{2g} + Z_2$$

$$= \frac{(24.525\times10^4)}{(1000\times9.81)} + \frac{(0.566)^2}{(2\times9.81)} + Z_1 = \frac{(9.81\times10^4)}{(1000\times9.81)} + \frac{(1.274)^2}{(2\times9.81)} + Z_2$$

$= 25 + 0.32 + Z_1 = 10 + 1.623 + Z_2$

$= Z_2 – Z_1 = 25.32 – 11.623 = 13.697$ or say 13.70m

The difference in datum head = **$Z_2 – Z_1 = 13.70$m**

TOPICS TO BE COVERED

- Problems on Momentum equation
- Applications
- Assignment Questions

Bernoulli's Equation

LECTURE 7

Problems

PROBLEM 1

- The water is flowing through a taper pipe of length 100m having diameters 600mm at the upper end and 300mm at the lower end, at the rate of 50lts/sec. the pipe has a slope of 1 in 30. Find the pressure at the lower end, if the pressure at the higher level is 19.62N/cm^2?

Sol: Given data

Length of pipe L = 100m

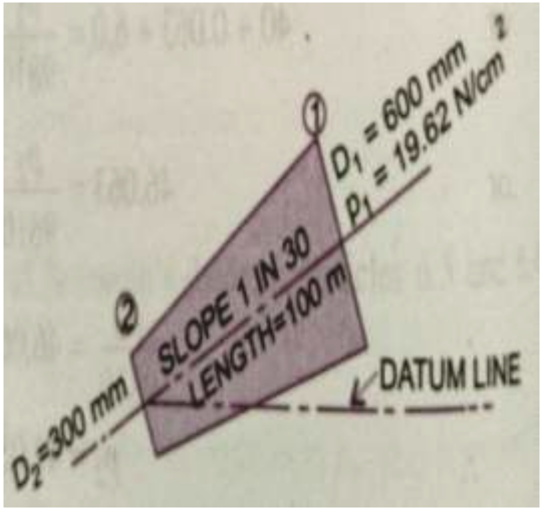
Dia. At the upper end D_1 = 600mm = 0.6

$$A_1 = \left(\frac{\pi}{4}\right) \times (0.6)^2 = 0.2827\text{m}^2$$

P_1 = 19.62N/cm^2 = 19.62 × 10^4 N/m^2

Dia. at the lower end D_2 = 300mm = 0.3m

$$A_2 = \left(\frac{\pi}{4}\right) \times (0.3)^2 = 0.07065\text{m}^2$$



Rate of flow Q = 50 Lts/sec = $\frac{50}{1000}$= 0.05 m^3/sec

Let the datum line is passing through the centre of the lower end.

Then Z_2 = o

As slope is 1 in 30 means $Z_1 = \left(\frac{1}{30}\right) \times 100 = \left(\frac{10}{3}\right) \mathrm{m}$

We also know that, $Q = A_1 V_1 = A_2 V_2$

$$V_1 = \frac{Q}{A_1} = \frac{0.05}{0.2827} = 0.177 \mathrm{m/sec}$$

$$V_2 = \frac{Q}{A_2} = \frac{0.05}{0.07065} = 0.707 \mathrm{m/sec}$$

Applying Bernoulli's equation at sections 1 and 2

$$\frac{P_1}{\rho g} + \frac{{V_1}^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{{V_2}^2}{2g} + Z_2$$

$$= \frac{(19.62\times10^4)}{1000\times9.81} + \frac{(0.177)^2}{2\times9.81} + \frac{10}{3} = \frac{P_2}{1000\times9.81} + \frac{(0.707)^2}{2\times9.81} + 0$$

$$= 20 + 0.001596 + 3.334 = \frac{P_2}{9810} + 0.0254$$

$$= 23.335 = \frac{P_2}{9810} + 0.0254$$

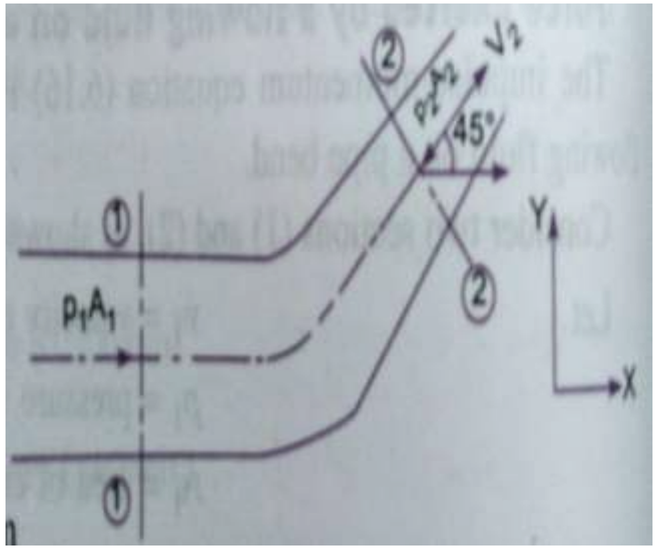
$$= \frac{P_2}{9810} = 23.335 - 0.0254 = 23.31$$

$$= P_2 = 23.31 \times 9810 = 228573 N/m^2$$

$P_2 = 22.857 N/cm^2$

PROBLEM 2

- A 45° reducing bend is connected to a pipe line, the diameters at inlet and out let of the bend being 600mm and 300mm respectively. Find the force exerted by the water on the bend, if the intensity of pressure at the inlet to the bend is 8.829N/cm² and rate of flow of water is 600 Lts/sec.



<u>Sol:</u> Given data

Angle of bend $\theta = 45°$

Dia. at inlet $D_1 = 600mm = 0.6m$

$A_1 = \left(\frac{\pi}{4}\right) \times (0.6)^2 = 0.2827\ m^2$

Dia. at outlet $D_2 = 300mm = 0.3m$

$A_2 = \left(\frac{\pi}{4}\right) \times (0.3)^2 = 0.07065\ m^2$

Applying Bernoulli's equation at sections 1 and 2, we get

$$\frac{P_1}{\rho g} + \frac{{V_1}^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{{V_2}^2}{2g} + Z_2$$

But $Z_1 = Z_2$, then $\frac{P_1}{\rho g} + \frac{{V_1}^2}{2g} = \frac{P_2}{\rho g} + \frac{{V_2}^2}{2g}$

$$= \frac{8.829\times 10^4}{1000\times 9.81} + \frac{(2.122)^2}{2\times 9.81} = \frac{\mathrm{P}_2}{1000\times 9.81} = \frac{(8.488)^2}{2\times 9.81}$$

$$= 9 + 0.2295 = \frac{\mathrm{P}_2}{9810} + 3.672$$

$$= \frac{\mathrm{P}_2}{9810} = 9.2295 - 3.672 = 5.5575 \text{ m of water}$$

$$P_2 = 5.5575 \times 9810 = 5.45 \times 10^4 \text{ N/m}^2$$

Force exerted on the bend in X and Y – directions

$F_x = \rho\, Q\, (V_1 - V_2 \text{ Cos } \theta) + P_1 A_1 - P_2 A_2 \text{ Cos } \theta$

$= 1000 \times 0.6\ (2.122 - 8.488 \text{ Cos}45°) + 8.829 \times 10^4 \times 0.2827 - 5.45 \times 10^4 \times 0.07065 \times \text{Cos}45°$

$= -\ 2327.9 + 24959.6 - 2720.3 = 24959.6 - 5048.2 = 19911.4\text{N}$

$F_x =$ 19911.4 N

$F_y = \rho\, Q\, (-V_2 \text{ Sin } \theta) - P_2 A_2 \text{ Sin } \theta$

$= 1000 \times 0.6\ (-\ 8.488 \text{ Sin}45°) - 5.45 \times 10^4 \times 0.07068 \text{ Sin}45°$

$= -\ 3601.1 - 2721.1 = -\ 6322.2\text{N}$

(- ve sign means Fy is acting in the down ward direction)

$F_y = -$ 6322.2N

Therefore the Resultant Force $F_R = \sqrt{F_x^2 + F_y^2} = \sqrt{(19911.4)^2 + (-6322.2)^2} = 20890.9N$

F_R = 20890.9 N

The angle made by resultant force with X – axis is Tan $\theta = \frac{F_x}{F_y}$

= (6322.2/19911.4) = 0.3175

θ= $\tan^{-1}$0.3175= $17^0 36'$

APPLICATIONS

- For sizing of pumps: Volute in the casing of centrifugal pumps converts velocity of fluid into pressure energy by increasing area of flow. The conversion of kinetic energy into pressure is according to Bernoulli's equation.
- Carburetor works on principle of Bernoulli's principle: the faster the air moves, the lower its static pressure and higher its dynamic pressure.
- Application of the Momentum Equation

➢ Force due to the flow of fluid round a pipe bend.

➢ Force on a nozzle at the outlet of a pipe.

➢ Impact of a jet on a plane surface.

ASSIGNMENT QUESTIONS

- A pipe line 300m long has a slope of 1 in 100 and tapers from 1.2m diameter at the high end to 0.6m at the low end. The discharge through the pipe is 5.4m^3/min. If the pressure at the high end is 70kPa, find the pressure at the lower end. Neglect losses.
- A pipe 1 of 450mm in diameter branches into two pipes (2&3) of diameter 300mm and 200mm. If average velocity in 450mm diameter pipe is 3/s. Find (i) Discharge through 450mm diameter pipe. (ii) Velocity in 200mm diameter pipe if the velocity in 300mm pipe is 2.5m/s.
- A pipe line ABC 200m long is laid on an upward slope 1 in 40. The length of the portion AB is 100m and its diameter is 100mm. At B the pipe section suddenly enlarges to 200mm diameter and remains so for the remainder of its length BC, 100m. A flow of 0.0m^3/s is pumped into the pipe at its lower end A and is discharged at the upper end C into closed tank. The pressure at the supply end is 200kN/m^2. What is the pressure at C?

- A 45^0 reducing bend is connected in a pipe line, the diameters at the inlet and outlet of the bend being 40cm and 20cm. Find the force exerted by water on the bend if the intensity of pressure at inlet of bend is $21.58 N/cm^2$. The rate of flow of water is 500litres/s.
- Derive Bernoulli's equation from Euler's equation of motion. What are the assumptions made in deriving Bernoulli's theorem?

LECTURE 8

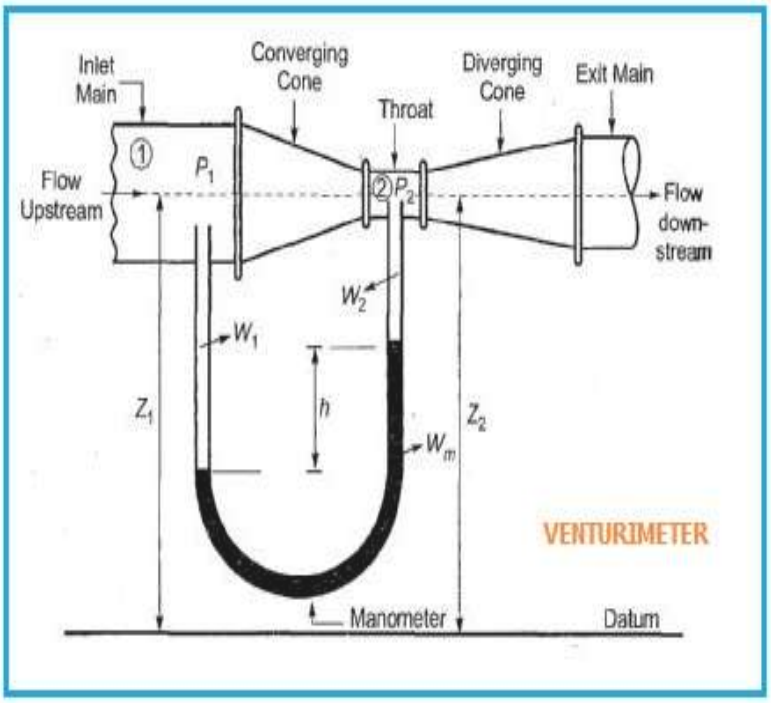
Measurement of flow

TOPICS TO BE COVERED

- Venturimeter
- Derivation of co-efficient of discharge for venturimeter
- Orificmeter

VENTURIMETER

- A venturimeter is a device used for measuring the rate of flow of fluid through a pipe.
- The basic principle on which venturimeter works is that by reducing the cross-sectional area of the flow passage, a pressure difference is created and the measurement of the pressure difference enables the determination of the discharge through the pipe.



- A venture meter consists of (1) an inlet section, followed by a converging cone (2) a cylindrical throat and (3) a gradually divergent cone.
- The inlet section of venture meter is the same diameter as that of the pipe which is followed by a convergent cone.
- The convergent cone is a short pipe, which tapers from the original size of the pipe to that of the throat of the venture meter.
- The throat of the venture meter is a short parallel – sided tube having its cross-sectional area smaller than that of the pipe.
- The divergent cone of the venture meter is a gradually diverging pipe with its cross-sectional area increasing from that of the throat to the original size of the pipe.
- At the inlet section and the throat i.e sections 1 and 2 of the venture meter pressure gauges are provided.

DERIVATION

- Let a_1 and a_2 be the cross-section areas at inlet and throat sections, at which P_1 and P_2 the pressures and velocities V_1 and V_2 respectively.
- Assuming the flowing fluid is incompressible and there is no loss of energy between section 1 and 2 and applying Bernoulli's equation between sections 1 and 2, we get,

$$\frac{P_1}{\omega} + \frac{{v_1}^2}{2g} + z_1 = \frac{P_2}{\omega} + \frac{{v_2}^2}{2g} + z_2$$

Where ω is the specific weight of flowing fluid.

- If the venturi meter is connected in a horizontal pipe, then $Z_1 = Z_2$, then

$$\frac{P_1}{\omega} + \frac{{v_1}^2}{2g} = \frac{P_2}{\omega} + \frac{{v_2}^2}{2g}$$

$$\frac{P_1}{\omega} - \frac{P_2}{\omega} = \frac{{v_2}^2}{2g} - \frac{{v_1}^2}{2g}$$

- In the above expression $\left(\frac{P_1}{\omega} - \frac{P_2}{\omega}\right)$ *is the pressure difference* between the pressure heads at section 1 and 2 , is known as venture head and is denoted by h

$$h = \frac{{v_2}^2}{2g} - \frac{{v_1}^2}{2g}$$

$$Q_{th} = a_1 v_1 = a_2 v_2 \quad ,$$

$$v_1 = \frac{Q_{th}}{a_1} \quad , \quad v_2 = \frac{Q_{th}}{a_2}$$

$$h = \frac{{Q_{th}}^2}{2g}\left(\frac{1}{{a_2}^2} - \frac{1}{{a_1}^2}\right)$$

$$Q_{th} = \frac{a_1 a_2 \sqrt{2gh}}{\sqrt{{a_1}^2 - {a_2}^2}}$$

$$Q = C_d Q_{th} = \frac{C_d a_1 a_2 \sqrt{2gh}}{\sqrt{{a_1}^2 - {a_2}^2}}$$

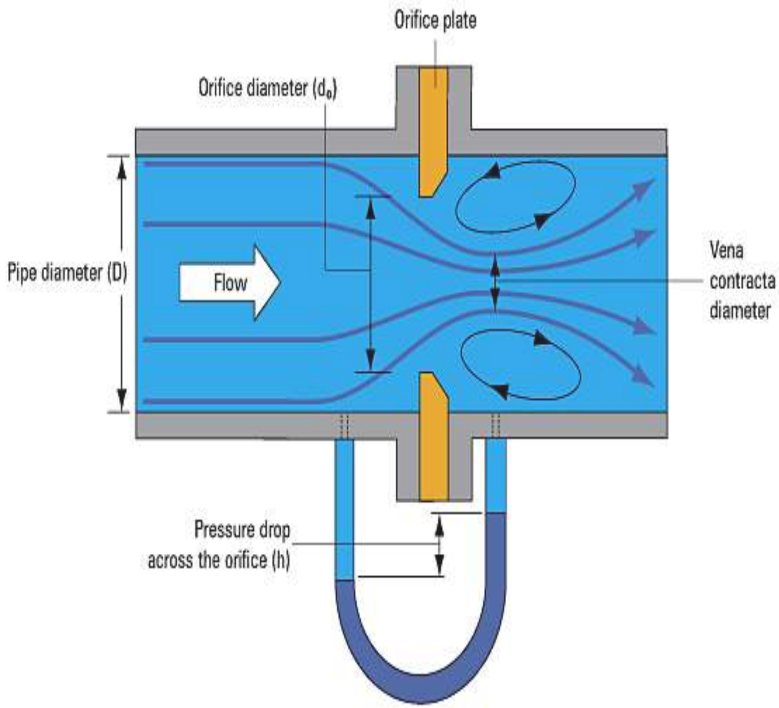
$$= C_d C \sqrt{h} \quad \left(\because C = \frac{a_1 a_2 \sqrt{2g}}{\sqrt{{a_1}^2 - {a_2}^2}} \right)$$

$$\boldsymbol{Q_{actual} = C_d C \sqrt{h}}$$

C_d = Co-efficient of discharge < 1

ORIFICEMETER

- An orifice meter is a simple device for measuring the discharge through pipes.
- Orifice meter also works on the same principle as that of venture meter i.e by reducing cross-sectional area of the flow passage, a pressure difference between the two sections is developed and the measurement of the pressure difference enables the determination of the discharge through the pipe.



- Orifice meter is a cheaper arrangement and requires smaller length and can be used where space is limited.

$$Q = C a_0 a_1 \frac{\sqrt{2gh}}{\sqrt{{a_1}^2 - {a_0}^2}}$$

- This gives the discharge through an orifice meter and is similar to the discharge through venture meter.
- The co-efficient C may be considered as the co-efficient of discharge of an orifice meter.
- The co-efficient of discharge for an orifice meter is smaller than that for a venture meter.
- This is because there are no gradual converging and diverging flow passages as in the case of venture meter, which results in a greater loss of energy and consequent reduction of the co-efficient of discharge for an orifice meter.

TOPICS TO BE COVERED

- Example problems on
  - Pitot tube
  - Venturimeter
  - Orificemeter
- Applications
- Assignment questions

LECTURE 9&10

Example Problems

PROBLEM 1

- At a sudden enlargement of a water main from 240mm to 480mm diameter, the hydraulic gradient rises by 10mm. Estimate the rate of flow.

Sol: Given Data

Dia. of smaller pipe D_1= 240mm =0.24m

Area $A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} (0.24)^2$

Dia. of larger pipe D_2 = 480mm = 0.48m

Area $A_2 = \frac{\pi}{4} D_2^2 = \frac{\pi}{4} (0.48)^2$

Rise of hydraulic gradient i.e. $\left(Z_2 + \frac{P_2}{\rho g}\right) - \left(Z_1 + \frac{P_1}{\rho g}\right)$ = 10mm $= \frac{10}{1000} m = \frac{1}{100} m$

Let the rate of flow = Q

Applying Bernoulli's equation to both sections i.e smaller and larger sections

$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$ + Head loss due to enlargement ______ (1)

But head loss due to enlargement, $h_e = \frac{(V_1 - V_2)^2}{2g}$ ________ (2)

From continuity equation, we have $A_1 V_1 = A_2 V_2$ $\quad V_1 = \frac{A_2 V_2}{V_1}$

$$V_1 = \frac{\frac{\pi}{4} D_2^{\ 2}\ V_2}{\frac{\pi}{4} D_1^{\ 2}} = \left(\frac{D_2}{D_1}\right)^2 \times V_2 = \left(\frac{0.48}{0.24}\right)^2 V_2 = 2^2\ V_2 = 4V_2$$

Substituting this value in equation (2), we get

$$h_e = \frac{(4V_2 - V_2)^2}{2g} = \frac{(3V_2)^2}{2g} = \frac{9V_2^{\ 2}}{2g}$$

Now substituting the value of h_e and V_1 in equation (1)

$$\frac{P_1}{\rho g} + \frac{(4V_2)^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^{\ 2}}{2g} + Z_2 + \frac{9V_2^{\ 2}}{2g}$$

$$\frac{16V_2^{\ 2}}{2g} - \frac{V_2^{\ 2}}{2g} - \frac{9V_2^{\ 2}}{2g} = \left(\frac{P_2}{\rho g} + Z_2\right) - \left(\frac{P_1}{\rho g} + Z_1\right)$$

But Hydraulic gradient rise = $\left(\frac{P_2}{\rho g} + Z_2\right) - \left(\frac{P_1}{\rho g} + Z_1\right) = \left(\frac{1}{100}\right)$ m

$$\frac{6{V_2}^2}{2g} = \left(\frac{1}{100}\right) \text{m} \qquad V_2 = \sqrt{\frac{2\times 9.81}{6\times 100}} = 0.1808 = 0.181 m/sec$$

Discharge, **Q** = $A_2\ V_2 = \frac{\pi}{4}\ {D_2}^2\ V_2$

$$= \frac{\pi}{4}(0.48)^2 \times 0.181 = 0.03275 m^3/sec$$

Q = 32.75Lts/sec

PROBLEM 2

- A 150mm dia. pipe reduces in dia. abruptly to 100mm dia. If the pipe carries water at 30lts/sec, calculate the pressure loss across the contraction. Take co-efficient of contraction as 0.6

Sol: Given Data

Dia. of larger pipe, D_1 = 150mm = 0.15m

Area of larger pipe, $A_1 = \frac{\pi}{4}(0.15)^2 = 0.01767 m^2$

Dia. of smaller pipe, D_2 = 100mm = 0.10m

Area of smaller pipe, $A_2 = \frac{\pi}{4}(0.10)^2 = 0.007854 m^2$

Discharge, Q = 30 lts/sec = $0.03 m^3/sec$

Co-efficient of contraction, C_C = 0.6

From continuity equation, we have $Q = A_1 V_1 = A_2 V_2$

$$V_1 = \frac{Q}{A_1} = \frac{0.03}{0.01767} = 1.697\ m/sec$$

$$V_2 = \frac{Q}{A_2} = \frac{0.03}{0.007854} = 3.82\ m/sec$$

Applying Bernoulli's equation before and after contraction

$$\frac{P_1}{\rho g} + \frac{V_1^{\ 2}}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^{\ 2}}{2g} + Z_2 + h_c \quad \_\_\_\_\_\_\_ \quad (1)$$

But $Z_1 = Z_2$ and h_c the head loss due to contraction is given by the equation

$$h_c = \left(\frac{V_2^{\ 2}}{2g}\right)\left[\left(\frac{1}{C_c} - 1\right)\right]^2 = \frac{(3.82)^2}{2\times 9.81}\left[\left(\frac{1}{0.6} - 1\right)\right]^2 = 0.33$$

Substituting these values in equation (1), we get

$$\frac{P_1}{\rho g} + \frac{(1.697)^2}{2\times 9.81} = \frac{P_2}{\rho g} + \frac{(3.82)^2}{2\times 9.81} + 0.33$$

$$\frac{P_1}{\rho g} + 0.1467 = \frac{P_2}{\rho g} + 0.7438 + 0.33$$

$$\frac{P_1}{\rho g} - \frac{P_2}{\rho g} = 0.7438 + 0.33 - 0.1467 = 0.9271\, m\ of\ Water$$

$$P_1 - P_2 = \rho g \times 0.9271 = 1000 \times 9.81 \times 0.9271 = 0.909 \times 10^4\ N/m^2$$

= **0.909 N/cm^2**

Pressure loss across contraction = **$P_1 - P_2$ = 0.909N/cm^2**

PROBLEM 3

- A pitot tube is placed in the centre of a 300mm pipe line has one end pointing upstream and other perpendicular to it. The mean velocity in the pipe is 0.80 of the central velocity. Find the discharge through the pipe, if the pressure difference between the two orifices is 60mm of water. Co-efficient of Pitot tube C_V =0.98

Sol: Given Data

Diameter of pipe = 300mm =0.3m

Difference of pressure head h = 60mm of water = 0.06m of water

Mean velocity $\bar{V}$ = 0.80 × central velocity

Central velocity = $C_v\sqrt{2gh} = 0.98 \times \sqrt{2 \times 9.81 \times 0.06} = 1.063 \frac{m}{sec}$

Mean velocity =0.8 × 1.063 = 0.8504m/sec

Discharge, Q = Area of pipe × Mean velocity = A × $\bar{V}$

$= \frac{\pi}{4}(0.3)^2 \times 0.8504$

Q = 0.06m³/sec

PROBLEM 4

- Find the velocity of flow of an oil through a pipe, when the difference of mercury level in a differential U-tube manometer connected to the two tappings of the pitot tube is 100mm. Co-efficient of pitot tube C = 0.98 and sp.gr.of oil =0.8.

Sol: Given Data

Difference of mercury level x = 100mm = 0.1m

Sp.gr. of oil = 0.8, $C_V = 0.98$

Difference of pressure head $h = x\left[\left(\frac{S_m}{S}\right) - 1\right] = 0.1\left[\left(\frac{13.6}{0.8}\right) - 1\right]$

= 1.6 m of oil

Velocity of flow = $C_V \sqrt{2 \times g \times h} = 0.98\sqrt{2 \times 9.81 \times 1.6}$

V = 5.49m/sec

PROBLEM 5

- A pitot tube is used to measure the velocity of water in a pipe. The stag nature pressure head is 6m and static pressure head is 5m. Calculate the velocity of flow. Co-efficient of pitot tube is 0.98.

Sol: Given Data

Stag nature pressure head, h_S =6m

Static pressure head, h_t =5m

$h = h_S - h_t = 6 - 5 = 1m$

Velocity of flow, $V = C_V \sqrt{2 \times g \times h}$

$= 0.98 \sqrt{2 \times 9.81 \times 1}$

V = 4.34 m/sec

PROBLEM 6

- A pitot tube is inserted in a pipe of 300mm diameter. The static pressure in the pipe is 100mm of mercury (Vacuum). The stagnation pressure at the centre of the pipe is recorded by Pitot tube is 0.981N/cm^2. Calculate the rate of flow of water through the pipe. The mean velocity of flow is 0.85 times the central velocity $C_V = 0.98$
- Sol: Given Data

Diameter of pipe d = 0.3m

Area of pipe a = $\frac{\pi}{4}(0.3)^2$ =0.07068m^2

Static pressure head = 100mm of mercury = $\frac{100}{1000} \times 13.6$ =1.36m of water

Stagnation pressure head = $\frac{0.981 \times 10^4}{1000} \times 9.81 = 1\ m$

h= Stagnation pressure head – static pressure head = 1 – (- 1.36) = 2.36m

Velocity at centre = $C_V \sqrt{2g\,h}$ = 0.98 $\sqrt{2 \times 9.81 \times 2.36}$ = 6.668m/sec

Mean velocity $\bar{V}$ = 0.85× 6.668 = 5.6678m/sec

Rate of flow of water = $\bar{V}$ × Area of pipe

= 5.6678 × 0.07068

Q = 0.4006m^3/sec

PROBLEM 7

Example 6.28. *A horizontal venturimeter with inlet diameter 200 mm and throat diameter 100 mm is used to measure the flow of water. The pressure at inlet is 0.18 N/mm² and the vacuum pressure at the throat is 280 mm of mercury. Find the rate of flow. The value of C_d may be taken as 0.98.*

Solution. Inlet diameter of venturimeter, $D_1 = 200 \text{ mm} = 0.2 \text{ m}$

$$\therefore \quad \text{Area of inlet, } A_1 = \frac{\pi}{4} \times 0.2^2 = 0.0314 \text{ m}^2$$

$$\text{Throat diameter, } D_2 = 100 \text{ mm} = 0.1 \text{ m}$$

$$\therefore \quad \text{Area of throat, } A_2 = \frac{\pi}{4} \times 0.1^2 = 0.00785 \text{ m}^2$$

$$\text{Pressure at inlet, } p_1 = 0.18 \text{ N/mm}^2 = 180 \text{ kN/m}^2$$

$$\therefore \quad \frac{p_1}{w} = \frac{180}{9.81} = 18.3 \text{ m}$$

Vacuum pressure at the throat,

$$\frac{p_2}{w} = -280 \text{ mm of mercury}$$

$$= -0.28 \text{ m of mercury} = -0.28 \times 13.6 = -3.8 \text{ m of water}$$

$$\text{Co-efficient of discharge, } C_d = 0.98$$

$$\therefore \quad \text{Differential head, } h = \frac{p_1}{w} - \frac{p_2}{w} = 18.3 - (-3.8) = 22.1 \text{ m}$$

Rate of flow, Q:

Using the relation,

$$Q = C_d \times \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \times \sqrt{2gh}, \text{ we have:}$$

$$= 0.98 \times \frac{0.0314 \times 0.00785}{\sqrt{(0.0314)^2 - (0.00785)^2}} \times \sqrt{2 \times 9.81 \times 22.1}$$

$$= \frac{0.000241}{0.0304} \times 20.82$$

PROBLEM 8

Example 6.39. *The following data relate to an orificemeter:*
Diameter of the pipe = 240 mm
Diameter of the orifice = 120 mm
Sp. gravity of oil = 0.88
Reading of differential manometer = 400 mm of mercury
Co-efficient of discharge of the meter = 0.65.
Determine the rate of flow of oil.

Solution. Diameter of the pipe $D_1 = 240 \text{ mm} = 0.24 \text{ m}$

$\therefore$ Area of the pipe, $A_1 = \frac{\pi}{4} \times 0.24^2 = 0.0452 \text{ m}^2$

Diameter of the orifice, $D_0 = 120 \text{ mm} = 0.12 \text{ m}$

$\therefore$ Area of the orifice, $A_0 = \frac{\pi}{4} \times 0.12^2 = 0.0113 \text{ m}^2$

Co-efficient of discharge, $C_d = 0.65$

Sp. gravity of oil, $S_0 = 0.88$

Reading of differential manometer, $y = 400$ mm of mercury $= 0.4$ m of mercury

$\therefore$ Differential head, $h = y\left[\frac{S_{hl}}{S_o} - 1\right]$

[where, S_{hl} = sp. gravity of heavier liquid = 13.6 (for mercury)]

$$= 0.4\left[\frac{13.6}{0.88} - 1\right] = 5.78 \text{ m of oil}$$

Discharge Q:

Using the relation, $Q = C_d \dfrac{A_0 . A_1 . \sqrt{2gh}}{\sqrt{A_1^2 - A_0^2}}$, we have:

$$Q = 0.65 \times \frac{0.0113 \times 0.0452 \times \sqrt{2 \times 9.81 \times 5.78}}{\sqrt{(0.0452)^2 - (0.0113)^2}}$$

APPLICATIONS

- The Venturimeter which has long been used in hydraulics is here applied to the measurement of volume flow of blood through vessels.
- In fluid flow, **friction loss** (or skin friction) is the loss of pressure or “head” that occurs in pipe or duct flow due to the effect of the fluid's viscosity near the surface of the pipe or duct.
- In mechanical systems such as internal combustion engines, the term refers to the power lost in overcoming the friction between two moving surfaces, a different phenomenon.
- Rigorous calculation of the pressure loss for flowing gases, based on as properties, flow, and piping configuration (pipe length, fittings, and valves). Results can be printed out or "cut and paste" into other applications.

ASSIGNMENT QUESTIONS

- Two reservoirs are connected by a pipe line consisting of two pipes, one of 15cm diameter and length 6m and the other diameter 22.5cm and 16m length. If the difference of water levels in the two reservoirs is 6m. Calculate the discharge and draw the energy gradient line. Take f=0.004
- A venturimeter has an area ratio of 9 to 1, the larger diameter being 300 mm. During the flow, the recorded pressure head in the larger section is 6.5m and that at the throat 4.25m. Take C d = 0.99, compute the discharge through the meter.
- A crude oil of kinematic viscosity 0.4 stoke is flowing through a pipe of diameter 300mm at the rate of 300litres/sec. Find the head lost due to friction for a length of 50m of the pipe.
- Derive Darcy Weisbach equation.

- The rate of flow of water through a horizontal pipe is $0.254 m^3/s$. The diameter of the pipe which is 200mm is suddenly enlarged to 400mm. The pressure intensity in the smaller pipe is $11.772 N/cm^2$. Determine i) Loss of head due to sudden contraction ii) pressure intensity in the larger pipe iii) power lost due to friction
- Explain boundary layer concept in detail.

TOPICS TO BE COVERED

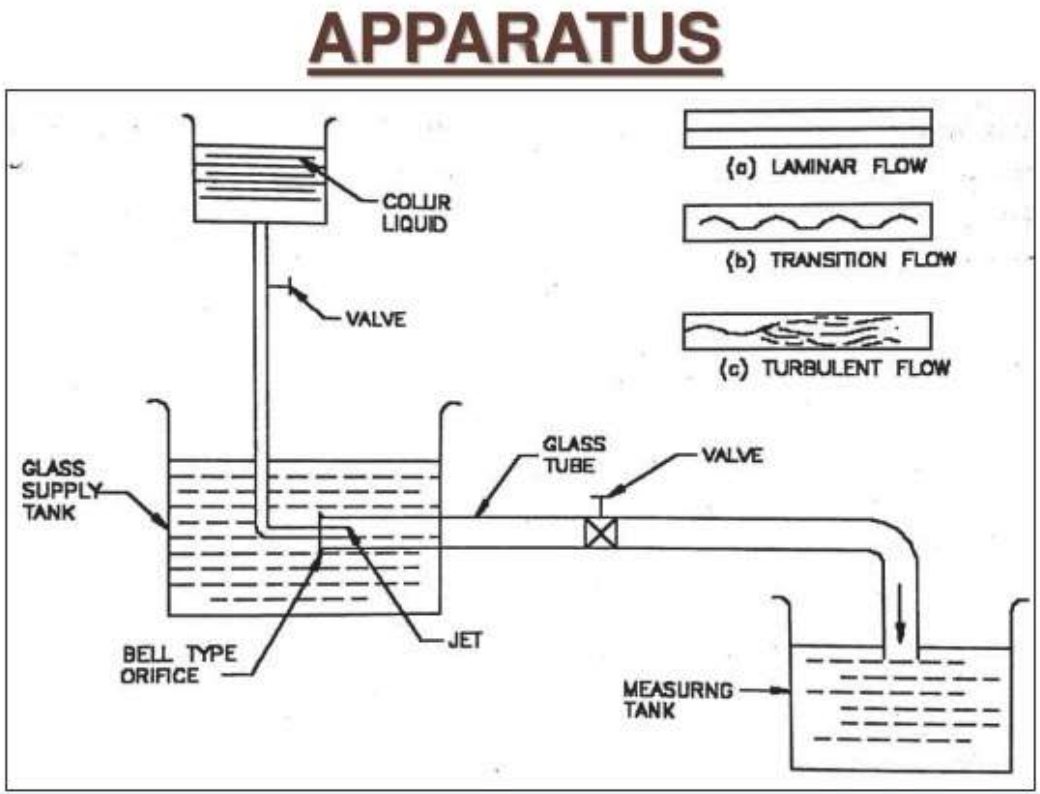
- Reynold's Experiment
- Reynold number

LECTURE 2

Reynold's Experiment

REYNOLD'S EXPERIMENT

REYNOLD'S APPARATUS



- It consists of a constant head tank filled with water, a small tank containing dye, a horizontal glass tube provided with a bell-mouthed entrance a and regulating valve.
- The water was made to flow from the tank through the glass tube in to the atmosphere and the velocity if flow was varied by adjusting the regulating valve.
- The liquid dye having the same specific weight as that of water was introduced in to the flow at the bell – mouth through a small tube.
- From the experiments it was disclosed that when the velocity of flow was low, the dye remained in the form of a straight line and stable filament passing through the glass tube so steady that it scarcely seemed to be in motion with increase in the velocity of flow a critical state was reached at which the filament of dye showed irregularities and began of waver.

- Further increase in the velocity of flow the fluctuations in the filament of dye became more intense and ultimately the dye diffused over the entire cross-section of the tube, due to intermingling of the particles of the flowing fluid
- Reynolds's deduced from his experiments that at low velocities the intermingling of the fluid particles was absent and the fluid particles moved in parallel layers or lamina, sliding past the adjacent lamina but not mixing with them, which is the laminar flow.
- At higher velocities the dye filament diffused through the tube it was apparent that the intermingling of fluid particles was occurring in other words the flow was turbulent.

- The velocity at which the flow changes from the laminar to turbulent for the case of a given fluid at a given temperature and in a given pipe is known as Critical Velocity.
- The state of flow in between these types of flow is known as transitional state or flow in transition.
- Reynolds discovered that the occurrence of laminar and turbulent flow was governed by the relative magnitudes of the inertia and the viscous forces.
- At low velocities the viscous forces become predominant and flow is viscous.
- At higher velocities of flow the inertial forces predominance over viscous forces.

REYNOLD'S NUMBER

- Reynolds related the inertia to viscous forces and arrived at a dimension less parameter.

$$R_e \ or \ N_e = \frac{\text{inertia force}}{\text{viscous force}} = \frac{F_i}{F_v}$$

- According to Newton's 2nd law of motion, the inertia force F_i is given by

F_i = mass × acceleration

= ρ × volume × acceleration (ρ = mass density)

$= \rho \times L^3 \times \frac{L}{T^2} = \rho L^2 V^2$ ---- (1) (L = Linear dimension)

- Similarly viscous force F_V is given by Newton's 2nd law of velocity as

$F_V = \tau \times \text{area}$ τ = shear stress

$= \mu \frac{dv}{dy} \times L^2 = \mu VL$ ---------------- (2)

V = Average Velocity of flow

μ = Viscosity of fluid

$$R_e \text{ or } N_R = \frac{\rho L^2 V^2}{\mu VL} = \frac{\rho VL}{\mu}$$

- In case of pipes L = D

- In case of flow through pipes

$$R_e = \frac{\rho DV}{\mu} \text{ or } \frac{VD}{\nu} \nu$$

Where μ/ρ = kinematic viscosity of the flowing liquid ν

- The Reynolds number is a very useful parameter in predicting whether the flow is laminar or turbulent.

➢ R_e < 2000 viscous / laminar flow

➢ R_e → 2000 to 4000 transient flow

➢ R_e > 4000 Turbulent flow

TOPICS TO BE COVERED

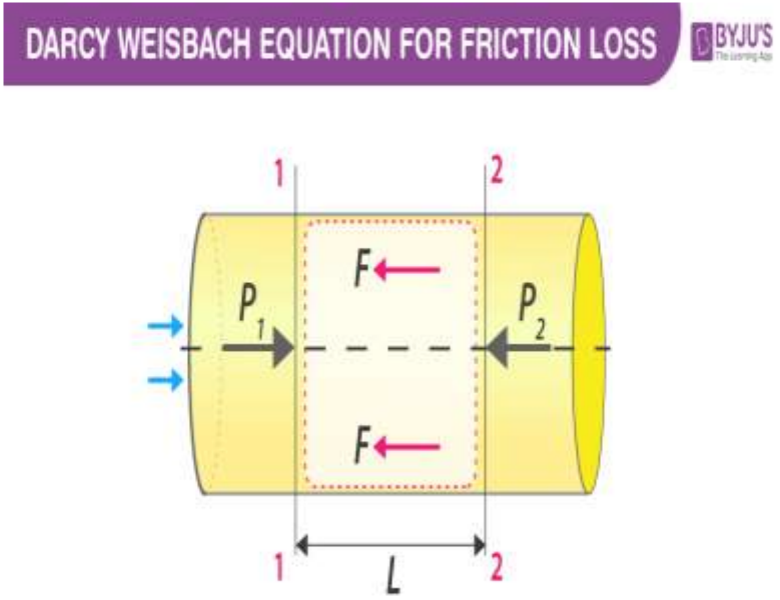
- Darcy Weisbach equation- Major loss
- Derivation
- Minor Losses in pipes

LECTURE 3

Losses in pipes

FRICTIONAL LOSS IN PIPE FLOW – DARCY WEISBACK EQUATION

- When a liquid is flowing through a pipe, the velocity of the liquid layer adjacent to the pipe wall is zero.
- The velocity of liquid goes on increasing from the wall and thus velocity gradient and hence shear stresses are produced in the whole liquid due to viscosity.
- This viscous action causes loss of energy, which is known as frictional loss.



DERIVATION

- Consider a uniform horizontal pipe having steady flow. Let 1-1, 2-2 are two sections of pipe.

Let P_1 = Pressure intensity at section 1-1

V_1 = Velocity of flow at section 1-1

L = Length of pipe between section 1-1 and 2-2

d = Diameter of pipe

f' = Fractional resistance for unit wetted area per a unit velocity

h_f = Loss of head due to friction

- And P_2,V_2 = are values of pressure intensity and velocity at section 2-2

- Applying Bernoulli's equation between sections 1-1 and 2-2

Total head at 1-1 = total head at 2-2 + loss of head due to friction between 1-1 and 2-2

$$\frac{P_1}{\rho g} + \frac{{V_1}^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{{V_2}^2}{2g} + Z_2$$

$Z_1 = Z_2$ as pipe is horizontal

$V_1 = V_2$ as dia. of pipe is same at 1-1 and 2-2

$$\frac{P_1}{\rho g} = \frac{P_2}{\rho g} + h_f \qquad \text{Or}$$

$$h_f = \frac{P_1}{\rho g} - \frac{P_2}{\rho g} \qquad \_\_\_\_\_\_\_\_\_\_ (1)$$

- But h_f is head is lost due to friction and hence the intensity of pressure will be reduced in the direction flow by frictional resistance.

- Now, Frictional Resistance = Frictional resistance per unit wetted area per unit velocity × Wetted Area × (velocity)^2

$F_1 = f' \times \pi d L \times V^2$ [∵Wetted area $=\pi d \times L$, Velocity = $V=V_1=V_2$]

$F_1 = f' \times pLV^2$ ____________ (2) [∵ πd = perimeter = p]

The forces acting on the fluid between section 1-1 and 2-2 are

Pressure force at section 1-1 = $P_1 \times A$ (where A = area of pipe)

Pressure force at section 2-2 = $P_2 \times A$

Frictional force = F_1

- Resolving all forces in the horizontal direction, we have

$$P_1 A - P_2 A - F_1 = 0$$

$(P_1 - P_2)A = F_1 = f' \times p \times L \times V^2$ from equation – (2)

$P_1 - P_2 = \frac{f' \times p \times L \times V^2}{A}$ But from equation (1) $P_1 - P_2 = \rho g h_f$

- Equating the value of $P_1 - P_{2,}$ we get

$$\rho g h_f = \frac{f' \times p \times L \times V^2}{A}$$

$$h_f = \frac{f'}{pg} \times \frac{P}{A} \times L \times V^2 \_\_\_\_\_\_\_\_ (3)$$

- In the equation (3) $\frac{P}{A} = \frac{\text{Wetted Permiter}}{\text{Area}} = \frac{\pi d}{\frac{\pi}{4} d^2} = \frac{\pi}{d}$

$$h_f = \frac{f'}{\rho g} \times \frac{4}{d} \times L \times V^2 = \frac{f'}{\rho g} \times \frac{4LV^2}{d}$$

Putting $\frac{f'}{\rho} = \frac{f}{2}$ Where f is known as co-efficient of friction.

- Equation (4) becomes as $h_f = \frac{4f}{2g} \times \frac{LV^2}{d}$

$$h_f = \frac{4fLV^2}{2gd}$$

- This Equation is known as Darcy – Weisbach equation, commonly used for finding loss of head due to friction in pipes
- Then f is known as a friction factor or co-efficient of friction which is a dimensionless quantity. f is not a constant but, its value depends upon the roughness condition of pipe surface and the Reynolds number of the flow.

MINOR LOSSES IN PIPES

- The loss of energy due to friction is classified as a major loss, because in case of long pipe lines it is much more than the loss of energy incurred by other causes.
- The minor losses of energy are caused on account of the change in the velocity of flowing fluids (either in magnitude or direction).
- In case of long pipes these loses are quite small as compared with the loss of energy due to friction and hence these are termed as ‘’minor losses ‘’
- Which may even be neglected without serious error, however in short pipes these losses may sometimes outweigh the friction loss.
- Some of the losses of energy which may be caused due to the change of velocity are:

MINOR LOSSES IN PIPES

- Loss of energy due to sudden enlargement, $h_e = \frac{(V_1 - V_2)^2}{2g}$
- Loss of energy due to sudden contraction, $h_c = 0.5 \frac{{V_2}^2}{2g}$
- Loss of energy at the entrance to a pipe, $h_i = 0.5 \frac{V^2}{2g}$
- Loss of energy at the exit from a pipe, $h_o = \frac{V^2}{2g}$
- Loss of energy due to gradual contraction or enlargement, $h_l = \frac{k(V_1 - V_2)^2}{2g}$
- Loss of energy in the bends, $h_b = \frac{kV^2}{2g}$
- Loss of energy in various pipe fittings, $h_l = \frac{kV^2}{2g}$

TOPICS TO BE COVERED

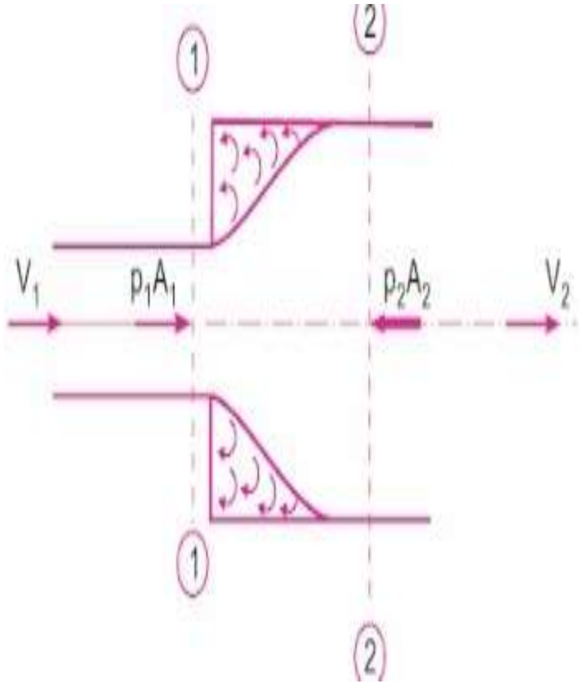
- Loss due to sudden enlargement
- Pipes in series
- Pipes in parallel

LECTURE 4

Minor Losses in Pipe

LOSS OF HEAD DUE TO SUDDEN ENLARGEMENT

- Consider a liquid flowing through a pipe which has sudden enlargement.
- Consider two sections 1-1 and 2-2 before and after enlargement.
- Due to sudden change of diameter of the pipe from D_1 to D_2.
- The liquid flowing from the smaller pipe is not able to fallow the abrupt change of the boundary.
- Thus the flow separates from the boundary and turbulent eddies are formed.
- The loss of head takes place due to the formation of these eddies



- Let p' = Pressure intensity of the liquid eddies on the area $(A_2 - A_1)$

  h_e = loss of head due to the sudden enlargement.

- Applying Bernoulli's equation at section 1-1 and 2-2

$\frac{p_1}{\rho g} + \frac{V_1{}^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2{}^2}{2g} + z_2 +$ Loss of head due to sudden enlargement

- But, $z_1 = z_2$ as pipe is horizontal

$$\frac{p_1}{\rho g} + \frac{V_1{}^2}{2g} = \frac{p_2}{\rho g} + \frac{V_2{}^2}{2g} + h_e$$

Or $h_e = \left(\frac{p_1}{\rho g} - \frac{p_2}{\rho g}\right) + \left(\frac{V_1{}^2}{2g} - \frac{V_2{}^2}{2g}\right)$ ________(1)

- The force acting on the liquid in the control volume in the direction of flow

$$F_x = p_1 A_1 + p'(A_2 - A_1) - p_2 A_2$$

- But experimentally it is found that $p' = p_1$

$$F_x = p_1 A_1 + p_1(A_2 - A_1) - p_2 A_2$$
$$= p_1 A_2 - p_2 A_2$$
$$= (p_1 - p_2)A_2 \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ (2)}$$

- Momentum of liquid/ second at section 1-1 = mass × velocity

$$= \rho A_1 V_1 \times V_1$$
$$= \rho A_1 {V_1}^2$$

- Momentum of liquid/ second at section 2-2 $\rho A_2 V_2 \times V_2 = \rho A_2 {V_2}^2$
- Change of momentum/second $= \rho A_2 {V_2}^2 - \rho A_1 {V_1}^2$ ________ (3)

- But from continuity equation, we have

$$A_1V_1 = A_2V_2$$

Or $$A_1 = \frac{A_2V_2}{V_1}$$

∴ Change of momentum/sec $= \rho A_2 {V_2}^2 - \rho \times \frac{A_2V_2}{V_1} \times {V_1}^2$

$$= \rho A_2 {V_2}^2 - \rho A_1 V_1 V_2$$

$$= \rho A_2 \left({V_2}^2 - V_1V_2\right) \text{\_\_\_\_\_\_\_\_\_\_\_\_} (4)$$

- Now the net force acting on the control volume in the direction of flow must be equal to rate of change of momentum per second. Hence equating equation (2) and equation (4)

$$(p_1 - p_2)A_2 = \rho A_2 \left({V_2}^2 - V_1V_2\right)$$

$$\frac{p_1 - p_2}{\rho} = {V_2}^2 - V_1V_2$$

- Dividing both sides by 'g' we have

$$\frac{p_1 - p_2}{\rho g} = \frac{{V_2}^2 - V_1 V_2}{g}$$

$$\text{Or } \frac{p_1}{\rho g} - \frac{p_2}{\rho g} = \frac{{V_2}^2 - V_1 V_2}{g}$$

- Substituting in equation (1)

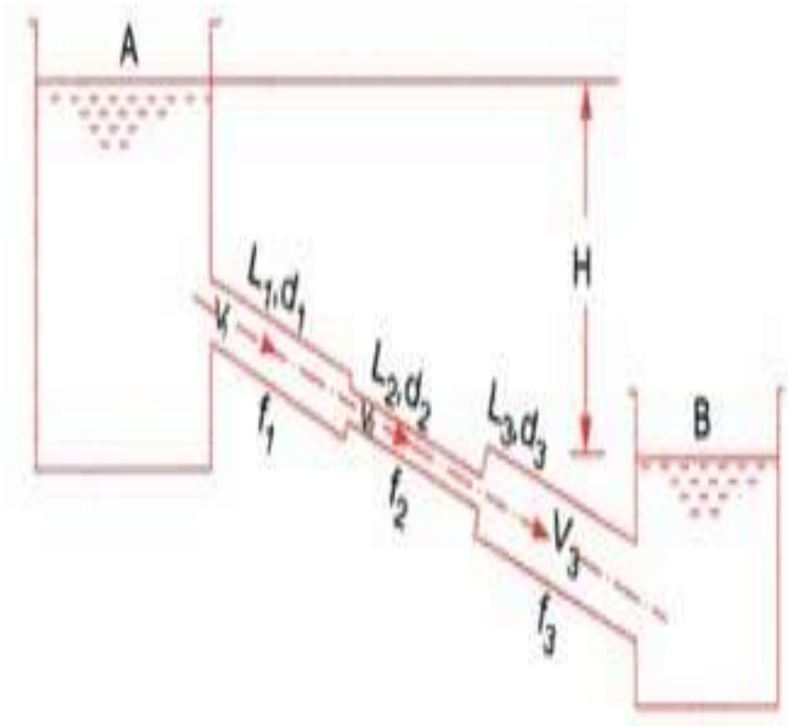
$$h_e = \frac{{V_2}^2 - V_1 V_2}{g} + \left(\frac{{V_1}^2}{2g} - \frac{{V_2}^2}{2g}\right) = \frac{2{V_2}^2 - 2V_1 V_2 + {V_1}^2 - {V_2}^2}{2g}$$

$$= \frac{{V_1}^2 + {V_2}^2 - 2V_1 V_2}{2g}$$

$$h_e = \frac{(V_1 - V_2)^2}{2g}$$

PIPES IN SERIES

- If a pipe line connecting two reservoirs is made up of several pipes of different diameters d_1, d_2, d_3, etc. and lengths L_1, L_2, L_3 etc. all connected in series (i.e. end to end), then the difference in the liquid surface levels is equal to the sum of the head losses in all the sections.
- Further the discharge through each pipe will be same.



$$H = \frac{0.5{V_1}^2}{2g} + \frac{4f_1L_1{V_1}^2}{2gd_1} + \frac{0.5{V_2}^2}{2g} + \frac{4f_2L_2{V_2}^2}{2gd_2} + \frac{0.5{V_3}^2}{2g} + \frac{4f_3L_3{V_3}^2}{2gd_3}$$

- Also, $Q = \left(\frac{\pi \times {d_1}^2}{4}\right) \times V_1 = \left(\frac{\pi \times {d_2}^2}{4}\right) \times V_2 = \left(\frac{\pi \times {d_3}^2}{4}\right) \times V_3$
- However if the minor losses are neglected as compared with the loss of head due to friction in each pipe, then

$$\boxed{H = \frac{4f_1L_1{V_1}^2}{2gd_1} + \frac{4f_2L_2{V_2}^2}{2gd_2} + \frac{4f_3L_3{V_3}^2}{2gd_3}}$$

- The above equation may be used to solve the problems of pipe lines in series.

- There are two types of problems which may arise for the pipe lines in series. Viz.

➢ Given a discharge Q to determine the head H and

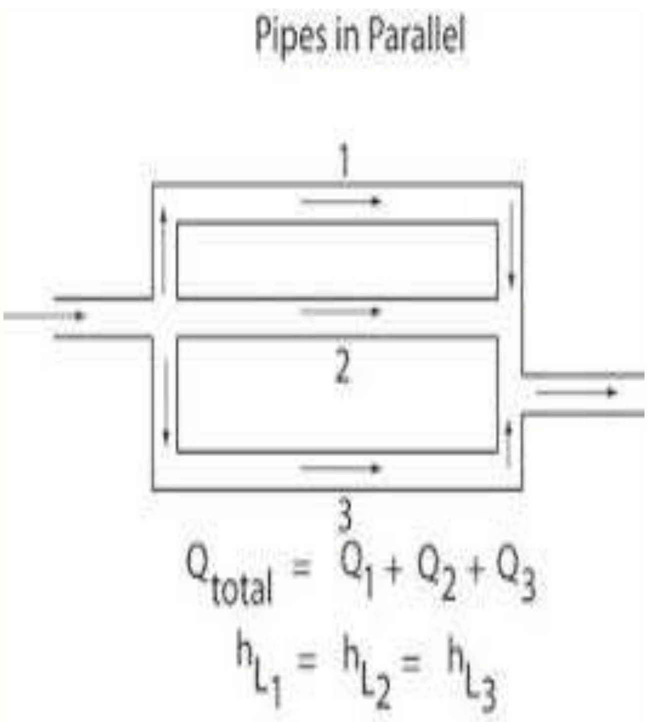
➢ Given H to determine discharge Q.

- If the co-efficient of friction is same for all the pipes i.e. $f_1 = f_2 = f_3$, then

$$H = \frac{4f_1}{2g}\left[\frac{L_1 {V_1}^2}{d_1} + \frac{L_2 {V_2}^2}{d_2} + \frac{L_3 {V_3}^2}{d_3}\right]$$

PIPES IN PARALLEL

- When a main pipeline divides in to two or more parallel pipes, which may again join together downstream and continue as main line, the pipes are said to be in parallel.
- The pipes are connected in parallel in order to increase the discharge passing through the main.
- It is analogous to parallel electric current in which the drop in potential and flow of electric current can be compared to head loss and rate of discharge in a fluid flow respectively.



- The rate of discharge in the main line is equal to the sum of the discharges in each of the parallel pipes.
- Thus, $Q = Q_1 + Q_2$
- The flow of liquid in pipes (1) and (2) takes place under the difference of head between the sections A and B and hence the loss of head between the sections A and B will be the same whether the liquid flows through pipe (1) or pipe (2).
- Thus if D_1, D_2 and L_1, L_2 are the diameters and lengths of the pipes (1) and (2) respectively, then the velocities of flow V_1 and V_2 in the two pipes must be such as to give

$$h_f = \frac{f L_1 {V_1}^2}{2 g d_1} = \frac{f L_2 {V_2}^2}{2 g d_2}$$

- Assuming same value of f for each parallel pipe

$$\frac{L_1 {V_1}^2}{2 g d_1} = \frac{L_2 {V_2}^2}{2 g d_2}$$

TOPICS TO BE COVERED

- Problem on Darcy Wiesbach equation
- Problem on Minor losses
- Problems on pipes in series & parallel

LECTURE 5&6

Example Problems

PROBLEM 1

- In a pipe of diameter 350 mm and length 75m water is flowing at a velocity of 2.8 m/s. Find the head lost due to friction by using Darcy Weisbach equation?

Solution. Diameter of the pipe, $D = 350 \text{ mm} = 0{\cdot}35 \text{ m}$

Length of the pipe, $L = 75 \text{ m}$

Velocity of flow, $V = 2{\cdot}8 \text{ m/s}$

Chezy's constant, $C = 55$

Kinematic viscocity of water, $\nu = 0{\cdot}012 \text{ stoke} = 0{\cdot}012 \times 10^{-4} \text{ m}^2\text{/s}$.

Head lost due to friction, h_f :

(*i*) **Darcy-Weisbach fomula :**

Darcy-Weisbach formula is given by:

$$h_f = \frac{4fLV^2}{D \times 2g}$$

where, f = coefficient of friction (a function of Reynolds number, Re)

$$Re = \frac{V \times D}{\nu} = \frac{2{\cdot}8 \times 0{\cdot}35}{0{\cdot}012 \times 10^{-4}} = 8{\cdot}167 \times 10^5$$

$$\therefore \quad f = \frac{0.0791}{(Re)^{1/4}} = \frac{0.0791}{(8.167 \times 10^5)^{1/4}} = 0.00263$$

$\therefore$ Head lost due to friction,

$$h_f = \frac{4 \times 0.00263 \times 75 \times (2.8)^2}{0.35 \times 2 \times 9.81} = \mathbf{0.9 \text{ m}} \quad \text{(Ans.)}$$

PROBLEM-2

Example 12.3. *In a pipe of 300 mm diameter and 800 m length an oil of specific gravity 0·8 is flowing at the rate of 0·45 m³/s. Find :*

(i) Head lost due to friction, and

(ii) Power required to maintain the flow.

Take kinematic viscosity of oil as 0·3 stoke.

Solution. Diameter of the pipe, D = 300 mm = 0·3 m

Length of the pipe, L = 800 m

Specific gravity of oil = 0·8

Kinematic viscosity of oil, ν = 0·3 stoke = $0{\cdot}3 \times 10^{-4}$ m²/s

Discharge, Q = 0·45 m³/s.

(i) Head lost due to friction, h_f :

$$\text{Velocity, } V = \frac{Q}{\text{Area}} = \frac{0.45}{\frac{\pi}{4} \times 0.3^2} = 6.366 \text{ m/s}$$

$$\therefore \quad \text{Reynolds number, } Re = \frac{V \times D}{\nu} = \frac{6.366 \times 0.3}{0.3 \times 10^{-4}} = 6.366 \times 10^4$$

$$\therefore \text{ Co-efficient of friction, } f = \frac{0.0791}{(Re)^{1/4}} = \frac{0.0791}{(6.366 \times 10^4)^{1/4}} = 0.00498$$

$$\therefore \quad h_f = \frac{4fLV^2}{D \times 2g} = \frac{4 \times 0.00498 \times 800 \times (6.366)^2}{0.3 \times 2 \times 9.81}$$

$$= 109.72 \text{ m (Ans.)}$$

(ii) Power required, P :

Power required to maintain the flow, $P = wQh_f$

where, $w = 0{\cdot}8 \times 9{\cdot}81 = 7{\cdot}848$ kN/m³

$h_f = 109{\cdot}72$ m, $Q = 0{\cdot}45$ m³/s

PROBLEM 3

Example 12.2. *Water flows through a pipe of diameter 300 mm with a velocity of 5 m/s. If the co-efficient of friction is given by* $f = 0.015 + \frac{0.08}{Re^{0.3}}$ *where Re is the Reynolds number, find the head lost due to friction for a length of 10 m. Take kinematic viscosity of water as 0·01 stoke.*

Solution. Diameter of the pipe, $D = 300 \text{ mm} = 0{\cdot}3 \text{ m}$

Velocity of water $V = 5$ m/s

Length of the pipe, $L = 10$ m

Viscosity of water, $\nu = 0{\cdot}01 \text{ stoke} = 0{\cdot}01 \times 10^{-4} \text{ m}^2/\text{s}$. $(\because 1 \text{ stoke} = 1 \text{ cm}^2/\text{s} = 1 \times 10^{-4} \text{ m}^2/\text{s})$

Head lost due to friction h_f :

Co-efficient of friction, $f = 0.015 + \frac{0.08}{(Re)^{0.3}}$

But, Reynolds number, $Re = \frac{\rho VD}{\mu} = \frac{VD}{\nu} = \frac{5 \times 0.3}{0.01 \times 10^{-4}} = 1.5 \times 10^6$

$\therefore$ $f = 0.015 + \frac{0.08}{(1.5 \times 10^6)^{0.3}} = 0.0161$

$\therefore$ Head lost due to friction,

$$h_f = \frac{4fLV^2}{D \times 2g} = \frac{4 \times 0.0161 \times 10 \times 5^2}{0.3 \times 2 \times 9.81}$$

$= $ **2.735 m (Ans.)**

PROBLEM 4

Example 12.13. *A horizontal pipe carries water at the rate of 0·04 m³/s. Its diameter, which is 300 mm reduces abruptly to 150 mm. Calculate the pressure loss across the contraction. Take the co-efficient of contraction = 0·62.*

Solution. Diameter of the large pipe, $D_1 = 300 \text{ mm} = 0{\cdot}3 \text{ m}$

$$\therefore \quad \text{Area, } A_1 = \frac{\pi}{4} \times 0.3^2 = 0.0707 \text{ m}^2$$

Diameter of the small pipe, $D_2 = 150 \text{ mm} = 0{\cdot}15 \text{ m}$

$$\therefore \quad \text{Area, } A_2 = \frac{\pi}{4} \times 0.15^2 = 0.1767 \text{ m}^2$$

Discharge, $Q = 0{\cdot}04 \text{ m}^3/\text{s}$.

Co-efficient of contraction, $C_c = 0{\cdot}62$

Pressure loss across the contraction, $(p_1 - p_2)$:

From continuity equation, we have:

$$A_1 V_1 = A_2 V_2 = Q$$

$$\therefore \quad V_1 = \frac{Q}{A_1} = \frac{0.04}{0.0707} = 0.566 \text{ m/s}$$

and,

$$V_2 = \frac{Q}{A_2} = \frac{0.04}{0.01767} = 2.26 \text{ m/s}$$

Applying Bernoulli's equation before and after contraction, we get:

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2 + h_c \qquad \text{...(i)}$$

But, $z_1 = z_2$ ...because the pipe is horizontal and head loss due to contraction (h_c) is given as :

$$h_c = \left[\frac{1}{C_c} - 1\right]^2 \frac{V_2^2}{2g} = \left[\frac{1}{0.62} - 1\right]^2 \times \frac{2.26^2}{2 \times 9.81} = 0.0978$$

$$\frac{p_1}{w} + \frac{0.566^2}{2 \times 9.81} = \frac{p_2}{w} + \frac{2.26^2}{2 \times 9.81} + 0.0978$$

$\therefore$

$$\frac{p_1}{w} - \frac{p_2}{w} = \frac{2.26^2}{2 \times 9.81} + 0.0978 - \frac{0.566^2}{2 \times 9.81}$$

$$= 0{\cdot}26 + 0{\cdot}0978 - 0{\cdot}016 = 0{\cdot}3418$$

Hence,

$$p_1 - p_2 = w \times 0{\cdot}3418 = 9{\cdot}81 \times 0{\cdot}3418$$

$$= \mathbf{3{\cdot}35\ kN/m^2}$$ **(Ans.)**

PROBLEM-5

Example 12.22. *Three pipes of diameters 300 mm, 200 mm and 400 mm and lengths 450 m, 255 m and 315 m respectively are connected in series. The difference in water surface levels in two tanks is 18 m. Determine the rate of flow of water if co-efficients of friction are 0·0075, 0·0078 and 0·0072 respectively considering :*

(*i*) *Minor losses also, and*

(*ii*) *Neglecting minor losses.*

Solution. Pipe 1 : $L_1 = 450$ m, $D_1 = 300$ mm $= 0·3$ m, $f_1 = 0·0075$

Pipe 2 : $L_2 = 255$ m, $D_2 = 200$ mm $= 0·2$ m, $f_2 = 0·0078$

Pipe 3 : $L_3 = 315$ m, $D_3 = 400$ mm $= 0·4$ m, $f_3 = 0·0072$

Difference of water level, $H = 18$ m.

(*i*) Considering minor losses :

Let V_1, V_2 and V_3 be the velocities in Ist, 2nd, and 3rd pipe respectively.

From continuity considerations, we have:

$$A_1V_1 = A_2V_2 = A_3V_3$$

$$\therefore \quad V_2 = \frac{A_1V_1}{A_2} = \frac{(\pi/4)\times D_1^2}{(\pi/4)\times D_2^2}\times V_1 = \frac{D_1^2}{D_2^2}\times V_1 = \left(\frac{0.3}{0.2}\right)^2 V_1 = 2.25\,V_1$$

and,
$$V_3 = \frac{A_1V_1}{A_3} = \frac{(\pi/4)\times D_1^2}{(\pi/4)\times D_3^2}\times V_1 = \frac{D_1^2}{D_3^2}\times V_1 = \left(\frac{0.3}{0.4}\right)^2 V_1 = 0.5625\,V_1$$

We know that:
$$H = \frac{0.5V_1^2}{2g} + \frac{4f_1L_1V_1^2}{D_1\times 2g} + \frac{0.5V_2^2}{2g} + \frac{4f_2L_2V_2^2}{D_2\times 2g} + \frac{(V_2-V_3)^2}{2g} + \frac{4f_3L_3V_3^2}{D_3\times 2g} + \frac{V_3^2}{2g} \qquad \text{...[Eqn. (12·9)]}$$

$$18 = \frac{0·5V_1^2}{2g} + \frac{4\times 0.0075\times 450\times V_1^2}{0.3\times 2g} + \frac{0.5\times(2.25\,V_1)^2}{2g} + \frac{4\times 0.0078\times 255\times(2.25V_1)^2}{0.2\times 2g} + \frac{(2.25\,V_1 - 0.5625\,V_1)^2}{2g} + \frac{4\times 0.0072\times 315\times(0.5625\,V_1)^2}{0.4\times 2g} + \frac{(0.5625\,V_1)^2}{2g}$$

$$18 = \frac{V_1^2}{2g}(0.5 + 45 + 2.53 + 201.4 + 2.847 + 7.176 + 0.316)$$

$$= 259.77\,\frac{V_1^2}{2g}$$

or,
$$V_1 = \sqrt{\frac{18\times 2\times 9.81}{\ldots}} = 1.166 \text{ m/s}$$

(*ii*) **Neglecting minor losses :**

We know that,
$$H = \frac{4f_1L_1V_1^2}{D_1 \times 2g} + \frac{4f_2L_2V_2^2}{D_2 \times 2g} + \frac{4f_3L_3V_3^2}{D_3 \times 2g} \quad \text{...[Eqn. (12·10)]}$$

$$18 = \frac{V_1^2}{2g}\left(\frac{4 \times 0.0075 \times 450}{0\cdot 3} + \frac{4 \times 0.0078 \times 255 \times 2.25^2}{0\cdot 2} + \frac{4 \times 0.0072 \times 315 \times (0.5625)^2}{0\cdot 4}\right)$$

$$= \frac{V_1^2}{2g}(45 + 201.4 + 7.176) = 253.57 \times \frac{V_1^2}{2g}$$

or,
$$V_1 = \sqrt{\frac{18 \times 2 \times 9.81}{253.57}} = 1.18 \text{ m}$$

∴ Discharge, $Q = A_1V_1 = (\pi/4) \times 0\cdot 3^2 \times 1\cdot 18 =$ **$0\cdot 0834$ m³/s** **(Ans.)**

PROBLEM-6

Example 12.24. *A piping system consists of three pipes arranged in series; the lengths of the pipes are 1200 m, 750 m and 600 m and diameters 750 mm, 600 mm and 450 mm respectively.*

(*i*) *Transform the system to an equivalent 450 mm diameter pipe, and*

(*ii*) *Determine an equivalent diameter for the pipe, 2550 m long.*

Solution. *Pipe 1*: L_1 = 200 m; D_1 = 750 mm = 0·75 m

Pipe 2: L_2 = 750 m; D_2 = 600 mm = 0·6 m

Pipe 3: L_3 = 600 m; D_3 = 450 mm = 0·45 m

(*i*) **Equivalent length, *L* :**

Diameter of the equivalent pipe, D = 450 mm = 0·45 m (*Given*)

Using the relation :

$$\frac{L}{D^5} = \frac{L_1}{D_1^5} + \frac{L_2}{D_2^5} + \frac{L_3}{D_3^5}$$

$$= \frac{1200}{(0.75)^5} + \frac{750}{(0.6)^5} + \frac{600}{(0.45)^5}, \text{ we have:}$$

$$\frac{L}{(0.45)^5} = 5056.8 + 9645 + 32515.4 = 47217.2$$

or, $$L = 47217{\cdot}2 \times (0{\cdot}45)^5 = \mathbf{871{\cdot}3\ m} \quad \textbf{(Ans.)}$$

(*ii*) **Equivalent diameter, *D* :**

Length of the equivalent pipe, $L = 2550$ m (*Given*)

Now, $$\frac{L}{D^5} = \frac{L_1}{D_1^5} + \frac{L_2}{D_2^5} + \frac{L_3}{D_3^5}$$

or, $$\frac{2550}{D^5} = \frac{1200}{(0.75)^5} + \frac{750}{(0.6)^5} + \frac{600}{(0.45)^5}$$

$$= 5056.8 + 9645 + 32515.4 = 47217.2$$

or, $$D = \left(\frac{2550}{47217.2}\right)^{1/5} = 0.5578 \text{ m or } \mathbf{557.8\ mm} \ \textbf{(Ans.)}$$

TOPICS TO BE COVERED

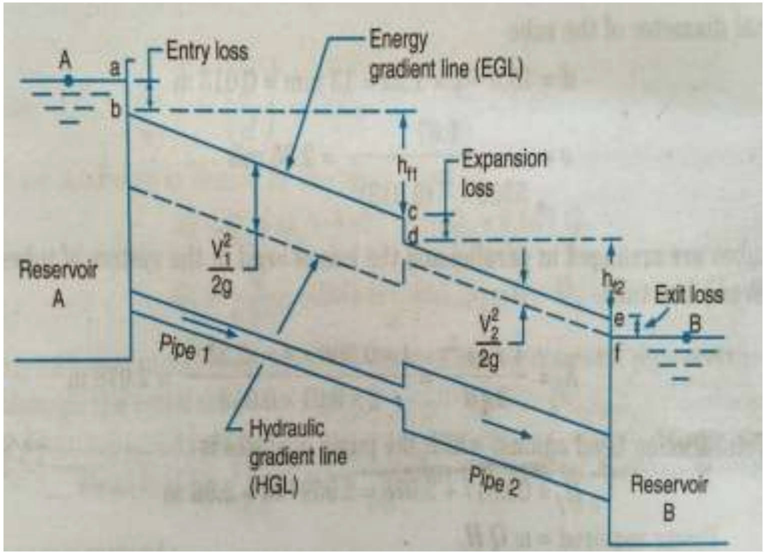
- Hydraulic Gradient line
- Total energy line
- Pitot tube

LECTURE 7

HGL & TEL

HYDRAULIC GRADIENT LINE AND TOTAL ENERGY LINE

- Consider a long pipe line carrying liquid from a reservoir A to reservoir B.
- At several points along the pipeline let piezometers be installed.
- The liquid will rise in the piezometers to certain heights corresponding to the pressure intensity at each section.
- The height of the liquid surface above the axis of the pipe in the piezometer at any section will be equal to the pressure head (p/w) at that section.



- On account of loss of energy due to friction, the pressure head will decrease gradually from section to section of pipe in the direction of flow.
- If the pressure heads at the different sections of the pipe are plotted to scale as vertical ordinates above the axis of the pipe and all these points are joined by a straight line, a sloping line is obtained, which is known as Hydraulic Gradient Line (H.G.L).
- Since at any section of pipe the vertical distance between the pipe axis and Hydraulic gradient line is equal to the pressure head at that section, it is also known as pressure line.
- Moreover if Z is the height of the pipe axis at any section above an arbitrary datum, then the vertical height of the Hydraulic gradient line above the datum at that section of pipe represents the piezometric head equal to $(p/w + z)$..

- Sometimes the Hydraulic gradient line is also known as piezometric head line.
- At the entrance section of the pipe for some distance the Hydraulic gradient line is not very well defined.
- This is because as liquid from the reservoir enters the pipe, a sudden drop in pressure head takes place in this portion of pipe.
- Further the exit section of pipe being submerged, the pressure head at this section is equal to the height of the liquid surface in the reservoir B and hence the hydraulic gradient line at the exit section of pipe will meet the liquid surface in the reservoir B.

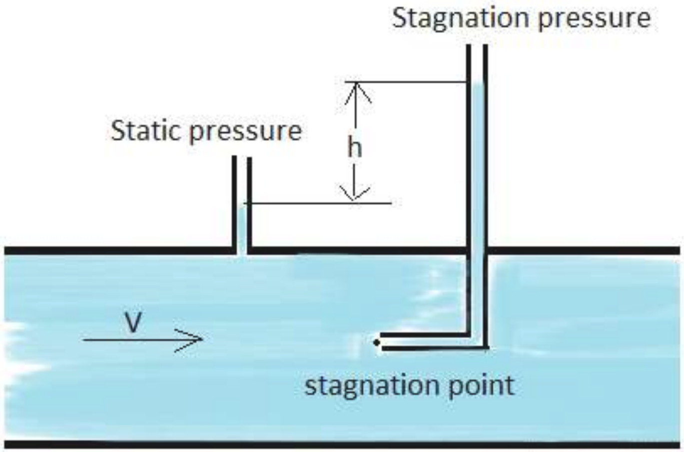
TOTAL ENERGY LINE

- If at different sections of pipe the total energy (in terms of head) is plotted to scale as vertical ordinate above the assumed datum and all these points are joined, then a straight sloping line will be obtained and is known as energy grade line or Total energy line (T.E.L).
- Since total energy at any section is the sum of the pressure head (p/w), datum head z and velocity head $\left(\frac{V^2}{2g}\right)$ and the vertical distance between the datum and hydraulic grade line is equal to the piezometric head (p/w + z), the energy grade line will be parallel to the hydraulic grade line, with a vertical distance between them equal to $\left(\frac{V^2}{2g}\right)$.

- At the entrance section of the pipe there occurs some loss of energy called “Entrance loss” equal to $h_L = 0.5\left(\frac{V^2}{2g}\right)$ and hence the energy grade line at this section will lie at a vertical depth equal to $0.5\left(\frac{V^2}{2g}\right)$ below the liquid surface in the reservoir A.

PITOT TUBE

- A Pitot tube is a simple device used for measuring the velocity of flow.
- The basic principle used in this is that if the velocity of flow at a particular point is reduced to zero, which is known as stagnation point, the pressure there is increased due to conversion of the kinetic energy in to pressure energy and by measuring the increase in pressure energy at this point,the velocity of flow may be determined.



- Simplest form of a pitot tube consists of a glass tube, large enough for capillary effects to be negligible and bent at right angles.
- A single tube of this type is used for measuring the velocity of flow in an open channel.
- The tube is dipped vertically in the flowing stream of fluid with its open end A directed to face the flow and other open end projecting above the fluid surface in the stream.
- The fluid enters the tube and the level of the fluid in the tube exceeds that of the fluid surface in the surrounding stream. This is so because the end A of the tube is a stagnation point, where the fluid is at rest, and the fluid approaching end A divides at this point and passes around tube.

- Since at stagnation point the kinetic energy is converted in to pressure energy, the fluid in the tube rises above the surrounding fluid surface by a height, which corresponds to the velocity of flow of fluid approaching end A of the tube.
- The pressure at the stagnation point is known as stagnation pressure.
- Consider a point 1 slightly upstream of end A and lying along the same horizontal plane in the flowing stream of velocity V.
- Now if the point 1 and A are at a vertical depth of h_o from the free surface of fluid and h is the height of the fluid raised in the pitot tube above the free surface of the liquid.

- Then by applying Bernoulli's equation between the point 1 and A, neglecting loss of energy,

we get $$h_o = \frac{V^2}{2g} = h_o + h$$

- (h_o + h) is the stagnation pressure head at a point A, which consists of static pressure head h_o and dynamic pressure head h.
- Simplifying the expression,

$$\frac{V^2}{2g} = h \quad \text{Or} \quad v = \sqrt{2gh} \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_} \quad (1)$$

- This equation indicates that the dynamic pressure head h is proportional to the square of the velocity of flow close to end A.
- Thus the velocity of flow at any point in the flowing stream may be determined by dipping the Pitot tube to the required point and measuring the height 'h' of the fluid raised in the tube above the free surface.

- The velocity of flow given by the above equation (1) is more than actual velocity of flow as no loss of energy is considered in deriving the above equation.
- When the flow is highly turbulent the Pitot tube records a higher value of h, which is higher than the mean velocity of flow.
- In order to take in to account the errors due to the above factors, the actual velocity of flow may be obtained by introducing a co-efficient C or C_V called Pitot tube co-efficient.
- So the actual velocity is given by

$$v = C\sqrt{2gh}$$

(Probable value of C is 0.98)