SREENIVASA INSTITUTE OF TECHNOLOGY AND MANAGEMENT STUDIES

(AUTONOMOUS)

DEPARTMENT OF MECHANICAL ENGINEERING

COURSE MATERIAL

Subject Name	Fluid Mechanics and Hydraulic Machines
Subject Code	23MEC241T
Semester	IV Semester
Academic Year	2025-26
Regulation	R23

Unit-IV

Basics of turbo machinery and Hydraulic Turbines

UNIT – IV (SYLLABUS)

Basics of Turbo Machinery:

- Hydrodynamic force of jets on stationary
- Jet on moving flat,
- jet on inclined, and curved vanes.

Hydraulic Turbines:

- Classification of turbines
- impulse and reaction turbines,
- Pelton wheel turbine,
- Francis turbine and Kaplan turbine
- draft tube.
- Performance of hydraulic turbines: characteristic curves, cavitation, surge tank, water hammer

COURSE OUTLINE

UNIT -4

LECTURE	LECTURE TOPIC	KEY ELEMENTS	LEARNING OBJECTIVES
1	Introduction to hydrodynamic force on jets	Derivation of force on Stationary- Flat, inclined & curved plate	Evaluate force exerted (B5)
2	Hydrodynamic force on jets	Derivation of force on Moving- Flat, inclined & curved plates	Evaluate force exerted (B5)
3	Example Problems on force on jets for stationary & moving plates		
4	Classification of turbines	Impulse & Reaction turbines	Understanding the types of turbines (B2)
5	Pelton wheel Turbine	Working principle, derivation of work done & η	Understand working principle (B2) Evaluate the efficiency (B5)
6	Francis & Kaplan turbine	Working principle, derivation of work done & η	Understand working principle (B2) Evaluate the efficiency (B5)
7	Hydraulic Design- Draft Tube theory	Functions & η	Evaluate the efficiency (B5)
8	Example Problems on turbines		
9	Geometric similarity	Derivation of Unit & specific quantities	Evaluate the unit quantities (B5)

TOPICS TO BE COVERED

- Derivation of force on Stationary-Flat, inclined & curved plate

LECTURE 1

Introduction to hydrodynamic force on jets

HYDRO-DYNAMIC FORCE OF JETS:

- The liquid comes out in the form of a jet from the outlet of a nozzle, the liquid is flowing under pressure.
- If some plate, which may be fixed or moving, is placed in the path of the jet, a force is exerted by the jet on the plate.
- This force is obtained by Newton's second law of motion or from Impulse – Momentum equation.

$$\mathbf{F = ma}$$

The following cases of impact of jet i.e. the force exerted by the jet on a plate will be considered.

1) Force exerted by the jet on a stationary plate, when

   a) Plate is vertical to the jet.
   b) Plate is inclined to the jet and
   c) Plate is curved

2) Force exerted by the jet on a moving plate, when

   a) Plate is vertical to the jet.
   b) Plate is inclined to the jet.
   c) Plate is curved.

JET ON A STATIONARY VERTICAL PLATE

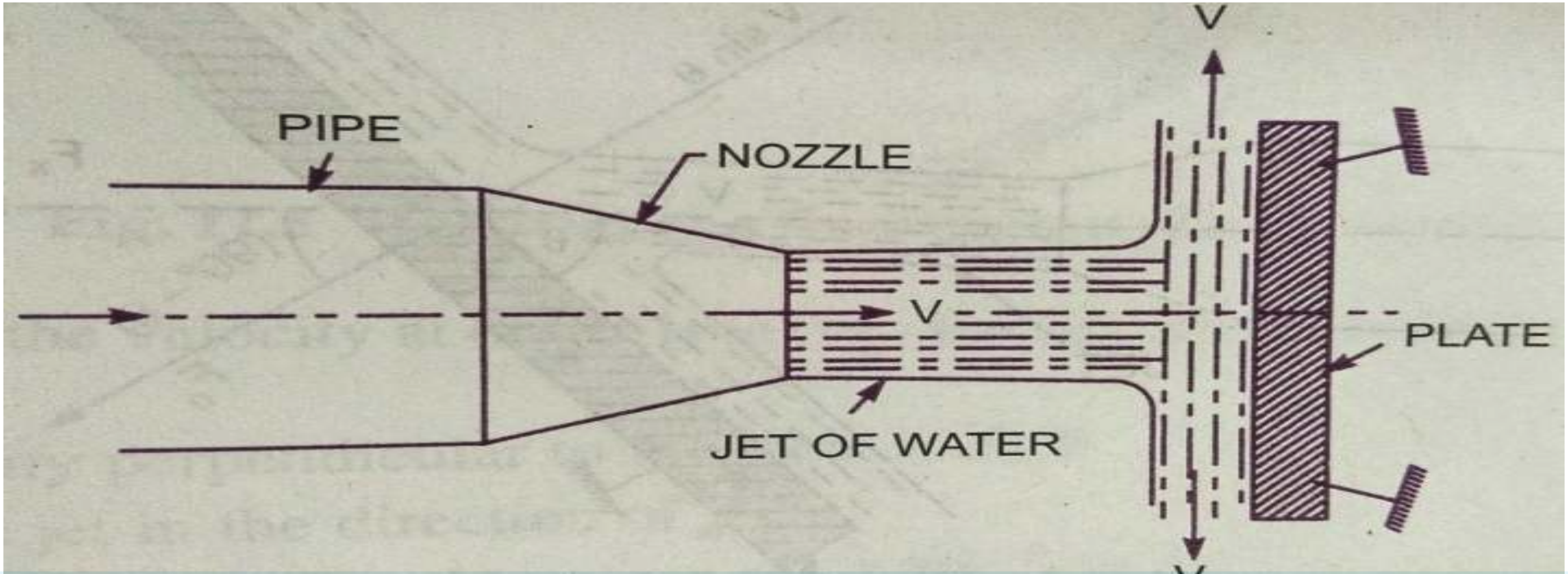
Consider a jet of water coming out from the nozzle, strikes a flat vertical plate.

Let V= Velocity of jet.

d = Diameter of jet.

a = Area of cross-section of jet. = d^2

The jet of water after striking the plate will move along the plate. But the plate is at right angles to the jet. Hence the jet after striking will be deflected through 90.



- The force exerted by the jet on the plate in the direction of jet,

  F_x = Rate of change of momentum in the direction of force.

$$= \frac{\text{Initial momentum} - \text{Final momentum}}{\text{Time}}$$

$$= \frac{\text{Mass} \times \text{Initial velocity} - \text{Mass} \times \text{Final velocity}}{\text{Time}}$$

$$= \frac{Mass}{Time}(\text{Initial velocity} - \text{Final velocity})$$

$$= (Mass/sec) \times (\text{Velocity of jet before striking} - \text{Final velocity of jet after striking})$$

$$= \rho \text{ a V (V} - \text{o)}$$

$$F_x = \rho a V^2$$

- For deriving the above equation, we have taken initial velocity minus final velocity and not final velocity minus initial velocity. If the force exerted on the jet is to be calculated, then final velocity minus initial velocity is to be taken

JET ON A STATIONARY INCLINED FLAT PLATE

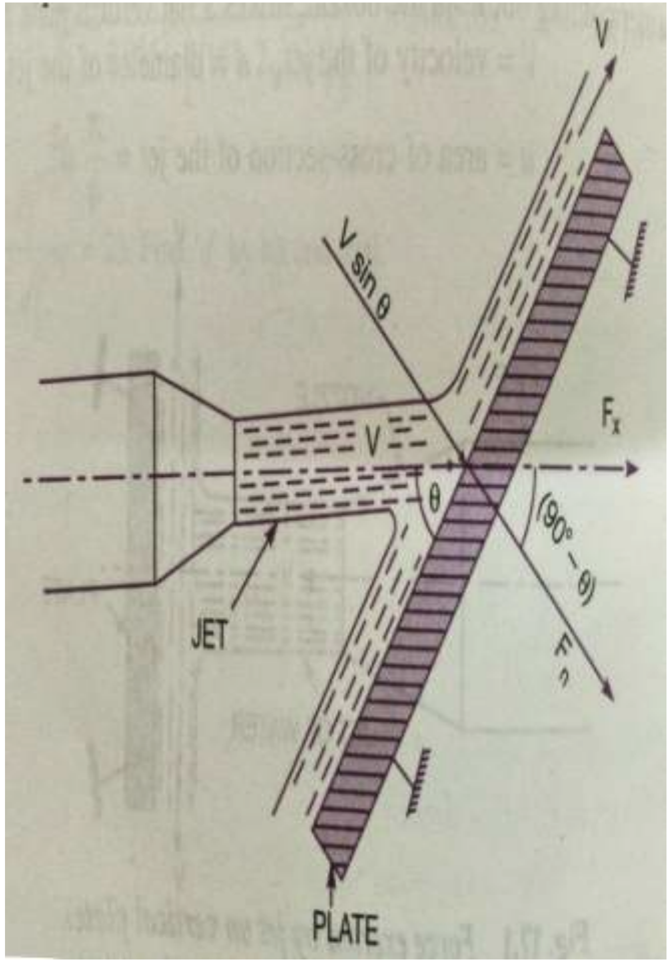
Let a jet of water coming out from the nozzle, strikes an inclined flat plate.

V = Velocity of jet in the direction of X

= Angle between the jet and plate.

a = Area of cross-section of jet.

Mass of water per second striking the plate = p a v



Let us find the force exerted by the jet on the plate in the direction normal to the plate. Let this force is represented by F_n.

Then

F_n = Mass of jet striking per second*(Initial velocity of jet before striking in the direction of n – Final velocity of jet after striking in the direction of n)

$= \rho$ a V (Vsin θ – 0) = ρ a V^2 sin θ -------------- (1)

This force can be resolved in two components, one in the direction of the jet and the other perpendicular to the direction of flow.

Then we have F_x = Component of F_n in the direction of flow.

$$F_x = F_n \cos(90-\theta) = F_n \sin\theta - \rho a V^2 \sin\theta \times \sin\theta$$

$$F_x = \rho a V^2 sin^2\theta \qquad \_\_\_\_\_\_\_\_\_\_ (1)$$

And F_y = Component of F_n in the direction perpendicular to the flow.

$$F_y = F_n \sin(90-\theta) = F_n \cos\theta = \rho a v^2 \sin\theta \times \cos\theta$$

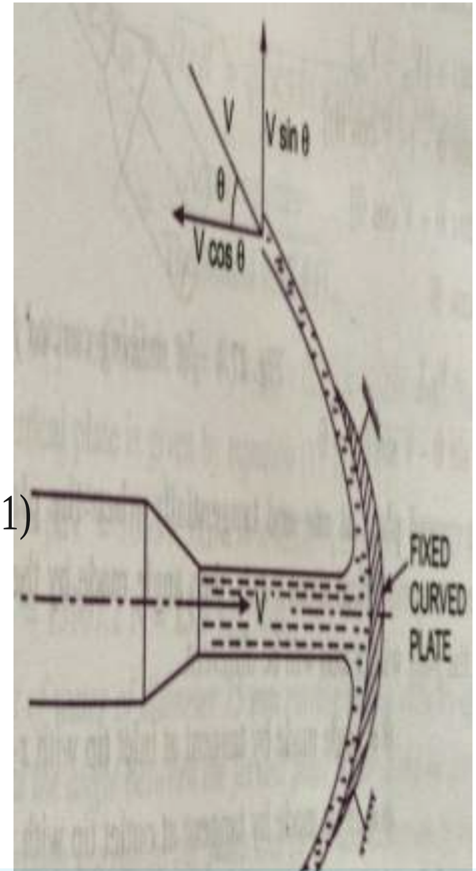
$$F_y = \rho a v^2 sin\theta cos\theta \qquad \_\_\_\_\_\_\_\_\_\_ (2)$$

JET STRIKES THE CURVED PLATE AT THE CENTRE

Component of velocity in the direction of jet = $-V\ Cos\theta$

(-ve sign is taken as the velocity at out let is in the opposite direction of the jet of water coming out from nozzle.)

Component of velocity perpendicular to the jet = V sinθ



Force exerted by the jet in the direction of the jet

$$F_x = \text{Mass per sec } (V_{1x} - V_{2x})$$

Where V_{1x} = Initial velocity in the direction of jet = V

V_{2x} = Final velocity in the direction of jet = $-V\ Cos\theta$

$F_x = \rho aV\,[V - (-V\ Cos\theta)] = \rho aV\,[V + V\ Cos\theta] = \rho aV^2\,(1 + Cos\theta)$ ---------- (1)

Similarly F_y = Mass per second $(V_{1y} - V_{2y})$

Where V_{1y} = Initial velocity in the direction of y = 0

V_{2y} = Final velocity in the direction of y = V sinθ

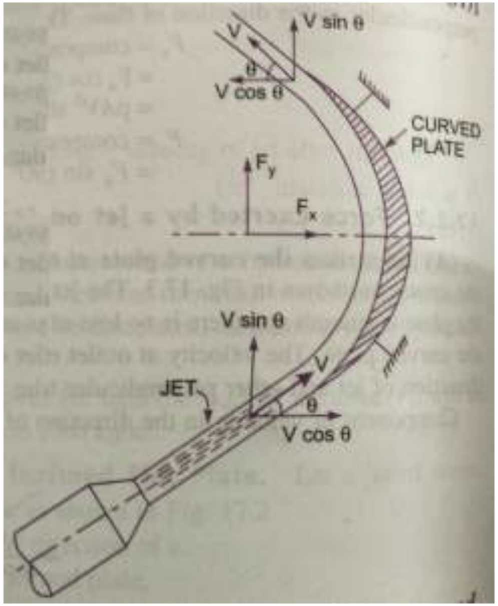
$F_y = \rho aV\,[0 - V\ Sin\theta] = -\rho aV^2\ Sin\theta$ ---------- (2)

JET STRIKES THE CURVED PLATE AT ONE END TANGENTIALLY WHEN THE PLATE IS SYMMETRICAL

Let V = Velocity of jet of water.

θ = Angle made by the jet with x–axis at the inlet tip of the curved plate.

If the plate is smooth and loss of energy due to impact is zero, then the velocity of water at the out let tip of the curved plate will be equal to V. The force exerted by the jet of water in the direction of x and y are



$$F_x = (\text{mass/sec}) \times (V_{1x} - V_{2x})$$

$$= \rho aV\,[V \cos\theta - (-V \cos\theta)]$$

$$= 2\,\rho aV^2 \cos\theta$$

$$F_y = \rho aV\,(V_{1y} - V_{2y}) = \rho aV\,[V \sin\theta - V \sin\theta] = 0$$

JET STRIKES THE CURVED PLATE AT ONE END TANGENTIALLY WHEN THE PLATE IS UN-SYMMETRICAL

When the curved plate is unsymmetrical about *x*- axis, then the angles made by tangents drawn at inlet and outlet tips of the plate with *x*- axis will be different.

Let θ = Angle made by tangent at the inlet tip with *x*-axis.

$\emptyset$ = Angle made by tangent at the outlet tip with *x*-axis

The two components of velocity at inlet are

$$V_{1x} = V\cos\theta \quad \text{and } V_{1y} = V\sin\theta$$

The two components of velocity at outlet are

$$V_{2x} = -V\cos\emptyset \quad \text{and } V_{2y} = V\sin\emptyset$$

The forces exerted by the jet of water in the directions of *x* and *y* are:

$$F_x = \rho aV\,(V_{1x} - V_{2x}) = \rho aV\,(V\,Cos\theta + V\,Cos\,\emptyset)$$

$$= \rho a V^2(cos\theta + cos\emptyset)$$

$$F_y = \rho aV\,(V_{1y} - V_{2y}) = \rho aV\,[V\sin\theta - V\sin\emptyset] = \boldsymbol{\rho}\,a\,V^2\,(\sin\boldsymbol{\theta} - Sin\,\emptyset)$$

TOPICS TO BE COVERED

- Flat vertical plate moving in the direction of jet
- Flat vertical plate moving in away from the jet
- Flat vertical plate moving in the direction of jet
- Curved plate moving in the direction of jet

LECTURE 2

Hydrodynamic force on jets on Moving Plates.

FORCE ON FLAT VERTICAL PLATE MOVING IN THE DIRECTION OF JET

Let V = Velocity of jet.

a = Area of cross-section of jet.

u = Velocity of flat plate.

- In this case, the jet does not strike the plate with a velocity v, but it strikes with a relative velocity, which is equal to the absolute velocity of jet of water minus velocity of the plate.

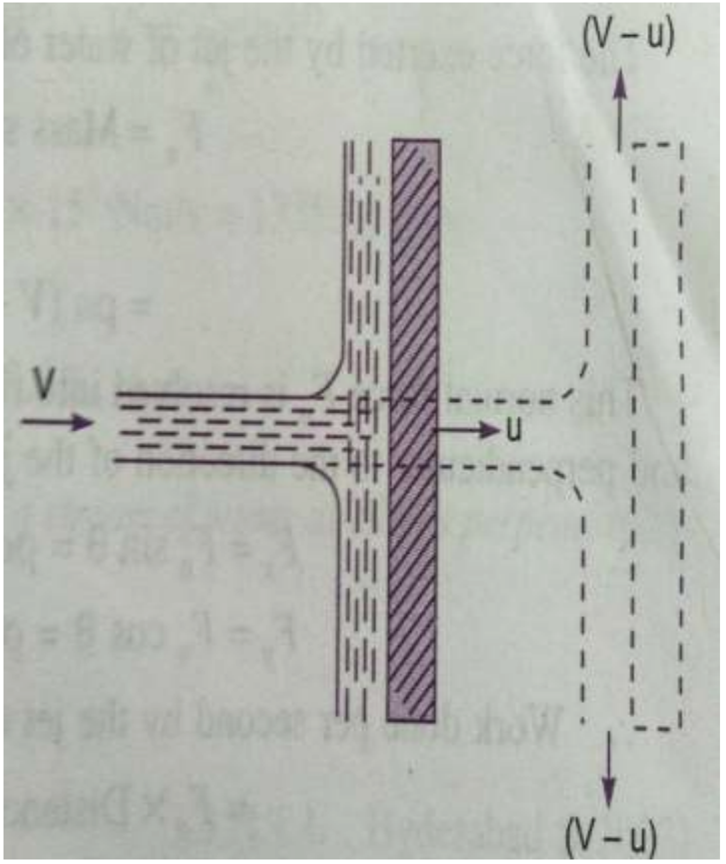
Hence relative velocity of the jet with respect to plate = V – u

Mass of water striking the plate per second= = a (V – u)

Force exerted by the jet on the moving in the direction of the plate

F_x = mass of water striking per second (Initial velocity with which water strikes – Final velocity)

= a (V – u)[(V – u) –0] = a $(V – u)^2$



Since final velocity in the direction of jet is zero.

In this, case the work will be done by the jet on the plate, as the plate is moving. For stationary plates, the work done is zero.

The work done per second by the jet on the plate =

$$= F_x\, u \quad = a(v - u)^2\, u \text{ ------------ } (2)$$

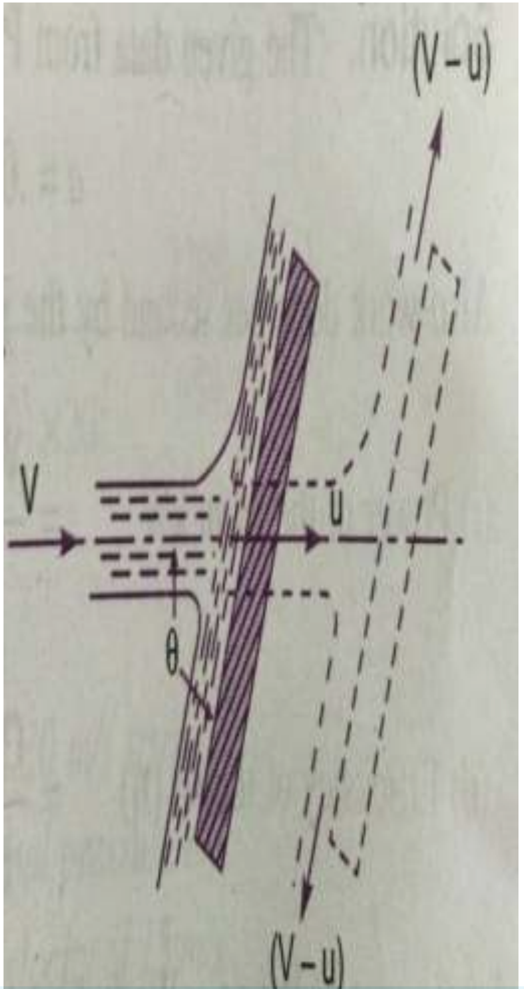
In the above equation (2), if the value of for water is taken in S.I units (i.e.1000kg/m^3) the work done will be in N m/s. The term is equal to Watt (W).

FORCE ON INCLINED PLATE MOVING IN THE DIRECTION OF JET

Let V = Absolute velocity of water.

u = Velocity of plate in the direction of jet.

a = Cross-sectional area of jet



θ = Angle between jet and plate.

Relative velocity of jet of water = $V - u$

The velocity with which jet strikes = $V - u$

Mass of water striking per second = ρ a (V – u)

If the plate is smooth and loss of energy due to impact of the jet is assumed zero, the jet of water will leave the inclined plate with a velocity equal to($V - u$).

The force exerted by the jet of water on the plate in the direction normal to the plate is given as

F_n = Mass striking per sec x (Initial velocity in the normal direction with which jet strikes – final velocity)

$= \rho$ a (V – u) [(V – u) sinθ − 0]

$= \boldsymbol{\rho}$ a $(V - u)^2$ sin$\boldsymbol{\theta}$

This normal force F_n is resolved in to two components, namely F_x and F_y in the direction of jet and perpendicular to the direction of jet respectively.

$$F_x = F_n \sin\theta = \rho a (V-u)^2 \sin^2\theta$$

$$F_y = F_n \cos\theta = \rho a (V-u)^2 \sin\theta\cos\theta$$

Work done per second by the jet on the plate

= F_x × Distance per second in the direction of x

= F_x × u = $\rho a(V-u)^2 \sin^2\theta$ × u

FORCE ON THE CURVED PLATE WHEN THE PLATE IS MOVING IN THE DIRECTION OF JET

Let V = absolute velocity of jet.

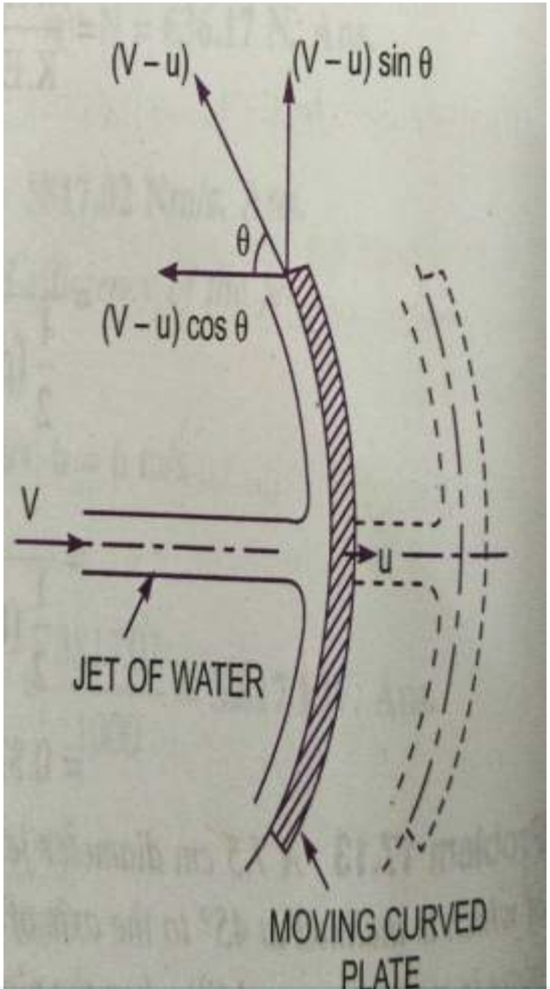
a = area of jet.

u = Velocity of plate in the direction of jet.

Relative velocity of jet of water or the velocity with which jet strikes the curved plate = V – u

If the plate is smooth and the loss of energy due to impact of jet is zero, then the velocity with which the

jet will be leaving the curved vane = (V – u)



This velocity can be resolved in to two components, one in the direction of jet and the other perpendicular to the direction of jet.

Component of the velocity in the direction of jet = - (V – u) Cosθ

(-ve sign is taken as at the out let, the component is in the opposite direction of the jet).

Component of velocity in the direction perpendicular to the direction of jet = (V – u) sinθ

Mass of water striking the plate = ρa × velocity with which jet strikes the plate.

$$= \rho a \; (V - u)$$

∴Force exerted by the jet of water on the curved plate in the direction of jet F_x

F_x = Mass striking per sec [Initial velocity with which jet strikes the plate in the direction of jet −Final velocity]

$$= \rho a(V-u)[(V-u)-(-(V-u)\cos\theta)]$$

$$= \rho a(V-u)^2[1+\cos\theta] \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_(1)}$$

Work done by the jet on the plate per second

= F_x ×Distance travelled per second in the direction of x

$$= F_x \times u = \rho a(V-u)^2\,[1+\cos\theta]\times u$$

$$= \rho a(V-u)^2 \times u[1+\cos\theta]$$

TOPICS TO BE COVERED

- PROBLEMS ON STATIONARY PLATES
- PROBLEMS ON MOVING PLATES

LECTURE 3

Example Problems on force on jets for stationary & moving plates

1. Water is flowing through a pipe at the end of which a nozzle is fitted. The diameter of the nozzle is 100 mm and the head of the water at the centre of the nozzle is 100m. Find the force exerted by the jet of water on a fixed vertical plate. The co-efficient of velocity is given as 0.95.

ANS:GIVEN :Diameter of nozzle d = 100mm =0.1m

Area of nozzle = $\frac{\pi}{4} \times (0.1)^2 = 0.00785 m^2$

Head of water H = 100m

Co-efficient of velocity= $C_v = 0.95$

Theoretical velocity of jet of water $V_{th} = \sqrt{2gh} = \sqrt{2 \times 9.81 \times 100} = 44.294$m/sec

But $C_v = \frac{Actual\ \ velocity}{Theoritical\ \ velocity}$

$\therefore$ Actual velocity of jet of water $= C_v \times V_{th} = 0.95 \times 44.294 =$ **42.08m/sec**

Force exerted on a fixed vertical plate

$$F = \rho a V^2 = 1000 \times 0.007854 \times (42.08)^2$$

F= 13907.2 N = 13.9 kN

2. A jet of water of diameter 75mm moving with a velocity of 25m/sec strikes a fixed plate in such a way that the angle between the jet and plate is 60. Find the force exerted by the jet on the plate

I. In the direction normal to the plate and

II. In the direction of the jet.

ANS:Given:

Diameter of the jet $d = 75mm = 0.075m$

Area of the jet $\frac{\pi}{4} \times (0.075)^2 = 0.004417 m^2$

Velocity of jet $V = 25m/sec$

Angle between jet and plate $\theta = 60°$

i) The force exerted by the jet of water in the direction normal to the plate

$$F_n = \rho a V^2 sin\theta = 1000 \times 0.004417 \times (25)^2 sin\, 60^0$$

$$\boldsymbol{F_n = 2390.7\, N}$$

ii) The force in the direction of jet

$$F_x = \rho a V^2 sin^2\, \theta = 1000 \times 0.004417 \times (25)^2 \times sin^2 60^0$$

$$\boldsymbol{F_x = 2070.4\, N}$$

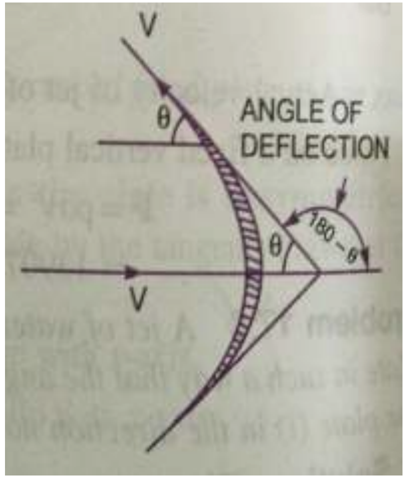
3. A jet of water of dia.50mm moving with a velocity of 40m/sec strikes a curved fixed symmetrical plate at the centre. Find the force exerted by the jet of water in the direction of jet, if the direction of jet is deflected through an angle of 120 at the out let of curved plate.

Ans) **Given:** Dia. of jet d = 0.05m

Area of jet $a = \frac{\pi}{4} \times (0.05)^2 = 0.001963 m^2$

Velocity of jet V = 40m/sec

Angle of deflection = $180 - \theta = 180 - 120 = 60°$



Force exerted by the jet on the curved plate in the direction of jet

$$F_x = \rho a V^2 [1 + cos\theta]$$

$$F_x = 1000 \times 0.001963 \times (40)^2 \times [1 + \cos 60^0]$$

4. A jet of water of dia. 75mm moving with a velocity of 30m/sec strikes a curved fixed plate tangentially at one end at an angle of 30 to the horizontal. The jet leaves the plate at an angle of 20^0 to the horizontal. Find the force exerted by the jet on the plate in the horizontal and vertical direction.

Given: Dia. of jet d = 75mm = 0.075m,

Area of jet $a = \frac{\pi}{4} \times (0.075)^2 = 0.004417\ m^2$

Velocity of jet V = 30m/sec

Angle made by the jet at inlet tip with the horizontal θ =30°

Angle made by the jet at out let tip with the horizontal Ø = 20°

The force exerted by the jet of water on the plate in horizontal direction F_x

$$F_x = \rho a V^2[\cos\theta + \cos Ø]$$

$$= 1000 \times 0.004417[\cos 30^0 + \cos 20^0] \times (30)^2$$

$$\boldsymbol{F_x = 7178.2\ N}$$

The force exerted by the jet of water on the plate in vertical direction F_y

$$F_y = \rho a V^2[sin\ \theta - sin\ Ø]$$

$$= 1000 \times 0.004417[\sin 30^0 - \sin 20^0] \times (30)^2$$

5. A nozzle of 50mm dia. delivers a stream of water at 20m/sec perpendicular to the plate that moves away from the plate at 5m/sec. Find:

i) The force on the plate.

ii) The work done and

iii) The efficiency of the jet.

Ans) **Given:** Dia. of jet d = 50mm = 0.05m,

Area of jet $a = \frac{\pi}{4}(0.05)^2 = 0.0019635\ m^2$

Velocity of jet V = 20m/sec,

Velocity of plate u = 5m/sec

i) The force on the plate $F_x = \rho a(V-u)^2$

$$F_x = 1000 \times 0.0019635 \times (20-5)^2$$

$$\boldsymbol{F_x = 441.78\ N}$$

ii) The work done by the jet $= F_x \times u$

$$= 441.78 \times 5$$

$$= \mathbf{2208.9\ Nm/s}$$

iii) The efficiency of the jet $\eta = \dfrac{Out\ put\ of\ jet}{Input\ of\ jet}$

$$= \frac{Work\ done/sec}{K.E\ of\ jet/sec} = \frac{F_x \times u}{\frac{1}{2} m v^2}$$

$$= \frac{F_x \times u}{\frac{1}{2}(\rho a V)V^2}$$

$$= \frac{2208.9}{\frac{1}{2}(1000 \times 0.0019635 \times 20) \times 20^2} = \frac{2208.9}{6540}$$

$$= \mathbf{0.3377} = \mathbf{33.77\%}$$

6. A 7.5cm dia. jet having a velocity of 30m/sec strikes a flat plate, the normal of which is inclined at 45 to the axis of the jet. Find the normal pressure on the plate: (i) When the plate is stationary and (ii) When the plate is moving with a velocity of 15m/sec away from the jet. Also determine the power and efficiency of the jet when the plate is moving.

Given: Dia. of the jet $d = 7.5cm = 0.075m$

Area of jet $a = \frac{\pi}{4}(0.075)^2 = 0.004417m^2$

Angle between jet and plate $\theta = 90° - 45° = 45°$

Velocity of jet $V = 30m/sec$

i) When the plate is stationary, the normal force F_n on the plate is

$$F_n = \rho a V^2 \sin\theta = 1000 \times 0.004417 \times (30)^2 \times \sin 45^0$$

$$= \mathbf{2810.96\ N}$$

ii) When the plate is moving with a velocity of 15m/sec away from the jet, the normal force on the plate F_n

$$F_n = \rho a (V-u)^2 \sin\theta = 1000 \times 0.004417 \times (30-15)^2 \times \sin 45^0$$

iii) The power and efficiency of the jet, when the plate is moving is obtained as

Work done /sec by the jet

= Force in the direction of jet x Distance moved by plate in the direction of jet/sec

$= F_x \times u$ Where $F_x = F_n \sin\theta = 702.74 \times \sin 45^0 = 496.9\ N$

Work done/ sec = 496.9 × 15 = 7453.5Nm/s

∴ Power in kW = $\frac{Work\ \ done\ \ /sec}{1000} = \frac{7453.5}{1000}$ = **7.453 kW**

Efficiency of jet $= \frac{Out\ \ put}{Input} = \frac{Work\ done\ per\ sec}{K.E\ of\ jet\ per\ sec}$

$$= \frac{7453.5}{\frac{1}{2}(\rho a V)\times V^2} = \frac{7453.5}{\frac{1}{2}\rho a V^3}$$

$$= \frac{7453.5}{\frac{1}{2}\times 1000 \times 0.004417 \times 30^3}$$

= 0.1249 ≃ 0.125 =12.5%

7. A jet of water of dia. 7.5cm strikes a curved plate at its centre with a velocity of 20m/sec. the curved plate is moving with a velocity of 8m/sec in the direction of the jet. The jet is deflected through an angle of 165. Assuming plate is smooth, find

1. Force exerted on the plate in the direction of jet.
2. Power of jet.
3. Efficiency of jet.

Given: Dia. of jet d = 7.5cm = 0.075m

Area of jet $a = \frac{\pi}{4}(0.075)^2 = 0.004417 m^2$

Velocity of jet V = 20m/sec

Velocity of plate u = 8m/sec

Angle made by the relative velocity at the out let of the plate $\theta = 180° - 165° = 15°$

i) Force exerted by the jet on the plate in the direction of jet

$$F_x = \rho a(V-u)^2(1+\cos\theta)$$

$$F_x = 1000 \times 0.004417 \times (20-8)^2[1+\cos 15^0]$$

$$= \mathbf{1250.38}\boldsymbol{N}$$

ii) Work done by the jet on the plate per second

$$= F_x \times u$$

$$= 1250.38 \times 8$$

$$= \mathbf{10003.04}\ \boldsymbol{Nm/s}$$

∴ Power of jet = $\frac{10003.04}{1000}$ = 10kW

iii)Efficiency of the jet = $\frac{Out\ put}{Input} = \frac{Work\ done\ per\ sec}{K.E\ of\ jet\ per\ sec}$

$$= \frac{1250.38 \times 8}{\frac{1}{2}\rho aV.V^2} = \frac{1250.38 \times 8}{\frac{1}{2} \times 1000 \times 0.004417 \times (20)20^3}$$

= 0.564 = 56.4%

8. A jet of water having a velocity of 40m/sec strikes a curved vane, which is moving with a velocity of 20m/sec. The jet makes an angle of 30 with the direction of motion of vane at inlet and leaves at an angle of 90 to the direction of motion of vane at out let. Draw velocity triangles at inlet and outlet and determine vane angles at inlet and outlet, so that the water enters and leaves the vanes without shock.

Ans)

Given: Velocity of jet $V_1 = 40$m/sec

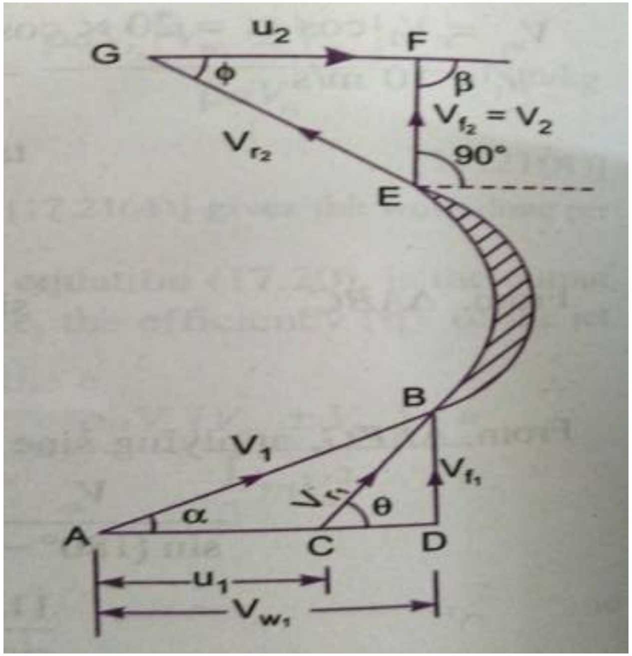
Velocity of vane $u_1 = 20$m/sec

Angle made by jet at inlet $\alpha = 30°$

Angle made by leaving jet = 90°

$\therefore \beta = 180° - 90° = 90°$

$u_1 = u_2 = u = 20$m/sec



Vane angles at inlet and outlet are θ and $\emptyset$

From Δ BCD we have $\tan\theta = \frac{BD}{CD} = \frac{BD}{AD-AC} = \frac{V_{f1}}{V_{w1}-u_1}$

Where $V_{f_1} = V_1 \sin\alpha = 40 \times \sin 30^0 = 20 m/s$

$V_{w_1} = V_1 \cos\alpha = 40 \times \cos 30^0 = 34.64\, m/s$

$u_1 = 20 m/s$

$\therefore \quad \tan\theta = \frac{20}{34.64-20} = \frac{20}{14.64} = 1.366 = \tan 53.79^0$

$\therefore \quad \boldsymbol{\theta = 53.79^0 \ or\ 53^0 47.4'}$

Also from Δ BCD we have $\sin\theta = \frac{V_{f1}}{V_{r1}}$ $or\ V_{r_1} = \frac{V_{f1}}{\sin\theta} = \frac{20}{\sin 53.79^0} = 24.78\, m/s$

$\therefore \quad V_{r_1} = 24.78 m/s$

But $V_{r_2} = V_{r_1} = 24.78$

Hence, From Δ EFG, $\cos\emptyset = \frac{u_2}{V_{r2}} = \frac{20}{24.78} = 0.8071 = \cos 36.18^0$

$\boldsymbol{\emptyset = 36.18^0 \ or\ 36^0 \ 10.8'}$

9. A stationary vane having an inlet angle of zero degree and an outlet angle of 25, receives water at a velocity of 50m/sec. Determine the components of force acting on it in the direction of jet velocity and normal to it. Also find the resultant force in magnitude and direction per unit weight of the flow

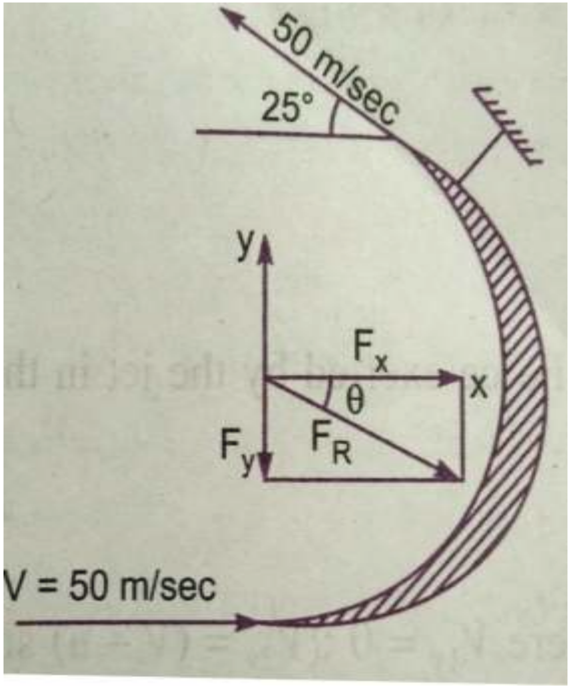
Given: Velocity of jet V = 50m/sec

Angle at outlet =25°

For the stationary vane, the force in the direction of jet.

$$F_x = Mass\ per\ sec \times [V_{1x} - V_{2x}]$$

Where V_{1x}= 50m/sec , $V_{2x} = -50\cos 25^0 = -45.315$



∴ Force in the direction of jet per unit weight of water F_x

$$F_x = \frac{Mass/\sec[50-(-45.315)]}{Weight\ of\ water\ /sec} = \frac{Mass/\sec[50+45.315]}{(mass/sec)\times g}$$

$$= \frac{95.315}{9.81} = 9.716\ N/N$$

Force exerted by the jet in perpendicular direction to the jet per unit weight of flow

$$V_{1y} = 0 \qquad V_{2y} = 50 \sin 25^0$$

$$F_y = \frac{Mass/\sec(V_{1y} - V_{2y})}{g \times\ mass\ per\ sec}$$

$$= \frac{(0-50\ Sin\,25°)}{g} = \frac{-50\ Sin\,25°}{9.81}$$

= **- 2.154 N**

- ve sign means the force F_y is acting in the downward direction.

∴ Resultant Force per unit weight of water $F_R = \sqrt{F_x{}^2 + F_y{}^2}$

$$F_R = \sqrt{(9.716)^2 + (2.154)^2} = \mathbf{9.952N}$$

The angle made by the Resultant Force with x- axis

F_y 2.154

10. A jet of water diameter 50mm moving with a velocity of 25m/sec impinges on a fixed curved plate tangentially at one end at an angle of 30 to the horizontal. Calculate the resultant force of the jet on the plate, if the jet is deflected through an angle of 50. Take g=10m/sec^2.

Given:

Dia. of jet d =50mm = 0.05m,

Area of jet $a = \frac{\pi}{4}(0.05)^2 = 0.0019635\ m^2$

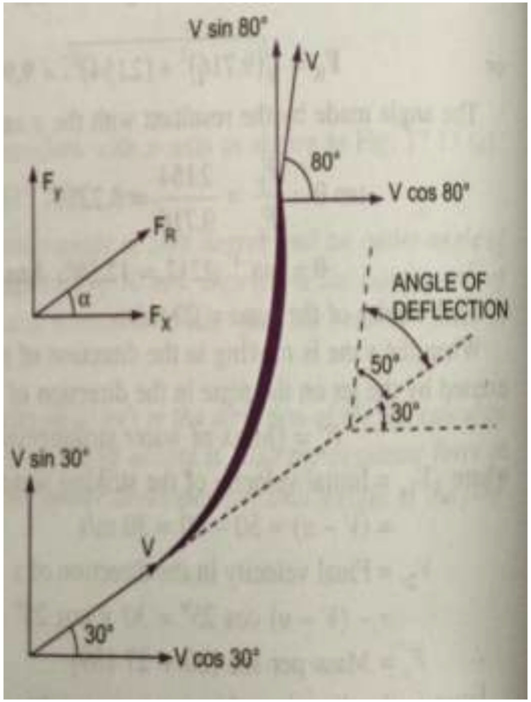
Velocity of jet V = 25m/sec,

Angle made by the jet at inlet with horizontal $\theta = 30°$

Angle of deflection = 50°

Angle made by the jet at the outlet with horizontal $\emptyset$

$$\emptyset = \theta + angle\ of\ deflection = 30°+50° = 80°$$



The Force exerted by the jet of water in the direction of x

$$F_x = \rho a V(V_{1x} - V_{2x})$$

Where $\rho = 1000$

$a = \frac{\pi}{4}(0.05)^2$ V = 25 m/s

$$V_{1x} = V\cos 30^0 = 25\cos 30^0$$

$$V_{2x} = V\cos 80^0 = 25\cos 80^0$$

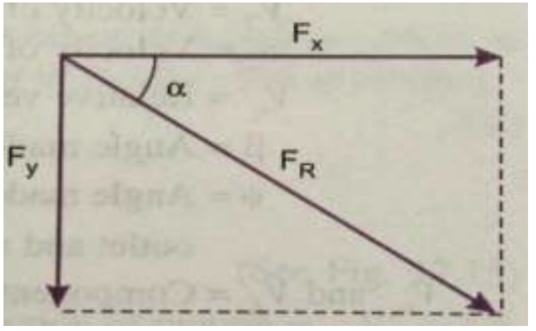
$$F_x = 1000 \times \frac{\pi}{4}(0.05)^2 \times 25[25\cos 30^0 - 25\cos 80^0] = \mathbf{849.7\ N}$$

The Force exerted by the jet of water in the direction of y

$$F_y = \rho a V\left(V_{1y} - V_{2y}\right)$$

$$F_y = 1000 \times \frac{\pi}{4}(0.05)^2 \times 25[25\sin 30^0 - 25\sin 80^0] = \mathbf{-594.9\ N}$$

- ve sign shows that Force F_y is acting in the downward direction.



The Resultant force $F_R = \sqrt{{F_x}^2 + {F_y}^2}$

$= \sqrt{(849.7)^2 + (594.9)^2}$

$= \mathbf{1037N}$

Angle made by the Resultant Force with the Horizontal

$\tan\alpha = \frac{F_y}{\ldots} = \frac{594.9}{\ldots} = 0.7$

LECTURE 4

CLASSIFICATION OF HYDRAULIC TURBINES:

TOPICS TO BE COVERED

- Impulse turbine and
- Reaction turbine

CLASSIFICATION OF HYDRAULIC TURBINES:

The Hydraulic turbines are classified according to the type of energy available at the inlet of the turbine, direction of flow through the vanes, head at the inlet of the turbine and specific speed of the turbine. The following are the important classification of the turbines.

1. According to the type of energy at inlet:
   (a) Impulse turbine and
   (b) Reaction turbine
2. According to the direction of flow through the runner:
   (a) Tangential flow turbine
   (b) Radial flow turbine.
   (c) Axial flow turbine
   (d) Mixed flow turbine

3. According to the head at inlet of the turbine:

(a) High head turbine

(b) Medium head turbine and

(c) Low head turbines.

4. According to the specific speed of the turbine:

(a) Low specific speed turbine

(b) Medium specific speed turbine

(c) High specific speed turbine.

- If at the inlet of turbine, the energy available is only kinetic energy, the turbine is known as **Impulse turbine**.
- As the water flows over the vanes, the pressure is atmospheric from inlet to outlet of the turbine. If at the inlet of the turbine, the water possesses kinetic energy as well as pressure energy, the turbine is known as **Reaction turbine**.
- As the water flows through runner, the water is under pressure and the pressure energy goes on changing in to kinetic energy. The runner is completely enclosed in an air-tight casing and the runner and casing is completely full of water.

- If the water flows along the tangent of runner, the turbine is known as **Tangential flow turbine**.
- If the water flows in the radial direction through the runner, the turbine is called **Radial flow turbine.**
- If the water flows from outward to inwards radially, the turbine is known as **Inward** radial flow turbine, on the other hand

- if the water flows radially from inward to outwards, the turbine is known as **outward** radial flow turbine.
- If the water flows through the runner along the direction parallel to the axis of rotation of the runner, the turbine is called **axial flow** turbine
- If the water flows through the runner in the radial direction but leaves in the direction parallel to the axis of rotation of the runner, the turbine is called **mixed flow** turbine.

TOPICS TO BE COVERED

- Working principle
- Derivation of work done & η

LECTURE 5

Pelton wheel Turbine:

PELTON WHEEL (TURBINE)

- It is a tangential flow impulse turbine. The water strikes the bucket along the tangent of the runner. The energy available at the inlet of the turbine is only kinetic energy. The pressure at the inlet and out let of turbine is atmospheric. This turbine is used for high heads and is named after L.A.Pelton an American engineer.
- The water from the reservoir flows through the penstocks at the out let of which a nozzle is fitted. The nozzle increases the kinetic energy of the water flowing through the penstock. At the outlet of the nozzle, the water comes out in the form of a jet and strikes the buckets (vanes) of the runner.

MAIN PARTS OF PELTON WHEEL TURBINE

- Nozzle and flow regulating arrangement (spear)
- Runner and Buckets.
- Casing and
- Breaking jet

1. **Nozzle and flow regulating arrangement:** The amount of water striking the buckets (vanes) of the runner is controlled by providing a spear in the nozzle. The spear is a conical needle which is operated either by hand wheel or automatically in an axial direction depending upon the size of the unit. When the spear is pushed forward in to the nozzle, the amount of water striking the runner is reduced. On the other hand, if the spear is pushed back, the amount of water striking the runner increases.
2. **Runner with buckets:** It consists of a circular disc on the periphery of which a number of buckets evenly spaced are fixed. The shape of the buckets is of a double hemispherical cup or bowl. Each bucket is divided in to two symmetrical parts by a dividing wall, which is known as splitter.

3. **Casing:** The function of casing is to prevent the splashing of the water and to discharge the water to tailrace. It also acts as safeguard against accidents. It is made of Cast Iron or fabricated steel plates. The casing of the Pelton wheel does not perform any hydraulic function.

4. **Breaking jet:** When the nozzle is completely closed by moving the spear in the forward direction, the amount of water striking the runner reduces to zero. But the runner due to inertia goes on revolving for a long time. To stop the runner in a short time, a small nozzle is provided, which directs the jet of water on the back of the vanes. This jet of water is called Breaking jet.

VELOCITY TRIANGLES AND WORK DONE FOR PELTON WHEEL

- The jet of water from the nozzle strikes the bucket at the splitter, which splits up the jet into two parts.
- These parts of the jet, glides over the inner surfaces and comes out at the outer edge.
- The splitter is the in let tip and outer edge of the bucket is the outlet tip of the bucket. The inlet velocity triangle is drawn at the splitter and outer velocity triangle is drawn at the outer edge of the bucket.

Let

H = Net head acting on the Pelton Wheel

$$= H_g - h_f$$

Where H_g = Gross Head

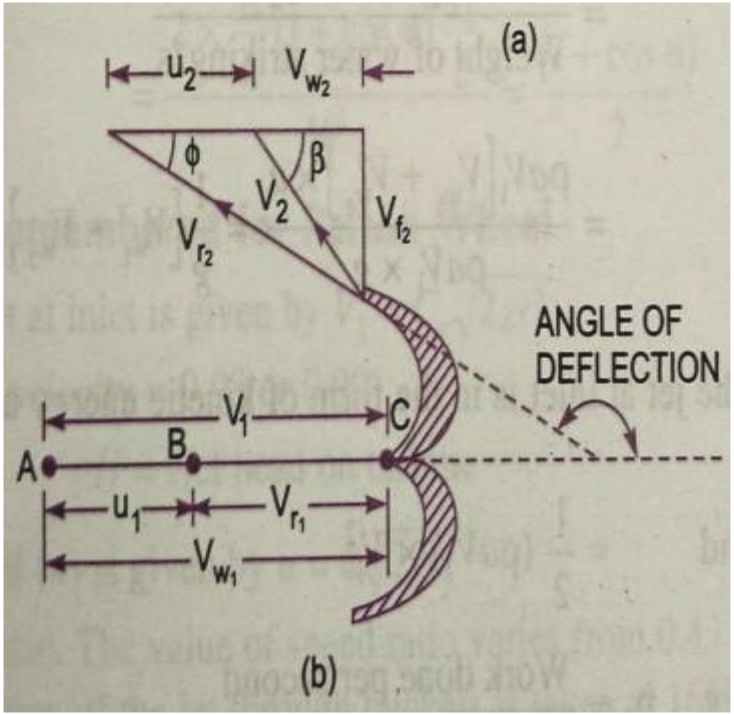
$$h_f = \frac{4fLV^2}{D^* \times 2g}$$

Where D^* =diameter of penstock,

D = Diameter of wheel,

d = Diameter of Jet,

N = Speed of the wheel in r.p.m



Then V_1 = Velocity of jet at inlet= $\sqrt{2gH}$

$$u = u_1 = u_2 = \frac{\pi DN}{60}$$

The Velocity Triangle at inlet will be a straight line where

$$V_{r_1} = V_1 - u_1 = V_1 - u$$

$$V_{w_1} = V_1 \qquad \alpha = 0^0 \qquad and \;\; \theta = 0^0$$

From the velocity triangle at outlet, we have

$$V_{r_2} = V_{r_1} \; and \; V_{w_2} = V_{r_2} \cos \emptyset - u_2$$

The force exerted by the Jet of water in the direction of motion is

$$F_x = \rho a V_1 \left[V_{w_1} + V_{w_2} \right] \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_} (1)$$

As the angle β is an acute angle, +ve sign should be taken. Also this is the case of series of vanes, the mass of water striking is $\rho a V_1$ and not $\rho a V_{r_1}$. In equation (1) ‘a’ is the area of the jet = $\frac{\pi}{4} d^2$

Now work done by the jet on the runner per second

$$= F_x \times u = \rho a V_1 \left[V_{w_1} + V_{w_2} \right] \times u \qquad Nm/s$$

Power given to the runner by the jet = $\frac{\rho a V_1 \left[V_{w_1} + V_{w_2} \right] \times u}{\;} \; kW$

Work done/s per unit weight of water striking/s $= \dfrac{\rho a V_1 [V_{w_1} + V_{w_2}] \times u}{\text{Weight of water} \frac{\text{striking}}{s}}$

$$= \frac{\rho a V_1 [V_{w_1} + V_{w_2}] \times u}{\rho a V_1 \times g} = \frac{1}{g}\left[V_{w_1} + V_{w_2}\right] \times u \qquad \text{\_\_\_\_\_\_\_\_ (3)}$$

The energy supplied to the jet at inlet is in the form of kinetic energy

$\therefore$ K.E. of jet per second $= \frac{1}{2} m V^2 = \frac{1}{2}(\rho a V_1) \times V_1{}^2$

$\therefore$ **Hydraulic efficiency,** $\eta_h = \dfrac{\text{Work done per second}}{\text{K.E. of jet per second}}$

$$= \frac{\rho a V_1 [V_{w_1} + V_{w_2}] \times u}{\frac{1}{2}(\rho a V_1) \times V_1{}^2}$$

$$= \frac{2[V_{w_1} + V_{w_2}] \times u}{V_1{}^2} \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_ (4)}$$

Now $V_{w_1} = V_1$ and $V_{r_1} = V_1 - u_1 = (V_1 - u)$

$\therefore$ $V_{r_2} = (V_1 - u)$

And $V_{w_2} = V_{r_2} \cos \emptyset - u_2$

$= V_{r_2} \cos \emptyset - u$

$= (V_1 - u) \cos \emptyset - u$

Substituting the values of V_{w_1} and V_{w_2} in equation (4)

$$\eta_h = \frac{2[V_1+(V_1-u)\cos\emptyset-u]\times u}{V_1^2} = \frac{2[V_1-u+(V_1-u)\cos\emptyset]\times u}{V_1^2}$$

$$= \frac{2(V_1-u)[1+\cos\emptyset]u}{V_1^2} \quad \text{\_\_\_\_\_\_\_\_\_\_} (5)$$

The efficiency will be maximum for a given value of V_1 when

$$\frac{d}{du}(\eta_h) = 0 \quad or \quad \frac{d}{du}\left[\frac{2u(V_1-u)[1+\cos\emptyset]}{V_1^2}\right] = 0$$

Or $\frac{(1+\cos\emptyset)}{V_1^2} \frac{d}{du}(2uV_1 - 2u^2) = 0$

Or $\frac{d}{du}[2uV_1 - 2u^2] = 0 \qquad \left(\because \frac{1+\cos\phi}{V_1^2} \neq 0\right)$

Or $2V_1 - 4u = 0$ Or $u = \frac{V_1}{2}$ __________ (6)

Equation (6) states that hydraulic efficiency of a Pelton wheel will be maximum when the velocity of the wheel is half the velocity of the jet water at inlet. The expression for maximum efficiency will be obtained by substituting the value of $u = \frac{V_1}{2}$ in equation (5)

$$\text{Max. } \eta_h = \frac{2\left(V_1-\frac{V_1}{2}\right)(1+\cos\emptyset)\times\frac{V_1}{2}}{V_1^2} = \frac{2\times\frac{V_1}{2}(1+\cos\emptyset)\frac{V_1}{2}}{V_1^2} = \frac{(1+\cos\emptyset)}{2}$$

RADIAL FLOW REACTION TURBINE

- In the Radial flow turbines water flows in the radial direction.
- The water may flow radially from outwards to inwards (i.e. towards the axis of rotation) or from inwards to outwards.
- If the water flows from outwards to inwards through the runner, the turbine is known as inwards radial flow turbine. And if the water flows from inwards to outwards, the turbine is known as outward radial flow turbine.

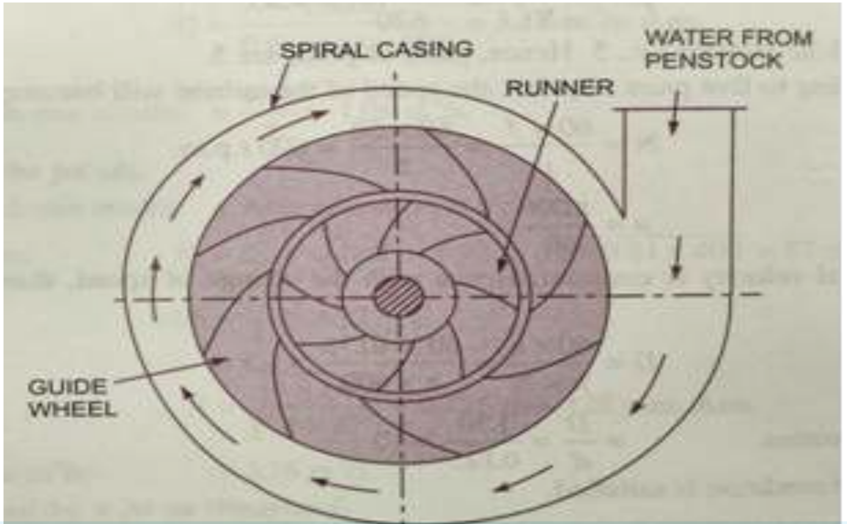
Main parts of a Radial flow Reaction turbine:

Casing

Guide mechanism

Runner and

Draft tube



1. **Casing:** in case of reaction turbine, casing and runner are always full of water. The water from the penstocks enters the casing which is of spiral shape in which area of cross-section one of the casing goes on decreasing gradually. The casing completely surrounds the runner of the turbine. The water enters the runner at constant velocity throughout the circumference of the runner.

2. **Guide Mechanism:** It consists of a stationary circular wheel all around the runner of the turbine. The stationary guide vanes are fixed on the guide mechanism. The guide vanes allow the water to strike the vanes fixed on the runner without shock at inlet. Also by suitable arrangement, the width between two adjacent vanes of guide mechanism can be altered so that the amount of water striking the runner can be varied.

3. **Runner:** It is a circular wheel on which a series of radial curved vanes are fixed. The surfaces of the vanes are made very smooth. The radial curved vanes are so shaped that the water enters and leaves the runner without shock. The runners are made of cast steel, cast iron or stain less steel. They are keyed to the shaft.

4. **Draft - Tube:** The pressure at the exit of the runner of a reaction turbine is generally less than atmospheric pressure. The water at exit can't be directly discharged to the tail race. A tube or pipe of gradually increasing area is used for discharging the water from the exit of the turbine to the tail race. This tube of increasing area is called draft-tube.

5. **Inward Radial Flow Turbine:** In the inward radial flow turbine, in which case the water from the casing enters the stationary guiding wheel. The guiding wheel consists of guide vanes which direct the water to enter the runner which consists of moving vanes. The water flows over the moving vanes in the inward radial direction and is discharged at the inner diameter of the runner. The outer diameter of the runner is the inlet and the inner diameter is the outlet.

VELOCITY TRIANGLES AND WORK DONE BY WATER ON RUNNER:

Work done per second on the runner by water

$$= \rho a V_1 \left[V_{w_1} u_1 \pm V_{w_2} u_2 \right]$$

$$= \rho Q \left[V_{w_1} u_1 \pm V_{w_2} u_2 \right] \quad \text{\_\_\_\_\_\_\_\_\_\_ (1)} \qquad (\because a V_1 = Q)$$

The equation represents the energy transfer per second to the runner.

Where V_{w_1} = Velocity of whirl at inlet

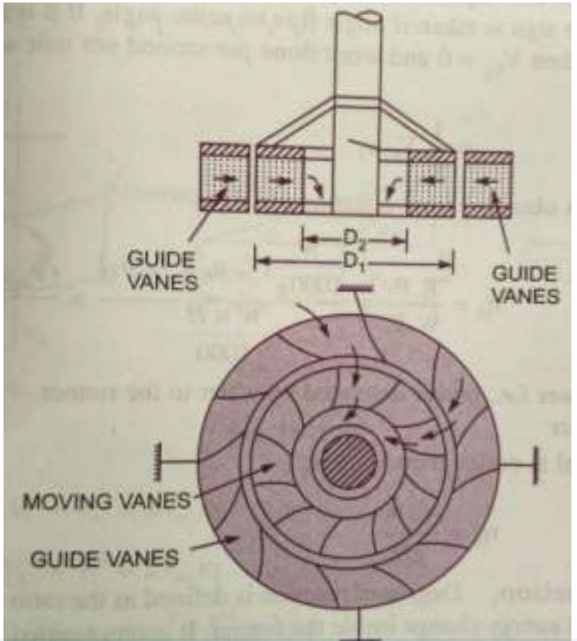
V_{w_2} = Velocity of whirl at outlet

u_1 = Tangential velocity at inlet

$= \frac{\pi D_1 \times N}{60}$, Where D_1 = Outer dia. Of runner,

u_2 = Tangential velocity at outlet

$= \frac{\pi D_2 \times N}{60}$



Where D_1 = Inner dia. Of runner,

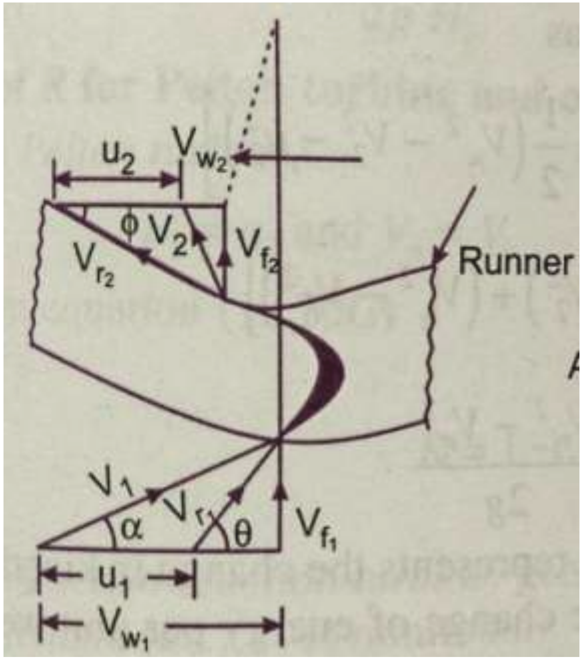
N = Speed of the turbine in r.p.m.

The work done per second per unit weight of water per second

$$= \frac{\text{work done per second}}{\text{weight of water striking per second.}}$$

$$= \frac{\rho Q\left[V_{w_1} u_1 \pm V_{w_2} u_2\right]}{\rho Q \times g}$$

$$= \frac{1}{g}\left[V_{w_1} u_1 \pm V_{w_2} u_2\right] \qquad \text{\_\_\_\_\_\_\_\_\_\_ (2)}$$



Equation (2) represents the energy transfer per unit weight/s to the runner. This equation is known by **Euler's equation.**

In equation +ve sign is taken if β is an acute angle,

-ve sign is taken if β is an obtuse angle.

If $\beta = 90^0$ then $V_{w_2} = 0$ and work done per second per unit weight of water striking/s

$$\text{Work done} = \frac{1}{g} V_{w_1} u_1$$

Hydraulic efficiency $\eta_h = \frac{R.P.}{W.P.} = \frac{\text{Power delivered to runner}}{\text{Power supplied at inlet}}$

$$= \frac{\frac{W}{1000 \times g}\left[V_{w_1} u_1 \pm V_{w_2} u_2\right]}{\frac{W \times H}{1000}} = \frac{\left(V_{w_1} u_1 \pm V_{w_2} u_2\right)}{gH} \quad \text{———— (3)}$$

Where R.P. = Runner Power i.e. power delivered by water to the runner

W.P. = Water Power

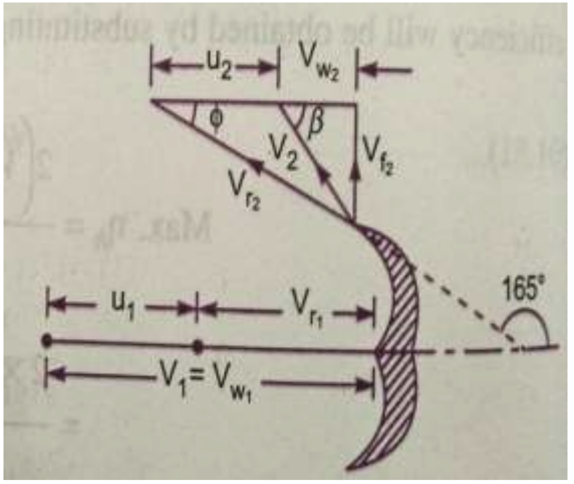
If the discharge is radial at outlet, then $V_{w_2} = 0$

$$\eta_h = \frac{V_{w_1} u_1}{gH}$$

1. A pelton wheel has a mean bucket speed of 10m/s with a jet of water flowing at the rate of 700lts/sec under a head of 30 m. the buckets deflect the jet through an angle of 160^0 calculate the power given by the water to the runner and hydraulic efficiency of the turbine? Assume co-efficient of velocity=0.98

ANS)

Given:



Speed of bucket $u = u_1 = u_2 = 10m/s$

Discharge Q = 700lt/sec=$0.7m^3$ /s

Head of water H = 30m

Angle deflection = 160^0

$\therefore Angle \quad \emptyset = 180 - 160 = 20^0$

Co-efficient of velocity $C_v = 0.98$

The velocity of jet $V_1 = C_v\sqrt{2gH} = 0.98\sqrt{2 \times 9.81 \times 30} = 23.77m/s$

$$V_{r_1} = V_1 - u_1 = 23.77 - 10 = 13.77 m/s$$

$$V_{w_1} = V_1 = \mathbf{23.77 m/s}$$

From the outlet velocity triangle

$$V_{r_2} = V_{r_1} = \frac{13.77m}{s}$$

$$V_{w_2} = V_{r_1} \cos \emptyset - u_2$$

$$= 13.77 \cos 20^0 - 10 = \mathbf{2.94 m/s}$$

Work done by the jet/sec on the runner is given by equation

$$= \rho a V_1 [V_{w_1} + V_{w_2}] \times u$$

$$= 1000 \times 0.7[23.77 + 2.94] \times 10$$

$$= \mathbf{186970 Nm/s}$$

Power given to the turbine $= \frac{186970}{1000} = \mathbf{186.97 kW}$

The hydraulic efficiency of the turbine is given by equation

$$\eta_h = \frac{2[V_{w_1} + V_{w_2}] \times u}{V_1{}^2} = \frac{2[23.77 + 2.94] \times 10}{23.77 \times 23.77} = \mathbf{0.9454}$$

2. A reaction turbine works at 450rpm under a head of 120m. its diameter at inlet is 120cm and flow area is 0.4m^2. The angles made by absolute and relative velocities at inlet are 20^0 and 60^0 respectively, with the tangential velocity. Determine

i) Volume flow rate **ii)** the power developed

iii) The hydraulic efficiency. Assume whirl at outlet is zero.

Ans **Given:** Speed of turbine N = 450rpm

Head H = 120m

Diameter of inlet D_1 = 120cm=1.2m

Flow area $\pi D_1 \times B_1 = 0.4m^2$

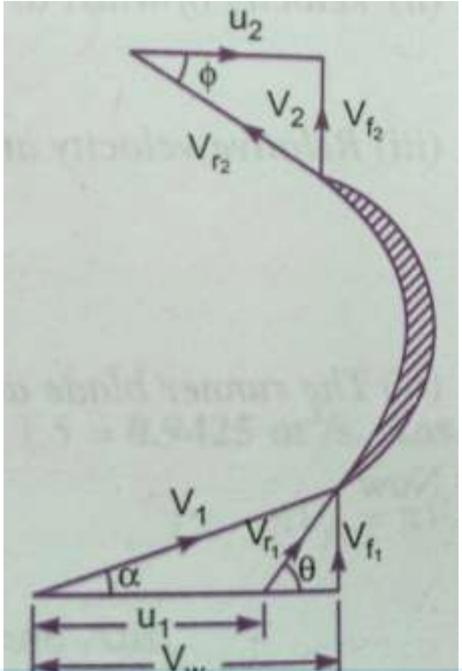
Angle made by absolute velocity $\alpha = 20^0$

Angle made by relative velocity $\theta = 60^0$

Whirl at outlet $V_{w_2} = 0$

Tangential velocity of the turbine at inlet

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 1.2 \times 450}{60} = 28.27 m/s$$



From inlet triangle $\tan \alpha = \frac{V_{f_1}}{\ldots}$

$$\text{Tan } 20^0 = \frac{V_{f_1}}{V_{w_1}} = 0.364,$$

$$V_{f_1} = 0.364 V_{w_1} \text{\_\_\_\_\_\_\_\_\_\_\_\_} (1)$$

Also $$\tan\theta = \frac{V_{f_1}}{V_{w_1} - u_1} = \frac{0.364 V_{w_1}}{V_{w_1} - 28.27} \qquad (\because \tan\theta = tan60^0 = 1.732)$$

$$1.732 = \frac{0.364 V_{w_1}}{V_{w_1} - 28.27}$$

$$0.364 V_{w_1} = 1.732(V_{w_1} - 28.27)$$

$$0.364 V_{w_1} = 1.732 V_{w_1} - 28.27 \times 1.732$$

$$V_{w_1}(1.732 - 0.364) = 48.96$$

$$V_{w_1} = \frac{48.96}{1.732 - 0.364} = \mathbf{35.79 m/s}$$

From equation (1) $$V_{f_1} = 0.364 V_{w_1} = 0.364 \times 35.79 = \mathbf{13.027 m/s}$$

i) Volume flow rate is given by equation as $Q = \pi D_1 B_1 V_{f_1}$

$$Q = 0.4 \times 13.027 = \mathbf{5.211 m^3/sec} \qquad (\because \pi D_1 B_1 = 0.4 m^2)$$

ii) Work done per second on the turbine is given by equation

$$= \rho Q [V_{w_1} \times u_1]$$

$$= 1000 \times 5.211[35.79 \times 28.27] = 5272.402 Nm/s$$

Power developed in $kW = \frac{work\ done\ per\ sec}{1000} = \frac{5272402}{1000} = \mathbf{5272.402 kW}$

iii) The hydraulic efficiency is given by equation

$$\eta_h = \frac{V_{w_1} \times u_1}{g \times H} = \frac{35.79 \times 28.27}{9.81 \times 120} = 0.8595$$

TOPICS TO BE COVERED

- Working principle
- Derivation of work done & η

LECTURE 6

FRANCIS & KAPLAN TURBINE:

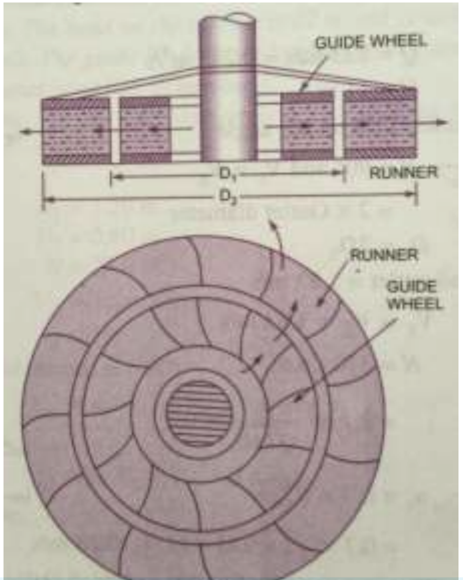
FRANCIS TURBINE

- The inward flow reaction turbine having radial discharge at outlet is known as Francis Turbine. The water enters the runner of the turbine in the radial direction at outlet and leaves in the axial direction at the inlet of the runner. Thus the Francis turbine is a mixed flow type turbine.
- The work done by water on the runner per second will be

$$=\rho Q\left[V_{w_1}u_1\right]$$

The work done per second per unit weight of water striking/sec $=\frac{1}{g}\left[V_{w_1}u_1\right]$

Hydraulic efficiency $\eta_h=\frac{V_{w_1}u_1}{gH}$



Important relations for Francis turbines:

1. The ratio of width of the wheel to its diameter is given as $n = \frac{B_1}{D_1}$. The value of n varies from 0.10 to 0.40
2. The flow ratio is given as

   Flow ratio $= \frac{V_{f_1}}{\sqrt{2gH}}$ and varies from 0.15 to 0.30

3. The speed ratio $= \frac{u_1}{\sqrt{2gH}}$ varies from 0.6 to 0.9

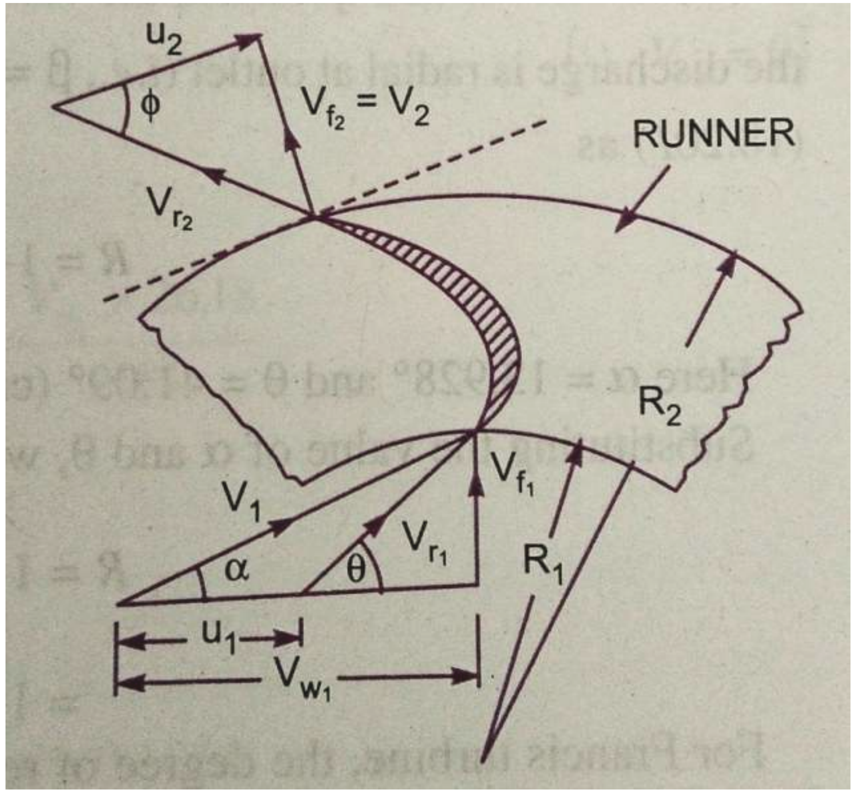
OUTWARD RADIAL FLOW REACTION TURBINE:

this case as the inlet of the runner is at the inner diameter of the runner, the tangential elocity at inlet will be less than that of an outlet. i.e.

$$u_1 < u_2 \qquad \text{As} \qquad D_1 < D_2$$

ll the working conditions flow through the nner blades without shock. As such eddy sses which are inevitable in Francis and ropeller turbines are almost completely iminated in a Kaplan turbine.

The discharge through the runner is btained as



$$Q = \frac{\pi}{4}\left(D_0^{\,2} - D_b^{\,2}\right) \times V_{f_1}$$

here D_0 = outer diameter of the runner

D_b = Diameter of the hub

V_{f_1} = Velocity of flow at inlet

IMPORTANT POINTS FOR KAPLAN TURBINE:

The peripheral velocity at inlet and outlet are equal.

$$u_1 = u_2 = \frac{\pi D_0 N}{60}$$

Where D_0 = Outer diameter of runner.

2. Velocity of flow at inlet and outlet are equal.

$$V_{f_1} = V_{f_2}$$

3. Area of flow at inlet = Area of flow at outlet

$$= \frac{\pi}{4}\left(D_0^{\,2} - D_b^{\,2}\right)$$

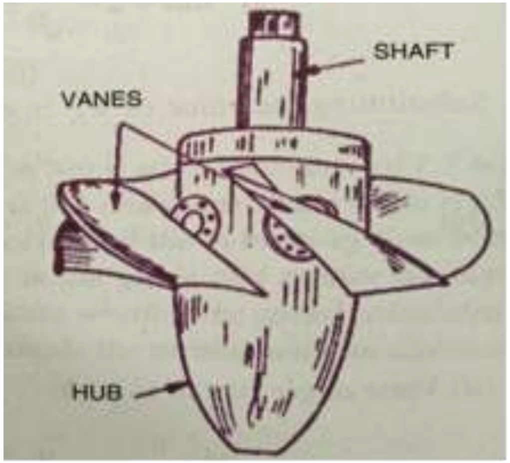
AXIAL FLOW REACTION TURBINE

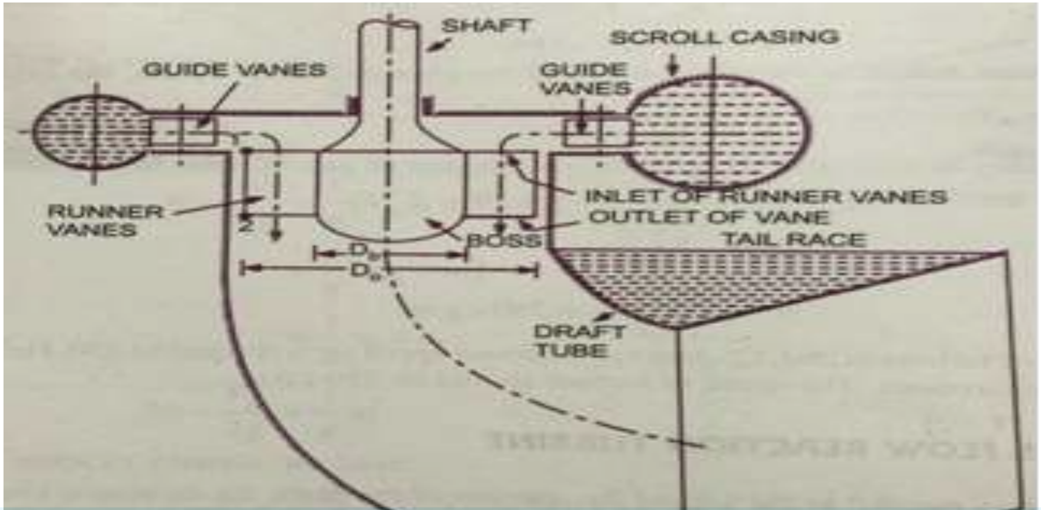
1. Propeller Turbine
2. Kaplan Turbine

KAPLAN TURBINE

- The main parts of the Kaplan turbine are:

1. Scroll casing
2. Guide vanes mechanism
3. Hub with vanes or runner of the turbine
4. Draft tube





WORKING PROPORTIONS OF KAPLAN TURBINE

The main dimensions of Kaplan Turbine runners are similar to Francis turbine runner. However the following are main deviations,

i. Choose an appropriate value of the ratio $n = \frac{d}{D}$, where d in hub or boss diameter and D is runner outside diameter. The value of n varies from 0.35 to 0.6

ii. The discharge Q flowing through the runner is given by

$$Q = \frac{\pi}{4}(D^2 - d^2)V_f = \frac{\pi}{4}(D^2 - d^2)\psi\sqrt{2gH}$$

The value of flow ratio ψ for a Kaplan turbine is 0.7

iii. The runner blades of the Kaplan turbine are twisted, the blade angle being greater at the outer tip than at the hub. This is because the peripheral velocity of the blades being directly proportional to radius. It will valy from section to section along the blade, and hence in order to have shock free entry and exit of water over the blades with angles varying from section to section will have to be designed.

A Francis turbine with an overall efficiency of 75% is required to produce 148.25kW power. It is working under a head of 7.62m. The peripheral velocity=0.26 and the radial velocity of flow at inlet is 0.96 . The wheel runs at 150rpm and the hydraulic losses in the turbine are 22% of the available energy. Assuming radial discharge determine

i) The guide blade angle **ii)** The wheel vane angle at inlet

iii) The diameter of the wheel at inlet, and **iv)** Width of the wheel at inlet

Ans

Given: Overall efficiency $\eta_0 = 75\% = 0.75$

Head H=7.62rpm

Power Produced S.P. = 148.25kW

Speed N= 150rpm

Hydraulic loses =22% of energy

Peripheral velocity $u_1 = 0.26\sqrt{2gh} = 0.26\sqrt{2 \times 9.81 \times 7.62} = 3.179 m/s$

Discharge at outlet = Radial

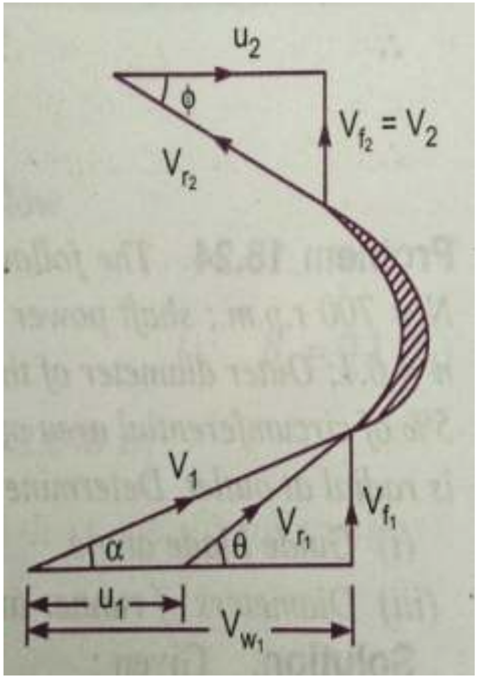
$V_{w_2} = 0 \qquad V_{f_2} = V_2$

The hydraulic efficiency $\eta_h = \frac{Total\ head\ at\ inlet - Hydraulic\ losses}{Heat\ at\ inlet}$

$$= \frac{H - 0.22H}{H} = 0.78$$

But $\eta_h = \frac{V_{w_1} u_1}{gH}$, $\frac{V_{w_1} u_1}{gH} = 0.78$

$$V_{w_1} = \frac{0.78 \times g \times H}{u_1} = \frac{0.78 \times 9.81 \times 7.62}{3.179} = \mathbf{18.34 m/s.}$$



i) The guide blade angle i.e. α From inlet velocity triangle

$$\tan\alpha = \frac{V_{f_1}}{V_{w_1}} = \frac{11.738}{18.34} = 0.64$$

$$\alpha = \tan^{-1}(0.64) = \mathbf{32.619^0} \quad or\ \mathbf{32^0 37'}$$

ii) The wheel angle at inlet (θ)

$$\tan\theta = \frac{V_{f_1}}{V_{w_1} - u_1} = \frac{11.738}{18.34 - 3.179} 0.774$$

$$\theta = \tan^{-1}(0.774) = \mathbf{37.74^0} \quad or\ \mathbf{37^0 44.4'}$$

iii) The diameter of wheel at inlet (D_1)

Using relation
$$u_1 = \frac{\pi D_1 N}{60}$$

$$D_1 = \frac{60 \times u_1}{\pi \times N} = \frac{60 \times 3.179}{\pi \times 150} = 0.4047m$$

iv) Width of the wheel at inlet (B_1)

$$\eta_0 = \frac{SP}{WP} = \frac{148.25}{WP}$$

But
$$WP = \frac{W \times H}{1000} = \frac{\rho \times g \times Q \times H}{1000} = \frac{1000 \times 9.81 \times Q \times 7.62}{1000}$$

$$\eta_0 = \frac{148.25 \times 1000}{1000 \times 9.81 \times Q \times 7.62}$$

$$Q = \frac{148.25 \times 1000}{1000 \times 9.81 \times 7.62 \times \eta_0} = \frac{148.25 \times 1000}{1000 \times 9.81 \times 7.62 \times 0.75} = 2.644m^3/s \quad (\because \ \eta_0 = 75\%)$$

Using equation
$$Q = \pi D_1 B_1 \times V_{f_1}$$

$$2.644 = \pi \times 0.4047 \times B_1 \times 11.738$$

$$B_1 = \frac{2.644}{\pi \times 0.4047 \times 11.738} = \mathbf{0.177m}$$

- A Kaplan turbine runner is to be designed to develop 7357.5kW shaft power. The net available head is 5.50m. Assume that the speed ratio is 2.09 and flow ratio is 0.68 and the overall efficiency is 60%. The diameter of boss is of the diameter of runner. Find the diameter of the runner, its speed and specific speed

Given: Shaft power P = 7357.5kW

Head H = 5.5m

Speed ratio $= \frac{u_1}{\sqrt{2gH}} = 2.09$

$$\therefore \quad u_1 = 2.09 \times \sqrt{2 \times 9.81 \times 5.5} = 21.71 m/s$$

Flow ratio $= \frac{V_{f_1}}{\sqrt{2gH}} = 0.68$

$$\therefore V_{f_1} = 0.68 \times \sqrt{2 \times 9.81 \times 5.5} = 7.064 m/s$$

Overall Efficiency $\eta_0 = 60\% = 0.60$

Diameter of boss $D_b = \frac{1}{3} \times D_0$

Using the relation $\eta_0 = \frac{Shaft\ power}{water\ power} = \frac{7357.5}{\frac{\rho g Q H}{1000}}$

$$0.60 = \frac{7357.5 \times 1000}{1000 \times 9.81 \times Q \times 5.5}$$

Discharge $$Q = \frac{7357.5 \times 1000}{1000 \times 9.81 \times 5.5 \times 0.60} = 227.27\ m^3/s$$

Using equation for discharge

$$Q = \frac{\pi}{4}\left[D_0{}^2 - D_b{}^2\right] \times V_{f_1}$$

$$227.27 = \frac{\pi}{4}\left[D_0{}^2 - \left(\frac{D_0}{3}\right)^2\right] \times V_{f_1}$$

$$227.27 = \frac{\pi}{4} \times \frac{8}{9} D_0{}^2 \times 7.064$$

$$D_0{}^2 = 227.27 \times \frac{4}{\pi} \times \frac{9}{8} \times \frac{1}{7.064}$$

$$D_0 = \mathbf{6.788}\boldsymbol{m}$$

$$D_b = \frac{1}{3} D_0 = \frac{6.788}{3} = \mathbf{2.262}\boldsymbol{m}$$

Using the relation $$u_2 = \frac{\pi D_0 N}{60} \qquad (\because\ u_1 = u_2)$$

$$N = \frac{60 \times u_1}{\pi D_0} = \frac{60 \times 21.71}{\pi \times 6.788} = \mathbf{61.08\ rpm}$$

The specific speed $$N_s = \frac{N\sqrt{P}}{\ldots} = \frac{61.08 \times \sqrt{7357.5}}{\ldots} = \mathbf{622\ rpm}$$

TOPICS TO BE COVERED

- Draft Tube
- Functions & efficiency

LECTURE 7

Hydraulic Design- Draft Tube theory:

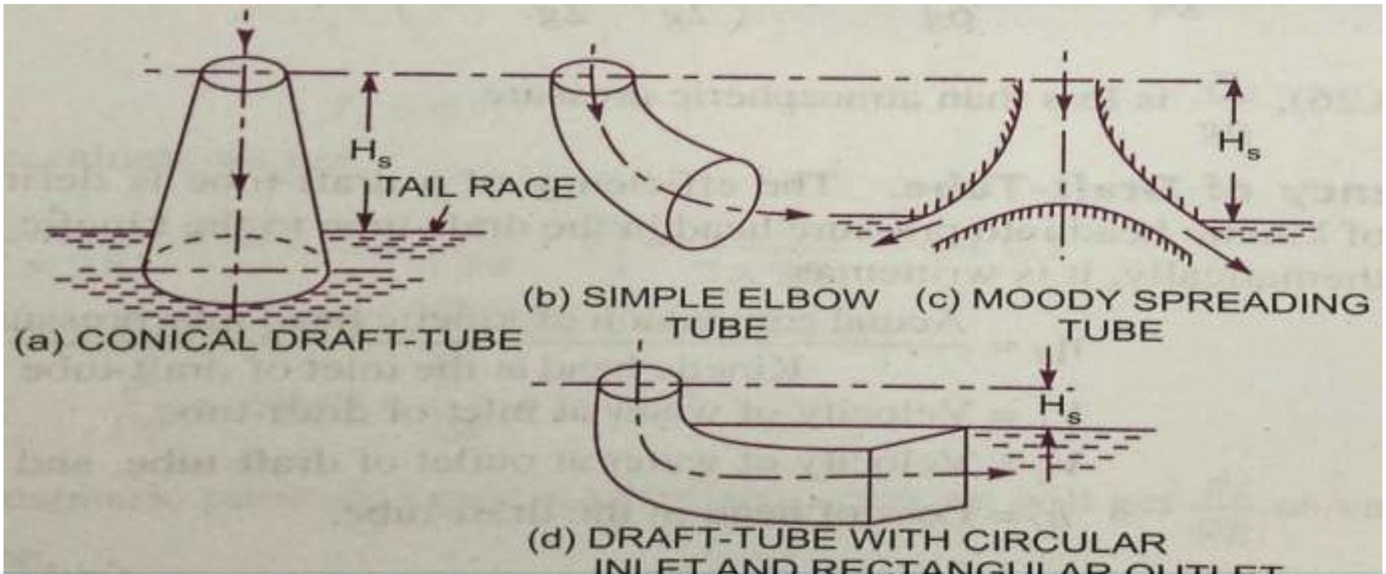
DRAFT TUBE:

- The draft tube is a pipe of gradually increasing area, which connects the outlet of the runner to the tail race.
- It is used for discharging water from the exit of the turbine to the tail race. This pipe of gradually increasing area is called a draft tube.
- One end of the draft tube is connected to the outlet of the runner and the other end is submerged below the level of water in the tail race.

TYPES OF DRAFT TUBE

- **Types of Draft Tube:**

1. Conical Draft Tube
2. Simple Elbow Tubes
3. Moody Spreading tubes
4. Elbow Draft Tubes with Circular inlet and rectangular outlet



DRAFT TUBE THEORY

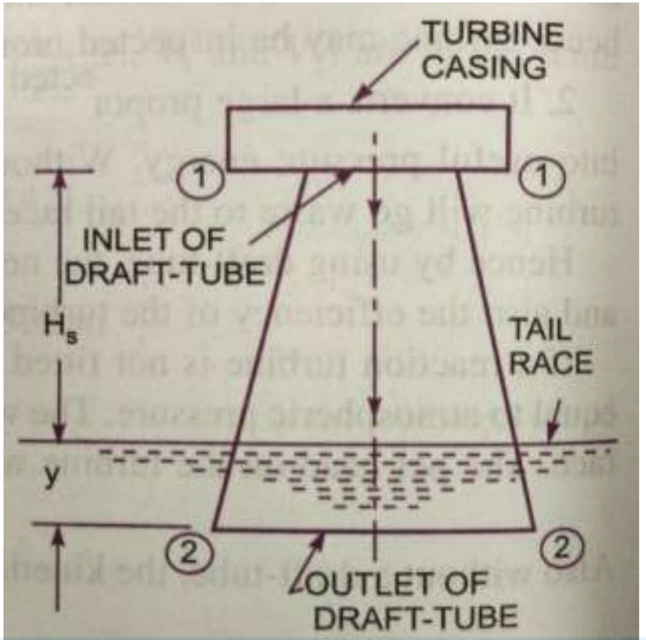
Consider a conical draft tube

H_s = Vertical height of draft tube above tail race

Y= Distance of bottom of draft tube from tail race.

Applying Bernoulli's equation to inlet section 1-1 and outlet section 2-2of the draft tube and taking section 2-2 a datum, we get

$$\frac{p_1}{\rho g} + \frac{V_1{}^2}{2g} + (H_s + y) = \frac{p_2}{\rho g} + \frac{V_2{}^2}{2g} + 0 + h_f$$



Where h_f = loss of energy between section 1-1 and 2-2.

But $\frac{p_2}{\rho g}$ = Atmospheric Pressure + y = $\frac{p_a}{\rho g} + y$

Substituting this value of $\frac{p_2}{\rho g}$ in equation (1) we get

$$\frac{p_1}{\rho g} + \frac{V_1{}^2}{2g} + (H_s + y) = \frac{p_a}{\rho g} + y + \frac{V_2{}^2}{2g} + h_f$$

$$\frac{p_1}{\rho g} + \frac{V_1{}^2}{2g} + H_s = \frac{p_a}{\rho g} + \frac{V_2{}^2}{2g} + h_f$$

$$\frac{p_1}{\rho g} = \frac{p_a}{\rho g} + \frac{V_2{}^2}{2g} + h_f - \frac{V_1{}^2}{2g} - H_s$$

$$\frac{p_1}{\rho g} = \frac{p_a}{\rho g} - H_s - \left[\frac{V_1{}^2}{2g} - \frac{V_2{}^2}{2g} - h_f\right] \quad \text{\_\_\_\_\_\_\_\_(2)}$$

Efficiency of Draft Tube: the efficiency of a draft tube is defined as the ratio of actual conversion of kinetic head in to pressure in the draft tube to the kinetic head at the inlet of the draft tube.

$$\eta_d = \frac{Actual\ conversion\ of\ Kinetic\ head\ in\ to\ Pressure\ head}{Kintic\ head\ at\ the\ inlet\ of\ draft\ tube}$$

Let V_1 = Velocity of water at inlet of draft tube

V_2 = Velocity of water at outlet of draft tube

h_f = Loss of head in the draft tube

Theoretical conversion of Kinetic head into Pressure head in

Draft tube $= \left[\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right]$

Actual conversion of Kinetic head into pressure head $= \left[\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right] - h_f$

Now Efficiency of draft tube $\eta_d = \frac{\left[\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right] - h_f}{\frac{V_1^2}{2g}}$