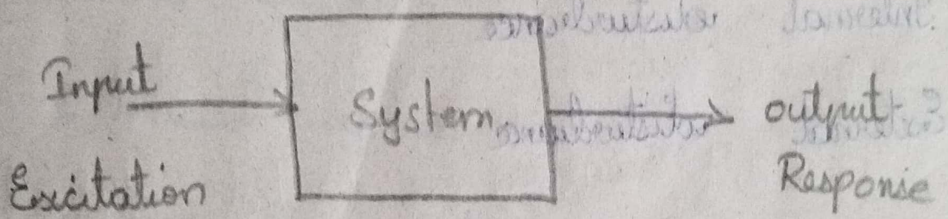
Linear Control System

UNIT - I

Control System Concepts :

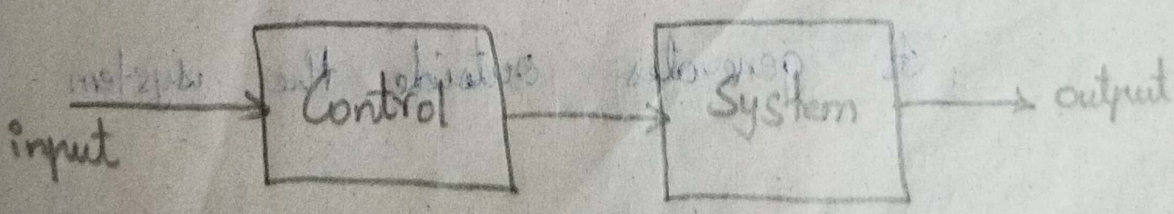
System :

The group of components are connected together to perform a specific function is called as system.



Control System :

The o/p is Controlled by i/p is called control system.



Regulate , Command , Manage = Control

Plant | Process | System:

The process under control is known as plant or system.

Disturbances:

It is a signal which effect the Value of output.

It is of two types:

1. Internal disturbance
2. External disturbance

1. Internal Disturbance:

It generates with in the system itself.

2. External Disturbance:

It generates outside the system

Classification of control system:

1. Based on components used:

- Mechanical system
- Electrical system
- Pneumatic system
- hydraulic system.

2. Based on differential equation that describes system:

- Linear system
- Non-Linear system.

- Linear system obeys superposition principle

In this we have two terms. Homogenity, Additivity

$$x(t) \longrightarrow y(t)$$

$$Ax(t) \longrightarrow AY(t). \longrightarrow \text{Superposition}$$

Homogenity:

$$x_1(t) \longrightarrow y_1(t)$$

$$x_2(t) \longrightarrow y_2(t)$$

$$x_1(t) + y_1(t) \longrightarrow x_2(t) + y_2(t)$$

Time Invarient :

1. Continuous

2. Discrete

Time Varient :

Corresponding change in input

3. Based on equation describing the system behaviour :

- Lumped system → ordinary differential eq
- Distributed system → Paritial differential eq.

4. Based on Number of i/p & o/p

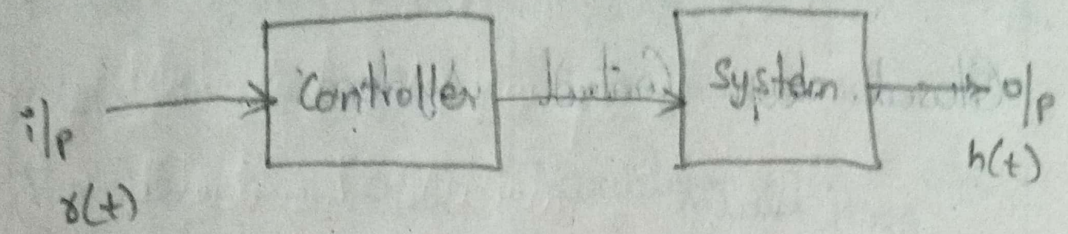
- SISO System → Single i/p Single o/p
- MIMO System → Multiple i/p & Multiple o/p

5. Based on feedback :

*** 2M

- Open loop Control System
- closed loop Control System

Open loop Control System:



- Changes in o/p which can't be corrected by i/p.
- Feedback is not present.

- Any system which does not correct the Variations in o/p is Called open loop Control System.
- There is no feedback.
- The Control action is independent of o/p.

Advantages:

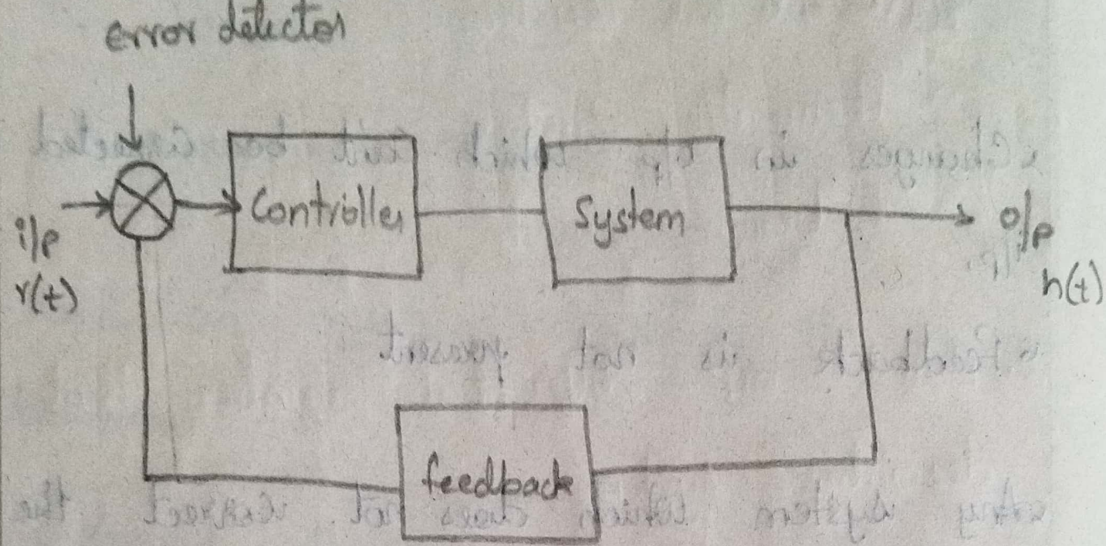
1. Simple
2. Easier to construct
3. Stable

Disadvantage:

1. Inaccurate and unreliable

2. The changes in o/p Cannot be corrected automatically.

Closed loop Control system:



Ex: Sensors

Advantages:

1. Accurate o/p and Reliable.

2. The Changes in o/p Can be Corrected automatically

Disadvantage:

1. Complex

2.

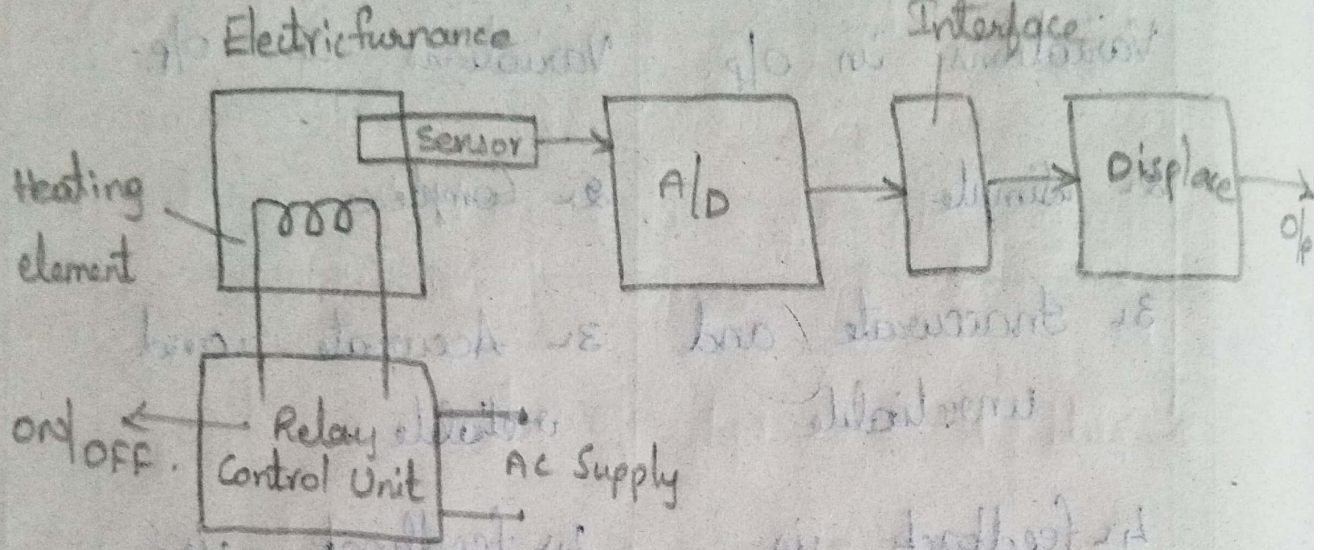
Open loop Control System	Closed loop Control System
1. Any System which does'nt Correct the Variations in o/p.	1. Any System which Can be correct the Variations in o/p.
2. Simple	2. Complex
3. Inaccurate and unreliable	3. Accurate and reliable.
4. feedback is not present	4. feedback is present
5. The changes in o/p Can't be Corrected automatically	5. The changes in o/p Can be Corrected automatically
6. Control action is independent on o/p	6. Control action is dependent on i/p

Examples of Control System:

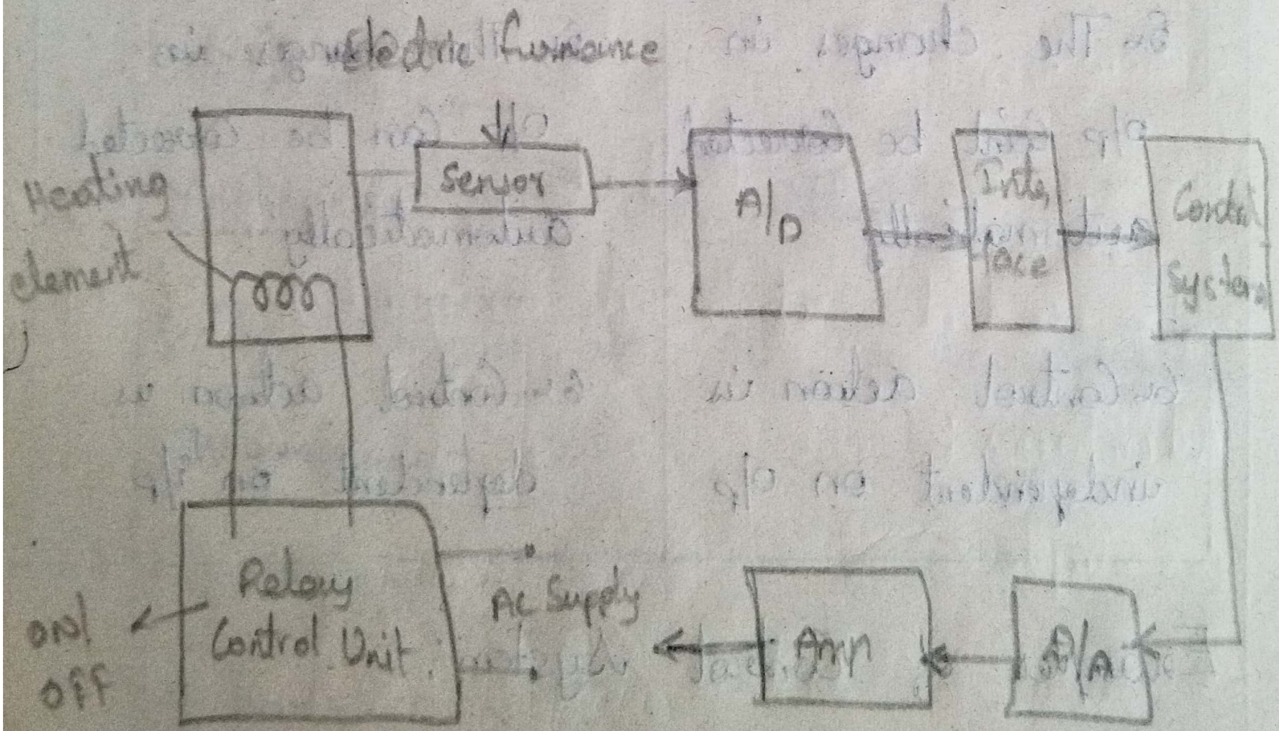
1. Temperature Control System.
2. Numerical control system.
3. Traffic Control System

4. Position Control System Using servo motor.

Open loop Control system:



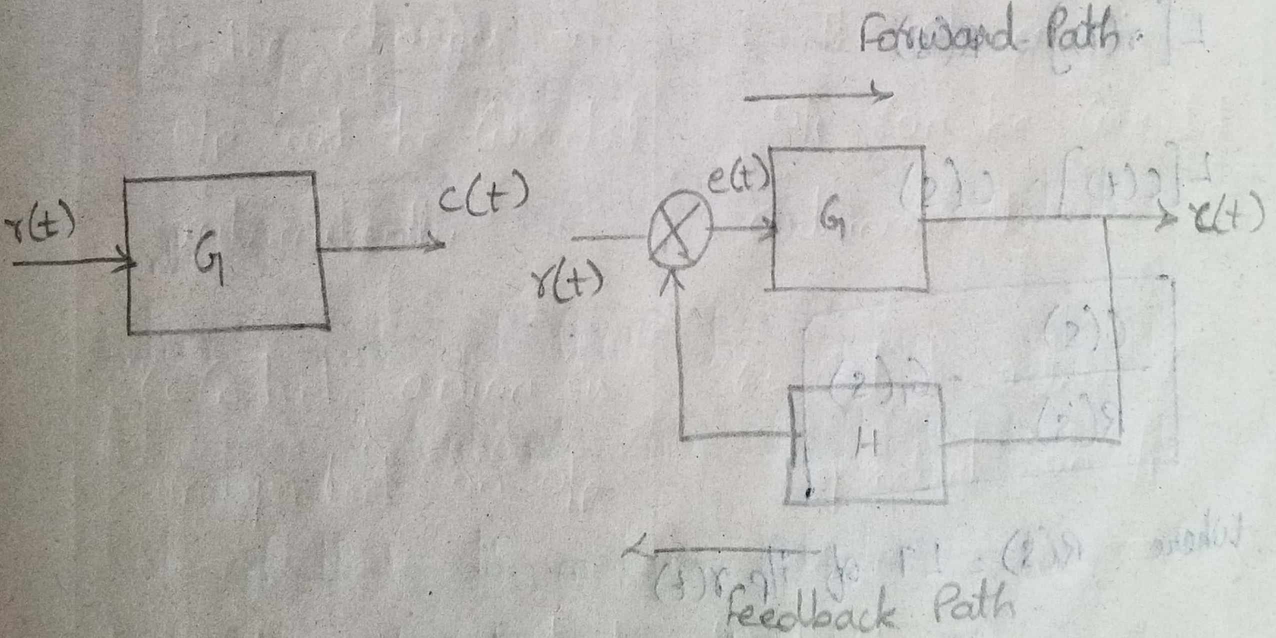
Closed loop Control System:



Feedback Characteristics : And Effect of

Feedback :

* Either o/p or Some part of o/p is fed back to i/p Side and utilised as system i/p is Called feedback.
* feedback improves the System performance
* It effects the system performance Characteristics such as gain, stability, sensitivity and disturbace/noise.



TF for -ve feedback

$$\boxed{T = \frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s) \cdot H(s)}}$$

Effect of feedback :

1. Overall gain :

Consider -ve feedback Transfer function

$$T = \frac{G}{1 + G \cdot H}$$

$$\downarrow\uparrow \text{Overall gain} \propto \frac{1}{1+GH} \uparrow\downarrow$$

2. Sensitivity :

For any system it will be less sensitive to effect of feedback on sensitivity.

For good control system it will be less sensitive to internal or external disturbances

$$\text{Sensitivity } (s) = \frac{\%\ \text{change in } T}{\%\ \text{Change in } G}$$

where $T \rightarrow$ Variable

$G \rightarrow$ Parameter

$$S = \frac{\frac{\partial T}{T} \times 100}{\frac{\partial G}{G} \times 100}$$

$$\boxed{S = \frac{\partial T}{\partial G} \times \frac{G}{T}} \longrightarrow (1)$$

It is denoted by S_G^T

$$T = \frac{G}{1+GH}$$

differentiate T.F wrt G

$$\frac{\partial T}{\partial G} = \frac{(1+GH)\frac{d}{dG}G - G\frac{d}{dG}(1+GH)}{(1+GH)^2}$$

$$\frac{\partial T}{\partial G} = \frac{1+\cancel{GH}-\cancel{GH}}{(1+GH)^2}$$

$$\frac{\partial T}{\partial G} = \frac{1}{(1+GH)^2}$$

Eq (1) $\Rightarrow S = \frac{\partial T}{\partial G} \times \frac{G}{T}$

$$S_G^T = \frac{1}{(1+GH)^{\cancel{2}}} \times (1+GH) \qquad T = \frac{G}{1+GH}$$

$$S_G^T = \frac{1}{1+GH} \qquad \therefore \frac{G}{T} = 1+GH$$

If there is any change in H there will be a change in sensitivity.

$$S \propto \frac{1}{1+GH}$$

Stability :

Stability refers to whether the system will follow the input command or not.

consider transfer function

$$T = \frac{G}{1+GH}$$

If $\quad H = -\frac{1}{G}$

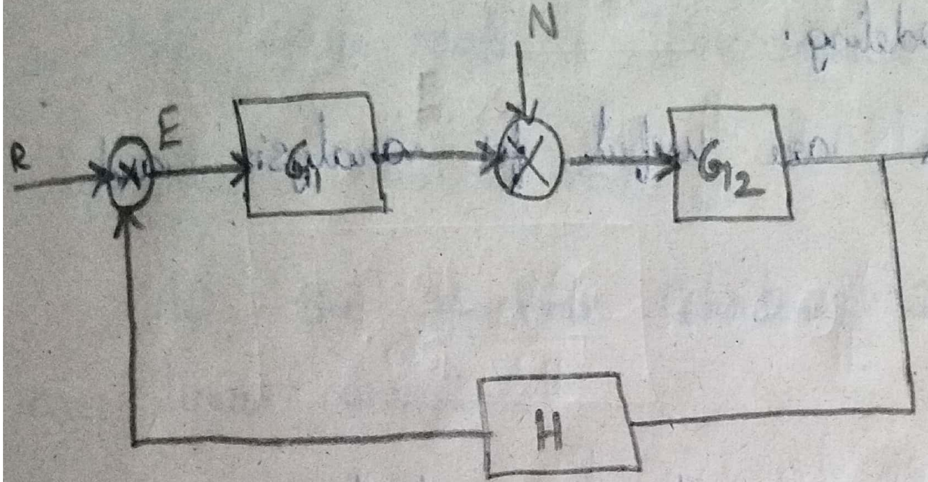
$$T = \frac{G}{1+G\left(\frac{1}{G}\right)}$$

$$T = \infty$$

The system is unstable

for this transfer function the system will be unstable.

Effect of feedback on Noise or disturbances :



* It is unwanted signal is called noise.

* If H=0 there is no feedback then output is

$$\boxed{C = G_2 N}$$

* If feedback element is added, then the noise can be reduce by a factor of

$1 + G_1 G_2 H$

$$\boxed{C = \frac{G_2 N}{1 + G_1 G_2 H}}$$

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Mathematical modeling of control systems:

The Control system can be represented with a set of mathematical equations is known as mathematical modeling.

This models are useful for analysis and design.

Analysis:

Input system are known output is unknown

Design:

Input output are known then system is unknown

* Dynamic relationship between input and output gives this model.

Types of Control Systems:

1> Mechanical Systems - Newton's law

2> Electrical Systems - Kirchoff's law

KCL KVL

Types of Mathematical model :

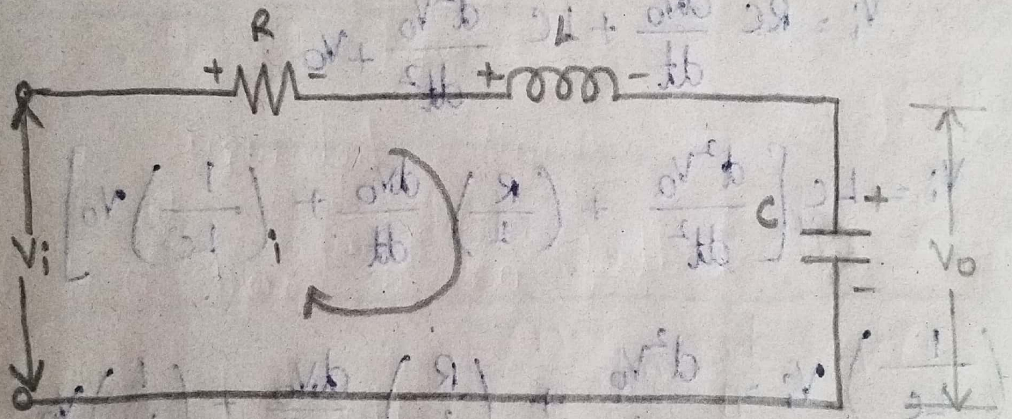
1. Differential Equation on model - Time domain

2. Transfer function model - Laplace domain

3. State space model - Time domain.

Differential Equation model :

All the systems considered in this model are linear invarient.



Apply KVL

$$V_i - iR - L\frac{di}{dt} - V_o = 0$$

$$V_i = iR + L\frac{di}{dt} + V_o \longrightarrow (1)$$

Current through the capacitor C is

$$i = C\frac{dV_o}{dt}$$

Current through the capacitor is

$$i = C\frac{dV_0}{dt}$$

Subsitute 'i' in eq ①

$$V_i = iR + L\frac{di}{dt} + V_0$$

$$V_i = C\frac{dV_0}{dt}R + LC\frac{d^2V_0}{dt^2} + V_0$$

$$V_i = RC\frac{dV_0}{dt} + LC\frac{d^2V_0}{dt^2} + V_0$$

$$V_i = LC\left[\frac{d^2V_0}{dt^2} + \left(\frac{R}{L}\right)\frac{dV_0}{dt} + \left(\frac{1}{LC}\right)V_0\right]$$

$$\boxed{\left(\frac{1}{LC}\right)V_i = \frac{d^2V_0}{dt^2} + \left(\frac{R}{L}\right)\frac{dV_0}{dt} + \left(\frac{1}{LC}\right)V_0} \rightarrow ②$$

Transfer function Model :

Solving differential equation is complex when the order increases to overcome this transfer function model is used

Consider eq ②

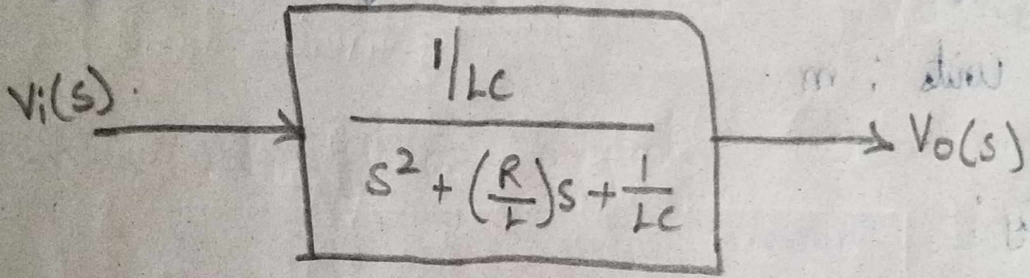
$$\left(\frac{1}{LC}\right)V_i = \frac{d^2V_0}{dt^2} + \left(\frac{R}{L}\right)\frac{dV_0}{dt} + \left(\frac{1}{LC}\right)V_0$$

Apply laplace transform on both sides

$$\frac{1}{LC} V_i(s) = s^2 V_o(s) + \left(\frac{R}{L}\right) s V_o(s) + \left(\frac{1}{LC}\right) V_o(s)$$

$$= V_o(s)\left[s^2 + \left(\frac{R}{L}\right) s + \frac{1}{LC}\right]$$

$$\therefore \frac{V_o(s)}{V_i(s)} = \frac{\frac{1}{LC}}{s^2 + \left(\frac{R}{L}\right) s + \frac{1}{LC}}$$



10M

Mathematical Modeling of Mechanical System:

Based on motion the mechanical system are classified into two types:

1. Translational Mechanical System
2. Rotational Mechanical System

Translational Mechanical System:

If the motion is in straight line then it is called translational Mechanical System.

Newton's second law:

It states that applied force is equal to Sum of the opposing forces.

Variables / parameters of translational system:

1. Displacement:

Change in position of object wrt time

$x(t)$. units : m

2. Velocity:

Rate of change of displacement wrt time

$$V = \frac{dx}{dt} \; m/s$$

3. Acceleration:

Rate of change of velocity 'a'

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

4. Force:

Push or pull of an object. It is denoted by 'F'.

Units = Joules/sec or N

Components or Elements of tranistational mechanical system:

There are three types

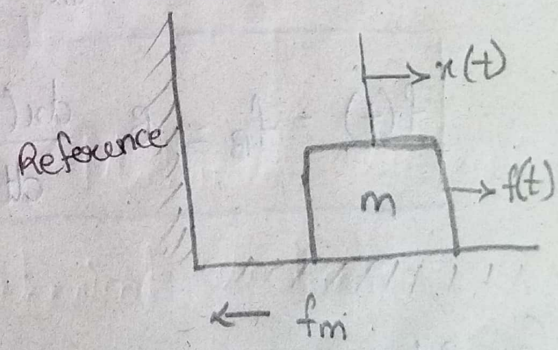
1. Mass (M)

2. Damper (B)

3. Spring (K)

1. Mass (m): Store KE

It is an inertia element it is represented by M.



$f_m \propto a$

$f_m = Ma$

Where M is Constant

$f(t) = f_m = Ma$.

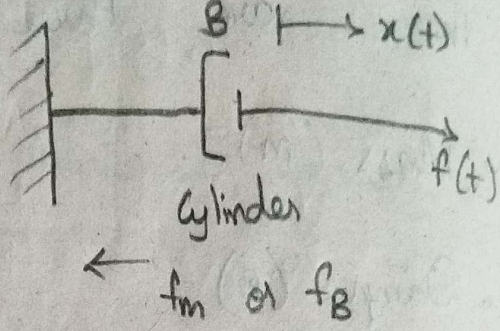
$f(t) = f(m)$ or $f_m = \frac{d^2x}{dt^2}$

2. Damper (B):

It is also called as dashpot.

Case 1: friction element

$f_B \propto V$



$$f_B = B\frac{dx(t)}{dt}$$

(or)

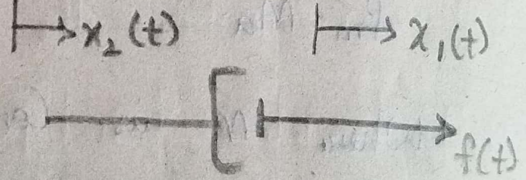
$$f_B = BV.$$

Where, B = damping Co-efficient.

By newton's second law.

$$\boxed{f(t) = f_B = B\frac{dx(t)}{dt}.}$$

Case 2:



$$f_B = B\frac{d}{dt}\left[x_1(t) - x_2(t)\right]$$

$$f_B = B\left[\frac{dx_1(t)}{dt} - \frac{dx_2(t)}{dt}\right]$$

$$f_B = B\left[V_1 - V_2\right]$$

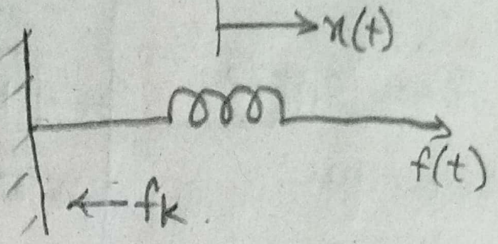
3. Spring (k) : Store PE.

It is the elastic element.

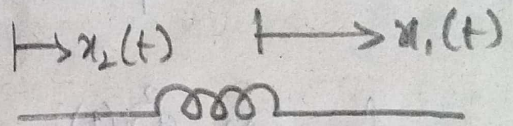
Case 1:

$f_k \propto x(t)$

$$\boxed{f_k = kx(t)}$$



Case 2: Two ends are free



$$f_k = k\left[x_1(t) - x_2(t)\right]$$

List of equations:

Mass = $f_m = M \dfrac{d^2x(t)}{dt^2}$

Damper = i. $f_B = B \dfrac{dx(t)}{dt}$

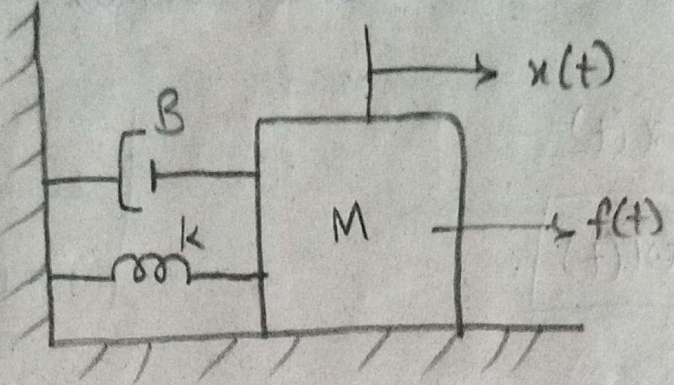
ii. $f_B = B \dfrac{d}{dt}\left[x_1(t) - x_2(t)\right]$

Spring = i. $f_k = kx(t)$

ii. $f_k = k.\left[x_1(t) - x_2(t)\right]$

Example :

three opposing forces are acting o



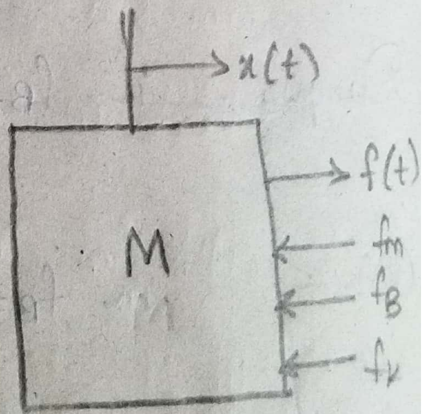
Mass Can be represented as nodes :

$$f_m = M \frac{d^2x(t)}{dt}$$

$$f_B = B . \frac{dx(t)}{dt} .$$

$$f_k = k\, x(t)$$

Free body diagram



input $= f(t)$

output $= x(t)$

By Newtons Second law

$$f(t) = f_m + f_B + f_k$$

$$f(t) = m \frac{d^2x(t)}{dt} + B \frac{dx(t)}{dt} + k . x(t)$$

Apply laplace transform on Bothsides

$\cancel{f(t)} =$

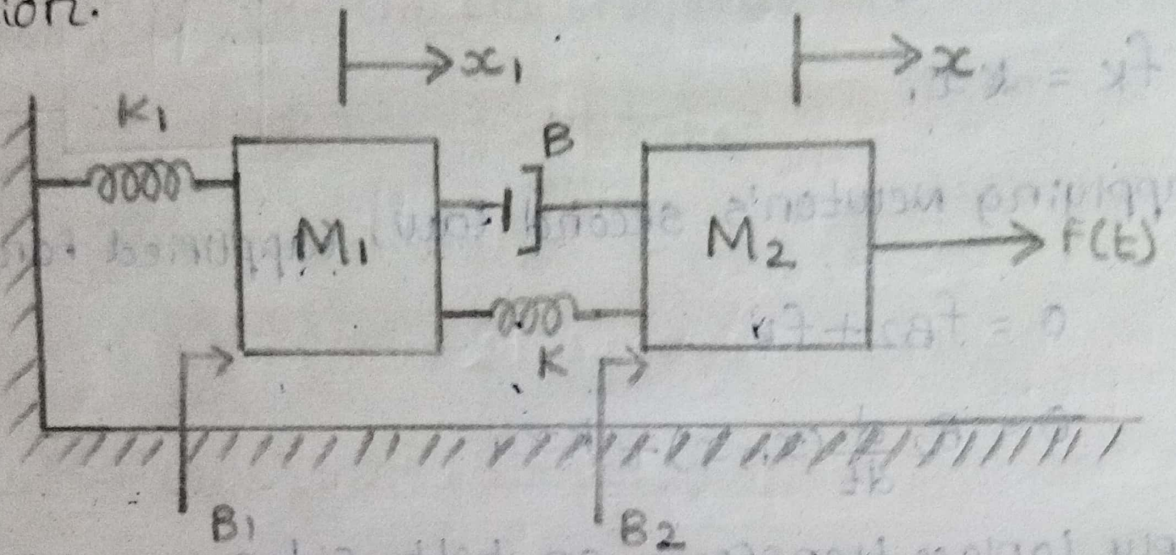
$$F(s) = Ms^2 X(s) + B X(s) + k X(s)$$

$$F(s) = X(s)\left[Ms^2 + Bs + k\right]$$

Transfer function:

$$\frac{X(s)}{F(s)} = \frac{1}{Ms^2 + Bs + k}$$

3. Write the differential equations Governing the mechanical systems shown in figure and determine the transfer function.



A. Input = f(t)

output = displacement (x)

Mass M_1 & M_2 acts as nodes

Consider Mass M_1,

Opposing forces acting on Mass M_1,

1. $f_{m_1} = M_1 \frac{d^2x_1}{dt^2}$

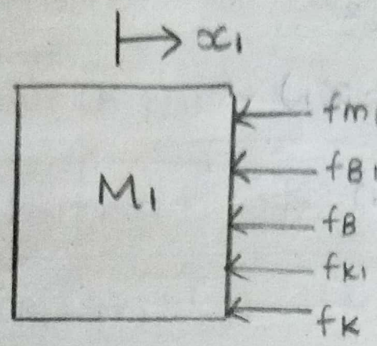
2. $f_{B_1} = B_1 \frac{dx_1}{dt}$

3. $f_{K_1} = K_1 x_1$

4. $f_B = B \frac{d}{dt}(x_1 - x)$

5. $f_K = K(x_1 - x)$.

Free body diagram for mass M_1 :-



By Newton's second Law,

Applied force = sum of opposing force

$$0 = f_{m1} + f_{B1} + f_B + f_{K1} + f_K$$

$$f_{m1} + f_{B1} + f_B + f_{K1} + f_K = 0$$

$$M_1 \frac{d^2 x_1}{dt^2} + B_1 \frac{dx_1}{dt} + B \frac{d}{dt}(x_1 - x) + K_1 x_1 + K(x_1 - x) = 0$$

Apply Laplace transform on both sides,

$$M_1 s^2 X_1(s) + B_1 s X_1(s) + Bs[X_1(s) - X(s)] + K_1 X_1(s) + K[X_1(s) - X(s)] = 0$$

$$M_1 s^2 X_1(s) + B_1 s X_1(s) + Bs X_1(s) - Bs X(s) + K_1 X_1(s) + K X_1(s) - K X(s) = 0$$

$$X_1(s)[M_1 s^2 + (B_1 + B)s + K_1 + K] - X(s)[Bs + K] = 0$$

$$X_1(s)[M_1 s^2 + (B_1 + B)s + K_1 + K] = X(s)[Bs + K]$$

$$X_1(s) = \frac{X(s)[Bs + K]}{[M_1 s^2 + (B_1 + B)s + K_1 + K]} \longrightarrow (1)$$

Now,

consider mass M_2, Now opposing forces acting on mass M_2

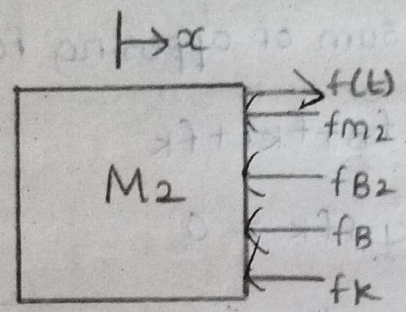
1. $f_{m2} = M_2 \frac{d^2x}{dt^2}$

2. $f_B = B \frac{d}{dt}(x - x_1)$

3. $f_K = K(x - x_1)$

4. $f_{B2} = B_2 \frac{dx}{dt}$

Free body diagram for Mass (M_2): -



By Newton's Second law,

Applied force = sum of opposing forces

$$f(t) = f_{m2} + f_{B2} + f_B + f_K$$

$$f(t) = M_2 \frac{d^2x}{dt^2} + B_2 \frac{dx}{dt} + B\frac{d}{dt}(x - x_1) + K(x - x_1)$$

Apply Laplace transform on both sides,

$$F(S) = M_2 S^2 X(S) + B_2 S X(S) + BS[X(S) - X_1(S)] + K[X(S) - X_1(S)]$$

$$F(S) = M_2 S^2 X(S) + B_2 S X(S) + BSX(S) - BSX_1(S) + K(X(S) - KX_1(S)$$

$$F(S) = X(S)[M_2 S^2 + (B_2 + B)S + K] - X_1(S)[BS + K] \rightarrow (2)$$

Sub eq (1) in (2)

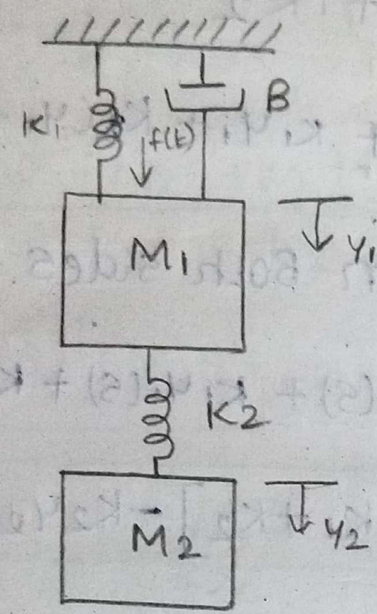
$$F(S) = X(S)[M_2 S^2 + (B_2 + B)S + K] - \frac{X(S)[BS + K]}{[M_1 S^2 + (B_1 + B)S + K_1 + K]} \times [BS + K]$$

$$F(S) = \frac{X(S)[(M_1 S^2 + (B_1 + B)S + K_1 + K][M_2 S^2 + (B_2 + B)S + K] - X(S)[BS + K]^2}{M_1 S^2 + (B_1 + B)S + K_1 + K}$$

$$F(S) = \frac{X(S)\left[\left[M_1 S^2 + (B_1 + B)S + K_1 + K\right]\left[M_2 S^2 + (B_2 + B)S + K\right] - \left[BS + K\right]^2\right]}{M_1 S^2 + (B_1 + B)S + K_1 + K}$$

$$\therefore \boxed{\frac{X(S)}{F(S)} = \frac{M_1 S^2 + (B_1 + B)S + K_1 + K}{\left[\left[M_1 S^2 + (B_1 + B)S + K_1 + K\right]\left[M_2 S^2 + (B_2 + B)S + K\right] - \left[BS + K\right]^2\right]}}$$

4. Determine the transfer function $\frac{Y_2(S)}{F(S)}$ of the system shown in Figure



A. Let,

Input = $f(t)$

output = displacement (y_2)

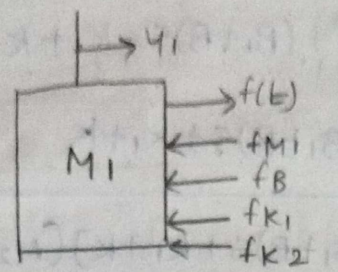
Now consider,

Mass M_1

opposing forces acting on Mass M_1

1. $f_{m_1} = M_1 \frac{d^2 y_1}{dt^2}$
2. $f_B = B \frac{dy_1}{dt}$
3. $f_{K_1} = K_1 y_1$
4. $f_{K_2} = K_2 (y_1 - y_2)$

Free body diagram for Mass M_1,



By Newton's second law,

Applied force = sum of opposing force

$$f(t) = f_{m1} + f_B + f_{K1} + f_{K2}$$

$$f(t) = M_1 \frac{d^2y_1}{dt^2} + B\frac{dy_1}{dt} + K_1 y_1 + K_2(y_1 - y_2)$$

Apply laplace transform on Both sides.

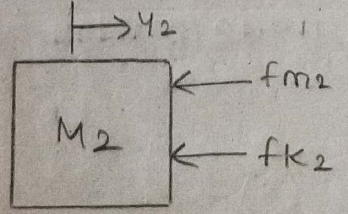
$$F(S) = M_1 S^2 Y_1(S) + BS\,Y_1(S) + K_1 Y_1(S) + K_2 Y_1(S) - K_2 Y_2(S)$$

$$\boxed{F(S) = Y_1(S)\left[M_1 S^2 + BS + K_1 + K_2\right] - K_2 Y_2(S)} \rightarrow ①$$

Now consider Mass M_2 opposing forces acting on mass M_2 are

1. $f_{m2} = M_2 \frac{d^2y_2}{dt^2}$
2. $f_{K2} = K_2(y_2 - y_1)$

Free body diagram for mass M_2,



By Newton's second law,

$$f_{m2} + f_{K2} = 0$$

$$M_2 \frac{d^2y_2}{dt^2} + K_2(y_2 - y_1) = 0$$

Apply laplace transform,

$$M_2 s^2 Y_2(s) + K_2 Y_2(s) - K_2 Y_1(s) = 0$$

$$Y_2(s)\left[M_2 s^2 + K_2\right] = K_2 Y_1(s)$$

$$\boxed{Y_1(s) = \frac{Y_2(s)\left[M_2 s^2 + K_2\right]}{K_2}} \rightarrow (2)$$

sub eq (2) in (1)

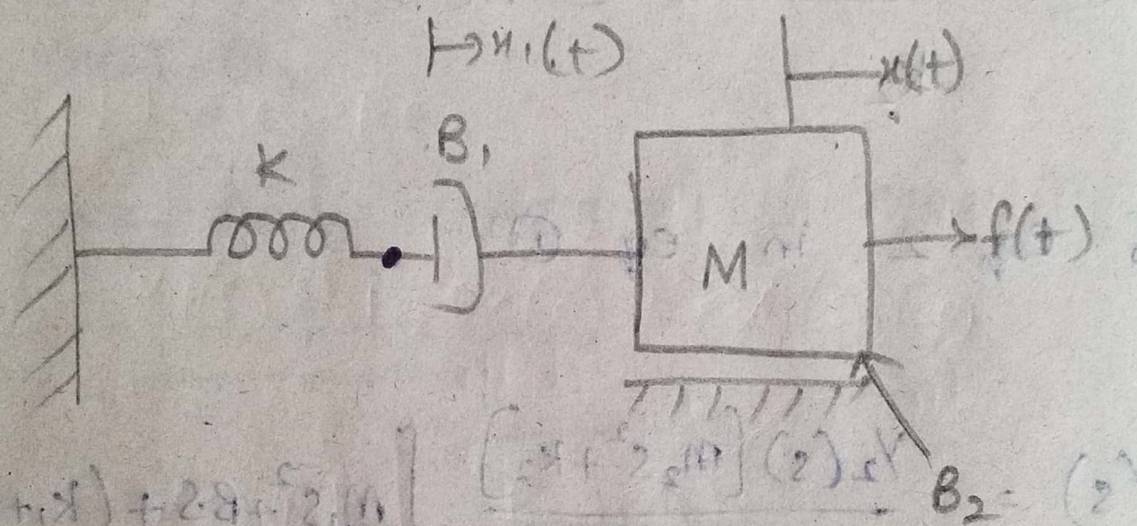
$$F(s) = \frac{Y_2(s)\left[M_2 s^2 + K_2\right]}{K_2}\left[M_1 s^2 + Bs + K_1 + K_2\right] - K_2 Y_2(s)$$

$$F(s) = \frac{Y_2(s)\left[M_2 s^2 + K_2\right]\left[M_1 s^2 + Bs + K_1 + K_2\right] - K_2^2 Y_2(s)}{K_2}$$

$$F(s) = \frac{Y_2(s)\left[\left[M_2 s^2 + K_2\right]\left[M_1 s^2 + Bs + K_1 + K_2\right] - K_2^2\right]}{K_2}$$

$$\therefore \boxed{\frac{Y_2(s)}{F(s)} = \frac{K_2}{\left[M_2 s^2 + K_2\right]\left[M_1 s^2 + Bs + K_1 - K_2\right] - K_2^2}}$$

3. Write the equations of motion in S domain for the System shown in figure Determine the transfer function of the system



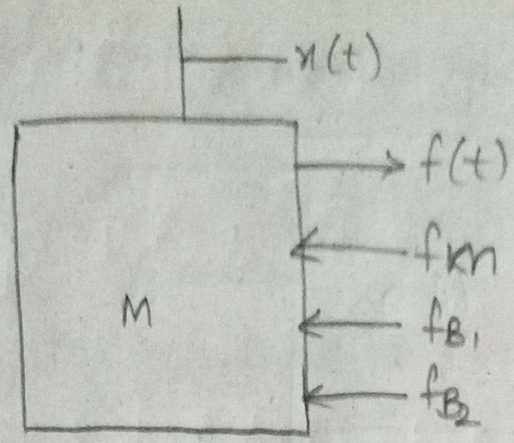
Sol:

$f(t)$ = input applied force

$x(t)$ = Output displacement

$\frac{X(s)}{F(s)}$ = Transfer function

free body diagram for Mass M



New ton's Second law.

$$f(t) = f_m + f_{B_1} + f_{B_2}$$

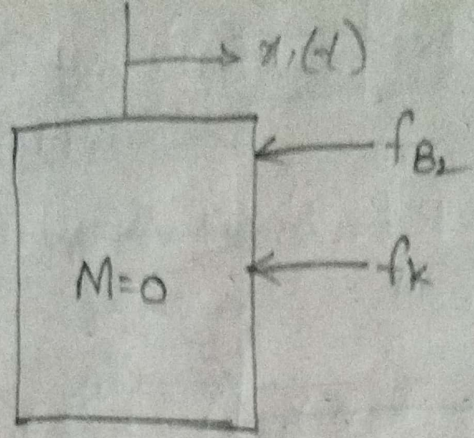
$$f(t) = \frac{m d^2 x(t)}{dt^2} + B_1 \frac{dx(t)}{dt} + B_2 \frac{d}{dt}[x(t) - x_1(t)]$$

Apply LT on Both Sides

$$= m s^2 X(s) + B_1 s X(s) + B_2 s X(x(t) - x_1(t))s$$

$$F(s) = ms^2 X(s) + B_1 s X(s) + B_2 s X(s) - B_2 s X_1(s).$$

$$\boxed{F(s) = X(s)[ms^2 + (B_1 + B_2)s] - B_2 s X_1(s)} \rightarrow (1)$$



Newton's second law,

$$f_{B_2} + f_k = 0$$

$$B_2 \frac{d}{dt}\left[x_1(t) - x(t)\right] + k\,x_1(t) = 0 .$$

Apply LT on BS

$$B_2 s\, X_1(s) - B_2 s X(s) + k X_1(s) = 0$$

$$X_1(s)\left[B_2 S \cancel{X_1(s)} + k\right] = B_2 S X(s)$$

$$\boxed{X_1(s) = \frac{B_2 S\, X(s)}{[B_2 S + k]}} \rightarrow (2)$$

Sub eq (2) in eq (1)

$$F(s) = X(s)\left[ms^2 + [B_1 + B_2]s\right] - B_2 S \frac{B_2 S X(s)}{[B_2 S + k]}$$

$$F(s) = X(s)\left[ms^2 + s(B_1+B_2)\right] - \frac{(B_2 S)^2 X(s)}{[B_2 S + K]}$$

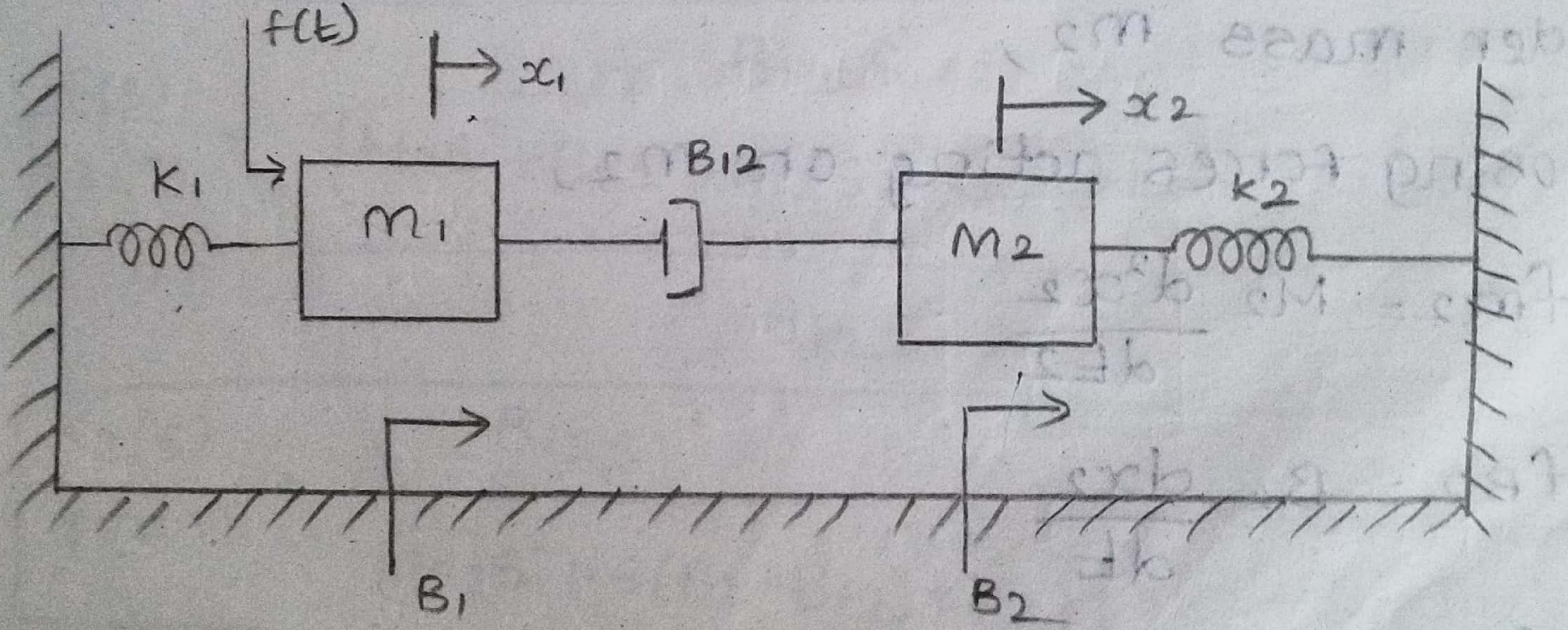
~~The~~ ~~T.F~~.

$$F(s) = \frac{[B_2 S + K]\, X(s)\left[ms^2 + (B_1+B_2)\right] - (B_2 S)^2 X(s)}{B_2 S + K}$$

The ~~T.K~~ T.F

$$\boxed{\frac{X(s)}{F(s)} = \frac{B_2 S + K}{\left[[B_2 S + K]\left[ms^2 + (B_1+B_2)\right] - \left[B_2 (S)\right]^2\right]}}$$

Determine the transfer function $\frac{X_1(S)}{F(S)}$ and $\frac{X_2(S)}{F(S)}$ for the system shown in figure :-



Here i/p is F(t) and o/p is displacement x_1

mass m_1 & m_2 acts as nodes,

consider mass m_1,

opposing forces acting on mass m_1

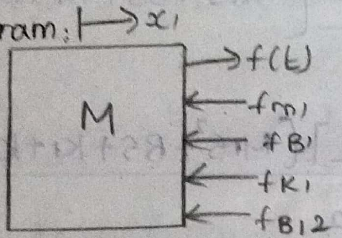
$$f_{m1} = M_1 \frac{d^2x_1}{dt^2}$$

$$f_{B1} = B_1 \frac{dx_1}{dt}$$

$$f_{K1} = K_1 x_1$$

$$f_{B12} = B_{12}\frac{d}{dt}(x_1 - x_2)$$

Free body diagram:



By Newton's second law,

$$f(t) = f_{m1} + f_{B1} + f_{K1} + f_{B12}$$

$$f(t) = M_1 \frac{d^2x_1}{dt^2} + B_1 \frac{dx_1}{dt} + K_1 x_1 + B_{12}\frac{d}{dt}(x_1 - x_2)$$

Apply laplace transform on both sides,

$$F(S) = M_1 S^2 X_1(S) + B_1 S X_1(S) + K_1 X_1(S) + B_{12} S X_1(S) - B_{12} S X_2(S)$$

$$F(S) = X_1(S)\left[M_1 S^2 + B_1 S + K_1 + B_{12}\right] - B_{12} S X_2(S) \rightarrow (1)$$

consider mass m_2,

opposing forces acting on m_2,

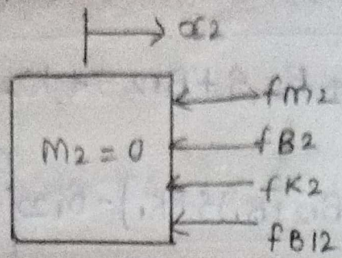
$$f_{m2} = M_2 \frac{d^2x_2}{dt^2}$$

$$f_{B2} = B_2 \frac{dx_2}{dt}$$

$$f_{K2} = K_2 x_2$$

$$f_{B12} = B_{12}(x_2 - x_1)$$

Free body diagram :-



By Newton's second law,

$$0 = f_{m2} + f_{B2} + f_{B12} + f_{K2}$$

$$0 = M_2 \frac{d^2 x_2}{dt^2} + B_2 \frac{dx_2}{dt} + B_{12} \frac{d}{dt}[x_2 - x_1] + K_2 x_2$$

Apply Laplace transform on both sides,

$$0 = M_2 S^2 X_2(S) + B_2 S X_2(S) + B_{12} S X_2(S) - B_{12} S X_1(S) + K_2 X_2(S)$$

$$0 = X_2(S)\left[M_2 S^2 + B_2 S + B_{12} S + K_2\right] - B_{12} S X_1(S) \rightarrow (2)$$

From (2)

$$X_1(S) = \frac{X_2(S)\left[M_2 S^2 + B_2 S + B_{12} S + K_2\right]}{B_{12} S} \rightarrow (3)$$

Sub (3) in (1)

$$F(S) = \frac{X_2(S)\left[M_2 S^2 + B_2 S + B_{12} S + K_2\right]}{B_{12} S}\left[M_1 S^2 + B_1 S + K_1 + B_{12} S\right] - B_{12} X_2(S)$$

$$F(S) = \frac{X_2(S)\left\{\left[M_2 S^2 + B_2 S + B_{12} S + K_2\right]\left[M_1 S^2 + B_1 S + K_1 + B_{12} S\right] - B_{12}^2 S^2\right\}}{B_{12} S}$$

$$\frac{F(S)}{X_2(S)} = \frac{\left[M_2 S^2 + B_2 S + B_{12} S + K_2\right]\left[M_1 S^2 + B_1 S + K_1 + B_{12} S\right] - B_{12}^2 S^2}{B_{12} S}$$

$$\therefore \boxed{\frac{X_2(S)}{F(S)} = \frac{B_{12} S}{\left[M_2 S^2 + S(B_2 + B_{12}) + K_2\right]\left[M_1 S^2 + (B_1 + B_{12}) S + K_1\right] - B_{12}^2 S^2}}$$

From ②

$$X_2(S) = \frac{B_{12}(S)X_1(S)}{M_2S^2 + B_2S + B_{12}S + K_2} \rightarrow ④$$

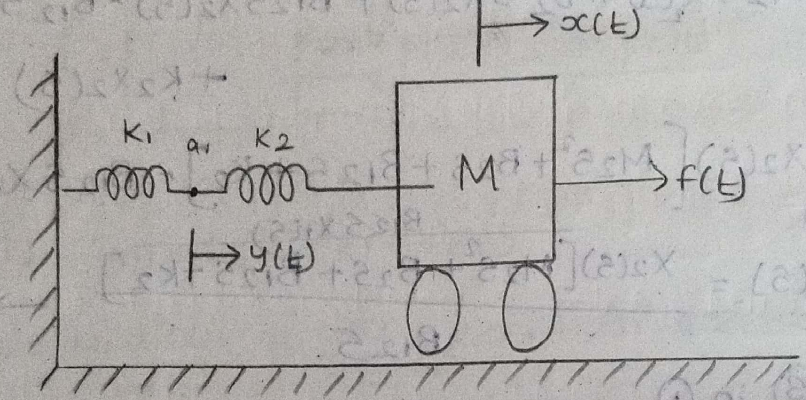
sub eq ④ in eq ①

$$F(S) = X_1(S)\left[M_1S^2 + (B_{12}+B_1)S + K_1\right] - B_{12}S\left[\frac{B_{12}(S)X_1(S)}{M_2S^2+B_2S+B_{12}S+K_2}\right]$$

$$F(S) = X_1(S)\left[M_1S^2+(B_{12}+B_1)S+K_1\right] - \left[\frac{(B_{12}S)^2 X_1(S)}{M_2S^2+(B_2+B_{12})S+K_2}\right]$$

$$F(S) = \frac{X_1(S)\left[M_1S^2+(B_{12}+B_1)S+K_1\right]\left[M_2S^2+(B_2+B_{12})S+K_2\right] - (B_{12}S)^2X_1(S)}{M_2S^2+(B_2+B_{12})S+K_2}$$

$$F(S) = \frac{X_1(S)\left[\left[M_1S^2+(B_{12}+B_1)S+K_1\right]\left[M_2S^2+(B_2+B_{12})S+K_2\right] - (B_{12}S)^2\right]}{M_2S^2+(B_2+B_{12})S+K_2}$$

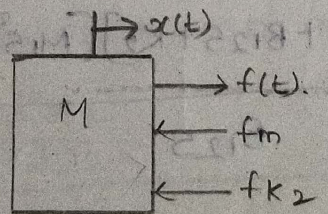


A. consider mass M,

opposing forces acting on Mass M are

1. $f_m = M\frac{d^2x}{dt^2}$
2. $f_{K_2} = K_2(x-y)$

Free body diagram for mass m,



By Newton's Second law,

$$f(t) = f_m + f_{K_2}$$

$$f(t) = M\frac{d^2x}{dt^2} + K_2(x-y)$$

Apply Laplace transform on both sides

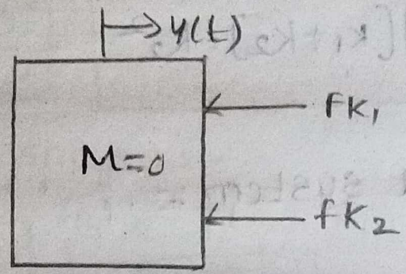
$$F(S) = MS^2X(S) + K_2X(S) - K_2Y(S)$$

$$F(S) = X(S)\left[MS^2 + K_2\right] - K_2Y(S) \rightarrow ①$$

consider a node a_1,

opposing forces acting on node a_1

Free body diagram,



opposing forces acting on mass M are

1. $f_{K_1} = K_1 y$
2. $f_{K_2} = K_2(y - x)$

By Newton's second law,

$$0 = f_{K_1} + f_{K_2}$$

$$K_1y + K_2(y - x) = 0$$

Apply Laplace transform on both sides,

$$K_1Y(S) + K_2Y(S) - K_2X(S) = 0$$

$$Y(S)\left[K_1 + K_2\right] - K_2X(S) = 0$$

$$Y(S)\left[K_1 + K_2\right] = K_2X(S)$$

$$\boxed{Y(S) = \frac{K_2X(S)}{K_1 + K_2}} \rightarrow ②$$

Sub eq ② in ①

$$F(S) = X(S)\left[MS^2 + K_2\right] - K_2\left[\frac{K_2 X(S)}{K_1 + K_2}\right]$$

$$F(S) = \frac{X(S)\left[MS^2 + K_2\right]\left[K_1 + K_2\right] - K_2^2}{K_1 + K_2}$$

The Transfer function is

$$\therefore \boxed{\frac{X(S)}{F(S)} = \frac{K_1 + K_2}{\left[MS^2 + K_2\right]\left[K_1 + K_2\right] - K_2^2}}$$

Rotational Mechanical System :-

Rotational Mechanical System moves above a Fixed axis

Parameters of Rotational Mechanical System:-

1. Angular displacement (θ): - Units: radians

2. Angular Velocity

$$\omega = \frac{d\theta}{dt} \quad \text{Units: rad/s}$$

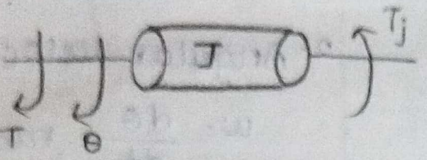
3. Angular acceleration (α) :-

$$\alpha = \frac{d^2\theta}{dt^2} \quad \text{units: rad/s}^2$$

4. Torque (T)

components (or) elements of Rotational Mechanical system:-

1. Moment of inertia of mass:-

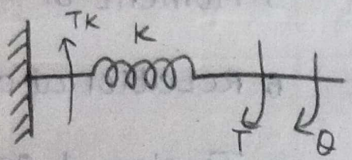


$T_j \propto a$

$T = T_j = J\dfrac{d^2\theta}{dt^2}$

$$\boxed{T_j = J\dfrac{d^2\theta}{dt^2}}$$

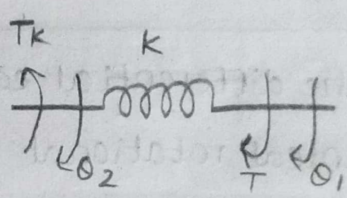
2. Torsional spring:-



$T_K \propto \theta$

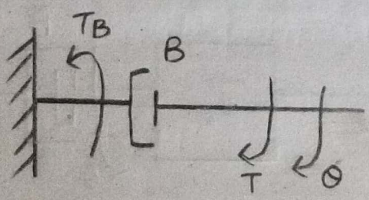
$$\boxed{T_K = K\theta}$$

If both ends are free,



$$\boxed{T_K = K(\theta_1 - \theta_2)}$$

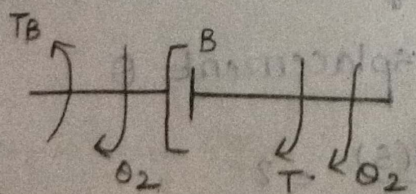
3. Damper (B):-



$T_B \propto \dfrac{d\theta}{dt}$

$T = T_B = B\dfrac{d\theta}{dt}$

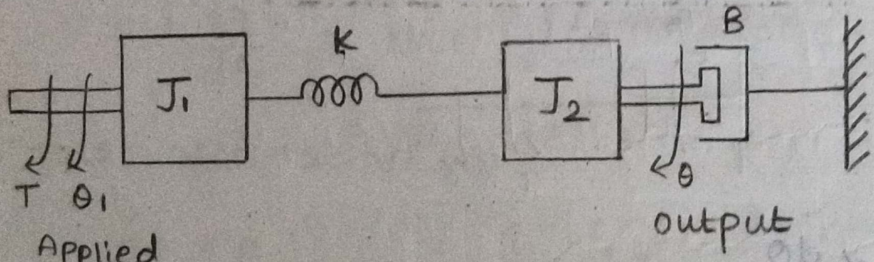
If Both the ends are free,



$T_B = B\dfrac{d}{dt}(\theta_1 - \theta_2)$

Translational Mechanical System	Rotational Mechanical system.
1. Displacement (x) units : meters	1. Displacement (θ) units: Radians.
2. Velocity (V) $V = \frac{dx(t)}{dt}$; m/s	2. Angular velocity (w):- $w = \frac{d\theta}{dt}$; rad/s
3. Accelaration (a) :- $a = \frac{d^2x(t)}{dt^2}$; m/s^2	3. Angular acceleration (α):- $\alpha = \frac{d^2\theta}{dt^2}$; rad/s^2
4. Force (F) : N	4. Torque (T)
5. Mass (M)	5. Moment of inertia of mass (J)
6. Damper (B)	6. Rotational damper (B)
7. Spring (K)	7. Torsional spring (K)

1. Write the differential equation governing the mechanical rotational system shown in figure. Determine the transfer function of the system.



A Consider moment of inertia,

Input = Applied torque T

output = angular displacement θ

$$\text{Transfer Function} = \frac{\theta(S)}{T(S)} = ?$$

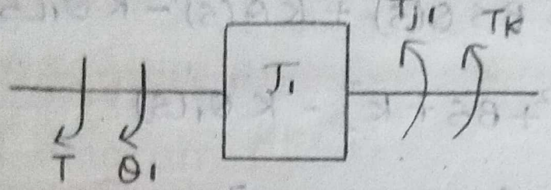
consider moment of inertia J_1,

opposing torques acting on J_1

1. $T_{j1} = J_1 \frac{d^2\theta_1}{dt^2}$

2. $T_K = K(\theta_1 - \theta)$

Free body diagram for J_1



By Newtons second law,

Applied torque = sum of opposing torques

$$T = T_{j1} + T_K$$

$$T = J_1 \frac{d^2\theta_1}{dt^2} + K(\theta_1 - \theta)$$

Apply laplace transform on both sides,

$$T(s) = J_1 s^2 \theta_1(s) + K\theta_1(s) - K\theta(s)$$

$$\boxed{T(s) = \theta_1(s)\left[J_1 s^2 + K\right] - K\theta(s)} \rightarrow (1)$$

Now consider moment of inertia J_2

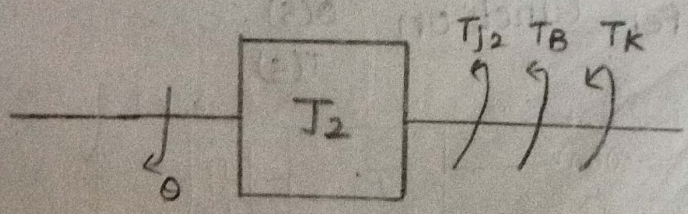
Opposing torques acting on J_2

1. $T_{j2} = J_2 \frac{d^2\theta}{dt^2}$

2. $T_B = B \frac{d\theta}{dt}$

3. $T_K = K(\theta - \theta_1)$

Free body diagram for J_2



By Newtons second law,

$$0 = T_{j2} + T_B + T_K$$

$$0 = J_2 \frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} + K(\theta - \theta_1)$$

Apply Laplace transform on both sides,

$$0 = J_2 s^2\theta(s) + Bs\,\theta(s) + K\theta(s) - K\theta_1(s)$$

$$0 = \theta(s)\left[J_2 s^2 + Bs + K\right] - K\theta_1(s)$$

$$K\theta_1(s) = \theta(s)\left[J_2 s^2 + B_2 s + K\right]$$

$$\boxed{\theta_1(s) = \frac{\theta(s)\left[J_2 s^2 + B_2 s + K\right]}{K}} \longrightarrow (2)$$

Sub eq (2) in (1)

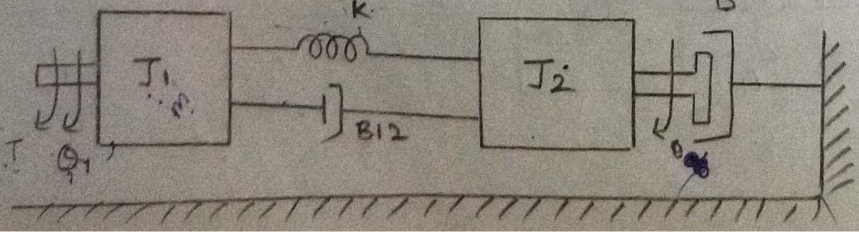
$$T(s) = \frac{\theta(s)\left[J_2 s^2 + Bs + K\right]}{K}\left[J_1 s^2 + K\right] - K\theta(s)$$

$$T(s) = \frac{\theta(s)\left[J_2 s^2 + Bs + K\right]\left[J_1 s^2 + K\right] - K^2\theta(s)}{K}$$

$$T(s) = \frac{\theta(s)\left[\left[J_2 s^2 + Bs + K\right]\left[J_1 s^2 + K\right] - K^2\right]}{K}$$

$$\therefore \boxed{\frac{\theta(s)}{T(s)} = \frac{K}{\left[J_2 s^2 + Bs + K\right]\left[J_1 s^2 + K\right] - K^2}}$$

2. Write the differential equation governing the mechanical rotational system shown in figure and determine the transfer function $\frac{\theta(s)}{T(s)}$



A. Input = Applied torque T

output = angular displacement θ

consider moment of Inertia J_1,

opposing torque acting on J_1

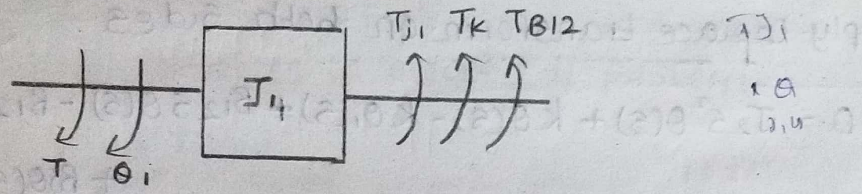
$\theta = x, y$
T = force
$J = m$
$B = b$
$K = k$

1. $T_{j1} = J_1 \frac{d^2\theta_1}{dt^2}$

2. $T_K = K(\theta_1 - \theta)$

3. $T_{B12} = B_{12} \frac{d}{dt}(\theta_1 - \theta)$

Free body diagram for J_1



By Newtons second law,

Applied torque = sum of opposing torques

$$T = T_{j1} + T_K + B_{12}$$

$$T = J_1 \frac{d^2\theta_1}{dt^2} + K(\theta_1 - \theta) + B_{12} \frac{d}{dt}(\theta_1 - \theta)$$

Apply laplace transform on both sides,

$$T(S) = J_1 S^2 \theta_1(S) + K\theta_1(S) - K\theta(S) + B_{12} S\, \theta_1(S) - B_{12} S\, \theta(S)$$

$$\boxed{T(S) = \theta_1(S)\left[J_1 S^2 + K + B_{12}\right] - \theta(S)\left[B_{12}S + K\right] \longrightarrow (1)}$$

Now consider moment of Inertia J_2,

opposing torque acting on J_2

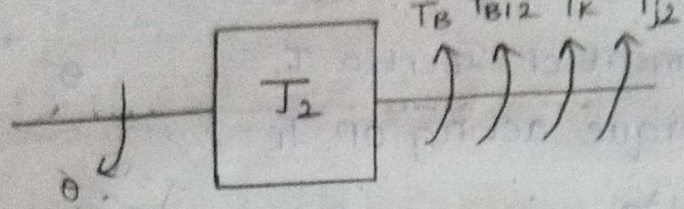
1. $T_{j2} = J_2 \frac{d^2\theta}{dt}$

2. $T_K = K(\theta - \theta_1)$

3. $T_{B12} = B_{12} \frac{d}{dt}(\theta - \theta_1)$

4. $T_B = B \frac{d\theta}{dt}$

Free body diagram



By Newtons second law,

$$0 = T_{j2} + T_K + T_{B12} + T_B$$

$$0 = J_2 \frac{d^2\theta}{dt^2} + K(\theta - \theta_1) + B_{12}\frac{d}{dt}(\theta - \theta_1) + B\frac{d\theta}{dt}$$

Apply laplace transform on both sides

$$0 = J_2 S^2 \theta(S) + K\theta(S) - K\theta_1(S) + B_{12}S\theta(S) - B_{12}S\theta_1(S) + BS\theta(S)$$

$$0 = \theta(S)\left[J_2S^2 + K + B_{12}S + BS\right] - \theta_1(S)\left[B_{12}S + K\right]$$

$$0 = \theta(S)\left[J_2S^2 + K + (B_{12}+B)S\right] - \theta_1(S)\left[B_{12}S + K\right] \rightarrow (2)$$

From eq (2)

$$\theta_1(S)\left[B_{12}S + K\right] = \theta(S)\left[J_2S^2 + K + (B_{12}+B)S\right]$$

$$\theta_1(S) = \frac{\theta(S)\left[J_2S^2 + K + (B_{12}+B)S\right]}{B_{12}S + K} \rightarrow (3)$$

Sub eq (3) in eq (1)

$$T(S) = \frac{\theta(S)\left[J_2S^2 + K + (B_{12}+B)S\right]}{B_{12}S + K}\left[J_1S^2 + K + B_{12}\right] - \theta(S)\left[B_{12}S + K\right]$$

$$T(S) = \frac{\theta(S)\left[J_2S^2 + K + (B_{12}+B)S\right]\left[J_1S^2 + K + B_{12}\right] - \theta(S)\left[B_{12}S + K\right]}{B_{12}S + K}$$

$$T(S) = \frac{\theta(S)\left[\left(J_2S^2 + K + (B_{12}+B)S\right]\left[J_1S^2 + K + B_{12}\right] - \left[B_{12}S + K\right]^2\right]}{B_{12}S + K}$$

The transfer function is,

$$\boxed{\frac{\theta(S)}{T(S)} = \frac{B_{12}S+K}{\left[J_2S^2+K+(B_{12}+B)S\right]\left[J_1S^2+K+B_{12}\right]-\left[B_{12}S+K\right]^2}}$$

Mathematical modeling of Electrical systems:-

The models of electrical systems can be obtained by using resistors, capacitors and inductors. The behaviour of electrical systems are governed by using ohm's law and Kirchoff's law.

There are three variables in electrical system

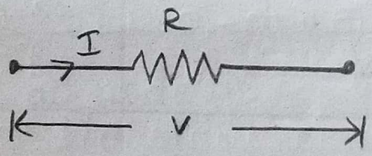
→ Voltage V → Input

→ current I → output

→ charge Q

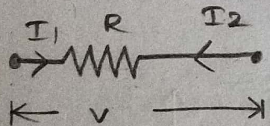
components or elements of Electrical system:-

1. Resistor 'R':-



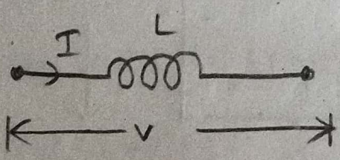
$$V = IR$$

$$I = \frac{V}{R}$$



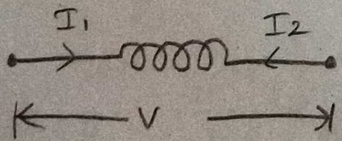
$$\boxed{V = (I_1 - I_2)R}$$

2. Inductor 'L':-



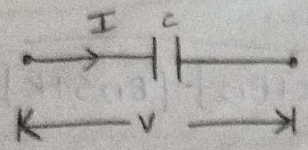
$$V = L\frac{dI}{dt}$$

$$I = \frac{1}{L}\int V\,dt$$



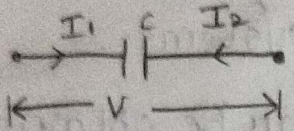
$$\boxed{V = L\frac{d}{dt}(I_1 - I_2)}$$

3. Capacitor 'c' :-

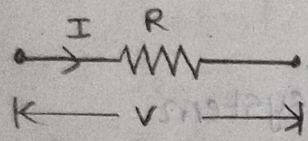
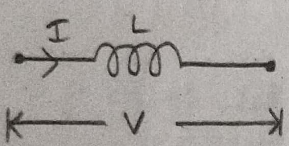
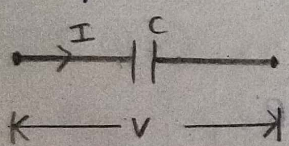


$$V = \frac{1}{c}\int I\,dt$$

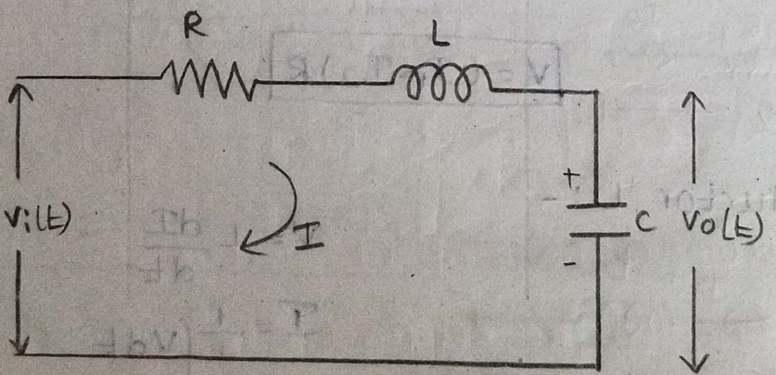
$$I = c\frac{dV}{dt}$$



$$\boxed{V = \frac{1}{c}\int (I_1 - I_2)\,dt}$$

Element.	Voltage across the element	current through acros the element
1. Resistor 	$V = IR$	$I = \frac{V}{R}$
2. Inductor 	$V = L\frac{dI}{dt}$	$I = \frac{1}{L}\int V\,dt$
3. capacitor 	$V = \frac{1}{c}\int I\,dt$	$I = c\frac{dV}{dt}$

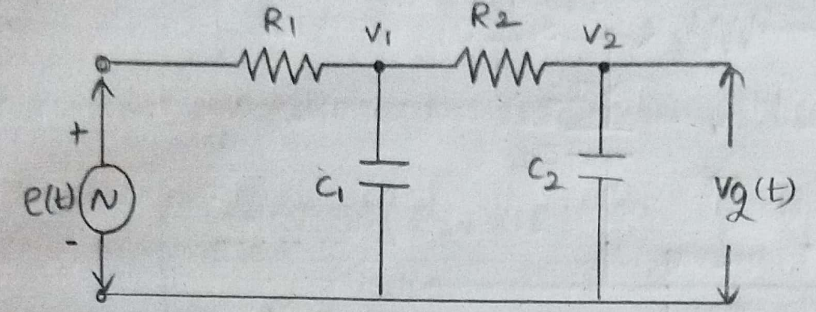
Example: 1



$$T.F = \frac{V_o(S)}{V_i(S)} = \frac{1/LC}{S^2 + S\frac{R}{L} + \frac{1}{LC}}$$

Example-2:

obtain the transfer function of the electrical network shown in figure.



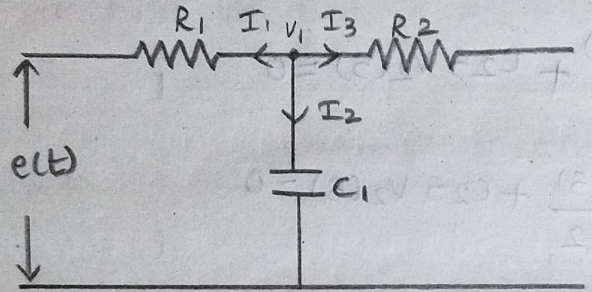
A. Input = e(t)

output = Vg(t)

The transfer function is $\frac{V_2(S)}{E(S)} = ?$

Take node1 as V_1 and node 2 as V_2

Ap



Apply KCL at node 1

sum of entering current = sum of leaving current

$$I_1 + I_2 + I_3 = 0$$

$$\frac{V_1 - e(t)}{R_1} + C_1\frac{dV_1}{dt} + \frac{V_1 - V_2}{R_2} = 0$$

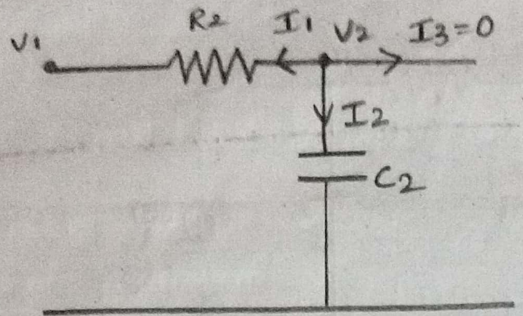
Apply laplace transform on both sides

$$\frac{V_1(S) - E(S)}{R_1} + C_1 S V_1(S) + \frac{V_1(S) - V_2(S)}{R_2} = 0$$

$$\frac{V_1(S)}{R_1} - \frac{E(S)}{R_1} + C_1 S V_1(S) + \frac{V_1(S)}{R_2} - \frac{V_2(S)}{R_2} = 0$$

$$V_1(S)\left[\frac{1}{R_1} + C_1 S + \frac{1}{R_2}\right] - \frac{V_2(S)}{R_2} = \frac{E(S)}{R_1}$$

$$\boxed{V_1(S)\left[\frac{R_2+R_1R_2C_1S+R_1}{R_1R_2}\right]-\frac{V_2(S)}{R_2}=\frac{E(S)}{R_1}} \longrightarrow (1)$$



Apply KCL at node V_2,

$$I_1+I_2=0$$

$$\frac{V_2-V_1}{R_2}+C_2\frac{dV_2}{dt}=0$$

Apply laplace transform on both sides

$$\frac{V_2(S)-V_1(S)}{R_2}+C_2S\,V_2(S)=0$$

$$\frac{V_2(S)}{R_2}-\frac{V_1(S)}{R_2}+C_2S\,V_2(S)=0$$

$$V_2(S)\left[\frac{1}{R_2}+C_2S\right]-\frac{V_1(S)}{R_2}=0$$

$$V_2(S)\left[\frac{1+R_2C_2S}{R_2}\right]=\frac{V_1(S)}{R_2}$$

$$V_1(S)=V_2(S)\left[\frac{1+R_2C_2S}{\cancel{R_2}}\right]\times\cancel{R_2}$$

$$\boxed{V_1(S)=V_2(S)\left[1+R_2C_2S\right]} \longrightarrow (2)$$

sub eq (2) in (1)

$$V_2(S)\left[1+R_2C_2S\right]\left[\frac{R_2+R_1R_2C_1S+R_1}{R_1R_2}\right]-\frac{V_2(S)}{R_2}=\frac{E(S)}{R_1}$$

$$V_2(S)\left[\frac{(1+C_2SR_2)(R_2+R_1R_2C_1S+R_1)}{R_1R_2}-\frac{1}{R_2}\right]=\frac{E(S)}{R_1}$$

$$E(S)=V_2(S)\left[\frac{(1+R_2C_2S)(R_2+R_1R_2C_1S+R_1)}{R_1R_2}-\frac{1}{R_2}\right]R_1$$

$$\therefore \boxed{\frac{V_2(S)}{E(S)}=\frac{1}{\left[\frac{(1+R_2C_2S)(R_2+R_1R_2C_1S)}{R_1R_2}-\frac{1}{R_2}\right]R_1}}$$

Analogy between Electrical and Mechanical systems :-

Analogous system :-

The systems have same differential equations or transfer function then it is said to be Analogous System.

For example the mechanical system is converted into electrical equivalent system then it is called as Electrical Analogous System.

Advantages of electrical Analog system :-

→ Standard symbols R, L, C

→ Simple laws i.e KCL and KVL

→ Easy to analyze

Types of Analogies :-

Depending on applied input may be either Voltage or current. There are 4 types.

$V=\frac{dx}{dt}$

$x=\int v\,dt$

1. Force - voltage (F-V) Analogy
2. Force - Current (F-I) Analogy

} - Translational system

3. Torque - voltage (T-V) Analogy
4. Torque - current (T-I) Analogy

} - Rotational system

1. Force-voltage (F-V) Analogy :-

The mathematical equations of translational mechanical systems compared with mesh equations of electrical systems. RLC components are connected in series

Translational Mechanical System.	Electrical Systems	Rotational Mechanical Systems
Force 'F'	voltage 'v' e(t)	Torque 'T'
Mass 'M'	Inductor 'L'	Moment of inertia 'J'
Damper 'B'	Resistor 'R'	Damper 'B'
Spring 'K'	capacitor '1/C'	Spring 'K'
Displacement 'x'	charge 'q'	Angular displacement 'θ'
Velocity 'v'	current 'I'	Angular velocity 'w'
newton law	Kirchoff's voltage law	

2. Force-current (F-I) Analogy :-

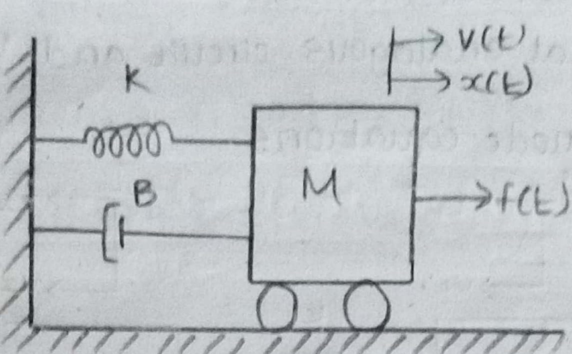
The mathematical equations of translational mechanical systems compared with node equations of electrical systems.

RLC components are connected in parallel.

Translational Mechanical System	Electrical system	Rotational Mechanical systems
Force 'F'	current 'I'	Torque 'T'
Mass 'M'	capacitor 'C'	Moment of inertia 'J'
Damper 'B'	Resistor '1/R'	Damper 'B'
Spring 'K'	Inductor '1/L'	Spring 'K'
Displacement 'x'	flux φ	Angular displacement 'θ'
Velocity 'v'	voltage 'v'	Angular velocity 'w'
Newton's law	Kirchoff's current law	

Problems :-

1. consider a translational mechanical system shown in Fig. Now we have to convert Mechanical to electrical



$$f(t) = M\frac{d^2x}{dt^2} + B\frac{dx}{dt} + Kx$$

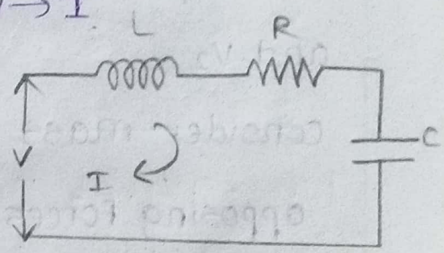
Replace displacement 'x' by velocity 'v'

$$f(t) = M\frac{dv}{dt} + Bv + K\int v\,dt \rightarrow ①$$

F-V Analogy:-

$f(t) \rightarrow V$, $M \rightarrow L$, $B \rightarrow R$, $K \rightarrow 1/C$, $v \rightarrow I$

$$\boxed{V = L\frac{dI}{dt} + RI + \frac{1}{C}\int I\,dt}$$

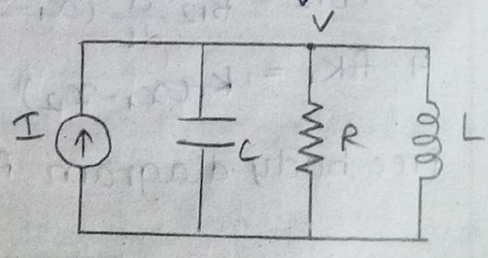


Force-current Analogy:-

$$f(t) = M\frac{dv}{dt} + Bv + K\int v\,dt$$

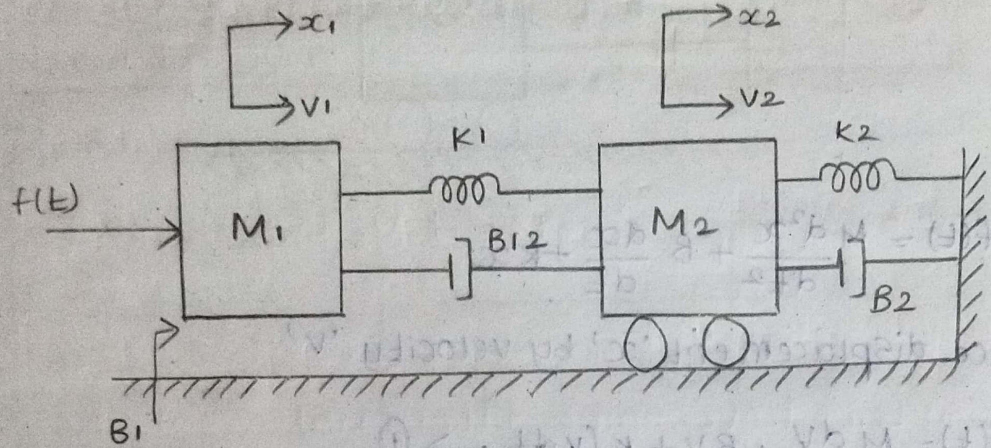
$f(t) \rightarrow I$, $M \rightarrow C$; $B \rightarrow 1/R$, $K \rightarrow 1/L$; $v \rightarrow V$

$$\boxed{I = C\frac{dv}{dt} + \frac{1}{R}V + \frac{1}{L}\int v\,dt}$$



Problems:-

1. write the differential equations governing the Mechanical system shown in figure. Draw the F-V and F-I electrical analogous circuit and verify by writing mesh and node equations.



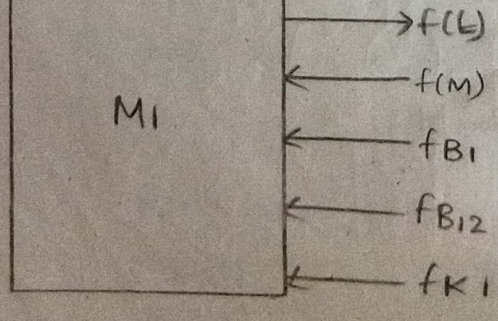
A. There are two masses M_1 and M_2 acts as nodes then two displacements x_1 and x_2 and two Velocity V_1 and V_2.

consider mass M_1,

opposing Forces acting on Mass M_1

1. $f(M) = M_1 \frac{d^2x_1}{dt^2}$
2. $fB_1 = B_1 \frac{dx}{dt}$
3. $fB_{12} = B_{12} \frac{d}{dt}(x_1 - x_2)$
4. $fK_1 = K_1(x_1 - x_2)$

Free body diagram for mass M_1,



By Newton's second law,

$$f(t) = f_{m1} + f_{B1} + f_{B12} + f_{K1}$$

$$f(t) = M_1 \frac{d^2 x_1}{dt^2} + B_1 \frac{dx_1}{dt} + B_{12} \frac{d}{dt}(x_1 - x_2) + K_1(x_1 - x_2)$$

Replace displacements x_1 and x_2 by velocity v_1 and v_2

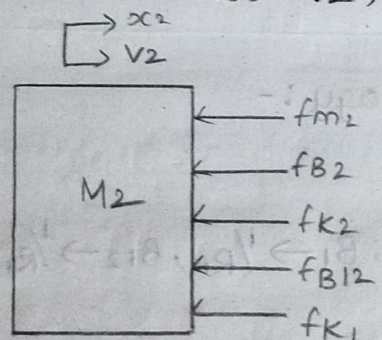
$$f(t) = M_1 \frac{dv}{dt} + B_1 V_1 + B_{12}(V_1 - V_2) + K_1 \int (V_1 - V_2)\, dt \rightarrow (1)$$

Consider mass M_2,

opposing forces acting on mass M_2;

1. $f_{m2} = M_2 \frac{d^2 x_2}{dt^2}$
2. $f_{B2} = B_2 \frac{dx}{dt}$
3. $f_{K2} = K_2 x_2$
4. $f_{B12} = B_{12} \frac{d}{dt}(x_2 - x_1)$
5. $f_{K1} = K_1(x_2 - x_1)$

Free body diagram for mass M_2,



By Newton's second law,

$$0 = M_2 \frac{d^2 x_2}{dt^2} + B_2 \frac{dx}{dt} + K_2 x_2 + B_{12} \frac{d}{dt}(x_2 - x_1) + K_1(x_2 - x_1)$$

Replace displacements x_1 and x_2 by velocity v_1 and v_2

$$0 = M_2 \frac{dV_2}{dt} + B_2 V_2 + K_2 \int V_2\, dt + B_{12}(V_2 - V_1) + K_1 \int (V_2 - V_1)\, dt$$

$\rightarrow (2)$

Force-voltage Analogy:-

consider eq ①

$f(t) \to V$, $M_1 \to L_1$, $B_1 \to R_1$, $B_{12} \to R_{12}$, $K_1 \to \frac{1}{C_1}$, $V_1 \to I_1$

$V_2 \to I_2$

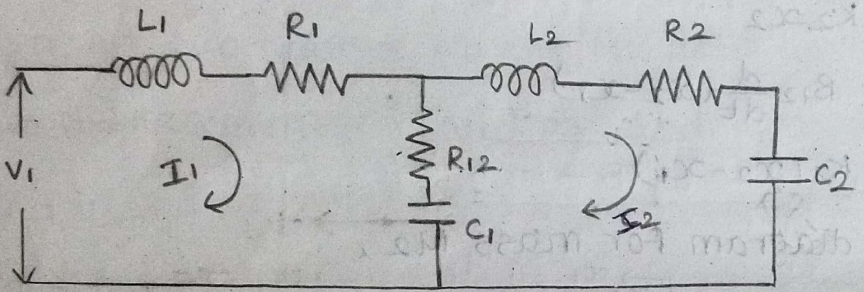
$$\boxed{V = L_1 \frac{dI_1}{dt} + R_1 I_1 + R_{12}(I_1 - I_2) + \frac{1}{C_1}\int (I_1 - I_2)\,dt} \to ③$$

consider eq ②

$M_2 \to L_2$, $B_2 \to R_2$, $B_{12} \to R_{12}$, $K_2 \to \frac{1}{C_2}$, $V_2 \to I_2$, $V_1 \to I_1$

$K_1 \to \frac{1}{C_1}$

$$\boxed{0 = L_2 \frac{dI_2}{dt} + R_2 I_2 + R_{12}(I_2 - I_1) + \frac{1}{C_2}\int I_2\,dt + \frac{1}{C_1}\int (I_2 - I_1)\,dt} \to ④$$



Force-current Analogy:-

consider eq ①

$f(t) \to I$, $M_1 \to C_1$, $B_1 \to 1/R_1$, $B_{12} \to 1/R_{12}$, $K_1 \to 1/L_1$, $V_1 \to V_1$

$V_2 \to V_2$

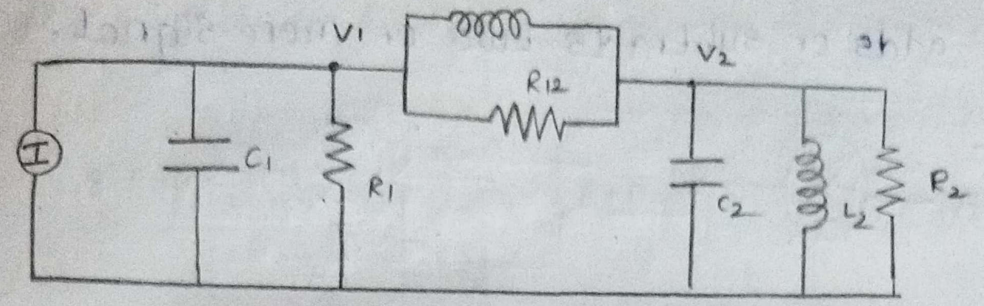
$$\boxed{I = C_1 \frac{dV_1}{dt} + \frac{1}{R_1} V_1 + \frac{1}{R_{12}}(V_1 - V_2) + \frac{1}{L_1}\int (V_1 - V_2)\,dt} \to ③$$

consider eq ②

$M_2 \to C_2$, $B_2 \to \frac{1}{R_2}$, $B_{12} \to 1/R_{12}$, $K_2 \to 1/L_2$, $V_2 \to V_2$,

$V_1 \to V_1$, $K_1 \to \frac{1}{L_2}$

$$\boxed{0 = C_2 \frac{dV_2}{dt} + \frac{1}{R_2} V_2 + \frac{1}{R_{12}}(V_2 - V_1) + \frac{1}{L_2}\int V_2\,dt + \frac{1}{L_1}\int (V_2 - V_1)\,dt} \to ④$$



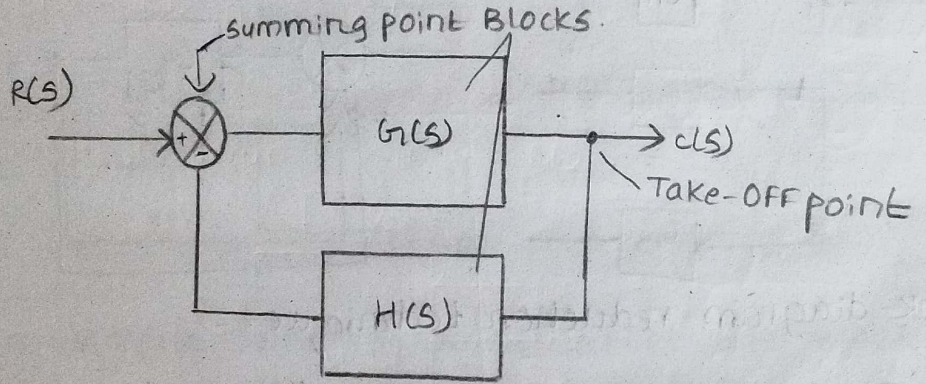
* Block diagram reduction technique:-

Block is a pictorial representation of functions of performed by each component and the flow of signal.

Block diagram representation is used to calculate overall transfer function of the system.

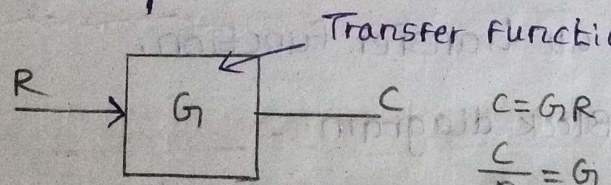
Elements of Block diagram:-

Consider closed loop control system



1. Block:-

It represent component or mathematical operations.



$C = GR$

$\frac{C}{R} = G$

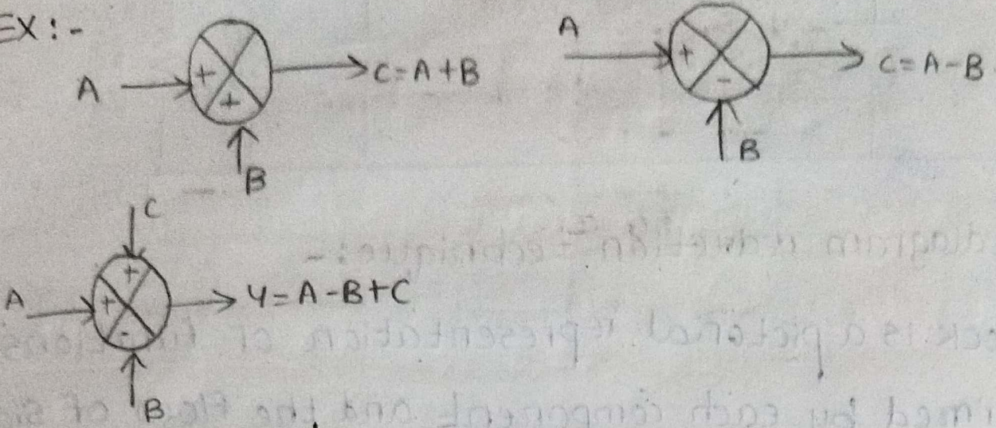
⇒ Arrows indicates flow of signal.

2. Summing point:-

It represented with a circle having cross mark inside of it. It produces algebraic sum of the inputs. Ex: Addition, Subtraction or combination of both.

It adds or subtracts two or more signal.

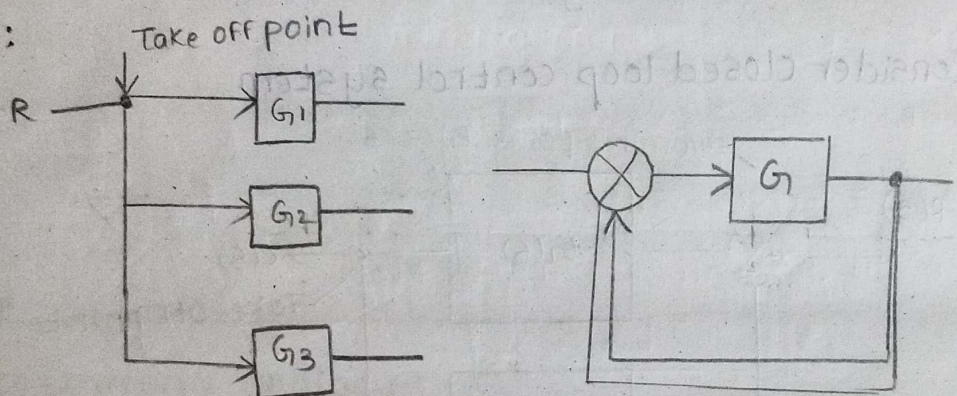
EX:-



3. Take-off point/Branch point :-

Giving same signal to multiple blocks or one block or summing point.

EX:

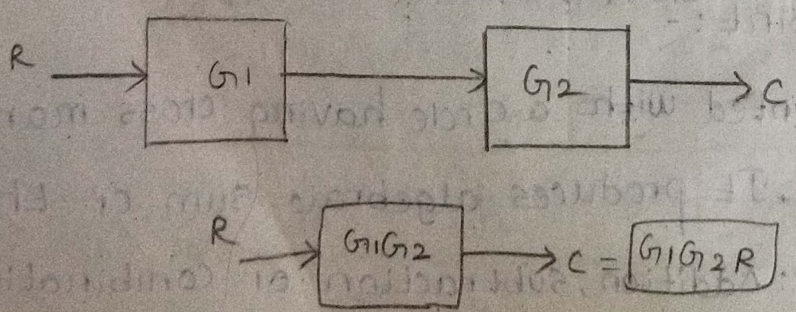


Block diagram reduction technique:-

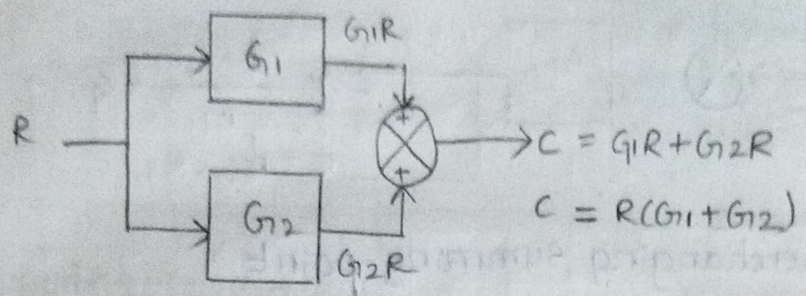
It is easy to reduce complex block diagram representation into single block by applying rules to find over all transfer function.

Rules of Block diagram:-

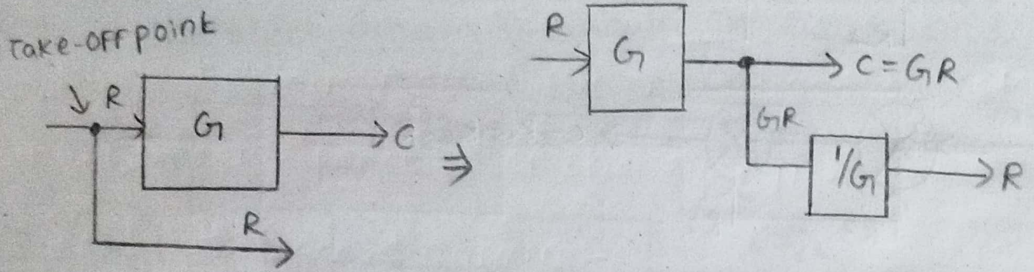
Rule 1:- combining the cascaded blocks (Series blocks)



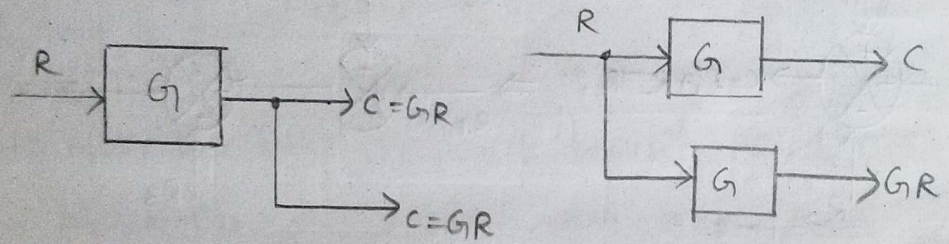
Rule 2:- Combining the parallel blocks



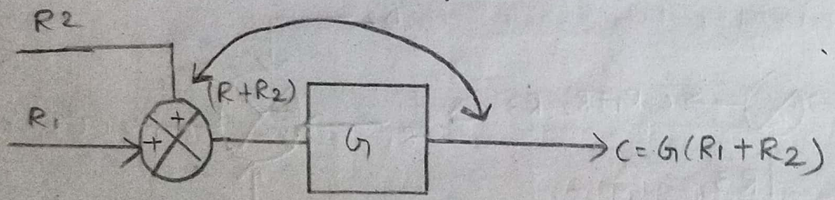
Rule 3:- Moving take-off point or branch point after the block.

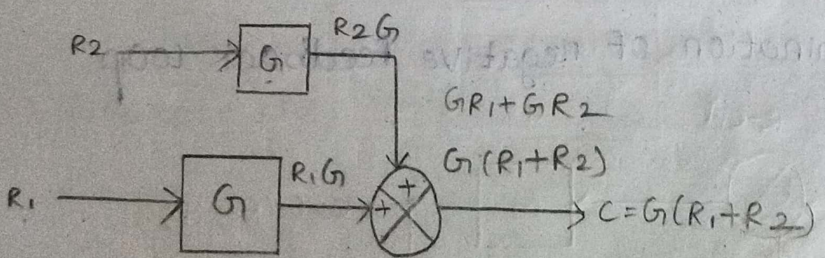


Rule 4:- Moving take-off point before the block

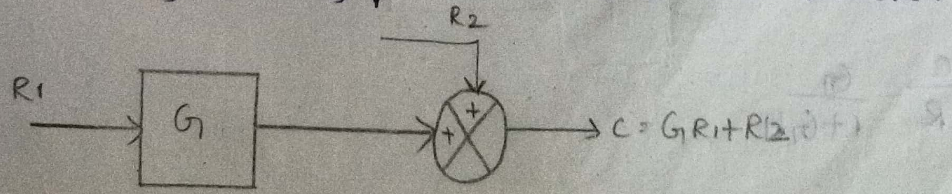


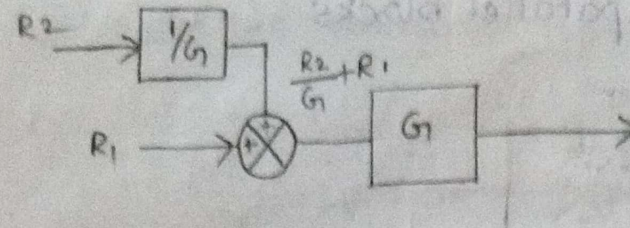
Rule 5:- Moving summing point after the block





Rule 6:- Moving suming point before the block.

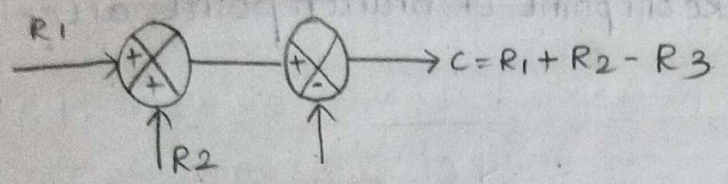


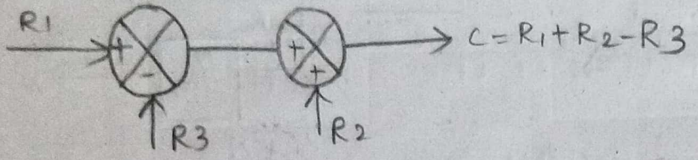


$$C = \frac{R_2 + R_1 G}{G} \times G$$

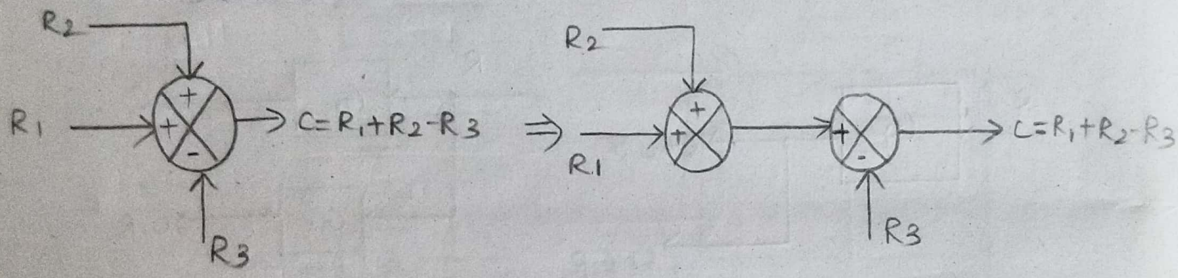
$$C = R_1 G + R_2$$

Rule 7:- Interchanging summing point

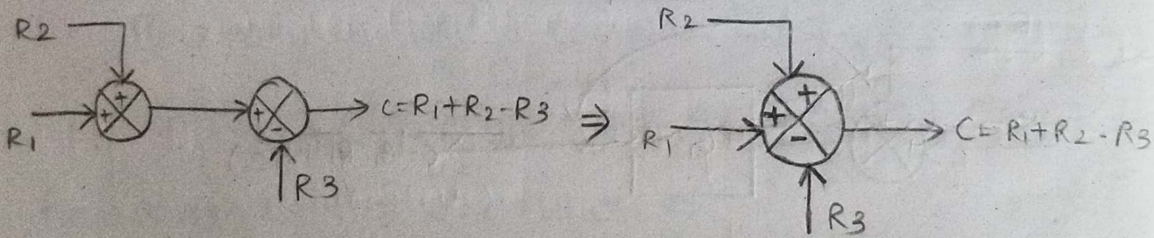




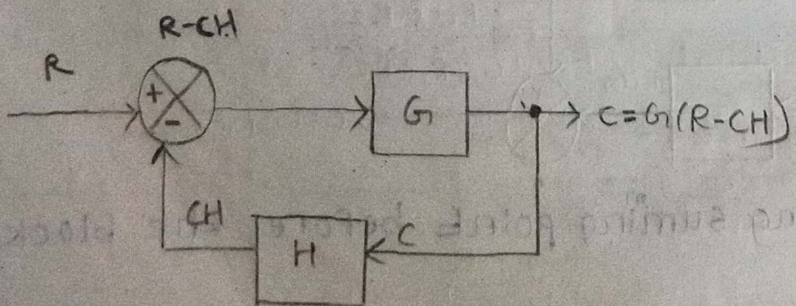
Rule 8:- splitting the summing points



Rule 9:- combining the summing points.

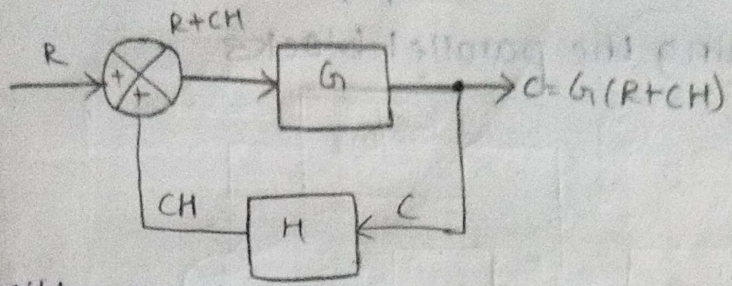


Rule 10:- Elimination of negative feedback loop



$$T.F = \frac{C}{R} = \frac{G}{1+GH}$$

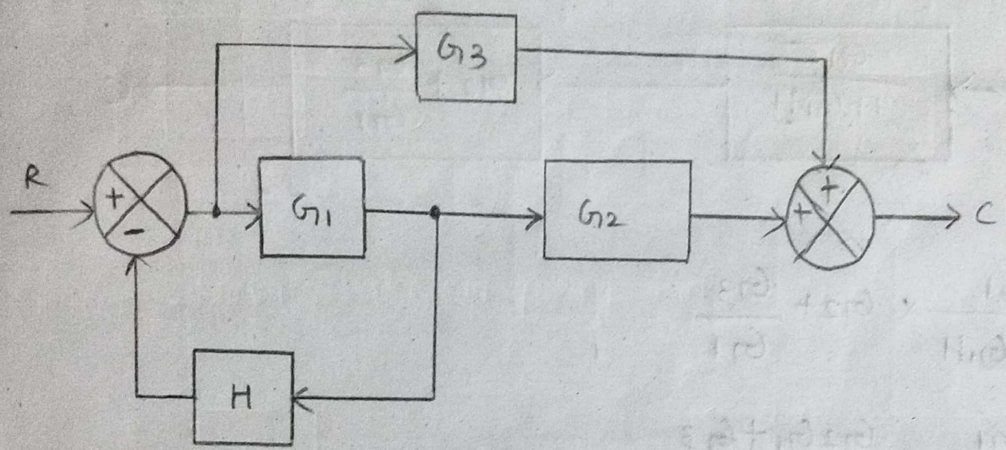
Rule 11 :- Elimination of positive feedback loop



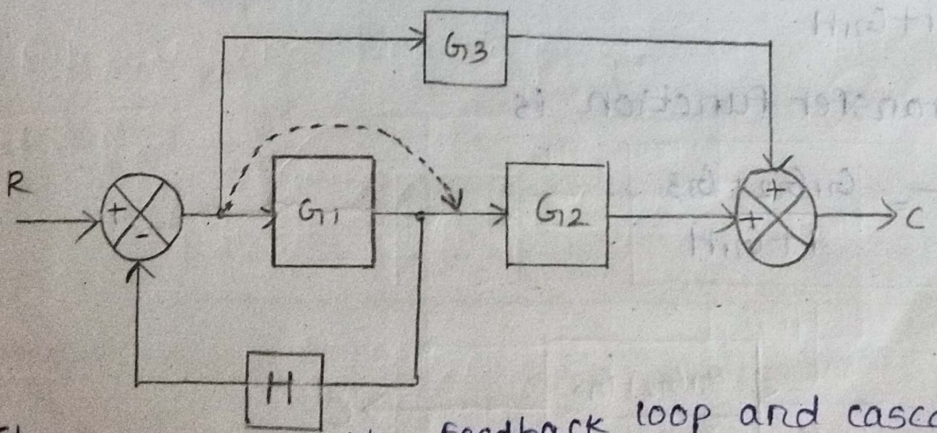
TF of positive feedback

$$T.F = \frac{C}{R} = \frac{G}{1-GH}$$

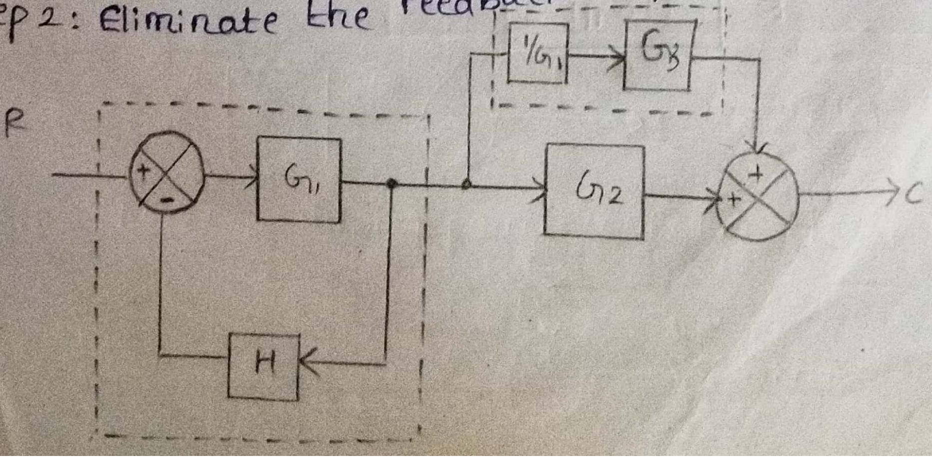
Ex: Reduce the block diagram shown in figure and find $\frac{C}{R}$.



A. Step 1 : Move the take-off point / branch point after the block G_1.

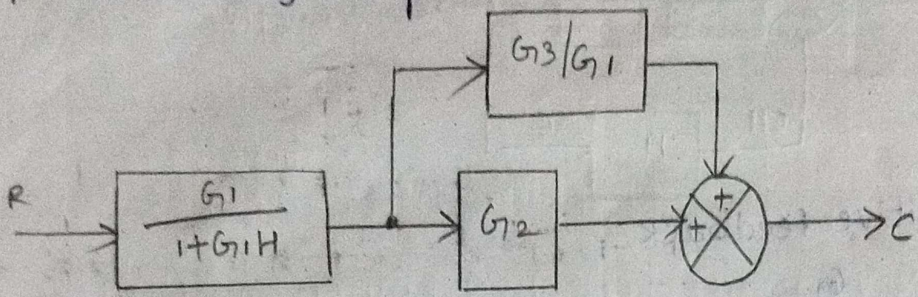


Step 2: Eliminate the feedback loop and cascaded the blocks.

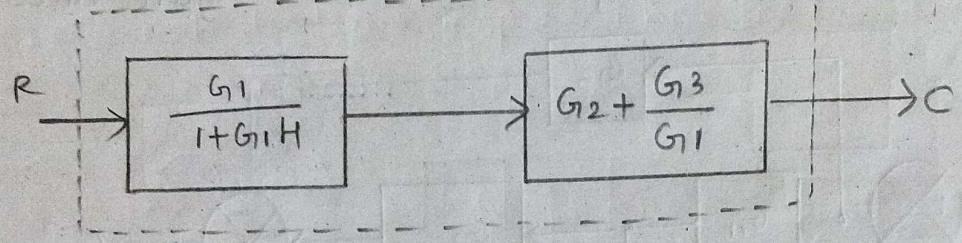


step2: Eliminate feedback loop and cascaded the blocks

step3: combining the parallel blocks



step4: combine cascaded blocks



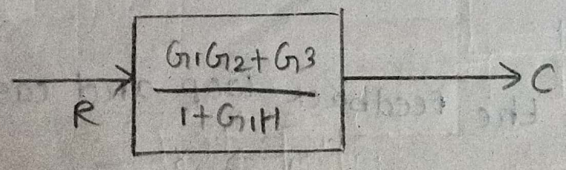
$$\frac{G_1}{1+G_1H} \times G_2 + \frac{G_3}{G_1}$$

$$\frac{\cancel{G_1}}{1+G_1H} \times \frac{G_2G_1 + G_3}{\cancel{G_1}}$$

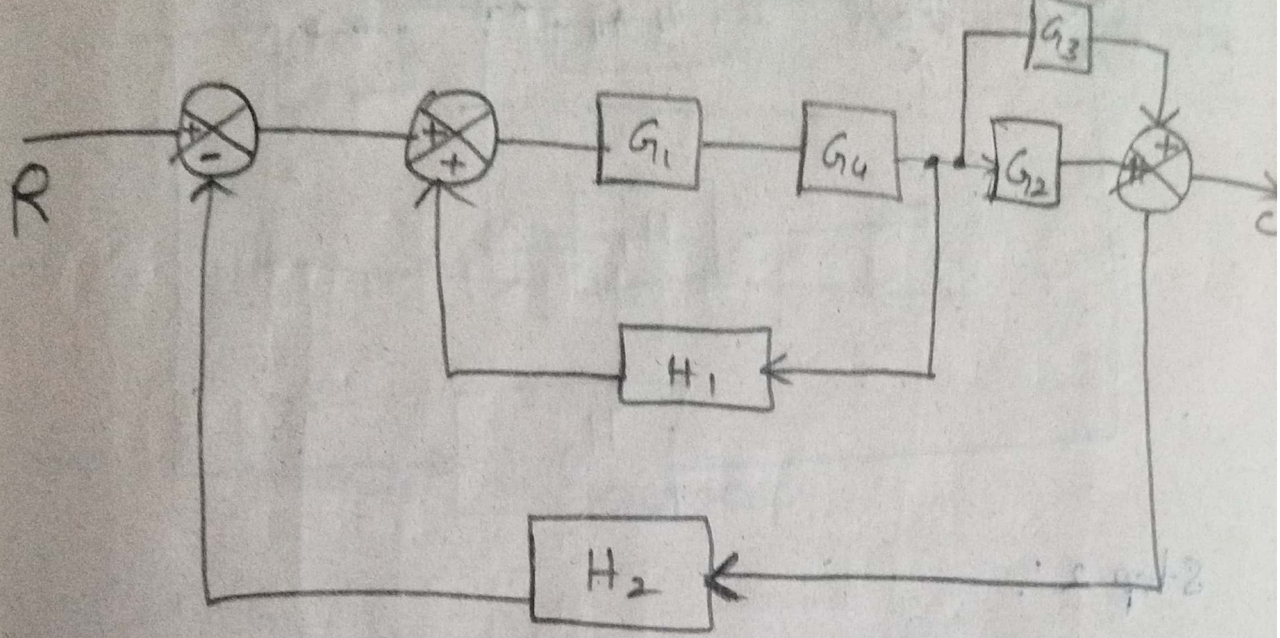
$$\frac{G_2G_1 + G_3}{1+G_1H}$$

Overall transfer function is

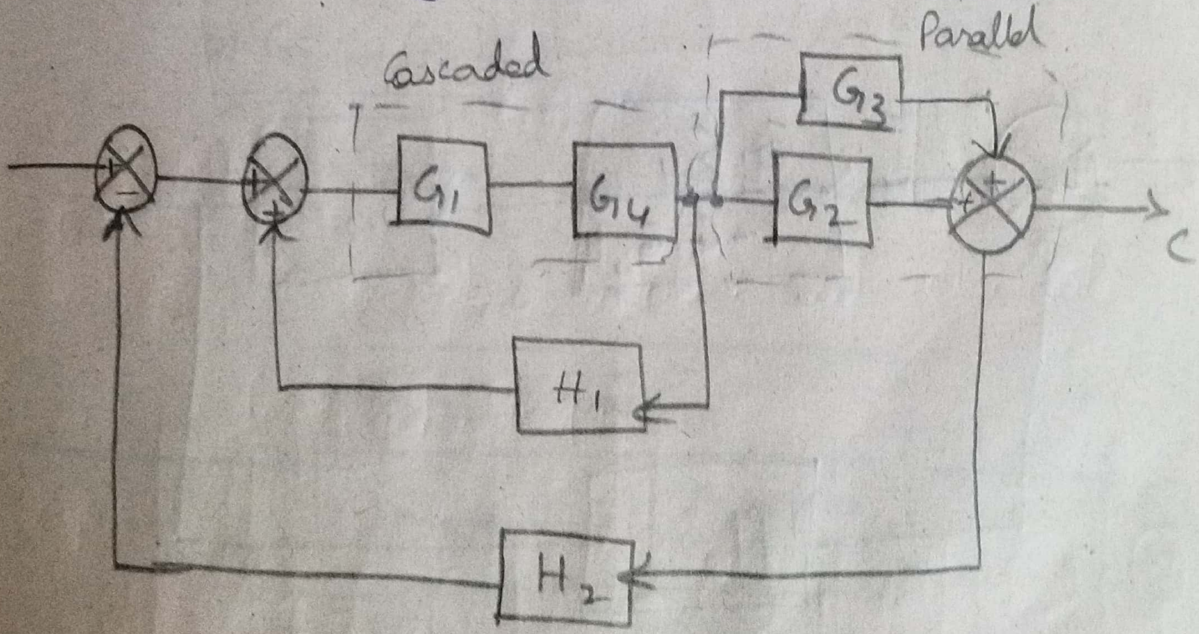
$$\boxed{\frac{C}{R} = \frac{G_1G_2 + G_3}{1+G_1H}}$$



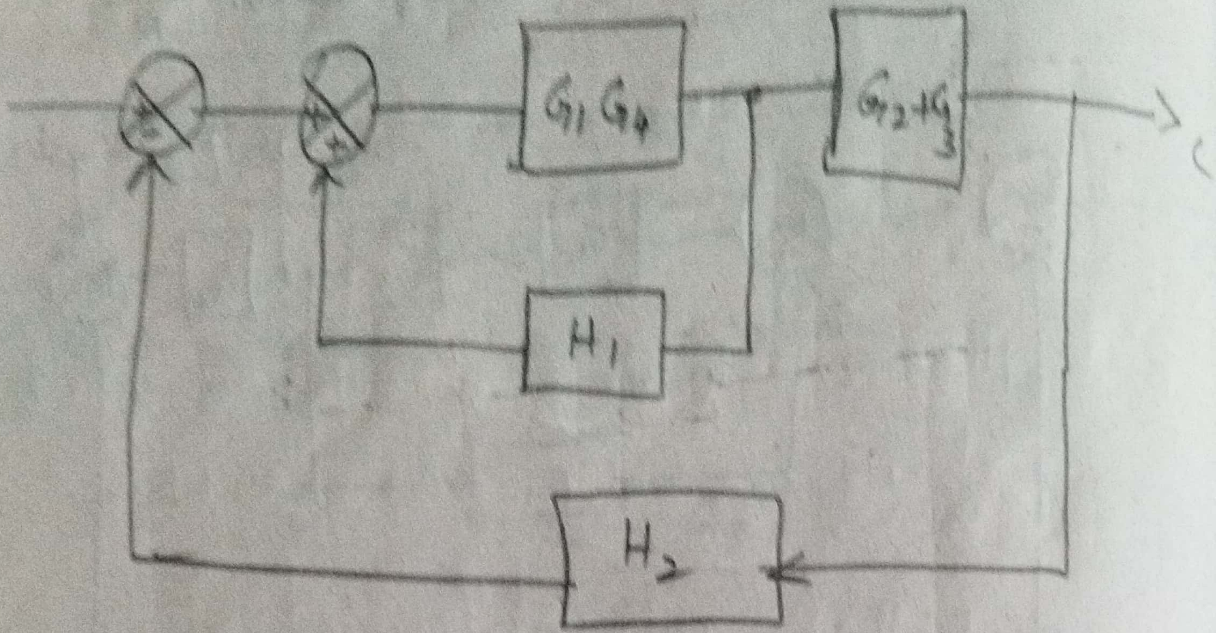
2. Reduce the block diagram shown in figure. Find the Transfer function $\frac{C}{R}$.



Step 1: Combining Cascaded & llel blocks.

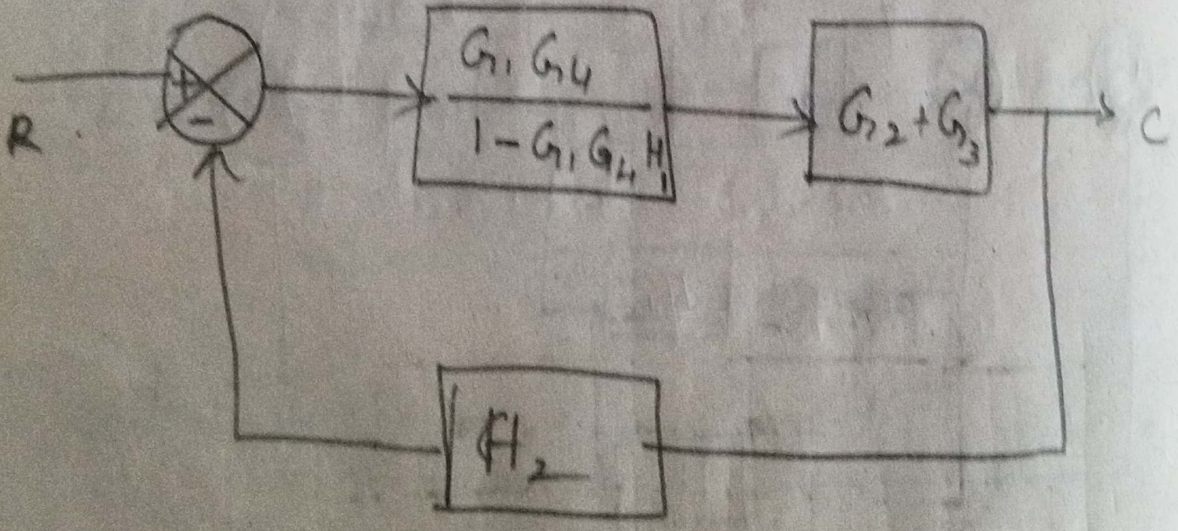


~~Step 2:~~

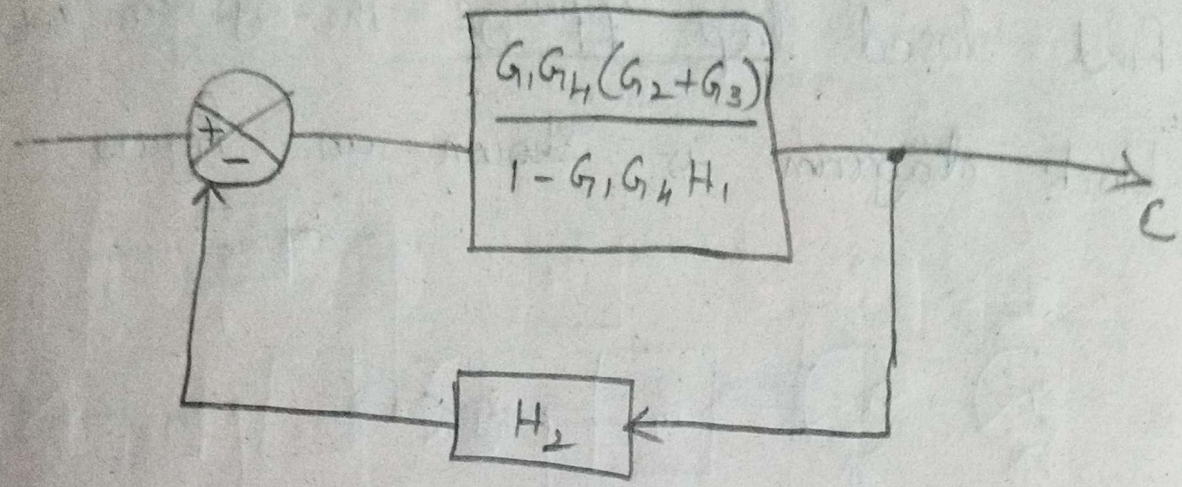


Step 2:

Eliminate feedback loop

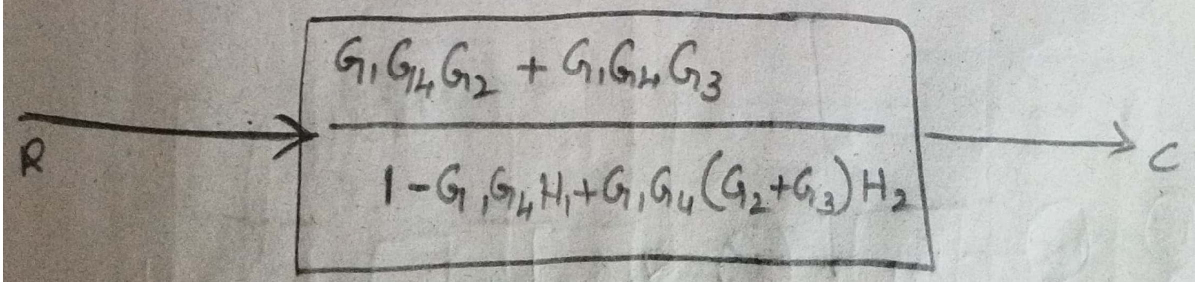


Step 3: Combining Cascaded block.



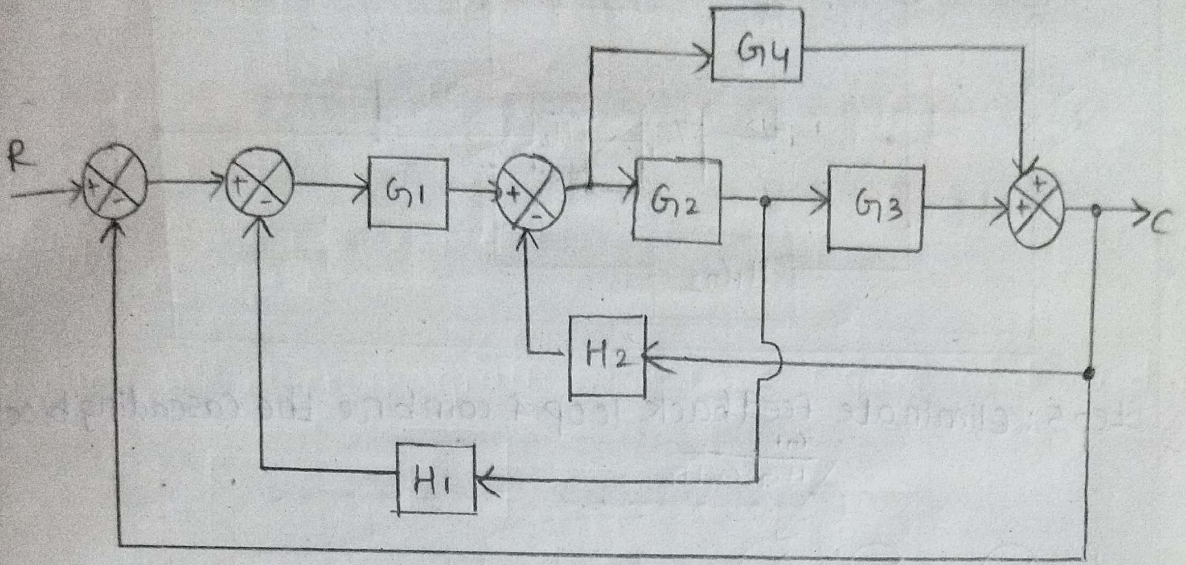
Step 4: Eliminate feedback loop.

$$\frac{\dfrac{G_1G_4(G_2+G_3)}{1-\cancel{G_1G_4}H_1}}{1+\dfrac{G_1G_4(G_2+G_3)}{1-\cancel{G_1}G_4H_1}\times H_2}.$$



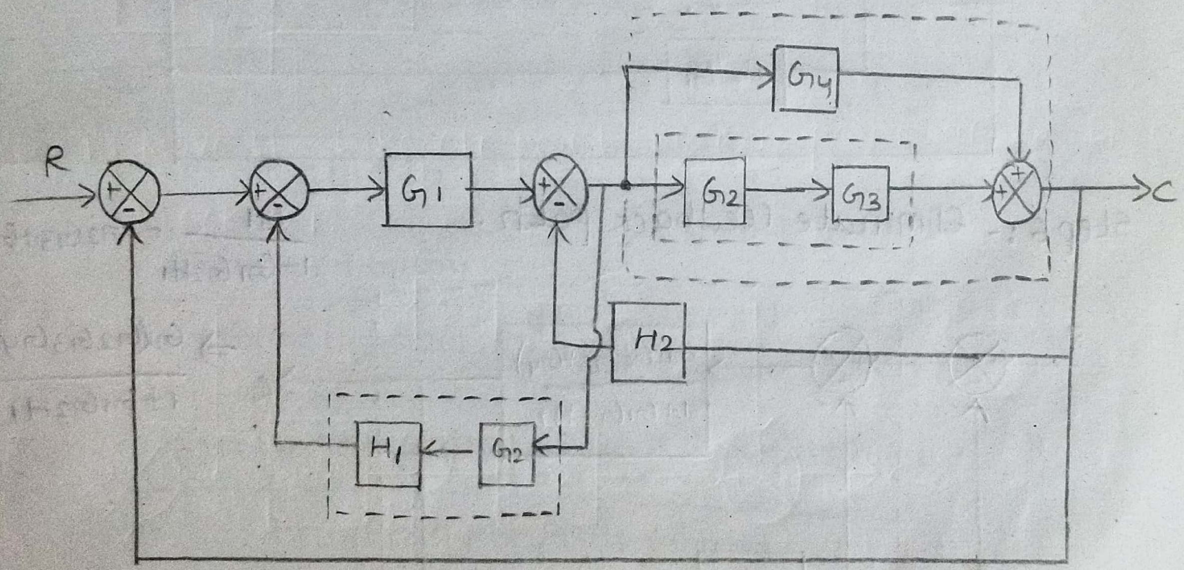
$$\frac{G_1G_4(G_2+G_3)}{1-G_1G_4H_1+G_1G_4(G_2+G_3)H_2}$$

Using block diagram reduction technique find closed loop transfer function of the system whose block diagram is shown in figure.

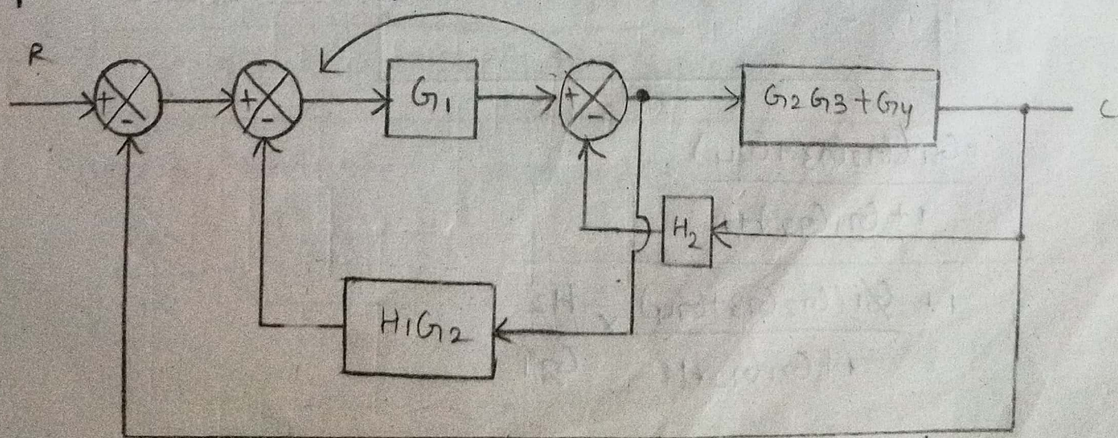


A. Step 1 : Move take-off point before the block G2

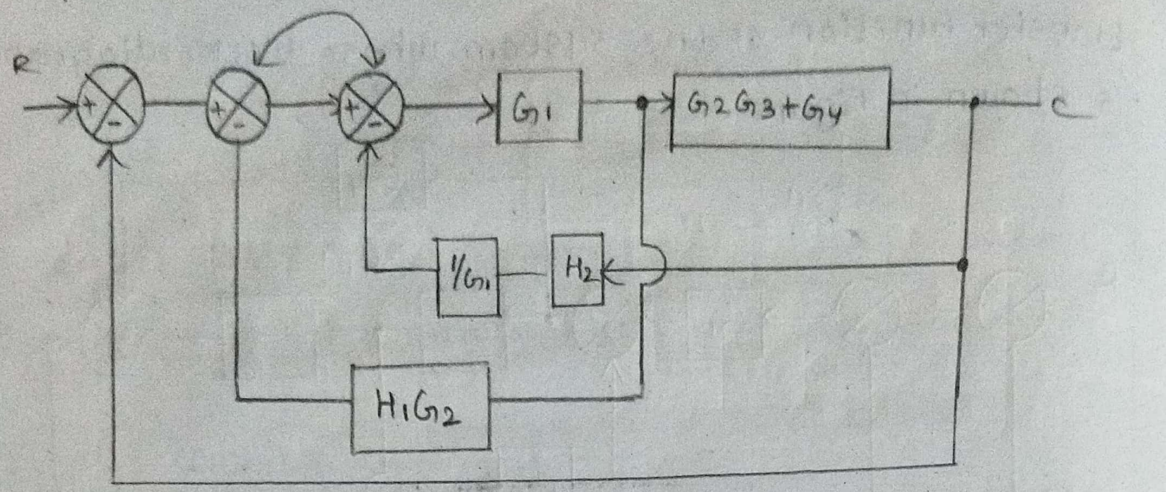
Step2: combine cascaded blocks and parallel blocks.



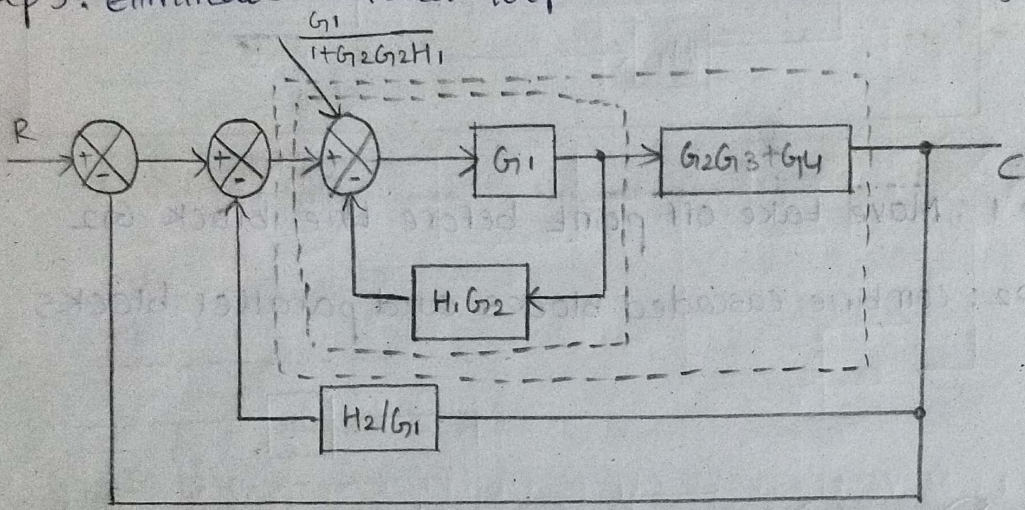
Step3: Move summing point before the block



Step 4:- Interchanging summing point & cascaded the Blocks



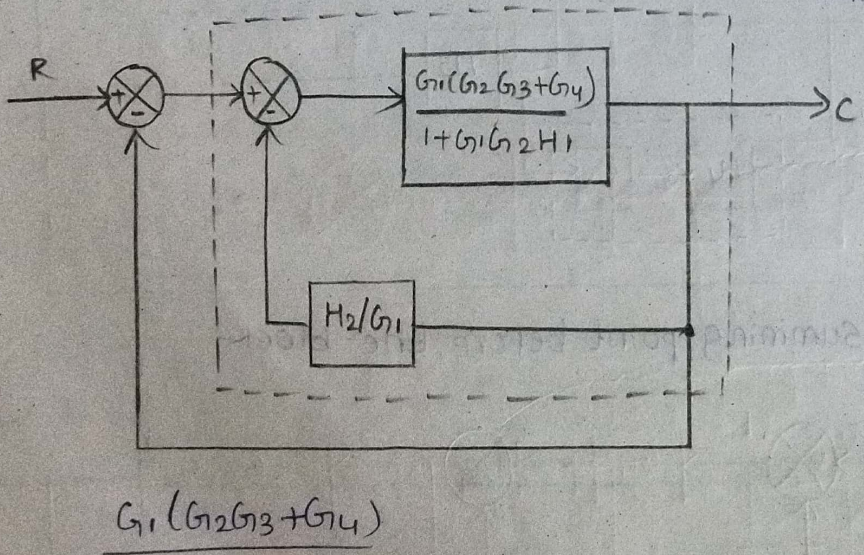
Step 5: Eliminate feedback loop & combine the cascading blocks



Step 6:- Eliminate feedback path.

$$\frac{G_1}{1+G_1G_2H_1} = G_2G_3+G_4$$

$$\Rightarrow \frac{G_1(G_2G_3G_4)}{1+G_1G_2H_1}$$

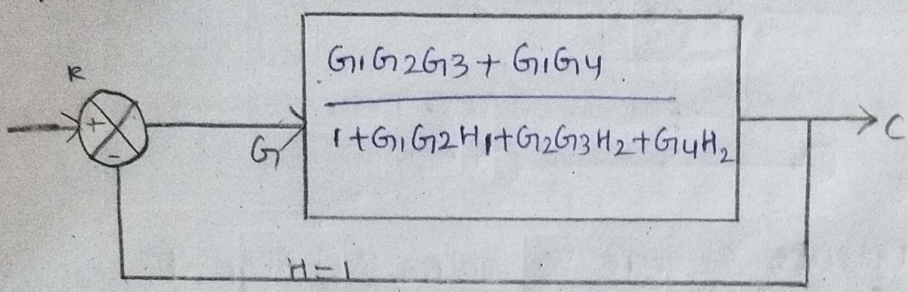


$$\frac{\dfrac{G_1(G_2G_3+G_4)}{1+G_1G_2H_1}}{1+\dfrac{\cancel{G_1}(G_2G_3+G_4)}{1+G_1G_2H_1}\times\dfrac{H_2}{\cancel{G_1}}}$$

$$\frac{\dfrac{G_1G_2G_3+G_1G_4}{\cancel{1+G_1G_2H_1}}}{\dfrac{1+G_1G_2H_1+G_2G_3H_2+G_4H_2}{\cancel{1+G_1G_2H_1}}}$$

$$\Rightarrow \frac{G_1G_2G_3+G_1G_4}{1+G_1G_2H_1+G_2G_3H_2+G_4H_2}$$

Step 7: Eliminate feedback path.

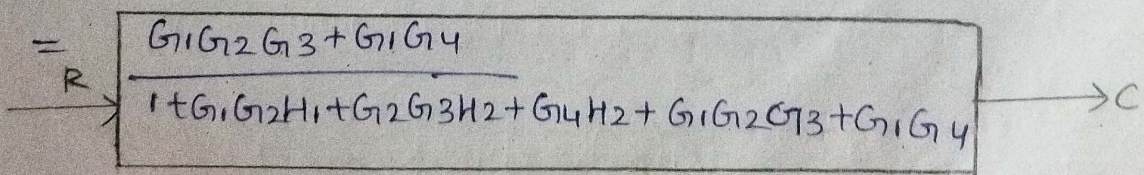


$$\frac{\dfrac{G_1G_2G_3+G_1G_4}{1+G_1G_2H_1+G_2G_3H_2+G_4H_2}}{1+\dfrac{G_1G_2G_3+G_1G_4}{1+G_1G_2H_1+G_2G_3H_2+G_4H_2}\ (1)}$$

Here H=1

$$=\frac{\dfrac{G_1G_2G_3+G_1G_4}{\cancel{1+G_1G_2H_1+G_2G_3H_2+G_4H_2}}}{\dfrac{1+G_1G_2H_1+G_2G_3H_2+G_4H_2+G_1G_2G_3+G_1G_4}{\cancel{1+G_1G_2H_1+G_2G_3H_2+G_4H_2}}}$$

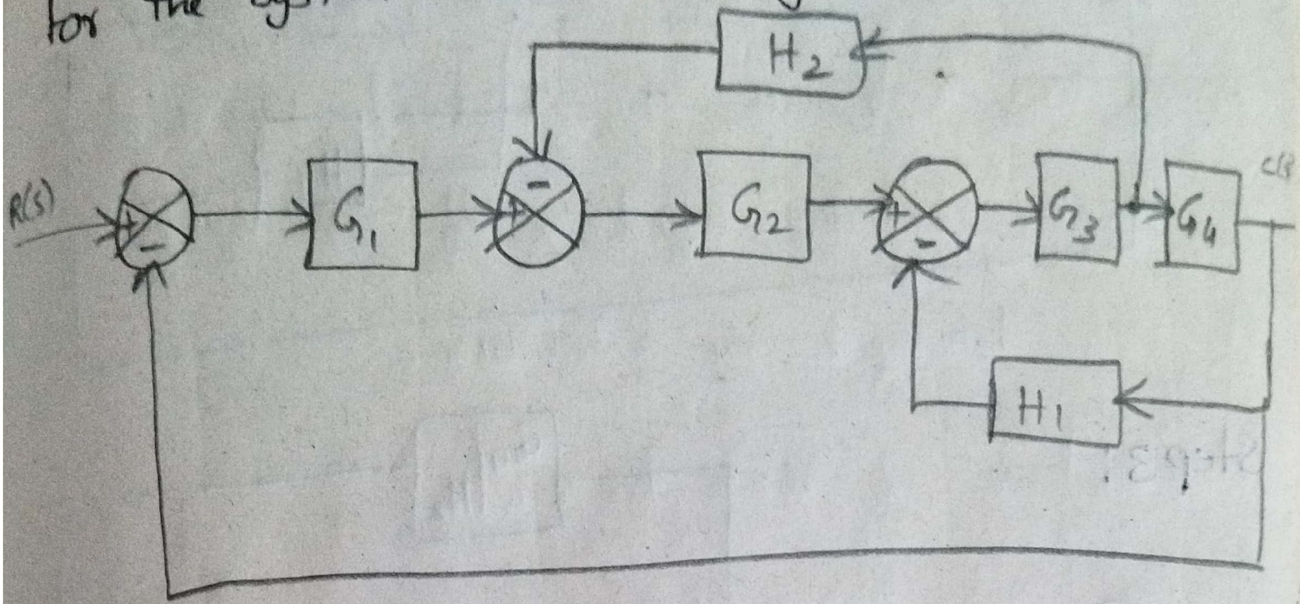
=



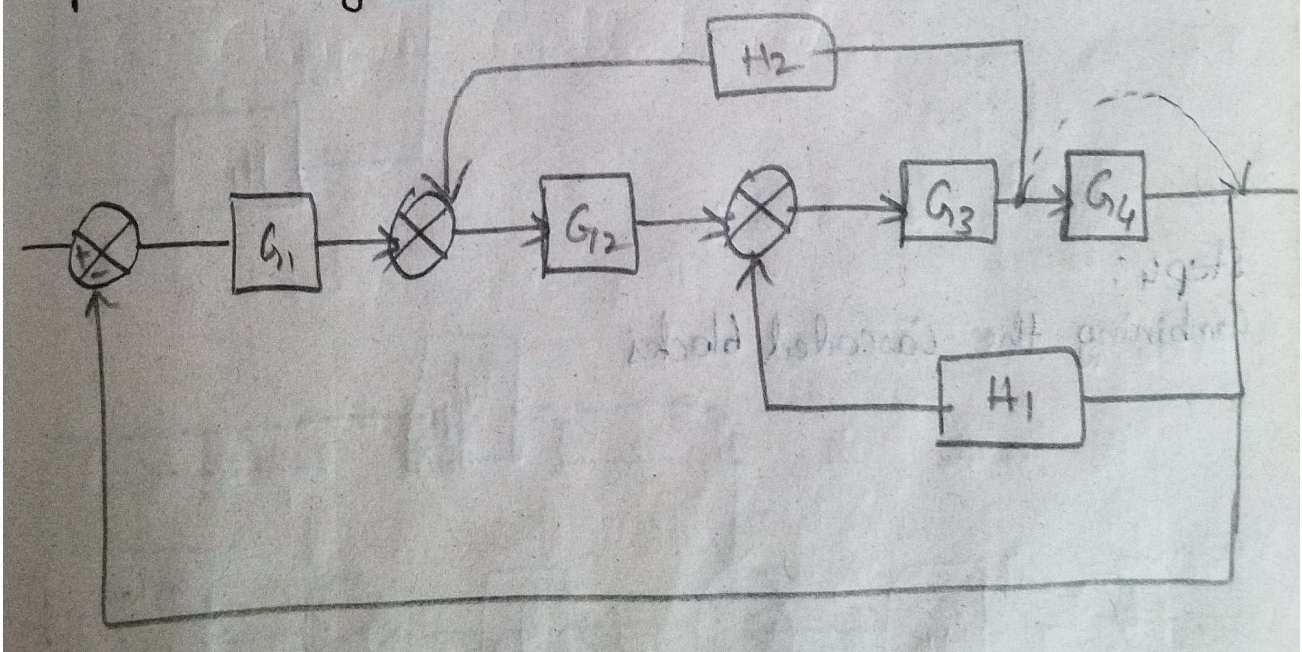
The over all transfer Function is

$$\frac{C}{R}=\frac{G_1G_2G_3+G_1G_4}{1+G_1G_2H_1+G_2G_3H_2+G_4H_2+G_1G_2G_3+G_1G_4}$$

Determine the overall Transfer function $\frac{C(s)}{R(s)}$ for the System shown in figure.



Step 1: Moving take off Point after the block



Step 2: Combining Cascaded blocks

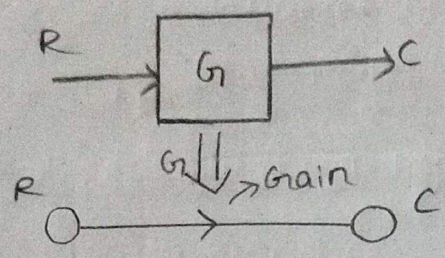
↑

multiplication

Signal Flow Graph (SFG):-

Signal flow graph is a graphical representation of control system. It was developed by a S.J. Mason.

→ The drawback of block diagram reduction technique is lengthy and time consuming.



→ Node represents system variables (or) signal

→ A line that joints two nodes is called branch. It consists an arrow that indicates the direction of signal or gain of that branch.

→ A signal flow graph consists of network in which nodes are connected by branches.

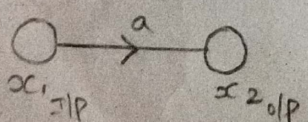
Elements of signal flow graph:-

Types of nodes:

1. Input node or source node
2. Output node or sink node
3. Mixed node or chain node
4. Dummy node.

Input node:

A node that has only outgoing branches

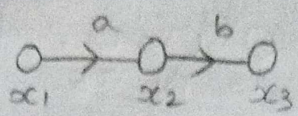


Output node:

A node that has only incoming branches.

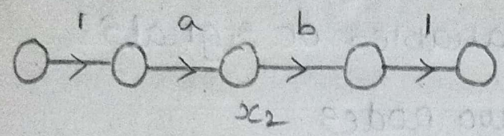
Mixed node :-

A mixed node have both incoming and outgoing branches.



Dummy node :-

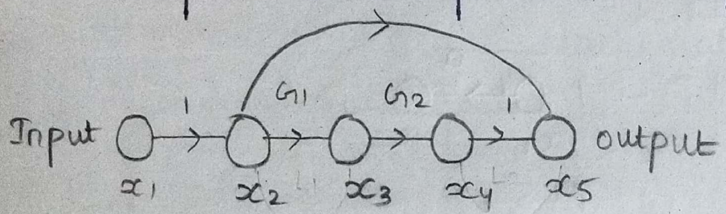
If there is no i/p or o/p node in SFG a node is created by adding a branch with gain one



Path :-

continuous traversal from input node to output node without repeating more than once.

1. Forward path: It is a path from input node to o/p node

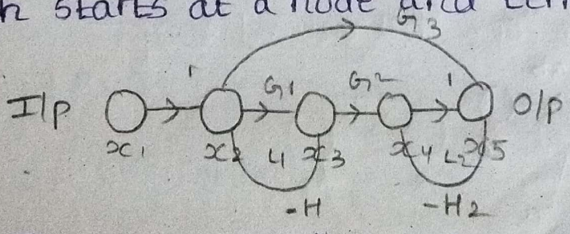


2. Forward path gain:

It is the product of branch gain of forward path

3. Feedback path/closed path/loop:

It is a path starts at a node and terminates at same node

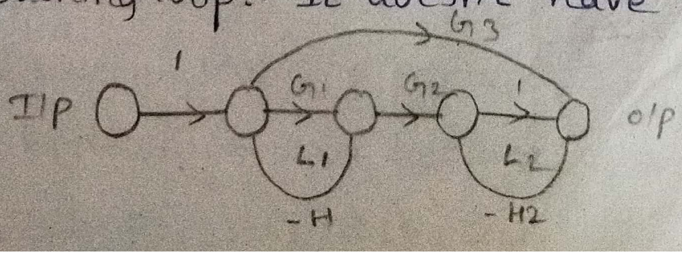


4. Loop gain: It is the product of branch gain of loop

$L_1 = -G_1H$

$L_2 = -H_2$

5. Non touching loop :- It doesn't have common node.



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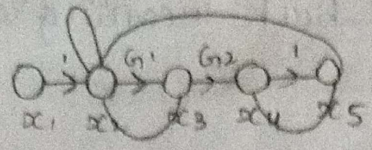
6. Self Loop :- It is feedback loop that has one node.



Properties of SFG :-

1. It is applicable to only linear systems.

2. By using algebraic equation SFG can be constructed

3. A Node representes a variables or signals.

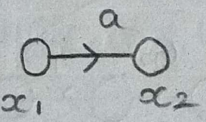
4. A line that joints two nodes.

5. A node that adds all incoming branches and transmit sum to all outgoing branches.

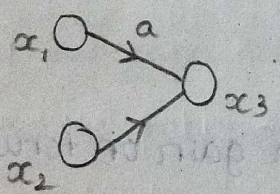
6. It is not unique to a system.

Rules of SFG :-

Rule 1: $x_2 = a x_1$

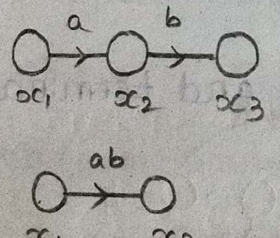


~~Rule~~ 2:

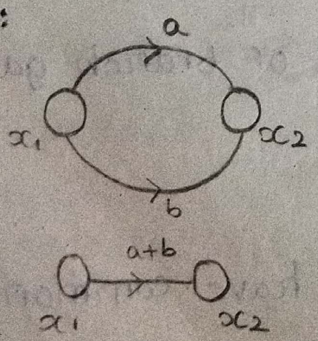


$x_3 = a x_1 + b x_2$

Rule 2:

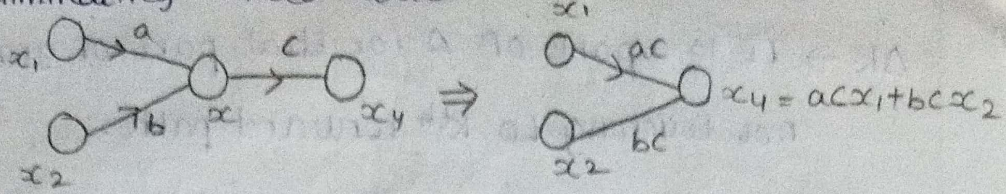


Rule 3:

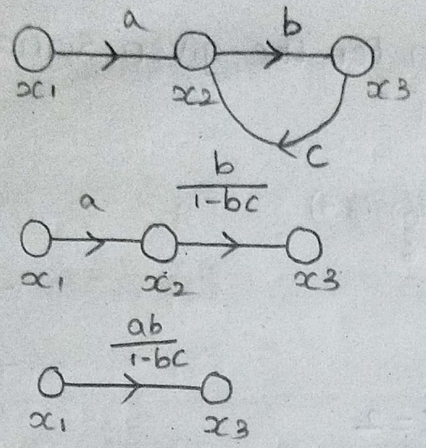


Eliminating mixed node

Rule 4:



Rule 5:



Mason's Gain Formula:-

It is used to determine over all transfer function or gain of signal flow graph. Mathematically it states that

over all gain = $\boxed{T = \frac{1}{\Delta} \sum_{K} P_K \Delta_K}$

where $T = T(S) = \frac{C(S)}{R(S)}$

Now $K \rightarrow$ Number of forward path

Ex: $K = 2$

$$T = \frac{1}{\Delta} P_1 \Delta_1 + P_2 \Delta_2$$

$P_K \rightarrow$ Gain of k^{th} forward path

$\Delta \rightarrow$ Determinant of SFG

where, $\Delta = 1 - \sum$ individual loop gain $+ \sum$ product of gain of all possible combination of two non-touching loop $- \sum$ product of gain of all possible combination of three non-touching loop $+ \sum$ product of gain of all possible combination of four non-touching loop.

$$\Delta = 1 - \sum L_1 + \sum L_2 - \sum L_3 + \sum L_4$$

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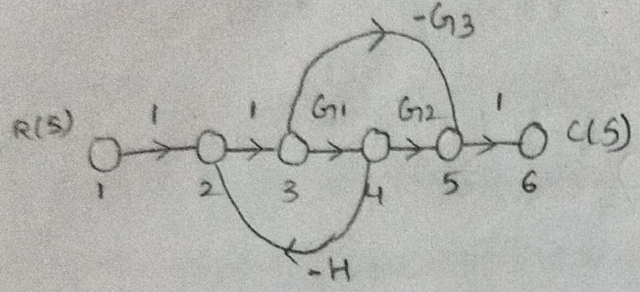
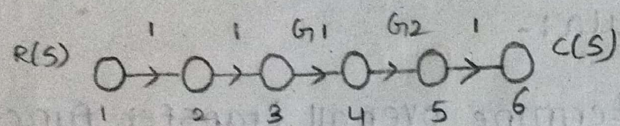
$\Delta K \rightarrow$ It is a part of Δ for that part of graph which is not touching to k^{th} forward path.

Problems:-

1. Find the transfer function for the given signal flow graph.

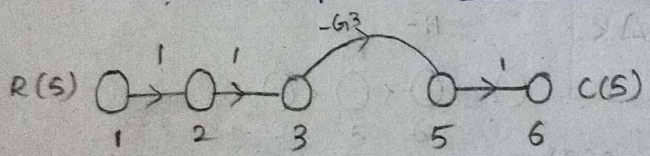


A. Step1: No. of forward path $K=2$



Forward path 1

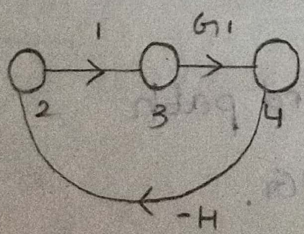
Gain of forward path 1 $\boxed{P_1 = G_1 G_2}$



Forward path 2

Gain of forward path 2 $\boxed{P_2 = -G_3}$

Step2: There are one individual loop



Gain of individual loop 1; $\boxed{P_{11} = -G_1 H}$

Step3: There is no non-touching loop, so there is no loop gain

Step 4: Calculate Δ

$\Delta = 1 - P_{11}$

$$\boxed{\Delta = 1 + G_1 H}$$

Step 5: Calculate ΔK

$\Delta_1 = 1 - 0 = 1$

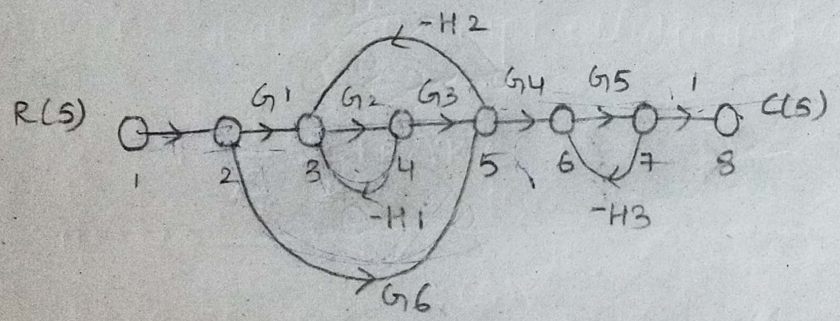
$\Delta_2 = 1 - 0 = 1$

Step 6: Calculate transfer function

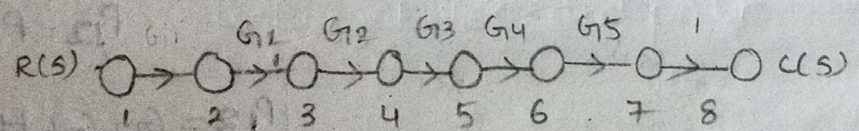
$$T = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta}$$

$$\boxed{T = \frac{G_1 G_2 - G_3}{1 + G_1 H}}$$

2. Find the overall transfer transfer function of the system whose signal flow graph is shown in figure.

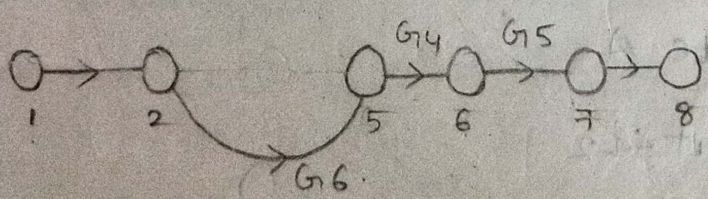


A. Step 1: No. of forward path $k = 2$



Forward path 1

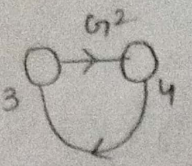
Gain of forward path $P_1 = \boxed{G_4 G_2 G_1 G_3 G_5}$



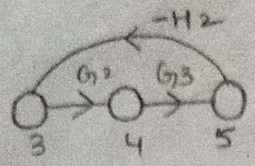
forward path 2

Gain of forward path 2 ; $\boxed{P_2 = G_6 G_4 G_5}$

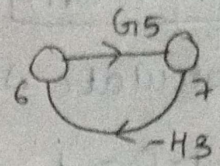
Step 2: Number of individual loops = 3



loop 1



loop 2



loop 3

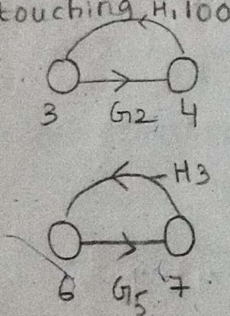
Loop gain of individual loop is $P_{11} = -G_2H_1$

$P_{21} = -G_2G_3H_2$

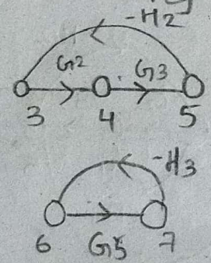
$P_{31} = -G_5H_3$

Step 3: There are two possible combination of two non-touching loops.

First combination of two non-touching H1 loop



Second combination of two non-touching loop



Gain of first combination of two non-touching loop =

$P_{12} = P_{11}P_{31}$

$\boxed{P_{12} = G_2G_5H_1H_3}$

Gain of second combination of two non-touching loop

$\boxed{P_{22} = G_2G_3G_5H_2H_3}$

Step 4: Calculate Δ

$\Delta = 1 - \Sigma L_1 + \Sigma L_2$

$\Delta = 1 - (P_{11} + P_{21} + P_{31}) + (P_{12} + P_{22})$

$\Delta = 1 - (-G_2H_1 - G_2G_3H_2 - G_5H_3) + (G_2G_5H_1H_3 + G_2G_3G_5H_2H_3)$

$\Delta = 1 + G_2H_1 + G_2G_3H_2 + G_5H_3 + G_2G_5H_1H_3 + G_2G_3G_5H_2H_3$

Step 5:- Calculate Δk

$\Delta_1 = 1 - 0 = 1$

$\Delta_2 = 1 + G_2 H_1$

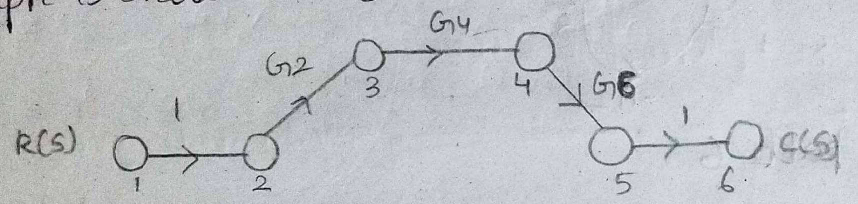
Step 6:

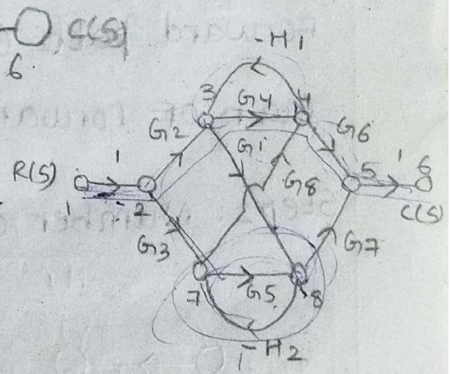
$$T = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta}$$

$$= \frac{G_1 G_2 G_3 G_4 G_5 (1) + G_4 G_6 G_5 (1 + G_2 H_1)}{1 + G_2 H_1 + G_2 G_3 H_2 + G_5 H_3 + G_2 G_5 H_1 H_3 + G_2 G_3 G_5 H_2 H_3}$$

$$= \frac{G_1 G_2 G_3 G_4 G_5 + G_4 G_6 G_5 + G_4 G_6 G_5 G_2 H_1}{1 + G_2 H_1 + G_2 G_3 H_2 + G_5 H_3 + G_2 G_5 H_1 H_3 + G_2 G_3 G_5 H_2 H_3}$$

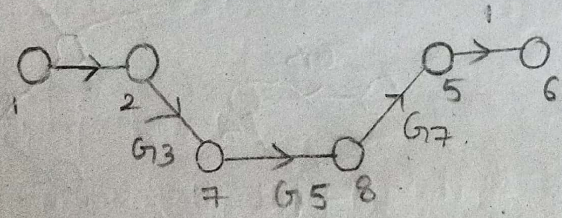
3. Find the overall gain of the system whose signal flow graph is shown in figure.

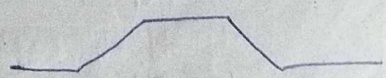




No. of forward path = 1

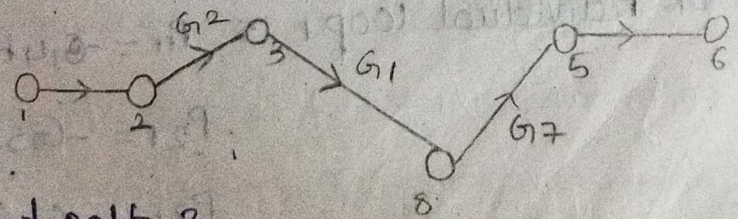
Gain of forward path 1 = $G_2 G_4 G_6$





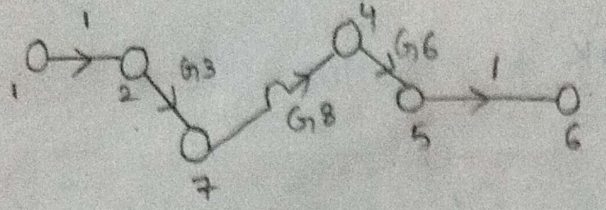
~~No of~~ Forward path 2

Gain of forward path 2 = $G_3 G_5 G_7$.



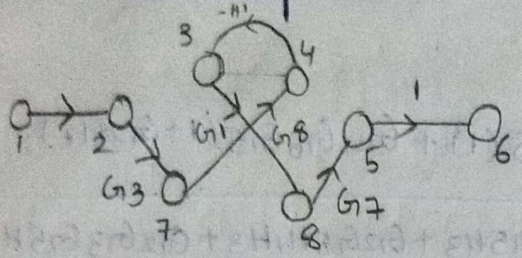
Forward path 3

Gain of forward path 3 = $G_2 G_1 G_7$



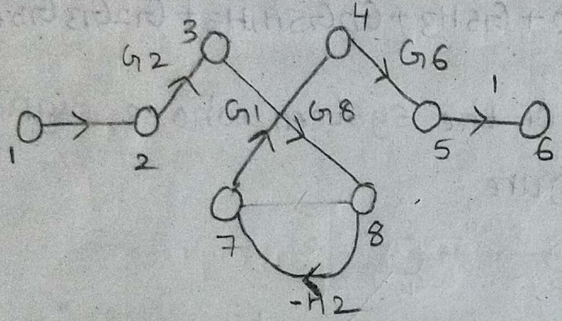
Forward path 4

Gain of Forward path 4 = G3G6G8.



Forward path 5

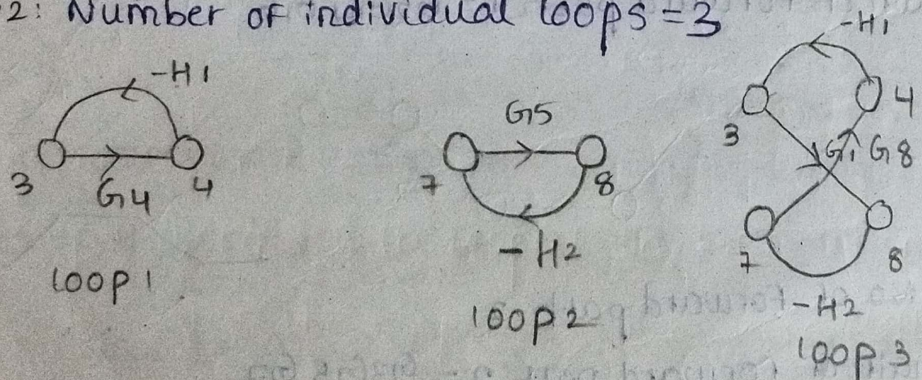
Gain of Forward path 5 = $G_1G_3G_8G_7H_1$



Forward path 6

Gain of forward path 6 = $-G_2G_1G_6G_8H_2$

Step 2: Number of individual loops = 3

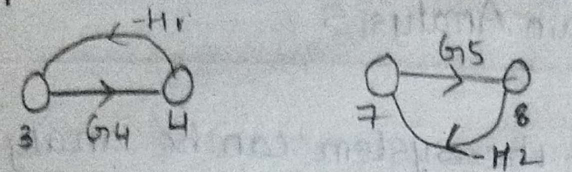


Loop gain of individual loop 1 is $P_{11} = -G_4H_1$

$P_{21} = -G_5H_2$

$P_{31} = G_1G_8H_1H_2$

Step 3: There are two non-touching loop



$P_{12} = P_{11} P_{21}$

$P_{12} = G_4 G_5 H_1 H_2$

Step 4: calculate Δ

$$\Delta = 1 - \Sigma L_1 + \Sigma L_2$$

$$\Delta = 1 - (P_{11} + P_{21} + P_{31}) + P_{12}$$

$$\Delta = 1 - (-G_4 H_1 - G_5 H_2 + G_1 G_8 H_1 H_2) + G_4 G_5 H_1 H_2$$

$$\Delta = 1 + G_4 H_1 + G_5 H_2 - G_1 G_8 H_1 H_2 + G_4 G_5 H_1 H_2$$

Step 5: calculate ΔK

$\Delta_1 = 1 + G_5 H_2$ $\quad$ $\Delta_4 = 1 - 0 = 1$

$\Delta_2 = 1 + G_4 H_1$ $\quad$ $\Delta_5 = 1 - 0 = 1$

$\Delta_3 = 1 - 0 = 1$ $\quad$ $\Delta_6 = 1 - 0 = 1$

Step 6 :-

Mason's Gain formula,

$$T = \frac{1}{\Delta} \sum_K P_K \Delta_K$$

$$T = \frac{P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3 + P_4\Delta_4 + P_5\Delta_5 + P_6\Delta_6}{\Delta}$$

$$= \frac{G_2 G_4 G_6 (1 + G_5 H_2) + G_3 G_5 G_7 (1 + G_4 H_1) + G_2 G_1 G_7 (1) + G_3 G_6 G_8 (1) + G_1 G_3 G_8 G_7 H_1 + (-G_2 G_1 G_6 G_8 H_2)}{1 + G_4 H_1 + G_5 H_2 - G_1 G_8 H_1 H_2 + G_4 G_5 H_1 H_2}$$

$$\therefore T = \frac{G_2 G_4 G_6 + G_2 G_4 G_6 G_5 H_2 + G_3 G_5 G_7 + G_3 G_5 G_7 G_4 H_1 + G_2 G_1 G_7 + G_3 G_6 G_8 + G_1 G_3 G_8 G_7 H_1 - G_2 G_1 G_6 G_8 H_2}{1 + G_4 H_1 + G_5 H_2 - G_1 G_8 H_1 H_2 + G_4 G_5 H_1 H_2}$$