Time Domain Analysis

The performance of the system can be analyses in two ways i) Time response Analysis

ii) Frequency response Analysis

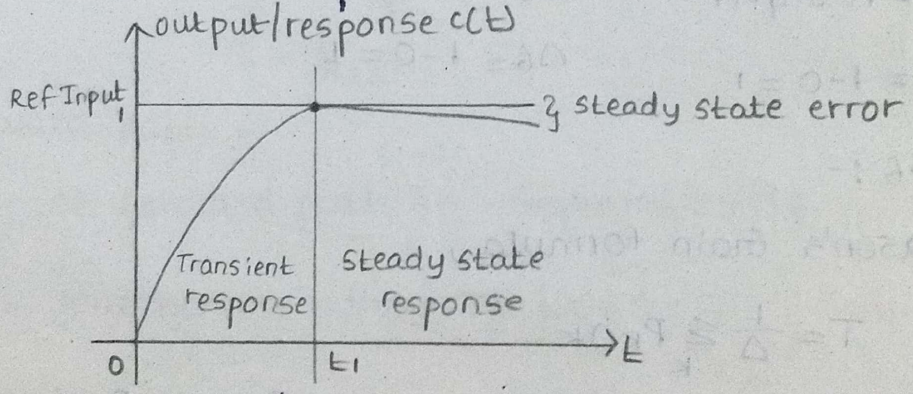
Time response :- It is the response of the system as a function of time when standard test input signal are applied to it.

It is denoted by $c(t)$.

Time response are of two types. They are

i) Transient response

ii) Steady state response



Transient Response :-

output changes with respect to time when time t_1 becomes large the response will be zero. Mathematically

$$\boxed{c(t) = c_t(t) + c_{ss}(t)}$$

$\lim_{t \to \infty} c_t(t) = 0$ Transient response may be exponential or oscillator.

Steady state Response :-

It is the part of total response after transient response died out.

EX: $c(t) = \underbrace{5}_{\text{SSR}} + \underbrace{e^{-2t}u(t)}_{\text{T.R}}$

SSR — steady state response; T.R — Transient response

Type and order of the system:-

The general form of transfer function is

$$\boxed{T(s) = \frac{C(s)}{R(s)} = \frac{b_0 s^m + b_1 s^{m-1} + b_2 s^{m-2} + \cdots + b_m}{a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \cdots + a_n}}$$

If the highest power of 's' in the denominator polynomial of the transfer function indicates order of the system.

EX: ~~when~~ If $n=0$ ~~or n=2~~ the system is zero order system

~~when~~ If $n=1$ the system is first order system

~~when~~ If $n=2$ the system is second order system

EX: $\frac{C(s)}{R(s)} = \frac{5}{4s+1}$

It is first order system.

EX: $\frac{C(s)}{R(s)} = \frac{10s}{s^2+4s+4}$

It is second order system.

EX: $\frac{C(s)}{R(s)} = \frac{10s^2}{3s^4+2s^2+4s+3}$

It is fourth order system.

$T(s) = K \cdot \frac{P(s)}{Q(s)}$

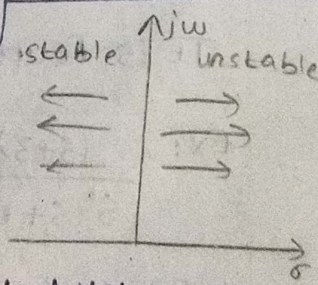
K = Constant

P(s) = Numerator polynomial

Q(s) = Denominator "

If $Q(s) = a_0 s^n + a_1 s^{n-1} + \cdots + a_{n-1}s + a_n$.

then n is order of system



Poles and zeroes of the system:-

Poles and zeroes determines the stability of the system. The poles lies on left hand side of system is stable. The poles lies on right hand side of system is unstable.

The general transfer function in factor form is

$$T(S) = \frac{C(S)}{R(S)} = \frac{K(S+Z_1)(S+Z_2)\cdots(S+Z_m)}{S^N(S+P_1)(S+P_2)\cdots(S+P_n)}$$

For the varies values of 'S'

Transfer function $\rightarrow 0$ or ∞

Poles :- The values of 'S' that makes transfer function indicates as 'X'

zero :- The values of 'S' that makes transfer function zero. zero can be represented as '0'.

Type of the system :-

No. of poles at origin. It is applicable to only open loop transfer function.

Ex: $\frac{50}{(S+1)(S+2)}$. $\frac{S+2}{S(S+1)}$

No pole at origin so it is type zero system.

It is second order system.

Ex: $G(S)\,H(S) = \frac{100(S+5S^2)}{S^3(1+7S)}$

It is type 3 system. There is 3 poles at origin.

It is fourth order system.

Ex: $\frac{(S+3)}{S^2(S+1)(S+2)}$

It is type 2 system. There is 2 poles at origin.

It is fourth order system.

NOTE:

order → open loop and closed loop system.

Type → open loop system.

$G(S)$ = open loop transfer function of open loop system.

$G(S)H(S)$ = open loop transfer function of closed loop

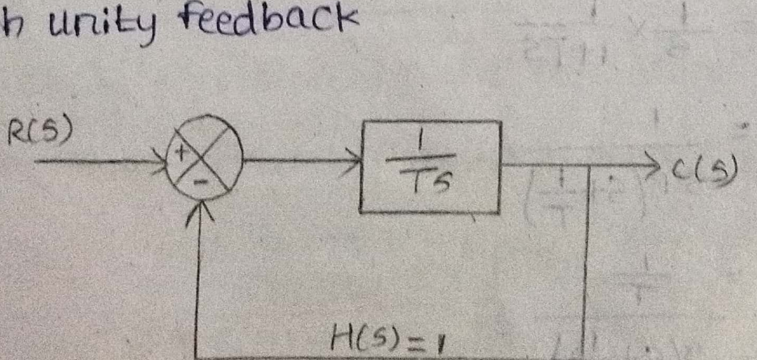
$$\frac{G(S)}{1+G(S)H(S)} = \text{closed loop transfer function.}$$

Standard Test signals:-

Signal	Time domain signal $r(t)$		Laplace transform of signal $r(s)$
Step $u(t)$	A		$\frac{A}{S}$
unit step	1		$\frac{1}{S}$
Ramp $r(t)$	At	ramp → step	$\frac{A}{S^2}$
unit Ramp	t	$r(t) = t\,u(t)$	$\frac{1}{S^2}$
Parabolic $p(t)$	$\frac{At^2}{2}$	$p(t) = \frac{t^2}{2}u(t)$	$\frac{A}{S^3}$
unit parabolic	$\frac{t^2}{2}$		$\frac{1}{S^3}$
Impulse $\delta(t)$	$\delta(t)$		1

First order response for unit step input:-

Consider first order closed loop control system with unity feedback



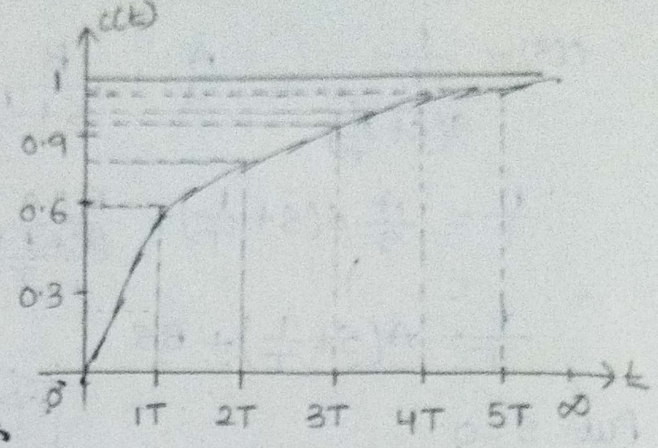
$L[1] = 1/s$

$L[t] = 1/s^2$

$L[t^2/2] = 1/s^3$

$L\left[\frac{1}{s+a}\right] = e^{-at}$

t	c(t)
0	0
1T	0.632
2T	0.865
3T	0.95
4T	0.9817
5T	0.993
∞	1



At time 5T the system response is attained the steady state. The response is purely exponential]

Time domain specifications of second order system:-

The performance characteristics of control system are specified in terms of time domain specification.

The time domain specifications are:

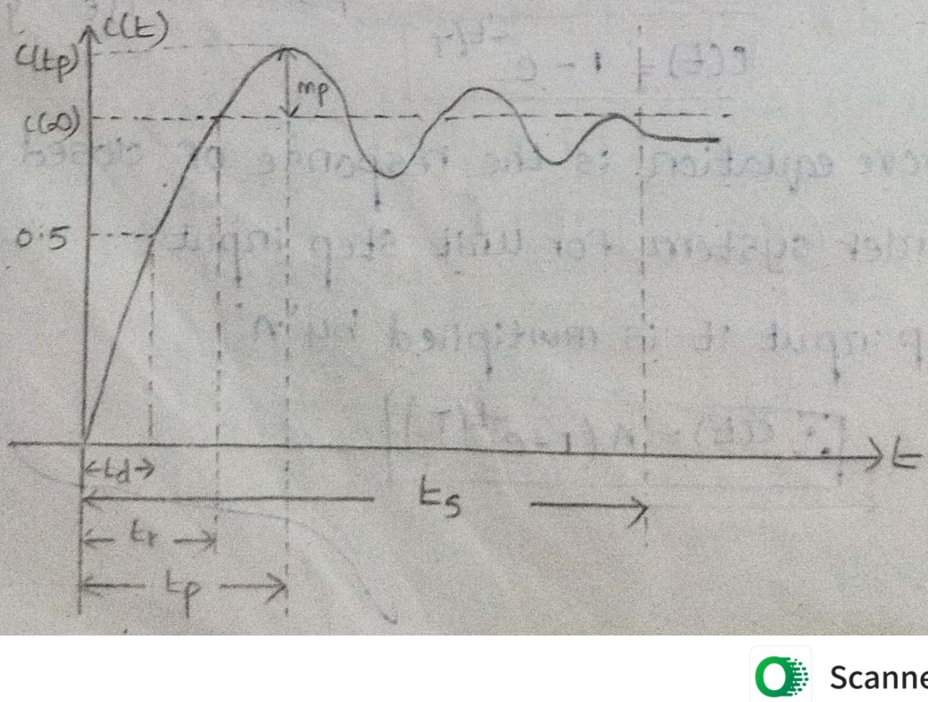
i) Delay time t_d

ii) Rise time t_r

iii) Peak time t_p

iv) Peak over shoot

v) settling time t_s



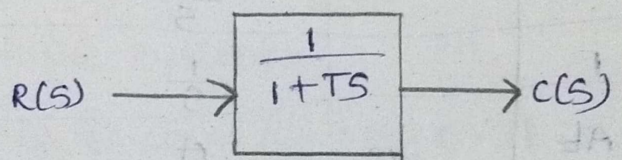
The closed loop transfer function of first order system is

$$T(S) = \frac{C(S)}{R(S)} = \frac{G(S)}{1+G(S)\,H(S)}$$

$$= \frac{\frac{1}{TS}}{1+\frac{1}{TS}(1)}$$

$$= \frac{1/\cancel{TS}}{\frac{TS+1}{\cancel{TS}}}$$

$$\frac{C(S)}{R(S)} = \frac{1}{TS+1} \longrightarrow (1)$$



In above equation the power of 's' in denominator is one. Hence it is called as first order system.

Now unit step input $r(t) = 1$

Laplace transform of unit $r(t)$ $L\{r(t)\} = R(S) = \frac{1}{S}$

The response in s domain is

From eq (1)

$$C(S) = R(S)\,\frac{1}{1+TS}$$

$$C(S) = \frac{1}{S} \times \frac{1}{1+TS}$$

$$C(S) = \frac{1}{ST\left(S+\frac{1}{T}\right)}$$

$$C(S) = \frac{\frac{1}{T}}{S\left(S+\frac{1}{T}\right)}$$

By taking partial fraction

$$C(S) = \frac{\frac{1}{T}}{S(S+\frac{1}{T})} = \frac{A}{S} + \frac{B}{S+\frac{1}{T}}$$

$$\frac{1}{T} = \frac{A}{\cancel{S}} \cancel{S}(S+\frac{1}{T}) + \frac{B}{\cancel{(S+\frac{1}{T})}} S\cancel{(S+\frac{1}{T})}$$

$$\frac{1}{T} = A(S+\frac{1}{T}) + BS$$

Put $S=0$

$$\frac{1}{T} = A(0+\frac{1}{T}) + B(0)$$

$$\cancel{\frac{1}{T}} = A(\cancel{\frac{1}{T}})$$

$$\boxed{A=1}$$

Put $S = -\frac{1}{T}$

$$\frac{1}{T} = A(\frac{-1}{T}+\frac{1}{T}) + B(\frac{-1}{T})$$

$$\frac{1}{T} = A(\frac{0}{T}) + B(\frac{-1}{T})$$

$$\cancel{\frac{1}{T}} = B(\cancel{\frac{-1}{T}})$$

$$\boxed{B=-1}$$

$$\boxed{C(S) = \frac{1}{S} - \frac{1}{S+\frac{1}{T}}}$$

error response

$e(t) = r(t) - c(t) = e^{-t/\tau}$

steady state error, e_{ss}

$$e_{ss} = \lim_{t\to\infty} e(t)$$

$$= \lim_{S\to 0} sE(s)$$

$$= 0$$

Apply inverse Laplace transform,

$$L^{-1}\{C(S)\} = C(t) = L^{-1}\left\{\frac{1}{S} - \frac{1}{S+\frac{1}{T}}\right\}$$

$$\because L\{e^{-at}\} = \frac{1}{S+a}$$

$$\boxed{C(t) = 1 - e^{-t/T}}$$

The above equation is the response of closed loop first order system for unit step input.

For step input it is multiplied by 'A'

$$\boxed{\therefore C(t) = A(1-e^{-t/T})}$$

i) Delay time (td):- It is defined as the time taken for the response to reach 50% for the final value at very first time

ii) Rise time (tr) :- It is defined as time taken for the response to rise from 0 to 100% for the very first time.

0% to 100% - under damped system

10% to 90% - over damped system

5% to 95% - critically damped system.

iii) Peak time (tp) :- It is the time taken for the response to reach peak value for very first time

iv) Peak over shoot :- It is defined as the deviation from final value. It can be expressed as

$$\% \text{ Peak overshoot} = \frac{c(tp) - c(\infty)}{c(\infty)} \times 100$$

$c(\infty)$ - Final value

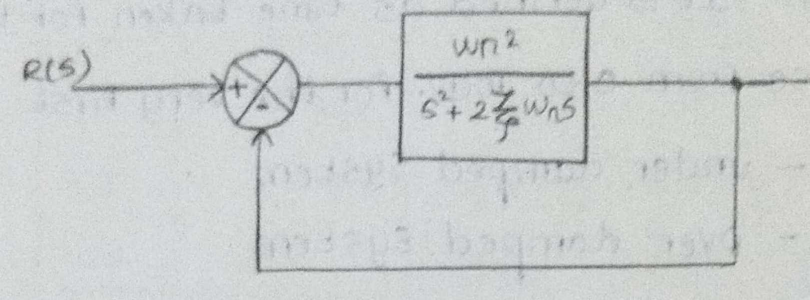
$c(tp)$ - maximum value.

v) Settling time (ts) :- It is defined as the time taken by the response to reach and stay within specified error.

It is usually expressed as percentage of final value

* Time response of second order system :-

The closed loop second order system is shown in figure



$\zeta \rightarrow$ zeta

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$

$$\frac{C(s)}{R(s)} = \frac{\dfrac{\omega_n^2}{s^2+2\zeta\omega_n s}}{1+\dfrac{\omega_n^2}{s^2+2\zeta\omega_n s}\times 1}$$

$$= \frac{\dfrac{\omega_n^2}{\cancel{s^2+2\zeta\omega_n s}}}{\dfrac{s^2+2\zeta\omega_n s+\omega_n^2}{\cancel{s^2+2\zeta\omega_n s}}}$$

The standard form of closed loop transfer function of second order system is

$$\therefore \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

In above equation the power of s in the denominator is '2' hence it is called as second order system.

where $\omega_n \rightarrow$ undamped natural frequency

$\xi \rightarrow$ damping factor or damping ratio.

Damping Ratio:-

It is the ratio of actual damping by critical damping.

characteristic Equation:-

$$\cancel{s^2+2\zeta\omega_n} \quad 1+G(s)H(s)=0$$

The characteristic equation of second order system is

$$s^2+2\zeta\omega_n s+\omega_n^2=0$$

The response $c(t)$ depends on value of ζ

case 1: $\zeta=0$ undamped system.

case 2: $0<\zeta<1$ under damped system

case 3: $\zeta=1$ critically damped system

case 4: $\zeta>1$ over damped system.

Roots or poles of characteristic equation are

$$s^2+2\zeta\omega_n s+\omega_n^2=0$$

$$a=1,\ b=2\zeta\omega_n,\ c=\omega_n^2$$

$$s_1,s_2=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-2\zeta\omega_n\pm\sqrt{(2\zeta\omega_n)^2-4\omega_n^2}}{2}$$

$$=\frac{-2\zeta\omega_n\pm\sqrt{4\zeta^2\omega_n^2-4\omega_n^2}}{2}$$

$$=\frac{\cancel{2}\left[-\zeta\omega_n\pm\omega_n\sqrt{\zeta^2-1}\right]}{\cancel{2}}$$

$$\boxed{s_1,s_2=-\zeta\omega_n\pm\omega_n\sqrt{\zeta^2-1}}$$

1. Response of undamped second order system for unit step input :-

The standard second order closed loop transfer function is

$$\boxed{\frac{C(S)}{R(S)} = \frac{\omega_n^2}{S^2+2\xi\omega_n S+\omega_n^2}} \longrightarrow ①$$

For undamped system, $\xi = 0$

$$\frac{C(S)}{R(S)} = \frac{\omega_n^2}{S^2+\omega_n^2}$$

when unit step input is applied,

$$r(t) = 1$$

$$\mathcal{L}\{r(t)\} = R(S) = \frac{1}{S}$$

Response in S-domain is,

$$C(S) = R(S) \times \frac{\omega_n^2}{S^2+\omega_n^2}$$

$$C(S) = \frac{1}{S} \times \frac{\omega_n^2}{S^2+\omega_n^2}$$

Apply partial fraction,

$$\boxed{C(S) = \frac{A}{S} + \frac{BS+C}{S^2+\omega_n^2}} \longrightarrow ②$$

$$\frac{\omega_n^2}{S(S^2+\omega_n^2)} = \frac{A}{S} + \frac{BS+C}{S^2+\omega_n^2}$$

$$\frac{\omega_n^2}{\cancel{S(S^2+\omega_n^2)}} = \frac{A(S^2+\omega_n^2)+(BS+C)S}{\cancel{S(S^2+\omega_n^2)}}$$

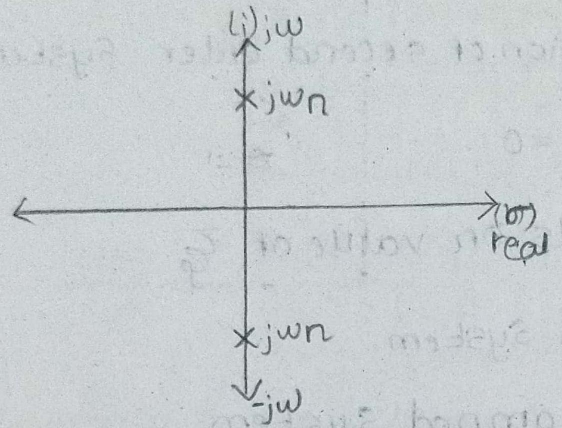
$$\omega_n^2 = AS^2 + A\omega_n^2 + BS^2 + CS$$

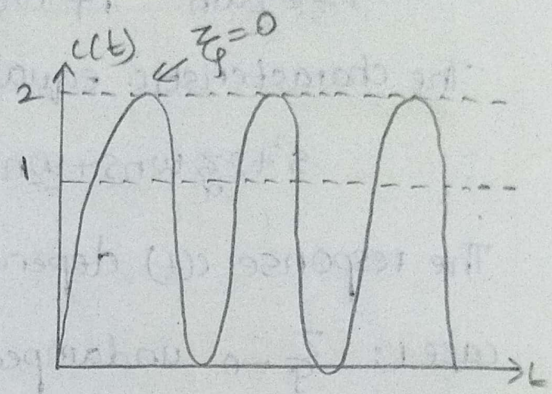
case 1:- when $\zeta = 0$ undamped system.

$$s_1, s_2 = \pm \omega_n \sqrt{-1} \qquad \sqrt{-1} = j$$

$$\boxed{s_1, s_2 = \pm j\omega_n}$$

∴ Roots are purely imaginary.





case 2: $0 < \zeta < 1$ (under damped system)

$$s_1, s_2 = -\zeta\omega_n \pm \omega_n \sqrt{(-1)(1-\zeta^2)}$$

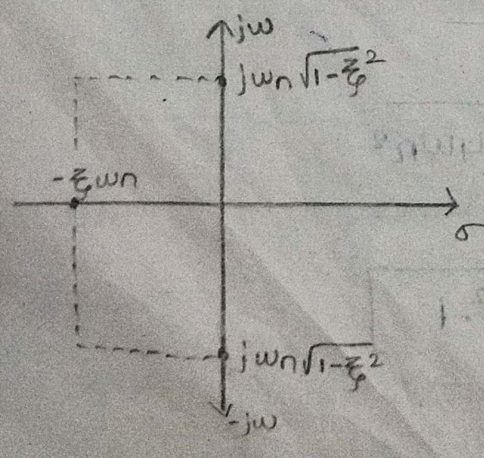
$$\boxed{s_1, s_2 = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}}$$

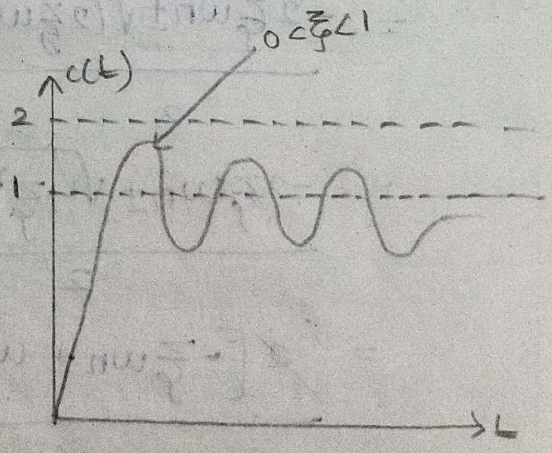
∴ Roots are complex conjugate and the system is under damped system.

Let $\boxed{\omega_d = \omega_n\sqrt{1-\zeta^2}}$

where ω_d = damped frequency

$$s_1, s_2 = -\zeta\omega_n \pm j\omega_d$$



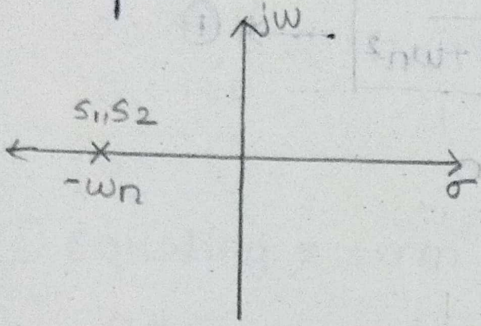


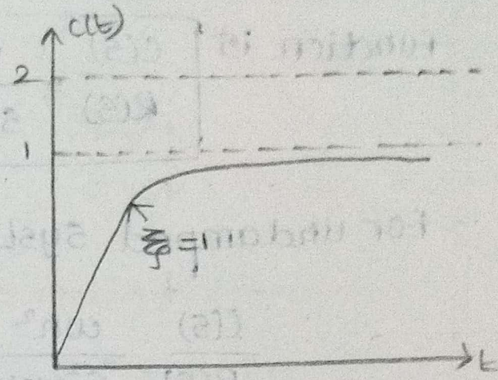
case 3:- $\xi = 1$

$$\boxed{s_1, s_2 = -\omega_n}$$

The system is stable.

∴ Roots are real and equal, the system is critically damped.

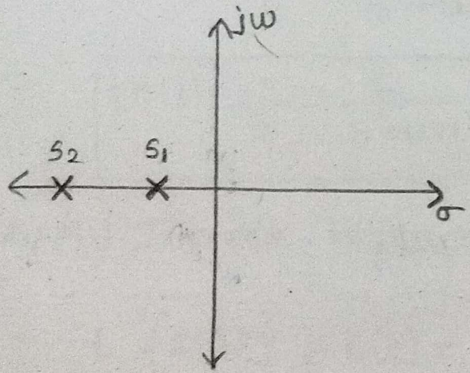


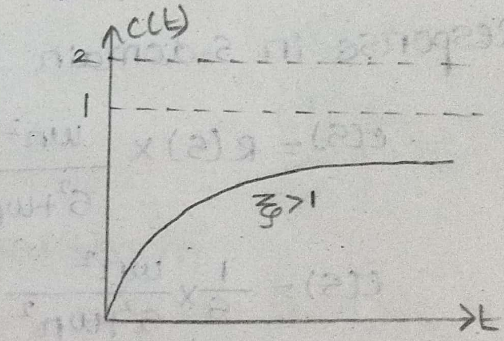


case 4: $\xi > 1$

$$\boxed{s_1, s_2 = -\xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1}}$$

∴ Roots are real and unequal.





∴ The system is stable and over damped.

Response of second order system for unit step input :-

Depending upon value of ξ

1. Response of undamped second order system
2. Response of underdamped second order system for unit step input.
3. Response of critically damped second order system for unit step input.
4. Response of over damped second order system for unit step input.

① Equating constant

$\cancel{\omega_n^2} = A\cancel{\omega_n^2}$

$\boxed{A = 1}$

② Equating s^2 term

$0 = A + B$

$0 = 1 + B$

$\boxed{B = -1}$

③ Equating s term

$0 = C$

$\boxed{C = 0}$

Substitute A, B & C values in eq ②

$$C(s) = \frac{1}{s} + \frac{(-1)s + 0}{s^2 + \omega_n^2}$$

$$L^{-1}\left\{\frac{1}{s}\right\} = 1$$

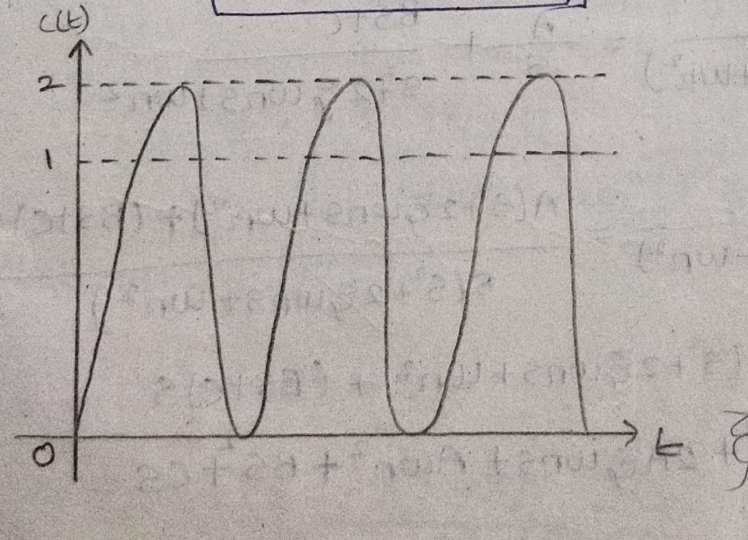
$$\boxed{C(s) = \frac{1}{s} - \frac{s}{s^2 + \omega_n^2}}$$

$$L^{-1}\left\{\frac{s}{s^2 + \omega^2}\right\} = \cos\omega t$$

Apply Inverse Laplace transform

$$L^{-1}\{C(s)\} = c(t) = L^{-1}\left\{\frac{1}{s} - \frac{s}{s^2 + \omega_n^2}\right\}$$

$$\boxed{c(t) = 1 - \cos\omega_n t}$$



$$C(s) = \frac{1}{s} - \left[\frac{s+\xi\omega_n}{(s+\xi\omega_n)^2+\omega_d^2} + \frac{\xi\omega_n}{(s+\xi\omega_n)^2+\omega_d^2}\right]$$

Multiply and divide by ω_d in third term.

$$C(s) = \frac{1}{s} - \left[\frac{s+\xi\omega_n}{(s+\xi\omega_n)^2+\omega_d^2} + \frac{\xi\omega_n\omega_d}{\omega_d(s+\xi\omega_n)^2+\omega_d^2}\right]$$

Apply Inverse Laplace transform

$$L^{-1}\{C(s)\} = L^{-1}\left\{\frac{1}{s}\right\} - L^{-1}\left\{\frac{s+\xi\omega_n}{(s+\xi\omega_n)^2+\omega_d^2}\right\} + L^{-1}\left\{\frac{\xi\omega_n\omega_d}{\omega_d(s+\xi\omega_n)^2+\omega_d^2}\right\}$$

$$c(t) = 1 - \left[e^{-\xi\omega_n t}\cos\omega_d t + \frac{\xi\omega_n}{\omega_d} L^{-1}\left\{\frac{\omega_d}{(s+\xi\omega_n)^2+\omega_d^2}\right\}\right]$$

$$c(t) = 1 - \left[e^{-\xi\omega_n t}\cos\omega_d t + \frac{\xi\omega_n}{\omega_d} e^{-\xi\omega_n t}\sin\omega_d t\right]$$

$$c(t) = 1 - \left[e^{-\xi\omega_n t}\cos\omega_d t + \frac{\xi\cancel{\omega_n}}{\cancel{\omega_n}\sqrt{1-\xi^2}} e^{-\xi\omega_n t}\sin\omega_d t\right]$$

$$c(t) = 1 - \left[e^{-\xi\omega_n t}\cos\omega_d t + \frac{\xi}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t}\sin\omega_d t\right]$$

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left[\sqrt{1-\xi^2}\cos\omega_d t + \xi\sin\omega_d t\right]$$

$\because \sin(A+B) = \sin A\cos B + \cos A\sin B$

$$\boxed{\sin(\omega_d t+\theta) = \sin\omega_d t\cos\theta + \cos\omega_d t\sin\theta}$$

where $\sin\theta = \sqrt{1-\xi^2}$

$\cos\theta = \xi$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\sqrt{1-\xi^2}}{\xi}$$

$$\boxed{\theta = \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)}$$

* 2. Response of underdamped second order system for unit step input.

The standard second order closed loop transfer function is

$$\boxed{\frac{C(S)}{R(S)} = \frac{\omega_n^2}{S^2+2\xi\omega_n S+\omega_n^2}} \rightarrow ①$$

For underdamped system, $0<\xi<1$

$$S_1, S_2 = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$$

when unit step input is applied,

$$r(t) = 1$$

$$L\{r(t)\} = R(S) = \frac{1}{S}$$

The Response in S-domain is;

$$C(S) = R(S) \times \frac{\omega_n^2}{S^2+2\xi\omega_n S+\omega_n^2}$$

$$C(S) = \frac{1}{S} \times \frac{\omega_n^2}{S^2+2\xi\omega_n S+\omega_n^2}$$

Apply partial fraction,

$$\boxed{C(S) = \frac{A}{S} + \frac{BS+C}{S^2+2\xi\omega_n S+\omega_n^2}} \rightarrow ②$$

$$\frac{\omega_n^2}{S(S^2+2\xi\omega_n S+\omega_n^2)} = \frac{A}{S} + \frac{BS+C}{S^2+2\xi\omega_n S+\omega_n^2}$$

$$\frac{\omega_n^2}{S(S^2+2\xi\omega_n S+\omega_n^2)} = \frac{A(S^2+2\xi\omega_n S+\omega_n^2)+(BS+C)S}{S(S^2+2\xi\omega_n S+\omega_n^2)}$$

$$\omega_n^2 = A(S^2+2\xi\omega_n S+\omega_n^2) + (BS+C)S$$

$$\omega_n^2 = AS^2 + 2A\xi\omega_n S + A\omega_n^2 + BS^2 + CS$$

① Equating constants

$\omega_n^2 = A\omega_n^2$

$\boxed{A = 1}$

② Equating s^2 term

$0 = A\cancel{s^2} + B\cancel{s^2}$

$0 = 1 + B$

$\boxed{B = -1}$

③ Equating s term

$0 = 2A\xi\omega_n + C$

$0 = 2\xi\omega_n + C$

$\boxed{C = -2\xi\omega_n}$

Substitute A, B and C values in eq ②

$$C(S) = \frac{1}{S} + \frac{(-1)(S) + (-2\xi\omega_n)}{(S^2 + 2\xi\omega_n S + \omega_n^2)}$$

$$C(S) = \frac{1}{S} - \left[\frac{S + 2\xi\omega_n}{S^2 + 2\xi\omega_n S + \omega_n^2}\right]$$

$\because L^{-1}\left\{\frac{1}{S}\right\} = 1$

$L^{-1}\left\{\frac{S+a}{(S+a)^2+\omega^2}\right\} = e^{-at}\cos\omega t$

~~Apply Inverse Laplace transform,~~

Add & sub $\xi^2\omega_n^2$ in Deno of second term,

$L^{-1}\left\{\frac{\omega}{(S+a)^2+\omega^2}\right\} = e^{-at}\sin\omega t$

$$C(S) = \frac{1}{S} - \left[\frac{S + 2\xi\omega_n}{\underbrace{S^2}_{a^2} + \underbrace{2\xi\omega_n S}_{2ab} + \omega_n^2 + \underbrace{\xi^2\omega_n^2}_{b^2} - \xi^2\omega_n^2}\right]$$

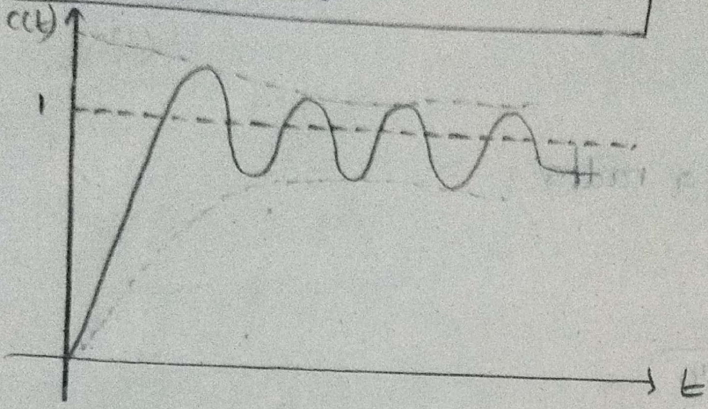
$$C(S) = \frac{1}{S} - \left[\frac{S + 2\xi\omega_n}{(S + \xi\omega_n)^2 + \omega_n^2[1 - \xi^2]}\right]$$

Let $\omega_d = \omega_n\sqrt{1 - \xi^2}$

$\omega_d^2 = \omega_n^2(1 - \xi^2)$

$$C(S) = \frac{1}{S} - \left[\frac{S + 2\xi\omega_n}{(S + \xi\omega_n)^2 + \omega_d^2}\right]$$

$$\boxed{c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta)}$$



Problems:

1. $c(t) = 1 + 0.2e^{-60t} - 1.2e^{-10t}$. Find closed loop T.F

* undamped frequency, damping

A. Apply Laplace transform to the given, $L\{e^{-at}\} = \frac{1}{s+a}$

Given,

$$c(t) = 1 + 0.2e^{-60t} - 1.2e^{-10t}$$

$$C(s) = \frac{1}{s} + 0.2\frac{1}{(s+60)} - 1.2\frac{1}{(s+10)}$$

$$C(s) = \frac{(s+60)(s+10) + 0.2(s)(s+10) - 1.2(s)(s+60)}{s(s+60)(s+10)}$$

$$C(s) = \frac{s^2 + \cancel{10s} + \cancel{60s} + 600 + 0.2s^2 + \cancel{2s} - 1.2s^2 - \cancel{72s}}{s(s+60)(s+10)}$$

$$C(s) = \frac{\cancel{s^2} + 600 + \cancel{0.2s^2} - \cancel{1.2s^2}}{s(s+60)(s+10)}$$

$$C(s) = \frac{600}{s(s+60)(s+10)}$$

$$R(s) = \frac{1}{s}, \quad C(s) = R(s) \times \frac{600}{\cancel{s}(s+60)(s+10)}$$

∴ The closed loop T.F

$$\boxed{\frac{C(s)}{R(s)} = \frac{600}{\cancel{s}(s+60)(s+10)}}$$

compare transfer function with $\frac{wn^2}{s^2+2\xi wns+wn^2}$

$wn^2=600$

$wn=\sqrt{600}$

$=24.49$ rad/s

$2\xi wn=70$

$\xi=\frac{70}{2wn}$

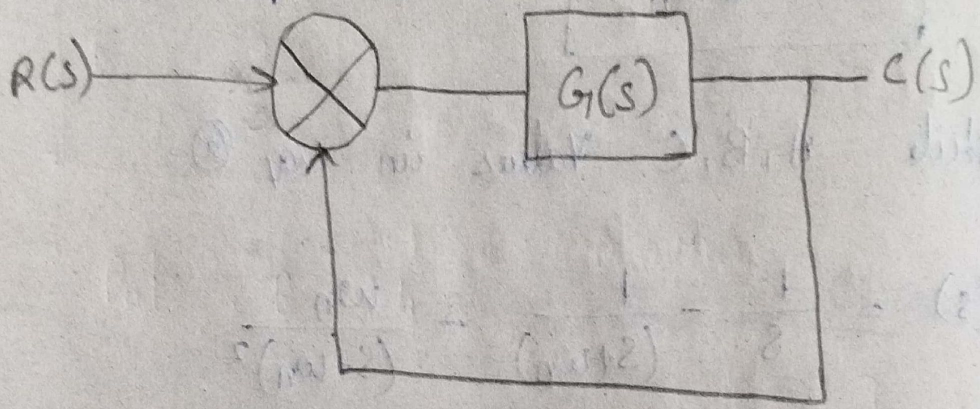
$\xi=\frac{70}{2\times 24.49}$

$\xi=1.43$

10M

1. Obtain the response of Unity feedback system whose open loop T.F $G(s) = \frac{4}{s(s+5)}$ and when the i/p is unit step.

closed loop with unity feedback.



$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s).H(s)}$$

$$= \frac{\frac{4}{s(s+5)}}{1+\frac{4}{s(s+5)}}$$

$$= \frac{\frac{4}{\cancel{s(s+5)}}}{\frac{s(s+5)+4}{\cancel{s(s+5)}}} = \frac{4}{s(s+5)+4}$$

$$\frac{C(s)}{R(s)} = \frac{4}{s(s+8)+4}$$

$$\boxed{\frac{C(s)}{R(s)} = \frac{4}{s^2+5s+4}}$$

Response, in s domain

$$C(s) = R(s) \times \frac{4}{s^2+5s+4}$$

$r(t) = 1$

$L\{r(t)\} = R(s) = \frac{1}{s}$

$$C(s) = R(s) \times \frac{4}{(s+1)(s+4)}$$

a b c
1 5 4

$$C(s) = \frac{1}{s} \times \frac{4}{(s+1)(s+4)}$$

$$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$\frac{-5 \pm \sqrt{25-16}}{2}$$

Apply partial fraction

$$\boxed{C(s) = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+4}} \rightarrow (1)$$

$$\frac{-5 \pm \sqrt{6}}{2}$$

$$\frac{-5 \pm 3}{2}$$

$$\frac{4}{s(s+1)(s+4)} = \frac{A(s+4)^2 + Bs(s+4) + Cs(s+1)}{s(s+1)(s+4)}$$

$$\frac{-5+3}{2}, \frac{-5-3}{2}$$

$$\frac{4}{s\cancel{(s+1)(s+4)}} = \frac{A(s+1)(s+4) + Bs(s+4) + Cs(s+1)}{s\cancel{(s+1)(s+4)}}$$

Put $s = 0$

$$4 = A(1)(4) + 0 + 0$$

$$4 = A(4)$$

$$\boxed{A = 1}$$

Put $B = -1$

$4 = \cancel{B(-}-B(-1+4) + C(-1+1)$

$4 = -B(3)$

$$\boxed{B = \frac{-4}{3}}$$

Put , $B = -4$

$4 = A(-4+1)(-4+4) + B(-4)[-4+4] - C4(-4+1)$

$4 = 0 - 0 - 4C(-3)$

$\cancel{4} = -\cancel{4}C(-3)$

$$\boxed{C = \frac{1}{3}}$$

Subsitute A, B, C Values in eq ①

$$C(S) = \frac{1}{S} - \frac{4}{3(S+1)} + \frac{1}{3(S+3)}$$

$S+a$

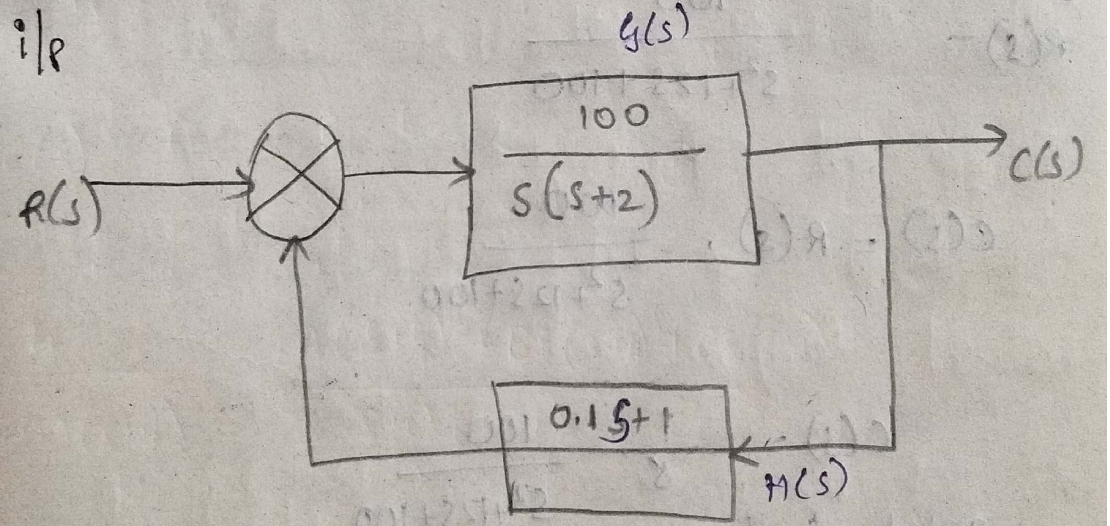
$L^{-1}\left[\frac{1}{S}\right] = 1$

$$L^{-1}\left[\frac{1}{(s+a)}\right] = e^{-at}$$

$$\cancel{L^{-1}\left\{\frac{1}{\ }\right.}$$

$$\boxed{c(t) = 1 - 4/3 \cdot e^{-t} + \frac{1}{3} e^{-4t}}$$

2. A positional Control System with Velocity feedback is shown in figure. What is the response of the system. for unit step i/p



$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s) \cdot H(s)}.$$

$$\frac{C(s)}{R(s)} = \frac{\frac{100}{s(s+2)}}{1 + \frac{100}{s(s+2)}(0.1s+1)}$$

$$\frac{\frac{100}{s(s+2)}}{}$$

$$= \frac{s(s+2)\ 100\,(0.1s+1)+100}{s(s+2)}$$

$$= \frac{\frac{100}{s(s+2)}}{\frac{s(s+2)\ 1.50+100+100}{s(s+2)}}$$

$$\frac{C(s)}{R(s)} = \frac{100}{s^2+12s+100}$$

$$C(s) = R(s) \cdot \frac{100}{s^2+12s+100}$$

$$C(s) = \frac{1}{s} \cdot \frac{100}{s^2+12s+100}$$

Partial fractions

$$\boxed{C(s) = \frac{A}{s} + \frac{Bs+C}{s^2+12s+100}} \longrightarrow (1)$$

$$= A(s^2+12s+100) + s(Bs+C)$$

$$C(s) = As^2 + 12As + 100A + Bs^2 + Cs$$

Compare Constants

$\cancel{A = 100}$

$100 = 100A$

$\boxed{A = 1}$

Compare s^2

$0 = A + B$

$\boxed{B = -1}$

Compare s

$0 = 12 + C$

$\boxed{C = -12}$

Substitute these values in eqn

$$C(s) = \frac{1}{s} - \frac{s+12}{s^2+12s+100}$$

$$C(s) = \frac{1}{s} - \frac{s+6+6}{s^2+12s+100}$$

36

64

$s^2+12s+36+64$

$$C(s) = \frac{1}{s} - \left[\frac{s+6}{(s+6)^2+8^2} + \frac{6}{(s+6)^2+8^2}\right]$$

mul & Divide by 8 for 3rd term

$$\frac{6 \quad 8}{8(s+6)^2+8^2} \Rightarrow \frac{a}{(s+a)^2+\omega^2} = e^{-at}\sin\omega t$$

$$L^{-1}\left[\frac{1}{s}\right] = 1$$

$$L^{-1}\left[\frac{s+a}{(s^2+a)^2+\omega^2}\right] = e^{-at}\cos\omega t$$

$$L^{-1}\left[\frac{a}{(s+a)^2+\omega^2}\right] = e^{-at}\sin\omega t$$

$$\boxed{c(t) = 1 - e^{-6t}\cos 8t + \frac{6}{8}e^{-6t}\sin 8t}$$

3. The response of servo mechanism is $c(t) = 1+0.2\,e^{-60t} - 1.2\,e^{-10t}$, when subject to a unit step i/p obtain an expression for closed loop T.f, Determine the undamped natural frequency and damping ratio.

Sol:

$$c(t) = 1+0.2e^{-60t} - 1.2e^{-10t}$$

$$L[e^{-at}] = \frac{1}{s+a}$$

$$L[1] = \frac{1}{s}$$

Apply laplace transform

$$c(s) = \frac{1}{s} + 0.2\frac{1}{(s+60)} - 1.2\frac{1}{(s+10)}$$

$$C(s) = \frac{(s+60)(s+10)+0.2+s(s+10)-1.2\,s(s+60)}{s(s+60)(s+10)}$$

$$C(s) = \frac{(s^2+10s+60s+600)+0.2+s^2+10s-1.2s^2+60s}{s(s+60)(s+10)}$$

$$C(s) = \frac{(s^2+70s+600)+0.2s^2+2s-1.2s^2-72s}{s(s+60)(s+10)}$$

$$C(s) = \frac{600}{s(s+60)(s+10)}$$

$$C(s) = \frac{1 \times 600}{s(s+60)(s+10)}$$

$$R(s) = 1/s$$

$$C(s) = R(s)\,\frac{600}{s^2+60s+10s+600}$$

$$= \frac{1}{s}\cdot\frac{600}{s^2+70s+100}$$

$$\frac{C(s)}{R(s)} = \frac{600}{s^2+70s+100} \Rightarrow \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

$$\omega_n^2 = 600$$

$$\omega_n = \sqrt{600} = \boxed{24.49\,\text{rad/sec}}$$

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$2\xi\omega_n s = 70s$

$$\xi = \frac{70}{2\xi\omega_n}$$

$$= \frac{70}{2\times 24.49}$$

$$\boxed{\xi = 1.42}$$

A Second order System is given by $G(s) = \frac{25}{s^2+6s+25}$

Find its rise time, Peak over shoot and settling time if subjected to unit step i/p.

$$\frac{25}{s^2+6s+25} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

$\omega_n^2 = 25$

$\omega_n = \sqrt{25}$

$\boxed{\omega_n = 5}$

$\omega_d = \omega_n\sqrt{1-\xi^2}$

$= 5\sqrt{1-(0.6)^2}$

$\omega_d = 4$ rad/sec

$2\xi\omega_n s = 6s$

$$\xi = \frac{6}{2\times 5}$$

$$\frac{6}{10} = 0.6$$

$\boxed{\xi = 0.6}$

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$$\theta = \tan^{-1} \frac{\sqrt{1-\xi^2}}{\xi}$$

$$\tan^{-1}\left(\frac{\sqrt{1-(0.6)^2}}{0.6}\right)$$

$$\tan^{-1}(1.33)$$

$$\theta = \cancel{52.43}\ 53.06$$

$$t_r = \frac{\pi - \theta}{\omega_d}$$

$$= \frac{3.14 - 53.06}{4} = 0.64 \text{ sec}$$

$$t_p = \frac{\pi}{\omega_d}$$

$$= \frac{\cancel{53.06}\,\pi}{4} = 0.785$$

$$\%\ M_p = \frac{e^{-\pi\xi}}{\sqrt{1-\xi^2}} \times 100$$

$$\frac{0.151}{0.8} \times 100$$
$$= 18.8$$

$$= \frac{e^{-\pi(0.6)}}{\sqrt{1-\cancel{\xi}(0.6)^2}} \times 100$$

$$= \frac{0.15}{0.8} \times 100 = 9.48$$

$$T_s = \frac{4}{\xi \omega_n} = \frac{4}{0.6 \times 5}$$

$$T_s = 1.33 \text{ sec}$$

A Unity feedback Control System has an openloop T.F $G(s) = \frac{10}{s(s+2)}$. Find the rise time, % over shoot, Peak time & Settling time for step i/p of 12 Units.

$$= \frac{G(s)}{1 + G(s) \cdot H(s)}$$

$$= \frac{\frac{10}{s(s+2)}}{1 + \frac{10}{s(s+2)} \times 12}$$

$$\frac{\frac{10}{s(s+2)}}{\frac{s(s+2) + 10 \times 12}{s(s+2)}} = \frac{10}{s^2 + 2s + 120}$$

$$\frac{10}{s^2+2s+10} \qquad \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

$$t_r = \frac{\pi-\theta}{\omega_d}$$

$$\theta = \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)$$

$\omega_n^2 = 10$

$\omega_n = \sqrt{10}$

$\boxed{\omega_n = 3.16 \text{ rad/sec}}$

$$t_r = \frac{\pi - 1.465}{2.99}$$

$\boxed{t_r = 0.562}$

$\omega_d = \omega_n\sqrt{1-\xi^2}$

$2\xi\omega_n s = 2s$

$$\xi = \frac{2}{2\omega_n} = \frac{1}{3.16}$$

$\boxed{\xi = 0.316}$

$$t_p = \frac{\pi}{\omega_d}$$

$$= \frac{\pi}{2.99}$$

$\boxed{t_p = 1.05 \text{ Sec}}$

$\omega_d = 3.16\sqrt{1-(0.316)^2}$

$\boxed{\omega_d = 2.99}$ rad/sec

$$\% m_p = \frac{e^{-\pi\xi}}{\sqrt{1-\xi^2}} \times 100$$

$$m_p = \frac{0.370}{0.94} \times 100$$

$\boxed{m_p = 39.36}$

$$T_s = \frac{4}{\xi\omega_n}$$

$$= \frac{4}{0.316 \times 2.99}$$

$T_s = 4.\cancel{22}$

Steady State error:

It is the Value of error Signal e(t) when $t \to \infty$ that is called as error signal error depends on nature of i/p, System or type of the system, non-linearity of Components.

$$\underset{t \to \infty}{Lt}\ e(t) = e_{ss}.$$

L.T:

$$\underset{s \to 0}{Lt}\ s\,E(s) = e_{ss}.$$

$$\boxed{E(s) = \frac{R(s)}{1 + G(s).H(s)}}$$

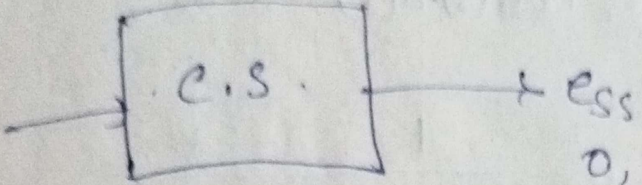
Eq in final Value theorem in L.Domain.

$$\underset{s \to 0}{Lt}\ s.E(s) = e_{ss}$$

$$\boxed{e_{ss} = \underset{s \to 0}{Lt}\ s.\frac{R(s)}{1+G(s).H(s)}}$$

Static Error Constants:

unit
ramp
~~impulse~~
parabolic



0, Constant
infinity

step → Type 0 → Constant — Positional error constant (k_p).

Ramp → Type 1 → Constant (k_v) velocity

Parabolic → Type 2 → Constant (k_a) acceleration

$$k_p = \lim_{s \to 0} G(s) \cdot H(s)$$

$$k_v = \lim_{s \to 0} s \cdot G(s) \cdot H(s)$$

$$k_a = \lim_{s \to 0} s^2 \cdot G(s) \cdot H(s)$$

where k_p, k_v, k_a are static error constants

Steady State error:

When unit step signal is applied.

$$e_{ss} = \lim_{s \to 0} s \cdot \frac{R(s)}{1 + G(s) \cdot H(s)}$$

$$R(s) = \frac{1}{s}$$

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$$e_{ss} = \lim_{s \to 0} s \cdot \frac{\frac{1}{s}}{1+G(s)\cdot H(s)}$$

$$\boxed{e_{ss} = \lim_{s \to 0} \frac{1}{1+G(s)\cdot H(s)}}$$

$$e_{ss} = \frac{1}{\lim_{s \to 0} 1+G(s)\cdot H(s)}$$

$$\boxed{e_{ss} = \frac{1}{1+K_p}}$$

Consider Type 0:

$$K_p = \lim_{s \to 0} G(s)\cdot H(s)$$

$$K_p = \lim_{s \to 0} \frac{K(s+Z_1)(s+Z_2)}{(s+P_1)(s+P_2)}$$

$$K_p = \frac{K(Z_1 Z_2)}{P_1 \cdot P_2}$$

K_p is Constant for Type 0.

Consider Type 1

$$K_p = \lim s \cdot G(s) \cdot H(s)$$

$$K_p = \underset{s \to 0}{Lt} \frac{k(s+z_1)(s+z_2)}{s(s+p_1)(s+p_2)} = \frac{1}{0} = \infty$$

$$\boxed{K_p = \infty}$$

$$e_{ss} = \frac{1}{1+K_p} = \frac{1}{\infty} = 0.$$

Static Constants	Types.
K_p	Type 0
K_v	Type 1
K_a	Type 2.

Static error Constant	Type 0	Type 1	Type 2.
K_p	Constant	∞	∞

Steady State error i/p	Type0	Type 1	Type 2.
Unit step	$\frac{1}{1+K_p}$	0	0

for Type 2 :

$$K_p = \infty \rightarrow \text{Static}$$

$$\frac{1}{1+\infty} = 0 \rightarrow \text{steady}$$

Velocity Error Constant :

Ramp i/p $\qquad R(s) = 1/s^2$

$$e_{ss} = \lim_{s \to 0} \frac{s \times 1/s^2}{1 + G(s).H(s)}$$

$$e_{ss} = \lim_{s \to 0} \frac{1}{s(1 + G(s).H(s))}$$

$$e_{ss} = \lim_{s \to 0} \frac{1}{s + sG(s)H(s)}$$

$$= \lim_{s \to 0} \frac{1}{sG(s).H(s)}$$

$$\boxed{e_{ss} = \frac{1}{K_v}}$$

for type 0

$$K_v = \lim_{s \to 0} sG(s).H(s) \quad \boxed{\therefore K_v = 0}$$

Steady state error = 1

Static Error Constant	Type 0	Type 1	Type 2
Kp	Constant	∞	∞
Kv	0	Constant	∞
Ka	0	0	Constant.

Steady State Error

i/p	Type 0	Type 1	Type 2
Unit Step	$\frac{1}{1+Kp}$	0	0
Ramp	∞	$\frac{1}{Kv}$	0
Parabolic	∞	∞	$1/Ka$

10m

Problem:

For servo mechanism with open loop T.F given below. explain what is type of i/p ~~of~~ Signal give rise to a constant steady state error and calculate their values.

a. $G(s) = \dfrac{20(s+2)}{s(s+1)(s+3)}$

b. $G(s) = \dfrac{10}{(s+2)(s+3)}$

c. $G(s) = \dfrac{10}{s^2(s+1)(s+2)}$

Assume unity feedback

i.e $H(s) = 1$

a. $G(s) = \dfrac{20(s+2)}{s(s+1)(s+3)}$.

$$e_{ss} = \frac{1}{K_v}$$

$$K_v = \lim_{s\to 0} s\,G(s)\,H(s) = \lim_{s\to 0} s.G(s)$$

$$= \lim_{s\to 0} \cancel{s} \times \frac{20(s+2)}{\cancel{s}\underbrace{(s+1)}_{0+1}\underbrace{(s+3)}_{0+3}}$$

$$\lim_{s\to 0} \frac{20(s+2)}{(s+1)(s+3)} = \frac{20(2)}{(1)\times(3)}$$

$$\boxed{K_v = \frac{40}{3}} = 13.33.$$

$$e_{ss} = \frac{1}{40/3}$$

$$e_{ss} = \frac{3}{40}$$

$$\boxed{e_{ss} = 0.075}$$

b) $G(s) = \frac{10}{(s+2)(s+3)}$ $\quad e_{ss} = \frac{1}{1+k_p}$

$$\lim_{s \to 0} G(s) \cdot H(s)$$

$$\lim_{s \to 0} G(s).$$

$$\lim_{s \to 0} \frac{10}{(s+2)(s+3)} = \frac{10}{6}$$

$$\boxed{k_p = \frac{5}{3}}$$

$$e_{ss} = \frac{1}{1+\frac{5}{3}}$$

$$= \frac{3}{8}$$

$$\boxed{e_{ss} = 0.375}$$

Q: $G(s) = \frac{10}{s^2(s+1)(s+2)}$

Type - 2. $\frac{1}{Ka}$

$H(s) = 1$

$\lim_{s \to 0} s^2 \cdot G(s) \cdot H(s)$

$\lim_{s \to 0} \cancel{s^2} \frac{10}{\cancel{s^2}(s+1)(s+2)}$

$\frac{\cancel{10}^{5}}{\cancel{2}_{1}} = 5.$

$\boxed{Ka = 5}$

$\frac{1}{Ka} = \frac{1}{5} = 0.2$.

2: for unity feedback Control System the open loop T.F $G(s) = \frac{10(s+2)}{s^2(s+1)}$. find

a: The position, Velocity and acceleration Constants.

b) The steady state error when the i/p is

$$R(s) = \frac{3}{s} - \frac{2}{s^2} + \frac{1}{3s^3}$$

Assume $H(s) = 1$

Position

a $G(s) = \frac{10(s+2)}{s^2(s+1)}$

$$K_p = \lim_{s \to 0} G(s)$$

$$\lim_{s \to 0} \frac{10(s+2)}{s^2(s+1)}$$

$\boxed{K_p = \infty}$

$$K_p = \frac{10(2)}{0} = \infty$$

Velocity error constant:

$$K_v = \lim_{s \to 0} s \frac{10(s+2)}{s^{\cancel{2}}(s+1)} = \frac{10(s+2)}{s(s+1)}$$

$K_v = \lim_{s \to 0} s . G(s)$

$$K_v = \frac{1}{0} = \frac{\cancel{20}}{1} \infty$$

$\boxed{K_v = \infty}$

acceleration error constant: $\lim_{s \to 0} s^2 G(s)$.

$$K_a = \lim_{s \to 0} s^2 \frac{10(s+2)}{s^2(s+1)} = \frac{10(2)}{1}$$

$\boxed{K_a = 20}$

b)

$$R(s) = \frac{3}{s} - \frac{2}{s^2} + \frac{1}{3s^3}$$

$$e_{ss} = \underset{s \to 0}{Lt}\; s \cdot \frac{R(s)}{1 + G(s)\cdot H(s)} \qquad \because H(s) = 1$$

$$\underset{s \to 0}{Lt}\; s \cdot \frac{3/s - 2/s^2 + 1/3s^3}{1 + \dfrac{10(s+2)}{s^2(s+1)}}$$

$$e_{ss} = \underset{s \to 0}{Lt}\; s \cdot \frac{\dfrac{3}{s} - \dfrac{2}{s^2} + \dfrac{1}{3s^3}}{\dfrac{s^2(s+1) + 10(s+2)}{s^2(s+1)}}$$

$$e_{ss} = \underset{s \to 0}{Lt}\; s \cdot \left(\frac{3}{s} - \frac{2}{s^2} + \frac{1}{3s^3}\right) \times \frac{s^2(s+1)}{s^2(s+1) + 10(s+2)}$$

$$e_{ss} = \underset{s \to 0}{Lt}\; s \left[\left(\frac{3}{s} \times \frac{s^2(s+1)}{s^2(s+1)+10(s+2)}\right) - \left(\frac{2}{s^2} \times \frac{s^2(s+1)}{s^2(s+1)+10(s+2)}\right) + \left(\frac{1}{3s^3} \times \frac{s^2(s+1)}{s^2(s+1) + 10(s+2)}\right)\right]$$

$10(0+2)$

$10(2)$

$e_{ss} = \lim_{s \to 0}$

Another Step:

$$R(s) = \frac{3}{s} - \frac{2}{s^2} + \frac{1}{3s^3}.$$

$\downarrow$ Kp Unit, $\downarrow$ Ramp, $\downarrow$ Parabola.

$$e_{ss_1} = \frac{1}{1+K_p} = \frac{3}{1+\infty} = \frac{3}{\infty} = 0.$$

$$e_{ss_2} = \frac{1}{K_v} = \frac{1}{\infty} = 0.$$

$$e_{ss_3} = \frac{1}{3K_a} = \frac{1}{3\times 20} = \frac{1}{60} = 0.0166.$$

$$e_{ss} = 0 - 0 + 0.0166.$$

Q. The open loop T.F of a servo system with unity feedback system is $G(s) = \frac{10}{s(0.1s+1)}$

Evaluate the static error constants of the