# UNIT-3

## STABILITY ANALYSIS IN TIME DOMAIN

### Concept of Stability:

**Definition:** The system is said to be stable when its o/p is under control is called stability otherwise the system is unstable.

### Bounded i/p Bounded o/p (BIBO):

- A stable system produces bounded o/p for given bounded i/p is called BIBO stability.
- If the system produces unbounded o/p for bounded input then it is unstable system.

### Types of Stability:

1. Absolute Stability.
2. Conditional Stability.
3. Relative Stability.
4. Marginal Stability.

### 1. Absolute Stability:

A System if the System output is Constant for all Variations

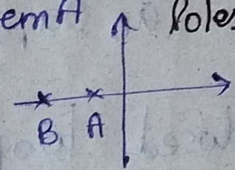
Examples:
All poles lies on left of LHS.

2. Conditional Stability:
The system is stable for bounded range.

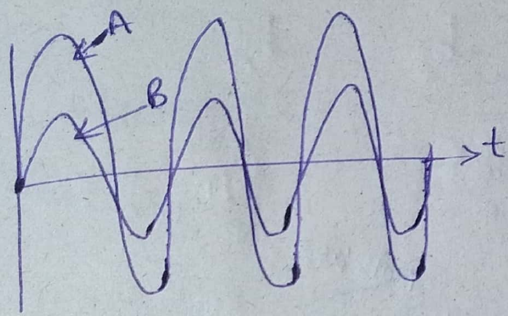
3. Relative stability:
It gives the degree of instability or how close it is to instability

System B is more stable than system A

Based on Poles




Based on response:




4. Marginal Stability:
for bounded i/p the o/p oscillates with constant amplitude and frequency their is marginal state

Necessary Conditions for stability:
The transfer function of any linear closed loop system is represented as

$$T(s) = \frac{C(s)}{R(s)} = \frac{b_0 s^m + b_1 s^{m-1} + b_m}{a_0 s^n + a_1 s^{n-1} + \cdots + a_n}$$

$$T(s) = \frac{N(s)}{n(s)}$$

The characteristic equation of closed loop System

$$\boxed{1+G(s)\cdot H(s)=0}$$

The characteristic equation is:

$$a_0 s^{\overset{\text{no. of roots}}{n}} + a_1 s^{n-1} + a_2 s^{n-2} + \cdots + a_n = 0$$

where,

$a_0, a_1, a_2 \ldots a_n$ are Co-efficient Constants

→ closed loop poles equal to roots of characteristic equation.

Condition:

first Condition: stable roots are, -ve, real parts.

All Co-efficient of characteristic equation polynomial must be +Ve.

Eg: $4s^2+4s+5=0 \Rightarrow$ stable
$4s^2-4s+5=0 \Rightarrow$ unstable

Second Condition:

All the powers of s must be present in the characteristic equation.

$$s+3s^2+4s+5=0.$$

↓

Power is missed for s term.

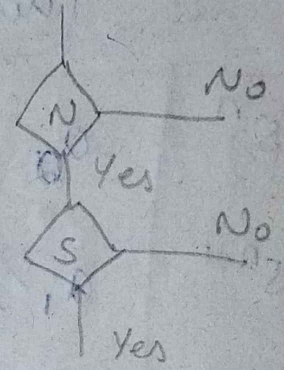
If any Condition is failed the System is unstable.

If any Co-efficient is Zero/-ve the System is unstable.

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Necessary Condition. → 2M

* Sufficient Condition.




SM

## Routh - Hurwitz - Stability criterion (RH Criterian):

also called as * Routh Stability criteria (or)

* Routh array method

Consider,

$n^{th}$ order characteristic equation

$$\boxed{a_0 s^n + a_1 s^{n-1} + \cdots + a_n = 0}$$

where, $a_0 > 0$.

Routh Suggested a method of tabulating all the co-efficients of the characteristic equation in a particular way. This tabulation given an array called Routh array.

$$\boxed{a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + a_3 s^{n-3} + \cdots + a_n = 0.}$$

$$\begin{array}{c|cccc}
s^n & a_0 & a_2 & a_4 & a_6 \\
s^{n-1} & a_1 & a_3 & a_5 & a_7 \\
s^{n-2} & b_1 & b_2 & b_3 & b_4 \\
s^{n-3} & c_1 & c_2 & c_3 & c_4 \\
\vdots & & & & \\
s^0 & a_n & & &
\end{array}$$

$$b_1 = \frac{a_1 a_2 - a_0 a_3}{a_1}$$

$$c_1 = \frac{b_1 a_3 - a_1 b_2}{b_1}$$

$$b_2 = \frac{a_1 a_4 - a_0 a_5}{a_1}$$

$$b_3 = \frac{a_1 a_6 - a_0 a_7}{a_1}$$

The ~~routh~~ Routh stability criterion can be stated as the necessary & sufficient condition is all the elements in the 1st column of the routh array are +ve then it is called as stable and the no. of sign changes ~~is equ~~ in the 1st column of routh array is equal to no. of ~~routes~~ ~~roots~~ lies on RHS [Right half of s-plane].

Conclusion:

1. Determine the stability.
2. Finding roots lies on LHS, RHS & imaginary axis,
3. Finding range of k for stability ~~as~~ k and marginal stability,

Eg: Examine the stability of given equations using Routh method.

$$s^3 + 6s^2 + 11s + 6 = 0$$

$a_0 \quad a_1 \quad a_2 \quad a_3$

(2) $s^3+4s^2+s+16=0$

Sol:

(1)

| | $C_1$ | $C_2$ | $C_3$ | |
|---|---|---|---|---|
| $s^3$ | 1 | 11 | 0 | Row-1 |
| $s^2$ | 6 | 6 | 0 | Row-2 |
| $s^1$ | 10 | 0 | | |
| $s^0$ | 16 | | | |

stable

$$s^1 = \frac{6\times 11 - 1\times 6}{6} = \frac{66-6}{6}$$

$$= \frac{\cancel{60}\ 10}{\cancel{6}\ 1}$$

$$\boxed{s^1 = 10}$$

$$s = \frac{6\times 0 - 1\times 0}{6} = \frac{0}{6} = 0$$

System is stable lies on LHS.

No. of Sign Changes = 0

(5) $s^3 + 4s^2 + s + 16 = 0$

$a_0 \quad a_1 \quad a_2 \quad a_3$

| | | | |
|---|---|---|---|
| $s^3$ | 1 | 1 | 0 |
| $s^2$ | 4 | 16 | 0 |
| $s^1$ | -3 | 0 | |
| $s^0$ | 16 | | |

1 + 1

unstable

$\frac{16}{4} \; 12$

$\frac{10}{4} \; 2.5$

It is unstable.

$s^1 = \frac{4 \times 1 - 1 \times 16}{4}$

No. of Sign changes = 2.

$= \frac{4 - 16}{4} = \frac{-12}{4} = -3$

It lies on RHS.

Given, characteristic equation no. of roots = ~~16~~4

$n = 4$

| | C-1 | C-2 | C-3 | |
|---|---|---|---|---|
| $s^4$ | 1 | 18 | 5 | R-1 |
| $s^3$ | 8 | 16 | 0 | R-2 |
| $s^2$ | 16 | 5 | 0 | R-3 |
| $s^1$ | 13.5 | 0 | 0 | |
| $s^0$ | 5 | | | |

↑ no. sign changes.

$$s^2 = \frac{8\times18 - 1\times11}{8}$$

$$\frac{144-16}{8} = \frac{144}{8}$$

$$s^2 = 16$$

$$s^2 = \frac{8\times5 - 1\times0}{8}$$

$$\frac{40-0}{8}$$

$$s^2 = 5$$

$$s^1 = \frac{16\times16 - 8\times5}{16}$$

$$\frac{256-40}{16}$$

$$s^1 = \frac{216}{16} = 13.5$$

$$s^0 = \frac{13\times5 - 16\times0}{13}$$

$$s^0 = \frac{65-0}{13} = 5$$

It is stable system. The colum has non-negative value system it is stable. All the 4 roots lies on the left half of s-plane.

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$b - s^3 + 8s^2 + 9s + 24 = 0$

No of roots are 4

i.e $n = 4$

| | | | |
|---|---|---|---|
| $s^3$ | 1 | 9 | 0 |
| $s^2$ | 8 | 24 | 0 |
| $s^1$ | 6 | 0 | 0 |
| $s^0$ | 24 | | |

No -ve Sign.

$$s^1 = \frac{8 \times 9 - 1 \times 24}{8}$$

$$s^0 = \frac{6 \times 24 - 8 \times 0}{6}$$

$$s^1 = \frac{72 - 24}{8}$$

$$= \frac{144 - 0}{6}$$

$$s^1 = \frac{48}{8}$$

$$s^0 = 24$$

$$\boxed{s^1 = 6}$$

$$s^1 = \frac{8 \times 0 - 1 \times 0}{8}$$

$$\boxed{0}$$

It is the stable System. The Column has non- -Ve Value System it is stable. All the 4 roots lies on the left half of s-plane.

Case 2 :

The elements of the row routh array

Eg :

| | | | |
|---|---|---|---|
| $s^5$ | $a_0$ | $a_2$ | $a_4$ |
| $s^4$ | $a_1$ | $a_3$ | $a_5$ |
| $s^3$ | 0 | 0 | 0 → Row of Zero ($Ro_z$) |

The next row Can't be determined here

The routh test fails

> There is a solution

* form a polynomial by using the Co-efficients of a row which is just above Row of Zero (Roz).

This polynomial form is called accelerces polynomial.

It is denoted by A(S).

↑ It mean accelary polynomial

$$A(s) = a_1 s^4 + a_3 s^2 + a_5$$

② Differentiate polynomial wrt to 's'

$$\frac{A(s)}{ds} = 4as^3 + 2a_3 s.$$

③ replace 0's with Co-efficient of differential equation.

④ Complete the routh table.

Note :

- We get ROZ is only Valid for odd row power of S.
- Accelerary Polynomial is always form even row.

⑤ Accelerary equation is a part of original characterstic equation. Hence the roots of accelerary equation is always symmetric to ~~equation~~ origin

1. Consider routh array & determine the Stability is $s^6 + 2s^5 + 8s^4 + 12s^3 + 20s^2 + 16s + 16$ also determine. The number of roots lying on the right half of s-plane left half of 's' plane on imaginary axis.

| | | | | |
|---|---|---|---|---|
| $s^6$ | 1 | 8 | 20 | 16 |
| $s^5$ | 2 | 12 | 16 | 0 |
| $s^4$ | 2 | 12 | 16 | 0 |
| $s^3$ | 8. | 24 | 0 | 0 |
| $s^2$ | 6 | 16 | 0 | 0 |
| $s^1$ | -82.11 | 0 | 0 | 0 |
| $s^0$ | 2.6 16 | | | |

$\frac{16-12}{2}$ $\frac{16}{12}$ $\frac{4}{2}$

$$s^4 = \frac{2\times20 - 1\times16}{2}$$

$$= \frac{40-16}{2} = \frac{24}{2}$$

$$\boxed{s^4 = 12} \rightarrow (2)$$

$$s^4 = \frac{2\times8 - 1\times12}{2}$$

$$= \frac{16-12}{2} = \frac{14}{2}$$

$$\boxed{s^4 = 7} \rightarrow (1)$$

Pance in

Routh test fail

Accelery Polynomial

$$A(s) = 2s^4 + 12s^2 + 16s$$

Differentiate A(s) with respect to s

$$\frac{A(s)}{ds} = 8s^3 + 24s + \cancel{16} = 0$$

Replace Row with Coefficient of $\frac{A(s)}{ds}$

$$s^2 = \frac{8 \times 12 - \cancel{8}2 \times 24}{8} = 6$$

$$= \frac{8 \times 16 - 2 \times 0}{8} = 16$$

There is no sign changes. no roots lies on LHS.

roots of accelary equation

$$2s^2 + 12s^2 + 16$$

$$2(s^4 + 6s^2 + 8) = 0$$

$$s^4 + 6s^2 + 8 = 0$$

Let $x = s^2$

$$x^2 + 6x + 8 = 0$$

$$(x+2) + 4(x+2) = 0$$

$(x+4)(x+2)$

$x=-4 \quad x=-2$

WKT $\quad s^2 = x$

$s^2 = -2$

$\boxed{s = \pm\sqrt{-2}}$

$\boxed{s = \pm j\sqrt{2}}$

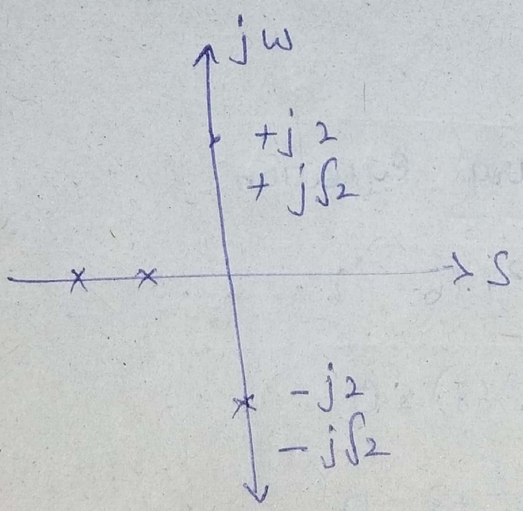
WKT $\quad s^2 = -4$

$\boxed{s = \pm\sqrt{-4}}$

$\boxed{s = \pm j2}$

We have 4 roots lies on imaginary

remaining 2 roots lies on LHS




The System is marginally Stable

no roots lies on RHS

$s^7 + 9s^6 + 24s^5 + 24s^4 + 24s^2 + 23s + 15 = 0$

| | | | | |
|---|---|---|---|---|
| $s^7$ | 1 | 24 | 24 | 23 |
| $s^6$ | 9 | 24 | 24 | 15 |
| $s^5$ | 21.3 | 21.3 | 21.3 | 0 |
| $s^4$ | 15 | 15 | 15 | 0 |
| $s^3$ | 0 | 0 | 0 | 0 |
| $s^2$ | | | | |
| $s^1$ | | | | |
| $s^0$ | | | | |

Row

Auxilairy Polynomial

$A(s) = 15s^4 + 15s^2 + 15$

Differentiate wrt to s

$\frac{dA(s)}{ds} = 60s^3 + 30s$

| | | | | |
|---|---|---|---|---|
| $s^7$ | 1 | 24 | 24 | 23 |
| $s^6$ | 9 | 24 | 24 | 15 |
| $s^5$ | 21.3 | 21.3 | 21.3 | 0 |
| $s^4$ | 15 | 15 | 15 | 0 |
| $s^3$ | ~~0~~ 60 | ~~0~~ 30 | 0 | 0 |
| $s^2$ | 7.5 | 15 | | |
| $s^1$ | -90 | 0 | | |
| $s^0$ | 15 | | | |

Number of Sign = 2.

2 roots lies on RHS.

* Roots of Auxiliary equation.

$15s^4 + 15s^2 + 15 = 0$

$15(s^4 + s^2 + 1) = 0$

$s^4 + s^2 + 1 = 0$

Let $x = s^2$

$x^2 + x + 1 = 0$

$x = \frac{-1 \pm \sqrt{1-4}}{2}$

$x = \frac{-1 \pm \sqrt{-3}}{2}$

$x = \frac{-1 \pm j\sqrt{3}}{2}$

roots are Conjugate.

~~The System is unstab~~

WKT $s^2 = x$.

$s^2 = \left(\frac{-1 + j\sqrt{3}}{2}\right)$

$s = \pm 0.5 \pm j0.866$

$s^2 = x$

$s^2 = \frac{-1 - j\sqrt{3}}{2}$

$s = 0.5 \pm j0.866$

The System is unstable

No. Sign changes = 2.

two roots lies on RHS and remaining five roots lies on LHS.

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## Finding Range of k:

Determine the range of k for stability of unity feedback System whose open loop transfer function is $G(s) = \frac{k}{s(s+1)(s+2)}$.

**Sol:**

The closed loop transfer function is

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s).H(s)}$$

$$= \frac{\frac{k}{s(s+1)(s+2)}}{1+\frac{k}{s(s+1)(s+2)} \times 1}$$

$$= \frac{\frac{k}{\cancel{s(s+1)(s+2)}}}{\frac{s(s+1)(s+2)+k}{\cancel{s(s+1)(s+2)}}}$$

$$\frac{C(s)}{R(s)} = \frac{k}{s(s+1)(s+2)+k}$$

characteristic equation is

$$s(s+1)(s+2) = 0$$

$$s(s^2+2s+s+2) + k = 0$$

$$s^3 + 3s^2 + 2s + k = 0$$

Routh array table

| | | | |
|---|---|---|---|
| $s^3$ | 1 | 2 | 0 |
| $s^2$ | 3 | k | 0 |
| $s^1$ | $\frac{6-k}{3}$ | 0 | |
| $s^0$ | k | | |

$$s^1 = \frac{(3\times2)-k}{3}$$

$$\boxed{s^1 = \frac{6-k}{3}}$$

for stability $k > 0$

$$\frac{6-k}{3} > 0$$

$$0 < k < 6$$

$$s^0 = \frac{\left(\frac{6-k}{3}\right)k - 0}{\frac{6-k}{3}}$$

$$\boxed{s^0 = k}$$

∴ Range of k is

$$\boxed{0 < k < 6}$$

The value of k is in the range $0 < k < 6$ for the system to be stable.

~~The Value of k~~

① The unity is characterised by the open loop T.f $G(s) = \frac{k(s+13)}{s(s+3)(s+7)}$ using Routh criterion. Calculate the range of k for the System to be stable.

② Using Routh criterion determine the stability of the System whose characteristic eq is $s^6 + s^5 - 2s^4 - 3s^3 - 7s^2 - 4s - 4 = 0$. find the no. of roots falling in the RHS plane and LHS Plane.

$\frac{G(s)}{1+G(s).H(s)}$

$k(s+13)$

① $G(s) = \dfrac{k(s+13)}{s(s+3)(s+7)}$

$$\frac{C(s)}{R(s)} = \frac{\cancel{k(s-}\ G(s)}{1+G(s)\cdot H(s)}$$

$$= \frac{\dfrac{k(s+13)}{s(s+3)(s+7)}}{1+\dfrac{k(s+13)}{s(s+3)(s+7)} \times 1}$$

$$\frac{C(s)}{R(s)} = \frac{k(s+13)}{s(s+3)(s+7)+k(s+13)}$$

C.E is

$$s(s+3)(s+7) + k(s+13) = 0$$

$$s(s^2+7s+3s+21) + k(s+13) = 0$$

$$s^3 + 7s^2 + 3s^2 + 21s + k(s+13) = 0.$$

$$s^3 + 10s^2 + 21s + k(s+13) = 0$$

$$s^3 + 10s^2 + 21s + ks + 13k = 0$$

$$10s^2 + 21ks + 13k = 0$$

$$s(21+k)$$

Advantages of RH criteria

1- Stability of System can be judge without Solving characteristic equation.

2- It gives number of roots with positive real part to find unstable system.

3- It helps to find range of values of k for System stability.

Limitations of Routh Stability:

1- It applicable to only linear Systems.

2- RH criteria is Valid only if all co-efficients are real.

3- It find only location of poles / roots.

4- It doesn't suggest any method for stabilising the unstable System.

~~nom~~ # Root Locus

Root locus technique uses an open loop transfer function to know the stability of closed loop Control system.

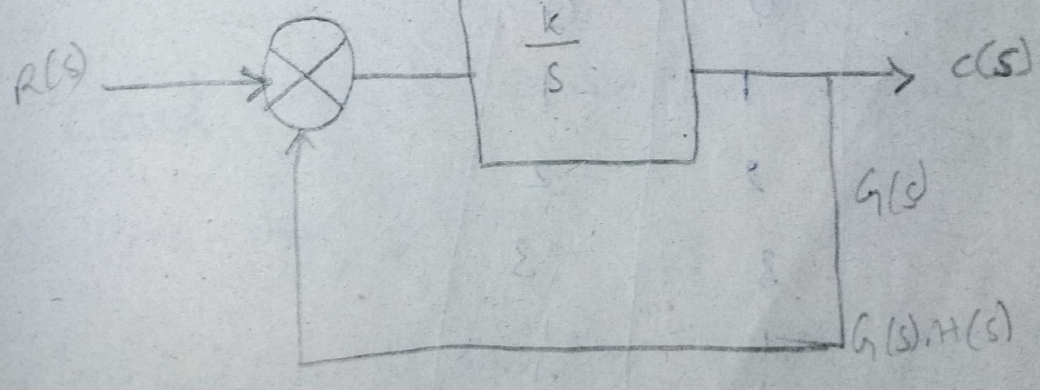
Root Locus :-

↓ roots of characteristic equation = closed loop ~~Path~~ poles

L→ Path.

* Root locus it is path of roots of characteristic equation (closed loop poles) when open loop gain 'k' varies from 0 to ∞.

Example :-



① method $G(s) = \frac{k}{s}$

$$\frac{G(s)}{1+G(s).H(s)} = \frac{\frac{k}{s}}{1+\frac{k}{s}\times 1}$$

$$\frac{\frac{k}{s}}{\frac{s+k}{s}} = \frac{k}{s+k}$$

$\boxed{s+k=0}$

② method. $1+G(s).H(s) = 0$

$G(s).H(s) = \frac{k}{s}$

$1 + k/s = 0$

$\frac{s+k}{s} = 0$

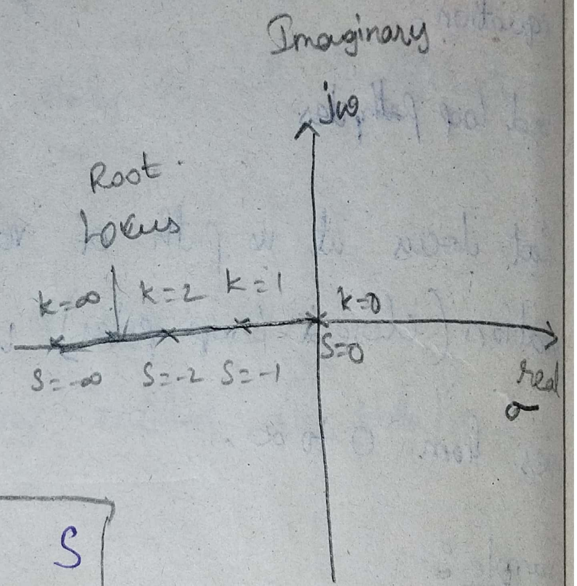
$\boxed{s+k=0}$

$\boxed{s=-k}$

roots of C.E ↗ ↑ Variable.

× → Pole

0 → zero




| k | S |
|---|---|
| 0 | 0 |
| 1 | -1 |
| 2 | -2 |
| 3 | -3 |
| ⋮ | ⋮ |
| ∞ | -∞ |

Root locus is the ~~RH~~ graphical representation of graphical poles when certain parameters changes.

In • root locus by adjusting the location of closed loops — by Varying 'k' to get desired performance of the system.

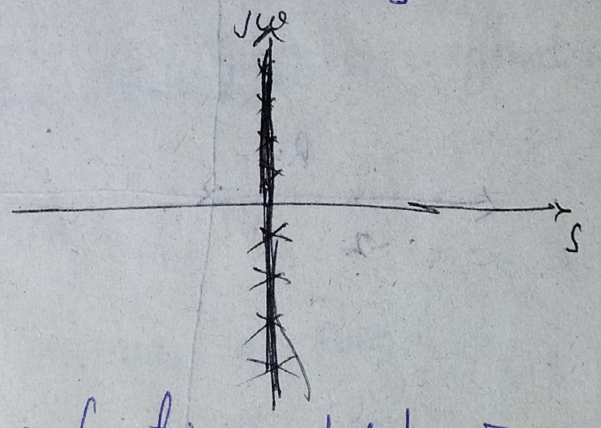
Closed loop Transfer function $\frac{k}{s^2+k}$.

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s).H(s)}.$$

$$s^2+k = 0$$

$$s^2 = -k \Rightarrow s = \pm\sqrt{-k}$$

$$s = \pm j\sqrt{k}$$




Closed loop is a function of poles, zeros of open loop System.

close loop poles = f( poles, zeros of open loop)

$$G(s) \cdot H(s) \Rightarrow k \frac{N(s)}{D(s)} \quad k \neq 0$$

$$1 + G(s) \cdot H(s) = 0$$

$$1 + k \frac{N(s)}{D(s)} = 0$$

$$D(s) + k N(s) = 0.$$

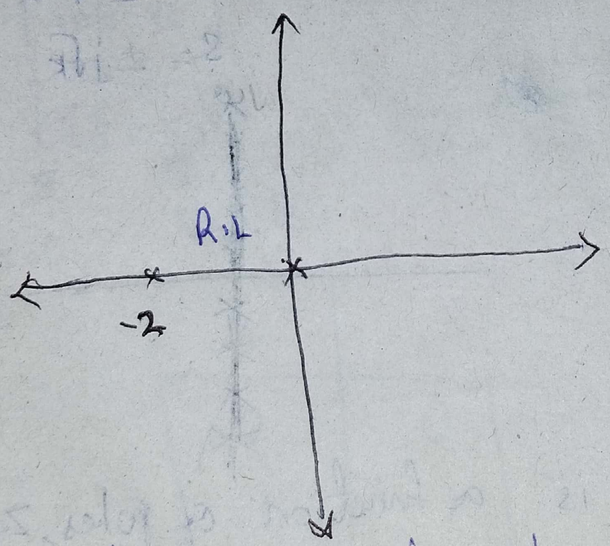
Rules of Construction of Root locus:-

Rule 1:

Root locus is Symmetrical above real axis

$$G(s) = \frac{k}{s(s+2)}.$$

$S = 0$ } Poles
$S = -2$



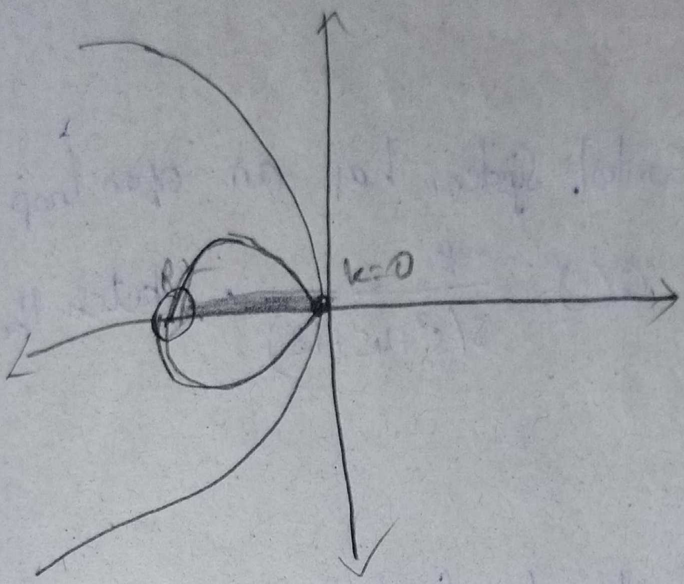

The number of open loop poles on Zeros toward its Right should be odd.

Rule 2:

Each branch of Root locus starts at open loop Pole corresponding to $k=0$.

Terminates/end at finite open loop zero or infinity.

i.e $K=\infty$




Rule 3: Branches of root locus.

No of branches terminates at $\infty$ equal $n-m$

No. of branches $= n-m$

$n$ = no. of Poles.

$m$ = no. of Zeros

$n=2$

$m=0$

$n-m$:

$2-0=2$

Condition:

If $n > m$; no of branches $= n$.

If $m > n$, " " " $= m$.

If $m = n$, " " " Can be m or n

Problems:

10M

A unity fdb Control System has an open loop transfer function $G(s) = \frac{k}{s(s^2+4s+13)}$. Sketch the root locus.

Sol:

The open loop transfer function is

$H(s) = 1$

$G(s) \cdot H(s)$

$$G(s) = \frac{K}{s(s^2+4s+13)}$$

Step 1:

Locating the poles and Zeros on s-plane

roots/Poles of open loop transfer function is

$s(s^2+4s+13) = 0 \quad \boxed{s = 0}$

$s^3 + 4s^2 + 13s = 0$

$a = 1 \quad b = 4 \quad c = 13$

The roots are $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\dfrac{-4 \pm \sqrt{16 - 52}}{2}$

$\begin{array}{r} 13 \\ \times 4 \\ \hline 52 \end{array}$

$\begin{array}{r} 16 \\ 52 \\ \hline 68 \end{array}$

$\dfrac{-4 \pm \sqrt{-\cancel{68}\,36}}{2}$

$\dfrac{-4 \pm \sqrt{-6}}{2} \qquad \dfrac{-4 \pm -6}{2}$

$\dfrac{-4}{2}, \quad \dfrac{-6}{2}$

$s = -2, \quad s = j3$

$\boxed{s = 0} \qquad \boxed{s = -2 \pm j3.}$

The open loop poles are $0, -2 \pm j3, -2 - j3$

three poles. $P_1 = 0$

$P_2 = -2 + j3$

$P_3 = -2 - j3.$

$\left.\begin{array}{l} P_2 = -2 + j3 \\ P_3 = -2 - j3 \end{array}\right\}$ Complex poles.

Number poles $n = 3$

Number of Zeros $m = 0$

Step 2: finding root locus on real axis

Take test point on -ve real axis to the right side of then the entire -ve real axis is root locus ∴ So entire -ve real axis is said to be root locus.

Step 3: finding angle of Asmptotes & Centroid.

for

$$n = 3$$

$$m = 0$$

$$n > m$$

No. of branches = 3.

Asmptotes gives direction of root locus branches.

No. of Asmptotes = $n - m$

$= 3 - 0$

$= 3$.

~~Asy~~ Asmptotes are straight line which means Zero at infinity. (0 at ∞)

Angle of Asmyptotes can be calculated as

$$= \frac{\pm 180^\circ (2q+1)}{n-m}$$

$q = 0, 1, 2, \ldots n-m.$

$3-0=3.$

$q = 0, 1, 2, 3.$

If $q=0$ Angle of Asmyptotes

$$= \frac{\pm 180^\circ (2(0)+1)}{3}$$

$$= \frac{\pm 180^\circ (1)}{3}$$

$= \pm 60^\circ.$

If $q=1$

$$= \frac{\pm 180^\circ (2(1)+1)}{3}$$

$= \pm 180^\circ$

If $q=2$

$$= \frac{\pm 180^\circ (4+1)}{3} = \pm 300^\circ$$

$= \mp 60^\circ$

If $q = 3$

$$= \frac{\pm 180^\circ (2(3)+1)}{3}$$

$$= \pm 420 = \pm 60^\circ$$

$$\boxed{\begin{aligned} \theta_1 &= \pm 60^\circ \\ \theta_2 &= \pm 180^\circ \\ \theta_3 &= \mp 60^\circ \end{aligned}}$$ because we have three branches.

$\theta_4 = \pm 60^\circ$

## Centroid:

Intersection point of Asmptotes.

$$\text{Centroid} = \frac{\text{Sum of Poles} - \text{Sum of Zeros}}{n - m}$$

$$= \frac{\cancel{3 - 0}}{\cancel{3}}$$

$$= \frac{0 + (-2+j3) + (-2-j3)}{3}$$

$$= \frac{-2-j3-2+j3}{3}$$

$$= \frac{-4}{3} = -1.333.$$

No of Poles Odd $\rightarrow$ Root locus forms on RHS.

" " " even $\rightarrow$ No Root locus forms.

Step 4: Finding break away point or break in point.

Rules:

Break point: The point at which two or more poles meet directly at any location.

Break away point: The point at which poles leave the real axis.

Break in point: The point at which poles enter into the real axis.

There is a Condition

Root locus branches enter or leave the real axis with an angle of $\frac{\pm 180^\circ}{n}$.

Condition for break away point or break in point available or not.

① Find characterstic equation.

② Find k.

③ Differentiate k wrt to s.

$$\frac{dk}{ds} = 0$$

④ Subsitute s value in k equation

⑤ If k = +ve or real, break away or break in point is available.

else if k = -ve or imaginary, break away or break in point is not available.

Find.

① Characterstic equation:

$$G(s) = \frac{k}{s(s^2+4s+13)} \qquad = 1+G(s).H(s)=0$$

$$1 + \frac{k}{s(s^2+4s+13)} = 0$$

$$s(s^2+4s+13)+k=0$$

③ Finding k Value:

$$s^3+4s^2+13s+k=0$$

$$\boxed{k=-s^3-4s^2-13s}$$

③ Differentiate k wrt to s.

$$\boxed{\frac{dk}{ds}=-3s^2-8s-13}$$

$$0=-(3s^2+8s+13)$$

$$\boxed{3s^2+8s+13=0}$$

$a=3$

$b=8$

$c=13$

$$\boxed{S=-1.33, \pm j1.6}$$

$$\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$\frac{13\times4\times3}{52}$

$4\times13\times3$

$\begin{array}{r} 64 \\ 52 \\ \hline 12 \end{array}$

$$=\frac{-8\pm\sqrt{64-156}}{6}$$

$$=\frac{-8\pm\sqrt{12}}{6}$$

$\frac{\cancel{14}\,7}{\cancel{6}\,3}$

$$=\frac{-8\pm6}{6}=\frac{-8+6}{6},\ \frac{-8-6}{6}$$

$$S=-\frac{1}{3},\ -\frac{7}{3}$$

$$\boxed{S=-1.33, \pm j1.6}$$

$13\times3$

$\begin{array}{r} 39\times4 \\ \hline 156 \end{array}$

$s^3+4s^2+13s+k$

(4) Sub s value in k eq

$$k = -s^3 - 8s^2 - 13s$$

$$k = -(-1.33+j1.6)^3 - 8(-1.33+j1.6)^2 - 13(-1.33+j1.6).$$

k ≠ +ve or real imaginary. So break away or breakin points are not available.

So Root locus doesn't have break away & break in Point.

Steps 5: finding angle of depature and angle of arrival

Angle of depature:

Calculated for only for Complex pole.

angle at which complex pole depart from its location.

The given system is having only Complex pole. So, Calculate angle of depature

Angle of departure = $180^\circ$ − Sum of angles from Complex pole A to other poles + Sum of angles from Complex pole A to other Zeors.

Angle of departure wrt to $P_2$ :

$$AOD \text{ at } P_2 = 180^\circ - \theta_1 + \theta_2$$

$$\theta_1 = 180^\circ - \tan^{-1}\left(\frac{a}{b}\right)$$

$$\theta_1 = 180^\circ - \tan^{-1}\left(\frac{3}{2}\right)$$

$P_2 \Rightarrow$ making an angle $\theta_1$ with $P_1$

$P_2 \Rightarrow$ " " " $\theta_2$ with $P_3$

$$\theta_1 = 180^\circ - \tan^{-1}\left(\frac{3}{2}\right)$$

$$\boxed{\theta_1 = 123.69^\circ}$$

$$\theta_2 = 90^\circ$$

$$= 180^\circ - (\theta_1 + \theta_2)$$

$$AOD \text{ to } P_2 = 180^\circ - (123.697 + 90^\circ) = -33.7^\circ$$

$\boxed{\text{AOD wrt } P_2 = -33.7}$

Angle of depature wrt to $P_3$ is

$\boxed{+33.7}$ $P_3$ is Complex conjugate of $P_2$.

~~Angle~~

Step 6: Finding the Crossing Point at imaginary axis.

Substitute $S = j\omega$ in characteristic equation

$$s^3 + 4s^2 + 13s + K = 0$$

$$(j\omega)^3 + 4(j\omega)^2 + 13(j\omega) + k = 0$$

$$\boxed{-j\omega^3 - 4\omega^2 + 13j\omega + k = 0}$$

Equate real part & imaginary part

→ Equate imaginary part with 0.

$$-\omega^3 + 13\omega = 0$$

$$+\cancel{\omega}(\omega^2 + 13) = 0 \qquad \omega = \sqrt{\ }$$

$$= \omega^2 + 13 = 0$$

$$\cancel{-}\omega^2 = \cancel{-}13$$

$$\omega = \pm\sqrt{13}.$$

$$\boxed{\omega = \pm 3.6}$$

→ Equate real part with

$$-4\omega^2 + k = 0$$

$$\therefore k = 4\omega^2.$$

$$k = 4 \times 13.$$

$$\boxed{k = 52}$$

The root locus crossing point on imaginary axis.

$$s = j\omega.$$

$$\boxed{s = \pm j3.6}$$

2m
Effect of adding Poles and Zeros to G(S). H(S). on root locus.

1. If we add poles & zeros the root locus can be shifted.

2. If adding pole to open loop T.F. then root locus branches will shift towards RHS. Therefore & damping ratio decreases the stability of the system decreases

3. If adding zero to open loop T.f then root locus branches will shift towards LHS. Therefor & ~~decreases~~ increases the stability of the system ~~decreases~~. increases.