UNIT - 4

Frequency ~~Domain~~ Response Analysis

Introduction :-

The system performance can be analysed by using time response or frequency response.

→ In time domain analysis the time response of the system can be obtained only if the transfer function is known earlier but it not always possible.

→ For higher order systems it is difficult to find roots or poles of the system

2M Advantages of frequency domain Analysis :-

→ Frequency response can be obtained even the transfer function is not known

→ Frequency response analysis is simple and accurate.

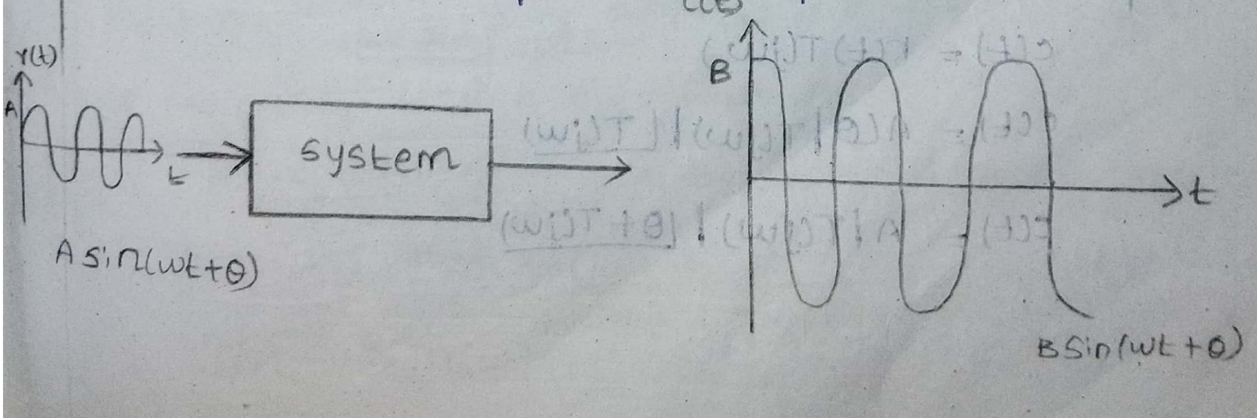
Frequency Response :-

→ Frequency Response is defined as the steady state response of the system to the sinusoidal input

→ Input → Sinusoidal

output → steady state

but differs in amplitude and phase.



Sinusoidal Transfer function :-

It is defined as the ratio of sinusoidal response to sinusoidal input.

$s \to j\omega$

$$T(s) \xrightarrow{s=j\omega} T(j\omega) = \frac{\text{Sinusoidal response}}{\text{Sinusoidal Input}}$$

$T(j\omega)$ ↓ Sinusoidal TF

→ Sinusoidal transfer function is also called as; frequency domain transfer function.

→ $T(j\omega)$ it is represented in magnitude of $T(j\omega)$ and phase angle of $T(j\omega)$.

$T(j\omega)$:
- Magnitude of $T(j\omega)$
- phase angle of $T(j\omega)$

Input,

$r(t) = A\sin(\omega t+\theta)$

$A\angle\theta$

Transfer function,

$$T(j\omega) = \underbrace{|T(j\omega)|}_{M}\ \underbrace{\angle T(j\omega)}_{\phi}$$

$r(t) = A\sin(\omega t+\theta)$ → [$T(j\omega) = |T(j\omega)|\angle T(j\omega)$] → $c(t)$

$A\angle\theta$

Output,

$c(t) = r(t)\,T(j\omega)$

$c(t) = A\angle\theta\,|T(j\omega)|\angle T(j\omega)$

$c(t) = A|T(j\omega)|\ \angle\theta + T(j\omega)$

Multiply   Addition

Frequency Response Representation:-

The frequency response analysis of a system is ~~obtained~~ used to determine gain and phase angle of the system at different frequencies. This can be analysied in two forms:

i) Tabular form

ii) Graphical form

Tabular form:-

The system output is connected to CRO and taking the readings, it is useful for representation of gain and phase angle and it is used for only limited data and even if the transfer function is not known.

Graphical form:-

There are various graphical techniques are available for frequency response analysis

Ex: Bode plot

polar plot

Frequency Domain Specifications:-

The performance and characteristics of the system in frequency domain are measured in terms of frequency domain specifications

$G(s) \xrightarrow{s=j\omega} G(j\omega)$ – open loop transfer function

$G(s)H(s) \xrightarrow{s=j\omega} G(j\omega)\,H(j\omega)$ – Loop transfer function

$T(s) \longrightarrow T(j\omega)$ – closed loop Transfer function

Frequency Response of second order system :-

The standard second order equation is

$$T(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

Replace $s \to j\omega$

$$T(j\omega) = \frac{\omega_n^2}{(j\omega)^2 + 2\xi\omega_n(j\omega) + \omega_n^2}$$

$$T(j\omega) = \frac{\omega_n^2}{-\omega^2 + j2\xi\omega\omega_n + \omega_n^2}$$

$$T(j\omega) = \frac{\omega_n^2}{\omega_n^2\left[1 - \left(\frac{\omega}{\omega_n}\right)^2 + j2\xi\frac{\omega}{\omega_n^2} \times \omega_n\right]}$$

$$T(j\omega) = \frac{1}{1 - \left(\frac{\omega}{\omega_n}\right)^2 + j2\xi\frac{\omega}{\omega_n}}$$

Let $u = \frac{\omega}{\omega_n}$

where u is normalized frequency

$$\boxed{T(j\omega) = \frac{1}{(1-u^2) + j2\xi u}}$$

($a + jb$)

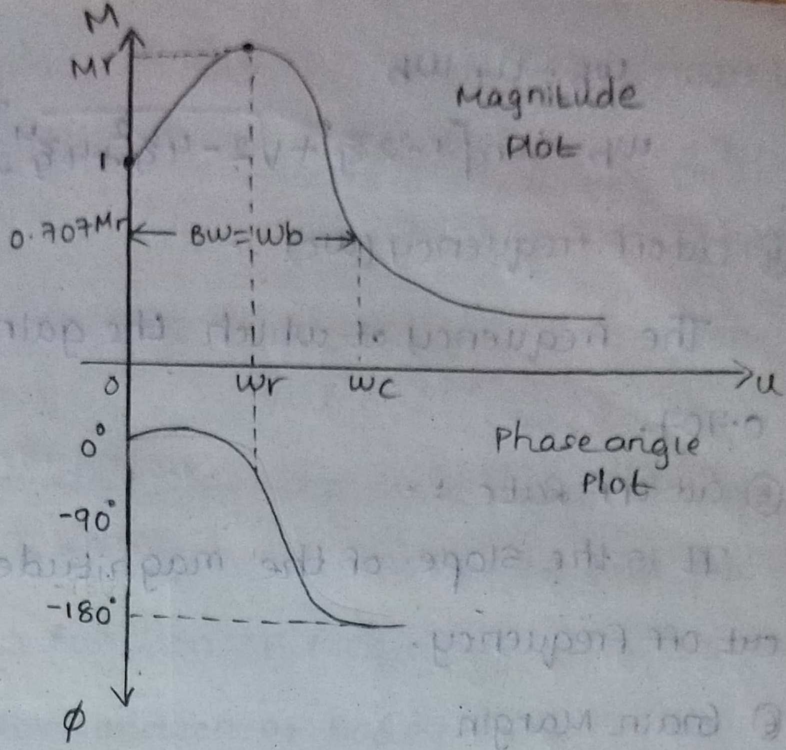
Magnitude $M = |T(j\omega)| = \frac{1}{\sqrt{(1-u^2)^2 + (2\xi u)^2}}$

Phase angle $\phi = \angle T(j\omega) = -\tan^{-1}\left(\frac{2\xi u}{1-u^2}\right)$

The steady state output is,

$$c(t) = \frac{1}{\sqrt{(1-u^2)^2 + (2\xi u)^2}} \sin\left(\omega t - \tan^{-1}\left(\frac{2\xi u}{1-u^2}\right)\right)$$

u	M	∅
0	1	0°
1	$\frac{1}{2\xi}$	-90°
∞	∞	-180°



Specifications :-

① Resonant peak (Mr) :-

It is maximum value of magnitude of closed loop transfer function. It indicates relative stability.

$$M_r = \frac{1}{2\xi\sqrt{1-\xi^2}}$$

② Resonant frequency (Wr) :-

The frequency at which maximum peak occurs is called Resonant frequency.

$$W_r = W_n\sqrt{1-2\xi^2}$$

③ Bandwidth ~~(BW)~~ :- (Wb) :-

It is the range of frequency at which the magnitude drops to 3dB down or 0.707Mr from its initial value.

$$BW \propto \frac{1}{t_r}$$

Bandwidth increases, rise time decrease then the system response will be fast

$\omega_b = \omega_n \omega_b$

$$\omega_b = \omega_n\left[1-2\xi^2+\sqrt{2-4\xi^2+4\xi^4}\right]^{1/2}$$

④. cutoff frequency (ω_c) :-

The frequency at which the gain becomes 0.707.

⑤ cut-off Rate :-

It is the slope of the magnitude curve near the cut-off frequency.

⑥ Gain Margin

⑦ phase margin

⑧ Gain crossover frequency

⑨ Phase cross over frequency

Frequency Response plots :-

There are various graphical techniques for frequency response analysis :

→ Bode plot - semilog graph
→ Polar plot (Nyquist plot) - polargraph & ordinary graph (0 to ∞ / -∞ to ∞)
→ Nichol's plot

} drawn for open loop system Performance & stability of closed loop system.

→ M and N circles plot
→ Nichol's chart

} unity feed back closed loop system

Bode Plot :-

→ Bode plot is a graphical representation of frequenc response. It consists of 2 plots

i) Magnitude plot
ii) phase angle plot

} as a function of ω

i) Magnitude plot :- It is a gain or magnitude of open loop transfer function as a function of frequency 'w'

$|G(jw)|_{dB} = 20 \log_{10} |G(jw)|$

ii) Phase angle plot :- It is a phase angle of open loop transfer function as a function of frequency 'w'.

Uses of Bodeplot :

It is used to determine the stability of closed loop transfer function by observing magnitude and phase angle plot as a function of frequency w.

Standard Transfer Function of Bode plot :-

$$G(S) = \frac{K(1+ST_1)(1+ST_2)}{S^N(1+ST_a)(1+ST_b)}$$

→ It is called as Bode form or Time constant form.

where K → constant gain

N → Type no. of poles at origin

where T_1, T_2 are time constant of zero

T_a, T_b are time constant of poles

→ $S = jw$

$$G(jw) = \frac{K(1+jwT_1)(1+jwT_2)}{(jw)^N(1+jwT_a)(1+jwT_b)}$$

General procedure :-

1. convert the given transfer function into frequency domain. $(S = jw)$
2. Find magnitude.
3. Express magnitude in dB
4. Find phase angle (ϕ)
5. plot magnitude and phase angle as a function of frequency.

Factors :-

1. K
2. $(j\omega)^{\pm N}$
3. $(1+j\omega T)^{\pm N}$
4. $\left[1+j2\xi\left(\frac{\omega}{\omega n}\right)-\left(\frac{\omega}{\omega n}\right)^2\right]^{\pm P}$

① K :-

$G(S) = K$

i) $S = j\omega$

$G(j\omega) = K$

ii) $|G(j\omega)| = \sqrt{K^2}$

$|G(j\omega)| = K$

iii) $|G(j\omega)|_{dB} = 20 \log K$

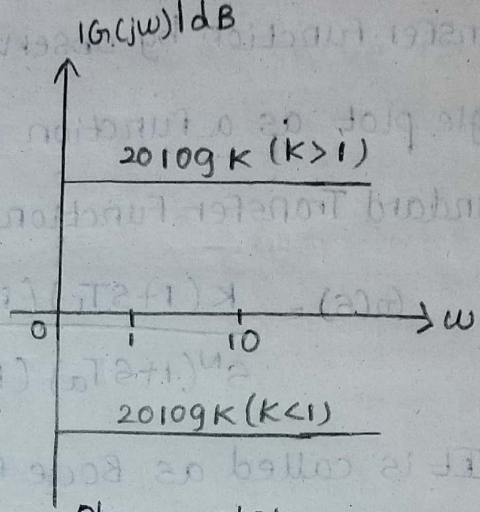
iv) $\phi = \tan^{-1}\left(\frac{0}{K}\right)$

$\boxed{\phi = 0°}$

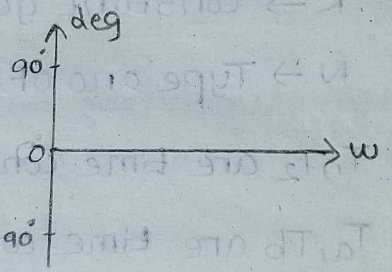
IF $K>1$ +ve

IF $K>1$ -ve

Magnitude Plot:



Phase plot:



② $(j\omega)^{\pm N}$

case(i): $(j\omega)^{+N}$

i) $G(j\omega) = (j\omega)^{+N}$

ii) $|G(j\omega)| = \sqrt{\omega^{2N}}$

$|G(j\omega)| = \omega^N$

iii) $|G(j\omega)|_{dB} = 20\log \omega^N$

$= 20N \log \omega$

$y = mx+c$

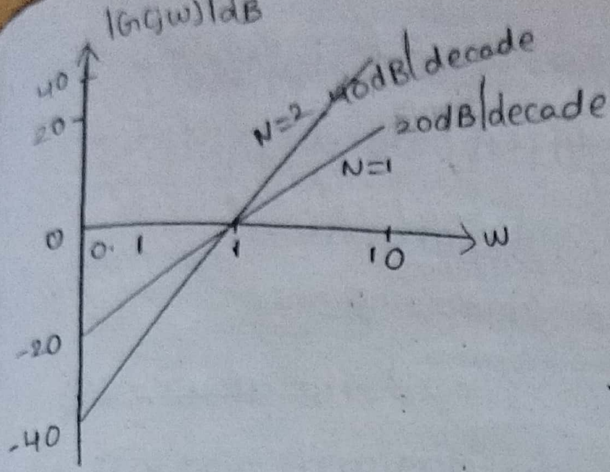
slope = $20N$ dB/decade

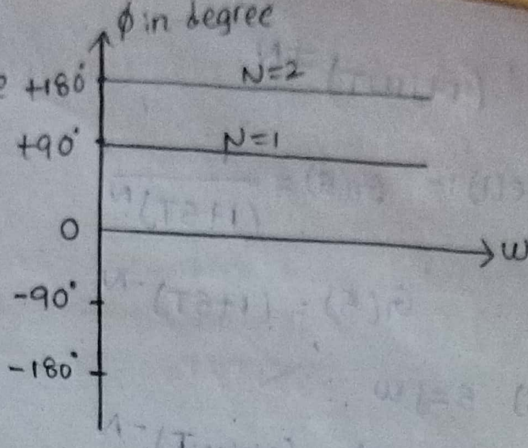
iv) $\phi = \tan^{-1}\left(\frac{\omega^N}{0}\right)$

$= N\tan^{-1}\left(\frac{\omega}{0}\right) \Rightarrow N\tan^{-1}(\infty) = 90°$

$\phi = N90°$

w	20log w
0.1	-20
1	0
10	20





case (ii): $(jw)^{-N}$

i) $G(jw) = \left(\frac{1}{jw}\right)^{-N}$

ii) $|G(jw)| = \sqrt{w^{2}}^{-N}$

$|G(jw)| = w^{-N}$

iii) $|G(jw)|_{dB} = +20 \log w^{-N}$

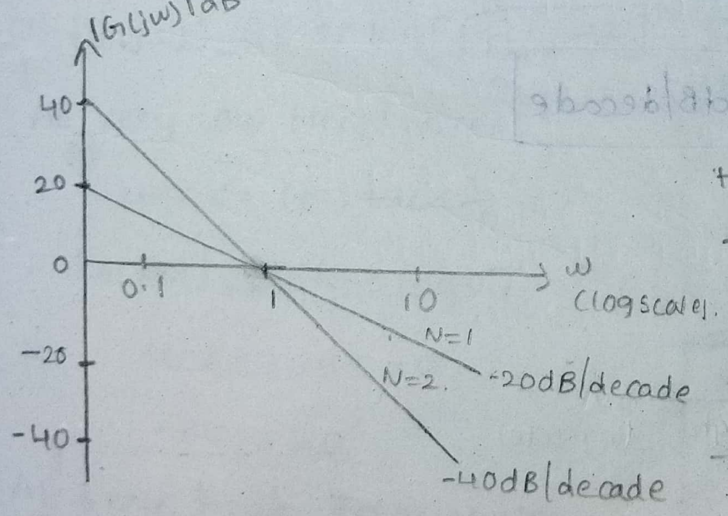
$= -20N \log w$

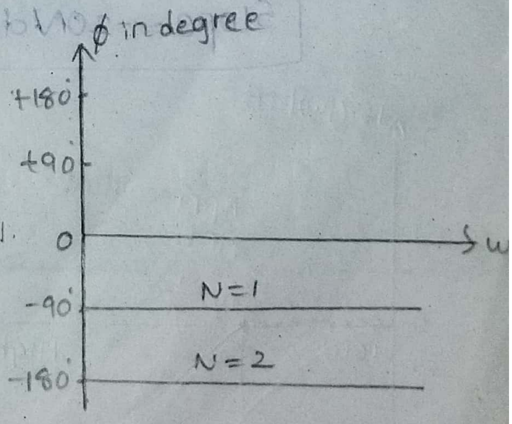
iv) $\phi = \tan^{-1}\left(\frac{w^{-N}}{0}\right)$

$= -N \tan^{-1}\left(\frac{w}{0}\right) = -N \tan^{-1}(\infty) = 90^{\circ}$

$\phi = N 90^{\circ}$

slope = $-20N$ dB/decade





③ $(1+j\omega T)^{\pm N}$

case(1):- $G(s) = \frac{1}{(1+sT)^N}$

$G(s) = (1+sT)^{-N}$

i) $s = j\omega$

$G(j\omega) = (1+j\omega T)^{-N}$

ii) $|G(j\omega)| = \left[\sqrt{1+(\omega T)^2}\right]^{-N}$

iii) $|G(j\omega)|_{dB} = 20\log\left[\sqrt{1+(\omega T)^2}\right]^{-N}$

$$\boxed{|G(j\omega)|_{dB} = -N\,20\log\sqrt{1+(\omega T)^2}} \longrightarrow ①$$

At very low frequencies $\omega T \ll 1$.

$|G(j\omega)|_{dB} = -20N\log\sqrt{1+(\omega T)^2} = 0$

At very high frequencies $\omega T \gg 1$

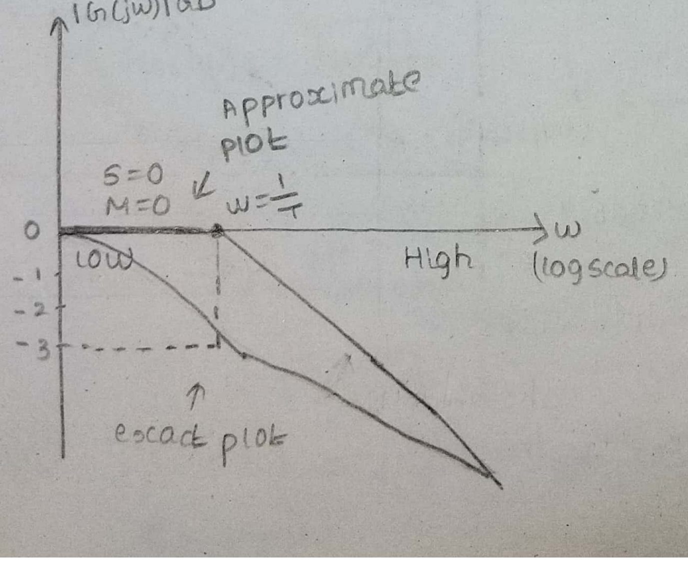
$|G(j\omega)|_{dB} = -20N\log\sqrt{(\omega T)^2}$ $\qquad \log ab = \log a + \log b$

$= -20N\log \omega T$

$|G(j\omega)|_{dB} = -20N\log\omega - 20N\log T$

$y = mx + c$

$$\boxed{\text{Slope} = -20N\ \text{dB/decade}}$$



sub $\omega = \frac{1}{T}$

$|G(j\omega)|_{dB} = -N 20 \log \sqrt{1+(1)^2}$

$= -N 20 \log \sqrt{2}$

$= -3$

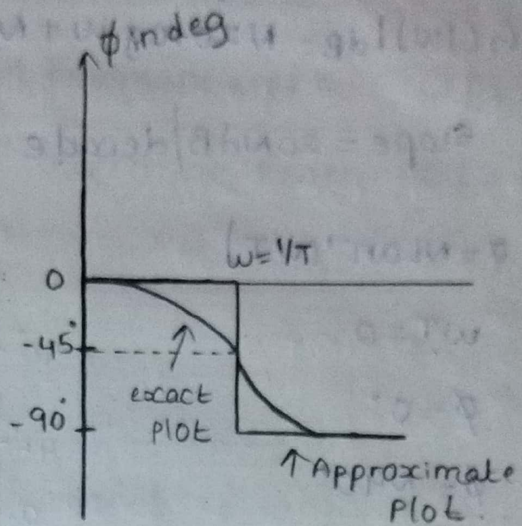
$\phi = -N \tan^{-1} \omega T$

At very Low frequency

$\boxed{\phi = 0^\circ}$

At very high frequency

$\boxed{\phi = -N 90^\circ}$



when $\omega = \frac{1}{T}$

$\phi = -N \tan^{-1}(1)$

$\phi = -45^\circ$

case (ii) :- $(1+j\omega T)^{+N}$

i) $G(j\omega) = (1+j\omega T)^{+N}$

ii) $|G(j\omega)| = \left(\sqrt{1^2+(\omega T)^2}\right)^N$

$|G(j\omega)| = \left(\sqrt{1+(\omega T)^2}\right)^N$

iii) $|G(j\omega)|_{dB} = 20 \log \left[\sqrt{1+(\omega T)^2}\right]^N$

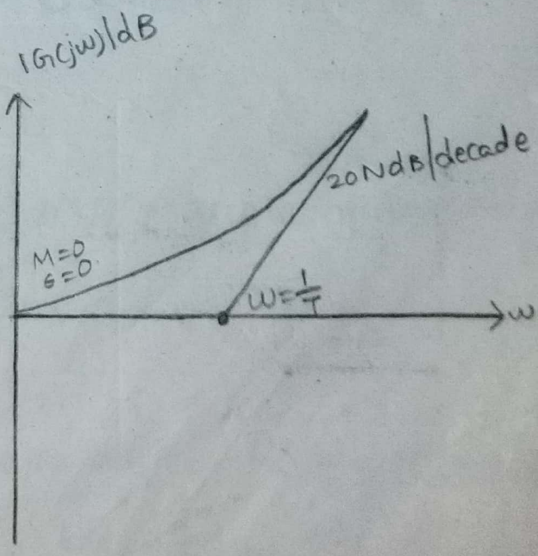
At very low frequencies

$\omega T << 1$ (or) $\omega << \frac{1}{T}$

$|G(j\omega)|_{dB} = N 20 \log \sqrt{1}$

Magnitude $= 0$

slope $= 0$



At very high frequencies,

$\omega T >> 1$ (or) $\omega >> \frac{1}{T}$

$|G(jw)|_{dB} = N 20 \log \sqrt{(wT)^2}$

$|G(jw)|_{dB} = N 20 \log wT$

$|G(jw)|_{dB} = N 20 \log w + N 20 \log T$

$y = mx + c$

slope = 20N dB/decade

At $w = \frac{1}{T}$

$|G(jw)|_{dB} = 20 N \log \sqrt{1+1}$

$= 20 N \log \sqrt{2}$

iv) $\phi = N \tan^{-1}(wT)$

$wT = 0$

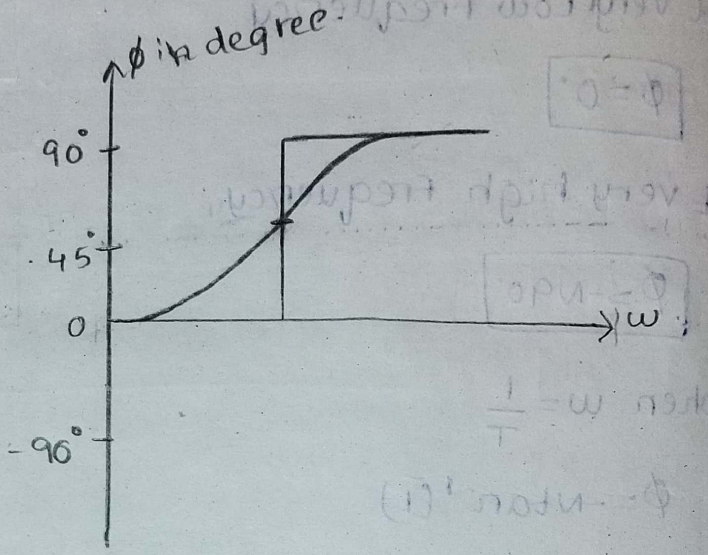
$\phi = 0°$

$\phi = N 90°$

At $w = T$

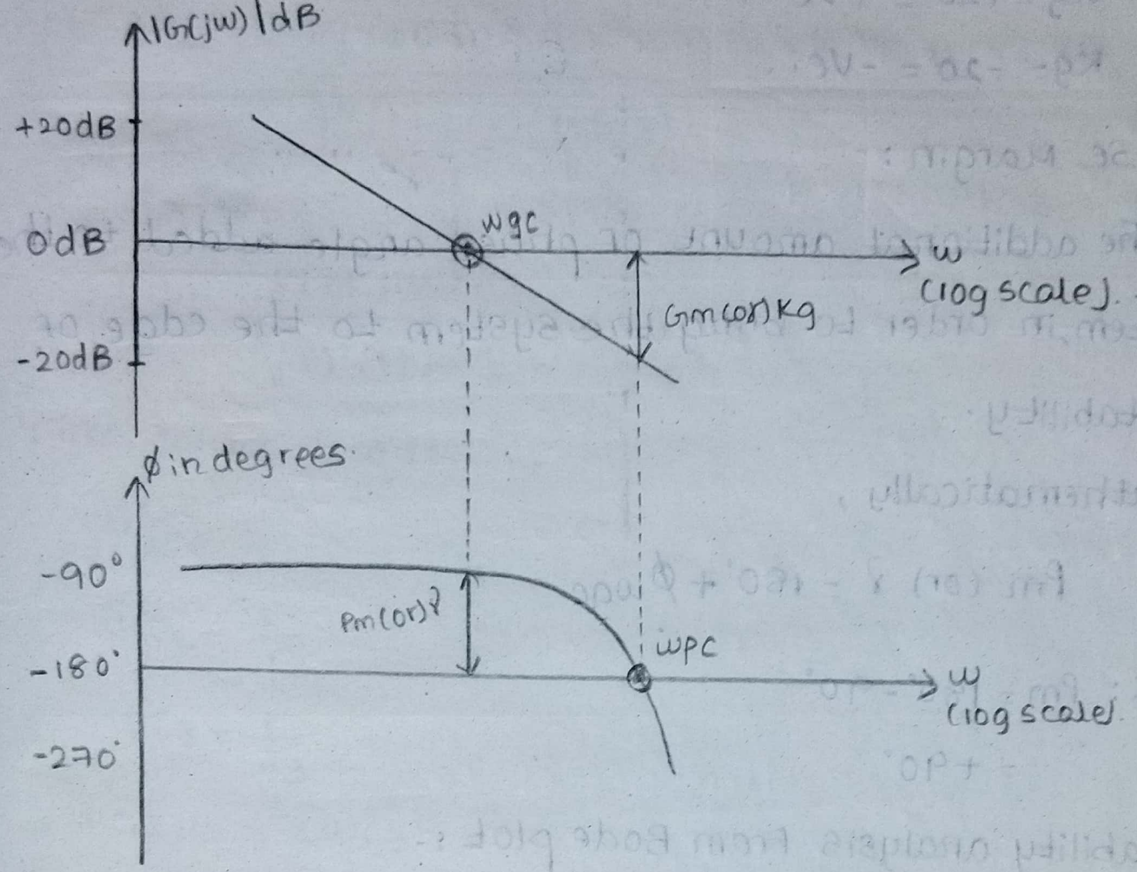
$\phi = N \tan^{-1}(1)$

$\phi = N 45°$



* Determination of frequency domain specifications from the Bode plot

The frequency domain specifications like Gain margin, phase margin, gain cross over frequency, phase cross over frequency, can be easily determine from bode plot



Gain cross over frequency (wgc):-

The frequency at which the gain of transfer function is unity or 0 dB.

Phase cross over frequency (wpc):-

The frequency at which the phase angle of transfer function is -180°

Gain margin :-

The additional amount of gain added to the system in order to bring the system to the edge of in instability

Mathematically,

$$G_m \text{ (or) } K_g = 20 \log \frac{1}{|G(j\omega_{pc})|}$$

$$\boxed{G_m \text{ (or) } K_g = -20 \log |G(j\omega_{pc})|}$$

Ex: $K_g = +20 = +ve$

$K_g = -20 = -ve$

Phase Margin :-

The additional amount of phase angle added to the system, in order to bring the system to the edge of instability.

Mathematically,

$$P_m \text{ (or) } \gamma = 180^\circ + \phi_{wgc}$$

EX: $P_m = 180^\circ - 90^\circ$

$= +90^\circ$

Stability analysis from Bode plot :-

① $\omega_{pc} > \omega_{gc}$

system is stable

$G_m, P_m = +ve$

② $\omega_{pc} < \omega_{gc}$ ∴ system is unstable

$G_m, P_m = -ve$

③ $\omega_{pc} = \omega_{gc}$

$G_m = 0 dB$

$P_m = 0^\circ$

Procedure for Bode plot:-

The Bode plot consists of two plots magnitude plot and phase angle plot.

step 1 :- convert the given transfer function into frequency domain or bode form or time constant form.

Ex-1: $G(S) = \frac{K(1+ST_1)^2}{S^2(1+ST_2)(1+ST_3)}$

$S = j\omega$

$G(j\omega) = \frac{K(1+j\omega T_1)^2}{(j\omega)^2(1+j\omega T_2)(1+j\omega T_3)}$

⇒ Find corner frequencies

$\omega c_1 = \frac{1}{T_1}$

$\omega c_2 = \frac{1}{T_2}$

$\omega c_3 = \frac{1}{T_3}$

$\omega c_1 < \omega c_2 < \omega c_3$

EX-2: $G(S) = \frac{(S+a)(S+b)}{(S+P)(S+q)}$

$G(S) = \frac{ab\left(1+\frac{S}{a}\right)\left(1+\frac{S}{b}\right)}{Pq\left(1+\frac{S}{P}\right)\left(1+\frac{S}{q}\right)}$

$\omega a < \omega b < \omega p < \omega q$

EX-3: $G(S) = \frac{K(1+ST_1)}{S(1+ST_2)\left(1+\frac{S^2}{\omega n^2}+2\xi\frac{S}{\omega n}\right)}$

$S = j\omega$

$$G(j\omega) = \frac{K(1+j\omega T_1)^2}{(j\omega)(1+j\omega T_2)\left(1+\frac{\omega^2}{\omega_n^2}+2\xi\frac{j\omega}{\omega_n}\right)}$$

$\omega_{c1} < \omega_{c2} < \omega_{c3} = \omega_n$

Step 2 :- Magnitude plot each term in the given transfer function can be identified then & enter in the table in the increasing order of corner frequency.

Term	corner frequency rad/sec	slope in dB/dec	change in slope dB/dec
$\frac{1}{(j\omega)^2}$	-	-40	-
$(1+j\omega T_1)^2$	$\omega_{c1} = \frac{1}{T_1}$	+40	$-40+40=0$
$\frac{1}{(1+j\omega T_2)}$	$\omega_{c2} = \frac{1}{T_2}$	-20	$0-20=-20$
$\frac{1}{1+j\omega T_3}$	$\omega_{c3} = \frac{1}{T_3}$	-20	$-20-20=-40$

Step3: choose two differeacies of frequencies other than corner frequency.

$\omega_l < \omega_{c1} < \omega_{c2} < \omega_{c3} < \omega_h$

choose ω_l which is less than lowest corner frequency

choose ω_h which is greater than highest corner frequency.

Step4: Calculate gain in dB at every corner frequency one by one by using the formula

$$\text{Gain at } \omega_y = \left[\text{change in slope from } \omega_x \text{ to } \omega_y \times \frac{\log \omega_y}{\log \omega_x}\right] + \text{gain at } \omega_x$$

EX: Gain at $\omega_{c2} = \left[\text{change in slope } \omega_{c1} \text{ to } \omega_{c2} \times \frac{\log \omega_{c2}}{\omega_{c1}}\right] + \text{gain at } \omega_{c1}$

Step 5: Mark the required range of frequency on x-axis (log scale) and range of dB on y-axis with proper units.

Step 6: Mark all the points obtained in Step 3 & 4 on graph and join the points. Mark the slope at every part of the graph.

Phase angle (ϕ):-

$$\phi = \angle G(j\omega) = -180^\circ + 2\tan^{-1}\omega T_1 - \tan^{-1}\omega T_2 - \tan^{-1}\omega T_3$$

Problems:

1. Sketch the bode plot for the following transfer function and determine the system gain K for the gain cross over frequency to be 5 rad/sec.

$$G(s) = \frac{Ks^2}{(1+0.2s)(1+0.02s)}$$

A. Step 1:

$$G(s) = \frac{Ks^2}{(1+0.2s)(1+0.02s)}$$

$s = j\omega$ and assume $K = 1$

$$G(j\omega) = \frac{K(j\omega)^2}{(1+0.2j\omega)(1+0.02j\omega)}$$

corner frequencies,

$$\omega c_1 = \frac{1}{0.2} = 5 \text{ rad/sec}$$

$$\omega c_2 = \frac{1}{0.02} = 50 \text{ rad/sec}$$

Step 2 :- Magnitude plot

Term	corner frequency rad/sec	slope in dB/dec	change in slope dB/sec
$(jw)^2$	-	40	-
$\frac{1}{1+0.2jw}$	5	-20	40-20=20
$\frac{1}{1+0.02jw}$	50	-20	20-20=0

Step 3 :- choose two different frequencies other than corner frequency.

$w_l < w_{c1} < w_{c2} < w_h$

0.5 5 50 100

$w_l = 0.5$ rad/sec

$w_h = 100$ rad/sec

Step 4 :- Gain calculation

Gain $A = 20\log|G(jw)|$

$A = 20\log|\text{First term}|$

$w_l < w_{c1}$

$A_{at}\ w = w_l = 0.5 = 20\log|(jw)^2|$

$A_{at}\ w_l = 0.5 = 20\log|(w)^2|$

$A_{at}\ w_l = 20\log(0.5)^2$

$\boxed{A_{at}\ w_l = -12dB}$

$A_{at}\ w_{c1} = 20\log(w)^2$

$= 20\log(5)^2$

$\boxed{A_{at}\ w_{c1} = 28dB}$

$A_{at}\ w_{c2} = \left[\text{change in slope } w_{c1} \text{ to } w_{c2} \times \frac{\log w_{c2}}{\log w_{c1}}\right] + A_{at}\ w_{c}$

A at $w_{c2} = \left[20 \log \frac{50}{5}\right] + 28$

$\boxed{A \text{ at } w_{c_2} = 48dB}$

A at $w_h = \left[\text{change in slope } w_{c_2} \text{ to } w_h \times \frac{\log w_h}{\log w_{c_2}}\right] + A$ at w_{c_2}

$= \left[0 \log \frac{100}{50}\right] + 48$

$\boxed{A \text{ at } w_h = 48dB}$

w	K=1 A in dB		K=0.0398 A in dB
0.5	-12	a	-40
5	28	b	0
50	48	c	20
100	48	d	20

Phase angle ϕ :-

Phase angle $\phi = \angle G(jw) = +180° - \tan^{-1}(0.2w) - \tan^{-1}(0.02w)$

w	ϕ in degrees	
0.5	173.7 ≃ 174	e
1	167.5 ≃ 168	f
5	129.2 ≃ 130	g
10	105.3 ≃ 106	h
50	50.7 ≃ 51	i
100	29.4 ≃ 30	j

calculation of K :-

$20 \log K = 0 dB$

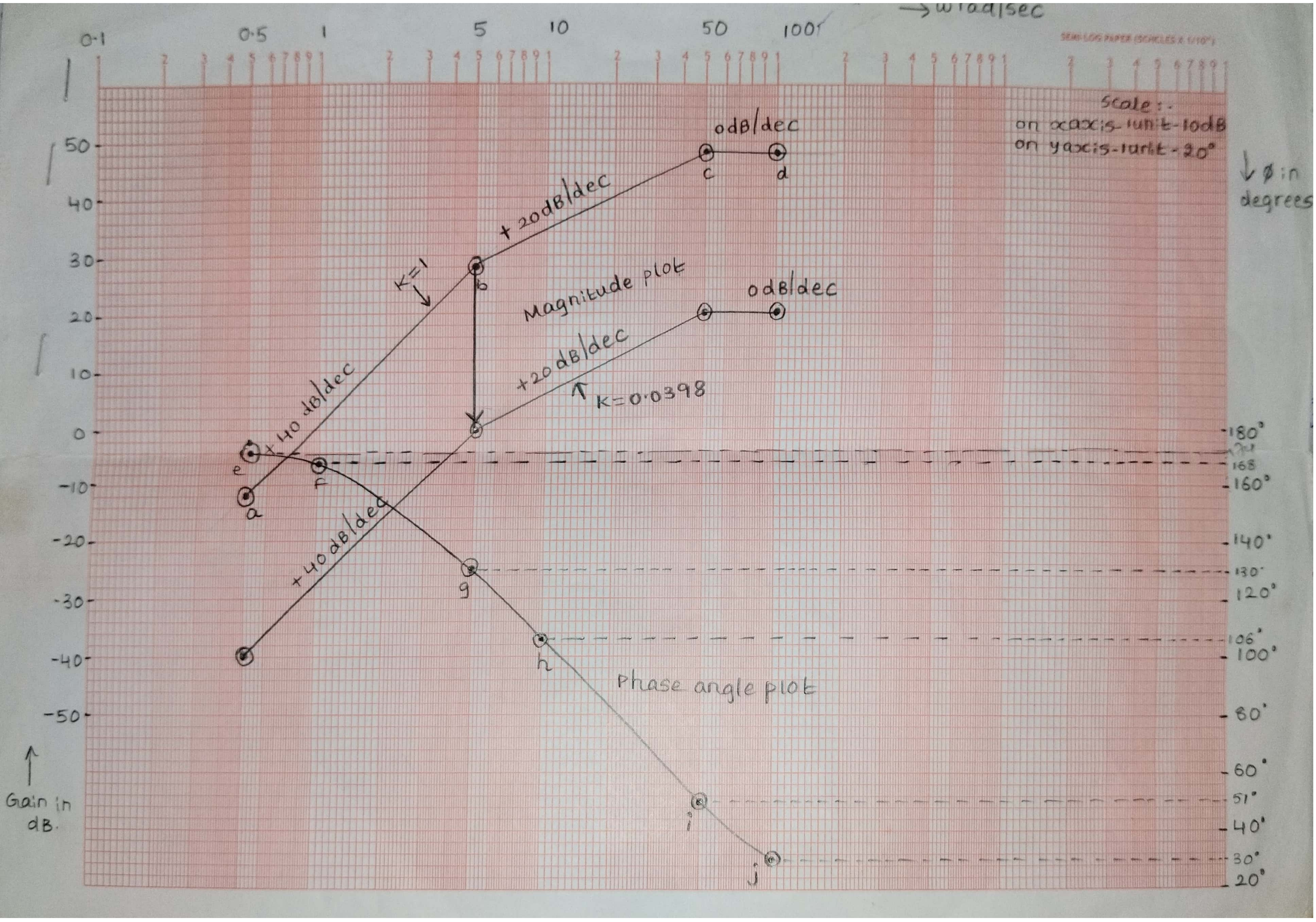
$20 \log K = -28$

$\log K = \frac{-28}{20}$

By anti logrithm,

$K = 10^{-28/20}$

$\boxed{K = 0.0398}$



→ ω rad/sec
0·1
0·5
1
5
10
50
100
SEMI-LOG PAPER (5CYCLES X 1/10")
Scale :-
on x axis - 1 unit - 10 dB
on y axis - 1 unit - 20°
↓ ϕ in degrees
odB/dec
odB/dec
+ 20dB/dec
+20 dB/dec
+ 40 dB/dec
+ 40 dB/dec
K=1
K=0·0398
Magnitude plot
Phase angle plot
a
b
c
d
e
f
g
h
i
j
50
40
30
20
10
0
-10
-20
-30
-40
-50
↑ Gain in dB.
180°
168
160°
140°
130°
120°
106°
100°
80°
60°
51°
40°
30°
20°

② Sketch the Bode plot for the following transfer function and determine phase margin and gain margin:

$$G(S) = \frac{75(1+0.2S)}{S(S^2+16S+100)}$$

A. The transfer function $G(S) = \frac{75(1+0.2S)}{S(S^2+16S+100)}$.

compare $S^2+16S+100$ with quadratic factor $S^2+2\xi\omega_n S+\omega_n^2$

$\omega_n^2 = 100 \Rightarrow \boxed{\omega_n = 10}$

$2\xi\omega_n S = 16S$

$\xi = \frac{16}{2\omega_n} = \frac{16}{2\times10}$

$\boxed{\xi = 0.8}$

$$G(S) = \frac{75(1+0.2S)}{S\times100\left(1+\frac{S^2}{100}+\frac{16S}{100}\right)}$$

$$\boxed{G(S) = \frac{0.75(1+0.2S)}{S(1+0.01S^2+0.16S)}}$$

$S = j\omega$

$$G(j\omega) = \frac{0.75(1+0.2j\omega)}{(j\omega)(1-0.01\omega^2+0.16j\omega)}$$

$\omega c_1 = \frac{1}{T_1} = \frac{1}{0.2} = 5$ rad/sec

$\omega c_2 = \omega_n = 10$ rad/sec

Step 2: Magnitude plot

Term	corner frequency rad/sec	slope dB/dec	change in slope dB/dec
$\frac{0.75}{j\omega}$	–	-20	–
$1+0.2j\omega$	$\omega c_1 = \frac{1}{0.2} = 5$	20	$-20+20=0$
$\frac{1}{1-0.01\omega^2+0.16j\omega}$	$\omega c_2 = \omega_n = 10$	-40	$0-40=-40$

Step 3 :- choose two different frequencies other than corner frequency

$\omega_l < \omega_{c1} < \omega_{c2} < \omega_h$

0.5   5   10   20

$\omega_l = 0.5$ rad/sec

$\omega_h = 20$ rad/sec

Step 4 :- Gain calculation.

A at $\omega_l = 0.5 = 20\log\left|\frac{0.75}{j\omega}\right|$

A at $\omega_l = 0.5 = 20\log\left(\frac{0.75}{\omega}\right)$

A at $\omega_l = 20\log\frac{0.75}{0.5}$

A at $\omega_l = 3.5$ dB

A at $\omega_{c1} = 5 = 20\log\left[\frac{0.75}{5}\right]$

A at $\omega_{c1} = -16.5$ dB

A at $\omega_{c2} = \left[\text{change in slope } \omega_{c1} \text{ to } \omega_{c2} \times \log\left(\frac{\omega_{c2}}{\omega_{c1}}\right)\right] +$ A at ω_{c1}

$= \left[0 \times \log\frac{10}{5}\right] + (-16.5)$

A at $\omega_{c2} = -16.5$ dB

A at $\omega_h = \left[\text{change in slope } \omega_{c2} \text{ to } \omega_h \times \log\left[\frac{\omega_h}{\omega_{c2}}\right]\right] + A$ at ω_{c2}

$= \left[-40 \times \log\frac{20}{10}\right] + (-16.5)$

A at $\omega_h = -28.5$ dB

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W	A in dB	
0.5	3.5	a
5	-16.5	b
10	-16.5	c
20	-28.5	d

Phase angle ϕ :-

phase angle $\phi = \angle G(j\omega) = -90° + \tan^{-1}(0.2\omega) - \tan^{-1}\left(\frac{0.16\omega}{1-0.01\omega^2}\right)$

$\omega < \omega_n$

$\phi = \angle G(j\omega) = -270° + \tan^{-1} 0.2\omega$ - (greater than 10) $\tan^{-1}\left(\frac{0.16\omega}{1-0.01\omega^2}\right)$

$\omega > \omega_n$

w	ϕ in degrees	
0.5	-88.9 ≃ -88	e
1	-87.9 ≃ -88	f
5	-91.8 ≃ -92	g
10	-116.6 ≃ -116	h
20	-147.3 ≃ -148	i
50	-167.3 ≃ -168	j
100	-173.7 ≃ 174	k

Phase Margin :

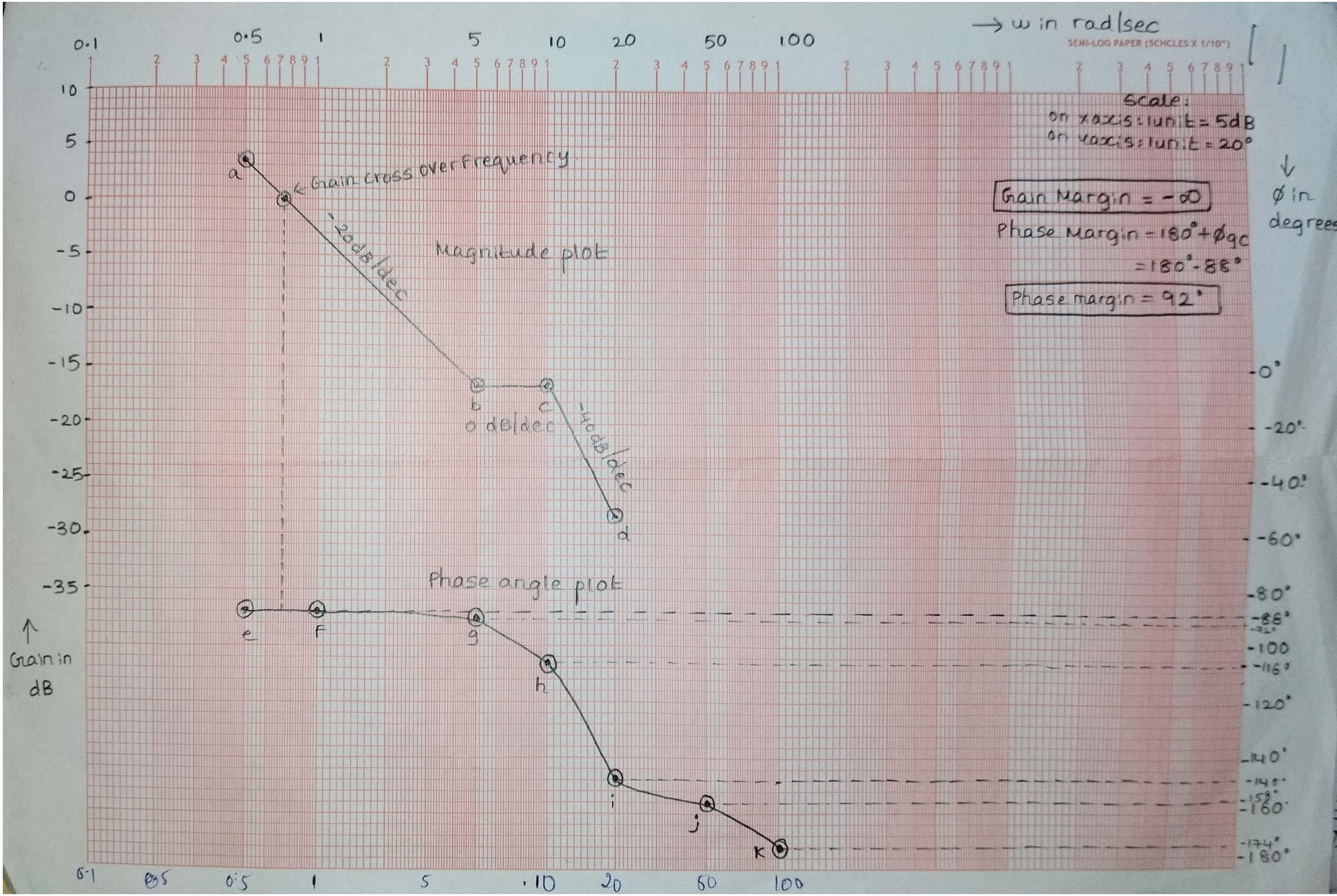
$PM = 180° + \phi gc$

$= 180° - 88°$

Phase angle Margin = 92°

Gain margin :-

Gain margin = -∞


→ ω in rad/sec
SEMI-LOG PAPER (5CHCLES X 1/10")
Scale:
On x axis: 1 unit = 5dB
On y axis: 1 unit = 20°
Gain Margin = −∞
Phase Margin = 180° + ∅gc
= 180° − 88°
Phase margin = 92°
↓ ∅ in degrees
Gain cross over Frequency
−20dB/dec
Magnitude plot
0 dB/dec
−40dB/dec
Phase angle plot
a
b
c
d
e
F
g
h
i
j
K
↑ Gain in dB
10
5
0
−5
−10
−15
−20
−25
−30
−35
0°
−20°
−40°
−60°
−80°
−88°
−100
−116°
−120°
−140°
−148°
−160°
−174°
−180°
0·1
0·5
1
5
10
20
50
100

*3. plot the bode diagram for the following transfer function and obtain gain and phase cross over frequencies

$$G(S) = \frac{10}{S(1+0.4S)(1+0.1S)}$$

A. The transfer function,

$$G(S) = \frac{10}{S(1+0.4S)(1+0.1S)}$$

step 1:

$S = j\omega$

$$G(j\omega) = \frac{10}{j\omega(1+0.4j\omega)(1+0.1j\omega)}$$

$$\omega c_1 = \frac{1}{T_1} = \frac{1}{0.4} = 2.5 \text{ rad/sec}$$

$$\omega c_2 = \frac{1}{T_2} = \frac{1}{0.1} = 10 \text{ rad/sec.}$$

Step 2: Magnitude plot

Term	corner frequency rad/sec	slope in dB/dec	change in slope in dB/dec
$\frac{10}{j\omega}$	-	-20	-
$\frac{1}{1+0.4j\omega}$	2.5	-20	-20-20=-40
$\frac{1}{1+0.1j\omega}$	10	-20	-40-20=-60

Step 3: choose two different frequencies other than corner frequency.

$\omega l < \omega c_1 < \omega c_2 < \omega h$

1 2.5 10 20

$\omega l = 1$ rad/sec

$\omega h = 20$ rad/sec

Step 4: Gain calculation

$A \text{ at } \omega_l = 1 = 20\log\left|\frac{10}{j\omega}\right|$

$= 20\log\left[\frac{10}{\omega}\right]$

$= 20\log(10)$

$A \text{ at } \omega_l = 20dB$

$A \text{ at } \omega_{c_1} = 2.5 = 20\log\left|\frac{10}{j\omega}\right|$

$= 20\log\left(\frac{10}{2.5}\right)$

$= 20\log(4)$

$A \text{ at } \omega_{c_1} = 12dB$

$A \text{ at } \omega_{c_2} = \left[\text{change in slope } \omega_{c_1} \text{ to } \omega_{c_2} \times \log\left(\frac{\omega_{c2}}{\omega_{c1}}\right)\right] + A \text{ at } \omega_{c_1}$

$= \left[-40 \times \log\left(\frac{10}{2.5}\right)\right] + 12$

$= -40\log(4) + 12.$

$A \text{ at } \omega_{c_2} = -12dB$

$A \text{ at } \omega_h = \left[\text{change in slope } \omega_{c_2} \text{ to } \omega_h \times \log\left(\frac{\omega_h}{\omega_{c_2}}\right)\right] + A \text{ at } \omega_{c_2}$

$= \left[-60 \times \log\left(\frac{20}{10}\right)\right] - 12$

$A \text{ at } \omega_h = -30dB$

ω	A in dB
1	20
2.5	12
10	-12
20	-30

Phase angle (ϕ) :-

Phase angle $\phi = \angle G(j\omega) = -90° - \tan^{-1}(0.4\omega) - \tan^{-1}(0.1\omega)$

w	ϕ in degrees
1	-117.5
1.5	-129.4
2.5	-149.0
5	-180
10	-210.9
15	-226.8
20	-236.3

Polar Plot :-

→ The disadvantage in bode plot is it consists 2 plots magnitude plot and phase angle plot as a function of frequency log w.

→ In polar plot it combines magnitude plot and phase angle plot into single plot as a function of frequency w

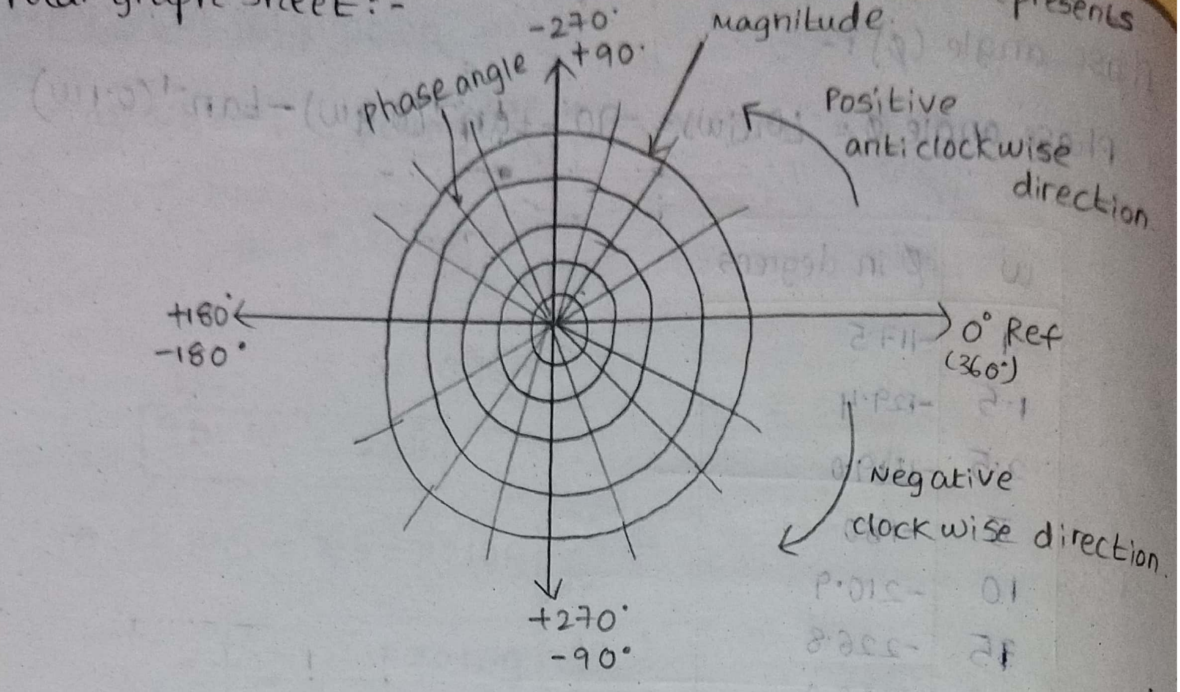
→ The polarplot can be plotted on either polar graph or an ordinary graph

Definition:

→ The polar plot of open loop transfer function is defined as a plot of magnitude $|G(j\omega)|$ Vs phase angle $\angle G(j\omega)$ drawn on polar coordinates as a function of w. w varies from 0 to ∞.

→ It is defined as real part of $G_R(j\omega)$ Vs imaginary part of $G_I(j\omega)$ as a function of frequency w.

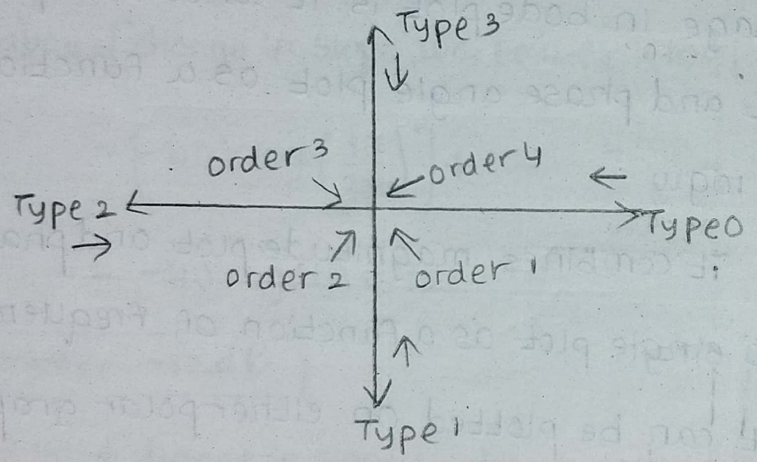
Polar graph sheet:-



Starting and Ending of polar plot:-

Start → Type

End (or) Terminate → order



Different diagrams of polar plot based on type and order:-

① $G(S) = \frac{1}{1+ST}$ Type - 0, Order - 1

$S = j\omega$

$G(j\omega) = \frac{1}{1+j\omega T}$

$M = |G(j\omega)| = \frac{1}{\sqrt{1+\omega^2 T^2}}$

$\phi = \angle G(j\omega) = -\tan^{-1}\omega T$

ω	M	ϕ
0	1	0°
∞	0	-90°

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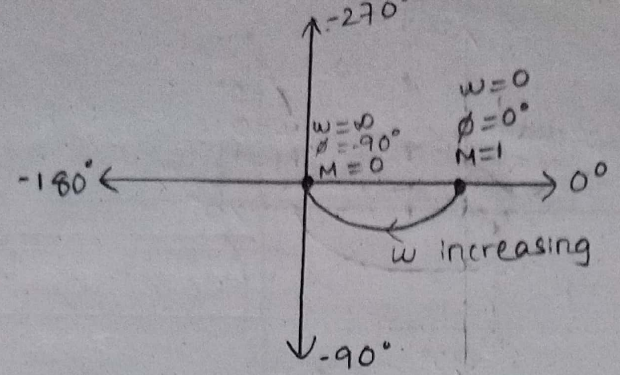


② $G(S) = \frac{1}{S(1+ST)}$ Type = 1
order = 2

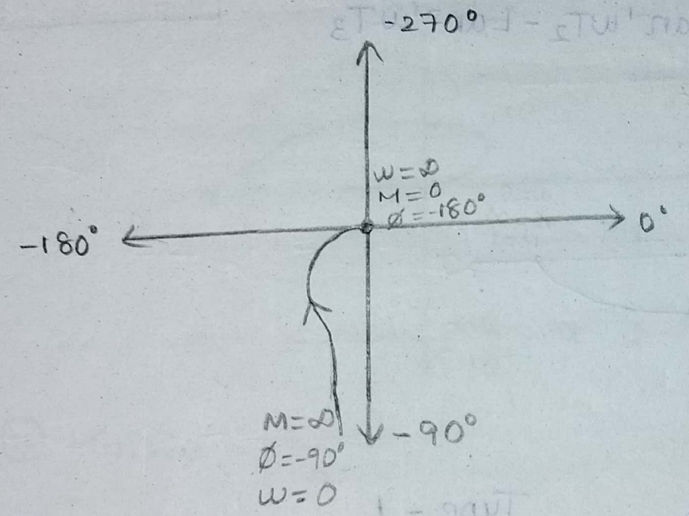
$S = j\omega$

$G(j\omega) = \frac{1}{j\omega(1+j\omega T)}$

$M = |G(j\omega)| = \frac{1}{\omega\sqrt{1+\omega^2 T^2}}$

ω	M	Ø
0	∞	-90°
∞	0	-180°

$\phi = \angle G(j\omega) = -90° - \tan^{-1}\omega T$



③ $G(S) = \frac{1}{(1+ST_1)(1+ST_2)}$ Type = 0
order = 2

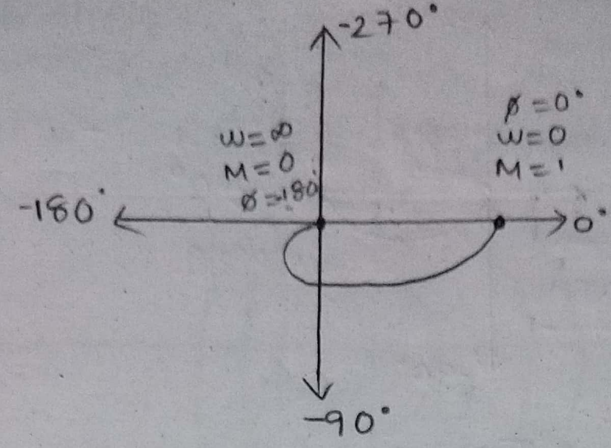
$S = j\omega$

$G(j\omega) = \frac{1}{(1+j\omega T_1)(1+j\omega T_2)}$

$M = |G(j\omega)| = \frac{1}{\sqrt{1+\omega^2 T_1^2}\sqrt{1+\omega^2 T_2^2}}$

ω	M	Ø
0	1	0°
∞	0	-180°

$\phi = \angle G(j\omega) = -\tan^{-1}(\omega T_1) - \tan^{-1}\omega T_2$



4) $G(S) = \dfrac{1}{(1+ST_1)(1+ST_2)(1+ST_2)}$ Type-0 order-3

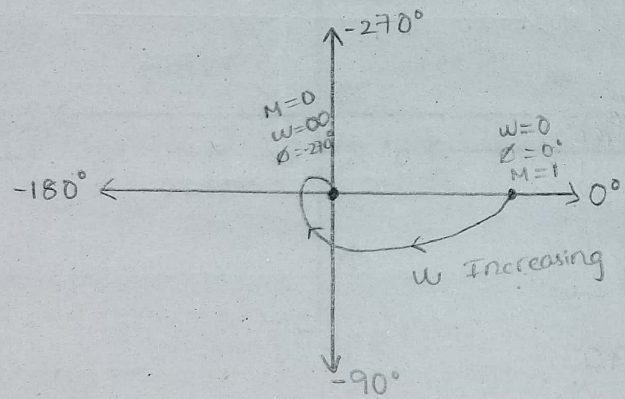
$S = j\omega$

$G(j\omega) = \dfrac{1}{(1+j\omega T_1)(1+j\omega T_2)(1+j\omega T_3)}$

$M = |G(j\omega)| = \dfrac{1}{\sqrt{1+\omega^2 T_1^2}\sqrt{1+\omega^2 T_2^2}\sqrt{1+\omega^2 T_3^2}}$

ω	M	ø
0	1	0°
∞	0	-270°

$\phi = \angle G(j\omega) = -\tan^{-1}(\omega T_1) - \tan^{-1}\omega T_2 - \tan^{-1}\omega T_3$



5) $G(S) = \dfrac{1}{S(1+ST_1)(1+ST_2)}$ Type-1 order-3

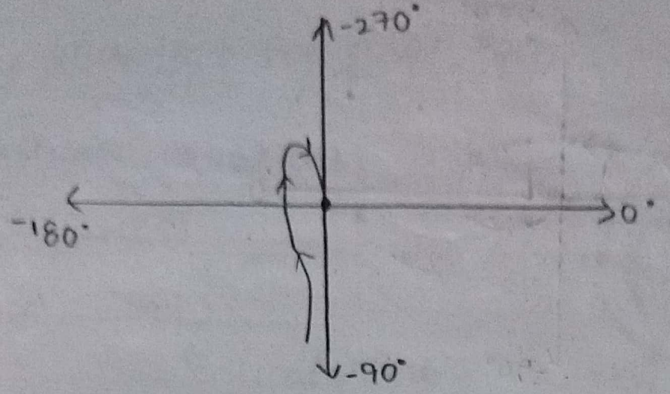
$S = j\omega$

$G(j\omega) = \dfrac{1}{j\omega(1+j\omega T_1)(1+j\omega T_2)}$

$M = |G(j\omega)| = \dfrac{1}{\omega\sqrt{1+\omega^2 T_1^2}\sqrt{1+\omega^2 T_2^2}}$

ω	M	ø
0	∞	-90°
∞	0	-270°

$\phi = \angle G(j\omega) = -90° - \tan^{-1}\omega T_1 - \tan^{-1}\omega T_2$



⑥ $G(S) = \frac{1}{S^2(1+ST_1)(1+ST_2)}$ Type = 2
order = 4

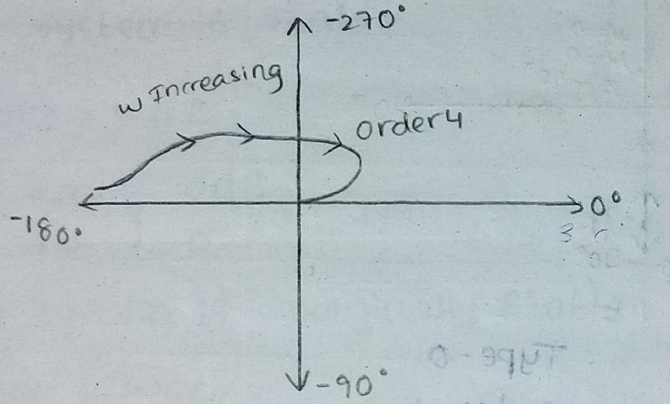
$S = j\omega$

$G(j\omega) = \frac{1}{(j\omega)^2(1+j\omega T_1)(1+j\omega T_2)}$

$M = |G(j\omega)| = \frac{1}{\omega^2\sqrt{1+\omega^2 T_1^2}\sqrt{1+\omega^2 T_2^2}}$

$\phi = \angle G(j\omega) = -180° - \tan^{-1}\omega T_1 - \tan^{-1}\omega T_2$

ω	M	Ø
0	∞	-180°
∞	0	-360°



⑦ $G(S) = \frac{1}{S^2(1+ST_1)(1+ST_2)(1+ST_3)}$ Type - 2
order : 5

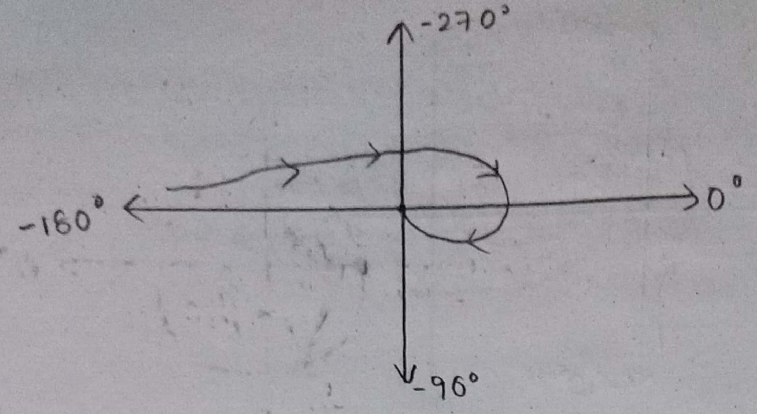
$S = j\omega$

$G(j\omega) = \frac{1}{(j\omega)^2(1+j\omega T_1)(1+j\omega T_2)(1+j\omega T_3)}$

$M = |G(j\omega)| = \frac{1}{\omega^2\sqrt{1+\omega^2 T_1^2}\sqrt{1+\omega^2 T_2^2}\sqrt{1+\omega^2 T_3^2}}$

$\phi = \angle G(j\omega) = -180° - \tan^{-1}\omega T_1 - \tan^{-1}\omega T_2 - \tan^{-1}\omega T_3$

ω	M	Ø
0	∞	-180°
∞	0	-450°



⑧ $G(S) = \frac{1}{S}$

Type - 1

order - 1

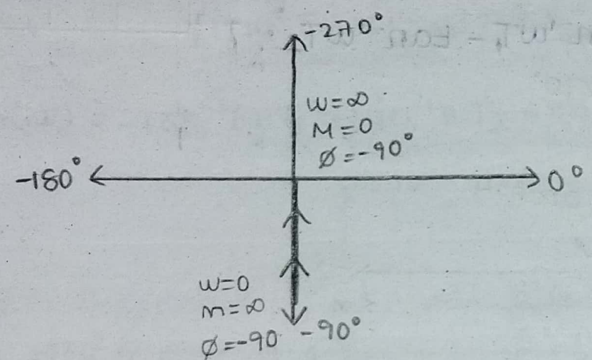
$S = j\omega$

$G(j\omega) = \frac{1}{j\omega}$

$M = |G(j\omega)| = \frac{1}{\omega}$

$\phi = \angle G(j\omega) = -\tan^{-1}\omega$

ω	M	∅
0	∞	-90°
∞	0	-90°



⑨ $G(S) = 1+ST$

Type - 0

Order - 1

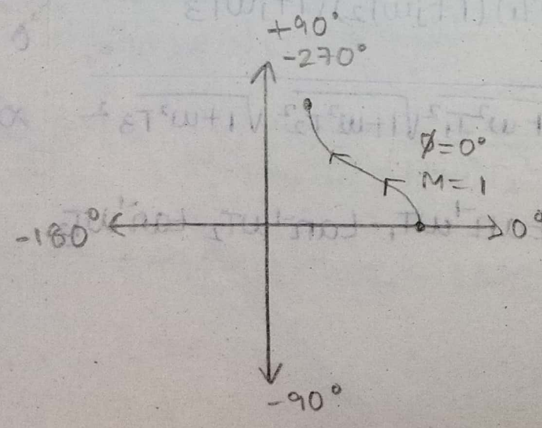
$S = j\omega$

$G(j\omega) = 1 + j\omega T$

$M = |G(j\omega)| = \sqrt{1+\omega^2 T^2}$

$\phi = \angle G(j\omega) = \tan^{-1}\omega T$

ω	M	∅
0	1	0°
∞	∞	90°



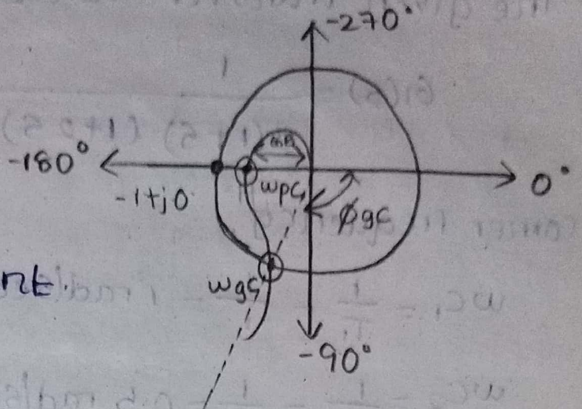
Stability analysis for polar form:-

characteristic equation,

$1 + G(S)H(S)$

$G(S)H(S) = -1 + j0$

↑critical point



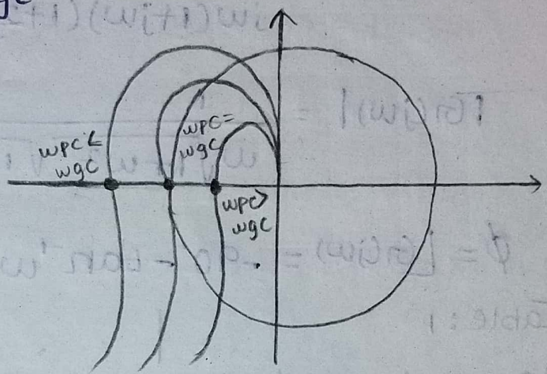
Gain margin (or) Kg $= \frac{1}{GB}$

Phase margin (or) $\gamma = 180° + \phi gc$

① wpc > wgc

Gm or Kg } +ve
Pm or γ

The system is stable



② wpc = wgc

Gm } 0dB
Pm } 0°

The system is marginally stable

③ wpc < wgc

Gm } -ve
Pm

The system is unstable.

① The open loop transfer function of a unity feedback system is given by $G(S) = \frac{1}{S^2(1+S)(1+2S)}$. Sketch the polar plot and determine the gain margin and phase margin.

A. The given transfer function,

$$G(s) = \frac{1}{s(1+s)(1+2s)}$$

Type = 1
order = 3

0/6

Corner frequency,

$$\omega c_1 = \frac{1}{T_1} = \frac{1}{1} = 1 \text{ rad/sec}$$

$$\omega c_2 = \frac{1}{T_2} = \frac{1}{2} = 0.5 \text{ rad/sec}$$

$\Rightarrow s = j\omega$

$$G(j\omega) = \frac{1}{j\omega(1+j\omega)(1+2j\omega)}$$

$$|G(j\omega)| = \frac{1}{\omega\sqrt{1+\omega^2}\sqrt{1+4\omega^2}}$$

$$\phi = \angle G(j\omega) = -90° - \tan^{-1}\omega - \tan^{-1}2\omega$$

Table: 1

Magnitude and phase angle of $G(j\omega)$ at various frequencies

ω rad/sec	0.35	0.4	0.45	0.5	0.6	0.7	1
$\|G(j\omega)\|$	2.2	1.8	1.5	1.2	0.9	0.7	0.3
$\angle G(j\omega)$ deg	-144	-150	-156	-162	-171	-179.5 ≃ -180	-198

Table: 2

Real and imaginary part of $G(j\omega)$ at various frequencies

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$$G(s) = \frac{1}{s^2(1+s)(1+2s)}$$

Determine phase and gain margin.

A. Given,

$$G(s) = \frac{1}{s^2(1+s)(1+2s)}$$

Type = 2
Order = 4

corner frequency,

$$\omega c_1 = \frac{1}{T_1} = \frac{1}{1} = 1 \text{ rad/sec}$$

$$\omega c_2 = \frac{1}{T_2} = \frac{1}{2} = 0.5 \text{ rad/sec}$$

$\Rightarrow s = j\omega$

$$G(j\omega) = \frac{1}{(j\omega)^2(1+j\omega)(1+2j\omega)}$$

$$|G(j\omega)| = \frac{1}{\omega^2\sqrt{1+\omega^2}\,\sqrt{1+4\omega^2}}$$

$$\phi = \angle G(j\omega) = -180^\circ - \tan^{-1}\omega - \tan^{-1}2\omega$$

Table: 1

Magnitude and phase angle of $G(j\omega)$ at various frequencies

ω rad/sec	0.35	0.4	0.45	0.5	0.6	0.7	1
$\lvert G(j\omega)\rvert$	6.3	4.5	3.3	2.5	1.5	0.9	0.3
$\angle G(j\omega)$ deg	126	119	114	108	99	90	71

<u>Compensation Techniques</u>: If the performance of a control system is not upto expectations as per desired specifications, then it is required that some change in the system is needed to obtain the desired performance. The change can be in the form of adjustment of forward path gain or inserting a compensating device in control systems.

For example, the steady state error in a control system can be reduced by increasing forward path gain, but on the other hand this increase in forward path gain results in making the system more oscillatory or sometimes unstable.

Thus the gain adjustment improves the steady state accuracy of the system at the cost of driving the system towards instability. In such cases a compensation network is introduced in the system. The compensation network can be introduced in forward path as shown in figure.

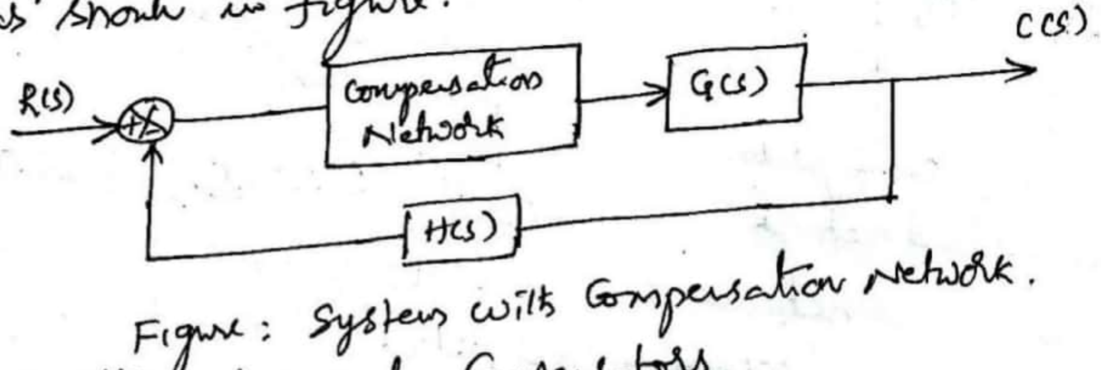

Figure: System with Compensation network.

There are three types of compensators

(1) phase lead compensator
(2) phase lag compensator
(3) Lead-lag compensator

① **phase-lead compensator**: For phase-lead network the output leads the input. Let us consider a phase lead network shown in figure (1)

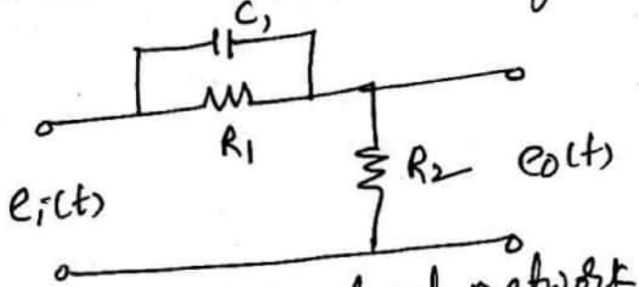

Figure: phase-lead network

The transfer function of phase lead network is given by $\frac{E_o(s)}{E_i(s)} = G(s) = \frac{\alpha(1+sT_1)}{(1+\alpha sT_1)}$

where $\alpha = \frac{R_2}{R_1+R_2} < 1$ and $T_1 = R_1C_1$

The sinusoidal transfer function $G(j\omega) = \frac{\alpha(1+j\omega T_1)}{(1+j\omega\alpha T)}$

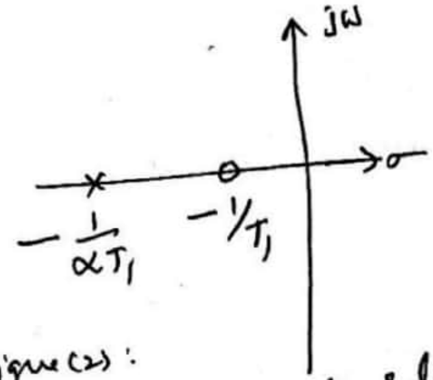

Figure (2): pole-zero configuration of phase-lead network

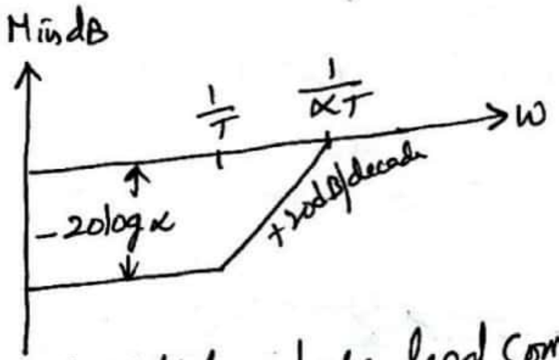

Fig: Bode plot of phase-lead compensator

The phase-lead network acts as a high pass filter. Thus it attenuates low frequencies and allows high frequencies. The phase-lead compensator increases the phase shift of the system. The phase-lead compensator shifts the gain cross over frequency to a higher value and therefore increases bandwidth, speed of the response and reduces overshoot but the steady state error does not show much improvement.

– **Phase-lag Compensator**: For phase-lag network, the output lags the input. The phase-lag network is shown in figure (1)

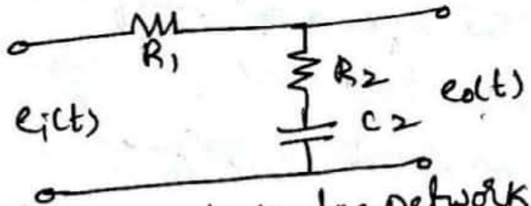

Figure : phase-lag network

The transfer function of phase-lag network is given by

$$\frac{E_0(S)}{E_i(S)} = \frac{1+ST_2}{1+S\beta T_2}$$

where $\beta = \frac{R_1+R_2}{R_2} > 1$; Time Constant $T_2 = R_2C_2$

The sinusoidal transfer function is given by

$$\frac{E_0(j\omega)}{E_i(j\omega)} = G(j\omega) = \frac{1+j\omega T_2}{1+j\omega\beta T_2}$$

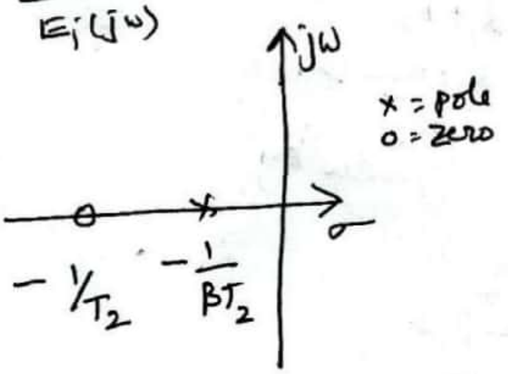

Figure : pole-Zero Configuration of phase-lag Compensator

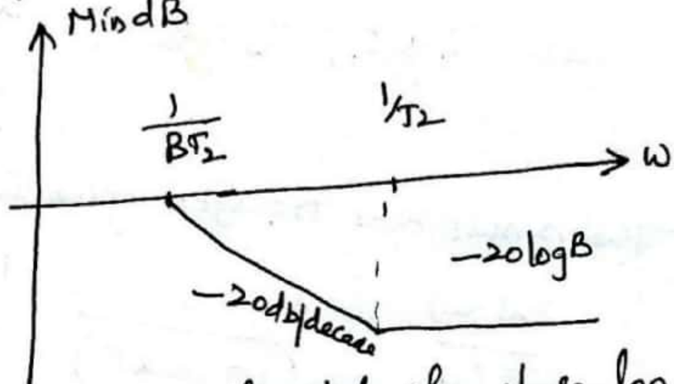

Figure : Bode plot of phase-lag Compensator

when the phase-lag network is introduced in cascade with forward transfer function, the phase-shift will be reduced. The phase-lag Compensator shifts the gain cross over frequency to lower value and thus decreases bandwidth and speed but improves the steady state error. The phase-lag compensator acts as a lowpass filter and thus allows low frequency signals and attenuates high frequency signals.

(3) **Lead-lag Compensator:** If phase-lead and phase-lag compensators are simultaneously used, then the speed of response and steady state error are simultaneously improved. The phase lead-lag network is shown in figure.

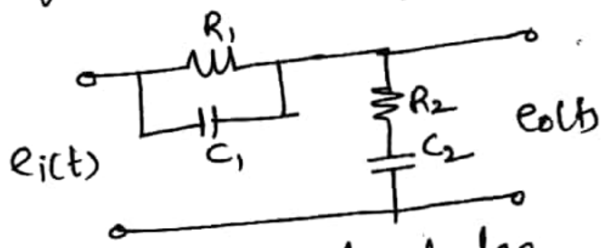

Figure: phase lead-lag network

The transfer function of lead-lag network is given by

$$\frac{E_o(s)}{E_i(s)} = \frac{\alpha(1+sT_1)}{(1+s\alpha T_1)} \frac{(1+sT_2)}{(1+s\beta T_2)}$$

where $T_1 = R_1C_1$; $T_2 = R_2C_2$; $\alpha = \frac{R_2}{R_1+R_2} < 1$

$$\beta = \frac{R_1+R_2}{R_2} > 1$$

The sinusoidal transfer function is given by

$$\frac{E_o(j\omega)}{E_i(j\omega)} = \frac{\alpha(1+j\omega T_1)}{(1+j\omega\alpha T_1)} \frac{(1+j\omega T_2)}{(1+j\omega\beta T_2)}$$

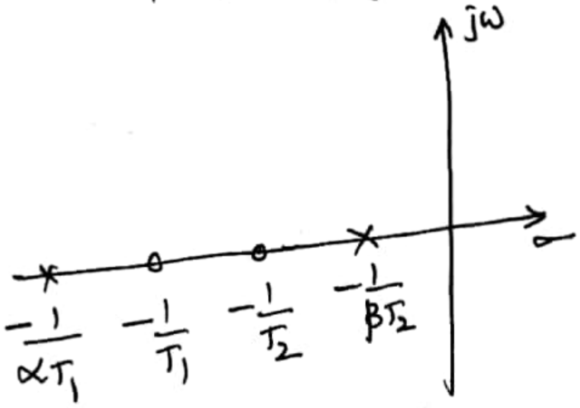

Figure: pole-zero pattern of Lead-lag Compensator

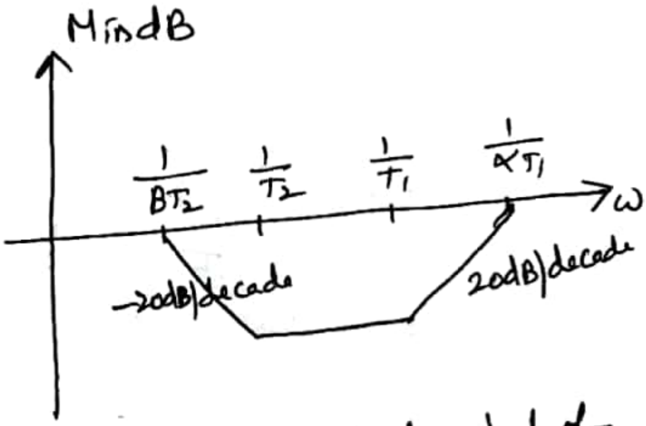

Figure: Magnitude plot of Lead-lag Compensator.