UNIT-5

State Space Analysis of Continuous System:

2M..

Advantages of State Space Analysis

1. Applicable to MIMO Systems.

2. Applicable to Linear time Varient & Linear time invarient systems.

3. Internal State of the system Can be determined.

2M

Concepts of state, State Variable and state model.

State:

It is the Condition of System at any instant of time t, where state is defined by set of Variables called state Variables.

State Variables:

State Variables are set of Variables that completely describe the behaviour of State & Condition of the system at any instant of time.

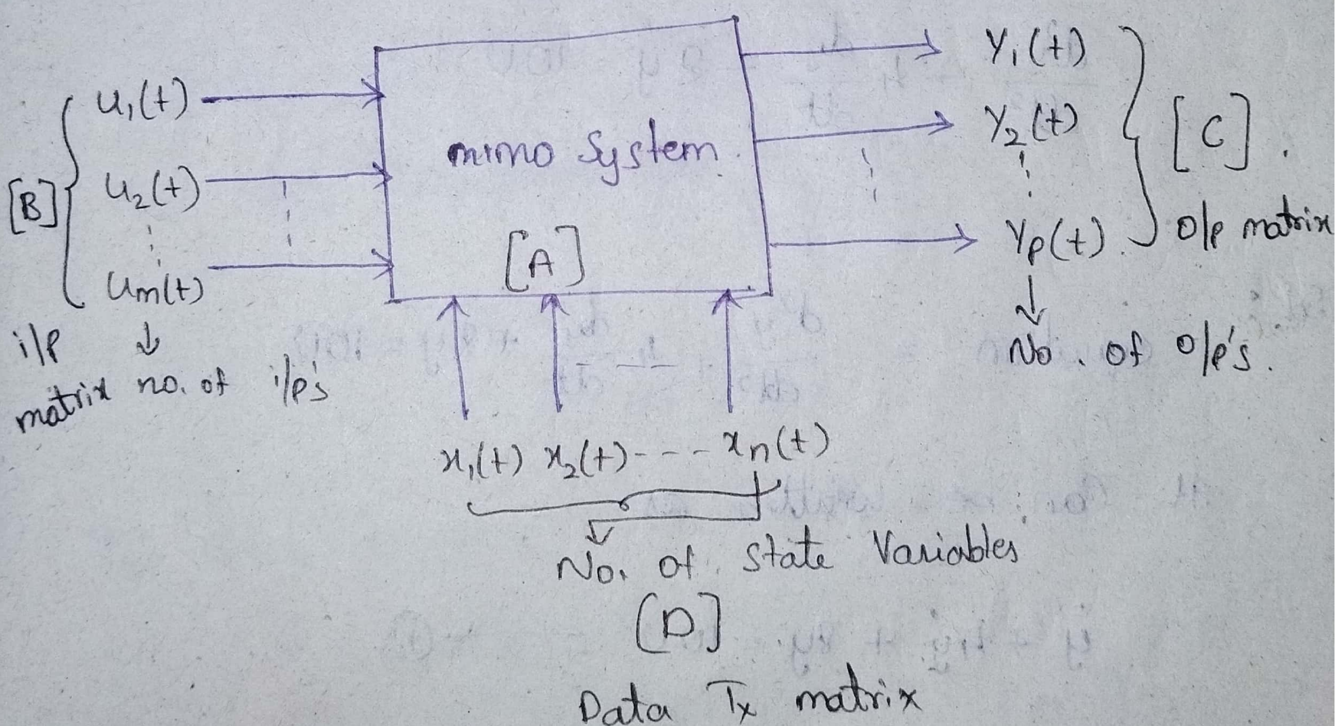
State Vector: 2m

The State Vector is a (nx1) Column matrix whose elements are State Variables of the System. It is denoted by $x(t)$.

State Space: 2m

It is defined as n dimensional space whose Co-ordinate axis represent as n state Variables that Completely determine the behaviour of the System.

State Model / State Space Model / State Space representation: 2m



State equation:

$$\dot{x}(t) = Ax(t) + BU(t)$$

$\dot{x}(t)$ - differentiation of state vector.

Output equation:

$$Y = Cx(t) + DU(t)$$

(Or)

$$\dot{x}(t) = AX + BU$$

$$Y = Cx + DU$$

10m

Obtain state space model for the given differential ~~given~~ model

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 8y = 10U.$$

Sol:

equation $= \frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 8y = 10U.$

It can be written as

$$\ddot{y} + 4\dot{y} + 8y = 10U \longrightarrow (1)$$

order of diff eq (n) = 2

No. of State Variables (n) = 2

The State Variables are

x_1, x_2,

Let

$$y = x_1$$

$$\frac{dy}{dt} = \dot{y} = x_2 = \dot{x}_1$$

$$\frac{d^2y}{dt^2} = \ddot{y} = \dot{x}_2$$

Subsitute $y = x_1$; $\dot{y} = x_2$ & $\ddot{y} = \dot{x}_2$ in eq ①

$$\dot{x}_2 + 4x_2 + 8x_1 = 10U$$

$$\boxed{\dot{x}_2 = -8x_1 - 4x_2 + 10U}$$

The state model

The state equations are

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -8x_1 - 4x_2 + 10U$$

The state equation in matrix form

$$\dot{X} = AX + BU$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{array}{cc} x_1 & x_2 \\ \hline \end{array}\!\!\!\! \begin{bmatrix} 0 & 1 \\ -8 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 10 \end{bmatrix} U$$

Output is

$y = x_1$

Output equation :

$Y = CX + DU$

$D = 0$

$Y = CX$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

2. $\frac{d^3y}{dt^3} + 6\frac{d^2y}{dt^2} + 11\frac{dy}{dt} + 6y + u = 0$

Sol:. $\frac{d^3y}{dt^3} + 6\frac{d^2y}{dt^2} + 11\frac{dy}{dt} + 6y + u = 0$

$\dddot{y} + 6\ddot{y} + 11\dot{y} + 6y + u = 0 \longrightarrow (i)$

order of diff eq = 3.

State Variables = 3.

The state Variables are

x_1, x_2, x_3

Let,

$y = x_1$

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$$\frac{dy}{dt} = \dot{y} = x_2 = \dot{x}_1$$

$$\ddot{y} = x_3 = \dot{x}_2$$

$$\dddot{y} = \dot{x}_3$$

Subsitute

$y = x_1$, $\dot{y} = x_2$, $\ddot{y} = x_3$, $\dddot{y} = \dot{x}_3$ in eq ①

$$\dot{x}_3 + 6\dot{x}_3 + 11x_2 + 6x_1 + u = 0$$

$$\dot{x}_3 = -6x_1 - 11x_2 - 6x_3 - u$$

The state model

The state equation

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = x_3$$

$$\dot{x}_3 = -6x_1 - 11x_2 - 6x_3 - u$$

$$\dot{x} = Ax + BU$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} U.$$

o/p is

$$y = x_1$$

o/p equation

$$Y = Cx + DU$$

$$D = 0$$

$$Y = Cx$$

$$\begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.$$

Obtain the state model of the system whose transfer function is given as $\frac{Y(s)}{U(s)} = \frac{24}{s^3+9s^2+26s+24}$

$$Y(s)\left[s^3+9s^2+26s+24\right] = 24\,U(s)$$

$$s^3 Y(s) + 9s^2 Y(s) + 26s\,Y(s) + 24\,Y(s) = 24\,U(s)$$

By taking Inverse laplace transform.

$$\frac{d^3y(t)}{dt} + 9\frac{d^2y}{dt} + 26\frac{dy}{dt} + 24\,y(t) = 24\,u(t)$$

$$\dddot{y} + 9\ddot{y} + 26\dot{y} + 24y = 24u \quad \text{--- (1)}$$

Order = 3

State Variables = 3

The State Variables = 3

x_1, x_2, x_3.

Let

$$y = x_1$$

$$\frac{dy}{dt} = \dot{y} = x_2 = \dot{x}_1$$

3) **

$$\frac{Y(s)}{U(s)} = \frac{10}{s^3+4s^2+2s+1}$$

$$Y(s)\left[s^3+4s^2+2s+1\right] = U(s)(10).$$

$$s^3Y(s) + 4s^2Y(s) + 2sY(s) + Y(s) = 10U(s)$$

By taking inverse Laplace transform

$$\frac{d^3y}{dt} + 4\frac{d^2y}{dt} + 2\frac{dy}{dt} + y(t) = 10u(t).$$

$$\dddot{y} + 4\ddot{y} + 2\dot{y} + y = 10u \longrightarrow (1)$$

Order = 3

state Variables = 3

The state Variables = 3

$x_1, x_2, x_3.$

$y = x_1$; $\frac{dy}{dt} = \dot{y} = x_2 = \dot{x}_1$; $\ddot{y} = x_3 = \dot{x}_2$

$\dddot{y} = \dot{x}_3$

Substitude $y = x_1$, $\dot{y} = x_2$, $\ddot{y} = x_3$, $\dddot{y} = \dot{x}_3$

$$\dot{x}_3 + 4x_3 + 2x_2 + x_1 = 10u.$$

$$\dot{x}_3 = -x_1 - 2x_2 - 4x_3 + 10u$$

The state model

The state equation

$\dot{x}_1 = x_2$

$\dot{x}_2 = x_3$

$\dot{x}_3 = -x_1 - 2x_2 - 4x_3 + 10u.$

$\dot{x} = Ax + BU.$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix} U.$$

o/p is $y = x_1$

o/p equation

$y = Cx + DU$ $\quad D = 0$

$Y = Cx + DU$

$$\begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.$$

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Ans $$\frac{Y(s)}{U(s)} = \frac{s^2+7s+2}{s^3+9s^2+26s+24}$$

Let.

$$\frac{Y(s)}{U(s)} = \frac{Y(s)}{C(s)} * \frac{C(s)}{U(s)}$$

$$\boxed{\frac{Y(s)}{C(s)} = s^2+7s+2} \longrightarrow ①$$

Numerator

$$\boxed{\frac{C(s)}{U(s)} = \frac{1}{s^3+9s^2+26s+24}} \longrightarrow ②$$

Consider eq ②.

$$C(s)\left[s^3+9s^2+26s+24\right] = 1\,U(s).$$

$$s^3 C(s) + 9s^2 C(s) + 26s\,C(s) + 24C(s) = U(s).$$

By taking ILT.

$$\frac{d^3c}{dt} + 9\frac{d^2c}{dt} + 26\frac{dc}{dt} + 24c(t) = u(t)$$

$$\dddot{c} + 9\ddot{c} + 26\dot{c} + 24c = u(t) \longrightarrow ③$$

order = 3 state Variables = 3

x_1, x_2, x_3

$c = x_1$

$\frac{dc}{dt} = \dot{c} = x_2 = \dot{x}_1$; $\dddot{c} = \dot{x}_3$

$\ddot{c} = x_3 = \dot{x}_2$;

sub $c = x_1, \quad \dot{c} = x_2, \quad \ddot{c} = x_3, \quad \dddot{c} = \dot{x}_3$ in eq ①

$$\dot{x}_3 + 9x_3 + 26x_2 + 24x_1 = u(t)$$

$$\dot{x}_3 = -24x_1 - 26x_2 - 9x_3 + u$$

$$\dot{x} = Ax + BU$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -24 & -26 & -9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} U$$

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = x_3$$

$$\dot{x}_3 = -24x_1 - 26x_2 - 9x_3 + u.$$

Consider equation ①

~~o/p~~ equation

$\cancel{Y = Cx}$

$$\frac{Y(s)}{C(s)} = s^2 + 7s + 2.$$

$$Y(s) = [s^2 + 7s + 2]\, C(s)$$

$$Y(s) = s^2 C(s) + 7s\, C(s) + 2\, C(s)$$

By ILT.

$$\boxed{y = \ddot{c} + 7\dot{c} + 2c} \longrightarrow ④$$

Sub, $c = x_1$,

$\dot{c} = x_2$ $\qquad y = x_3 + 7x_2 + 2x_1$

$\ddot{c} = x_3$.

The o/p eq

$$y = \begin{bmatrix} 2 & 7 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

2M.

State Transistion Model Matrix :

(Or)

Solution of Time - Invarient State equation

2M

Homogenous State equation:

$$\dot{x}(t) = Ax(t) + B\cancel{u}(t)^{\to 0}$$

$$\dot{x}(t) = Ax(t)$$

taking L.T

$$Sx(s) = Ax(s) + x(0)$$

$$Sx(s) - Ax(s) = x(0)$$

$$x(s)\left[SI - A\right] = x(0)$$

I = identity matrix

$$x(s) = \frac{x(0)}{(SI-A)}$$

$$\boxed{x(s) = \left[SI - A\right]^{-1} x(0)}$$

$$\boxed{x(s) = \phi(s)\, x(0)}$$

$\phi(S)$ = State transition matrix

$$\boxed{\phi(s) = [sI - A]^{-1}}$$

Resolvant matrix

$$\phi(t) = L^{-1}\left[[sI - A]^{-1}\right]$$

2m

Properties of State transition matrix:

$\phi(t) = e^{At}$

1. $\phi(0) = e^{A(0)} = I$ unity matrix

2. $\phi(-t) = [\phi(t)]^{-1}$

3. $\phi(t_1+t_2) = [e^{At_1} e^{At_2}]$

$= \phi(t_1)\,\phi(t_2)$

4. $\phi(nt) = [\phi(t)]^n$

5. $\phi(t_2-t_1)\,\phi(t_1-t_0) = \phi(t_2-t_0)$.

Determine the state transistion matrix for the system $\dot{x} = Ax + BU$ with $A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$

Sol:

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$$

The State transition matrix

$$\phi(t) = L^{-1}\left\{[sI - A]^{-1}\right\}$$

$$[SI-A] = S\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} S & 0 \\ 0 & S \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$$

$$[SI-A] = \begin{bmatrix} S & -1 \\ 2 & S+3 \end{bmatrix}$$

$$[SI-A]^{-1} = \frac{\text{Adj } A}{\text{Determinant } A}$$

$$= \frac{\text{Adj } (SI-A)}{|SI-A|}$$

$$\text{adj}\,[SI-A] = \begin{bmatrix} S+3 & 1 \\ -2 & S \end{bmatrix}$$

$$\det\,|SI-A| = S(S+3)+2$$

$$= S^2+3S+2$$

$$[SI-A]^{-1} = \frac{\begin{bmatrix} S+3 & +1 \\ -2 & S \end{bmatrix}}{S^2+3S+2}$$

$$[SI-A]^{-1} = \begin{bmatrix} \frac{S+3}{S^2+3S+2} & \frac{1}{\ } \end{bmatrix}$$

$$[SI-A]^{-1} = \begin{bmatrix} \frac{S+3}{S^2+3S+2} & \frac{1}{S^2+3S+2} \\ \frac{-2}{S^2+3S+2} & \frac{S}{S^2+3S+2} \end{bmatrix}$$

$$[SI-A]^{-1} = \begin{bmatrix} \frac{S+3}{(S+1)(S+2)} \text{ (I)} & \frac{1}{(S+1)(S+2)} \text{ (II)} \\ \frac{-2}{S^2+3S+2} \text{ (III)} & \frac{S}{(S+1)(S+2)} \text{ (IV)} \end{bmatrix}$$

Apply Partial fraction. for each & every term.

$$[SI-A]^{-1} = \begin{bmatrix} \frac{A}{(S+1)} + \frac{B}{(S+2)} & \frac{A}{(S+1)} + \frac{B}{(S+2)} \\ \frac{A}{(S+1)} + \frac{B}{(S+2)} & \frac{A}{(S+1)} + \frac{B}{(S+2)} \end{bmatrix} \rightarrow (1)$$

$$\frac{S+3}{(S+1)(S+2)} = \frac{A}{(S+1)} + \frac{B}{S+2}$$

$$\frac{S+3}{\cancel{(S+1)(S+2)}} = \frac{A(S+2)+B(S+1)}{\cancel{(S+1)(S+2)}}$$

$$S+3 = A(S+2) + B(S+1)$$

B term $S = -1$

$$-1+3 = A(-1+2)$$

$$2 = A(1)$$

$$\boxed{A=2}$$

A term -2

$(-2+3) = B(-2+1)$

$1 = B(-1)$

$\boxed{B = -1.}$

(II) $1 = A(s+2) + B(s+1)$

$1 = A(-1+2)$

$\boxed{A = 1}$

$1 = B(-2+1)$

$\boxed{B = -1}$

(III) $-2 = A(s+2) + B(s+1)$

$-2 = A(-1+2)$

$-2 = B(-2+1)$

$\boxed{A = -2}$

$\boxed{B = 2}$

(11) $S = A(S+2) + B(S+1)$

$\boxed{A = -1}$ $\boxed{B = 2}$

Sub A & B Values in eq (1)

$$[S-A]^{-1} = \begin{bmatrix} \left(\frac{2}{(S+1)} - \frac{1}{(S+2)}\right) & \frac{1}{S+1} - \frac{1}{S+2} \\ \frac{-2}{(S+1)} + \frac{2}{(S+2)} & \frac{-1}{(S+1)} + \frac{2}{(S+2)} \end{bmatrix}$$

$$L^{-1}\left[\frac{1}{S+a}\right] = e^{-at}$$

$$\phi(t) = L^{-1}\left[[SI-A]^{-1}\right]$$

$$\phi(t) = \begin{bmatrix} 2e^{-t} - e^{2t} & e^{-t} - e^{-2t} \\ -2e^{-t} + 2e^{-2t} & -e^{-t} + 2e^{-2t} \end{bmatrix}$$

Eg:
Homogenous

$$x(t) = \phi(t)\,x(0)$$

Eg : $x(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

$$x(t) = \begin{bmatrix} 2e^{-t} - e^{2t} & e^{-t} - e^{-2t} \\ -2e^{-t} + 2e^{-2t} & -e^{-t} + 2e^{-2t} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$x(t) = \begin{bmatrix} 0 & e^{-t} - 2e^{-2t} \\ 0 & -e^{-t} + 2e^{-2t} \end{bmatrix},$$

Example:

$$A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \qquad x(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\phi(t) = L^{-1}\left[[SI - A]^{-1} \right]$$

$$[SI - A] = S \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} S & 0 \\ 0 & S \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

$$[SI - A] = \begin{bmatrix} S-1 & 0 \\ -1 & S-1 \end{bmatrix} \qquad \det A = (S-1)^2 - 0$$

$$[SI - A]^{-1} = \frac{\operatorname{Adj} A}{\det A} = \frac{\operatorname{Adj}(SI - A)}{|SI - A|}$$

$$\frac{\operatorname{adj}(SI - A)}{\det A} = \frac{\begin{bmatrix} S-1 & 0 \\ +1 & S-1 \end{bmatrix}}{(S-1)^2}$$

$$[SI-A]^{-1} = \begin{bmatrix} \frac{\cancel{S-1}}{(S-1)^2} & 0 \\ \frac{1}{(S-1)^2} & \frac{\cancel{S-1}}{(S-1)^2} \end{bmatrix}$$

$$(SI-A)^{-1} = \begin{bmatrix} \frac{1}{S-1} & 0 \\ \frac{1}{(S-1)^2} & \frac{1}{S-1} \end{bmatrix}$$

Apply inverse laplace transform.

$$\cancel{[SI-A]^{-1}} \quad \phi(t) = \left[(SI-A)^{-1}\right]$$

$$(SI-A) = L^{-1}\begin{bmatrix} \frac{1}{S-1} & 0 \\ \frac{1}{(S-1)^2} & \frac{1}{S-1} \end{bmatrix}$$

$$\phi(t) = \begin{bmatrix} e^t & 0 \\ te^t & e^t \end{bmatrix}.$$

solution of homogenous eq

$x(t) = \phi(t) \cdot x(0);$

$$x(t) = \begin{bmatrix} e^t & 0 \\ te^t & e^t \end{bmatrix} \times \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ e^t \end{bmatrix}.$$

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2m

Contrdalitity & Obsewebility

2m

Kalman's test $\qquad Ax+BU$

$$Q_c\left[B \quad AB \quad A^2B \,---\, A^{n-1}B\right]$$

$|Q_c| \neq 0$

Then the System is Completely Contrable.

Test Contrablility for the given System.

$$\dot{x} = \underbrace{\begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}}_{A} x + \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{B} u, \quad y = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{C} x$$

Sol:

$$\dot{x} = \underbrace{\begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}}_{2\times 2} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u, \quad y = \begin{bmatrix} 1 & 0 \end{bmatrix} x$$

for $n=2$, the Contrabiliity matrix is

$$Q_c = \left[B \quad AB\right]$$

$$AB = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} (0+1) \\ 0-3 \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

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$Q_C = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ -3 \end{bmatrix}$

$Q_c = \begin{bmatrix} 0 & 1 \\ 1 & -3 \end{bmatrix}$

$\det = \begin{vmatrix} \overset{a}{0} & \overset{b}{1} \\ \underset{c}{1} & \underset{d}{-3} \end{vmatrix}$ $\qquad ad - bc$

$\cancel{-3-1} = 0 - 1$

$= -1$

$Q_C = -1$

$|Q_c| \neq 0$ System is in Control.

observability:

$$2m. \begin{cases} Q_0 = \left[C^T \; A^T C^T \; (A^T)^2 C^T - - - (A^T)^{n-1} C^T \right] \\ |Q_0| \neq 0 \rightarrow \text{Completely observable.} \end{cases}$$

Q) Test observalibility of System to described by

$A = \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$, $C = [1 \; 0]$

for $n = 2$

$Q_0 = \left[C^T \; A^T C^T \right]$

$C^T = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$$A^T = \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix}$$

$$A^T C^T = \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -2 \\ 0 \end{bmatrix}$$

$$Q_0 = \begin{bmatrix} C^T & A^T C^T \end{bmatrix}$$

$$Q_0 = \begin{bmatrix} 1 & -2 \\ 0 & 0 \end{bmatrix}$$

$$|Q_0| = ad - bc = 0$$

$\Downarrow$

System is unobservable.

Consider unity feedback system with plant/system/process. $\dot{X} = AX + BU$

$Y = CX$ where

$$A = \begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix}; \; B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}; \; C = \begin{bmatrix} 1 & 1 \end{bmatrix}$$

Test Controblility & observalibility of system whose state space representation is given.

sol: Controbability

for $n = 2$.

$$Q_c = [B \quad AB].$$

$$AB = \begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} ; \begin{bmatrix} 2+1 \\ -1+2 \end{bmatrix} \quad \cancel{\begin{matrix} 1+1 \\ 2+2 \end{matrix}}$$

$$= \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$Q_c = \begin{bmatrix} 1 & 3 \\ 1 & 1 \end{bmatrix}$$

$$|Q_c| = \begin{array}{cc} a & b \\ \left| \begin{matrix} 1 & 3 \\ 1 & 1 \end{matrix} \right| \\ c & d \end{array} \qquad \begin{matrix} ad - bc \\ 1 - 3 \\ \boxed{-2} \end{matrix}$$

$$|Q_c| = -3.$$

$\therefore |Q_c| \neq 0 \Rightarrow$ System is completely Controlable.

Observability :

for $n = 2$

$$Q_0 = \left[C^T \; A^T C^T \right]$$

$$C^T = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}$$

$$A^T C^T = \begin{bmatrix} 2 & -1 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad \begin{matrix} -2+1. \\ 1+2. \end{matrix}$$

$$= \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

$$Q_0 \neq 0$$

Test obs & con described by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -3 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$Y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$