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UNIT-1 - WAVEGUIDES

INTRODUCTION TO MICRO WAVES

→ Micro waves are defined in terms of their wavelength.

→ Micro waves are Electro Magnetic waves whose frequency range varies from 1000MHz to 1000GHz (or) 1GHz to 1THz

→ The wave length (λ) of 'cm' waves at micro wave frequencies are very short typically from a few tens of cms to a fraction of a mm.

→ The micro wave is a signal that has a wavelength of 1 foot or less i.e., $\lambda \le 30.48$ cms.

→ Micro waves behave like rays of light than ordinary radio waves because of the higher ~~border of~~ frequency range of micro wave range lies on the Infrared and Visible light regions or (of) spectrum.

→ Due to this unique behaviour, micro wave frequencies are classified seperately from the radio waves.

→ As a wave, it is characterised by a velocity, wavelength and frequency.

Microwave region and Band designations:-

>30 kHz → Radio freq. range

FREQUENCY RANGE	BAND DESIGNATION
3 Hz – 30 Hz	Ultra low frequency (ULF)
30 Hz – 300 Hz	Extra low frequency (ELF)
300 Hz – 3000 Hz (3 kHz)	Voice Frequency (VF) (Telephone/ base band frequencies)
3 kHz – 30 kHz	Very Low Frequency (VLF)
30 kHz – 300 kHz	Low Frequency (LF)
300 kHz – 3000 kHz (3 MHz)	Medium Frequency (MF)
3 MHz – 30 MHz	High Frequency (HF)
30 MHz – 300 MHz	Very high Frequency (VHF)
300 MHz – 3000 MHz (3 GHz)	Ultra High frequency (UHF)
3 GHz – 30 GHz	Super High Frequency (SHF)
30 GHz – 300 GHz	Extra High Frequency (EHF)
>300 GHz – 430 THz 300 GHz – 3000 GHz (3 THz) (300 THz (3 to 300 THz))	infrared Frequency
430 THz – 1000 THz	visible light

Standard Band Designations for microwave freq. as per military stds:-

BAND DESIGNATION	FREQUENCY RANGE (GHz)
UHF	0.3 – 3.0
L	1.1 – 1.7
LS	1.7 – 2.6
S	2.6 – 3.9
C	3.9 – 8.0
** $\boxed{X}$	$\boxed{8.0 - 12.5}$ (Microwave lab)
Ku	12.5 – 18.0
K	18 – 26
Ka	26 – 40
Q	33 – 50
U	40 – 60
M	50 – 75
E	60 – 90
F	90 – 140
G	140 – 220
R	220 – 325
Sub millimeter	> 300

millimeter

L–Ka – mw Band
mm
Sub millimeter

Applications of Micro waves :-

→ Microwaves have a broad range of applications in modern technology like long distance commn systems, RADAR, Navigatio military etc., and also in the following areas,

i) Tele commn :-

* International telephone
* Television
* Space commn
* Telemetry commn link for railways
* Defence

ii) RADARs :-

* Detect air craft
* Track (or) guide supersonic missiles.
* Air traffic control
* Police Speed Detectors etc,

iii) Identifying objects (or) personnel by non-conta method :-

iv) Light generated charge carriers in a microwave semiconductor makes it possibl to create a new micro wave devices, switches, phase shifters, tuning elements etc., uses heat property of micro waves.

a) Micro wave oven (2.45GHz, 600W)

b) Drying machines are used in food, textile, paper industries, ...

c) <u>Food processing industries</u> :- pre cooling, cooking, ~~moisture~~ pasteurising, sterility, roasting food grains etc.,

d) Rubber industry / plastic / chemical industries.

e) <u>Biomedical app^{n}s</u> :- Deep Electro Magnetic heating for treatment of cancer

<u>Advantages of Micro Waves</u> :-

→ There are some unique advantages of Micro waves over low frequencies.

1) Increased Band Width availability.
2) Improved directive properties.
3) Fading effect and Reliability
4) Transparency property of micro ~~mi~~waves.
5) power requirements.

1) <u>Increased Band Width availability</u> :-

→ μwaves have large B.W of 1GHz − 1000GHz compared to medium wave, short wave and Ultra High Freq. waves.

→ The advantage of large BW is the range of inf. channels will be a small of carrier freq. and more inf. can be transmitted in the μwave freq. ranges.

→ The μwave region i.e., 1000GHz contain 1000 sections of the freq. band and any of these 1000 sections may be used to transmit the s/ls of TV, Radio and other commⁿs.

→ Generally the B.W of speech s/l is 4KHz music is 10-15KHz, TV is 5-7MHz, telegraph channel is 120-240Hz.

2) Improved directive properties:-

→ As freq. ↑, directivity ↑ and beam width ↓.

→ The beam width of radiation, $\theta = \frac{\lambda}{D}$

→ At low freq. bands the size of the antenna becomes very large, then we get the short "beams" of radiation.

→ At μwave freq. antenna size (diameter) of several wave lengths wide is needed to get the smaller beam width for the directived beam.

Fading effect :- The transmitted signal strength reduc when the signal ~~tow~~ passes through the diff. mediums or objects before it reaches the RCL

4) Transparency property of ~~P.~~ micro waves :-

→ The μwave freq. band ranging from 300 MH to 10GHz are capable of freely propagatin through the ionized layers surrounding t earth as well as through the atmosphere.

→ The presence of such a transparent windo in a μwave band facilitates the study of μwave radiation from the sun and stars

5) Power requirements :- The T^x and R^x power requirements are very low at μwave freq. compared to Short wave band.

Field theory :-

i) Static field (Time invariant)

- ~~Electro~~ electric static field
- Static magnetic field

} independent to each other

beam width is given by,

$$B = \frac{140^\circ}{D/\lambda} = \frac{140^\circ \lambda}{D}$$

where, B = Beam width in degrees.
D = Diameter of the antenna in cms
λ = wavelength in cms.

→ At 30GHz (λ = 1cm) for 1° beam width, D = ?

Given,

$B = 1^\circ$, $\lambda = 1cm$

$$\Rightarrow B = \frac{140^\circ \lambda}{D} \Rightarrow D = \frac{140^\circ \lambda}{B}$$

$$= \frac{140^\circ (1cm)}{1^\circ}$$

$$D = 140cm$$

→ At 300MHz, λ = 100cm, 1° beam width, D = ?

$$D = \frac{140^\circ \lambda}{B} = \frac{140^\circ \times 100cm}{1^\circ}$$

$$D = 140 mts$$

From the above calculations it is clear that antenna size is small for μwave freq.

<u>Fading effect and reliability</u>:-

→ Due to Line of sight propagation and high freq. there is less fading effect. Hence μwave comm is more reliable.

densities at that point.

Maxwell Equations for time varying fields.

1) $\nabla \times \overline{H} = J + \frac{\partial \overline{D}}{\partial t}$

H – Magnetic field vector

2) $\nabla \times \overline{E} = -\frac{\partial B}{\partial t}$

E – electric field vector

3) $\nabla \cdot \overline{D} = \rho_v$

4) $\nabla \cdot \overline{B} = 0$

(in free space, volume charge density $\rho_v = 0$ and conductivity σ i.e. zero

Maxwell Equations ~~for~~ in freespace / air:-

1) $\nabla \times \overline{H} = \frac{\partial \overline{D}}{\partial t}$ → Ampere's circuit law

2) $\nabla \times \overline{E} = \frac{\partial B}{\partial t}$ → Faraday's law

3) $\nabla \cdot \overline{D} = 0$ → Gauss's law for E-field

4) $\nabla \cdot \overline{B} = 0$ → Gauss's law for M-field

where, J – Conduction current density $(J = \sigma E)$

$\frac{\partial D}{\partial t}$ = displacement current density.

ρ_v – charge density

$\overline{D}$ = Electric flux density

$\overline{B}$ = Magnetic flux density.

ii) Dynamic field (Time variant)

- → Dynamic electric field
- → Dynamic magnetic field

} interdependent.

→ Time varying electric and magnetic fields together produces an EM wave travelling in the space with the velocity of light.

→ The path of the EM wave depends on the freq. of the s/l, and atmospheric conditions.

→ Both the electric and magnetic fields and the direction of propagation are $\perp^r$ to each other ~~and~~ or orthogonal to each other.

→ The eqn's describing the relationships b/w the time varying electric and magnetic fields are known as Maxwell's equation.

→ These eqns. can be represented in differential and integral forms.

→ These eqns. are nothing but a set of 4 expressions from Ampere's ckt law, Faraday's law and Gauss law for E field and Guass law for magnetic field.

→ Maxwell eqns. in point form or differential form explains the chqr. of different field vectors at a given point to each other and as well as to the charge and current quantity.

→ The divergence operation can be reperesented by the use of mathematical operator called dell operator (∇) which is a vector operator.

→ In Electrostatics, Gauss law is applied to the differential volume element to develop the concept of divergence.

→ In Electro-magnetostatics, the Ampere's ckt law is applied to the differential surface element to develop the concept of a curl.

→ The curl of $\vec{H}$ is indicated by $\nabla \times \vec{H}$, which is the cross product of operator ∇ and $\vec{H}$

<u>Wave Equations</u>:- (Electric field) - E

→ Assume all the field vectors vary w.r.t. time in a sinusoidal manner then we have,

$$E = E_0 \cdot e^{j\omega t}.$$

where, E_0 = max. value of field intensity.
ω = angular frequency = $2\pi f$
freq. of sinusoidal variations

→ eqn. $\frac{\partial \bar{E}}{\partial t} = E_0 \cdot e^{j\omega t} \cdot j\omega \longrightarrow (2)$

$$\frac{\partial E}{\partial t} = Ej\omega = j\omega E$$

From the above eqn. we can define an operator,

$$\frac{\partial}{\partial t} = j\omega$$

→ Differenting eqn. (2) again partially wrt time,

$$\frac{\partial^2 E}{\partial t^2} = j\omega E_0 \cdot e^{j\omega t} \cdot j\omega$$

$$= j^2 \omega^2 E_0 \cdot e^{j\omega t}$$

$$\frac{\partial^2 E}{\partial t^2} = -\omega^2 E_0 \cdot e^{j\omega t} = -\omega^2 E \longrightarrow (3)$$

From this, we can define an operator

$$\frac{\partial^2}{\partial t^2} = -\omega^2$$

→ Consider a medium, which doesn't contain any free charges and is non-conducting. i.e., medium is air/free space, then $\rho = 0, \sigma = 0$

From the maxwell's (1) eqn.

$$\nabla \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t} = \sigma \bar{E} + \frac{\partial}{\partial t}(\varepsilon \bar{E})$$

$\rightarrow$ eqn.

$$\frac{\partial \bar{E}}{\partial t} = E_0 \cdot e^{j\omega t} \cdot j\omega \longrightarrow (2)$$

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$$\nabla \times H = \bar{J} + \frac{\partial \bar{D}}{\partial t} = \sigma \bar{E} + \frac{\partial}{\partial t}(\varepsilon \bar{E})$$

∂t

$$= \sigma E + \frac{\partial}{\partial t}(\epsilon E)$$ $\left\{ \because D = \varepsilon E \right.$

$\because J = \sigma E$

$$= 0 + \frac{\partial}{\partial t}(\varepsilon E)$$ $\left. \because \sigma = 0, \rho = 0 \right\}$

$$\nabla \times H = \frac{\partial}{\partial t}(\varepsilon E)$$

$$\nabla \times H = j\omega \varepsilon E \longrightarrow (4)$$ $\because \frac{\partial}{\partial t} = j\omega$

From Maxwell's (2) eqn.

$$\nabla \times E = \frac{-\partial B}{\partial t}$$

$$= -\frac{\partial}{\partial t} \mu H$$

$$\nabla \times E = -j\omega \mu H \longrightarrow (5)$$

$B = \mu H$

$\mu = \mu_0 \mu_r$

(permeability)

μ_0 = absolute permeability

μ_r = relative permeability

$\mu_r = 1$ in the air/space

Taking curl of $\nabla \times E$ we get,

$$\nabla \times \nabla \times E = \nabla \times (-j\omega \mu H)$$

$$= -j\omega\mu (\nabla \times H)$$

$$= -j\omega\mu (j\omega \varepsilon E)$$

$$= \omega^2 \mu \varepsilon E \longrightarrow (6)$$

WKT,

$$\nabla \times \nabla \times E = \nabla(\nabla \cdot E) - \nabla^2 E$$

$$\nabla \times \nabla \times E = \omega^2 \mu \varepsilon E \longrightarrow (7)$$

$$\nabla(\nabla \cdot \bar{E}) - \nabla^2 \bar{E} = \omega^2 \mu \varepsilon \bar{E}$$

$\nabla \cdot H = 0$ in eqn. (10)

Maxwell's III eqn.,

$$\nabla \cdot D = \rho = 0$$

$$\nabla \cdot \varepsilon E = 0 \text{ (or) } \varepsilon(\nabla \cdot E) = 0$$

$$\varepsilon \neq 0 \text{ then } \nabla \cdot E = 0$$

$$\nabla(\nabla \cdot E) - \nabla^2 E = \nabla(0) - \nabla^2 E = \omega^2 \mu \varepsilon E$$

$$\Rightarrow \nabla^2 E = -\omega^2 \mu \varepsilon E \rightarrow (8)$$

Taking curl of $\nabla \times H$

we know,

$$\nabla \times \nabla \times H = \nabla \times (j\omega\varepsilon E)$$

$$= j\omega\varepsilon(\nabla \times E)$$

$$= j\omega\varepsilon(-j\omega\mu H)$$

$$\nabla \times \nabla \times H = \omega^2 \varepsilon \mu H \rightarrow (9)$$

From the vector analysis,

$$\nabla \times \nabla \times H = \nabla \cdot (\nabla \cdot H) - \nabla^2 H = \omega^2 \mu \varepsilon H \rightarrow (1$$

From Maxwell's IV eqn.

$$\nabla \cdot B = 0 \qquad (B = \mu H)$$

$$\nabla \cdot \mu H = 0$$

$$\mu(\nabla \cdot H) = 0$$

$$\mu \neq 0 \text{ then } \nabla \cdot H = 0$$

Substitute $\nabla \cdot H = 0$ in eqn. (10)

$$\nabla \cdot (\nabla \cdot H) - \nabla^2 H = \omega^2 \varepsilon \mu H$$

$$\nabla \cdot (0) - \nabla^2 H = \omega^2 \varepsilon \mu H$$

$$-\nabla^2 H = \omega^2 \varepsilon \mu H$$

$$\nabla^2 H = -\omega^2 \varepsilon \mu H \longrightarrow (11)$$

→ Resolving Electric field (E) into 3 $\perp^{r}$ directions then we have,

$$\nabla^2 E = -\omega^2 \varepsilon \mu E$$

$$\cancel{\nabla^2 H = -\omega^2 \varepsilon \mu H}$$

$$\Rightarrow \nabla^2 (E_x \hat{i} + E_y \hat{j} + E_z \hat{k}) = -\omega^2 \varepsilon \mu (E_x \hat{i} + E_y \hat{j} + E_z \hat{k})$$

where, $\hat{i}, \hat{j}$ and $\hat{k}$ are unit vectors along x, y and z directions.

Equating the co-efficients we get,

$$\nabla^2 . E_x = -\omega^2 \varepsilon \mu E_x$$

$$\nabla^2 . E_y = -\omega^2 \varepsilon \mu E_y$$

$$\nabla^2 . E_z = -\omega^2 \varepsilon \mu E_z$$

⟶ Solns. for these three eqns. are same. But, the boundary conditions are different.

Similarly, $\nabla^2 H = -\omega^2 \mu \varepsilon H$

and $\nabla^2 H_x = -\omega^2 \mu \varepsilon H_x$

$\nabla^2 H_y = -\omega^2 \mu \varepsilon H_y$

$\nabla^2 H_z = -\omega^2 \mu \varepsilon H_z$

In general wave eqns. can be written as,

$$\boxed{\begin{aligned} \nabla^2 E &= -\omega^2 \varepsilon \mu E \\ \nabla^2 H &= -\omega^2 \varepsilon \mu H \end{aligned}}$$

Replacing $-\omega^2$ by $\frac{\partial^2}{\partial t^2}$ we get,

$$\nabla^2 E = \frac{\partial^2}{\partial t^2} \varepsilon \mu E$$

$$\nabla^2 E = \varepsilon \mu \cdot \frac{\partial^2 E}{\partial t^2} \quad \text{and} \quad \nabla^2 H = \mu \varepsilon \cdot \frac{\partial^2 H}{\partial t}$$

where, $\mu = \mu_0 \mu_r$

$\mu_0 = 4\pi \times 10^{-7}$ Henry/mt in free spc

$\mu_r \doteq 1$

and $\varepsilon = \varepsilon_0 \varepsilon_r$

ε_0 = permittivity in free space

$= 8.854 \times 10^{-12}$ F/m (Farads/mt)

$= \frac{1}{36\pi} \times 10^{-9}$ F/m

$$\mu \varepsilon = (\mu_0 \mu_r)(\varepsilon_0 \varepsilon_r) \cdots$$

$$= (\mu_0 \varepsilon_0)(\mu_r \varepsilon_r)$$

$$= (4\pi \times 10^{-7} H/m)\left(\frac{1}{36\pi} \times 10^{-9} F/m\right)(1 \cdot 1)$$

$$= (4\pi \times 10^{-7} H/m)\left(\frac{1}{36\pi} \times 10^{-9} F/m\right)$$

$$= \frac{1}{(3 \times 10^{8} m/sec)^2}$$

$$\mu \varepsilon = \frac{1}{c^2} \Rightarrow c^2 = \frac{1}{\mu\varepsilon}$$

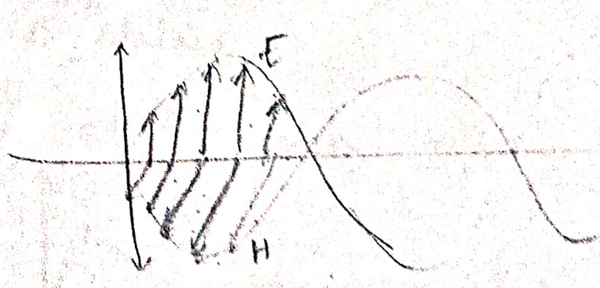
$$\Rightarrow \quad c = \frac{1}{\sqrt{\mu \varepsilon}}$$

$c^2 = \frac{1}{\mu\varepsilon}$

$\mu\varepsilon = \frac{1}{c^2}$

$$\rightarrow \nabla^2 E = \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} \quad \text{and} \quad \nabla^2 H = \frac{1}{c^2} \cdot \frac{\partial^2 H}{\partial t^2}$$

→ These 2 eqns. represents the waves propagating through free space with a velocity = velocity of light



(EM spectrum dia HW)

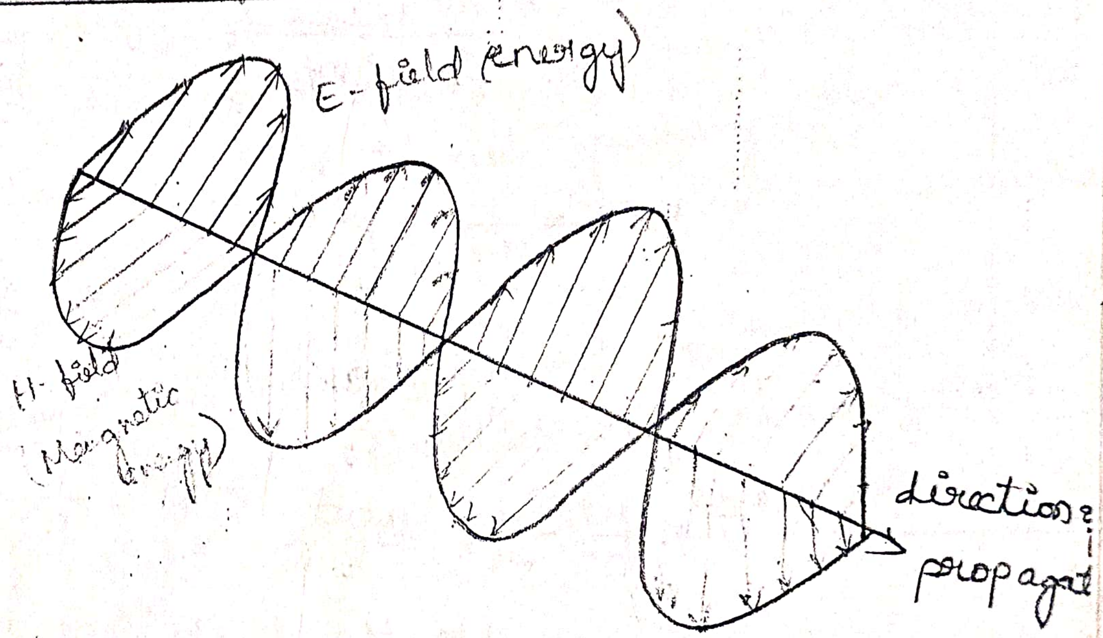
Electro Magnetic Spectrum :-

ELF	SLF	VLF	LF	MF	HF	VHF	UHF	SHF	EHF	Infra red	Light	X-rays

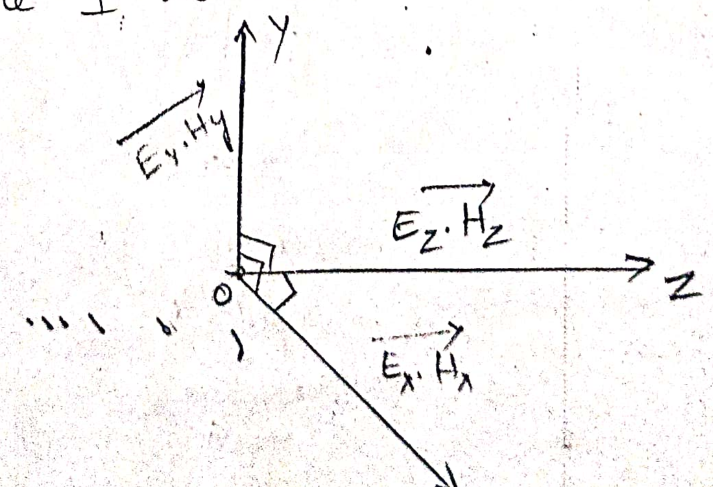
300Hz 30KHz 3MHz 300MHz 30GHz 430THz 10^{18} Hz

30Hz 3KHz 300kHz 30MHz 3GHz 300GHz 1000THz

EM Energy (Signal) :-



Wave defns :- The direction of electric and magnetic fields components along 3 mutual directions of X, Y, Z are $\perp^r$ to each other.



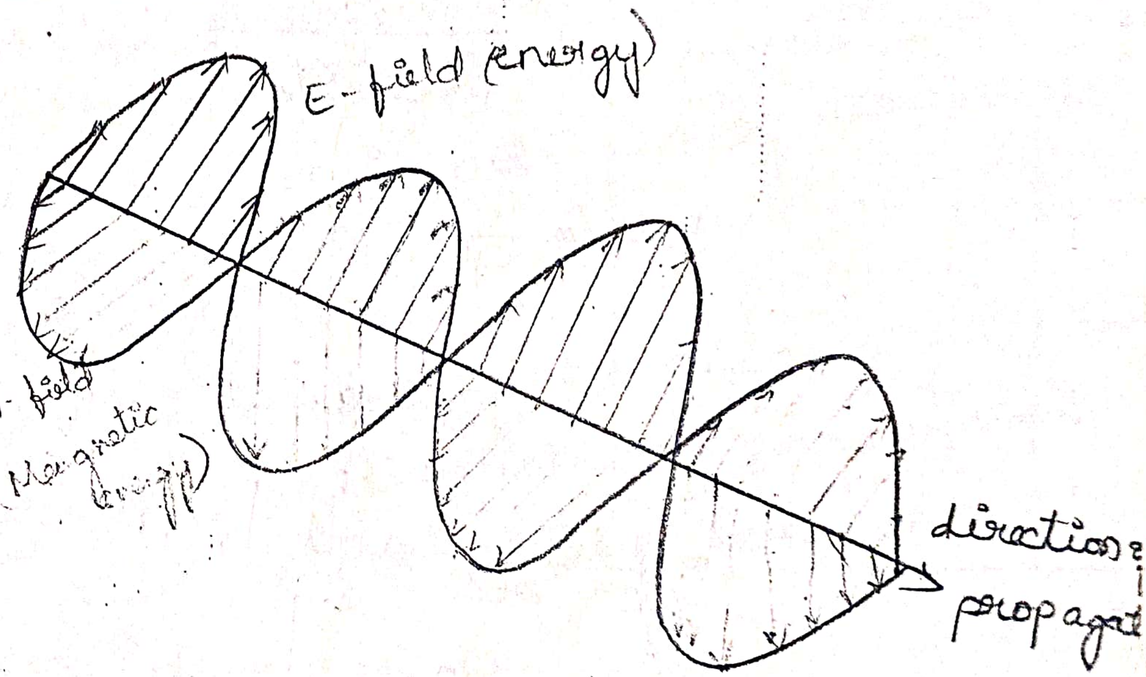
$E_z \neq 0, H_z \neq 0.$

Electro Magnetic spectrum :-

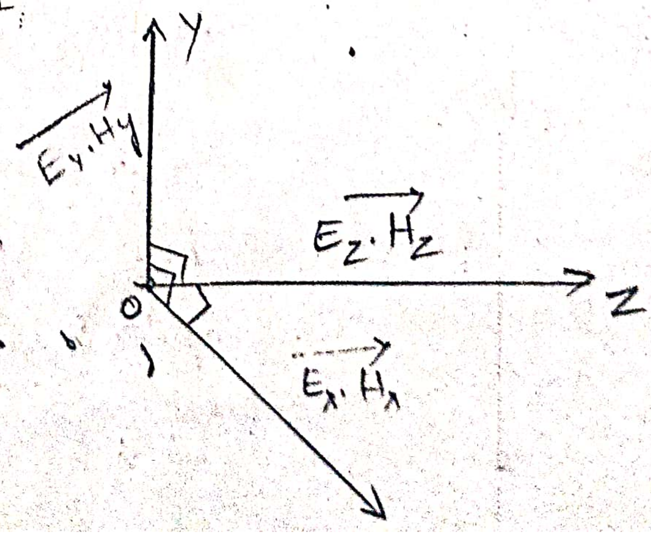
ELF	SLF	VLF	LF	MF	HF	VHF	UHF	SHF	EHF	Infra red	light	X-rays
300Hz		30kHz		3MHz		300MHz		30GHz		430THz		10^{16} Hz

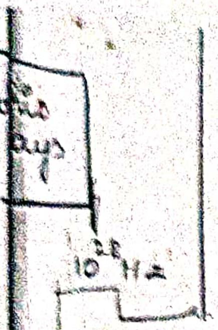
30Hz 3kHz 300kHz 30MHz 3GHz 300GHz 1000THz

EM Energy (Signal):-



Wave defns :- The direction of electric and magnetic fields components along 3 mutual directions of X, Y, Z are $\perp^r$ to each other.





i) TE wave (Transverse Electric wave)

ii) TM wave (" Magnetic wave)

iii) TEM wave (" Electro " ")

iv) HE wave (Hybrid wave)

TE wave: The electric field is purely transverse to the direction of propagation and Magnetic field is not purely transverse.

$$\Rightarrow E_Z = 0,\ H_Z \neq 0$$

TM wave: The magnetic field is purely transverse to the direction of propagation and E field is not purely transverse.

$$\Rightarrow E_Z \neq 0,\ H_Z = 0.$$

TEM waves: Here both the Electric and magnetic fields are transverse to the direction of propagation and there is no z-directed component.

$$\Rightarrow E_Z = 0,\ H_Z = 0$$

HE wave: Here neither E nor magnetic fields are purely transverse to the direction of propagation

$$\Rightarrow E_Z \neq 0,\ H_Z \neq 0.$$

In order to overcome the losses of the transmission lines, we [illegible] guides

<u>Wave guides</u>:-

1) At freq. higher than 3GHz, transmission of EM waves along transmission lines and cables becomes difficult due to losses occurs in the conductors and as well as in the supporting dielectrics.

2) A metallic tube can be used to transmit EM wave at these freq.

3) A hollow metallic tube of uniform cross section for transmitting EM waves for successive reflections from the inner walls of the tube is called a wave guide.

These wave guides are used in UHF and μwave regions, is an alternative to the T^* lines.

4. TEM waves are not exist but TE and TM waves can exist in a wave guide.

5. pwr loss will be present in the walls of due to roughness. In order to reduce the pwr loss, inner walls are coated with either Au/Ag to improve the conductivity

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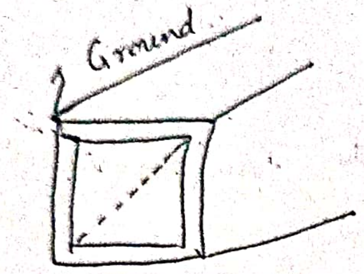
6. Generally the WG is manufactured by using Cu.

Comparison of waveguide with Transmission line:-

Wave guide	Tx line
1 wave travelling in a wave guide has phase velocity and will be attenuated to the Tx line.	1. signal will be attenuated for long distance trans-mission.
2. The wave will be reflected when it reaches the end of wave guide unless the load impedence is adjusted to absorb the wave.	2. Signal will be reflected.
3. Any irregularity in a wave-guide produces reflection.	3. Irregularity in a Tx line produces reflections.
4. Reflections are eliminated by proper impedence matching should be maintained	
5. Standing waves exist.	5. Standing waves exist.
6. Any wave having frequency greater than cut off frequency (fc) will be propagated.. Hence the	6. It allows all the frequencies.

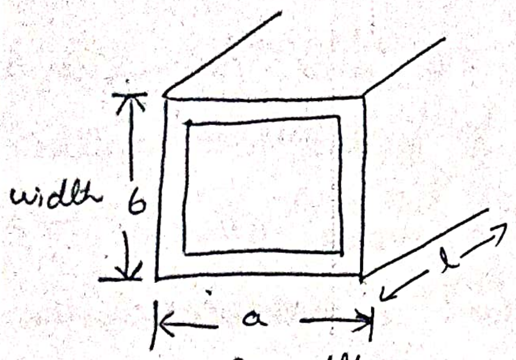
7. The whole body of the waveguide acts as ground.  wave propagates through multiple reflections from the walls of wave guide.	
8. propagation of waves in wave guide is in accordance with the field theory.	8. Signal transmission is in accordance with circuit theory.
9. One end is closed, standing waves are formed. If both the ends are closed signal will bounce back and forth b/w two ends results in resonance. It will be used in cavity resonators.	9. One end closed, standing waves will be formed.

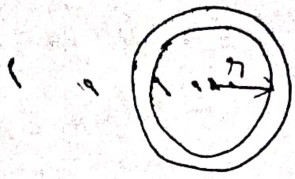
Types of waveguides:

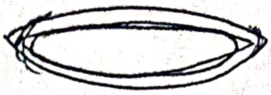
Any shape of the waveguide can support the electro-magnetic fields.

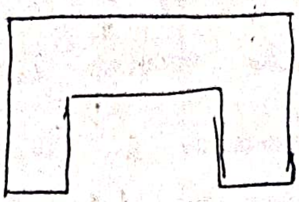
1. Rectangular wave guide :- military, labour.
2. Circular wave guide (Twist the waves). RADARS.
3. ... wave guide (flexible ...

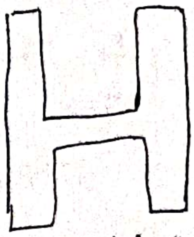
4. Single sredged wave guide.
5. Double bridged wave guide.






Elliptical wave guide


Single ridged

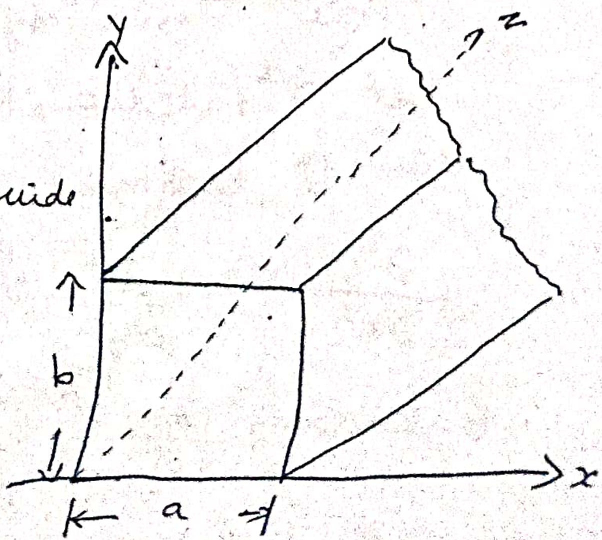

double ridged.

* Reduces the waveguide dimensions and increases critical wave length, increases attenuation, reduced phase velocity.

Propagation of waves in a Rectangular wave guide:-

1. Generally a wave guide is filled with air as dielectric.

2. Consider a rectangular wave guide situated in a rectangle Co-ordinate system with its



z direction of wave propagation

breadth along x- axis and width along y- axis, u
propagates along z- axis.

3. The wave equation for TE and TM waves are g
by $\Delta^2 H_z = -\omega^2 \mu \varepsilon H_z$ for TE wave $[E_z = 0] \rightarrow ①$

$\Delta^2 E_z = -\omega^2 \mu \varepsilon E_z$ for TM wave $(H_z = 0) \rightarrow 2$

Expanding $\Delta^2 E_z$ in rectangular co-ordinate system.

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \frac{\partial^2 E_z}{\partial z^2} = -\omega^2 \mu \varepsilon E_z \rightarrow ③$$

4. The figure, the wave is propagating in z- direction, u
gets the new operator in rectangular co- ordinate system.

$$\frac{\partial^2}{\partial z^2} = \gamma^2 \text{ operator.}$$

Substitute operator in ③, we get

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \gamma^2 E_z = -\omega^2 \mu \varepsilon E_z$$

$$\frac{\partial^2}{\partial x^2} E_z + \frac{\partial^2}{\partial y^2} E_z + E_z [\gamma^2 + \omega^2 \mu \varepsilon] = 0 \rightarrow ④$$

Let $\gamma^2 + \omega^2 \mu \varepsilon = h^2$ be a constant.
then ④ becomes.

$$\frac{\partial^2}{\partial x^2} E_z + \frac{\partial^2}{\partial y^2} E_z + E_z h^2 = 0 \rightarrow \text{for TM wave} - ⑤$$

5. Similarly $\frac{\partial^2}{\partial x^2} H_z + \frac{\partial^2}{\partial y^2} H_z + H_z h^2 = 0 \rightarrow$ for TE wave - ⑥

By solving ⑤ and ⑥ equations by partial differen equation we get the solution for H_z and E_z.

6. By using maxwell's equations it is possible to find out the various components like E_x, E_y, H_x and ___ in x and y directions.

7. From Maxwell's first equation.

$$\nabla \times H = j\omega \varepsilon E$$

expanding $\nabla \times H$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix} = j\omega\varepsilon(\hat{i}E_x + \hat{j}E_y + \hat{k}E_z)$$

8) replacing $\frac{\partial}{\partial z} = -\gamma$ (operator) then we get.

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & -\gamma \\ H_x & H_y & H_z \end{vmatrix} = j\omega\varepsilon\left[\hat{i}E_x + \hat{j}E_y + \hat{k}E_z\right]$$

9) equating the coefficients of vectors i, j, k we get.

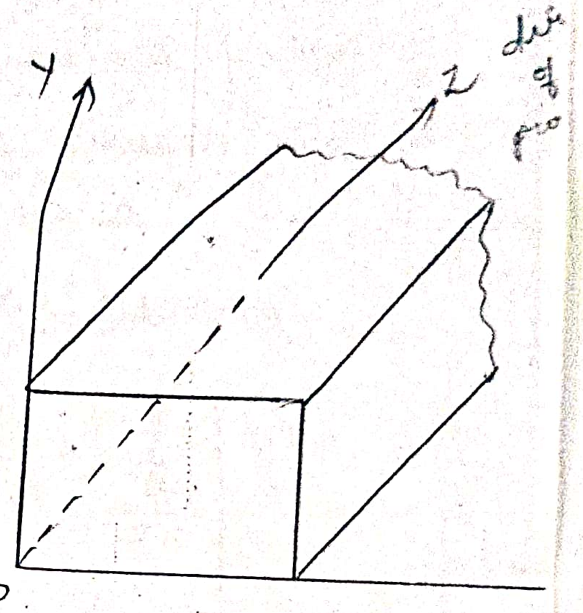
$$\frac{\partial H_z}{\partial y} + \gamma H_y = j\omega\varepsilon E_x \longrightarrow (7)$$

$$\frac{\partial H_z}{\partial x} + \gamma H_x = -j\omega\varepsilon E_y \longrightarrow (8)$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega\varepsilon E_z \longrightarrow (9)$$

~~brea~~ Propagation of waves in Rectangular wave guides:-

10) Similarly from Maxwell's II eqn.



$$\nabla \times E = -j\omega\mu H$$

Expanding $\nabla \times E$,

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & -\gamma \\ E_x & E_y & E_z \end{vmatrix}$$

$\because \frac{\partial}{\partial z} = -\gamma$ (operator)

$$= -j\omega\mu\left[\hat{i}\, H_x + \hat{j}\, H_y + \hat{k}\, H_z\right]$$

Expanding and equating the co-efficients of $\hat{i}, \hat{j}, \hat{k}$;

$$\frac{\partial E_z}{\partial y} + \gamma E_y = -j\omega\mu H_x \longrightarrow (10)$$

$$\frac{\partial E_z}{\partial x} + \gamma E_x = -j\omega\mu H_y \longrightarrow (11)$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu H_z \longrightarrow (12)$$

By using (7) and (11) eqns., we get expression for E_x

$$(7) \Rightarrow \frac{\partial H_z}{\partial y} + \gamma H_y = j\omega\varepsilon E_x$$

$$(11) \Rightarrow \partial E_z + \gamma E_x = j\omega\mu H_y$$

$$H_y = \frac{1}{j\omega\mu}\frac{\partial E_z}{\partial x} + \frac{1}{j\omega\mu}\gamma E_x$$

$$\frac{\partial H_z}{\partial y} + \gamma\left(\frac{1}{j\omega\mu}\cdot\frac{\partial E_z}{\partial x} + \frac{1}{j\omega\mu}\cdot\gamma E_x\right) = j\omega\varepsilon E_x$$

$$\frac{\partial H_z}{\partial y} + \left(\frac{\gamma}{j\omega\mu}\cdot\frac{\partial E_z}{\partial x} + \frac{\gamma^2}{j\omega\mu}\cdot E_x\right) = j\omega\varepsilon E_x$$

$$\cancel{\frac{\partial H_z}{\partial y}} \; E_x\left[\frac{\gamma^2}{j\omega\mu} - j\omega\varepsilon\right] = -\left[\frac{\gamma}{j\omega\mu}\cdot\frac{\partial E_z}{\partial x} + \frac{\partial H_z}{\partial y}\right]$$

$$E_x\left[\frac{\gamma^2+\omega^2\mu\varepsilon}{j\omega\mu}\right] = -\left[\frac{\gamma}{j\omega\mu}\cdot\frac{\partial E_z}{\partial x} + \frac{\partial H_z}{\partial y}\right]$$

$$E_x = \frac{-\gamma}{\cancel{j\omega\mu}}\cdot\frac{\partial E_z}{\partial x}\times\frac{\cancel{j\omega\mu}}{\gamma^2+\omega^2\varepsilon\mu} - \frac{j\omega\mu}{\gamma^2+\omega^2\mu\varepsilon}\cdot\frac{\partial H_z}{\partial y}$$

$$E_x = \frac{-\gamma}{h^2}\cdot\frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2}\cdot\frac{\partial H_z}{\partial y}$$

where, $\boxed{h^2 = \gamma^2 + \omega^2\mu\varepsilon}$

$|||^{ly}$, $$E_y = \frac{-\gamma}{h^2}\cdot\frac{\partial E_z}{\partial y} + \frac{j\omega\mu}{h^2}\cdot\frac{\partial H_z}{\partial x}$$

$$H_x = \frac{-\gamma}{h^2}\cdot\frac{\partial H_z}{\partial x} + \frac{j\omega\varepsilon}{h^2}\cdot\frac{\partial E_z}{\partial y}$$

$$H_y = \frac{-\gamma}{h^2}\cdot\frac{\partial H_z}{\partial y} - \frac{j\omega\varepsilon}{h^2}\cdot\frac{\partial E_z}{\partial x}$$

$E_z = H_z = 0$ (As the field travels in z-direction)

For a TEM wave,

$E_z = H_z = 0$, all the field components (i.e., E_x, E_y, H_x and $H_y = 0$) along the X, and Y-directions disappears, and hence a TEM wave cannot exist inside wave guide.

TE and TM mode:-

Mode :- The EM wave inside a WG can have ∞ no. of patterns which are called as modes.

The EM wave is ⊥r to each other. At the surface of the conductor, no electric field parallel to the surface, but it must be ⊥r the surface.

llly no mag. field is ⊥r to the surface it must be llel to the surface.

There are 2 types of modes,

i) TE mode and

ii) TM mode.

TE mode:-

The e⁻ field is always transferred transverse to the direction of propagation is called transverse e⁻ / TE mode i.e., no e⁻ field in the direction of propagation

TM mode:- Here the mag. field is always transverse to the direction of propagation is called TM field/mode i.e., no mag. field in the direction of propagation

i.e., $E_z \neq 0, H_z = 0$

Propagation of TM waves in Rectangular WG:-

WKT, for TM wave $H_z = 0$, $E_z \neq 0$

The eqn. for TM wave is,

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + h^2 E_z = 0 \longrightarrow (1)$$

~~For~~ It is the partial D.E. By solving this eqn., we get different field components like E_x, E_y, H_x and H_y by using Variable Seperation method.

Let us assume $E_z = XY \longrightarrow (2)$

where 'X' is a pure fn. of x only
'Y' is a pure fn. of y only

and X, Y are independent variables.

$$\Rightarrow \frac{\partial^2 (XY)}{\partial x^2} = Y \cdot \frac{\partial^2 (X)}{\partial x^2} \text{ and } \frac{\partial^2 (XY)}{\partial y^2} = X \cdot \frac{\partial^2 Y}{\partial y^2}$$

$$\Rightarrow Y \cdot \frac{\partial^2 X}{\partial x^2} + X \cdot \frac{\partial^2 Y}{\partial y^2} + h^2 \cdot XY = 0 \longrightarrow (3)$$

$\div XY$ on both sides we get,

$$\frac{Y}{XY}\cdot\frac{\partial^2 X}{\partial x^2}+\frac{X}{XY}\cdot\frac{\partial^2 Y}{\partial y^2}+\frac{h^2 XY}{XY}=0$$

$$\frac{1}{X}\cdot\frac{\partial^2 X}{\partial x^2}+\frac{1}{Y}\cdot\frac{\partial^2 Y}{\partial y^2}+h^2=0 \longrightarrow (4)$$

$\frac{1}{X}\cdot\frac{\partial^2 X}{\partial x^2}$ is a pure func. of x only and

$\frac{1}{Y}\cdot\frac{\partial^2 Y}{\partial y^2}$ is a pure func. of y only.

X and Y are independent variables.

We use the variable Seperation method to solve eqn. (4)

→ let, $\frac{1}{X}\cdot\frac{\partial^2 X}{\partial x^2}=-B^2$ and $\frac{1}{Y}\cdot\frac{\partial^2 Y}{\partial y^2}=-A^2$

where $-A^2$ and $-B^2$ are constants.

Sub. $-A^2$ and $-B^2$ in eqn. (4) we get

$$-B^2-A^2+h^2=0$$

$$\Rightarrow \boxed{h^2=A^2+B^2}$$

→ A^2 and B^2 are ordinary 2nd order D.Es.

The solns. of A^2 and B^2 are given by,

$$X \cancel{A} = C_1\cos Bx + C_2 \sin Bx \longrightarrow (5)$$

$$Y = C_3 \cos Ay + C_4 \sin Ay \longrightarrow (6)$$

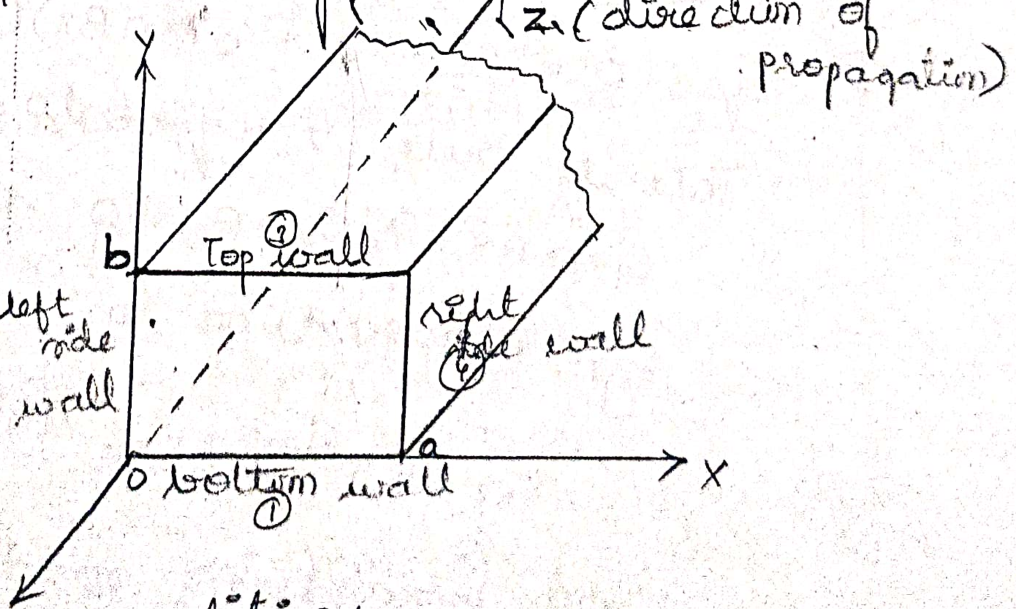
evaluated by ~~to~~ applying the boundary conditions.

→ The complete soln. of E_z in terms of x and y is given by,

$$E_z = XY$$

$$E_z = (C_1 \cos Bx + C_2 \sin Bx)(C_3 \cos Ay + C_4 \sin Ay) \quad \text{(7)}$$

Boundary conditions:- The entire surface of the rectangular wave guide acts as a gnd ~~for~~ or a short ckt for e⁻ field $E_z = 0$, all along the boundary walls of the W/G. The W/G has four walls and it has four boundary conditions.



1st boundary condition:-

(Bottom wall or bottom plane)

$E_z = 0$ at $y = 0 \; \forall \; x \to 0$ to a

2nd boundary condition :-

$E_z = 0$ at $x = 0$ $\forall$ $y \to 0$ to b

3rd boundary :-

$E_z = 0$ at $y = b$ $\forall$ $x \to 0$ to a

4th boundary :-

$E_z = 0$ at $x = a$ $\forall$ $y \to 0$ to b

$\rightarrow$ Sub. 1st boundary condition in eqn ⑦

⑦ $\Rightarrow$ $E_z = (C_1 \cos Bx + C_2 \sin Bx)(C_3 \cos Ay + C_4 \sin Ay)$

$E_z = 0, y = 0, x \to 0$ to a

$\Rightarrow$ $0 = (C_1 \cos Bx + C_2 \sin Bx)[C_3(1) + C_4(0)]$

$0 = (C_1 \cos Bx + C_2 \sin Bx)(C_3)$

This is true $\forall$ $x \to 0$ to a.

$C_1 \cos Bx + C_2 \sin Bx \neq 0$ and $C_3 = 0$

$\therefore$ Eqn. ⑦ reduces to

$$E_z = (C_1 \cos Bx + C_2 \sin Bx)(C_4 \sin Ay) \rightarrow ⑧$$

$\rightarrow$ Sub. 2nd boundary condition in eqn ⑧

$0 = [C_1(1) + C_2(0)](C_4 \sin Ay)$

$0 = C_1 C_4 \sin Ay$

$C_1 = 0, \quad C_4 \sin Ay \neq 0.$

$\therefore$ Eqn (8) reduces to,

$$E_z = (C_2 \sin Bx)(C_4 \sin Ay) \longrightarrow (9)$$

Sub. (3) boundary condition in eqn. (9)

$$0 = (C_2 \sin Bx)(C_4 \sin Ab)$$

$$0 = C_2 C_4 \sin Bx \cdot \sin Ab$$

$C_2, C_4, \sin Bx \neq 0$ then,

$$\sin Ab = 0$$

$Ab =$ a multiple of π

$$Ab = n\pi, \quad n = 0, 1, 2 \ldots\ldots$$

$$\Rightarrow A = \frac{n\pi}{b} \longrightarrow (10)$$

Sub. (4) boundary condition in eqn. (9)

$$0 = (C_2 \sin Ba)(C_4 \sin Ay)$$

$\Rightarrow C_2 \neq 0, \; C_4 \neq 0, \; \sin Ay \neq 0$ then,

$$\sin aB = 0$$

$$\Rightarrow aB = m\pi, \quad m = 0, 1, 2 \ldots$$

$$\Rightarrow B = \frac{m\pi}{a} \longrightarrow (11)$$

Sub. (10) and (11) in eqn. (9) we get,

$$E_z = \left[C_2 \cdot \sin\left(\frac{m\pi}{a}\right)x\right]\left[C_4 \sin\left(\frac{n\pi}{b}\right)y\right]$$

$$E_z = C_2 C_4 \left(\sin\left(\frac{m\pi}{a}\right)x\right) \cdot \left(\sin\left(\frac{n\pi}{b}\right)y\right)$$

$$E_z = C_2 C_4 \cdot \sin\left(\frac{m\pi}{a}\right)x \cdot \sin\left(\frac{n\pi}{b}\right)y \cdot e^{j\omega t} \cdot e^{-\gamma z} \rightarrow ($$

$e^{j\omega t}$ – sinusoidal variation of the wave wrt time 't'

$e^{-\gamma z}$ – propagation along the z-direct

Let, $C_2 C_4 = C$ (new constant)

$$\Rightarrow E_z = C \cdot \sin\left(\frac{m\pi}{a}\right)x \cdot \sin\left(\frac{n\pi}{b}\right)y \cdot e^{j\omega t - \gamma z} \rightarrow (1$$

WKT,

$$E_x = \frac{-\gamma}{h^2} \cdot \frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2} \cdot \frac{\partial H_z}{\partial y}$$

$$E_y = \frac{-\gamma}{h^2} \cdot \frac{\partial E_z}{\partial y} + \frac{j\omega\mu}{h^2} \cdot \frac{\partial H_z}{\partial x}$$

$$H_x = \frac{-\gamma}{h^2} \cdot \frac{\partial H_z}{\partial x} + \frac{j\omega\varepsilon}{h^2} \cdot \frac{\partial E_z}{\partial y}$$

$$H_y = \frac{-\gamma}{h^2} \cdot \frac{\partial H_z}{\partial y} - \frac{j\omega\mu}{h^2} \cdot \frac{\partial E_z}{\partial x}$$

For TM wave, $H_z = 0$

Sub. $H_z = 0$ in E_x

then, $E_x = \frac{-\gamma}{h^2} \cdot \frac{\partial E_z}{\partial x}$

$$\frac{\partial E_z}{\partial x} = \frac{\partial}{\partial x}\left[C \cdot \sin\left(\frac{m\pi}{a}\right)x \cdot \sin\left(\frac{n\pi}{b}\right)y \cdot e^{j\omega t - \gamma z}\right]$$

$$\frac{\partial E_z}{\partial x} = C\sin\left(\frac{n\pi}{b}\right)\cdot y \cdot e^{j\omega t - \gamma z} \cdot \cos\left(\frac{m\pi}{a}\right)x \cdot \left(\frac{m\pi}{a}\right).$$

$$\cancel{E_z = C\sin\left(\frac{m\pi}{a}\right)x \cdot \sin\left(\frac{n\pi}{b}\right)y\, e^{j\omega t - \gamma z}}$$

$$E_x = \frac{-\gamma}{h^2} \cdot \frac{\partial E_z}{\partial x}$$

$$E_x = \frac{-\gamma}{h^2}\left[C\sin\left(\frac{n\pi}{b}\right)y \cdot e^{j\omega t - \gamma z} \cdot \cos\left(\frac{m\pi}{a}\right)x \cdot \left(\frac{m\pi}{a}\right) \cdot e^{j\omega t - \gamma z}\right]$$

$$\boxed{E_x = \frac{-\gamma}{h^2} C \cdot \frac{m\pi}{a} \sin\left(\frac{n\pi}{b}\right)y \cdot \cos\left(\frac{m\pi}{a}\right)x \cdot e^{j\omega t - \gamma z}}$$

For TM wave, $H_z = 0$, Sub. $H_z = 0$ in E_y.

$$\Rightarrow E_y = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial y}$$

$$\frac{\partial E_z}{\partial y} = \frac{\partial}{\partial y}\left[C \cdot \sin\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y \cdot e^{j\omega t - \gamma z}\right]$$

$$\frac{\partial E_z}{\partial y} = C \sin\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)y \cdot \frac{n\pi}{b} \cdot e^{j\omega t - \gamma z}$$

$$\Rightarrow E_y = \frac{-\gamma}{h^2}\left[C \sin\left(\frac{m\pi}{a}\right)x \cdot \cos\left(\frac{n\pi}{b}\right)y \cdot \frac{n\pi}{b} e^{j\omega t - \gamma z}\right]$$

$$\boxed{E_y = \frac{-\gamma}{h^2} C \cdot \left(\frac{n\pi}{b}\right) \sin\left(\frac{m\pi}{a}\right)x \cdot \cos\left(\frac{n\pi}{b}\right)y \cdot e^{j\omega t - \gamma z}}$$

Sub. $H_z = 0$ in, H_x

$$\Rightarrow H_x = \frac{j\omega\varepsilon}{h^2} \cdot \frac{\partial E_z}{\partial y}$$

$$\Rightarrow \boxed{H_x = \frac{j\omega\varepsilon}{h^2}\left[C \cdot \frac{n\pi}{b} \sin\left(\frac{m\pi}{a}\right)x \cdot \cos\left(\frac{n\pi}{b}\right) \cdot y \cdot e^{j\omega t - \gamma z}\right.}$$

Sub. $H_z = 0$ in H_y

$$\Rightarrow H_y = -\frac{j\omega\varepsilon}{h^2} \cdot \frac{\partial E_z}{\partial x}$$

$$\Rightarrow \boxed{H_y = -\frac{j\omega\varepsilon}{h^2} C \cdot \frac{m\pi}{a} \cdot \sin\left(\frac{n\pi}{b}\right)y \cdot \cos\left(\frac{m\pi}{a}\right)x \cdot e^{j\omega t - \gamma z}}$$

Various TM modes in $\square^{ar}$ WG :-

1) Depending on the values of 'm' and 'n' various modes can be formed in TM wave

2) Generally modes are represented by,

TM_{mn}

$m=0$, $n=0$ $\Rightarrow$ TM_{00} mode :-

$\longrightarrow$ Sub $m=n=0$ in E_x, E_y, H_x and H_y eqns. The values of E_x, E_y, H_x and H_y becomes Zero and hence TM_{00} mode doesnot exist in rectangular WG.

TM_{01} mode $(m=0, n=1)$:-

→ Sub. $m=0, n=1$ in E_x, E_y, H_x and H_y eqns.
E_x, E_y, H_x and H_y becomes zero, hence TM_{01} doesnot exist in rectangular WG.

TM_{10} mode $(m=1, n=0)$:-

→ E_x, E_y, H_x and H_y becomes zero, hence TM_{10} doesnot exists in □^ar WG.

TM_{11} mode $(m=1, n=1)$:-

→ $E_x, E_y, H_x, H_y \neq 0$, hence TM_{11} exists in rectangular WG.

→ If $m \geq 1$ and $n \geq 1$, then TM_{mn} mode exists in rectangular WG.

Nature of Wave guide :-

→ Wave guide acts as a ~~Low~~ High pass filter ckt

char. of ~~HPF~~ W/G :-

i) Cut off frequency (f_c) of a WG.

ii) Guided wavelength (λ_g)

iii) Group and phase velocity.

Cut off freq. of a WG (f_c):-

The freq at which propagation constant γ becomes zero is defined as the cutoff freq (or) Threshold freq (f_c).

WKT, $h^2 = \gamma^2 + \omega^2\mu\varepsilon$

and $h^2 = A^2 + B^2$

$$h^2 = \gamma^2 + \omega^2\mu\varepsilon = A^2 + B^2$$

$$= \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$\gamma^2 + \omega^2\mu\varepsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$\gamma^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\varepsilon$$

$$\gamma = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\varepsilon}$$

Propagation constant, $\gamma = \alpha + j\beta$

where, α = attenuation constant

β = phase constant.

$$\alpha + j\beta = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\varepsilon}$$

At low frequencies,

$$\omega^2\mu\varepsilon < \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

then, γ becomes real and positive, i.e, the wave is completely attenuated and there is no phase change. The wave cannot propagate.

At high frequencies,

$$\omega^2 \mu \varepsilon > \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

then, γ - becomes imaginary, phase constant β exists and wave propagates.

At $f = f_c$, $\omega = \omega_c$, $\gamma = 0$

$$2\pi f = 2\pi f_c$$

$$\gamma^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega_c^2 \mu \varepsilon$$

$$0 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega_c^2 \mu \varepsilon$$

$$\Rightarrow \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 = \omega_c^2 \mu \varepsilon$$

$$\omega_c^2 = \frac{1}{\mu \varepsilon}\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]$$

$$\omega_c = \sqrt{\frac{1}{\mu \varepsilon}\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]}$$

$$2\pi f_c = \frac{1}{\sqrt{\mu \varepsilon}} \cdot \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

$\left(\frac{1}{\sqrt{\mu\varepsilon}} = c\right)$

$$f_c = \frac{c}{2\pi}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

$$f_c = \frac{c\cancel{\pi}}{2\cancel{\pi}}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2}$$

$$f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2}$$

then the cutoff wavelength (λ_c) is,

$$\lambda_c = \frac{c}{f_c} = \frac{c}{\frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2}}$$

$$= \cancel{c} \times \frac{2}{\cancel{c}} \times \frac{1}{\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2}}$$

$$\lambda_c = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2}}$$

$$= \frac{2}{\sqrt{\frac{m^2b^2+n^2a^2}{a^2b^2}}}$$

$$\boxed{\lambda_c = \frac{2ab}{\sqrt{m^2b^2+n^2a^2}}}$$

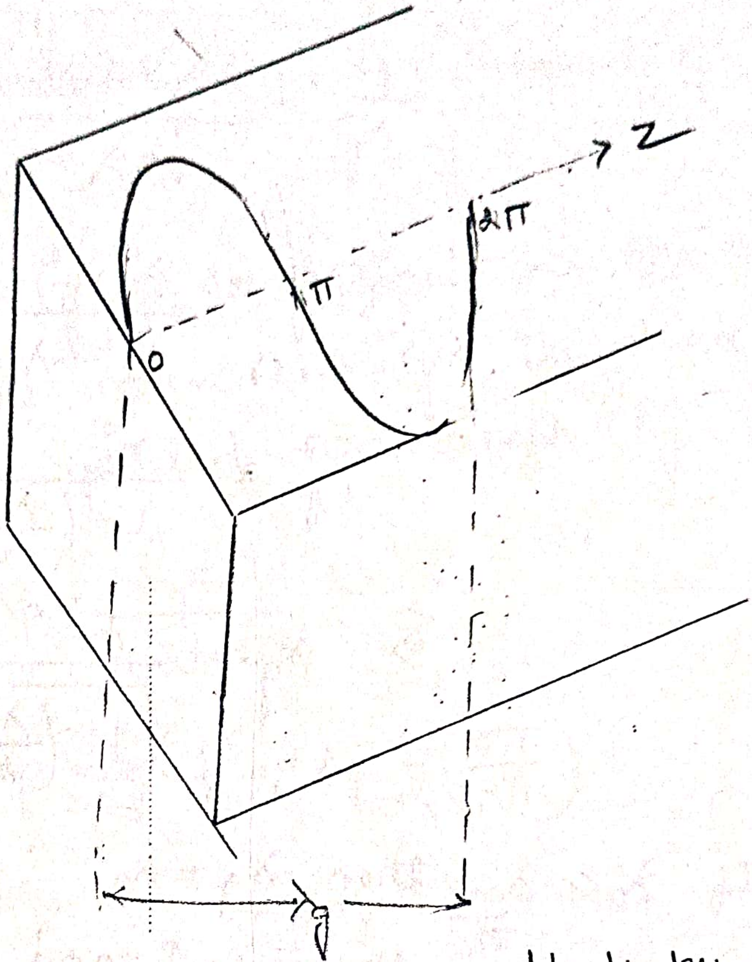
All wavelengths $>\lambda_c$ are attenuated and $<\lambda_c$ are allowed to propagate inside the wave guide.

$$\lambda_{c(m,n)} = \frac{2ab}{\sqrt{m^2b^2+n^2a^2}}$$

of

from

$n(\lambda_0)$



1) It is defined as the dist. travelled by the wave in order to produce a phase shift of 2π radians.

2) It is related to the 'phase' constant 'β' by,

$$\lambda_g = \frac{2\pi}{\beta}$$

3) wave length in the WG (λ_g) is different from the " " " free space (λ_0)

4) It is related by the free space wavelength (λ_0) and cut off ~~f~~ wavelength (λ_c) as

$$\frac{1}{\lambda_g^2} = \frac{1}{\lambda_0^2} - \frac{1}{\lambda_c^2}$$

$$\frac{1}{\lambda_g^2} = \frac{\lambda_c^2 - \lambda_0^2}{\lambda_0^2 \lambda_c^2}$$

$$\Rightarrow \lambda_g^2 = \frac{\lambda_0^2 \lambda_c^2}{\lambda_c^2 - \lambda_0^2}$$

$$\lambda_g^2 = \frac{\lambda_0^2}{1 - \left(\frac{\lambda_0}{\lambda_c}\right)^2}$$

$$\Rightarrow \lambda_g = \frac{\lambda_0}{\sqrt{1 - \left(\frac{\lambda_0}{\lambda_c}\right)^2}}$$

If $\lambda_0 \ll \lambda_c$ then $\lambda_g = \lambda_0$

If $\lambda_0 = \lambda_c$, then $\lambda_g \uparrow$ and reaches infinity.

when $\lambda_0 > \lambda_c$, λ_g is imaginary and no propagation of wave in the WG.

<u>Phase Velocity (V_p):-</u>

In the WG, wave propagates when guided wavelength λ_g is greater than the free space " λ_0, then the velocity of wave propagation is

$$V_p = \lambda \cdot f$$

∴ in the waveguide, $v_p = \lambda_g f$

→ Speed of light $= \lambda_0 \cdot f$

and phase velocity (V_p) is > speed of light

i.e., $\lambda_g > \lambda_0$

→ Phase velocity (V_p) is defined as the rate at which the wave changes its phase in terms of the guided wavelength (λ_g).

$$V_p = \frac{\lambda_g}{\text{unit time}}$$

$$V_p = \lambda_g \cdot f = \lambda_g \cdot f \left(\frac{2\pi}{2\pi}\right)$$

$$= 2\pi f \cdot \frac{\lambda_g}{2\pi}$$

$$V_p = \frac{\lambda_g}{2\pi} \cdot \omega$$

$$V_p = \frac{\omega}{(2\pi/\lambda_g)}$$

$$\boxed{V_p = \frac{\omega}{\beta}}$$

WKT,

$$\gamma^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2 \mu \varepsilon$$

for wave will be propagated,

$$\gamma = \alpha + j\beta$$

$$\gamma = 0 + j\beta = j\beta$$

$$\gamma^2 = (j\beta)^2 = -\beta^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\varepsilon \longrightarrow (1)$$

Group velocity (v_g):

At $f = f_c$, $\omega = \omega_c$, $\gamma = 0$

$$\Rightarrow 0 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega_c^2\mu\varepsilon$$

$$\Rightarrow \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 = \omega_c^2\mu\varepsilon \longrightarrow (2)$$

$$(1)-(2) \Rightarrow (j\beta)^2 = \omega_c^2\mu\varepsilon - \omega^2\mu\varepsilon$$

$$\Rightarrow \beta^2 = (\omega^2 - \omega_c^2)\mu\varepsilon.$$

$$\Rightarrow \beta = \sqrt{\mu\varepsilon(\omega^2 - \omega_c^2)}$$

$$v_p = \frac{\omega}{\beta} = \frac{\omega}{\sqrt{\mu\varepsilon}\cdot\sqrt{\omega^2 - \omega_c^2}} \qquad c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$= \frac{c\cdot\omega}{\sqrt{\omega^2\left[1 - \left(\frac{\omega_c}{\omega}\right)^2\right]}}$$

$$= \frac{c}{\sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2}}$$

$$= \frac{c}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$$

$$\lambda_0 = \frac{c}{f} \Rightarrow f = \frac{c}{\lambda_0}$$

$$\frac{f_c}{f} = \frac{c}{\lambda_c} \cdot \frac{\lambda_0}{c}$$

$$= \frac{\lambda_0}{\lambda_c}$$

$$\Rightarrow \boxed{V_p = \frac{c}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}}}$$

Group velocity (V_g) :-

→ It is defined as the rate at which the wave propagates through the WG. It is given by,

$$V_g = \frac{d\omega}{d\beta}$$

→ Generally carrier s/l's velocity is equal to the speed of light.

→ But the carrier s/l will be modulated then the modulated s/l's envelope velocity is < the speed of light

→ The velocity of modulation envelope in the WG is called the Group velocity (V_g)

V_g in terms of λ_0 and λ_c:

$$V_g = c \cdot \sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}$$

$$\lambda_0 = \frac{c}{f} \rightarrow f = \frac{c}{\lambda_0}$$

$$\frac{f_c}{f} = \frac{c}{\lambda_c} \cdot \frac{\lambda_0}{c}$$

$$= \frac{\lambda_0}{\lambda_c}$$

$$\Rightarrow \boxed{V_p = \frac{c}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}}}$$

<u>Group velocity (V_g):-</u>

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V_g in terms of λ_0 and λ_c:-

$$V_g = c \cdot \sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}$$

** $\boxed{V_p \cdot V_g = C^2}$

Propagation of TE waves in $\square^{ar}$ WG :-

for TE wave, $E_z = 0$, $H_z \neq 0$.

→ The wave eqn. for a TE wave is given as,

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + h^2 H_z = 0 \longrightarrow (1)$$

→ Eqn. (1) is a partial D.E which is solved to get the field components E_x, E_y and H_x, H_y by variable seperation method.

Assume a complete soln. $H_z = XY$

where, X = pure fn. of x only

Y = " " y "

Sub. $H_z = XY$ in eqn. (1)

$$\Rightarrow \frac{\partial^2 (XY)}{\partial x^2} + \frac{\partial^2 (XY)}{\partial y^2} + h^2 (XY) = 0 \longrightarrow (2)$$

$\div XY$,

$$\frac{Y}{XY} \cdot \frac{\partial^2 X}{\partial x^2} + \frac{X}{XY} \cdot \frac{\partial^2 Y}{\partial y^2} + h^2 = 0$$

$$\frac{1}{X} \cdot \frac{\partial^2 X}{\partial x^2} + \frac{1}{Y} \cdot \frac{\partial^2 Y}{\partial y^2} + h^2 = 0 \longrightarrow (3)$$

Sum of these 2 fn. is constant and hence

$$H_x = \frac{-\gamma}{h^2} \cdot \frac{\partial H_z}{\partial x} + \frac{j\omega\varepsilon}{h^2} \cdot \frac{\partial E_z}{\partial y}$$

$$H_y = \frac{-\gamma}{h^2} \cdot \frac{\partial H_z}{\partial y} - \frac{j\omega\varepsilon}{h^2} \cdot \frac{\partial E_z}{\partial x}$$

For TE wave, $E_z = 0$

Sub. $E_z = 0$ in E_x

$$E_x = \frac{-j\omega\mu}{h^2} \cdot \frac{\partial H_z}{\partial y}$$

$$E_x = \frac{-j\omega\mu}{h^2} \cdot \frac{\partial}{\partial y}\left[C\cos\left(\frac{m\pi}{a}\right)x \cdot \cos\left(\frac{n\pi}{b}\right)y \cdot e^{j\omega t - \gamma z}\right]$$

$$E_x = \frac{-j\omega\mu}{h^2} \cdot C\cos\left(\frac{m\pi}{a}\right)x \cdot \left[-\sin\left(\frac{n\pi}{b}\right)y \cdot \left(\frac{n\pi}{b}\right)\right] e^{j\omega t - \gamma z}$$

$$E_x = \frac{j\omega\mu}{h^2} C\left(\frac{n\pi}{b}\right)\cos\left(\frac{m\pi}{a}\right)x \cdot \sin\left(\frac{n\pi}{b}\right)y \, e^{j\omega t - \gamma z}$$

Sub $E_z = 0$ in E_y

$$E_y = +\frac{j\omega\mu}{h^2} \cdot \frac{\partial H_z}{\partial x}$$

$$= \frac{j\omega\mu}{h^2} \cdot \frac{\partial}{\partial x}\left[C\cos\left(\frac{m\pi}{a}\right)x \cdot \cos\left(\frac{n\pi}{b}\right)y \cdot e^{j\omega t - \gamma z}\right]$$

$$= \frac{j\omega\mu}{h^2} \cdot C\left[-\sin\left(\frac{m\pi}{a}\right)x \cdot \frac{m\pi}{a}\right]\cos\left(\frac{n\pi}{b}\right)y \, e^{j\omega t - \gamma z}$$

$$E_y = \frac{-j\omega\mu}{h^2} C \cdot \left(\frac{m\pi}{a}\right) \sin\left(\frac{m\pi}{a}\right)x \cdot \cos\left(\frac{n\pi}{b}\right)y \, e^{j\omega t - \gamma z}$$

Sub $E_z = 0$ in H_x

$$H_x = \frac{-\gamma}{h^2} \cdot \frac{\partial H_z}{\partial x}$$

$$= \frac{-\gamma}{h^2} \cdot \frac{\partial}{\partial x}\left[C \cdot \cos\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)y \cdot e^{j\omega t - \gamma z}\right]$$

$$= \frac{-\gamma}{h^2} C \cdot \left[-\sin\left(\frac{m\pi}{a}\right)x \cdot \frac{m\pi}{a}\right]\cos\frac{n\pi}{b}y \cdot e^{j\omega t - \gamma z}$$

$$H_x = \frac{\gamma}{h^2} C\left(\frac{m\pi}{a}\right)\sin\left(\frac{m\pi}{a}\right)x \cdot \cos\left(\frac{n\pi}{b}\right)y \cdot e^{j\omega t - \gamma z}$$

Sub $E_z = 0$ in H_y

$$H_y = \frac{-\gamma}{h^2} \cdot \frac{\partial H_z}{\partial y}$$

$$= \frac{-\gamma}{h^2} \cdot \frac{\partial}{\partial y}\left[C\cos\left(\frac{m\pi}{a}\right)x \cdot \cos\left(\frac{n\pi}{b}\right)y \cdot e^{j\omega t - \gamma z}\right]$$

$$= \frac{-\gamma}{h^2} C \cdot \cos\left(\frac{m\pi}{a}\right)x \left[-\sin\left(\frac{n\pi}{b}\right)y \, \frac{n\pi}{b}\right] e^{j\omega t - \gamma z}$$

$$H_y = \frac{+\gamma}{h^2} C\left(\frac{n\pi}{b}\right)\cos\left(\frac{m\pi}{a}\right)x \cdot \sin\left(\frac{n\pi}{b}\right)y \cdot e^{j\omega t - \gamma z}$$

TE modes in rectangular wg :-

→ $TE_{(mn)}$ is the general mode and speci[fic] modes are given by selecting the values 'm' and 'n'.

WKT,

$$E_x = \frac{j\omega\mu}{h^2} C\left(\frac{n\pi}{b}\right)\cos\left(\frac{m\pi}{a}\right)x \cdot \sin\left(\frac{n\pi}{b}\right)y\, e^{(j\omega t-\gamma z)}$$

$$E_y = \frac{-j\omega\mu}{h^2} C\left(\frac{m\pi}{a}\right)\sin\left(\frac{m\pi}{a}\right)x \cdot \cos\left(\frac{n\pi}{b}\right)y\, e^{(j\omega t-\gamma z)}$$

$$H_x = +\frac{\gamma}{h^2} C\left(\frac{m\pi}{a}\right)\sin\left(\frac{m\pi}{a}\right)x \cdot \cos\left(\frac{n\pi}{b}\right)y \cdot e^{(j\omega t-\gamma z)}$$

$$H_y = -\frac{\gamma}{h^2} C\left(\frac{n\pi}{b}\right)\cos\left(\frac{m\pi}{a}\right)x \cdot \sin\left(\frac{n\pi}{b}\right)y \cdot e^{(j\omega t-\gamma z)}$$

For TE_{00} mode $(m=0, n=0)$

E_x, E_y, H_x and $H_y = 0$

Hence this mode doesnot exist.

For TE_{01} mode $(m=0, n=1)$

$E_x \neq 0,\ E_y = 0,\ H_x = 0,\ H_y \neq 0$

TE_{01} mode exists.

For TE_{10} mode $(m=1, n=0)$

$E_x = 0,\ E_y \neq 0,\ H_x \neq 0,\ H_y = 0$

E_y and H_x exists

TE modes in rectangular wg :-

→ $TE_{(mn)}$ is the general mode and speci[al] modes are given by selecting the values 'm' and 'n'.

WKT,

$$E_x = \frac{j\omega\mu}{h^2} C\left(\frac{n\pi}{b}\right) \cos\left(\frac{m\pi}{a}\right)x . \sin\left(\frac{n\pi}{b}\right)y \; e^{(j\omega t - \gamma z)}$$

$$E_y = \frac{-j\omega\mu}{h^2} C\left(\frac{m\pi}{a}\right) \sin\left(\frac{m\pi}{a}\right)x . \cos\left(\frac{n\pi}{b}\right)y \; e^{(j\omega t - \gamma z)}$$

$$H_x = +\frac{\gamma}{h^2} C\left(\frac{m\pi}{a}\right) \sin\left(\frac{m\pi}{a}\right)x . \cos\left(\frac{n\pi}{b}\right)y \cdot e^{(j\omega t - \gamma z)}$$

$$H_y = -\frac{\gamma}{h^2} C\left(\frac{n\pi}{b}\right) \cos\left(\frac{m\pi}{a}\right)x . \sin\left(\frac{n\pi}{b}\right)y \cdot e^{(j\omega t - \gamma z)}$$

For TE_{00} mode $(m=0, n=0)$

E_x, E_y, H_x and $H_y = 0$

Hence this mode doesnot exist.

For TE_{01} mode $(m=0, n=1)$

$E_x \neq 0, \; E_y = 0, \; H_x = 0, \; H_y \neq 0$

TE_{01} mode exists.

For TE_{10} mode $(m=1, n=0)$

$E_x = 0, \; E_y \neq 0, \; H_x \neq 0, \; H_y = 0$

E_y and H_x exists

$$0 = \frac{-\gamma}{h^2}\cdot\frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2}\cdot\frac{\partial H_z}{\partial y}$$

$$\Rightarrow \quad \frac{-j\omega\mu}{h^2}\cdot\frac{\partial H_z}{\partial y} = 0 \quad (\text{where, } E_z = 0)$$

$$\Rightarrow \frac{\partial H_z}{\partial y} = 0$$

$$\frac{\partial}{\partial y}\left[(C_1\cos Bx + C_2\sin Bx)(C_3\cdot\cos Ay + C_4\sin Ay)\right] = 0.$$

$$(C_1\cos Bx + C_2\sin Bx)(-AC_3\sin Ay + AC_4\cdot\cos Ay) = 0.$$

$$A(C_1\cos Bx + C_2\sin Bx)(-C_3\cdot\sin A(0) + C_4\cdot\cos A(0)) = 0.$$

$$A(C_1\cos Bx + C_2\sin Bx)(C_4) = 0$$

$$C_1\cos Bx + C_2\sin Bx \neq 0; \quad C_4 = 0$$

$$\therefore H_z = (C_1\cos Bx + C_2\sin Bx)(C_3\cos Ay)$$

Sub. III boundary condition in (5)

$$\Rightarrow \quad 0 = \frac{-\gamma}{h^2}\cdot\frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2}\cdot\frac{\partial H_z}{\partial y}$$

$$\frac{-j\omega\mu}{h^2}\cdot\frac{\partial H_z}{\partial y} = 0 \Rightarrow \frac{\partial H_z}{\partial y} = 0 \Rightarrow \frac{\partial}{\partial y}\left[(C_1\cos Bx + C_2\sin Bx)(C_3\cos Ay)\right] = 0$$

$$(C_1\cos Bx + C_2\sin Bx)(-AC_3\cdot\sin Ay) = 0.$$

$$(C_1\cos Bx + C_2\sin Bx)(-AC_3\cdot\sin Ab) = 0.$$

Here, $C_1\cos Bx + C_2\sin Bx \neq 0$; $\sin Ab = 0$

$$Ab = n\pi \Rightarrow A = n\pi/b$$

Sub. II boundary condition in (6)

$$0 = -\frac{\gamma}{h^2}\cdot\frac{\partial E_z}{\partial y} + \frac{j\omega\mu}{h^2}\cdot\frac{\partial H_z}{\partial x} \Rightarrow \frac{j\omega\mu}{h^2}\cdot\frac{\partial H_z}{\partial x} = 0.$$

$$\frac{\partial}{\partial x}\left[(C_1 \cos Bx + C_2 \sin Bx)(C_3 \cos Ay)\right] = 0.$$

$$(-BC_1 \sin Bx + BC_2 \cos Bx)(C_3 \cos Ay) = 0$$

$$(0 + C_2)(C_3 \cos Ay) = 0 \Rightarrow C_2 = 0; \; C_3 \cos Ay \neq 0.$$

$$\therefore H_z = (C_1 \cos Bx)(C_3 \cos Ay) \longrightarrow 7$$

Sub. IV boundary condition in 6

$$0 = -\frac{\gamma}{h^2} \cdot \frac{\partial E_z}{\partial y} + \frac{j\omega\mu}{h^2} \cdot \frac{\partial H_z}{\partial x} \Rightarrow \frac{j\omega\mu}{h^2} \cdot \frac{\partial H_z}{\partial x} = 0.$$

$$\frac{\partial}{\partial x}\left[(C_1 \cos Bx)(C_3 \cos Ay)\right] =$$

$$(-BC_1 \sin Bx)(C_3 \cos Ay) = 0 \Rightarrow (C_1 \sin Ba)(C_3 \cos$$

$$C_3 \cos Ay \neq 0; \; \sin Ba = 0$$

$$Ba = m\pi \Rightarrow \boxed{B =}$$

Sub A, B values in 7

$$H_z = \left(C_1 \cos \frac{m\pi}{a} x\right)\left(C_3 \cos \frac{n\pi}{b} y\right)$$

$$H_z = C_1 C_3 \cdot \cos \frac{m\pi}{a} x \cdot \cos \frac{n\pi}{b} y \cdot e^{j\omega t} \cdot e^{-\gamma z}$$

where, $e^{j\omega t}$ - sinusoidal variation of wave wrt time

$e^{-\gamma z}$ - propagation along z-direction.

C_1, C_3 - new constant 'C'.

$$H_z = C \cos\left(\frac{m\pi}{a}\right)x \cdot \cos\left(\frac{n\pi}{b}\right)y \cdot e^{j\omega t - \gamma z}$$

WKT,

$$E_x = \frac{-\gamma}{h^2} \cdot \frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2} \cdot \frac{\partial H_z}{\partial y}$$

$$E_y = \frac{-\gamma}{h^2} \cdot \frac{\partial E_z}{\partial y} + \frac{j\omega\mu}{h^2} \cdot \frac{\partial H_z}{\partial x}$$

since X and Y are independent variable.

let, $\frac{1}{X}\cdot\frac{\partial^2 X}{\partial x^2} = -B^2$ and $\frac{1}{Y}\cdot\frac{\partial^2 Y}{\partial y^2} = -A^2$

$\Rightarrow \quad h^2 = A^2 + B^2$

$X = C_1 \cos Bx + C_2 \sin Bx$

$Y = C_3 \cos Ay + C_4 \sin Ay$

$\rightarrow (4)$

$\Rightarrow \quad H_z = XY$

$H_z = (C_1 \cos Bx + C_2 \sin Bx)(C_3 \cos Ay + C_4 \sin Ay)$

where, C_1, C_2, C_3 and C_4 are constants which can be evaluated by appling boundary conditions.

Boundary conditions :-

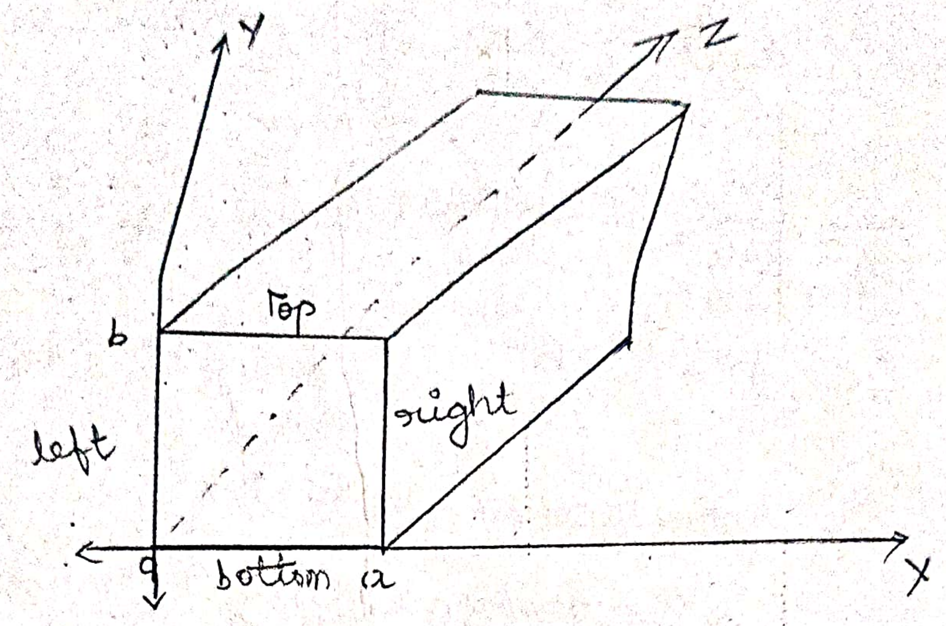
$\parallel^{ar}$ to TM wave, TE wave also has 4 boundary conditions. The entire surface of □ar WG acts as Short ckt (or) gnd for e^- field but we have components along X and Y directions.

$E_z = 0$ but we have components along X and Y directions.

$E_z = 0$, X and Y directions.

$E_x = 0$, all along bottom and top walls.

$E_y = 0$, all along left and right wall.



I (bottom wall) :-

$E_x = 0$ at $y = 0$ $\forall$ $x \to 0$ to a

II (left wall) :-

$E_y = 0$ at $x = 0$ $\forall$ $y \to 0$ to b

III (Top wall) :-

$E_x = 0$ at $y = b$ $\forall$ $x \to 0$ to a

IV (right wall) :-

$E_y = 0$, at $x = a$ $\forall$ $y \to 0$ to b.

From the propagation of waves in $\square^{ar}$ wg

$$E_x = -\frac{\gamma}{h^2} \cdot \frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2} \cdot \frac{\partial H_z}{\partial y} \longrightarrow (5)$$

$$E_y = -\frac{\gamma}{h^2} \cdot \frac{\partial E_z}{\partial y} + \frac{j\omega\mu}{h^2} \cdot \frac{\partial H_z}{\partial x} \longrightarrow (6)$$

Sub. 1st boundary condition in E_x

[illegible]

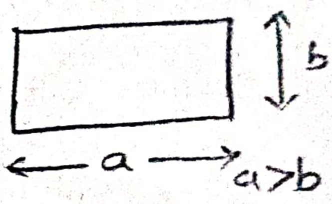
$E_x \neq 0$, $E_y \neq 0$, $H_x \neq 0$, $H_y \neq 0$

All the components exists

For $m \geq 1$, and $n \geq 1$ the modes will be existing in TE mode.

Dominant mode: It is the mode in which the cutoff wavelength is max.

Dominant mode in TM_{mn} for $\square^{ar}$ W/G:-



TM_{11} mode ($m=1, n=1$):-

$$\lambda_c(m,n) = \frac{2ab}{\sqrt{m^2b^2+n^2a^2}}$$

Consider, $a=2, b=1$.

$$\lambda_{c(11)} = \frac{2ab}{\sqrt{b^2+a^2}} \Rightarrow \lambda_{c(1,1)} = 1.788$$

TM_{12} mode ($m=1, n=2$):-

$$\lambda_c(1,2) = \frac{2ab}{\sqrt{b^2+4a^2}} \Rightarrow \lambda_{c(1,2)} = 0.97$$

TM_{21} mode ($m=2, n=1$):-

$$\lambda_{c(2,1)} = \frac{2ab}{\sqrt{4b^2+a^2}} \Rightarrow \lambda_{c(2,1)} = 1.414$$

$\lambda_{c(11)} > \lambda_{c(12)}$

$\lambda_{c(11)} > \lambda_{c(21)}$

Hence, TM_{11} mode is the dominant mode.

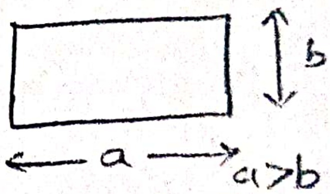
$E_x \neq 0$, $E_y \neq 0$, $H_x \neq 0$, $H_y \neq 0$

All the components exists

For $m \geq 1$, and $n \geq 1$ the modes will be existing in TE mode.

Dominant mode: It is the mode in which the cutoff wavelength is max.

Dominant mode in TM_{mn} for $\square^{ar}$ W/G :-



TM_{11} mode $(m=1, n=1)$:-

$$\lambda_c(m,n) = \frac{2ab}{\sqrt{m^2b^2+n^2a^2}}$$

Consider, $a=2, b=1$.

$$\lambda_{c(11)} = \frac{2ab}{\sqrt{b^2+a^2}} \Rightarrow \lambda_{c(1,1)} = 1.788$$

TM_{12} mode $(m=1, n=2)$:-

$$\lambda_c(1,2) = \frac{2ab}{\sqrt{b^2+4a^2}} \Rightarrow \lambda_{c(1,2)} = 0.97$$

TM_{21} mode $(m=2, n=1)$:-

$$\lambda_{c(2,1)} = \frac{2ab}{\sqrt{4b^2+a^2}} \Rightarrow \lambda_{c(2,1)} = 1.414$$

$\lambda_{c(11)} > \lambda_{c(12)}$

$\lambda_{c(11)} > \lambda_{c(21)}$

Hence, TM_{11} mode is the dominant mode.

Dominant mode in TE_{mn} for □^{r} w/G

$TE_{(0,1)}$ mode :-

(Consider, $a = 2$, $b = 1$)

$$\lambda_c(m,n) = \frac{2ab}{\sqrt{m^2 b^2 + n^2 a^2}}$$

$$\lambda_c(0,1) = 2b$$

$$\lambda_c(0,1) = \frac{2(2)(1)}{\sqrt{0 + 1(4)}}$$

$$= 2$$

$TE_{(1,0)}$ mode :-

$$\lambda_c(1,0) = 2a$$

$$\lambda_c(1,0) = \frac{2(2)(1)}{\sqrt{1 + 0}}$$

$$= 4$$

$TE_{(1,1)}$ mode :-

$$\lambda_c(1,1) = \frac{2ab}{\sqrt{b^2 + a^2}}$$

$a = 2$, $b = 1$

$$\lambda_c(1,1) = \frac{4}{\sqrt{1+4}}$$

$$= 1.788$$

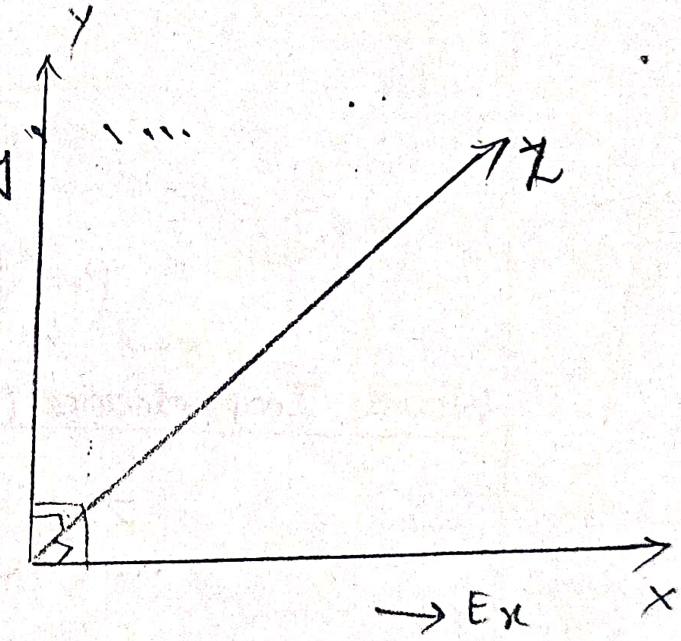
Here, $TE_{(1,0)}$ mode is the dominant mode in TE waves.

same cut-off freq. are called degnerate modes.

→ Depending upon the values of m,n and a,b we can find the dominant and degenerate modes

Wave Impedance (Z_z):

It is defined as the ratio of the strength of the e⁻ field in one transverse direction to the strength of the mag. field in the other transverse direction.



$$Z_z = \frac{E_x}{H_y} \text{ (or) } \frac{-E_y}{H_x}$$

$$Z_z = \sqrt{\frac{E_x^2 + E_y^2}{H_x^2 + H_y^2}}$$

Wave Impedance for a TM wave in R.W.G.

$$Z_z = Z_{TM} = \frac{E_x}{H_y} = \frac{-\frac{\gamma}{h^2}\cdot\frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2}\cdot\frac{\partial H_z}{\partial y}}{-\frac{\gamma}{h^2}\cdot\frac{\partial H_z}{\partial y} - \frac{j\omega\varepsilon}{h^2}\cdot\frac{\partial E_z}{\partial x}}$$

For a TM wave, $H_z = 0, E_z \neq 0$

$$\gamma = j\beta$$

$$Z_z = \frac{\frac{-\gamma}{h^2} \cdot \frac{\partial E_z}{\partial x}}{\frac{-j\omega\varepsilon}{h^2} \cdot \frac{\partial E_z}{\partial x}}$$

$$= \frac{-\gamma}{-j\omega\varepsilon} = \frac{\gamma}{j\omega\varepsilon} = \frac{j\beta}{j\omega\varepsilon}$$

$$\boxed{Z_z = \frac{\beta}{\omega\varepsilon}}$$

$$\beta = \sqrt{\omega^2\mu\varepsilon - \omega_c^2\mu\varepsilon}$$

<u>Wave Impedance (Z_z) for a TM wave in □ar,</u>

$$Z_{TM} = \frac{\beta}{j\omega\varepsilon}$$

$$\beta = \sqrt{\omega^2\mu\varepsilon - \omega_c^2\mu\varepsilon}$$

$$Z_{TM} = \frac{\sqrt{\omega^2\mu\varepsilon - \omega_c^2\mu\varepsilon}}{\omega\varepsilon}$$

$$= \frac{\sqrt{\mu\varepsilon} \cdot \sqrt{\omega^2 - \omega_c^2}}{\omega\varepsilon}$$

$$= \frac{\sqrt{\mu\varepsilon}}{\omega\varepsilon} \sqrt{\omega^2\left(1 - \left(\frac{\omega_c}{\omega}\right)^2\right)}$$

$$= \sqrt{\frac{\mu}{\varepsilon}} \sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2}$$

Given,

$$\lambda_g = 4cm$$

$$\lambda_0 = \frac{c}{f} = \frac{3\times10^{10}}{9\times10^{9}} = 3.33cms.$$

WKT,

$$\lambda_g = \frac{\lambda_0}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}}$$

$$\Rightarrow 4 = \frac{3.33}{\sqrt{1-\left(\frac{3.33}{\lambda_c}\right)^2}}$$

$$\Rightarrow 16 = \frac{11.0889}{\left[1-\left(\frac{3.33}{\lambda_c}\right)^2\right]} \Rightarrow 16\left[1-\left(\frac{3.33}{\lambda_c}\right)^2\right] = 11.0889$$

$$1-\left(\frac{3.33}{\lambda_c}\right)^2 = 0.693$$

$$\frac{3.33}{\lambda_c} = 0.554$$

$$\boxed{\lambda_c = 6.01cms}$$

WKT,

$$\lambda_c = 2a.$$

$$\Rightarrow 6.01 = 2a$$

$$\Rightarrow \boxed{a = 3.005cms}$$

A □ar w/G has dimensions of a=1.5cms, b=1cm calculate the amount of attenuation if freq= 6GHz.

$$\lambda_0 = \frac{3\times10^{10}}{6\times10^{9}} = 5cms.$$

$$\lambda_c(1,0) = \frac{2ab}{\sqrt{b^2m^2+n^2a^2}} = 2a = 3cms.$$

cut off freq $= \frac{c}{\lambda_{c(1,0)}} = \frac{3\times10^{10}}{3} = 10GHz$

Thus the 6GHz s/l will not propagate through the w/g and will be attenuated ($f_c > f_o$)

Attenuation constant (α) $= \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\varepsilon}$

For TE_{10} mode, $m=1, n=0$

$\mu = 4\pi\times10^{-7}$

$\varepsilon = 8.857\times10^{-12}$

$$\alpha = \sqrt{\left(\frac{\pi}{1.5}\right)^2 + 0 - (12\pi\times10^9)^2(4\pi\times10^{-7})(8.857\times10^{-12})}$$

$\omega = 2\pi f = 2\pi(6\times10^9)$

$$Z_{TE} = \frac{\eta}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}} \quad \because \frac{f_c}{f} = \frac{\lambda_0}{\lambda_c}$$

$$Z_{TE} = \frac{\eta}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}}$$

$Z_{TE} > \eta$ as $\lambda_0 < \lambda_c$ for wave propagation.

<u>Wave impedance (Z_z) for a TEM wave in □ar W/G:-</u>

For TEM waves, cut off freq = 0 and wave impedance for TEM wave is the free space impedance (i.e., 377Ω)

$$Z_z(TEM) = \eta$$

$\because E_z = 0$ and $H_z = 0$ for TEM waves.

Q) A □ar W/G has dimension $a = 4cm$, $b = 3cm$ as its sectional dimensions. Find all the modes which will propagate at 5000MHz

<u>Soln:-</u> The condition for the wave propagation along a guide is $\lambda_0 < \lambda_c$ (or) $\lambda_c > \lambda_0$

Given, frequency (f) = 5000MHz

$$\lambda_0 = \frac{c}{f} = \frac{3\times10^8}{5000\times10^6}$$

$$= 6 cms.$$

$$\lambda_c(m,n) = \frac{2ab}{\sqrt{m^2b^2+n^2a^2}}$$

$a = 4\,cm$
$b = 3\,cm$

$$\lambda_c(1,1) = \frac{2(4)(3)}{\sqrt{16+9}}$$

$$= \frac{24}{5} = 4.8\,cm$$

$$\lambda_c(1,0) = \frac{2(4)(3)}{\sqrt{4^2}} = 6\,cm$$

$$\lambda_c(0,1) = \frac{2(4)(3)}{\sqrt{3^2}} = 8\,cm$$

$$\lambda_c(0,0) = \infty$$

Q) $\square^{ar}$ W/G has dimensions, 2.5×5cms. determine the guided wavelength (λ_g), phase constant (β) and phase velocity V_p at wavelength of 4.5cms for the dominant mode.

TE_{10} mode is the dominant mode.

Given, $a = 5\,cms$
$b = 2.5\,cm$

for TE_{10}, $\lambda_c(1,0) = 2a = 10\,cm$

$\lambda_0 = 4.5\,cm$

$$\lambda_g = \frac{\lambda_0}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}}$$

$$= \frac{1}{\sqrt{1-\left(\frac{4.5}{10}\right)^2}}$$

$= 5.037 cms.$

$\longrightarrow \beta = \frac{2\pi}{\lambda g}$

$= \frac{2\pi}{5.05} = 1.24$

$\longrightarrow V_p = \lambda g \cdot f$

$\lambda_0 = \frac{c}{f} \Rightarrow f = \frac{c}{\lambda_0} = \frac{3 \times 10^{10}}{4.5} = 6.66\ GHz$

$V_p = \lambda g \cdot f$

$= 5.0\cancel{50} \times 6.66 \times 10^9 \times 10^{-2}$

$= 3.36 \times 10^8$

3) A $\square^{ar}$ w/g with a dimension of $\cancel{3}\times\cancel{2}$ cms (3×2) operates in the TM_{11} mode at 10GHz. Determine the char. wave impedance.

$\lambda_0 = \frac{c}{f} = \frac{3 \times 10^{10}}{10 \times 10^9} = 3\ cms$

$a = 3cms,\ b = 2\ cms$

For TM_{11}, $\lambda_c(m,n) = \frac{2ab}{\sqrt{m^2b^2 + n^2a^2}}$ $\therefore \lambda_c = \frac{2(3)(2)}{\sqrt{4+9}}$

$\lambda_c = 3.328\ cm$

Given, ... $b = 1cm$

$\lambda_0 = \frac{3\times10^{10}}{}$

$$Z_{TM} = \eta \cdot \sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}$$

$$= 377 \cdot \sqrt{1-\left(\frac{3}{3.328}\right)^2}$$

$$= 163.2\,\Omega$$

Q) Consider a W/G of $a=8, b=4$, given critical wavelength of TE_{10} mode $= 16cms$, $TM_{11} = 7.16cms$, $TM_{21} = 5.6cms$. What modes are propagated at a free space wavelength of λ_0 (i) 10cms (ii) 5cms.

λ_c of $TE_{10} > \lambda_0 \Rightarrow 16 > 10cms \Rightarrow TE_{10}$ propagates.

λ_c of $TM_{11} < \lambda_0 \Rightarrow 7.16 < 10cms \Rightarrow TM_{11}$ doesnot propagates.

λ_c of $TM_{21} < \lambda_0 \Rightarrow 5.6 < 10 \Rightarrow TM_{21}$ doesnot propagate

$\lambda_0 = 5cm$

λ_c of $TE_{10} > \lambda_0 \Rightarrow 16cms > 5cms \Rightarrow TE_{10}$ propagates

λ_c of $TM_{11} > \lambda_0 \Rightarrow 7.16 > 5cms \Rightarrow TM_{11}$ propagates.

λ_c of $TM_{21} > \lambda_0 \Rightarrow 5.6 > 5cms \Rightarrow TM_{21}$ "

Q) The dimensions of a W/G are 2.5X1 cm. The freq. is 8.6GHz. Find the fol.

i) possible modes ii) cut off freq. iii) λ_g

Given, a = 2.5 cm
b = 1 cm

$$\lambda_0 = \frac{3\times10^{10}}{8.6\times10^9}$$

$= ~~3.529~~$ cms.

$= 3.488$ cms.

i) $TE_{00} \rightarrow$ doesnot exist

TE_{01},

$$\lambda_c = \frac{2ab}{\sqrt{m^2b^2+n^2a^2}}$$

$$= \frac{2(2.5)(1)}{\sqrt{0+(2.5)^2}} = 2 \text{ cms}$$

TE_{10}:-

$$\lambda_c = \frac{2ab}{\sqrt{m^2b^2+n^2a^2}}$$

$$= \frac{2(2.5)(1)}{\sqrt{1+0}} = 5 \text{cms}.$$

TE_{11}:-

$$\lambda_c = \frac{2(2.5)(1)}{\sqrt{(2.5)^2+1^2}} = 1.856 \text{cms}.$$

For TE_{10} mode, $\lambda_c > \lambda_0$ hence, waves will be propagated

For TE_{01} and TE_{11}, $\lambda_c < \lambda_0 \Rightarrow$ waves will not propagate

$$\lambda_g = \frac{\lambda_0}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}}$$

$$= \frac{3.488}{\sqrt{1-\left(\frac{3.488}{5}\right)^2}} = 4.868 \text{ cms}.$$

Q) Determine the cut-off wavelength for the dominant mode in a □ar w/g of breadth for a 2.5GHz s/l propagated in this w/g in the dominant mode. Calculate λ_g, group and phase velocities.

$$\rightarrow \lambda_0 = \frac{3\times10^{10}}{2.5\times10^9} = 12 \text{ cms.}$$

Given, $a = 10\text{cms.}$

λ_c for TE_{10} mode $= 2a$

$\lambda_c = 20\text{cms.}$

$$\rightarrow \lambda_g = \frac{\lambda_0}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}} = \frac{12}{\sqrt{1-\left(\frac{12}{20}\right)^2}} = 15\text{cms.}$$

$$\rightarrow V_p = \frac{c}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}} = 3.75\times10^{10}\text{cms/sec}$$

$$\rightarrow V_g = c\cdot\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2} = 2.4\times10^{10}\text{ cms/sec}$$

$$\left(V_p \cdot V_g = c^2\right)$$

Q) The dominant mode is propagated in a air filled □ar w/g, the guided wavelength for a freq. of 9000MHz is 4cm. Calculate breadth of the wave guide.

$$= \frac{\frac{-\gamma}{h^2} \cdot \frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2} \cdot \frac{\partial H_z}{\partial y}}{\frac{-\gamma}{h^2} \cdot \frac{\partial H_z}{\partial y} - \frac{j\omega\varepsilon}{h^2} \cdot \frac{\partial E_z}{\partial x}}$$

For TE waves, $E_z = 0$, $\gamma = j\beta$

$$\therefore Z_{TE} = \frac{\frac{-j\omega\mu}{h^2} \cdot \cancel{\frac{\partial H_z}{\partial y}}}{\frac{-\gamma}{h^2} \cdot \cancel{\frac{\partial H_z}{\partial y}}}$$

$$= \frac{j\omega\mu}{\gamma} = \frac{j\omega\mu}{j\beta}$$

$$Z_{TE} = \frac{\omega\mu}{\beta}.$$

WKT, $\beta = \sqrt{\mu\varepsilon} \cdot \sqrt{\omega^2 - \omega_c^2}$

$$\Rightarrow Z_{TE} = \frac{\omega\mu}{\sqrt{\mu\varepsilon} \cdot \sqrt{\omega^2 - \omega_c^2}}$$

$$= \sqrt{\frac{\mu^2}{\mu\varepsilon}} \cdot \frac{\omega}{\sqrt{\omega^2 - \omega_c^2}}$$

$$= \sqrt{\frac{\mu}{\varepsilon}} \cdot \frac{\omega}{\sqrt{\omega^2\left(1 - \left(\frac{\omega_c}{\omega}\right)^2\right)}}$$

$$= \sqrt{\frac{\mu}{\varepsilon}} \cdot \frac{1}{\sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2}}$$

$$= \eta \cdot \frac{1}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$$

$$= \sqrt{\frac{\mu}{\varepsilon}} \cdot \sqrt{\left[1-\left(\frac{f_c}{f}\right)^2\right]}$$

$$\therefore Z_{TM} = \sqrt{\frac{\mu}{\varepsilon}} \cdot \sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

$$Z_{TM} = \sqrt{\frac{\mu}{\varepsilon}} \cdot \sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}$$

For air dielectric, $\sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0 \cdot \mu_r}{\varepsilon_0 \cdot \varepsilon_r}} = \sqrt{\frac{4\pi \times 10^{-7}}{\frac{1}{36}\pi \times 10^{-9}}}$

$$= \sqrt{4\pi \times 36\pi \times 10^2}$$

$$= 2(6)(\pi)(10)$$

$$= 120\pi$$

$$= 377\,\Omega = \eta$$

where, η is the intrinsic impedance of free space.

$$Z_{TM} = \eta \cdot \sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}$$

For wave propagation, $\lambda_0 < \lambda_c$ and $Z_{TM} < \eta$

~~wave~~ <u>Wave impedance (Z_z) for a TE wave in</u> <u>$\square$ar W/G</u>:-

Wave impedance $(Z_z) = Z_{TE}$

$$Z_{TE} = \frac{E_x}{H_y}$$

MICROWAVE TRANSMISSION LINES - II

Power transmission in a □ar W/G :-

→ The power transmitted through a W/G can be calculated by means of complex Poynting theorem.

→ The power transmitted through a W/G is given by,

$$P_{tr} = \oint P.ds = \oint \frac{1}{2}(E \times H) ds$$

→ For loss less dielectric, the time avg power flows through a □ar W/G.

$$P_{tr} = \frac{1}{2Z_z} \int_a |E|^2 . da$$

$$= \frac{Z_z}{Z} \int_a |H|^2 . da$$

where, $Z_z = \frac{E_x}{H_y} = \frac{-E_y}{H_x}$

$$|E|^2 = |E_x|^2 + |E_y|^2$$

$$|H|^2 = |H_x|^2 + |H_y|^2$$

→ For TM_{mn} mode, the avg power transmitted through a □ar W/G of dimensions a & b is given by,

$$P_{tor} = \frac{1}{2\eta\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}} \int_0^{a}\int_0^{} |E_x|^2 + |E_y|^2 \, dx\, dy$$

For TE_{mn} mode, $Z_Z = \frac{W}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}}$

$$P_{tor} = \frac{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}}{2W} \int_0^{b}\int_0^{a} |E_x|^2 + |E_y|^2 \, dx\, dy$$

~~Power~~ losses in a W/G :-

Mainly power losses in a W/G due to

i) attenuation below the cutoff freq.

ii) attenuation due to dissipation within the W/G walls.

iii) dielectric within the W/G.

i) Attenuation below cutoff freq :-

At freq. below the cutoff freq ($f < f_c$) the propagation constant has only attenuati term. and phase constant (β) becomes imaginary.

Hence, $\beta = \frac{2\pi}{\lambda_g}$

$$\lambda_g = \frac{\lambda}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$\Rightarrow \beta = \frac{2\pi}{\ldots} = \frac{2\pi}{\lambda}\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

The attenuation constant for a wave travelling in an unbounded lossy dielectric is given by,

$$\alpha = \frac{\sigma}{2} \cdot \sqrt{\frac{\mu}{\varepsilon}}$$

$$\alpha = \eta \cdot \frac{\sigma}{2} \qquad \left(\because \eta = \sqrt{\frac{\mu}{\varepsilon}}\right)$$

where, σ = conductivity

→ for TE mode, the attenuation constant cause by low loss dielectric in □ar w/G is given by,

$$\hookrightarrow * \alpha_{g(TE)} = \frac{\sigma\eta}{2\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

→ for TM mode,

$$* \alpha_{g(TM)} = \frac{\sigma\eta}{2} \cdot \sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

iii) Losses caused by the w/G walls:-

The mag. of e^- and magnetic fields when they are propagating through a lossy w/G is given by,

$$|E| = |E_0| e^{-\alpha_g \cdot z}$$

$$|H| = |H_0| \cdot e^{-\alpha_g \cdot z}$$

where, E_0 is electric field intensity at $z=0$

~~z=0~~ H_0 is magnetic field " " $z=0$

The time avg. power flows through a low loss

$w/_{\sigma}$ decreases proportional to $e^{-2\alpha_g \cdot z}$.

The transmitted power,

$$P_{tr} = (P_{tr} + P_{loss}) \cdot e^{-2\alpha_g \cdot Z}$$

$$P_{tr} = P_{tr} \cdot e^{-2\alpha_g Z} + P_{loss} \cdot e^{-2\alpha_g \cdot Z}$$

$$P_{tr}(1 - e^{-2\alpha_g \cdot z}) = P_{loss} \cdot e^{-2\alpha_g \cdot z}$$

$$\frac{P_{loss}}{P_{tr}} = \frac{1 - e^{-2\alpha_g \cdot Z}}{e^{-2\alpha_g \cdot Z}}$$

$$= \frac{1}{e^{-2\alpha_g \cdot Z}} - 1$$

$$\boxed{\frac{P_{loss}}{P_{tr}} + 1 = e^{2\alpha_g \cdot Z}}$$

If the power loss is very greater than P_{tr}

$(P_{loss} \gg P_{tr})$ then $2\alpha_g \cdot z \ll 1$

$$\frac{P_{loss}}{P_{tr}} + 1 = 1 + 2\alpha_g \cdot z$$

$$\Rightarrow \alpha_g = \frac{P_{loss}}{P_{tr} \cdot 2z}$$

where, z is the direction of propagation

then $\boxed{\alpha_g = \frac{P_{loss}}{2 \cdot P_{tr}}}$

loss

$$= \frac{R_s}{2}\int_S |H_t|^2 \cdot ds$$

R_s - Surface resistance

H_t - tangential component of magnetic intensity at W/G walls.

P_{tr} - transmitted pwr per unit length

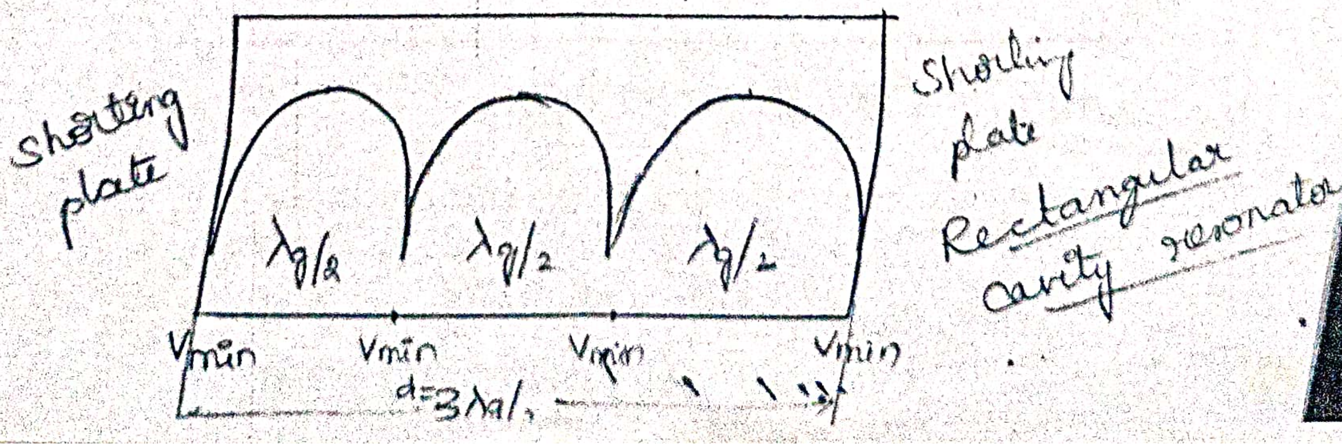
Cavity Resonators :- 1) If one end of the w/G is closed, using a shorting plate, there will be reflections, hence standing waves are formed.

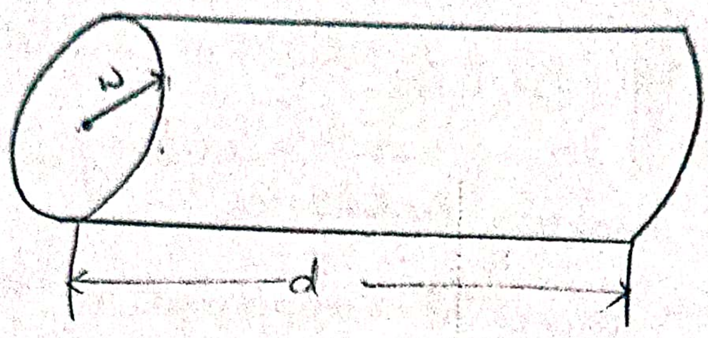
2) If the other end is also closed at a distance of the multiples of $\frac{\lambda g}{2}$, then the hollow space (air) (vacuum) so formed can support a s/l which can bounce back and forth b/w the 2 ends or plates resulting in resonance.

3) It is the basic principle of cavity resonators.

4) The hollow space is called cavity and the resonator acts as the cavity resonator.

5)





Circular cavity resonator.

5) The μwave cavity resonator is similar to a tuned ckt, which has a resonant freq. of

$$f_0 = \frac{1}{2\pi\sqrt{LC}}.$$

6) The cavity resonator can resonate at only one part. freq. ||^{ar} to ||el resonant ckt

7) For a given resonator and mode $a, b, m, n, \lambda_0, \lambda_c$ are fixed values.

$$\left[\text{where, } \lambda_0 = \frac{c}{f_0}\right]$$

and f is also a constant value which is equal to f_0

f_0 is the resonant freq. of cavity resonator

Expression for f_0 in a $\square^{ar}$ cavity resonator:-

WKT, For a $\square^{ar}$ W/G,

$$h^2 = \gamma^2 + \omega^2\mu\varepsilon$$

Also, $h^2 = A^2 + B^2$

$$h^2 = (m\pi)^2 + (n\pi)^2$$

$$\omega^2\mu\varepsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \gamma^2 \longrightarrow ①$$

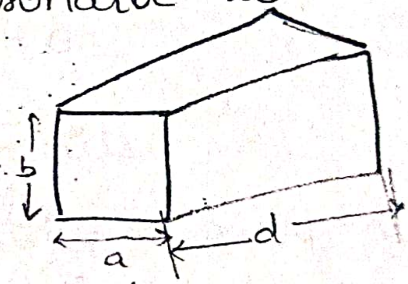
→ For wave propagation, $\gamma = j\beta$, $\gamma^2 = -\beta^2$

$$\omega^2\mu\varepsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \beta^2 \longrightarrow ②$$

→ If a wave exists in a cavity resonator, the wave must follow the phase change corresponding to λ_g.

$$\Rightarrow \beta = \frac{2\pi}{\lambda_g}$$

→ The condition for the resonator to resonant is $\beta = \frac{p\pi}{d}$



where, p - constant

It's values varies from $p = 0$ to ∞

It indicates half wave variation along z-direction

d - length of the resonator.

$\beta = \frac{p\pi}{d}$, $f = f_0$; $\omega = 2\pi f_0 = \omega_0$

$$\omega_0^2\mu\varepsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{d}\right)^2$$

$$\omega_0^2 = \frac{1}{\mu\varepsilon}\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{d}\right)^2\right]$$

$$\omega_0 = \sqrt{\frac{1}{\mu\varepsilon}\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{d}\right)^2\right]}$$

$$f_0 = \frac{1}{2\pi\sqrt{\mu\varepsilon}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{d}\right)^2}$$

$$= \frac{c}{2\pi} \cdot \pi \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{d}\right)^2} \qquad \frac{1}{\sqrt{\mu\varepsilon}} = c$$

$$\therefore f_0 = \frac{c}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{d}\right)^2}$$

Mode of p Expression for f_0 in a circular cavity resonator

→ In a circular cavity resonator, circular 'n' plates are used to short the ends.

$a \rightarrow$ radius of the w/G,

$d \rightarrow$ length / height of the w/G.

→ The condition for resonance in circular cavity resonator, is

$$\beta = \frac{p\pi}{d}$$

→ For circular ~~cavity resonator~~ w/G,

$$h^2 = \gamma^2 + \omega^2 \mu\varepsilon$$

$$h^2_{nm} = \left(\frac{P_{nm}}{a}\right)^2$$

$$\omega^2 \mu\varepsilon = \left(\frac{P_{nm}}{a}\right)^2 - \gamma^2$$

→ For wave propagation, $\gamma = j\beta$ and for $\beta = \frac{p\pi}{d}$, $\omega = \omega_0$

$$\omega_0^2 \mu\varepsilon = \left(\frac{P_{nm}}{a}\right)^2 - (j\beta)^2$$

$$= \left(\frac{P_{nm}}{a}\right)^2 + \beta^2$$

$$= \left(\frac{\dots}{a}\right) + \left(\frac{\dots}{d}\right)$$

$$\omega_0^2 = \frac{1}{\mu\varepsilon}\left[\left(\frac{P_{nm}}{a}\right)^2 + \left(\frac{P\pi}{d}\right)^2\right]$$

$$\omega_0 = \sqrt{\frac{1}{\mu\varepsilon}\left[\left(\frac{P_{nm}}{a}\right)^2 + \left(\frac{P\pi}{d}\right)^2\right]}$$

$$2\pi f_0 = \frac{\cancel{2\pi}\,1}{\sqrt{\mu\varepsilon}} \cdot \sqrt{\left(\frac{P_{nm}}{a}\right)^2 + \left(\frac{P\pi}{d}\right)^2}$$

$$\boxed{f_0 = \frac{c}{2\pi} \cdot \sqrt{\left(\frac{P_{nm}}{a}\right)^2 + \left(\frac{P\pi}{d}\right)^2}} \qquad \left(\because c = \frac{1}{\sqrt{\mu\varepsilon}}\right)$$

App^{n}s of cavity resonators :-

1) They can be used as tuned circuits.
2) In UHF tubes like Reflex Klystron, Cavity magnetron ...
3) In RADARs as duplexers.
4) Cavity wave meters.

→ In □ar w/g the modes are represented by, TE_{mn} (or) TM_{mn}, but in □ar cavity resonators, the modes are represented as, TE_{mnp} (or) TM_{mnp}.

→ In circular w/g, the modes are represented as, TE_{nm} (or) TM_{nm}, but in circular cavity resonators, the modes are represented as, TE_{nmp} (or) TM_{nmp}.

Field expressions for TMmnp and TEmnp modes

▭ar Ca.Re (Cavity Resonator):-

a) TM waves :-

(write derivation)

$$E_z(TM_{mnp}) = C\cdot\sin\left(\frac{m\pi}{a}\right)x\cdot\sin\left(\frac{n\pi}{b}\right)y\ \cos\left(\frac{p\pi}{d}\right)z\cdot e^{j\omega t-}$$

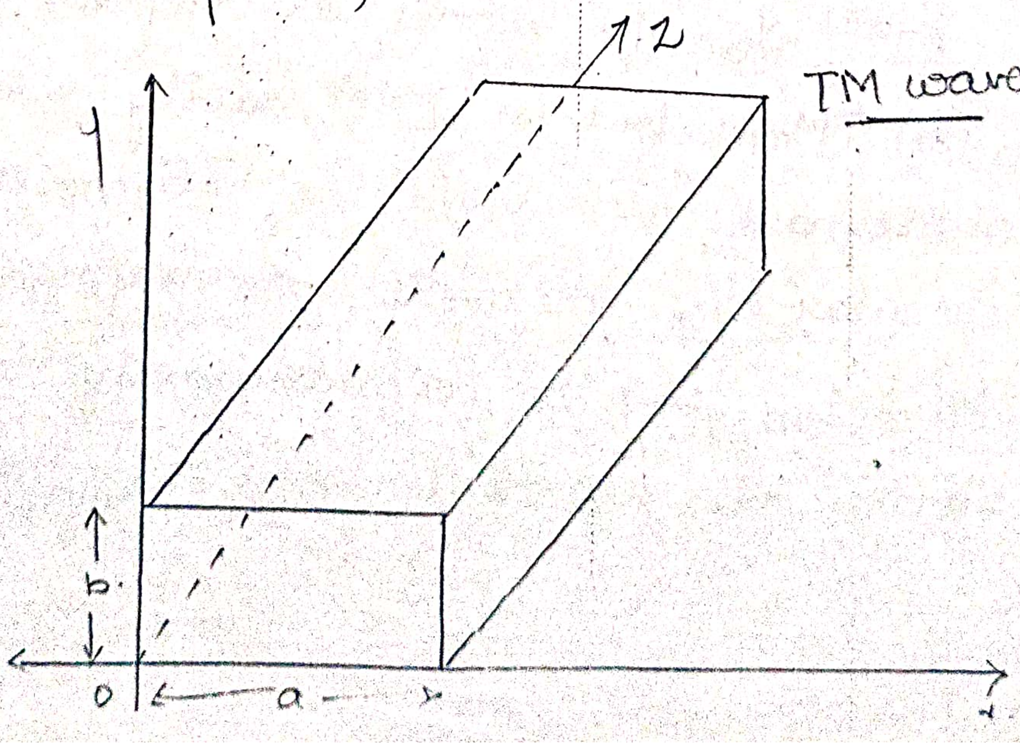
b) TE waves :-

$$H_z(TE_{mnp}) = C\cdot\cos\left(\frac{m\pi}{a}\right)x\cdot\cos\left(\frac{n\pi}{b}\right)y\cdot\sin\left(\frac{p\pi}{d}\right)z\cdot e^{j\omega t-}$$

where, m = 0, 1, 2, 3 ..... represents the no. of half wave variations in the X-direction.

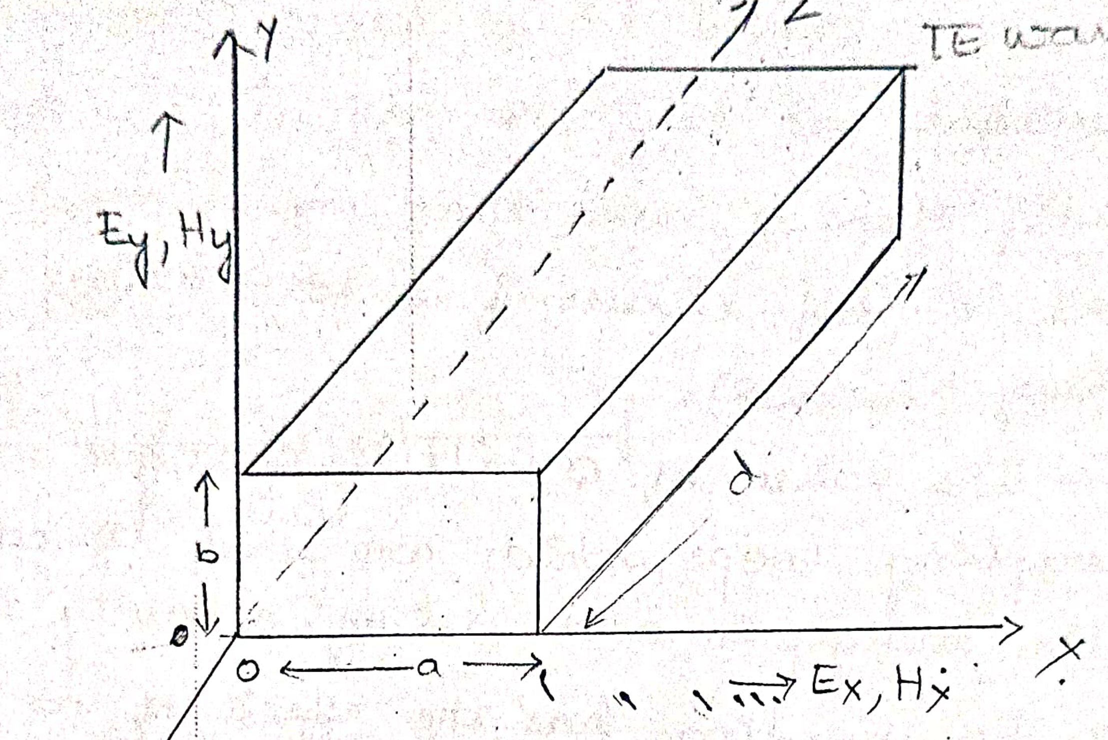
n = 0, 1, 2, ... represents the no. of half wave variations in the Y-direction

p = 1, 2 ...... " " " z-direct



TM wave :-

TE wave :-



Q-factor and coupling co-efficients for cavity resonator :-

- over coupling
- under "
- critical "

→ The Q-factor of any resonant or anti-resonant ckt is a measure of freq. selectivity and is defined by the eqn.

$$Q = \frac{\omega_0 . W}{P}$$

where, ω_0 = resonant freq.

W = max. energy stored.

P = avg. power loss / power dissipated

→ $$Q = \frac{2\pi . (\text{Max. energy stored per cycle})}{\text{avg. power loss per cycle.}}$$

→ For a perfect or ideal cavity resonator,

$Q = \infty$ and $P = 0$.

→ It is proved for a cavity resonator that it is resonant at only one freq.

→ If there is more than one resonant freq, there will be diff. values of Q for various values of freq.

→ The value of Q will be changed with the coupling loops, which are used to couple/connect the energy in and out of a cavity resonator.

→ This coupling has the effect of an imperfectly reflecting wall and so load the cavity or produces the finite termination.

→ The coupling b/w the cavity and coupling paths is known as the loaded Q_L.

It is given by;

$$\frac{1}{Q_L} = \frac{1}{Q_0} + \frac{1}{Q_{ext}}$$

Q_0 – Quality factor of an unloaded cavity.

Q_{ext} – Q-factor due to external ohmic losses.

$$\rightarrow Q_0 = \frac{\omega_0 L}{R}.$$

$$Q_{ext} = \frac{Q_0}{k} = \frac{\omega_0 L}{R \cdot k}$$

k – coupling coefficient of cavity

$$Q_L = \frac{Q_0 \cdot 1}{1+k}$$

i) critically coupled cavity $(K=1)$,

ii) over " ", $(K>1)$,

iii) under " " $(K<1)$.

Critically coupled cavity :- (Resonator matched to the generator)

$$\therefore Q_{ext} = Q_0$$

$$\frac{1}{Q_L} = \frac{1}{Q_0} + \frac{1}{Q_{ext}}$$

$$\frac{1}{Q_L} = \frac{1}{Q_0} + \frac{1}{Q_0}$$

$$\frac{1}{Q_L} = \frac{2}{Q_0} \Rightarrow \boxed{Q_L = \frac{Q_0}{2}}$$

Over coupled cavity :- $(K>1)$

The cavity terminals are at a voltage max. (V_{max}) and the impedance is Standing Wave Ratio (SWR)

$$SWR = \rho \quad \text{and} \quad K = \rho$$

$$Q_L = \frac{Q_0}{1+\rho}$$

(Strip lines, Integrated microwave ckts)

Under coupled cavity :- $(K<1)$

The cavity terminals are at V_{min} and the impedance is inverse of SWR.

$$SWR = \frac{1}{\rho} \quad \text{and} \quad K = \frac{1}{\rho}$$

$$\boxed{Q_L = \frac{Q_0 \cdot \rho}{1+\rho}}$$

Dominant mode in Cavity Resonator :-

→ The mode with lowest resonant freq. is dominant mode. The dominant modes in cavity resonator are TE_{101} and TM_{111}

Degenerate mode in cavity resonator :-

→ If the 2 modes have same resonant freq. then that 2 modes are said to be in degenerate mode.