SREENIVASA INSTITUTE OF TECHNOLOGY AND MANAGEMENT STUDIES::CHITTOOR

Department of Science & Humanities

B Tech- I Semester

23BSC214	PROBABILITY AND COMPLEX VARIABLES	L	T	P	C
	(ECE)	3	1	-	4

PRE-REQUISITES: Set Theory, Permutations and Combinations, Calculus, Real Analysis, Linear Algebra

COURSE EDUCATIONAL OBJECTIVES:

1. To introduce the basic concepts of probability and random variables
2. To introduce the basic concepts of multiple Random variables
3. To Understand concepts of operations on multiple random variables
4. To analyze the functions of complex variable with a review of elementary complex Functions and to learn continuity, differentiability and analyticity of a complex function
5. To understand the Taylor and Laurent expansion with their use in finding out the residue and improper integral

UNIT I : Probability & Random Variable 09

Probability through Sets and Relative Frequency: Experiments and Sample Spaces, Discrete and Continuous Sample Spaces, Events, Probability Definitions and Axioms, Joint Probability, Conditional Probability, Total Probability, Bayes' Theorem, Independent Events.

Random variables: Definition of a random variable - conditions for a function to be a random variable- discrete and continuous random variables - Mixed Random Variable. Distribution and Density functions and their properties: Gaussian random variable - Other distributions and density functions (Binomial, Poisson, Uniform, Exponential and Rayleigh).

UNIT II : Operations on Random variable 09

One Random Variable - Expectation: Expected value of a Random variable - Expected value of a function of a Random variable.

Moments: Moments about the origin, Central moments, Variance and Skew Functions that Give Moments: characteristic function - Moment generating functions.

Multiple Random Variables: Vector Random Variables, Joint Distribution and its properties: Joint distribution function - properties of joint distribution - Marginal Distribution Functions.

Joint density and its properties : joint density function - properties of Joint density - Marginal density functions. Conditional distribution and density - point conditioning. Statistical Independence.

UNIT III : Operations on Multiple Random variables 09

Expected Value of a Function of Random Variables: Joint Moments about the Origin - Joint Central Moments. Joint Characteristic Functions.

Jointly Gaussian Random Variables: Two Random Variables case - Properties of Gaussian random variables.

UNIT IV:Complex Variable – Differentiation 09

Introduction to functions of complex variable-concept of Limit & continuity-Differentiation, Cauchy-Riemann equations, analytic functions harmonic functions, finding harmonic conjugate-construction of analytic function by MilneThomson method.

UNIT V:Complex Variable – Integration 09

Line integral - Cauchy's integral theorem(Simple Case), Cauchy Integral formula,
Power series expansions: Taylor's series,Laurent's series, zeros of analytic functions, singularities.Residues, Cauchy Residue theorem (without proof).

SREENIVASA INSTITUTE OF TECHNOLOGY AND MANAGEMENT STUDIES::CHITTOOR

COURSE OUTCOMES:

Onsuccessfulcompletionofthecourse,studentswillbeableto		Pos
CO1	Understand the concepts of Probability, Random Variables and their characteristics	**PO1, PO2, P03**
CO2	Learn how to deal with multiple random variables, conditional probability, joint distribution and statistical independence.	**PO1, PO2, P03**
CO3	Formulate and solve the engineering problems involving random variables.	**PO1, PO2, P03**
CO4	Analyze limit, continuity and differentiation of functions of complex variables and Understand Cauchy-Riemann equations, analytic functions and various properties of analytic functions.	**PO1, PO2, P03**
CO5	Understand Cauchy theorem, Cauchy integral formulas and apply these to evaluate complex contour integrals. Classify singularities and poles; find residues and evaluate complex integrals using the residue theorem.	**PO1, PO2, P03**

TEXT BOOKS:

1. Peyton Z. Peebles, "Probability, Random Variables & Random Signal Principles", 4th Edition, TMH, 2002.
2. B.S.Grewal,HigherEngineeringMathematics, KhannaPublishers,2017, 44th Edition

REFERENCE BOOKS:

1. Athanasios Papoulis and S. Unnikrishna Pillai, "Probability, Random Variables and Stochastic Processes", 4th Edition, PHI, 2002
2. Erwin Kreyszig, Advanced Engineering Mathematics, Wiley India
3. Henry Stark and John W.Woods, "Probability and Random Processes with Application to Signal Processing," 3rd Edition, Pearson Education, 2002.
4. B.V.Ramana, Higher Engineering Mathematics, Mc Graw Hill publishers.

ONLINE LEARNING RESOURCES:

1. https://onlinecourses.nptel.ac.in/noc20_ma50/preview
2. https://onlinecourses.nptel.ac.in/noc21_ma66/preview#:~:text=This%20course%20provides%20random%20variable,and%20simple%20Markovian%20queueing%20models.

CO\PO	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12
CO.1	3	3	3	-	-	-	-	-	-	-	-	-
CO.2	3	3	3	-	-	-	-	-	-	-	-	-
CO.3	3	3	3	-	-	-	-	-	-	-	-	-
CO.4	3	3	3	-	-	-	-	-	-	-	-	-
CO.5	3	3	3	-	-	-	-	-	-	-	-	-
CO*	**3**	**3**	**3**	**-**	**-**	**-**	**-**	**-**	**-**	**-**	**-**	-

Syllabus

unit-1 :-

Probability and Random Variable

* Probability through sets and Relative frequency :

Experiments and sample spaces, Discrete and continuous sample spaces, Events, probability definitions and Axioms, joint probability, conditional probability, total probability, Baye's theorem, Independent events.

* Random variables :

Definition of a random variable - conditions for a function to be a random variable - discrete and continuous random variables - Mixed Random variable.

* Distribution and Density functions and their properties:

Gaussian random variable - other distributions and density functions (Binomial, poisson, uniform, Exponential and Rayleigh).

unit-2 :-

operations on Random variable

* one Random variable - Expectation:

Expected value of a Random variable - Expected value of a function of a Random variable.

* Moments :

Moments about the origin, central moments, variance and skew functions that give moments : characteristic function Moment generating functions.

* Multiple Random variables :

Vector Random Variables, joint distribution and its properties : Joint distribution function - properties of Joint distribution - Marginal distribution Functions.

* Joint density and its properties:

Joint density function - properties of Joint density - Marginal density functions. conditional distribution and density - point conditioning. statistical Independence.

Unit - 3:-

Operations on Multiple Random variables

* Expected value of a function of Random variables:

Joint moments about the origin - Joint central moments. Joint characteristic functions.

* Jointly Gaussian Random variables:

Two Random variables case - properties of Gaussian random variables.

Unit - 4:-

Complex variable - Differentiation

Introduction to functions of complex variable - concept of Limit & Continuity - differentiation, cauchy - Riemann equations, analytic functions harmonic functions, finding harmonic conjugate - construction of analytic function by Milne Thomson Method.

Unit - 5:-

Complex variable - Integration

Line integral - cauchy's integral theorem (simple case), cauchy Integral formula, power series expansions: Taylor's series, Laurent's series, zeros of analytic functions, singularities. Residues, cauchy Residue theorem (without proof).

Probability and Random Variables

Galileo (1564-1642), an Italian Mathematician was the first to attempt at a quantitative measure of probability while dealing with some problems related to the theory of dice in gambling but the first foundation of the Mathematical theory of probability two was laid in the Mid 17^{th} century by two french Mathematicians B. pascal (1623-1662) and P. Fermat (1601-1665). while solving a no.of problems given by French gambler and Nobelman De-mere to pascal.

Note:-

1) Dice - 6 faces

2) cards - total 52

* Diamond - ◇ - 13 A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K

* Heart - ♡ - 13 colours $\longrightarrow$ 2 [Black & Red]

26 + 26

* spade - ♤ - 13

* clubs - ♧ - 13

Definitions:-

Probability:-

The chance of getting a success or failure is called a probability.

Random Experiment:-

If an experiment is conducted any no.of times under essentially identical conditions there is a set of all possible outcomes associated with it. If the result is not outcomes. The experiment is called Random experiment.

Ex:- i) Tossing a coin

ii) Throwning a card from a pack of 52 cards.

Sample space:-

The set of all possible outcomes is called sample space.

Ex:- i) when a coin is tossed, Sample space is

$$S = \{H, T\}$$

ii) when a dice is thrown, sample space is

$$S = \{1, 2, 3, 4, 5, 6\}$$

Discrete sample space:-

If a sample space contains finite set of elements then it is said to be discrete sample space.

continuous sample space:-

If a sample space contains uncountable infinite set of elements then it is said to be continuous sample space.

Trial:-

conducting an experiment once is called Trial.

outcome:-

The Result of a trial in a Random experiment is called a outcome.

Event:-

Every non-Empty subset of a sample space in a Random Experiment is called a event.

Simple event :-

An event in a trial that can not be further split is called a simple event.

Exhaustive Event:-

The total no. of possible outcomes of an experiment is known as Exhaustive event.

Favourable event :-

The no. of events which favour the happening of the event are known as favourable events.

Dependent event :-

Two events are said to be dependent if the happening of one event effect the happening of other is called dependent event.

Independent event :-

Two events are said to be independent if the happening of one event does not effect the happening of other is called Independent event.

classical definition of probability:-

In a Random experiment, if n is the no. of Exhaustive cases and m is the no. of favourable cases of an event A then the probability of event A is denoted by P(A) defined by

$$\boxed{P(A) = \frac{n(A)}{n(S)} = \frac{m}{n}}$$

* $0 \le P(A) \le 1$

1) Find the probability of getting a head in tossing a coin

A tossing a coin sample space $S = \{H, T\}$

$\therefore n(S) = 2$

Let A be the event of geeting a head

$A = \{H\}$

$\therefore n(A) = 1$

Therefore required probability

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{2} = 0.5$$

2) Find the probability of getting only one head in tossing two coins.

A tossing two coins, Sample space

$S = \{HH, HT, TH, TT\}$

$\therefore n(S) = 4$

Let A be the event of getting only one head

$A = \{HT, TH\}$

$n(A) = 2$

$\therefore$ Required probability

$$P(A) = \frac{n(A)}{n(S)} = \frac{2}{4} = \frac{1}{2} = 0.5$$

3) In a single thrown two dice, find the probability of throwing a sum

i) 10

ii) which is a perfect square

when two dice are thrown, sample space

$$S = \left\{ \begin{matrix} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\ (2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) \\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) \\ (4,1) & (4,2) & (4,3) & (4,4) & (4,5) & (4,6) \\ (5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) \\ (6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6) \end{matrix} \right\}$$

$\therefore n(S) = 6 \times 6 = 36$

i) The sum of the numbers $= 10$

Let A be the event of getting sum of the numbers $= 10$

$A = \{(4,6), (5,5) (6,4)\}$

$n(A) = 3$

$\therefore$ Required probability $p(A) = \dfrac{n(A)}{n(S)} = \dfrac{3}{36} = \dfrac{1}{12} = 0.083$

ii) The sum of the numbers is a perfect square

Let B be the event of getting sum of the numbers 4, 9 are

we know that perfect squares between 2 and 12 are 4 and 9 that is $4 = 2^2$, $9 = 3^2$.

$$B = \left\{ \begin{matrix} (1,3) (2,2) (3,1) \\ (3,6) (4,5) (5,4) (6,3) \end{matrix} \right\}$$

$n(B) = 7$

$\therefore$ Required probability $P(B) = \dfrac{n(B)}{n(S)} = \dfrac{7}{36} = 0.19$

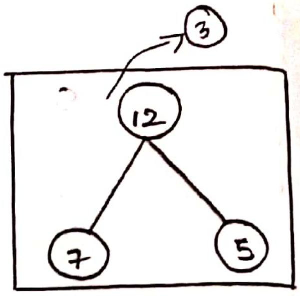
4) Three light bulbs are selected at random from 12 bulbs out of which 5 are defective. Find the Probability that i) all are defective ii) one is defective iii) two are defective

Scanned with CamScanner

Total no. of bulbs = 12.

No. of bulbs to be selected = 3.

$\therefore n(S) = {}^{12}C_3$



i) Let A be the event of getting all are defective bulbs

$n(A) = {}^{5}C_3$

$${}^{n}C_r = \frac{n!}{(n-r)!\,r!}$$

$\therefore$ Required probability

$${}^{n}C_n = {}^{n}C_0 = 1$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{{}^{12}C_3}{{}^{5}C_3}$$

$${}^{n}C_1 = n$$

$$P(A) = \frac{\frac{5!}{2! \times 3!}}{\frac{12!}{9! \times 3!}} = \frac{1}{22} = 0.045$$

ii) Let B be the event of getting one is defective

$$n(B) = {}^{5}C_1 \times {}^{7}C_2 = 5 \times {}^{7}C_2$$

$\therefore$ Required probability

$$P(B) = \frac{n(B)}{n(S)} = \frac{5 \times {}^{7}C_2}{{}^{12}C_3}$$

$$P(B) = \frac{5 \times \frac{7!}{5! \times 2!}}{\frac{12!}{9! \times 3!}} = \frac{21}{44} = 0.477$$

iii) Let C be the event of getting two are defective

$$n(C) = {}^{5}C_2 \times {}^{7}C_1 = {}^{5}C_2 \times 7$$

$\therefore$ Required probability $P(C) = \frac{n(C)}{n(S)} = \frac{{}^{5}C_2 \times 7}{{}^{12}C_3} = \frac{7}{22} = 0.318$

Scanned with CamScanner

5) Three cards are drawn from a pack of 52 cards find the probability that

i) three are spades

ii) 2 spade and 1 diamond

iii) one spade and 1 diamond and 1 heart

Total no. of cards = 52

No. of cards to be selected = 3

$\therefore n(S) = {}^{52}C_3$

i) Let A be the event of getting three are spades

$n(A) = {}^{13}C_3$

$\therefore$ Required probability

$$P(A) = \frac{n(A)}{n(S)} = \frac{{}^{13}C_3}{{}^{52}C_3} = \frac{11}{850} = 0.012$$

ii) Let B be the event of getting 2 spade and 1 diamond

$n(B) = {}^{13}C_2 \times {}^{13}C_1 = {}^{13}C_2 \times 13$

$\therefore$ Required probability

$$P(B) = \frac{n(B)}{n(S)} = \frac{{}^{13}C_2 \times 13}{{}^{52}C_3} = 0.045$$

iii) Let C be the event of getting 1 spade, 1 diamond and 1 heart.

$n(C) = {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1$

$\therefore$ Required probability

$$P(C) = \frac{n(C)}{n(S)} = \frac{{}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1}{{}^{52}C_3} = \frac{13^3}{{}^{52}C_3} = 0.099$$

Scanned with CamScanner

Axioms of probability:-

If A and B are any two events in a sample space,

i) $0 \leq P(A) \leq 1$

ii) $P(S) = 1$ (i.e) total probability $= 1$

iii) If A and B are mutually exclusive events then

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \quad \left(P(A \cap B) = P(\phi) = 0 \right)$$

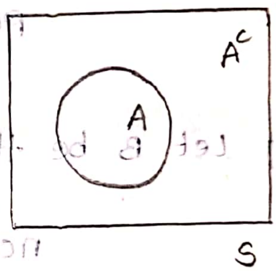
$$P(A \cup B) = P(A) + P(B)$$

iv) probability of complementary events i.e show that

$$P(A^c) = P(\bar{A}) = 1 - P(A)$$

$$S = A \cup A^c$$

$$P(S) = P(A \cup A^c)$$



$$1 = P(A) + P(A^c)$$

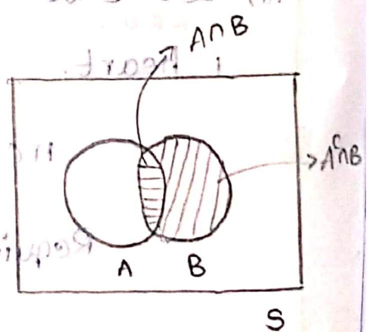
$$1 - P(A) = P(A^c)$$

$$\therefore \boxed{P(A^c) = 1 - P(A)}$$

v) For any two events A and B,

Prove that $P(A^c \cap B) = P(B) - P(A \cap B)$

Given A and B are any two event in a sample space 'S'.



we find that $A \cap B$ and $A^c \cap B$ are disjoint events.

$$P[(A \cap B) \cup (A^c \cap B)] = P(A \cap B) + P(A^c \cap B)$$

$$P(B) = P(A \cap B) + P(A^c \cap B)$$

$P(B) - P(A \cap B) = P(A^c \cap B)$

$$\therefore \boxed{P(A^c \cap B) = P(B) - P(A \cap B)}$$

vi) Addition theorem of probability :-

statement :-

For any two events A and B,

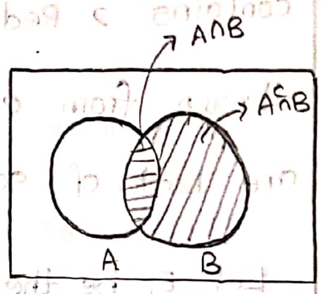
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Proof :-

Given

A and B are any two events in a sample space 's'.

we find that A and $A^c \cap B$ are mutually exclusive events. in sample space 's'.



$$P[A \cup (A^c \cap B)] = P(A) + P(A^c \cap B)$$

$$= P(A) + P(B) - P(A \cap B)$$

$$\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$

* vii) Addition theorem of three events :-

For any three event A, B and C,

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$$

Conditional probability :-

If A, B are any two events of an experiment then the conditional probability of occurance of A when event B is occured is given by $P(A/B) = \dfrac{P(A \cap B)}{P(B)}$, $P(B) > 0$.

Note :-

i) If A and B are independent events

$$P(A \cap B) = P(A) \cdot P(B)$$

then $$P(A/B) = \frac{P(A)\,\cancel{P(B)}}{\cancel{P(B)}} = P(A)$$

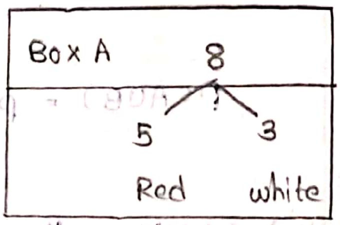
ii) If A and B are mutually exclusive events then P(A) is less than equal to B component

$$P(A) \le P(B^c)$$

iii) If A and B are independent events A^c and B^c are also independent events.

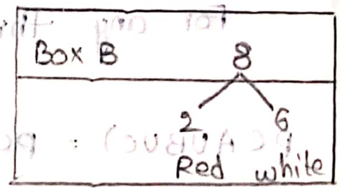
Box A contains 5 Red and 3 white marbles and box B contains 2 Red and 6 white marbles. If a marble is thrown from each box, what is the probability that they are both of same colour.

Let E_1 be the event of getting Red marble from box A



$$P(E_1) = \frac{1}{2} \times \frac{5c_1}{8c_1} = \frac{1}{2} \times \frac{5}{8} = \frac{5}{16}$$

Let E_2 be the event of getting Red marble from box B



$$P(E_2) = \frac{1}{2} \times \frac{2c_1}{8c_1} = \frac{1}{\cancel{2}} \times \frac{\cancel{2}}{8} = \frac{1}{8}$$

∴ The probability that both the marbles are red colour $P(E_1 \cap E_2) = P(E_1) \cdot P(E_2) = \frac{5}{16} \times \frac{1}{8} = 0.039 \longrightarrow ①$

Let E_3 be the event of getting white marble from box A.

$P(E_3) = \frac{1}{2} \times \frac{3_{C_1}}{8_{C_1}} = \frac{1}{2} \times \frac{3}{8} = \frac{3}{16}$

Let E_4 be the event of getting white marble from box B

$$P(E_4) = \frac{1}{2} \times \frac{6_{C_1}}{8_{C_1}} = \frac{1}{\cancel{2}} \times \frac{\cancel{6}^{3}}{8} = \frac{3}{8}$$

$\therefore$ The probability that both are white colour

$$P(E_3 \cap E_4) = P(E_3)\, P(E_4)$$

$$= \frac{3}{16} \times \frac{3}{8}$$

$$P(E_3 \cap E_4) = 0.070$$

$\therefore$ The probability that both are of same colour

$$P(E_1 \cap E_2) + P(E_3 \cap E_4) = 0.039 + 0.070 = 0.109$$

8) The probabilities that students A, B, C, D solve a problem on $\frac{1}{3}$, $\frac{2}{5}$, $\frac{1}{5}$ and $\frac{1}{4}$ respectively. If all of them try to solve the problem, what is the probability that the problem is solved.

The probability of solving the problem by a students A, B, C, D are $P(A) = \frac{1}{3}$ (from given data)

$$P(B) = \frac{2}{5}$$

$$P(C) = \frac{1}{5}$$

$$P(D) = \frac{1}{4}$$

The probability of not solving the problem by that students A, B, C, D are

$$P(\bar{A}) = 1 - P(A) = 1 - \frac{1}{3} = \frac{2}{3}$$

$P(\bar{B}) = 1 - P(B)$ $\quad$ $P(\bar{C}) = 1 - P(C)$ $\quad$ $P(\bar{D}) = 1 - P(D)$

$= 1 - \frac{2}{5}$ $\quad$ $= 1 - \frac{1}{5}$ $\quad$ $= 1 - \frac{1}{4}$

$= \frac{3}{5}$ $\quad$ $= \frac{4}{5}$ $\quad$ $= \frac{3}{4}$

The probability that the problem does not solved when A, B, C, D try together (independently)

$$P(\bar{A} \cap \bar{B} \cap \bar{C} \cap \bar{D}) = P(\bar{A}) \cdot P(\bar{B}) \cdot P(\bar{C}) \cdot P(\bar{D})$$

$$= \frac{2}{3} \times \frac{3}{5} \times \frac{4}{5} \times \frac{3}{4}$$

$$= \frac{6}{25}$$

Probability of solving the problem if A, B, C, D try independently $= 1 - P(\bar{A} \cap \bar{B} \cap \bar{C} \cap \bar{D})$

$$= 1 - \frac{6}{25}$$

$$= \frac{19}{25}$$

$$= 0.76$$

3) A can hit a Target 3 times in 5 shots, B can hit a target 2 times in 5 shots, C hits a target 3 times in 4 shots. Find the probability the target being hit when all of them try.

Let $P(A)$, $P(B)$ and $P(C)$ denote the probability hitting the target.

According to given data

$P(A) = \frac{3}{5}$ $\quad$ $P(B) = \frac{2}{5}$ $\quad$ and $\quad$ $P(C) = \frac{3}{4}$

Probability of not hitting the target by A, B, C

$P(\bar{A}) = 1 - P(A)$ $\qquad P(\bar{B}) = 1 - P(B)$ $\qquad P(\bar{C}) = 1 - P(C)$

$= 1 - \frac{3}{5}$ $\qquad = 1 - \frac{2}{5}$ $\qquad = 1 - \frac{3}{4}$

$= \frac{2}{5}$ $\qquad = \frac{3}{5}$ $\qquad = \frac{1}{4}$.

The probability of not hitting the target if A, B, C try together (independently)

$$P(\bar{A} \cap \bar{B} \cap \bar{C}) = P(\bar{A}) \cdot P(\bar{B}) \cdot P(\bar{C})$$

$$= \frac{\cancel{2}}{5} \times \frac{3}{5} \times \frac{1}{\cancel{4}_2}$$

$$= \frac{3}{50}$$

The probability of hitting the target $= 1 - P(\bar{A} \cap \bar{B} \cap \bar{C})$

$$= 1 - \frac{3}{50}$$

$$= \frac{47}{50}.$$

Baye's Theorem:-

Statement:-

Let $E_1, E_2, \ldots, E_n$ are n mutually exclusive and exhaustive events such that $P(E_i) > 0$ $(i = 1, 2, \ldots, n)$ in a sample space 'S' and A is any other event in S intersecting with every E_i such that $P(A) > 0$.

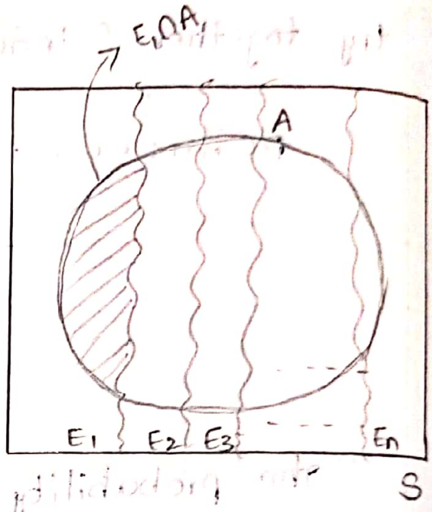
If E_i is any of the events of $E_1, E_2, \ldots, E_n$ then

$$\boxed{P(E_i/A) = \frac{P(E_i)\, P(A/E_i)}{\sum_{i=1}^{n} P(E_i)\, P(A/E_i)}}$$

Proof:-

Given $E_1, E_2, \ldots, E_n$ are mutually exclusive and exhaustive events in a sample space 'S'

Also given A is any non empty set which intersect each E_i which is shown in the figure.



we know that

$$S = E_1 \cup E_2 \cup \cdots \cup E_n$$

$$A = A \cap S = A \cap [E_1 \cup E_2 \cup \cdots \cup E_n]$$

$$A = (A \cap E_1) \cup (A \cap E_2) \cup \cdots \cup (A \cap E_n)$$

$$P(A) = P[(A \cap E_1) \cup (A \cap E_2) \cup \cdots \cup (A \cap E_n)]$$

$$P(A) = P(A \cap E_1) + P(A \cap E_2) + \cdots + P(A \cap E_n)$$

$$\therefore P(E_i/A) = \frac{P(E_i \cap A)}{P(A)}$$

$$P(A/B) = \frac{P(A \cap B)}{P(B)} \qquad P(B/A) = \frac{P(B \cap A)}{P(A)}$$

$$P(E_i/A) = \frac{P(E_i \cap A)}{P(A \cap E_1) + P(A \cap E_2) + \cdots + P(A \cap E_n)} \rightarrow ①$$

$$\therefore P(A/E_i) = \frac{P(A \cap E_i)}{P(E_i)}$$

$$P(A \cap E_i) = P(E_i)\, P(A/E_i)$$

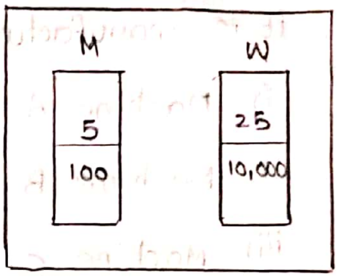
$$\therefore \text{eq } ① \Rightarrow P(E_i/A) = \frac{P(E_i)\, P(A/E_i)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2) + \cdots + P(E_n)P(A/E_n)}$$

$$P(E_i/A) = \frac{P(E_i)\, P(A/E_i)}{\sum_{i=1}^{n} P(E_i)\, P(A/E_i)}$$

1) Suppose 5 men out of 100 and 25 women out of 10,000 are colour blind person. is selected at random what is the probability of the person being male.

Given that

5 men out of 100 and 25 women out of 10,000 are colour blind.



A colour blind person is selected at random.

The probability that the selected person is male

$$P(M) = \frac{1}{2}$$

The probability that the selected person is female

$$P(W) = \frac{1}{2}$$

Let B represent a blind person $P(B/M) = \frac{5}{100}$ (from given data)

$$P(B/W) = \frac{25}{10000}$$

According to Baye's theorem, the probability that the selected person is male.

$$P(M/B) = \frac{P(M)\,P(B/M)}{P(M)\,P(B/M) + P(W)\,P(B/W)}$$

$$= \frac{\frac{1}{2}\left(\frac{5}{100}\right)}{\frac{1}{2}\left(\frac{5}{100}\right) + \frac{1}{2}\left(\frac{25}{10000}\right)}$$

$$= \frac{\left(\frac{5}{100}\right)}{\left(\frac{5}{100} + \frac{25}{10000}\right)}$$

$$P(M/B) = 0.047$$

2) In a bolt factory Machines A, B, C manufacture 20%, 30% and 50% of the total of their output and 6%, 3% and 2% are defective. A bolt is ~~thrown~~ drawn at random and found to be defective. find the probabilities that it is manufacture from

i) Machine A

ii) Machine B

iii) Machine C

Let $P(A)$, $P(B)$ and $P(C)$ denote the producing capacity of bolts from Machines A, B and C

A	B	C
20%	30%	50%
6%	3%	2%

From the given data

$P(A) = 20\% = 0.2$

$P(B) = 30\% = 0.3$

$P(C) = 50\% = 0.5$

Let D denote the defective bolt, From the given data

$P(D/A) = 6\% = 0.06$

$P(D/B) = 3\% = 0.03$

$P(D/C) = 2\% = 0.02$

i) probability of getting defective bolt from Machine A

$$P(A/D) = \frac{P(A)\,P(D/A)}{P(A)\,P(D/A) + P(B)\,P(D/B) + P(C)\,P(D/C)}$$

$$= \frac{0.2 \times 0.06}{0.2 \times 0.06 + 0.3 \times 0.03 + 0.5 \times 0.02}$$

$$= \frac{0.2 \times 0.06}{0.031}$$

$$= 0.387$$

ii) probability of getting defective bolt from Machine B

$$P(B/D) = \frac{P(B)\,P(D/B)}{P(B)\,P(D/B) + P(A)\,P(D/A) + P(C)\,P(D/C)}$$

$$= \frac{0.3 \times 0.03}{0.3 \times 0.03 + 0.2 \times 0.06 + 0.5 \times 0.02}$$

$$= \frac{0.009}{0.031}$$

$$= 0.290$$

iii) probability of getting defective bolt from Machine C

$$P(C/D) = \frac{P(C)\,P(D/C)}{P(A)\,P(D/A) + P(B)\,P(D/B) + P(C)\,P(D/C)}$$

$$= \frac{0.5 \times 0.02}{0.2 \times 0.06 + 0.3 \times 0.03 + 0.5 \times 0.02}$$

$$= \frac{0.01}{0.031}$$

$$= 0.322$$

3. A business man go to hotels x, y, z 20%, 50%, 30% of the time respectively. It is known that 5%, 4%, 8% of the rooms in x, y, z hotels have faculty plumbings what is the probability that business mans having faculty plumbing is assigned to hotel z.

Let the probabilities of business man going to hotels

$$= \frac{0.2 \times 0.06}{0.031}$$

$$= 0.387$$

ii) probability of getting defective bolt from Machine B

$$P(B/D) = \frac{P(B)\,P(D/B)}{P(B)\,P(D/B) + P(A)\,P(D/A) + P(C)\,P(D/C)}$$

$$= \frac{0.3 \times 0.03}{0.3 \times 0.03 + 0.2 \times 0.06 + 0.5 \times 0.02}$$

$$= \frac{0.009}{0.031}$$

$$= 0.290$$

iii) probability of getting defective bolt from Machine C

$$P(C/D) = \frac{P(C)\,P(D/C)}{P(A)\,P(D/A) + P(B)\,P(D/B) + P(C)\,P(D/C)}$$

$$= \frac{0.5 \times 0.02}{0.2 \times 0.06 + 0.3 \times 0.03 + 0.5 \times 0.02}$$

$$= \frac{0.01}{0.031}$$

$$= 0.322$$

3. A business man go to hotels x, y, z 20%, 50%, 30% of the time respectively. It is known that 5%, 4%, 8% of the rooms in x, y, z hotels have faculty plumbings what is the probability that business mans having faculty plumbing is assigned to hotel z.

Let the probabilities of business man going to hotels

X, Y, Z are denoted by $p(x)$, $p(y)$, $p(z)$ respectively.

Given data

$p(x) = 20\% = 0.2$ $\quad p(y) = 50\% = 0.5$ $\quad p(z) = 30\% = 0.3$

X	Y	Z
20%	50%	30%
5%	4%	8%

Let E denotes the event of faculty plumbing then

$P(E/x) = 5\% = 0.05$

$P(E/y) = 4\% = 0.04$

$P(E/z) = 8\% = 0.08$

∴ The probability that the business mans room having faculty plumbing is assigned to hotel z.

$$P(Z/E) = \frac{p(z)\,p(E/z)}{p(x)\,p(E/x) + p(y)\,p(E/y) + p(z)\,p(E/z)}$$

$$= \frac{0.3 \times 0.08}{0.2\times 0.05 + 0.5 \times 0.04 + 0.3 \times 0.08}$$

$$= 0.444$$

4. Company B_1, B_2, B_3 produce 30% and 45%, 25% of the cars respectively it is known that 2%, 3% and 2% of the cars produced from V_1, V_2 and V_3 are defective

a) what is the probability that a car purchased is defective.

b) If a car purchase is found to be defective what is the probability that this cars is produced by company B_c.

From the given data

$P(B_1) = 30\% = 0.3$

$P(B_2) = 45\% = 0.45$

$P(B_3) = 25\% = 0.25$

B_1	B_2	B_c
30%	45%	25%
2%	3%	2%

Let D denotes defective cars

$P(D/B_1) = 2\% = 0.02$

$P(D/B_2) = 3\% = 0.03$

$P(D/B_3) = 2\% = 0.02$

i) To find $P(D)$:-

$$P(D) = P(B_1)\,P(D/B_1) + P(B_2)\,P(D/B_2) + P(B_3)\,P(D/B_3)$$

$$= 0.3 \times 0.02 + 0.45 \times 0.03 + 0.25 \times 0.02$$

$$= 0.0245$$

ii) To find probability of $P(B_3/D)$:-

$$P(B_3/D) = \frac{P(B_3)\,P(D/B_3)}{P(B_1)\,P(D/B_1) + P(B_2)\,P(D/B_2) + P(B_3)\,P(D/B_3)}$$

$$= \frac{0.25 \times 0.02}{0.3 \times 0.02 + 0.45 \times 0.03 + 0.25 \times 0.02}$$

$$= \frac{0.25 \times 0.02}{0.0245} = 0.204$$

5. Suppose 3 companies x, y, z produce TV's. x produce twice as many as y while y and z produce the same number. It is known that 2% of x, 2% of y, 4% of z are defective all the TV's produced and kept in one shape then one tv is selected at random

a) what is the probability that the TV is defective

b) suppose a tv is selected it is defective what is the probability that this TV is produced by company?

Let $p(x)$, $p(y)$ & $p(z)$ denotes the producing capacity of TV's by the companies x, y, z

From the given data

TV's	X	Y	Z
X	2%	2%	4%

$$p(x) = 2p(y) \; ; \; p(y) = p(z)$$

we know that

Total probability = 1

$$p(x) + p(y) + p(z) = 1$$

$$2p(y) + p(y) + p(y) = 1$$

$$4p(y) = 1$$

$$p(y) = \frac{1}{4} = 0.25$$

$$p(x) = 2p(y) = 2\left(\frac{1}{4}\right) = \frac{1}{2} = 0.5$$

$$p(z) = p(y) = \frac{1}{4} = 0.25$$

Let D denotes the defective TV's

$$p(D/x) = 2\% = 0.02$$

$$p(D/y) = 2\% = 0.02$$

$$p(D/z) = 4\% = 0.04$$

a) To find $p(D)$:-

$$p(D) = p(x)\,p(D/x) + p(y)\,p(D/y) + p(z)\,p(D/z)$$

$$= 0.5 \times 0.02 + 0.25 \times 0.02 + 0.25 \times 0.04$$

$$= 0.025$$

b) To find $P(X/D)$:-

$$P(X/D) = \frac{P(X)\,P(D/X)}{P(X)\,P(D/X) + P(Y)\,P(D/Y) + P(Z)\,P(D/Z)}$$

$$= \frac{0.5 \times 0.02}{0.025}$$

$$= 0.4$$

Random variables:-

A Random variable X can be consider to be a function that maps all events to the sample space into points on the real axes, that is

$$X : S \longrightarrow R.$$

Ex:- An experiment consist of tossing two coins, let the Random variable be a function X. Such that no. of heads shown so X maps the real numbers of the events showing no head as zero, the event any one head as one and both heads as two 1.

$\therefore$ Random variable $X = \{0, 1, 2\}$

i.e

$X = \{x_1, x_2, x_3\}$

Types of Random variables:-

i) Discrete Random variables (D.R.V):-

A discrete random variable is one having only discrete values.

Ex:- consider a discrete sample space S

$$S = \{1, 2, 3, 4\}$$

we define random variable $X = S^2 = \{1, 4, 9, 16\}$

ii) Continuous Random variable (C.R.V):-

A continuous random variable is one having a continuous range of values.

Ex:- A temperature of some region is a continous random variable T it is always exists $-T_1$ to T_1 in the range.

iii) Mixed random variable (M.R.V):-

A Mixed random variable is one for which some of its values are discrete and same are discrete.

probability distribution function:-

A concept of PDF describes the probabilistic behaviour of a Random variable. It defines the probability $P\{X \leq x\}$ of the event $\{X \leq x\}$ for all values of random variable X upto the value x is also called cumulative probability distribution function of the random variable X. It is denoted by $\boxed{F_X(x) = P\{X \leq x\}}$

Ex:- Tossing 3 coins, X is a discrete random variable of the event showing the no.of tails.

$X = \{0,1,2,3\}$

Favourable outcomes of x_1 is $\{HHH\} = 1$ $\quad 2^3$

$P(x_1) = \frac{1}{8}$ $\quad \left\{ \begin{matrix} (HHH) & (HHT) & (HTH) & (THH) \\ (TTT) & (TTH) & (THT) & (HTT) \end{matrix} \right\}$

$P(x_2) = \frac{3}{8}$ (Getting one tail)

$P(x_3) = \frac{3}{8}$ (Getting two tails)

$P(x_4) = \frac{1}{8}$ (Getting three tails)

$\therefore$ The distribution function is given as,

$F_X(x_1) = P\{X \le x_1\} = P(x_1) = \frac{1}{8}$

$F_X(x_2) = P\{X \le x_2\} = P(x_1) + P(x_2) = \frac{1}{8} + \frac{3}{8} = \frac{4}{8}$

$F_X(x_3) = P\{X \le x_3\} = P(x_1) + P(x_2) + P(x_3)$

$= \frac{1}{8} + \frac{3}{8} + \frac{3}{8}$

$F_X(x_3) = \frac{7}{8}$

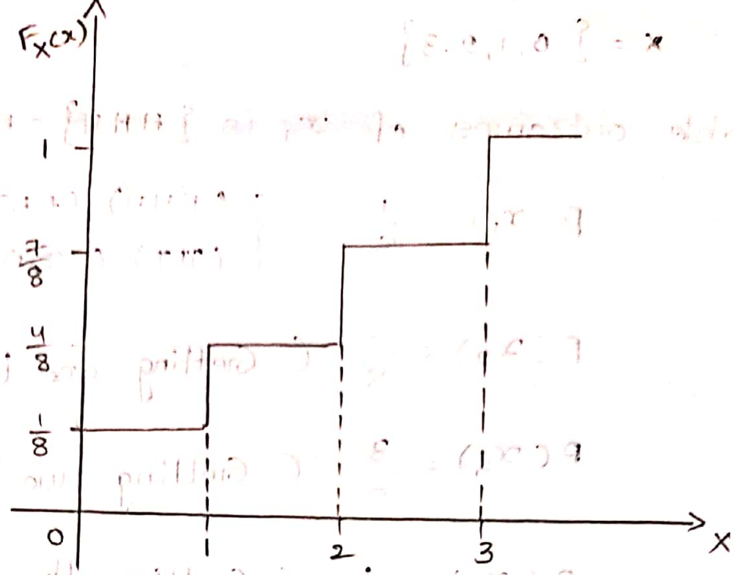
$F_X(x_4) = P\{X \le x_4\} = P(x_1) + P(x_2) + P(x_3) + P(x_4)$

$= \frac{1}{8} + \frac{3}{8} + \frac{3}{8} + \frac{1}{8}$

$= \frac{8}{8}$

$\therefore F_X(x_4) = P\{X \le x_4\} = 1$

X	0	1	2	3
$P(x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$
$F_X(x)$	$\frac{1}{8}$	$\frac{4}{8}$	$\frac{7}{8}$	1



Properties of distribution function :-

i) $F_X(-\infty) = 0$

ii) $F_X(\infty) = 1$

iii) $0 \le F_X(x) \le 1$

iv) $F_X(x_1) \le F_X(x_2)$ if $x_1 < x_2$

v) $P\{x_1 \le X \le x_2\} = F_X(x_2) - F_X(x_1)$

Expression for distribution function :-

Discrete Random variable:-

If X is a discrete random variable, the distribution function $F_X(x)$ is defined by

$$F_X(x) = \sum_{i=1}^{N} p(x_i)\, u(x - x_i)$$

where $u(x) = \begin{cases} 1 & x \ge 0 \\ 0 & x < 0 \end{cases}$

continuous Random Variable:-

If X is a continuous random variable, the distribution function $F_X(x)$ is defined by

$$F_X(x) = \int_{-\infty}^{x} f_X(x)\,dx$$

where $f_X(x)$ is the probability density function

probability density function :-

The probability density function of the random variable X is defined as the values of probabilites that a given value of x. It is a derivative of the distribution function $F_X(x)$ the probability density function is denoted by $f_X(x)$.

$$f_X(x) = \frac{d}{dx}\{F_X(x)\}$$

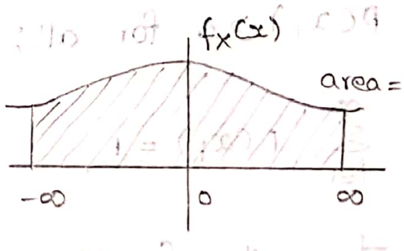
where x is a real number $-\infty \leq x \leq \infty$

we also called $f_X(x)$ simply as density function of x.

properties of probability density function :-

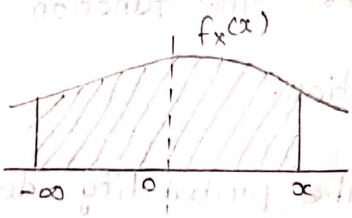
If $f_X(x)$ is a p.d.f of a random variable X then

* $0 \leq f_X(x)$ for all x

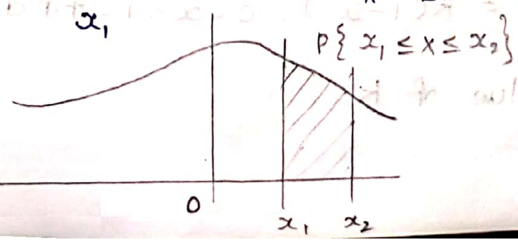


* $\int_{-\infty}^{\infty} f_X(x)\,dx = 1$

* $F_X(x) = \int_{-\infty}^{x} f_X(x)\,dx$



* $P\{x_1 \leq X \leq x_2\} = \int_{x_1}^{x_2} f_X(x)\,dx = F_X(x_2) - F_X(x_1)$



Discrete Random variable:-

The density function for discrete random variable is given by

$$f_X(x) = \sum_{i=1}^{n} p(x_i)\,\delta(x-x_i)$$

continuous Random variable:-

for continuous random variable density function is given by

$$f_X(x) = \frac{d}{dx} F_X(x) \quad , \quad -\infty < x < \infty$$

probability Mass function:-

consider a discrete random variable x in a Sample space with infinite no. of possible outcomes

i.e $\quad x = \{x_1, x_2, \dots, x_n, \dots \infty\}$

If the probability of x, $p(x_i)$, $i = 1, 2, \dots \infty$ Satisfy the following conditions.

i) $p(x_i) \geq 0$ for all;

ii) $\sum_{i=1}^{\infty} p(x_i) = 1$

Then the function $p(x)$ is called probability Mass function.

1) If the probability density of a random variable is given by $F_X(x) = k(1-x^2)$, $0 < x < 1$. find

i) The value of k

ii) $F_X(x)$

Given p.d.f is $f_X(x) = k(1-x^2), \ 0 \leq x < 1$

i) To find 'k' value:-

From the properties of p.d.f,

$$\int_{-\infty}^{\infty} f_X(x)\, dx = 1$$

$$\int_{0}^{1} k(1-x^2)\, dx = 1$$

$$k\int_{0}^{1} (1-x^2)\, dx = 1$$

$$k\left[x - \frac{x^3}{3}\right]_0^1 = 1$$

$$k\left[\left(1-\frac{1^3}{3}\right) - \left(0-\frac{0^3}{3}\right)\right] = 1$$

$$k\left[\frac{2}{3}\right] = 1$$

$$\boxed{k = \frac{3}{2}}$$

ii) To find $F_X(x)$:-

we know that $F_X(x) = \int_{-\infty}^{x} f_X(x)\, dx$

$$F_X(x) = \int_{0}^{x} k(1-x^2)\, dx$$

$$= \frac{3}{2}\int_{0}^{x} (1+x^2)\, dx$$

$$= \frac{3}{2}\left[x - \frac{x^3}{3}\right]_0^x$$

$$= \frac{3}{2}\left[\left(x - \frac{x^3}{3}\right) - (0-0)\right]$$

$$F_X(x) = \frac{3}{2}\left[x - \frac{x^3}{3}\right], \quad 0 < x < 1$$

a) Find the value of constant 'k' such that effects of

$$f_X(x) = \begin{cases} k\, x^2(1-x^3), & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

is a proper density of a continuous random variables.

Given

$$f_X(x) = \begin{cases} k\, x^2(1-x^3), & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

From the properties of probability density function

$$\int_{-\infty}^{\infty} f_X(x)\, dx = 1$$

$$\int_0^1 k\, x^2(1-x^3)\, dx = 1$$

$$k \int_0^1 (x^2 - x^5)\, dx = 1$$

$$k \left[\frac{x^3}{3} - \frac{x^6}{6}\right]_0^1 = 1$$

$$k \left[\left(\frac{1}{3} - \frac{1}{6}\right) - (0-0)\right] = 1$$

$$\frac{k}{3}\left[1 - \frac{1}{2}\right] = 1$$

$$\frac{k}{3}\left[\frac{1}{2}\right] = 1$$

$$\boxed{k = 6}$$

Find a Constant $b>0$ so that the function $f_X(x)$.

$$f_X(x) = \begin{cases} \frac{1}{10} e^{3x}, & 0 \le x \le b \\ 0, & \text{otherwise} \end{cases}$$

is a valid probability density function.

Given

$$f_X(x) = \begin{cases} \frac{1}{10} e^{3x}, & 0 \le x \le b \\ 0, & \text{otherwise} \end{cases}$$

From the properties of p.d.f

$$\int_{-\infty}^{\infty} f_X(x)\, dx = 1$$

$$\int_0^b \frac{1}{10} e^{3x}\, dx = 1$$

$$\frac{1}{10} \int_0^b e^{3x}\, dx = 1$$

$$\frac{1}{10} \left[\frac{e^{3x}}{3} \right]_0^b = 1$$

$$\frac{1}{10} \left[\frac{e^{3b}}{3} - e^0 \right] = 1$$

$$\frac{1}{30} \left[e^{3b} - 1 \right] = 1$$

$$e^{3b} - 1 = 30$$

$$e^{3b} = 30 + 1$$

$$e^{3b} = 31$$

Take 'log' on both sides

$$\log (e^{3b}) = \log 31$$

$$3b \log_e e = \log 31$$

$$3b = \log 31$$

$$b = \frac{1}{3} \log 31$$

$$\cancel{b = 2.904 \approx 3}$$

$$b = 1.144$$

4. Random variable x has the discrete variable in the set $\{-1, -0.5, 0.7, 1.5, 3\}$ the corresponding probabilites are $\{0.1, 0.2, 0.1, 0.4, 0.2\}$. plot it's distribution function.

From the given data

x	-1	-0.5	0.7	1.5	3
P(x)	0.1	0.2	0.1	0.4	0.2

From the definition of probability distribution function

$$F_X(x) = p\{X \le x\}$$

$$F_X(-1) = p\{X \le -1\} = 0.1$$

$$F_X(-0.5) = p\{X \le -0.5\} = 0.1 + 0.2 = 0.3$$

$$F_X(0.7) = p\{X \le 0.7\} = 0.1 + 0.2 + 0.1 = 0.4$$

$$F_X(1.5) = p\{X \le 1.5\} = 0.1 + 0.2 + 0.1 + 0.4 = 0.8$$

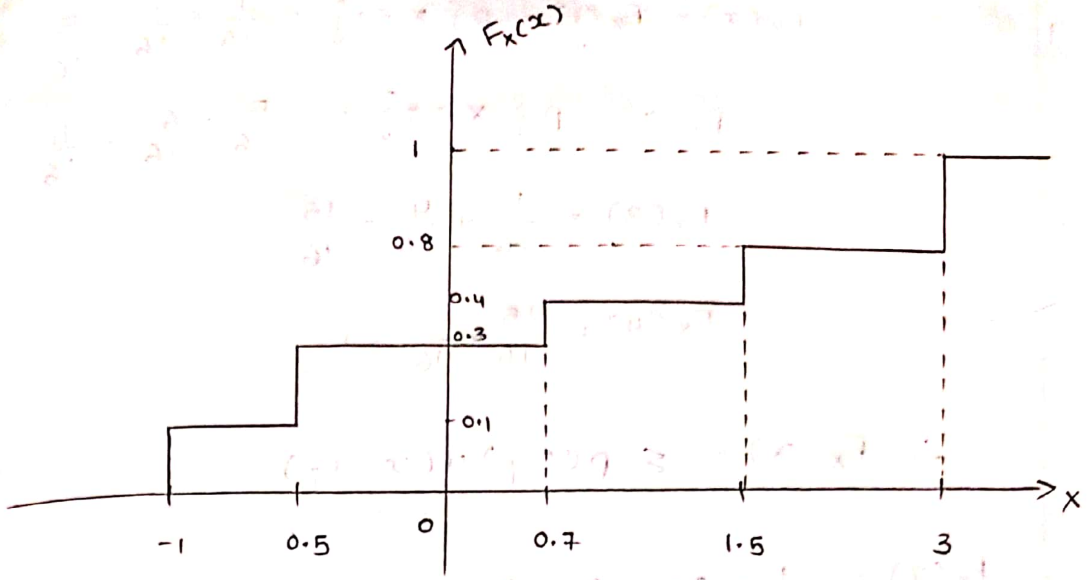
$$F_X(3) = p\{X \le 3\} = 0.1 + 0.2 + 0.1 + 0.4 + 0.2 = 1$$

The probability distribution function can also expressed as

$$F_X(x) = \sum_{i=1}^{n} p(x_i)\, u(x - x_i)$$

$$\therefore\ F_X(x) = (0.1)\,u(x+1) + (0.2)\,u(x+0.5) + (0.1)\,u(x-0.7)$$
$$+ (0.4)\,u(x-1.5) + (0.2)\,u(x-3)$$

Scanned with CamScanner



5. consider the experiment of tossing 4 coins the random variable x is associated with no.of tails, find and sketch the cumulative distribution function of x.

consider the experiment of tossing 4 coins, sample space

$$S = \begin{cases} (HHHH) \\ (HHHT), (HHTH), (HTHH), (THHH) \\ (HHTT), (TTHH), (HTTH), (THHT), (HTHT), (THTH) \\ (HTTT), (THTT), (TTHT), (TTTH) \\ (TTTT) \end{cases}$$

Let x be a discrete random variable show the no.of tails.

$$X = \{0, 1, 1, 1, 1, 2, 2, 2, 2, 2, 2, 3, 3, 3, 3, 4\}$$

x	0	1	2	3	4
$p(x)$	$\frac{1}{16}$	$\frac{4}{16}$	$\frac{6}{16}$	$\frac{4}{16}$	$\frac{1}{16}$

From the definition of PDF

$$F_X(x) = p\{X \le x\}$$

$$F_X(0) = p\{X \le 0\} = \frac{1}{16}$$

$$F_X(x) = F_X(1) = P\{X \le 1\} = \frac{1}{16} + \frac{4}{16} = \frac{5}{16}$$

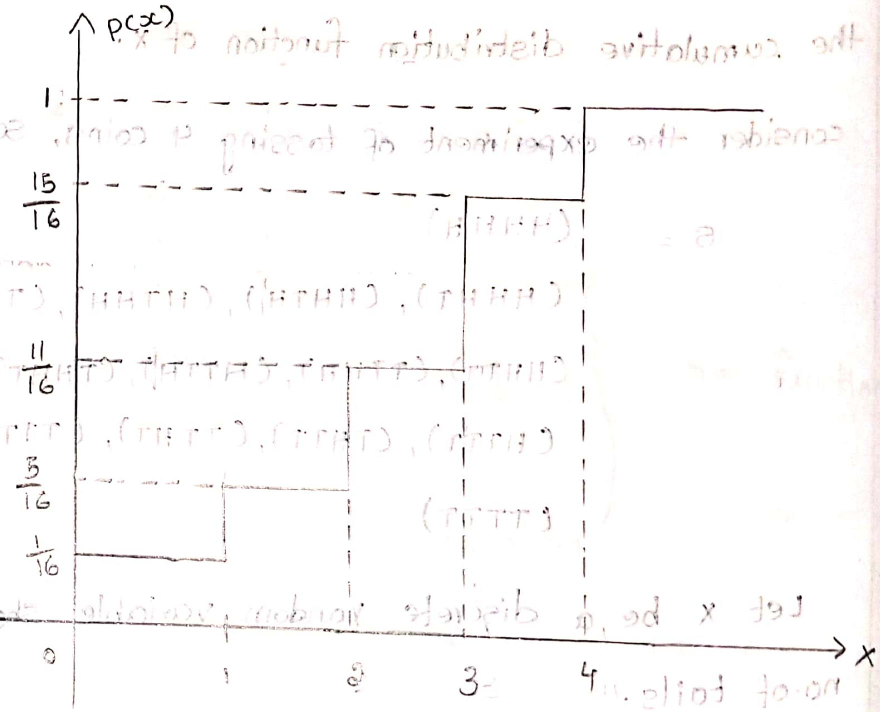
$$F_X(2) = P\{X \le 2\} = \frac{5}{16} + \frac{6}{16} = \frac{11}{16}$$

$$F_X(3) = \frac{11}{16} + \frac{4}{16} = \frac{15}{16}$$

$$F_X(4) = \frac{15}{16} + \frac{1}{16} = 1$$

$$\therefore F_X(x) = \Sigma\, p(x_i)\, u(x - x_i)$$

$$F_X(x) = \frac{1}{16}u(x-0) + \frac{4}{16}u(x-1) + \frac{6}{16}u(x-2) + \frac{4}{16}u(x-3) + \frac{1}{16}u(x-4)$$



6. The probability Mass function of x is given the table

X	0	1	2	3	4	5	6
P(x)	k	3k	5k	7k	9k	11k	13k

Find i) the value of k

ii) $P(X < 4)$

iii) $P(X \ge 5)$

iv) $P(3 < X \leq 6)$

v) The minimum value of k such that $P(X \leq 2) > 0.3$.

) To find 'k' value:-

From the properties of probability Mass function,

$$\sum_{i=1}^{n} P(x_i) = 1$$

$$k + 3k + 5k + 7k + 9k + 11k + 13k = 1$$

$$49k = 1$$

$$k = \frac{1}{49}$$

ii) To find $P(X<4)$:-

X	0	1	2	3	4	5	6
$P(x)$	$\frac{1}{49}$	$\frac{3}{49}$	$\frac{5}{49}$	$\frac{7}{49}$	$\frac{9}{49}$	$\frac{11}{49}$	$\frac{13}{49}$
$F_X(x)$	$\frac{1}{49}$	$\frac{4}{49}$	$\frac{9}{49}$	$\frac{16}{49}$	$\frac{25}{49}$	$\frac{36}{49}$	1

$$P(X<4) = P(X \leq 3) = F_X(3) = \frac{16}{49} \simeq 0.326$$

iii) To find $P(X \geq 5)$:-

$$P(X \geq 5) = 1 - P(X<4)$$

$$= 1 - F_X(4)$$

$$= 1 - \frac{25}{49}$$

$$= 0.48$$

iv) To find $P(3 < X \leq 6)$:-

$$P(3<X\leq 6) = F_X(6) - F_X(3)$$

$$= 1 - \frac{16}{49} = 0.673$$

Scanned with CamScanner

v) To find 'K' such that $P(X \leq 2) > 0.3$:-

$$P(X \leq 2) > 0.3$$

$$F_X(2) > 0.3$$

$$(K + 3K + 5K) > 0.3$$

$$9K > 0.3$$

$$K > \frac{0.3}{9}$$

$$K > \frac{3}{90}$$

$$K > 0.033$$

7. consider the probability density function $f(x) = ae^{-b|x|}$ where x is a Random variable values range from $-\infty < x < \infty$. Find

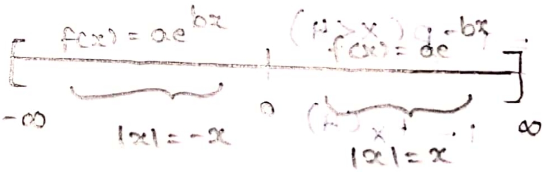
i) CDF (i.e) $F_X(x)$

ii) Relation between a and b

iii) The probability that the outcome x lies between 1 and 2

Given

$$f(x) = ae^{-b|x|}, \quad -\infty < x < \infty$$



$$f(x) = \begin{cases} ae^{bx}, & -\infty < x < 0 \\ ae^{-bx}, & 0 < x \leq \infty \end{cases}$$

i) To find $F_X(x)$:-

we know that

$$F_X(x) = \int_{-\infty}^{x} f_X(x)\, dx$$

$$= \int_{-\infty}^{0} f_X(x)\,dx + \int_{0}^{x} f_X(x)\,dx$$

$$= \int_{-\infty}^{0} ae^{bx}\,dx + \int_{0}^{x} ae^{-bx}\,dx$$

$$= a\left[\left(\frac{e^{bx}}{b}\right)_{-\infty}^{0} + \left(\frac{e^{-bx}}{-b}\right)_{0}^{x}\right]$$

$$= \frac{a}{b}\left[\{1-0\} - \{e^{-bx}-1\}\right]$$

$$F_X(x) = \frac{a}{b}\left[2 - e^{-bx}\right]$$

ii) Relation between a and b:-

we know that

$$F_X(\infty) = 1$$

$$\frac{a}{b}\left[2 - e^{-\infty}\right] = 1$$

$$\frac{a}{b}\left[2-0\right] = 1$$

$$2\frac{a}{b} = 1$$

$$\boxed{2a = b}$$

iii) To find $p(1 \le x \le 2)$:-

$$p(1 \le x \le 2) = F_X(2) - F_X(1)$$

$$= \frac{a}{b}\left[2 - e^{-2b}\right] - \frac{a}{b}\left[2 - e^{-b}\right]$$

$$= \frac{a}{b}\left[\cancel{2} - e^{-2b} - \cancel{2} + e^{-b}\right]$$

$$= \frac{a}{b}\left[e^{-b} - e^{-2b}\right] = \frac{1}{2}\left[e^{-b} - e^{-2b}\right]$$

Scanned with CamScanner

8. If the probability density of a random variable is given by

$$f(x) = \begin{cases} x & , 0 < x < 1 \\ 2-x & , 1 < x < 2 \end{cases}$$

Find the probabilities that random variable having this probability density will take on a value

i) between 0.2 and 0.8

ii) between 0.6 and 1.2

Given

probability density function is

$$f(x) = \begin{cases} x & , 0 < x < 1 \\ 2-x & , 1 < x < 2 \end{cases}$$

i) To find $p(0.2 \leq x \leq 0.8)$:-

$$P(0.2 \leq x \leq 0.8) = \int_{0.2}^{0.8} f_x(x)\, dx$$

$$= \int_{0.2}^{0.8} x\, dx$$

$$= \left[\frac{x^2}{2}\right]_{0.2}^{0.8}$$

$$= \frac{1}{2}\left[(0.8)^2 - (0.2)^2\right]$$

$$= \frac{1}{2}\left[0.64 - 0.04\right]$$

$$= [0.6]\frac{1}{2}$$

$$= 0.3$$

Scanned with CamScanner

ii) To find $p(0.6 \leq x \leq 1.2)$:-

$$p(0.6 \leq x \leq 1.2) = \int_{0.6}^{1.2} f_x(x)\, dx$$

$$= \int_{0.6}^{1} x\, dx + \int_{1}^{1.2} 2-x\, dx$$

$$= \left[\frac{x^2}{2}\right]_{0.6}^{1} + \left[2x - \frac{x^2}{2}\right]_{1}^{1.2}$$

$$= \frac{1}{2}\left[-(0.6)^2 + 1\right] + \left[\left(2(1.2) - \frac{(1.2)^2}{2}\right) - \left(2 - \frac{1}{2}\right)\right]$$

$$= 0.41$$

9. If the probability density of a random variable is given by

$$f_x(x) = \begin{cases} c\exp(-\frac{x}{4}), & 0 \leq x < 1 \\ 0 & \text{otherwise} \end{cases}$$

Find, the value that 'c' must have and also find $F_x(0.5)$

Given

$$f_x(x) = \begin{cases} c e^{-\frac{x}{4}}, & 0 \leq x < 1 \\ 0, & \text{otherwise} \end{cases}$$

i) To find 'c' value :-

From the properties of probability density function

$$\int_{-\infty}^{\infty} f_x(x)\, dx = 1$$

$$\int_{0}^{1} c e^{-\frac{x}{4}}\, dx = 1$$

$$c\int_0^1 e^{-\frac{x}{4}}\,dx = 1$$

$$c\left[\frac{e^{-\frac{x}{4}}}{-\frac{1}{4}}\right]_0^1 = 1$$

$$\div\ 4c\left[e^{-\frac{1}{4}}-1\right] = 1$$

$$c = \frac{1}{4\left[1-e^{-\frac{1}{4}}\right]}$$

$$c = 1.13$$

ii) To find $F_X(0.5)$:-

$$F_X(0.5) = \int_{-\infty}^{0.5} f_X(x)\,dx$$

$$= \int_0^{0.5} c e^{-\frac{x}{4}}\,dx$$

$$= 1.13\int_0^{0.5} e^{-\frac{x}{4}}\,dx$$

$$= 1.13\left[\frac{e^{-\frac{x}{4}}}{-\frac{1}{4}}\right]_0^{0.5}$$

$$= -4(1.13)\left[e^{-\frac{x}{4}}\right]_0^{0.5}$$

$$= -4(1.13)\left[e^{-\frac{0.5}{4}}-1\right]$$

$$= 0.53$$

10. A random variable x has the distribution function $F_X(x) = \sum_{n=1}^{12} \frac{n^2}{650} u(x-n)$ find the probability

a) $p\{-\infty < x \leq 6.5\}$

b) $p\{x > 4\}$

c) $p\{6 < x \leq 9$

Given

the probability Distribution function is

$$F_X(x) = \sum_{n=1}^{12} \frac{n^2}{650} u(x-n)$$

i.e

$$F_X(x) = \sum_{n=1}^{12} \frac{n^2}{650}$$

a) To find $p\{-\infty < x \leq 6.5\}$:-

$$p\{-\infty < x \leq 6.5\} = F_X(6.5)$$

$$= F_X(6)$$

$$= \sum_{n=1}^{6} \frac{n^2}{650}$$

$$= \frac{1}{650} \sum_{n=1}^{6} n^2$$

$$= \frac{1}{650}\left[\frac{n(n+1)(2n+1)}{6}\right]_{n=6}$$

$$= \frac{1}{650}\left[\frac{\cancel{6}(7)(13)}{\cancel{6}}\right]$$

$$= 0.14$$

b) To find $p\{x > 4\}$:-

$$p\{x > 4\} = 1 - p\{x \leq 3\}$$

$$= 1 - F_X(3)$$

$$= 1 - \sum_{n=1}^{3} \frac{n^2}{650}$$

$$= 1 - \frac{1}{650}\left[1^2+2^2+3^2\right]$$

$$= 1 - \frac{1}{650}\left[1+4+9\right]$$

$$= 1 - \frac{14}{650}$$

$$= 0.978$$

c) To find $p\{6 < x \leq 9\}$:-

$$p\{6 < x \leq 9\} = F_X(9) - F_X(6)$$

$$= \sum_{n=1}^{9} \frac{n^2}{650} - \sum_{n=1}^{6} \frac{n^2}{650}$$

$$= \frac{1}{650}\left[\sum_{n=1}^{9} n^2 - \sum_{n=1}^{6} n^2\right]$$

$$= \frac{1}{650}\left[\left(\frac{n(n+1)(2n+1)}{6}\right)_{n=9} - \left(\frac{n(n+1)(2n+1)}{6}\right)_{n=6}\right]$$

$$= \frac{1}{650}\left[\left(\frac{9(10)(19)}{6}\right) - \left(\frac{\cancel{6}(7)(13)}{\cancel{6}}\right)\right]$$

$$= \frac{1}{650}\left[285 - 91\right]$$

$$= \frac{194}{650} = 0.298$$

Examples of distribution and density functions:-

1) The Gaussian density function (or) Normal density function:-

The Gaussian density function for a random variable x is given by

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma_X^2}} e^{-\frac{(x-\mu_x)^2}{2\sigma_X^2}} \quad \text{for all } x$$

and $F_X(x) = \frac{1}{\sqrt{2\pi\sigma_X^2}} \int_{-\infty}^{x} e^{-\frac{(x-\mu_x)^2}{2\sigma_X^2}} dx$ for all x

2) uniform density function:-

The uniform probability density function is defined as

$$f_X(x) = \begin{cases} \frac{1}{b-a} & , a \le x \le b \\ 0 & ; \text{otherwise} \end{cases}$$

The uniform distribution function is given by

$$F_X(x) = \frac{x-a}{b-a}$$

$$F_X(a) = 0 \quad , \quad F_X(b) = 1$$

3) Exponential probability density function:-

The e.p.d.f for a contincious random variable X is defined as

$$f_X(x) = \begin{cases} \frac{1}{b} e^{-\frac{(x-a)}{b}} & , x > a \\ 0 & , x < a \end{cases}$$

where a & b are real constants $-\infty < a < \infty$ and $b > 0$.

probability distribution function is given by

$$F_X(x) = \begin{cases} 0 & , x < a \\ 1 - e^{-\frac{(x-a)}{b}} & , x > a \\ 1 & x = \infty \end{cases}$$

4) Rayleigh probability density function:-

The Rayleigh probability density function of a random variable X is defined as

$$f_X(x) = \begin{cases} \frac{2}{b}(x-a) e^{-\frac{(x-a)^2}{b}} & , x \ge a \\ 0 & , x < a \end{cases}$$

where a & b are real constants then $-\infty < a < \infty$ & $b > 0$

probability distribution function is given by,

$$F_X(x) = \begin{cases} 0 & , x < a \\ 1-e^{-\frac{(x-a)^2}{b}} & , x \geq a \\ 1 & , x = \infty \end{cases}$$

5) Binomial probability density function:-

Consider an experiment having only two possible outcomes per trial, such as one or zero, yes or No, tail or head etc

If the experiment is repeated for 'n' trials then the Binomial probability density function, is given by

$$f_X(x) = \sum_{k=0}^{n} {}^{n}C_k \, p^k (1-p)^{n-k} \delta(x-k)$$

And the Binomial distribution function is

$$F_X(x) = \sum_{k=0}^{n} {}^{n}C_k \, p^k (1-p)^{n-k} u(x-k)$$

6) poisson probability density function:-

The poisson probability density function for a discrete random variable X is given by

$$f_X(x) = e^{-b} \sum_{k=0}^{\infty} \frac{b^k}{k!} \delta(x-k)$$

where $b>0$

And the distribution function is given by

$$F_X(x) = e^{-b} \sum_{k=0}^{\infty} \frac{b^k}{k!} u(x-k)$$