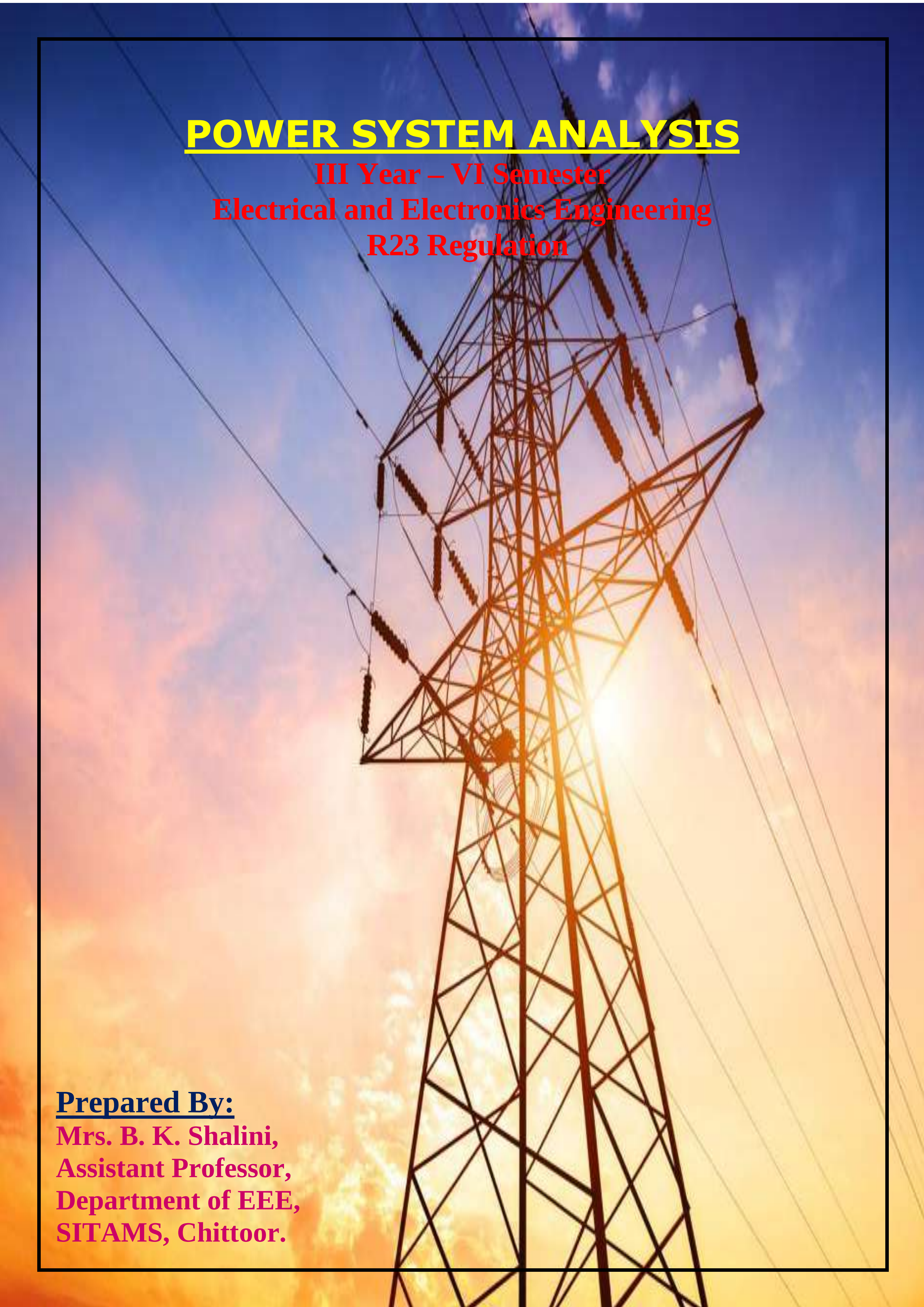

<u>POWER SYSTEM ANALYSIS</u>

III Year – VI Semester

Electrical and Electronics Engineering

R23 Regulation

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SYLLABUS

UNIT-1: PER-UNIT SYSTEM AND YBUS FORMATION:

Per-Unit representation of Power system elements - Per-Unit equivalent reactance network of a three phase Power System - Graph Theory: Definitions, Bus Incidence Matrix, Y Bus formation by Direct and Singular Transformation Methods, Numerical Problems.

UNIT -2: FORMATION OF ZBUS:

Formation of Z Bus: Partial network, Algorithm for the Modification of ZBus Matrix for addition element for the following cases: Addition of element from a new bus to reference, Addition of element from a new bus to an old bus, Addition of element between an old bus to reference and Addition of element between two old busses - Modification of Z Bus for the changes in network.

UNIT-3: POWER FLOW ANALYSIS:

Static load flow equations – Load flow solutions using Gauss Seidel Method: Algorithm and Flowchart. Acceleration Factor, Load flow Solution for Simple Power Systems (Max. 3-Buses): Newton Raphson Method in Polar Co-Ordinates Form: Load Flow Solution- Jacobian Elements, Algorithm and Flowchart. Decoupled and Fast Decoupled Methods. - Comparison of Different Methods

UNIT-4: SHORT CIRCUIT STUDIES:

Short Circuit Current and MVA Calculations, Fault levels, Application of Series Reactors. Symmetrical Component Theory: Positive, Negative and Zero sequence components, Positive, Negative and Zero sequence Networks. Symmetrical Fault Analysis: LLLG faults with and without fault impedance, Unsymmetrical Fault Analysis: LG, LL and LLG faults with and without fault impedance, Numerical Problems.

UNIT-5: STABILITY ANALYSIS:

Elementary concepts of Steady State, Dynamic and Transient Stabilities. Derivation of Swing Equation, Power Angle Curve and Determination of Steady State Stability. Determination of Transient Stability by Equal Area Criterion, Application of Equal Area Criterion, Critical Clearing Angle Calculation. Numerical methods for solution of swing equation - Methods to improve Stability - Application of Auto Reclosing and Fast Operating Circuit Breakers.

OBJECTIVES

UNIT–1: PER-UNIT SYSTEM AND YBUS FORMATION

- Understand the basic elements of a power system and their representation.
- Learn the concept and advantages of the Per-Unit (p.u.) system in power system calculations.
- Calculate per-unit quantities for different power system components.
- Understand the representation of a three-phase power system network in per-unit form.
- Learn the formation of Y-Bus matrix using Direct Inspection method.
- Apply Bus Incidence Matrix and Singular Transformation method for Y-Bus formation.

UNIT–2: FORMATION OF ZBUS

- Understand the concept of bus impedance matrix (Zbus) in power systems.
- Learn the step-by-step algorithm for Zbus formation from partial networks.
- Construct Zbus for different cases such as:
    a. Addition of a branch from a new bus to reference bus
    b. Addition of a branch from a new bus to an existing bus
    c. Addition of a branch from an existing bus to reference bus
    d. Addition of a branch between two existing buses
- Modify the Zbus matrix when network elements are added or changed.
- Apply Zbus formation methods to solve numerical problems.

UNIT–3: POWER FLOW ANALYSIS

- Understand the importance of load flow studies in power system operation and planning.
- Formulate static load flow equations for power system networks.
- Learn the Gauss–Seidel method for load flow solution.
- Study acceleration factor and convergence characteristics.
- Understand Newton–Raphson method in Polar Coordinates.
- Learn Jacobian matrix formation and algorithm steps.
- Compare Gauss–Seidel, Newton–Raphson, Decoupled and Fast Decoupled methods.

UNIT–4: SHORT CIRCUIT STUDIES

- Understand the types and causes of faults in power systems.
- Calculate short circuit current and fault MVA.
- Study the application of series reactors in fault current limitation.
- Learn Symmetrical Component Theory.
- Understand positive, negative and zero sequence networks.
- Analyse symmetrical faults (LLLG) and unsymmetrical faults (LG, LL, LLG).
- Perform fault analysis with and without fault impedance.

UNIT–5: STABILITY ANALYSIS

- Understand the concept of power system stability.
- Differentiate steady state, dynamic and transient stability.
- Derive and understand the swing equation.
- Study power angle curve and steady state stability limit.
- Understand Equal Area Criterion for transient stability analysis.
- Calculate critical clearing angle and time.
- Study methods for improving system stability such as auto-reclosing and fast circuit breakers.

INTRODUCTION

Power systems are large interconnected networks that generate, transmit, and distribute electrical energy to consumers. The analysis of power systems is essential to ensure reliable, efficient, and stable operation under normal and abnormal conditions. Power System Analysis deals with the study of electrical networks to determine voltage levels, power flow, fault currents, and system stability.

This course introduces the fundamental concepts and analytical techniques used in the study of power systems.

The first unit focuses on the **Per-Unit system**, which simplifies calculations in power systems by expressing electrical quantities in normalized form. It also discusses the formation of the **bus admittance matrix (Ybus)**, which is an important tool used in power system analysis.

The second unit deals with the **formation of the bus impedance matrix (Zbus)** using systematic algorithms and methods for modifying the matrix when new elements are added to the network.

The third unit focuses on **Power Flow Analysis**, which is used to determine the voltage magnitude, phase angle, and power flow in different buses of the system. Various numerical methods such as **Gauss–Seidel and Newton–Raphson methods** are used to solve load flow problems.

The fourth unit discusses **Short Circuit Studies**, which analyse different types of faults that may occur in power systems. It introduces **symmetrical components and sequence networks** used to analyse both symmetrical and unsymmetrical faults.

The fifth unit deals with **Power System Stability**, which studies the ability of the system to maintain synchronism under disturbances. Concepts such as **swing equation, power angle curve, and equal area criterion** are used to analyse system stability.

Overall, this course provides essential knowledge and analytical tools required for planning, operation, and protection of modern power systems.

Unit-I

Per unit Systems and Symmetrical Component Theory

Per unit System Representation, advantages, Per unit equivalent reactance representation of power system components. Symmetrical component theory - Voltages, currents and Impedances: Sequence representation of power system components - Generators, transformers, transmission line, load and Networks.

① Per Unit System Representation:-

For the analysis of electrical machines, or electrical machine system, different values are required, thus, per unit system provides the value for voltages, currents, power, Impedence, and admittance.

The "per unit system" also makes the calculations easier as all the values are taken in the same unit. The per-unit system is mainly used in circuit where variation in voltage occurs.

Definition: The per-unit value of any quantity is defined as the ratio of actual value in any unit to the base or reference value in the same unit.

* Any quantity is converted into per unit quantity by dividing the numeral value by the chosen base value of the same dimension.

* The per-unit value is dimensionless.

$$\boxed{\text{per unit value} = \frac{\text{Actual Value in any unit}}{\text{Base or reference value in the same unit}}}$$

* It is usual to assume the base value as given below

→ Base Voltage = rated Voltage of the machine

→ Base Current = rated current of the machine

→ Base Impedance = $\dfrac{\text{Base Voltage}}{\text{Base current}}$

→ Base power = Base Voltage * Base Current

* Firstly the values of Base power and the Base Voltages are selected, and their choice automatically fixes the other Base Values.

As, per unit KV = $\dfrac{\text{Actual KV}}{\text{Base KV}} = \dfrac{KV_{actual}}{KV_B}$

So, Base current $I_B = \dfrac{\text{Base KVA}}{\text{Base KV}} = \dfrac{KVA_B}{KV_B}$ → (1)

per unit current $I_{pu} = \dfrac{\text{Actual Value of current}}{\text{Base current}}$ → (2)

Substitute eq (1) in eq (2), we get

per unit current $I_{pu} = \dfrac{\text{Actual Value of Current}}{KVA_B / KV_B}$

$= \dfrac{\text{Actual Value of current} \times KV_B}{KVA_B}$

Base Impedance $Z_B = \dfrac{\text{Base KV} \times 1000}{\text{Base current}}$ → (3)

Substitute eq (1) in eq (3), we get

Base Impedance $Z_B = \dfrac{KV_B \times 1000}{KVA_B / KV_B}$

$$\boxed{Z_B = \frac{(KV_B)^2 \times 1000}{KVA_B}} \rightarrow (4)$$

Base power = KVA_B

Now, $Z_{pu} = \frac{\text{Actual Impedence}}{\text{Base Impedence}}$ → (5)

Substitute eq (4) in eq (5), we get per unit Impedance Value.

$$\therefore Z_{pu} = \frac{\text{Actual Impedance}}{\frac{(KV_B)^2 \times 1000}{KVA_B}}$$

$$\boxed{Z_{pu} = \frac{\text{Actual Impedence} \times (KVA_B \text{ or Base power})}{(KV_B)^2 \times 1000}}$$

Advantages of per unit Systems:

There are mainly two advantages of using the per unit system.

* The parameters of the rotating electrical machines and the transformer lie roughly in the same range of numerical values irrespective of their ratings if expressed in a per-unit system of ratings.
* It relieves the analyst of the need to refer circuit quantities to one or the other side of the transformer, making the calculations easy.

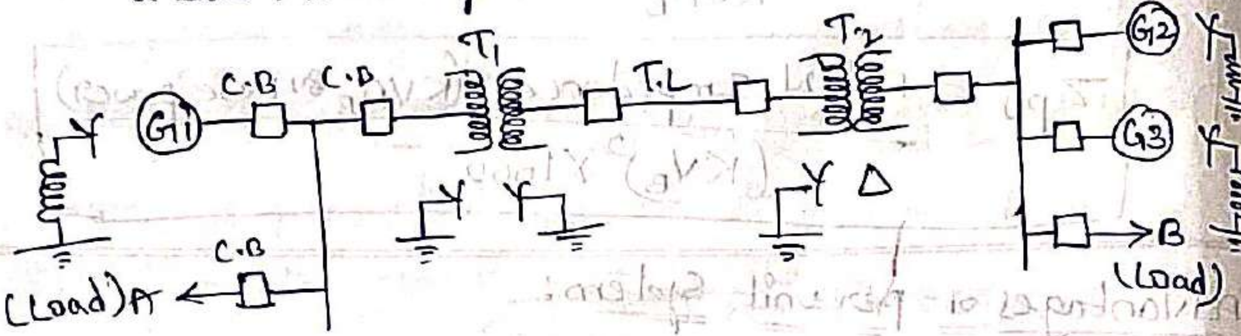
Other advantages of per unit system may include

* Manufactures usually specify the Impedance values of equipments in per unit of the equipments rating. If any data is not available, it is easier to assume its per unit value than its numerical value.
* per unit data Representation yields Important information about relative magnitudes.

② Per unit equivalent reactance Representation of power system components :-

A one-line diagram of a power system shows the main connections and arrangements of components. Any particular component may or may not be shown depending on the information required in a system study.

Example: Circuit diagram which has C.B's need not be shown in a Load Flow study but are a must for a protection study.



One line Representation of a simple power system

Reactance diagram & approximations made in reactance diagram:

The Reactance diagram is the simplified equivalent circuit of power system in which the various components are represented by their reactances. The reactance diagram can be obtained from Impedance diagram if all the resistive components are Neglected.

"The Reactance diagram is used for Fault calculations".

Approximation:

(i) The Neutral Reactances are Neglected.

(ii) The shunt branches in equivalent circuit of transformers are Neglected.

(iii) The Resistances are Neglected.

(iv) All static loads are Neglected.

(v) The capacitance of Transmission Lines are Neglected.

Procedure to Form Reactance Diagram from Single Line Diagram:

1. Select a base power KVA_b or MVA_b
2. Select a base Voltage KV_b
3. The Voltage conversion is achieved by means of Transformer KV_b on LT Section

$$= \frac{KV_b \text{ on HT section} \times \text{LT Voltage Rating}}{\text{HT Voltage Rating}}$$

where, LT - Low Tension [Commercial, Residential supply]
HT - High Tension [Industrial Loads]

4. When specified Reactances of a components is in ohms

$$* \text{ p.u reactance} = \frac{\text{Actual Reactance}}{\text{Base Reactance}}$$

Specified Reactance of a components is in p.u

$$X_{p.u,new} = X_{p.u,old} * \frac{(KV_{b,old})^2}{(KV_{b,new})^2} * \frac{MVA_{b,new}}{MVA_{b,old}}$$

Example:

→ The single line diagram of an unloaded power system is shown in the figure. The generator Transformer ratings are as follows.

$G1 = 20MVA, 11KV, X'' = 25\%$

$G2 = 30MVA, 18KV, X'' = 25\%$

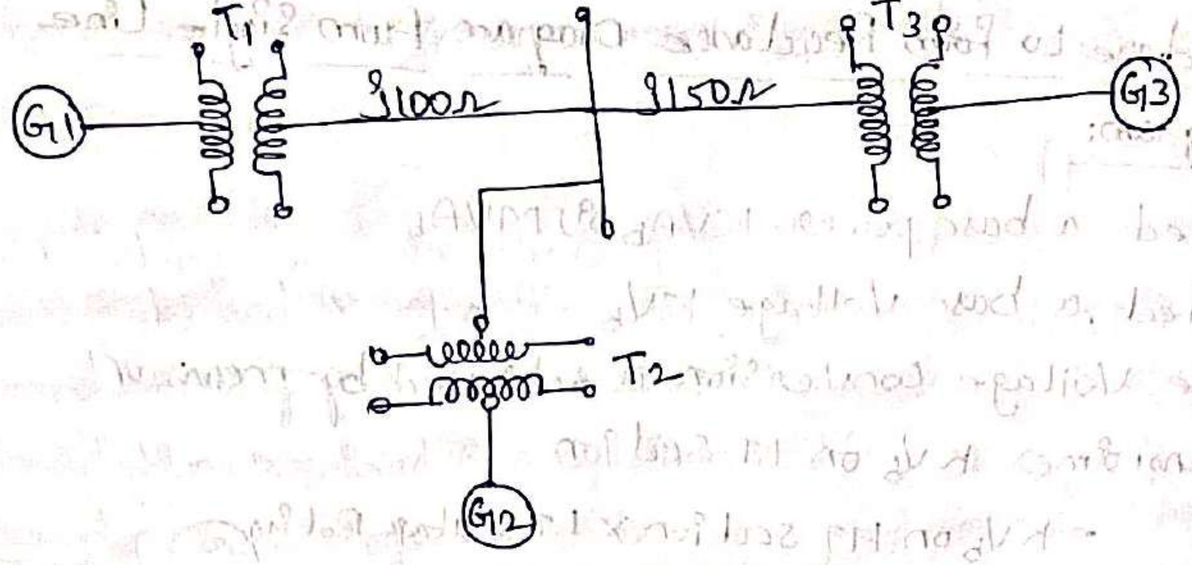
$G3 = 30MVA, 20KV, X'' = 21\%$

$T1 = 25MVA, 220/13.8KV (\Delta/Y), X = 15\%$

$T2 =$ 3 single phase units each rated $10MVA, 127/18KV (Y/\Delta), X = 15\%$

$T3 = 15MVA, 220/20KV (Y/\Delta), X = 15\%$

Draw the reactance diagram using a base of 50MVA and 11KV on the generator 1.



Solution:

Base $MVA_{b,new} = 50MVA$

Base $KV_{b,new} = 11KV$ (generator side)

(i) Reactance of Generator G_1:

$KV_{b,old} = 11KV$ $\quad KV_{b,new} = 11KV$

$MVA_{b,old} = 20MVA$ $\quad MVA_{b,new} = 50MVA$

$X_{p.u,old} = 0.25 p.u$

The New p.u. reactance of Generator (G_1) =

$$X_{p.u,old} \times \frac{(KV_{b,old})^2}{(KV_{b,new})^2} \times \frac{(MVA_{b,new})}{(MVA_{b,old})}$$

$$= 0.25 \times \left(\frac{11}{11}\right)^2 \times \left(\frac{50}{20}\right) = j0.63 p.u$$

(ii) Reactance of Transformer T_1:

$KV_{b,old} = 11KV$ $\quad KV_{b,new} = 11KV$

$MVA_{b,old} = 25MVA$ $\quad MVA_{b,new} = 50MVA$

$X_{p.u,old} = 0.15 p.u$

The New p.u reactance of Transformer

$$(T_1) = X_{pu,old} \times \frac{(KV_{b,old})^2}{(KV_{b,new})^2} \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.15 \times \left(\frac{11}{11}\right)^2 \times \left(\frac{50}{25}\right) = j0.3 p.u$$

(iii) Reactance of Transmission Line:

It is connected to the HT side of the Transformer T1

Base KV on HT side of T1 = Base KV on LT side × $\frac{\text{HT Voltage rating}}{\text{LT Voltage rating}}$

$$= 11 \times \frac{220}{11} = 220\,KV$$

Actual Impedance $X_{actual} = 100\Omega$

Base Impedance $X_{base} = \frac{(KV_{b,new})^2}{MVA_{b,new}} = \frac{(220)^2}{50} = 968\Omega$

p.u reactance of 100Ω, T.L = $\frac{\text{Actual Reactance}}{\text{Base Reactance}} = \frac{100}{968} = 0.1\,p.u$

p.u reactance of 150Ω, T.L = $\frac{150}{968} = 0.15\,p.u$

(iv) Reactance of Transformer T2:

$KV_{b,old} = 127\sqrt{3} = 220\,KV$ $\quad KV_{b,new} = 220\,KV$

$MVA_{b,old} = 10 \times 3 = 30\,MVA$ $\quad MVA_{b,new} = 50\,MVA$

$X_{p.u,old} = 0.15\,p.u$

The New p.u reactance of T2 $= 0.15 \times \left(\frac{220}{220}\right)^2 \times \frac{50}{30}$

$$= 0.25\,p.u$$

(v) Reactance of Generator G2:

It is connected to the LT side of the Transformer T2.

Base KV on LT side of Transformer T2 = Base KV on HT side × $\frac{\text{LT Voltage rating}}{\text{HT Voltage rating}}$

$$(T2) = 220 \times \frac{18}{220} = 18\,KV$$

$KV_{b,old} = 18\,KV$ $\quad KV_{b,new} = 18\,KV$

$MVA_{b,old} = 30\,MVA$ $\quad MVA_{b,new} = 50\,MVA$

$X_{p.u,old} = 0.25\,p.u$

The new pu reactance of Generator (G2) $= 0.25 \times \left(\frac{18}{18}\right)^2 \times \frac{50}{30}$

$= j0.142$ p.u

(vi) Reactance of Transformer (T3):

$KV_{b,old} = 20KV$ $KV_{b,new} = 20KV$

$MVA_{b,old} = \cancel{20}\,15\,MVA$ $MVA_{b,new} = 50MVA$

$X_{pu,old} = 0.15$ p.u

The new Reactance of Transformer (T3) $= 0.15 \times \left(\frac{20}{20}\right)^2 \times \frac{50}{15}$

$= j0.15$ p.u

(vii) Reactance of Generator (G3):

It is connected to LT side of Transformer T3

Base KV on LT side of T3 = Base KV on HT side $\times$ $\frac{\text{LT rated Voltage}}{\text{HT rated Voltage}}$

$= 220 \times \frac{20}{220} = 20KV$

$KV_{b,old} = 20KV$ $KV_{b,new} = 20KV$

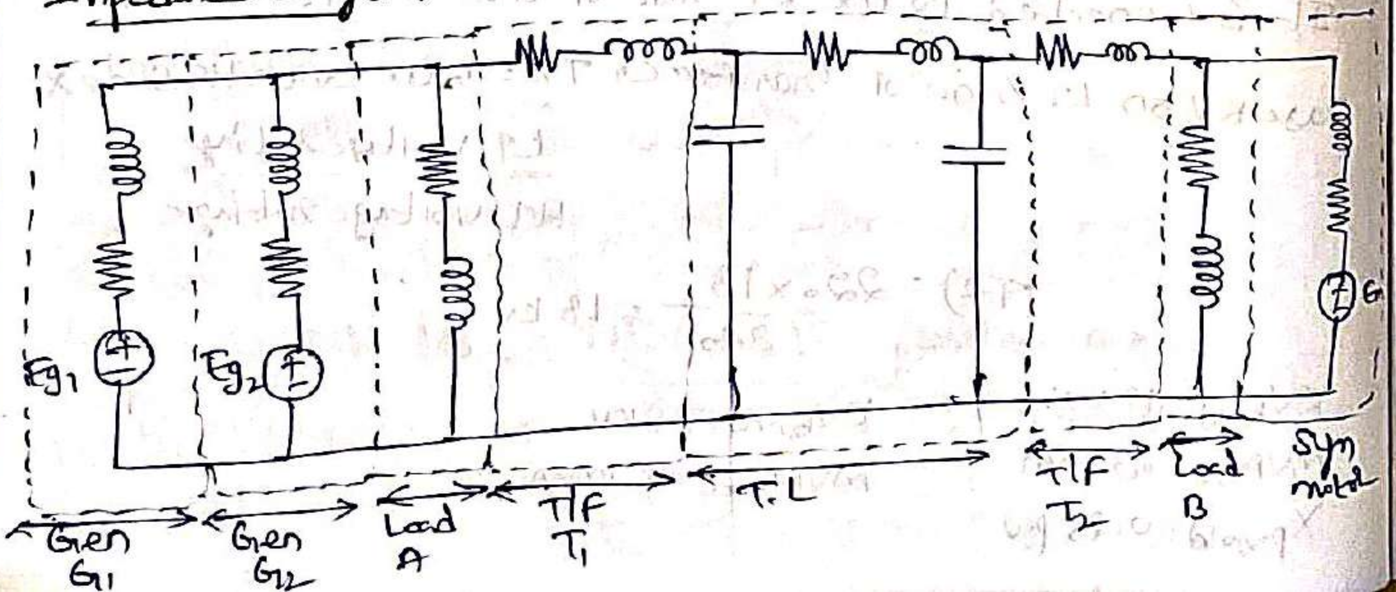
$MVA_{b,old} = 30MVA$ $MVA_{b,new} = 50MVA$

$X_{pu,old} = 0.21$ p.u

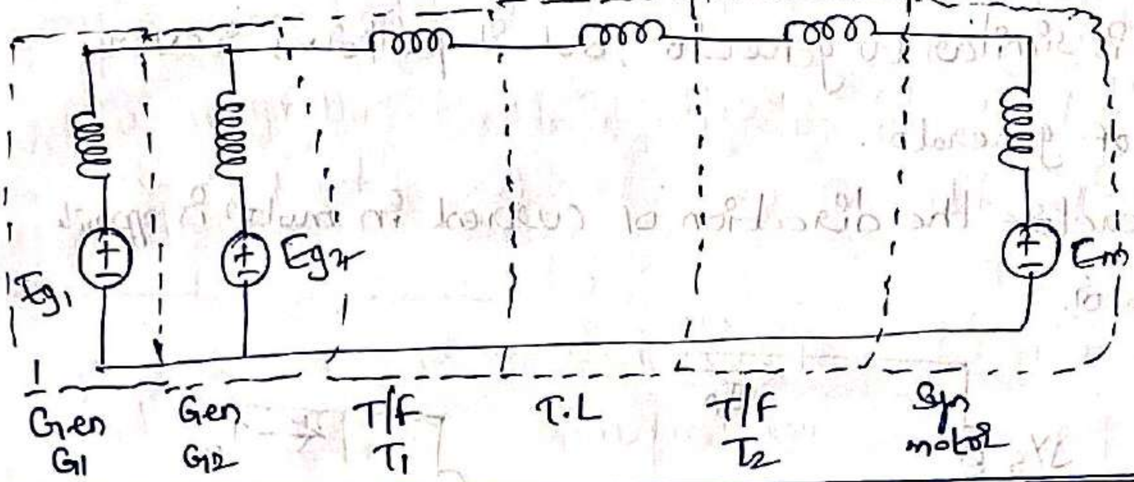
The new Reactance of Generator (G3) $= 0.21 \times \left(\frac{20}{20}\right)^2 \times \left(\frac{50}{30}\right)$

$= j0.35$ pu

Impedance Diagram



Reactance Diagram



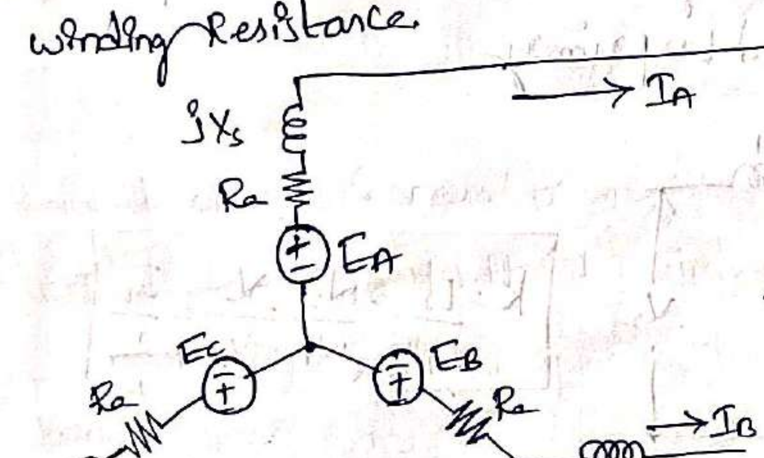
⊛ Equivalent circuit components of power systems:

The need of Equivalent circuit is to perform many operations like load flow analysis, fault level calculations etc..

The various components of power system are generators, Transformers, Transmission lines, Synchronous motors, resistive, reactive loads.

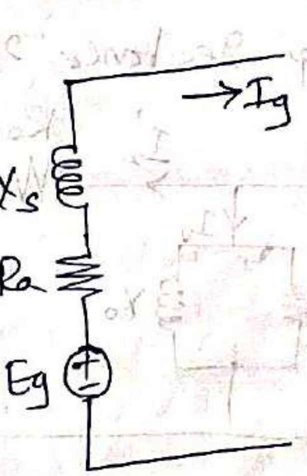
(i) Equivalent circuit of generator:

It consist of source representing induced Emf/phase, A series reactance represents armature reactance and leakage reactance & series resistance represents armature winding Resistance.



3-φ Equivalent circuit

⇔

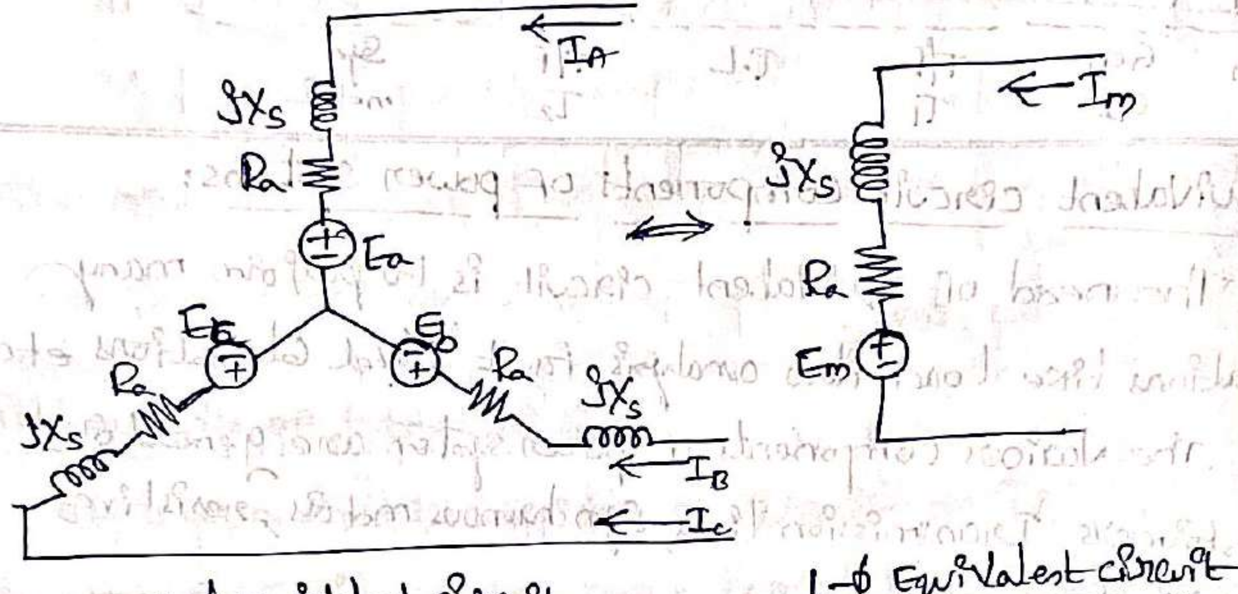


1-φ Equivalent circuit

(ii) Equivalent circuit of synchronous motor:

It is similar to generator, but it performs reverse action of generator.

Therefore the direction of current in motor is opposite to generator.

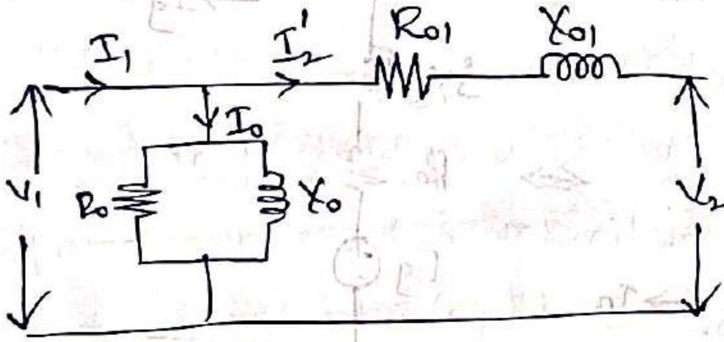


3-ϕ Equivalent circuit

1-ϕ Equivalent circuit

(iii) Equivalent circuit of Transformer:

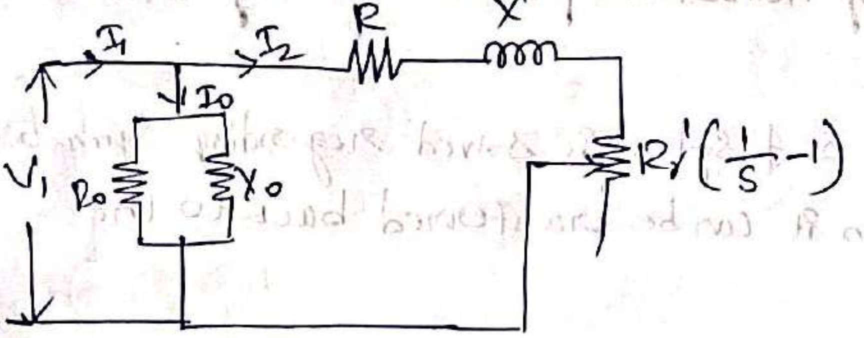
It consist of shunt branches to represents magnetising current and core losses, series Resistances represents winding Resistance referred to primary and series reactances represents Leakage reactance referred to primary



$$\boxed{K = \frac{E_2}{E_1} = \frac{N_2}{N_1} = \frac{V_2}{V_1} = \frac{I_1}{I_2}}$$

(iv) Equivalent circuit of Induction motor:

It is similar to Transformer Equivalent circuit, but for Induction motor it requires rotating Transformer.



Here $R_r'\left(\frac{1}{s}-1\right)$ = Resistance representing Load

$R = R_s + R_r'$

$X = X_s + X_r'$

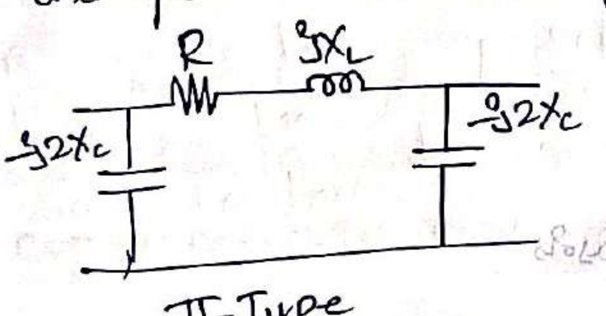
$R_s \cdot X_s$ = R & X of stator

$R_r \cdot X_r$ = R & X of Rotor

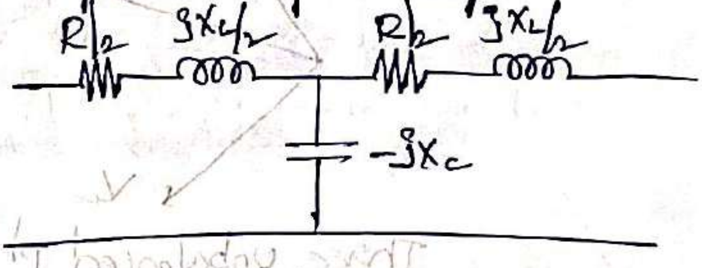
S = slip ring

(V) Equivalent circuit of Transmission lines:

The Transmission line can be represented by its R, L, C. The elements R, X_L and X_c are resistance, Inductive reactances and capacitance reactances per phase respectively.



Π-Type



T-Type

③ Symmetrical component Theory:-

When the system is unbalanced the voltages, currents and the phase Impedances are in general unequal. Such system can be solved by a symmetrical per phase Technique, known as the method of Symmetrical components.

This method is also called a "three component method."

The method of Symmetrical components simplified the problems of the unbalanced 3-φ system.

It is used for any number of phases but mainly used for 3-ϕ system.

The unbalanced 3-ϕ system is solved regarding symmetrical components, and then it can be transferred back to the actual circuit.

The balanced set of components can be given as follows

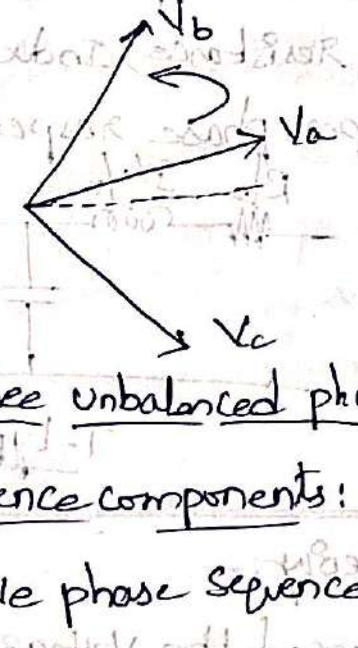
(i) positive sequence components

(ii) Negative sequence components

(iii) zero sequence components.

Consider an unbalanced voltage phasor system shown in the figure. Suppose that the phasors are represented by V_a, V_b and V_c and their phase sequence is V_a, V_b and V_c.

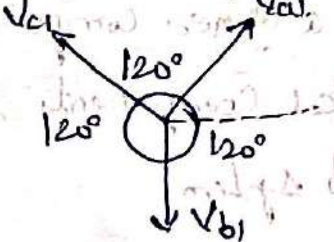
The phase sequence of the +ve components is V_a, V_b and V_c and the phase sequence of -ve components is V_a, V_c and V_b.



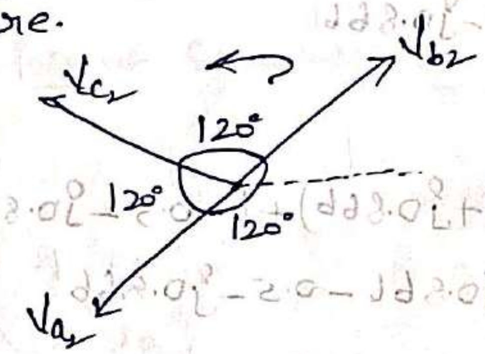
Three unbalanced phasors

(i) positive sequence components:

In positive phase sequence components, the set of three phasors are equal in magnitude, spaced 120° apart from each other and having the same phase sequence as the original unbalanced phasors. The positive sequence components of the unbalanced three phase system is shown below.

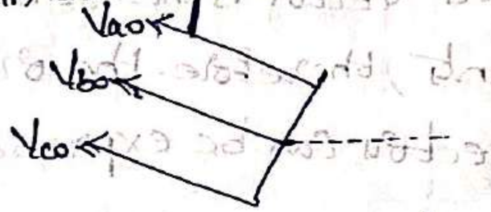


(ii) Negative Sequence component: the set of the three phasors are equal in magnitude, spaced 120° apart from each other and having the phase sequence opposite to that of the original phasors. The Negative phase sequence is shown in the figure.



(iii) Zero sequence components:

In zero phase sequence components, the set of three phasors is equal in magnitude to zero phase displacement from each other. The zero phase sequence component is shown in the figure.



The 3-ϕ balanced System is a special case of a general 3-ϕ system in which zero and Negative sequence components are zero.

→ from Vector Diagram of Symmetrical components

* on rotating the Vector V_{a1} by 120° in Anticlockwise direction we get V_{c1}.
* on rotating the Vector V_{a1} by 240° in Anticlockwise direction we get V_{b1}.
* On rotating the Vector V_{a2} by 120° in Anticlockwise direction we get V_{b2}.
* on rotating the Vector V_{a2} by 240° in Anticlockwise direction we get V_{c2}.

The operator 'a' is defined as

$$a = 1\angle 120^\circ = 1e^{j\frac{2\pi}{3}} = \cos\frac{2\pi}{3} + j\sin\frac{2\pi}{3} = -0.5 + j0.866$$

Since $a = 1\angle 120^\circ = -0.5 + j0.866$

$a^2 = 1\angle 240^\circ = -0.5 - j0.866$

$a^3 = 1\angle 360^\circ = 1$

$\therefore 1 + a + a^2 = 1 + (-0.5 + j0.866) + (-0.5 - j0.866)$

$= 1 - 0.5 + j0.866 - 0.5 - j0.866$

$= 0$

$\therefore \boxed{1 + a + a^2 = 0}$

Computation of unbalanced vector from their symmetrical components:

Each of original unbalanced vector is the sum of +ve, -ve, zero sequence components, therefore the original unbalanced 3-ϕ voltage vectors can be expressed interms of their components.

$V_a = V_{a0} + V_{a1} + V_{a2}$

$V_b = V_{b0} + V_{b1} + V_{b2}$

$V_c = V_{c0} + V_{c1} + V_{c2}$

From the vector diagram, the relation between symmetrical components.

$V_{b0} = V_{a0}$; $V_{b1} = a^2 V_{a1}$; $V_{b2} = aV_{a2}$

$V_{c0} = V_{a0}$; $V_{c1} = aV_{a1}$; $V_{c2} = a^2 V_{a2}$

Substitute values in above equation

$V_a = V_{a0} + V_{a1} + V_{a2}$

$V_b = V_{a0} + a^2 V_{a1} + aV_{a2}$

$V_c = V_{a0} + aV_{a1} + a^2 V_{a2}$

Matrix form of the above equation is

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix}$$

<u>Computation of Symmetrical Components of unbalanced Vectors:</u>

$$V = A\,V_{symmetrical} \longrightarrow (1)$$

where, $V = \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$; $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}$; $V_{sy} = \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix}$

Multiply eq (1) by A^{-1}

$$A^{-1}V = V_{sy}$$

$$V_{sy} = A^{-1}V \longrightarrow (2)$$

where $A^{-1} = \dfrac{\text{Adjoint of } A}{\text{Determinant of } A}$

Δ = Determinant of A

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{vmatrix} = 1(a^4 - a^2) - 1(a^2 - a) + 1(a - a^2)$$

$$= a^4 - a^2 - a^2 + a + a - a^2$$

$$= a^4 - 3a^2 + 2a$$

$$= a^3 \cdot a - 3a^2 + 2a$$

$$= 1 \cdot a - 3a^2 + 2a \quad \left[\text{where } a^3 = 1\right]$$

$$= a - 3a^2 + 2a$$

$$= 3a - 3a^2 = 3(a - a^2)$$

$$\therefore \boxed{\Delta = 3(a - a^2)}$$

Let Δ_{ij} = Co-factor of A_{ij}

$\Delta_{11} = a^4 - a^2 = a^3 \cdot a - a^2 = a - a^2$

$\Delta_{12} = -(a^2 - a) = a - a^2$

$\Delta_{13} = a - a^2$

$\Delta_{21} = -(a^2 - a) = a - a^2$

$\Delta_{22} = a^2 - 1$

$\Delta_{23} = -(a-1) = 1 - a$

$\Delta_{31} = a - a^2$

$\Delta_{32} = -(a-1) = 1 - a$

$\Delta_{33} = a^2 - 1$

Now $A^{-1} = \dfrac{\text{Adjoint of } A}{\text{Determinant of } A}$

$$A^{-1} = \frac{1}{\Delta}\begin{bmatrix} \Delta_{11} & \Delta_{12} & \Delta_{13} \\ \Delta_{21} & \Delta_{22} & \Delta_{23} \\ \Delta_{31} & \Delta_{32} & \Delta_{33} \end{bmatrix}^T$$

$$A^{-1} = \frac{1}{\Delta}\begin{bmatrix} \Delta_{11} & \Delta_{21} & \Delta_{31} \\ \Delta_{12} & \Delta_{22} & \Delta_{32} \\ \Delta_{13} & \Delta_{23} & \Delta_{33} \end{bmatrix}$$

$$A^{-1} = \frac{1}{3(a-a^2)}\begin{bmatrix} a-a^2 & a-a^2 & a-a^2 \\ a-a^2 & a^2-1 & 1-a \\ a-a^2 & 1-a & a^2-1 \end{bmatrix}$$

$$A^{-1} = \frac{a-a^2}{3(a-a^2)}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \frac{a^2-1}{a-a^2} & \frac{1-a}{a-a^2} \\ 1 & \frac{1-a}{a-a^2} & \frac{a^2-1}{a-a^2} \end{bmatrix} \rightarrow (3)$$

From eq (3)

$$\frac{a^2-1}{a-a^2} = \frac{a^2-a^3}{a-a^2} = \frac{a(a-a^2)}{a-a^2} = a$$

$$\frac{1-a}{a-a^2} = \frac{a^2(1-a)}{a^2(a-a^2)} = \frac{a^2(1-a)}{a^3-a^4} = \frac{a^2(1-a)}{a^3(1-a)} = \frac{a^2(1-a)}{1-a}$$

$$= a^2$$

Substitute values in eq (3)

$$A^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \rightarrow (4)$$

Substitute eq (4) in eq (2)

$$V_{sy} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} V$$

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \rightarrow (5)$$

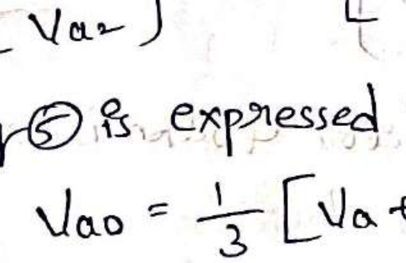
Eq (5) is expressed in independent linear equations

$$V_{a0} = \frac{1}{3}\left[V_a + V_b + V_c\right]$$

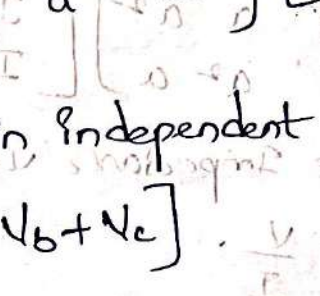
$$V_{a1} = \frac{1}{3}\left[V_a + aV_b + a^2V_c\right]$$

$$V_{a2} = \frac{1}{3}\left[V_a + a^2V_b + aV_c\right]$$

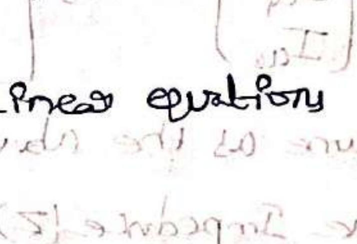
<u>Symmetrical components of unbalanced current vectors:</u>



<u>+ve sequence components</u>



<u>-ve sequence components</u>



<u>zero sequence components</u>

From the Voltage equation of symmetrical components we get current components

$$I_a = I_{a0} + I_{a1} + I_{a2}$$

$$I_b = I_{a0} + a^2 I_{a1} + a I_{a2}$$

$$I_c = I_{a0} + a I_{a1} + a^2 I_{a2}$$

The unbalanced symmetrical components current vectors

$$I_{a0} = \frac{1}{3}[I_a + I_b + I_c]$$

$$I_{a1} = \frac{1}{3}[I_a + a I_b + a^2 I_c]$$

$$I_{a2} = \frac{1}{3}[I_a + a^2 I_b + a I_c]$$

$$\begin{bmatrix} I_0 \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

Same as the above Impedance vector equation is defined where Impedance $(Z) = \frac{V}{I}$.

Problems:-

① The Voltage a/c 3-ϕ balanced load are $V_a = 300\angle 20^\circ$ V, $V_b = 360\angle 90^\circ$ V & $V_c = 500\angle -140^\circ$ V. Determine the Symmetrical components of Voltages. phase sequence is abc.

Sol: Given data

$$V_a = 300\angle 20^\circ = 281.91 + j102.61 \text{ V}$$

$$V_b = 360\angle 90^\circ = 0 + j360 \text{ V}$$

$$V_c = 500\angle -140^\circ = -383.02 - j321.39 \text{ V}$$

we know that

$$V_{a0} = \frac{1}{3}\left[V_a + V_b + V_c\right] \longrightarrow ①$$

$$V_{a1} = \frac{1}{3}\left[V_a + aV_b + a^2V_c\right] \longrightarrow ②$$

$$V_{a2} = \frac{1}{3}\left[V_a + a^2V_b + aV_c\right] \longrightarrow ③$$

Now

$aV_b = 1\angle 120^\circ \times 360\angle 90^\circ$

$= 360\angle 210$

$= -311.77 - j180^\circ V$

$a^2V_b = 1\angle 240 \times 360\angle 90^\circ$

$= 360\angle 330$

$= 311.77 - j180^\circ V$

$aV_c = 1\angle 120^\circ \times 500\angle -140$

$= 500\angle -20^\circ$

$= 469.85 - j171.01 V$

$a^2V_c = 1\angle 240^\circ \times 500\angle -140^\circ$

$= 500\angle 100^\circ$

$= -86.82 + j492.40 V$

Substitute above values in eq①, ② ③ respectively

$$V_{a0} = \frac{1}{3}\left[281.91 + j102.61 + 0 + j360 - 383.02 - j321.39\right]$$

$= -33.70 + j47.07$

$= 57.89\angle 126^\circ V$

$$V_{a1} = \frac{1}{3}\left[281.91 + j102.61 - 311.77 - j180 - 86.82 + j492.40\right]$$

$= -38.89 + j138.34$

$= 143.700\angle 1.06^\circ V$

$V_{a2} = \frac{1}{3}[281.91 + j102.61 + 311.77 - j180 + 469.85 - j171.01]$

$= 364.05 \angle -13° \, V$

we know that

$V_{a0} = V_{b0} = V_{c0}$

zero sequence

$V_{a0} = 57.89 \angle 126° \, V$

$V_{b0} = 57.89 \angle 126° \, V$

$V_{c0} = 57.89 \angle 126° \, V$

we know that

$V_{b1} = a^2 V_{a1}, \; V_{c1} = a V_{a1}$

+ve sequence

$V_{a1} = 143.70 \angle 106° \, V$

$V_{b1} = a^2 V_{a1}$

$= 1 \angle 240 \times 143.70 \angle 106°$

$= 143.70 \angle 346° \, V$

$V_{c1} = a V_{a1}$

$= 1 \angle 120 \times 143.70 \angle 106°$

$= 143.70 \angle 226° \, V$

we know that

$V_{b2} = a V_{a2} \; ; \; V_{c2} = a^2 V_{a2}$

-ve sequence

$V_{a2} = 364.05 \angle -13° \, V$

$V_{b2} = a V_{a2} = 1 \angle 120° \times 364.05 \angle -13$

$= 364.05 \angle 107° \, V$

$V_{c2} = a^2 V_{a2} = 1 \angle 240° \times 364.05 \angle -13$

$= 364.05 \angle 227° \, V$

② Calculate $Z_{p.u}(new)$ by using the following data:
A 3-φ generator having 100 MVA, 200 MVA with ratings 22 kV, 33 kV, the per unit Impedance of generator is 20.

Sol: $$Z_{p.u}(new) = Z_{p.u}(old) \times \frac{kV_b^2(old)}{kV_b^2(new)} \times \frac{MVA_b(new)}{MVA_b(old)}$$

$$= 20 \times \frac{(22)^2}{(33)^2} \times \frac{200}{100}$$

$$Z_{p.u}(new) = 17.78\ \Omega$$

③ The symmetrical components of phase fault current in a 3-φ unbalanced system are $I_{a0} = 350\angle 90^\circ$ A, $I_{a1} = 600\angle -90^\circ$ A, $I_{a2} = 250\angle 90^\circ$ A. Determine the phase currents I_a, I_b, I_c.

Sol: $$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$$

$$I_a = I_{a0} + I_{a1} + I_{a2}$$

$$I_b = I_{a0} + a^2 I_{a1} + a I_{a2}$$

$$I_c = I_{a0} + a I_{a1} + a^2 I_{a2}$$

Given Data

$I_{a0} = 350\angle 90^\circ = 0 + 350j$

$I_{a1} = 600\angle -90^\circ = 0 - 600j$

$I_{a2} = 250\angle 90^\circ = 0 + 250j$

Now

$a I_{a1} = 1\angle 120 \times 600\angle -90^\circ = 600\angle 30^\circ = 519.62 + j300$

$a^2 I_{a1} = 1\angle 240 \times 600\angle -90^\circ = 600\angle 150^\circ = -519.62 + j300$

$a I_{a2} = 1\angle \overset{120}{\cancel{240}} \times 250\angle 90^\circ = 250\angle 210 = -216.51 - j125$

$a^2 I_{a2} = 1\angle 240 \times 250\angle 90 = 250\angle 330 = 216.51 - j125$

$I_a = 0 + j350 + 0 - j600 + 0 + j250$

$= 0$

$I_b = 0 + j350 + (-519.62 + j300) - 216.51 - j125$

$= -736.13 + 525j$

$= 904.16\angle 145^\circ A$

$I_c = j350 + 519.62 + j300 + 216.51 - j125$

$= 736.13 + 525j$

$= 904.16\angle 35^\circ A$

---

(4) Sequence Representation of power system components:

(i) Generator:

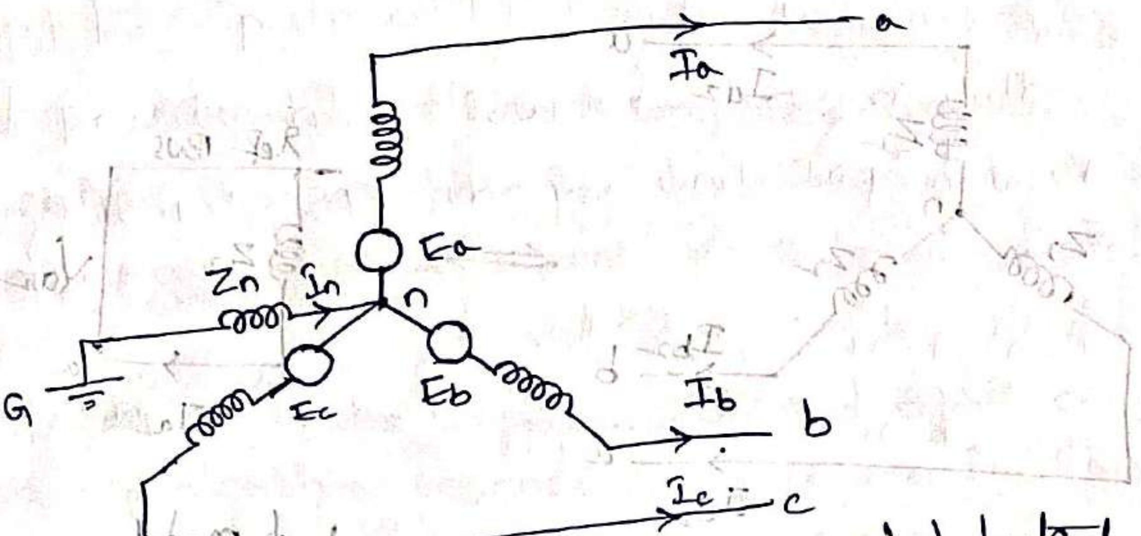
→ Figure shows an unloaded synchronous machine (generator or motor) grounded through a reactor (impedance Z_n).

→ E_a, E_b, and E_c are the induced emf's of three phases.

→ When a fault takes place at machine terminals, currents I_a, I_b and I_c flows in the lines.

→ whenever the faults involve in ground, currents $I_n = I_a + I_b + I_c$ flows to neutral from ground via Z_n.

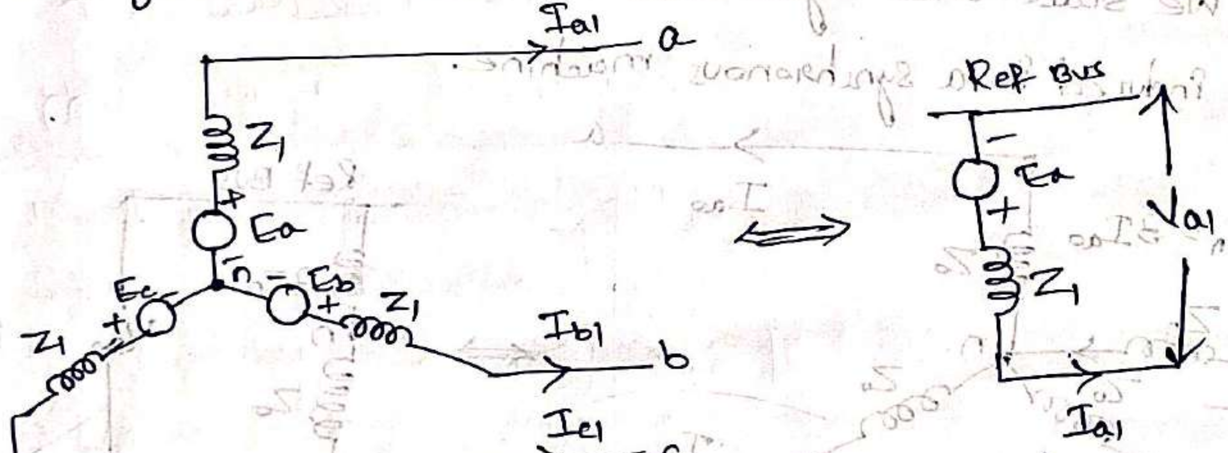
→ unbalanced line currents are I_{a1}, I_{a2}, I_{a0}



3-ϕ synchronous generator with grounded Neutral

(a) +ve sequence Impedance and Network:

Since a synchronous machine is designed with symmetrical windings, it induces emf's of positive sequence only, i.e. no negative (or) zero sequence voltages are induced in it.



3-ϕ model

1-ϕ model

Fig: +ve sequence Network of Synchronous Machine

$$\boxed{V_{a1} = E_a - Z_1 I_{a1}}$$

(b) -ve sequence Impedance and Networks:

Synchronous machine has zero Negative sequence Induced Voltages. with the flow of -ve sequence currents in the stator a rotating field is created which rotates in the Opposite direction to that of +ve sequence field.

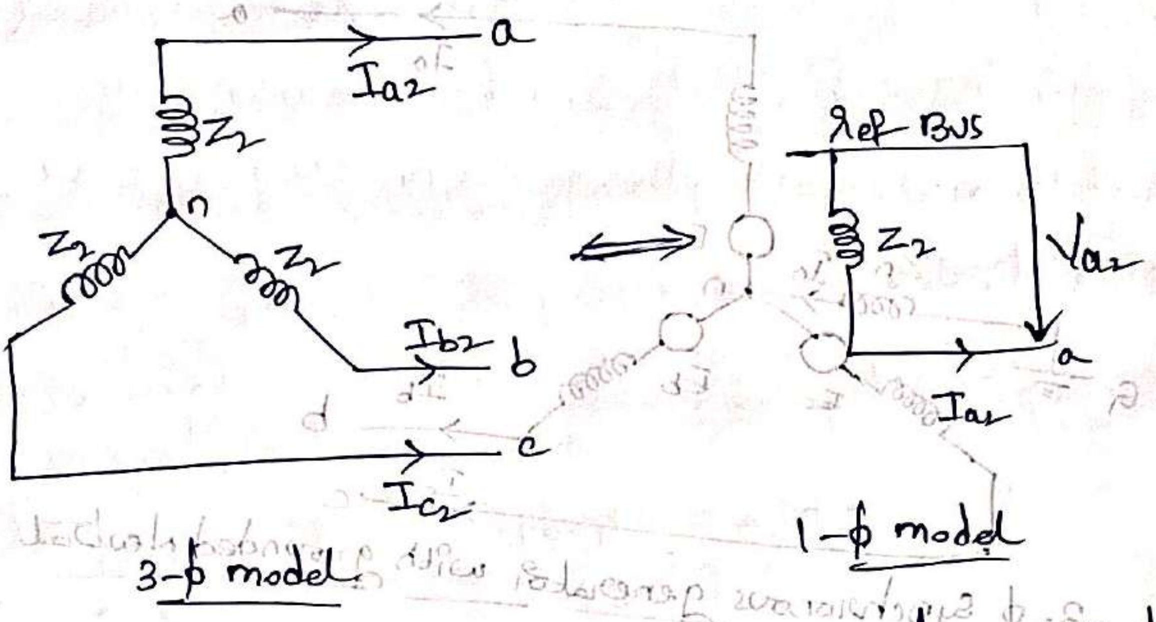


Fig: -ve sequence network of a synchronous machine

$$\boxed{V_{a2} = -Z_2 I_{a2}}$$

(c) Zero Sequence Impedance and Network:

We state once again that no zero sequence voltages are induced in a synchronous machine.

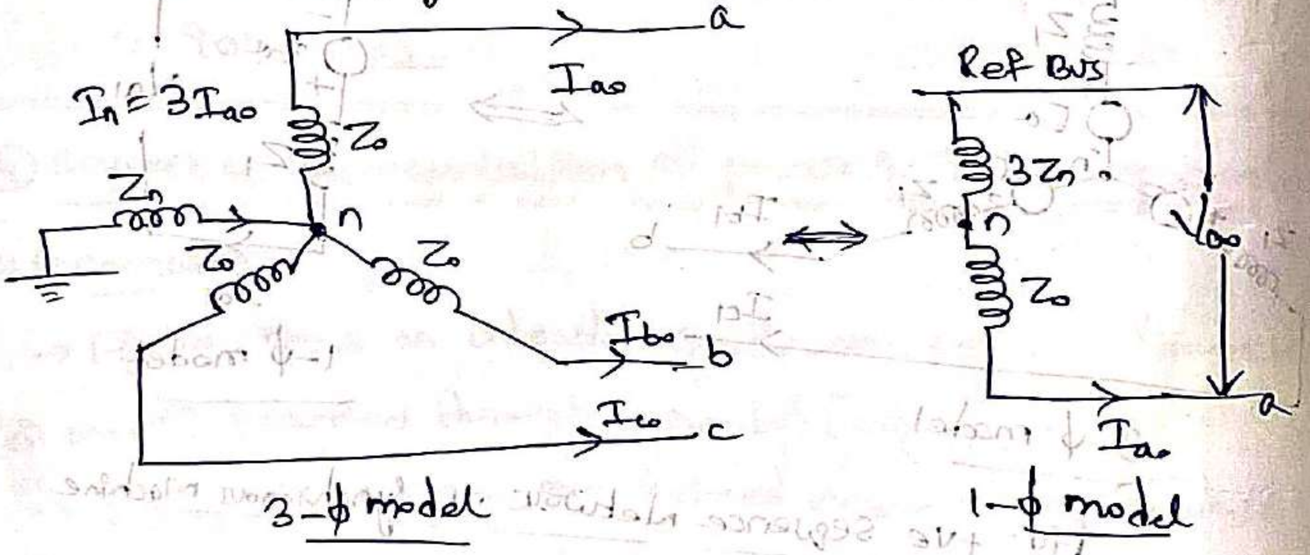


Fig: Zero sequence Network of a synchronous machine

$$V_{a0} = -3Z_n I_{a0} - Z_0 I_{a0}$$

$$V_{a0} = -(3Z_n + Z_0) I_{a0}$$

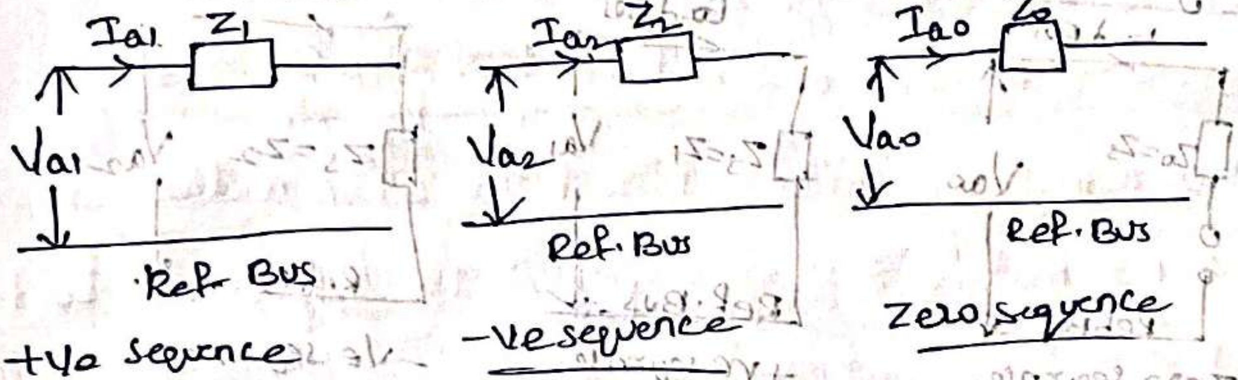
$$\boxed{V_{a0} = -Z_0 I_{a0}}$$

where, $Z_0 = 3Z_n + Z_0$

(ii) Transmission Lines:

A Fully Transposed 3-ϕ Line is completely symmetrical and therefore the per phase impedance offered by it is independent of the phase sequence of Balanced set of currents.

In other words, the Impedances offered by it to positive and Negative sequence currents are identical.



(iii) Transformer:

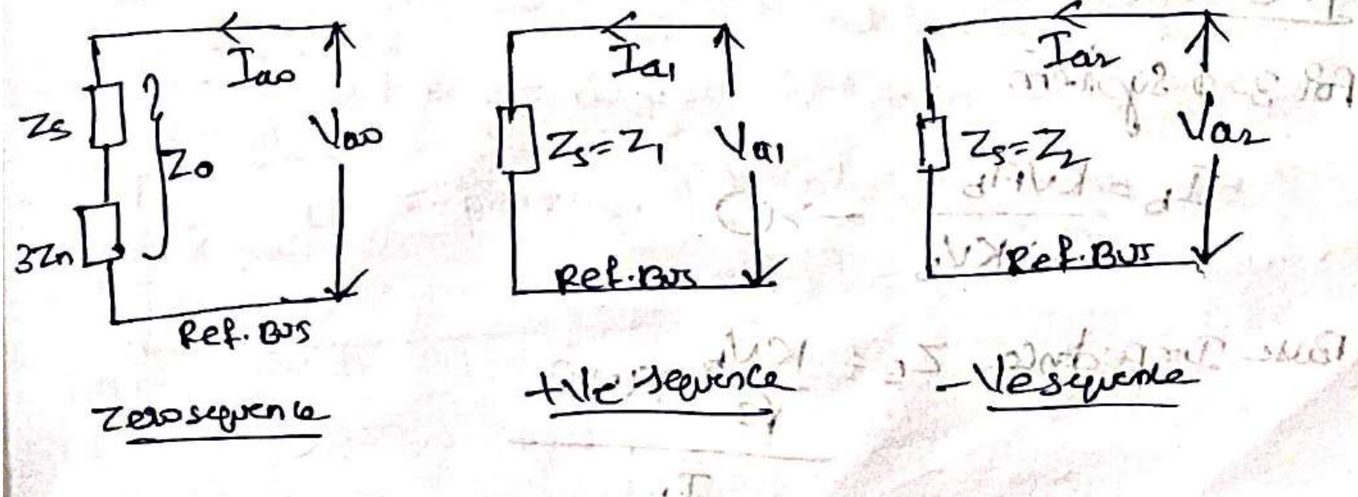
When the supplied voltage is Balanced, the +ve and -ve sequence of linear symmetrical, static devices are identical. Therefore in a Transformer the positive and negative sequence are Identical.

Zero sequence Impedance may slightly change.

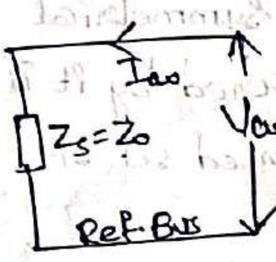
(iv) Loads:

Sequence Networks of a 3-ϕ Loads:

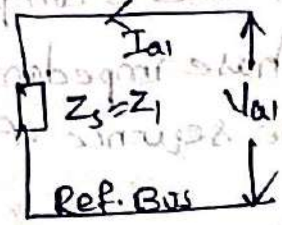
(a) Star connected Load grounded through Z_n:



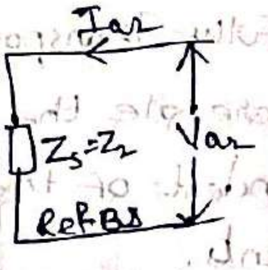
(b) Solidly Grounded 3-ϕ Load:



zero sequence

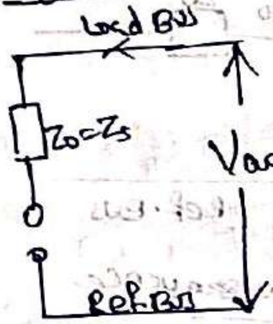


+ve sequence

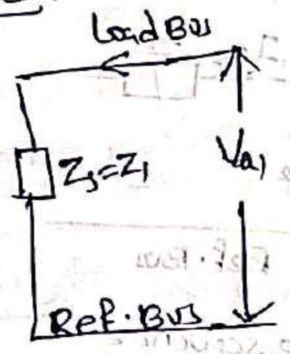


–ve sequence

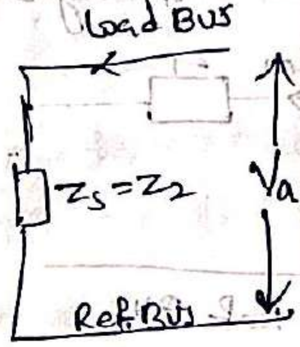
(c) Ungrounded 3-ϕ Load:



Zero sequence

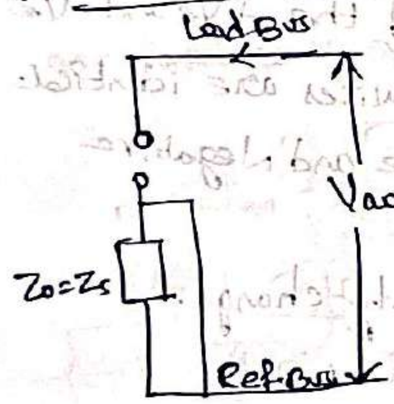


+ve sequence

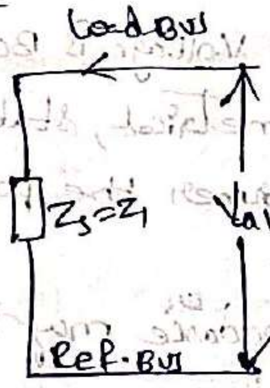


–ve sequence

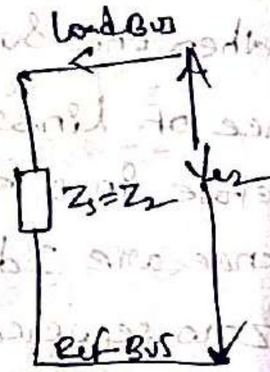
(d) Delta (Δ) Connected Load:



Zero sequence



+ve sequence



–ve sequence

Topic (1) - Continuation

For 3-ϕ system

$$I_b = \frac{KVA_b}{\sqrt{3} \times KV_b} \longrightarrow (1)$$

Base Impedance $Z_B = \dfrac{\dfrac{KV_b}{\sqrt{3}} \times 1000}{I_b}$

$$Z_B = \frac{KV_b \times 1000}{\sqrt{3}\, I_b} \rightarrow ②$$

Substitute eq ① in eq ②

$$Z_B = \frac{KV_b \times 1000}{\sqrt{3} \times \frac{KVA_b}{\sqrt{3} \times KV_b}}$$

$$Z_B = \frac{(KV_b)^2 \times 1000}{KVA_b}$$

$$\boxed{Z_B = \frac{(KV_b)^2}{MVA_b}} \rightarrow ③$$

Changing the base of per unit quantities:-

$$\text{per unit Impedance } (Z_{p.u}) = \frac{Z_a}{Z_b} = \frac{Z_a}{\frac{(KV_b)^2}{MVA_b}} = \frac{Z_a \times MVA_b}{(KV_b)^2} \rightarrow ④$$

from eq ④, $Z_a = Z_{p.u} \times \frac{KV_b^2}{MVA_b}$ (old)

$$\text{Now } Z_{p.u,new} = \frac{Z_a \times MVA_{b\,new}}{KV^2_{b\,new}} \rightarrow ⑤$$

Substitute 'Z_a' in eq ⑤

$$Z_{p.u,new} = Z_{p.u\,old} \times \frac{KV^2_{b(old)}}{MVA_{b(old)}} \times \frac{MVA_{b(new)}}{KV^2_{b(new)}}$$

$$\boxed{Z_{p.u(new)} = Z_{p.u(old)} \times \frac{KV_b^2(old)}{KV_b^2(new)} \times \frac{MVA_{b(new)}}{MVA_{b(old)}}}$$

References:

- Power System Analysis by A. Nagoor Kani, Second Edition, CBS Publisher & Distributors Pvt. Ltd.
- Modern Power System Analysis by D. P. Kothari and I. J. Nagarath, Fourth Edition, Tata McGraw-Hill.

Case Study – Application of Per Unit System in a 3-Bus Power System:

In a power system network consisting of three buses, generators, transformers, and transmission lines operate at different voltage and power levels. Direct analysis using actual values (ohms, volts, amperes) becomes complicated because of different ratings. To simplify calculations, engineers use the Per Unit (p.u.) System, where all quantities are expressed as a fraction of chosen base values.

In this case study, a **100 MVA base** and **11 kV base voltage** are selected. The reactances of generators and transmission lines are converted into **per-unit values**. After conversion, the network elements become easier to compare and analyze because their values fall within a **small numerical range (usually 0–1 p.u.)**.

Using the per-unit reactance values, the **bus admittance matrix (Y-Bus)** is formed using the **direct inspection method**. The Y-Bus matrix represents the relationship between bus currents and bus voltages in the system. It helps engineers perform **load flow studies, fault analysis, and stability studies** efficiently. This approach significantly reduces computational complexity and eliminates errors caused by transformer turns ratios. Therefore, the **per-unit system and Y-Bus matrix formation are fundamental tools in power system analysis.**

Outcome of Case Study:

- Simplifies multi-voltage level system analysis
- Reduces calculation complexity
- Helps in formation of Y-Bus matrix for network analysis
- Widely used in load flow and fault analysis studies

Unit-1 Outcomes:

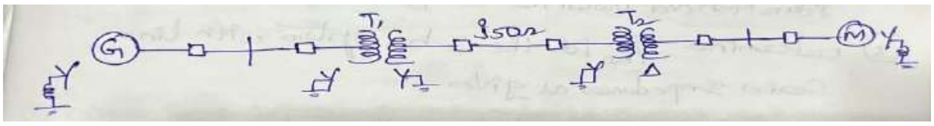
- **Understand** the concept and importance of the **per-unit system** in power systems.
- **Convert** electrical quantities into **per-unit values** using suitable base quantities.
- **Apply graph theory** to represent power system networks.
- **Construct** the **Bus Incidence Matrix** for a given network.
- **Form** the **Y-Bus Admittance Matrix** using direct and singular transformation methods.

2 Marks Questions:

1. Define Per Unit System in power systems.
2. What are the advantages of using the per unit system?
3. Write the expression for per unit impedance.
4. Define base quantities in the per unit system.
5. What are the commonly selected base quantities in power system analysis?
6. Define Bus Admittance Matrix (Y-Bus).
7. What is a Bus Incidence Matrix?
8. List the properties of the Y-Bus matrix.
9. What are the methods used for Y-Bus formation?
10. What is the difference between direct inspection method and singular transformation method?

10 Marks Questions:

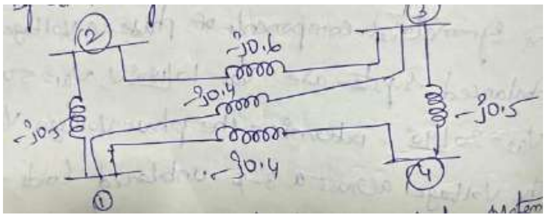
1. A. Derive the Per unit Equivalent Reactance of a 3-ϕ Power Systems
   B. Write the Advantages of Per Unit System
2. Sketch the reactance diagram for the Power System shown in the figure. The ratings of the Generator, Motor and Transformer are given below. Neglect the Resistance and use a base of 100MVA, 220KV.



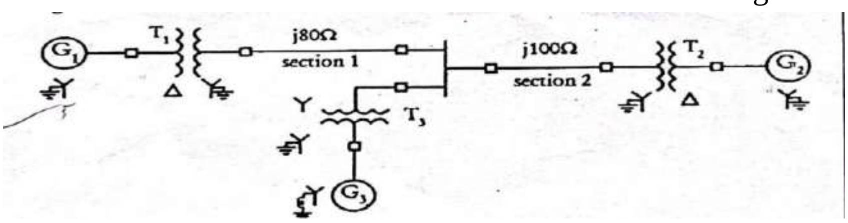
Generator: 50MVA, 25KV, $X^{II} = 20\%$

Synchronous Motor: 40MVA, 11KV, , $X^{II} = 30\%$
Y-Y Transformer: 40MVA, 33/220KV, X=15%
Y-Δ Transformer: 30MVA, 11/220KV (Δ/Y), X=15%

3. Examine the formation of Y_{Bus} Matrix using Direct Inspection Method.
4. For the Network shown in the figure form the Bus Admittance matrix. Determine the reduced Admittance matrix by eliminating Node-4. The Values marked in P.U

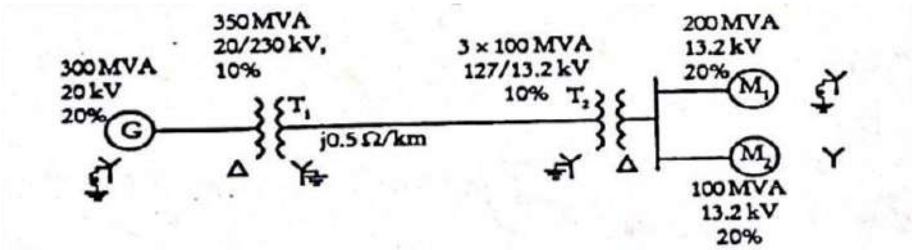


5. The Single Line diagram of an Unloaded Power System is shown in figure. Reactance's of the two sections of the transmission line are shown in the diagram.



The generators and transformers are rated as follows:
Generator G1= 20MVA, 11KV, $X^{II} = 25\%$
Generator G2= 30MVA, 18KV, $X^{II} = 25\%$
Generator G3= 30MVA, 20KV, $X^{II} = 21\%$
Transformer T1= 25MVA, 13.8/220KV, X=15%
Transformer T2= 30MVA, 220/18KV, X=15%
Transformer T3= 35MVA, 220/22KV, X=15%
Draw the Impedance diagram with all reactance marked in per unit. Choose a base of 50MVA, 11KV in the circuit of Generator G1.

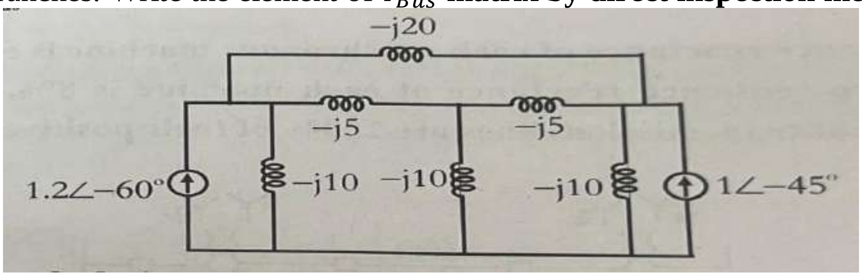
6. A 300MVA, 20KV, 3-Phase generator has a sub-transient reactance of 20%. The Generator supplies 2 synchronous motors through a 64Km transmission line having transformer at both ends as shown in figure. In this T1 is a 3-phase transformer and T2 is made of 3 single phase transformers of rating 100MVA, 127/13.2KV, 10% reactance. The reactance of the transmission line is 0.5 Ω/Km. Draw the reactance diagram with all the reactance's marked in PU. Select the generator rating as a base value.



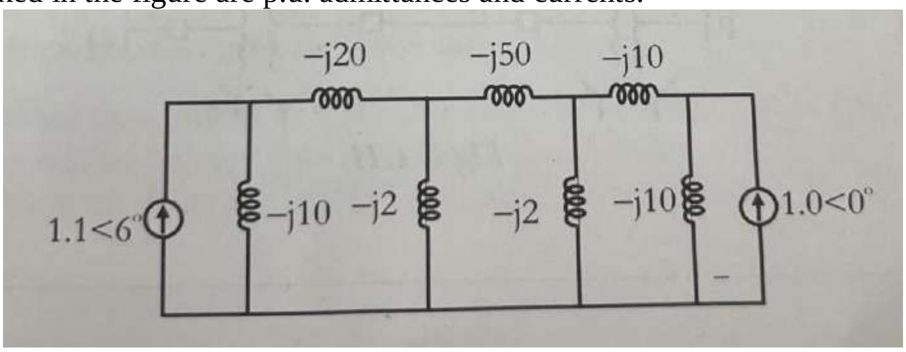
7. A 15 MVA, 10.5 kV, 3-phase generator has a synchronous reactance of 0.2 p.u. and it is connected to a transmission line through a transformer rated 15 MVA, 33/11 kV with X=0.15 p.u.

(i) Calculate the p.u. reactance by taking generator rating as a base value.
(ii) Calculate the p.u. reactance by taking transformer rating as a base value.

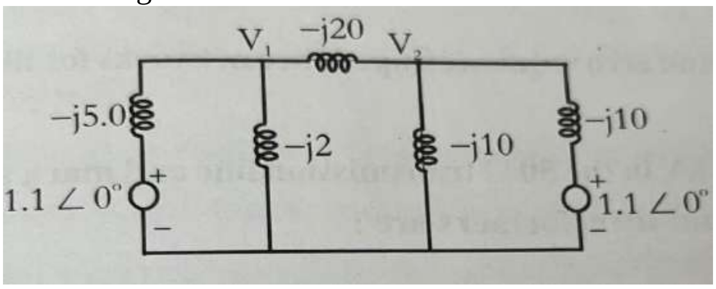
8. For the network shown in the figure. Give the total number of elements, nodes, buses and branches. Write the element of Y_{Bus} **matrix by direct inspection method.**



9. Determine the reduced admittance matrix by eliminating nodes (3) and (4). Values marked in the figure are p.u. admittances and currents.



10. A. What is Impedance and reactance diagram.
    B. Solve the node Voltages.



11. Derive the Node Equation and Bus Admittance Matrix and give its Solution to Obtain final Bus voltage equation.
12. Determine the Y bus matrix by Direct inspection method for Line specifications as mentioned below

Line p-q	Impedance (p.u)	Half line charging admittance (p.u)
1-2	0.04+j0.02	j0.05
1-4	0.05+j0.03	j0.07
1-3	0.025+j0.06	j0.08
2-4	0.08+j0.015	j0.05
3-4	0.035+j0.045	j0.02

Unit-2

Power System Network Matrices

Bus admittance matrix - Direct Inspection method, Bus Impedance matrix - formation of Z-Bus matrix for partial network, algorithm for the modification of bus impedance matrix - addition of element from a new bus to reference, new bus to an old bus, between an old bus & reference and between two old buses.

① Bus admittance matrix - Direct Inspection method :-

→ Y-Bus matrix is very Important to carryout the Load Flow study.

→ There are various methods are available to update the Y-Bus matrix.

→ Comparing to other methods Direct Inspection method is quite simple. Because it takes shorter time to compute Y-Bus matrix.

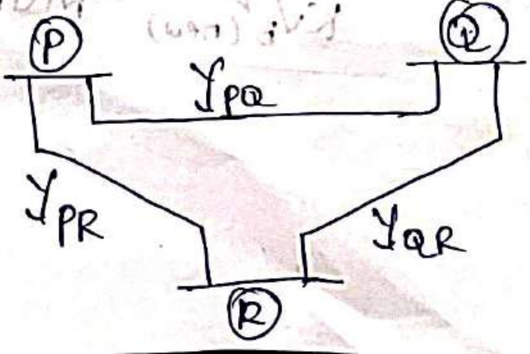
→ Direct Inspection method is applicable for the power system with mutual coupling as well as without regulating Transformer.

Steps to calculate Y-Bus matrix :

① Diagonal elements are the summation of all the elements connected to the Bus.

② Off Diagonal elements are negative of actual element present between two Buses.

Example



$$Y_{BUS} = \begin{bmatrix} Y_{pp} & Y_{pq} & Y_{pr} \\ Y_{qp} & Y_{qq} & Y_{qr} \\ Y_{rp} & Y_{rq} & Y_{rr} \end{bmatrix}_{3\times3}$$

where $n = 3$ = Number of Buses.

Now for diagonal elements from the figure, we get

$$Y_{pp} = y_{pq} + y_{pr}$$

$$Y_{qq} = y_{pq} + y_{qr}$$

$$Y_{rr} = y_{pr} + y_{qr}$$

Similarly for off-diagonal elements, we get

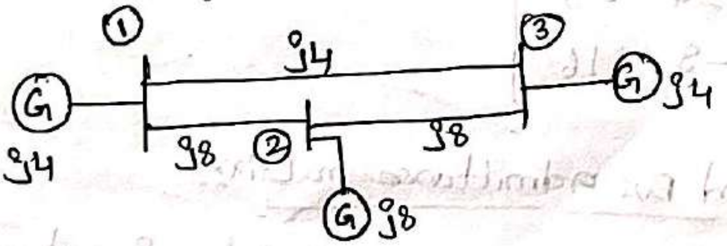
$$Y_{pq} = Y_{qp} = -y_{pq}$$

$$Y_{pr} = Y_{rp} = -y_{pr}$$

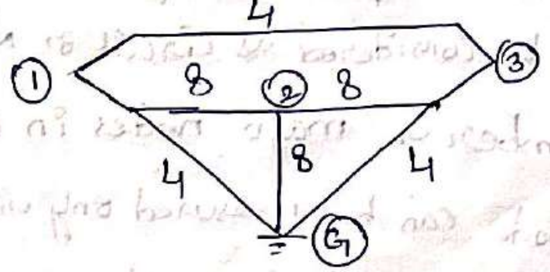
$$Y_{qr} = Y_{rq} = -y_{qr}$$

Examples: [problems]

① Obtain the bus admittance matrix for the admittance Network shown by using Direct Inspection method.



Sol:



Number of nodes $(n) = 3$

Diagonal elements are Y_{11}, Y_{22}, Y_{33}

off diagonal elements are $Y_{12}, Y_{13}, Y_{21}, Y_{23}, Y_{31}, Y_{32}$

Now, $Y_{Bus} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix}_{3\times3}$

$$\left.\begin{aligned} Y_{11} &= Y_{13} + Y_{12} + Y_{1G} \\ &= 4+8+4 \\ &= 16 \\ Y_{22} &= Y_{2G} + Y_{21} + Y_{23} \\ &= 8+8+8 \\ &= 24 \\ Y_{33} &= Y_{31} + Y_{3G} + Y_{32} \\ &= 4+4+8 \\ &= 16 \end{aligned}\right] \rightarrow \text{Diagonal elements.}$$

$$\left.\begin{aligned} Y_{12} &= Y_{21} = -8 \\ Y_{13} &= Y_{31} = -4 \\ Y_{23} &= Y_{32} = -8 \end{aligned}\right] \rightarrow \text{Off-Diagonal elements.}$$

$$\therefore Y_{Bus} = \begin{bmatrix} 16 & -8 & -4 \\ -8 & 24 & -8 \\ -4 & -8 & 16 \end{bmatrix}$$

① Node Equations and Bus Admittance matrix:-

When the power system is represented by impedance/reactance diagram, it can be considered as circuit or Network.

Let N be the number of major nodes in a circuit. Since the voltage of a node can be measured only with respect to a reference point, one of the node is considered as reference node.

Now the network will have (N-1) independent Voltages.

Let $V_1, V_2, V_3 \ldots V_n$ = Node Voltages of nodes $1, 2, 3 \ldots n$

$I_{11}, I_{22}, I_{33} \ldots I_{nn}$ = Sum of current sources connected

Y_{jj} = Sum of admittances connected to node-j

Y_{jk} = Negative of sum of admittances connected to node-j and node-k.

Now the n-number of nodal equations for N-bus system will be in the form shown below (Here $n = N-1$)

$$Y_{11}V_1 + Y_{12}V_2 + Y_{13}V_3 + \cdots + Y_{1n}V_n = I_{11}$$
$$Y_{21}V_1 + Y_{22}V_2 + Y_{23}V_3 + \cdots + Y_{2n}V_n = I_{22}$$
$$Y_{31}V_1 + Y_{32}V_2 + Y_{33}V_3 + \cdots + Y_{3n}V_n = I_{33}$$
$$\vdots \qquad \vdots$$
$$Y_{n1}V_1 + Y_{n2}V_2 + Y_{n3}V_3 + \cdots + Y_{nn}V_n = I_{nn}$$

The above equations can be arranged in matrix form, we get

$$\begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & \cdots & Y_{1n} \\ Y_{21} & Y_{22} & Y_{23} & \cdots & Y_{2n} \\ Y_{31} & Y_{32} & Y_{33} & \cdots & Y_{3n} \\ \vdots & \vdots & \vdots & & \vdots \\ Y_{n1} & Y_{n2} & Y_{n3} & \cdots & Y_{nn} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ \vdots \\ V_n \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ \vdots \\ I_n \end{bmatrix} \longrightarrow (1)$$

The above matrix can also be written as

$$\boxed{YV = I} \longrightarrow (2)$$

In power systems the Y-matrix is designated as Y_{bus}, and called "Bus-admittance matrix."

The Node Voltages are called 'Bus Voltages'.

Therefore eq(2) can be written as

$$\boxed{Y_{Bus}V = I} \longrightarrow (3)$$

$\therefore$ Bus admittance matrix

$$Y_{bus} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & \cdots & Y_{1n} \\ Y_{21} & Y_{22} & Y_{23} & \cdots & Y_{2n} \\ Y_{31} & Y_{32} & Y_{33} & \cdots & Y_{3n} \\ \vdots \\ Y_{n1} & Y_{n2} & Y_{n3} & \cdots & Y_{nn} \end{bmatrix} \longrightarrow (4)$$

→ The bus admittance matrix Y_{BUS} is symmetrical around the principal diagonal.

→ The admittances $Y_{11}, Y_{22}, Y_{33} \cdots, Y_{nn}$ are called "Self-Admittances" at the buses.

→ The other admittances are called "Mutual-Admittances".

→ In general,

Y_{jj} = Sum of all admittances connected to bus-j

Y_{Kj} (or) Y_{jK} = Negative of sum of all admittances connected between bus-j and bus-K.

Solution of Bus Voltages-

Consider the eq (3) for N-Bus system

$$Y_{BUS} V = I \longrightarrow (5)$$

Multiply both sides by Y_{BUS}^{-1} we get

$$Y_{BUS}^{-1} \cdot Y_{BUS} V = Y_{BUS}^{-1} I \qquad [a^m \times a^n = a^{m+n}]$$

$$[a^0 = 1]$$

$$Y_{BUS}^{-1+1} \cdot V = Y_{BUS}^{-1} I$$

$$Y_{BUS}^{0} \cdot V = Y_{BUS}^{-1} I$$

$$\boxed{V = Y_{BUS}^{-1} I} \longrightarrow (6)$$

$$\therefore Y_{BUS}^{-1} = \frac{\text{Adjoint of } Y_{BUS}}{\text{Determinant of } Y_{BUS}} \qquad \left[a^{-1} = \frac{\text{Adjoint of a}}{\text{Determinant of a}}\right]$$

Let Δ = Determinant of Y_{BUS}

Δ_{ij} = Cofactor of Y_{jK}

$$\text{Now, Adjoint of } Y_{BUS} = \begin{bmatrix} \Delta_{11} & \Delta_{12} & \Delta_{13} & \cdots & \Delta_{1n} \\ \Delta_{21} & \Delta_{22} & \Delta_{23} & \cdots & \Delta_{2n} \\ \Delta_{31} & \Delta_{32} & \Delta_{33} & \cdots & \Delta_{3n} \\ \vdots \\ \Delta_{n1} & \Delta_{n2} & \Delta_{n3} & \cdots & \Delta_{nn} \end{bmatrix}^T$$

$$\therefore \text{Adjoint of } Y_{BUS} = \begin{bmatrix} \Delta_{11} & \Delta_{12} & \Delta_{13} & \cdots & \Delta_{1n} \\ \Delta_{21} & \Delta_{22} & \Delta_{23} & \cdots & \Delta_{2n} \\ \Delta_{31} & \Delta_{32} & \Delta_{33} & \cdots & \Delta_{3n} \\ \vdots \\ \Delta_{1n} & \Delta_{2n} & \Delta_{3n} & \cdots & \Delta_{nn} \end{bmatrix}^T \longrightarrow (7)$$

$$\therefore Y_{BUS}^{-1} = \frac{\begin{bmatrix} \Delta_{11} & \Delta_{12} & \Delta_{13} & \cdots & \Delta_{1n} \\ \Delta_{21} & \Delta_{22} & \Delta_{23} & \cdots & \Delta_{2n} \\ \Delta_{31} & \Delta_{32} & \Delta_{33} & \cdots & \Delta_{3n} \\ \vdots \\ \Delta_{1n} & \Delta_{2n} & \Delta_{3n} & \cdots & \Delta_{nn} \end{bmatrix}^T}{\Delta} \longrightarrow (8)$$

[$\because$ Transpose each value]

Substitute eq (8) in eq (6)

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ \vdots \\ V_n \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} \Delta_{11} & \Delta_{12} & \Delta_{13} & \cdots & \Delta_{1n} \\ \Delta_{21} & \Delta_{22} & \Delta_{23} & \cdots & \Delta_{2n} \\ \Delta_{31} & \Delta_{32} & \Delta_{33} & \cdots & \Delta_{3n} \\ \vdots \\ \Delta_{1n} & \Delta_{2n} & \Delta_{3n} & \cdots & \Delta_{nn} \end{bmatrix}^T \begin{bmatrix} I_{11} \\ I_{22} \\ I_{33} \\ \vdots \\ I_{nn} \end{bmatrix} \longrightarrow (9)$$

Now Express eq (9) in n-number of Linear Independent equations.

$$V_1 = \frac{1}{\Delta}\left[\Delta_{11} I_{11} + \Delta_{21} I_{22} + \Delta_{31} I_{33} + \cdots \Delta_{n1} I_{nn}\right]$$

$$V_2 = \frac{1}{\Delta}\left[\Delta_{12} I_{11} + \Delta_{22} I_{22} + \Delta_{32} I_{33} + \cdots \Delta_{n2} I_{nn}\right]$$

$$V_3 = \frac{1}{\Delta}\left[\Delta_{13} I_{11} + \Delta_{23} I_{22} + \Delta_{33} I_{33} + \cdots \Delta_{n3} I_{nn}\right]$$

$$\vdots$$

$$V_n = \frac{1}{\Delta}\left[\Delta_{1n} I_{11} + \Delta_{2n} I_{22} + \Delta_{3n} I_{33} + \cdots \Delta_{nn} I_{nn}\right]$$

In general the K^{th}-bus Voltage is given by

$$V_K = \frac{1}{\Delta}\left[\Delta_{1K} I_{11} + \Delta_{2K} I_{22} + \Delta_{3K} I_{33} + \cdots + \Delta_{nK} I_{nn}\right] \rightarrow (10)$$

$$\therefore \boxed{V_K = \frac{1}{\Delta}\sum_{j=1}^{n} \Delta_{jK} I_{jj}}$$

where Δ = Determinant of Y_{bus} matrix

I_{jj} = Sum of currents sources injecting current to node-j.

Δ_{jK} = Co-factor of the element Y_{jK}.

Bus or node elimination by matrix algebra:-

Consider Basic equation

$$Y_{Bus} V = I \longrightarrow (1)$$

→ Now the matrices in eq (1) can be partitioned using the guidelines given below.

(i) The column matrices V and I are re-arranged such that the element associated with buses to be eliminated are in the lower rows of the matrices.

(ii) The square matrix Y_{Bus} is re-arranged such that the elements associated with buses to be eliminated are in the last rows and columns of the matrix.

(iii) The bus admittance matrix is partitioned so that elements identified only with nodes to be eliminated are separated from the other elements by horizontal and vertical lines.

→ consider a bus admittance matrix of order (5x5). Now the matrix eq (1) for n=5 written.

Let us assume that the buses 4 and 5 does not have any current source and so they can be eliminated.

$$\left[\begin{array}{ccc|cc} Y_{11} & Y_{12} & Y_{13} & Y_{14} & Y_{15} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} & Y_{25} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} & Y_{35} \\ \hline Y_{41} & Y_{42} & Y_{43} & Y_{44} & Y_{45} \\ Y_{51} & Y_{52} & Y_{53} & Y_{54} & Y_{55} \end{array}\right] \left[\begin{array}{c} V_1 \\ V_2 \\ V_3 \\ \hline V_4 \\ V_5 \end{array}\right] = \left[\begin{array}{c} I_{11} \\ I_{22} \\ I_{33} \\ \hline I_{44} \\ I_{55} \end{array}\right] \rightarrow (2)$$

Let us partition the matrix (2) and define the sub-matrices

$$K = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix};\ L = \begin{bmatrix} Y_{14} & Y_{15} \\ Y_{24} & Y_{25} \\ Y_{34} & Y_{35} \end{bmatrix};\ L^T = \begin{bmatrix} Y_{41} & Y_{42} & Y_{43} \\ Y_{51} & Y_{52} & Y_{53} \end{bmatrix}$$

$$M = \begin{bmatrix} Y_{44} & Y_{45} \\ Y_{54} & Y_{55} \end{bmatrix};\ V_A = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix};\ V_x = \begin{bmatrix} V_4 \\ V_5 \end{bmatrix}$$

$$I_A = \begin{bmatrix} I_{11} \\ I_{22} \\ I_{33} \end{bmatrix};\ I_x = \begin{bmatrix} I_{44} \\ I_{55} \end{bmatrix}$$

where,

K = Submatrix composed of self and mutual admittances identified only with buses to be retained.

M = Submatrix composed of self and mutual admittances identified only with buses to be eliminated.

L = Submatrix composed of only mutual admittances between bus to be retained and eliminated.

L^T = Transpose of L

I_x = submatrix composed of the currents entering the buses to be eliminated.

V_x = Submatrix composed of the voltages of the buses to be eliminated.

∴ using the submatrices as defined above, the matrix eq (2) can be rewritten as

$$\begin{bmatrix} K & L \\ L^T & M \end{bmatrix} \begin{bmatrix} V_A \\ V_x \end{bmatrix} = \begin{bmatrix} I_A \\ I_x \end{bmatrix} \longrightarrow (3)$$

Equation form of above matrix

$$KV_A + LV_x = I_A \longrightarrow (4)$$

$$L^T V_A + MV_x = I_x \longrightarrow (5)$$

from eq (5), we get

$$MV_x = I_x - L^T V_A \longrightarrow (6)$$

Here all the I_x are zero, because the buses to be eliminated does not have any current source.

$$MV_x = -L^T V_A \rightarrow (7)$$

On premultiplying eq (7) by M^{-1} we get

$$V_x = -M^{-1} L^T V_A \rightarrow (8)$$

Substitute eq (8) in eq (4)

$$KV_A + L\left[-M^{-1}L^T V_A\right] = I_A$$

$$V_A \left[K - LM^{-1}L^T\right] = I_A$$

$$V_A Y_{Bus,new} = I_A \longrightarrow (9)$$

where $Y_{Bus,new} = K - LM^{-1}L^T \longrightarrow (10)$

Consider a bus admittance matrix of order $(n \times n)$, in which the n^{th} bus has to be eliminated.

$$Y_{Bus} = \begin{bmatrix} \overbrace{\begin{matrix} Y_{11} & Y_{12} & \cdots & Y_{1(n-1)} \\ Y_{21} & Y_{22} & \cdots & Y_{2(n-1)} \\ \vdots & & & \\ Y_{(n-1)1} & Y_{(n-2)2} & \cdots & Y_{(n-1)(n-1)} \end{matrix}}^{K} & \overbrace{\begin{matrix} Y_{1n} \\ Y_{2n} \\ \\ Y_{(n-1)n} \end{matrix}}^{L} \\ \underbrace{\begin{matrix} Y_{n1} & Y_{n2} & \cdots & Y_{n(n-1)} \end{matrix}}_{L^T} & \underbrace{Y_{nn}}_{M} \end{bmatrix} \rightarrow (11)$$

where $K = \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1(n-1)} \\ Y_{21} & Y_{22} & \cdots & Y_{2(n-1)} \\ \vdots & & & \\ Y_{(n-1)1} & Y_{(n-2)2} & \cdots & Y_{(n-1)(n-1)} \end{bmatrix}$; $L = \begin{bmatrix} Y_{1n} \\ Y_{2n} \\ \vdots \\ Y_{(n-1)n} \end{bmatrix}$

$L^T = \begin{bmatrix} Y_{n1} & Y_{n2} & \cdots & Y_{n(n-1)} \end{bmatrix}$; $M = Y_{nn}$; $M^{-1} = \frac{1}{Y_{nn}}$

Substitute above values in eq (10)

$$Y_{bus,new} = \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1(n-1)} \\ Y_{21} & Y_{22} & \cdots & Y_{2(n-1)} \\ \vdots & & & \\ Y_{(n-1)1} & Y_{(n-2)2} & \cdots & Y_{(n-1)(n-1)} \end{bmatrix} - \begin{bmatrix} Y_{1n} \\ Y_{2n} \\ \vdots \\ Y_{(n-1)n} \end{bmatrix} \begin{bmatrix} \frac{1}{Y_{nn}} \end{bmatrix} \begin{bmatrix} Y_{n1} & Y_{n2} & \cdots & Y_{n(n-1)} \end{bmatrix}$$

$$Y_{bus,new} = \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1(n-1)} \\ Y_{21} & Y_{22} & \cdots & Y_{2(n-1)} \\ \vdots & & & \\ Y_{(n-1)1} & Y_{(n-2)2} & \cdots & Y_{(n-1)(n-1)} \end{bmatrix} - \frac{1}{Y_{nn}} \begin{bmatrix} Y_{n1}Y_{1n} & Y_{1n}Y_{n2} & \cdots & Y_{1n}Y_{n(n-1)} \\ Y_{2n}Y_{n1} & Y_{2n}Y_{n2} & \cdots & Y_{2n}Y_{n(n-1)} \\ \vdots & & & \\ Y_{(n-1)n}Y_{n1} & Y_{(n-1)n}Y_{n2} & \cdots & Y_{(n-1)n}Y_{n(n-1)} \end{bmatrix} \rightarrow (12)$$

$$\therefore \boxed{Y_{jK,new} = Y_{jK} - \frac{Y_{jn}Y_{nK}}{Y_{nn}}} \rightarrow (13)$$

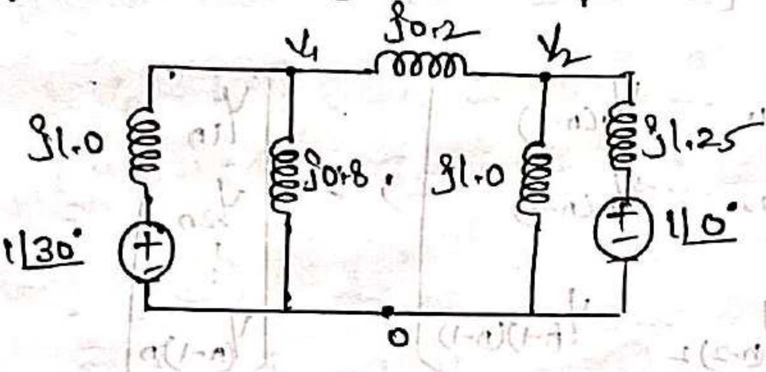
for $j = 1, 2, 3 \ldots, (n-1)$ and

$K = 1, 2, 3 \ldots, (n-1)$.

Where Y_{jK}, Y_{jn}, Y_{nK} and Y_{nn} are elements of original or given bus admittance matrix.

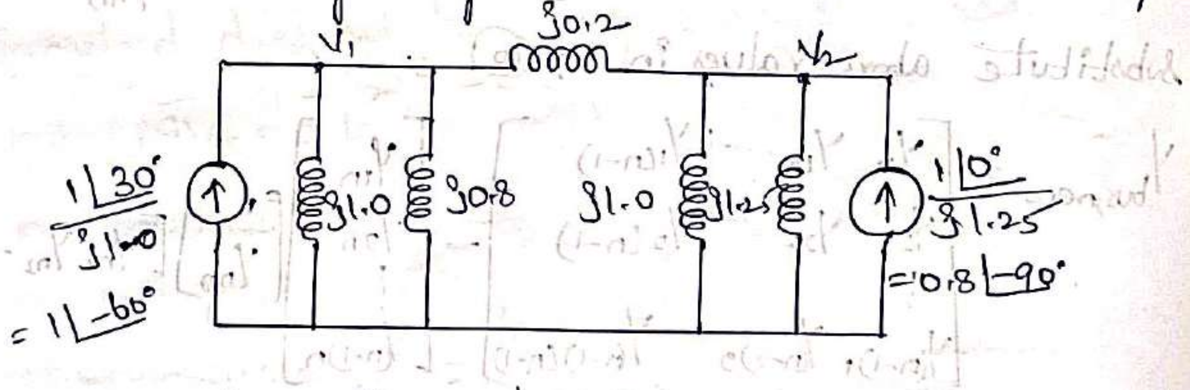
<u>Problems:</u>

① Solve the node voltages V_1 and V_2 in the network shown in the figure. The voltages and impedances are in p.u



$Z = \frac{1}{Y}$

<u>Sol:</u> The voltage sources in the network are converted into current source by using source transformation technique



Here we have two nodes, $n = 2$

Hence matrix form is

$$\begin{bmatrix} \frac{1}{j1} + \frac{1}{j0.8} + \frac{1}{j0.2} & -\frac{1}{j0.2} \\ -\frac{1}{j0.2} & \frac{1}{j0.2} + \frac{1}{j1.0} + \frac{1}{j1.25} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} \frac{1\angle 30°}{j1} \\ \frac{1\angle 0°}{1.25\angle 90°} \end{bmatrix}$$

$$\begin{bmatrix} -j1 - j1.25 - j5 & j5 \\ j5 & -j5 - j1 - j0.8 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} \frac{1\angle 30°}{j1} \\ \frac{1\angle 0}{1.25\angle 90°} \end{bmatrix}$$

$$\begin{bmatrix} -j7.25 & j5 \\ j5 & -j6.8 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 1\angle -60^\circ \\ 0.8\angle -90^\circ \end{bmatrix}$$

$$\Delta = \begin{vmatrix} -j7.25 & j5 \\ j5 & -j6.8 \end{vmatrix} = (-j7.25)(-j6.8) - (j5)(j5)$$

$$= j^2 49.3 - j^2 25$$

$$= -49.3 + 25$$

$$= 24.3$$

In node basis analysis, K^{th} node Voltage V_K is given by

$$V_K = \frac{1}{\Delta} \sum_{j=1}^{n} \Delta_{jK} I_{jj}$$

where Δ_{jK} = Cofactor of Y_{jK}.

Δ = Determinant of Y

I_{jj} = Sum of currents sources connected to node j.

Here $n=2$

$\therefore$ Cofactor $\begin{bmatrix} -j6.8 & -j5 \\ -j5 & -j7.25 \end{bmatrix}$

$$\therefore V_1 = \frac{1}{\Delta}\left[\Delta_{11} I_{11} + \Delta_{21} I_{22}\right]$$

$$= \frac{-1}{24.3}\left[(-j6.8 \times 1\angle -60^\circ) + (-j5 \times 0.8\angle -90^\circ)\right]$$

$$= \frac{-1}{24.3}\left[(6.8\angle -90^\circ \times 1\angle -60^\circ) + (5\angle -90^\circ \times 0.8\angle -90^\circ)\right]$$

$$= \frac{-1}{24.3}\left[6.8\angle -150^\circ + 4\angle -180^\circ\right]$$

$$= \frac{-1}{24.3}\left[-5.88 - 3.4j - 4\right]$$

$$= \frac{-1}{24.3}\left[-9.88 - 3.4j\right]$$

$$= \frac{9.88}{24.3} + j\frac{3.4}{24.3}$$

$$= 0.4066 + j0.1399$$

$$= 0.429 \underline{|18.98} = 0.43 \underline{|19^\circ} \text{ p.u}$$

$$\therefore V_2 = \frac{1}{\Delta}\left[\Delta_{12} I_{11} + \Delta_{22} I_{22}\right]$$

$$= \frac{-1}{24.3}\left[(-j5 \times 1\underline{|-60^\circ}) + (-j7.25 \times 0.8\underline{|-90^\circ})\right]$$

$$= \frac{-1}{24.3}\left[(5\underline{|-90^\circ} \times 1\underline{|-60^\circ}) + (7.25\underline{|-90^\circ} \times 0.8\underline{|-90^\circ})\right]$$

$$= \frac{-1}{24.3}\left[5\underline{|-150^\circ} + 5.8\underline{|-180^\circ}\right]$$

$$= \frac{-1}{24.3}\left[-4.33 - 2.5j - 5.8\right]$$

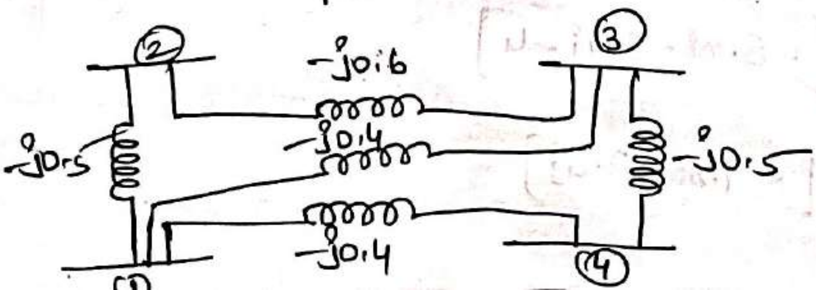
$$= \frac{-1}{24.3}\left[-10.13 - 2.5j\right]$$

$$= \frac{-10.13}{-24.3} + \frac{2.5j}{24.3}$$

$$= 0.416 + 0.10288j$$

$$= 0.428\underline{|13.89} = 0.43\underline{|14^\circ} \text{ p.u}$$

(2) For the Network shown in the figure, form the bus admittance matrix by eliminating node (4). The values are marked in p.u



Sol: The Y_{bus} matrix of the Network is,

Number of nodes = 4

$$Y_{bus} = \begin{bmatrix} -j0.5-j0.4-j0.4 & j0.5 & j0.4 & j0.4 \\ j0.5 & -j0.5-j0.6 & j0.6 & 0 \\ j0.4 & j0.6 & -j0.6-j0.5-j0.4 & j0.5 \\ j0.4 & 0 & j0.5 & -j0.5-j0.4 \end{bmatrix}$$

$$Y_{Bus} = \begin{bmatrix} -j1.3 & j0.5 & j0.4 & j0.4 \\ j0.5 & -j1.1 & j0.6 & 0 \\ j0.4 & j0.6 & -j1.5 & j0.5 \\ j0.4 & 0 & j0.5 & -j0.9 \end{bmatrix}$$

The elements of New Bus admittance matrix after eliminating the 4^{th} row and 4^{th} column is given by

$$Y_{jk,new} = Y_{jk} - \frac{Y_{jn}Y_{nk}}{Y_{nn}}$$

where $n=4$; $j=1,2,3$ and $k=1,2,3$

The Bus admittance matrix is symmetrical,

$\therefore Y_{kj,new} = Y_{jk,new}$

Case(i): $j=1, k=1, n=4$

$$Y_{11,new} = Y_{11} - \frac{Y_{14}Y_{41}}{Y_{44}}$$

$$= -j1.3 - \frac{(j0.4)(j0.4)}{-j0.9}$$

$$= -j1.3 + \frac{(0.16j^2)}{+j0.9}$$

$$\boxed{Y_{11,new} = -j1.3 + j0.17 = -j1.13}$$

Case(ii): $j=1, k=2, n=4$

$$Y_{12,new} = Y_{12} - \frac{Y_{14}Y_{42}}{Y_{44}}$$

$$\boxed{Y_{12,new} = j0.5 - \frac{(j0.4)(0)}{-j0.9} = j0.5}$$

Case(iii): $j=1, k=3, n=4$

$$Y_{13,new} = Y_{13} - \frac{Y_{14}Y_{43}}{Y_{44}}$$

$$= j0.4 - \frac{(j0.4)(j0.5)}{-j0.9}$$

$$\boxed{Y_{13,new} = j0.622}$$

Case(iv): $j=2, k=1, n=4$

$$Y_{21,new} = Y_{21} - \frac{Y_{24}Y_{41}}{Y_{44}}$$

$$= +j0.5 - \frac{(0)(j0.4)}{-j0.9}$$

$$\boxed{Y_{21,new} = j0.5}$$

Case(v): $j=2, k=2, n=4$

$$Y_{22,new} = Y_{22} - \frac{Y_{24}Y_{42}}{Y_{44}}$$

$$= -j1.1 - \frac{(0)(0)}{-j0.9}$$

$$\boxed{Y_{22,new} = -j1.1}$$

Case(vi): $j=2, k=3, n=4$

$$Y_{23,new} = Y_{23} - \frac{Y_{24}Y_{43}}{Y_{44}}$$

$$= j0.6 - \frac{(0)(j0.5)}{-j0.9}$$

$$\boxed{Y_{23,new} = j0.6}$$

Case (vii): $i=3, K=1, n=4$

$$Y_{31,new} = Y_{31} - \frac{Y_{34}Y_{41}}{Y_{44}}$$

$$= j0.4 - \frac{(j0.5)(j0.4)}{-j0.9}$$

$$= j0.4 + \frac{j^2 0.2}{j0.9}$$

$$= j0.4 + j0.222$$

$$\boxed{Y_{31,new} = j0.622}$$

Case (viii): $i=3, K=2, n=4$

$$Y_{32,new} = Y_{32} - \frac{Y_{34}Y_{42}}{Y_{44}}$$

$$= j0.6 - \frac{(j0.5)(0)}{-j0.9}$$

$$\boxed{Y_{32,new} = j0.6}$$

Case (ix): $i=3, K=3, n=4$

$$Y_{33,new} = Y_{33} - \frac{Y_{34}Y_{43}}{Y_{44}}$$

$$= -j1.5 - \frac{(j0.5)(j0.5)}{-j0.9}$$

$$= -j1.5 + \frac{j^2 0.25}{j0.9}$$

$$= -j1.5 + j0.277$$

$$\boxed{Y_{33,new} = -j1.223}$$

∴ The reduced bus admittance matrix after eliminating 4th row is given by

$$Y_{Bus} = \begin{bmatrix} -j1.2 & j0.5 & j0.622 \\ j0.5 & -j1.1 & j0.6 \\ j0.622 & j0.6 & -j1.223 \end{bmatrix}$$

② Bus Impedance matrix - formation of Z-Bus matrix for Partial Network:

Consider the node basis matrix equation representing the power system

$$Y_{Bus} V = I \longrightarrow ①$$

Multiply both sides of eq ① by Y_{Bus}^{-1}, we get

$$Y_{Bus}^{-1} Y_{Bus} V = Y_{Bus}^{-1} I$$

$$V = Y_{Bus}^{-1} I \longrightarrow ②$$

Now the elements of Y_{Bus}^{-1} will be impedances and so the matrix Y_{Bus}^{-1} can be represented by an impedance matrix called Bus impedance matrix, Z_{Bus}.

$$Z_{Bus} = Y_{Bus}^{-1} \longrightarrow ③$$

∴ substitute eq ③ in eq ②

and $\boxed{V = Z_{Bus} I} \longrightarrow ④$

Since the bus admittance matrix is symmetrical, the bus Impedance matrix is also symmetrical around the principle Diagonal.

In bus Impedance matrix, the elements on the main diagonal are called "Driving point Impedance" of the buses & nodes.

The off-Diagonal elements are called the 'Transfer Impedances' of the buses & nodes.

③ Algorithm for the modification of Bus Impedance matrix:

(or)

Modification of an existing Bus Impedance matrix:

Let us denote the original Z_{bus} of a system with n number of independent buses as Z_{orig}.

When a branch of Impedances Z_b is added to the system the Z_{orig} gets modified.

The branch Impedances Z_b can be added to the original system in the following four different ways.

$$\begin{bmatrix} N-R \\ N-E \\ E-R \\ E-E \end{bmatrix}$$

Case-1: Adding a branch of Impedance Z_b from a new bus-p to the reference bus.

Case-2: Adding a branch of Impedance Z_b from a new bus-p to an existing bus-q.

Case-3: Adding a branch of Impedance Z_b from an existing bus-q to reference bus.

Case-4: Adding a branch of Impedance Z_b between two existing buses h and q.

Case 1: Adding Z_b from a new-bus-p to the Reference Bus:-

→ Consider a n-bus system as shown in the figure.

→ Let us add a bus-p through an Impedance Z_b to the Reference bus.

→ The addition of a bus will increase the order of the Bus Impedances.

→ In this case the elements of $(n+1)^{th}$ column and row are all zeros expect Diagonal.

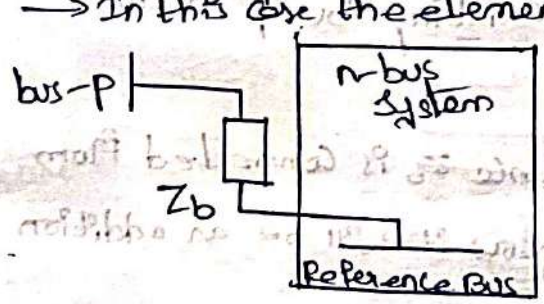


fig: Adding a new bus through an Impedance to reference BUS

$$Z_{bus,new} = \left[\begin{array}{c|c} Z_{orig} & \begin{matrix} 0 \\ 0 \\ \vdots \\ 0 \end{matrix} \\ \hline \begin{matrix} 0 & 0 & \cdots & 0 \end{matrix} & Z_b \end{array}\right] \rightarrow ①$$

→ The Diagonal elements is the added Branch Impedance Z_b.

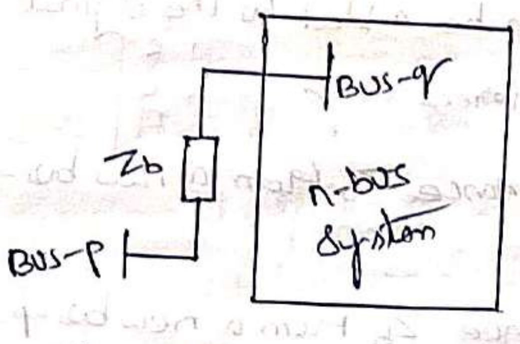
→ The elements of original Z_{bus} matrix are not altered.

Case 2: Adding Z_b from a new bus-p to an existing Bus-q

→ Consider a n-bus system as shown in the figure.

→ In which a new-Bus-p is added through an Impedance Z_b to an existing Bus-q.

→ The addition of a bus will increase the order of the bus Impedance matrix by one.



$$Z_{bus,new} = \left[\begin{array}{cccc|c} & & & & Z_{1q} \\ & Z_{orig} & & & Z_{2q} \\ & & & & \vdots \\ & & & & Z_{nq} \\ \hline Z_{q1} & Z_{q2} & \cdots & Z_{qn} & Z_{qq}+Z_b \end{array}\right] \rightarrow (1)$$

Fig: Adding a New-Bus through an Impedance to an existing Bus

→ In this elements of $(n+1)^{th}$ column are the elements of q^{th} column and the elements of $(n+1)^{th}$ row are the elements q^{th} row.

→ The diagonal elements are given by sum of $Z_{qq}+Z_b$

→ The elements of original Z_{bus} matrix are not altered.

Case 3: Adding Z_b from an existing bus-q to the reference Bus

→ Consider a n-bus system shown in the figure, in which an Impedance Z_b is added from an existing bus-q to the reference bus.

→ Let us consider as if it is impedance Z_b is connected from a new bus-p and existing bus-q. Now it will be an addition as that of case-2.

→ The new bus Impedance matrix order $(n+1)$ can be formed as that of case-2

→ Then we can short-circuit the bus-q to the reference bus.

→ This is equivalent to eliminating $(n+1)^{th}$ bus [i.e., bus-p in this case) and so Bus Impedance matrix has to be modified by eliminating $(n+1)^{th}$ row and $(n+1)^{th}$ column.

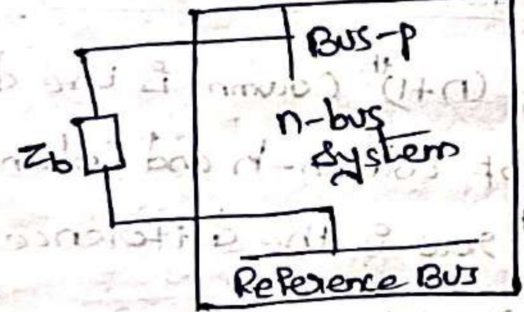


Fig: Adding an Impedance between existing & reference bus

→ The reduced bus Impedance matrix can be formed by a procedure similar to that of Bus elimination in Bus admittance matrix.

$$\therefore Z_{jk,act} = Z_{jk} - \frac{Z_{j(n+1)} Z_{(n+1)k}}{Z_{(n+1)(n+1)}}$$

Note:

(i) $Z_{jk,act}$ is the Impedance corresponding to row-j and column-k of actual new bus Impedance matrix.

(ii) $Z_{jk}, Z_{(n+1)k}, Z_{j(n+1)}, Z_{(n+1)(n+1)}$ are Impedance of new bus Impedance matrix of order $(n+1)$.

(iii) Since Bus Impedance matrix is symmetrical

$$Z_{jk,act} = Z_{kj,act}$$

Case 4: Adding Z_b between two existing buses h and q

→ Consider a n-bus system shown in the figure, in which Impedance Z_b is added in between two existing buses h and q.

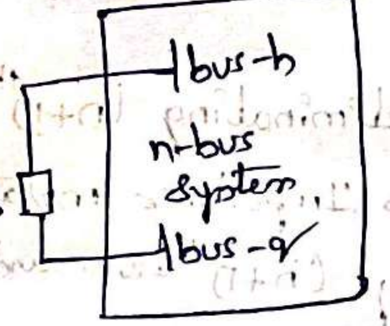


Fig: Adding an Impedance between bus-h and bus-q

→ Here the elements of $(n+1)^{th}$ column is the difference between the elements of column-h and column-q.

→ The elements of $(n+1)^{th}$ row is the differience between the elements of row-h and row-q.

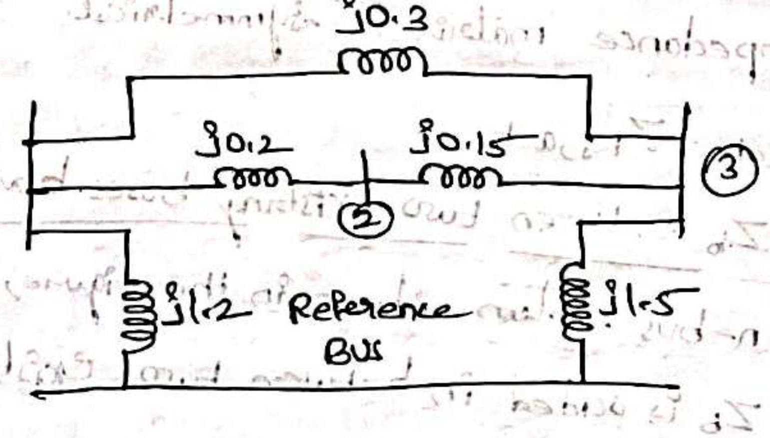
→ The diagonal elements is given by $Z_{(n+1)(n+1)}$

$$Z_{bus,new} = \left[\begin{array}{cccc|c} & & & & Z_{1h}-Z_{1q} \\ & & & & Z_{2h}-Z_{2q} \\ & Z_{orig} & & & \vdots \\ & & & & Z_{nh}-Z_{nq} \\ \hline Z_{h1}-Z_{q1} & Z_{h2}-Z_{q2} & \cdots & Z_{hn}-Z_{qn} & Z_{(n+1)(n+1)} \end{array}\right]$$

$$\therefore Z_{(n+1)(n+1)} = Z_b + Z_{hh} + Z_{qq} - 2Z_{hq}$$

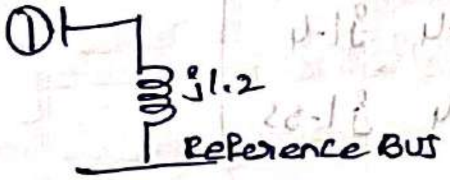
Problems:

(1) Determine Z_{bus} for the system whose reactance diagram is shown in the figure, where the Impedances is given in p.u. preserve all the three nodes.



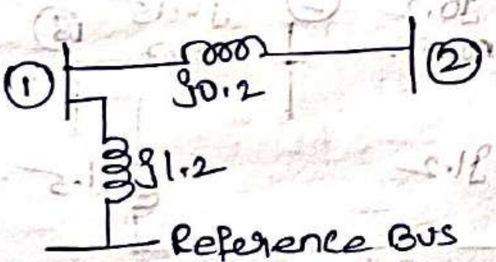
Sol:

Step-1: Consider the branch with Impedance $j1.2$ p.u. Connected between bus-1 and reference as shown in the figure. The system shown in the figure has a single bus and so the order of Bus Impedance matrix is on, as shown below



$\therefore Z_{bus} = [j1.2]$

Step-2: Connect bus-2 to bus-1 through an Impedance $j0.2$ as shown in the figure. This is $\boxed{\text{case-2}}$ modification and so order of matrix increase by one.



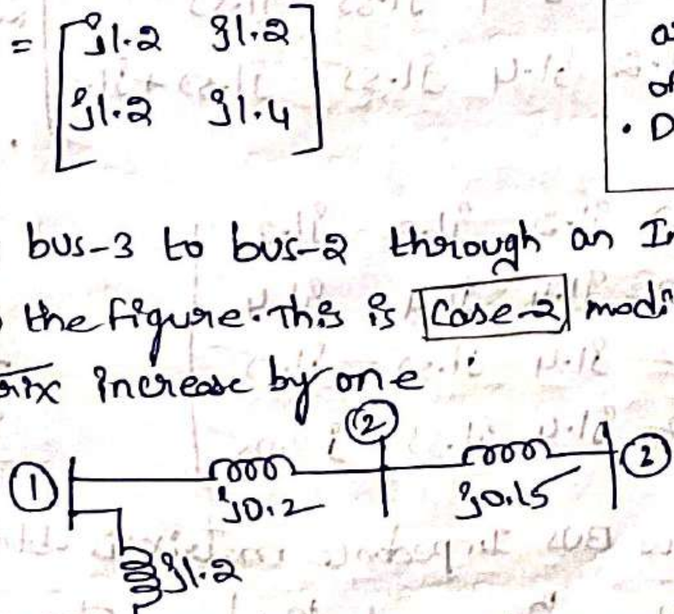
Reference purpose only

Note: • In New Bus Impedance matrix the elements of 1st column are copied as elements of 2nd column
• The elements of 1st row are copied as elements of 2nd row.
• Diagonal elements is $Z_{11}+Z_b$

$$\therefore Z_{Bus} = \begin{bmatrix} j1.2 & j1.2 \\ j1.2 & j1.2+j0.2 \end{bmatrix}$$

$$= \begin{bmatrix} j1.2 & j1.2 \\ j1.2 & j1.4 \end{bmatrix}$$

Step3: Connect bus-3 to bus-2 through an Impedance $j0.15$ as shown in the figure. This is $\boxed{\text{case-2}}$ modification and so order of matrix increase by one



$$Z_{BUS} = \begin{bmatrix} j1.2 & j1.2 & j1.2 \\ j1.2 & j1.4 & j1.4 \\ j1.2 & j1.4 & j1.4 + j0.15 \end{bmatrix}$$

$$= \begin{bmatrix} j1.2 & j1.2 & j1.2 \\ j1.2 & j1.4 & j1.4 \\ j1.2 & j1.4 & j1.55 \end{bmatrix}$$

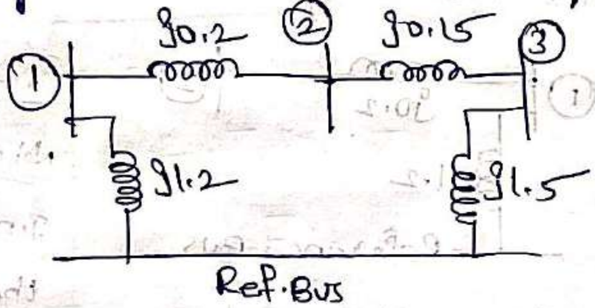
Reference purpose only

Note:

- In the new matrix the elements of 2^{nd} row are copied as element of 3^{rd} row.
- Diagonal element is given by $Z_{22} + Z_b$

Step 4: Connect the impedance $j1.5$ from BUS (3) to reference bus as shown in the figure. This is $\boxed{\text{Case-3}}$ modification. In case-2 and then the last row and column are eliminated by node elimination Technique.

~~Zbus~~



$$Z_{BUS} = \begin{bmatrix} j1.2 & j1.2 & j1.2 & j1.2 \\ j1.2 & j1.4 & j1.4 & j1.4 \\ j1.2 & j1.4 & j1.55 & j1.55 \\ j1.2 & j1.4 & j1.55 & j1.55 + j1.5 \end{bmatrix}$$

$$= \begin{bmatrix} j1.2 & j1.2 & j1.2 & j1.2 \\ j1.2 & j1.4 & j1.4 & j1.4 \\ j1.2 & j1.4 & j1.55 & j1.55 \\ j1.2 & j1.4 & j1.55 & j3.05 \end{bmatrix}$$

Reference purpose only

Note:

- In the new matrix 3^{rd} column copied as 4^{th} column, 3^{rd} row copied as 4^{th} row
- Diagonal element is $Z_{33} + Z_b$

The actual new BUS Impedance matrix is obtained by eliminating the 4^{th} row and 4^{th} column. The element Z_{jk} of the actual new bus Impedance matrix is given by

$$\therefore Z_{jK,act} = Z_{jK} - \frac{Z_{j(n+1)} Z_{(n+1)K}}{Z_{(n+1)(n+1)}}$$

where $n=3$; $j=1,2,3$ and $K=1,2,3$

Note:

Cases formation

$$\begin{bmatrix} jk & jk & jk \\ 11 & 12 & 13 \\ jk & jk & jk \\ 21 & 22 & 23 \\ jk & jk & jk \\ 31 & 32 & 33 \end{bmatrix}$$

C-1: $j=1, K=1, n=3$

$$Z_{11,act} = Z_{11} - \frac{Z_{1(3+1)} Z_{(3+1)1}}{Z_{(3+1)(3+1)}}$$

$$= Z_{11} - \frac{Z_{14} Z_{41}}{Z_{44}}$$

$$= j1.2 - \frac{(j1.2)(j1.2)}{j3.05}$$

$$= j1.2 - \frac{(1.44j^2)}{j3.05} \qquad [j^2=-1]$$

$$= j1.2 - \frac{(\cancel{-}1.44)}{\cancel{j}3.05}$$

$$= j1.2 - j0.472$$

$$\boxed{Z_{11,act} = j0.728}$$

C-2: $j=1, K=2, n=3$

$$Z_{12,act} = Z_{12} - \frac{Z_{1(3+1)} Z_{(3+1)2}}{Z_{(3+1)(3+1)}}$$

$$= Z_{12} - \frac{Z_{14} Z_{42}}{Z_{44}}$$

$$= j1.2 - \frac{(\cancel{j}1.2)(j1.4)}{\cancel{j}3.05}$$

$$= j1.2 - \frac{j1.68}{3.05}$$

$$= j1.2 - j0.55$$

$$\boxed{Z_{12,act} = j0.65}$$

<u>case-3</u>: $j=1, k=3, n=3$

$$Z_{13,act} = Z_{13} - \frac{Z_{1(3+1)}Z_{(3+1)3}}{Z_{(3+1)(3+1)}}$$

$$= Z_{13} - \frac{Z_{14}Z_{43}}{Z_{44}}$$

$$= j1.2 - \frac{(j1.2)(j1.55)}{j3.05}$$

$$= j1.2 - j0.609$$

$$\boxed{Z_{13,act} = j0.591}$$

<u>case-4</u>: $j=2, k=1, n=3$

$$\boxed{Z_{21,act} = Z_{12,act} = j0.65}$$

<u>case-5</u>: $j=2, k=2, n=3$

$$Z_{22,act} = Z_{22} - \frac{Z_{2(3+1)}Z_{(3+1)2}}{Z_{(3+1)(3+1)}}$$

$$= Z_{22} - \frac{Z_{24}Z_{42}}{Z_{44}}$$

$$= j1.4 - \frac{(j1.4)(j1.4)}{j3.05}$$

$$= j1.4 - j0.642$$

$$\boxed{Z_{22,act} = j0.758}$$

<u>case-6</u>: $j=2, k=3, n=3$

$$Z_{23,act} = Z_{23} - \frac{Z_{2(3+1)}Z_{(3+1)(3)}}{Z_{(3+1)(3+1)}}$$

$$= Z_{23} - \frac{Z_{24}Z_{43}}{Z_{44}}$$

$$= j1.4 - \frac{(j1.4)(j1.55)}{j3.05}$$

$= j1.4 - j0.711$

$$\boxed{Z_{23,act} = j0.689}$$

<u>case 7:</u> $j=3, K=1, n=3$

$$\boxed{Z_{31,act} = Z_{13,act} = j0.591}$$

<u>case 8:</u> $j=3, K=2, n=3$

$$\boxed{Z_{32,act} = Z_{23,act} = j0.689}$$

<u>case 9:</u> $j=3, K=3, n=3$

$$Z_{33,act} = Z_{33} - \frac{Z_{3(3+1)} Z_{(3+1)3}}{Z_{(3+1)(3+1)}}$$

$$= Z_{33} - \frac{Z_{34} Z_{43}}{Z_{44}}$$

$$= j1.55 - \frac{(\cancel{j}1.55)(j1.55)}{\cancel{j}3.05}$$

$$= j1.55 - j0.787$$

$$\boxed{Z_{33,act} = j0.763}$$

$$\therefore Z_{BUS} = \begin{bmatrix} j0.728 & j0.685 & j0.591 \\ j0.685 & j0.758 & j0.689 \\ j0.591 & j0.689 & j0.763 \end{bmatrix}$$

<u>Step-5:</u> Connect the Impedance $j0.3$ between bus-1 and bus-3 as shown in figure. This is $\boxed{\text{Case-4}}$ modification.

<u>Note:</u> • The elements of <u>4th column</u> are obtained by substracting the elements of <u>3rd column</u> from <u>1st column</u>.

• The elements of <u>4th row</u> are obtained by substracting the elements of <u>3rd row</u> from <u>1st row</u>.

• Diagonal element $Z_{44} = Z_b + Z_{11} + Z_{33} - 2Z_{13}$

Now, $Z_{44} = j0.3 + j0.728 + j0.763 - 2(j0.591)$

$$Z_{44} = j0.609$$

$$\therefore Z_{bus} = \begin{bmatrix} j0.728 & j0.65 & j0.591 & j0.728 - j0.591 \\ j0.65 & j0.758 & j0.689 & j0.65 - j0.689 \\ j0.591 & j0.689 & j0.763 & j0.591 - j0.763 \\ j0.728 - j0.591 & j0.65 - j0.689 & j0.591 - j0.763 & j0.609 \end{bmatrix}$$

$$Z_{bus} = \begin{bmatrix} j0.728 & j0.65 & j0.591 & j0.137 \\ j0.65 & j0.758 & j0.689 & -j0.039 \\ j0.591 & j0.689 & j0.763 & -j0.172 \\ j0.137 & -j0.039 & -j0.172 & j0.609 \end{bmatrix}$$

Since the modification does not add a new bus, the 4^{th} row and column has to be eliminated using node elimination Technique

$$Z_{jk,act} = Z_{jk} - \frac{Z_{j(n+1)} Z_{(n+1)k}}{Z_{(n+1)(n+1)}}$$

where $n=3$, $j=1,2,3$ and $k=1,2,3$

C-1: $j=1, k=1, n=3$

$$Z_{11,act} = Z_{11} - \frac{Z_{1(3+1)} Z_{(3+1)1}}{Z_{(3+1)} Z_{(3+1)}}$$

$$= Z_{11} - \frac{Z_{14} Z_{41}}{Z_{44}}$$

$$= j0.728 - \frac{(j0.137)(j0.137)}{j0.609}$$

$= j0.728 - j0.031$

$$\boxed{Z_{11,act} = j0.697}$$

C-2: $j=1, k=2, n=3$

$$Z_{12,act} = Z_{12} - \frac{Z_{1(3+1)} Z_{(3+1)2}}{Z_{(3+1)(3+1)}}$$

$$= Z_{12} - \frac{Z_{14} Z_{42}}{Z_{44}}$$

$$= j0.65 - \frac{(j0.137)(-j0.039)}{j0.609}$$

$$= j0.65 + j8.77\times10^{-3}$$

$$\boxed{Z_{12,act} = j0.658}$$

C-3: $j=1, k=3, n=3$

$$Z_{13,act} = Z_{13} - \frac{Z_{1(3+1)} Z_{(3+1)3}}{Z_{(3+1)(3+1)}}$$

$$= Z_{13} - \frac{Z_{14} Z_{43}}{Z_{44}}$$

$$= j0.591 - \frac{(j0.137)(-j0.172)}{j0.609}$$

$$= j0.591 + j0.038$$

$$\boxed{Z_{13,act} = j0.629}$$

C-4: $j=2, k=1, n=3$

$$\boxed{Z_{21,act} = Z_{12,act} = j0.658}$$

C-5: $j=2, K=2, n=3$

$$Z_{22,act} = Z_{22} - \frac{Z_{2(3+1)} Z_{(3+1)2}}{Z_{(3+1)(3+1)}}$$

$$= j0.758 - \frac{Z_{24} Z_{42}}{Z_{44}}$$

$$= j0.758 - \frac{(-j0.039)(-j0.039)}{j0.609}$$

$$= j0.758 - j2.497 \times 10^{-3}$$

$$\boxed{Z_{22,act} = j0.755}$$

C-6: $j=2, K=3, n=3$

$$Z_{23,act} = Z_{23} - \frac{Z_{2(3+1)} Z_{(3+1)3}}{Z_{(3+1)(3+1)}}$$

$$= Z_{23} - \frac{Z_{24} Z_{43}}{Z_{44}}$$

$$= j0.689 - \frac{(-j0.039)(-j0.172)}{j0.609}$$

$$= j0.689 - j0.011$$

$$\boxed{Z_{23,act} = j0.678}$$

C-7: $j=3, K=1, n=3$

$$\boxed{Z_{31,act} = Z_{13,act} = j0.629}$$

C-8: $j=3, K=2, n=3$

$$\boxed{Z_{32,act} = Z_{23,act} = j0.678}$$

C-9: $j=3, k=3, n=3$

$$Z_{33,act} = Z_{33} - \frac{Z_{3(3+1)} Z_{(3+1)3}}{Z_{(3+1)(3+1)}}$$

$$= Z_{33} - \frac{Z_{34} Z_{43}}{Z_{44}}$$

$$= j0.763 - \frac{(-\cancel{j}0.172)(-j0.172)}{\cancel{j}0.609}$$

$$= j0.763 - j0.048$$

$$\boxed{Z_{33,act} = j0.715}$$

$$\therefore Z_{Bus} = \begin{bmatrix} j0.697 & j0.658 & j0.629 \\ j0.658 & j0.755 & j0.678 \\ j0.629 & j0.678 & j0.715 \end{bmatrix}$$

References:

- Power System Analysis by A. Nagoor Kani, Second Edition, CBS Publisher & Distributors Pvt. Ltd.
- Modern Power System Analysis by D. P. Kothari and I. J. Nagarath, Fourth Edition, Tata McGraw-Hill.

Case Study

Topic: Formation of Z-Bus Matrix

Z-Bus Matrix Formation for Network Modification

In practical power systems, transmission networks frequently change due to the addition of new buses, transmission lines, or system expansion. These changes require recalculation of the system parameters to maintain proper power flow and stability analysis.

Consider a power system where initially two buses are connected by a transmission line. The system operator decides to add a new bus to supply power to a growing industrial area. With the addition of this new bus and its connecting transmission line, the system impedance matrix must be updated.

Instead of recomposing the entire impedance matrix from the beginning, the Z-Bus building algorithm is used. This algorithm modifies the existing Z-Bus matrix step-by-step when new elements are added to the network.

Different cases are considered during Z-Bus formation, such as:

- Addition of a branch from a **new bus to reference bus**
- Addition of a branch from a **new bus to an existing bus**
- Addition of a branch from an **existing bus to reference bus**
- Addition of a branch between **two existing buses**

By applying the Z-Bus modification algorithm, engineers can efficiently update the network parameters without performing complete recalculations. This approach saves time, computational effort, and improves system analysis efficiency.

This method is widely used in power system fault analysis, load flow studies, and network planning.

Unit-2 Outcomes:

- Understand the **concept of Bus Impedance Matrix (Z-Bus)**.
- Explain the **steps involved in Z-Bus formation**.
- Identify different **cases involved in network modification** during Z-Bus building.

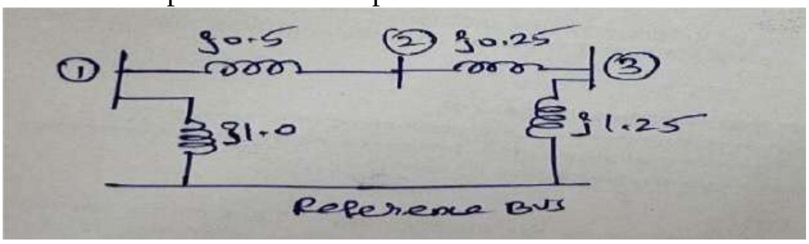
- Apply the **Z-Bus building algorithm for addition of elements in a power system network**.
- Analyze how **changes in network configuration affect system impedance matrix**.

2 Marks Questions:

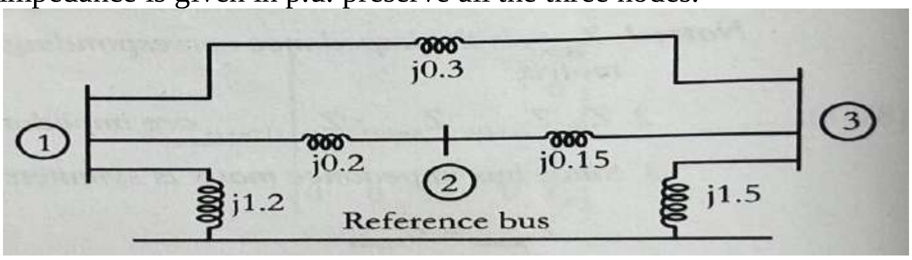
1. Define Bus Impedance Matrix (Z-Bus).
2. What is the significance of Z-Bus matrix in power system analysis?
3. List the different cases involved in Z-Bus matrix formation.
4. What is meant by a reference bus in Z-Bus formation?
5. What is the difference between Y-Bus and Z-Bus matrices?
6. What is meant by the addition of a branch from a new bus to the reference bus?
7. What is the procedure for adding a branch between an existing bus and a new bus in Z-Bus building algorithm?
8. What is meant by modification of Z-Bus matrix?
9. State the advantages of using the Z-Bus building algorithm.
10. Why is the Z-Bus matrix symmetrical?

10 Marks Questions:

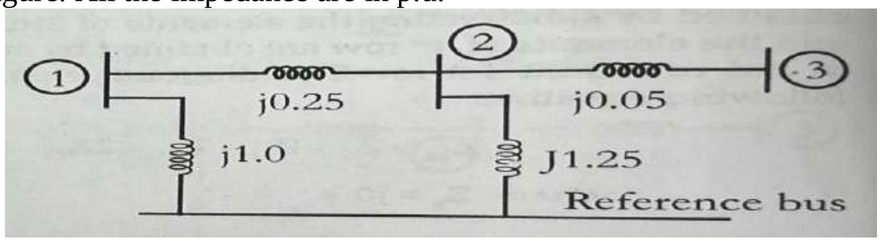
1. Develop an Algorithm for the bus impedance matrix formation and modification
2. Find the bus impedance matrix for the system whose reactance diagram is shown in the figure. All the impedances are in p.u.



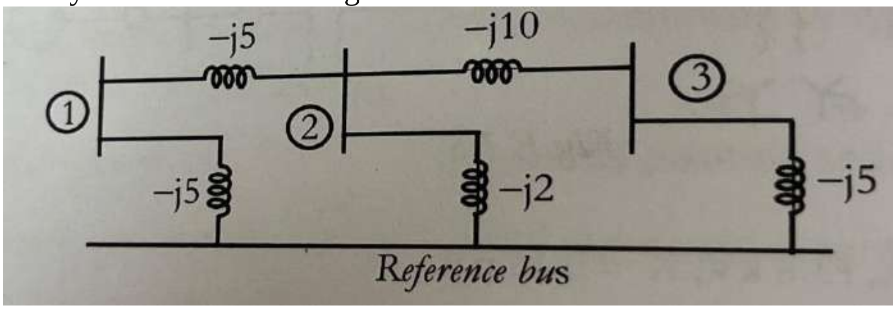
3. Determine Z bus for the system whose reactance diagram is shown in the figure. Where the impedance is given in p.u. preserve all the three nodes.



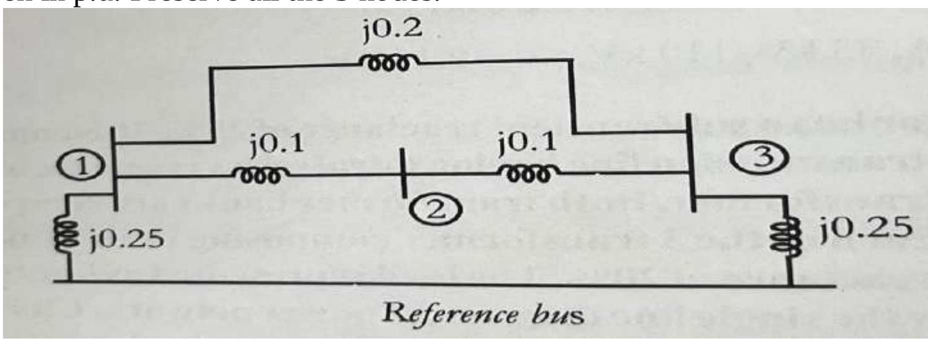
4. Find the bus impedance matrix for the system whose reactance diagram is shown in the figure. All the impedance are in p.u.



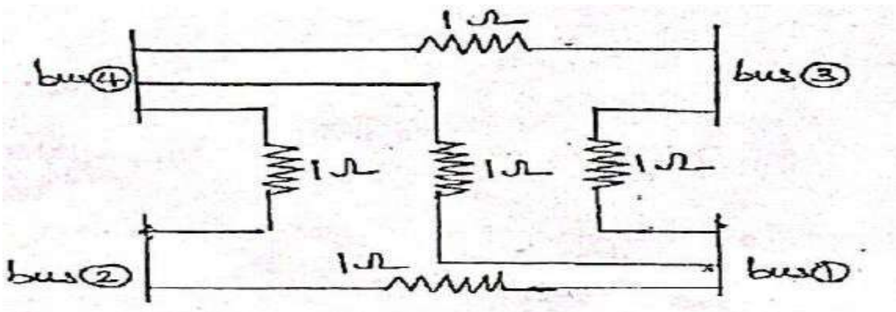
5. A. Write the four ways of adding an impedance to an existing system so as to modify bus impedance matrix.
   B. For the system shown in the figure determine Z bus.



6. Determine the Z bus for the network shown in the figure. Where the impedances are given in p.u. Preserve all the 3 nodes.



7. Find the bus impedance matrix for the 4-bus system shown in the figure. Consider bus-4 as reference bus.



UNIT-3 - POWER FLOW STUDIES

→ INTRODUCTION :-

The load flow studies involves the solution of power system network under steady state condition.

* The Solution can be obtain based on load, node voltages and reactive power of generators.
* A load flow study of power system generally requires the following steps.
  1) Representation of complete network by single line diagram
  2) Determining the impedence diagram using the information in single line diagram.
  3) formation of network equations.
  4) Solution of network equation.

Types of Buses :-

→ In power system the bus is nothing but meeting point of Various Components.

→ The generator with feed energy to the buses and load will ~~drop~~ draw energy from the buses.

→ In the network of power system the buses becomes nodes.

→ Each bus is acting as one node and hence voltage can be Specified for each bus.

→ Therefore each bus in a power system is associated with four quantities. They are real power, reactive Power, magnitude of voltage and phase angle.

→ The buses of a power system is classified into 3 types

They are 1) Load bus (or) PQ Bus
2) Generator bus (or) PV Bus (or) Voltage Controlled B
3) Slack bus (or) Refference Bus (or) Swing Bus.

Bus-Type	Quantities Specified	Quantities to be obtained
Load Bus	P, Q	$\lvert V \rvert$, δ
Generator Bus	P, $\lvert V \rvert$	Q, δ
Slack Bus	$\lvert V \rvert$, δ	P, Q

Load Bus :-

When real and reactive Components of power are specified quantities, the load flow equation can be solved to find magnitude of Voltage and phase angle of Voltage. In load bus the voltage is allow to varry within the permissible limits ($\pm 5\%$).

Generator Bus :-

When real power and magnitude of voltages are Specified quantities, the load flow equation can be Solved to find reactive power and Phase angle of voltage. Usually for generator bus reactive power limit will be Specified.

Slack Bus :-

When magnitude of voltage and phase angle of voltages are Specified quantities, the load flow equation can be Solved to find real and reactive Components of power.

→ The Slack bus is the reference bus for load flow Solution and usually one of the generator bus is selected as Slack bus.

→ In a power System, the total power generated will be equals to Sum of power consume by loads and losses.

$$\therefore \begin{pmatrix} \text{Sum of complex} \\ \text{power of generator} \end{pmatrix} = \begin{pmatrix} \text{Sum of complex} \\ \text{power of loads} \end{pmatrix} + \begin{pmatrix} \text{Total (complex) Power} \\ \text{losses in Transmission} \\ \text{line} \end{pmatrix}$$

Derivation of static load flow equations.

The node Basic matrix equation of a n-bus system is given by

$$Y_{Bus} \cdot V = I \longrightarrow ①$$

Where Y_{Bus} = Bus admittance matrix of order $(n \times n)$

V = Bus (or) node voltage matrix of order $(n \times n)$

I = Source current matrix of order $(n \times n)$

$\therefore$ eq ①, can be written as ;

$$\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_p \\ \vdots \\ I_n \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1p} & \cdots & Y_{1n} \\ Y_{21} & Y_{22} & \cdots & Y_{2p} & \cdots & Y_{2n} \\ \vdots & \vdots & \cdots & & & \\ Y_{p1} & Y_{p2} & \cdots & Y_{pp} & \cdots & Y_{pn} \\ \vdots & \vdots & & & & \\ Y_{n1} & Y_{n2} & \cdots & Y_{np} & \cdots & Y_{nn} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_p \\ \vdots \\ V_n \end{bmatrix} \longrightarrow ②$$

Where V_p = Voltage across Bus-p

I_p = Current Injected at Bus-p

From eq ②: The Current I_p can be expressed as

$$I_p = Y_{p1}V_1 + Y_{p2}V_2 + Y_{p3}V_3 + \cdots\cdots Y_{pp}V_p \cdots Y_{pn}V_n$$

$$I_p = \sum_{q=1}^{p-1} Y_{pq}V_q + Y_{pp}V_p + \sum_{q=p+1}^{n} Y_{pq}V_q \longrightarrow ③$$

Now,

$$S_p = P_p + jQ_p$$

where, S_p = Complex power of bus-p

P_p = Real power of bus-p

Q_p = Reactive power of bus-p

$$S_p = V_p I_p^*$$

where $V_p I_p^* = P_p + jQ_p \longrightarrow ④$

The load flow problem can be handled more conveniently by use of I_p rather than I_p^*.

Therefore take Conjugate of eq ④

$$(V_p I_p^*)^* = (P_p + jQ_p)^*$$

$$V_p^* I_p = P_p - jQ_p$$

$$V_p^* I_p = P_p - jQ_p$$

$$I_p = \frac{P_p - jQ_p}{V_p^*} \longrightarrow (5)$$

Substitute eq (5) in eq (3)

$$\boxed{\frac{P_p - jQ_p}{V_p^*} = \sum_{q=1}^{p-1} Y_{pq} V_q + Y_{pp} V_p + \sum_{q=p+1}^{n} Y_{pq} V_q} \longrightarrow (6)$$

Model-1 :- Gauss Seidal method :-

Eq (6) can be rewritten as,

$$Y_{pp} V_p = \frac{P_p - jQ_p}{V_p^*} - \sum_{q=1}^{p-1} Y_{pq} V_q - \sum_{q=p+1}^{n} Y_{pq} V_q$$

$$V_p = \frac{1}{Y_{pp}} \left[\frac{P_p - jQ_p}{V_p^*} - \sum_{q=1}^{p-1} Y_{pq} V_q - \sum_{q=p+1}^{n} Y_{pq} V_q \right] \longrightarrow (7)$$

$$V_p = \frac{1}{Y_{pp}} \left[\frac{P_p - jQ_p}{V_p^*} - \sum_{\substack{q=1 \\ q \neq p}}^{n} Y_{pq} V_q \right] \longrightarrow (8)$$

Eq (7) & (8) are called load flow equation of Gauss seidal method.

Model-2 :- Newton Raphson method :-

From eq (6),

$$\frac{P_p - jQ_p}{V_p^*} = \sum_{q=1}^{n} Y_{pq} V_q \longrightarrow (9)$$

$$P_p - jQ_p = V_p^* \left[\sum_{q=1}^{n} Y_{pq} V_q \right] \longrightarrow (10)$$

Let
$$\left.\begin{aligned} V_p &= e_p + jf_p \\ V_q &= e_q + jf_q \\ Y_{pq} &= G_{pq} - jB_{pq} \end{aligned}\right\} \longrightarrow (11)$$

Where e_p, f_p = Real and Imaginary part of V_p

e_q, f_q = Real and Imaginary part of V_q

G_{pq}, B_{pq} = Conductance & Susceptance of the admittance Y_{pq}

Substitute eq (11) in eq (10).

$$P_p - jQ_p = (e_p + jf_p)^* \left[\sum_{q=1}^{n} (G_{pq} - jB_{pq})(e_q + jf_q) \right]$$

$$P_p - jQ_p = e_p - jf_p \left[\sum_{q=1}^{n} G_{pq}e_q + G_{pq}jf_q - jB_{pq}e_q - jB_{pq}jf_q \right]$$

$$P_p - jQ_p = e_p - jf_p \left[\sum_{q=1}^{n} G_{pq}e_q + jG_{pq}f_q - jB_{pq}e_q - j^2B_{pq}f_q \right]$$

$$P_p - jQ_p = e_p - jf_p \left[\sum_{q=1}^{n} G_{pq}e_q + jG_{pq}f_q - jB_{pq}e_q + B_{pq}f_q \right] \quad \boxed{\because j^2 = -1}$$

$$P_p - jQ_p = e_p - jf_p \left[\sum_{q=1}^{n} (G_{pq}e_q + B_{pq}f_q) + j(G_{pq}f_q - B_{pq}e_q) \right]$$

$$P_p - jQ_p = \left[\sum_{q=1}^{n} (e_p - jf_p) \left[(G_{pq}e_q + B_{pq}f_q) + j(G_{pq}f_q - B_{pq}e_q) \right] \right]$$

$$P_p - jQ_p = \left[\sum_{q=1}^{n} e_p(G_{pq}e_q + B_{pq}f_q) + je_p(G_{pq}f_q - B_{pq}e_q) - jf_q(G_{pq}e_q + B_{pq}f_q) - j^2f_p(G_{pq}f_q - B_{pq}e_q) \right]$$

$$P_p - jQ_p = \left[\sum_{q=1}^{n} e_p(G_{pq}e_q + B_{pq}f_q) + je_p(G_{pq}f_q - B_{pq}e_q) - jf_q(G_{pq}e_q + B_{pq}f_q) + f_p(G_{pq}f_q - B_{pq}e_q) \right]$$

$$P_p - jQ_p = \left\{ \sum_{q=1}^{n} \left[e_p(G_{pq}e_q + B_{pq}f_q) + f_q(G_{pq}f_q - B_{pq}e_q) \right] + j\left[e_p(G_{pq}f_q - B_{pq}e_q) - f_q(G_{pq}e_q + B_{pq}f_q) \right] \right\} \longrightarrow (12)$$

Comparing L.H.S and R.H.S :-

$$P_p = \sum_{q=1}^{n} \left[e_p(G_{pq}e_q + B_{pq}f_q) + f_q(G_{pq}f_q - B_{pq}e_q) \right]$$

$$Q_p = \sum_{q=1}^{n} \left[f_q(G_{pq}e_q + B_{pq}f_q) - e_q(G_{pq}f_q - B_{pq}e_q) \right]$$

Load Flow solution by Gauss-Seidal Method.

$$V_p = \frac{1}{Y_{pp}} \left[\frac{P_p - jQ_p}{V_p^*} - \sum_{q=1}^{p-1} Y_{pq}V_q - \sum_{q=p+1}^{n} Y_{pq}V_q \right] \longrightarrow (1)$$

* The Gauss-seidal method is an iterative algorithm for solving a set of non-linear low flow equation.
* the non-linear low flow equation are given in eq ①.

where $P = 1, 2, 3, \ldots n$

* If we consider k no. of iteration then $(k+1)^{th}$ iteration can be given as V_p^{k+1}.

$$V_p^{k+1} = \frac{1}{Y_{PP}} \left[\frac{P_P - jQ_P}{(V_p^k)^*} - \sum_{q=1}^{P-1} Y_{pq} V_q^{(k+1)} - \sum_{q=P+1}^{n} Y_{pq} V_q^k \right]$$

ALGORITHM FOR GUASS SEIDAL METHOD :-

Step 1 :- Assume a flat voltage profile 1+j0 b for all buses except the slack bus.

Step 2 :- Assume a suitable value of ε called Convergence criterian.

Step 3 :- Set iteration count, $K=0$ and assumed voltage profile of the buses are denoted as $V_1^0, V_2^0, \ldots V_n^0$

Step 4 : Set the bus count $P = 1$

Step 5 : Check the for slack bus, if it is a slack bus go to step 12 otherwise continue.

Step 6 : Check for generator bus, if it is generator bus go to next step else go to step 9.

Step 7 : Temporarily set $|V_p^k| = |V_p|_{spec}$ is specified reactive power of generator bus.

$$Q_{P,cal}^{k+1} = (-1) \times \mathrm{Img}\left[(V_p^k)^* \left[\sum_{q=1}^{P+1} Y_{pq} V_q^{k+1} + \sum_{q=P}^{n} Y_{pq} V_q^k \right] \right]$$

The calculated reactive power may be within specified limit or it may violated the limits. If the calculated reactive power is within the specified limit, then consider the bus as generator bus and set $Q_P = Q_{P cal}^{k+1}$, for this iteration and go to step 8. If the calculated reactive power violates the specified limit then treat the bus as load bus.

if $Q_P^{k+1}, cal \leq Q_{P.min}$ then $Q_P = Q_{P.min}$

or $Q_P^{k+1}, cal \geq Q_{P.max}$ then $Q_P = Q_{P.max}$

Since the bus is treated as load bus (V_p^k) need not be replaced by $|V_p|_{spec}$. go to step 9

Step 8 : For generator bus the phase voltage of the bus can be calculated as

$$V_{p\,temp}^{k+1} = \frac{1}{Y_{pp}}\left[\frac{P_p - jQ_p}{(V_p^k)^*} - \sum_{q=1}^{p-1} Y_{pq} V_q^{k+1} - \sum_{q=p+1}^{n} Y_{pq} V_q^k\right]$$

$$\delta_p^{k+1} = \tan^{-1}\left[\frac{\text{Im part of } V_p^{k+1}\text{, temp}}{\text{real part of } V_p^{k+1}\text{, temp}}\right]$$

Now the $(k+1)^{th}$ iteration voltage of the generator bus is given by $V_p^{k+1} = |V_p|_{spec} \angle \delta_p^{k+1}$

Step 9 : For load bus, V_p^{k+1} is calculated as

$$V_p^{k+1} = \frac{1}{Y_{pp}}\left[\frac{P_p - jQ_p}{(V_p^k)^*} - \sum_{q=1}^{p-1} Y_{pq} V_q^{k+1} - \sum_{q=p+1}^{n} Y_{pq} V_q^k\right]$$

Step 10 : An acceleration factor, α, can be used for faster Convergence.

$$V_{p,acc}^{k+1} = V_p^k + \alpha(V_p^{k+1} - V_p^k)$$

then $V_p^{k+1} = V_p^{k+1}, acc$

Step 11 : Calculate the change in bus-p voltage

$$\Delta V_p^{k+1} = V_p^{k+1} - V_p^k$$

Step 12 : Repeat steps 5 to 11 until all the bus voltages have been calculated. For this, increment the bus count by 1 until the bus count is n.

Step 13 : Find the largest amoung $\Delta V_1^{k+1}, \Delta V_2^{k+2} \ldots \Delta V_n^{k+1}$. Let this be $|\Delta V_{max}|$. If $|\Delta V_{max}|$ is less than ε, then move to the next step, else increment the iteration count and go to step 4.

Step 14 : Calculate the line flow and slack bus power using the bus voltages.

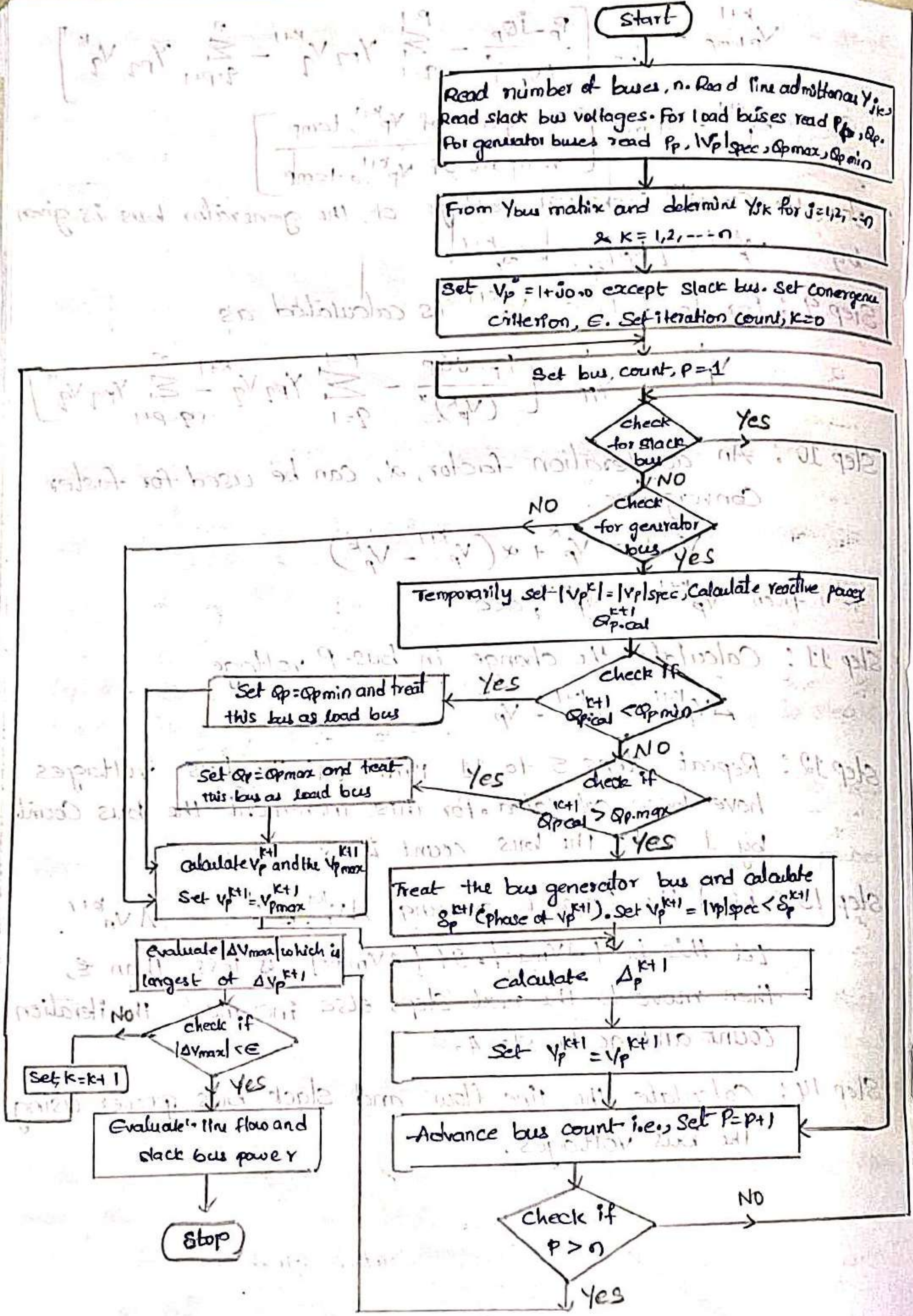
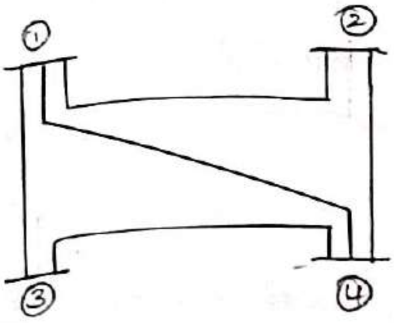


Fig : FLOW CHART FOR LOAD FLOW SOLUTION BU GAUSS-SEIDEL METHOD.

* Generators are Connected to all the four buses, while loads are at bus 2 and bus 3



Bus	P	Q	V	Remarks
1	—	—	1.05∠0	Slack
2	0.5	-0.2	-	PQ
3	-1.0	0.5	-	PQ
4	0.3	-0.1	-	PQ

Line	R	X
1-2	0.05	0.15
1-3	0.10	0.30
1-4	0.20	0.40
2-4	0.10	0.30
3-4	0.05	0.15

Determine the bus voltages at the end of first iteration using gause seidel method.

Sol :- Iteration method are used to solve non-linear equation.. Where in iteration method we start with an assumption.

* Iterations are done to get the closure values to the actual value.

* From the given data line impedances are ;

$Z_{12} = 0.05 + j0.15$

$Z_{13} = 0.16 + j0.30$

$Z_{14} = 0.20 + j0.40$

$Z_{24} = 0.10 + j0.30$

$Z_{34} = 0.05 + j0.15$

* From the line impedances we are going to calculate the line Admittances.

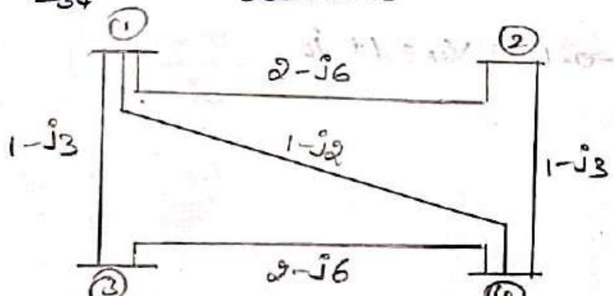
$Y_{12} = \frac{1}{Z_{12}} = \frac{1}{0.05 + j0.15} \Rightarrow \boxed{Y_{12} = 2 - j6}$

$Y_{13} = \frac{1}{Z_{13}} = \frac{1}{0.10 + j0.30} \Rightarrow \boxed{Y_{13} = 1 - j3}$

$Y_{14} = \frac{1}{Z_{14}} = \frac{1}{0.20 + j0.40} \Rightarrow \boxed{Y_{14} = 1 - j2}$

$Y_{24} = \frac{1}{Z_{24}} = \frac{1}{0.10 + j0.30} \Rightarrow \boxed{Y_{24} = 1 - j3}$

$Y_{34} = \frac{1}{Z_{34}} = \frac{1}{0.05 + j0.15} \Rightarrow \boxed{Y_{34} = 2 - j6}$



Total number of Buses (n) = 4
Hence matrix formation is 4×4

$$Y_{BUS\,(4\times4)} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix}_{4\times4}$$

$\therefore Y_{11} = 2-j6+1-j3+1-j2 \Rightarrow Y_{11} = 4-j11$

$Y_{12} = 2-j6 = Y_{21}$

$Y_{13} = 1-j3 = Y_{31}$

$Y_{14} = 1-j2 = Y_{41}$

$Y_{22} = 2-j6+1-j3 = 3-j9 = Y_{22}$

$Y_{23} = 0 = Y_{32}$

$Y_{24} = 1-j3 = Y_{42} = Y_{24}$

$Y_{33} = 2-j6+1-j3 \Rightarrow Y_{33} = 3-j9$

$Y_{34} = 2-j6 = Y_{43}$

$Y_{44} = 2-j6+1-j3+1-j2 \Rightarrow Y_{44} = 4-j11$

To found the matrix off diagonal elements give negative of actual value

$$Y_{Bus} = \begin{bmatrix} 4-j11 & -2+j6 & -1+j3 & -1+j2 \\ -2+j6 & 3-j9 & 0 & -1+j3 \\ -1+j3 & 0 & 3-j9 & -2+j6 \\ -1+j2 & -1+j3 & -2+j6 & 4-j11 \end{bmatrix}$$

From given data;

For bus-1 :- $V_1 = 1.05\angle 0°$ (or) $1.05+j0$ [slack bus] , P, Q = ?

For Bus-2 : $P=0.5,\ Q=-0.2,\ V_2 = 1+j0$

for Bus-3 :- $P=-1.0,\ Q=0.5,\ V_3 = 1+j0$

for Bus-4 :- $P=0.3,\ Q=-0.1,\ V_4 = 1+j0$

0^{th} iteration

$V_1^0 = 1.05 + j0$

$V_2^0 = 1 + j0$

$V_3^0 = 1 + j0$

$V_4^0 = 1 + j^0$

1^{st} Iteration

$V_1' = 1.05 + j0$

$V_2' = 1.03 + 0.0566j$

$V_3' = 1.033 - j0.1166$

$V_4' = 1.035 - j0.02$

The Bus 1 is a slack bus and so, its voltages remain same for all iterations.

We know that ; $$V_p = \frac{1}{Y_{pp}}\left[\frac{P_p - jQ_p}{(V_p)^*} - \sum_{\substack{q=1 \\ q\neq p}}^{n} Y_{pq} V_q\right]$$ $1.03 + j0.005$

for ; $p = 2$, $q = 1, 2, 3, 4$.

$$\Rightarrow V_2' = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{(V_2)^*} - Y_{21}V_1' - Y_{23}V_3^0 - Y_{24}V_4^0\right]$$

$$V_2' = \frac{1}{3 - j9}\left[\frac{0.5 + j0.2}{1 - j0} - (-2 + j6)(1.05 + j0) - (0)(1 + j0) - (-1 + j3)(1 + j0)\right]$$

$$\boxed{V_2' = 1.03 + 0.0566j}\ V$$

for $P = 3$, $q = 1, 2, 3, 4$ $\quad 1.033 + j0.1166$

$$\Rightarrow V_3' = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{(V_3)^*} - Y_{31}V_1' - Y_{32}V_2' - Y_{34}V_4^0\right]$$

$$\Rightarrow V_3' = \frac{1}{3 - j9}\left[\frac{-1.0 - j0.5}{1 - j0} - (-1 + j3)(1.05 + j0) - (0)(1.03 + 0.0566j) - (-2 + j6)(1 + j0)\right]$$

$$\boxed{V_3' = 1.033 - j0.1166}$$

for $P = 4$, $q = 1, 2, 3, 4$ $(P \neq q)$:-

$$\Rightarrow V_4' = \frac{1}{Y_{44}}\left[\frac{P_4 - jQ_4}{(V_4)^*} - Y_{41}V_1 - Y_{42}V_2 - Y_{43}V_3\right]$$

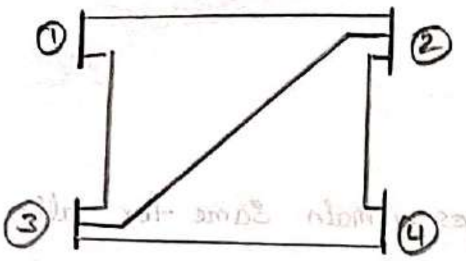
Sub. best value of voltages :-

$$\Rightarrow V_4' = \frac{1}{Y_{44}}\left[\frac{P_4 - jQ_4}{(V_4^0)^*} - Y_{41}V_1' - Y_{42}V_2' - Y_{43}V_3'\right]$$

$$\Rightarrow V_4' = \frac{1}{4 - j11}\left[\frac{0.3 + j0.1}{1 - j0} - (-1 + j2)(1.05 + j0) - (-1 + j3)(1.03 + 0.0566j) - (-2 + j6)(1.033 - j0.1166)\right]$$

$$\boxed{V_4' = 1.035 - j0.02}$$

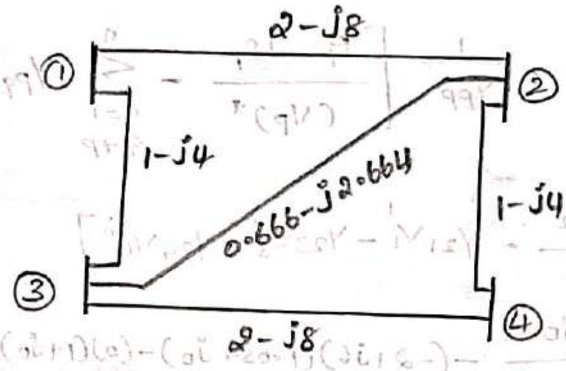
* The System data for load flow solution are given in tables. Determine the voltage at the end of 1^{st} iteration by gauss seidal method.



Bus Code	Admittance
1-2	2-j8
1-3	1-j4
2-3	0.666-j2.664
2-4	1-j4
3-4	2-j8

Bus code	P	Q	V	Remarks
1	–	–	1.06∠0	slack
2	0.5	0.2	–	PQ
3	0.4	0.3	–	PQ
4	0.3	0.1	–	PQ

Sol :-



Total no. of buses $n = 4$

Hence matrix formation is 4×4

$$Y_{Bus\,(4\times4)} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{34} & Y_{44} \end{bmatrix}_{4\times4}$$

$\therefore\ Y_{11} = 2-j8 + 1-j4 = 3-j12$

$Y_{12} = 2-j8 = Y_{21}$

$Y_{13} = 1-j4 = Y_{31}$

$Y_{14} = 0 = Y_{41}$

$Y_{22} = 2-j8 + 1-j4 + 0.666-j2.664 = 3.666 - j14.664$

$Y_{23} = 0.666 - j2.664 = Y_{32}$

$Y_{24} = 1-j4 = Y_{42}$

$Y_{33} = 1-j4 + 2-j8 + 0.666 - j2.664 = 3.666 - j14.664$

$Y_{34} = 2-j8 = Y_{43}$

$Y_{44} = 1-j4 + 2-j8 = 3-j12$

To found the matrix off diagonal elements give negative of actual value.

$$Y_{BUS} = \begin{bmatrix} 3-j12 & -2+j8 & -1+j4 & 0 \\ -2+j8 & 3.666-j14.664 & -0.666+j2.664 & -1+j4 \\ -1+j4 & -0.666+j2.664 & 3.666-j14.664 & -2+j8 \\ 0 & -1+j4 & -2+j8 & 3-j12 \end{bmatrix}$$

From given data ;

For bus 1 ; $V_1 = 1.06\angle 0^\circ$ (or) $1.06 + j0$ [slack bus] $P, Q = ?$

for bus 2 ; $P_2 = 0.5$, $Q_2 = 0.2$, $V_2 = 1 + j0$

for bus 3 ; $P_3 = 0.4$, $Q_3 = 0.3$, $V_3 = 1 + j0$

for bus 4 ; $P_4 = 0.3$, $Q_3 = 0.1$, $V_4 = 1 + j0$

0th Iteration :-

$V_1^0 = 1.06\angle 0^\circ$ (or) $1.06 + j0$

$V_2^0 = 1 + j0$

$V_3^0 = 1 + j0$

$V_4^0 = 1 + j0$

1st Iteration :-

$V_1^1 = 1.06 + j0$

$V_2^1 = 1.05 + 0.028j$

$V_3^1 = 1.051 + 0.0259j$

$V_4^1 = 1.06 + 0.048j$

W.K.T ; $$V_p = \frac{1}{Y_{PP}} \left[\frac{P_p - jQ_p}{(V_p)^*} - \sum_{\substack{q=1 \\ q \neq p}}^{n} Y_{pq} V_q \right]$$

For $P = 2$; $q = 1, 3, 4$ $P \neq q$

$$\Rightarrow V_2 = \frac{1}{Y_{22}} \left[\frac{P_2 - jQ_2}{(V_2)^*} - Y_{21} V_1^1 - Y_{23} V_3^0 - Y_{24} V_4^0 \right]$$

$$V_2^1 = \frac{1}{3.666 - j14.664} \left[\frac{0.5 - j0.2}{1 - j0} - (-2+j8)(1.06+j0) - (-0.666+j2.664)(1+j0) - (-1+j4)(1+j0) \right]$$

$$\boxed{V_2^1 = 1.05 + 0.028j} \text{ V}$$

For $P = 3$; $q = 1, 2, 4$ $P \neq q$

$$\Rightarrow V_3^1 = \frac{1}{Y_{33}} \left[\frac{P_3 - jQ_3}{(V_3)^*} - Y_{31} V_1^1 - Y_{32} V_2^1 - Y_{34} V_4^0 \right]$$

$$V_3^1 = \frac{1}{3.666 - j14.664} \left[\frac{0.4 - 0.3}{1 - j0} - (-1+j4)(1.06+j0) - (-0.666+j2.664)(1.005 + 0.0028j) - (-2+j8)(1+j0) \right]$$

$$\boxed{V_3' = 1.051 + 0.0259j}\ V_{//}$$

For $P = 4$; $q = 1, 2, 3$; $P \neq q$

$$V_4' = \frac{1}{Y_{44}}\left[\frac{P_4 - jQ_4}{(V_4)^*} - Y_{41}V_1' - Y_{42}V_2' - Y_{43}V_3'\right]$$

$$= \frac{1}{3-j12}\left[\frac{0.3-j0.1}{1-j0} - (0)(1.06+j0) - (-1+j4)(1.05+0.026j) - (-2+j8)(1.051+0.0259j)\right]$$

$$\boxed{V_4' = 1.06 + 0.048j}\ V_{//}$$

$\alpha = 1.6$ $\boxed{\left[V_{p\,acc}^{k+1} = V_p^k + \alpha\left[V_p^{k+1} - V_p^k\right]\right]} \longrightarrow (1)$

$$V_2'acc = V_2^0 + \alpha(V_2' - V_2^0) = 1 + j0 + 1.6\left[(1.05 + j0.026) - (1 + j0)\right]$$

$$\boxed{V_2'acc = 1.08 + j0.048}$$

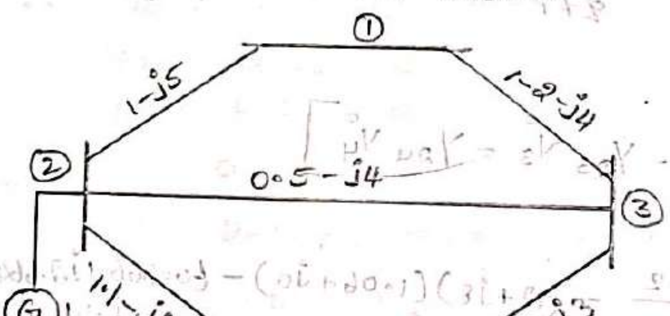
$$V_3'acc = V_3^0 + \alpha(V_3' - V_3^0) = 1 + j0 + 1.6\left[(1.051 + j0.025) - (1 + j0)\right]$$

$$\boxed{V_3'acc = 1.08 + j0.082}$$

$$V_4'acc = V_4^0 + \alpha(V_4' - V_4^0) = 1 + j0 + 1.6\left[(1.06 + j0.048) - (1 + j0)\right]$$

$$\boxed{V_4'acc = 1.096 + j0.076}$$

* For the system given in the figure. Determine the voltages at end of 1st iteration by Gauss-seidal method. $\alpha = 1$, Bus specified are given into the table.



Bus Code	P	Q	V	Remarks
1	-	-	$1.06\angle 0$	slack
2	0.5	$0.1 < Q_2 < 1$	1.04	PV
3	0.4	0.3	-	PQ
4	0.2	0.1	-	PQ

Sol :-

Total no. of buses $n = 4$

Hence matrix formation is 4×4

$$Y_{(BUS)}(4\times4) = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix}_{4\times4}$$

$\therefore$ $Y_{11} = 1-j5+1.2-j4 = 2.2-j9$

$Y_{12} = Y_{21} = 1-j5$

$Y_{13} = Y_{31} = 1.2-j4$

$Y_{14} = Y_{41} = 0$

$Y_{22} = 1-j5+1.1-j2+0.5-j4 = 2.6-j_{11}$

$Y_{23} = Y_{32} = 0.5-j4$

$Y_{24} = Y_{42} = 1.1-j2$

$Y_{33} = 1.2-j4+1.2-j3+0.5-j4 = 2.9-j_{11}$

$Y_{34} = Y_{43} = 1.2-j3$

$Y_{44} = 1.1-j2+1.2-j3 = 2.3-j5$

$$Y_{BUS} = \begin{bmatrix} 2.2-j9 & -1+j5 & -1.2+j4 & 0 \\ -1+j5 & 2.6-j_{11} & -0.5+j4 & -1.1+j_2 \\ -1.2+j4 & -0.5+j4 & 2.9-j4 & -1.2+j3 \\ 0 & -1.1+j2 & -1.2+j3 & 2.3-j5 \end{bmatrix}$$

0th iteration

$V_1^0 = 1.06\angle 0$ (or) $1.06+j0$

$V_2^0 = 1.04$

$V_3^0 = 1+j_0$

$V_4^0 = 1+j_0$

1st Iteration

$V_1' = 1.06+j0$

$V_2' = 1.048+0.035j$

$V_3' = 1.075+j0.082$

$V_4' = 1.096+j0.062$

For generator Bus.

$$Q_{p\,cal}^{k+1} = -1 * Im \left\{ (V_p^k)^* \left[\sum_{q=1}^{n-1} Y_{pq} V_q^{k+1} - \sum_{q=p}^{n} Y_{pq} V_q^k \right] \right\}$$

For Bus-2 Generator Bus [k=0]

$$Q_2' = -1 * Im \left\{ (V_2^0) \left[Y_{21}V_1 + Y_{22}V_2 + Y_{23}V_3 + Y_{24}V_4 \right] \right\}$$

$$= 1 \times Im \{ 1.04 [(-1+j5)(1.06+j0) + (2.6-j_{11})(1.04) + (-0.5+j4)(1+j_0) + (-1.1-j_2)(1+j0) \}$$

$$= 1 \times Im [0.045-0.0145j]$$

$$= -1 \times (-0.0145)$$

$$\boxed{Q_2'cal = 0.145}$$

$P_2 = 0.5,\ Q_2 = 0.145,\ V_2^{0} = 1.04 + j0$

$$V_2' = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{(V_2^{0})^*} - Y_{21}V_1 - Y_{23}V_3 - Y_{24}V_4\right]$$

$$= \frac{1}{2.6-j11}\left[\frac{0.5-j0.145}{1.04-j0} - (-1+j5)(1.06+j0) - (0.5+j4)(1+j0) - (-1.1+j2)(1+j0)\right]$$

$V_2' = 1.048 + 0.035$ (or) $1.048\angle 2.05$

$$V_3' = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{(V_3^{0})^*} - Y_{31}V_1' - Y_{32}V_2' - Y_{34}V_4^{0}\right]$$

$$= \frac{1}{2.9-j11}\left[\frac{0.4-j0.3}{1+j0} - (-1.2+j9)(1.06+j0) - (-0.5-j4)(1.048+0.081) - (-1.2+j3)(1+j0)\right]$$

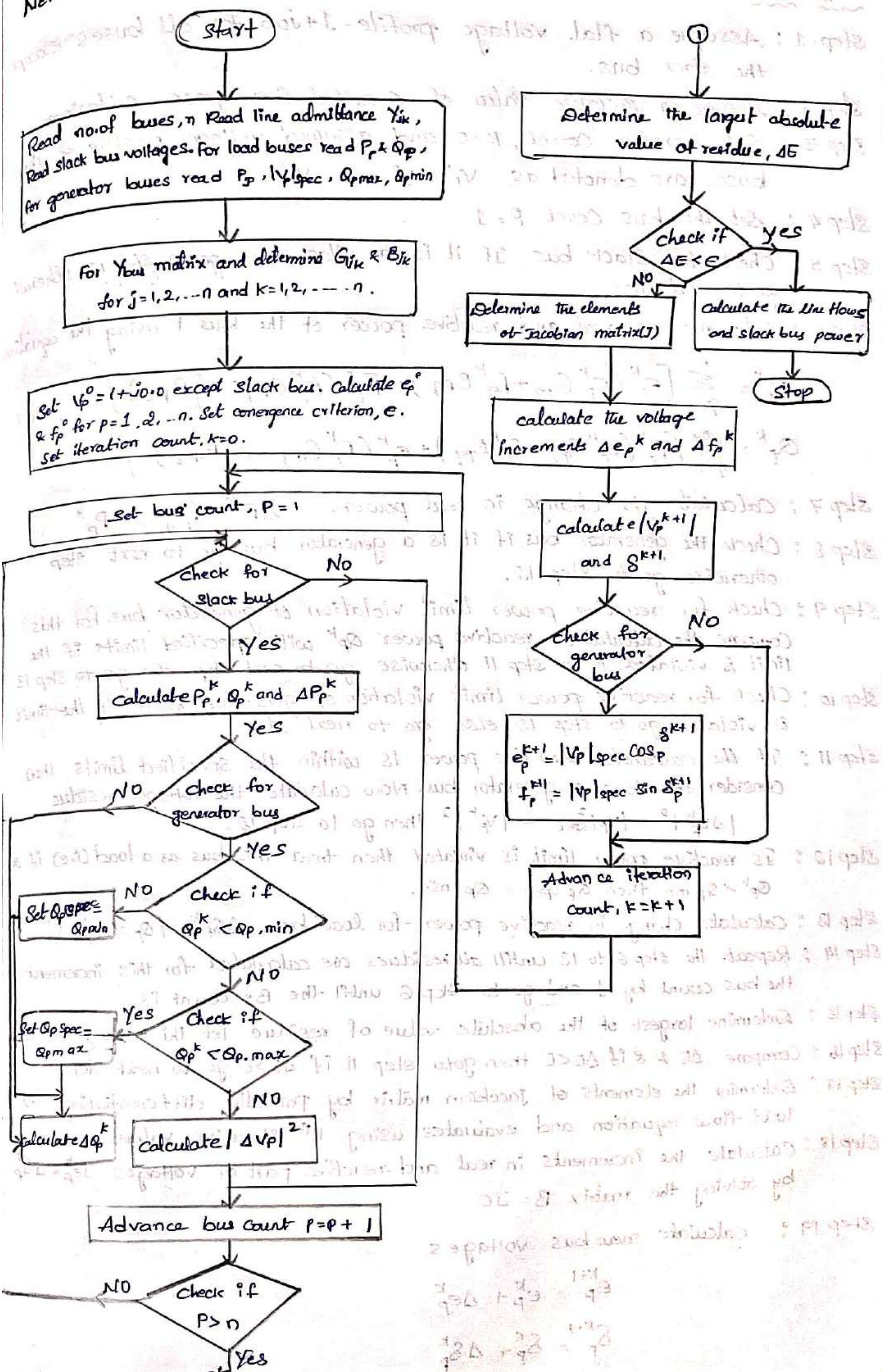
$$\boxed{V_3' = 1.075 + j0.0382}$$

$$V_4' = \frac{1}{Y_{44}}\left[\frac{P_4 - jQ_4}{(V_4^{0})^*}\left(-Y_{41}V_1' - Y_{42}V_2' - Y_{43}V_3'\right)\right]$$

$$= \frac{1}{2.8-j11}\left[\frac{0.2-j0.11}{1+j0}(-0)(1.06+j0) - (-1.1+j2) - (1.04+j0.035) - (-1.2+j3)(1.0753+j0.055)\right]$$

$$\boxed{V_4' = 1.096 + j0.062}$$

Newton- Raphson Method for load flow.



Algorithm :-

Step-1 : Assume a flat voltage profile- 1+j0 for all buses except the slack bus.

step-2 : Assume a suitable value of ε called Convergence Criterion

Step 3 : Set iteration count, K=0 and assumed voltage profile of the buses are denoted as $V_1^0, V_2^0, \ldots V_n^0$

Step 4 : Set the bus count P=1.

Step 5 : Check for slack bus, If it is a slack bus go to step 13 otherwise go to next step.

Step 6 : Calculate the real and reactive power of the bus P using the equation

$$P_p^k = \sum_{q=1}^{n} \left[e_p^k (e_p^k G_{pq} + f_q^k B_{pq}) + F_q^k (F_q^k G_{pq} - e_q^k B_{pq})\right]$$

$$Q_p^k = \sum_{q=1}^{n} \left[F_p^k (e_p^k G_{pq} + F_q^k B_{pq}) + e_p^k (F_q^k G_{pq} - e_q^k B_{pq})\right]$$

Step 7 ; Calculate the change in real power. $\Delta P^k = P_{p\,spec} - P_p^k$

Step 8 : Check the generator bus if it is a generator bus go to next step otherwise go to step 12.

Step 9 : Check for reactive power limit violation of generator bus. for this Compare the calculated reactive power Q_p^k with specified limits if the limit is violated go to step 11 otherwise go to next step else go to step 13.

Step 10 ; Check for reactive power limit violation of generator buses if the limit is violated go to step 12 else go to next step.

Step 11 : If the calculated reactive power is within the specified limits then consider this bus as generator bus. Now calculate the voltage residue.
$|\Delta V_p^k|^2 = |V_p|^2_{spec} - |V_p^k|^2$ then go to step 12.

Step 12 : Is reactive power limit is violated then treat this bus as a load (i.e) if a $Q_p^k < Q_{p\,min}$ then $Q_{p\,spec} = Q_{p,\,min}$.

Step 13 : Calculate change in reactive power for load bus. $\Delta Q_p^k = |Q_{p\,spec}| - Q_p^k$

Step 14 : Repeat the step 6 to 13 untill all residues are calculated for this increment the bus count by 1 and go to step 6 until the Bs count is

Step 15 : Determine largest of the absolute value of residue let this change be ΔE.

Step 16 : Compare ΔE & ε if $\Delta E < E$ then goto step 11 if $\Delta E > E$ go to next step

Step 17 : Determine the elements of jacobian matrix by partially differentiating the load flow equation and evaluates using k^{th} iteration values.

Step 18 : Calculate the increments in real and reactive part of voltages Δe_p^k & $\Delta \delta_p^k$ by solving the matrix B = JC

Step 19 : Calculate new bus voltages

$$e_p^{k+1} = e_p^k + \Delta e_p^k$$

$$\delta_p^{k+1} = \delta_p^k + \Delta \delta_p^k$$

$$|V_p^{k+1}| = \sqrt{(e_p^{k+1})^2 + (\delta_p^{k+1})^2} \quad \& \quad \delta_p^{k+1} = \tan^{-1}\left[\frac{e_p^{k+1}}{\delta_p^{k+1}}\right]$$

Therefore $\quad V_p^{k+1} = |V_p^{k+1}| \angle \delta_p^{k+1}$

step 20 : Advance iteration count $k = k+1$ & go to step 5

step 21 : Calculate the line flows and stop the program.

Perform an iteration of Newton Raphson load flow method and determine the power flow solution for the given system. Take base MVA as 100, assume $V_1 = 1.05 \angle 0°$ per unit

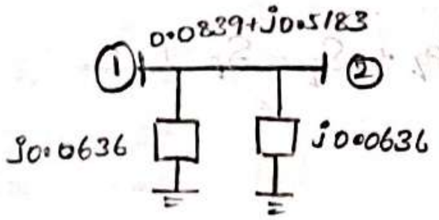
Line	Bus From	Bus To	R(p.u)	X(p.u)	Half line admittance charging (Y'P/2)-pu
1	1	2	0.0839	0.5183	0.0636

Bus	PL	QL
1	90	20
2	30	10

Sol:- From the given data, No. of buses = 2

$\therefore \boxed{n=2}$

Impedence $(z) = R + jX = 0.0839 + j0.5183$



step 1 : Form Y_{Bus} ; $\quad Y_{Bus} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}_{2\times 2}$

$$Y_{11} = \frac{1}{0.0839 + j0.5183} + j0.0636$$

$\boxed{Y_{11} = 0.3044 - j1.8164}$

$$Y_{12} = Y_{21} = -\left[\frac{1}{0.0839 + j0.5183}\right]$$

$\boxed{Y_{12} = Y_{21} = -0.3044 + j1.88}$

$\boxed{Y_{22} = 0.3044 - j1.8164}$

$$Y_{BUS} = \begin{bmatrix} 0.3044 - j1.8164 & -0.3044 + j1.88 \\ -0.3044 + j1.88 & 0.3044 - j1.8164 \end{bmatrix}$$

Convert Y_{Bus} into polar form;

$$Y_{Bus} = \begin{bmatrix} 1.842 \angle -1.405 & 1.904 \angle 1.731 \\ 1.904 \angle 1.731 & 1.842 \angle -1.405 \end{bmatrix} = \begin{bmatrix} |Y_{11}| \angle \theta_{11} & |Y_{12}| \angle \theta_{12} \\ |Y_{21}| \angle \theta_{21} & |Y_{22}| \angle \theta_{22} \end{bmatrix}$$

Step-2: Consider first bus as slack bus

$V_1^0 = 1.05 \angle 0^\circ$ (or) $1.05 + j0$

Assume the value for Bus (2)

$V_2^0 = 1 + j0$ (or) $1 \angle 0^\circ$

$|V_2| \angle \delta_2$

$$[X]^0 = \begin{bmatrix} \delta_2 \\ |V_2| \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Step-3: Calculate P_2^{cal}, Q_2^{cal}, ΔP_2, ΔQ_2

To calculate Real Power P_2^{cal}

$$P_p^{cal} = \sum_{q=1}^{n} |V_p| |Y_{pq}| |V_q| \cos(\theta_{pq} + \delta_q - \delta_p)$$

We know, $p = 2$; $q = 1, 2$; $n = 1$

$$P_2^{cal} = \left[|V_2| |Y_{21}| |V_1| \cos(\theta_{21} + \delta_1 - \delta_2) + |V_2| |Y_{22}| |V_2| \cos(\theta_{22} - \delta_2 - \delta_2) \right]$$

$$P_2^{cal} = \left[1.0 \times 1.904 \times 1.05 \cos(1.731 + 0 - 0) + 1.0 \times 1.842 \times 1.0 \cos(-1.405) \right]$$

$$\boxed{P_2^{cal} = -0.0149 \cong -0.015}$$

$P_2\ spec = P_{G_2} - P_L$

$P_2\ spec = 0 - \frac{30}{100} = -0.3$ p.u

$\therefore \Delta P_2 = P_{2,spec} - P_{2cal}$

$= -0.3 - (-0.015)$

$$\boxed{\Delta P_2 = -0.285}$$

To calculate Reactive Power Q_2^{cal}

$$Q_p^{cal} = -\sum_{q=1}^{n} |V_p||Y_{pq}||V_q| \sin(\theta_{pq} + \delta_q - \delta_p)$$

Here $p=2$; $n=2$, $q=1,2$

$$Q_2^{cal} = -\sum_{q=1}^{2} |V_2||Y_{21}||V_1| \sin(\theta_{21} + \delta_1 - \delta_2) + (-|V_2||Y_{22}||V_2| \sin(\theta_{22} + \delta_2 - \delta_2)]$$

$$Q_2^{cal} = -[(1.0 \times 1.904 \times 1.05 \sin(1.731 + 0 - 0) + (1.0 \times 1.842 \times 1.0 \sin(-1.405)]$$

$$\boxed{Q_2^{cal} = -0.156}$$

$$Q_{2,spec} = Q_{G_2} - Q_L = 0 - \frac{10}{100} = -0.1 \text{ p.u}$$

$$\Delta Q_2 = Q_{2,spec} - Q_2^{cal}$$

$$= -0.1 - (-0.156)$$

$$\boxed{\Delta Q_2 = 0.056}$$

Step-4 :- Form Jacobin matrix for Bus-2 :-

$$\begin{bmatrix} \frac{\partial P_2}{\partial \delta_2} & \frac{\partial P_2}{\partial |V_2|} \\ \frac{\partial Q_2}{\partial \delta_2} & \frac{\partial Q_2}{\partial |V_2|} \end{bmatrix} \begin{bmatrix} \Delta \delta_2 \\ \Delta |V_2| \end{bmatrix} = \begin{bmatrix} \Delta P_2 \\ \Delta Q_2 \end{bmatrix}$$

① To find $\frac{\partial P_2}{\partial \delta_2}$ we have

$$\frac{\partial P_p}{\partial \delta_p} = \sum_{\substack{q=1 \\ \neq p}}^{n} |V_p||Y_{pq}||V_q| \sin(\theta_{pq} + \delta_q - \delta_p)$$

$p=2$, $q=1$, $n=2$

$$\frac{\partial P_2}{\partial \delta_2} = |V_2||Y_{21}||V_1| \sin(\theta_{21} + \delta_1 - \delta_2) = 1.0 \times 1.904 \times 1.05 \times \sin(1.731 + 0 - 0)$$

$$\boxed{\frac{\partial P_2}{\partial \delta_2} = 1.973}$$

② To find $\frac{\partial Q_2}{\partial \delta_2}$ we have

$$\frac{\partial Q_p}{\partial \delta_p} = \sum_{\substack{q=1 \\ \neq p}}^{n} |V_p||Y_{pq}||V_q| \cos(\theta_{pq}+\delta_q-\delta_p)$$

$p=2,\ q=1,\ n=2$;

$$\frac{\partial Q_2}{\partial \delta_2} = \sum_{\substack{q=1 \\ \neq p}}^{n} |V_2||Y_{21}||V_1| \cos(\theta_{21}+\delta_1-\delta_2) = 1.0 \times 1.904 \times 1.05 \cos(1.731+0-0)$$

$$\boxed{\frac{\partial Q_2}{\partial \delta_2} = -0.3189}$$

③ To find $\frac{\partial P_2}{\partial |V_2|}$ we have ;

$$\frac{\partial P_p}{\partial |V_p|} = 2|V_p||Y_{pp}|\cos\theta_{pp} + \sum_{\substack{q=1 \\ \neq p}}^{n} |V_q||Y_{pq}|\cos(\theta_{pq}+\delta_q-\delta_p)$$

$P=2,\ q=1,\ n=2$

$$\frac{\partial P_2}{\partial |V_2|} = 2|V_2||Y_{22}|\cos(\overset{\theta_{22}}{-1.405}) + |V_1||Y_{21}|\cos(\theta_{21}+\delta_1-\delta_2)$$

$$= 2\times 1.0 \times 1.842 \times \cos(-1.4051) + [1.05 \times 1.804 \times \cos(1.731+0-0)]$$

$$\boxed{\frac{\partial P_2}{\partial |V_2|} = 0.289}$$

④ To find $\frac{\partial Q_2}{\partial |V_2|}$ we have ;

$$\frac{\partial Q_p}{\partial |V_p|} = -2|V_p||Y_{pp}|\sin\theta_{pp} - \sum_{\substack{q=1 \\ \neq p}}^{n} |V_q||Y_{pq}|\sin(\theta_{pq}+\delta_q-\delta_p)$$

$P=2,\ q=1,\ n=2.$

$$\frac{\partial Q_2}{\partial |V_2|} = -2|V_2||Y_{22}|\sin\theta_{22} - |V_1||Y_{21}|\sin(\theta_{21}+\delta_1-\delta_2)$$

$$\frac{\partial Q_2}{\partial |V_2|} = -2\times 1.0 \times 1.842 \times \sin(-1.405) - 1.05 \times 1.804 \times \sin(1.731+0-0)$$

$$\boxed{\frac{\partial Q_2}{\partial |V_2|} = 1.659}$$

Matrix form :-

$$\begin{bmatrix} 1.973 & 0.289 \\ -0.3189 & 1.659 \end{bmatrix} \begin{bmatrix} \Delta \delta_2 \\ \Delta |V_2| \end{bmatrix} = \begin{bmatrix} -0.285 \\ 0.056 \end{bmatrix}$$

Mode
Matrix - 6
Mahi A - 1
2x2 - 5
AC
Shif + 4, 1
Mat A - 3 ⟹ 2, 2, 6
AC
Shif + 4,
MTA = 3x
Shif + 4
M B - 4
=

$$\frac{\Delta \delta_2}{\Delta |V_2|} = \begin{bmatrix} -0.285 \\ 0.056 \end{bmatrix} \begin{bmatrix} 1.973 & 0.289 \\ -0.3189 & 1.659 \end{bmatrix}^{-1}$$

$x^{-1} =$

$$\frac{\Delta \delta_2}{\Delta |V_2|} = \begin{bmatrix} -0.285 \\ 0.056 \end{bmatrix} \begin{bmatrix} 0.4929 & -0.085 \\ 0.0947 & 0.5862 \end{bmatrix}$$

$$\frac{\Delta \delta_2}{\Delta |V_2|} = \begin{bmatrix} -0.145 \\ 0.00583 \end{bmatrix}$$

$$x^1 = x^0 + \Delta x$$

$$x^1 = \begin{bmatrix} \delta_2 \\ |V_2| \end{bmatrix} + \begin{bmatrix} \Delta \delta_2 \\ \Delta |V_2| \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} + \begin{bmatrix} -0.145 \\ 0.00583 \end{bmatrix}$$

$$\boxed{x^1 = \begin{bmatrix} -0.145 \\ 1.00583 \end{bmatrix}}$$

$$V_2 = 1 + j0$$

$$\boxed{V_2 = 1.00583 \angle -0.145}$$

DECOUPLED LOAD FLOW ANALYSIS :-

In Newton raphson method the half off the elements in the Jacobian matrix represents week load load flow and hence decoupled load flow analysis came into existance.

$$[\Delta P] = [H][\Delta \delta] \longrightarrow ①$$

real power Jacobian matrix = H
reactive " " = L

$$[\Delta Q] = [L]\left[\frac{\Delta |V|}{|V|}\right] \longrightarrow ②$$

Where $H_{ij} = L_{ij} = |V_i||V_j| \, (G_{ij} \sin \delta_{ij} - B_{ij} \cos \delta_{ij}) \longrightarrow ③$
$i \neq j$

and H_{ii} is given as,

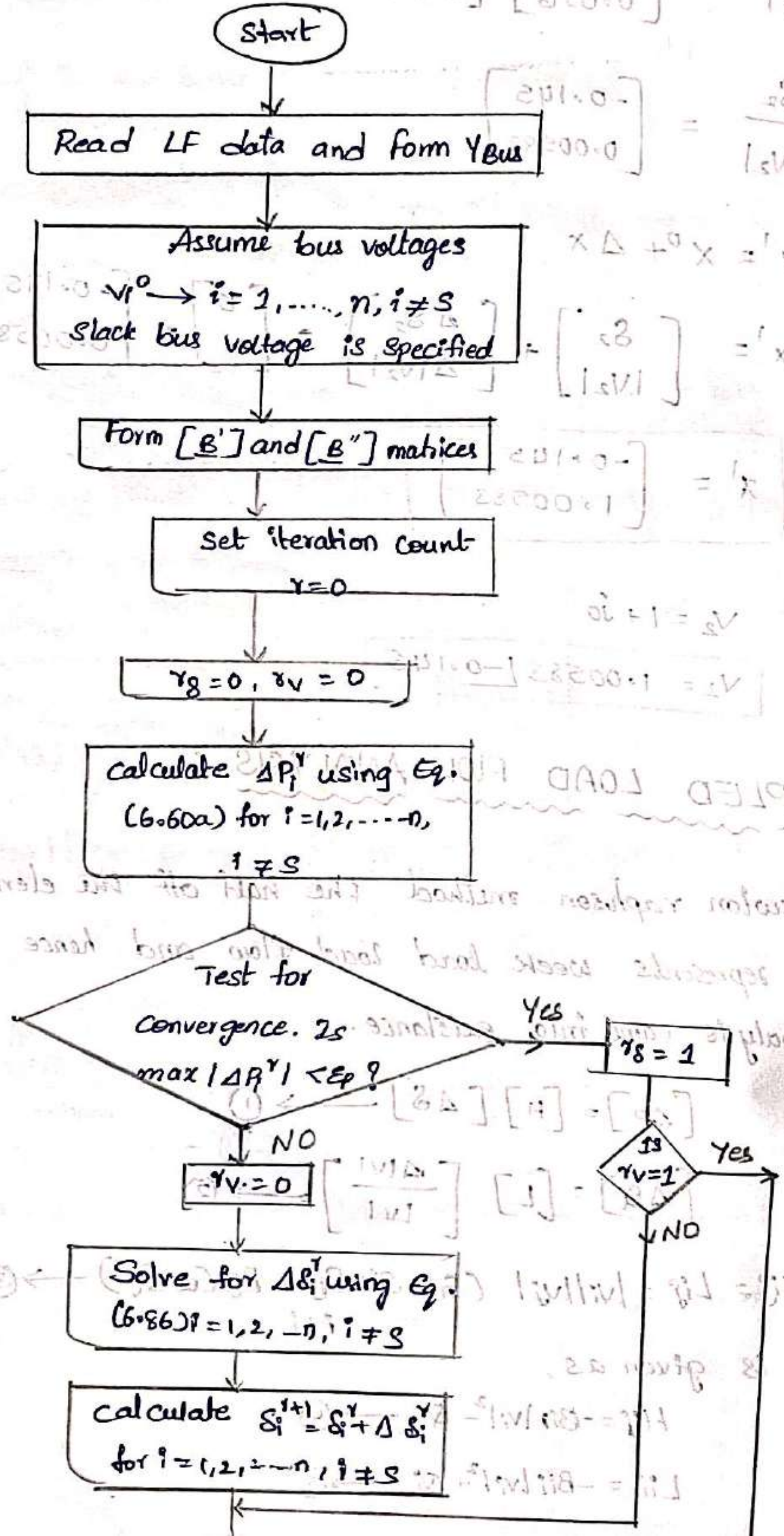
$$H_{ii} = -B_{ii}|V_i|^2 - Q_i \longrightarrow ④$$

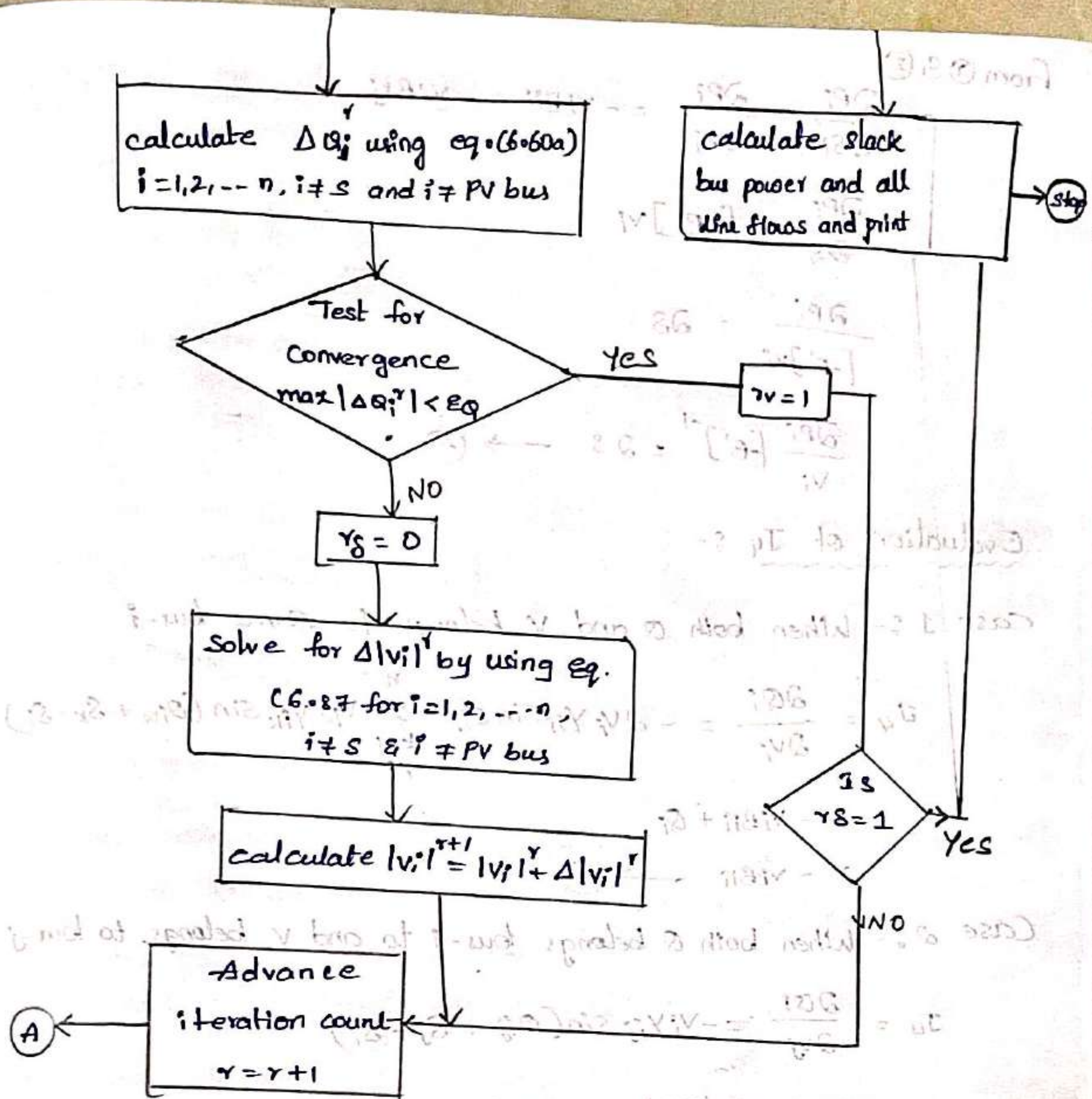
$$L_{ii} = -B_{ii}|V_i|^2 + Q_i \longrightarrow ⑤$$

equation ① & ② can be solved simultaneously by using Jacobian matrix. By updating H & L matrices in each itteration.

* A better aproach is to conduct each itteration by solving eq ① followed by eq ②.
  This will result in the faster load flow study.

FAST DECOUPLED METHOD :-





Evaluation of Jacobian Sub-Matrices J_1 & J_4

Evaluation of J_1 :-

$$J_1 = \frac{\partial P_i}{\partial \delta_i} = -V_i \sum_{\substack{k=1 \\ \neq i}}^{n} V_k Y_{ik} \sin(\theta_{ik} + \delta_k - \delta_i)$$

$$= -V_i V_i Y_{ii} \sin(\theta_{ii} + \delta_i - \delta_i) - V_i \sum_{\substack{k=1 \\ \neq i}}^{n} V_k Y_{ik} \sin(\theta_{ik} + \delta_k - \delta_i)$$

$$= -V_i^2 Y_{ii} \sin(\theta_{ii}) - V_i \sum_{\substack{k=1 \\ \neq i}}^{n} V_k Y_{ik} \sin(\theta_{ik} + \delta_k - \delta_i)$$

$$= -V_i B_{ii} \longrightarrow (1)$$

$(\because V_i^2 = V_i = 1;\ Q_i << V_i B_{ii})$

Case: 2 :- $P = i$, $\delta = j$

$$J_1 = \frac{\partial P_i}{\partial \delta_j} = -V_i V_j Y_{ij} \sin(\theta_{ij} + \delta_j - \delta_i)$$

$(\because \delta_i - \delta_j = 0;\ V_j = 1)$

$$= -V_i V_j Y_{ij} \sin(\theta_{ij})$$

$$= -V_i B_{ij} \longrightarrow (2)$$

From ① & ②

$$\frac{\partial P_i}{\partial \delta_i} = \frac{\partial P_i}{\partial \delta_j} = -V_i B_{ii} = -V_i B_{ij}$$

$$\frac{\partial P_i}{\partial \delta} = [-B'] V_i$$

$$\frac{\partial P_i}{[-B'] V_i} = \partial \delta$$

$$\frac{\partial P_i}{V_i} [-B']^{-1} = \partial \delta \longrightarrow (A)$$

Evaluation of J_4 :-

Case-1 :- When both Q and V belongs to same bus-i

$$J_4 = \frac{\partial Q_i}{\partial V_i} = -2 V_i Y_{ii} \sin \theta_{ii} - \sum_{\substack{k=1 \\ \neq i}}^{n} V_k Y_{ik} \sin(\theta_{ik} + \delta_k - \delta_i)$$

$$= -V_i B_{ii} + Q_i$$

$$= -V_i B_{ii} \longrightarrow (3)$$

Case-2 :- When both Q belongs bus-i to and V belongs to bus-j

$$J_4 = \frac{\partial Q_i}{\partial V_j} = -V_i Y_{ij} \sin(\theta_{ij} + \delta_j - \delta_i)$$

$$= -V_i Y_{ij} \sin(\theta_{ij}) \approx -V_i B_{ij} \longrightarrow (4)$$

From ③ & ④

$$\frac{\Delta Q_i}{\Delta V} = -[B''] V_i$$

$$\Delta V = -[B'']^{-1} \frac{\Delta Q_i}{V_i} \longrightarrow (B)$$

From (A) & (B)

$$\Delta \delta = -[B']^{-1} \frac{\Delta P_i}{V_i}$$

$$\Delta V = -[B'']^{-1} \frac{\Delta Q_i}{V_i}$$

* The above two equations are FDLF equations, by evaluating these equations in every iteration.
* The unknown states, i.e. δs and Vs are obtained.
* Here B' & B'' are susceptance.
  * δs = B' corresponds to susceptances (Both PV & PQ buses)
  * Vs = B'' corresponds to susceptances (only PQ buses)

COMPARISION OF DIFFERENT LOAD FLOW METHODS

Sl. No.	GS METHOD	NR METHOD	FDLF METHOD
1.	Rectangular coordinates are preferred for solution	Polar Coordinates are Preffered for solution	Polar Coordinates are Preffered for solution.
2.	More no.of iteration are required to get the acceptable solution.	Less no.of iteration are required to get the acceptable solution.	Less no.of iteration are required to get the acceptable solution
3.	The computation time per iteration will be less due to less no.of mathematical operation.	The Computation time per iteration is more i.e 8 times than GS method	The computation time per iteration is more i.e 2 to 3 times than GS method and 5 times than NR method.
4.	The number of iterations increases as the size of the system increases.	The number of iterations independent of the size of the system.	The number of iterations independent of the size of the system.
5.	Require large number of iterations to reach Convergence	Require less number of iterations to reach Convergence	Require more number of iterations than N.R method.
6.	Computation time per iteration is less	Computation time per iteration is more	Computation time per iteration is less
7.	The number of iterations required for convergence increases with size of the system.	The number of iterations are independent of the size of the system.	The number of iterations are do~~…~~ not dependent of the size of the system.
8.	Less memory requirements	More memory requirements	Less memory requirements than N.R. method.

References:

- Power System Analysis by A. Nagoor Kani, Second Edition, CBS Publisher & Distributors Pvt. Ltd.
- Modern Power System Analysis by D. P. Kothari and I. J. Nagarath, Fourth Edition, Tata McGraw-Hill.

Case Study

Topic: Load Flow Studies

Load Flow Analysis in a Power Distribution Network

In a practical power system, electricity generated at power plants must be transmitted and distributed efficiently to different consumers. As the demand for electricity increases in urban and industrial areas, system operators need to determine the voltage magnitude, voltage angle, real power, and reactive power at different buses in the network.

Consider a three-bus power system consisting of one slack bus, one PV bus, and one PQ bus. Due to increasing industrial load at one bus, the voltage level starts dropping. To analyze the condition of the system and maintain proper voltage levels, engineers perform load flow analysis.

Using methods such as Gauss-Seidel Method, Newton-Raphson Method, and Fast Decoupled Load Flow Method, the voltage magnitude and phase angle at each bus are calculated. The results help engineers identify voltage violations, line losses, and power flow in transmission lines.

After the analysis, corrective actions such as reactive power compensation or transformer tap changing are implemented to maintain voltage within acceptable limits.

This case study demonstrates how load flow studies help maintain reliable and efficient operation of power systems.

Unit-3 Outcomes:

1. Understand the **concept of load flow analysis in power systems**.
2. Identify different **types of buses such as slack bus, PV bus, and PQ bus**.
3. Explain the **Gauss-Seidel method for load flow calculations**.
4. Describe the **Newton-Raphson method used in power flow studies**.
5. Analyze the **importance of load flow studies in system planning and operation**.

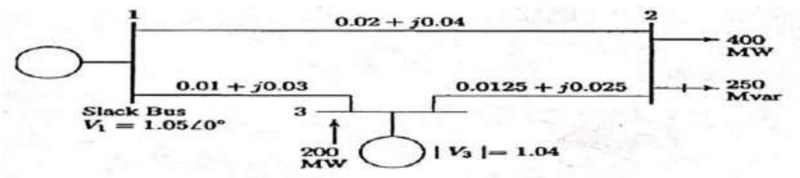
2 Marks Questions:

1. Define load flow analysis.
2. What is a slack bus?

3. What is a PQ bus in load flow studies?
4. What is a PV bus?
5. List the different load flow methods used in power system analysis.
6. State the advantages of the Newton-Raphson method.
7. What are the disadvantages of the Gauss-Seidel method?
8. What is meant by convergence in load flow studies?
9. Why is load flow analysis important in power systems?
10. What is the difference between real power and reactive power?

10 Marks Questions:

1. Discuss in detail about Gauss Seidel load flow analysis algorithm and give steps for its implementation when PV buses are also present in the system. Develop a flow chart for it.
2. Figure shows the one-line diagram of a simple three-bus power system. Line impedances are marked in per unit on a 100 MVA base. Obtain the power flow solution by Newton Raphson method for the first iteration.

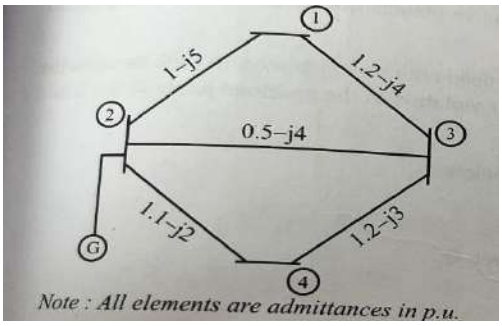


3. Illustrate the step-by-step procedure for load flow solutions using Gauss Seidel method, if PV and PQ buses are present along with slack bus.
4. Demonstrate various steps involved in the load flow analysis using Fast Decoupled Method.
5. A. What is Load flow or Power flow study?
   B. Derive the Load Flow Equation using Gauss Seidel method.
6. Explain the key differences between the Gauss Seidel, Newton Raphson, and Fast Decoupled Load flow methods. Highlight their advantages?
7. The System data for a load flow solution are given in tables. Determine the voltages at the end of first iteration by Gauss-seidel method. Take acceleration factor $\alpha = 1.6$.

Line Admittances	
Bus Code	**Admittance**
1-2	2-j8
1-3	1-j4
2-3	0.666-j2.664
2-4	1-j4
3-4	2-j8

Bus Specifications				
Bus Code	**P**	**Q**	**V**	**Remarks**
1	-	-	1.06<0	Slack
2	0.5	0.2	-	PQ
3	0.4	0.3	-	PQ
4	0.3	0.1	-	PQ

8. For the system shown in the figure determine the voltages at the end of first iteration by Gauss-Seidel method. Take $\alpha = 1$ and bus specifications are given in the table.



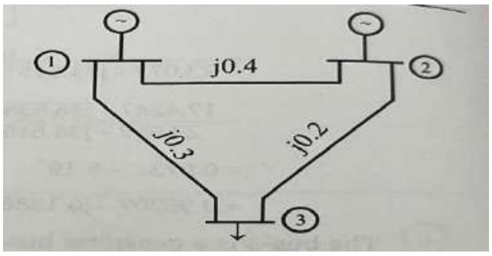
9. Figure shows a three-bus power system with impedance connected across.

Bus-1: Slack bus, V=1.05<0 p.u.

Bus-2: PV bus, |V|=1.0 p.u., Pg=3 p.u.

Bus-3: PQ bus, P=4 p.u, Q=2 p.u.

Carry out one iteration of load flow solution using Gauss Seidel method. Neglect limits on reactive power generation.



Bus Specifications				
Bus Code	**P**	**Q**	**V**	**Remarks**
1	-	-	1.06<0	Slack
2	0.5	0.1≤Q2≤1	1.04	PV
3	0.4	0.3	-	PQ
4	0.2	0.1	-	PQ

4. FAULT ANALYSIS

Series fault - open circuited faults
Shunt fault - short circuited faults

Symmetrical / Un symmetrical

Symmetrical	Un symmetrical	
L-L-L	L-G	with and
L-L-L-G	L-L	without field impedence
	L-L-G	

Introduction :-

Under normal operating condition the power system network in operate with stand parameter like voltage, current, power, etc...

* Whenever fault occurs the abnormality can be noticed in the power system network.
* The faults basic classified into two types;
    * Series faults
    * Shunt faults
* The shunt type fault are further classified into symmetrical and un-symmetrical.
* Symmetrical faults are L-L-L, L-L-L-G.
* Unsymmetrical faults are L-G, L-L, L-L-G.
* Symmetrical fault in power system are the faults that effect all 3-ø equal, resulting in a balance system.
* The unsymmetrical faults of the faults that effect the phases unequal, resulting in unbalanced sytem.

Advantage of Symmetrical fault :-

→ All the 3-ø are effected equal, maintaing in a balance Condition.
→ Symmetrical fault rare faults
→ Due to balance natural symmetrical fault are early to analysis

Review of Symmetrical Components of unbalance voltages and currents

* V_a, V_b, V_c are 3-ϕ unbalanced Voltage.
* V_{a1}, V_{b1}, V_{c1} are +ve sequence components of voltage
* V_{a2}, V_{b2}, V_{c2} are -ve sequence components of voltages.
* V_{a0}, V_{b0}, V_{c0} are zero sequence components of voltages

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} \longrightarrow (1)$$

* Now, the unbalanced voltages can be related to sequence component has;

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \longrightarrow (2)$$

$V_{a0} = V_{b0} = V_{c0} \longrightarrow$ zero sequence

$V_{a1} = V_{a0}$ $\quad$ $V_{a2} = V_{a0}$

$V_{b1} = a^2 V_{a1}$ $\quad$ $V_{b2} = aV_{a2}$

$V_{c1} = aV_{a1}$ $\quad$ $V_{c2} = a^2 V_{a2}$

$\downarrow$ +ve $\quad$ $\downarrow$ -ve

* Now, the unbalanced current can be related to sequence component as;

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} \longrightarrow (1)$$

$$\begin{bmatrix} I_{a0} \\ I_{b0} \\ I_{c0} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \longrightarrow (2)$$

$$I_{a0} = I_{b0} = I_{c0}$$

$$I_{a1} = I_{a0} \qquad I_{a2} = I_{a0}$$

$$I_{b1} = a^2 I_{a1} \qquad I_{b2} = a I_{a2}$$

$$I_{c1} = a I_{a1} \qquad I_{c2} = a^2 I_{a2}$$

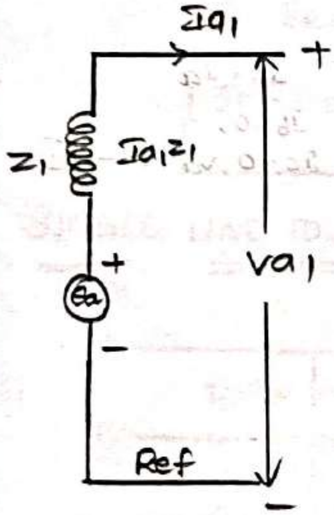
Review of Sequence Network of a generator.

Let

Z_1 = +ve impedence of a generator

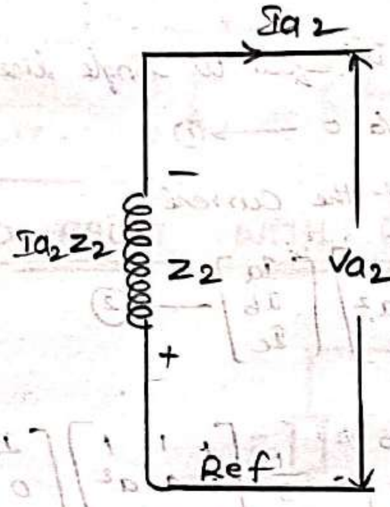
Z_2 = −ve impedence of a generator

Z_0 = Zero impedence of a generator



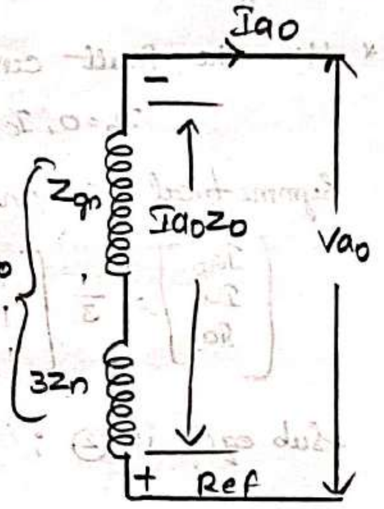
+ve Sequence Network of generator

$$V_{a1} = E_a - I_{a1}Z_1$$



−ve Sequence Network of generator

$$V_{a2} = -I_{a2}Z_2$$

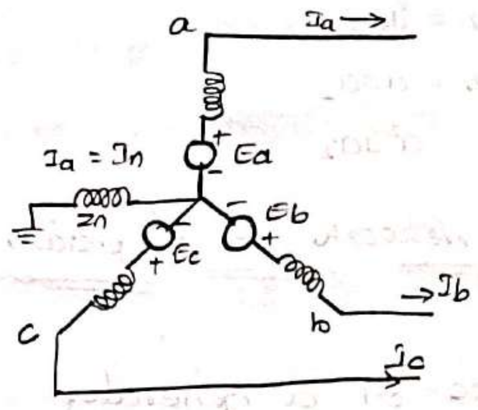


Zero Sequence Network of generator

$$V_{a0} = -I_{a0}Z_0$$

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$$

SINGLE LINE-TO-GROUND FAULT ON AN UNLOADED GENERATOR



The circuit diagram shows single line to ground fault without considering fault impedence.

* Here the fault current is equal to single line current $I_f = I_a$

$$I_b = 0,\ I_c = 0,\ V_a = 0 \longrightarrow (1)$$

$I_b = 0,$
$I_c = 0, V_a = 0 \longrightarrow (1)$

Symmetrical Components of the Current.

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \longrightarrow (2)$$

Sub eq(1) in (2) ;

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} I_a \\ 0 \\ 0 \end{bmatrix} \longrightarrow (3)$$

$$I_{a0} = \frac{1}{3} I_a,\ I_{a1} = \frac{1}{3} I_a,\ I_{a2} = \frac{1}{3} I_a ;$$

$$I_{a0} = I_{a1} = I_{a2} = \frac{1}{3} I_a \longrightarrow (4)$$

From the sequence network of generator :

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix}\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} \longrightarrow (5)$$

Now substitute $\left.\begin{matrix} I_{a0} = I_a, \\ I_{a2} = I_{a1} \end{matrix}\right\} \longrightarrow (5)$

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix}\begin{bmatrix} I_{a1} \\ I_{a1} \\ I_{a1} \end{bmatrix} \longrightarrow (6)$$

$$V_{a0} = -Z_0 I_{a1} \longrightarrow (7)$$

$$V_{a1} = E_a - Z_1 I_{a1} \longrightarrow (8)$$

$$V_{a2} = -Z_2 I_{a1} \longrightarrow (9)$$

Adding (7), (8), (9) and make it equals to zero

$\therefore V_{a0} + V_{a1} + V_{a2} = 0$

$\Rightarrow -Z_0 I_{a1} + E_a - Z_1 I_{a1} - Z_2 I_{a1} = 0$

$I_{a1}\left[-Z_0 - Z_1 - Z_2\right] + E_a = 0$

$I_{a1}\left[-(Z_0 + Z_1 + Z_2)\right] = -E_a$

$$I_{a1} = \frac{-E_a}{-(Z_0+Z_1+Z_2)}$$

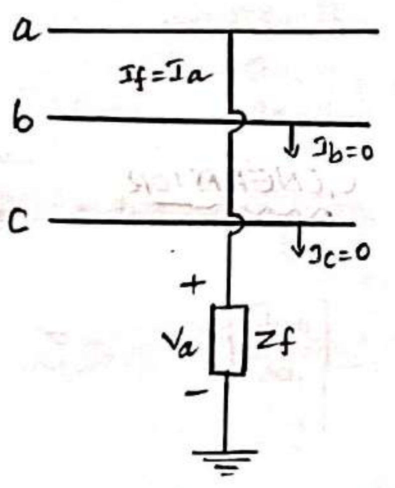
$$\boxed{I_{a1} = \frac{E_a}{Z_0+Z_1+Z_2}} \longrightarrow (10)$$

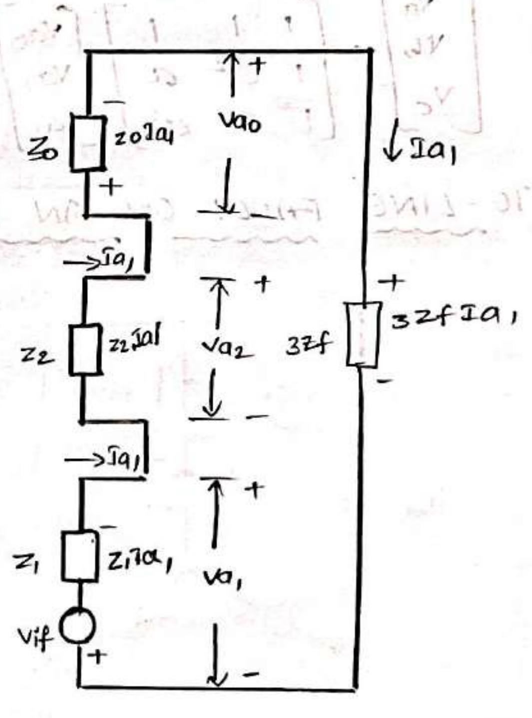
from eq (4) $I_{a1} = \frac{1}{3} I_a$

$I_a = 3 I_{a1}$

$$\boxed{I_f = I_a = 3\left[\frac{E_a}{Z_0+Z_1+Z_2}\right]} \quad \therefore \text{ from eq (10)}$$

SINGLE LINE TO GROUND FAULT WITH FIELD IMPEDENCE





With field impedence the voltages of Symmetrical components is given as

$$V_{a0} + V_{a1} + V_{a2} = 3Z_f I_{a1} \longrightarrow (12)$$

where $$I_{a1} = \frac{V_{pf}}{(Z_0+Z_1+Z_2)+3Z_f} \longrightarrow (13)$$

Substitute eq (13) in (12)

$$V_{a0} + V_{a1} + V_{a2} = \frac{3V_{pf} Z_f}{(Z_0+Z_1+Z_2)+3Z_f} \longrightarrow (14)$$

$$I_{a1} = \frac{1}{3} I_a$$

$$I_a = I_f = 3I_{a1}$$

$$I_f = 3 \left[\frac{V_{pf}}{(Z_0+Z_1+Z_2)+3Z_f} \right]$$

$$\text{Fault current, } I_f = I_a = 3I_{a1} = \frac{3V_{pf}}{(Z_1+Z_2+Z_0)+3Z_f} \longrightarrow (15)$$

The symmetrical components of phase-a voltage are calculated by writing Kirchoff's voltage law equations;

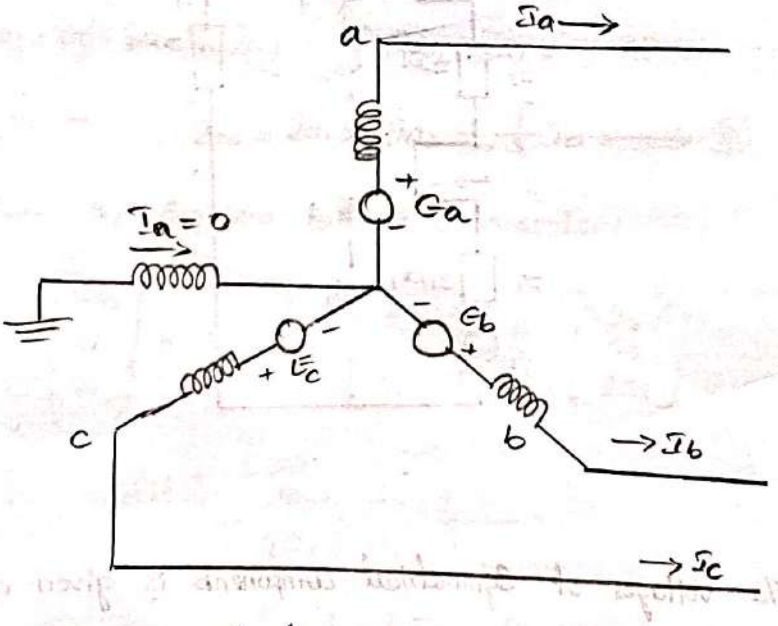
$$V_{a1} = V_{pf} - Z_1 I_{a1}$$

$$V_{a2} = -Z_2 I_{a1}$$

$$V_{a0} = -Z_0 I_{a1}$$

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix}$$

LINE-TO-LINE FAULT ON AN UNLOADED GENERATOR



The circuit diagram for a line-to-line fault on an unloaded, Y-connected generator, phase-b and phase-c are shorted. Now the fault current $I_f = I_b = -I_c$. Since the generator is unloaded the current in phase-a is zero.

The conditions at the fault are expressed by the following equation

$V_b = V_c$; $I_a = 0$;

$I_b + I_c = 0 \Rightarrow I_b = -I_c \longrightarrow (1)$

With $V_c = V_b$, the symmetrical components of voltages are given by,

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \longrightarrow (2)$$

On multiplying row 2 & 3 of eq (2) we get,

$$V_{a1} = \frac{1}{3}(V_a + aV_b + a^2V_b) \longrightarrow (3)$$

$$V_{a2} = \frac{1}{3}(V_a + a^2V_b + aV_b) \longrightarrow (4)$$

from eq (3) & (4) we can write

$$V_{a1} = V_{a2}$$

The symmetrical components of current are given by

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \longrightarrow (5)$$

on substituting $I_b = -I_c$ and $I_a = 0$ in eq (5) we get,

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} 0 \\ -I_c \\ I_c \end{bmatrix} \longrightarrow (6)$$

on multiplying eq (6) we get,

$$I_{a0} = \frac{1}{3}(-I_c + I_c) = 0$$

$$I_{a1} = \frac{1}{3}(-aI_c + a^2I_c) \longrightarrow (7)$$

$$I_{a2} = \frac{1}{3}(-a^2I_c + aI_c) = -\frac{1}{3}(-aI_c + a^2I_c) \longrightarrow (8)$$

from eq (7) & (8) we can write

$$I_{a2} = -I_{a1}$$

From the sequence networks of the generator we get ;

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} \longrightarrow (9)$$

on substituting $I_{a0} = 0$ & $I_{a2} = -I_{a1}$ in eq (9) we get,

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix} \begin{bmatrix} 0 \\ I_{a1} \\ -I_{a1} \end{bmatrix} \longrightarrow (10)$$

on multiplying eq (10) we get,

$$V_{a0} = 0$$

$$V_{a1} = E_a - Z_1 I_{a1} \longrightarrow (11)$$

$$V_{a2} = Z_2 I_{a1} \longrightarrow (12)$$

from eq (3) we known that $V_{a1} = V_{a2}$.

Hence, on equating (11) & (12) we get,

$$E_a - Z_1 I_{a1} = Z_2 I_{a1}$$

(or) $I_{a1}(Z_1 + Z_2) = E_a$

$$\therefore I_{a1} = \frac{E_a}{Z_1 + Z_2} \longrightarrow (13)$$

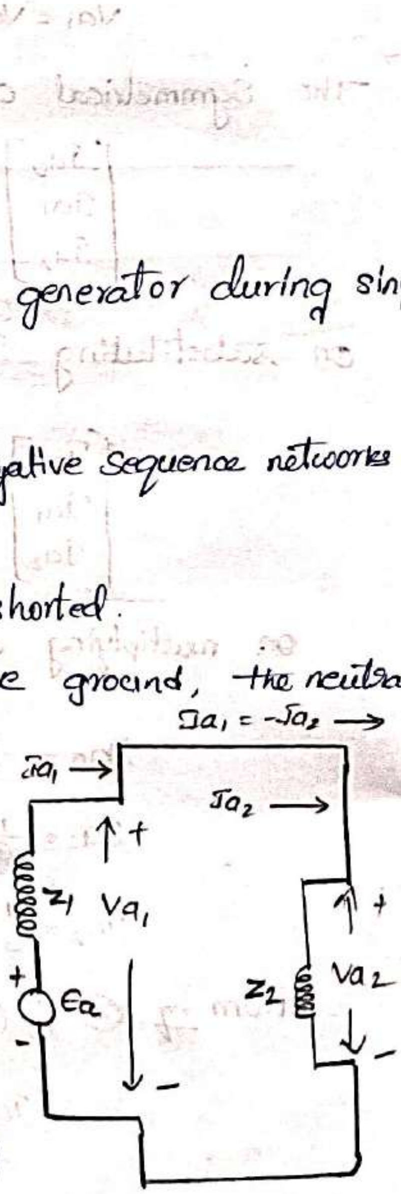
Using eq (13), the equivalent circuit of generator during single line to ground fault is drawn.

→ Since $V_{a1} = V_{a2}$, here the positive and negative sequence networks of the generator must be in parallel.

→ since $V_{a0} = 0$, the zero sequence network is shorted.

→ Since this type of fault does not involve ground, the neutral current $I_n = 0$.

Here the fault current, $I_f = I_b = -I_c$



The unbalanced currents I_a, I_b and I_c are related to the symmetrical components of currents ;

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} \longrightarrow (14)$$

from above equation, we get,

$$I_b = I_{a0} + a^2 I_{a1} + a I_{a2}$$

on substituting the conditions $I_{a0} = 0$ & $I_{a2} = -I_{a1}$,

we get;

$$I_b = a^2 I_{a1} - a I_{a2} = I_{a1}(a^2 - a)$$

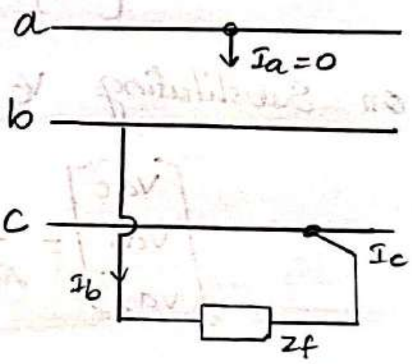
$\therefore$ Fallt current, $I_f = I_b = I_{a1}(a^2 - a)$

LINE-TO-LINE FAULT WITH FIELD IMPEDENCE

A line-to-line fault at point F in a power system between phases b and c through a fault impedence Z_f. can be represented by connecting three stubs.

→ The currents and voltages at the fault can be expressed as;

$$I_a = 0 \text{ and } I_b = -I_c \rightarrow ①$$



also, $V_b - V_c = I_b Z_f$;

$$\therefore V_c = V_b - I_b Z_f \rightarrow ②$$

The symmetrical components of the currents after substituting $I_a = 0$ and $I_c = -I_b$ we get,

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} 0 \\ I_b \\ -I_b \end{bmatrix} \rightarrow ③$$

on multiplying eq ③ we, get.

$$I_{a0} = \frac{1}{3}(I_b - I_b) = 0$$

$$I_{a1} = \frac{1}{3}(a I_b - a^2 I_b)$$

$$I_{a2} = \frac{1}{3}(a^2 I_b - a I_b) = -I_{a1} \longrightarrow ④$$

The line currents can be obtained from the following matrix equation, after substituting;

$$I_{a0} = 0 \text{ and } I_{a2} = -I_{a1}$$

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}\begin{bmatrix} 0 \\ I_{a1} \\ -I_{a1} \end{bmatrix}$$

From row-2 of matrix above equation we get,

$$I_b = a^2 I_{a1} - a I_{a1} = I_{a1}(a^2 - a)$$

$$= I_{a1}\left(-\frac{1}{2} - j\frac{\sqrt{3}}{2} + \frac{1}{2} - j\frac{\sqrt{3}}{2}\right) = -j\sqrt{3}\, I_{a1}$$

The symmetrical components of phase voltages are

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

on substituting $V_c = V_b - I_b Z_f$ in above eqn we get,

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} V_a \\ V_b \\ V_b - I_b Z_f \end{bmatrix}$$

From row-2 of matrix in above eqn. we get,

$$V_{a1} = \frac{1}{3}\left[V_a + aV_b + a^2 V_b - a^2 I_b Z_f\right]$$

$$\therefore 3V_{a1} = V_a + V_b(a + a^2) - a^2 Z_f I_b$$

From row-3 of matrix in above eqn we get.

$$V_{a2} = \frac{1}{3}\left[V_a + a^2 V_b + aV_b - a I_b Z_f\right]$$

$$\therefore 3V_{a2} = V_a + V_b(a^2 + a) - a Z_f I_b \rightarrow (5)$$

on subtracting eq (5) from above eqn; we get.

$$3V_{a1} - 3V_{a2} = V_a + V_b(a + a^2) - a^2 Z_f I_b - V_a(a^2 + a) + a Z_f I_b$$

$$= +(a^2 + a) Z_f I_b$$

$$= \left(\frac{1}{2} + j\frac{\sqrt{3}}{2} - \frac{1}{2} + j\frac{\sqrt{3}}{2}\right) Z_f I_b = j\sqrt{3} Z_f I_b$$

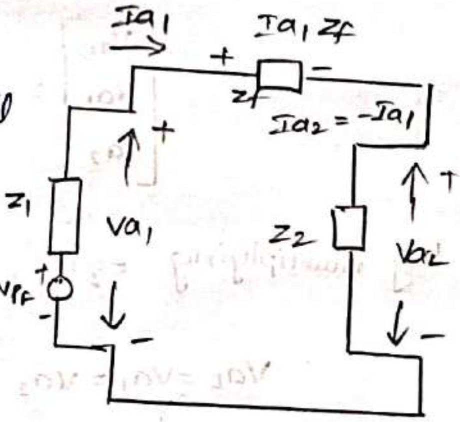
$$\therefore V_{a1} - V_{a2} = \frac{1}{3} j\sqrt{3} Z_f I_b = j \frac{1}{\sqrt{3}} Z_f I_b$$

on substituting for I_b, we get.

$$V_{a1} = V_{a2}$$

$$V_{a1} - V_{a2} = j\frac{1}{\sqrt{3}} Z_f(-j\sqrt{3} I_{a1}) = Z_f I_{a1}$$

$$\therefore V_{a1} - V_{a2} = Z_f I_{a1} \rightarrow (6)$$



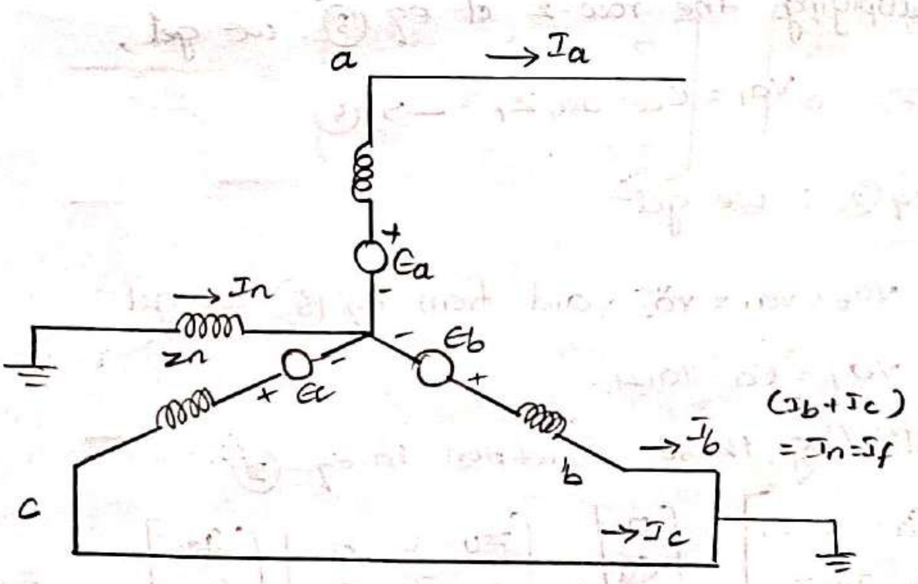
The equations (4) & (6) suggest parallel connection of positive & negative sequence networks through a series impedance Z_f.

$\therefore$ the equivalent circuit of power system under line-to line fault can be drawn.

$$I_{a1} = \frac{V_{PF}}{Z_1 + Z_2 + Z_f}$$

The fault current, $I_f = I_b = -j\sqrt{3} I_{a1}$

DOUBLE LINE-TO-GROUND FAULT ON AN UNLOADED GENERATOR



The circuit diagram for a double line-to ground fault on an unloaded Y connected generator having a grounded neutral.

→ The faulted phases are b and c.

Now, the fault current, $I_f = I_b + I_c$.

Since the generator is unloaded the current in phase-a is zero.

The Conditions at the fault are expressed by the following equations.

$$V_b = 0, \quad V_c = 0, \quad I_a = 0$$

With $V_b = V_c = 0$, the symmetrical components of voltage are given by.

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} V_a \\ 0 \\ 0 \end{bmatrix} \rightarrow (1)$$

by multiplying eq (1); we get,

$$V_{a0} = V_{a1} = V_{a2} = \frac{V_a}{3} \longrightarrow (2)$$

From the Sequence networks of the generator we get the following matrix equation.

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix}\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} \longrightarrow (3)$$

On multiplying the row-2 of eq (3), we get,

$$V_{a1} = E_a - I_{a1} Z_1 \longrightarrow (4)$$

From eq (2); we get

$V_{a0} = V_{a1} = V_{a2}$ and from eq (4) we get;

$V_{a1} = E_a - I_{a1} Z_1$.

On substituting these condition in eq (3):

$$\begin{bmatrix} E_a - I_{a1} Z_1 \\ E_a - I_{a1} Z_1 \\ E_a - I_{a1} Z_1 \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix}\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} \longrightarrow (5)$$

On rearranging the eq ⑤ we get,

$$\begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix} \begin{bmatrix} I_{a_0} \\ I_{a_1} \\ I_{a_2} \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} E_a - I_{a_1}Z_1 \\ E_a - I_{a_1}Z_1 \\ E_a - I_{a_1}Z_1 \end{bmatrix} \longrightarrow ⑥$$

Let $Z = \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix}$

$$\therefore Z^{-1} = \begin{bmatrix} \frac{1}{Z_0} & 0 & 0 \\ 0 & \frac{1}{Z_1} & 0 \\ 0 & 0 & \frac{1}{Z_2} \end{bmatrix} \longrightarrow ⑦$$

To find Z^{-1} :-

$$Z^{-1} = \frac{\cancel{\text{Adjoint}} \text{ of } Z}{\text{Determinant of } Z} = \frac{Z_{cof}^T}{\Delta Z}$$

$$\Delta Z = \begin{vmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{vmatrix} = Z_0 Z_1 Z_2$$

$$Z_{cof}^T = \begin{bmatrix} Z_1Z_2 & 0 & 0 \\ 0 & Z_0Z_2 & 0 \\ 0 & 0 & Z_0Z_1 \end{bmatrix}^T = \begin{bmatrix} Z_1Z_2 & 0 & 0 \\ 0 & Z_0Z_2 & 0 \\ 0 & 0 & Z_0Z_1 \end{bmatrix}$$

$$\therefore Z^{-1} = \frac{1}{Z_0Z_1Z_2} = \begin{bmatrix} Z_1Z_2 & 0 & 0 \\ 0 & Z_0Z_2 & 0 \\ 0 & 0 & Z_0Z_1 \end{bmatrix} = \begin{bmatrix} \frac{1}{Z_0} & 0 & 0 \\ 0 & \frac{1}{Z_1} & 0 \\ 0 & 0 & \frac{1}{Z_2} \end{bmatrix}$$

on premultiplying the eq ⑥ by Z^{-1} we get-

$$\begin{bmatrix} I_{a_0} \\ I_{a_1} \\ I_{a_2} \end{bmatrix} = \begin{bmatrix} \frac{1}{Z_0} & 0 & 0 \\ 0 & \frac{1}{Z_1} & 0 \\ 0 & 0 & \frac{1}{Z_2} \end{bmatrix} \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} \frac{1}{Z_0} & 0 & 0 \\ 0 & \frac{1}{Z_1} & 0 \\ 0 & 0 & \frac{1}{Z_2} \end{bmatrix} \begin{bmatrix} E_a - I_{a_1}Z_1 \\ E_a - I_{a_1}Z_1 \\ E_a - I_{a_1}Z_1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} I_{a_0} \\ I_{a_1} \\ I_{a_2} \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{E_a}{Z_1} \\ 0 \end{bmatrix} - \begin{bmatrix} \frac{E_a - I_{a_1}Z_1}{Z_0} \\ \frac{E_a - I_{a_1}Z_1}{Z_1} \\ \frac{E_a - I_{a_1}Z_1}{Z_2} \end{bmatrix} \longrightarrow ⑧$$

on multiplying above eqn we, get ;

$$I_{a0} = -\left(\frac{E_a - I_{a1}Z_1}{Z_0}\right) = -\frac{E_a}{Z_0} + \frac{I_{a1}Z_1}{Z_0}$$

$$I_{a1} = \frac{E_a}{Z_1} - \left(\frac{E_a - I_{a1}Z_1}{Z_1}\right) = \frac{E_a}{Z_1} - \frac{E_a}{Z_1} + \frac{I_{a1}Z_1}{Z_1} = I_{a1}$$

$$I_{a2} = -\left(\frac{E_a - I_{a1}Z_1}{Z_2}\right) = -\frac{E_a}{Z_2} + \frac{I_{a1}Z_1}{Z_2}$$

Here $I_a = 0$, on expressing I_a by its symmetrical components we get,

$$I_a = I_{a0} + I_{a1} + I_{a2} = 0$$

on substituting for I_{a0}, I_{a1} and I_{a2} from above equations:

$$-\frac{E_a}{Z_0} + \frac{I_{a1}Z_1}{Z_0} + I_{a1} - \frac{E_a}{Z_2} + \frac{I_{a1}Z_1}{Z_2} = 0$$

$$I_{a1}\left(\frac{Z_1}{Z_0} + 1 + \frac{Z_1}{Z_2}\right) = \frac{E_a}{Z_0} + \frac{E_a}{Z_2}$$

$$I_{a1}\left(1 + Z_1\left(\frac{1}{Z_0} + \frac{1}{Z_2}\right)\right) = E_a\left(\frac{1}{Z_0} + \frac{1}{Z_2}\right)$$

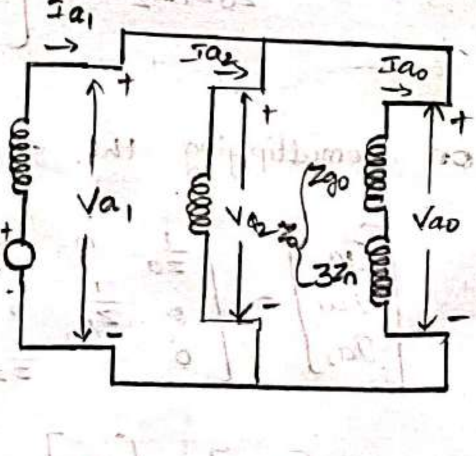
$$I_{a1}\left(1 + Z_1\left(\frac{Z_0 + Z_2}{Z_0 Z_2}\right)\right) = E_a\left(\frac{Z_0 + Z_2}{Z_0 Z_2}\right)$$

on multiplying the above eqn throughout by $\frac{Z_0 Z_2}{Z_0 + Z_2}$ we get,

$$I_{a1}\left(\frac{Z_0 Z_2}{Z_0 + Z_2} + Z_1\right) = E_a$$

$$\therefore\ I_{a1} = \frac{E_a}{Z_1 + \frac{Z_0 Z_2}{Z_0 + Z_2}} \longrightarrow (9)$$

using above eqn. the equivalent circuit of generator under double line-to-ground fault can be drawn.



→ Using this fault condition the sequence networks should be connected in parallel

→ Since the positive, negative and zero sequence voltages are equal during this fault.

→ Absence of a ground connection at the generator no current can flow into the ground at the fault.

Here fault current, $I_f = I_b + I_c$

$$I_{a1} = \frac{E_a}{Z_1 + \frac{Z_0 Z_2}{Z_0 + Z_2}}$$

$$I_{a0} = -\frac{E_a}{Z_0} + \frac{I_{a1} Z_1}{Z_0}$$

$$I_{a2} = -\frac{E_a}{Z_2} + \frac{I_{a1} Z_1}{Z_2}$$

Alternatively the I_{a1} can be calculated using eq (9) and then I_{a2} & I_{a0} can be calculated using current division rule;

$$I_{a2} = -I_{a1} \times \frac{Z_0}{Z_0 + Z_2} \quad \& \quad I_{a0} = -(I_{a1} - I_{a2})$$

The unbalanced current I_a, I_b and I_c are related to the symmetrical components of currents by the following equation.

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$$

From above eqn we get,

$$I_b = I_{a0} + a^2 I_{a1} + a I_{a2}$$

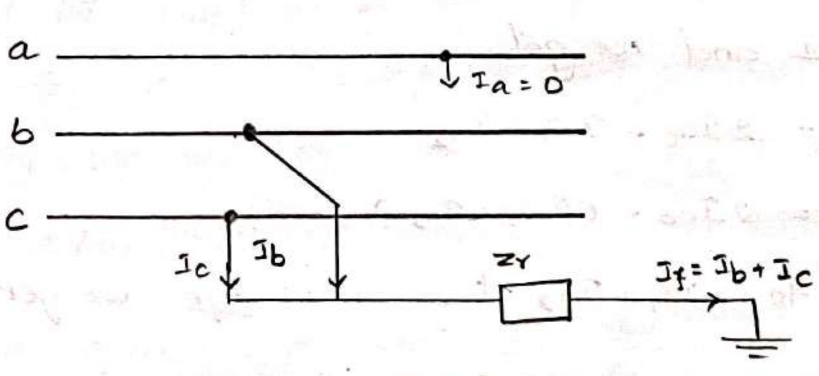
$$I_c = I_{a0} + a I_{a1} + a^2 I_{a2}$$

$\therefore$ Fault current, $I_f = I_b + I_c = I_{a0} + a^2 I_{a1} + a I_{a2} + I_{a0} + a I_{a1} + a^2 I_{a2}$

$$= 2I_{a0} + (a + a^2) I_{a1} + (a + a^2) I_{a2}$$

$$= 2I_{a0} + (a + a^2)(I_{a1} + I_{a2})$$

DOUBLE LINE - TO - GROUND FAULT WITH FIELD IMPEDENCE



A double line-to-ground fault at point F in a power system through a fault impedence Z_f can be respresented by connecting three stubs.

→ The current and voltage conditions at the faults are

$$I_a = 0$$

$$V_b = V_c = Z_f\,(I_b + I_c)$$

The line currents are given by

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} \rightarrow ①$$

From row-1 of eq ① we get,

$$I_a = I_{a0} + I_{a1} + I_{a2}$$

From above eq we get $I_a = 0$

$$\therefore I_{a0} + I_{a1} + I_{a2} = 0 \text{ (or) } I_{a0} = -(I_{a1} + I_{a2})$$

From row-2 and 3 of eq ① we get,

$$I_b = I_{a0} + a^2 I_{a1} + I_{a2}$$

$$I_c = I_{a0} + a I_{a1} + a^2 I_{a2}$$

$$I_b + I_c = I_{a0} + a^2 I_{a1} + a I_{a2} + I_{a0} + a I_{a1} + a^2 I_{a2}$$

$$\therefore \quad = 2 I_{a0} + (a^2 + a) I_{a1} + (a^2 + a) I_{a2}$$

W.K.T, $1 + a + a^2 = 0$

$$\therefore a + a^2 = -1$$

From above eqn and we get,

$$I_b + I_c = 2 I_{a0} - I_{a1} - I_{a2}$$

$$= 2 I_{a0} - (I_{a1} + I_{a2})$$

on substituting for $I_{a1} + I_{a2}$ from above eqn we get,

$$I_b + I_c = 2 I_{a0} - (-I_{a0}) = 3 I_{a0}$$

The symmetrical components of voltages after substituting $V_c = V_b$ are given by.

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} V_a \\ V_b \\ V_b \end{bmatrix}$$

From row-1 of above eqn we get,

$$V_{a0} = \frac{1}{3}[V_a+V_b+V_b] = \frac{1}{3}[V_a+2V_b] \rightarrow (2)$$

From row-2 and 3 in above eqn, we get.

$$V_{a1} = V_{a2} = \frac{1}{3}[V_a+aV_b+a^2V_b] \qquad (\therefore a+a^2=-1)$$

$$= \frac{1}{3}(V_a+(a+a^2)V_b = \frac{1}{3}(V_a-V_b) \rightarrow (3)$$

from eq (2) & (3), we can write,

$$V_{a0}-V_{a1} = \frac{1}{3}(V_a+2V_b) - \frac{1}{3}(V_a-V_b)$$

$$= \frac{1}{3}(V_a+2V_b-V_a+V_b) = V_b$$

$\therefore V_{a0} - V_{a1} = V_b$

on substituting for V_b value in above eqn:

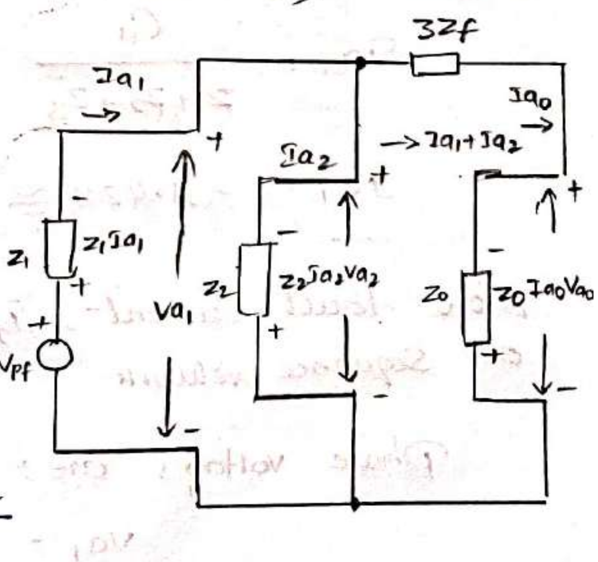
$$V_{a0} - V_{a1} = Z_f(I_b+I_c)$$

on substituting (I_b+I_c) from I_b+I_c value in above eqn, we get,

$$V_{a0} - V_{a1} = Z_f 3 I_{a0}$$

$$\therefore V_{a0} = V_{a1} + 3Z_f I_{a0} \qquad (\because V_{a1} = V_{a2})$$

Also, $V_{a0} = V_{a2} + 3Z_f I_{a0}$



→ Suggest the connection of sequence networks for a double line-to ground fault.

From fig; we can write,

$$I_{a1} = V_{pf} / Z_1 + \frac{Z_2(Z_0+3Z_f)}{Z_2+Z_0+3Z_f}$$

$V_{a1} = V_{pf} - Z_1 I_{a1}$; $V_{a2} = V_{a1}$; $I_{a2} = \frac{V_{a2}}{Z_2}$

$I_{a0} = -(I_{a1}+I_{a2})$

$\therefore$ Fault current, $I_f = I_b + I_c = 3I_{a0}$

Problem:-

* A salient pole generator without damper is rated 20 MVA 13.8 kV and has a direct axis subtransient reactance of 0.25/unit. The negative and zero sequence reactance are 0.35 and 0.10 /unit respectively. The neutral of generator is grounded. Determine the subtransient current in the generator and line-to-line voltages per subtraction conditions when a single line to ground fault occurs at the generator terminals with generator operating at rated voltage.

Sol Given Data;

$MVA_b = 20\,MVA$

$kV_b = 13.8\,kV$

$Z_1 = j0.25\,p.u$

$Z_2 = j0.35\,p.u$

$Z_0 = j0.10\,p.u$

$$\text{Base Current} = \frac{\text{Base kVA}}{\sqrt{3}\,kV} = \frac{20 \times 1000}{\sqrt{3} \times 13.8} = 836.7\,A.$$

Single line to ground :-

$I_{a0} = I_{a1} = I_{a2}$

W.K.T;

$$I_{a1} = \frac{E_a}{Z_1+Z_2+Z_3} = \frac{1\angle 0}{j0.25+j0.35+j0.1}$$

$I_{a1} = -j1.428 \cong -j1.43\,p.u.$

Now fault current; $I_f = I_a = 3 \times I_{a1} = 3 \times (-j1.43) = -4.285j$ of sequence Network.

Phase voltages are;

$V_{a1} = E_a - I_{a1} Z_1 = 1 - (-j1.43)(j1.25)$

$V_{a1} = 0.643\,p.u$

$$V_{a2} = -I_{a2} Z_2 = -(-j1.43)(j0.35) = -0.5 \text{ p.u}$$

$$V_{a0} = -I_{a0} Z_0 = -(-j1.43)(j0.1) = -0.143 \text{ p.u}$$

Phase voltage :-

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix}$$

$$V_a = V_{a0} + V_{a1} + V_{a2}$$

$$= -0.143 + 0.643 - 0.5$$

$$\boxed{V_a = 0}$$

$a^2 = 1\angle 240 = -0.5 - j0.8$

$a = 1\angle 120 = -0.5 + j0.866$

$$V_b = V_{a0} + a^2 V_{a1} + a V_{a2}$$

$$= -0.143 + 1\angle 240 \times 0.643 + 1\angle 120 \times (-0.5)$$

$$= -0.143 + (-0.5 - j0.86) \times 0.643 + (-0.5) + j0.86) \times (-0.5)$$

$$\boxed{V_b = -0.214 - 0.982j}$$

$$V_c = V_{a0} + a V_{a1} + a^2 V_{a2}$$

$$= 0.143 + (-0.5 + j0.866)(0.643) + (-0.5 - j0.866)(-0.5)$$

$$\boxed{V_c = 0.214 + 0.98j}$$

Phase to line voltages are ;

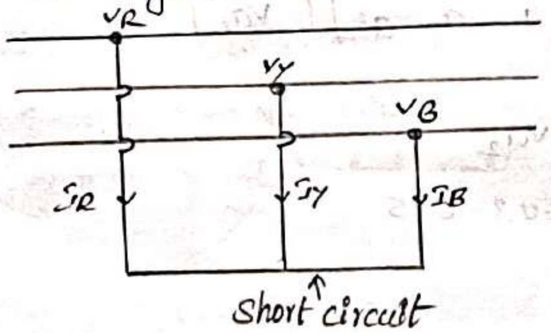
$$V_{ab} = V_a - V_b = 0 - (-0.214 - j0.982) = 0.214 + 0.98j$$

$$V_{bc} = V_b - V_c = (-0.214 - 0.982j) - (-0.214 + 0.98j) = 0 - 1.96j$$

$$V_{ca} = V_c - V_a = (-0.214 + 0.98j) - 0 = -0.214 + 0.98j$$ //

Symmetrical Fault Analysis

* The fault on the power system which gives rise to symmetrical current is called a symmetrical fault.
* The symmetrical fault occurs when all the three conductors of a 3-φ line are brought together simultaneously into a short circuit condition as shown in the figure.



Symmetrical fault Analysis (contd....)

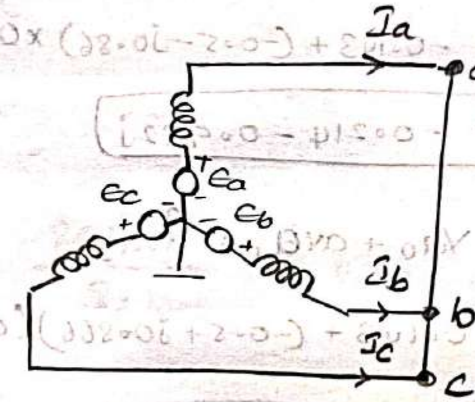
3-phase fault :-

* The boundary conditions are

$V_a = V_b = V_c$

$I_a + I_b + I_c = 0$

* The symmetrical fault conditions are analyzed on per phase basis using Thevenin's Theorem or Bus Impedence matrix



Sequence Components :-

→ An unbalanced system of 'n' related vectors can be resolved into 'n' system of balanced vectors called symmetrical components of original vectors.

→ In a 3-φ system, the 3 unbalanced vectors either V_a, V_b, V_c or I_a, I_b, I_c can be resolved into 3 balanced system of vectors. The vectors of the balanced system are called symmetrical components of the original system.

→ The symmetrical components of 3-φ system are as follows:

Positive-Sequence Components

Negative-Sequence Components

Zero-Sequence Components

Sequence Components (contd....)

1. Positive Sequence Components :-

* Equal in magnitude
* 120 degrees phase angles exists with same phase sequence of original vectors.
* Occurs before and after fault.

Importance :- Relay and circuit breaker operates on positive sequence components.

2. Negative Sequence Components :-

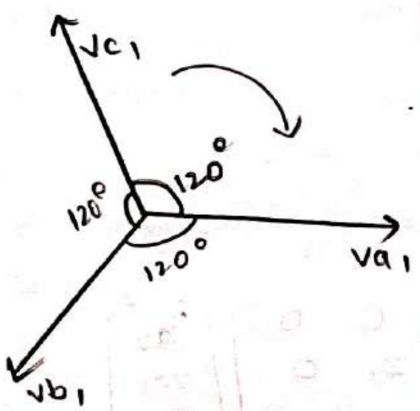
* Equal in magnitude
* 120 degrees phase angle exists with opposite phase sequence of original vectors.
* Occurs only during faults.

Importance :- Synchronous Generator is protected from unbalanced condition by using negative sequence relay.

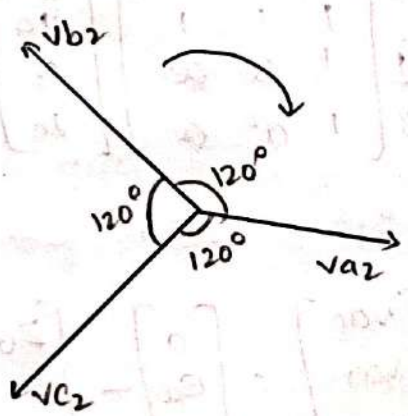
3. Zero Sequence Components :-

* Equal in magnitude, No phase difference
* Occurs only when neutral is grounded and fault occured with grounded.

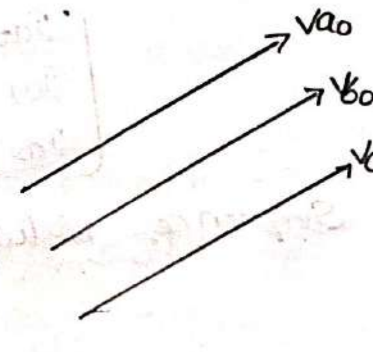
Importance :- Zero Sequence Components are used in the calculation of leakage flux.



+ve sequence Component



-ve sequence Component



Zero Sequence Component

The operator 'a' is defined as

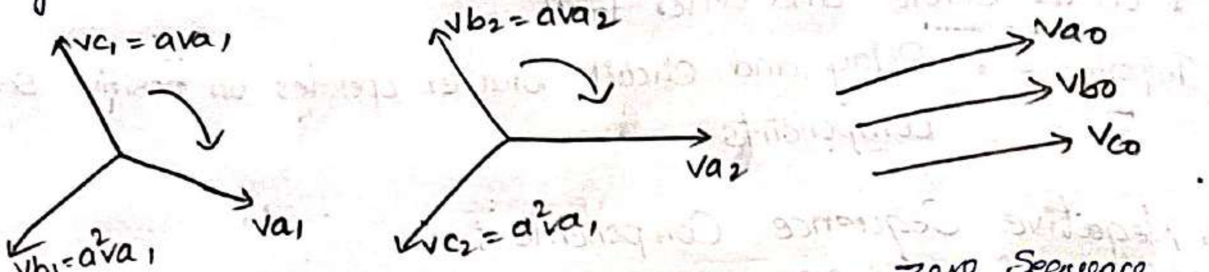
$$a = \cos 120^\circ + j\sin 120^\circ = -0.5 + j0.8666$$

$$a^2 = \cos 120^\circ - j\sin 120^\circ = -0.5 - j0.8666$$

$$a^3 = 1$$

$$1 + a + a^2 = 0$$

The Symmetrical Component for Voltages are derived as follows:



+ve Sequence Component

-ve Sequence component

Zero Sequence Component

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix}$$

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

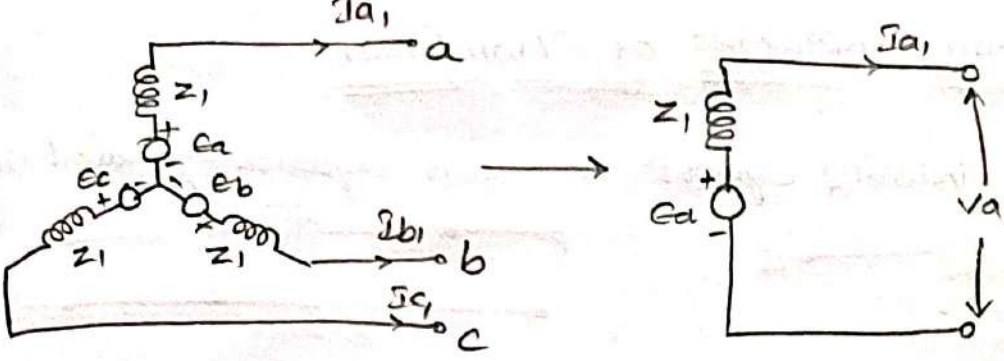
The Symmetrical Components for current can be expressed as follows;

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$$

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

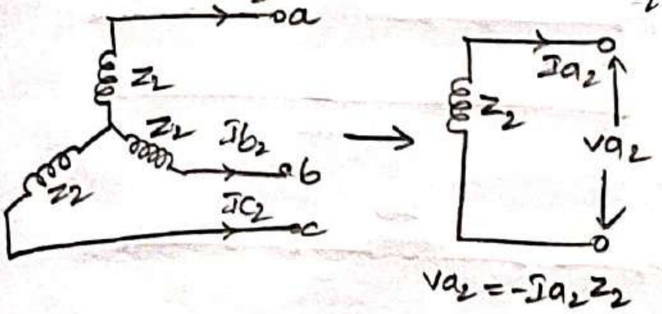
Sequence Networks ;-

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$$



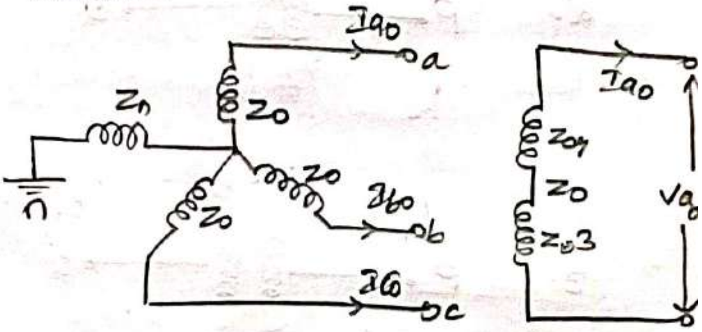
$V_{a1} = E_a - I_{a1} Z_1$

(a) Positive Sequence Network



$V_{a2} = -I_{a2} Z_2$

(b) Negative Sequence Network

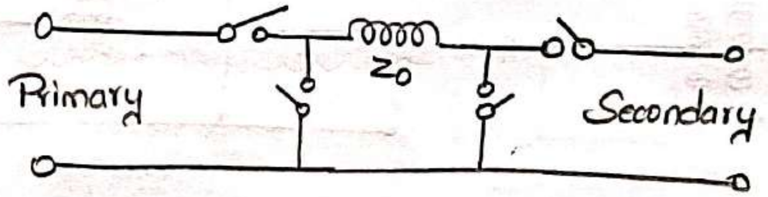


$V_{a0} = -I_{a0} Z_0$

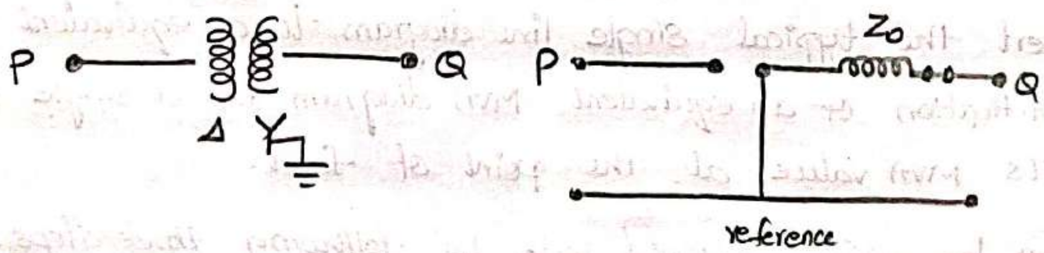
(c) Zero Sequence Network.

Zero Sequence Networks of Transformer

→ Series and Shunt Switch- Connections for Delta and Star windings of Transformers are represented as follows.



→ Consider a Δ/Y Transformer Connected with star grounded as shown in the following fig.

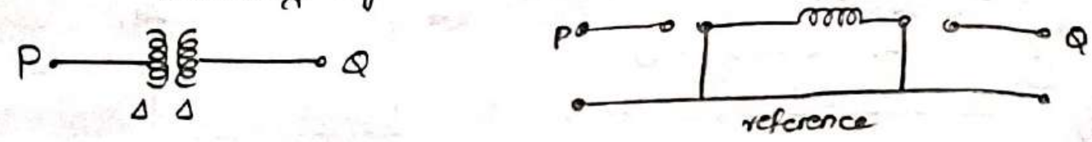


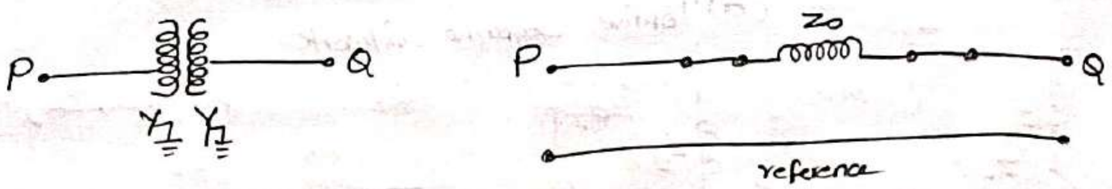
→ Since, the primary is delta connected, the shunt switch of primary side is coming closed and the series switch is left open.

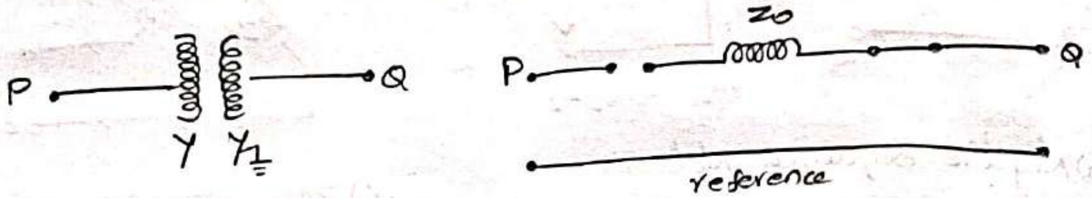
→ Secondary is star grounded, therefore the series switch is closed and the shunt switch is left open.

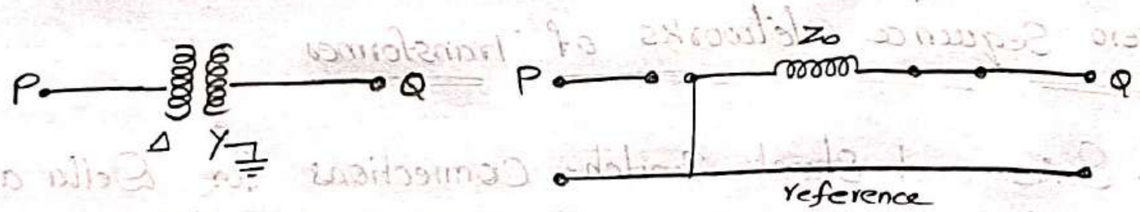
Zero Sequence Networks of Transformer

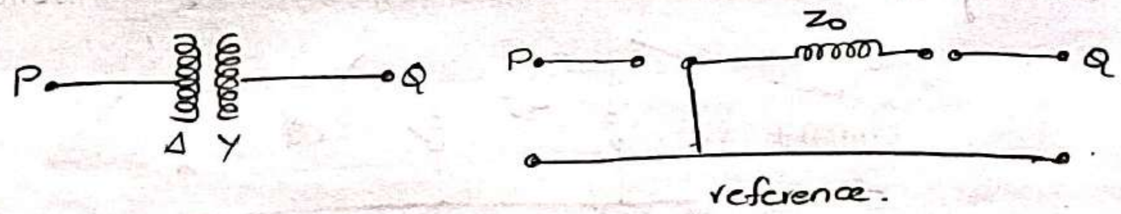
Winding Symbol — Zero Sequence equivalent circuit

1.


2.


3.


4.


5.


Short Circuit Current Calculation using MVA method :-

1. Convert the typical single line diagram to a equivalent MVA diagram.
2. Simplification of an equivalent MVA diagram into a single short-circuits MVA value at the point of fault.

This can be easily achieved with the following three steps.

Step-1 :- Convert all single line components to short circuit MVA's.

In practical, the MVA method is used by seperating the circuit into components and calculating each component with its own infinite bus.

~~Each~~ 8

Equipment such as generators, motors, transformers, etc..., is normally given their own MVA and impedence or reactance ratings.

This short circuit MVA of each component in the given SLD is equal to its MVA rating divided by its own per unit impedence or reactance.

Step-2 : Combine individual MVA values.

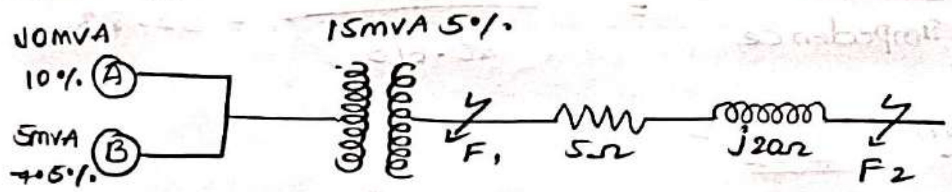
1) Series MVA's are combined as resistances in parallel
2) Parallel MVA's are added arithmetically.

Step-3 : Reduce MVA diagram into a single short-circuits MVA value at the point of fault.

Reduce MVA diagram by simplifying the equivalent MVA diagram using the MVA quantities obtained in the previous step.

Problem :-

1. Consider an example power system network as shown in the below SLD.



one line Diagram

SLD Components Data :-

1. Generator - A : 10MVA, 10% reactance
2. Generator - B : 5MVA, 7.5% reactance
3. Transformer : 15MVA, 5% reactance, 11/33 kV.
4. Transmission Line : Impedence $Z = 5+j20$ ohms.

For this Network find the short circuit MVA and fault current values fed to the symmetrical fault between phases if it occurs at points F1 and F2 that is

1. At the high voltage terminals of the transformer F1.
2. At the load end of the transmission line F2.

Step - 1 :-

Sol :- Convert all single line components in the given SLD to short circuit MVA's.

1. Generator - A : 10 MVA, 10 % reactance

Short circuit of MVA of Generator-A $MVA_1 = \dfrac{MVA}{\text{Sub-transient reactance of generator in per unit}}$

$$\Rightarrow MVA_1 = \frac{10}{0.1} = 100.$$

2. Generator - B :- 5MVA, 7.5 % reactance.

$$\Rightarrow MVA_2 = \frac{5}{0.075} = 66.67$$

3. Transformer : 15 MVA, 5 % reactance, $\frac{11}{33\,KV}$

$$\Rightarrow MVA_3 = \frac{15}{0.05} = 300$$

4. Transmission Line :-

Impedence $Z = 5 + j20$ ohms

$Z = Sqrt(5*5 + 20 \times 20) = Sqrt(25+400) = sqrt(425)$

$Z = 20.615$ ohms

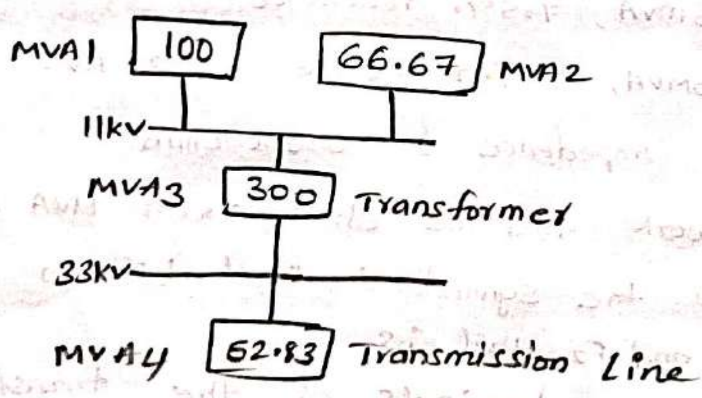
Voltage rating of Transmission line $= 33kv$.

$$\Rightarrow MVA4 = \frac{kv^2}{\text{Impedence}} = \frac{33 \times 33}{20.615} = 52.83$$

SLD Equivalent MVA Diagram :-

Using the above short circuit MVA values of each components in the SLD, draw the MVA diagram as shown below.

Generators



Step-2 :- Combine Individual MVA values.

Two generators are connected in parallel.

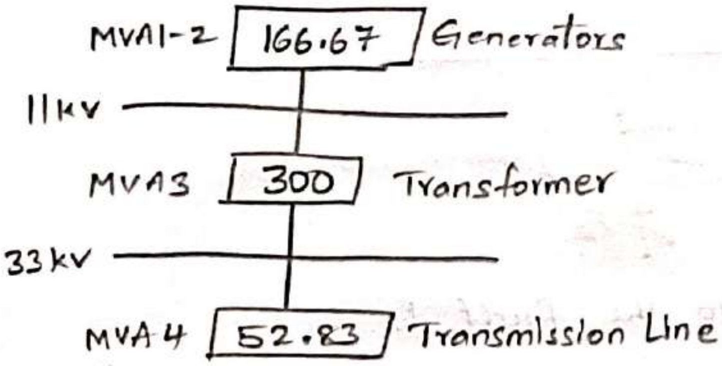


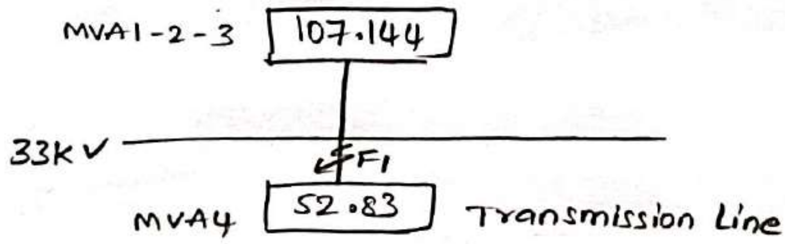
Fig : Combined MVA Diagram.

Combined $MVA1\text{-}2 = MVA1 + MVA2 = 100 + 66.67 = 166.67$

Step-3:- Reduce MVA diagram into a single short-circuits MVA value at the point of fault to find SC MVA and SC current values.

1. Short circuit Circuit MVA and short circuit current calculation for fault F1 :

MVA1-2 is in series with MVA-3.



Reduced MVA Diagram for fault-F1

Total short circuit MVA up to the fault F1 =

$$\Rightarrow \text{Combined } MVA1\text{-}2\text{-}3 = \frac{(MVA1\text{-}2 \times MVA3)}{(MVA1\text{-}2) + (MVA3)}$$

$$MVA1\text{-}2\text{-}3 = \frac{(166.67 \times 300)}{(166.67 + 300)} = 107.144$$

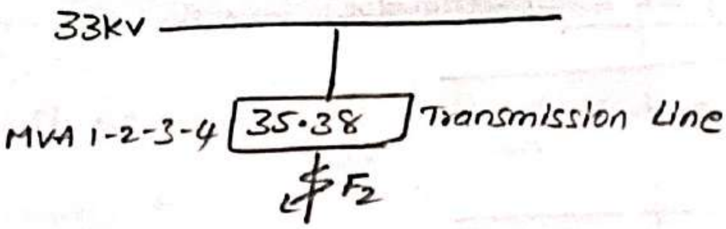
→ Total short circuit MVA up to the fault $F_1 = 107.144$

→ Short circuit Current at $F_1 = \dfrac{\text{Total short circuit MVA up to the fault} \times 1000}{(1.732 \times kV)}$

$$= \frac{107.144 \times 1000}{1.732 \times 33} = 1874.584$$

2. Short circuit MVA and short circuit current calculation for fault F_2

MVA1-2-3 and MVA-4 are in Series



Total S.C. MVA upto to the fault F_2 ;

$$\text{Combined MVA1-2-3-4} = \frac{(\text{MVA1-2-3} \times \text{MVA4})}{(\text{MVA1-2-3} + \text{MVA4})}$$

$$\text{MVA1-2-3-4} = \frac{(107.144 \times 52.83)}{(107.144 + 52.83)} = 35.38.$$

∴ i) Total short circuit MVA up to the fault $F2 = 35.38$

$$\text{ii) S.C current at } F_2 = \frac{\text{Total S.C MVA upto the fault} \times 1000}{(1.732 \times kV)}$$

$$= \frac{35.38 \times 1000}{(1.732 \times 33)} = 619\ A.$$

References:

- Power System Analysis by A. Nagoor Kani, Second Edition, CBS Publisher & Distributors Pvt. Ltd.
- Modern Power System Analysis by D. P. Kothari and I. J. Nagarath, Fourth Edition, Tata McGraw-Hill.

Case Study

Topic: Symmetrical Fault Analysis

Three-Phase Fault in a Transmission System

In a large power system network, transmission lines carry electricity from generating stations to load centres. Sometimes unexpected disturbances such as lightning strikes, insulation failure, or equipment malfunction may cause faults in the system.

Consider a situation where a three-phase symmetrical fault occurs at a bus in a transmission network. This type of fault causes a sudden increase in fault current, which may damage generators, transformers, and transmission equipment if not cleared quickly.

To analyse this condition, engineers perform symmetrical fault analysis using The venin's equivalent circuit. The system is simplified into an equivalent network consisting of a voltage source and impedance. Using this equivalent circuit, the fault current and fault level at the faulted bus can be calculated.

Based on the calculated fault current, appropriate circuit breakers and protective relays are selected to isolate the fault quickly and protect the system.

This case study highlights the importance of fault analysis in designing protection systems and ensuring safe operation of power systems.

Unit-4 Outcomes:

- Understand the **concept of faults in power systems**.
- Classify different **types of faults occurring in power systems**.
- Explain the **analysis of symmetrical (three-phase) faults**.
- Apply **Thevenin's theorem to calculate fault current**.
- Determine **fault levels and their importance in protection system design**.

2 Marks Questions:

1. Define fault in a power system.
2. What is a symmetrical fault?
3. List the different types of faults in power systems.
4. What is a three-phase fault?

5. Why a symmetrical fault is considered severe?
6. What is fault current?
7. What is meant by fault level?
8. State the importance of fault analysis.
9. What is Thevenin's equivalent circuit?
10. Why are circuit breakers required in power systems?

<u>10 Marks Questions:</u>

1. Discuss in detail about the Sequence impedance and network of generators, transmission lines, transformers and Loads.
2. Obtain the expression for fault current for single line to ground (LG) fault taken place through impedance Zf in phase 'a' at bus 'k' of a power system. Draw the connections of the Thevenin equivalent of the sequence networks.
3. Deduce and draw the sequence network for LLG fault at the terminals of unloaded generators.
4. A 25 MVA, 13.2 KV alternator with solidly grounded neutral has a sub transient reactance of 0.25 p.u. the negative and zero sequence reactance are 0.35 and 0.01 p.u. respectively. If a line to ground fault occurs at the terminals of the alternator, determine the fault current and line-line voltages at the fault.
5. Determine the symmetrical components of the unbalanced 3-phase currents, Ia0=10<0 Amps, Ia1=12<230 Amps and Ia2=10<130 Amps. Calculate the Sequence Components.
6. The Symmetrical Components of Phase-a voltage is a 3-phase unbalanced system are Va0=10<180 V, Va1=50<0 V, Va2=20<90. Determine the phase voltages Va, Vb, Vc.
7. The Voltage across a 3-phase unbalanced loads are Va=300<20 V, Vb=360<90 V, Vc=500<-140 V. Determine the Symmetrical Components of Voltages, Phase Sequences abs.
8. Obtain the expression for Symmetrical Component of Voltages from the Unsymmetrical 3-phase System.
9. Derive the expression for fault current in double line to ground fault (LLG – without fault impedance) on unloaded Generator. Draw an equivalent network showing the inter connection of sequence network for LLG fault.
10. A 11 KV, 100 MVA alternator having a sub-transient reactance of 0.25 p.u is supplying a 50MVA motor having a sub transient reactance of 0.2 p.u through a transmission line. The Line reactance of 0.05 p.u on a base of 100 MVA. The Motor is drawing 40MW at 0.8 p.f leading with a terminal of 10.95 KV, when a 3-phase fault occurs at the generator terminals. Calculate the total current in generator and motor under fault conditions.

CHAPTER - 5

STABILITY ANALYSIS

5.1 INTRODUCTION

The term stability refers to stable operation of the synchronous machines connected to a power system when they are subjected to sudden disturbances. Hence we can say that the stability is the ability of power system to return to stable operation when it is subjected to a disturbance. The instability refers to the loss of synchronism (or falling out of step) of synchronous machines in the power system. The stability studies are classified into three types depending upon the nature of the disturbance. They are Transient, Dynamic and Steady state stability.

The study of steady state stability is concerned with the determination of upper limit of loading synchronous machines before loosing synchronism. In these studies it is assumed that the machine is loaded gradually.

In a power system, small disturbances will continuously occur due to variations in load, changes in the speed of prime mover, fault in certain part of system, etc.,. These small disturbances may excite the system into a state of natural oscillations.

In dynamically stable systems the amplitude of the oscillations will be small and they die out quickly. In dynamically unstable systems the amplitude of the oscillations is very large and they exist for a long time. This type of unstable behaviour may create serious problems in power system operation. (i.e., it may lead to unnecessary frequent load shedding).

Large disturbances may occur in a power system due to switching heavy loads, switching long transmission lines, major faults, etc., The sudden large disturbances are characterized by large changes in the speed of the rotor of synchronous machine, large changes in power angle and fast change in power transfer. These characteristic may lead to loss of synchronism. This type of instability is called transient instability. Hence the aim of transient stability is to determine whether the system will remain in synchronism or not following a sudden or major disturbances, such as transmission line fault, sudden heavy loads, tripping of generating units or switching lines with heavy loads.

The concepts of stability can be better understood by considering a single synchronous machine connected to infinite bus. It is equivalent to two machine system. A practical power system may have a number of machines running in synchronism.

For theoretical analysis the multimachine system can be converted to an equivalent single machine system connected to infinite bus. Alternatively, the stability study of a multimachine system can be carried on a computer.

The dynamic behaviour of a power system have the following basic features.

1. A synchronous machine has a maximum limit of power transfer when remains in synchronism. When the power transfer exceeds the limit it cannot stay in synchronism.
2. The synchronous machine connected to an infinite bus is basically a spring-inertia oscillatory system. The inertia is due to the mechanical part (rotor) and the spring action is due to the synchronous tie with infinite bus.
3. If δ is the angular displacement of the rotor in electrical radians then the power transfer is proportional to sin δ. Hence the mathematical equation governing the system dynamics is nonlinear.

In order to simplify the computational task in stability studies, the following three assumptions are made in all stability studies.

1. The dc-offset currents and harmonic components are neglected. The currents and voltages are assumed to have fundamental component alone.
2. The symmetrical components are used for the representation of unbalanced faults.
3. It is assumed that the machine speed variations will not affect the generated voltage.

5.2 DYNAMICS OF SYNCHRONOUS MACHINE ROTOR

List of terms used for the analysis of the dynamics of a synchronous machine.

E_{ke} = Kinetic energy of the rotor in MJ (Mega Joules)
J = Moment of inertia of the rotor in kg - m^2
ω_{sm} = Synchronous angular speed of the rotor in mech.rad/sec.
ω_s = Synchronous angular speed of rotor in elect.rad/sec.
p = Number of poles in synchronous machine
M = Moment of inertia of rotor in MJ-s/elec. rad or MJ-s/mech rad.
S = Power rating of machine in MVA
H = Inertia constant in MJ/MVA or MW-s/MVA.
f = Frequency in cycles/sec or Hz.
δ_m = Angular displacement of rotor with respect to synchronously rotating reference frame in mech.rad.
δ = Angular displacement of rotor with respect to synchronously rotating reference frame in elect rad.
θ_m = Angular displacement of rotor with respect to a stationary axis in mech.rad.
θ = Angular displacement of rotor with respect to a stationary axis in elect.rad.
t = Time in seconds.
T_m = Mechanical torque at the shaft of rotor (supplied by prime mover) in N-m.
T_e = Net electromagnetic torque in N-m.
T_a = Net accelerating torque in N-m.
P_m = Mechanical power input in p.u.
P_e = Electrical power output in p.u.

Note : *Here the mechanical losses in the rotating system and electrical losses in the stator of synchronous machine are neglected.*

The kinetic energy (in MJ) of the rotor of a synchronous machine is given by equ (5.1).

$$E_{ke} = \frac{1}{2} J\omega_{sm}^2 \times 10^{-6} \quad .....(5.1)$$

The mechanical and electrical angular speeds are related to the number of poles in synchronous machine as shown in equ(5.2).

$$\omega_s = \frac{p}{2}\,\omega_{sm} \qquad \text{(or)} \qquad \omega_{sm} = \frac{2}{p}\,\omega_s \quad .....(5.2)$$

On substituting for ω_{sm} from equ (5.2) in equ (5.1) we get,

$$E_{ke} = \frac{1}{2} J\left(\frac{2}{p}\right)^2 \omega_s^2 \times 10^{-6}$$

Let, $E_{ke} = \frac{1}{2}M\omega_s$ .....(5.3)

where, $M = J\left(\frac{2}{p}\right)^2 \omega_s \times 10^{-6}$ .....(5.4)

Here M is the moment of intertia in MJ-s/elec.rad. This moment of inertia is used popularly in stability studies.

Another useful constant which is popularly used in stability studies is the inertia constant, H. It is defined as the ratio of stored kinetic energy in megajoules at synchronous speed to the machine rating in megavoltampere.

$$H = \frac{\text{Store kinetic energy in MJ at synchronous speed}}{\text{Machine rating in MVA}}$$

$$\therefore\ H = \frac{E_{ke}}{S} \quad .....(5.5)$$

On substituting for E_{ke} from equ (5.1) in equ(5.5) we get,

$$H = \frac{\frac{1}{2}J\omega_{sm}^2}{S} \quad .....(5.6)$$

On substituting for E_{ke} from equ(5.3) in equ(5.5) we get,

$$H = \frac{\frac{1}{2}M\omega_s}{S} \quad .....(5.7)$$

$$\therefore\ M = \frac{2HS}{\omega_s} \quad .....(5.8)$$

We know that, $\omega_s = 2\pi f$

$$\therefore M = \frac{2HS}{2\pi f} = \frac{HS}{\pi f} \quad \text{(in MJ - s / elect.rad)} \quad .....(5.9)$$

In equ(5.9) the moment of inertia, M is obtained in the units of MJ-s/elect.rad. Sometimes it is required in M J-s/elect.degree. In this case the π radians in equ(5.9) should be replaced by 180° as shown in equ(5.10).

$$\therefore\ M = \frac{HS}{180\,f} \quad \text{(in MJ - s / elect.deg)} \quad .....(5.10)$$

The value of M can be expressed in per unit by selecting a base for MVA.

Let, S_b = Base MVA

Now p.u. value of $M = M_{p.u.} = \dfrac{SH/\pi f}{S_b}$ or $\dfrac{SH/180f}{S_b}$ .....(5.11)

If the machine rating S is chosen as base value, then $S = S_b$, in this case the equation 5.11 can be written as shown in equ (5.12)

$$\left.\begin{matrix}\text{p.u. value of M with machine}\\ \text{rating as base MVA}\end{matrix}\right\} = M_{p.u} = \frac{H}{\pi f} \text{ or } \frac{H}{180f} \qquad .....(5.12)$$

Note : *Many authors treat M and H as two different inertia constants in different units.*

5.3 SWING EQUATION

The rotor of a synchronous machine is subjected to two torques, T_e and T_m, which are acting in opposite directions as shown in fig 5.1.

where, T_e = Net electrical or electromechanical torque in N-m.

T_m = Mechanical or shaft torque supplied by the prime mover in N-m.

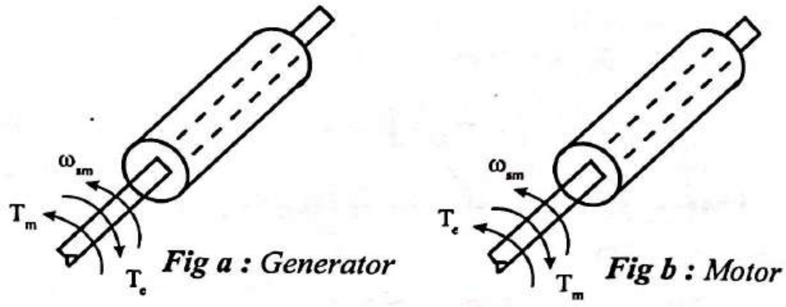


Fig 5.1 : Torque acting on rotor of synchronous machine

Under steady state operating condition the T_e and T_m are equal and the machine runs at constant speed, which is called synchronous speed. If there is a difference between the two torques then the rotor will have an accelerating or deaccelerating torque, denoted as T_a.

$$\therefore T_a = T_m - T_e \qquad .....(5.13)$$

Here T_m & T_e are positive for generators and T_m & T_e are negative for motors.

Let θ_m = Angular displacement of rotor with respect to stationary reference axis.

δ_m = Angular displacement of rotor with respect to synchronously rotating reference axis.

By Newton's second law of motion, we can say that the accelerating torque, T_a is directly proportional to angular acceleration and the constant of proportionality is the moment of inertia, J.

$$\therefore\ T_a \,\alpha\, \frac{d^2\theta_m}{dt^2} \quad \text{(or)} \quad T_a = J\frac{d^2\theta_m}{dt^2} \qquad .....(5.14)$$

On substituting for T_a from equ(5.14) in (5.13) we get,

$$J\frac{d^2\theta_m}{dt^2} = T_m - T_e \qquad .....(5.15)$$

The angular displacements θ_m and δ_m are related to synchronous speed by the following equation [equ(5.16)]

$$\theta_m = \omega_{sm}t + \delta_m \qquad .....(5.16)$$

On differentiating the equ(5.16) with respect to time, t we get,

$$\frac{d\theta_m}{dt} = \omega_{sm} + \frac{d\delta_m}{dt} \qquad .....(5.17)$$

On differentiating the equ(5.17) with respect to time, t we get,

$$\frac{d^2\theta_m}{dt^2} = \frac{d^2\delta_m}{dt^2} \quad .....(5.18)$$

From equ(5.17) we can say that the rotor angular velocity $d\theta_m/dt$ is constant and equal to ω_{sm} (synchronous speed) only if $d\delta_m/dt$ is zero. Hence $d\delta_m/dt$ represents the deviation of the rotor speed from synchronism.

From equation (5.15) and (5.18) we can write, $J\frac{d^2\delta_m}{dt^2} = T_m - T_e$ .....(5.19)

Let, $P_{m,act}$ = Shaft power input to the machine neglecting losses (in MW)

$P_{e,act}$ = Electrical power developed in rotor (in MW)

We know that, power, $P = \frac{2\pi NT}{60} = \omega T$

$$\therefore\ P_{m,act} = \omega_{sm}T_m \quad \text{(or)} \quad T_m = \frac{P_{m,act}}{\omega_{sm}} \quad .....(5.20)$$

$$P_{e,act} = \omega_{sm}T_e \quad \text{(or)} \quad T_e = \frac{P_{e,act}}{\omega_{sm}} \quad .....(5.21)$$

On substituting for T_m and T_e from equation (5.20) and (5.21) in equ(5.19) we get,

$$J\frac{d^2\delta_m}{dt^2} = \frac{P_{m,act}}{\omega_{sm}} - \frac{P_{e,act}}{\omega_{sm}}$$

$$J\omega_{sm}\frac{d^2\delta_m}{dt^2} = P_{m,act} - P_{e,act} \quad .....(5.22)$$

The inertia constant, H is defined as the ratio of stored kinetic energy in MJ to the machine rating in MVA

Let, H = Inertia constant in MJ/MVA

S = Power rating of machine in MVA

$$\text{Now, } H = \frac{\frac{1}{2}J\omega_{sm}^2}{S} \quad .....(5.23)$$

$$\therefore\ J\omega_{sm} = \frac{2HS}{\omega_{sm}} \quad .....(5.24)$$

(Note : The kinetic energy $= \frac{1}{2}J\omega_{sm}^2$ *)*

From equations (5.22) and (5.24) we can write

$$\frac{2HS}{\omega_{sm}}\frac{d^2\delta_m}{dt^2} = P_{m,act} - P_{e,act} \quad .....(5.25)$$

We know that, $\omega_{sm} = \frac{2}{p}\omega_s$ and $\delta_{sm} = \frac{2}{p}\delta$ .....(5.26)

where p = Number of poles in synchronous machine.

On substituting for ω_{sm} and δ_m from equ(5.26) in equ(5.25) we get,

$$\frac{2HS}{2\omega_s/p}\frac{d^2(2\delta/p)}{dt^2} = P_{m,act} - P_{e,act}$$

$$\frac{2HS}{\omega_s}\frac{d^2\delta}{dt^2} = P_{m,act} - P_{e,act} \quad .....(5.27)$$

On substituting, $\omega_s = 2\pi f$ in equ(5.27) we get,

$$\frac{2HS}{2\pi f}\frac{d^2\delta}{dt^2} = P_{m,act} - P_{e,act}$$

$$\therefore \frac{HS}{2\pi f}\frac{d^2\delta}{dt^2} = P_{m,act} - P_{e,act} \quad .....(5.28)$$

In equ(5.28) the powers $P_{m,act}$ and $P_{e,act}$ are in MW. If the machine MVA rating S is chosen as base value then p.u. value of power is given by,

$$\text{p.u. value of mechanical power, } P_m = \frac{P_{m,act}}{S}$$

$$\text{p.u. value of mechanical power, } P_e = \frac{P_{e,act}}{S}$$

From equ(5.28) we get

$$\frac{H}{\pi f}\frac{d^2\delta}{dt^2} = \frac{P_{m,act}}{S} - \frac{P_{e,act}}{S}$$

$$\frac{H}{\pi f}\frac{d^2\delta}{dt^2} = P_m - P_e \quad .....(5.29)$$

The equation (5.29) is called swing equation. It is the fundamental equation which governs the dynamics of the synchronous machine rotor. The swing equation is a second order differential equation. In the next section it is proved that the electrical power, P_e depends on the sine of angle δ, hence the swing equation is a nonlinear second order differential equation.

5.4 POWER ANGLE EQUATION

The equation relating the electrical power generated (P_e) to the angular displacement of the rotor (δ) is called power angle equation. The power angle equation can be derived using the transient model of the generator, because for stability studies the transient model of the generator is used. The transient model of a generator, is shown in fig 5.2. It consists of the transient internal emf in series with transient reactance X_d'.

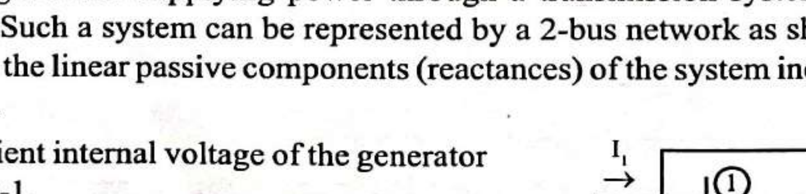


Fig 5.2 : Transient circuit model of generator used for stability studies

Consider a single generator supplying power through a transmission system to a load or to a large system at other end. Such a system can be represented by a 2-bus network as shown in fig 5.3. The rectangular box represents the linear passive components (reactances) of the system including the transient reactance of the generator.

Here E_1' = Transient internal voltage of the generator at bus-1.

E_2' = Voltage at the receiving end (This may be the voltage at infinite bus or transient internal voltage of synchronous motor at bus-2).

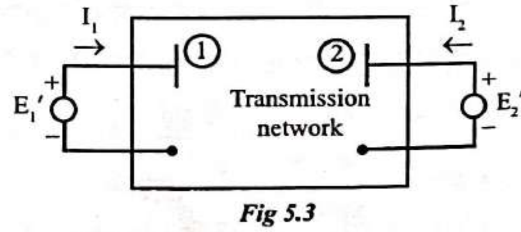


Fig 5.3

The node basis matrix equation of 2-bus system of fig5.3. can be written as shown below

$$\mathbf{I} = \mathbf{Y}_{bus}\,\mathbf{V} \qquad .....(5.30)$$

In the expanded form the equation (5.30) can be written as,

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} E_1^{\prime} \\ E_2^{\prime} \end{bmatrix} \qquad .....(5.31)$$

where, I_1 & I_2 are the currents injected by the sources E_1' & E_2' respectively to the system.

From the row-1 of matrix equation (5.31) we get,

$$I_1 = Y_{11}\,E_1' + Y_{12}\,E_2' \qquad .....(5.32)$$

The complex power S_1 injected to bus-1 is given by

$$S_1 = P_1 + j\,Q_1 = E_1'\,I_1^* \qquad .....(5.33)$$

On substituting for I_1 from equ(5.32) in equ(5.33) we get,

$$\begin{aligned} P_1 + j\,Q_1 &= E_1'\,(\,Y_{11}\,E_1' + Y_{12}\,E_2'\,)^* \\ &= E_1'\,(\,Y_{11}^*\,(\,E_1')^* + Y_{12}^*\,(\,E_2')^*\,) \\ &= (E_1')\,(\,E_1')^*\;Y_{11}^* + Y_{12}^*\,E_1'(\,E_2')^* \\ &= |E_1'|^2\,Y_{11}^* + Y_{12}^*\,E_1'(\,E_2')^* \end{aligned} \qquad .....(5.34)$$

The quantities E_1' , E_2', Y_{11} and Y_{12} in equation (5.39) are complex quantities and so they can be expressed in polar coordinates (or in rectangular coordinates) as shown below.

Let $\quad E_1' = |E_1'|\,\angle\delta_1 \qquad\qquad Y_{11} = G_{11} + jB_{11} = |Y_{11}|\angle\theta_{11}$

$\qquad E_2' = |E_2'|\,\angle\delta_2 \qquad\qquad Y_{12} = G_{12} + jB_{12} = |Y_{12}|\angle\theta_{12}$

Using the polar or rectangular coordinates of the complex quantities involved in equation(5.34) it can be written as shown below.

$$\begin{aligned} P_1 + j\,Q_1 &= |E_1'|^2\,(\,G_{11} + jB_{11}\,)^* + (Y_{12}\,\angle\theta_{12})^*\,|E_1'|\angle\delta_1\,(\,|E_2'|\,\angle\delta_2\,)^* \\ &= |E_1'|^2\,(\,G_{11} - jB_{11}\,) + |Y_{12}|\,\angle{-\theta_{12}})\,|E_1'|\angle\delta_1\,|E_2'|\,\angle{-\delta_2} \\ &= |E_1'|^2\,(\,G_{11} - jB_{11}\,) + |E_1'||E_2'|\,|Y_{12}|\,\angle(\delta_1 - \delta_2 - \theta_{12}) \\ &= |E_1'|^2\,(\,G_{11} - jB_{11}\,) + |E_1'||E_2'|\,|Y_{12}|\cos\,(\delta_1 - \delta_2 - \theta_{12}) \\ &\qquad + j|E_1'||E_2'|\,|Y_{12}|\sin\,(\delta_1 - \delta_2 - \theta_{12}) \end{aligned} \qquad .....(5.35)$$

On equating the real and imaginary part of equation (5.35) we get,

$$P_1 = |E_1'|^2\,G_{11} + |E_1'|\,|E_2'||Y_{12}|\cos\,(\delta_1 - \delta_2 - \theta_{12}) \qquad .....(5.36)$$

$$Q_1 = -|E_1'|^2\,B_{11} + |E_1'|\,|E_2'||Y_{12}|\sin\,(\delta_1 - \delta_2 - \theta_{12}) \qquad .....(5.37)$$

The equ(5.36) and (5.37) are equations of real and reactive power injected by a generator at bus-1.

Let, $\quad \delta = \delta_1 - \delta_2$

$\qquad \gamma = \theta_{12} - \pi/2 \qquad\qquad$ (and $\theta_{12} = \gamma + \pi/2$)

$\qquad P_c = |E_1'|^2\,G_{11}$

$\qquad P_{max} = |E_1'||E_2'||Y_{12}|$

$\qquad P_1 = P_e$

By using the terms defined above, the real power P_1 (i.e., equ 5.36) can be written as shown below.

$$P_e = P_c + P_{max} \cos(\delta - \gamma - \pi/2)$$

$$\therefore P_e = P_c + P_{max} \sin(\delta - \gamma) \quad .....(5.38)$$

The equation (5.38) is called power angle equation. In this equation the term P_c represents power loss in the system and P_{max} represents the maximum real power that can be delivered by the generator to an infinite bus.

Note : Here the bus-2 is considered as infinite bus and so $\delta_2 = 0$. Therefore $\delta = \delta_1$

The power angle equation can be further simplified by considering the network as purely reactive network (by neglecting the resistances).

When resistances are neglected $G_{11} = 0$, $\theta_{12} = \pi/2$ and so $\gamma = 0$. On substituting these conditions in equ(5.38) we get,

$$P_e = P_{max} \sin\delta \quad .....(5.39)$$

where $P_{max} = \dfrac{|E_1'||E_2'|}{X_{12}}$

and X_{12} = Transfer reactance between bus-1 & 2.

The equation (5.39) is called simplified power angle equation. The graph of (or plot of) P_e as a function δ is called the power angle curve. A typical power angle curve is shown in fig (5.4).

On substituting the expression for P_e from equ(5.39) in the swing equation (i.e., equ(5.29), we get.

$$\frac{H}{\pi f}\frac{d^2\delta}{dt^2} = P_m - P_{max}\sin\delta \quad .....(5.40)$$

The equation (5.40) is the swing equation in which the electrical power is expressed as a function of δ.

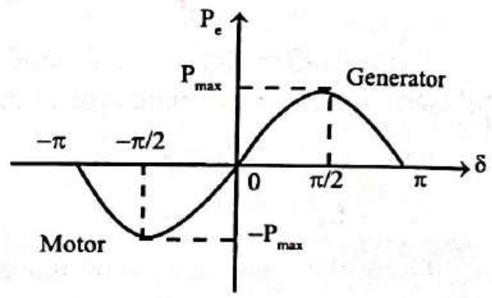


Fig 5.4 : Power angle curve of synchronous machine

5.5 STEADY STATE STABILITY

In steady state every synchronous machine has a limit for power transfer to a receiving system. The steady state limit of a machine or transmitting system is defined as the maximum power that can be transmitted to the receiving system without loss of synchronism.

Let us consider a single synchronous machine delivering power to a large system through a transmitting system. The power angle equation of such a system have been developed in section 5.4, and it can be used for the analysis of steady state stability if the transient emfs are replaced by steady state emfs.

Let $|E|$ = Magnitude of steady state internal emf of synchronous machine.

$|V|$ = Magnitude of voltage of receiving system.

X = Transfer reactance between the synchronous machine and receiving system.

Now, $P_{max} = \dfrac{|E||V|}{X}$ .....(5.41)

$\therefore$Real power injectd by machine to system, $P_e = P_{max} \sin\delta$ .....(5.42)

Let the system be operating with steady power transfer with a torque angle δ_0. In this operating condition, let the electrical power output be P_{e0}. Now, the mechanical power input P_m is equal to P_{e0} under ideal conditions.

With the power input (P_m) remaining same let us assume that the electrical power output increases by a small amount ΔP. Now the torque angle change by a small amount $\Delta\delta$. Therefore the new value of torque angle is $(\delta_0 + \Delta\delta)$. The electrical power output for this new torque angle can be obtained from equ (5.42)

$$\left.\begin{array}{l}\text{The electrical power output}\\ \text{for a torque angle of } (\delta_0+\Delta\delta)\end{array}\right\} = P_{e0} + \Delta P = P_{max}\sin(\delta_0 + \Delta\delta)$$

$$= P_{max}\,[\sin\delta_0 \cos\Delta\delta + \cos\delta_0 \sin\Delta\delta) \qquad .....(5.43)$$

Since $\Delta\delta$ is a small incremental displacement from δ_0,

$$\sin\Delta\delta \approx \Delta\delta \quad \text{and} \quad \cos\Delta\delta \approx 1 \qquad .....(5.44)$$

$$\therefore P_{e0} + \Delta P = P_{max}\sin\delta_0 + (P_{max}\cos\delta_0)\,\Delta\delta \qquad .....(5.45)$$

From equation (5.42) when $\delta = \delta_0$ we get,

$$P_e = P_{e0} = P_m \sin\delta_0 \qquad .....(5.46)$$

From equations (5.45) and (5.46) we can write,

$$\Delta P = (P_{max}\cos\delta_0)\,\Delta\delta \qquad .....(5.47)$$

The swing equation for the system discussed here, have been developed in section (5.3) [The swing equation is given by equ 5.29]. The nonlinear swing equation can be linearized about the operating point for steady state stability analysis. The nonlinear swing equation is

$$\frac{H}{\pi f}\frac{d^2\delta}{dt^2} = P_m - P_e \qquad .....(5.48)$$

We know that the p.u. value of $M = H/\pi f$. Hence the equation (5.48) cane be written as

$$M\frac{d^2\delta}{dt^2} = P_m - P_e \qquad .....(5.49)$$

For a torque of $\delta = (\delta_0 + \Delta\delta)$, The electrical power P_e in the equ(5.49) should be replaced by $(P_{e0} + \Delta P)$.

$$\therefore\ M\frac{d^2(\delta_0 + \Delta\delta)}{dt^2} = P_m - (P_{e0} + \Delta P) \qquad .....(5.50)$$

Since δ_0 is constant and $P_m = P_{e0}$, the equation (5.51) can be written as,

$$M\frac{d^2\Delta\delta}{dt^2} = -\,\Delta P \qquad .....(5.51)$$

On substituting for ΔP from equ(5.47) in equ(5.50) we get,

$$M\frac{d^2\,\Delta\delta}{dt^2} = -\,(P_{max}\cos\delta_0)\,\Delta\delta$$

$$M\frac{d^2}{dt^2}\Delta\delta + P_{max}\cos\delta_0\,\Delta\delta = 0 \qquad .....(5.52)$$

Let $\frac{d^2}{dt^2} = x^2$ and $P_{max}\cos\delta_0 = C$

$$\therefore\ Mx^2\,\Delta\delta + C\,\Delta\delta = 0$$

$$(Mx^2 + C)\,\Delta\delta = 0$$

In equ(5.53) since $\Delta\delta \neq 0$, $M\,x^2 + C = 0$ .....(5.54)

The equation (5.54) is the characteristic equation of the system for small changes. The stability is determined by the roots of the characteristic equation.

$$\therefore\ x^2 = -\frac{C}{M} \qquad \text{(or)} \qquad x = \pm\sqrt{-\frac{C}{M}} \qquad .....(5.55)$$

Case-i : When C is positive, (i.e, $P_{max}\cos\delta_0 > 0$

In this case the roots are purely imaginary and conjugate. Hence the system behaviour is oscillatory about δ_0. In this analysis the resistances in the system have been neglected. If we include the resistances in the analysis then the roots will be complex conjugate and the response will be damped oscillatory. Therefore in a practical system the system is stable for small increment in power provided, $P_{max}\cos\delta_0 > 0$.

Case-ii : When C is negative, (i.e, $P_{max}\cos\delta_0 < 0$

In this case the roots are real and equal in magnitude. One of the root is positive and the other one is negative. Due to the positive root the torque angle increases without bound when there is a small increment in power and the machine will loose synchronism. Hence the machine becomes unstable for small changes in power provided $P_{max}\cos\delta_0 < 0$.

Steady state limit

The term $P_{max}\cos\delta_0$ decides the steady state stability of the system and so it is called synchronizing coefficient or stiffness of synchronous machine. From the power angle curve for generator action shown in fig 5.5, we get the range of δ_0 as 0 to π. (The fig 5.5 is drawn by using the equation $P_e = P_{max}\sin\delta$).

When $0 \le \delta_0 \le \pi/2$; $P_{max}\cos\delta_0$ and P_e are positive

When $\delta_0 = \pi/2$; $P_{max}\cos\delta_0 = 0$ and $P_e = P_{max}$

When $\pi/2 < \delta_0 \le \pi$; $P_{max}\cos\delta_0$ is negative an P_e is positive

From the above discussion we can say that the synchronizing co-efficient ($P_{max}\cos\delta_0$) and real power injected to the system (P_e) are positive when δ is in the range of 0 to $\pi/2$. Therefore the maximum power that can be transmitted without loss of stability occurs for $\delta = \delta_0 = \pi/2 = 90°$. The maximum power transmitted is P_{max}.

$$\text{where, } P_{max} = \frac{|E||V|}{X}$$

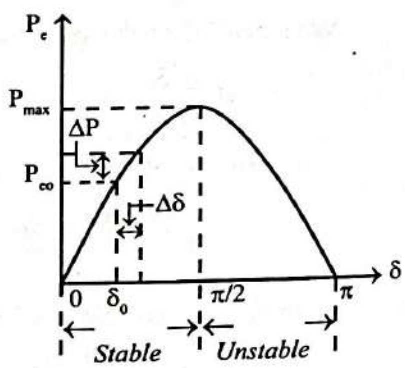


Fig 5.5 : Power angle curve for steady state operation of generator

The fig 5.5 shows the stable and unstable steady state operating regions of a generator. The concepts developed in this section is also applicable for power transfer from one system to another system if the transmitting system is represented by single equivalent generator.

In stable operating region of the system the damping should be sufficient to reduce the oscillations developed due to small changes in loads. If the oscillations exists for a long time then it may pose a problem to system security.

Practically, the system has to be operated below the steady state stability limit. This limit can be improved by reducing the reactance X or by increasing the voltages at sending end and/or at receiving end. The reactance can be reduced by introducing series capacitors in the transmission line. Alternatively the line reactance can be reduced by using two parallel transmission lines. (The two parallel lines improves the reliability of the system).

5.6 TRANSIENT STABILITY

The transient stability of a system is concerned with the study of system behaviour for large disturbances. The short circuits (fault conditions) and switching heavy loads can be treated as large disturbances. They, are associated with large change in torque angle, δ.

The dynamics of the system under transient state are governed by the nonlinear swing equation developed in section 5.3 [equation 5.29]. Since the changes in δ is very large in the transient state, the swing equation cannot be linearized for a general solution and so solution has to be obtained by using any one of the numerical techniques. The most popular methods available for the solution of swing equation are point-by-point method and Runga-Kutta method.

The transient stability of a single machine connected to infinite bus bar can be easily determined by a simple criterion called equal area criterion.

The computational task involved in transient stability studies can be understood by considering, the transient state of a practical system. Consider a single machine system shown in fig 5.6, feeding energy through a transmission line to an infinite bus. Let the circuit breakers shown in the figure 5.6 be autoreclosure type. In this type the circuit breaker will open its contact upon sensing a fault and after a small time it will close it contacts, if the fault still exist then again it will open its contact to permanently disconnect the faulty part.

This feature is useful in clearing transient faults. Because the transient fault exists for a small time and it gets cleared when the circuit is opened. Then the circuit can be closed for normal operation.

Most of the autoreclosure circuit breakers will open and close the contacts twice before permanently disconnecting the circuit. In the majority of faults the first reclosure will be successful. Hence the system stability is improved by using autoreclosure circuit breakers.

Let a fault occurs at the sending end of the transmission line in the system shown in fig 5.6. Let the fault be transient in nature and cleared in the transient state. Hence the circuit breaker will close the circuit automatically after the fault is cleared. The transient stability study for this situation involves the following steps.

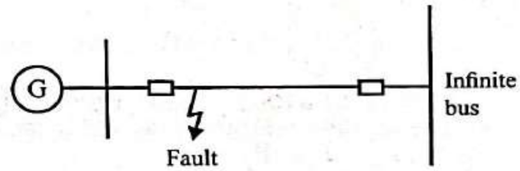


Fig 5.6 : A single generator feeding energy to infinite bus.

1. Calculate the transient internal emf and torque angle δ_0 using the prefault load currents. (Here δ_0 is the torque angle corresponding to prefault operating point).

2. Determine an equation for power during the fault condition. Let this equation be $P_e(\delta)$. If the fault is 3-phase fault then power transferred to infinite bus is zero and the entire power goes to fault.

3. Calculate $\delta(t)$ for various time instant by solving the swing equation using a numerical technique. The initial value of δ for this solution in δ_0. (Here $\delta(t)$ is the torque angle expressed as a function of time).

4. Let us assume that the fault is cleared when the circuit breaker open its contact the first time. Now $P_e(\delta) = 0$. Continue calculating $\delta(t)$ by taking previous step value as initial condition.

5. Let us assume that the circuit breaker close its contact and power feeding to infinite bus is resumed. For this situation find $P_e(\delta)$ and continue to calculate $\delta(t)$.
6. Now examine the variations of $\delta(t)$. If $\delta(t)$ goes through a maximum value and starts to reduce then the system is a stable system. On the otherhand if $\delta(t)$ remains increasing for a specified length of time then the system is considered unstable.

5.7 EQUAL AREA CRITERION

The transient stability analysis of simple system can be performed by using a simple criterion called equal area criterion.

During the transient state of a power system we may come across the following two situations for changes in δ (torque angle) with respect to time.

i) The δ may increase to a maximum value and then decrease to a stable value. The system is considered as stable.

ii) The δ may keep on increasing indefinitely. In this case the system is unstable.

The above facts can be stated as a stability criterion as given below.

i) The system is stable if, $\frac{d\delta}{dt} = 0$ at some time instant.

ii) The system is unstable if, $\frac{d\delta}{dt} > 0$ for a sufficiently long time (typically 1 second or more)

For a single machine-infinite bus bar system, the stability criterion stated above can be converted to a simple condition as shown below.

Consider the swing equation of a generator connected to infinite bus (which is derived in section 5.3, equ 5.29).

$$\frac{H}{\pi f}\frac{d^2\delta}{dt^2} = P_m - P_e \quad \text{.....(5.56)}$$

Let there be a change in P_e due to a large disturbance, with P_m remaining constant.

Now, $P_m - P_e = P_a$ .....(5.57)

where P_a is the accelerating power.

Also we know that p.u value of $M = H/\pi f$. Hence the equ(5.56) can be written as,

$$M\frac{d^2\delta}{dt^2} = P_a$$

$$\therefore \frac{d^2\delta}{dt^2} = \frac{P_a}{M} \quad \text{.....(5.58)}$$

On multiplying the equ(5.58) by $2\frac{d\delta}{dt}$ we get,

$$2\frac{d\delta}{dt}\frac{d^2\delta}{dt^2} = 2\frac{d\delta}{dt}\frac{P_a}{M}$$

$$2\frac{d\delta}{dt}\frac{d}{dt}\frac{d\delta}{dt} = \frac{2}{M}P_a\frac{d\delta}{dt}$$

$$2\frac{d}{dt}\left(\frac{d\delta}{dt}\right)^2 = \frac{2}{M}P_a\frac{d\delta}{dt}$$

$$\therefore 2d\left(\frac{d\delta}{dt}\right)^2 = \frac{2}{M}P_a\,d\delta \quad \text{.....(5.59)}$$

On integrating the equation (5.59) we get

$$\left(\frac{d\delta}{dt}\right)^2 = \frac{2}{M}\int_{\delta_0}^{\delta} P_a \, d\delta \qquad .....(5.60)$$

where δ_0 is the initial value of torque angle or rotor angle

On taking square root of equ(5.60) we get,

$$\frac{d\delta}{dt} = \sqrt{\frac{2}{M}\int_{\delta_0}^{\delta} P_a \, d\delta} \qquad .....(5.61)$$

For a stable system $\frac{d\delta}{dt} = 0$, at a particular time instance. There fore for a stable system the equation (5.61) can be written as

$$\sqrt{\frac{2}{M}\int_{\delta_0}^{\delta} P_a \, d\delta} = 0 \qquad .....(5.62)$$

The equation (5.62) is zero if the integral of P_a is zero. Hence we can say that,

$$\text{for } \frac{d\delta}{dt} = 0, \text{ the term } \int_{\delta_0}^{\delta} P_a \, d\delta = 0 \qquad .....(5.63)$$

The physical meaning of integration is the estimation of the area under the curve. Hence the integral of P_a equal to zero refers to zero area. Hence the condition of stability can be stated as : the system is stable if the area under $P_a - \delta$ curve reduces to zero at some value of δ. This is possible only if the positive (accelerating) area under $P_a - \delta$ curve is equal to the negative (deaccelerating) area under $P_a - \delta$ curve for a finite change in δ. Hence this stability criterion is called equal area criterion of stability.

The equal area criterion of stability can be applied to any type of disturbances that may occur in a single machine-infinite bus bar system. The transient stability study for a sudden change in mechanical input using equal area criterion in presented in this section.

Transient stability analysis for a sudden change in mechanical input

Consider a single generator feeding energy to infinite bus as shown in fig 5.7. The electrical power transmitted by the generator is given by

$$P_e = \frac{|E'||V|}{X} \sin\delta = P_{max} \sin\delta \qquad .....(5.64)$$

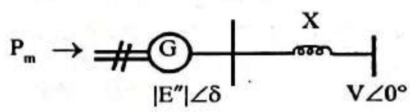


Fig 5.7.

where, $P_{max} = \frac{|E'||V|}{X}$

Let the generator be operating in steady state with a torque angle δ_0. At this condition the mechanical power input is P_{m0} and electrical power output is P_{e0}. Under ideal conditions $P_{m0} = P_{e0}$. Hence from equ(5.64) for this operating condition we can write,

$$P_{m0} = P_{e0} = P_{max} \sin\delta_0 \qquad .....(5.65)$$

The power angle curve of the generator is shown in fig 5.8. In this the steady state operating point as described by equ(5.65) is the point-a.

Let the mechanical input to the generator rotor be suddenly increased to P_{m1} (by some adjustment in prime mover). Since the mechanical power is more than electrical power, the generator will have an accelerating power, P_a given by

$$P_a = P_{m1} - P_e \quad .....(5.66)$$

where $P_e = P_{max} \sin \delta$

Due to accelerating power the rotor speed increases and so the rotor angle also increases. This results in increased electrical power generation. Therefore the operating point will move upwards along the power angle curve. At point-b again the mechanical power P_{m1} equals the electrical power P_{e1}, where p_{e1} is the electrical power output corresponding to torque angle δ_1.

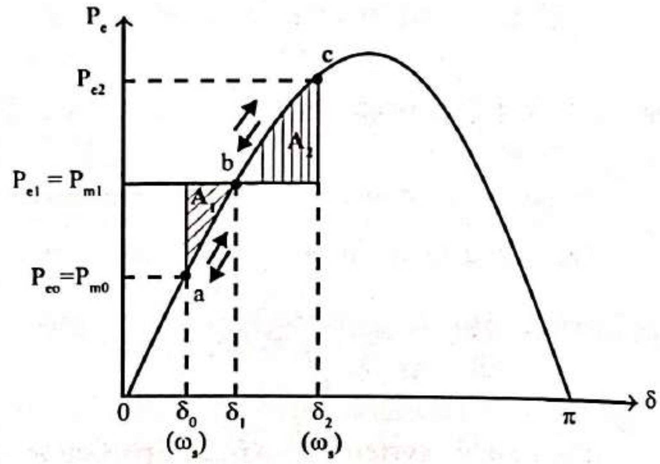


Fig 5.8 : Power angle curve for sudden increase in mechanical input to generator

Now the rotor angle cannot stay at this point, because the inertia of the rotor will make the rotor to oscillate with respect to point-b. Hence the torque angle will continue to increase till point-c, when the operating point moves from b to c, the electrical power is more than mechanical power. Therefore the power, P_a given by equ(5.66) is negative and it is called deaccelerating power.

In this region (i.e., from point-b to c) the rotor angle δ increases but the rotor speed decreases due to deaccelerating power. The point-c is decided by the damping of the system (because the oscillations in the rotor are reduced by damping). At point-c the speed of rotor will be equal to synchronous speed.

***Note :** At point-a the speed is synchronous speed (ω_s). From point a to b the speed increases and then from point b to c the speed decreases. Once again at point-c the speed is equal to synchronous speed (ω_s). Thus the rotor oscillates between point-a and point-c before settling to point-b.*

In fig 5.8, the area A_1 is the acceleration area and area A_2 is the deacceleration area. The equal area criterion says that, the system is stable if,

$$\int_{\delta_0}^{\delta_2} P_a \, d\delta = 0 \quad .....(5.67)$$

To satisfy equ(5.67) the acceleration area A_1, should be equal to deacceleration area A_2. When the oscillations die out the system will settle to a new state. In this new steady state, $P_{m1} = P_{e1}$.

$$\therefore P_{m1} = P_{e1} = P_{max} \sin \delta_1 \quad .....(5.68)$$

The areas A_1 and A_2 can be evaluated as shown below.

$$A_1 = \int_{\delta_0}^{\delta_1} (P_{m1} - P_e) \, d\delta \quad .....(5.69)$$

$$A_2 = \int_{\delta_1}^{\delta_2} (P_e - P_{m1}) \, d\delta \quad .....(5.70)$$

where, $P_e = P_{max} \sin \delta$

From the above discussions we can say that there is a upper limit for increase in mechanical power input P_m. As the mechanical power is increased, a limiting condition is finally reached at a point where the area A_1 equals the area above the $P_{ml,max}$ line as shown in fig 5.9. The corresponding δ can be $\delta_{1,max}$.

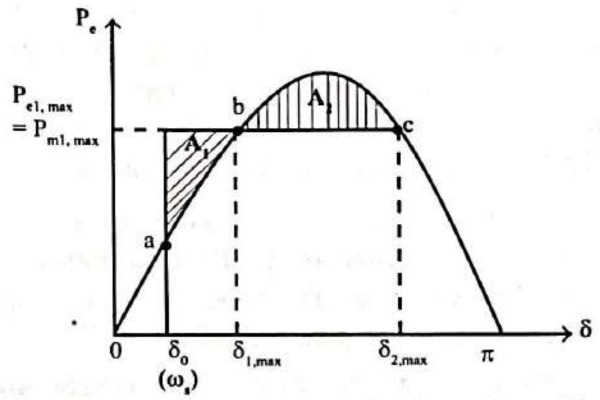


Fig 5.9 : Transient stability limit for sudden change in mechanical input

Under this condition δ_2 takes a maximum value of $\delta_{2,max}$.

Here, $\delta_{2,max} = \pi - \delta_{1,max}$ .....(5.71)

From equ(5.64) we can write,

$$P_{el,max} = P_{ml,max} = P_{max} \sin \delta_{1,max}$$

$$\therefore \sin \delta_{1,max} = \frac{P_{ml,max}}{P_{max}} \quad \text{(or)} \quad \delta_{1,max} = \sin^{-1}\left(\frac{P_{ml,max}}{P_{max}}\right) \qquad .....(5.72)$$

From equations(5.71) and (5.72) we can write,

$$\delta_{2,max} = \pi - \sin^{-1}\left(\frac{P_{ml,max}}{P_{max}}\right) \qquad .....(5.72)$$

From fig 5.9 we can say that, any further increase in $P_{ml,max}$ will make the area A_2 less than the area A_1. This means that the acceleration power is more than the deacceleration power. Hence the system will have an excess kinetic energy which causes δ to increase beyond point-c. If the δ increases beyond point-c the deacceleration power changes to acceleration power and so the system become unstable.

> ***Note :** It is important to note that the system will remain stable even though the rotor may oscillate beyond $\delta = 90°$, as long as the equal area criterion is met. Hence the condition of $\delta = 90°$ for stability is applicable only for steady state stability and not for transient stability.*

Clearing time and clearing angle

Consider a single machine system shown in fig 5.10. Let the mechanical input be P_m and the machine is operating in steady state with torque angle, δ_0. The power angle curve for this system is shown in fig 5.11. The operating point is shown as point-a.

Let a three phase fault occur at point F in the system. Now P_e=0 and the operating point drops to b. This means that the power transferred to infinite bus is zero and the entire power generated is flowing through fault. Now the operating point moves along bc.

Let the fault be transient in nature and so the fault be cleared by opening of the circuit breaker at point-c, where $\delta = \delta_c$ and the corresponding time be t_c. Here t_c is called clearing time and δ_c is called clearing angle.

It is assumed that the circuit breaker closes its contact immediately after opening. Hence the normal operation is restored. Now the operating point shifts to point-d. The rotor now deaccelerates and the operating point moves along de.

For this transient state, if an angle δ_1, can be found such that, $A_2 = A_1$, then the system is found to be stable. The stable system may finally settles down to the steady operating point-a in an oscillatory manner due to damping in the system.

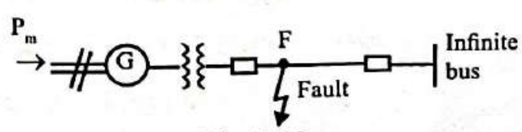


Fig 5.10.

In the above discussion it is assumed that the fault is cleared at δ_c, but if the fault clearing is delayed then the angle δ_1 continue to increase to an upper limit, δ_{max}. This corresponds to a point where equal areas for A_1 and A_2 can be found for a given P_m as shown in fig 5.12.

For this situation the fault would have been cleared at an angle δ_{cc}, as shown in fig 5.12. This angle δ_{cc} is called critical clearing angle. The time corresponding to this angle is called critical clearing time, t_{cc}.

If the fault is not cleared within critical time, then δ_1 would increase to a value greater than δ_{max}. In such a situation the area A_2 will be less than the area A_1 and so the system would be unstable.

For a 3-phase fault in simple systems, the equations for δ_{cc} and t_{cc} can be obtained as shown below.

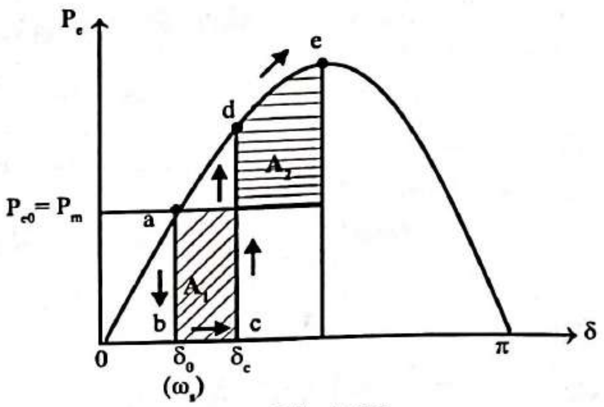


Fig 5.11

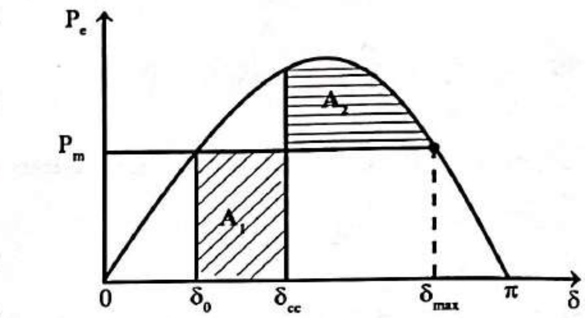


Fig 5.12 : Critical clearing angle

From fig 5.12, we get

$$\delta_{max} = \pi - \delta_0 \quad .....(5.73)$$

Under steady state, for a given δ_0, $P_m = P_e$ and it is constant

$$\therefore P_m = P_e = P_{max} \sin \delta_0 \quad .....(5.74)$$

The acceleration power, $P_a = P_m - P_e$ .....(5.75)

When a three phase fault occurs, $P_e = 0$.

$$\therefore P_a = P_m = \text{constant} \quad .....(5.76)$$

The acceleration area A_1 can be evaluated by integrating P_a of equ (5.76) from $\delta = \delta_0$ to $\delta = \delta_{cc.}$

$$\therefore \; A_1 = \int_{\delta_0}^{\delta_{cc}} P_m \, d\delta = P_m [\delta]_{\delta_0}^{\delta_{cc}} = P_m (\delta_{cc} - \delta_0) \quad .....(5.77)$$

When the power feeding is resumed after the fault, $P_e = P_{max} \sin \delta$.

Now, $P_a = P_e - P_m = P_{max} \sin \delta - P_m$ .....(5.78)

The deacceleration area A_2 can be evaluated by integrating P_a of equ(5.78) from $\delta = \delta_{cc}$ to $\delta = \delta_{max}$.

$$\therefore \; A_2 = \int_{\delta_{cc}}^{\delta_{max}} (P_{max} \sin \delta - P_m) d\delta = [- P_{max} \cos \delta - P_m \delta]_{\delta_{cc}}^{\delta_{max}}$$

$$= [- P_{max} \cos \delta_{max} - P_m \delta_{max} + P_{max} \cos \delta_{cc} + P_m \delta_{cc}]$$

$$= P_{max} (\cos \delta_{cc} - \cos \delta_{max}) - P_m (\delta_{max} - \delta_{cc}) \quad .....(5.79)$$

For a stable system $A_1 = A_2$. Hence the equations (5.77) and (5.79) can be equated to solve δ_{cc}.

On equating (5.77) and (5.79) we get

$$P_{max} (\cos \delta_{cc} - \cos \delta_{max}) - P_m (\delta_{max} - \delta_{cc}) = P_m (\delta_{cc} - \delta_0)$$

$$\therefore P_{max} \cos \delta_{cc} = P_m \delta_{cc} - P_m \delta_0 + P_{max} \cos \delta_{max} + P_m \delta_{max} - P_m \delta_{cc}$$

$$P_{max} \cos \delta_{cc} = P_m (\delta_{max} - \delta_0) + P_{max} \cos \delta_{max}$$

$$\therefore \cos \delta_{cc} = \frac{P_m}{P_{max}} (\delta_{max} - \delta_0) + \cos \delta_{max}$$

$$\text{(or)} \quad \delta_{cc} = \cos^{-1}\left[\frac{P_m}{P_{max}} (\delta_{max} - \delta_0) + \cos \delta_{max}\right] \qquad .....(5.80)$$

The equation (5.80) can be used to estimate the value of critical clearing angle, δ_{cc}.

Consider the swing equation of single machine system,

$$\frac{H}{\pi f} \frac{d^2\delta}{dt^2} = P_m - P_e \qquad .....(5.81)$$

During a three phase fault, $P_e = 0$

$$\therefore \frac{H}{\pi f} \frac{d^2\delta}{dt^2} = P_m$$

$$\frac{d^2\delta}{dt^2} = \frac{\pi f \, P_m}{H} \qquad .....(5.82)$$

On integrating equ(5.82) twice we get

$$\delta = \frac{\pi f}{2H} P_m t^2 + \delta_0 \qquad .....(5.83)$$

where δ_0 is the integral constant

In equ(5.83) when $\delta = \delta_{cc}$, $t = t_{cc}$.

$$\therefore \text{ at } t = t_{cc}, \qquad \delta_{cc} = \frac{\pi f}{2H} P_m t_{cc}^2 + \delta_0$$

$$\frac{\pi f}{2H} P_m t_{cc}^2 = \delta_{cc} - \delta_0$$

$$\therefore t_{cc} = \sqrt{\frac{2H (\delta_{cc} - \delta_0)}{\pi f \, P_m}} \qquad .....(5.84)$$

The equation (5.84) can be used to estimate the value of critical clearing time, t_{cc}.

Note : Since $P_e = 0$, it is possible to find an explicit solution for δ by integrating the swing equation.

5.8 SOLUTION OF SWING EQUATION BY POINT-BY-POINT METHOD

Consider the swing equation of a power system.

$$\frac{H}{\pi f} \frac{d^2\delta}{dt^2} = P_m - P_e \qquad .....(5.85)$$

We know that, $M = \dfrac{H}{\pi f}$ .....(5.86)

$$P_e = P_{max} \sin \delta \qquad .....(5.87)$$

$$P_a = P_m - P_e = P_m - P_{max} \sin \delta \qquad .....(5.88)$$

$$\therefore\ M\frac{d^2\delta}{dt} = P_a$$

$$\frac{d^2\delta}{dt} = \frac{P_a}{M} \qquad .....(5.89)$$

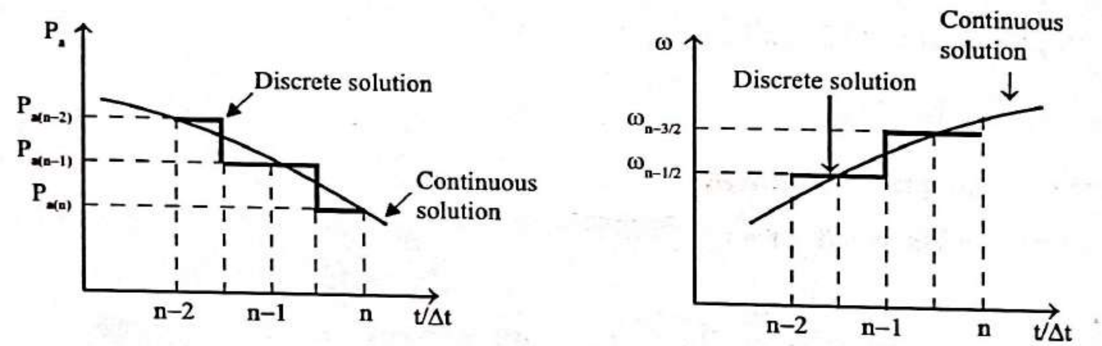
The equation (5.89) in a nonlinear equation. During transient state the δ is a function of time, t and so it can be denoted as δ(t). In point-by-point method, the solution of δ(t) is obtained by dividing the time into small equal values of Δt. (i.e., the entire time (range) of interest is divided into number of small equal interval Δt).

The accelerating power and the change in speed are also continuous function of time. They are discretized as follows,

1. The accelerating power P_a computed at the beginning of an interval is assumed to remain constant from the middle of the preceding interval to the middle of the interval being considered.

2. The angular velocity is assumed constant throughout any interval. This constant value is the value corresponding to the midpoint of concerned interval.

The discretization of P_a and ω are shown in fig 5.13.



***Note :** The n represents discretized time instant and it pertains to the end of the interval.*

Fig 5.13 : Discretization of P_a and ω.

For each discrete interval the values of P_a, ω and δ are calculated as shown below.

The solution starts from the initial condition values, which corresponds to a stable operating point. Let δ_0 be the angle corresponding to initial operating point.

Let us assume that, the values ω and δ for $(n-1)^{th}$ interval are known

δ_{n-1} = The value of δ at the end of $(n-1)^{th}$ interval

$\omega_{n-1/2}$ = The value of ω at the end of $(n-1)^{th}$ interval

$P_{a(n-1)}$ = Value of P_a at the end of $(n-1)^{th}$ interval.

Now from equ(5.89) we get,

$$P_{a(n-1)} = P_m - P_{max}\sin\delta_{n-1} \qquad .....(5.90)$$

The equ (5.89) can be written in the modified form as shown in equ(5.91)

$$\frac{d\omega}{dt} = \frac{P_a}{M} \qquad .....(5.91)$$

where, $\omega = \frac{d\delta}{dt}$ and $\frac{d\omega}{dt} = \frac{d^2\delta}{dt^2}$

For small changes in δ, the equation (5.91) can be linearized as shown below

$$\frac{\Delta\omega}{\Delta t} = \frac{P_a}{M}$$

$$\therefore\ \Delta\omega = \frac{\Delta t}{M} P_a \quad .....(5.92)$$

Let $\omega_{n-3/2}$ = The value of ω at the end of n^{th} interval.

For calculating n^{th} interval value of ω, put, $\Delta\omega = \omega_{n-1/2} - \omega_{n-3/2}$ and $P_a = P_{a(n-1)}$ in equ(5.92).

$$\therefore\ \omega_{n-1/2} - \omega_{n-3/2} = \frac{\Delta t}{M} P_{a(n-1)}$$

$$\therefore\ \omega_{n-3/2} = \omega_{n-1/2} - \frac{\Delta t}{M} P_{a(n-1)} \quad .....(5.93)$$

For small changes in δ, we can write

$$\omega = \frac{\Delta\delta}{\Delta t} \quad .....(5.94)$$

To solve for change in δ in $(n-1)^{th}$ interval put $\Delta\delta = \Delta\delta_{n-1}$ and $\omega = \omega_{n-3/2}$ in equ(5.94)

$$\therefore \Delta\delta_{n-1} = \Delta t\ \omega_{n-3/2} \quad .....(5.95)$$

Similarly for change in δ in n^{th} interval put $\Delta\delta = \Delta\delta_n$ and $\omega = \omega_{n-1/2}$ in equ(5.94)

$$\therefore \Delta\delta_n = \Delta t\ \omega_{n-1/2} \quad .....(5.96)$$

Let δ_n = The value of δ at the end of n^{th} interval.

Now, $\delta_n = \delta_{n-1} + \Delta\delta_n$ .....(5.97)

The above process of computation is repeated to obtain $P_{a(n)}$, $\Delta\delta_{(n+1)}$ and $\delta_{(n+1)}$. The solution of δ(t) is thus obtained in discrete form over the desired length of time. The normal desired length of time is 0.5 sec.

The continuous form of solution is obtained by drawing a smooth curve through discrete values as shown in fig 5.14.

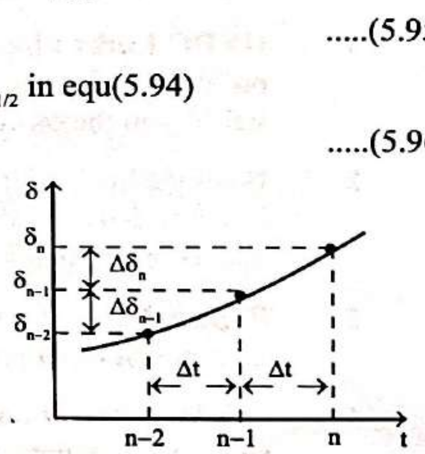


Fig 5.14 : Solution of swing equation by point-by-point method

5.9 METHODS OF IMPROVING TRANSIENT STABILITY

The transient stability depends on the type and location of a fault. Let us consider the most severe type of fault, i.e., 3-phase fault and the following discussions are applicable for transient conditions due to three phase fault.

An increase in the inertia constant M of a single machine connected to infinite bus, reduces the angle through which the rotor swings in a given time interval. Hence stability can be improved by increasing M.

However this cannot be employed in practice because of economic reasons. Also, increasing M will have an undesirable effect of slowing down the response of the speed governor loop.

It can be proved that for a given clearing angle, the acceleration area decreases but the deacceleration area increases as the maximum power limit of a power angle curve is raised which improves the transient stability limit of the system. The above conclusion suggest the following methods of improving the transient stability limit of a power system.

1. Increase of system voltages, use of AVR (Automatic Voltage Regulators)
2. Use of high speed excitation systems.
3. Reduction in system transfer reactance.
4. Use of high speed reclosing breakers.

During fault, the voltage reduction of all buses at the generator terminals are sensed by the automatic voltage regulators which help to restore the generator voltage. Modern exciter systems having solid state controls which quickly respond to bus voltage reduction.

The transfer reactance can be reduced to improve the stability limit. Incidentally this also raises system voltage profile. The reactance of a transmission line can be decreased (i) by reducing the conductor spacing and (ii) by increasing conductor diameter.

Compensation for line reactance by seires capacitors is an effective and economical method of increasing stability limit specially for transmission distances of more than 350 km. Switched series capacitors simultaneously decrease fluctuation of load voltages and raise the transient stability limit to a value almost equal to the steady state limit.

The transfer reactance can also to reduced by increasing the number of parallel lines between transmission points. Rapid switching and isolation of unhealthy lines followed by reclosing also improves stability margins.

Some of the recent methods of improving stability are,

1. **HVDC Links :** Increased use of HVDC links employing thyristors would deviate stability problem. A dc link is asynchronous. There is no risk of a fault in one system causing loss of stability in the other system.

2. Breaking Resistors : For improving stability where clearing is delayed or large load is suddenly lost, a resistive load called a breaking resistor is connected at or near the generator bus. This load compensates for some of the reduction of load on the generators.

3. **Bypass Valving :** In this method, the stability of a unit is improved by decreasing the mechanical input power to the turbine.

4. **Full load Rejection Technique :** Fast valving combined with high-speed clearing time will sufficient to maintain stability in most cases. However, there are still situations where stability is difficult to maintain. To remedy these situations, a full load rejection scheme could be utilized after the unit is spearated from the system. To do this, the unit has to be equipped with a large steam bypass system. After system has recovered from the shock caused by the fault, the unit could be resynchronized and reloaded. The main disadvantage is the extra cost of the large bypass system.

EXAMPLE 5.1

The generator shown in fig 5.1.1. is delivering power to infinite bus. Take $|V_t| = 1.1.$ p.u. Find the maximum power that can be transferred when

(i) The system is healthy

(ii) One line is open

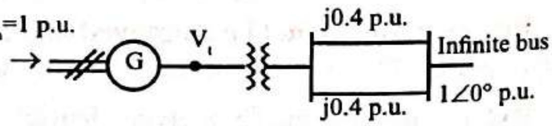


Fig 5.1.1.

SOLUTION

The reactance diagram of the system is shown in fig 5.1.2.

Here, $V_t = |V_t| \angle\theta$ and $V = 1\angle 0°$ p.u.

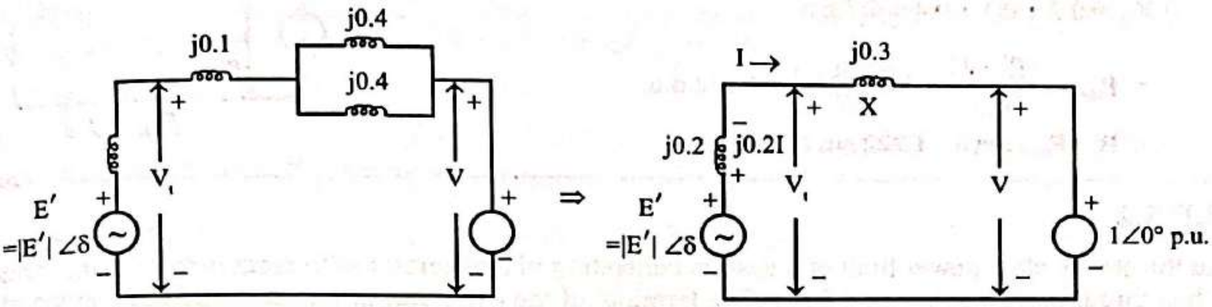


Fig 5.1.2

Let electrical power transferred at steady state be equal to P_m.

$\therefore P_e = P_m = 1$ p.u.

We know that,

$$P_e = \frac{|V_t||V|}{X}\sin\theta$$

$$\therefore\ \theta = \sin^{-1}\left[\frac{P_e X}{|V_t||V|}\right] = \sin^{-1}\left[\frac{1\times 0.3}{1.1\times 1}\right] = 15.8°$$

$\therefore V_t = |V_t| \angle\theta = 1.1 \angle 15.8° = 1.058 + j0.3$

$$\text{Current, } I = \frac{V_t - V}{jX} = \frac{1.058 + j0.3 - 1}{j0.3} = \frac{0.058 + j0.3}{0.3\angle 90°} = \frac{0.3056\angle 79°}{0.3\angle 90°} = 1.0187\angle -11° \text{p.u.}$$

In fig 5.1.2. using KVL we can write,

The transient internal voltage, $E' = j0.2\, I + V_t$

$$= j0.2 \times 1.0187 \angle -11° + 1.1\angle 15.8°$$

$$= 0.2 \angle 90° \times 1.0187 \angle -11° + 1.058 + j0.3$$

$$= 0.2037 \angle 79° + 1.058 + j0.3$$

$$= 0.039 + j0.2 + 1.058 + j0.3$$

$$= 1.097 + j0.5 = 1.2056 \angle 24.5° \text{ p.u.}$$

$\therefore |E'| = 1.2056$ p.u. and $\delta = \delta_0 = 24.5°$

case (i) Healthy system

Let X_{12} be the reactance between the transient internal emf and the source representing infinite bus.

From fig 5.1.2., $X_{12} = 0.5$ p.u.

$$\therefore\ P_{max} = \frac{|E'||V|}{X_{12}} = \frac{1.2056\times 1}{0.5} = 2.4112 \text{ p.u.}$$

$$\therefore\ P_e = P_{max}\sin\delta = 2.4112\sin\delta$$

Case (ii) When one line is open

The reactance diagram when one line is open is shown in fig 5.1.3.

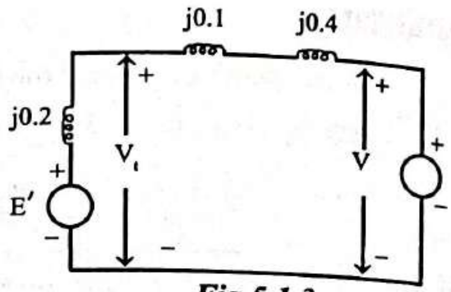


Fig 5.1.3

Let X_{12} = Transfer reactance, from fig 5.1.3.

$$X_{12} = 0.2 + 0.1 + 0.4 = 0.7 \text{ p.u.}$$

$$\therefore\ P_{max} = \frac{|E'||V|}{X_{12}} = \frac{1.2056 \times 1}{0.7} = 1.722 \text{ p.u.}$$

$$\therefore\ P_e = P_{max} \sin\delta = 1.722 \sin\delta$$

EXAMPLE 5.2

Find the steady state power limit of a system consisting of a generator with reactance 0.6 p.u., connected to an infinite bus through a reactance of 0.8 p.u. The terminal of the generator is 1.15 p.u. and the voltage of infinite bus is $1\angle0°$ p.u.

SOLUTION

The reactance diagram of the system is shown in fig 5.2.1.

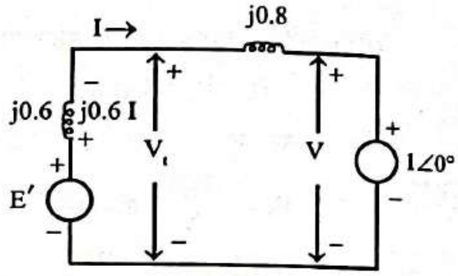


Fig 5.2.1

Given that, $V = 1\angle0°$ and $|V_t| = 1.15$.

Let, $V_t = 1.15\ \angle\theta$

From fig 5.2.1. we get,

$$I = \frac{V_t - V}{j0.8}$$

$$E' = j0.6\,I + V_t = j0.6\left(\frac{V_t - V}{j0.8}\right) + V_t = 0.75\,(V_t - V) + V_t = 1.75\,V_t - 0.75\,V$$

$$= 1.75 \times 1.15\angle\theta - 0.75 \times 1\angle0° = 2.0125\angle\theta - 0.75 = 2.0125\cos\theta + j2.0125\sin\theta - 0.75$$

$$= (2.0125\cos\theta - 0.75) + j\,(2.0125\sin\theta) \quad .....(5.2.1)$$

Consider the transient internal emf in polar form.

$$\text{i.e., } E' = |E'|\ \angle\delta = |E'|\cos\delta + j\,|E'|\sin\delta \quad .....(5.2.2)$$

When the steady state power limit is reached, the value of δ is 90°, $\cos\delta = 0$ and so the real part of equ(5.2.2) is zero.Therefore at steady state limit, the real part of equ(5.2.1) is also zero.

$$\therefore 2.0125\cos\theta - 0.75 = 0$$

$$2.0125\cos\theta = 0.75$$

$$\theta = \cos^{-1}\left(\frac{0.75}{2.0125}\right) = 68.1°$$

$$\therefore V_t = |V_t|\ \angle\theta = 1.15\ \angle\ 68.1° \text{ p.u.}$$

$$\text{Now, } I = \frac{V_t - V}{j0.8} = \frac{1.15\angle68.1° - 1\angle0°}{j0.8} = \frac{0.4289 + j1.067 - 1}{0.8\angle90°}$$

$$= \frac{-0.5711 + j1.067}{0.8\angle90°} = \frac{1.21\angle118.2°}{0.8\angle90°} = 1.5125\angle28.2° = 1.333 + j0.715 \text{ p.u.}$$

$$E' = j0.6\,I + V_t = 0.6\angle90° \times 1.5125\angle28.2° + 1.15\angle68.1°$$

$$= 0.9075\ \angle118.2° + 1.15\angle68.1° = -\,0.4288 + j0.8 + 0.4289 + j1.067$$

$$= 0.0001 + j1.867 = 1.867\angle90° \text{p.u.}$$

$$\text{Steady state power limit, } P_{max} = \frac{|E'||V|}{0.6 + 0.8} = \frac{1.867 \times 1}{1.4} = 1.3336 \text{ p.u.}$$

EXAMPLE 5.3

A synchronous generator having a reactance of 1 p.u. is connected to an infinite bus through a transmission system with a reactance of 0.7 p.u. The generator is running on no-load with a voltage of 1.1 p.u. Take H = 4.5. MW-s/MVA. The voltage of infinite bus is $1\angle 0°$ p.u. and its frequency is 50 Hz.

Calculate the frequency of natural oscillations if the machine is suddenly loaded to (i) 60% and (ii) 75% of its maximum power limit. Neglect the resistance and machine damping.

SOLUTION

Maximum power limit, $P_{max} = \dfrac{|E||V|}{X}$

Given that, $|E| = 1.1$ p.u. ; $|V| = 1$ p.u. ; $X = (1 + 0.7)$ p.u.

$$\therefore\ P_{max} = \frac{1.1 \times 1}{1.7} = 0.6471 \text{ p.u.}$$

The inertia constant, $M = H/\pi f$ in p.u.

Given that, H = 4.5 MW-s/MVA and f = 50 Hz.

$$\therefore\ M = \frac{4.5}{\pi \times 50} = 0.0287 \text{ p.u.}$$

Case (i) : 60% loading

The electrical power, $P_e = P_{max} \sin \delta$

For 60% loading, let $\delta = \delta_1$ and $P_e = P_{e1}$

It is given that, $\dfrac{P_{e1}}{P_{max}} = 60\% = 0.6$

$$\therefore\ P_{e1} = P_{max} \sin \delta_1 \quad \text{(or)} \quad \sin \delta_1 = \frac{P_{e1}}{P_{max}} = 0.6$$

$$\therefore\ \delta_1 = \sin^{-1} 0.6 = 36.9° \text{ (elect. deg)}$$

The characteristic equation of the system during small disturbances is given by

$$Mx^2 + C = 0$$

where, $C = P_{max} \cos \delta_1$

The natural frequency of oscillation of the system for small disturbances is given by the roots of characteristic equation.

The root of characteristic equation, $x = \sqrt{\dfrac{-C}{M}} = \pm j\sqrt{\dfrac{P_{max} \cos \delta_1}{M}}$

$$= \pm j\sqrt{\frac{0.6471 \times \cos 36.9°}{0.0287}} = \pm j4.25$$

$$\therefore\ \text{Natural frequency of oscillation} = 4.25 \text{ rad / sec} = \frac{4.25}{2\pi} = 0.6764 \text{ Hz}$$

Case (ii) : 75 % loading

The electrical power, $P_e = P_{max} \sin \delta$

For 75% loading, let $\delta = \delta_2$ and $P_e = P_{e2}$

It is given that, $\dfrac{P_{e2}}{P_{max}} = 75\% = 0.75$

$$\therefore\ P_{e2} = P_{max} \sin \delta_2 \quad \text{(or)} \quad \sin \delta_2 = \frac{P_{e2}}{P_{max}} = 0.85$$

$$\therefore\ \delta_2 = \sin^{-1} 0.75 = 48.6° \text{ (elect. deg)}$$

The characteristic equation of the system during small disturbances is given by

$$Mx^2 + C = 0$$

where, $C = P_{max} \cos \delta_2$

The natural frequency of oscillation of the system for small disturbances is given by the roots of characteristic equation.

The root of characteristic equation, $x = \sqrt{\frac{-C}{M}} = \pm j\sqrt{\frac{P_{max} \cos \delta_2}{M}}$

$$= \pm j\sqrt{\frac{0.6471 \times \cos 48.6^\circ}{0.0287}} = \pm j3.86$$

$\therefore$ Natural frequency of oscillation $= 3.86$ rad / sec $= \frac{3.86}{2\pi} = 0.6143$ Hz.

EXAMPLE 5.4

Two power stations A and B are located close together. Station A has four identical generator sets each rated 100 MVA and having an inertia constant of 9 MJ/MVA whereas the station B has 3 sets each rated 200 MVA, 4 MJ/MVA. Calculate the inertia constant of a single equivalent machine on a base of 100 MVA.

SOLUTION

Let us assume that the machines are swinging coherently (i.e., working in unison). For two machines swinging coherently the equivalent inertia constant, H_{eq} is given by

$$H_{eq} = \frac{H_{1,mach} S_{1,mach}}{S_{sys}} + \frac{H_{2,mach} S_{2,mach}}{S_{sys}}$$

where, S_{sys} = MVA rating of system (or specified base)

$S_{1,mach}$ & $H_{1,mach}$ = MVA rating and inertia constant of machine-1.

$S_{2,mach}$ & $H_{2,mach}$ = MVA rating and inertia constant of machine-2.

The equation(5.4.1) can be extended to any number of machines swinging coherently.

Station-A

The station-A has four machines of identical rating

$\therefore$ Equivalent intertia constant of station A $= H_A = 4\left(\frac{H_{mach} S_{mach}}{S_{sys}}\right)$

Given that, $H_{mach} = 9$ MJ/MVA ; $S_{mach} = 100$ MVA and $S_{sys} = 100$ MVA

$$\therefore H_A = 4\left(\frac{9 \times 100}{100}\right) = 36 \text{ MJ / MVA}$$

Station-B

The station-B has three machines of identical rating

$\therefore$ Equivalent intertia constant of station B $= H_B = 3\left(\frac{H_{mach} S_{mach}}{S_{sys}}\right)$

Given that, $H_{mach} = 4$ MJ/MVA ; $S_{mach} = 200$ MVA and $S_{sys} = 100$ MVA

$$\therefore H_B = 3\left(\frac{4 \times 200}{100}\right) = 24 \text{ MJ / MVA}$$

H_{eq} of system

The inertia constants of station-A and station-B are calculated on a common system base MVA (of 100 MVA). Hence the equivalent inertia constant of the system is given by the sum of inertia constants of the two stations.

$\therefore$Equivalent inertia constant of the system $= H_{eq} = H_A + H_B = 36 + 24 = 60$ MJ/MVA

EXAMPLE 5.5

A 2-pole 50 Hz, 11 kV turbo alternator has a ratio of 100 MW, power factor 0.85 lagging. The rotor has a moment of inertia of 10,000 kg m². Calculate H and M.

GIVEN DATA

pole, p = 2	Rated voltage, V = 11 kv
frequency, f = 50 Hz	Rated power, P = 100 MW
power factor = 0.85 lag	Moment of inertia, J = 10,000 kg m²

SOLUTION

Synchronous speed in rps, $n_s = \frac{2f}{p} = \frac{2 \times 50}{2} = 50$ rps

Synchronous speed in rad / sec, $\omega_s = 2\pi n_s = 2\pi \times 50 = 314.16$ elect.rad / sec

(Note : One revolution is equal to 2π radians)

Intertia constant, $M = J\left(\frac{2}{p}\right)^2 \omega_s \times 10^{-6}$ in MJ - s / elect.rad

$\therefore\ M = 10,000 \times \left(\frac{2}{2}\right)^2 \times 314.16 \times 10^{-6} = 3.146$ MJ - s / elect.rad

Let us choose the rated voltage and MVA as base values.

$\therefore kV_b = 11$ kV and $MVA_b = 117.65$ MVA

Intertia constant, M in p.u., $M_{pu} = \frac{\text{M in MJ - s / elect.rad}}{MVA_b} = \frac{3.1416}{117.65} = 0.0267$ p.u.

The inertia constant, $H = \pi f M_{pu} = \pi \times 50 \times 0.0267 = 4.194$ MW - s / MVA

RESULT

Inertia constant, M = 3.1416 MJ-s/elect.rad (or) 0.0267 p.u.

Inertia constant, H = 4.194 MW-s/MVA.

5.10 SHORT-ANSWER QUESTIONS

Q5.1 Define stability

The stability of a system is defined as the ability of power system to return to stable (synchronous) operation when it is subjected to a disturbance.

Q5.2 Define steady state stability

The steady state stability is defined as the ability of a power system to remain stable (i.e, without loosing synchronism) for small disturbances.

Q5.3 Define transient stability

The transient stability is defined as the ability of a power system to remain stable (i.e., without loosing synchronism) for large disturbances.

Q5.4 What is steady state stability limit?

The steady state stability limit is the maximum power that can be transmitted by a machine (or transmitting system) to a receiving system without loss of synchronism. In steady state the power transferred by synchronous machine (or power system) is always less than the steady state stability limit.

References:

- Power System Analysis by A. Nagoor Kani, Second Edition, CBS Publisher & Distributors Pvt. Ltd.
- Modern Power System Analysis by D. P. Kothari and I. J. Nagarath, Fourth Edition, Tata McGraw-Hill.

Case Study

Topic: Transient Stability

Transient Stability in a Power System During a Fault

In a power system, generators operate in synchronism to supply electricity to loads. Stability refers to the ability of the system to maintain synchronism when subjected to disturbances.

Consider a **large power system where a generator is connected to an infinite bus through a transmission line**. During normal operation, the generator supplies power steadily. Suddenly, a **three-phase fault occurs on the transmission line**, which causes a sudden reduction in electrical power output.

Because the **mechanical power input remains constant**, the generator rotor begins to accelerate and the **power angle increases**. If the fault is cleared quickly by circuit breakers, the generator may return to its stable operating condition. However, if the **fault clearing time is too long**, the generator may lose synchronism with the system.

Engineers analyse this situation using the **swing equation and equal area criterion** to determine whether the system remains stable after the disturbance. Proper **protective relays and fast circuit breakers** help maintain system stability by clearing faults quickly.

This case study highlights the importance of **transient stability analysis in maintaining reliable power system operation**.

Unit-5 Outcomes:

- Understand the **concept of power system stability**.
- Explain different **types of stability such as steady state, dynamic and transient stability**.
- Derive and apply the **swing equation for stability analysis**.
- Analyze **transient stability using the equal area criterion**.
- Determine **critical clearing angle and critical clearing time** in power systems.

2 Marks Questions:

1. Define power system stability.
2. What are the different types of stability in power systems?
3. What is steady state stability?
4. What is transient stability?
5. Write the swing equation of a synchronous machine.
6. What is power angle (δ)?
7. What is the equal area criterion?
8. Define critical clearing angle.
9. Define critical clearing time.
10. What is meant by dynamic stability?

10 Marks Questions:

1. A. Define Steady State Stability, Transient Stability.

   B. Examine the methods to improve transient stability
2. State and explain equal area criterion and discuss how you will apply it to find the maximum additional load that can be suddenly added.
3. How can the transient stability of the system be improved? Discuss the traditional as well as new approach to the problems.
4. A Generator is operating at 50 Hz delivers 1 p.u. power to an infinite bus through a transmission circuit in which resistance is ignored. A fault takes place reducing the maximum power was 2.0 p.u and after the clearance of fault, it is 1.5 p.u. by the use of equal area criterion, determine the critical clearing angle.
5. A. Define Power Angle.

   B. Derive the Expression for Swing equations for the Synchronous Machine?
6. A. State Equal Area Criterion.

   B. Derive the Expression for Critical Clearing time for a SMIB
7. A. Define Synchronizing power co-efficient and explain its significance.

   B. Draw and explain Power Angle curve for Synchronous Machine.
8. Derive the Power flow equation for a single machine connected to infinite bus without losses. Also explain power angle curve.
9. Explain in detailed about Clearing Time and Clearing Angle.
10. What is Equal Area criterion? Discuss the application of Equal area Criterion for the System Stability when a sudden change in Mechanical Input.