II B.Tech. - II Semester

23ESC241 **EM WAVES AND TRANSMISSION LINES**

L	T	P	C
2	1	-	3

PRE-REQUISITES: Engineering Mathematics

COURSE EDUCATIONAL OBJECTIVES:

1. To understand and analyze different laws and theorems of electrostatic fields.
2. To study and analyze different laws and theorems of magnetostatic fields.
3. To analyze the Maxwell's equations in different forms.
4. To learn the concepts of wave theory and its propagation through various mediums.
5. To get exposure to the properties of transmission lines.

UNIT –1: ELECTROSTATICS (9)

Review of Co-ordinate Systems, Coulomb's Law, Electric Field Intensity, Electric Flux Density, Gauss Law and Applications, Electric Potential, Maxwell's Two Equations for Electrostatic Fields, Energy Density, Illustrative Problems. Convection and Conduction Currents, Dielectric Constant, Poisson's and Laplace's Equations; Capacitance – Parallel Plate, Coaxial Capacitors, Illustrative Problems.

UNIT –2: MAGNETOSTATICS (9)

Biot-Savart Law, Ampere's Circuital Law and Applications, Magnetic Flux Density, Maxwell's Two Equations for Magnetostatic Fields, Magnetic Scalar and Vector Potentials, Forces due to Magnetic Fields, Ampere's Force Law, Inductances and Magnetic Energy, Illustrative Problems.

Maxwell's Equations (Time Varying Fields): Faraday's Law and Transformer EMF, Inconsistency of Ampere's Law and Displacement Current Density, Maxwell's Equations in Different Final Forms and Word Statements, Conditions at a Boundary Surface, Illustrative Problems.

UNIT –3: EM WAVE CHARACTERISTICS (9)

Wave Equations for Conducting and Perfect Dielectric Media, Uniform Plane Waves – Definition, All Relations Between E & H, Sinusoidal Variations, Wave Propagation in Lossy dielectrics, lossless dielectrics, free space, wave propagation in good conductors, skin depth, Polarization & Types, Illustrative Problems.
Reflection and Refraction of Plane Waves – Normal and Oblique Incidences, for both Perfect Conductor and Perfect Dielectrics, Brewster Angle, Critical Angle and Total Internal Reflection, Surface Impedance, Poynting Vector and Poynting Theorem, Illustrative Problems.

UNIT –4: TRANSMISSION LINES - I (9)

Types, Parameters, T & π Equivalent Circuits, Transmission Line Equations, Primary & Secondary Constants, Expressions for Characteristic Impedance, Propagation Constant, Phase and Group Velocities, Infinite Line, Lossless lines, distortion less lines, Illustrative Problems.

UNIT –5: TRANSMISSION LINES - II (9)

Input Impedance Relations, Reflection Coefficient, VSWR, Average Power, Shorted Lines, Open Circuited Lines, and Matched Lines, Low loss radio frequency and UHF Transmission lines, UHF Lines as Circuit Elements, Smith Chart – Construction and Applications, Quarter wave transformer, Single Stub Matching, Illustrative Problems.

Total Hours: 45

COURSE OUTCOMES:

On successful completion of the course, students will be able to		Pos
CO1	Learn the concepts of wave theory and its propagation through various mediums. (L2)	PO1, PO2, P03, PO4
CO2	Understand the properties of transmission lines and their applications. (L2)	PO1, PO2, P03, PO4
CO3	Apply the laws & theorems of electrostatic fields to solve the related problems (L3)	PO1, PO2, P03, PO4
CO4	Gain proficiency in the analysis and application of magnetostatic laws and theorems (L4).	PO1, PO2, P03, PO4
CO5	Analyze Maxwell's equations in different forms. (L4)	PO1, PO2, P03, PO4

TEXT BOOKS:

1. Elements of Electromagnetics, Matthew N.O. Sadiku, 4th Edition, Oxford University Press, 2008.
2. Electromagnetic Waves and Radiating Systems, E.C. Jordan and K.G. Balmain, 2nd Edition, PHI, 2000.

REFERENCE BOOKS:

1. Electromagnetic Field Theory and Transmission Lines, G. S. N. Raju, 2nd Edition, Pearson Education, 2013.
2. Engineering Electromagnetics, William H. Hayt Jr. and John A. Buck, 7th Edition, Tata McGraw Hill, 2006.
3. Electromagnetics, John D. Krauss, 3rd Edition, McGraw Hill, 1988.
4. Networks, Lines, and Fields, John D. Ryder,2nd Edition, PHI publications, 2012.

REFERENCE WEBSITE:

1. https://nptel.ac.in/courses/112/103/112103109/
2. https://nptel.ac.in/courses/122/104/122104015/
3. https://www.digimat.in/nptel/courses/video/112106180/L01.html
4. https://nptel.ac.in/courses/112/106/112106286/
5. https://nptel.ac.in/courses/112/105/112105164/
6. https://nptel.ac.in/courses/112/103/112103108/

CO-PO MAPPING:

CO\PO	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12
CO.1	3	2	2	2	-	-	-	-	-	-	-	-
CO.2	3	2	2	2	-	-	-	-	-	-	-	-
CO.3	3	2	2	2	-	-	-	-	-	-	-	-
CO.4	3	2	2	2	-	-	-	-	-	-	-	-
CO.5	3	2	2	2	-	-	-	-	-	-	-	-
CO*	**3**	**2**	**2**	**2**	**-**	**-**	**-**	**-**	**-**	**-**	**-**	**-**

UNIT – I
ELECTROSTATICS

Contents

- Review of coordinate system
- Coulomb's Law
- Electric Field Intensity - Fields due to Different Charge Distributions
- Electric Flux Density
- Gauss Law and Applications
- Electric Potential
- Relations Between E and V
- Maxwell's Equations for Electrostatic Fields
- Continuity Equation
- Relaxation Time
- Poisson's and Laplace's Equations
- Illustrative Problems.

Introduction:
Electrostatics, as the name implies, is the study of stationary electric charges. Electrostatics is the study of electric charges at rest. It involves the interaction between charged particles and the forces and fields they create. Coulomb's law is a fundamental principle in electrostatics that describes the force between two-point charges.

Vector Algebra is a part of algebra that deals with the theory of vectors and vector spaces.

Most of the physical quantities are either scalar or vector quantities.

Scalar Quantity:

Scalar is a number that defines magnitude. Hence a scalar quantity is defined as a quantity that has magnitude only. A scalar quantity does not point to any direction i.e. a scalar quantity has no directional component.

For example, when we say, the temperature of the room is 30^o C, we don 't specifies the direction.

Hence examples of scalar quantities are mass, temperature, volume, speed etc.

A scalar quantity is represented simply by a letter – A, B, T, V, S.

Vector Quantity:

A Vector has both a magnitude and a direction. Hence a vector quantity is a quantity that has both magnitude and direction.

Examples of vector quantities are force, displacement, velocity, etc.

$\vec{A}, \vec{V}, \vec{B}, \vec{F}$

A vector quantity is represented by a letter with an arrow over it or a bold letter.

Unit Vectors:

When a simple vector is divided by its own magnitude, a new vector is created known as the unit vector. A unit vector has a magnitude of one. Hence the name - unit vector.

A unit vector is always used to describe the direction of respective vector.

$$\mathbf{a}_A = \frac{\vec{A}}{|A|} \Rightarrow \vec{A} = |A|\,\mathbf{a}_A$$

Hence any vector can be written as the product of its magnitude and its unit vector. Unit Vectors along the co-ordinate directions are referred to as the base vectors. For example unit vectors along X, Y and Z directions are ax, ay and az respectively.

Position Vector / Radius Vector ($\overline{OP}$):

A Position Vector / Radius vector define the position of a point(P) in space relative to the origin(O). Hence Position vector is another way to denote a point in space.

$$\overline{OP} = x\bar{a}_x + y\bar{a}_y + z\bar{a}_z$$

Displacement Vector

Displacement Vector is the displacement or the shortest distance from one point to another.

Vector Multiplication

When two vectors are multiplied the result is either a scalar or a vector depending on how they are multiplied. The two important types of vector multiplication are:

- Dot Product/Scalar Product (A.B)
- Cross product (A x B)

1. DOT PRODUCT (A. B):

Dot product of two vectors A and B is defined as:

$$\bar{A}.\bar{B} = |\bar{A}|\,|\bar{B}|\cos\theta_{AB}$$

Where θ_{AB} is the angle formed between A and B.
Also θ_{AB} ranges from 0 to π i.e. $0 \le \theta_{AB} \le \pi$
The result of A.B is a scalar, hence dot product is also known as Scalar Product.

Properties of Dot Product:

1. If A = (Ax, Ay, Az) and B = (Bx, By, Bz) then

$$\bar{A}.\bar{B} = AxBx + AyBy + AzBz$$

2. $\bar{A}.\bar{B}$= |A| |B|, if $\cos\theta_{AB}$=1 which means $\theta_{AB} = 0^0$

This shows that A and B are in the same direction or we can also say that A and B are parallel to each other.

3. $\bar{A}.\bar{B}$ = - |A| |B|, if $\cos\theta_{AB}$=-1 which means $\theta_{AB} = 180^0$.
This shows that A and B are in the opposite direction or we can also say that A and B are antiparallel to each other.

4. $\bar{A}.\bar{B}$ = 0, if $\cos\theta_{AB}$=0 which means $\theta_{AB} = 90^0$.
This shows that A and B are orthogonal or perpendicular to each other.

5. Since we know the Cartesian base vectors are mutually perpendicular to each other, we have

$$\bar{a}_x.\bar{a}_x = \bar{a}_y.\bar{a}_y = \bar{a}_z.\bar{a}_z = 1 \text{ and } \bar{a}_x.\bar{a}_y = \bar{a}_y.\bar{a}_z = \bar{a}_z.\bar{a}_x = 0$$

2. Cross Product (A X B):

Cross Product of two vectors A and B is given as:

$$\bar{A} X \bar{B} = |\bar{A}|\,|\bar{B}| \sin\theta_{AB}\, \bar{a}_N$$

Where θ_{AB} is the angle formed between A and B and $\bar{a}_N$ is a unit vector normal to both A and B. Also θ ranges from 0 to π i.e. $0 \leq \theta_{AB} \leq \pi$

The cross product is an operation between two vectors and the output is also a vector.

Properties of Cross Product:

1. If A = (Ax, Ay, Az) and B = (Bx, By, Bz) then,

$$\mathbf{A} * \mathbf{B} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ \mathbf{A}_x & \mathbf{A}_y & \mathbf{A}_z \\ \mathbf{B}_x & \mathbf{B}_y & \mathbf{B}_z \end{vmatrix}$$

The resultant vector is always normal to both the vector A and B.

2. $\bar{A} X \bar{B} = 0$, if $\sin\theta_{AB} = 0$ which means $\theta_{AB} = 0^0$ or 180^0;
This shows that A and B are either parallel or antiparallel to each other.

6. 3. $\bar{A} X \bar{B} = |\bar{A}|\,|\bar{B}|\,\bar{a}_N$, if $\sin\theta_{AB} = 0$ which means $\theta_{AB} = 90^0$.
This shows that A and B are orthogonal or perpendicular to each other.

4. Since we know the Cartesian base vectors are mutually perpendicular to each other, we have

$$\bar{a}_x X\, \bar{a}_x = \bar{a}_y\, X\, \bar{a}_y = \bar{a}_z X \bar{a}_z = 0$$

$$\bar{a}_x X\, \bar{a}_y = \bar{a}_z\ ,\bar{a}_y\, X\, \bar{a}_z = \bar{a}_x\ ,\ \bar{a}_z X\, \bar{a}_x = \bar{a}_y$$

CO-ORDINATE SYSTEMS:

Co-Ordinate system is a system of representing points in a space of given dimensions by coordinates, such as the Cartesian coordinate system or the system of celestial longitude and latitude.

In order to describe the spatial variations of the quantities, appropriate coordinate system is required. A point or vector can be represented in a curvilinear coordinate system that may be orthogonal or non-orthogonal. An orthogonal system is one in which the coordinates are mutually perpendicular to each other.

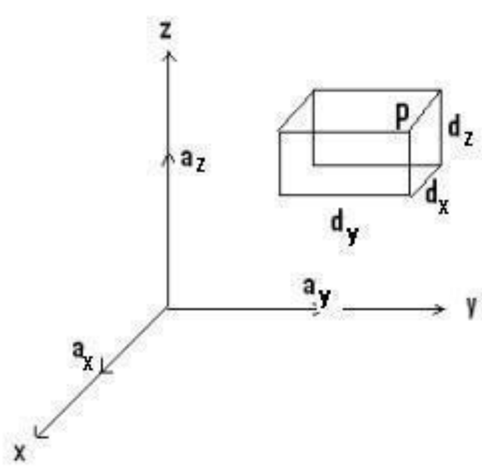
The different co-ordinate system available are:

- Cartesian or Rectangular co-ordinate system. (Example: Cube, Cuboid)
- Circular Cylindrical co-ordinate system. (Example: Cylinder)
- Spherical co-ordinate system. (Example: Sphere)

The choice depends on the geometry of the application.

A set of 3 scalar values that define position and a set of unit vectors that define direction form a co-ordinate system. The 3 scalar values used to define position are called co-ordinates. All coordinates are defined with respect to an arbitrary point called the origin.

1. Cartesian Co-ordinate System / Rectangular Co-ordinate System (x,y,z)



A Vector in Cartesian system is represented as (Ax, Ay, Az) Or

$$\bar{A} = A_x\bar{a}_x + A_y\bar{a}_y + A_z\bar{a}_z$$

Where $\bar{a}_x$, $\bar{a}_y$ and $\bar{a}_z$ are the unit vectors in x, y, z direction respectively

Range of the variables:

It defines the minimum and the maximum value that x, y and z can have in Cartesian system.
$-\infty \leq x,y,z \leq \infty$

Differential Displacement / Differential Length (dl):

It is given as

$$\overline{dl} = dx\bar{a}_x + dy\bar{a}_y + dz\bar{a}_z$$

Differential length for a line parallel to x, y and z axis are respectively given as:

dl = $dx\bar{a}_x$---(For a line parallel to x-axis).
dl = $dy\bar{a}_y$ ---(For a line Parallel to y-axis).
dl = $dz\bar{a}_z$ ---(For a line parallel to z-axis).

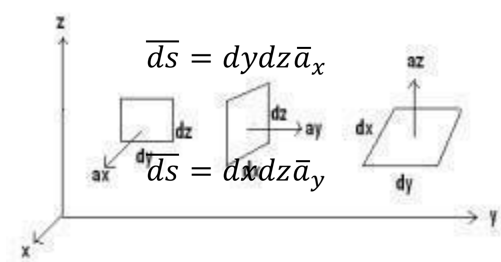
If there is a wire of length L in z-axis, then the differential length is given as dl = dz az. Similarly, if the wire is in y-axis, then the differential length is given as dl = dy ay.

Differential Normal Surface (ds):

Differential surface is basically a cross product between two parameters of the surface. The differential surface (area element) is defined as

$$\overline{ds} = ds\bar{a}_N$$

Where$\bar{a}_N$, is the unit vector perpendicular to the surface.



For the 1st figure, $\overline{ds} = dydz\bar{a}_x$

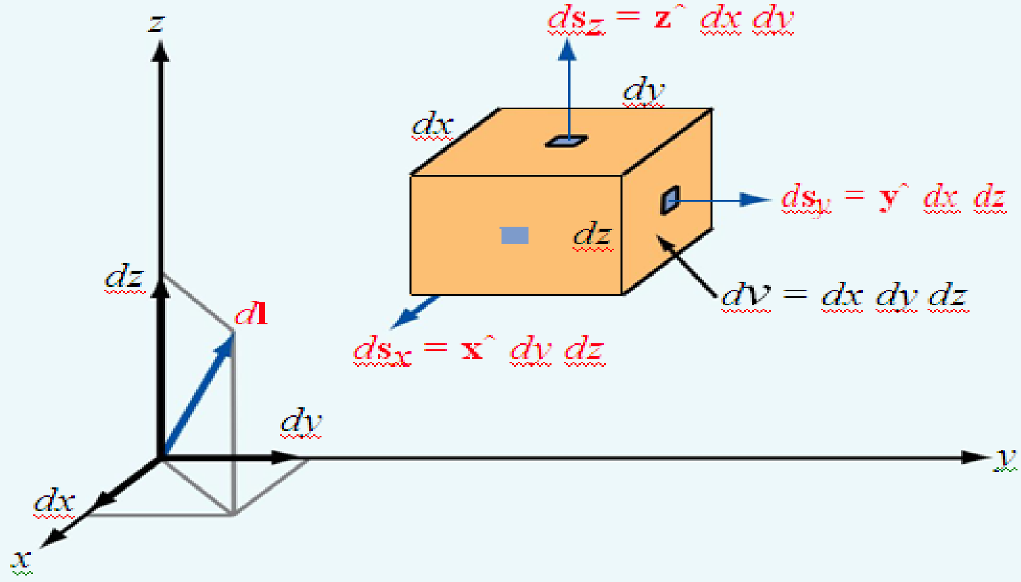
2nd figure, $\overline{ds} = dxdz\bar{a}_y$

3rd figure, $\overline{ds} = dxdy\bar{a}_z$

Differential Volume:

The differential volume element (dv) can be expressed in terms of the triple product.

$$dv = dxdydz$$



Differential length, area, and volume in Cartesian coordinates.

2. Circular Cylindrical Co-ordinate System

A Vector in Cylindrical system is represented as (A_r, A_ϕ, A_z) or

$$\bar{A} = A_r\bar{a}_r + A_\emptyset\bar{a}_\emptyset + A_z\bar{a}_z$$

Where$\bar{a}_r$, $\bar{a}_\emptyset$ and $\bar{a}_z$ are the unit vectors in r, Φ and z directions respectively.

The physical significance of each parameter of cylindrical coordinates:

1. The value r indicates the distance of the point from the z-axis. It is the radius of the cylinder.
2. The value Φ, also called the azimuthal angle, indicates the rotation angle around the z-axis. It is basically measured from the x axis in the x-y plane. It is measured anti clockwise.
3. The value z indicates the distance of the point from z-axis. It is the same as in the Cartesian system. In short, it is the height of the cylinder.

Range of the variables:

It defines the minimum and the maximum values of r, Φ and z.

$0 \leq r \leq \infty$
$0 \leq \Phi \leq 2\pi$
$-\infty \leq z \leq \infty$

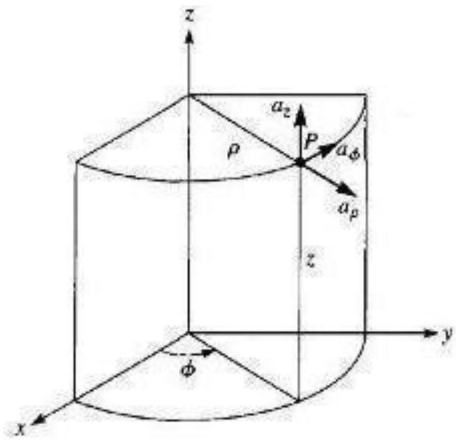


Figure shows Point P and Unit vectors in Cylindrical Co-ordinate System.

Differential Displacement / Differential Length (dl):

It is given as

$\overline{dl} = dr\bar{a}_r + rd\varphi\bar{a}_\varphi + dz\bar{a}_z$

Differential length for a line parallel to r, Φ and z axis are respectively given as:

dl = $dr\bar{a}_r$---(For a line parallel to r-direction).
dl = $rd\varphi\bar{a}_\varphi$ ---(For a line Parallel to Φ-direction).
dl = $dz\bar{a}_z$ ---(For a line parallel to z-axis).

Differential Normal Surface (ds):

Differential surface is basically a cross product between two parameters of the surface. The differential surface (area element) is defined as

$$\overline{ds} = ds\bar{a}_N$$

Where$\bar{a}_N$, is the unit vector perpendicular to the surface.

This surface describes a circular disc. Always remember- To define a circular disk we need two parameter one distance measure and one angular measure. An angular parameter will always give a curved line or an arc.

In this case dΦ is measured in terms of change in arc. Arc is given as:
Arc= radius * angle

$$\overline{ds} = rdrd\varphi\bar{a}_z$$
$$\overline{ds} = drdz\bar{a}_\varphi$$
$$\overline{ds} = rdrd\varphi\bar{a}_r$$

Differential Volume:
The differential volume element (dv) can be expressed in terms of the triple product.

$$dv = rdrd\varphi dz$$

3. Spherical coordinate System:

Spherical coordinates consist of one scalar value (r), with units of distance, while the other two scalar values (θ, Φ) have angular units (degrees or radians).

A Vector in Spherical System is represented as (A_r ,A_Θ, A_Φ) or

$$\bar{A} = A_r\bar{a}_r + A_\theta\bar{a}_\theta + A_\varphi\bar{a}_\varphi$$

Where $\bar{a}_r$, $\bar{a}_\theta$ and $\bar{a}_\varphi$ are the unit vectors in r, θ and Φ direction respectively.

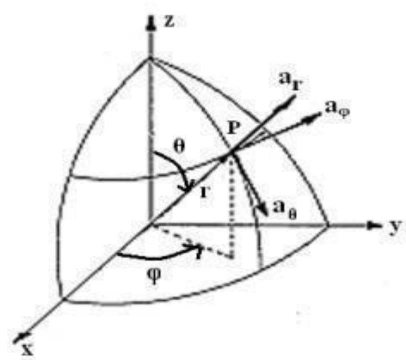
The physical significance of each parameter of spherical coordinates:

1. The value r expresses the distance of the point from origin (i.e. similar to altitude). It is the radius of the sphere.
2. The angle θ is the angle formed with the z- axis (i.e. similar to latitude). It is also called the co-latitude angle. It is measured clockwise.
3. The angle Φ, also called the azimuthal angle, indicates the rotation angle around the z-axis (i.e. similar to longitude). It is basically measured from the x axis in the x-y plane. It is measured counter-clockwise.

Range of the variables:

It defines the minimum and the maximum value that r, θ and υ can have in spherical co-ordinate system.

$0 \leq r \leq \infty$
$0 \leq \theta \leq \pi$
$0 \leq \Phi \leq 2\pi$



Differential length:
It is given as

$$\overline{dl} = dr\bar{a}_r + rd\theta\bar{a}_\theta + r\sin\theta\, d\varphi\bar{a}_\varphi$$

Differential length for a line parallel to r, θ and Φ axis are respectively given as:

dl = $dr\bar{a}_r$--(For a line parallel to r axis)

dl = $rd\theta\bar{a}_\theta$---(For a line parallel to θ direction)

dl = $r\sin\theta\, d\varphi\bar{a}_\varphi$ --(For a line parallel to Φ direction)

Differential Normal Surface (ds): Differential surface is basically a cross product between two parameters of the surface.
The differential surface (area element) is defined as

$$\overline{ds} = ds\bar{a}_N$$

Where $\bar{a}_N$, is the unit vector perpendicular to the surface.

$$\overline{ds} = rdrd\theta\bar{a}_\varphi$$

$$\overline{ds} = r^2\sin\theta\, d\varphi d\theta\bar{a}_r$$

$$\overline{ds} = r\sin\theta\, dr d\varphi \bar{a}_\theta$$

Differential Volume:

The differential volume element (dv) can be expressed in terms of the triple product.

$$dv = r^2 \sin\theta\, dr d\varphi d\theta$$

Coordinate transformations:

Coordinate transformations

Transformation	Coordinate Variables	Unit Vectors	Vector Components
Cartesian to cylindrical	$r = \sqrt[+]{x^2+y^2}$ $\phi = \tan^{-1}(y/x)$ $z = z$	$\hat{\mathbf{r}} = \hat{\mathbf{x}}\cos\phi + \hat{\mathbf{y}}\sin\phi$ $\hat{\boldsymbol{\phi}} = -\hat{\mathbf{x}}\sin\phi + \hat{\mathbf{y}}\cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{z}}$	$A_r = A_x\cos\phi + A_y\sin\phi$ $A_\phi = -A_x\sin\phi + A_y\cos\phi$ $A_z = A_z$
Cylindrical to Cartesian	$x = r\cos\phi$ $y = r\sin\phi$ $z = z$	$\hat{\mathbf{x}} = \hat{\mathbf{r}}\cos\phi - \hat{\boldsymbol{\phi}}\sin\phi$ $\hat{\mathbf{y}} = \hat{\mathbf{r}}\sin\phi + \hat{\boldsymbol{\phi}}\cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{z}}$	$A_x = A_r\cos\phi - A_\phi\sin\phi$ $A_y = A_r\sin\phi + A_\phi\cos\phi$ $A_z = A_z$
Cartesian to spherical	$R = \sqrt[+]{x^2+y^2+z^2}$ $\theta = \tan^{-1}[\sqrt[+]{x^2+y^2}/z]$ $\phi = \tan^{-1}(y/x)$	$\hat{\mathbf{R}} = \hat{\mathbf{x}}\sin\theta\cos\phi + \hat{\mathbf{y}}\sin\theta\sin\phi + \hat{\mathbf{z}}\cos\theta$ $\hat{\boldsymbol{\theta}} = \hat{\mathbf{x}}\cos\theta\cos\phi + \hat{\mathbf{y}}\cos\theta\sin\phi - \hat{\mathbf{z}}\sin\theta$ $\hat{\boldsymbol{\phi}} = -\hat{\mathbf{x}}\sin\phi + \hat{\mathbf{y}}\cos\phi$	$A_R = A_x\sin\theta\cos\phi + A_y\sin\theta\sin\phi + A_z\cos\theta$ $A_\theta = A_x\cos\theta\cos\phi + A_y\cos\theta\sin\phi - A_z\sin\theta$ $A_\phi = -A_x\sin\phi + A_y\cos\phi$
Spherical to Cartesian	$x = R\sin\theta\cos\phi$ $y = R\sin\theta\sin\phi$ $z = R\cos\theta$	$\hat{\mathbf{x}} = \hat{\mathbf{R}}\sin\theta\cos\phi + \hat{\boldsymbol{\theta}}\cos\theta\cos\phi - \hat{\boldsymbol{\phi}}\sin\phi$ $\hat{\mathbf{y}} = \hat{\mathbf{R}}\sin\theta\sin\phi + \hat{\boldsymbol{\theta}}\cos\theta\sin\phi + \hat{\boldsymbol{\phi}}\cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{R}}\cos\theta - \hat{\boldsymbol{\theta}}\sin\theta$	$A_x = A_R\sin\theta\cos\phi + A_\theta\cos\theta\cos\phi - A_\phi\sin\phi$ $A_y = A_R\sin\theta\sin\phi + A_\theta\cos\theta\sin\phi + A_\phi\cos\phi$ $A_z = A_R\cos\theta - A_\theta\sin\theta$
Cylindrical to spherical	$R = \sqrt[+]{r^2+z^2}$ $\theta = \tan^{-1}(r/z)$ $\phi = \phi$	$\hat{\mathbf{R}} = \hat{\mathbf{r}}\sin\theta + \hat{\mathbf{z}}\cos\theta$ $\hat{\boldsymbol{\theta}} = \hat{\mathbf{r}}\cos\theta - \hat{\mathbf{z}}\sin\theta$ $\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}$	$A_R = A_r\sin\theta + A_z\cos\theta$ $A_\theta = A_r\cos\theta - A_z\sin\theta$ $A_\phi = A_\phi$
Spherical to cylindrical	$r = R\sin\theta$ $\phi = \phi$ $z = R\cos\theta$	$\hat{\mathbf{r}} = \hat{\mathbf{R}}\sin\theta + \hat{\boldsymbol{\theta}}\cos\theta$ $\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}$ $\hat{\mathbf{z}} = \hat{\mathbf{R}}\cos\theta - \hat{\boldsymbol{\theta}}\sin\theta$	$A_r = A_R\sin\theta + A_\theta\cos\theta$ $A_\phi = A_\phi$ $A_z = A_R\cos\theta - A_\theta\sin\theta$

Vector relations in the three common coordinate systems.

	Cartesian Coordinates	Cylindrical Coordinates	Spherical Coordinates
Coordinate variables	x, y, z	r, ϕ, z	R, θ, ϕ
Vector representation $\mathbf{A} =$	$\hat{\mathbf{x}}A_x + \hat{\mathbf{y}}A_y + \hat{\mathbf{z}}A_z$	$\hat{\mathbf{r}}A_r + \hat{\boldsymbol{\phi}}A_\phi + \hat{\mathbf{z}}A_z$	$\hat{\mathbf{R}}A_R + \hat{\boldsymbol{\theta}}A_\theta + \hat{\boldsymbol{\phi}}A_\phi$
Magnitude of A $\|\mathbf{A}\| =$	$\sqrt[+]{A_x^2 + A_y^2 + A_z^2}$	$\sqrt[+]{A_r^2 + A_\phi^2 + A_z^2}$	$\sqrt[+]{A_R^2 + A_\theta^2 + A_\phi^2}$
Position vector $\overrightarrow{OP_1} =$	$\hat{\mathbf{x}}x_1 + \hat{\mathbf{y}}y_1 + \hat{\mathbf{z}}z_1$, for $P(x_1, y_1, z_1)$	$\hat{\mathbf{r}}r_1 + \hat{\mathbf{z}}z_1$, for $P(r_1, \phi_1, z_1)$	$\hat{\mathbf{R}}R_1$, for $P(R_1, \theta_1, \phi_1)$
Base vectors properties	$\hat{\mathbf{x}}\cdot\hat{\mathbf{x}} = \hat{\mathbf{y}}\cdot\hat{\mathbf{y}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{z}} = 1$ $\hat{\mathbf{x}}\cdot\hat{\mathbf{y}} = \hat{\mathbf{y}}\cdot\hat{\mathbf{z}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{x}} = 0$ $\hat{\mathbf{x}}\times\hat{\mathbf{y}} = \hat{\mathbf{z}}$ $\hat{\mathbf{y}}\times\hat{\mathbf{z}} = \hat{\mathbf{x}}$ $\hat{\mathbf{z}}\times\hat{\mathbf{x}} = \hat{\mathbf{y}}$	$\hat{\mathbf{r}}\cdot\hat{\mathbf{r}} = \hat{\boldsymbol{\phi}}\cdot\hat{\boldsymbol{\phi}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{z}} = 1$ $\hat{\mathbf{r}}\cdot\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}\cdot\hat{\mathbf{z}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{r}} = 0$ $\hat{\mathbf{r}}\times\hat{\boldsymbol{\phi}} = \hat{\mathbf{z}}$ $\hat{\boldsymbol{\phi}}\times\hat{\mathbf{z}} = \hat{\mathbf{r}}$ $\hat{\mathbf{z}}\times\hat{\mathbf{r}} = \hat{\boldsymbol{\phi}}$	$\hat{\mathbf{R}}\cdot\hat{\mathbf{R}} = \hat{\boldsymbol{\theta}}\cdot\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\phi}}\cdot\hat{\boldsymbol{\phi}} = 1$ $\hat{\mathbf{R}}\cdot\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\theta}}\cdot\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}\cdot\hat{\mathbf{R}} = 0$ $\hat{\mathbf{R}}\times\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\phi}}$ $\hat{\boldsymbol{\theta}}\times\hat{\boldsymbol{\phi}} = \hat{\mathbf{R}}$ $\hat{\boldsymbol{\phi}}\times\hat{\mathbf{R}} = \hat{\boldsymbol{\theta}}$
Dot product $\mathbf{A}\cdot\mathbf{B} =$	$A_xB_x + A_yB_y + A_zB_z$	$A_rB_r + A_\phi B_\phi + A_zB_z$	$A_RB_R + A_\theta B_\theta + A_\phi B_\phi$
Cross product $\mathbf{A}\times\mathbf{B} =$	$\begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{r}} & \hat{\boldsymbol{\phi}} & \hat{\mathbf{z}} \\ A_r & A_\phi & A_z \\ B_r & B_\phi & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{R}} & \hat{\boldsymbol{\theta}} & \hat{\boldsymbol{\phi}} \\ A_R & A_\theta & A_\phi \\ B_R & B_\theta & B_\phi \end{vmatrix}$
Differential length $d\mathbf{l} =$	$\hat{\mathbf{x}}\,dx + \hat{\mathbf{y}}\,dy + \hat{\mathbf{z}}\,dz$	$\hat{\mathbf{r}}\,dr + \hat{\boldsymbol{\phi}}r\,d\phi + \hat{\mathbf{z}}\,dz$	$\hat{\mathbf{R}}\,dR + \hat{\boldsymbol{\theta}}R\,d\theta + \hat{\boldsymbol{\phi}}R\sin\theta\,d\phi$
Differential surface areas	$d\mathbf{s}_x = \hat{\mathbf{x}}\,dy\,dz$ $d\mathbf{s}_y = \hat{\mathbf{y}}\,dx\,dz$ $d\mathbf{s}_z = \hat{\mathbf{z}}\,dx\,dy$	$d\mathbf{s}_r = \hat{\mathbf{r}}r\,d\phi\,dz$ $d\mathbf{s}_\phi = \hat{\boldsymbol{\phi}}\,dr\,dz$ $d\mathbf{s}_z = \hat{\mathbf{z}}r\,dr\,d\phi$	$d\mathbf{s}_R = \hat{\mathbf{R}}R^2\sin\theta\,d\theta\,d\phi$ $d\mathbf{s}_\theta = \hat{\boldsymbol{\theta}}R\sin\theta\,dR\,d\phi$ $d\mathbf{s}_\phi = \hat{\boldsymbol{\phi}}R\,dR\,d\theta$
Differential volume $dV =$	$dx\,dy\,dz$	$r\,dr\,d\phi\,dz$	$R^2\sin\theta\,dR\,d\theta\,d\phi$

Del operator:

Del is a vector differential operator. The del operator will be used in for differential operations throughout any course on field theory. The following equation is the del operator for different coordinate systems.

$$\nabla = \frac{\partial}{\partial x}\hat{a}_x + \frac{\partial}{\partial y}\hat{a}_y + \frac{\partial}{\partial z}\hat{a}_z = \nabla_{x,y,z}$$

$$\nabla = \frac{\partial}{\partial \rho}\hat{a}_\rho + \frac{1}{\rho}\frac{\partial}{\partial \phi}\hat{a}_\phi + \frac{\partial}{\partial z}\hat{a}_z$$

$$\nabla = \frac{\partial}{\partial r}\hat{a}_r + \frac{1}{r}\frac{\partial}{\partial \theta}\hat{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}\hat{a}_\phi$$

Gradient of a Scalar:

• The gradient of a scalar field, V, is a vector that represents both the magnitude and the direction of the maximum space rate of increase of V.

$$\nabla V = \frac{\partial V}{\partial x}\hat{a}_x + \frac{\partial V}{\partial y}\hat{a}_y + \frac{\partial V}{\partial z}\hat{a}_z = \nabla V_{x,y,z}$$

• To help visualize this concept, take for example a topographical map. Lines on the map represent equal magnitudes of the scalar field. The gradient vector crosses map at the location where the lines packed into the most dense space and perpendicular (or normal) to them. The orientation (up or down) of the gradient vector is such that the field is increased in magnitude along that direction.

-Fundamental properties of the gradient of a scalar field

- The magnitude of gradient equals the maximum rate of change in V per unit distance
- Gradient points in the direction of the maximum rate of change in V
- Gradient at any point is perpendicular to the constant V surface that passes through that point
- The projection of the gradient in the direction of the unit vector **a**, is

$$\nabla V \cdot \hat{a}$$

- and is called the directional derivative of V along **a**. This is the rate of change of V in the direction of **a**.
- If **A** is the gradient of V, then V is said to be the scalar potential of **A.**

Divergence of a Vector:

The divergence of a vector, **A**, at any given point P is the outward flux per unit volume as volume shrinks about P.

$$div\vec{A} = \nabla \cdot \vec{A} = \lim_{\Delta v \to 0} \frac{\oint_s \vec{A} \cdot d\vec{S}}{\Delta v}$$

The divergence of a vector field is a scalar field. The divergence is generally denoted by “div”. The divergence of a vector field can be calculated by taking the scalar product of the vector operator applied to the vector field

$$\nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

→ **Rectangular Coordinate System**

$$\nabla \cdot \vec{A} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

→ **Cylindrical Coordinate System**

$$\nabla \cdot \vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(A_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

→ Spherical Coordinate System

Curl of a Vector:

The curl of a vector, **A** is an axial vector whose magnitude is the maximum circulation of **A** per unit area as the area tends to zero and whose direction is the normal direction of the area when the area is oriented to make the circulation maximum.

-Curl of a vector in each of the three primary coordinate systems are,

Cartesian

$$\nabla \times \vec{A} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \left[\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right]\hat{a}_x - \left[\frac{\partial A_z}{\partial x} - \frac{\partial A_x}{\partial z}\right]\hat{a}_y + \left[\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right]\hat{a}_z$$

Cylindrical

$$\nabla \times \vec{A} = \frac{1}{\rho}\begin{vmatrix} \hat{a}_\rho & \rho\hat{a}_\phi & \hat{a}_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_\rho & \rho A_\phi & A_z \end{vmatrix} = \left[\frac{1}{\rho}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right]\hat{a}_\rho - \left[\frac{\partial A_z}{\partial \rho} - \frac{\partial A_\rho}{\partial z}\right]\hat{a}_\phi + \frac{1}{\rho}\left[\frac{\partial(\rho A_\phi)}{\partial \rho} - \frac{\partial A_\rho}{\partial \phi}\right]\hat{a}_z$$

Spherical

$$\nabla \times \vec{A} = \frac{1}{r^2 \sin\theta}\begin{vmatrix} \hat{a}_r & r\hat{a}_\theta & r\sin\theta\hat{a}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_r & rA_\theta & r\sin\theta A_\phi \end{vmatrix}$$

$$\nabla \times \vec{A} = \frac{1}{r\sin\theta}\left[\frac{\partial(A_\phi \sin\theta)}{\partial \theta} - \frac{\partial A_\theta}{\partial \phi}\right]\hat{a}_r - \frac{1}{r}\left[\frac{\partial(rA_\phi)}{\partial r} - \frac{1}{\sin\theta}\frac{\partial A_r}{\partial \phi}\right]\hat{a}_\theta + \frac{1}{r}\left[\frac{\partial(rA_\theta)}{\partial r} - \frac{\partial A_r}{\partial \theta}\right]\hat{a}_\phi$$

Divergence Theorem:

- The divergence theorem states that the total outward flux of a vector field, **A**, through the closed surface, S, is the same as the volume integral of the divergence of **A**.
- This theorem is easily shown from the equation for the divergence of a vector field.

$$\vec{A} = A_1\hat{a}_1 + A_2\hat{a}_2 + A_3\hat{a}_3$$

$$div\vec{A} = \nabla \cdot \vec{A} = \lim_{\Delta v \to 0} \frac{\oint_s \vec{A} \cdot d\vec{S}}{\Delta v}$$

$$\int_v \nabla \cdot \vec{A}\,dv = \oint_s \vec{A} \cdot d\vec{S}$$

Stokes Theorem:

- Stokes theorem states that the circulation of a vector field A, around a closed path, L is equal to the surface integral of the curl of A over the open surface S bounded by L. This theorem has been proven to hold as long as **A** and the curl of **A** are continuous along the closed surface S of a closed path L.

• This theorem is easily shown from the equation for the curl of a vector field.

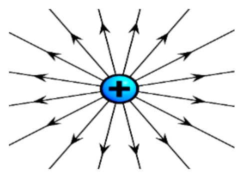
$$\vec{A} = A_1\hat{a}_1 + A_2\hat{a}_2 + A_3\hat{a}_3$$

$$curl\vec{A} = \nabla \times \vec{A} = \left(\lim_{\Delta S \to 0} \frac{\oint_L \vec{A} \cdot d\vec{l}}{\Delta S} \right)_{max} \hat{a}_n$$

$$\oint_L \vec{A} \cdot d\vec{l} = \oint_s \left(\nabla \times \vec{A}\right) \cdot d\vec{S}$$

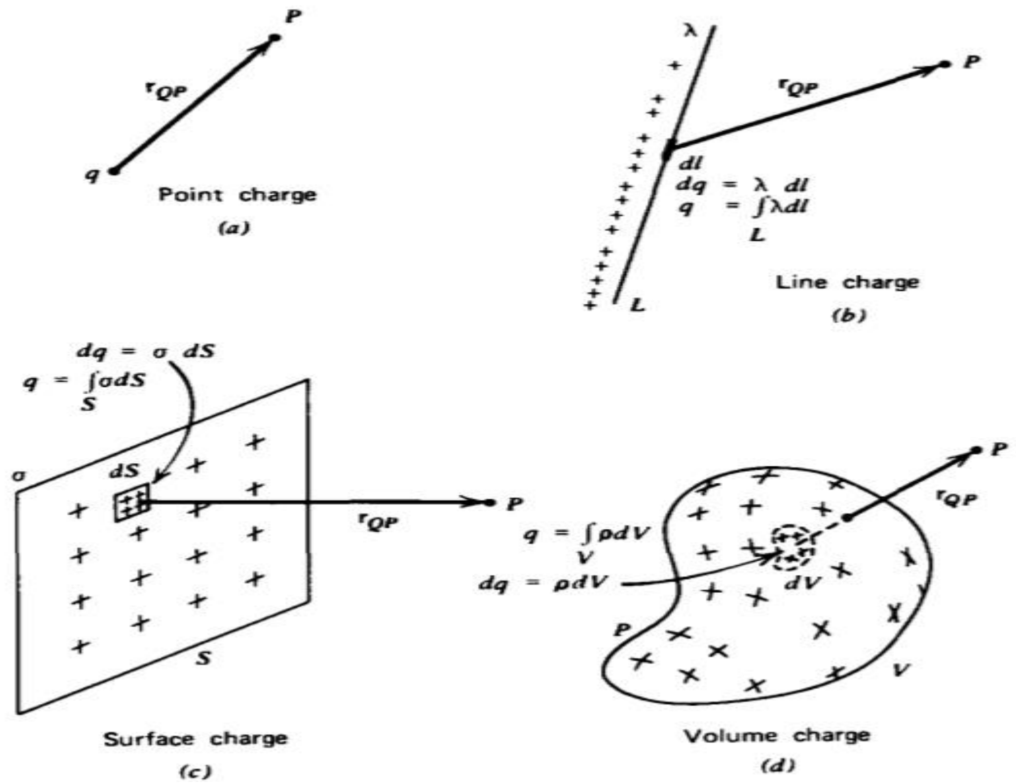
Types of Charge Distributions:

Point charge: When size of a body is much smaller than the distance under consideration, then the size of the body may be ignored and the charged body is called point charge.

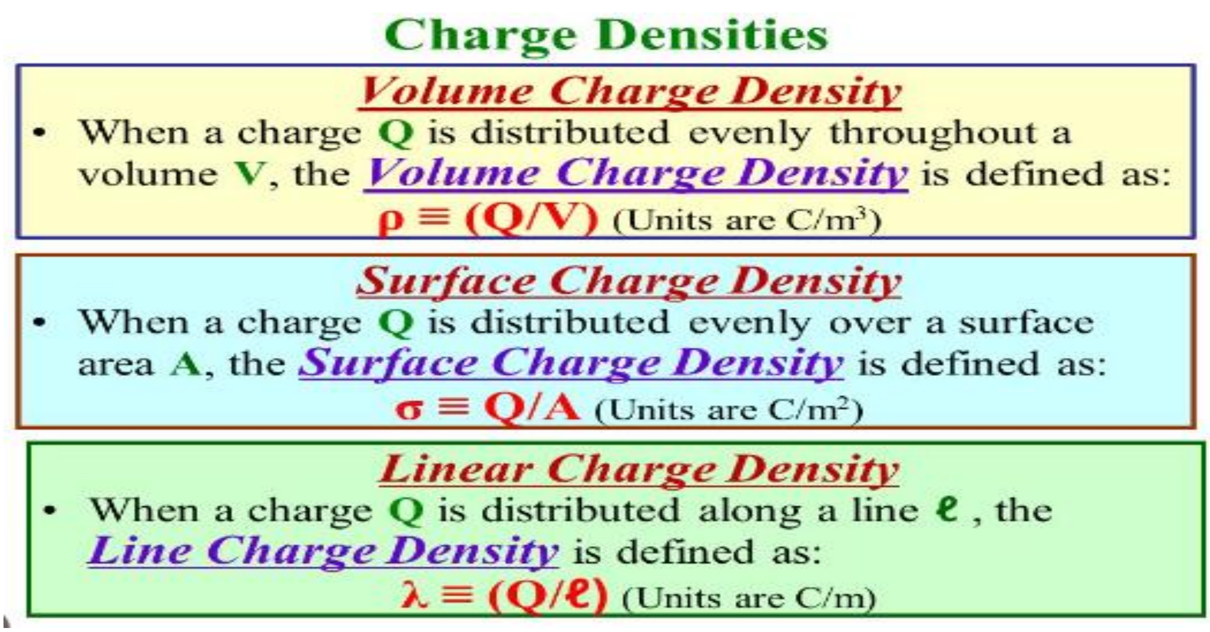


The continuous load distribution system is a system in which the charge is uniformly distributed over the conductor. For a continuous charging device, the infinite number of charges is closely packed and there is no space between them. Unlike the discrete charging system, the continuous load distribution in the conductor is uninterrupted and continuous. There are 3 types of continuous charge distribution system -

- Linear Charge Distribution
- Surface Charge Distribution
- Volume Charge Distribution



Charge distributions. (a) Point charge; (b) Line charge; (c) Surface charge; (d) Volume charge.



Coulomb's Law

Coulomb's Law states that the force between two-point charges Q1and Q2 is directly proportional to the product of the charges and inversely proportional to the square of the distance between them.

A point charge is a charge that occupies a region of space which is negligibly small compared to the distance between the point charge and any other object.
Point charge is a hypothetical charge located at a single point in space. It is an idealized model of a particle having an electric charge. Mathematically, ,

$$F = \frac{kQ_1Q_2}{R^2} \qquad k = \frac{1}{4\pi\varepsilon_0}$$

where k is the proportionality constant.
In SI units, Q1 and Q2 are expressed in Coulombs(C) and R is in meters.
Force F is in Newtons (N) and , ε_0 is called the permittivity of free space.

(We are assuming the charges are in free space. If the charges are any other dielectric medium, we will use $\varepsilon = \varepsilon_0 \varepsilon_r$ instead where ε_r is called the relative permittivity or the dielectric constant of the medium).
Therefore

$$F = \frac{1}{4\pi\varepsilon_0} \frac{Q_1Q_2}{R^2} \quad \text{....................... (1)}$$

As shown in the Figure 1 let the position vectors of the point charges Q1and Q2 are given by $\vec{r_1}$ and $\vec{r_2}$. Let $\vec{F_{12}}$ represent the force on Q1 due to charge Q2.

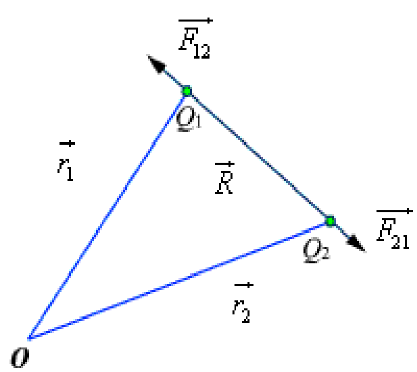


Fig 1: Coulomb's Law

The charges are separated by a distance of $R = \left|\vec{r_1} - \vec{r_2}\right| = \left|\vec{r_2} - \vec{r_1}\right|$. We define the unit vectors as

$\widehat{a_{12}} = \frac{\left(\vec{r_2} - \vec{r_1}\right)}{R}$ and $\widehat{a_{21}} = \frac{\left(\vec{r_1} - \vec{r_2}\right)}{R}$ $\vec{F_{12}}$ can be defined as

$$\overrightarrow{F_{12}} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R^2}\widehat{a_{12}} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R^2}\frac{(\vec{r_2}-\vec{r_1})}{\left|\vec{r_2}-\vec{r_1}\right|^3}.$$

Similarly the force on Q_1 due to charge Q_2 can be calculated and if $\overrightarrow{F_{21}}$ represents this force then we can write $\overrightarrow{F_{21}} = -\overrightarrow{F_{12}}$

When we have a number of point charges, to determine the force on a particular charge due to all other charges, we apply principle of superposition. If we have N number of charges $Q_1, Q_2, \ldots\ldots Q_N$ located respectively at the points represented by the position vectors $\vec{r_1}, \vec{r_2}, \ldots\ldots \vec{r_N}$, the force experienced by a charge Q located at $\vec{r}$ is given by,

$$\vec{F} = \frac{Q}{4\pi\varepsilon_0}\sum_{i=1}^{N}\frac{Q_i(\vec{r}-\vec{r_i})}{\left|\vec{r}-\vec{r_i}\right|^3}$$

Field:

A field is a function that specifies a particular physical quantity everywhere in a region. Depending upon the nature of the quantity under consideration, the field may be a vector or a scalar field. Example of scalar field is the electrostatic potential in a region while electric or magnetic fields at any point is the example of vector field.

Static Electric Fields:

Electrostatics can be defined as the study of electric charges at rest. Electric fields have their sources in electric charges. The fundamental & experimentally proved laws of electrostatics are Coulomb's law & Gauss's theorem.

Electric Field:

Electric field due to a charge is the space around the unit charge in which it experiences a force. Electric field intensity or the electric field strength at a point is defined as the force per unit charge.

Mathematically,

E = F / Q

OR

F = E Q

The force on charge Q is the product of a charge (which is a scalar) and the value of the electric field (which is a vector) at the point where the charge is located. That is

$$\vec{E} = \lim_{Q\to 0}\frac{\vec{F}}{Q} \quad \text{or,} \quad \vec{E} = \frac{\vec{F}}{Q}$$

The electric field intensity E at a point r (observation point) due a point charge Q located at $\vec{r}\,'$ (source point) is given by:

$$\vec{E} = \frac{Q(\vec{r} - \vec{r}\,')}{4\pi\varepsilon_0 \left|\vec{r} - \vec{r}\,'\right|^3}$$

For a collection of N point charges $Q_1, Q_2, \ldots\ldots Q_N$ located at $\vec{r_1}, \vec{r_2}, \ldots\ldots \vec{r_N}$, the electric field intensity at point $\vec{r}\,'$ is obtained as

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{N}\frac{Q_k(\vec{r} - \vec{r_i})}{\left|\vec{r} - \vec{r_i}\right|^3}$$

The expression (6) can be modified suitably to compute the electric filed due to a continuous distribution of charges.

In figure 2 we consider a continuous volume distribution of charge *(t)* in the region denoted as the source region.

For an elementary charge $dQ = \rho(\vec{r}\,')dv'$, i.e. considering this charge as point charge, we can write the field expression as:

$$d\vec{E} = \frac{dQ(\vec{r} - \vec{r}\,')}{4\pi\varepsilon_0 \left|\vec{r} - \vec{r}\,'\right|^3} = \frac{\rho(\vec{r}\,')dv'(\vec{r} - \vec{r}\,')}{4\pi\varepsilon_0 \left|\vec{r} - \vec{r}\,'\right|^3}$$

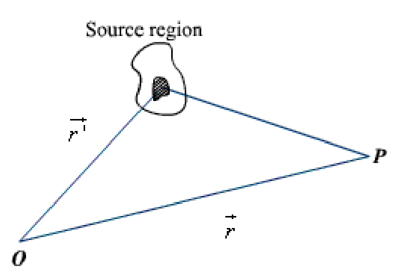


Fig 2: Continuous Volume Distribution of Charge

When this expression is integrated over the source region, we get the electric field at the point P due to this distribution of charges. Thus the expression for the electric field at P can be written as:

$$\overrightarrow{E(r)} = \int_V \frac{\rho(\vec{r'})(\vec{r} - \vec{r'})}{4\pi\varepsilon_0 \left|\vec{r} - \vec{r'}\right|^3} dv'$$ ....volume charge...........................

Similar technique can be adopted when the charge distribution is in the form of a line charge density or a surface charge density.

$$\overrightarrow{E(r)} = \int_L \frac{\rho_L(\vec{r'})(\vec{r} - \vec{r'})}{4\pi\varepsilon_0 \left|\vec{r} - \vec{r'}\right|^3} dl'$$ ....line charge ....

$$\overrightarrow{E(r)} = \int_S \frac{\rho_s(\vec{r'})(\vec{r} - \vec{r'})}{4\pi\varepsilon_0 \left|\vec{r} - \vec{r'}\right|^3} ds'$$ ....surface charge......................

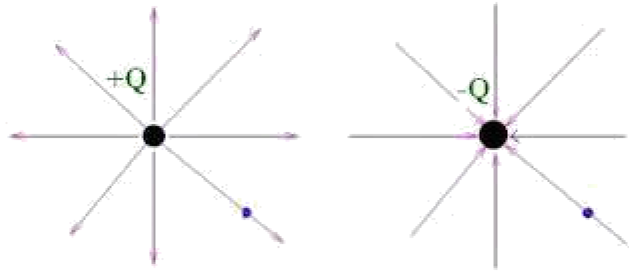
Electric Lines of Forces:

Electric line of force is a pictorial representation of the electric field.

Electric line of force (also called Electric Flux lines or Streamlines) is an imaginary straight or curved path along which a unit positive charge tends to move in an electric field.

Properties Of Electric Lines of Force:

1. Lines of force start from positive charge and terminate either at negative charge or move to infinity.
2. Similarly, lines of force due to a negative charge are assumed to start at infinity and terminate at the negative charge.



3. The number of lines per unit area, through a plane at right angles to the lines, is proportional to the magnitude of E. This means that, where the lines of force are close together, E is large and where they are far apart E is small.
4. If there is no charge in a volume, then each field line which enters it must also leave it.
5. If there is a positive charge in a volume then more field lines leave it than enter it.
6. If there is a negative charge in a volume then more field lines enter it than leave it.
7. Hence, we say Positive charges are sources and Negative charges are sinks of the field.
8. These lines are independent on medium.
9. Lines of force never intersect i.e. they do not cross each other.
10. Tangent to a line of force at any point gives the direction of the electric field E at that point.

Electric flux density:

As stated earlier electric field intensity or simply 'Electric field' gives the strength of the field at a particular point. The electric field depends on the material media in which the field is being considered. The flux density vector is defined to be independent of the material media (as we'll see that it relates to the charge that is producing it). For a linear isotropic medium under consideration; the flux density vector is defined as:

Electric flux density is defined as the amount of flux passes through unit surface area in the space imagined at right angle to the direction of electric field. The expression of electric field at a point is given by

$$E = \frac{Q}{4\pi\epsilon_0\epsilon_r r^2}$$

Where, Q is the charge of the body by which the field is created. R is the distance of the point from the center of the charged body.

$$\vec{D} = \varepsilon\vec{E}$$

We define the electric flux as

$$\psi = \int_S \vec{D}.d\vec{s}$$

Solved problems:

Problem1:

Find the charge in the volume defined by $0 \le x \le 1$ m, $0 \le y \le 1$ m, and $0 \le z \le 1$ m if $\rho = 30x^2y$ (μC/m^3). What change occurs for the limits $-1 \le y \le 0$ m?

Since $dQ = \rho\, dv$,

$$Q = \int_0^1\int_0^1\int_0^1 30x^2y\,dx\,dy\,dz = 5\ \mu\text{C}$$

For the change in limits on y,

$$Q = \int_0^1\int_{-1}^0\int_0^1 30x^2y\,dx\,dy\,dz = -5\ \mu\text{C}$$

Problem-2

Three point charges, $Q_1 = 30$ nC, $Q_2 = 150$ nC, and $Q_3 = -70$ nC, are enclosed by surface S. What net flux crosses S?

Since electric flux was defined as originating on positive charge and terminating on negative charge, part of the flux from the positive charges terminates on the negative charge.

$$\Psi_{net} = Q_{net} = 30 + 150 - 70 = 110\ \text{nC}$$

Problem-3

A point charge, $Q = 30$ nC, is located at the origin in cartesian coordinates. Find the electric flux density **D** at (1, 3, −4) m.

Referring to Fig. 3.12,

$$\mathbf{D} = \frac{Q}{4\pi R^2}\mathbf{a}_R$$

$$= \frac{30\times10^{-9}}{4\pi(26)}\left(\frac{\mathbf{a}_x + 3\mathbf{a}_y - 4\mathbf{a}_z}{\sqrt{26}}\right)$$

$$= (9.18\times10^{-11})\left(\frac{\mathbf{a}_x + 3\mathbf{a}_y - 4\mathbf{a}_z}{\sqrt{26}}\right)\ \text{C/m}^2$$

or, more conveniently, $D = 91.8$ pC/m^2.

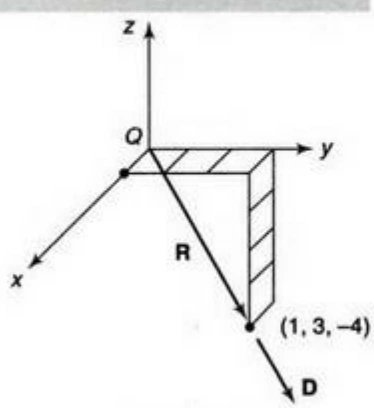


Fig. 3.12

Problem-4

Given that $\mathbf{D} = 10x\mathbf{a}_x$ (C/m^2), determine the flux crossing a 1-m^2 area that is normal to the x axis at $x = 3$ m.

Since **D** is constant over the area and perpendicular to it,

$$\Psi = DA = (30 \text{ C/m}^2)(1 \text{ m}^2) = 30 \text{ C}$$

Problem-5

Given the vector field $\mathbf{A} = 5x^2\left(\sin\frac{\pi x}{2}\right)\mathbf{a}_x$, find div **A** at $x = 1$.

$$\text{div } \mathbf{A} = \frac{\partial}{\partial x}\left(5x^2 \sin\frac{\pi x}{2}\right) = 5x^2\left(\cos\frac{\pi x}{2}\right)\frac{\pi}{2} + 10x\sin\frac{\pi x}{2} = \frac{5}{2}\pi x^2\cos\frac{\pi x}{2} + 10x\sin\frac{\pi x}{2}$$

and $\text{div } \mathbf{A}\big|_{x=1} = 10.$

Problem-6

Given that $\mathbf{D} = (10r^3/4)\mathbf{a}_r$ (C/m^2) in the region $0 < r \le 3$ m in cylindrical coordinates and $\mathbf{D} = (810/4r)\mathbf{a}_r$ (C/m^2) elsewhere, find the charge density.

For $0 < r \le 3$ m,

$$\rho = \frac{1}{r}\frac{\partial}{\partial r}\left(\frac{10r^4}{4}\right) = 10r^2 \text{ (C/m}^3)$$

and for $r > 3$ m,

$$\rho = \frac{1}{r}\frac{\partial}{\partial r}(810/4) = 0$$

Problem-8

Ex. *A charge $Q_1 = -20\mu C$ is located at P (− 6, 4, 6) and a charge $Q_2 = 50\mu C$ is located at R (5, 8, − 2) in a free space. Find the force exerted on Q_2 by Q_1 in vector form. The distances given are in metres.*

Sol. : From the co-ordinates of P and R , the respective position vectors are –

$$\bar{P} = -6\bar{a}_x + 4\bar{a}_y + 6\bar{a}_z$$

and

$$\bar{R} = 5\bar{a}_x + 8\bar{a}_y - 2\bar{a}_z$$

The force on Q_2 is given by,

$$\bar{F}_2 = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R_{12}^2}\bar{a}_{12}$$

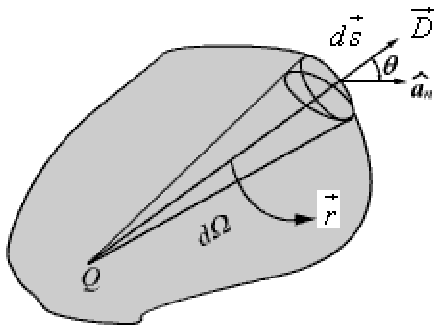
$$\bar{R}_{12} = \bar{R}_{PR} = \bar{R} - \bar{P} = [5-(-6)]\,\bar{a}_x + (8-4)\,\bar{a}_y + [-2-(6)\bar{a}_z]$$

$$= 11\bar{a}_x + 4\bar{a}_y - 8\bar{a}_z$$

$$\therefore \quad |R_{12}| = \sqrt{(11)^2 + (4)^2 + (-8)^2} = 14.1774$$

Gauss's Law:

Gauss's law is one of the fundamental laws of electromagnetism and it states that the total electric flux through a closed surface is equal to the total charge enclosed by the surface.



Let us consider a point charge *Q* located in an isotropic homogeneous medium of dielectric constant. The flux density at a distance *r* on a surface enclosing the charge is given by

$$\vec{D} = \varepsilon\vec{E} = \frac{Q}{4\pi r^2}\hat{a}_r$$

If we consider an elementary area ***ds***, the amount of flux passing through the elementary area is given by

$$d\psi = \vec{D}.ds = \frac{Q}{4\pi r^2} ds\cos\theta$$

But $\frac{ds\cos\theta}{r^2} = d\Omega$, is the elementary solid angle subtended by the area $d\vec{s}$ at the location of *Q*.

Therefore, we can write $d\psi = \frac{Q}{4\pi} d\Omega$

For a closed surface enclosing the charge, we can write $\psi = \oint_S d\psi = \frac{Q}{4\pi}\oint_S d\Omega = Q$

which can seen to be same as what we have stated in the definition of Gauss's Law.

Hence we have,

$$Q_{enc} = \oint_S D \cdot ds = \int_V \rho_V \, dv$$

Applying Divergence theorem we have,

$$\oint_S D \cdot ds = \int_V \nabla \cdot D \, dv$$

Comparing the above two equations, we have

$$\int_V \nabla \cdot D \, dv = \int_V \rho_V \, dv$$

This equation is called the 1st Maxwell's equation of electrostatics.

Application of Gauss's Law:

Gauss's law is particularly useful in computing $\vec{E}$ or $\vec{D}$ where the charge distribution has some symmetry. We shall illustrate the application of Gauss's Law with some examples.

1. $\vec{E}$ **due to an infinite line charge**

As the first example of illustration of use of Gauss's law, let consider the problem of determination of the electric field produced by an infinite line charge of density $_L$C/m. Let us consider a line charge positioned along the *z*-axis as shown in Fig. 4(a) (next slide). Since the line charge is assumed to be infinitely long, the electric field will be of the form as shown in Fig. 4(b) (next slide).

If we consider a close cylindrical surface as shown in Fig. 2.4(a), using Gauss's theorm we can write,

$$\rho_L l = Q = \oint_S \varepsilon_0 \vec{E}.\vec{ds} = \int_{S_1} \varepsilon_0 \vec{E}.\vec{ds} + \int_{S_2} \varepsilon_0 \vec{E}.\vec{ds} + \int_{S_3} \varepsilon_0 \vec{E}.\vec{ds}$$

Considering the fact that the unit normal vector to areas S_1 and S_3 are perpendicular to the electric field, the surface integrals for the top and bottom surfaces evaluates to zero. Hence we can write, $\rho_L l = \varepsilon_0 E.2\pi\rho l$

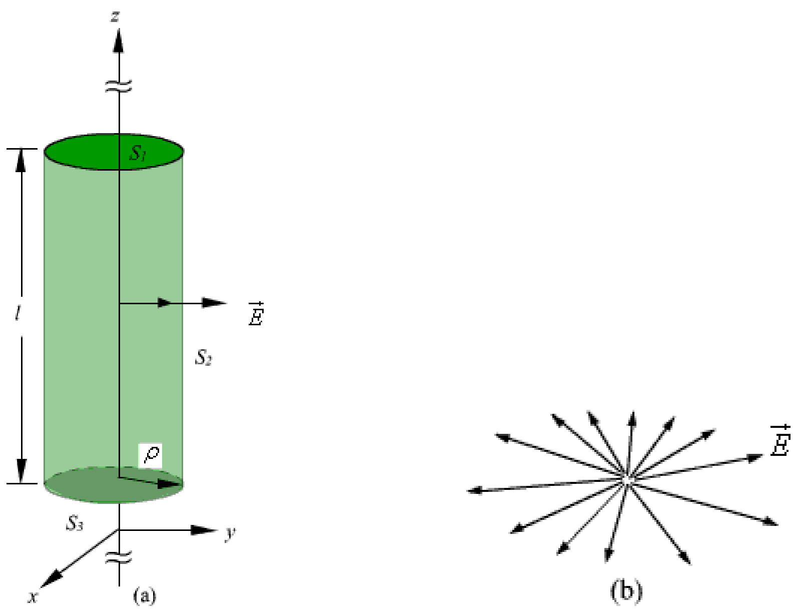


Fig 4: Infinite Line Charge

$$\vec{E} = \frac{\rho_L}{2\pi\varepsilon_0\rho}\hat{a}_\rho$$

2. Infinite Sheet of Charge

As a second example of application of Gauss's theorem, we consider an infinite charged sheet covering the *x-z* plane as shown in figure 5. Assuming a surface charge density of ρ_s for the infinite surface charge, if we consider a cylindrical volume having sides Δs placed symmetrically as shown in figure 5, we can write:

$$\oint_s \vec{D}\cdot d\vec{s} = 2D\Delta s = \rho_s \Delta s$$

$$\therefore \qquad \vec{E} = \frac{\rho_s}{2\varepsilon_0}\hat{a}_y$$

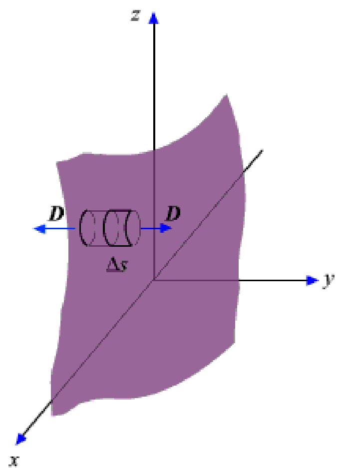


Fig 5: Infinite Sheet of Charge

It may be noted that the electric field strength is independent of distance. This is true for the infinite plane of charge; electric lines of force on either side of the charge will be perpendicular to the sheet and extend to infinity as parallel lines. As number of lines of force per unit area gives the strength of the field, the field becomes independent of distance. For a finite charge sheet, the field will be a function of distance.

3. Uniformly Charged Sphere

Let us consider a sphere of radius r0 having a uniform volume charge density of rv C/m3. To determine $\vec{D}$ everywhere, inside and outside the sphere, we construct Gaussian surfaces of radius r < r0 and r > r0 as shown in Fig. 6 (a) and Fig. 6(b).

For the region $r \leq r_0$; the total enclosed charge will be

$$Q_{en} = \rho_v \frac{4}{3}\pi r^3$$

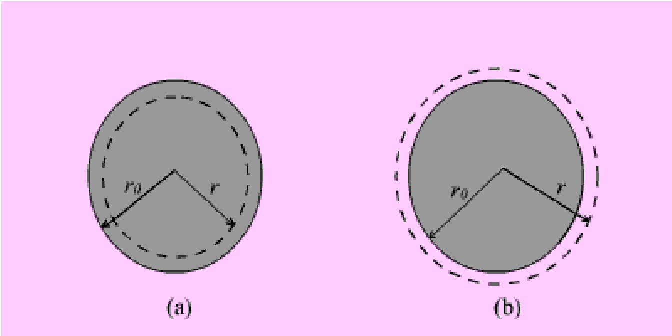


Fig 6: Uniformly Charged Sphere

By applying Gauss's theorem,

$$\oint_s \vec{D}\cdot d\vec{s} = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} D_r r^2 \sin\theta d\theta d\phi = 4\pi r^2 D_r = Q_{en}$$

Therefore

$$\vec{D} = \frac{r}{3}\rho_v \hat{a}_r \qquad 0 \le r \le r_0$$

For the region $r \ge r_0$; the total enclosed charge will be

$$Q_{en} = \rho_v \frac{4}{3}\pi r_0^3$$

By applying Gauss's theorem,

$$\vec{D} = \frac{r_0^3}{3r^2}\rho_v \hat{a}_r \qquad r \ge r_0$$

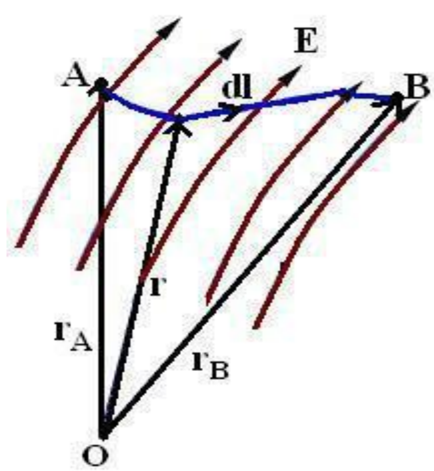
Electric Potential / Electrostatic Potential (V):

If a charge is placed in the vicinity of another charge (or in the field of another charge), it experiences a force. If a field being acted on by a force is moved from one point to another, then work is either said to be done on the system or by the system.

Say a point charge Q is moved from point A to point B in an electric field E, then the work done in moving the point charge is given as:

WA→B = - ∫AB (F . dl) = - Q ∫AB(E . dl)

where the – ve sign indicates that the work is done on the system by an external agent.



The work done per unit charge in moving a test charge from point A to point B is the electrostatic potential difference between the two points(VAB).

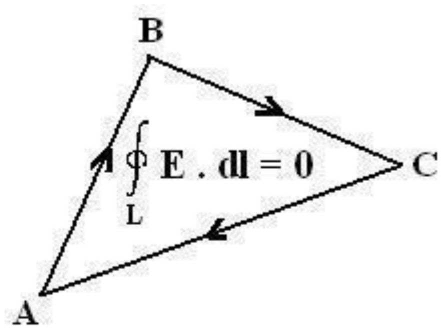
VAB = WA→B / Q

- ∫AB(E . dl)

- ∫InitialFinal (E . dl)

If the potential difference is positive, there is a gain in potential energy in the movement, external agent performs the work against the field. If the sign of the potential difference is negative, work is done by the field.

The electrostatic field is conservative i.e. the value of the line integral depends only on end points and is independent of the path taken.



- Since the electrostatic field is conservative, the electric potential can also be written as:

$$V_{AB} = -\int_A^B \overline{E} . \overline{dl}$$

$$V_{AB} = -\int_A^{p_0} \overline{E} . \overline{dl} - \int_{p_0}^B \overline{E} . \overline{dl}$$
$$V_{AB} = -\int_{p_0}^B \overline{E} . \overline{dl} + \int_{p_0}^A \overline{E} . \overline{dl}$$

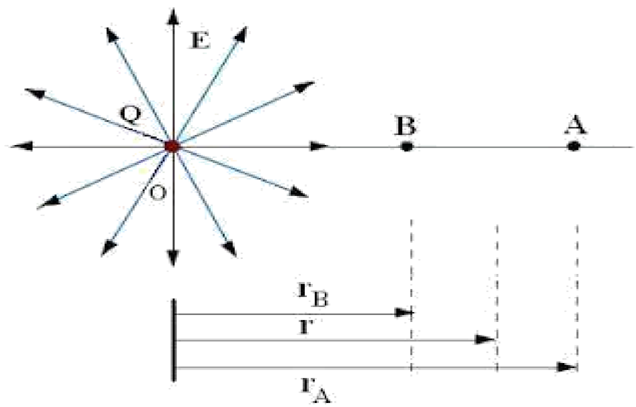
$$V_{AB} = V_B - V_A$$

Thus, the potential difference between two points in an electrostatic field is a scalar field that is defined at every point in space and is independent of the path taken.

- The work done in moving a point charge from point A to point B can be written as:

WA→B = - Q [V_B – V_A] = $-Q\int_A^B \bar{E}\,.\,\bar{dl}$

- Consider a point charge Q at origin O.



Now if a unit test charge is moved from point A to Point B, then the potential difference between them is given as:

$$V_{AB} = -\int_A^B \mathbf{E}\,.\,\mathbf{dl} = -\int_{r_A}^{r_B} \mathbf{E}\,.\,\mathbf{dl} = -\int_{r_A}^{r_B} \frac{Q}{4\pi\varepsilon r^2}\,\mathbf{a}_r \cdot dr\,\mathbf{a}_r$$

$$= \frac{Q}{4\pi\varepsilon}\left(\frac{1}{r_B} - \frac{1}{r_A}\right) = V_B - V_A$$

- Electrostatic potential or Scalar Electric potential (V) at any point P is given by:

$$V = -\int_{P_0}^{P} \bar{E}\,.\,\bar{dl}$$

The reference point Po is where the potential is zero (analogues to ground in a circuit).
The reference is often taken to be at infinity so that the potential of a point in space is defined as

$$V = -\int_{\infty}^{P} \bar{E}\,.\,\bar{dl}$$

Basically, potential is considered to be zero at infinity. Thus potential at any point (rB = r) due to a point charge Q can be written as the amount of work done in bringing a unit positive charge from infinity to that point (i.e. rA → ∞)

Electric potential (V) at point r due to a point charge Q located at a point with position vector r1 is given as:

$$V = \frac{Q}{4\pi\varepsilon|r - r_1|}$$

Similarly for N point charges Q1, Q2 ….Qn located at points with position vectors r1, r2, r3…..rn, theelectric potential (V) at point r is given as:

$$V = \frac{1}{4\pi\varepsilon}\sum_{k=1}^{N}\frac{Q_k}{|r - r_k|}$$

The charge element dQ and the total charge due to different charge distribution is given as:

dQ = ρldl → Q = ∫L (ρldl) → (Line Charge)

$$V = \frac{Q}{4\pi\varepsilon r}$$

dQ = ρsds → Q = ∫S (ρsds) → (Surface Charge)

dQ = ρvdv → Q = ∫V (ρvdv) → (Volume Charge)

$$V = \int_L \frac{\rho_L \, dl}{4\pi\varepsilon|r - r_1|} \quad \textbf{(Line Charge)}$$

$$V = \int_S \frac{\rho_S \, ds}{4\pi\varepsilon|r - r_1|} \quad \textbf{(Surface Charge)}$$

$$V = \int_V \frac{\rho_V \, dv}{4\pi\varepsilon|r - r_1|} \quad \textbf{(Volume Charge)}$$

Second Maxwell's Equation of Electrostatics:

The work done per unit charge in moving a test charge from point A to point B is the electrostatic potential difference between the two points(V_{AB}).

$V_{AB} = V_B – V_A$

Similarly,

$V_{BA} = V_A – V_B$

Hence it‘s clear that potential difference is independent of the path taken. Therefore

$V_{AB} = - V_{BA}$

$V_{AB} + V_{BA} = 0$

$\int_{AB} (E \cdot dl) + [- \int_{BA} (E \cdot dl)] = 0$

$$\oint_L \mathbf{E} \cdot \mathbf{dl} = 0$$

The above equation is called the second Maxwell‘s Equation of Electrostatics in integral form.. The above equation shows that the line integral of Electric field intensity (E) along a closed path is equal to zero.

In simple words—No work is done in moving a charge along a closed path in an electrostatic field.

Applying Stokes‘Theorem to the above Equation, we have:

$$\oint_L \mathbf{E} \cdot \mathbf{dl} = \int_S (\nabla \times \mathbf{E}) \cdot \mathbf{ds} = 0$$

$$\text{--->} \nabla \times \mathbf{E} = 0$$

If the Curl of any vector field is equal to zero, then such a vector field is called an Irrotational or Conservative Field. Hence an electrostatic field is also called a conservative field.

The above equation is called the second Maxwell ‘s Equation of Electrostatics in differential form.

Relationship Between Electric Field Intensity (E) and Electric Potential (V):

Since Electric potential is a scalar quantity, hence dV (as a function of x, y and z variables) can be written as:

$$dV = \frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy + \frac{\partial V}{\partial z}dz$$

$$\left(\frac{\partial V}{\partial x}a_x + \frac{\partial V}{\partial y}a_y + \frac{\partial V}{\partial z}a_z\right)\cdot\left(dx\,a_x + dy\,a_y + dz\,a_z\right) = -E\cdot dl$$

$$\nabla V \cdot dl = -E\cdot dl \quad \text{--->} \quad \boxed{E = -\nabla V}$$

Hence the Electric field intensity (E) is the negative gradient of Electric potential (V).
The negative sign shows that E is directed from higher to lower values of V i.e. E is opposite to the direction in which V increases.

Properties of Materials and Steady Electric Current:

Electric field can not only exist in free space and vacuum but also in any material medium. When an electric field is applied to the material, the material will modify the electric field either by strengthening it or weakening it, depending on what kind of material it is.

Materials are classified into 3 groups based on conductivity / electrical property:

- Conductors (Metals like Copper, Aluminum, etc.) have high conductivity ($\sigma >> 1$).
- Insulators / Dielectric (Vacuum, Glass, Rubber, etc.) have low conductivity ($\sigma << 1$).
- Semiconductors (Silicon, Germanium, etc.) have intermediate conductivity.

Conductivity (σ) is a measure of the ability of the material to conduct electricity. It is the reciprocal of resistivity (ρ). Units of conductivity are Siemens/meter and mho.

The basic difference between a conductor and an insulator lies in the amount of free electrons available for conduction of current. Conductors have a large amount of free electrons whereas insulators have only a few number of electrons for conduction of current. Most of the conductors obey ohm‘s law. Such conductors are also called ohmic conductors.

Due to the movement of free charges, several types of electric current can be caused. The different types of electric current are:

- Conduction Current.
- Convection Current.
- Displacement Current.

Electric current:

Electric current (I) defines the rate at which the net charge passes through a wire of cross-sectional surface area S.

Mathematically,

If a net charge ΔQ moves across surface S in some small amount of time Δt, electric current(I) is defined as:

$$I = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}$$

How fast or how speed the charges will move depends on the nature of the material medium.

Current density:

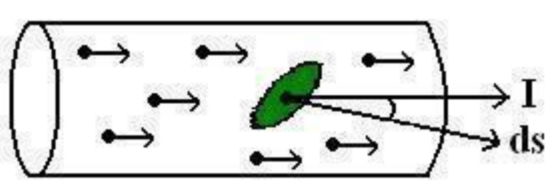
Current density (J) is defined as current ΔI flowing through surface ΔS.

Imagine surface area ΔS inside a conductor at right angles to the flow of current. As the area approaches zero, the current density at a point is defined as:

$$J = \lim_{\Delta s \to 0} \frac{\Delta I}{\Delta S}$$

The above equation is applicable only when current density (J) is normal to the surface.

In case if current density(J) is not perpendicular to the surface, consider a small area ds of the conductor at an angle θ to the flow of current as shown:



In this case current flowing through the area is given as:

$dI = J\, dS \cos\theta = J\, .\, dS$ and $I = \int_S \bar{J} . \overline{ds}$

Where angle θ is the angle between the normal to the area and direction of the current.
From the above equation it's clear that electric current is a scalar quantity.

CONTINUITY EQUATION:

The continuity equation is derived from two of Maxwell's equations. It states that the divergence of the current density is equal to the negative rate of change of the charge density,

$$\nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t}.$$

Derivation

One of Maxwell's equations, Ampère's law, states that

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}.$$

Taking the divergence of both sides results in

$$\nabla \cdot \nabla \times \mathbf{H} = \nabla \cdot \mathbf{J} + \frac{\partial \nabla \cdot \mathbf{D}}{\partial t},$$

but the divergence of a curl is zero, so that

$$\nabla \cdot \mathbf{J} + \frac{\partial \nabla \cdot \mathbf{D}}{\partial t} = 0. \qquad (1)$$

Another one of Maxwell's equations, Gauss's law, states that

$$\nabla \cdot \mathbf{D} = \rho.$$

Substitute this into equation (1) to obtain

$$\nabla \cdot \mathbf{J} + \frac{\partial \rho}{\partial t} = 0,$$

which is the continuity equation.

Relaxation Time

Relaxation time can be defined as the time taken by electron to attain an average velocity which is 1/e times its value.

The different physics interfaces involving only the scalar electric potential can be interpreted in terms of the charge relaxation process. The fundamental equations involved are *Ohm's law* for the conduction current density

$$\mathbf{J}_c = \sigma \mathbf{E}$$

the *equation of continuity*

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{J}_c = 0$$

and *Gauss' law*

$$\nabla \cdot (\varepsilon \mathbf{E}) = \rho$$

By combining these, one can deduce the following differential equation for the space charge density in a homogeneous medium

$$\frac{\partial \rho}{\partial t} + \frac{\sigma}{\varepsilon}\rho = 0$$

This equation has the solution

$$\rho(t) = \rho_0 e^{-t/\tau}$$

were

$$\tau = \frac{\varepsilon}{\sigma}$$

is called the charge relaxation time.

LAPLACE'S AND POISSON'S EQUATIONS:

A useful approach to the calculation of electric potentials is to relate that potential to the charge density which gives rise to it. The electric field is related to the charge density by the divergence relationship

$$\nabla \cdot E = \frac{\rho}{\varepsilon_0}$$

E = electric field

ρ = charge density

ε_0 = permittivity

and the electric field is related to the electric potential by a gradient relationship

$$E = -\nabla V$$

Therefore the potential is related to the charge density by Poisson's equation

$$\nabla \cdot \nabla V = \nabla^2 V = \frac{-\rho}{\varepsilon_0}$$

In a charge-free region of space, this becomes LaPlace's equation

$$\nabla^2 V = 0$$

This mathematical operation, the divergence of the gradient of a function, is called the Laplacian. Expressing the Laplacian in different coordinate systems to take advantage of the symmetry of a charge distribution helps in the solution for the electric potential V. For example, if the charge distribution has spherical symmetry, you use the Laplacian in spherical polar coordinates.

Since the potential is a scalar function, this approach has advantages over trying to calculate the electric field directly. Once the potential has been calculated, the electric field can be computed by taking the gradient of the potential.

Solved problems:

Problem1:

Three point charges, $Q_1 = 30$ nC, $Q_2 = 150$ nC, and $Q_3 = -70$ nC, are enclosed by surface S. What net flux crosses S?

Since electric flux was defined as originating on positive charge and terminating on negative charge, part of the flux from the positive charges terminates on the negative charge.

$$\Psi_{net} = Q_{net} = 30 + 150 - 70 = 110 \text{ nC}$$

Problem-2

What electric field intensity and current density correspond to a drift velocity of 6.0×10^{-4} m/s in a silver conductor?

For silver $\sigma = 61.7$ MS/m and $\mu = 5.6 \times 10^{-3}$ m²/V · s.

$$E = \frac{U}{\mu} = \frac{6.0 \times 10^{-4}}{5.6 \times 10^{-3}} = 1.07 \times 10^{-1} \text{ V/m}$$

$$J = \sigma E = 6.61 \times 10^{6} \text{ A/m}^2$$

Problem-3

Find the current in the circular wire shown in Fig. 6.6 if the current density is $\mathbf{J} = 15(1 - e^{-1000r})\mathbf{a}_z$ (A/m²). The radius of the wire is 2 mm.

A cross section of the wire is chosen for S. Then

$$dI = \mathbf{J} \cdot d\mathbf{S} = 15(1 - e^{-1000r})\mathbf{a}_z \cdot r\,dr\,d\phi\,\mathbf{a}_z$$

and

$$I = \int_0^{2\pi}\int_0^{0.002} 15(1 - e^{-1000r})r\,dr\,d\phi = 1.33 \times 10^{-4} \text{ A} = 0.133 \text{ mA}$$

Any surface S which has a perimeter that meets the outer surface of the conductor all the way around will have the same total current, $I = 0.133$ mA, crossing it.

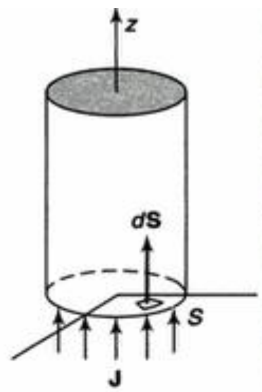


Fig. 6.6

Problem-4

Determine the relaxation time for silver, given that $\sigma = 6.17 \times 10^{7}$ S/m. If charge of density ρ_0 is placed within a silver block, find ρ after one, and also after five, time constants.

Since $\varepsilon \approx \varepsilon_0$,

$$\tau = \frac{\varepsilon}{\sigma} = \frac{10^{-9}/36\pi}{6.17 \times 10^{7}} = 1.43 \times 10^{-19} \text{ s}$$

Therefore

at $t = \tau$: $\rho = \rho_0 e^{-1} = 0.368\rho_0$

at $t = 5\tau$: $\rho = \rho_0 e^{-5} = 6.74 \times 10^{-3}\rho_0$