UNIT – IV
Transmission Lines - I:

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Introduction:

A transmission line is used for the transmission of electrical power from generating substation to the various distribution units. It transmits the wave of voltage and current from one end to another. The transmission line is made up of a conductor having a uniform cross-section along the line. Air act as an insulating or dielectric medium between the conductors.

Fig. Transmission Lines

Types of Transmission Lines

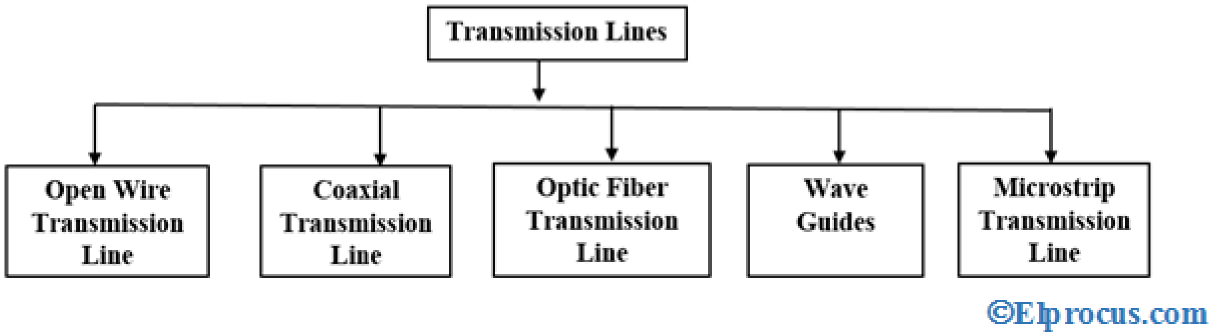
The different types of transmission lines include the following.

Open Wire Transmission Line

It consists pair of parallel conducting wires separated by a uniform distance. The two-wire transmission lines are very simple, low cost and easy to maintain over short distances and these lines are used up to 100 MHz Another name of an open-wire transmission line is a parallel wire transmission line.

Coaxial Transmission Line

The two conductors placed coaxially and filled with dielectric materials such as air, gas or solid. The frequency increases when losses in the dielectric increases, the dielectric is polyethylene. The coaxial cables are used up to 1 GHz. It is a type of wire which carries high-frequency signals with low losses and these cables are used in CCTV systems, digital audios, in computer network connections, in internet connections, in television cables, etc.



Optic Fiber Transmission Line

The first optical fiber invented by Narender Singh in 1952. It is made-up of silicon oxide or silica, which is used to send signals over a long distance with little loss in signal and at the speed of light. The optic fiber cables used as light guides, imaging tools, lasers for surgeries, used for data transmission and also used in a wide variety of industries and applications.

Microstrip Transmission Lines

The microstrip transmission line is a Transverse Electromagnetic (TEM) transmission line invented by Robert Barrett in 1950.

Wave Guides

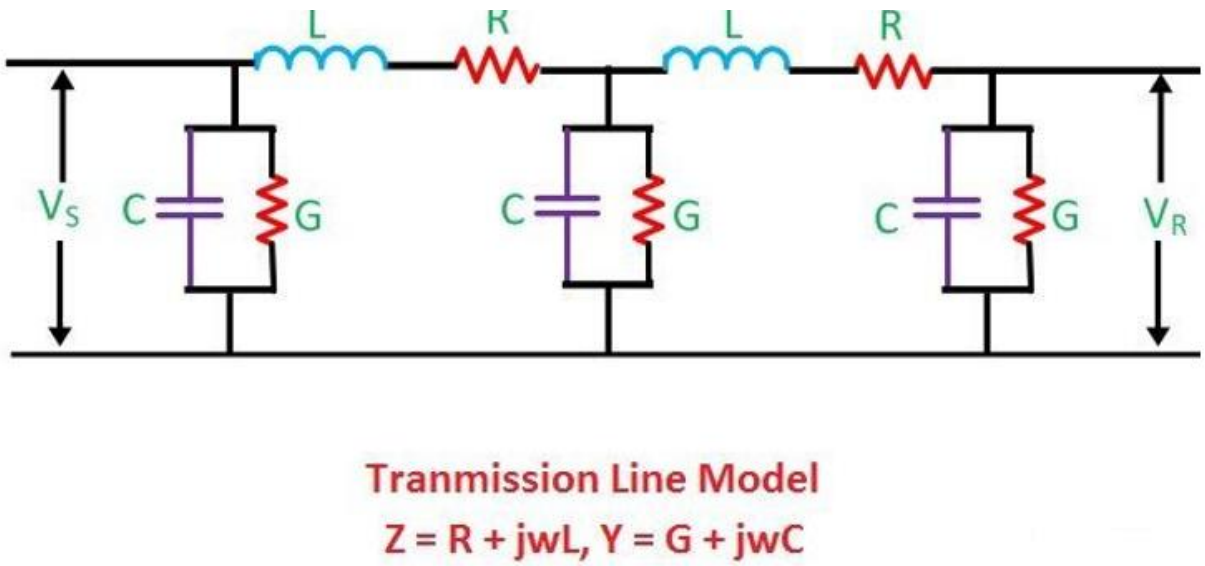
Waveguides are used to transmit electromagnetic energy from one place to another place and they are usually operating in dominant mode. The various passive components such as filter, coupler, divider, horn, antennas, tee junction, etc. Waveguides are used in scientific instruments to measure optical, acoustic ad elastic properties of materials and objects. There are two types of waveguides are Metal waveguides and dielectric waveguides. The waveguides are used in optical fiber communication, microwave ovens, space crafts, etc.
Applications

The applications of transmission line are

- Power transmission line
- Telephone lines
- Printed circuit board
- Cables
- Connectors (PCI, USB)

Parameters of transmission line (Primary Constants):

The performance of transmission line depends on the parameters of the line. The transmission line has mainly four parameters, resistance, inductance, capacitance and shunt conductance. These parameters are uniformly distributed along the line. Hence, it is also called the distributed parameter of the transmission line.



The inductance and resistance form series impedance whereas the capacitance and conductance form the shunt admittance. Some critical parameters of transmission line are explained below in detail

Line inductance – The current flow in the transmission line induces the magnetic flux. When the current in the transmission line changes, the magnetic flux also varies due to which emf induces in the circuit. The magnitude of inducing emf depends on the rate of change of flux. Emf produces in the transmission line resist the flow of current in the conductor, and this parameter is known as the inductance of the line.

Line capacitance – In the transmission lines, air acts as a dielectric medium. This dielectric medium constitutes the capacitor between the conductors, which store the electrical energy, or increase the capacitance of the line. The capacitance of the conductor is defined as the present of charge per unit of potential difference.

Capacitance is negligible in short transmission lines whereas in long transmission; it is the most important parameter. It affects the efficiency, voltage regulation, power factor and stability of the system.

Shunt conductance – Air act as a dielectric medium between the conductors. When the alternating voltage applies in a conductor, some current flow in the dielectric medium because of dielectric imperfections. Such current is called leakage current. Leakage current depends on the atmospheric condition and pollution like moisture and surface deposits.

Shunt conductance is defined as the flow of leakage current between the conductors. It is distributed uniformly along the whole length of the line. The symbol Y represented it, and it is measured in Siemens.

Primary & Secondary Constants:

The primary line constants are the resistance, inductance, conductance, and capacitance per unit length of the transmission line.

However, the term "secondary line constants" is not commonly used. It is normally known as "quaternary parameters" or "quaternary constants" used in telecommunication line analysis. These parameters extend the analysis of transmission lines beyond the primary parameters by including additional effects, such as radiation and shunt capacitance. Quaternary parameters are also used to model the behavior of transmission lines at higher frequencies.

Propagation Constant Definition:

Electromagnetic waves propagate in a sinusoidal fashion. **The measure of the change in amplitude and phase per unit distance is called the propagation constant.** Denoted by the Greek letter γ. The terminologies like Transmission function, Transmission constant, Transmission parameter, Propagation coefficient, and Propagation parameter are synonymous with this quantity. Sometimes α and β are collectively referred to as Propagation or Transmission parameters.

The propagation constant can be mathematically expressed as:

$$\gamma = \alpha + j\beta$$

Where:

α (alpha) represents the attenuation constant, which measures the rate of amplitude decay of the signal as it travels through the medium. It is a real number and is usually measured in Nepers per unit length or decibels per unit length.

β (beta) represents the phase constant, which determines the phase shift experienced by the signal as it propagates through the medium. It is an imaginary number and is usually measured in radians per unit length.

The magnitude of the propagation constant (γ) gives the overall rate of signal decay, while the argument or phase angle of the propagation constant (arg(γ)) gives the phase shift experienced by the signal.

Propagation Constant of a Transmission Line:

The propagation constant for any conducting lines (like copper lines) can be calculated by relating the primary line parameters.

$$\gamma = \sqrt{ZY}$$

Where, $Z = R + i\omega L$ is the series impedance of line per unit length.

$Y = G + i\omega C$ is the shunt admittance of line per unit length.

Characteristic Impedance (Z_o)

As we already discussed that primary constants are very significant in transmission lines, they make characteristic impedance (Z_o) a very significant parameter as well, because characteristic impedance (Z_o) involves all four of the primary constants in its expression.

What is Characteristic Impedance?

Characteristic impedance can be defined as the ratio of amplitude of voltage to the amplitude of current of a unidirectional wave travelling from source to load along a uniform transmission line in the absence of reflections.

It may also be defined as a square root of the ratio of series impedance of a line to its shunt admittance.

$$Z_o = \sqrt{\frac{Z}{Y}}$$

Where,

Z = R + jwL (series impedance per unit length per phase)

Y = G + jwC (shunt admittance per unit length per phase)

R, L, G and C are the primary constants of a transmission line, and the above expression confirms that characteristics of a transmission line are described by primary line constants.

Transmission Line Equations

Let us take the equivalent circuit of the transmission line, for this we are going to take the simplest form of transmission line which is two wirelines. These two wirelines are made up of two conductors separated by a dielectric medium usually air medium, which is shown in the below figure

If we pass a current (I) through the conductor-1, will find that there is a magnetic field around the current-carrying wire of a conductor-1 and the magnetic field can be illustrated using series inductor due to the current flow in the conductor-1, there should be a voltage drop across the conductor-1, which can be illustrated by a series of resistance and inductor. The setup of the two-wireline conductor can be made to a capacitor. The capacitor in the figure will always be loosy to illustrate that we have added conductor G. The total setup i.e, series resistance an inductor, parallel capacitor, and conductor make up an equivalent circuit of a transmission line.

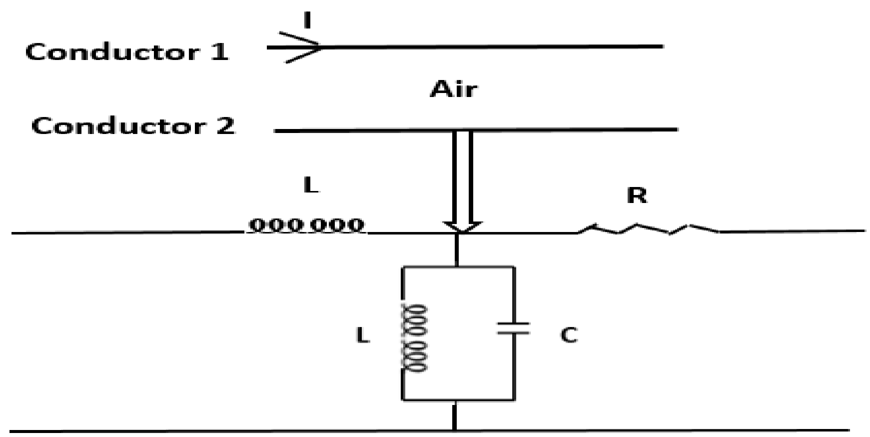


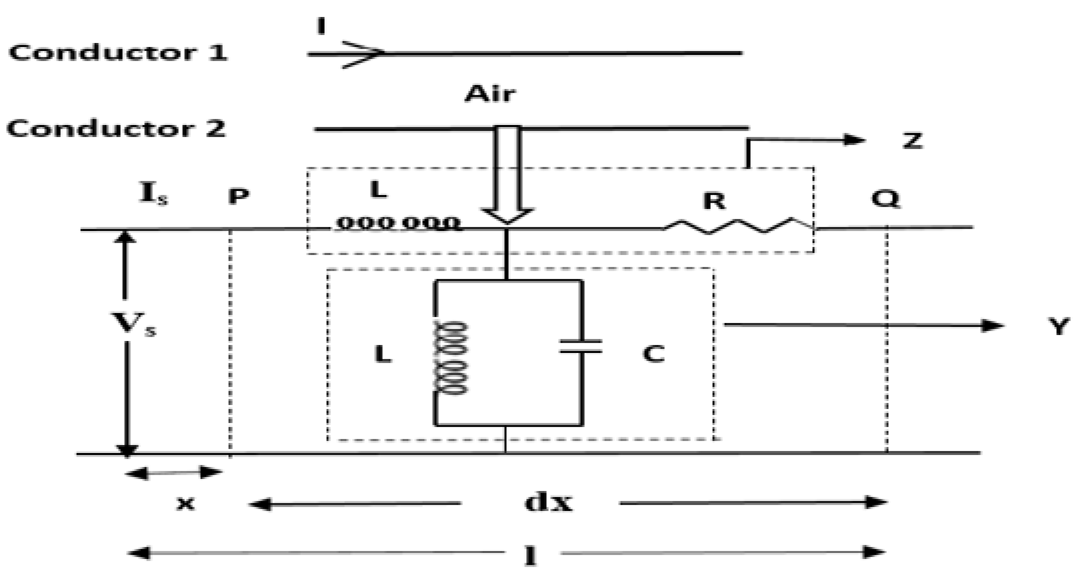
Fig.equivalent_circuit_of_a_transmission_line_1

The inductor and resistance put together in the above figure can be called as series impedance, which is expressed as

$$\mathbf{Z = R+j\omega L}$$

The parallel combination of capacitance and conductor n the above figure can be expressed as

$$\mathbf{Y = G+j\omega c}$$



Where l – length

I_s – Sending end current

V_s – Sending end voltage

dx – element length

x – a distance of dx from sending end

At a point, ‘p’ take current(I) and voltage(v) and at a point, ‘Q’ take I+dV and V+dV

The change in voltage for the length PQ is the

$$V-(V + dV) = (R + j\omega L)\, dx * I$$
$$V-V-dv = (R + j\omega L)\, dx * I$$
$$-dv/dx = (R + j\omega L) * I \;\ldots\ldots\ldots\ldots\ldots\ldots\; eq(1)$$
$$I-(I + dI) = (G + j\omega c)dx * V$$
$$I - I+dI = (G + j\omega c)dx * V$$
$$-dI/dx = (G + j\omega c) * V \;\ldots\;\ldots\ldots\ldots\ldots\ldots\; eq(2)$$

Differentiating eq(1) and (2) with respect to dx will get

$$-d^2v/dx^2 = (R + j\omega L) * dI/dx \;\ldots\ldots\ldots\ldots\ldots\ldots\; eq(3)$$
$$-d^2I/dx^2 = (G + j\omega c) * dV/dx \;\ldots\;\ldots\ldots\ldots\ldots\ldots\; eq(4)$$

Substituting eq(1) and (2) in eq(3) and (4) will get

$$-d^2v/dx^2 = (R + j\omega L)\,(G + j\omega c)\, V \;\ldots\ldots\ldots\ldots\ldots\ldots\; eq(5)$$
$$-d^2I/dx^2 = (G + j\omega c)\,(R + j\omega L)\, I \;\ldots\;\ldots\ldots\ldots\ldots\ldots\; eq(6)$$
$$\text{Let } P^2 = (R + j\omega L)\,(G + j\omega c) \;\ldots\;\ldots\ldots\ldots\ldots\ldots\; eq(7)$$

Where P – propagation constant

Substitute d/dx = P in eq(6) and (7)

$$-d^2v/dx^2 = P^2V \;\ldots\ldots\ldots\ldots\ldots\ldots\; eq(8)$$
$$-d^2I/dx^2 = P^2I \;\ldots\;\ldots\ldots\ldots\ldots\ldots\; eq(9)$$

General solution is

$$V = Ae^{px} + Be^{-px} \;\ldots\;\ldots\ldots\ldots\ldots\ldots\; eq(10)$$
$$I = Ce^{px} + De^{-px} \;\ldots\;\ldots\ldots\ldots\ldots\ldots\; eq(11)$$

Where A, B C and D are constants

Differentiating eq(10) and (11) with respect to ‘x’ will get

$$-dv/dx = P\,(Aepx - Be\text{-}px\,) \;\ldots\ldots\ldots\ldots\ldots\ldots\; eq(12)$$
$$-dI/dx = P\,(Cepx - De\text{-}px) \;\ldots\;\ldots\ldots\ldots\ldots\ldots\; eq(13)$$

Substitute eq(1) and (2) in eq(12) and (13) will get

$$-(R + j\omega L) * I = P\,(\,Ae^{px} + Be^{-px}\,) \;\ldots\ldots\ldots\ldots\ldots\ldots\; eq(14)$$
$$-(\,G + j\omega c) * V = P\,(Ce^{px} + De^{-px}\,) \;\ldots\ldots\ldots\ldots\ldots\ldots\; eq(15)$$

Substitute ‘p’ value in eq(14) and (15) will get

$$I = -p/\,R + j\omega L * (Ae^{px} + Be^{-px})$$
$$= \sqrt{}G + j\omega c\,/\,R + j\omega L * (Ae^{px} + Be^{-px}) \;\ldots\ldots\ldots\ldots\ldots\ldots\; eq(16)$$
$$V = -p/\,G + j\omega c * (Ce^{px} + De^{-px}\,)$$
$$= \sqrt{}R + j\omega L\,/\,G + j\omega c * (Ce^{px} + De^{-px}\,) \;\ldots\ldots\ldots\ldots\ldots\ldots\; eq(17)$$
$$\text{Let } Z_0 = \sqrt{}R + j\omega L\,/\,G + j\omega c$$

Where $Z_0$is the characteristic impedenc

Substitute boundary conditions x=0, $V=V_S$ and $I=I_S$ in eq(16) and (17) will get

$$\mathbf{I_S = A+B \;\ldots\ldots\ldots\ldots\ldots\ldots\; eq(18)}$$
$$\mathbf{V_S = C+D \;\ldots\ldots\ldots\ldots\ldots\ldots\; eq(19)}$$
$$\mathbf{I_S Z_0 = -A+B \;\ldots\ldots\ldots\ldots\ldots\ldots\; eq(20)}$$
$$\mathbf{V_S / Z_0 = -C+D \;\ldots\ldots\ldots\ldots\ldots\ldots\; eq(21)}$$

From (20) will get A and B values

$$\mathbf{A = V_S - I_S Z_0}$$
$$\mathbf{B = V_S + I_S Z_0}$$

From eq(21) will get C and D values

$$\mathbf{C = (I_S - V_S / Z_0) / 2}$$
$$\mathbf{D = (I_S + V_S / Z_0) / 2}$$

Substitute A, B, C and D values in eq(10) and (11)

$$\mathbf{V = (V_S - I_S Z_0)\, e^{px} + (V_S + I_S Z_0) e^{-px}}$$
$$\mathbf{= V_S (e^{px} + e-px/2) - I_S Z¬0(e^{px} - e^{-px}/2)}$$
$$\mathbf{= V_S \cosh x - I_S Z_0 \sinh x}$$

Similarly

$$\mathbf{I = (I_S - V_S Z_0)\, e^{px} + (V_S / Z_0 + I_S / 2) e^{-px}}$$
$$\mathbf{= I_S (e^{px} + e^{-px}/2) - V_S / Z_0 (e^{px} - e^{-px}/2)}$$
$$\mathbf{= I_S \cosh x - V_S / Z_0 \sinh x}$$

Thus $V = V_S \cosh x - I_S Z_0 \sinh x$

$I = I_S \cosh x - V_S / Z_0 \sinh x$

Equation of transmission line in terms of sending end parameters are derived

Phase and Group Velocities:

Phase velocity is the speed at which a point of constant phase moves through a medium. In simple terms, it's like tracing the path of a ruffling wave crest or trough marking a constant phase in the wave.

In physics, phase velocity can be calculated by using the simple formula:

$$v_p = \frac{\omega}{k}$$

Where:

- ω indicates phase velocity
- v_p is the angular frequency of the wave
- k is the wave number

It's worth mentioning that phase velocity depends on the medium the wave passes through. In some media, the phase velocity might change, leading to phenomena such as refraction.

Group Velocity

Group velocity is defined as the derivative of the wave's angular frequency with respect to its wave number. It can be mathematically expressed as:

$$v_g = \frac{d\omega}{dk}$$

Relation Between Group Velocity and Phase Velocity

The Group Velocity and Phase Velocity relation can be mathematically written as-

$$V_g = V_p + k\frac{dV_p}{dk}$$

Where,

- V_g is the group velocity.
- V_p is the phase velocity.
- k is the angular wavenumber.

The group velocity is directly proportional to phase velocity. This means-

- When group velocity increases, proportionately phase velocity will also increase.
- When phase velocity increases, proportionately group velocity will also increase.

For the amplitude of wave packet let-

- ω is the angular velocity given by $\omega=2\pi f$
- k is the angular wave number given by

$$k = \frac{2\pi}{\lambda}$$

- t is time
- x be the position
- V_p phase velocity
- V_g be the group velocity

The phase velocity of a wave is given by the following equation:

$v_p = \frac{\omega}{k}$

.....(eqn 1)

Rewriting the above equation, we get:

$\omega = kv_p$

.....(eqn 2)

Differentiating (eqn 2) w.r.t k we obtain,

$\frac{dw}{dk} = v_p + k\frac{dv_p}{dk}$

.....(eqn 3)

As

$v_g = \frac{dw}{dk}$

(eqn 3) reduces to:

$v_g = v_p + k\frac{dv_p}{dk}$

The above equation signifies the relationship between the phase velocity and the group velocity.

Infinite Line Concepts:

A finite line is a line having a finite length on the line. It is a line, which is terminated, in its characteristic impedance (ZR=Z0), so the input impedance of the finite line is equal to the characteristic impedance (Zs=Z0).

An infinite line is a line in which the length of the transmission line is infinite. A finite line, which is terminated in its characteristic impedance, is termed as infinite line. So for an infinite line, the input impedance is equivalent to the characteristic impedance.

Figure: infinite line

→ The ratio of the voltage applied to the current flowing will give the input impedance of an infinite line. This input impedance is known as **characteristic impedance** of the line and is denoted by 'Z_0'.

Therefore $Z_0 = \frac{V_{si}}{I_{si}}$

Where V_{si} is sending end voltage of an infinite line and I_{si} is sending end current of an infinite line.

Current at any point at a distance 'x' from the sending end is given bv

$I = ce^{Px} + de^{-Px}$ ⟶ 1

The values of 'c' and'd' now determined by considering an infinite line.

The values of 'c' and'd' now determined by considering an infinite line. At sending of an infinite line $x = 0$ and $I = I_{si}$ applying these conditions we get

$I_{si} = c + d$

However at the receiving end of the infinite line $x = \infty$ and $I = 0$.

Applying these conditions to same equation

$0 = c \times \infty + 0$

$c \times \infty = 0$

Thus either $c = 0$ or $\infty = 0$ but ∞ can not equal to zero.

Therefore $c = 0$

When $c = 0$, $I_{si} = d$

Putting these values in equation (1), we get

$I = I_{si}e^{-Px}$

This equation gives current at any point of an infinite line.

Similarly the voltage at any point of an infinite line can be given as

$V = V_{si}e^{-Px}$

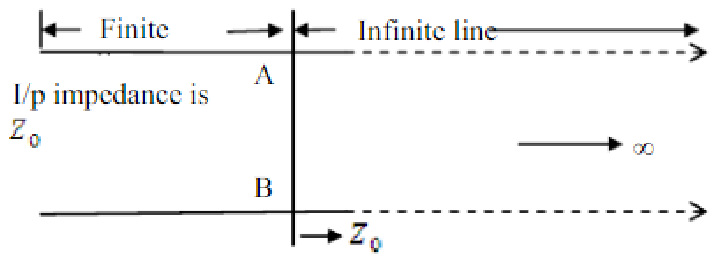
Infinite line is equivalent to a finite line terminated in its Z_0

→ If a finite length of line is joined with a similar kind of infinite line, their total input impedance is the same as that of infinite itself.

→ A finite line terminated by its Z_0, behaves as an infinite line.

→ Consider a line of length 'l' terminated in its characteristic impedance Z_0.

Let the voltage and current at the termination be V_R and I_R respectively.



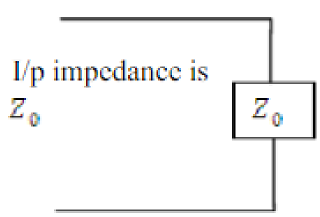


Fig: equivalent of an infinite line.

Therefore $\frac{V_R}{I_R} = Z_0$

We know that voltage and current equations at a point distance 'x' from the sending end in terms of sending end voltage and current is given by

$$v = V_s \cosh Px - I_s Z_0 \sinh Px$$

$$I = -\frac{1}{Z_0}[V_s \sinh Px - I_s Z_0 \cosh Px]$$

At $x = l, v = V_R$ and $I = I_R$ we get

$$V_R = V_s \cosh Pl - I_s Z_0 \sinh Pl \longrightarrow 1$$

$$I_R = -\frac{1}{Z_0}[V_s \sinh Pl - I_s Z_0 \cosh Pl] \longrightarrow 2$$

By dividing equation (1) and (2)

$$\frac{V_R}{I_R} = -\frac{(V_s \cosh Pl - I_s Z_0 \sinh Pl)}{\frac{1}{Z_0}[V_s \sinh Pl - I_s Z_0 \cosh Pl]}$$

Since $\frac{V_R}{I_R} = Z_0 =>$ $1 = \frac{(V_s \cosh Pl - I_s Z_0 \sinh Pl)}{(I_s Z_0 \cosh Pl - V_s \sinh Pl)}$

$$V_s \cosh Pl - I_s Z_0 \sinh Pl = I_s Z_0 \cosh Pl - V_s \sinh Pl$$

$$V_s(\cosh Pl + \sinh Pl) = I_s Z_0(\cosh Pl + \sinh Pl)$$

$$\frac{V_s}{I_s} = Z_0$$

But $\frac{V_s}{I_s}$ is the input impedance of the line. Therefore

$$Z_0 = Z_{in}$$

Therefore the input impedance of a finite line terminated in its characteristic impedance Z_0 is the characteristic impedance of line.

By definition the input impedance of an infinite line is the characteristic impedance of the line. Therefore a finite line terminated in its Z_0, is equivalent to an infinite line as both will have an input impedance Z_0.

Distortion - Condition for Distortion less Transmission and Minimum Attenuation:

It is desirable, however to know the condition on the line parameters that allows propagation without distortion. The line having parameters satisfy this condition is termed as a distortion less line.

The condition for a distortion less line was first investigated by Oliver Heaviside. Distortion less condition can help in designing new lines or modifying old ones to minimize distortion.

A line, which has neither frequency distortion nor phase distortion is called a distortion less line.

Condition for a distortion less line

The condition for a distortion less line is RC=LG. Also,

a) The attenuation constant _ should be made independent of frequency. α = RG
b) The phase constant _ should be made dependent of frequency. β = ω LC
c) The velocity of propagation is independent of frequency.
V=1 / LC

For the telephone cable to be distortion less line, the inductance value should be increased by placing lumped inductors along the line.

For a perfect line, the resistance and the leakage conductance value were neglected. The conditions for a perfect line are R=G=0. Smooth line is one in which the load is terminated by its characteristic impedance and no reflections occur in such a line. It is also called as flat line.

The distortion Less Iine

If a line is to have neither frequency nor delay distortion, then attenuation constant and velocity of propagation cannot be function of frequency.

Then the phase constant be a direct function of frequency

$$\beta = \sqrt{\frac{\omega^2 LC - RG + \sqrt{(RG - \omega^2 LC) + \omega^2 (LG + CR)}}{2}}$$

The above equation shows that if the the term under the second radical be reduced to equal

$(RG + \omega^2 LC)^2$

Then the required condition for ß is obtained. Expanding the term under the internal radical and forcing the equality gives

$R^2G^2 - 2\omega^2 LCRG + \omega^4 L^2C^2 + \omega^2 L^2G^2 + 2\omega^2 LCRG + \omega^2 CR^2 = (RG + \omega^2 LC)^2$

This reduces to

$2\omega^2 LCRG + \omega^2 L^2G^2 + \omega^2 CR^2 = 0$

$(LG - CR)^2 = 0$

Therefore, the condition that will make phase constant a direct form is

$LG = CR$

A hypothetical line might be built to fulfill this condition. The line would then have a value of ß obtained by use of the above equation.

Already we know that the formula for the phase constant

$$\beta = \omega LC$$

Then the velocity of propagation will be $v = 1/ LC$

This is the same for the all frequencies, thus eliminating the delay distortion.

May be made independent of frequency if the term under the internal radical is forced to reduce to $(RG + \omega LC)^2$

Analysis shows that the condition for the distortion less line $LG = CR$, will produce the desired result, so that it is possible to make attenuation constant and velocity independent of frequency simultaneously. Applying the condition LG= RC to the expression for the attenuation gives $\alpha = RG$

This is the independent of frequency, thus eliminating frequency distortion on a line. To achieve

$LG = CR$

Require a very large value of L, since G is small. If G is intentionally increased, attenuation are increased, resulting in poor line efficiency.

To reduce R raises the size and cost of the conductors above economic limits, so that the hypothetical results cannot be achieved.

Propagation constant is as the natural logarithm of the ratio of the sending end current or voltage to the receiving end current or voltage of the line. It gives the manner in the wave is propagated along a line and specifies the variation of voltage and current in the line as a function of distance. Propagation constant is a complex quantity and is expressed as $\gamma = \alpha + j\,\beta$.

The real part is called the attenuation constant, whereas the imaginary part of propagation constant is called the phase constant.