UNIT-V
Transmission Lines - II:

Contents:

- SC and OC Lines
- Input Impedance Relations
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Input Impedance Relations

- The input impedance of a transmission line is the impedance seen by any signal entering it. It is caused by the physical dimensions of the transmission line and its downstream circuit elements.
- If a transmission line is ideal, there is no attenuation to the signal amplitudes and the propagation constant turns out to be purely imaginary.
- When the transmission line length is infinite, the input impedance is equal to the characteristic impedance.

Calculating the Input Impedance

Consider a lossless, high-frequency transmission line where the voltage and currents are given by equations 1 and 2, with the input impedance, characteristic impedance, and load impedance as Zin, Z0, and ZL, respectively.

$$V(z) = V^{+}\left(e^{-j\beta z} + \Gamma e^{j\beta z}\right) \quad (1)$$

$$I(z) = \frac{V^{+}}{Z_0}\left(e^{-j\beta z} - \Gamma e^{j\beta z}\right) \quad (2)$$

Γ - Reflection co-effcient

As the transmission line is ideal, there is no attenuation to the signal amplitudes and the propagation constant turns out to be purely imaginary. Let’s define the output terminals with axis point z=0 and input terminals z=-L. Our objective is to find the impedance of the circuit when looking from Z=-L:

$$Z_{in}(z) = \frac{V(z)}{I(z)} = \frac{V^{+}\left(e^{-j\beta z} + \Gamma e^{j\beta z}\right)}{V^{+}\left(e^{-j\beta z} - \Gamma e^{j\beta z}\right)} Z_0 \quad (3)$$

$$Z_{in}(z) = \left[\frac{e^{-j\beta z} + \Gamma e^{j\beta z}}{e^{-j\beta z} - \Gamma e^{j\beta z}}\right] Z_0 \quad (4)$$

The input impedance is the ratio of input voltage to the input current and is given by equation 3. By substituting equation 5 into equation 4, we can obtain the input impedance, as given in equation 6:

$$\Gamma = \frac{Z_L - Z_0}{Z_0 + Z_L} \quad (5)$$

$$Z_{in}(-L) = Z_0\left[\frac{Z_L + jZ_0 \tan(\beta L)}{Z_0 + jZ_L \tan(\beta L)}\right] \quad (6)$$

From equation 6, we can conclude that the input impedance of the transmission line depends on the load impedance, characteristic impedance, length of the transmission line, and the phase constant of the signals propagating through it.

It is already a known fact that the characteristic impedance Z0 is dependent on the distributed parameters of the transmission line, such as resistance, inductance, capacitance, and conductance (as given by equation 7), which are usually defined per unit length. Whenever any change is made in the circuit, the input impedance changes.

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \quad (7)$$

The relationship between the characteristic impedance and input impedance can be deduced for certain transmission lines. In the derivation of the input impedance equation, we have considered the finite length of the transmission line. When the transmission line length is infinite, then the input impedance of the transmission line is equal to the characteristic impedance. Whenever the transmission line of finite length is terminated by a load impedance that is equal to the characteristic impedance, there is no reflection of signals (according to equation 7). In this case, the input impedance equals characteristic impedance.

OPEN AND SHORT-CIRCUITED LINES

As limited cases it is convenient to consider lines terminated in open circuit or short circuit, that is with $ZR = \infty$or $ZR = 0$.

First, let us consider the question at hand: What is the input impedance when the transmission line is open- or short-circuited?

For a short circuit, $Z_L = 0, \Gamma = -1$, so we find

$$Z_{in}(l) = Z_0 \frac{1 + \Gamma e^{-j2\beta l}}{1 - \Gamma e^{-j2\beta l}}$$
$$= Z_0 \frac{1 + e^{-j2\beta l}}{1 - e^{-j2\beta l}}$$

(3.16.1)

Multiplying numerator and denominator by $e^{+j\beta l}$, we obtain

$$Z_{in}(l) = Z_0 \frac{e^{+j\beta l} - e^{-j\beta l}}{e^{+j\beta l} + e^{-j\beta l}}$$

(3.16.2)

Now we invoke the following trigonometric identities:

$$\cos\theta = \frac{1}{2}\left[e^{+j\theta} + e^{-j\theta}\right]$$

(3.16.3)

$$\sin\theta = \frac{1}{j2}\left[e^{+j\theta} - e^{-j\theta}\right]$$

(3.16.4)

Employing these identities, we obtain:

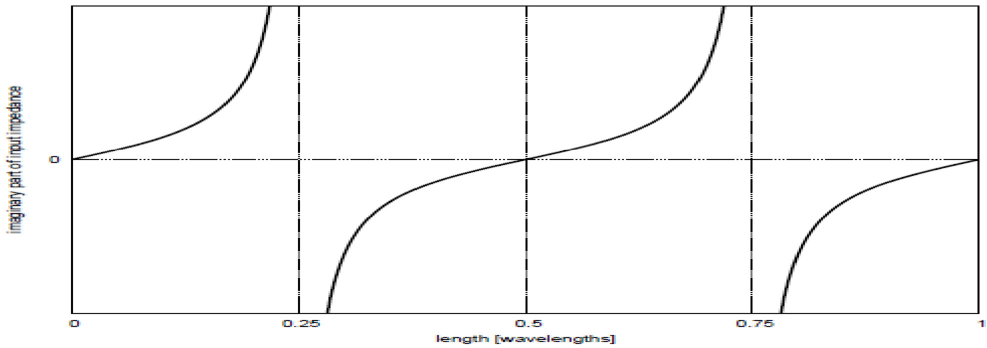
$$Z_{in}(l) = Z_0 \frac{j2\,(\sin\beta l)}{2\,(\cos\beta l)}$$

(3.16.5)

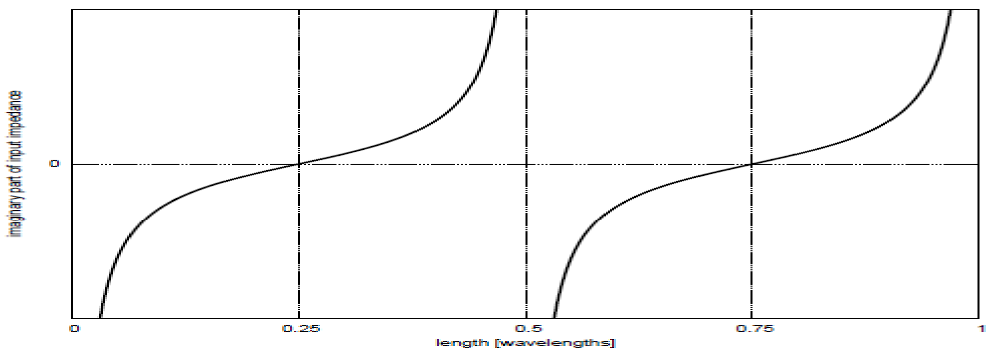
and finally:

$$\boxed{Z_{in}(l) = +jZ_0 \tan\beta l}$$

(3.16.6)



(a) Short-circuit termination ($Z_L = 0$).



(b) Open-circuit termination ($Z_L \to \infty$).

(b) Open-circuit termination ($Z_L \to \infty$).

For an open circuit termination, $Z_L \to \infty, \Gamma = +1$, and we find

$$Z_{in}(l) = Z_0 \frac{1 + \Gamma e^{-j2\beta l}}{1 - \Gamma e^{-j2\beta l}}$$
$$= Z_0 \frac{1 + e^{-j2\beta l}}{1 - e^{-j2\beta l}}$$

(3.16.7)

Following the same procedure detailed above for the short-circuit case, we find

$$\boxed{Z_{in}(l) = -jZ_0 \cot \beta l}$$

(3.16.8)

Figure 3.16.1 (b) shows the result for open-circuit termination. As expected, $Z_{in} \to \infty$ for $l = 0$, and the same $\lambda/2$ periodicity is observed. What is of particular interest now is that at $l = \lambda/4$ we see $Z_{in} = 0$. In this case, the transmission line has transformed the *open* circuit termination into a *short* circuit.

And for the short circuit case ZR =0., so that

$$Z_s = Z_0 \tanh \gamma l$$

Before the open circuit case is considered, the input impedance should be written. The input impedance of the open circuited line of length l, with $ZR = \infty$, is

$$Z_{oc} = Z_0 \coth \gamma l$$

By multiplying the above two equations it can be seen that

$$Z_0 = Z_{oc}Z_{sc}$$

This is the same result as was obtained for a lumped network. The above equation supplies a very valuable means of experimentally determining the value of z0 of a line.

Also from the same two equations

$$\tanh \gamma l = \sqrt{\frac{Z_{sc}}{Z_{oc}}}$$

$$\gamma l = \tanh^{-1} \sqrt{\frac{Z_{sc}}{Z_{oc}}}$$

Use of this equation in experimental work requires the determination of the hyperbolic tangent of a complex angle. If

Reflection coefficient:

A reflection coefficient, sometimes called reflection parameter, defines how much energy is reflected from the load to the source of the RF systems. A reflection coefficient is also known as s11 parameter. By definition, a reflected coefficient is a ration of the reflected wave and the incident wave of the electric field strength. In the literature it is presented with the capital Greek letter gamma (Γ).

The mismatch of a load Z_L to a source Z_0 results in a reflection coefficient of:
$\Gamma=(Z_L-Z_0)/(Z_L+Z_0)$

Note that the load can be a complex (real and imaginary) impedance. If you can't remember in which order the numerator is subtracted (did we just say "Z_L-Z_0" or Z_0-Z_L"?), you can always figure it out by remembering that a short circuit (Z_L=0) is on the left side of the Smith chart (angle = -180 degrees) which means Γ=-1 in this case, which means that the minus sign belongs in front of Z_0.

The magnitude of the reflection coefficient is given by:
ρ=mag(Γ)
For cases where Z_L is a real number,
ρ=abs((Z_L-Z_0)/(Z_L+Z_0))
Note that "abs" means "absolute value" here. VSWR can be calculated from the magnitude of the reflection coefficient:
VSWR=(1+ρ)/(1-ρ)
For cases where Z_L is real, with a little algebra you'll see there are two cases for VSWR, calculated from load impedance:
For $Z_L<Z_0$: VSWR=Z_0/Z_L
For $Z_L>Z_0$: VSWR=Z_L/Z_0

VSWR:

VSWR is an abbreviation for Voltage Standing Wave Ratio or sometimes in literature just SWR (Standing Wave Ratio). The value of VSWR presents the power reflected from the load to the source. It is often used to describe how much power is lost from the source (usually a High Frequency Amplifier) through a transmission line (usually a coaxial cable) to the load (usually an antenna).

How to express VSWR using voltage?
By the definition, VSWR is the ratio of the highest voltage (the maximum amplitude of the standing wave) to the lowest voltage (the minimum amplitude of the standing wave) anywhere between source and load.

VSWR = |V(max)| / |V(min)|

V(max) = the maximum amplitude of the standing wave
Vmin) = the minimum amplitude of the standing wave

What is the ideal value of a VSWR?
The value of an ideal VSWR is 1:1 or shortly expressed as 1. In this case the reflected power from the load to the source is zero.

How to express VSWR using an impedance?
By the definition, VSWR is the ratio of the load impedance and source impedance.

The reflection coefficient can also be expressed in terms of the characteristic impedance of the inner conductor and the matched load impedance as follows:

$$\Gamma = (Z_L - Z_O)/(Z_L + Z_O) \qquad (Eq.\ 5)$$

Where
Z_L is the matched load impedance.
Z_O is the characteristic impedance of the inner conductor.

Substituting *(Eq.5)* into *(Eq.2)*, to obtain VSWR in terms of Z_L and Z_O:

$$VSWR = \frac{\left[1 + \left|\frac{ZL - ZO}{ZL + ZO}\right|\right]}{\left[1 - \left|\frac{ZL - ZO}{ZL + ZO}\right|\right]}$$

$$VSWR = \frac{[ZL + ZO + |ZL - ZO|]}{[ZL + ZO - |ZL - ZO|]} \qquad (Eq.6)$$

Solving *(Eq.6)* for,

Case 1: if $Z_L > Z_O$ then $|Z_L - Z_O| = Z_L - Z_O$

$$\therefore VSWR = \frac{[ZL + ZO + ZL - ZO]}{[ZL + ZO - ZL + ZO]}$$

$$\therefore VSWR = \frac{ZL}{ZO} \qquad (Eq.7)$$

Case 2: if $Z_L < Z_O$ then $|Z_L - Z_O| = Z_O - Z_L$

$$\therefore VSWR = \frac{[ZL + ZO + ZO - ZL]}{[ZL + ZO - ZO + ZL]}$$

$$\therefore VSWR = \frac{ZO}{ZL} \qquad (Eq.8)$$

ZL = the load impedance
Zo = the source impedance

How to express a VSWR using reflection and forward power?
By the definition VSWR is equal to

VSWR = 1 + √(Pr/Pf) / 1 – √(Pr/Pf)

where:

Pr = Reflected power
Pf = Forward power

Smith Chart:

The Smith Chart has been in use since the 1930s as a method to solve various RF design problems - notably impedance matching with series and shunt components - and it provides a convenient way to find these solutions without the use of a calculator. In order to understand the construction of the chart, you'll need to understand high school algebra and the basics of complex numbers, as well as have a basic understanding of impedance in electronic circuits. That said, even if you don't fully understand the derivation below, you can still use the chart to help you with your own design. By taking the standard reflection coefficient formula and manipulating it so that it provides us with the equations for circles of various radii, we'll be able to construct the basic Smith Chart. That's all the Smith Chart really is: a collection of circles, each one centered in a different place in (or outside) the plot, and each one representing either **constant resistance** or **constant reactance**

Deriving the Smith Chart

Once we get past the derivation, there will be a few simplified images showing how those equations can be used and combined to get the final product. Let's get started by writing the equation for the reflection coefficient of a load impedance, given a source impedance:

$$\Gamma = \frac{Z_{source} - Z_{load}}{Z_{source} + Z_{load}}$$

The reflection coefficient is just the ratio of the *complex* amplitude of a reflected wave to the amplitude of the incident wave. This is the main equation we'll be using, but there will be some quick transformations to it. First, we'll want to simplify it a little by normalizing the equation with respect to Z_{load}, dividing each term on the right side:

$$\Gamma = \frac{\frac{Z_{source}}{Z_{load}} - \frac{Z_{load}}{Z_{load}}}{\frac{Z_{source}}{Z_{load}} + \frac{Z_{load}}{Z_{load}}}$$

$$\Gamma = \frac{Z_O - 1}{Z_O + 1} \qquad Z_O = \frac{Z_{source}}{Z_{load}}$$

At this point, recall that Z_o, being an impedance of complex value, can be represented in the form R + jX. Since the reflection coefficient (which is currently in polar form) can also be represented in rectangular coordinates (we'll use A + jB for it), the above formula can be transformed into this:

$$A + jB = \frac{R + jX - 1}{R + jX + 1}$$

Great! At this point we've got the equation in the form we need to start constructing the Smith Chart. The next step - solving for the real and imaginary parts of the equation - is probably the most difficult part of the entire derivation, and even then you only need to understand the concept of complex conjugates to do it. Let's go ahead and split it into real and imaginary components, first by multiplying by the complex conjugate (it helps if you separate the existing real and imaginary parts using brackets as shown below):

$$A + jB = \frac{(R-1) + jX}{(R+1) + jX} \cdot \frac{(R+1) - jX}{(R+1) - jX}$$

$$A + jB = \frac{R^2 - 1 + X^2 + 2jX}{(R+1)^2 + X^2}$$

At this point we can separate the real and imaginary components. After that, there will be two final simplifications to do before we'll have the equations to draw the Smith Chart. Here are the separated real and imaginary parts (we'll call them Equations 1 and 2):

$$A = \frac{R^2 - 1 + X^2}{(R+1)^2 + X^2} \text{ (Equation 1)}$$

$$B = \frac{2X}{(R+1)^2 + X^2} \text{ (Equation 2)}$$

Finally, you will want to do just a *little* more algebra (tedious, I know). Solving the real component, A, for X^2, you will get Equation 3:

$$X^2 = \frac{A(R+1)^2 - R^2 + 1}{1 - A} \text{ (Equation 3)}$$

You can substitute this into Equation 2 to get the first of our two final equations, which allows us to determine the circles of constant resistance (Equation 4):

$$(A - \frac{R}{R+1})^2 + B^2 = (\frac{1}{R+1})^2 \text{ (Equation 4)}$$

Does that look familiar? It's a circle, with a radius of $\frac{1}{R+1}$ and a center of $(\frac{R}{R+1},\ 0)$. By varying the value of R in this equation, you can draw each of the circles in the Smith Chart.

Similarly, solving for R (I used Equation 2) will get you solutions that look like this:

$$R = \frac{\sqrt{-BX(BX-2)} - B}{B}$$

$$R = \frac{\sqrt{-BX(BX-2)} - B}{B}$$

which, when substituted and simplified into Equation 1, will get you this result (Equation 5):

$$(A-1)^2 + (B - \frac{1}{X})^2 = (\frac{1}{X})^2 \text{ (Equation 5)}$$

1

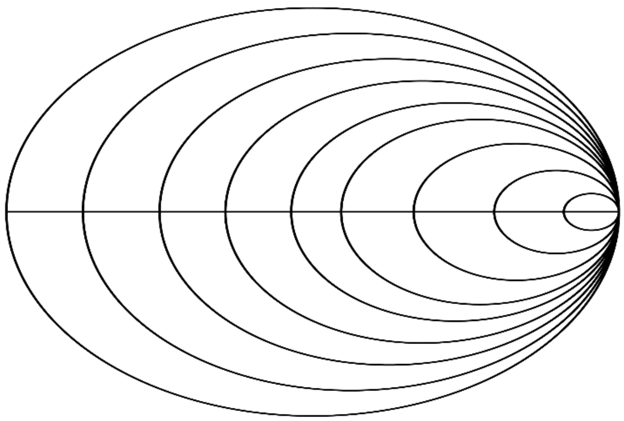
Just like the previous result, this is a circle with radius 1/X but this time there are two sets of circles (more on that in a bit), with centers at (1,1/X)These are circles (they appear as arcs on the diagram) of constant reactance. Now you should see how the standard Smith Chart is drawn; it consists of constant resistance circles graphed together with the constant reactance arcs. Below you'll find some simplified images of both equations graphed separately and combined. But first, let's talk about how to interpret the Smith Chart and its physical relevance.

There is quite a bit of information to obtain from analyzing the equations we've derived. Here are just a few things of note:

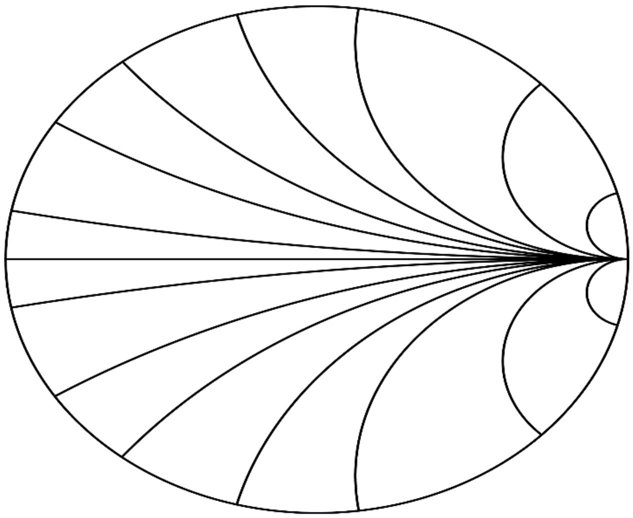
- At infinite R and X, both types of circles converge to the same location (typically shown on a Smith Chart at the far right or far left side of the diagram). This is at the point (1, 0).
- Setting R = 0 will result in a circle centered at (0, 0) on your chart with a radius of 1, which is the "boundary" of the chart.
- Approaching X = 0 results in an infinite radius; this is represented by a line crossing the center of the chart. How do we interpret this? This is often called the **real axis**. In terms of reactances, lines above the real axis in the chart (the positive arcs from the second derived equation) represent inductive reactances, while those below (negative arcs) represent capacitive reactances.

- What happens if R < 0? The standard Smith Chart doesn't provide much detail about this, but situations with R lying outside the boundary suggest oscillation in any would-be circuit (which is pretty handy to know).
- Based on the knowledge we now have on resistance and reactance on the chart, we know that every point represents a series combination of resistance and reactance (R + jX). This'll help us when we want to do some plotting

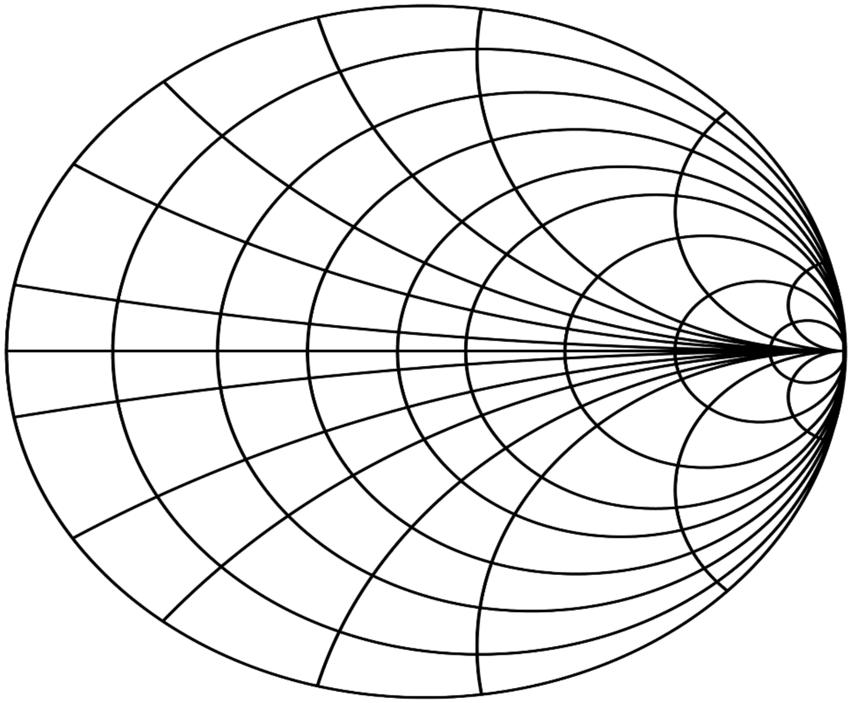
Constant Resistance Circles:



Constant Reactance Arcs:



Smith Chart:



Constant resistance and reactance circles plotted together

Applications of Smith Charts:

Smith charts find applications in all areas of RF Engineering. Some of the most popular application includes;

- **Impedance calculations** on any transmission line, on any load.
- **Admittance calculations** on any transmission line, on any load.
- Calculation of the length of a short-circuited piece of transmission line to provide a required capacitive or inductive reactance.
- **Impedance matching.**
- Determining VSWR among others.