UNIT 1

PRECESSION

COURSE OBJECTIVES

To study about gyroscope and its effects during precession motion of moving vehicles.

COURSE OUTCOMES

Knowledge acquired about Gyroscope and its precession motion.

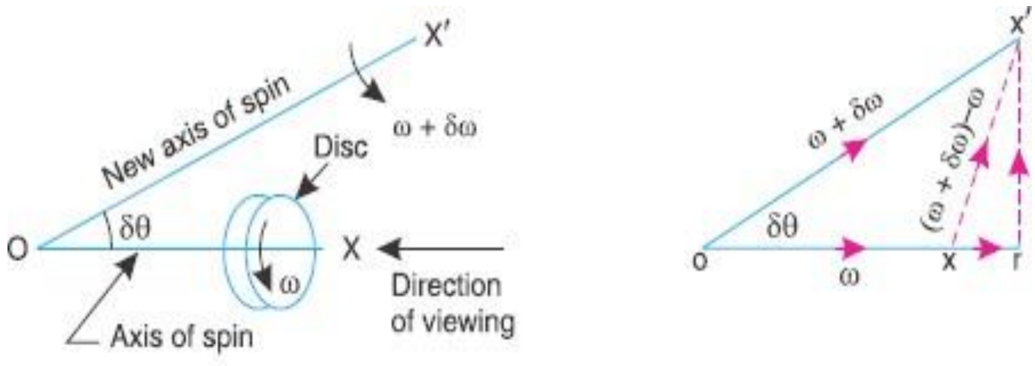
Introduction

When a body moves along a curved path with a uniform linear velocity, a force in the direction of centripetal acceleration (known as centripetal force) has to be applied externally over the body, so that it moves along the required curved path. This external force applied is known as **active force.**

When a body, itself, is moving with uniform linear velocity along a circular path, it is subjected to the centrifugal force* radially outwards. This centrifugal force is called **reactive force.** The action of the reactive or centrifugal force is to tilt or move the body along radially outward direction.

Precessional Angular Motion

We have already discussed that the angular acceleration is the rate of change of angular velocity with respect to time. It is a vector quantity and may be represented by drawing a vector diagram with the help



(*a*) (*b*)

of right hand screw rule.

Consider a disc, as shown in Fig (*a*), revolving or spinning about the axis *OX* (known as **axis of spin**) in anticlockwise when seen from the front, with an angular velocity ω in a plane at right angles to the paper.

After a short interval of time δt, let the disc be spinning about the new axis of spin *OX* ′(at an angle δθ) with an angular velocity (ω+ δω). Using the right hand screw rule, initial angular velocity of the disc (ω) is represented by vector *ox*; and the final angular velocity of the disc (ω+ δω) is represented by vector *ox*′ as shown in Fig. 14.1 (*b*). The vector *xx*′ represents the change of angular velocity in time δt *i.e.* the angular

Component of angular acceleration in the direction of *ox*,

$$\alpha_t = \frac{xr}{\delta t} = \frac{or - ox}{\delta t} = \frac{ox' \cos\delta\theta - ox}{\delta t}$$

$$= \frac{(\omega + \delta\omega)\ \cos\delta\theta - \omega}{\delta t} = \frac{\omega\cos\delta\theta + \delta\omega\cos\delta\theta - \omega}{\delta t}$$

Since δθ is very small, therefore substituting $\cos\delta\theta = 1$, we have

$$\alpha_t = \frac{\omega + \delta\omega - \omega}{\delta t} = \frac{\delta\omega}{\delta t}$$

acceleration of the disc. This may be resolved into two components.

One parallel to *ox* and the other perpendicular to *ox.*

In the limit, when $\delta t \to 0$,

$$\alpha_t = \underset{\delta t \to 0}{\text{Lt}} \left(\frac{\delta \omega}{\delta t} \right) = \frac{d\omega}{dt}$$

Component of angular acceleration in the direction perpendicular to ox,

$$\alpha_c = \frac{rx'}{\delta t} = \frac{ox' \sin \delta\theta}{\delta t} = \frac{(\omega + \delta\omega) \sin \delta\theta}{\delta t} = \frac{\omega \sin \delta\theta + \delta\omega . \sin \delta\theta}{\delta t}$$

Since $\delta\theta$ in very small, therefore substituting $\sin \delta\theta = \delta\theta$, we have

$$\alpha_c = \frac{\omega . \delta\theta + \delta\omega . \delta\theta}{\delta t} = \frac{\omega . \delta\theta}{\delta t}$$

...(Neglecting $\delta\omega . \delta\theta$, being very small)

In the limit when $\delta t \to 0$,

$$\alpha_c = \underset{\delta t \to 0}{\text{Lt}} \frac{\omega . \delta\theta}{\delta t} = \omega \times \frac{d\theta}{dt} = \omega . \omega_P \qquad \ldots \left(\text{Substituting } \frac{d\theta}{dt} = \omega_P \right)$$

∴ Total angular acceleration of the disc

$= \text{vector } xx' = \text{vector sum of } \alpha_t \text{ and } \alpha_c$

$$= \frac{d\omega}{dt} + \omega \times \frac{d\theta}{dt} = \frac{d\omega}{dt} + \omega . \omega_P$$

Where dθ/dt is the angular velocity of the axis of spin about a certain axis, which is perpendicular to the plane in which the axis of spin is going to rotate. This angular velocity of the axis of spin (i.e.dθ/dt) is known as angular velocity of precession and is denoted by ω_P. The axis, about which the axis of spin is to turn, is known as axis of precession. The angular motion of the axis of spin about the axis of precession is known as precessional angular motion.

Gyroscopic Couple

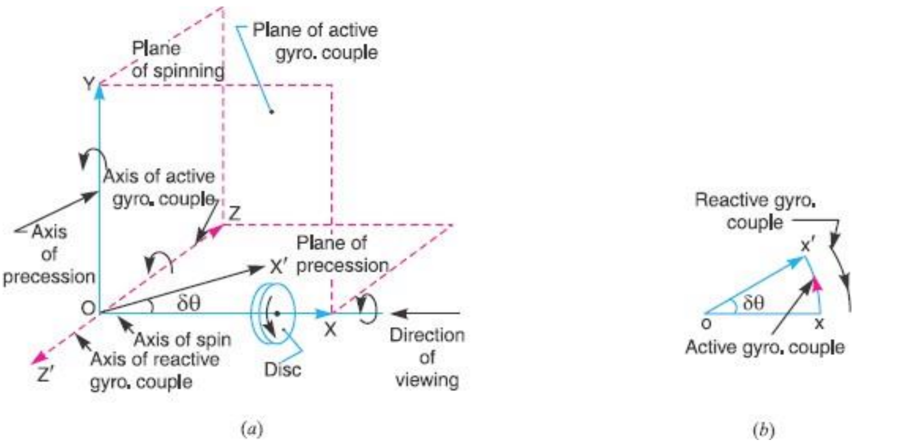
Consider a disc spinning with an angular velocity ω rad/s about the axis of

spin OX, in anticlockwise direction when seen from the front, as shown in Fig. 14.2 (a). Since the plane in which the disc is rotating is parallel to the plane YOZ, therefore it is called plane of spinning. The plane XOZ is a horizontal plane and the axis of spin rotates in a plane parallel to the horizontal plane about an axis OY. In other words, the axis of spin is said to be rotating or processing about an axis OY. In other words, the axis of spin is said to be rotating or processing about an axis OY (which is perpendicular to both the axes OX and OZ) at an angular velocity _P rap/s. This horizontal plane XOZ is called plane of precession and OY is the axis of precession.

Let *I* = Mass moment of inertia of the disc about *OX,* and ω= Angular velocity of the disc.

Angular momentum of the disc = *I.* ω

Since the angular momentum is a vector quantity, therefore it may be represented by the vector *ox*′ , as shown in Fig. 14.2 (*b*). The axis of spin *OX* is also rotating anticlockwise when seen from the top about the axis *OY.* Let the axis *OX* is turned in the plane *XOZ* through a small angle δθ radians to the position *OX′* , in time δt seconds. Assuming the angular velocity ω to be constant, the angular momentum will now be represented by vector *ox′*.



$$= \vec{ox'} - \vec{ox} = \vec{xx'} = \vec{ox}.\delta\theta \qquad \text{...(in the direction of } \vec{xx'}\text{)}$$
$$= I.\ \omega.\delta\theta$$

Change in angular momentum and rate of change of angular momentum

$$= I.\omega \times \frac{\delta\theta}{dt}$$

$$C = \underset{\delta t \to 0}{\text{Lt}}\ I.\omega \times \frac{\delta\theta}{\delta t} = I.\omega \times \frac{d\theta}{dt} = I.\omega.\omega_P \qquad \ldots\left(\because \frac{d\theta}{dt} = \omega_P\right)$$

Since the rate of change of angular momentum will result by the application of a couple to the disc,

Therefore the couple applied to the disc causing precession,

Where $\boldsymbol{\omega_P}$ = Angular velocity of precession of the axis of spin or the speed of rotation of the axis of spin about the axis of precession *OY.*

The couple $\boldsymbol{I.\omega.\omega_P}$**,** in the direction of the vector *xx′* (representing the change in angular momentum) is the *active gyroscopic couple,* which has to be applied over the disc when the axis of spin is made to rotate with angular velocity ω_P about the axis of precession. The vector *xx′* lies in the plane *XOZ* or the horizontal plane. In case of a very small displacement δθ, the vector *xx′* will be perpendicular to the vertical plane *XOY.* Therefore the couple causing this change in the angular momentum will lie in the plane *XOY.* The vector *xx′*, as shown in Fig(*b*), represents an anticlockwise couple in the plane *XOY.* Therefore, the plane *XOY* is called the *plane of active gyroscopic couple* and the axis *OZ* perpendicular to the plane *XOY,* about which the couple acts, is called the axis of active gyroscopic couple.

When the axis of spin itself moves with angular velocity ω_P, the disc is subjected to *reactive couple* whose magnitude is same (*i.e.* $I.\omega.\omega_P$) but opposite in direction to that of active couple. This reactive couple to which the disc is subjected when the axis of spin rotates about the axis of precession is known as *reactive gyroscopic couple.* The axis of the reactive gyroscopic couple is represented by *OZ*′ in Fig(*a*).

The gyroscopic couple is usually applied through the bearings which support the shaft. The bearings will resist equal and opposite couple.

The gyroscopic principle is used in an instrument or toy known as gyroscope. The gyroscopes are installed in ships in order to minimize the rolling and pitching effects of waves. They are also used in aero planes, monorail cars, gyrocompasses etc.

Effect of the Gyroscopic Couple on an aero plane:

The top and front view of an aero plane is shown in Fig (*a*). Let engine or propeller rotates in the clockwise direction when seen from the rear or tail end and the aero plane takes a turn to the left.

Let ω= Angular velocity of the engine in rad/sec,

m = Mass of the engine and the propeller in kg,

k = Its radius of gyration in metre

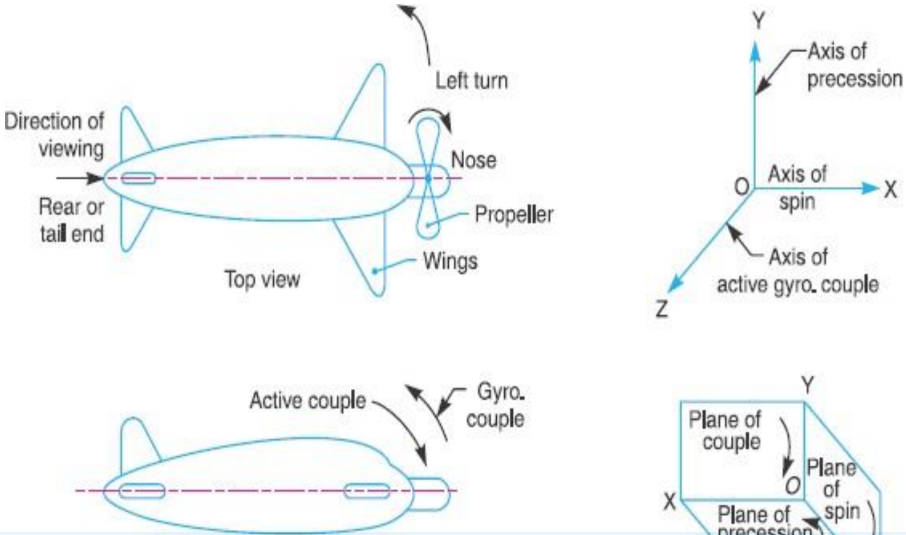
I = Mass moment of inertia of the engine and the propeller in kg-m^2

= $\boldsymbol{mk^2}$

v = Linear velocity of the aero plane in m/s,

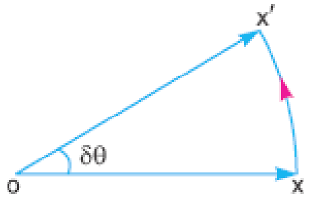
R = Radius of curvature in metres, and

ω_p = Angular velocity of precession=V/R

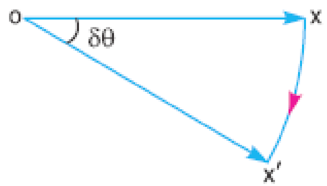


Before taking the left turn, the angular momentum vector is represented by *ox*. When it takes left turn, the active gyroscopic couple will change the direction of the angular momentum vector from *ox* to *ox*′ as shown in Fig(*a*). The vector *xx*′*,* in the limit, represents the change of angular momentum or the active gyroscopic couple and is perpendicular to *ox.* Thus the plane of active gyroscopic couple *XOY* will be perpendicular to *xx*′, *i.e.* vertical in this case, as shown in Fig (*b*). By applying right hand screw rule to vector *xx*′*,* we find that the direction of active gyroscopic couple is clockwise as shown in the front view of Fig (*a*).

In other words, for left hand turning, the active gyroscopic couple on the aero plane in the axis *OZ* will be clockwise as shown in Fig (*b*).The reactive gyroscopic couple.(equal in magnitude of active gyroscopic



(*a*) Aeroplane taking left turn.



(*b*) Aeroplane taking right turn.

couple) will act in the opposite direction (*i.e.* in the anticlockwise direction) and the effect of this couple is, therefore, to raise the nose and dip the tail of the aero plane.

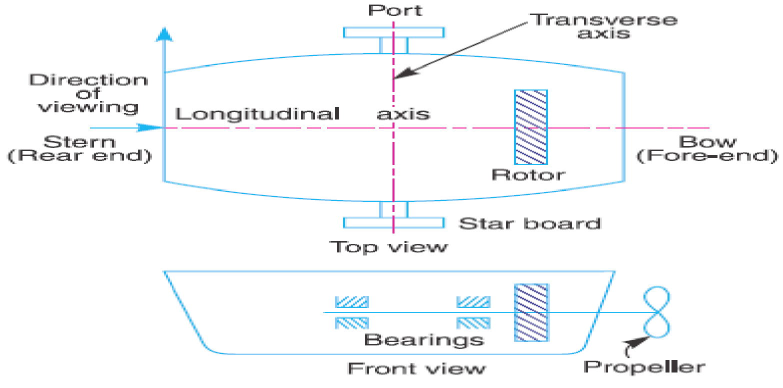
Terms Used in a Naval Ship.

The top and front views of a naval ship are shown in Fig 14.7. The fore end of the ship is called ***bow*** and the rear end is known as ***stern*** or ***aft****.* The left hand and right hand sides of the ship, when viewed from the stern are called ***port*** and ***star-board*** respectively. We shall now discuss the effect of gyroscopic couple on the naval ship in the following three cases:

Steering,

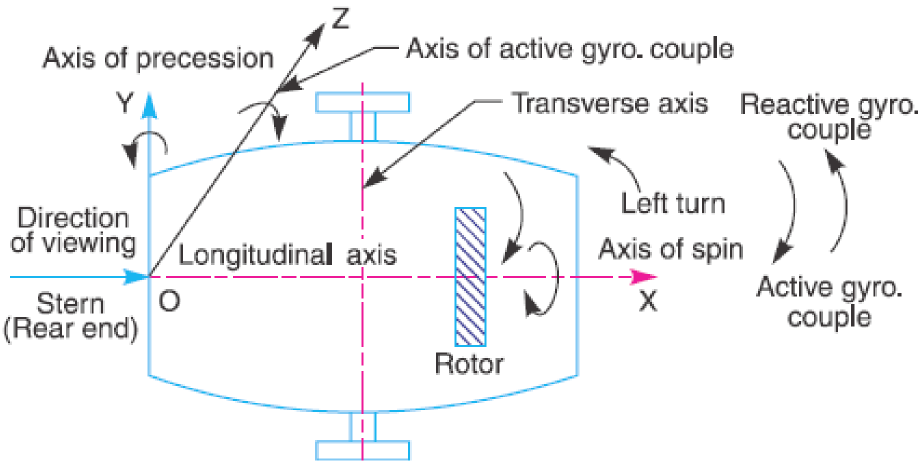
Pitching,

Rolling.

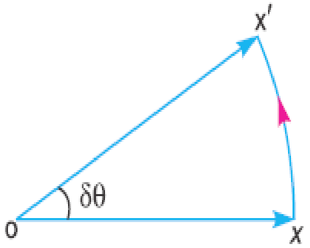


Effect of Gyroscopic Couple on a Naval Ship during Steering

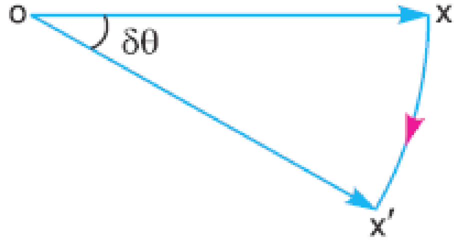
Steering is the turning of a complete ship in a curve towards left or right, while it moves forward. Consider the ship taking a left turn, and rotor rotates in the clockwise direction when viewed from the stern, as shown in Fig. The effect of gyroscopic couple on a naval ship during steering taking left or right turn may be obtained in the similar way as for an aeroplane.



When the rotor of the ship rotates in the clockwise direction when viewed from the stern, it will have its angular momentum vector in the direction ox as shown in Fig(a). As the ship steers to the left, the active gyroscopic couple will change the angular momentum vector from ox to ox′. The vector xx′ now represents the active gyroscopic couple and is perpendicular to ox. Thus the plane of active gyroscopic couple is perpendicular to xx′ and its direction in the axis OZ for left hand turn is clockwise as shown in Fig. The reactive gyroscopic couple of the same magnitude will act in the opposite direction (i.e. in anticlockwise direction). The effect of this reactive gyroscopic couple is to raise the bow and lower the stern.



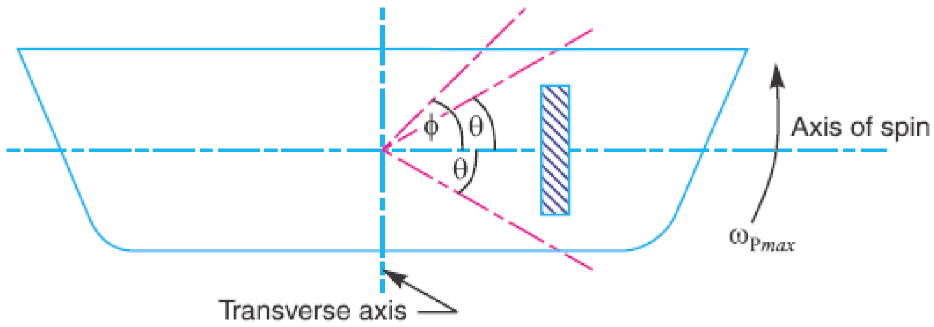
(*a*) Streeing to the left



(*b*) Streeing to the right

Effect of Gyroscopic Couple on a Naval Ship during Pitching

Pitching is the movement of a complete ship up and down in a vertical plane about transverse axis, as shown in Fig(a). In this case, the transverse axis is the axis of precession. The pitching of the ship is assumed to take place with simple harmonic motion i.e. the motion of the axis of spin about transverse axis is simple harmonic.



(*a*) Pitching of a naval ship

Let   *I* = Moment of inertia of the rotor in kg-m^2, and ω= Angular velocity of the rotor inrad/s.

Minimum gyroscopic couple,

$$C_{max} = I.\ \omega.\ \omega_{Pmax}$$

When the pitching is upward, the effect of the reactive gyroscopic couple, as shown in Fig.(*b*), will try to move the ship toward star-board. On the other hand, if the pitching is downward, the effect of the reactive gyroscopic couple, as shown in Fig(*c*), is to turn the ship towards port side.

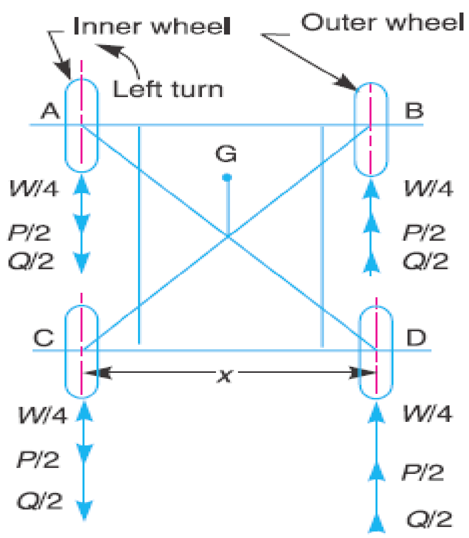
Effect of Gyroscopic Couple on a Naval Ship during Rolling

We know that, for the effect of gyroscopic couple to occur, the axis of precession should always be perpendicular to the axis of spin. If, however, the axis of precession becomes parallel to the axis of spin, there will be no effect of the gyroscopic couple acting on the body of the ship.

In case of rolling of a ship, the axis of precession (*i.e.* longitudinal axis) is always parallel to the axis of spin for all positions. Hence, there is no effect of the gyroscopic couple acting on the body of a ship.

Stability of a Four Wheel Drive Moving in a Curved Path

Consider the four wheels *A*, *B*, *C* and *D* of an automobile locomotive taking a turn towards left as shown in Fig. The wheels *A* and *C* are inner wheels, whereas *B* and *D* are outer wheels. The centre of gravity (*C.G.*) of the vehicle lies vertically above the road surface.



Let m = Mass of the vehicle inkg,

W = Weight of the vehicle in newtons = $m.g$, r_W = Radius of the wheels in metres,

R = Radius of curvature in metres ($R > r_W$),

h = Distance of centre of gravity, vertically above the roadsurfacein metres,

x = Width of track in metres,

I_W = Mass moment of inertia of one of the wheels in kg-m^2,

ω_W= Angular velocity of the wheels or velocity of spin in rad/s,

I_E = Mass moment of inertia of the rotating parts of the engine in kg-m2

ω_E = Angular velocity of the rotating parts of the engine in rad/s,

G = Gear ratio =ω_E/ω_W

v = Linear velocity of the vehicle in m/s = $\boldsymbol{\omega_W.r_W}$

A little consideration will show that the weight of the vehicle (*W*) will be equally distributed over the four wheels which will act downwards. The reaction between each wheel and the road surface of the same magnitude will act upwards.

Therefore Road reaction over each wheel = *W*/4 = *m.g*/4 newtons

Let us now consider the effect of the gyroscopic couple and centrifugal couple on the vehicle.

∴ Angular displacement of the axis of spin from mean position after time t seconds,

$$\theta = \phi \sin \omega_1 . t$$

ϕ = Amplitude of swing *i.e.* maximum angle turned from the mean position in radians, and

ω_1 = Angular velocity of S.H.M.

$$= \frac{2\pi}{\text{Time period of S.H.M. in seconds}} = \frac{2\pi}{t_p} \text{ rad/s}$$

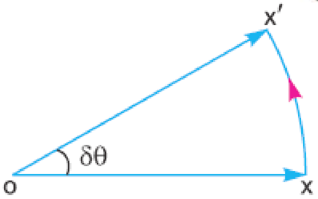
Angular velocity of precession,

$$\omega_P = \frac{d\theta}{dt} = \frac{d}{dt}(\phi \sin \omega_1 . t) = \phi \omega_1 \cos \omega_1 t$$

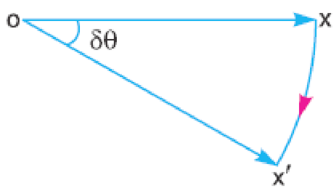
The angular velocity of precession will be maximum, if $\cos \omega_1 . t = 1$.

∴ Maximum angular velocity of precession,

$$\omega_{Pmax} = \phi . \omega_1 = \phi \times 2\pi / t_p \quad \text{...(Substituting } \cos \omega_1 . t = 1)$$



(*b*) Pitching upward



(*c*) Pitching downward

Effect of the gyroscopic couple

Since the vehicle takes a turn towards left due to the precession and other rotating parts, therefore a gyroscopic couple will act.

We know that velocity of precession, $\omega_P = v/R$

Gyroscopic couple due to 4 wheels,

$$C_W = 4\, I_W . \omega_W . \omega_P$$

and gyroscopic couple due to the rotating parts of the engine,

$$C_E = I_E . \omega_E . \omega_P = I_E . G . \omega_W . \omega_P$$

Net gyroscopic couple,

$$C = C_W \pm C_E = 4\, I_W . \omega_W . \omega_P \pm I_E . G . \omega_W . \omega_P$$

$= \omega_W.\omega_P (4\ I_W \pm G.I_E)$

The ***positive*** sign is used when the wheels and rotating parts of the engine rotate in the same direction. If the rotating parts of the engine revolve in opposite direction, then ***negative*** sign is used.

Due to the gyroscopic couple, vertical reaction on the road surface will be produced. The reaction will be vertically upwards on the outer wheels and vertically downwards on the inner wheels.

Let the magnitude of this reaction at the two outer or inner wheels be *P* new tons. Then

$P \times x = C$ or $P = C/x$

Vertical reaction at each of the outer or inner wheels,

$P/2 = C/2x$

Effect of the centrifugal couple

Since the vehicle moves along a curved path, therefore centrifugal force will act outwardly at the centre of gravity of the vehicle. The effect of this centrifugal force is also to overturn the vehicle.

$$F_C = \frac{m \times v^2}{R}$$

We know that centrifugal force,

The couple tending to overturn the vehicle or overturning couple,

$$C_O = F_C \times h = \frac{m.v^2}{R} \times h$$

This overturning couple is balanced by vertical reactions, which are vertically up wards on the outer wheels and vertically downwards on the inner wheels. Let the magnitude of this reaction at the two outer

$$Q \times x = C_O \quad \text{or} \quad Q = \frac{C_O}{x} = \frac{m.v^2.h}{R.x}$$

or inner wheels be Q. Then

Vertical reaction at each of the outer or inner wheels,

Total vertical reaction at each of the outer wheel,

$$\frac{Q}{2} = \frac{m.v^2.h}{2R.x}$$

$$P_O = \frac{W}{4} + \frac{P}{2} + \frac{Q}{2}$$

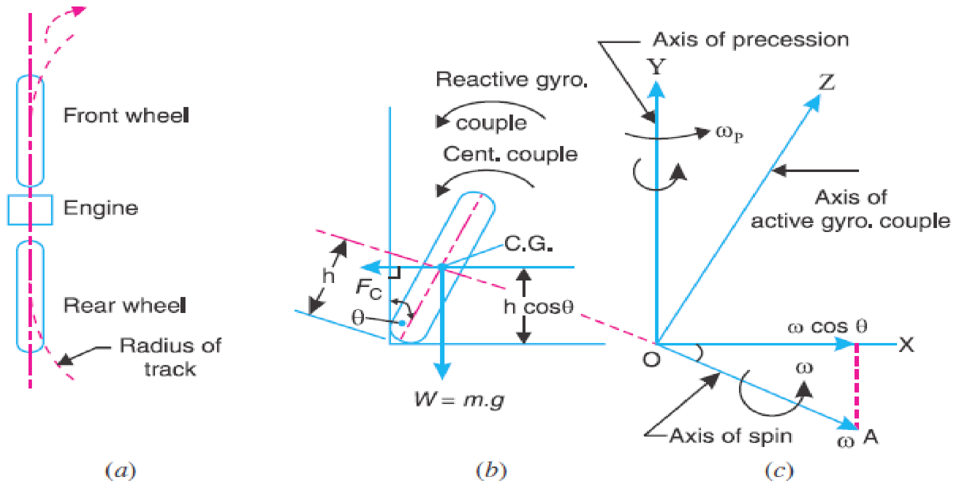
Total vertical reaction at each of the inner wheel

$$P_I = \frac{W}{4} - \frac{P}{2} - \frac{Q}{2}$$

A little consideration will show that when the vehicle is running at high speeds, P_I may bezero or even negative. This will cause the inner wheels to leave the ground thus tending to overturn the automobile. In order to have the contact between the inner wheels and the ground, the sum of P/2 and Q/2 must be less than W/4.

Stability of a Two Wheel Vehicle Taking a Turn

Consider a two wheel vehicle (say a scooter or motor cycle) taking a right turn as shown in fig.



Let

m = Mass of the vehicle and its rider in kg,

W = Weight of the vehicle and its rider in Newton's = $m.g$, h = Height of the centre of gravity of the vehicle and rider, r_W = Radius of the wheels,

R = Radius of track or curvature,

I_W = Mass moment of inertia of each wheel,

I_E = Mass moment of inertia of the rotating parts of the v engine, ω_W = Angular velocity of the wheels,

ω_E= Angular velocity of the engine,

G = Gear ratio = ω_E / ω_W,

v = Linear velocity of the vehicle = $\omega_W \times r_W$,

θ=Angle of wheel. It is inclination of the vehicle to the vertical for equilibrium.

Let us now consider the effect of the gyroscopic couple and centrifugal couple on the vehicle,

Effect of gyroscopic couple

$$\omega_E = G.\omega_W = G \times \frac{v}{r_W}$$

$$\therefore \text{ Total } \quad (I \times \omega) = 2\, I_W \times \omega_W \pm I_E \times \omega_E$$

$$= 2\, I_W \times \frac{v}{r_W} \pm I_E \times G \times \frac{v}{r_W} = \frac{v}{r_W}(2\, I_W \pm G.I_E)$$

We know that $v = \omega_W \times r_W$ or $\omega_W = v / r_W$

Velocity of precession, $\omega_P = v / R$

A little consideration will show that when the wheels move over the curved path, the vehicle is always inclined at an angle θ with the vertical plane as shown in Fig (*b*). This angle is known as ***angle of heel.*** In other words, the axis of spin is inclined to the horizontal at an angle θ, as shown in Fig (*c*). Thus the angular

Momentum vector ***Iω*** due to spin is represented by *OA* inclined to *OX* at an angle θ. But the precession

Gyroscopic couple,

$$C_1 = I.\omega \cos\theta \times \omega_P = \frac{v}{r_W}(2\, I_W \pm G.I_E)\cos\theta \times \frac{v}{R}$$

$$= \frac{v^2}{R.r_W}(2\, I_W \pm G.I_E)\cos\theta$$

axis is vertical. Therefore the spin vector is resolved along *OX*.

Effect of centrifugal couple

We know that centrifugal force,

$$F_C = \frac{m.v^2}{R}$$

This force acts horizontally through the centre of gravity (*C.G.*) along the outward direction.

$$C_2 = F_C \times h\cos\theta = \left(\frac{m.v^2}{R}\right) h\cos\theta$$

Centrifugal couple,

Since the centrifugal couple has a tendency to overturn the vehicle, therefore Total overturning couple,

$$C_O = \text{Gyroscopic couple} + \text{Centrifugal couple}$$

$$= \frac{v^2}{R.r_W}(2\,I_W + G.I_E)\cos\theta + \frac{m.v^2}{R}\times h\cos\theta$$

$$= \frac{v^2}{R}\left[\frac{2\,I_W + G.I_E}{r_W} + m.h\right]\cos\theta$$

We know that balancing couple **=** ***m.g.h*** **sinθ**

The balancing couple acts in clockwise direction when seen from the front of the vehicle. Therefore for stability, the overturning couple must be equal to the balancing couple,*i.e*.

$$\frac{v^2}{R}\left(\frac{2\,I_W + G.I_E}{r_W} + m.h\right)\cos\theta = m.g.h\sin\theta$$

From this expression, the value of the angle of heel (θ) may be determined, so that the vehicle does not skid.

PROBLEMS

Example 1.

A uniform disc of diameter 300 mm and of mass 5 kg is mounted on one end of an arm of length 600 mm. The other end of the arm is free to rotate in a universal bearing. If the disc rotates about the arm with a speed of 300 r.p.m. clockwise, looking from the front, with what speed will it process about the vertical axis?

Solution. Given: d = 300 mm or r = 150 mm = 0.15 m ;m = 5 kg ; l = 600 mm = 0.6 m
N= 300 r.p.m. or ω= 2π× 300/60 = 31.42 rad/s

We know that the mass moment of inertia of the disc, about an axis through its centre of gravity and perpendicular to the plane of disc,

$I = m.r^{2/}2 = 5(0.15)2/2 = 0.056$ kg-m^2

couple due to mass of disc,

$C = m.g.l = 5 \times 9.81 \times 0.6 = 29.43$ N-m

Let ω_P = Speed of precession.

We know that couple (C),

$29.43 = I.\omega.\omega_P = 0.056 \times 31.42 \times \omega_P = 1.76\ \omega_P$

$\omega_P = 29.43/1.76 = 16.7$ rad/s

Example 2.

An aeroplane makes a complete half circle of 50 metres radius, towards left, when flying at 200 km per hr. The rotary engine and the propeller of the plane has a mass of 400 kg and a radius of gyration of 0.3 m. The engine rotates at 2400 r.p.m. clockwise when viewed from the rear. Find the gyroscopic couple on the aircraft and state its effect on it.

Solution: Given :$R = 50$ **m ;** $v = 200$ **km/hr = 55.6 m/s ;** $m = 400$ **kg ;** $k = 0.3$ **m ;** $N =$

2400 r.p.m. or $\omega = 2\pi \times 2400/60 = 251$ **rad/s**

We know that mass moment of inertia of the engine and the propeller,

$I = m.k^2 = 400(0.3)2 = 36$ **kg-m²**

Angular velocity of precession, $\omega_P = v/R = 55.6/50 = 1.11$ rad/s

We know that gyroscopic couple acting on the aircraft,

$C = I.\ \omega.\ \omega_P = 36 \times 251.4 \times 1.11 = 100\ 46$ N-m=10.046 kN-m

When the aeroplane turns towards left, the effect of the gyroscopic couple is to lift the nose upwards and tail downwards.

Example 3 : The turbine rotor of a ship has a mass of 8 tonnes and a radius of gyration0.6 m.It rotates at 1800 r.p.m. clockwise, when looking from the stern. Determine the gyroscopic couple, if the ship travels at 100 km/hr and steer to the left in a curve of 75 m radius.

Solution. Given: $m = 8$ t = 8000 kg ;$k = 0.6$ m ; $N = 1800$ r.p.m. or $\omega = 2\pi \times 1800/60 = 188.5$ rad/s ;
$v = 100$ km/h = 27.8 m/s ; $R = 75$ m

We know that mass moment of inertia of the rotor,

$I = m.k^2 = 8000\ (0.6)2 = 2880$ kg-m²

Angular velocity of precession,

$\omega_p = v / R = 27.8 / 75 = 0.37$ rad/s We know that gyroscopic

Couple,

$C = I.\omega.\omega_P = 2880 \times 188.5 \times 0.37 = 200\ 866$ N-m

= 200.866 kN-m

When the rotor rotates in clockwise direction when looking from the stern and the ship steers to the left, the effect of the reactive gyroscopic couple is to raise the bow and lower the stern.

INDUSTRIAL APPLICATIONS OF GYROSCOPIC

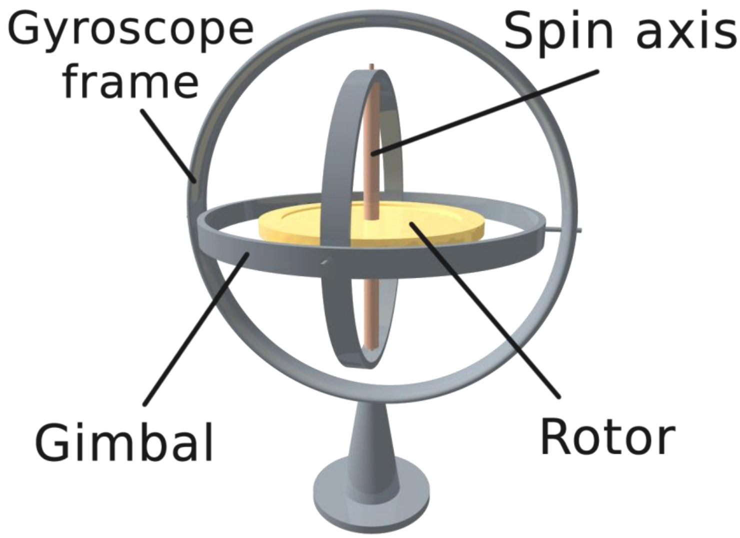
1. Motor car

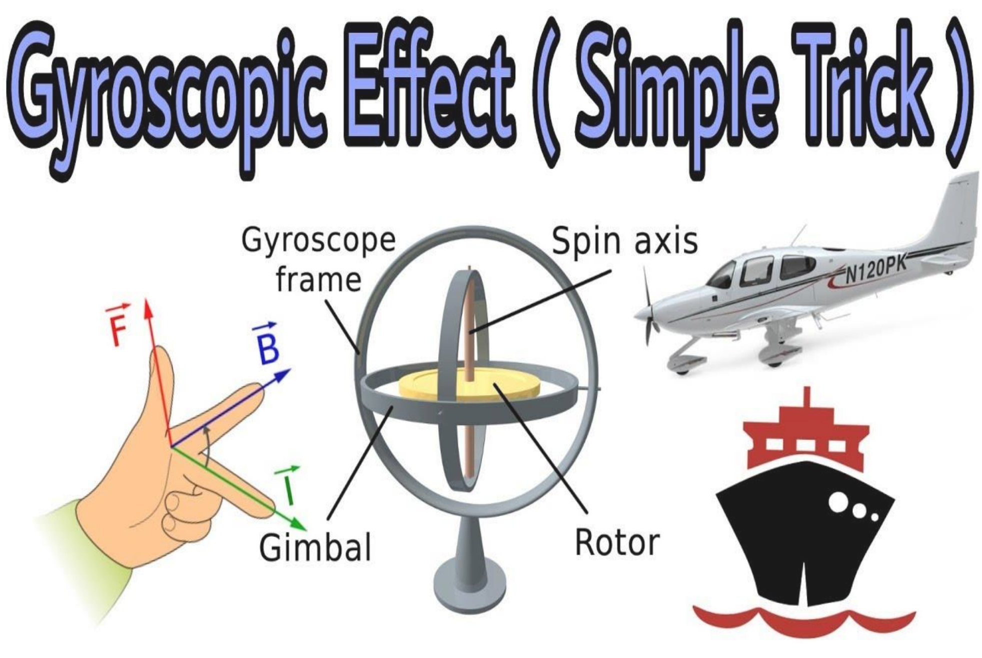
2. Motor cycle

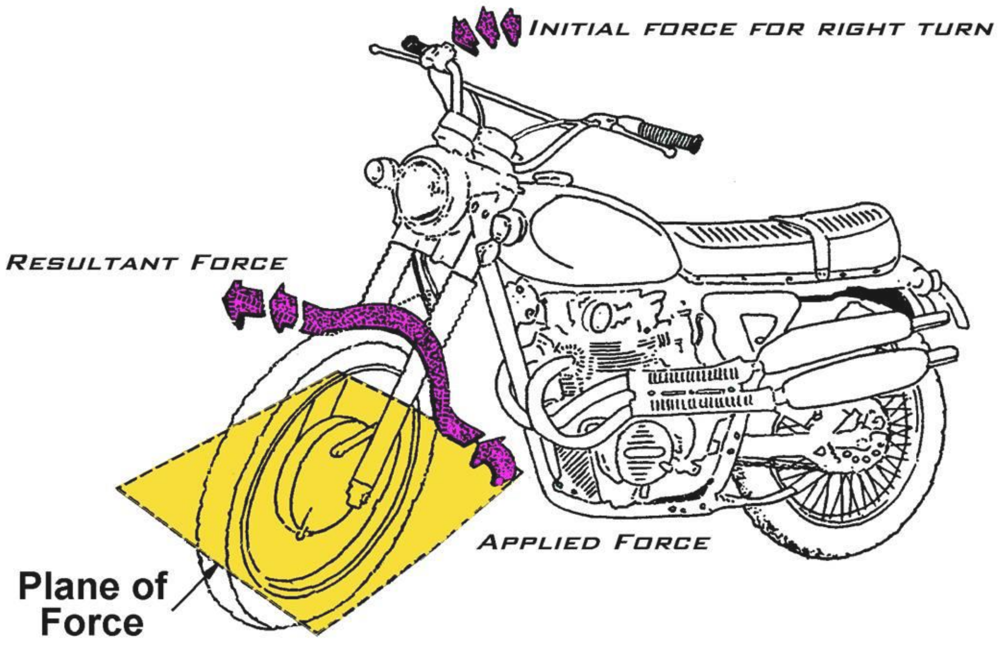
3. Aero planes

4. Ships

5. Applications of gyroscopes include inertial navigation systems, such as in the Hubble Telescope, or inside the steel hull of a submerged submarine. Due to their precision, gyroscopes are also used in gyrotheodolites to maintain direction in tunnel mining.






INITIAL FORCE FOR RIGHT TURN
RESULTANT FORCE
APPLIED FORCE
Plane of
Force

TUTORIAL QUESTIONS

1. What do you understand by gyroscopic couple? Derive a formula for its magnitude.
2. Explain the effect of the gyroscopic couple on the reaction of the four wheels of a vehicle negotiating a curve.
3. Discuss the effect of the gyroscopic couple on a two wheeled vehicle when taking a turn.
4. What will be the effect of the gyroscopic couple on a disc fixed at a certain angle to a rotating shaft?
5. A flywheel of mass 10 kg and radius of gyration 200 mm is spinning about its axis, which is horizontal and is suspended at a point distant 150 mm from the plane of rotation of the flywheel. Determine the angular velocity of precession of the flywheel. The spin speed of flywheel is 900 r.p.m.
6. Find the angle of inclination with respect to the vertical of a two wheeler negotiating a turn. Given : combined mass of the vehicle with its rider 250 kg ; moment of inertia of the engine flywheel 0.3 kg-m2 ; moment of inertia of each road wheel 1 kg-m2 ; speed of engine flywheel 5 times that of road wheels and in the same direction ; height of centre of gravity of rider with vehicle 0.6 m ; two wheeler speed 90 km/h ; wheel radius 300 mm ; radius of turn 50 m.
7. A pair of locomotive driving wheels with the axle, have a moment of inertia of 180 kg-m2. The diameter of the wheel treads is 1.8 m and the distance between wheel centres is 1.5 m. When the locomotive is travelling on a level track at 95 km/h, defective ballasting causes one wheel to fall 6 mm and to rise again in a total time of 0.1 s. If the displacement of the wheel takes place with simple harmonic motion, find: 1. The gyroscopic couple set up, and 2. The reaction between the wheel and rail due to this couple.
8. A four-wheeled trolley car of total mass 2000 kg running on rails of 1.6 m gauge, rounds a curve of 30 m radius at 54 km/h. The track is banked at 8°. The wheels have an external diameter of 0.7 m and each pair with axle has a mass of 200 kg. The radius of gyration for each pair is 0.3 m. The height of centre of gravity of the car above the wheel base is 1 m. Determine, allowing for centrifugal force and gyroscopic couple actions, the pressure on each rail.
9. A horizontal axle AB, 1 m long, is pivoted at the midpoint C. It carries a weight of 20 N at A and a wheel weighing 50 N at B. The wheel is made to spin at a speed of 600 r.p.m in a clockwise direction looking from its front. Assuming that the weight of the flywheel is uniformly distributed around the rim whose mean diameter is 0.6 m; calculate the angular velocity of precession of the system around the vertical axis through C.

ASSIGNMENT QUESTIONS

1. Explain the effect of the gyroscopic couple on the reaction of the four wheels of a vehicle negotiating a curve
2. Discuss the effect of the gyroscopic couple on a two wheeled vehicle when taking a turn.
3. What will be the effect of the gyroscopic couple on a disc fixed at a certain angle to a rotating shaft?
4. A uniform disc of 150 mm diameter has a mass of 5 kg. It is mounted centrally in bearings which maintain its axle in a horizontal plane. The disc spins about it axle with a constant speed of 1000 r.p.m. while the axle precesses uniformly about the vertical at 60 r.p.m. The directions of rotation are as shown in Fig. 14.3. If the distance between the bearings is 100 mm, find the resultant reaction at each bearing due to the mass and gyroscopic effects.
5. An aeroplane makes a complete half circle of 50 metres radius, towards left, when flying at 200 km per hr. The rotary engine and the propeller of the plane has a mass of 400 kg and a radius of gyration of 0.3 m. The engine rotates at 2400 r.p.m. clockwise when viewed from the rear. Find the gyroscopic couple on the aircraft and state its effect on it.
6. The turbine rotor of a ship has a mass of 8 tonnes and a radius of gyration 0.6 m. It rotates at 1800 r.p.m. clockwise, when looking from the stern. Determine the gyroscopic couple, if the ship travels at 100 km/hr and steer to the left in a curve of 75 m radius.
7. The heavy turbine rotor of a sea vessel rotates at 1500 r.p.m. clockwise looking from the stern, its mass being 750 kg. The vessel pitches with an angular velocity of 1 rad/s. Determine the gyroscopic couple transmitted to the hull when bow is rising, if the radius of gyration for the rotor is 250 mm. Also show in what direction the couple acts on the hull?
8. The turbine rotor of a ship has a mass of 3500 kg. It has a radius of gyration of 0.45 m and a speed of 3000 r.p.m. clockwise when looking from stern. Determine the gyroscopic couple and its effect upon the ship:

   1. when the ship is steering to the left on a curve of 100 m radius at a speed of 36 km/h.
   2. when the ship is pitching in a simple harmonic motion, the bow falling with its maximum velocity. The period of pitching is 40 seconds and the total angular displacement between the two extreme positions of pitching is 12 degrees.

9. A ship propelled by a turbine rotor which has a mass of 5 tonnes and a speed of 2100 r.p.m. The rotor has a radius of gyration of 0.5 m and rotates in a clockwise direction when viewed from the stern. Find the gyroscopic effects in the following conditions:

1. The ship sails at a speed of 30 km/h and steers to the left in a curve having 60 m radius.
2. The ship pitches 6 degree above and 6 degree below the horizontal position. The bow is descending with its maximum velocity. The motion due to pitching is simple harmonic and the periodic time is 20 seconds.
3. The ship rolls and at a certain instant it has an angular velocity of 0.03 rad/s clockwise when viewed from stern.

Determine also the maximum angular acceleration during pitching. Explain how the direction of motion due to gyroscopic effect is determined in each case.

UNIT 2

STATIC AND DYNAMICS FORCE ANALYSIS OF PLANNER MECHANISMS /FRICTION IN MACHINE MECHANISMS

Course Objectives

To understand the force-motion relationship in components subjected to external forces and analysis of standard mechanisms.

Course Outcomes

Able to predict the force analysis in mechanical system and able to solve the problem.

Introduction:

Then the system can be treated as static, which permits application of. Techniques of static force analysis. Dynamic force analysis is the evaluation of input forces or torques and joint forces. Considering motion of members. Evaluation of the inertia force /torque is explained first.

The inertia force is an imaginary force, which when acts upon a rigid body, brings it in an equilibrium position. It is numerically equal to the accelerating force in magnitude, but opposite in direction. Mathematically,

Inertia force = – Accelerating force = – m.a

Where m = Mass of the body, and

a = Linear acceleration of the centre of gravity of the body.

Similarly, the inertia torque is an imaginary torque, which when applied upon the rigid body, brings it in equilibrium position. It is equal to the accelerating couple in magnitude but opposite in direction.

Resultant Effect of a System of Forces Acting on a Rigid Body:

Consider a rigid body acted upon by a system of forces. These forces may be reduced to a single resultant force F whose line of action is at a distance h from the centre of gravity G. Now let us assume two equal and opposite forces (of magnitude F) acting through G, and parallel to the resultant force, without influencing the effect of the resultant force F, as shown in Fig. 15.1. A little consideration will show that the body is now subjected to a couple (equal to F × h) and a force; equal and parallel to the resultant force F passing through G. The force F through G causes linear acceleration of the c.g. and the moment of the couple (F × h) causes angular acceleration of the body about an axis passing through G and perpendicular to the point in which the couple acts.

α = Angular acceleration of the rigid body due to couple,

h = Perpendicular distance between the force and centre of gravity of the body,

m = Mass of the body,

k = Least radius of gyration about an axis through G, and

I = Moment of inertia of the body about an axis passing through its centre of gravity and perpendicular to the point in which the couple acts = m.k2

We know that Force,

F = Mass × Acceleration = m.a ...(i) and

$F.h = m.k^2.\alpha = I.\alpha$

D-Alembert's Principle

Consider a rigid body acted upon by a system of forces. The system may be reduced to a single resultant force acting on the body whose magnitude is given by the product of the mass of the body and the linear acceleration of the centre of mass of the body. According to Newton's second law of motion. F = m.a ...(i)

Where F = Resultant force acting on the body,

m = Mass of the body, and

a = Linear acceleration of the centre of mass of the body

The equation (i) may also be written as:

F – m.a = 0

A little consideration will show, that if the quantity – m.a be treated as a force, equal, opposite and with the same line of action as the resultant force F, and include this force with the system of forces of which F is the resultant, then the complete system of forces will be in equilibrium. This principle is known as D-Alembert's principle. The equal and opposite force – m.a is known as reversed effective force or the inertia force (briefly written as FI). The equation (ii) may be written as F + FI = 0 ... (iii)

Thus, D-Alembert's principle states that the resultant force acting on a body together with the reversed effective force (or inertia force), are in equilibrium. This principle is used to reduce a dynamic problem into an equivalent static problem.

Friction in Machine Elements

Screw Friction

The screws, bolts, studs, nuts etc. are widely used in various machines and structures for temporary fastenings. These fastenings have screw threads, which are made by cutting a continuous helical groove on a cylindrical surface. If the threads are cut on the outer surface of a solid rod, these are known as external threads. But if the threads are cut on the internal surface of a hollow rod, these are known as internal threads. The screw threads are mainly of two types i.e. V-threads and square threads. The V-threads are stronger and offer more frictional resistance to motion than square threads. Moreover, the V-threads have an advantage of preventing the nut from slackening. In general, the V threads are used for the purpose of tightening pieces together e.g. bolts and nuts etc. But the square threads are used in screw jacks, vice screws etc. The following terms are important for the study of screw

Helix. It is the curve traced by a particle, while describing a circular path at a uniform speed and advancing in the axial direction at a uniform rate. In other words, it is the curve traced by a particle while moving along a screw thread.

Pitch. It is the distance from a point of a screw to a corresponding point on the next thread, measured parallel to the axis of the screw

Lead. It is the distance; a screw thread advances axially in one turn.

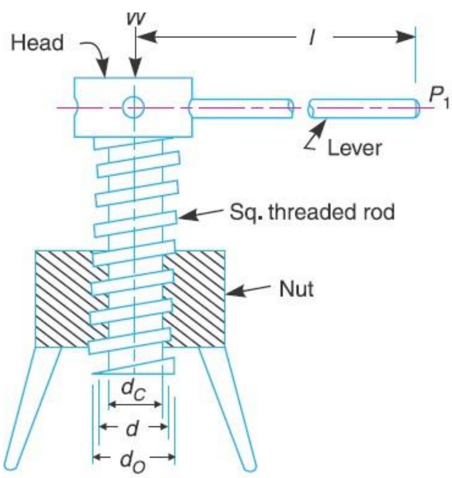
Depth of thread. It is the distance between the top and bottom surfaces of a thread (also known as crest and root of a thread).

Single-threaded screw. If the lead of a screw is equal to its pitch. It is known as single threaded screw.

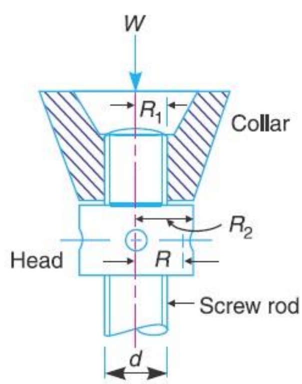
Lead = Pitch × Number of threads

Helix angle. It is the slope or inclination of the thread with the horizontal.

The screw jack is a device, for lifting heavy loads, by applying a comparatively smaller effort at its handle. The principle, on which a screw jack works, is similar to that of an inclined plane.



(*a*) Screw jack.



(*b*) Thrust collar.

Fig (*a*) shows a common form of a screw jack, which consists of a square threaded rod (also called screw rod or simply screw) which fits into the inner threads of the nut. The load, to be raised or lowered, is placed on the head of the square threaded rod which is rotated by the application of an effort at the end of the lever for lifting or lowering the load.

Torque Required to Lifting the Load by a Screw Jack :

If one complete turn of a screw thread by imagined to be unwound, from the body of the screw and developed, it will form an inclined plane as shown in Fig (*a*).

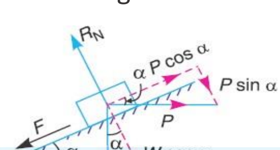
Let p = Pitch of the screw,

d = Mean diameter of the screw, α= Helix angle,

P = Effort applied at the circumference of the screw to lift the load,

W = Load to be lifted, and

μ = Coefficient of friction, between the screw and nut = tan ø, Where ϕ is the friction angle.



From the geometry of the Fig(*a*), we find that

$$\tan\alpha = p/\pi d$$

Since the principle on which a screw jack works is similar to that of an inclined plane, therefore the force applied on the lever of a screw jack may be considered to be horizontal as shown in Fig(*b*).

Since the load is being lifted, therefore the force of friction ($F = \mu.R_N$) will act downwards. All the forces acting on the screw are shown in Fig(*b*). Resolving the forces along the plane,

$P\cos\alpha = W\sin\alpha + F = W\sin\alpha + \mu.R_N$ ***(i)***

and resolving the forces perpendicular to the plane,

$R_N = P\sin\alpha + W\cos\alpha$ ***(ii)*** Substituting this value of R_N in equation(*i*),

$P\cos\alpha = W\sin\alpha + \mu\,(P\sin\alpha + W\cos\alpha)$

$= W\sin\alpha + \mu\,P\sin\alpha + \mu\,W\cos\alpha$

$P\cos\alpha - \mu\,P\sin\alpha = W\sin\alpha + \mu\,W\cos\alpha$

$P\,(\cos\alpha - \mu\sin\alpha) = W\,(\sin\alpha + \mu\cos\alpha)$

$$P = W \times \frac{\sin\alpha + \mu\cos\alpha}{\cos\alpha - \mu\sin\alpha}$$

Substituting the value of $\mu = \tan\phi$ in the above equation, we get

$$P = W \times \frac{\sin\alpha + \tan\phi\cos\alpha}{\cos\alpha - \tan\phi\sin\alpha}$$

Multiplying the numerator and denominator by $\cos\phi$,

$$P = W \times \frac{\sin\alpha\cos\phi + \sin\phi\cos\alpha}{\cos\alpha\cos\phi - \sin\alpha\sin\phi} = W \times \frac{\sin(\alpha + \phi)}{\cos(\alpha + \phi)}$$

$= W\tan(\alpha + \phi)$

$$T_1 = P \times \frac{d}{2} = W\tan(\alpha + \phi)\frac{d}{2}$$

Torque required to overcoming friction between the screw and nut,

When the axial load is taken up by a thrust collar or a flat surface, as shown in Fig (*b*),so that the load does not rotate with the screw, then the torque required to overcome friction at the collar,

$$T_2 = \mu_1 . W\left(\frac{R_1 + R_2}{2}\right) = \mu_1 . W . R$$

R_1 and R_2 = Outside and inside radii of the collar,

R = Mean radius of the collar, and

μ_1 = Coefficient of friction for the collar.

$$T = T_1 + T_2 = P \times \frac{d}{2} + \mu_1 . W . R$$

Total torque required to overcome friction (*i.e.* to rotate the screw),

If an effort P_1 is applied at the end of a lever of arm length l, then the total torque required to overcome friction must be equal to the torque applied at the end of the lever, *i.e.*

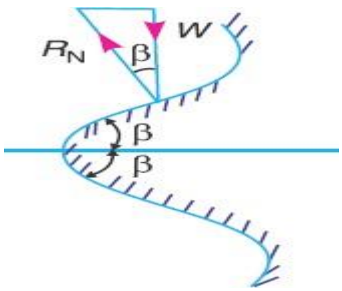
$$T = P \times \frac{d}{2} = P_1 . l$$

Friction of a V-thread

The normal reaction in case of a square threaded screw is

$R_N = W \cos\alpha$, where α= Helix angle.

But in case of V-thread (or acme or trapezoidal threads), the normal reaction between the screw and nut is increased because the axial component of this normal reaction must be equal to the axial load.



W, as shown in Fig.

Let 2β= Angle of the V-thread, and

$$R_N = \frac{W}{\cos\beta}$$

$$\text{frictional force, } F = \mu . R_N = \mu \times \frac{W}{\cos\beta} = \mu_1 . W$$

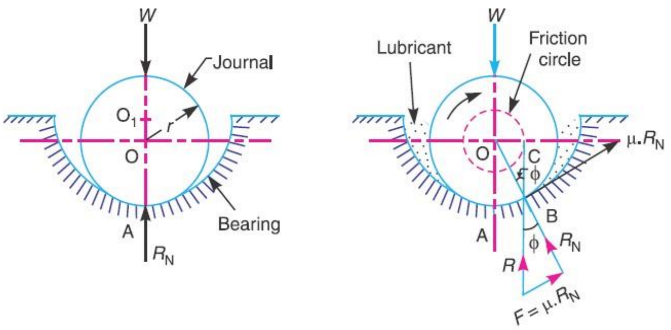
β= Semi-angle of the V-thread.

Friction in Journal Bearing-Friction Circle

A journal bearing forms a turning pair as shown in Fig (*a*). The fixed outer element of a turning pair is

$$\frac{\mu}{\cos\beta} = \mu_1, \text{ known as virtual coefficient of friction.}$$

called a ***bearing*** and that portion of the inner element (*i.e.* shaft) which fits in the bearing is called a ***journal.*** The journal is slightly less in diameter than the bearing, in order to permit the free movement of the journal in a bearing.



(*a*) (*b*)

When the bearing is not lubricated (or the journal is stationary), then there is a line contact between the two elements as shown in Fig (a). The load W on the journal and normal reaction R_N (equal to W) of the bearing acts through the centre. The reaction RN acts vertically upwards at point A. This point A is known as **seat** or **point of pressure.**

Now consider a shaft rotating inside a bearing in clockwise direction as shown in Fig(*b*). The lubricant between the journal and bearing forms a thin layer which gives rise to a greasy friction. Therefore, the reaction *R* does not act vertically upward, but acts at another point of pressure *B*. This is due to the fact that when shaft rotates, a frictional force $F = \mu R_N$ acts at the circumference of the shaft which has a tendency to rotate the shaft in opposite direction of motion and this shifts the point *A* to point *B*. In order that the rotation may be maintained, there must be a couple rotating the shaft.

Let ø= Angle between *R* (resultant of *F* and *R*N) and R_N,

μ = Coefficient of friction between the journal and bearing,

T = Frictional torque in N-m, and

r = Radius of the shaft in meters.

For uniform motion, the resultant force acting on the shaft must be zero and the resultant turning moment on the shaft must be zero. In other words,

$R = W$, and $T = W \times OC = W \times OB \sin\phi = W.r\sin\phi$

Since ø is very small, therefore substituting $\sin\phi = \tan\phi$

$T = W.r\tan\phi = \mu.W.r \quad (\mu = \tan\phi)$

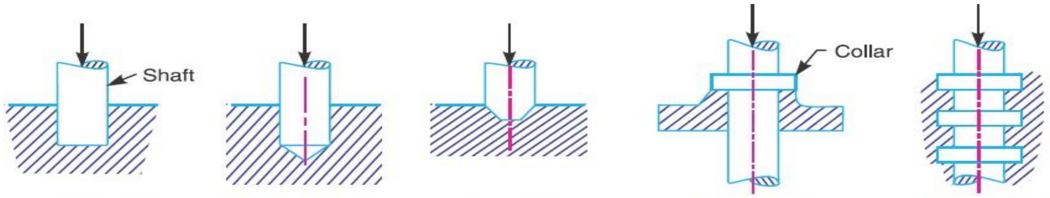
If the shaft rotates with angular velocity ω rad/s, then power wasted in friction,

$P = T\omega = T \times 2\pi N/60$ watts Where N = Speed of the shaft in r.p.m.

Friction of Pivot and Collar Bearing

The rotating shafts are frequently subjected to axial thrust. The bearing surfaces such as pivot and collar bearings are used to take this axial thrust of the rotating shaft. The propeller shafts of ships, the shafts of steam turbines, and vertical machine shafts are examples of shafts which carry an axial thrust. The bearing surfaces placed at the end of a shaft to take the axial thrust are known as pivots. The pivot may have a flat surface or conical surface as shown in Fig. 10.16 (a) and (b) respectively. When the cone is truncated, it is then known as truncated or trapezoidal pivot as shown in Fig (c).

The collar may have flat bearing surface or conical bearing surface, but the flat surface is most commonly used. There may be a single collar, as shown in Fig (*d*) or several collars along the length of a shaft, as shown in Fig(*e*) in order to reduce the intensity of pressure.



(*a*) Flat pivot. (*b*) Conical pivot. (*c*) Truncated pivot. (*d*) Single flat collar. (*e*) Multiple flat collar.

In modern practice, ball and roller thrust bearings are used when power is being transmitted and when thrusts are large as in case of propeller shafts of ships.

A little consideration will show that in a new bearing, the contact between the shaft and bearing may be good over the whole surface. In other words, we can say that the pressure over the rubbing surfaces is uniformly distributed. But when the bearing becomes old, all parts of the rubbing surface will not move with the same velocity, because the velocity of rubbing surface increases with the distance from the axis of the bearing. This means that wear may be different at different radii and this causes to alter the distribution of pressure. Hence, in the study of friction of bearings, it is assumed hat

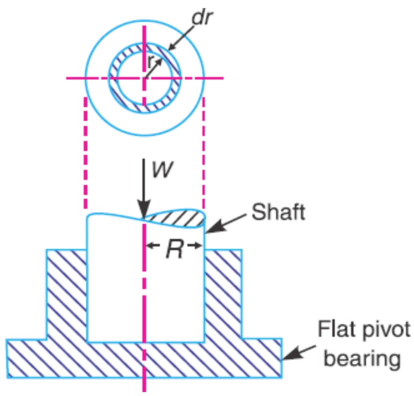
The pressure is uniformly distributed throughout the bearing surface, and

The wear is uniform throughout the bearing surface.

Flat Pivot Bearing:

When a vertical shaft rotates in a flat pivot bearing (known as **foot step bearing**), as shown in Fig., the sliding friction will be along the surface of contact between the shaft and the bearing.

Let W =Load transmitted over the bearing surface,



R =Radius of bearing surface,

p =Intensity of pressure per unit area of bearing Surface between rubbing surfaces, and

μ =Coefficient of friction.

We will consider the following two cases:

1. When there is a uniform pressure
2. When there is a uniform wear

Considering uniform pressure

$$p = \frac{W}{\pi R^2}$$

When the pressure is uniformly distributed over the bearing area, then

Consider a ring of radius r and thickness $dr$of the bearing area. Area of bearing surface, $A = 2\pi r.dr$

Load transmitted to the ring,

$\delta W= p \times A = p \times 2\pi r.dr$ **(*i*)**

Frictional resistance to sliding on the ring acting tangentially at radius r, $F_r= \mu.\delta W= \mu\ p \times 2\pi\ r.dr=2\pi\mu.p.r.dr$

Frictional torque on the ring,

$T_r = F_r \times r= 2\pi\ \mu\ p\ r.dr\times r = 2\ \pi\ \mu\ pr^2dr$ **(*ii*)**

Integrating this equation within the limits from 0 to *R* for the total frictional torque on the pivot bearing.

$$\therefore \text{ Total frictional torque, } T = \int_0^R 2\pi\mu\, p r^2\, dr = 2\pi\mu\, p \int_0^R r^2\, dr$$

$$= 2\pi\mu\, p \left[\frac{r^3}{3}\right]_0^R = 2\pi\mu\, p \times \frac{R^3}{3} = \frac{2}{3} \times \pi\mu.p.R^3$$

$$= \frac{2}{3} \times \pi\mu \times \frac{W}{\pi R^2} \times R^3 = \frac{2}{3} \times \mu.W.R \qquad \ldots\left(\because\ p = \frac{W}{\pi R^2}\right)$$

When the shaft rotates at ω rad/s, then power lost in friction,

$$P = T.\omega = T \times 2\pi N/60 \qquad \ldots(\because\ \omega = 2\pi N/60)$$

N = Speed of shaft in r.p.m.

Considering uniform wear

We have already discussed that the rate of wear depends upon the intensity of pressure and the velocity of rubbing surfaces (*v*). It is assumed that the rate of wear is proportional to the product of intensity of pressure and the velocity of rubbing surfaces (*i.e. p.v.*). Since the velocity of rubbing surfaces increases with the distance (*i.e.* radius *r*) from the axis of the bearing, therefore for uniform Wear

$$p.r = C \text{ (a constant)} \quad \text{or} \quad p = C/r$$

and the load transmitted to the ring,

$$\delta W = p \times 2\pi r.dr \qquad \text{...[From equation } (i)]$$

$$= \frac{C}{r} \times 2\pi r.dr = 2\pi C.dr$$

∴ Total load transmitted to the bearing

$$W = \int_0^R 2\pi\, C.dr = 2\pi\, C[r]_0^R = 2\pi C.R \quad \text{or} \quad C = \frac{W}{2\pi R}$$

We know that frictional torque acting on the ring,

$$T_r = 2\pi\mu\, p\, r^2\, dr = 2\pi\mu \times \frac{C}{r} \times r^2\, dr \qquad \text{...}\left(\because\ p = \frac{C}{r}\right)$$

$$= 2\pi\, \mu.C.r\, dr \qquad \text{...}(iii)$$

∴ Total frictional torque on the bearing,

$$T = \int_0^R 2\pi\, \mu.C.r.dr = 2\pi\mu.C\left[\frac{r^2}{2}\right]_0^R$$

$$= 2\pi\mu.C \times \frac{R^2}{2} = \pi\mu.C.R^2$$

$$= \pi\mu \times \frac{W}{2\pi R} \times R^2 = \frac{1}{2} \times \mu.W.R \qquad \text{...}\left(\because\ C = \frac{W}{2\pi R}\right)$$

PROBLEMS

Example 1.A vertical shaft 150 mm in diameter rotating at 100 r.p.m. rests on a flat end foot step bearing. The shaft carries a vertical load of 20 kN. Assuming uniform pressure distribution and coefficient of friction equal to 0.05, estimate power lost in friction.

Solution. Given :*D* = 150 mm or *R* = 75 mm = 0.075 m ; *N* = 100 r.p.m or ω= 2 δ × 100/60= 10.47 rad/s ; *W* = 20 kN = 20 × 103 N ; μ = 0.05

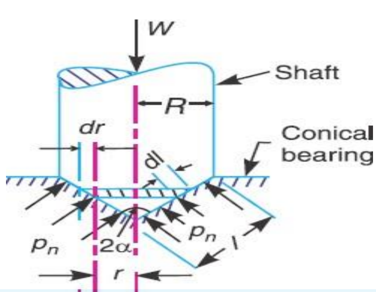
We know that for uniform pressure distribution, the total frictional torque,

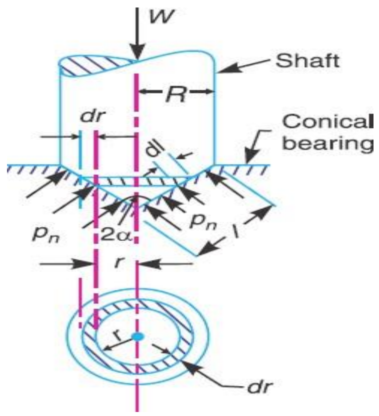
$$T = \frac{2}{3} \times \mu.W.R = \frac{2}{3} \times 0.05 \times 20 \times 10^3 \times 0.075 = 50 \text{ N-m}$$

∴ Power lost in friction,

$$P = T.\omega = 50 \times 10.47 = 523.5 \text{ W} \quad \textbf{Ans.}$$

Conical Pivot Bearing





The conical pivot bearing supporting a shaft carrying a load *W* is shown in Fig. Let

P_n = Intensity of pressure normal to the cone,

α = Semi angle of the cone,

μ = Coefficient of friction between the shaft and the bearing,

R = Radius of the shaft.

Consider a small ring of radius *r* and thickness *dr.*

Let *dl* is the length of ring along the cone, such that

$dl = dr \operatorname{cosec} \alpha$

Area of the ring, $A = 2\pi r.dl = 2\pi r.dr \operatorname{cosec} \alpha$ ($dl = dr \operatorname{cosec}\alpha$)

Considering uniform pressure

We know that normal load acting on the ring, δW_n= Normal pressure ×Area

$= p_n \times 2\pi r.dr \operatorname{cosec} \alpha$ vertical load acting on the ring,

δW= Vertical component of $\delta W_n = \delta W_n.\sin\alpha$ Total vertical load transmitted to the bearing,

$$W = \int_0^R p_n \times 2\pi r.dr = 2\pi\, p_n \left[\frac{r^2}{2}\right]_0^R = 2\pi\, p_n \times \frac{R^2}{2} = \pi R^2.p_n$$

$$p_n = W / \pi R^2$$

$$T_r = F_r \times r = 2\pi\mu.p_n.\operatorname{cosec}\alpha.r.dr \times r = 2\pi\mu.p_n \operatorname{cosec}\alpha.r^2.dr$$

We know that frictional force on the ring acting tangentially at radius *r*,

The vertical load acting on the ring is also given by δW= Vertical component of p_n× Area of the ring

$$= p_n \sin\alpha \times 2\pi r.dr.\text{cosec}\,\alpha = p_n \times 2\pi r.dr$$

Integrating the expression within the limits from 0 to *R* for the total frictional torque on the conical pivot bearing.

$$T = \int_0^R 2\,\pi\,\mu.p_n\ \text{cosec}\,\alpha.r^2\ dr = 2\,\pi\mu.p_n.\text{cosec}\,\alpha\left[\frac{r^3}{3}\right]_0^R$$

$$= 2\pi\mu.p_n.\text{cosec}\,\alpha \times \frac{R^3}{3} = \frac{2\pi\ R^3}{3} \times \mu.p_n.\text{cosec}\,\alpha \qquad ...(i)$$

Total frictional torque:

$$T = \int_0^R 2\,\pi\,\mu.p_n\ \text{cosec}\,\alpha.r^2\ dr = 2\,\pi\mu.p_n.\text{cosec}\,\alpha\left[\frac{r^3}{3}\right]_0^R$$

$$= 2\pi\mu.p_n.\text{cosec}\,\alpha \times \frac{R^3}{3} = \frac{2\pi\ R^3}{3} \times \mu.p_n.\text{cosec}\,\alpha \qquad ...(i)$$

Substituting the value of $p_n$in equation (*i*),

$$T = \frac{2\pi R^3}{3} \times \pi \times \frac{W}{\pi R^2} \times \text{cosec}\,\alpha = \frac{2}{3} \times \mu.W.R.\ \text{cosec}\,\alpha$$

Considering uniform wear

In Fig. let p_r be the normal intensity of pressure at a distance *r* from the central axis. We know that, in case of uniform wear, the intensity of pressure varies inversely with the distance.

$p_r.r = C$ (a constant) or $p_r = C/r$

$$\delta W = p_r \times 2\pi r.dr = \frac{C}{r} \times 2\pi r.dr = 2\pi C.dr$$

The load transmitted to the ring,

Total load transmitted to the bearing,

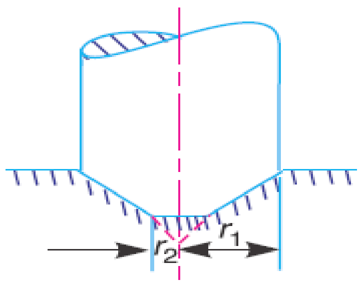
$$W = \int_0^R 2\pi C.dr = 2\pi C\,[r]_0^R = 2\pi C.R \quad \text{or} \quad C = \frac{W}{2\pi R}$$

We know that frictional torque acting on the ring, Total frictional torque acting on the bearing,

$$T = \pi\mu \times \frac{W}{2\pi R} \times \text{cosec } \alpha . R^2 = \frac{1}{2} \times \mu . W . R \text{ cosec } \alpha = \frac{1}{2} \times \mu . W . l$$

Substituting the value of *C*, we have

Trapezoidal or Truncated Conical Pivot Bearing



Area of the bearing surface,

$$A = \pi[(r_1)^2 - (r_2)^2]$$

∴ Intensity of uniform pressure,

$$p_n = \frac{W}{A} = \frac{W}{\pi[(r_1)^2 - (r_2)^2]} \quad \text{...}(i)$$

If the pivot bearing is not conical, but a frustum of a cone with *r*1 and *r*2, the external and internal radius respectively as shown in Fig, then

Considering uniform pressure

The total torque acting on the bearing is obtained by integrating the value of T_r, within the limits r_1 and r_2.

$$T = \int_{r_2}^{r_1} 2\pi\mu . p_n \text{ cosec } \alpha . r^2 . dr = 2\pi\mu . p_n . \text{cosec } \alpha \left[\frac{r^3}{3}\right]_{r_2}^{r_1}$$

$$= 2\pi\mu . p_n . \text{cosec } \alpha \left[\frac{(r_1)^3 - (r_2)^3}{3}\right]$$

Total torque acting on the bearing,

$$W = \int_{r_2}^{r_1} 2\pi \, C . dr = 2\pi C [r]_{r_2}^{r_1} = 2\pi C (r_1 - r_2)$$

$$C = \frac{W}{2\pi(r_1 - r_2)}$$

$$T = 2\pi\mu \times \frac{W}{\pi[(r_1)^2 - (r_2)^2]} \times \text{cosec}\ \alpha\left[\frac{(r_1)^3 - (r_2)^3}{3}\right]$$

$$= \frac{2}{3} \times \mu.W.\text{cosec}\ \alpha\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right]$$

Substituting the value of p_n from equation **(*i*),**

Considering uniform wear the load transmitted to the ring, $\delta W = 2\pi C.dr$

Total load transmitted to the ring,

We know that the torque acting on the ring, considering uniform wear, is Total torque acting on the bearing,

$$T = \int_{r_2}^{r_1} 2\pi\ \mu.C\ \text{cosec}\ \alpha.r.dr = 2\pi\ \mu.C.\text{cosec}\ \alpha\left[\frac{r^2}{2}\right]_{r_2}^{r_1}$$

$$= \pi\ \mu.C.\text{cosec}\ \alpha\left[(r_1)^2 - (r_2)^2\right]$$

Substituting the value of C from equation **(*ii*),** we get

$$T = \pi\mu \times \frac{W}{2\pi(r_1 - r_2)} \times \text{cosec}\ \alpha\ [(r_1)^2 - (r_2)^2]$$

$$= \frac{1}{2} \times \mu.W\ (r_1 + r_2)\ \text{cosec}\ \alpha = \mu.W.R\ \text{cosec}\ \alpha$$

$$R = \text{Mean radius of the bearing} = \frac{r_1 + r_2}{2}$$

PROBLEMS

Example 1.A conical pivot supports a load of 20 kN, the cone angle is 120º and the intensity of normal pressure is not to exceed 0.3 N/mm2. The external diameter is twice the internal diameter. Find the outer and inner radii of the bearing surface. If the shaft rotates at 200 r.p.m. and the coefficient of friction is 0.1, find the power absorbed in friction. Assume uniform pressure.

Solution: Given: W = 20 kN = 20 × 103 N ; 2 α= 120º or α= 60º ; p_n= 0.3 N/mm^2 ;N = 200 r.p.m. or ω= 2π× 200/60 = 20.95 rad/s ; μ =0.1

Outer and inner radii of the bearing surface.

Let r_1 and r_2 = Outer and inner radii of the bearing surface, in mm. Since the external diameter is twice the internal diameter, therefore

$r_1 = 2\, r_2$

$$0.3 = \frac{W}{\pi[(r_1)^2 - (r_2)^2]} = \frac{20 \times 10^3}{\pi[(2r_2)^2 - (r_2)^2]} = \frac{2.12 \times 10^3}{(r_2)^2}$$

$$(r_2)^2 = 2.12 \times 10^3 / 0.3 = 7.07 \times 10^3 \quad \text{or} \quad r_2 = 84 \text{ mm} \quad \textbf{Ans.}$$

$$r_1 = 2\, r_2 = 2 \times 84 = 168 \text{ mm} \quad \textbf{Ans.}$$

We know that intensity of normal pressure (p_n),

Power absorbed in friction

$$T = \frac{2}{3} \times \mu.W.\text{cosec}\,\alpha \left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right]$$

$$= \frac{2}{3} \times 0.1 \times 20 \times 10^3 \times \text{cosec}\,60^\circ = \left[\frac{(168)^3 - (84)^3}{(168)^2 - (84)^2}\right] \text{N-mm}$$

$$= 301760 \text{ N-mm} = 301.76 \text{ N-m}$$

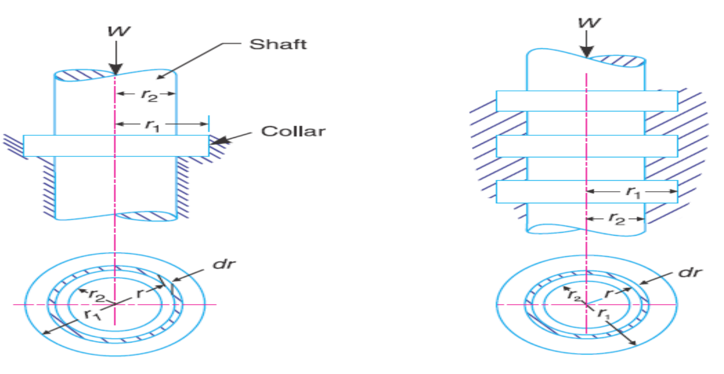
We know that total frictional torque (assuming uniform pressure),

Power absorbed in friction

$P = T.\omega = 301.76 \times 20.95 = 6322$ W = 6.322 kW

Flat Collar Bearing

We have already discussed that collar bearings are used to take the axial thrust of the rotating hafts. There may be a single collar or multiple collar bearings as shown in Fig.(*a*) and (*b*)respectively. The collar bearings are also known as ***thrust bearings.*** The friction in the collar bearings may be found as discussed below:



(*a*) Single collar bearing

(*b*) Multiple collar bearing.

Consider a single flat collar bearing supporting a shaft as shown in Fig(*a*).

Let r_1 = External radius of the collar,

r_2 = Internal radius of the collar.

Area of the bearing surface,

$$A = \pi\,[(r_1)^2 - (r_2)^2]$$

Considering uniform pressure

When the pressure is uniformly distributed over the bearing surface, then the intensity of pressure,

$$p = \frac{W}{A} = \frac{W}{\pi[r_1)^2 - (r_2)^2]} \qquad \ldots(i)$$

The frictional torque on the ring of radius r and thickness dr,

$$T_r = 2\pi\mu.p.r^2.dr$$

Integrating this equation within the limits from r_2 to r_1 for the total frictional torque on the collar.

Total frictional torque,

$$T = \int_{r_2}^{r_1} 2\pi\mu.p.r^2.dr = 2\pi\mu.p\left[\frac{r_3}{3}\right]_{r_2}^{r_1} = 2\pi\mu.p\left[\frac{(r_1)^3 - (r_2)^3}{3}\right]$$

$$T = 2\pi\mu \times \frac{W}{\pi[(r_1)^2 - (r_2)^2]}\left[\frac{(r_1)^3 - (r_2)^3}{3}\right]$$

$$= \frac{2}{3} \times \mu.W\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right]$$

Substituting the value of p from equation **(*i*),**

Considering uniform wear

The load transmitted on the ring, considering uniform wear is,

$$\delta W = p_r.2\pi r.dr = \frac{C}{r} \times 2\pi r.dr = 2\pi C.dr$$

$$W = \int_{r_2}^{r_1} 2\pi C.dr = 2\pi C[r]_{r_2}^{r_1} = 2\pi C(r_1 - r_2)$$

$$C = \frac{W}{2\pi(r_1 - r_2)} \qquad \ldots(ii)$$

Total load transmitted to the collar,

We also know that frictional torque on the ring; we also know that frictional torque on the ring,

$$T_r = \mu.\delta W.r = \mu \times 2\pi C.dr.r = 2\pi\mu.C.r.dr$$

Total frictional torque on the bearing,

$$T = \int_{r_2}^{r_1} 2\pi\mu C.r.dr = 2\pi\mu.C\left[\frac{r^2}{2}\right]_{r_2}^{r_1} = 2\pi\mu.C\left[\frac{(r_1)^2 - (r_2)^2}{2}\right]$$

$$= \pi\mu.C[(r_1)^2 - (r_2)^2]$$

Substituting the value of *C* from equation ***(ii)*,**

$$T = \pi\mu \times \frac{W}{2\pi(r_1 - r_2)}[(r_1)^2 - (r_2)^2] = \frac{1}{2} \times \mu.W(r_1 + r_2)$$

PROBLEMS

Example 1.A thrust shaft of a ship has 6collars of 600 mm external diameter and 300 mm internal diameter. The total thrust from the propeller is 100 kN. If the coefficient of friction is 0.12 and speed of the engine90 r.p.m., find the power absorbed in friction at the thrust block, assuming.

1. Uniform pressure

2. Uniform wear.

Solution. Given: n = 6 ; d_1 = 600 mm or r_1 = 300mm ;d_2 = 300 mm or r_2 = 150 mm ;

W = 100 kN= 100 × 10^3 N ;

μ = 0.12 ; N = 90 r.p.m. or ω= 2 π× 90/60 = 9.426 rad/s

Power absorbed in friction, assuming uniform pressure

We know that total frictional torque transmitted,

Power absorbed in friction,

$P = T\omega$= 2800 × 9.426 = 26 400 W = 26.4 kW

Power absorbed in friction assuming uniform wear

We know that total frictional torque transmitted,

$$T = \frac{1}{2} \times \mu.W(r_1 + r_2) = \frac{1}{2} \times 0.12 \times 100 \times 10^3\ (300 + 150)\ \text{N-mm}$$

$$= 2700 \times 10^3\ \text{N-mm} = 2700\ \text{N-m}$$

Power absorbed in friction,

$P = T.\omega$ = 2700 × 9.426 = 25 450 W = 25.45 kW

Example 2.A shaft has a number of a collars integral with it. The external diameter of the collars is 400 mm and the shaft diameter is 250 mm. If the intensity of pressure is 0.35 N/mm^2(uniform) and the coefficient of friction is 0.05, estimate power absorbed when the shaft runs at 105 r.p.m. carrying a load of 150 kN. Number of collars required.

Solution. Given: d1 = 400 mm or r_1 = 200 mm ; d_2 = 250 mm or r_2 = 125 mm ; p = 0.35N/mm^2 ; μ = 0.05 ; N = 105 r.p.m or

$\omega = 2\pi \times 105/60 = 11$ rad/s ; W = 150 kN = 150×10^3 N

Power absorbed

We know that for uniform pressure, total frictional torque transmitted

$$T = \frac{2}{3} \times \mu.W \left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right] = \frac{2}{3} \times 0.05 \times 150 \times 10^3 \left[\frac{(200)^3 - (125)^3}{(200)^2 - (125)^2}\right] \text{N-mm}$$

$$= 5000 \times 248 = 1240 \times 10^3 \text{ N-mm} = 1240 \text{ N-m}$$

Power absorbed,

$P = T\omega = 1240 \times 11 = 13640$ W = 13.64 kW

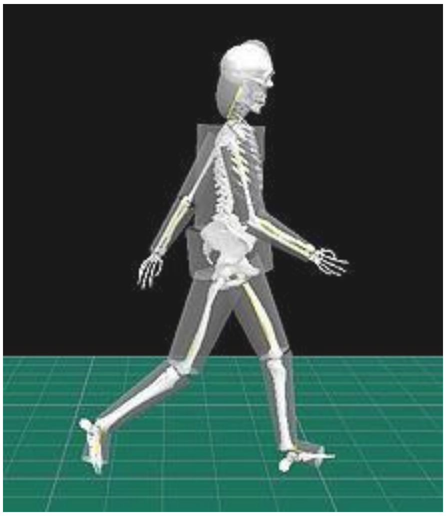
Number of collarsrequired

Let n = Number of collarsrequired.

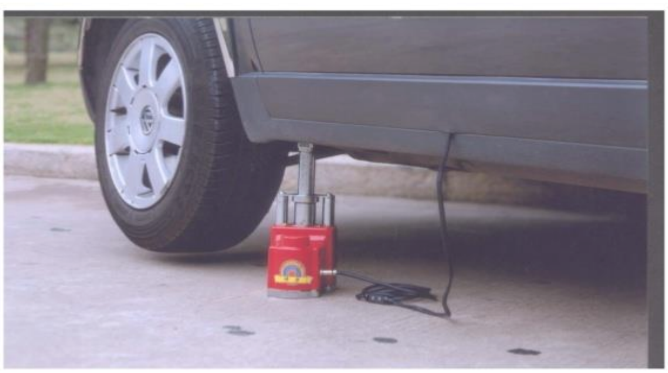
We know that the intensity of uniform pressure (p),

INDUSTRIAL APPLICATIONS

1. Human body modeled as a system of rigid bodies of geometrical solids. Representative bones were added for better visualization of the walking person.



2. Screw jack



In a ***bottle jack*** the piston is vertical and directly supports a bearing pad that contacts the object being lifted. With a single action piston the lift is somewhat less than twice the collapsed height of the jack, making it suitable only for vehicles with a relatively high clearance. For lifting structures such as houses the hydraulic interconnection of multiple vertical jacks through valves enables the even distribution of forces while enabling close control of the lift.

In a ***floor jack*** a horizontal piston pushes on the short end of a bellcrank , with the long arm providing the vertical motion to a lifting pad, kept horizontal with a horizontal linkage. Floor jacks usually include castors and wheels, allowing compensation for the arc taken by the lifting pad. This mechanism provide a low profile when collapsed, for easy maneuvering underneath the vehicle, while allowing considerable extension

.

TUTORIAL QUESTIONS

1. Discuss briefly the various types of friction experienced by a body
2. Derive from first principles an expression for the effort required to raise a load with a screw jack taking friction into consideration.
3. What is meant by the expression 'friction circle'? Deduce an expression for the radius of friction circle in terms of the radius of the journal and the angle of friction.
4. Derive from first principles an expression for the friction moment of a conical pivot assuming (i) Uniform pressure, and (ii) Uniform wear.
5. Find the force required to move a load of 300 N up a rough plane, the force being applied parallel to the plane. The inclination of the plane is such that a force of 60 N inclined at 30º to a similar smooth plane would keep the same load in equilibrium. The coefficient of friction is 0.3.
6. A square threaded screw of mean diameter 25 mm and pitch of thread 6 mm is utilised to lift a weight of 10 kN by a horizontal force applied at the circumference of the screw. Find the magnitude of the
7. Force if the coefficient of friction between the nut and screw is 0.02.
8. A bolt with a square threaded screw has mean diameter of 25 mm and a pitch of 3 mm. It carries an axial thrust of 10 kN on the bolt head of 25 mm mean radius. If μ = 0.12, find the force required at the end of a spanner 450 mm long, in tightening up the bolt.

ASSIGNMENT QUESTIONS

1. The thrust of a propeller shaft in a marine engine is taken up by a number of collars integral with the shaft which is 300 mm in diameter. The thrust on the shaft is 200 kN and the speed is 75 r.p.m. Taking μ constant and equal to 0.05 and assuming intensity of pressure as uniform and equal to 0.3 N/mm2, find the external diameter of the collars and the number of collars required, if the power lost in friction is not to exceed 16 kW.
2. A shaft has a number of a collars integral with it. The external diameter of the collars is 400 mm and the shaft diameter is 250 mm. If the intensity of pressure is 0.35 N/mm2 (uniform) and the coefficient of friction is 0.05, estimate: 1. power absorbed when the shaft runs at 105 r.p.m. carrying a load of 150 kN and 2. number of collars required.
3. A thrust shaft of a ship has 6 collars of 600 mm external diameter and 300 mm internal diameter. The total thrust from the propeller is 100 kN. If the coefficient of friction is 0.12 and speed of the engine 90 r.p.m., find the power absorbed in friction at the thrust block, assuming l. uniform pressure and 2. Uniform wear.
4. A conical pivot bearing supports a vertical shaft of 200 mm diameter. It is subjected to a load of 30 kN. The angle of the cone is 120º and the coefficient of friction is 0.025. Find the power lost in friction when the speed is 140 r.p.m., assuming 1. uniform pressure ; and 2. uniform wear.
5. A conical pivot supports a load of 20 kN, the cone angle is 120º and the intensity of normal pressure is not to exceed 0.3 N/mm2. The external diameter is twice the internal diameter. Find the outer and inner radii of the bearing surface. If the shaft rotates at 200 r.p.m. and the coefficient of friction is 0.1, find the power absorbed in friction. Assume uniform pressure.
6. A vertical shaft 150 mm in diameter rotating at 100 r.p.m. rests on a flat end footstep bearing. The shaft carries a vertical load of 20 kN. Assuming uniform pressure distribution and coefficient of friction equal to 0.05, estimate power lost in friction.
7. Derive the Resultant Effect of a System of Forces Acting on a Rigid Body ?
8. Explain the D-Alembert's Principle?
9. Explain Velocity and Acceleration of the Reciprocating Parts in Engines?
10. The crank and connecting rod of a reciprocating engine are 200 mm and 700 mm respectively. The crank is rotating in clockwise direction at 120 rad/s. Find with the help of Klein's construction: 1. Velocity and acceleration of the piston, 2. Velocity and acceleration of the midpoint of the connecting rod, and 3. Angular velocity and angular acceleration of the connecting rod, at the instant when the crank is at 30° to I.D.C. (inner dead centre).

UNIT 3

CLUTCHES,BRAKE/DYNAMOMETERS/ TURNING MOMENT DIAGRAM & FLY WHEEL

Course Objectives

Able to learn about the working of Clutches, Brakes, Dynamometers and Fly wheel.

Course Outcomes

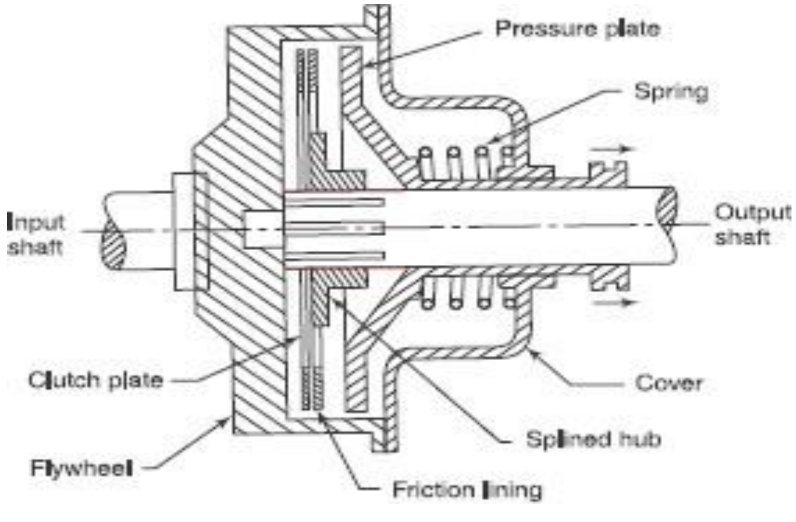
The student will learn about the kinematics and dynamic analysis of machine elements.

FRICTION CLUTCHES

A clutch is a device used to transmit the rotary motion of one shaft to another when desired. The axes of the two shafts are coincident. In friction clutches, the connection of the engine shaft to the gear box shaft is affected by friction between two or more rotating concentric surfaces. The surfaces can be pressed firmly against one another when engaged and the clutch tends to rotate as a single unit.

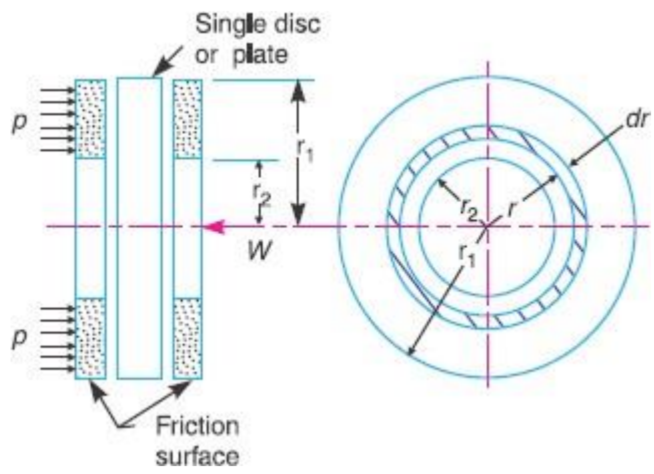
SINGLE PLATE CLUTCH (DISC CLUTCH)

A disc clutch consists of a clutch plate attached to a splined hub which is free to slide axially on splines cut on the driven shaft. The clutch plate is made of steel and has a ring of friction lining on each side. The engine shaft supports a rigidly fixed flywheel. A spring-loaded pressure plate presses the clutch plate firmly against the flywheel when the clutch is engaged. When disengaged, the springs press against a cover attached to the flywheel. Thus, both the flywheel and the pressure plate rotate with the input shaft. The movement of the clutch pedal is transferred to the pressure plate through a thrust bearing. Figure 8.13 shows the pressure plate pulled back by the release levers and the friction linings on the clutch plate are no longer in contact with the pressure plate or the flywheel. The flywheel rotates without driving the clutch plate and thus, the driven shaft.



When the foot is taken off the clutch pedal, the pressure on the thrust bearing is released. As a result, the springs become free to move the pressure plate to bring it in contact with the clutch plate. The clutch plate slides on the splined hub and is tightly gripped between the pressure plate and the fly wheel. The friction between the linings on the clutch plate, and the flywheel on one side and the pressure plate on the other, cause the clutch plate and hence, the driven shaft to rotate. In case the resisting torque on the drive shaft exceeds the torque at the clutch, clutch slip will occur.

Torque transmitted by plate or disc clutch



The following notations are used in the derivation T= Torque transmitted by the clutch

P= intensity of axial pressure

r1&r2=external and internal radii of friction faces

μ= co-efficient of friction

Consider an elemental ring of radius r and thickness dr Friction surface = 2πrdr

Axial force on the dw= pressure *area

= P*2πrdr

Frictional force acting on the ring tangentially at radius r Fr= μdw=μ*p*2πrdr

Frictional torque acting on the ring $T_r=F_r*r=\mu p*2\pi r*dr*r=2\pi\mu pr^2dr$

Considering uniform pressure

When the pressure is uniformly distributed over the entire area of the friction face, then the intensity o pressure,

$P=W/\pi[(r^2-(r)^2]$ (i)

Where *W* = Axial thrust with which the contact or friction surfaces are held together.

We have discussed above that the frictional torque on the elementary ring of radius *r* and thickness *dr* is

$T_r = 2\pi\mu.p.r^2dr$

Integrating this equation within the limits from r_2 to r_1 for the total frictional torque.

Therefore total frictional torque acting on the friction surface or on the clutch,

$$T = \int_{r_1}^{r_2} 2\pi\mu.p.r^2.dr = 2\pi\mu p\left[\frac{r_3}{3}\right]_{r_2}^{r_1} = 2\pi\mu p\left[\frac{(r_1)^3-(r_2)^3}{3}\right]$$

Substituting the value of p from equation (i),

$$T = 2\pi\mu \times \frac{W}{\pi[(r_1)^2 - (r_2)^2]} \times \frac{(r_1)^3 - (r_2)^3}{3}$$

$$= \frac{2}{3} \times \mu.W \left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2} \right] = \mu.W.R$$

R = Mean radius of friction surface

$$= \frac{2}{3} \left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2} \right]$$

2. Considering uniform wear

Let p be the normal intensity of pressure at a distance r from the axis of the Clutch. Since the intensity of pressure varies inversely with the distance, therefore

$p.r. = C$ (a constant) or $p = C/r$

and the normal force on the ring,

$$\delta W = p.2\pi r.dr = \frac{C}{r} \times 2\pi C.dr = 2\pi C.dr$$

∴ Total force acting on the friction surface,

$$W = \int_{r_2}^{r_1} 2\pi C\, dr = 2\pi C [r]_{r_2}^{r_1} = 2\pi C (r_1 - r_2)$$

$$C = \frac{W}{2\pi(r_1 - r_2)}$$

We know that the frictional torque acting on the ring,

$$T_r = 2\pi\mu.p r^2.dr = 2\pi\mu \times \frac{C}{r} \times r^2.dr = 2\pi\mu.C.r.dr$$

Total frictional torque on the friction surface,

$$T = \int_{r_2}^{r_1} 2\pi\mu.C.r.dr = 2\pi\mu.C \left[\frac{r^2}{2} \right]_{r_2}^{r_1} = 2\pi\mu.C \left[\frac{(r_1)^2 - (r_2)^2}{2} \right]$$

$$= \pi\mu.C[(r_1)^2 - (r_2)^2] = \pi\mu \times \frac{W}{2\pi(r_1 - r_2)} [(r_1)^2 - (r_2)^2]$$

$$= \frac{1}{2} \times \mu.W (r_1 + r_2) = \mu.W.R$$

R = Mean radius of the friction surface = $(r_1 + r_2)/2$

Multiple plate clutches

In a multi-plate clutch, the number of frictional linings and the metal plates is Increased which increases the capacity of the clutch to transmit torque. Figure8.14 shows a simplified diagram of a multi-plate clutch. The friction rings are splined on their outer circumference and engage with corresponding splines on the flywheel. They are free to slide axially.

The Friction material thus, rotates with the flywheel and the engine shaft. The Number of friction rings depends

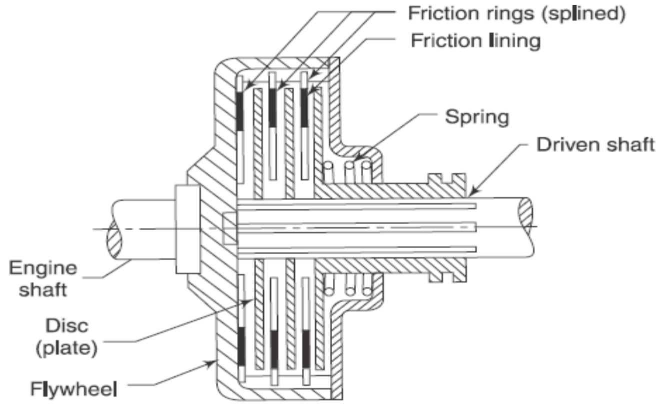


Fig. 8.14

The driven shaft also supports discs on the splines which rotate with the driven shaft and can slide axially. If the actuating force on the pedal is removed, a spring presses the discs into contact with the friction rings and the torque is transmitted between the engine shaft and the driven shaft. If n is the total number of plates both on the driving and the driven members, the number of active surfaces will be $n - 1$.

Let n_1 = Number of discs on the driving shaft, and

n_2 = Number of discs on the driven shaft.

Number of pairs of contact surfaces, $n = n_1 + n_2 - 1$

And total frictional torque acting on the friction surfaces or on the clutch,

$T = n.\mu.W.R$

Where R = Mean radius of the friction surfaces

$$= \frac{2}{3}\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right]$$

$$= \frac{r_1 + r_2}{2}$$

PROBLEMS

Example1. Determine the maximum, minimum and average pressure in plate clutch when the axial force is 4 kN. The inside radius of the contact surface is 50 mm and the outside radius is 100 mm. Assume uniform wear.

Solution.

Given: W = 4 kN = 4 × 103 N, r_2 = 50 mm ;r_1 = 100mm

Maximum pressure

Let p_{max} = Maximum pressure.

Since the intensity of pressure is maximum at the inner radius (r_2), therefore

$p_{max} \times r2 = C$ or $C = 50\ p_{max}$

We know that the total force on the contact surface (W),

4 × 103 = 2 π C ($r_1 - r_2$) = 2 π× 50 p_{max} (100 – 50) = 15 710 p_{max}

P_{max} = 4 × 103/15 710 = 0.2546 N/mm^2

Minimum pressure

Let p_{min} = Minimum pressure.

Since the intensity of pressure is minimum at the outer radius (r_1), Therefore $P_{min} \times r_1 = C$ or $C = 100\ p_{min}$

We know that the total force on the contact surface (W),

4 × 103 = 2 π C ($r_1 - r_2$) = 2π × 100 p_{min} (100 – 50) = 31 420 p_{min}

P_{min} = 4 × 103/31 420 = 0.1273 N/mm^2

Average pressure

We know that average pressure,

$$p_{av} = \frac{\text{Total normal force on contact surface}}{\text{Cross-sectional area of contact surfaces}}$$

$$= \frac{W}{\pi[(r_1)^2 - (r_2)^2]} = \frac{4 \times 10^3}{\pi[(100)^2 - (50)^2]} = 0.17 \text{ N/mm}^2$$

Example2. A single plate clutch, with both sides effective, has outer and inner diameters 300 mm and 200 mm respectively. The maximum intensity of pressure at any point in the contact surfaceis not to exceed 0.1 N/mm2.Ifthe coefficient of friction is 0.3, determine the power transmitted by a clutch at a speed 2500r.p.m.

Solution.Given: d_1 = 300 mm or r_1 = 150 mm ;d_2 = 200 mm or r_2 = 100 mm ;

p = 0.1 N/mm^2; μ = 0.3 ;N= 2500 r.p.m. or ω= 2π× 2500/60 = 261.8 rad/s

Since the intensity of pressure (p) is maximum at the inner radius (r_2), therefore for uniform

$p.r_2 = C$ or $C = 0.1 \times 100 = 10$ N/mm

We know that the axial thrust,

$W = 2\,\pi\, C\,(r_1 - r_2) = 2\,\pi \times 10\,(150 - 100) = 3142$ N

$$R = \frac{r_1 + r_2}{2} = \frac{150 + 100}{2} = 125 \text{ mm} = 0.125 \text{ m}$$

Mean radius of the friction surfaces for uniform wear,

We know that torque transmitted,

$T = n.\mu.W.R = 2 \times 0.3 \times 3142 \times 0.125 = 235.65$ N-m

Power transmitted by a clutch,

$P = T^*\omega = 235.65 \times 261.8 = 61\,693$ W $= 61.693$ kW

CONE CLUTCH

A cone clutch, as shown in Fig. 10.24, was extensively used in automobiles but now-a-days it has been replaced completely by the disc clutch

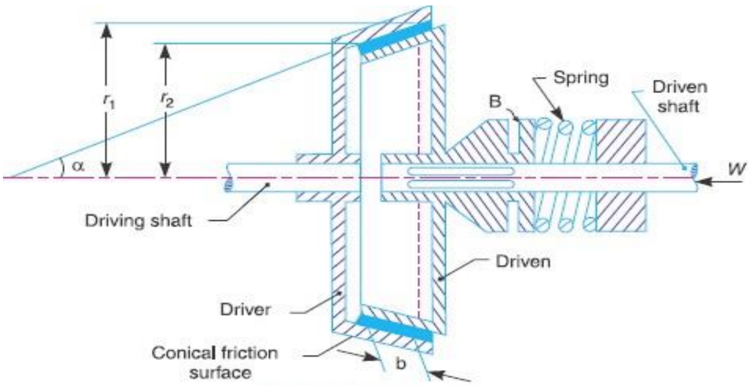


Fig. 10.24. Cone clutch.

It consists of one pair of friction surface only. In a cone clutch, the driver is keyed to the driving shaft by a sunk key and has an inside conical surface or face which exactly fits into the outside conical surface of the driven.

The driven member resting on the feather key in the driven shaft, maybe shifted along the shaft by a forked lever provided at *B*, in order to engage the clutch by bringing the two conical surfaces in contact. Due to the frictional resistance set up at this contact surface, the torque is transmitted from one shaft to another. In some cases, a spring is placed around the driven shaft in contact with the hub of the driven. This spring holds the clutch faces in contact and maintains the pressure between them, and the forked lever is used only for disengagement of the clutch. The contact surfaces of the clutch may be metal to metal contact, but more often the driven member is lined with some material like wood, leather, cork or asbestos etc. The material of the clutch faces (*i.e.* contact surfaces) depends upon the allowable normal pressure and the coefficient of friction.

Consider a pair of friction surface as shown in Fig. Since the area of contact of a pair of friction surface is a frustum of a cone, therefore the torque transmitted by the cone clutch maybe determined in the similar manner as discussed.

Let p_n = Intensity of pressure with which the conical friction surfaces are held together (*i.e.* normal pressure between contact surfaces),

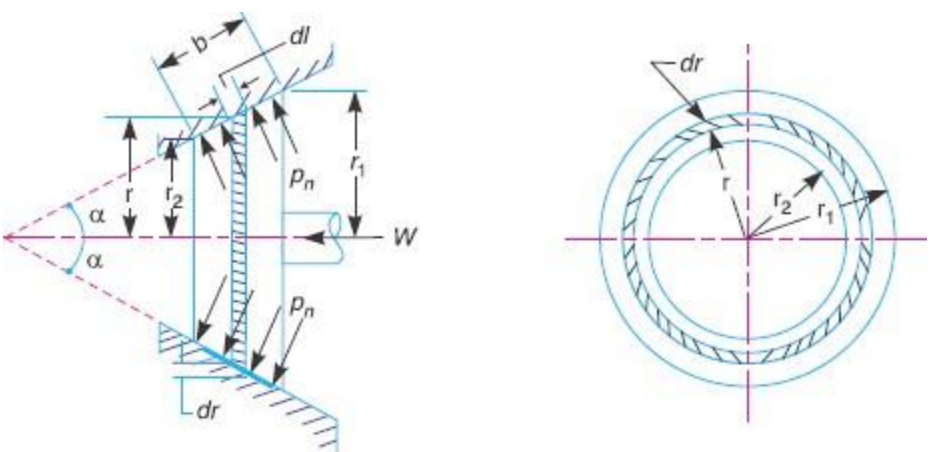
r_1 and r_2 = Outer and inner radius of friction surfaces respectively

R = Mean radius of the friction surface=$(r_1 + r_2)/2$

α= Semi angle of the cone (also called face angle of the cone) or the angle of the friction surface with the axis of the clutch,

μ = Coefficient of friction between contact surfaces, and

b = Width of the contact surfaces (also known as face width or clutch face).



Consider a small ring of radius r and thickness dr, as shown in Fig. 10.25 (*b*).

Let dl is length of ring of the friction surface, such that

$dl = dr.\text{cose } \alpha$ Area of the ring= $A = 2\pi\, r.dl = 2\pi r.dr \text{cosec } \alpha$

We shall consider the following two cases :

When there is a uniform pressure and when there is a uniform wear.

Considering uniform pressure

We know that normal load acting on the ring,

δW_n = Normal pressure × Area of ring = $p_n \times 2\,\pi r.dr.\text{cosec } \alpha$ The axial load acting on the ring,

δW= Horizontal component of δW_n (*i.e.* in the direction of W)

$= \delta W_n \times \sin \alpha = p_n \times 2\pi\, r.dr.\ \text{cosec } \alpha \times \sin \alpha = 2\pi \times p_n .r.dr$

Total axial load transmitted to the clutch or the axial spring force required,

$$W = \int_{r_2}^{r_1} 2\pi p_n .r.dr = 2\pi p_n \left[\frac{r^2}{2}\right]_{r_2}^{r_1} = 2\pi p_n \left[\frac{(r_1)^2 - (r_2)^2}{2}\right]$$

$$= \pi p_n \left[(r_1)^2 - (r_2)^2\right]$$

$$p_n = \frac{W}{\pi[(r_1)^2 - (r_2)^2]}$$

We know that frictional force on the ring acting tangentially at radius r, $F_r = \mu.\delta W_n = \mu.p_n \times 2\pi\, r.dr.\text{cosec}\, \alpha$ Frictional torque acting on the ring,

Integrating this expression within the limits from r_2 to r_1 for the total frictional torque on the clutch

$\therefore$ Total frictional torque,

$$T = \int_{r_2}^{r_1} 2\pi\mu.p_n.\text{cosec}\,\alpha.r^2.dr = 2\pi\mu\, p_n.\text{cosec}\,\alpha\left[\frac{r^3}{3}\right]_{r_2}^{r_1}$$

$$= 2\pi\mu\, p_n.\text{cosec}\,\alpha\left[\frac{(r_1)^3 - (r_2)^3}{3}\right]$$

Substituting the value of p_n from equation (*i*), we get

$$T = 2\pi\mu \times \frac{W}{\pi[(r_1)^2 - (r_2)^2]} \times \text{cosec}\,\alpha\left[\frac{(r_1)^3 - (r_2)^3}{3}\right]$$

$$= \frac{2}{3} \times \mu.W.\text{cosec}\,\alpha\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right]$$

Considering uniform wear

In Fig., let $p_r$be the normal intensity of pressure at a distance r from the axis of the clutch. We know that, in case of uniform wear, the intensity of pressure varies inversely with thedistance.

$P_r.r = C$ (a constant) or $p_r = C / r$

We know that the normal load acting on the ring,

δW_n = Normal pressure × Area of ring = $p_r \times 2\pi r.dr \text{cosec}\alpha$ The axial load acting on the ring ,

$\delta W = \delta W_n \times \sin\alpha = p_r.2\pi\, r.dr.\text{cosec}\,\alpha.\sin\alpha = p_r \times 2\pi\, r.dr$

$$= \frac{C}{r} \times 2\pi r.dr = 2\pi C.dr$$

$\therefore$ Total axial load transmitted to the clutch,

$$W = \int_{r_2}^{r_1} 2\pi C.dr = 2\pi C[r]_{r_2}^{r_1} = 2\pi C(r_1 - r_2)$$

$$C = \frac{W}{2\pi(r_1 - r_2)}$$

We know that frictional force acting on the ring,

$F_r = \mu.\delta W_n = \mu.p_r \times 2\pi r \times dr\text{cosec}\alpha$ Frictional torque acting on the ring,

$$= \mu \times \frac{C}{r} \times 2\pi r^2.dr.\text{cosec}\ \alpha = 2\pi\mu.C\ \text{cosec}\ \alpha \times r\ dr$$

∴ Total frictional torque acting on the clutch,

$$T = \int_{r_2}^{r_1} 2\pi\mu.C.\text{cosec}\ \alpha.r\,dr = 2\pi\mu.C.\text{cosec}\ \alpha\left[\frac{r^2}{2}\right]_{r_2}^{r_1}$$

$$= 2\pi\mu.C.\text{cosec}\ \alpha\left[\frac{(r_1)^2 - (r_2)^2}{2}\right]$$

Substituting the value of C from equation (*i*), we have

$$T = 2\pi\mu \times \frac{W}{2\pi(r_1 - r_2)} \times \text{cosec}\ \alpha\left[\frac{(r_1)^2 - (r_2)^2}{2}\right]$$

$$= \mu.W\ \text{cosec}\ \alpha\left(\frac{r_1 + r_2}{2}\right) = \mu.W.R\ \text{cosec}\ \alpha$$

$$R = \frac{r_1 + r_2}{2} = \text{Mean radius of friction surface}$$

PROBLEMS

Example 1.An engine developing 45 kW at 1000 r.p.m. is fitted with a cone clutch built inside the flywheel. The cone has a face angle of 12.5º and a maximum mean diameter of 500 mm. The coefficient of friction is 0.2. The normal pressure on the clutch face is not to exceed 0.1 N/mm 2. Determine: 1. the axial spring force necessary to engage to clutch, and 2. the face widthrequired.

Solution.Given :P = 45 kW = 45 × 103 W ; N = 1000 r.p.m. or ω= 2π× 1000/60 = 104.7rad/s ; α= 12.5º ; D = 500 mm or R = 250 mm = 0.25 m ; μ = 0.2 ;

p_n=0.1N/mm^2

Axial spring force necessary to engage theclutch

First of all, let us find the torque (T) developed by the clutch and the normal load (W_n) acting on the friction surface.

We know that power developed by the clutch (P),

45 × 103 = $T\omega$= T × 104.7 or T = 45 × 10^3/104.7 = 430 N-m

We also know that the torque developed by the clutch (T), 430 = $\mu.W_n.R$ = 0.2 × W_n× 0.25 = 0.05 W_n

W_n = 430/0.05 = 8600 N

Axial spring force necessary to engage the clutch,

$W_e = W_n$ (sin α+ μ cosα)

= 8600 (sin 12.5º + 0.2 cos 12.5º) = 3540 N

Face widthrequired

Let b = Face widthrequired

We know that normal load acting on the friction surface (W_n), $8600 = p_n \times 2\pi R.b = 0.1 \times 2\pi \times 250 \times b = 157\,b$

$b = 8600/157 = 54.7$ mm

Example 2. A conical friction clutch is used to transmit 90 kW at 1500 r.p.m. The semi cone angle is 20º and the coefficient of friction is 0.2. If the mean diameter of the bearing surface is375 mm and the intensity of normal pressure is not to exceed 0.25 N/mm2, find the dimensions of the conical bearing surface and the axial load required.

Solution.Given: $P = 90$ kW $= 90 \times 10^3$ W ;$N = 1500$ r.p.m. or $\omega = 2\pi \times 1500/60 = 156$rad/s ; $\alpha = 20º$; $\mu = 0.2$; $D = 375$ mm or $R = 187.5$ mm ; $p_n = 0.25$ N/mm^2

Dimensions of the conical bearing surface

Let r_1 and r_2 = External and internal radii of the bearing surface respectively,

b = Width of the bearing surface in mm, and

T = Torque transmitted.

We know that power transmitted (P), $90 \times 103 = T\omega = T \times 156$

$T = 90 \times 103/156 = 577$ N-m $= 577 \times 10^3$ N-mm

The torque transmitted (T),

$577 \times 103 = 2\pi \mu\, p_n.R^2.b = 2\pi \times 0.2 \times 0.25\,(187.5)^2 b = 11\,046\,b$

$b = 577 \times 10^3/11\,046 = 52.2$ mm

We know that $r_1 + r_2 = 2R = 2 \times 187.5 = 375$ mm.............. i

$r1 - r2 = b \sin \alpha = 52.2 \sin 20º = 18$ mm...................... ii

From equations (*i*) and (*ii*),

$r_1 = 196.5$ mm, and $r_2 = 178.5$ mm

Axial load required

Since in case of friction clutch, uniform wear is considered and the intensity of pressure is maximum at the minimum contact surface radius ($r2$), therefore

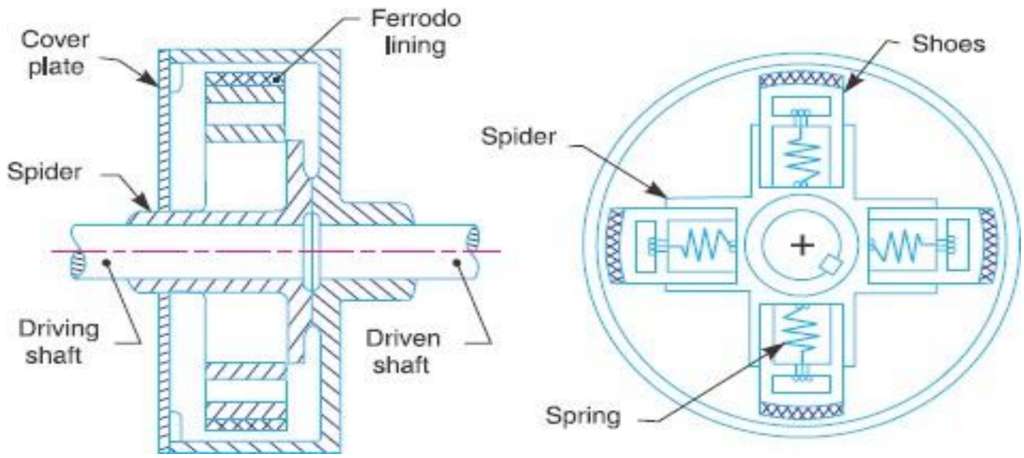
$p_n.r_2 = C$ (a constant) or $C = 0.25 \times 178.5 = 44.6$ N/mm

We know that the axial load required,

$W = 2\pi C\,(r_1 - r_2) = 2\pi \times 44.6\,(196.5 - 178.5) = 5045$ N

Centrifugal Clutch

The centrifugal clutches are usually incorporated into the motor pulleys. It consists of a number of shoes on the inside of a rim of the pulley, as shown in Fig. 10.28. The outer surface of the shoes is covered with



a friction material. These shoes, which can move radially in guides, are held

Against the boss (or spider) on the driving shaft by means of springs. The springs exert a radially inward force which is assumed constant. The mass of the shoe, when revolving, causes it to exert a radially outward force (*i.e.* centrifugal force). The magnitude of this centrifugal force depends upon the speed at which the shoe is revolving. A little consideration will show that when the centrifugal force is less than the spring force, the shoe remains in the same position as when the driving shaft was stationary, but when the centrifugal force is equal to the spring force, the shoe is just floating. When the centrifugal force exceeds the spring force, the shoe moves outward and comes into contact with the driven member and presses against it. The force with which the shoe presses against the driven member is the difference of the centrifugal force and the spring force. The increase of speed causes the shoe to press harder and enables more torque to be transmitted. In order to determine the mass and size of the shoes, the following procedure is adopted:

Mass of the shoes

Consider one shoe of a centrifugal clutch as shown in Fig Let m = Mass of eachshoe,

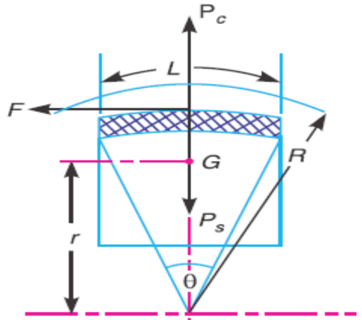


Fig. 10.29. Forces on a shoe of centrifugal clutch.

n = Number of shoes,

r = Distance of centre of gravity of the shoe from the centre of the spider,

R = Inside radius of the pulley rim,

N = Running speed of the pulley in r.p.m.,

ω = Angular running speed of the pulley in rad/s

= $2\pi N/60$ rad/s,

ω_1 = Angular speed at which the engagement begins to take place, and

$\propto$ = Coefficient of friction between the shoe and rim.

We know that the centrifugal force acting on each shoe at the running speed,

*$P_c = m\,.\omega^2.r$

and the inward force on each shoe exerted by the spring at the speed at which engagement begins to take place,

$P_s = m\,(\omega_1)^2 r$

∴ The net outward radial force (*i.e.* centrifugal force) with which

The shoe presses against the rim at the running speed = $P_c - P_s$

The frictional force acting tangentially on each shoe, F = $\propto\,(P_c - P_s)$

∴ Frictional torque acting on each shoe, = $F \times R = \propto\,(P_c - P_s)\,R$

Total frictional torque transmitted,

$T = \propto\,(P_c - P_s)\,R \times n = n.F.R$

From this expression, the mass of the shoes (m) may be evaluated. Size of the shoes

Let l = Contact length of theshoes, b = Width of theshoes,

R = Contact radius of the shoes. It is same as the inside radius of the rim of the pulley.

θ = Angle subtended by the shoes at the centre of the spider in radians.

p = Intensity of pressure exerted on the shoe. In order to ensure reason-able life, the intensity of pressure may be taken as 0.1 N/mm^2.

We know that $\theta = l/R$ rad or $l = \theta.R$

∴ Area of contact of the shoe, $A = l.b$

The force with which the shoe presses against the rim

$A \times p = l.b.p$

Since the force with which the shoe presses against the rim at the running speed is ($P_c - P_s$), therefore

$$l.b.p = P_c - P_s$$

PROBLEMS

Example 1.

A centrifugal clutch is to transmit 15 kW at 900 r.p.m. The shoes are four in number. The speed at which the engagement begins is 3/4th of the running speed. The inside radius of the pulley rim is 150 mm and the centre of gravity of the shoe lies at 120 mm from the centre of the spider. The shoes are lined with Ferrodo for which the coefficient of friction may be taken as 0.25. Determine: 1. Mass of the shoes, and 2. Size of the shoes, if angle subtended by the shoes at the centre of the spider is 60º and the pressure exerted on the shoes is $0.1 N/mm^2$.

Solution.:

Given: P = 15 kW = 15×10^3 W; N = 900 r.p.m. or ω = 25 × 900/60 = 94.26 rad/s ; n = 4 ; R = 150 mm = 0.15 m ; r = 120 mm = 0.12 m ; $\propto$ = 0.25

Since the speed at which the engagement begins (*i.e.* ω_1) is 3/4th of the running speed (*i.e.* ω), therefore

$$\omega_1 = \frac{3}{4}\,\omega = \frac{3}{4} \times 94.26 = 70.7 \text{ rad/s}$$

Let T = Torque transmitted at the running speed.

We know that power transmitted (P),

$= T.\omega = T \times 94.26$ or $T = 15 \times 10^3/94.26 =$

15 × 10 =159 N-m

Mass of the shoes

Let m = Mass of the shoes in kg.

We know that the centrifugal force acting on each shoe,

$P_c = m.\omega^2.r = m\,(94.26)^2 \times 0.12 = 1066\,m$ N

the inward force on each shoe exerted by the spring *i.e*. the centrifugal force at the engagement speed ω_1,

$P_s = m\,(\omega_1)^2 r = m\,(70.7)^2 \times 0.12 = 600\,m$ N

∴ Frictional force acting tangentially on each shoe,

$F = \propto (P_c - P_s) = 0.25\,(1066\,m - 600\,m)\ 116.5\,m$ N

We know that the torque transmitted (T),

$159 = n.F.R = 4 \times 116.5\,m \times 0.15 = 70\,m$ or $m = 2.27$ kg

Size of theshoes

Let l = Contact length of shoes in mm,

b = Width of the shoes in mm,

θ Angle subtended by the shoes at the centre of the spider in radians

= 60º = π/3 rad, and

p = Pressure exerted on the shoes in N/mm^2 = 0.1 N/mm^2

We know that $l = \theta . R = \frac{\pi}{3} \times 150 = 157.1$ mm

$$l.b.p = P_c - P_s = 1066\,m - 600\,m = 466\,m$$

$$\therefore \quad 157.1 \times b \times 0.1 = 466 \times 2.27 = 1058$$

b = 1058/157.1 × 0.1 = 67.3 mm

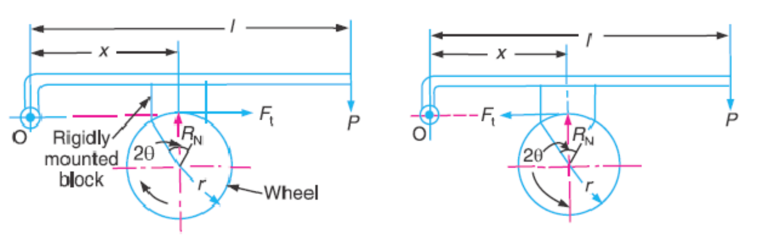
BRAKES AND DYNAMOMETERS

A *brake* is a device by means of which artificial frictional resistance is applied to a moving machine member, in order to retard or stop the motion of a machine. In the process of performing this function, the brake absorbs either kinetic energy of the moving member or potential energy given up by objects being lowered by hoists, elevators etc

Single Block or Shoe Brake

A single block or shoe brake is shown in Fig. It consists of a block or shoe which is pressed against the rim of a revolving brake wheel drum. The block is made of a softer material than the rim of the wheel. This type of a brake is commonly used on railway trains and tram cars. The friction between the block and the wheel causes a tangential braking force to act on the wheel, which retard the rotation of the wheel. The block is pressed against the wheel by a force applied to one end of a lever to which the block is rigidly fixed as shown in Fig. 19.1. The other end of the lever is pivoted on a fixed fulcrum *O*

Let P = Force applied at the end of thelever



(*a*) Clockwise rotation of brake wheel (*b*) Anticlockwise rotation of brake wheel.

R_N= Normal force pressing the brake block on the wheel,

r = Radius of the wheel,

2θ = Angle of contact surface of the block,

μ = Coefficient of friction, and

F_t = Tangential braking force or the frictional force acting at the contact surface of the block and the wheel

If the angle of contact is less than 60°, then it may be assumed that the normal pressure between the block and the wheel is uniform. In such cases, tangential braking force on the wheel,

$F_t = \mu.R_N$...(*i*)

The braking torque, $T_B = F_t.r = \mu.R_N.r$ ...(*ii*)

Let us now consider the following three cases:

Case1. When the line of action of tangential braking force (Ft) passes through the fulcrum *O* of the lever, and the brake wheel rotates clockwise as shown in Fig. 19.1(*a*), then for equilibrium, taking moments about the fulcrum.

$$R_N \times x = P \times l \text{ or } R_N = \frac{P \times l}{x}$$

Braking torque,

$$T_B = \mu.R_N.r = \mu \times \frac{P.l}{x} \times r = \frac{\mu.P.l.r}{x}$$

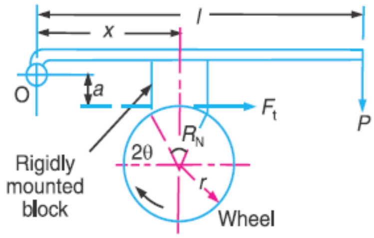
O, wehave

It may be noted that when the brake wheel rotates anticlockwise as shown in Fig. 19.1 (*b*), then the braking torque is same, *i.e.*

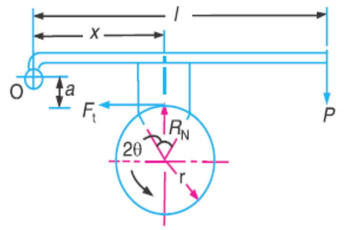
$$T_B = \mu.R_N.r = \frac{\mu.P.l.r}{x}$$

Case2. When the line of action of the tangential braking force (Ft) passes through a distance '*a*' below the fulcrum *O*, and the brake wheel rotates clockwise as shown in Fig.

When the brake wheel rotates anticlockwise, as shown in Fig. 19.2 (*b*), then for equilibrium,



(*a*) Clockwise rotation of brake wheel.



(*b*) Anticlockwise rotation of brake wheel.

(*a*), then for equilibrium, taking moments about the fulcrum *O*, Case 3.When the lie of action of the tangential braking force (*Ft*) passes through a distance ‘*a*’ above the fulcrum *O*, and the brake wheel rotates clockwise as shown in Fig.

(*a*), then fore equilibrium, taking moments about the fulcrum *O*, wehave

$$R_N.x = P.l + F_t.a = P.l + \mu.R_N.a$$

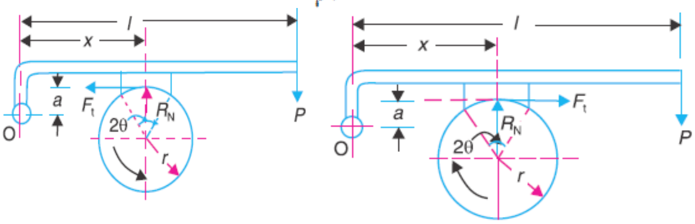
$$R_N\,(x - \mu.a) = P.l \qquad \text{or} \qquad R_N = \frac{P.l}{x - \mu.a}$$

$$T_B = \mu.R_N.r = \frac{\mu.P.l.r}{x - \mu.a}$$

$R_N.x = P.l + F_t.a = P.l + \mu.R_N.a$

$R_N\,(x - \mu.a) = P.l$

$$R_N = \frac{P.l}{x - \mu.a}$$



(*b*) Anticlockwise rotation of brake wheel. (*a*) Clockwise rotation of brake wheel.

$$T_B = \mu.R_N.r = \frac{\mu.P.l.r}{x - \mu.a}$$

When the brake wheel rotates anticlockwise as shown in Fig. 19.3 (*b*), then for equilibrium, taking moments about the fulcrum *O*, we have

$R_N \times x + F_t \times a = P.l \;\; R_N \times x + \mu.R_N \times a = P.l$

$$T_B = \mu.R_N.r = \frac{\mu.P.l.r}{x + \mu.a}$$

Pivoted Block or Shoe Brake : We have discussed in the previous article that when the angle of contact is less than 60°, then it may be assumed that the normal pressure between the block and the wheel is uniform. But when the angle of contact is greater than 60°, then the unit pressure normal to the surface of contact is less at the ends than at the centre.

Instead of being rigidly attached to the lever. This gives uniform wear of the brake lining in the direction of the applied force. The braking torque or a pivoted block

$$T_B = F_t \times r = \mu'.R_N.r$$

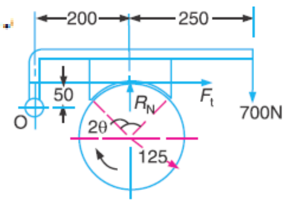
$$\mu' = \text{Equivalent coefficient of friction} = \frac{4\mu \sin\theta}{2\theta + \sin 2\theta},$$

$$\mu = \text{Actual coefficient of friction.}$$

PROBLEMS

Example1. A single block brake is shown in Fig. 19.5.The diameter of the drum is 250 mm and the angle of contact is90°. If the operating force of 700 N is applied at the end of alever and the coefficient of friction

$$R_N = \frac{P.l}{x + \mu.}$$



All dimensions in mm.

Fig. 19.5

between the drum and the lining is0.35,Determine the torque that may be transmitted by the

Solution. Given: d = 250 mm or r = 125 mm ; 2θ = 90° = π / 2 rad ; P = 700 N ; μ = 0.35

Since the angle of contact is greater than 60°, therefore equivalent coefficient of friction,

$$\mu' = \frac{4\mu \sin\theta}{2\theta + \sin 2\theta} = \frac{4 \times 0.35 \times \sin 45°}{\pi/2 + \sin 90°} = 0.385$$

R_N = Normal force pressing the block to the brake drum, and

F_t = Tangential braking force = $\mu'.R_N$

Taking moments about the fulcrum O, we have

$$700(250+200) + F_t \times 50 = R_N \times 200 = \frac{F_t}{\mu'} \times 200 = \frac{F_t}{0.385} \times 200 = 520\ F_t$$

520 F_t – 50F_t = 700 × 450 or F_t = 700 × 450/470 = 670 N

We know that torque transmitted by the blockbrake,

T_B = F_t ×r = 670 × 125 = 8 3750 N-mm =83.75N-m

Example 2.A bicycle and rider of mass 100 kg are travelling at the rate of 16 km/h on a level road. A brake is applied to the rear wheel which is 0.9 m in diameter and this is the only resistance acting. How far will the bicycle travel and how many turns will it make before it comes to rest? The pressure applied on the brake is 100 N and μ = 0.05.

Solution.Given: m = 100 kg, v = 16 km / h = 4.44 m / s ;D = 0.9 m ; R

N = 100 N; μ = 0.05

Distance travelled by the bicycle before it comes to rest

Let x = Distance travelled (in meters) by the bicycle before it comes to rest.

We know that tangential braking force acting at the point of contact of the brake wheel,

$F_t = \mu . R_N = 0.05 \times 100 = 5$ N

$= F_t \times x = 5 \times x = 5x$ N-m (i)

We know that kinetic energy of the bicycle

$$= \frac{m.v^2}{2} = \frac{100(4.44)^2}{2}$$
$$= 986 \text{ N-m} \quad \ldots (ii)$$

In order to bring the bicycle to rest, the work done against friction must be equal to kinetic energy of the bicycle. Therefore equating equations (*i*) and (*ii*),

$5x = 986$ or $x = 986/5 = 197.2$ m

Number of revolutions made by the bicycle before it comes to rest

Let N = Required number of revolutions.

We know that distance travelled by the bicycle (x), $197.2 = \pi DN = \pi \times 0.9N = 2.83N$
$N = 197.2 / 2.83 = 70$

Double Block or Shoe Brake

When a single block brake is applied to a rolling wheel, an additional load is thrown on the shaft bearings due to the normal force (RN). This produces bending of the shaft. In order to overcome this drawback, a double block or shoe brake, as shown in Fig. 19.9, is used. It consists of two brake blocks applied at the opposite ends of a diameter of the wheel which eliminate or reduces the unbalanced force on the shaft. The brake is set by a spring which pulls the upper ends of the brake arms together. When a force P is applied to the bell crank lever, the spring is compressed and the brake is released. This type of brake is often used on electric cranes and the force P is produced by an electromagnet or solenoid.

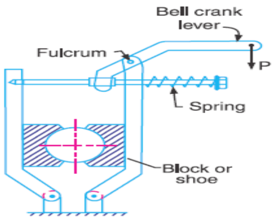


Fig. 19.9. Double block or shoe brake.

In a double block brake, the braking action is doubled by the use of two blocks and these blocks may be operated practically by the same force which will operate one. In case of double block or shoe brake, the braking torque is given by

$T_B = (F_{t1} + F_{t2})\, r$

Where F_{t1} and F_{t2} are the braking forces on the two blocks.

Internal Expanding Brake

An internal expanding brake consists of two shoes $S1$ and $S2$ as shown in Fig. 19.24. The outer surface of the shoes are lined with some friction material (usually with Ferodo) to increase the coefficient of friction and to prevent wearing away of the metal. Each shoe is pivoted at one end about a fixed fulcrum $O1$ and $O2$ and made to contact a cam at the other end. When the cam rotates, the shoes are pushed outwards against the rim of the drum. The friction between the shoes and the drum produces the braking torque and hence reduces the speed of the drum. The shoes are normally held in off position by a spring as shown in Fig. 19.24. The drum encloses the entire mechanism to keep out dust and moisture. This type of brake is commonly used in motor cars and light trucks.

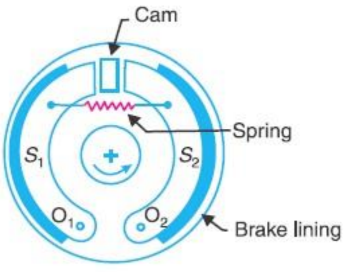


Fig. 19.24. Internal expanding brake.

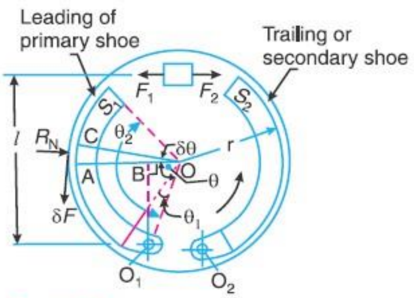


Fig. 19.25. Forces on an internal expanding brake.

We shall now consider the forces acting on such a brake, when the drum rotates in the anticlockwise direction as shown in Fig. 19.25. It may be noted that for the anticlockwise direction, the left hand shoe is known as *leading* or *primary shoe* while the right hand shoe is known as *trailing* or *secondaryshoe*.

Let r = Internal radius of the wheelrim,

b = Width of the brake lining,

p_1 = Maximum intensity of normal pressure,

p_N = Normal pressure,

F_1 = Force exerted by the cam on the leading shoe, and

F_2 = Force exerted by the cam on the trailing shoe.

Consider a small element of the brake lining

AC subtending an angle δθ at the centre. Let *OA* makes an angle θ with OO_1 as shown in Fig. 19.25. It is assumed that the pressure distribution on the shoe is nearly uniform, however the friction lining wears out more at the free end. Since the shoe turns about*O*1, therefore the rate of wear of the shoe lining at *A* will be proportional to the radial displacement of that point. The rate of wear of the shoe lining varies directly as the perpendicular distance from $O_1$to *OA*, *i.e*.

O_1B. From the geometry of the figure,

$O_1B = OO_1 \sin\theta$

Normal pressure at *A*,

$$p_N \propto \sin\theta \quad \text{or} \quad p_N = p_1 \sin\theta$$

∴ Normal force acting on the element,

$$\delta R_N = \text{Normal pressure} \times \text{Area of the element}$$

$$= p_N(b.r.\delta\theta) = p_1 \sin\theta(b.r.\delta\theta)$$

and total braking torque about *O* for whole of one shoe,

$$T_B = \mu\, p_1 b\, r^2 \int_{\theta_1}^{\theta_2} \sin\theta\, d\theta = \mu\, p_1 b\, r^2 [-\cos\theta]_{\theta_1}^{\theta_2}$$

$$= \mu\, p_1 b r^2 (\cos\theta_1 - \cos\theta_2)$$

Moment of normal force δR_N of the element about the fulcrum O_1,

$$\delta M_N = \delta R_N \times O_1B = \delta R_N (OO_1 \sin\theta)$$

$$= p_1 \sin\theta(b.r.\delta\theta)(OO_1 \sin\theta) = p_1 \sin^2\theta(b.r.\delta\theta)OO_1$$

∴ Total moment of normal forces about the fulcrum O_1,

$$= p_1.b.r.OO_1 \int_{\theta_1}^{\theta_2} \frac{1}{2}(1-\cos 2\theta)d\theta \qquad \ldots\left[\because \sin^2\theta = \frac{1}{2}(1-\cos 2\theta)\right]$$

$$= \frac{1}{2} p_1.b.r.OO_1 \left[\theta - \frac{\sin 2\theta}{2}\right]_{\theta_1}^{\theta_2}$$

$$= \frac{1}{2} p_1.b.r.OO_1 \left[\theta_2 - \frac{\sin 2\theta_2}{2} - \theta_1 + \frac{\sin 2\theta_1}{2}\right]$$

$$= \frac{1}{2} p_1.b.r.OO_1 \left[(\theta_2 - \theta_1) + \frac{1}{2}(\sin 2\theta_1 - \sin 2\theta_2)\right]$$

$$M_N = \int_{\theta_1}^{\theta_2} p_1 \sin^2\theta(b.r.\delta\theta)OO_1 = p_1.b.r.OO_1 \int_{\theta_1}^{\theta_2} \sin^2\theta\, d\theta$$

Now for leading shoe, taking moments about the fulcrum O_1,

$$F_1 \times l = M_N - M_F$$

and for trailing shoe, taking moments about the fulcrum O_2,

$$F_2 \times l = M_N + M_F$$

$$\delta M_F = \delta F \times AB = \delta F \, (r - OO_1 \cos\theta) \qquad \ldots (\because AB = r - OO_1 \cos\theta)$$

$$= \mu \, p_1 \sin\theta (b.r.\delta\theta)\,(r - OO_1 \cos\theta)$$

$$= \mu . p_1 . b.r (r \sin\theta - OO_1 \sin\theta \cos\theta)\delta\theta$$

$$= \mu . p_1 . b.r \left(r \sin\theta - \frac{OO_1}{2} \sin 2\theta \right) \delta\theta \qquad \ldots (\because 2\sin\theta\cos\theta = \sin 2\theta$$

$\therefore$ Total moment of frictional force about the fulcrum O_1,

$$M_F = \mu \, p_1 b \, r \int_{\theta_1}^{\theta_2} \left(r \sin\theta - \frac{OO_1}{2} \sin 2\theta \right) d\theta$$

$$= \mu \, p_1 b \, r \left[-r \cos\theta + \frac{OO_1}{4} \cos 2\theta \right]_{\theta_1}^{\theta_2}$$

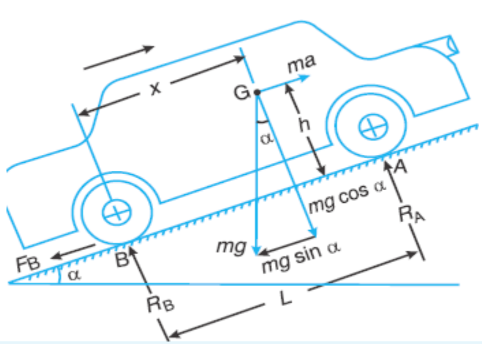
$$= \mu \, p_1 b \, r \left[-r \cos\theta_2 + \frac{OO_1}{4} \cos 2\theta_2 + r \cos\theta_1 - \frac{OO_1}{4} \cos 2\theta_1 \right]$$

$$= \mu \, p_1 b \, r \left[r (\cos\theta_1 - \cos\theta_2) + \frac{OO_1}{4} (\cos 2\theta_2 - \cos 2\theta_1) \right]$$

Braking of a Vehicle

In a four wheeled moving vehicle, the brakes may be applied to the rear wheels only, the front wheels only, and all the four wheels. In all the above mentioned three types of braking, it is required to determine the retardation of the vehicle when brakes are applied. Since the vehicle retards, therefore it is a problem of dynamics. But it may be reduced to an equivalent problem of statics by including the inertia force in the system of forces actually applied to the vehicle. The inertia force is equal and opposite to the braking force causing retardation.

Now, consider a vehicle moving up an inclined plane, as shown in Fig.



Let α = Angle of inclination of the plane to the horizontal,

m = Mass of the vehicle in kg (such that its weight is $m.g$ newtons),

h = Height of the C.G. of the vehicle above the road surface in metres,

x = Perpendicular distance of C.G. from the rear axle in metres,

L = Distance between the centres of the rear and front wheels of the vehicle in metres,

R_A = Total normal reaction between the ground and the front wheels in newtons,

R_B = Total normal reaction between the ground and the rear wheels in newtons,

μ = Coefficient of friction between the tyres and road surface, and

a = Retardation of the vehicle in m/s2.

We shall now consider the above mentioned three cases of braking, one by one. In all these cases, the braking force acts in the opposite direction to the direction of motion of the vehicle.

When the brakes are applied to the rear wheels only

It is a common way of braking the vehicle in which the braking force acts at the rear wheels only.

Let F_B =Total braking force(in newtons) acting at the rear wheels due to the application of the brakes. Its maximum value is $\mu.R_B$.

The various forces acting on the vehicle are shown in Fig. For the equilibrium of the vehicle, the forces acting on the vehicle must be in equilibrium.

Resolving the forces parallel to the plane,

$F_B + m.g.\sin\alpha = m.a$ . . . (*i*)

Resolving the forces perpendicular to the plane,

$R_A + R_B = m\, g \cos\alpha$………(*ii*)

Taking moments about G, the centre of gravity of the vehicle

$F_B \times h + R_B \times x = R_A (L - x)$ (*iii*)

Substituting the value of $F_B = \mu.R_B$, and $R_A = m.g\cos\alpha - R_B$ [from equation (*ii*)] in the above expression, we have

$\mu.R_B \times h + R_B \times x = (m.g\cos\alpha - R_B)(L - x)$ $R_B(L + \mu.h) = m.g\cos\alpha(L - x)$

$$R_B = \frac{m.g\cos\alpha(L - x)}{L + \mu.h}$$

and

$$R_A = m.g\cos\alpha - R_B = m.g\cos\alpha - \frac{m.g\cos\alpha(L - x)}{L + \mu.h}$$

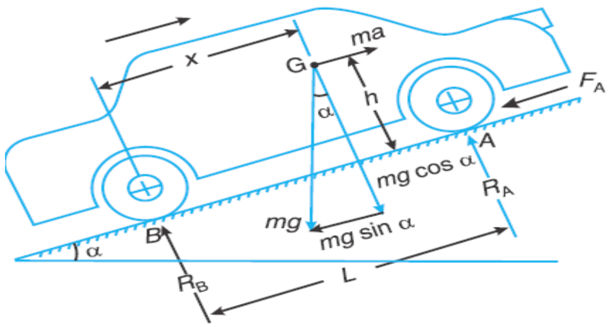
$$= \frac{m.g\cos\alpha(x + \mu.h)}{L + \mu.h}$$

We know from equation (*i*),

$$a = \frac{F_B + m.g\sin\alpha}{m} = \frac{F_B}{m} + g\sin\alpha = \frac{\mu.R_B}{m} + g\sin\alpha$$

When the brakes are applied to front wheelsonly

It is a very rare way of braking the vehicle, in which the braking force acts at the front wheels only.



Let F_A = Total braking force (in newtons) acting at the front wheels due to the application of brakes. Its maximum value is $\mu.R_A$.

The various forces acting on the vehicle are shown in Fig. Resolving the forces parallel to theplane,

$F_A + m.g\sin\alpha = m.a$ . . . (*i*)

Resolving the forces perpendicular to the plane,

$R_A + R_B = m.g\cos\alpha$ . . . (*ii*)

Taking moments about *G*, the centre of gravity of the vehicle,

$F_A \times h + R_B \times x = R_A (L - x)$

Substituting the value of $F_A = \mu.R_A$ and $R_B = m.g\cos\alpha - R_A$ [from equation (*ii*)] in the above expression, we have

$\mu.R_A \times h + (m.g\cos\alpha - R_A)\, x = R_A (L - x)$ $\mu.R_A \times h + m.g\cos\alpha \times x = R_A \times L$

$$\therefore \qquad R_A = \frac{m.g\cos\alpha \times x}{L - \mu.h}$$

and

$$R_B = m.g\cos\alpha - R_A = m.g\cos\alpha - \frac{m.g\cos\alpha \times x}{L - \mu.h}$$

$$= m.g\cos\alpha\left(1 - \frac{x}{L - \mu.h}\right) = m.g\cos\alpha\left(\frac{L - \mu.h - x}{L - \mu.h}\right)$$

We know from equation (*i*),

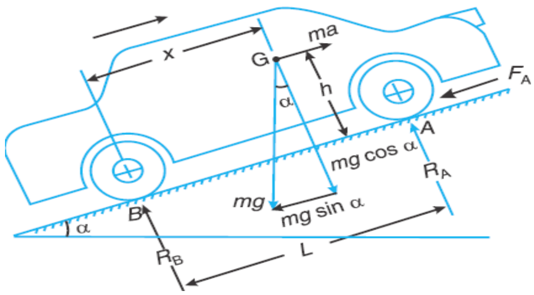
$$a = \frac{F_A + m.g\sin\alpha}{m} = \frac{\mu.R_A + m.g\sin\alpha}{m}$$

$$= \frac{\mu.m.g\cos\alpha \times x}{(L - \mu.h)m} + \frac{m.g\sin\alpha}{m} \qquad \text{. . . (Substituting the value of } R_A\text{)}$$

$$= \frac{\mu.g\cos\alpha \times x}{L - \mu.h} + g\sin\alpha$$

When the brakes are applied to all the fourwheels

This is the most common way of braking the vehicle, in which the braking force acts on both the rear and front wheels.



Let F_A = Braking force provided by the front wheels = $\mu.R_A$,and

F_B = Braking force provided by the rear wheels = $\mu.R_B$.

Little consideration will show that when the brakes are applied to all the four wheels, the braking distance (i.e. the distance in which the vehicle is brought to rest after applying the brakes) will be the least. It is due to this reason that the brakes are applied to all the four wheels. The various forces acting on the vehicle are shown in fig.

Resolving the forces parallel to the plane, $F_A+F_B+m.g\sin\alpha=m.a$.......(i)

Resolving forces vertical to the plane

$R_A + R_B = m.g\cos\alpha$. . .(*ii*)

Taking moments about *G*, the centre of gravity of the vehicle, $(F_A + F_B)\, h + R_B \times x = R_A(L - x)$.......(iii)

Substituting the value of $F_A = \mu.R_A$, $F_B = \mu.R_B$ and $R_B = m.g\cos\alpha - R_A$

[From equation (*ii*)] in the above expression,

$\mu\,(R_A + R_B)\, h + (m\, g\cos\alpha - R_A)\, x = R_A(L - x)$

$\mu\,(R_A + mg\cos\alpha - R_A)h + (mg\cos\alpha - R_A)x = R_A(L - x)$ $\mu.m.g\cos\alpha \times h + m.g\cos\alpha \times x = R_A \times L$

$$R_A = \frac{m.g\cos\alpha(\mu.h + x)}{L}$$

$$R_B = m.g\cos\alpha - R_A = m.g\cos\alpha - \frac{mg\cos\alpha(\mu.h + x)}{L}$$

$$= m.g\cos\alpha\left[1 - \frac{\mu.h + x}{L}\right] = m.g\cos\alpha\left(\frac{L - \mu.h - x}{L}\right)$$

Now from equation (i), $\mu.R_A + \mu R_B + m.g\sin\alpha = m.a$

$\mu(R_A + R_B) + m.g\sin\alpha = m.a$

$\mu.m.g.\cos\alpha + m.g\sin\alpha = m.a$ . . . [From equation (ii)]

$a = g(\mu.\cos\alpha + \sin \alpha)$

PROBLEMS

Example 1. A car moving on a level road at a speed 50 km/h has a wheel base 2.8metres, distance of C.G. from ground level 600 mm, and the distance of C.G. from rear wheels 1.2metres. Find the distance travelled by the car before coming to rest when brakes are applied,To the rear wheels, To the front wheels, and To all the four wheels. The coefficient of friction between the tyres and the road may be taken as0.6.

Solution.

Given :u = 50 km/h = 13.89 m/s ; L = 2.8 m ; h = 600 mm = 0.6 m ; x = 1.2 m ; μ = 0.6 Let s = Distance travelled by the car before coming to rest.

When brakes are applied to the rearwheels

Since the vehicle moves on a level road, therefore retardation of the car,

$$a = \frac{\mu.g(L-x)}{L+\mu.h} = \frac{0.6\times 9.81(2.8-1.2)}{2.8+0.6\times 0.6} = 2.98 \text{ m/s}^2$$

We know that for uniform retardation,

$$s = \frac{u^2}{2a} = \frac{(13.89)^2}{2\times 2.98} = 32.4 \text{ m}$$

When brakes are applied to the frontwheels

Since the vehicle moves on a level road, therefore retardation of the car,

$$a = \frac{\mu.g.x}{L-\mu.h} = \frac{0.6\times 9.18\times 1.2}{2.8-0.6\times 0.6} = 2.9 \text{ m/s}^2$$

We know that for uniform retardation,

When the brakes are applied to all the fourwheels

Since the vehicle moves on a level road, therefore retardation of the car,

$a = g.\mu = 9.81\times 0.6 = 5.886 \text{ m/s}_2$

We know that for uniform retardation,

$$s = \frac{u^2}{2a} = \frac{(13.89)^2}{2\times 5.886} = 16.4 \text{ m}$$

Example2. A vehicle moving on a rough plane inclined at 10° with the horizontal at a speed of 36 km/h has a wheel base 1.8 metres. The centre of gravity of the vehicle is 0.8 metre from the rear wheels and

0.9 metre above the inclined plane. Find the distance travelled by the vehicle before coming to rest and the time taken to do so when

The vehicle moves up the plane,and

The vehicle moves down theplane.

The brakes are applied to all the four wheels and the coefficient of friction is 0.5.

Solution.

Given : α= 10°; u = 36 km / h = 10 m / s ; L = 1.8 m ; x = 0.8 m ; h = 0.9 m ; μ = 0.5 Let s = Distance travelled by the vehicle before coming to rest,and

t = Time taken by the vehicle in coming to rest.

When the vehicle moves up the plane and brakes are applied to all the four wheel

Since the vehicle moves up the inclined plane, therefore retardation of the vehicle,

$a = g\,(\mu \cos\alpha + \sin\alpha)$

$= 9.81\,(0.5\cos 10° + \sin 10°) = 9.81(0.5\times0.9848 + 0.1736) = 6.53\ m/s^2$

We know that for uniform retardation,

$$s = \frac{u^2}{2a} = \frac{(10)^2}{2\times6.53} = 7.657\ \text{m}$$

and final velocity of the vehicle (v),

$0 = u + a.t = 10 - 6.53t$ (Minus sign due toretardation)

$t = 10 / 6.53 = 1.53$

When the vehicle moves down the plane and brakes are applied to all the four wheels

Since the vehicle moves down the inclined plane, therefore retardation of the vehicle,

$a = g\,(\mu \cos\alpha - \sin\alpha)$

$= 9.81(0.5\cos 10° - \sin 10°) = 9.81(0.5\times0.9848 - 0.1736) = 3.13\ m/s^2$

We know that for uniform retardation,

$$s = \frac{u^2}{2a} = \frac{(10)^2}{2\times3.13} = 16\ \text{m}$$

and final velocity of the vehicle (v),

$0 = u + a.t = 10 - 3.13t$ . . . (Minus sign due toretardation)

$t = 10/3.13 = 3.2$ s

DYNAMOMETER

A dynamometer is a brake but in addition it has a device to measure the frictional resistance. Knowing the frictional resistance, we may obtain the torque transmitted and hence the power of the engine.

Types of Dynamometers

Absorptiondynamometers,

Transmissiondynamometers.

In the *absorption dynamometers*, the entire energy or power produced by the engine is absorbed by the friction resistances of the brake and is transformed into heat, during the process of measurement. But in the *transmission dynamometers*, the energy is not wasted in friction but is used for doing work. The energy or power produced by the engine is transmitted through the dynamometer to some other machines where the power developed is suitably measured.

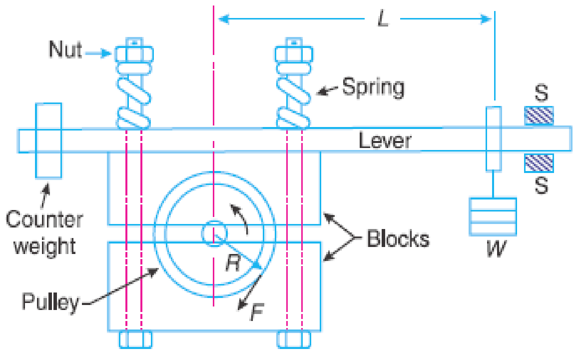
Classification of Absorption Dynamometers

Prony brakedynamometer, 2.Rope brakedynamometer.

rony brakedynamometer

A simplest form of an absorption type dynamometer is a prony brake dynamometer, as shown in Fig. It consists of two wooden blocks placed around a pulley fixed to the shaft of an engine whose power is required to be measured. The blocks are clamped by means of two bolts and nuts, as shown in Fig.

A helical spring is provided between the nut and the upper block to adjust the pressure on the pulley to control its speed. The upper block has a long lever attached to it and carries a weight *W* at its outer end. A counter weight is placed at the other end of the lever which balances the brake when unloaded. Two stops S, S are provided to limit the motion of thelever.



When the brake is to be put in operation, the long end of the lever is loaded with suitable weights *W* and the nuts are tightened until the engine shaft runs at a constant speed and the lever is in horizontal position. Under these conditions, the moment due to the weight *W* must balance the moment of the frictional resistance between the blocks and the pulley.

Let *W* = Weight at the outer end of the lever innewtons,

L = Horizontal distance of the weight *W* from the centre of the pulley inmetres,

F = Frictional resistance between the blocks and the pulley in newtons,

R = Radius of the pulley in metres, and

N = Speed of the shaft in r.p.m.

We know that the moment of the frictional resistance or torque on the shaft,

$T = W.L = F.R$ N-m

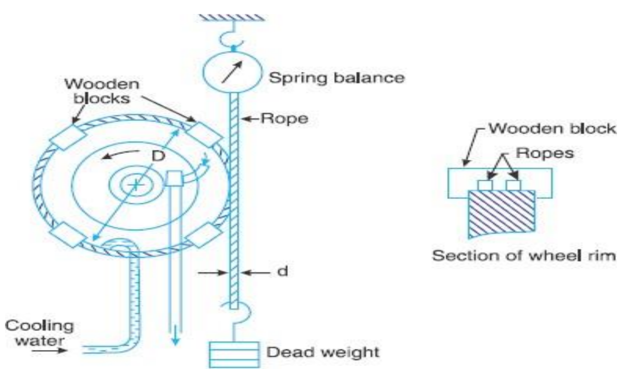
Work done in one revolution Work done in one revolution
= Torque × Angle turned in radians
= $T \times 2\pi$ N-m
_ Work done per minute

= $T \times 2\pi N$ N-m
We know that brake power of the engine

$$B.P. = \frac{\text{Work done per min.}}{60} = \frac{T \times 2\pi N}{60} = \frac{W.L \times 2\pi\ N}{60} \text{ watts}$$

Rope Brake Dynamometer

It is another form of absorption type dynamometer which is most commonly used for measuring the brake power of the engine. It consists of one, two or more ropes wound around the flywheel or rim of a pulley fixed rigidly to the shaft of an engine. The upper end of the ropes is attached to a spring balance while the lower end of the ropes is kept in position by applying a dead weight as shown in Fig.19.32. In



order to prevent the slipping of the rope over the flywheel, wooden blocks are placed at intervals around the circumference of the flywheel.

In the operation of the brake, the engine is made to run at a constant speed. The frictional torque, due to the rope, must be equal to the torque being transmitted by the engine.

Let *W* = Dead load innewtons,
S = Spring balance reading in newtons, *D* = Diameter of the wheel in metres, *d* = diameter of rope in metres, and
N = Speed of the engine shaft in r.p.m.

Net load on the brake= $(W - S)$ N

We know that distance moved in one revolution= $\pi (D + d)$ m Work done per revolution= $(W - S)$ _ $(D + d)$N-m
Work done per minute= $(W - S)\, \pi\, (D + d)\, N$N-m

Brake power of the engine,

$$B.P = \frac{\text{Work done per min}}{60} = \frac{(W - S)\,\pi\,(D + d)N}{60} \text{ watts}$$

,
If the diameter of the rope (*d*) is neglected, then brake power of the engine

$$B.P. = \frac{(W - S)\,\pi D\, N}{60} \text{ watts}$$

Example 1.In a laboratory experiment, the following data were recorded with rope brake: Diameter of the flywheel 1.2 m; diameter of the rope12.5 mm; speed of the engine 200 r.p.m.; dead load on the brake 600 N; spring balance reading 150 N. Calculate the brake power of the engine.

Solution.Given : D = 1.2 m ; d = 12.5 mm
= 0.0125 m ; N = 200 r.p.m ; W = 600 N ; S = 150 N
We know that brake power of the engine,

$$B.P. = \frac{(W - S)\,\pi\,(D + d)N}{60} = \frac{(600 - 150)\,\pi\,(1.2 + 0.0125)200}{60} = 5715 \text{ W}$$

Classification of Transmission Dynamometers

Epicyclic-traindynamometer,

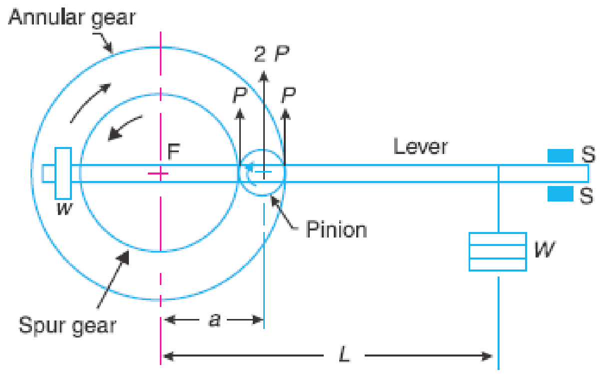
Belt transmission dynamometer,and

Torsiondynamometer

Epicyclic-train Dynamometer

An epicyclic-train dynamometer, as shown in Fig. 19.33, consists of a simple epicyclic train of gears, *i.e*. a spur gear, an annular gear (a gear having internal teeth) and a pinion. The spur gear is keyed to the engine shaft (*i.e*. driving shaft) and rotates in anticlockwise direction. The annular gear is also keyed to the driving shaft and rotates in clockwise direction. The pinion or the intermediate gear meshes with both the spur and annular gears. The pinion revolves freely on a lever which is pivoted to the common axis of the driving and driven shafts. A weight *w* is placed at the smaller end of the lever in order to keep it in position. A little consideration will show that if the

friction of the pinon which the pinion rotates is neglected, then the tangential effort *P* exerted by the spur gear on the pinion and the tangential reaction of the annular gear on the pinion areequal.



Since these efforts act in the upward direction as shown, therefore total upward force on the lever acting through the axis of the pinion is 2*P*. This force tends to rotate the lever about its fulcrum and it is balanced by a dead weight *W* at the end of the lever. The stops *S*, *S* are provided to control the movement of the lever.

$2P \times a = W.L$ or $P = W.L/2a$

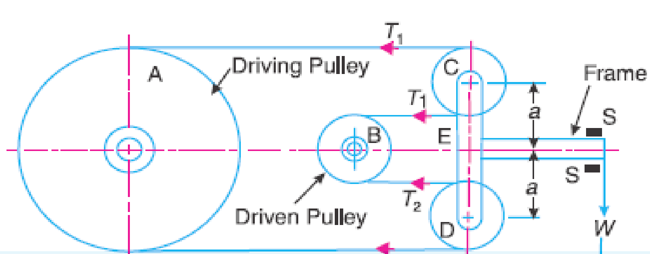
R = Pitch circle radius of the spur gear in metres, and

N = Speed of the engine shaft in r.p.m. Torque transmitted, $T = P.R$

Belt Transmission Dynamometer-Froude or Throneycroft Transmission Dynamometer

power transmitted $= \dfrac{T \times 2\pi N}{60} = \dfrac{P.R \times 2\pi N}{60}$ watts

When the belt is transmitting power from one pulley to another, the tangential effort on the driven pulley is equal to the difference between the tensions in the tight and slack sides of the belt. A belt dynamometer is introduced to measure directly the difference between the tensions of the belt, while it isrunning.



A belt transmission dynamometer, as shown in Fig, is called a Froude or Throneycroft transmission dynamometer. It consists of a pulley *A*(called driving pulley) which is rigidly fixed to the shaft of an engine whose power is required to be measured. There is another pulley *B* (called driven pulley) mounted on another shaft to which the power from pulley *A* is transmitted. The pulleys *A* and *B* are connected by means of a continuous belt passing round the two loose pulleys *C* and *D* which are mounted on a *T*-shaped frame. The frame is pivoted at *E* and its movement is controlled by two stops *S,S*. Since the tension in the tight side of the belt ($T1$) is greater than the tension in the slack side of the belt ($T2$), therefore the total force acting on the pulley *C* (*i.e.* $2T1$) is greater than the total force acting on the pulley *D* (*i.e.* $2T2$). It is thus obvious that the frame causes movement about *E* in the anticlockwise direction. In order to balance it, a weight *W* is applied at a distance *L* from *E* on the frame as shown inFig.

Now taking moments about the pivot *E*, neglecting friction, $2\,T_1 \times a = 2T_2 \times a + W\,L$

$$T_1 - T_2 = \frac{W.L}{2a}$$

Let *D* = diameter of the pulley *A* inmetres,

N = Speed of the engine shaft in r.p.m.

Work done in one revolution = $(T_1 - T_2)\pi D$ N-m work done per minute = $(T_1 - T_2)\pi\,DN$ N-m

$$\therefore \text{ Brake power of the engine, } B.P. = \frac{(T_1 - T_2)\pi\,DN}{60} \text{ watts}$$

Torsion Dynamometer

A torsion dynamometer is used for measuring large powers particularly the power transmitted along the propeller shaft of a turbine or motor vessel. A little consideration will show that when the power is being transmitted, then the driving end of the shaft twists through a small angle relative to the driven end of the shaft. The amount of twist depends upon many factors such as torque acting on the shaft (*T*), length of the shaft (*l*), diameter of the shaft (*D*) and modulus of rigidity (*C*) of the material of the shaft. We know that the torsion equationis

$$\frac{T}{J} = \frac{C.\theta}{l}$$

where θ = Angle of twist in radians, and

J = Polar moment of inertia of the shaft.

For a solid shaft of diameter *D*, the polar moment of inertia

$$J = \frac{\pi}{32} \times D^4$$

and for a hollow shaft of external diameter *D* and internal diameter *d*, the polar moment of inertia,

$$J = \frac{\pi}{32}(D^4 - d^4)$$

From the above torsion equation,

$$T = \frac{C.J}{l} \times \theta = k.\theta$$

Where $k = C.J/l$ is a constant for a particular shaft. Thus, the torque acting on the shaft is proportional to the angle of twist. This means that if the angle of twist is measured by some means, then the torque and hence the power transmitted may be determined. We know that the power transmitted P= 2πNT/60watts,

Where N is the speed in r.p.m.

PROBLEMS

Example 1. A torsion dynamometer is fitted to a propeller shaft of a marine engine. It is found that the shaft twists 2° in a length of 20 metres at 120 r.p.m. If the shaft is hollow with 400 mm external diameter and 300 mm internal diameter, find the power of the engine. Take modulus of rigidity for the shaft material as 80 GPa.

Solution.

Given : _ = 2° = 2 × _/180 = 0.035 rad ; l = 20 m ; N = 120 r.p.m. ; D = 400 mm= 0.4 m ;

d = 300 mm = 0.3 m ;C = 80 GPa = 80 × 109 N/m^2

We know that polar moment of inertia of the shaft

$$J = \frac{\pi}{32}(D^4 - d^4) = \frac{\pi}{32}\left[(0.4)^4 - (0.3)^4\right] = 0.0017\text{m}^4$$

and torque applied to the shaft,

$$T = \frac{C.J}{l} \times \theta = \frac{80 \times 10^9 \times 0.0017}{20} \times 0.035 = 238 \times 10^3 \text{ N-m}$$

We know that power of the engine,

$$P = \frac{T \times 2\pi N}{60} = \frac{238 \times 10^3 \times 2\pi \times 120}{60} = 2990 \times 10^3 \text{ W} = 2990 \text{ kW}$$

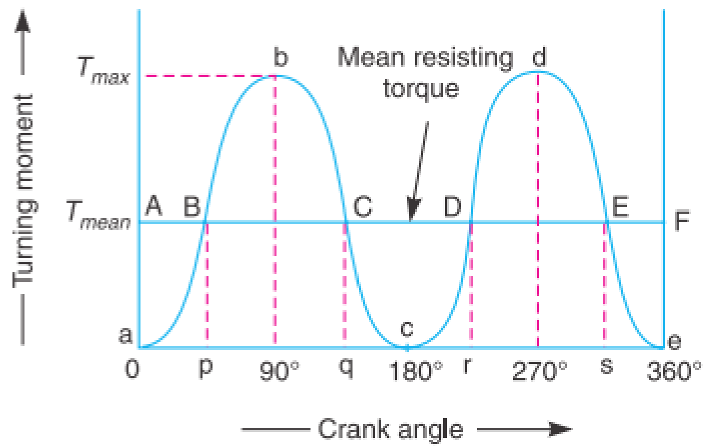
Turning Moment and Flywheel

The turning moment diagram (also known as crank- effort diagram) is the graphical representation of the turning moment or crank-effort for various positions of the crank. It is plotted on cartesian co-ordinates, in which the turning moment is taken as the ordinate and crank angle as abscissa.

Turning diagram for a single Cylinder Double acting Steam Engine:

A turning moment diagram for a single cylinder double acting steam engine is shown in Fig. 16.1. The vertical ordinate represents the turning moment and the horizontal ordinate represents the crank angle. the turning moment on the crankshaft,

$$T = F_P \times r \left(\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}} \right)$$



Turning moment diagram for a single cylinder, double acting steam engine.

FP = Piston effort, r = Radius of crank,

n = Ratio of the connecting rod length and radius of crank, and θ = Angle turned by the crank from inner dead centre.

From the above expression, we see that the turning moment (T) is zero, when the crank angle (θ) is zero. It is maximum when the crank angle is 90° and it is again zero when crank angle is 180°.This is shown by the curve abc in Fig. 16.1 and it represents the turning moment diagram for outstroke. The curve cde is the turning moment diagram for instroke and is somewhat similar to the curve abc.

Notes: 1. When the turning moment is positive (i.e. when the engine torque is more than the mean resisting torque) as shown between points B and C (or D and E) in Fig. 16.1, the crankshaft accelerates and the work is done by the steam.

When the turning moment is negative (i.e. when the engine torque is less than the mean resisting torque) as shown between points C and D in Fig. 16.1, the crankshaft retards and the work is done on the steam

. T = Torque on the crankshaft at any instant, and

Tmean=Mean resisting torque

Then accelerating torque on the rotating parts of the engine

= T – Tmean

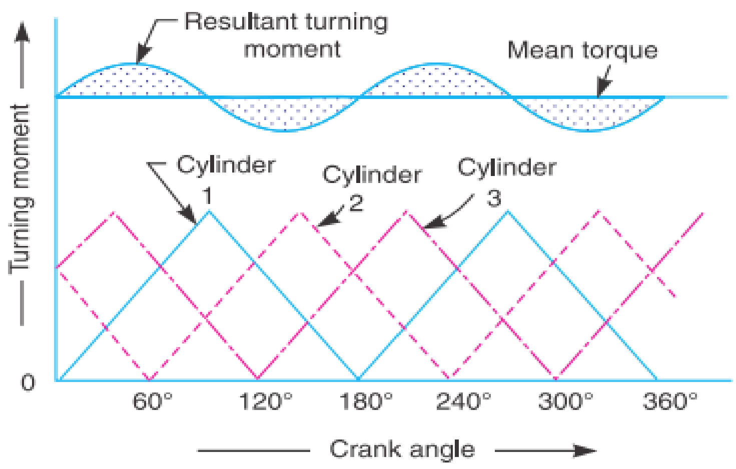
If (T –Tmean) is positive, the flywheel accelerates and if (T – Tmean) is negative, then the flywheel retards.

Turning Moment Diagram for a Four Stroke Cycle Internal Combustion Engine:

A turning moment diagram for a four stroke cycle internal combustion engine is shown in Fig. 16.2. We know that in a four stroke cycle internal combustion engine, there is one working stroke after the crank has turned through two revolutions, i.e. 720° (or 4 π radians).

Turning moment diagram for a four stroke cycle internal combustion engine..

Turning Moment Diagram for a Multi-cylinder Engine:



Turning moment diagram for a multi-cylinder engine.

Fluctuation of Energy:

Let the energy in the flywheel at A = E, then from Fig.

Let the energy in the flywheel at A = E, then from Fig. 16.4, we have

Energy at B = E + a1

Energy at C = E + a1– a2

Energy at D = E + a1 – a2 + a3

Energy at E = E + a1 – a2 + a3 – a4

Energy at F = E + a1 – a2 + a3 – a4 + a5

Energy at G = E + a1 – a2 + a3 – a4 + a5 – a6 = Energy at A (i.e. cycle repeats after G)

Let us now suppose that the greatest of these energies is at B and least at E. Therefore,

Maximum energy in flywheel

= E + a

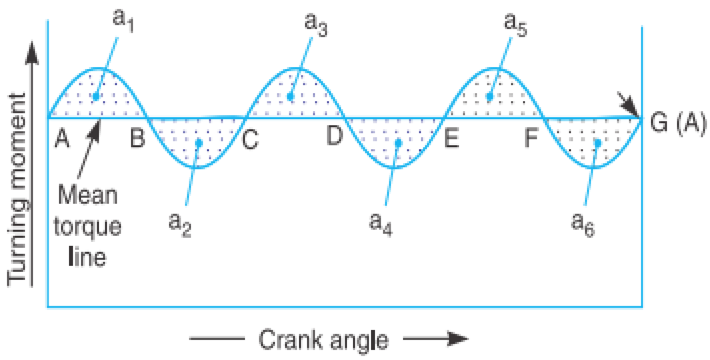
Minimum energy in the flywheel

= E + a1 – a2 + a3 – a4

∴ Maximum fluctuation of energy,

Δ E = Maximum energy – Minimum energy

=(E + a1) – (E + a1 – a2 + a3 – a4) = a2 – a3 + a4



Determination of maximum fluctuation of energy.

Coefficient of Fluctuation of Energy:

It may be defined as the ratio of the maximum fluctuation of energy to the work done per cycle. Mathematically, coefficient of fluctuation of energy,

$$C_E = \frac{\text{Maximum fluctuation of energy}}{\text{Work done per cycle}}$$

Work done per cycle = Tmean × θ

T mean = Mean torque, and

θ = Angle turned (in radians), in one revolution.

=2π, in case of steam engine and two stroke internal combustion engines

=4π, in case of four stroke internal combustion engines.

The mean torque (Tmean) in N-m may be obtained by using the following relation

$T_{mean} = \frac{P*60}{2\pi N}$

Coefficient of Fluctuation of Speed:

The difference between the maximum and minimum speeds during a cycle is called the maximum fluctuation of speed. The ratio of the maximum fluctuation of speed to the mean speed is called the coefficient of fluctuation of speed.

N1 and N2 = Maximum and minimum speeds in r.p.m. during the cycle, and

N = Mean speed in r.p.m $\frac{N1+N2}{2}$

Coefficient of fluctuation of speed

$$C_s = \frac{N_1 - N_2}{N} = \frac{2(N_1 - N_2)}{N_1 + N_2}$$

$$= \frac{\omega_1 - \omega_2}{\omega} = \frac{2(\omega_1 - \omega_2)}{\omega_1 + \omega_2} \quad \text{...(In terms of angular speeds)}$$

$$= \frac{v_1 - v_2}{v} = \frac{2(v_1 - v_2)}{v_1 + v_2} \quad \text{...(In terms of linear speeds)}$$

Note. The reciprocal of the coefficient of fluctuation of speed is known as coefficient of steadiness and is denoted by m

$$m = \frac{1}{C_s} = \frac{N}{N_1 - N_2}$$

Flywheel

A flywheel used in machines serves as a reservoir, which stores energy during the period when the supply of energy is more than the requirement, and releases it during the period when the requirement of energy is more than the supply.

- In case of steam engines, internal combustion engines, reciprocating compressors and pumps, the energy is developed during one stroke and the engine is to run for the whole cycle on the energy produced during this one stroke.
- For example, in internal combustion engines, the energy is developed only during expansion or power stroke which is much more than the engine load and no energy is being developed during suction, compression and exhaust strokes in case of four stroke engines and during compression in case of two stroke engines.
- The excess energy developed during power stroke is absorbed by the flywheel and releases it to the crankshaft during other strokes in which no energy is developed, thus rotating the crankshaft at a uniform speed.
- A little consideration will show that when the flywheel absorbs energy, its speed increases and when it releases energy, the speed decreases. Hence a flywheel does not maintain a constant speed, it simply reduces the fluctuation of speed.
- In other words, a flywheel controls the speed variations caused by the fluctuation of the engine turning moment during each cycle of operation.
- In machines where the operation is intermittent like punching machines, shearing machines, rivetting machines, crushers, etc., the flywheel stores energy from the power source during the greater portion of the operating cycle and gives it up during a small period of the cycle. Thus, the energy from the power source to the machines is supplied practically at a constant rate throughout the operation.

Energy stored in Flywheel:

When a flywheel absorbs energy, its speed increases and when it gives up energy, its speed decreases.

Let m = Mass of the flywheel in kg

k = Radius of gyration of the flywheel in metres,

I = Mass moment of inertia of the flywheel about its axis of rotation in kg-m2 = m.k2,

N 1 and N2 = Maximum and minimum speeds during the cycle in r.p.m., ω1 and ω2 = Maximum and minimum angular speeds during the cycle in rad/s,

$N =$ Mean speed in r.p.m. $= \frac{N_1 + N_2}{2}$

$\omega =$ Mean angular speed during the cycle in rad/s $= \frac{\omega_1 + \omega_2}{2}$

$C_S =$ Coefficient of fluctuation of speed $= \frac{N_1 - N_2}{N}$ or $\frac{\omega_1 + \omega_2}{\omega}$

We know that the mean kinetic energy of the flywheel,

$$E = \frac{1}{2} I.\omega^2 = \frac{1}{2} m.k^2.\omega^2$$

As the speed of the flywheel changes from ω_1 to ω_2, the maximum fluctuation of energy,

ΔE = Maximum K.E. – Minimum K.E.

$$= \frac{1}{2} I.(\omega_1)^2 - \frac{1}{2} I.(\omega_2)^2 = \frac{1}{2} I\left[(\omega_1)^2 - (\omega_2)^2\right]$$

$$= \frac{1}{2} I(\omega_1 + \omega_2)(\omega_1 - \omega_2) = I.\omega(\omega_1 - \omega_2) \quad \text{..................(i)}$$

$$= I.\omega^2\left(\frac{\omega_1 - \omega_2}{\omega}\right)$$

$$= I.\omega^2 C_S = m.k^2.\omega^2 C_S \quad \text{....................(ii)}$$

$$= 2.E.C_S \quad \text{................(iii)}$$

he radius of gyration (k) may be taken equal to the mean radius of the rim (R), because the thickness of rim is very small as compared to the diameter of rim. Therefore, substituting k = R, in equation (ii), we have

$$\Delta E = m.R^2.\omega^2.C_S = m.v^2.C_S$$

v = Mean linear velocity (*i.e.* at the mean radius) in m/s

Problems:

The mass of flywheel of an engine is 6.5 tonnes and the radius of gyration is 1.8 metres. It is found from the turning moment diagram that the fluctuation of energy is 56 kN-m. If the mean speed of the engine is 120 r.p.m., find the maximum and minimum speeds.

Given : m = 6.5 t = 6500 kg ; k = 1.8 m ; Δ E = 56 kN-m = 56 × 103 N-m ; N = 120 r.p.m.

Let N1 and N2 = Maximum and minimum speeds respectively. We know that fluctuation of energy (Δ E),

$$56 \times 10^3 = \frac{\pi^2}{900} \times m.k^2 . N (N_1 - N_2) = \frac{\pi^2}{900} \times 6500 (1.8)^2\ 120 (N_1 - N_2)$$

$$= 27\ 715 (N_1 - N_2)$$

$\therefore \quad N_1 - N_2 = 56 \times 10^3 / 27\ 715 = 2$ r.p.m. ...(*i*)

We also know that mean speed (N),

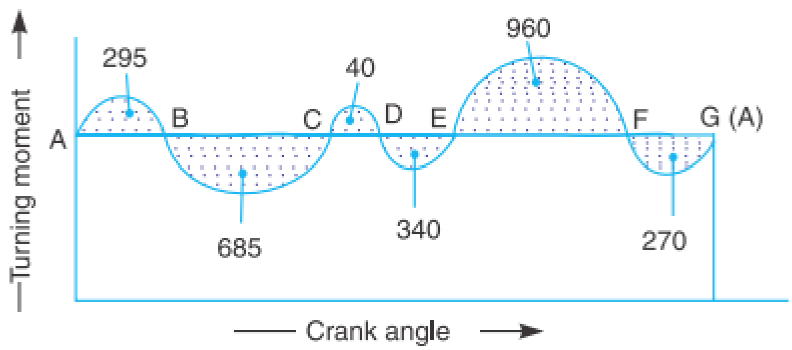
$$120 = \frac{N_1 + N_2}{2} \text{ or } N_1 + N_2 = 120 \times 2 = 240 \text{ r.p.m.} \quad ...(ii)$$

From equations (*i*) and (*ii*),

$N_1 = 121$ r.p.m., and $N_2 = 119$ r.p.m. **Ans.**

2. The turning moment diagram for a petrol engine is drawn to the following scales : Turning moment, 1 mm = 5 N-m ; crank angle, 1 mm = 1°. The turning moment diagram repeats itself at every half revolution of the engine and the areas above and below the mean turning moment line taken in order are 295, 685, 40, 340, 960, 270 mm2. The rotating parts are equivalent to a mass of 36 kg at a radius of gyration of 150 mm. Determine the coefficient of fluctuation of speed when the engine runs at 1800 r. p . m.

Given : m = 36 kg ; k = 150 mm = 0.15 m ; N = 1800 r.p.m. or ω = 2 π × 1800/60 = 188.52 rad /s



Since the turning moment scale is 1 mm = 5 N-m and crank angle scale is 1 mm = 1° = π /180 rad, therefore,

1 mm2 on turning moment diagram

$$= 5 \times \frac{\pi}{180} = \frac{\pi}{36} \text{ N-m}$$

Let the total energy at A = E,

Energy at B = E + 295

... (Maximum energy)

Energy at C = E + 295 – 685 = E – 390

Energy at D = E – 390 + 40 = E – 350

Flywheel of an electric motor.

Energy at E = E – 350 – 340 = E – 690 ...(Minimum energy)

Energy at F = E – 690 + 960 = E + 270

Energy at G = E + 270 – 270 = E = Energy at A

We know that maximum fluctuation of energy,

Δ E = Maximum energy – Minimum energy =(E + 295) – (E – 690) = 985 mm^2

$$= 985 \times \frac{\pi}{36} = 86\ \text{N-m} = 86\ \text{J}$$

Let C_S = Coefficient of fluctuation of speed.

We know that maximum fluctuation of energy (ΔE),

$$86 = m.k^2\,\omega^2.C_S = 36 \times (0.15)^2 \times (188.52)^2\, C_S = 28\,787\, C_S$$

$$\therefore \quad C_S = 86 / 28\,787 = 0.003 \text{ or } 0.3\% \quad \textbf{Ans.}$$

Dimensions of the Flywheel Rim

Consider a rim of the flywheel as shown in Fig

D = Mean diameter of rim in metres,

R = Mean radius of rim in metres, A = Cross-sectional area of rim in m2, ρ=Density of rim material in kg/m3, N = Speed of the flywheel in r.p.m.,

ω = Angular velocity of the flywheel in rad/s, v = Linear velocity at the mean radius in m/s

= ω .R = π D.N/60, and

σ = Tensile stress or hoop stress in N/m2 due to the centrifugal force

Consider a small element of the rim as shown shaded in Fig. 16.17. Let it subtends an angle δθ at the centre of the flywheel.

Volume of the small element = A × R.δθ

∴ Mass of the small element

dm = Density × volume = ρ.A.R.δθ

nd centrifugal force on the element, acting radially outwards,

$dF = dm.\omega^2.R = \rho.A.R^2.\omega^2.\delta\theta$

Vertical component of dF= dF.sin θ = ρ.A.R2.ω2.δθ.sin θ

Total vertical upward force tending to burst the rim across the diameter X Y.

$$= \rho.A.R^2.\omega^2 \int_0^{\pi} \sin\theta.d\theta = \rho.A.R^2.\omega^2\, [-\cos\theta]_0^{\pi}$$

$$= 2\rho.A.R^2.\omega^2 \qquad \ldots (i)$$

This vertical upward force will produce tensile stress or hoop stress (also called centrifugal stress or circumferential stress), and it is resisted by 2P, such that

$$2P = 2\sigma.A \quad \text{. . . }(ii)$$

Equating equations (*i*) and (*ii*),

$$2.\rho.A.R^2.\omega^2 = 2\sigma.A$$

$$\sigma = \rho.R^2.\omega^2 = \rho.v^2 \quad \text{....}(\because v = \omega.R)$$

$$\therefore \quad v = \sqrt{\frac{\sigma}{\rho}} \quad \text{...}(iii)$$

We know that mass of the rim,

$$m = \text{Volume} \times \text{density} = \pi\, D.A.\rho$$

$$\therefore \quad A = \frac{m}{\pi.D.\rho} \quad \text{...}(iv)$$

From equations (iii) and (iv), we may find the value of the mean radius and cross-sectional area of the rim.

Note: If the cross-section of the rim is a rectangular, then

A = b × t

where

b = Width of the rim, and t = Thickness of the rim.

Problem: The turning moment diagram for a multi-cylinder engine has been drawn to a scale of 1 mm to 500 N-m torque and 1 mm to 6° of crank displacement. The intercepted areas between output torque curve and mean resistance line taken in order from one end, in sq. mm are

– 30, + 410, – 280, + 320, – 330, + 250, – 360, + 280, – 260 sq. mm, when the engine is running at 800 r.p.m.

The engine has a stroke of 300 mm and the fluctuation of speed is not to exceed ± 2% of the mean speed. Determine a suitable diameter and cross-section of the flywheel rim for a limiting value of the safe centrifugal stress of 7 MPa. The material density may be assumed as 7200 kg/m3. The width of the rim is to be 5 times the thickness.

Given : N = 800 r.p.m. or ω = 2π × 800 / 60 = 83.8 rad/s; *Stroke = 300 mm ; σ = 7 MPa = 7 × 106 N/m^2 ; ρ = 7200 kg/m^3

Since the fluctuation of speed is ± 2% of mean speed, therefore total fluctuation of speed,

ω1 – ω2 = 4% ω = 0.04 ω and coefficient of fluctuation of speed,

$$C_S = \frac{\omega_1 - \omega_2}{\omega} = 0.04$$

Diameter of the flywheel rim

D = Diameter of the flywheel rim in metres, and v = Peripheral velocity of the flywheel rim in m/s.

We know that centrifugal stress (σ),

7 × 106 = ρ.v^2 = 7200 v2 or v^2 = 7 × 106/7200 = 972.2

v = 31.2 m/s

∴ We know that $v = \pi D.N/60$

$D = v \times 60 / \pi N = 31.2 \times 60/\pi \times 800 = 0.745$ m

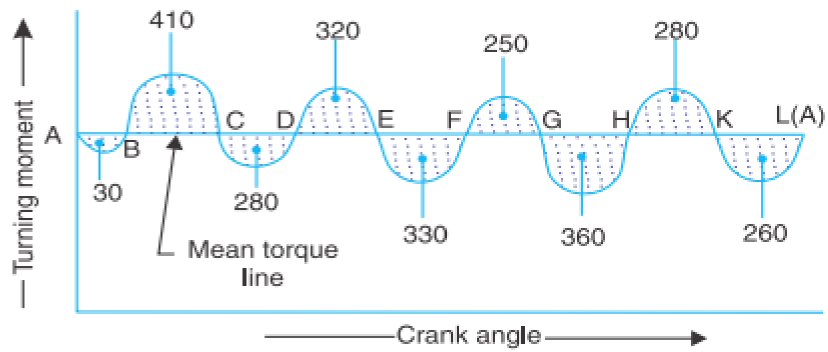
Cross-section of the flywheel rim

t = Thickness of the flywheel rim in metres, and b = Width of the flywheel rim in metres = 5 t

∴ Cross-sectional area of flywheel rim,

$A = b.t = 5\,t \times t = 5\,t^2$

First of all, let us find the mass (m) of the flywheel rim. The turning moment diagram is



Since the turning moment scale is 1 mm = 500 N-m and crank angle scale is 1 mm = 6° = π /30 rad, therefore 1 mm2 on the turning moment diagram

$= 500 \times \pi / 30 = 52.37$ N-m

Let the energy at A = E, then referring to Fig

Energy at B = E – 30 . . . (Minimum energy)

Energy at C = E – 30 + 410 = E + 380

Energy at D = E + 380 – 280 = E + 100

Energy at E = E + 100 + 320 = E + 420 . . . (Maximum energy)

Energy at F = E + 420 – 330 = E + 90

Energy at G = E + 90 + 250 = E + 340

Energy at H = E + 340 – 360 = E – 20

Energy at K = E – 20 + 280 = E + 260

Energy at L = E + 260 – 260 = E = Energy at A

We know that maximum fluctuation of energy,

ΔE = Maximum energy – Minimum energy

$=(E + 420) - (E - 30) = 450\ mm^2 = 450 \times 52.37 = 23\,566$ N-m

We also know that maximum fluctuation of energy (ΔE),

$23\,566 = m.v^2.CS = m \times (31.2)^2 \times 0.04 = 39\,m$

$m = 23566 / 39 = 604$ kg

∴ We know that mass of the flywheel rim (m),

$604 = \text{Volume} \times \text{density} = \pi\, D.A.\rho$

$= \pi \times 0.745 \times 5t^2 \times 7200 = 84\,268\ t^2$

$t^2 = 604 / 84\,268 = 0.007\,17\ m^2$ or $t = 0.085$ m = 85 mm Ans. $b = 5t = 5 \times 85 = 425$ mm

INDUSTRIAL APPLICATIONS

1. **Mining & motor vehicles**

TUTORIAL QUESTIONS

1. Define ‘inertia force’ and ‘inertia torque’.
2. Derive an expression for the inertia force due to reciprocating mass in reciprocating engine, neglecting the mass of the connecting rod.
3. Describe with a neat sketch the working of a single plate friction clutch.
4. Which of the two assumptions-uniform intensity of pressure or uniform rate of wear, would you make use of in designing friction clutch and why ?
5. Determine the maximum, minimum and average pressure in plate clutch when the axial force is 4 kN. The inside radius of the contact surface is 50 mm and the outside radius is 100 mm. Assume uniform wear.
6. A single plate clutch, with both sides effective, has outer and inner diameters 300 mm and 200 mm respectively. The maximum intensity of pressure at any point in the contact surface is not to exceed 0.1 N/mm2. If the coefficient of friction is 0.3, determine the power transmitted by a clutch at a speed 2500 r.p.m.
7. A cone clutch with cone angle 20º is to transmit 7.5 kW at 750 r.p.m. The normal intensity of pressure between the contact faces is not to exceed 0.12 N/mm2. The coefficient of friction is 0.2. If face width is 15 th of mean diameter, find : 1. the main dimensions of the clutch, and 2. axial force required while running
8. An engine flywheel has a mass of 6.5 tonnes and the radius of gyration is 2 m. If the maximum and minimum speeds are 120 r. p. m. and 118 r. p. m. respectively, find maximum fluctuation of energy.
9. In a turning moment diagram, the areas above and below the mean torque line taken in order are 4400, 1150, 1300 and 4550 mm2 respectively. The scales of the turning moment diagram are:Turning moment, 1 mm = 100 N-m ; Crank angle, 1 mm = 1°Find the mass of the flywheel required to keep the speed between 297 and 303 r.p.m., if the radius of gyration is 0.525 m.
10. A steam engine runs at 150 r.p.m. Its turning moment diagram gave the following area measurements in mm2 taken in order above and below the mean torque line: 500, – 250, 270, – 390, 190, – 340, 270, – 250 The scale for the turning moment is 1 mm = 500 N-m, and for crank angle is 1mm = 5°. The fluctuation of speed is not to exceed ± 1.5% of the mean, determine the cross-section of the rim of the flywheel assumed rectangular with axial dimension equal to 1.5 times the radial dimension. The hoop stress is limited to 3 MPa and the density of the material of the flywheel is 7500 kg/m3.
11. Draw the turning moment diagram of a single cylinder double acting steam engine.

12. Explain the turning moment diagram of a four stroke cycle internal combustion engine.
13. Define the terms 'coefficient of fluctuation of energy' and 'coefficient of fluctuation of speed', in the case of flywheels.

ASSIGNMENT QUESTIONS

1. Describe with a neat sketch a centrifugal clutch and deduce an equation for the total torque transmitted.
2. Describe with a neat sketch the working of a single plate friction clutch.
3. Establish a formula for the maximum torque transmitted by a single plate clutch of external and internal radii r1 and r2, if the limiting coefficient of friction is μ and the axial spring load is W. Assume that the pressure intensity on the contact faces is uniform.
4. A centrifugal clutch has four shoes which slide radially in a spider keyed to the driving shaft and make contact with the internal cylindrical surface of a rim keyed to the driven shaft. When the clutch is at rest, each shoe is pulled against a stop by a spring so as to leave a radial clearance of 5 mm between the shoe and the rim. The pull exerted by the spring is then 500 N. The mass centre of the shoe is 160 mm from the axis of the clutch.If the internal diameter of the rim is 400 mm, the mass of each shoe is 8 kg, the stiffness of each spring is 50 N/mm and the coefficient of friction between the shoe and the rim is 0.3 ; find the power transmitted by the clutch at 500 r.p.m.
5. A centrifugal clutch is to transmit 15 kW at 900 r.p.m. The shoes are four in number. The speed at which the engagement begins is 3/4th of the running speed. The inside radius of the pulley rim is 150 mm and the centre of gravity of the shoe lies at 120 mm from the centre of the spider. The shoes are lined with Ferrodo for which the coefficient of friction may be taken as 0.25. Determine : 1. Mass of the shoes, and 2. Size of the shoes, if angle subtended by the shoes at the centre of the spider is 60º and the pressure exerted on the shoes is 0.1 N/mm2.
6. The contact surfaces in a cone clutch have an effective diameter of 75 mm. The semi-angle of the cone is 15º. The coefficient of friction is 0.3. Find the torque required to produce slipping of the clutch if an axial force applied is 180 N. This clutch is employed to connect an electric motor running uniformly at 1000 r.p.m. with a flywheel which is initially stationary. The flywheel has a mass of 13.5 kg and its radius of gyration is 150 mm. Calculate the time required for the flywheel to attain full speed and also the energy lost in the slipping of the clutch.
7. What is the function of a flywheel? How does it differ from that of a governor?
8. A punching machine makes 25 working strokes per minute and is capable of punching 25 mm diameter holes in 18 mm thick steel plates having an ultimate shear strength 300 MPa. The punching operation takes place during 1/10th of a revolution of the crankshaft.Estimate the power needed for the driving motor, assuming a mechanical efficiency of 95 percent. Dtetermine suitable dimensions for the rim cross-section of the flywheel, having width equal to twice thickness. The flywheel is to revolve at 9 times the speed of the crankshaft. The permissible

coefficient of fluctuation of speed is 0.1. The flywheel is to be made of cast iron having a working stress (tensile) of 6 MPa and density of 7250 kg/m3. The diameter of the flywheel must not exceed 1.4 m owing to space restrictions. The hub and the spokes may be assumed to provide 5% of the rotational inertia of the wheel.

9. The turning moment diagram for a multi-cylinder engine has been drawn to a scale of 1 mm to 500 N-m torque and 1 mm to 6° of crank displacement. The intercepted areas between output torque curve and mean resistance line taken in order from one end, in sq. mm are– 30, + 410, – 280, + 320, – 330, + 250, – 360, + 280, – 260 sq. mm, when the engine is running at 800 r.p.m. The engine has a stroke of 300 mm and the fluctuation of speed is not to exceed ± 2% of the mean speed. Determine a suitable diameter and cross-section of the flywheel rim for a limiting value of the safe centrifugal stress of 7 MPa. The material density may be assumed as 7200 kg/m3. The width of the rim is to be 5 times the thickness.
10. The equation of the turning moment curve of a three crank engine is (5000 + 1500 sin 3 θ) N-m, where θ is the crank angle in radians. The moment of inertia of the flywheel is 1000 kg-m2 and the mean speed is 300 r.p.m. Calculate : 1. power of the engine, and 2. the maximum fluctuation of the speed of the flywheel in percentage when (i) the resisting torque is constant, and (ii) the resisting torque is (5000 + 600 sin θ) N-m.

UNIT 4

BALANCING AND VIBRATION

Course Objectives

To study about the balancing, unbalancing of rotating masses and the effect of Dynamics of undesirable vibrations

Course Outcomes

Ability to understand the importance of balancing and implications of computed results in dynamics to improve the design of a mechanism

Balancing of Rotating Masses

The high speed of engines and other machines is a common phenomenon now-a- days. It is, therefore, very essential that all the rotating and reciprocating parts should be completely balanced as far as possible. If these parts are not properly balanced, the dynamic forces are set up. These forces not only increase the loads on bearings and stresses in the various members, but also produce unpleasant and even dangerous vibrations. In this chapter we shall discuss the balancing of unbalanced forces caused by rotating masses, in order to minimize pressure on the main bearings when an engine is running.

We have already discussed, that whenever a certain mass is attached to a rotating shaft, it exerts some centrifugal force, whose effect is to bend the shaft and to produce vibrations in it. In order to prevent the effect of centrifugal force, another mass is attached to the opposite side of the shaft, at such a position so as to balance the effect of the centrifugal force of the first mass. This is done in such a way that the centrifugal force of both the masses aremade to be equal and opposite. The process of providing the second mass in order to counteract the effect of the centrifugal force of the first mass, is called ***balancing of rotatingmasses*.**

The following cases are important from the subject point of view:

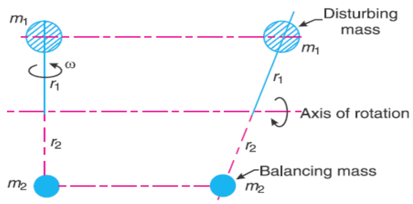
- Balancing of a single rotating mass by a single mass rotating in the same plane.
- Balancing of a single rotating mass by two masses rotating in different planes.
- Balancing of different masses rotating in the same plane.
- Balancing of different masses rotating in different planes.
- We shall now discuss these cases, in detail, in the following pages.

Balancing of a Single Rotating Mass By a Single Mass Rotating in the Same Plane

Consider a disturbing mass m_1 attached to a shaft rotating at ω rad/s as shown in Fig. Let r_1 be the radius of rotation of the mass m_1 (*i.e*. distance between the axis of rotation of the shaft and the centre of gravity of the mass m_1).We know that the centrifugal force exerted by the mass m_1 on the shaft,

$$F_{C1} = m_1 \cdot \omega^2 \cdot r_1$$

This centrifugal force acts radially outwards and thus produces bending moment on the shaft. In order to counteract the effect of this force, a balancing mass (m_2) may be attached in the same plane of rotation as that of disturbing mass (m_1) such that the centrifugal forces due to the two masses are equal and opposite



Let $r2$ = Radius of rotation of the balancing mass $m2$ (*i.e.* distance between the axis of rotation of the shaft and the centre of gravity of mass m_2).

∴ Centrifugal force due to mass m_2

$$F_{C2} = m_2 \cdot \omega^2 \cdot r_2$$

Equating equations **(*i*)** and **(*ii*),**

$$m_1.\omega^2 \cdot r_1 = m_2 \cdot \omega^2 \cdot r_2 \quad \text{or} \quad m_1 \cdot r_1 = m_2 \cdot r_2$$

Balancing of a Single Rotating Mass By Two Masses Rotating in Different Planes

We have discussed in the previous article that by introducing a single balancing mass in the same plane of rotation as that of disturbing mass, the centrifugal forces are balanced. In other words, the two forces are equal in magnitude and opposite in direction. But this type of arrangement for balancing gives rise to a couple which tends to rock the shaft in its bearings. Therefore in order to put the system in complete balance, two balancing masses are placed in two different planes, parallel to the plane of rotation of the disturbing mass, in such a way that they satisfy the following two conditions of equilibrium.

The net dynamic force acting on the shaft is equal to zero. This requires that the line of action of three centrifugal forces must be the same. In other words, the centre of the masses of the system must lie on the axis of rotation. This is the condition for ***static balancing*.** The net couple due to the dynamic forces acting on the shaft is equal to zero. In other words, the algebraic sum of the moments about any point in the plane must be zero.

The conditions **(1)** and **(2)**together give ***dynamic balancing*.** The following two possibilities may arise while attaching the two balancing masses:

The plane of the disturbing mass may be in between the planes of the two balancing masses, and the plane of the disturbing mass may lie on the left or right of the two planes containing the balancing masses.

We shall now discuss both the above cases one by one.

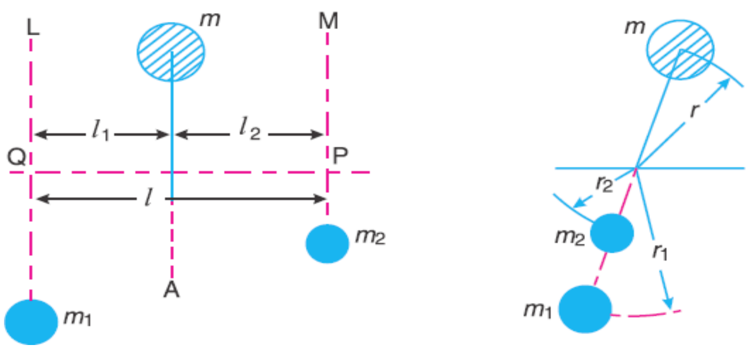
When the plane of the disturbing mass lies in between the planes of the two balancing masses

Consider a disturbing mass *m* lying in a plane *A* to be balanced by two rotating masses m_1 and m_2 lying in two different planes *L* and *M* as shown in Fig. Let r, r_1 and r_2 be the radii of rotation of the masses in planes *A*, *L* and *M* respectively.

Let l_1 = Distance between the planes *A* an d*L*,

l_2 = Distance between the planes *A* and *M*, and

l = Distance between the planes *L* and *M*



We know that the centrifugal force exerted by the mass *m* in the plane *A*,

$$F_C = m \cdot \omega^2 \cdot r$$

Similarly, the centrifugal force exerted by the mass m_1 in the plane *L*,

$$F_{C1} = m_1 \cdot \omega^2 \cdot r_1$$

and, the centrifugal force exerted by the mass m_2 in the plane *M*,

$$F_{C2} = m_2 \cdot \omega^2 \cdot r_2$$

Since the net force acting on the shaft must be equal to zero, therefore the centrifugal force on the disturbing mass must be equal to the sum of the centrifugal forces on the balancing masses,therefore

$$F_C = F_{C1} + F_{C2} \quad \text{or} \quad m \cdot \omega^2 \cdot r = m_1.\omega^2 \cdot r_1 + m_2 \cdot \omega^2 \cdot r_2$$

$$m \cdot r = m_1 \cdot r_1 + m_2 \cdot r_2 \qquad \ldots (i)$$

Now in order to find the magnitude of balancing force in the plane *L* (or the dynamic force at the bearing *Q* of a shaft), take moments about *P* which is the point of intersection of the plane *M* and the axis of rotation .Therefore

$$F_{C1} \times l = F_C \times l_2 \quad \text{or} \quad m_1 \cdot \omega^2 \cdot r_1 \times l = m \cdot \omega^2 \cdot r \times l_2$$

$$m_1 \cdot r_1 \cdot l = m \cdot r \cdot l_2 \quad \text{or} \quad m_1 \cdot r_1 = m \cdot r \times \frac{l_2}{l}$$

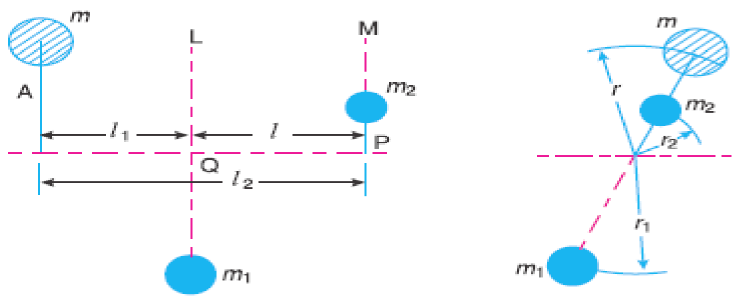
Similarly, in order to find the balancing force in plane *M* (or the dynamic force at the bearing *P* of a shaft), take moments about *Q* which is the point of intersection of the plane *L* and the axis of rotation.Therefore

$$F_{C2} \times l = F_C \times l_1 \quad \text{or} \quad m_2 \cdot \omega^2 \cdot r_2 \times l = m \cdot \omega^2 \cdot r \times l_1$$

$$m_2 \cdot r_2 \cdot l = m \cdot r \cdot l_1 \quad \text{or} \quad m_2 \cdot r_2 = m \cdot r \times \frac{l_1}{l}$$

It may be noted that equation **(*i*)** represents the condition for static balance,

When the plane of the disturbing mass lies on one end of the planes of the balancing masses



In this case, the mass m lies in the plane A and the balancing masses liein the planes L and M, as shown in Fig. As discussed above, the following conditions must be satisfied in order to balance the system,i.e.

$$F_C + F_{C2} = F_{C1} \quad \text{or} \quad m \cdot \omega^2 \cdot r + m_2 \cdot \omega^2 \cdot r_2 = m_1 \cdot \omega^2 \cdot r_1$$

$$m \cdot r + m_2 \cdot r_2 = m_1 . r_1 \qquad \ldots (iv)$$

Now, to find the balancing force in the plane *L* (or the dynamic force at the bearing *Q* of a shaft), take moments about *P* which is the point of intersection of the plane *M* and the axis of rotation. Therefore

$$F_{C1} \times l = F_C \times l_2 \quad \text{or} \quad m_1 \cdot \omega^2 \cdot r_1 \times l = m \cdot \omega^2 \cdot r \times l_2$$

$$m_1 \cdot r_1 \cdot l = m \cdot r \cdot l_2 \quad \text{or} \quad m_1 \cdot r_1 = m \cdot r \times \frac{l_2}{l} \qquad \ldots (v)$$

Similarly, to find the balancing force in the plane *M* (or the dynamic force at the bearing *P* of a shaft), take moments about *Q* which is the point of intersection of the plane *L* and the axis of rotation.Therefore

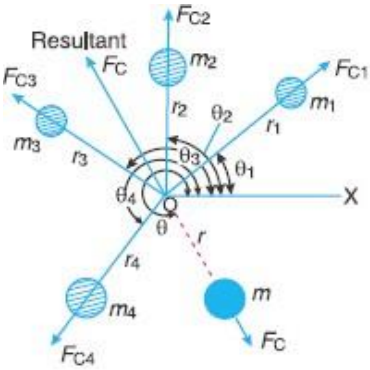
$$F_{C2} \times l = F_C \times l_1 \quad \text{or} \quad m_2 \cdot \omega^2 \cdot r_2 \times l = m \cdot \omega^2 \cdot r \times l_1$$

$$m_2 \cdot r_2 \cdot l = m \cdot r \cdot l_1 \quad \text{or} \quad m_2 \cdot r_2 = m \cdot r \times \frac{l_1}{l} \qquad \ldots (vi)$$

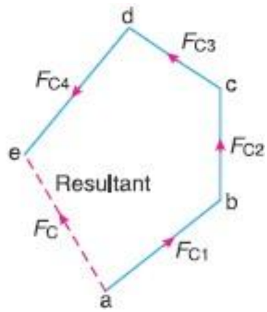
Balancing of Several Masses Rotating in the Same Plane

Consider any number of masses (say four) of magnitude m_1, m_2, m_3 and m_4 at distances of r_1, r_2, r_3 and r_4 from the axis of the rotating shaft. Let θ_1, θ_2, θ_3 and θ_4 be the angles of these masses with the horizontal line *OX*, as shown in Fig. Let these masses rotate about an axis through *O* and perpendicular to the plane of paper, with a constant angular velocity of ω rad/s.

The magnitude and position of the balancing mass may be found out analytically or graphically as discussed below:



(*a*) Space diagram.



(*b*) Vector diagram.

Analytical method

The magnitude and direction of the balancing mass may be obtained, analytically, as discussed below:

First of all, find out the centrifugal force*(or the product of the mass and its radius of rotation) exerted by each mass on the rotatingshaft.

Resolve the centrifugal forces horizontally and vertically and find their sums, i.e. ΣH and ΣV .We know that Sum of horizontal components of the centrifugalforces,

$\Sigma H = m_1 \cdot r_1 \cos \theta_1 + m_2 \cdot r_2 \cos \theta_2 + \ldots\ldots$

sum of vertical components of the centrifugal forces,

$\Sigma V = m_1 \cdot r_1 \sin \theta_1 + m_2 \cdot r_2 \sin \theta_2 + \ldots\ldots$

Magnitude of the resultant centrifugal force,

$$F_C = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

If θ is the angle, which the resultant force makes with the horizontal,then

$\tan \theta = \Sigma V / \Sigma H$

The balancing force is then equal to the resultant force, but in **oppositedirection.**

Now find out the magnitude of the balancing mass, suchthat

$F_C = m \cdot r$ where m = Balancing mass,and

r = Its radius of rotation.

Graphicalmethod

The magnitude and position of the balancing mass may also be obtained graphically as discussed below:

First of all, draw the space diagram with the positions of the several masses,

Find out the centrifugal force (or product of the mass and radius of rotation) exerted by each mass on the rotating shaft.

Now draw the vector diagram with the obtained centrifugal forces (or the product of the masses and their radii of rotation), such that ab represents the centrifugal force exerted by the mass m_1 (or $m_1.r_1$) in magnitude and direction to some suitable scale. Similarly, draw bc, cd and deto represent centrifugal forces of other masses m_2, m_3 and m_4 (or $m_2.r_2$, $m_3.r_3$and $m_4.r_4$).

Now, as per polygon law of forces, the closing side ae represents the resultant force in magnitude and direction.

The balancing force is, then, equal to the resultant force, but in **opposite direction.**

Now find out the magnitude of the balancing mass (m) at a given radius of rotation (r), such that

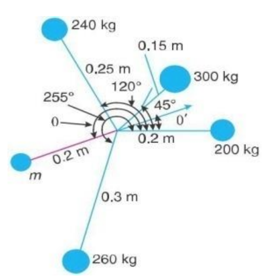
$$m \cdot \omega^2 \cdot r = \text{Resultant centrifugal force}$$

$$m.r = \text{Resultant of } m_1.r_1,\ m_2.r_2,\ m_3.r_3 \text{ and } m_4.r_4$$

Example 1. Four masses m_1, m_2, m_3 and m_4 are 200 kg, 300 kg, 240 kg and 260 kg respectively. The corresponding radii of rotation are 0.2 m, 0.15 m, 0.25 m and 0.3 m respectively and the angles between successive masses are 45°, 75° and 135°. Find the position and magnitude of the balance mass required, if its radius of rotation is 0.2 m.

Solution. Given : m_1 = 200 kg ; m_2 = 300 kg ; m_3 = 240 kg ; m_4 = 260 kg ; r_1 = 0.2 m ;r_2 = 0.15 m ; r_3 = 0.25 m ; r_4 = 0.3 m ; θ_1 = 0° ; θ_2 = 45° ; θ_3 = 45° + 75° =120° ; θ_4 = 45° + 75° + 135° = 255° ;r = 0.2 m

Let m = Balancing mass,and



F = The angle which the balancing mass makes with m_1.

Since the magnitude of centrifugal forces are proportional to the product of each mass and its radius, therefore

$m_1 \cdot r_1$ = 200 · 0.2 = 40 kg-m $m_2 \cdot r_2$ = 300 · 0.15 = 45 kg-m $m_3 \cdot r_3$ = 240 · 0.25 = 60 kg-m $m_4 \cdot r_4$ = 260 · 0.3 = 78 kg-m

The problem may, now, be solved either analytically or graphically. But we shall solve the problem by both the methods one by one.

Analytical method

Resolving $m_1.r_1$, $m_2.r_2$, $m_3.r_3$ and $m_4.r_4$ horizontally,

$\Sigma H = m_1 \cdot r_1 \cos \theta_1 + m_2 \cdot r_2 \cos \theta_2 + m_3 \cdot r_3 \cos \theta_3 + m_4 \cdot r_4 \cos \theta_4$

$= 40 \cos 0° + 45 \cos 45° + 60 \cos 120° + 78 \cos 255°$

$= 40 + 31.8 - 30 - 20.2 = 21.6$ kg-m

Now resolving vertically,

$\Sigma V = m_1 \cdot r_1 \sin \theta_1 + m_2 \cdot r_2$

$\sin \theta_2 + m_3 \cdot r_3 \sin \theta_3 + m_4 \cdot r_4 \sin \theta_4$

$= 40 \sin 0° + 45 \sin 45° + 60 \sin 120° + 78 \sin 255°$

$= 0 + 31.8 + 52 - 75.3 = 8.5$ kg-m

$\therefore$ Resultant, $R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(21.6)^2 + (8.5)^2} = 23.2$ kg-m

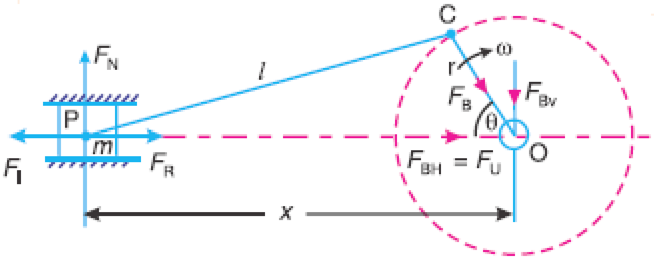
$m \cdot r = R = 23.2$ or $m = 23.2 / r = 23.2 / 0.2 = 116$ kg

Since θ' is the angle of the resultant R from the horizontal mass of 200 kg, therefore the angle of the balancing mass from the horizontal mass of 200 kg,

$= 180° + 21.48° = 201.48°$ **Ans.**

Balancing of Reciprocating Masses

The various forces acting on the reciprocating parts of an engine. The resultant of all the forces acting on the body of the engine due to inertia forces only is known as ***unbalanced force*** or ***shaking force***. Thus if the resultant of all the forces due to inertia effects is zero, then there will be no unbalanced force, but even then an unbalanced couple or shaking couple will be present Consider a horizontal reciprocating engine mechanism as shown in Fig.



F_R = Force required to accelerate the reciprocating parts,

F_B = Force acting on the crankshaft bearing or main bearing

Since F_R and F_I are equal in magnitude but opposite in direction, therefore they balance each other. The horizontal component of F_B (*i.e.* F_{BH}) acting along the line of reciprocation is also equal and opposite to F_I. This force $F_{BH} = F_U$ is an unbalanced force or shaking force and required to be properly balance.

The force on the sides of the cylinder walls (F_N) and the vertical component of F_B (*i.e.* F_{BV}) are equal and opposite and thus form a shaking couple of magnitude $F_N \times x$ or $F_{BV} \times x$.

From above we see that the effect of the reciprocating parts is to produce a shaking force and a shaking couple. Since the shaking force and a shaking couple vary in magnitude and direction during the engine cycle, therefore they cause very objectionable vibrations.

Thus the purpose of balancing the reciprocating masses is to eliminate the shaking force and a shaking couple. In most of the mechanisms, we can reduce the shaking force and a shaking couple by adding appropriate balancing mass, but it is usually not practical to eliminate them completely. In other words, the reciprocating masses are only partially balanced.

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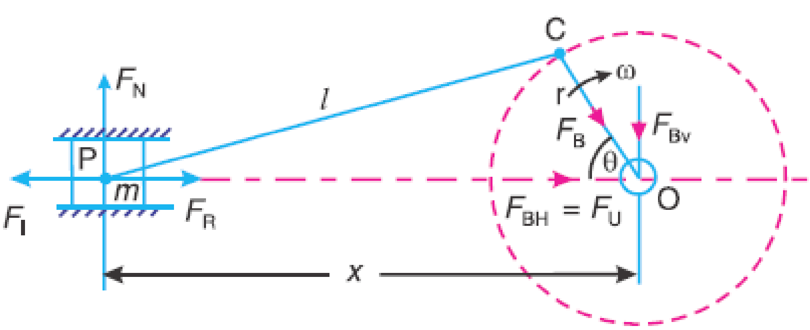

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Primary and Secondary Unbalanced Forces of Reciprocating Masses

Consider a reciprocating engine mechanism as shown inFig. Let m = Mass of the reciprocating parts,

l = Length of the connecting rod PC, r = Radius of the crank OC,

θ = Angle of inclination of the crank with the line of stroke PO, ω = Angular speed of the crank,

n = Ratio of length of the connecting rod to the crank radius = l / r.

$$a_R = \omega^2 \cdot r\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$$

Inertia force due to reciprocating parts or force required to accelerate the reciprocating parts.

$$F_I = F_R = \text{Mass} \times \text{acceleration} = m \cdot \omega^2 \cdot r\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$$

We have discussed in the previous article that the horizontal component of the force exerted on the crank shaft bearing (*i.e.* F_{BH}) is equal and opposite to inertia force (F_I). This force is an unbalanced one and is denoted by F_U.

Unbalanced force,

□

$$F_U = m \cdot \omega^2 \cdot r\left(\cos\theta + \frac{\cos 2\theta}{n}\right) = m.\omega^2 \cdot r\cos\theta + m \cdot \omega^2 \cdot r \times \frac{\cos 2\theta}{n} = F_P + F_S$$

The expression $(m \cdot \omega^2 \cdot r\cos\theta)$ is known as ***primary unbalanced force*** and $\left(m \cdot \omega^2 \cdot r \times \frac{\cos 2\theta}{n}\right)$ is called ***secondary unbalanced force***.

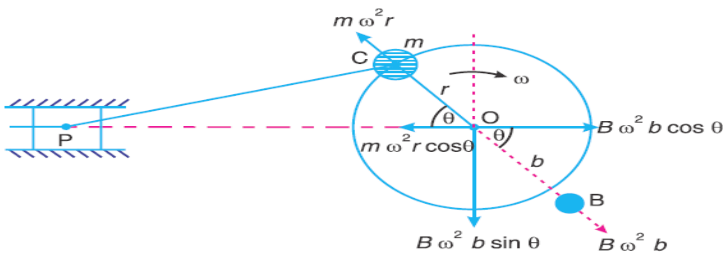
$\therefore$ Primary unbalanced force, $F_P = m \cdot \omega^2 \cdot r\cos\theta$

and secondary unbalanced force, $F_S = m \cdot \omega^2 \cdot r \times \frac{\cos 2\theta}{n}$

Partial Balancing of Unbalanced Primary Force in a Reciprocating Engine

$$(m \cdot \omega^2 \cdot r \cos\theta)$$

The primary unbalanced force may be considered as the component of the centrifugal force produced by a rotating mass *m* placed at the crank radius *r*, as shown inFig.



The primary force acts from *O* to *P* along the line of stroke. Hence, balancing of primary force is considered as equivalent to the balancing of mass *m* rotating at the crank radius *r*. This is balanced by having a mass *B* at a radius *b*, placed diametrically opposite to the crank pin *C*.

We know that centrifugal force due to mass *B*,

$$= B \cdot \omega^2 \cdot b$$

Horizontal component of this force acting in opposite direction of primary force

$$= B \cdot \omega^2 \cdot b \cos\theta$$

The primary force is balanced, if

$$B \cdot \omega^2 \cdot b \cos\theta = m \cdot \omega^2 \cdot r \cos\theta \quad \text{or} \quad B.b = m.r$$

A little consideration will show, that the primary force is completely balanced if

B.b = m.r, but the centrifugal force produced due to the revolving mass B, has also a vertical component (perpendicular to the line of stroke) of magnitude $B \cdot \omega^2 \cdot b \sin\theta$. This force remains unbalanced

The maximum value of this force is equal to $B \cdot \omega^2 \cdot b$ when θ is 90° and 270°, which is same as the maximum value of the primary force $m \cdot \omega^2 \cdot r$

let a fraction ‘*c*’ of the reciprocating masses is balanced, such that *c.m.r*= *B.b*

$\therefore$ Unbalanced force along the line of stroke

$$= m \cdot \omega^2 \cdot r\cos\theta - B \cdot \omega^2 \cdot b\cos\theta$$

$$= m \cdot \omega^2 \cdot r\cos\theta - c \cdot m \cdot \omega^2 \cdot r\cos\theta \qquad \ldots (\because\ B.b = c.m.r)$$

$$= (1-c)m \cdot \omega^2 \cdot r\cos\theta$$

and unbalanced force along the perpendicular to the line of stroke

$$= B \cdot \omega^2 \cdot b\sin\theta = c \cdot m \cdot \omega^2 \cdot r\sin\theta$$

$\therefore$ Resultant unbalanced force at any instant

$$= \sqrt{\left[(1-c)m \cdot \omega^2 \cdot r\cos\theta\right]^2 + \left[c \cdot m \cdot \omega^2 \cdot r\sin\theta\right]^2}$$

$$= m \cdot \omega^2 \cdot r\sqrt{(1-c)^2\cos^2\theta + c^2\sin^2\theta}$$

Partial Balancing of Locomotives

The locomotives, usually, have two cylinders with cranks placed at right angles to each other in order to have uniformity in turning moment diagram. The two cylinder locomotives maybe classified as:

Inside cylinder locomotives

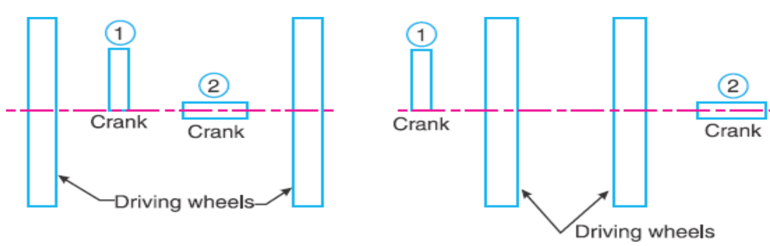
Outside cylinder locomotives.

In the ***inside cylinder locomotives***, the two cylinders are placed in between the planes of two driving wheels as shown in Fig(*a*) ; whereas in the ***outside cylinder locomotives***, the two cylinders are placed outside the driving wheels, one on each side of the driving wheel, as shown in

Fig(*b*). The locomotives may be

Single or uncoupled locomotives

Coupled locomotives



(*a*) Inside cylinder locomotives. (*b*) Outside cylinder locomotives.

A ***single*** or ***uncoupled locomotive*** is one, in which the effort is transmitted to one pair of the wheels only; whereas in ***coupled locomotives,*** the driving wheels are connected to the leading and trailing wheel by an outside coupling rod.

We have discussed in the previous article that the reciprocating parts are only partially balanced. Due to this partial balancing of the reciprocating parts, there is an unbalanced primary force along the line of stroke and also an unbalanced primary force perpendicular to the line of stroke. The effect of an unbalanced primary force along the line of stroke is to produce;

Variation interactive force along the line of stroke Swaying couple.

The effect of an unbalanced primary force perpendicular to the line of stroke is to produce variation in pressure on the rails, which results in hammering action on the rails. The maximum magnitude of the unbalanced force along the perpendicular to the line of stroke is known as a ***hammer blow***. We shall now discuss the effects of an unbalanced primary force in the following articles.

Variation of Tractive Force

The resultant unbalanced force due to the two cylinders, along the line of stroke, is known as ***tractive force***. Let the crank for the first cylinder be inclined at an angle θ with the line of stroke, as shown in Fig. Since the crank for the second cylinder is at right angle to the first crank, therefore the angle of inclination for the second crank will be (90° +θ).

Let m = Mass of the reciprocating parts per cylinder,and

c = Fraction of the reciprocating parts to be balanced.

We know that unbalanced force along the line of stroke for cylinder 1

$=2\,(1-c)m.\omega^2.r\cos\theta$

Similarly, unbalanced force along the line of stroke for cylinder 2,

$= (1-c)m.\omega^2 r\cos(90° + \theta)$

As per definition, the tractive force,

Definition, the tractive force,

FT = Resultant unbalanced force along the line of stroke

$=(1-c)m.\omega^2.r\cos\theta+(1-c)m.\omega^2 r\cos(90° + \theta)\;(1-c)m.\omega^2.r(\cos\theta - \sin\theta)$

The tractive force is maximum or minimum when (cosθ – sin θ) is maximum or minimum. For (cosθ– sinθ) to be maximum orminimum,

$$\frac{d}{d\theta}(\cos\theta - \sin\theta) = 0 \quad \text{or} \quad -\sin\theta - \cos\theta = 0 \quad \text{or} \quad -\sin\theta = \cos\theta$$

$$\therefore \quad \tan\theta = -1 \quad \text{or} \quad \theta = 135° \quad \text{or} \quad 315°$$

Thus, the tractive force is maximum or minimum when θ = 135° or 315°. Maximum and minimum value of the tractive force or the variation in tractiveforce

$$= \pm(1-c)\,m.\omega^2.r(\cos 135° - \sin 135°) = \pm\sqrt{2}\,(1-c)\,m.\omega^2.r$$

Balancing of Primary Forces of Multi-cylinder In-line Engines

The multi-cylinder engines with the cylinder centrelines in the same plane and on the same side of the centreline of the crankshaft are known as **In-line engines.** The following two conditions must be satisfied

The algebraic sum of the primary forces must be equal to zero. In other words, the primary force polygon must*close

The algebraic sum of the couples about any point in the plane of the primary forces must be equal to zero. In other words, the primary couple polygon mustclose.

We have already discussed, that the primary unbalanced force due to the reciprocating masses is equal to the component, parallel to the line of stroke, of the centrifugal force produced by the equal mass placed at the crankpin and revolving with it. Therefore, in order to give the primary balance of the reciprocating parts of a multi-cylinder engine, it is convenient to imagine the reciprocating masses to be transferred to their respective crank pins and to treat the problem as one of revolving masses.

Balancing of Secondary Forces of Multi-cylinder In-line Engines

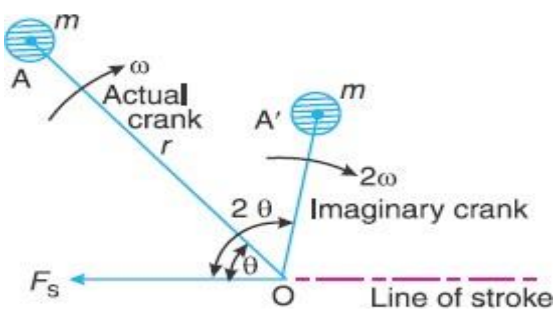
When the connecting rod is not too long (*i.e.* when the obliquity of the connecting rod is considered), then the secondary disturbing force due to the reciprocating mass arises.

$$F_S = m.\omega^2.r \times \frac{\cos 2\theta}{n}$$

This expression may be written as

$$F_S = m.(2\omega)^2 \times \frac{r}{4n} \times \cos 2\theta$$

As in case of primary forces, the secondary forces may be considered to be equivalent to the component, parallel to the line of stroke, of the centrifugal force produced by an equal mass placed at the imaginary crank of length *r* / 4*n* and revolving at twice the speed of the actual crank (*i.e.* 2ω) as shown in Fig.



Thus, in multi-cylinder in-line engines, each imaginary secondary crank with a mass attached to the crankpin is inclined to the line of stroke at twice the angle of the actual crank. The values of the secondary forces and couples may be obtained by considering the revolving mass. This is done in the similar way as discussed for primary forces. The following two conditions must be satisfied in order to give a complete secondary balance of an engine:

The algebraic sum of the secondary forces must be equal to zero. In other words, the secondary force polygon must close, and The algebraic sum of the couples about any point in the plane of the secondary forces must be equal to zero. In other words, the secondary couple polygon must close.

Example1. A four cylinder vertical engine has cranks 150 mm long. The planes of rotation of the first, second and fourth cranks are 400 mm, 200 mm and 200 mm respectively from the third crank and their reciprocating masses are 50 kg, 60 kg and 50 kg respectively. Find the mass of the reciprocating parts for the third cylinder and the relative angular positions of the cranks in order that the engine may be in complete primary balance.

Solution. Given $r_1 = r_2 = r_3 = r_4$ = 150 mm = 0.15 m ; m_1 = 50 kg ; m_2 = 60 kg ; m_4 = 50 kg

in order to give the primary balance of the reciprocating parts of a multi-cylinder engine, the problem may be treated as that of revolving masses with the reciprocating masses transferred to their respective crank pins.

The position of planes is shown in Fig(*a*). Assuming the plane of third cylinder as the reference plane, the data may be tabulated as given in Table.

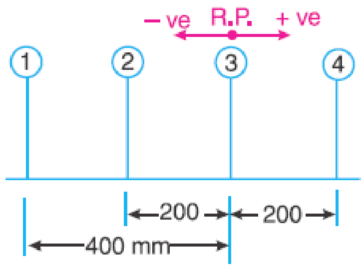
Plane (1)	*Mass* (*m*) *kg* (2)	*Radius* (*r*) *m* (3)	*Cent. force* $\div \omega^2$ (*m.r*) *kg-m* (4)	*Distance from plane 3(l) m* (5)	*Couple* $\div \omega^2$ (*m.r.l*) *kg-m*^{2} (6)
1	50	0.15	7.5	– 0.4	– 3
2	60	0.15	9	– 0.2	– 1.8
3(*R.P.*)	m_3	0.15	$0.15m_3$	0	0
4	50	0.15	7.5	0.2	1.5

First of all, the angular position of cranks 2 and 4 are obtained by drawing the couple polygon from the data given in Table (column 6). Assume the position of crank 1 in the horizontal direction as shown in Fig (*b*),

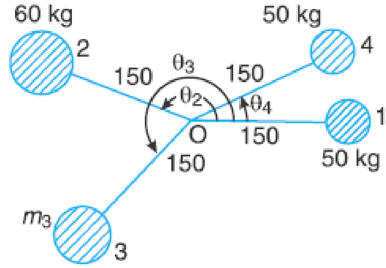
The couple polygon, as shown in Fig(*c*), is drawn as discussed below:

Draw vector o' a' in the horizontal direction (i.e. parallel to O_1) and equal to – 3 kg-m^2, to some suitablescale.

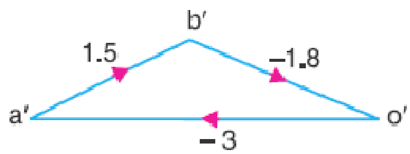
From point o'and a', draw vectors o' b' and a'b' equal to – 1.8 kg-m2 and 1.5 kg- m^{2}respectively. These vectors intersect atb'.



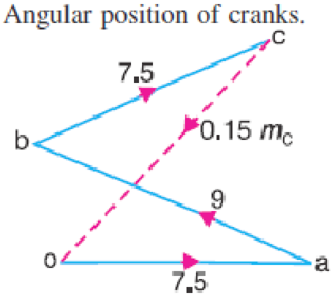
(*a*) Position of planes.



(*b*) Angular position of cranks.



(*c*) Couple polygon.

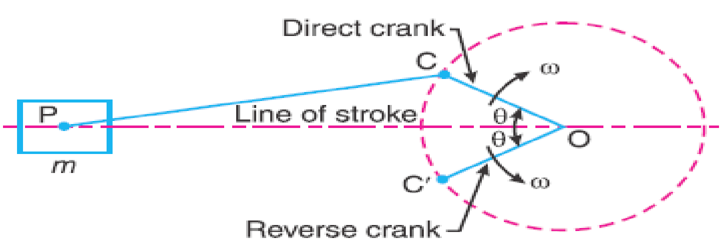


(*d*) Force polygon.

Now in Fig. 22.17 (*b*), draw *O*2 parallel to vector *o*′ *b*′ and O_4 parallel to vector *a*′ *b*′. By measurement, we find that the angular position of crank 2 from crank 1 in the anticlockwise directions $\theta_2 = 160°$and the angular position of crank 4 from crank 1 in the anticlockwise direction is$\theta 4 = 26°$In order to find the mass of the third cylinder (m_3) and its angular position, draw the force polygon, to some suitable scale, as shown in Fig (*d*), from the data given in Table(column 4). Since the closing side of the force polygon (vector *co*) is proportional to 0.15 m_3, therefore by measurement,$0.15 m_3 = 9$ kg-m or $m_3 = 60$ kgNow draw *O*3 in Fig 22.17 (*b*), parallel to vector *co*. By measurement, we find that the angular position of crank 3 from crank 1 in the anticlockwise direction is $\theta_3 = 227°$

Balancing of Radial Engines (Direct and Reverse Cranks Method)

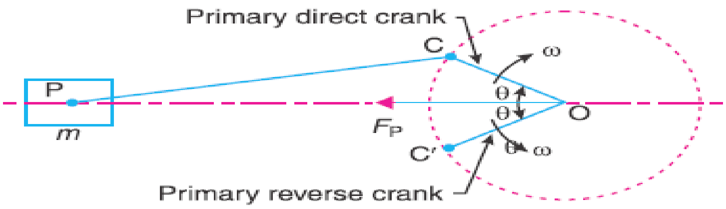
The method of direct and reverse cranks is used in balancing of radial or *V*-engines, in which the connecting rods are connected to a common crank. Since the plane of rotation of the various cranks (in radial or *V*-engines) is same, therefore there is no unbalanced primary or secondary couple.



Consider a reciprocating engine mechanism as shown in Fig. 22.27. Let the crank *OC* (known as the direct crank) rotates uniformly at ω radians per second in a clockwise direction. Let at any instant the crank makes an angle θ with the line of stroke *OP*. The indirect or reverse crank *OC*′ is the image of the direct crank *OC*, when seen through the mirror placed at the line of stroke .A little consideration will show that when the direct crank revolves in a clockwise direction, the reverse crank will revolve in the anticlockwise direction. We shall now discuss the primary and secondary forces due to the mass (*m*) of the reciprocating parts at *P*.

Considering the primary forces

We have already discussed that primary force is 2 $m.\omega^2.r\cos\theta$. This force is equal to the component of the centrifugal force along the line of stroke, produced by a mass (*m*) placed at the crank pin *C*. Now let us suppose that the mass (*m*) of the reciprocating parts is divided into two parts, each equal to *m* /2.



It is assumed that *m* / 2 is fixed at the ***direct crank*** (termed as ***primary direct crank***) pin *C* and *m* / 2 at the ***reverse crank*** (termed as ***primary reverse crank***) pin *C′*, as shown in Fig. We know that the centrifugal force acting on the primary direct and reverse crank

$$= \frac{m}{2} \times \omega^2.r$$

∴ Component of the centrifugal force acting on the primary direct crank

$$= \frac{m}{2} \times \omega^2.r\cos\theta \qquad \text{. . . (in the direction from } O \text{ to } P)$$

and, the component of the centrifugal force acting on the primary reverse crank

$$= \frac{m}{2} \times \omega^2.r\cos\theta \qquad \text{. . . (in the direction from } O \text{ to } P)$$

∴ Total component of the centrifugal force along the line of stroke

$$= 2 \times \frac{m}{2} \times \omega^2.r\cos\theta = m.\omega^2.r\cos\theta = \text{Primary force, } F_P$$

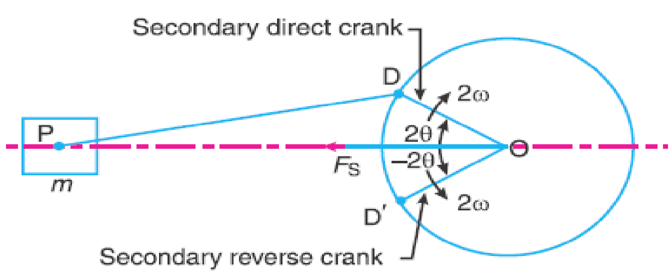
Hence, for primary effects the mass m of the reciprocating parts at P may be replaced by two masses at C and C′ each of magnitudem/2.

Considering secondary forces

We know that the secondary force

$$= m(2\omega)^2 \frac{r}{4n} \times \cos 2\theta = m.\omega^2 r. \times \frac{\cos 2\theta}{n}$$

In the similar way as discussed above, it will be seen that for the secondary effects, the mass (m) of the reciprocating parts may be replaced by two masses (each m/2) placed at D and D′ such that OD = OD′ = r/4n. The crank OD is the secondary direct crank and rotates at 2π rad/s in the clockwise direction, while the crank OD′ is the secondary reverse crank and rotates at 2πrad/s in the anticlockwise direction as shown in Fig.

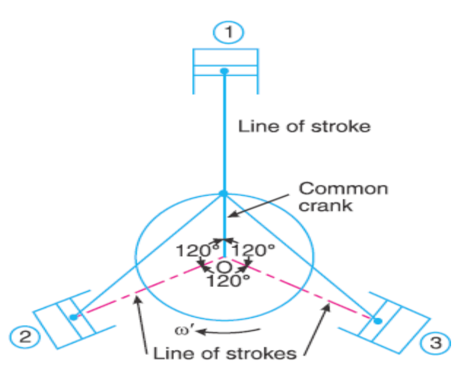


PROBLEMS

Example 1. The three cylinders of an air compressor have their axes 120° to one another, and their connecting rods are coupled to a single crank. The stroke is 100 mm and the length of each connecting rod is 150 mm. The mass of the reciprocating parts per cylinder is 1.5kg. Find the maximum primary and secondary forces acting on the frame of the compressor when running at 3000 r.p.m. Describe clearly a method by which such forces may be balanced.

Solution. Given :*L* = 100 mm or *r* = *L* / 2 = 50 mm = 0.05 m ; *l* = 150 mm = 0.15 m ;*m* = 1.5 kg ; *N* = 3000 r.p.m. or ω = 2π × 3000/60 = 314.2 rad/s

The position of three cylinders is shown in Fig. Let the common crank be along the inner dead centre of



cylinder 1. Since common crank rotates clockwise, therefore θ is positive when measured clockwise.

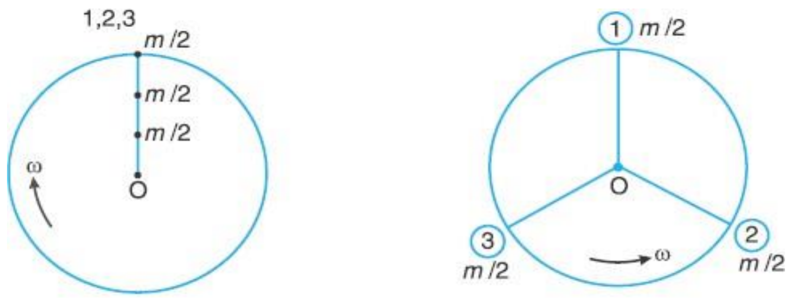
Maximum primary force acting on the frame of the compressor

The primary direct and reverse crank positions as shown in Fig.(*a*) and (*b*), are obtained as discussed below :

Since θ = 0° for cylinder 1, therefore both the primary direct and reverse cranks will coincide with the common crank. Since θ = ±120° for cylinder 2, therefore the primary direct crank is 120° clockwise and the primary reverse crank is 120°anti-clockwise from the line of stroke of cylinder 2.

Since θ = ± 240° for cylinder 3, therefore the primary direct crank is 240° clockwise and the primary reverse crank is 240° anti-clockwise from the line of stroke of cylinder 3.From Fig.(*b*), we see that the primary reverse cranks form a balanced system. Therefore there is no unbalanced primary force due to the reverse cranks. From Fig (*a*), we see that the resultant primary force is

$$\therefore \text{ Maximum primary force } = \frac{3m}{2}\times\omega^2.r = \frac{3\times1.5}{2}(314.2)^2 0.05 = 11\,106 \text{ N} = 11.106 \text{ kN}$$



(*a*) Direct primary cranks. (*b*) Reverse primary cranks.

equivalent to the centrifugal force of a mass 3 *m*/2 attached to the end of thecrank.

maximum primary force may be balanced by a mass attached diametrically opposite to the crank pin and

$$B_1.b_1 = \frac{3m}{2}\times r = \frac{3\times1.5}{2}\times0.05 = 0.1125 \text{ N-m}$$

rotating with the crank, of magnitude B_1 at radius b_1 such that

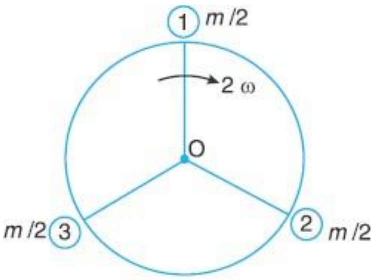
Maximum secondary force acting on the frame of the compressor

The secondary direct and reverse crank positions as shown in Fig (*a*) and (*b*), are obtained as discussed below:

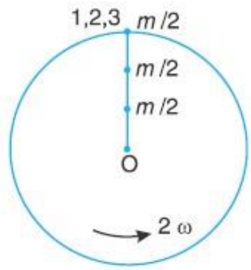
Since θ = 0° and 2 θ = 0° for cylinder 1, therefore both the secondary direct and reverse cranks will coincide with the commoncrank.

Since θ = ±120° and 2 θ = ± 240° for cylinder 2, therefore the secondary direct crank is 240° clockwise and the secondary reverse crank is 240° anticlockwise from the line of stroke of cylinder2.

Since θ = ± 240° and 2 θ = ± 480°, therefore the secondary direct crank is 480° or 120°clockwise and the secondary reverse crank is 480° or 120° anti-clockwise from the line of stroke of cylinder3.



(*a*) Direct secondary cranks.



(*b*) Reverse secondary cranks.

From Fig (*a*), we see that the secondary direct cranks form a balanced system. Therefore there is no unbalanced secondary force due to the direct cranks. From Fig (*b*), we see that the resultant secondary force is equivalent to the centrifugal force of a mass 3 $m/2$attached at a crank radius of $r/4n$ and rotating at a speed of 2ωrad/s in the opposite direction to the crank.

∴ Maximum secondary force

$$= \frac{2m}{2}(2\omega)^2\left(\frac{r}{4n}\right) = \frac{3\times1.5}{2}(2\times314.2)^2\left[\frac{0.05}{4\times0.15/0.05}\right]\text{N}$$

...(∵ $n = l/r$)

= 3702 N **Ans.**

This maximum secondary force may be balanced by a mass B_2 at radius b_2, attached diametrically opposite to the crankpin, and rotating anti-clockwise at twice the crank speed, such that

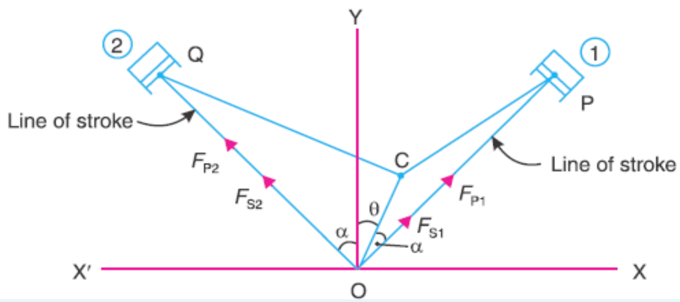
$$B_2.b_2 = \frac{3m}{2}\times\frac{r}{4n} = \frac{3\times1.5}{2}\times\frac{0.05}{4\times0.15/0.05} = 0.009\ 375\ \text{N-m}$$

Balancing of V-engines

Consider a symmetrical two cylinder *V*-engine as shown in Fig. 22.33,

The common crank *OC* is driven by two connecting rods *PC* and *QC*. The lines of stroke

OP and *OQ* are inclined to the vertical *OY*, at an angle α as shown in Fig.



that inertia force due to reciprocating parts of cylinder 1, along the line of stroke

the inertia force due to reciprocating parts of cylinder 2, along the line of stroke

$$= m.\omega^2.r\left[\cos(\alpha-\theta)+\frac{\cos 2(\alpha+\theta)}{n}\right]$$

The balancing of *V*-engines is only considered for primary and secondary forces* as discussed below :

Considering primary forces

We know that primary force acting along the line of stroke of cylinder 1, $F_{P1} = m.\omega^2.r \cos(\alpha - \theta)$

Component of F_{P1} along the vertical line OY

$= F_{P1} \cos\alpha = m.\omega^2 r.\cos(\alpha-\theta)\cos\alpha$ **(i)**

and component of *F*P1 along the horizontal line *OX*

$= F_{P1} \sin\alpha = m.\omega^2 r.\sin(\alpha-\theta)\sin\alpha$ **(ii)**

Similarly, primary force acting along the line of stroke of cylinder 2, $F_{P2} = m.\omega^2.r \cos(\alpha + \theta)$

Component of F_{P2} along the vertical line *OY*

$F_{P2=}\cos\alpha == m.\omega^2 r.\cos(\alpha+\theta)\cos\alpha$ **(iii)**

and component of F_{P2} along the horizontal line *OX′*

$$F_{PV} = (i) + (iii) = m.\omega^2.r\cos\alpha\,[\cos(\alpha-\theta)+\cos(\alpha+\theta)]$$

$$= m.\omega^2.r\cos\alpha\times 2\cos\alpha\cos\theta$$

$$\ldots\ [\because\ \cos(\alpha-\theta)+\cos(\alpha+\theta)=2\cos\alpha\cos\theta]$$

$$= 2\,m.\omega^2.r\cos^2\alpha.\cos\theta$$

$= F_{P2} \sin\alpha = m.\omega^2 r.\sin(\alpha+\theta)\sin\alpha$ **(iv)**

Total component of primary force along the vertical line *OY*

total component of primary force along the horizontal line *OX*

$$F_{PH} = (ii) - (iv) = m.\omega^2.r\sin\alpha[\cos(\alpha-\theta) - \cos(\alpha+\theta)]$$

$$= m.\omega^2.r\sin\alpha \times 2\sin\alpha\sin\theta$$

$$\ldots \quad [\because \cos(\alpha-\theta) - \cos(\alpha+\theta) = 2\sin\alpha\sin\theta]$$

$$= 2m.\omega^2.r\sin^2\alpha.\sin\theta$$

$\therefore$ Resultant primary force,

$$F_P = \sqrt{(F_{PV})^2 + (F_{PH})^2}$$

$$= 2m.\omega^2.r\sqrt{(\cos^2\alpha.\cos\theta)^2 + (\sin^2\alpha.\sin\theta)^2} \quad \ldots (v)$$

Considering secondary forces

We know that secondary force acting along the line of stroke of cylinder 1,

$$F_{S1} = m.\omega^2.r \times \frac{\cos 2(\alpha-\theta)}{n}$$

Component of F_{S1} along the vertical line *OY*

$$= F_{S1}\cos\alpha = m.\omega^2.r \times \frac{\cos 2(\alpha-\theta)}{n} \times \cos\alpha \quad \ldots (ix)$$

Component of F_{S1} along the horizontal line *OX*

$$= F_{S1}\sin\alpha = m.\omega^2.r \times \frac{\cos 2(\alpha-\theta)}{n} \times \sin\alpha \quad \ldots (x)$$

Similarly, secondary force acting along the line of stroke of cylinder 2,

$$F_{S2} = m.\omega^2 r \times \frac{\cos 2(\alpha+\theta)}{n}$$

Component of F_{S2} along the vertical line *OY*

$$= F_{S2}\cos\alpha = m.\omega^2.r \times \frac{\cos 2(\alpha+\theta)}{n} \times \cos\alpha \quad \ldots (xi)$$

$$= F_{S2}\sin\alpha = m.\omega^2.r \times \frac{\cos 2(\alpha+\theta)}{n} \times \sin\alpha \quad \ldots (xii)$$

Component of F_{S2} along the horizontal line *OX'*

Total component of secondary force along the vertical line *OY*,

$$F_{SV} = (ix) + (xi) = \frac{m}{n} \times \omega^2.r\cos\alpha\,[\cos 2(\alpha-\theta) + \cos 2(\alpha+\theta)]$$

$$= \frac{m}{n} \times \omega^2.r\cos\alpha \times 2\cos 2\alpha\cos 2\theta = \frac{2m}{n} \times \omega^2.r\cos\alpha.\cos 2\alpha\cos 2\theta$$

$$F_{SH} = (x) - (xii) = \frac{m}{n} \times \omega^2.r \sin\alpha [\cos 2(\alpha - \theta) - \cos 2(\alpha + \theta)]$$

$$= \frac{m}{n} \times \omega^2.r \sin\alpha \times 2 \sin 2\alpha . \sin 2\theta$$

$$= \frac{2m}{n} \times \omega^2.r \sin\alpha . \sin 2\alpha . \sin 2\theta$$

total component of secondary force along the horizontal line *OX*,

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Resultant secondary force,

$$F_S = \sqrt{(F_{SV})^2 + (F_{SH})^2}$$

$$= \frac{2m}{n} \times \omega^2.r \sqrt{(\cos\alpha . \cos 2\alpha . \cos 2\theta)^2 + (\sin\alpha . \sin 2\alpha . \sin 2\theta)^2}$$

Example 1.A vee-twin engine has the cylinder axes at right angles and the connecting rods operate a common crank. The reciprocating mass per cylinder is 11.5 kg and the crank radius is 75 mm. The length of the connecting rod is 0.3 m. Show that the engine may be balanced for primary forces by means of a revolving balancemass.

If the engine speed is 500 r.p.m. What is the value of maximum resultant secondary force?

Solution. Given : $2\theta = 90°$ or $\alpha = 45°$; $m = 11.5$ kg ; $r = 75$ mm $= 0.075$ m ; $l = 0.3$ m ;$N = 500$ r.p.m. or $\omega = 2\pi \times 500 / 60 = 52.37$ rad/s

We know that resultant primary force,

$$F_P = 2m.\omega^2.r \sqrt{(\cos^2\alpha \cos\theta)^2 + (\sin^2\alpha \sin\theta)^2}$$

$$= 2m.\omega^2.r \sqrt{(\cos^2 45° \cos\theta)^2 + (\sin^2 45° \sin\theta)^2}$$

$$= 2m.\omega^2.r \sqrt{\left[\frac{\cos\theta}{2}\right]^2 + \left[\frac{\sin\theta}{2}\right]^2} = m.\omega^2.r$$

Since the resultant primary force $m.\omega^2.r$ is the centrifugal force of a mass m at the crank radius r when rotating at ω rad / s, therefore, the engine may be balanced by a rotating balance mass.

Maximum resultant secondary force

We know that resultant secondary force,

$$F_S = \sqrt{2} \times \frac{m}{n} \times \omega^2.r \sin 2\theta \qquad \ldots (\text{When } 2\alpha = 90°)$$

This is maximum, when sin 2 θ is maximum *i.e*. when sin 2 θ = ± 1 or θ = 45° or 135°. Maximum resultant secondary force,

$$F_{S_{max}} = \sqrt{2} \times \frac{m}{n} \times \omega^2 . r \qquad \text{. . . (Substituting } \theta = 45^\circ \text{)}$$

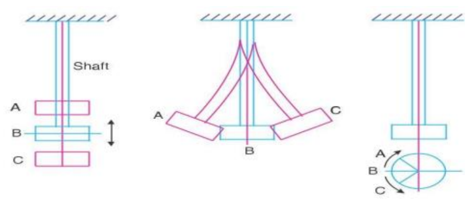
$$= \sqrt{2} \times \frac{11.5}{0.3/0.075} (52.37)^2 0.075 = 836 \text{ N } \textbf{Ans.} \qquad \text{. . . } (\because n = l / r)$$

Types of Free Vibrations

The following three types of free vibrations are important from the subject point of view :

1. Longitudinal vibrations, **2.** Transverse vibrations, and **3.** Torsional vibrations.

Consider a weightless constraint (spring or shaft) whose one end is fixed and the other end carrying a heavy disc, as shown in Fig. This system may execute one of the three above mentioned types of



vibrations

(*a*) Longitudinal vibrations. (*b*) Transverse vibrations. (*c*) Torsional vibrations.

1. ***Longitudinal vibrations*.** When the particles of the shaft or disc moves parallel to the axis of the shaft, as shown in Fig. (*a*), then the vibrations are known as ***longitudinal vibrations*.** In this case, the shaft is elongated and shortened alternately and thus the tensile and compressive stresses are induced alternately in the shaft.

***2 .Transverse vibrations*.** When the particles of the shaft or disc move approximately perpendicular to the axis of the shaft, as shown in Fig (*b*), then the vibrations are known as ***transverse vibrations*.**In this case, the shaft is straight and bent alternately and bending stresses are induced in theshaft

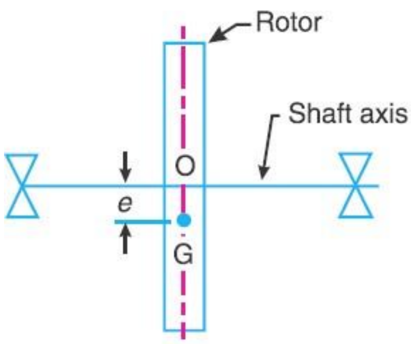
Torsional vibrations*. When the particles of the shaft or disc move in a circle about the axis of the shaft, as shown in Fig. (*c*), then the vibrations are known as ***torsional vibrations*.** In this case, the shaft is twisted and untwisted alternately and the torsional shear stresses are induced in these shaft.

Critical or Whirling Speed of a Shaft

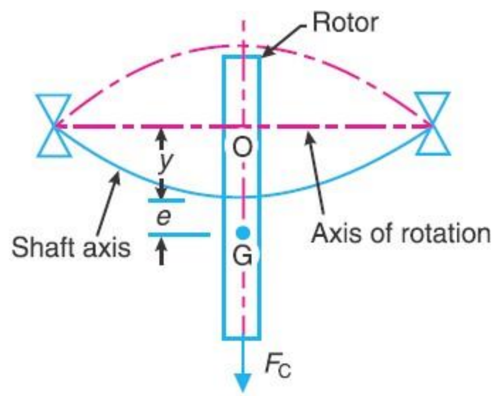
A rotating shaft carries different mountings and accessories in the form of gears, pulleys, etc. When the gears or pulleys are put on the shaft, the centre of gravity of the

Pulley or gear does not coincide with the centerline of the bearings or with the axis of the shaft, when the shaft is stationary. This means that the centre of gravity of the pulley or gear is at a certain distance from the axis of rotation and due to this, the shaft is subjected to centrifugal force.

This force will bent the shaft which will further increase the distance of centre of gravity of the pulley or gear from the axis of rotation. This correspondingly increases the value of centrifugal force, which further increases the distance of centre of gravity from the axis of rotation. This effect is cumulative and ultimately the shaft fails. The bending of shaft not only depends upon the value of eccentricity (distance between centre of gravity of the pulley and the axis of rotation) but also depends upon the speed at which the shaft rotates **The speed at which the shaft runs so that the additional deflection of the shaft from the axis of rotation becomes infinite, is known as *critical* or *whirlingspeed*.**



(*a*) When shaft is stationary.



(*b*) When shaft is rotating.

Consider a shaft of negligible mass carrying a rotor, as shown in Fig.(*a*). The point *O* is on the shaft axis and *G* is the centre of gravity of the rotor. When the shaft is stationary, the centreline of the bearing and the axis of the shaft coincides. Fig.(*b*) shows the shaft when rotating about the axis of rotation at a uniform speed of ωrad/s.

Let m = Mass of the rotor,

e = Initial distance of centre of gravity of the rotor from the centre line of the bearing or shaft axis, when the shaft is stationary,

y = Additional deflection of centre of gravity of the rotor when the shaft starts rotating at

ω rad/s, and s = Stiffness of the shaft *i.e.* the load required per unit deflection of the shaft.

Since the shaft is rotating at ω rad/s, therefore centrifugal force acting radially outwards through *G* causing the shaft to deflect is given by The shaft behaves like a spring.

$$F_C = m.\omega^2 (y+e)$$

Therefore the force resisting the deflection *y*, =*s.y*For the equilibrium position

$$m.\omega^2 (y+e) = s.y$$

$$m.\omega^2.y + m.\omega^2.e = s.y \quad \text{or} \quad y(s - m.\omega^2) = m.\omega^2.e$$

$$y = \frac{m.\omega^2.e}{s - m.\omega^2} = \frac{\omega^2.e}{s/m - \omega^2}$$

We know that circular frequency,

$$\omega_n = \sqrt{\frac{s}{m}} \quad \text{or} \quad y = \frac{\omega^2.e}{(\omega_n)^2 - \omega^2}$$

$$\omega_n = \sqrt{\frac{s}{m}} \quad \text{or} \quad y = \frac{\omega^2.e}{(\omega_n)^2 - \omega^2}$$

A little consideration will show that when $\omega > \omega_n$, the value of *y* will be negative and the shaft deflects is

$$y = \pm \frac{\omega^2 e}{(\omega_n)^2 - \omega^2} = \frac{\pm e}{\left(\frac{\omega_n}{\omega}\right)^2 - 1} = \frac{\pm e}{\left(\frac{\omega_c}{\omega}\right)^2 - 1}$$

the opposite direction as shown dotted in Fig

We see from the above expression that when $\omega_n = \omega_c$ the value of *y* becomes infinite. Therefore ω_c is the **critical or whirling speed.**

$$\omega_c = \omega_n = \sqrt{\frac{s}{m}} = \sqrt{\frac{g}{\delta}} \text{ Hz}$$

If N_c is the critical or whirling speed in r.p.s., then

$$2\pi N_c = \sqrt{\frac{g}{\delta}} \quad \text{or} \quad N_c = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}} = \frac{0.4985}{\sqrt{\delta}} \text{ r.p.s.}$$

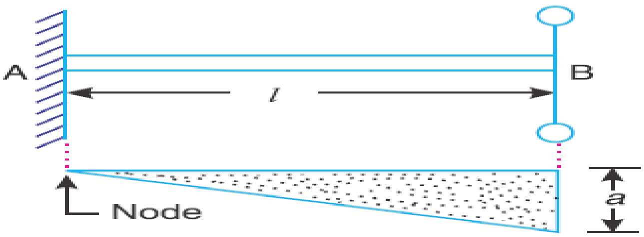
δ = Static deflection of the shaft in metres.

Hence the critical or whirling speed is the same as the natural frequency of transverse vibration but its unit will be revolutions per second.

Free Torsional Vibrations of a Single Rotor System

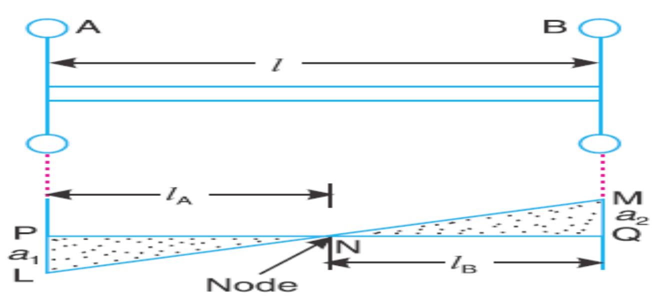
$$f_n \quad \frac{1}{2}\sqrt{\frac{q}{I}} \quad \frac{1}{2}\sqrt{\frac{C.J}{l.I}}$$

A shaft is fixed at one end and carrying a rotor at the free end.The natural frequency of torsional vibration.



Free Torsional Vibrations of a Two Rotor System

Consider two rotor systems. It consists of a shaft with two rotors at its ends. In these system torsional vibrations occurs only when the two rotors A and B move in opposite directions .It may be noted that the two rotors must have same frequency. The node lies at Point N. This point can be safely assumed as fixed end of the shaft may be considered as two separate shafts NP and NQ each fixed to one of its ends and carrying rotors at the free end



l=Length of the shaft l_A=Length of the part NP' l_B=Length of the part NQ

I_A=Mass moment of inertia of rotor A I_B=Mass moment of inertia of rotor B d= diameter of the shaft

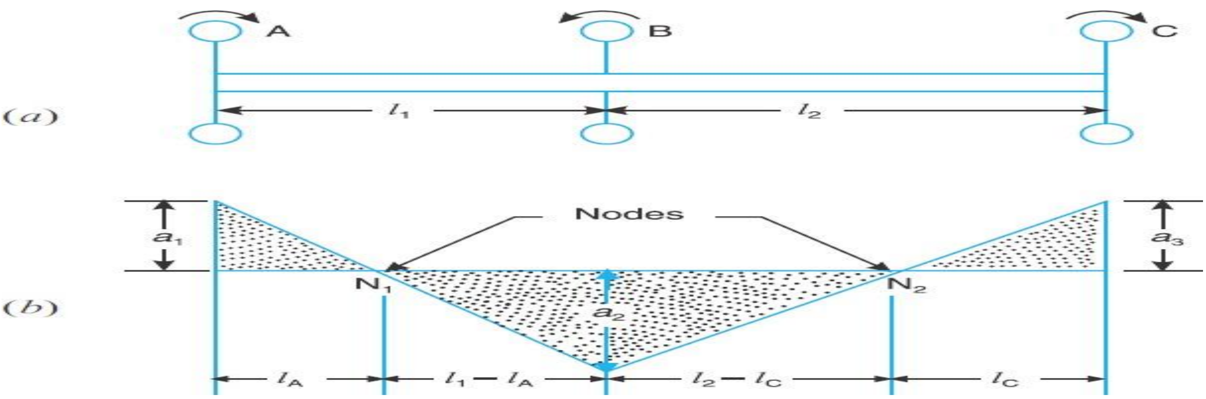
J=polar moment of inertia of shaft C = Modulus of rigidity for shaft

then natural frequency for Rotor A

$$f_{nA} \quad \frac{1}{2}\sqrt{\frac{C.J}{l_A.I_A}}$$

$$f_{nB} \quad \frac{1}{2} \sqrt{\frac{C.J}{l_B.I_B}}$$

Natural frequency for Rotor …



Free Tensional Vibrations of a Three Rotor System

Since $f_{nA} = f_{nB} = f_{nC}$,

$$\frac{1}{2} \sqrt{\frac{C.J}{l_A.I_A}} \quad \frac{1}{2} \sqrt{\frac{C.J}{l_C.I_C}} \qquad \text{or} \qquad l_A.I_A = l_C.I_C$$

$$l_A \quad \frac{l_C.I_C}{I_A}$$

Now equating equations

$$\frac{1}{2} \sqrt{\frac{C.J}{I_B} \quad \frac{1}{l_1 \quad l_A} \quad \frac{1}{l_2 \quad l_C}} \qquad \frac{1}{2} \sqrt{\frac{C.J}{l_C.I_C}}$$

INDUSTRIAL APPLICATIONS

Industrial Electrical Engineering and Automation

Applications

- **Optics, Photonics and Measuring Technology**
- Image stabilization
- Scanning microscopy
- Auto focus systems
- Interferometry
- Fiber optic alignment & switching
- Fast mirror scanners
- Adaptive and active optics
- Laser tuning
- Mirror positioning
- Holography
- Stimulation of vibrations
- **Disk Drive**
- MR head testing
- Pole tip recession
- Disk spin stands
- Vibration cancellation
- **Microelectronics**
- Nano-metrology
- Wafer and mask positioning
- Critical Dimensions measurement
- Microlithography
- Inspection systems
- Vibration cancellation
- **Precision Mechanics and Mechanical Engineering**
- Vibration cancellation
- Structural deformation
- Out-of-roundness grinding, drilling, turning
- Tool adjustment
- Wear correction
- Needle valve actuation
- Micro pumps
- Linear drives
- Piezo hammers
- Knife edge control in extrusion tools
- Micro engraving systems
- Shock wave generation
- **Life Science, Medicine, Biology**
- Patch-clamp drives
- Gene technology
- Micro manipulation
- Cell penetration
- Micro dispensing devices
- Audiophysiological stimulation
- Shock wave generation

TUTORIAL QUESTIONS

1. Explain clearly the terms ‘static balancing’ and ‘dynamic balancing’. State the necessary conditions to achieve them. and Discuss how a single revolving mass is balanced by two masses revolving in different planes.
2. Four masses A, B, C and D are attached to a shaft and revolve in the same plane. The masses are 12 kg, 10 kg, 18 kg and 15 kg respectively and their radii of rotations are 40 mm, 50 mm, 60 mm and 30 mm. The angular position of the masses B, C and D are 60°, 135° and 270° from the mass A. Find the magnitude and position of the balancing mass at a radius of 100 mm.
3. Four masses A, B, C and D revolve at equal radii and are equally spaced along a shaft. The mass B is 7 kg and the radii of C and D make angles of 90° and 240° respectively with the radius of B. Find the magnitude of the masses A, C and D and the angular position of A so that the system may be completely balanced.
4. The cranks and connecting rods of a 4-cylinder in-line engine running at 1800 r.p.m. are 60 mm and 240 mm each respectively and the cylinders are spaced 150 mm apart. If the cylinders are numbered 1 to 4 in sequence from one end, the cranks appear at intervals of 90° in an end view in the order 1-4-2-3. The reciprocating mass corresponding to each cylinder is 1.5 kg. Determine : 1. Unbalanced primary and secondary forces, if any, and 2. Unbalanced primary and secondary couples with reference to central plane of the engine.
5. A cantilever shaft 50 mm diameter and 300 mm long has a disc of mass 100 kg at its free end. The Young's modulus for the shaft material is 200 GN/m2 . Determine the frequency of longitudinal and transverse vibrations of the shaft.

ASSIGNMENT QUESTIONS

1. Explain the method of balancing of different masses revolving in the same plane.
2. A, B, C and D are four masses carried by a rotating shaft at radii 100, 125, 200 and 150 mm respectively. The planes in which the masses revolve are spaced 600 mm apart and the mass of B, C and D are 10 kg, 5 kg, and 4 kg respectively. Find the required mass A and the relative angular settings of the four masses so that the shaft shall be in complete balance.
3. **Example 21.3.** *Four masses A, B, C and D as shown below are to be completely balanced.*

	A	B	C	D
Mass (kg)	—	30	50	40
Radius (mm)	180	240	120	150

The planes containing masses B and C are 300 mm apart. The angle between planes containing B and C is 90°. B and C make angles of 210° and 120° respectively with D in the same sense. Find :

1. The magnitude and the angular position of mass A ; and

2. The position of planes A and D.

4. The three cranks of a three cylinder locomotive are all on the same axle and are set at 120°. The pitch of the cylinders is 1 metre and the stroke of each piston is 0.6 m. The reciprocating masses are 300 kg for inside cylinder and 260 kg for each outside cylinder and the planes of rotation of the balance masses are 0.8 m from the inside crank. If 40% of the reciprocating parts are to be balanced, find : 1. the magnitude and the position of the balancing masses required at a radius of 0.6 m ; and 2. the hammer blow per wheel when the axle makes 6 r.p.s.
5. The following data refer to two cylinder locomotive with cranks at 90° : Reciprocating mass per cylinder = 300 kg ; Crank radius = 0.3 m ; Driving wheel diameter = 1.8 m ; Distance between cylinder centre lines = 0.65 m ; Distance between the driving wheel central planes = 1.55 m. Determine : 1. the fraction of the reciprocating masses to be balanced, if the hammer blow is not to exceed 46 kN at 96.5 km. p.h. ; 2. the variation in tractive effort ; and 3. the maximum swaying couple.
6. A four cylinder vertical engine has cranks 150 mm long. The planes of rotation of the first, second and fourth cranks are 400 mm, 200 mm and 200 mm respectively from the third crank and their reciprocating masses are 50 kg, 60 kg and 50 kg respectively. Find the mass of the reciprocating parts for the third cylinder and the relative angular positions of the cranks in order that the engine may be in complete primary balance.
7. A shaft of length 0.75 m, supported freely at the ends, is carrying a body of mass 90 kg at 0.25 m from one end. Find the natural frequency of transverse vibration. Assume E = 200 GN/m2 and shaft diameter = 50 mm
8. Calculate the whirling speed of a shaft 20 mm diameter and 0.6 m long carrying a mass of 1 kg at its mid-point. The density of the shaft material is 40 Mg/m3 , and Young's modulus is 200 GN/m2 . Assume the shaft to be freely supported
9. Define, in short, free vibrations, forced vibrations and damped vibrations.
10. Discuss the effect of inertia of the shaft in longitudinal and transverse vibrations.

UNIT 5

GOVERNORS

Course Objectives

To understand the working principles of different type governors and its characteristics

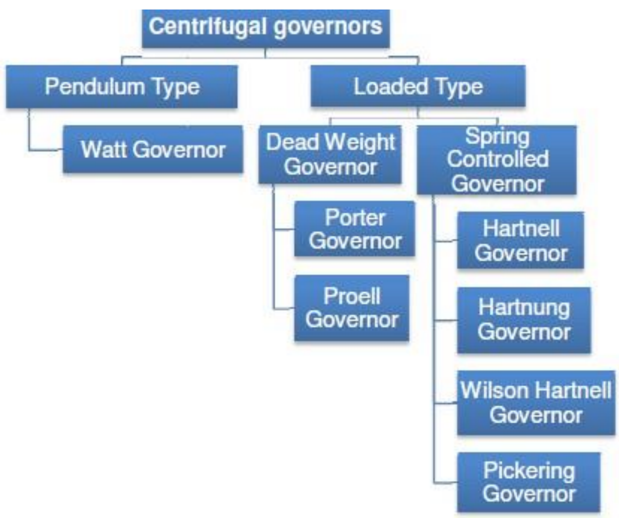
Course Outcomes

Student gets the exposure of different governors and its working principle

INTRODUCTION

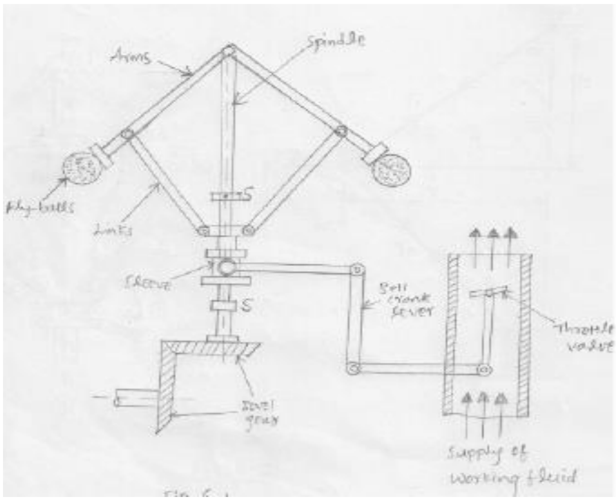
The function of a governor is to regulate the mean speed of an engine, when there are variations in the load *e.g.* when the load on an engine increases, its speed decreases, Therefore it becomes necessary to increase the supply of working fluid. On the other hand, when the load on the engine decreases, its speed increases and thus less working fluid is required. The governor automatically controls the supply of working fluid to the engine with the varying load conditions and keeps the mean speed within certain limits. A little consideration will show, that when the load increases, the configuration of the governor changes and a valve is moved to increase the supply of the working fluid ***conversely*****,** when the load decreases, the engine speed increases and the governor decreases the supply of working fluid.

Classifications of the governor



Centrifugal Governors

The centrifugal governors are based on the balancing of centrifugal force on the rotating balls by an equal and opposite radial force, known as the controlling force*.It consists of two balls of equal mass, which are attached to the arms as shown in Fig. These balls are known as governor balls or fly balls. The balls revolve with a spindle, which is driven by the engine through bevel gears. The upper ends of the arms are pivoted to the spindle, so that the balls may rise up or fall down as they revolve about the vertical axis. The arms are connected by the links to a sleeve, which is keyed to the spindle. This sleeve revolves with the spindle but can slide up and down. The balls and the sleeve rise when the spindle speed increases, and falls when the speed decreases. In order to limit the travel of the sleeve in upward and downward directions, two stops *S*, *S* are provided on the spindle. The sleeve is connected by a bell crank lever to a throttle valve. The supply of the working fluid decreases when the and the governor speed decreases. This results in the decrease of centrifugal force on the balls. Hence the balls move inwards and the sleeve moves downwards. The downward movement of the sleeve operates a throttle valve at the other end of the bell crank lever to



increase the supply of working fluid and thus the engine speed is increased.

In this case, the extra power output is provided to balance the increased load. When the load on the engine decreases, the engine and the governor speed increases, which results in the increase of centrifugal force on the balls. Thus the balls move outwards and the sleeve rises upwards. This upward movement of the sleeve reduces the supply of the working fluid and hence the speed is decreased. In this case, the power output is reduced.

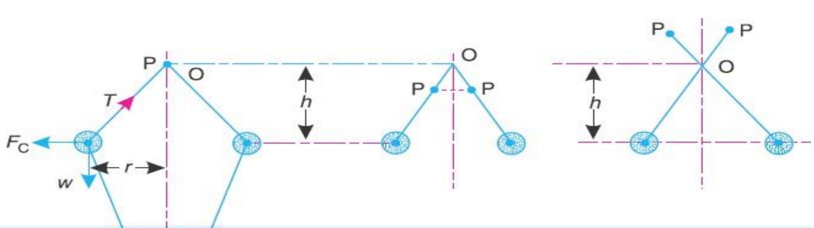
Watt Governor

The simplest form of a centrifugal governor is a Watt governor, as shown in Fig. It is basically a conical pendulum with links attached to a sleeve of negligible mass. The arms of the governor may be connected to the spindle in the following three ways governor may be connected to the spindle in the following three ways:

The pivot *P*,may be on the spindle axis as shown in Fig.(*a*).

The pivot *P*, may be offset from the spindle axis and the arms when produced intersect at*O*, as shown in Fig.(*b*).

The pivot *P*, may be offset, but the arms cross the axis at *O*, as shown in Fig(a)



Let m = Mass of the ball inkg,

w = Weight of the ball in newtons = $m.g$, T = Tension in the arm in newtons,

ω = Angular velocity of the arm and ball about the spindle axis in rad/s,

r = Radius of the path of rotation of the ball *i.e.* horizontal distance from the centre of the ball to the spindle axis in metres,

F_C = Centrifugal force acting on the ball in newtons = $m.\omega^2.r$, and

h = Height of the governor in metres.

It is assumed that the weight of the arms, links and the sleeve are negligible as compared to the weight of the balls. Now, the ball is in equilibrium under the action of the centrifugal force (FC) acting on the ball,the tension (T) in the arm, and the weight(w) of the ball.

Taking moments about point O, we have

$F_C \times h = w \times r = m.g.r$or

$m.\omega^2.r.h=m.g.r$ or $h = g/\omega^2$

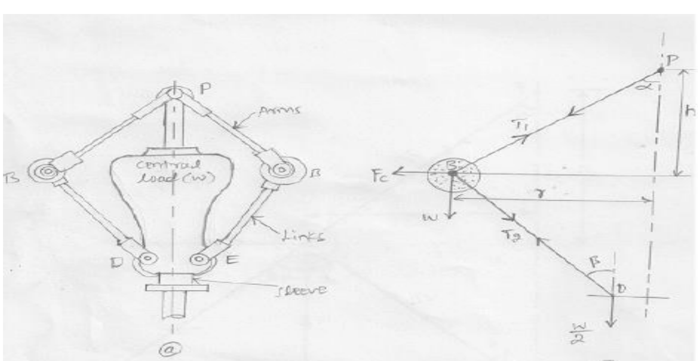
When g is expressed in m/s^2 and ω in rad/s, then h is in metres. If N is the speed in r.p.m., then

$\omega = 2\pi N/60$

$$h = \frac{9.81}{(2\pi N/60)^2} = \frac{895}{N^2} \text{ metres}$$

Porter Governor

The Porter governor is a modification of a Watt's governor, with central load attached to the sleeve as shown in Fig (a). The load moves up and down the central spindle. This additional downward force increases the speed of revolution required to enable the balls to rise to any predetermined level. Consider the forces acting on one-half of the governor as shown in Fig (b)



Let m = Mass of each ball inkg,

w = Weight of each ball in newtons = $m.g$, M = Mass of the central load in kg

h = Height of governor in metres ,

N = Speed of the balls in r.p.m .,

ω= Angular speed of the balls in rad/s

= $2\pi N/60$ rad/s,

F_C = Centrifugal force acting on the ball in newtons = $m.\omega^2.r$,

T_1 = Force in the arm innewtons,

T_2 = Force in the link innewtons,

α= Angle of inclination of the arm (or upper link) to the vertical, and

β= Angle of inclination of the link (or lower link) to the vertical.

Though there are several ways of determining the relation between the height of the governor (h) and the angular speed of the balls (ω), yet the following two methods are important from the subject point of view.

Method of resolution of forces Instantaneous centre method

Method of resolution of forces

Considering the equilibrium of the forces acting at D, we have

$$T_2 \cos\beta = \frac{W}{2} = \frac{M.g}{2}$$

$$T_2 = \frac{M.g}{2\cos\beta}$$

Again, considering the equilibrium of the forces acting on B. The point B is in equilibrium under the action of the following forces, as shown in Fig(b).

The weight of ball ($w = m.g$),

The centrifugal force (F_C),

The tension in the arm (T_1), and

The tension in the link (T_2).

Resolving the forces vertically,

$$T_1 \cos\alpha = T_2 \cos\beta + w = \frac{M.g}{2} + m.g \quad \ldots (ii)$$

Resolving the forces horizontally,

$$T_1 \sin\alpha + T_2 \sin\beta = F_C$$

$$T_1 \sin\alpha + \frac{M.g}{2\cos\beta} \times \sin\beta = F_C \quad \ldots \left(\because T_2 = \frac{M.g}{2\cos\beta}\right)$$

$$T_1 \sin\alpha + \frac{M.g}{2} \times \tan\beta = F_C$$

$$T_1 \sin\alpha = F_C - \frac{M.g}{2} \times \tan\beta \quad \ldots (iii)$$

Dividing equation (iii) by equation (ii),

$$\frac{T_1 \sin\alpha}{T_1 \cos\alpha} = \frac{F_C - \frac{M.g}{2} \times \tan\beta}{\frac{M.g}{2} + m.g}$$

or $$\left(\frac{M.g}{2} + m.g\right)\tan\alpha = F_C - \frac{M.g}{2}\times\tan\beta$$

$$\frac{M.g}{2} + m.g = \frac{F_C}{\tan\alpha} - \frac{M.g}{2}\times\frac{\tan\beta}{\tan\alpha}$$

Substituting $\frac{\tan\beta}{\tan\alpha} = q$, and $\tan\alpha = \frac{r}{h}$, we have

Instantaneous centre method

$$\frac{M.g}{2} + m.g = m.\omega^2.r\times\frac{h}{r} - \frac{M.g}{2}\times q \qquad \ldots(\because F_C = m.\omega^2.r)$$

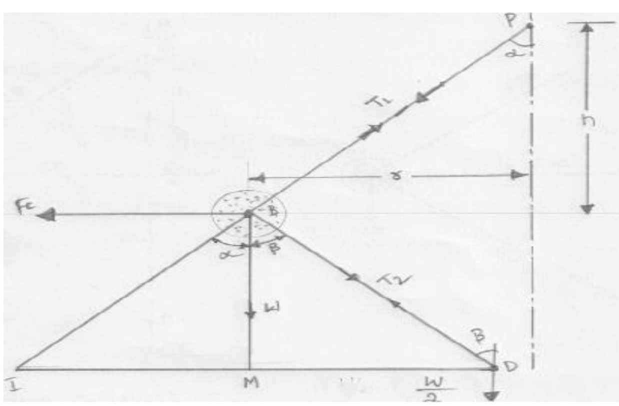
$$m.\omega^2.h = m.g + \frac{M.g}{2}(1+q)$$

$$\omega^2 = \left[m.g + \frac{Mg}{2}(1+q)\right]\frac{1}{m.h} = \frac{m + \frac{M}{2}(1+q)}{m}\times\frac{g}{h}$$

$$\left(\frac{2\pi N}{60}\right)^2 = \frac{m + \frac{M}{2}(1+q)}{m}\times\frac{g}{h}$$

$$N^2 = \frac{m + \frac{M}{2}(1+q)}{m}\times\frac{g}{h}\left(\frac{60}{2\pi}\right)^2 = \frac{m + \frac{M}{2}(1+q)}{m}\times\frac{895}{h} \qquad \ldots(v)$$

In this method, equilibrium of the forces acting on the link *BD* are considered. The instantaneous centre*I* lies at the point of intersection of *PB* produced and a line through *D* perpendicular to the spindle axis, as shown in Fig. 18.4.Taking moments about the point*I*,



$$F_C \times BM = w \times IM + \frac{W}{2} \times ID$$

$$= m.g \times IM + \frac{M.g}{2} \times ID$$

$$F_C = m.g \times \frac{IM}{BM} + \frac{M.g}{2} \times \frac{ID}{BM}$$

$$= m.g \times \frac{IM}{BM} + \frac{M.g}{2}\left(\frac{IM + MD}{BM}\right)$$

$$= m.g \times \frac{IM}{BM} + \frac{M.g}{2}\left(\frac{IM}{BM} + \frac{MD}{BM}\right)$$

$$= m.g \tan\alpha + \frac{M.g}{2}(\tan\alpha + \tan\beta)$$

$$\therefore\ m.\omega^2.r \times \frac{h}{r} = m.g + \frac{M.g}{2}(1+q)$$

$$h = \frac{m.g + \frac{M.g}{2}(1+q)}{m} \times \frac{1}{\omega^2} = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{g}{\omega^2}$$

... (Same as before)

When $\tan\alpha = \tan\beta$ or $q = 1$, then

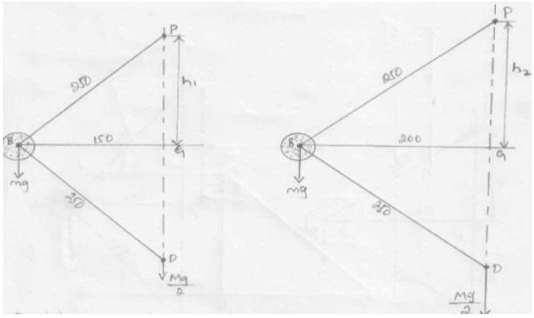
$$h = \frac{m + M}{m} \times \frac{g}{\omega^2}$$

PROBLEMS

Example1. A Porter governor has equal arms each 250 mm long and pivoted on the axis of rotation. Each ball has a mass of 5 kg and the mass of the central load on the sleeve is 25 kg. The radius of rotation of the ball is 150 mm when the governor begins to lift and 200 mm when the governor is at maximum speed. Find the minimum and maximum speeds and range of speed of the governor.

Solution. Given :*BP* = *BD* = 250 mm = 0.25 m ; *m* = 5 kg ; *M* = 15 kg ;

r_1 = 150 mm= 0.15m; r_2 = 200 mm = 0.2 m



The minimum and maximum positions of the governor are shown in Fig.(*a*) and (*b*)respectively.

Minimum speed when r_1 = BG = 0.15 m

Let N_1 = Minimumspeed.

From Fig.(*a*), we find that height of the governor,

$$h_1 = PG = \sqrt{(PB)^2 - (BG)^2} = \sqrt{(0.25)^2 - (0.15)^2} = 0.2 \text{ m}$$

We know that

$$(N_1)^2 = \frac{m + M}{m} \times \frac{895}{h_1} = \frac{5 + 15}{5} \times \frac{895}{0.2} = 17\,900$$

$N1$ = 133.8 r.p.m.

Maximum speed when r_2 = BG = 0.2 m Let $N2$ = Maximum speed.

From Fig(*b*), we find that height of the governor,

$$h_2 = PG = \sqrt{(PB)^2 - (BG)^2} = \sqrt{(0.25)^2 - (0.2)^2} = 0.15 \text{ m}$$

$$(N_2)^2 = \frac{m+M}{m} \times \frac{895}{h_2} = \frac{5+15}{5} \times \frac{895}{0.15} = 23\,867$$

$N2$ = 154.5 r.p.m.

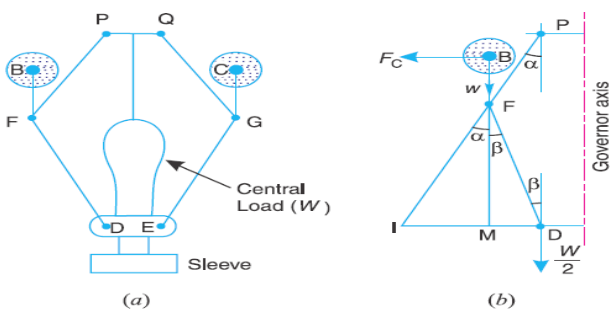
Range of speed

We know that range of speed= $N_2 - N_1$ = 154.4 – 133.8 = 20.7 r.p.m.

Proell Governor

The Proell governor has the balls fixed at *B* and *C* to the extension of the links *DF* and *EG*, as shown in Fig(*a*). The arms *FP* and *GQ* are pivoted at *P* and *Q* respectively.

Consider the equilibrium of the forces on one-half of the governor as shown in Fig (*b*).The instantaneous centre (*I*) lies on the intersection of the line *PF* produced and the line from *D* drawn perpendicular to the spindle axis. The perpendicular *BM* is drawn on *ID*.



Taking moments about *I*,

$$F_C \times BM = w \times IM + \frac{W}{2} \times ID = m.g \times IM + \frac{M.g}{2} \times ID \qquad \ldots (i)$$

$$\therefore \qquad F_C = m.g \times \frac{IM}{BM} + \frac{M.g}{2}\left(\frac{IM + MD}{BM}\right) \qquad \ldots (\because \; ID = IM + MD)$$

$$F_C = \frac{FM}{BM}\left[m.g \times \frac{IM}{FM} + \frac{M.g}{2}\left(\frac{IM}{FM} + \frac{MD}{FM}\right)\right]$$

$$= \frac{FM}{BM}\left[m.g \times \tan\alpha + \frac{M.g}{2}(\tan\alpha + \tan\beta)\right]$$

$$= \frac{FM}{BM} \times \tan\alpha\left[m.g + \frac{M.g}{2}\left(1 + \frac{\tan\beta}{\tan\alpha}\right)\right]$$

We know that $F_C = m.\omega^2 r$; $\tan\alpha = \frac{r}{h}$ and $q = \frac{\tan\beta}{\tan\alpha}$

$$\therefore \quad m.\omega^2.r = \frac{FM}{BM} \times \frac{r}{h}\left[m.g + \frac{M.g}{2}(1+q)\right]$$

and

$$\omega^2 = \frac{FM}{BM}\left[\frac{m + \frac{M}{2}(1+q)}{m}\right]\frac{g}{h} \qquad \ldots (ii)$$

Substituting $\omega = 2\pi N/60$, and $g = 9.81$ m/s^2, we get

$$N^2 = \frac{FM}{BM}\left[\frac{m + \frac{M}{2}(1+q)}{m}\right]\frac{895}{h} \qquad \ldots (iii)$$

PROBLEMS

Example 1.A Proell governor has equal arms of length 300 mm. The upper and lower ends of the arms are pivoted on the axis of the governor. The extension arms of the lower links are each 80 mm long and parallel to the axis when the radii of rotation of the balls are 150 mm and200 mm. The mass of each ball is 10 kg and the mass of the central load is 100 kg. Determine the range of speed of the governor.

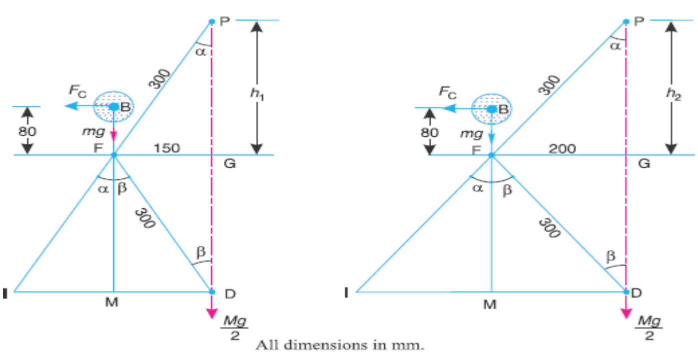
Solution. Given :$PF = DF = 300$ mm ; $BF = 80$ mm ; $m = 10$ kg ; $M = 100$ kg ;

$r_1 = 150$ mm; $r_2 = 200$ mm

First of all, let us find the minimum and maximum speed of the governor. The minimum and maximum position of the governor is shown in Fig (a) Let

N_1 = Minimum speed when radius of rotation, $r_1 = FG = 150$ mm;

N_2 = Maximum speed when radius of rotation ,$r_2 = FG = 200$mm.



(a) Minimum position. (a) Maximum position.

From Fig(a), we find that height of the governor,

$$FM = GD = PG = 260 \text{ mm} = 0.26 \text{ m}$$

$$BM = BF + FM = 80 + 260 = 340 \text{ mm} = 0.34 \text{ m}$$

We know that $(N_1)^2 = \frac{FM}{BM}\left(\frac{m+M}{m}\right)\frac{895}{h_1}$ . . . ($\therefore \alpha = \beta$ or $q = 1$)

$$= \frac{0.26}{0.34}\left(\frac{10+100}{10}\right)\frac{895}{0.26} = 28\,956 \quad \text{or} \quad N_1 = 170 \text{ r.p.m.}$$

Now from Fig. 18.13 (*b*), we find that height of the governor,

$$h_2 = PG = \sqrt{(PF)^2 - (FG)^2} = \sqrt{(300)^2 - (200)^2} = 224 \text{ mm} = 0.224 \text{ m}$$

$$FM = GD = PG = 224 \text{ mm} = 0.224 \text{ m}$$

$\therefore$ $BM = BF + FM = 80 + 224 = 304 \text{ mm} = 0.304 \text{ m}$

We know that $(N_2)^2 = \frac{FM}{BM}\left(\frac{m+M}{m}\right)\frac{895}{h_2}$ . . . ($\because \alpha = \beta$ or $q = 1$)

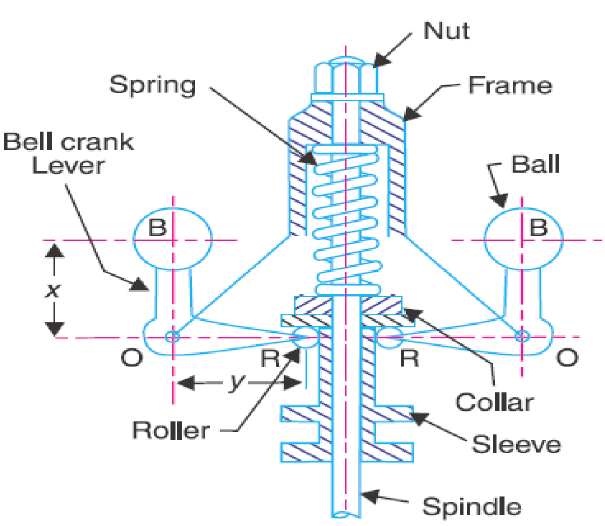
$$= \frac{0.224}{0.304}\left(\frac{10+100}{10}\right)\frac{895}{0.224} = 32\,385 \quad \text{or} \quad N_2 = 180 \text{ r.p.m.}$$

We know that range of speed

$$= N_2 - N_1 = 180 - 170 = 10 \text{ r.p.m.}$$ **Ans.**

Hartnell Governor

A Hartnell governor is a spring loaded governor as shown in Fig. It consists of two bell crank levers pivoted at the points *O*, *O* to the frame. The frame is attached to the governor spindle and therefore rotates with it. Each lever carries a ball at the end of the vertical arm *OB* and a roller at the end of the horizontal arm *OR*. A helical spring in compression provides equal downward forces on the two rollers through a collar on the sleeve. The spring force may be adjusted by screwing a nut up or down on the sleeve.



m = Mass of each ball in kg,

M = Mass of sleeve in kg,

r_1 = Minimum radius of rotation in metres,

ω_1 = Angular speed of the governor at minimum radius in rad/s, ω_2 = Angular speed of the governor at maximum radius in rad/s, S_1 = Spring force exerted on the sleeve at ω_1 in newtons,

S_2 = Spring force exerted on the sleeve at ω_2 in newtons, F_{C1} = Centrifugal force at ω_1 in newtons = $m(\omega_1)^2 r_1$, F_{C2} = Centrifugal force at ω_2 in newtons = $m(\omega_2)^2 r_2$,

s = Stiffness of the spring or the force required to compress the spring by one mm,

x = Length of the vertical or ball arm of the lever in metres,

y = Length of the horizontal or sleeve arm of the lever in metres, and

r = Distance of fulcrum O from the governor axis or the radius of rotation when thegovernor is in mid-position, in metres.

Consider the forces acting at one bell crank lever. The minimum and maximum position is shown in Fig. Let h be the compression of the spring when the radius of rotation changes from r_1 to r_2.

For the minimum position *i.e.* when the radius of rotation changes from r to r_1, as shown in

$$\frac{h_1}{y} = \frac{a_1}{x} = \frac{r - r_1}{x}$$

Fig (a), the compression of the spring or the lift of sleeve h_1 is given by

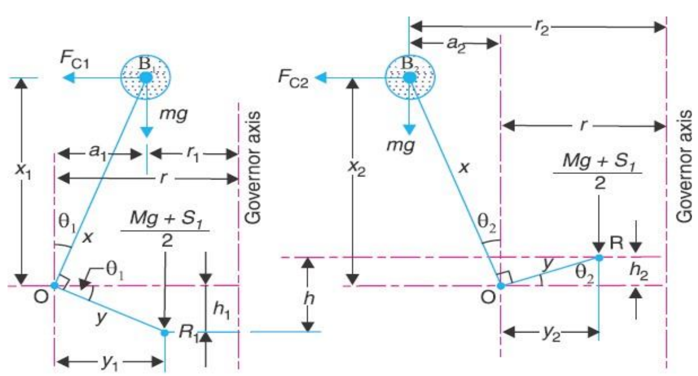
Similarly, for the maximum position *i.e.* when the radius of rotation changes from r to r_2, as shown in Fig (b), the compression of the spring or lift of sleeve h_2 is given by

$$\frac{h_2}{y} = \frac{a_2}{x} = \frac{r_2 - r}{x}$$

Adding equations (*i*) and (*ii*),

$$\frac{h_1 + h_2}{y} = \frac{r_2 - r_1}{x} \quad \text{or} \quad \frac{h}{y} = \frac{r_2 - r_1}{x} \qquad \ldots (\because h = h_1 + h_2)$$

$$\therefore \qquad h = (r_2 - r_1)\frac{y}{x} \qquad \ldots (iii)$$



(a) Minimum position. (b) Maximum position.

$$S_2 - S_1 = h.s, \quad \text{and} \quad h = (r_2 - r_1)\frac{y}{x}$$

$$s = \frac{S_2 - S_1}{h} = 2\left(\frac{F_{C2} - F_{C1}}{r_2 - r_1}\right)\left(\frac{x}{y}\right)^2 \quad \ldots (ix) \qquad \ldots (iv)$$

Again for maximum position, taking moments about point *O*, we get

$$\frac{M.g + S_2}{2} \times y_2 = F_{C2} \times x_2 + m.g \times a_2$$

$$M.g + S_2 = \frac{2}{y_2}(F_{C2} \times x_2 + m.g \times a_2) \quad \ldots (v)$$

Subtracting equation ***(iv)*** from equation ***(v)*,**

$$S_2 - S_1 = \frac{2}{y_2}(F_{C2} \times x_2 + m.g \times a_2) - \frac{2}{y_1}(F_{C1} \times x_1 - m.g \times a_1)$$

We know that

$$S_2 - S_1 = h.s, \quad \text{and} \quad h = (r_2 - r_1)\frac{y}{x}$$

$$\therefore \quad s = \frac{S_2 - S_1}{h} = \left(\frac{S_2 - S_1}{r_2 - r_1}\right)\frac{x}{y}$$

Neglecting the obliquity effect of the arms (*i.e.* $x_1 = x_2 = x$, and $y_1 = y_2 = y$) and the moment due to weight of the balls (*i.e.* *m.g*), we have for minimum position,

$$\frac{M.g + S_1}{2} \times y = F_{C1} \times x \quad \text{or} \quad M.g + S_1 = 2F_{C1} \times \frac{x}{y} \quad \ldots (vi)$$

Similarly for maximum position,

$$\frac{M.g + S_2}{2} \times y = F_{C2} \times x \quad \text{or} \quad M.g + S_2 = 2F_{C2} \times \frac{x}{y} \quad \ldots (vii)$$

Subtracting equation ***(vi)*** from equation ***(vii)*,**

$$S_2 - S_1 = 2(F_{C2} - F_{C1})\frac{x}{y} \quad \ldots (viii)$$

PROBLEMS

Example1. A Hartnell governor having a central sleeve spring and two right-angled bell crank levers moves between 290 r.p.m. and 310 r.p.m. for a sleeve lift of 15 mm. The sleeve arm sand the ball arms are 80 mm and 120 mm respectively. The levers are pivoted at 120 mm governor axis at the lowest equilibrium speed. Determine loads on the spring at the lowest and the highest equilibrium speeds ,and Stiffness of the spring.

Solution. Given : $N_1 = 290$ r.p.m. or $\omega_1 = 2\pi \times 290/60 = 30.4$ rad/s ;$N_2 = 310$ r.p.m. or $\omega_2 = 2\pi \times 310/60 = 32.5$ rad/s ; h = 15 mm = 0.015 m ;y = 80 mm = 0.08 m ; x = 120 mm =0.12 m ; r = 120 mm = 0.12 m ; m = 2.5 kgLoads on the spring at the lowest and highest equilibrium speeds

Let S= Spring load at lowest equilibrium speed,and

S_2 = Spring load at highest equilibrium speed.

Since the ball arms are parallel to governor axis at the lowest equilibrium speed (*i.e.* atN_1 = 290 r.p.m.), as shown in Fig(*a*), therefore

$r = r_1 = 120 \text{ mm} = 0.12 \text{ m}$

We know that centrifugal force at the minimum speed,

$F_{C1} = m\,(\omega_1)2\, r_1 = 2.5\,(30.4)2\; 0.12 = 277$ N

Now let us find the radius of rotation at the highest equilibrium speed, *i.e.* at N_2 =310 r.p.m. The position of ball arm and sleeve arm at the highest equilibrium speed is shown in Fig(*b*).

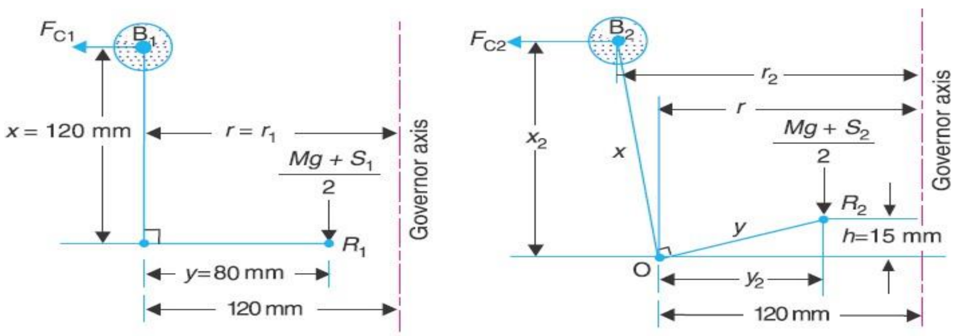
Let r_2 = Radius of rotation at N_2 = 310r.p.m.

We know that $h = (r_2 - r_1)\dfrac{y}{x}$

$$r_2 = r_1 + h\left(\frac{x}{y}\right) = 0.12 + 0.015\left(\frac{0.12}{0.08}\right) = 0.1425 \text{ m}$$

Centrifugal force at the maximum speed,

$F_{C2} = m\,(\omega_2)^2 r_2 = 2.5 \times (32.5)2 \times 0.1425 = 376$ N



(*a*) Lowest position. (*b*) Highest position.

Neglecting the obliquity effect of arms and the moment due to the weight of the balls, we have for lowest position,

$$M.g + S_1 = 2F_{C1} \times \frac{x}{y} = 2 \times 277 \times \frac{0.12}{0.08} = 831 \text{ N}$$

$$S_2 = 831 \text{ N } \textbf{Ans.} \qquad (\because M = 0)$$

and for highest position,

$$M.g + S_2 = 2F_{C2} \times \frac{x}{y} = 2 \times 376 \times \frac{0.12}{0.08} = 1128 \text{ N}$$

$$\therefore \quad S_1 = 1128 \text{ N } \textbf{Ans.} \qquad (\because M = 0)$$

Stiffness of thespring

We know that stiffness of the spring,

$$s = \frac{S_2 - S_1}{h} = \frac{1128 - 831}{15} = 19.8 \text{ N/mm}$$

Hartung Governor

A spring controlled governor of the Hartung type is shown in Fig. (*a*). In this type of governor, the vertical arms of the bell crank levers are fitted with spring balls which compress against the frame of the governor when the rollers at the horizontal arm press against the sleeve.

Let S = Springforce,

F_C = Centrifugal force,

M = Mass on the sleeve, and

$x$and y = Lengths of the vertical and horizontal arm of the bell crank lever respectively.

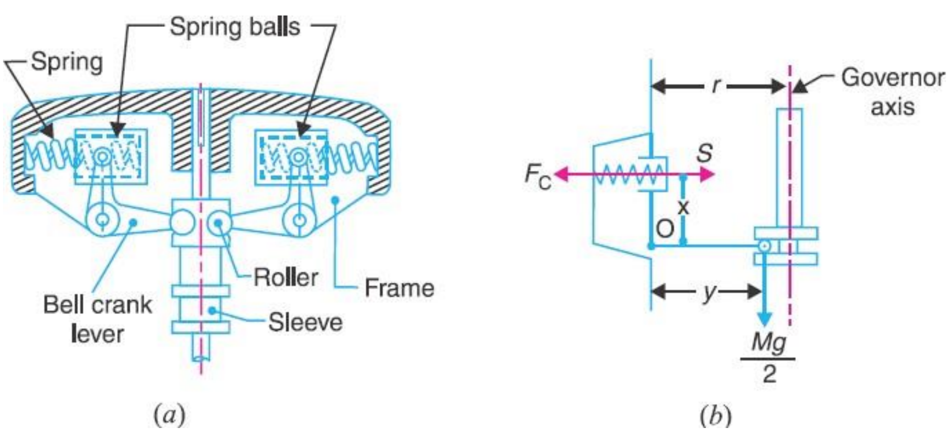


Fig (*a*) and (*b*) show the governor in mid-position. Neglecting the effect of obliquity of the arms, taking moments about the fulcrum O,

$$F_C \times x = S \times x + \frac{M.g}{2} \times y$$

Sensitiveness of Governors

Consider two governors *A* and *B* running at the same speed. When this speed increases or decreases by a certain amount, the lift of the sleeve of governor *A* is greater than the lift of the sleeve of governor *B*. It is then said that the governor *A* is more sensitive than the governor *B*.

In general, the greater the lift of the sleeve corresponding to a given fractional change in speed, the greater is the sensitiveness of the governor. It may also be stated in another way that for a given lift of the sleeve, the sensitiveness of the governor increases as the speed range decreases. This definition of sensitiveness may be quite satisfactory when the governor is considered as an independent mechanism. But when the governor is fitted to an engine, the practical requirement is simply that the change of equilibrium speed from the full load to the no load position of the sleeve should be as small a fraction as possible of the mean equilibrium speed.

The actual displacement of the sleeve is immaterial, provided that it is sufficient to change the energy supplied to the engine by the required amount. For this reason, the sensitiveness is defined as the **ratio of the difference between the maximum and minimum equilibrium speeds to the mean equilibrium speed**

Let N_1 = Minimum equilibriumspeed,

N_2 = Maximum equilibrium speed,and

N = Mean equilibriumspeed=N_1+N_2/2

∴ Sensitiveness of the governor

$$= \frac{N_2 - N_1}{N} = \frac{2(N_2 - N_1)}{N_1 + N_2}$$

$$= \frac{2(\omega_2 - \omega_1)}{\omega_1 + \omega_2} \quad \text{. . . (In terms of angular speeds)}$$

Stability of Governors

A governor is said to be ***stable*** when for every speed within the working range there is a definite configuration *i.e*. there is only one radius of rotation of the governor balls at which the governor is in equilibrium. For a stable governor, if the equilibrium speed increases, the radius of governor balls must alsoincrease.

Isochronous Governors

A governor is said to be ***isochronous*** when the equilibrium speed is constant (*i.e*. range of speed is zero) for all radii of rotation of the balls within the working range, neglecting friction. The isochronisms is the stage of infinite sensitivity.

Let us consider the case of a Porter governor running at speeds *N*1 and *N*2 r.p.m.

$$(N_1)^2 = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{895}{h_1} \quad \ldots (i)$$

$$(N_2)^2 = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{895}{h_2} \quad \ldots (ii)$$

For isochronisms, range of speed should be zero *i.e.* $N_2 - N_1 = 0$ or $N_2 = N_1$. Therefore from equations **(*i*)** and **(*ii*)**, $h_1 = h_2$, which is impossible in case of a Porter governor. Hence a ***Porter governor cannot beisochronous.***

Now consider the case of a Hartnell governor running at speeds N_1 and N_2 r.p.m.

$$M.g + S_1 = 2\,F_{C1} \times \frac{x}{y} = 2 \times m \left(\frac{2\pi N_1}{60}\right)^2 r_1 \times \frac{x}{y} \quad \ldots (iii)$$

$$M.g + S_2 = 2\,F_{C2} \times \frac{x}{y} = 2 \times m \left(\frac{2\pi N_2}{60}\right)^2 r_2 \times \frac{x}{y} \quad \ldots (iv)$$

$$\frac{M.g + S_1}{M.g + S_2} = \frac{r_1}{r_2}$$

For isochronisms, $N_2 = N_1$. Therefore from equations **(*iii*)** and **(*iv*),**

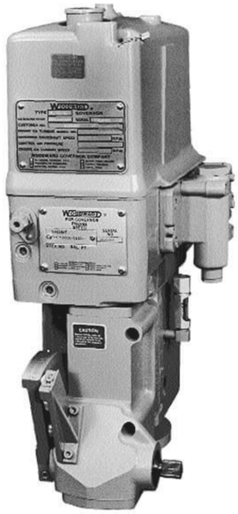
Hunting

A governor is said to be ***hunt*** if the speed of the engine fluctuates continuously above and below the mean speed. This is caused by a too sensitive governor which changes the fuel supply by a large amount when a small change in the speed of rotation takes place. For example, when the load on the engine increases, the engine speed decreases and, if the governor is very sensitive, the governor sleeve immediately falls to its lowest position. This will result in the opening of the control valve wide which will supply the fuel to the engine in excess of its requirement so that the engine speed rapidly increases again and the governor sleeve rises to its highest position. Due to this movement of the sleeve, the control valve will cut off the fuel supply to the engine and thus the engine speed begins to fall once again. This cycle is repeated indefinitely. Such a governor may admit either the maximum or the minimum amount of fuel. The effect of this will be to cause wide fluctuations in the engine speed or in other words, the engine will hunt

INDUSTRIAL APPLICATIONS

Diesel Generators

Gas Engine Governors



In steam Turbines

TUTORIAL QUESTIONS

1. Explain types of governors?
2. A Porter governor has all four arms 250 mm long. The upper arms are attached on the axis of rotation and the lower arms are attached to the sleeve at a distance of 30 mm from the axis. The mass of each ball is 5 kg and the sleeve has a mass of 50 kg. The extreme radii of rotation are 150 mm and 200 mm. Determine the range of speed of the governor.
3. A Proell governor has equal arms of length 300 mm. The upper and lower ends of the arms are pivoted on the axis of the governor. The extension arms of the lower links are each 80 mm long and parallel to the axis when the radii of rotation of the balls are 150 mm and 200 mm. The mass of each ball is 10 kg and the mass of the central load is 100 kg. Determine the range of speed of the governor.
4. In a spring loaded Hartnell type governor, the extreme radii of rotation of the balls are 80 mm and 120 mm. The ball arm and the sleeve arm of the bell crank lever are equal in length. The mass of each ball is 2 kg. If the speeds at the two extreme positions are 400 and 420 r.p.m., find : 1. the initial compression of the central spring, and 2. the spring constant.
5. In a spring controlled governor of the type, as shown in Fig. 18.24, the mass of each ball is 1.5 kg and the mass of the sleeve is 8 kg. The two arms of the bell crank lever are at right angles and their lengths are OB = 100 mm and OA = 40 m m. The distance of the fulcrum O of each bell crank lever from the axis of rotation is 50 mm and minimum radius of rotation of the governor balls is also 50 m m. The corresponding equilibrium speed is 240 r.p.m. and the sleeve is required to lift 10 mm for an increase in speed of 5 per cent. Find the stiffness and initial compression of the spring
6. A spring loaded governor of the Wilson-Hartnell type is shown in Fig 18.50. Two balls each of mass 4 kg are connected across by two springs A. The stiffness of each spring is 750 N/m and a free length of 100 mm. The length of ball arm of each bell crank lever is 80 mm and that of sleeve arm is 60 mm. The lever is pivoted at its mid-point. The speed of the governor is 240 r.p.m. in its mean position and the radius of rotation of the ball is 80 mm. If the lift of the sleeve is 7.5 mm for an increase of speed of 5%, find the required stiffness of the auxiliary spring B.