Pbm 5 Solve by graphical method maximize $z = 6x_1 + 9x_2$ subject to the constrains $x_1 + x_2 \leq 12$, $x_1 + 5x_2 \leq 45$, $3x_1 + x_2 \leq 30$ and non negative constrains $x_1, x_2 \geq 0$

Sol:- Replace all inequalities constrains into equations

$\therefore$ we have

$$x_1 + x_2 = 12 \quad \text{---} (1)$$
$$x_1 + 5x_2 = 45 \quad \text{---} (2)$$
$$3x_1 + x_2 = 30 \quad \text{---} (3)$$

By considering eqn (1)

$x_1 + x_2 = 12$ ——— (1)

put $x_1 = 0$ $(0, 12)$

put $x_2 = 0$ $(12, 0)$

$\therefore$ co-ordinates of equation (1)

$(0, 12)$ and $(12, 0)$

Similarly :

By considering eqn (2)

$x_1 + 5x_2 = 45$

put $x_1 = 0$ $(0, 9)$

put $x_2 = 0$ $(45, 0)$

coordinates of equation (2)

$(0, 9)$ and $(45, 0)$

By considering eqn (3)

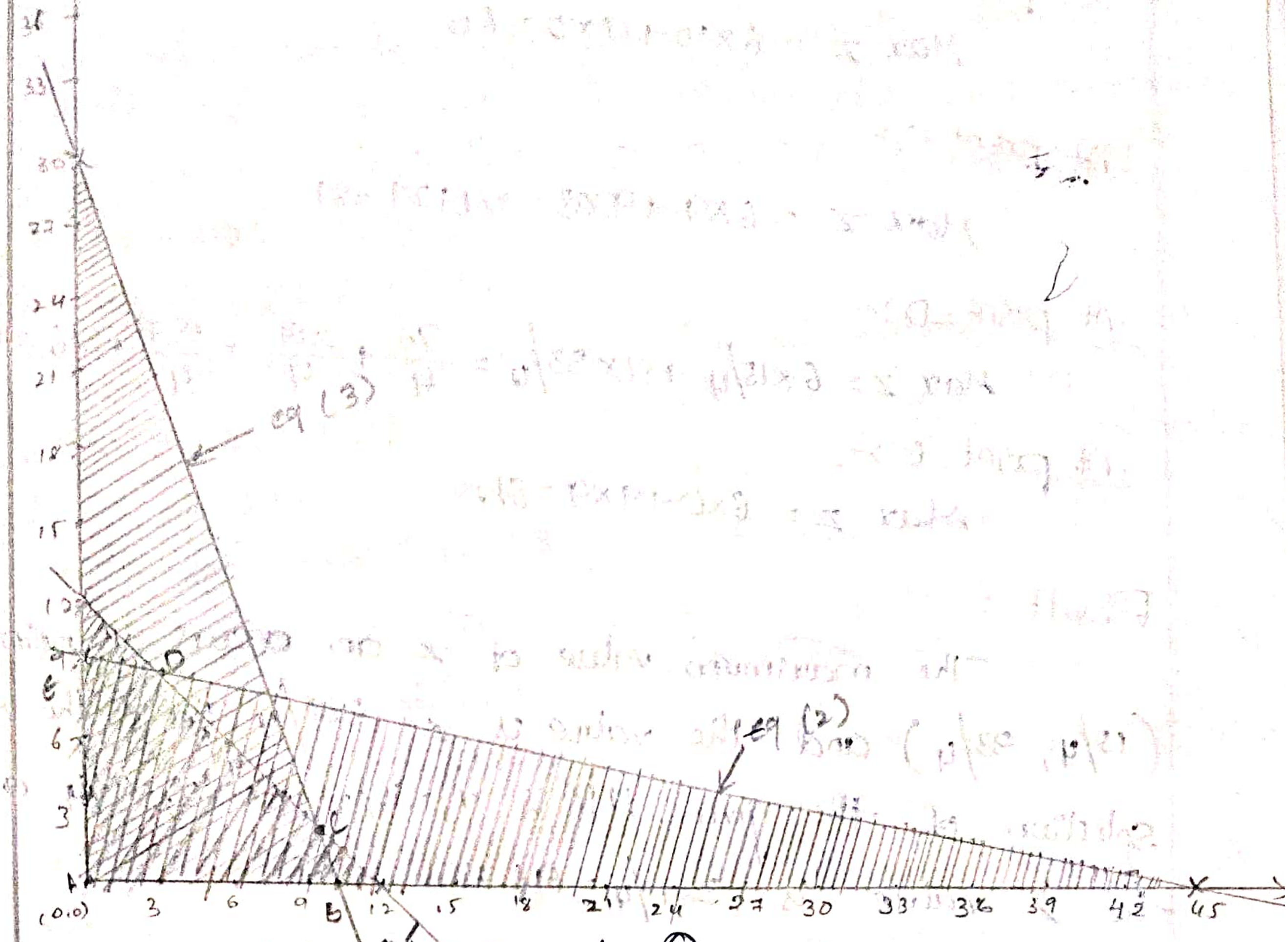
$3x_1 + x_2 = 30$

put $x_1 = 0$ $(0, 30)$ and

$x_2 = 0$ $(10, 0)$

coordinates of equation (3)

$(0, 30)$ and $(10, 0)$



considering from eq(1) and eq(3)

$x_1+x_2=12 \longrightarrow (1)$

$3x_1+x_2=30 \longrightarrow (3)$

$-2x_1=-18$

$\therefore \boxed{x_1=9}$

from eqn(1) $\Rightarrow x_1+x_2=12$

$9+x_2=12$

$x_2=12-9$

$\boxed{x_2=3} \Rightarrow (9,3)$

Considering from eq(1) and (2)

$x_1+x_2=12 \longrightarrow (1)$

$\underset{(-)}{}x_1 \underset{(-)}{+} 5x_2 \underset{(-)}{=} 45 \longrightarrow (3)$

$-4x_2=-33$

$\boxed{x_2=33/4}$

from eq(1)

$\Rightarrow x_1=12-33/4$

$=\frac{48-33}{4}=15/4$

max $Z = 6x_1+9x_2$

At point B :-

$Max\ z = 6\times10 + 9\times0 = 60$

At point C :-

$Max\ Z = 6\times9 + 9\times3 = 54 + 27 = 81$

At point D :-

$Max\ z = 6\times 15/4 + 9\times 33/4 = \frac{90}{4} + \frac{297}{4} = \frac{387}{4} = 96.75$

At point E :-

$Max\ z = 6\times0 + 9\times9 = 81.$

Result :-

The maximum value of z an occurs at point $(15/4, 33/4)$ and the value is $z = 387/4$ Hence the optima solution of the given LPP is $x_1 = 15/4$ and $x_2 = 33/4$ and the z value is $387/4 = 96.75$

* Properties of Linear Programming Solution :-

1) Feasible Solution :-

If all the constrains of the given linear programming model are satisfies the solution of the model then that solution is known as feasable solution.

2) Optimal solution :-

If there is no other superior solution to the solution obtain for a given linear programming model then the solution obtain is treated as the optimal solution.

3) Alternate optimal solution :- (Multiple optimal solution)

For some linear programming model there may be more than one combination of values of the decision variables yielding the best objective function values. Such Combination are the values of the decision variables are known as Alternate optimal solution.

4) Unboundedht Solution :-

For some linear programming model the objective function value can be increased (or) decreased infinitely with out any limitation such solution is known as unboundedht solution.

5) Infeasible Solution :-

If there is no combination of values of the decision variables satisfing all the constrains of the Linear programming model then that model is said to have Infeasable solution. This means that there is no solution for the given model which can be implement.

6) Degenerate solution :-

In LPP, intersection of two constrains will define a corner point of that feasable region. But if more than two constrains pass through any one of the corner points of the feasable region, excess constrains will not serve any purpose and therefore, act as ~~re-boundent~~ re-boundent constrains. Under such situation De-generasy will occurs. This means that some iterations will be carried out in simplex method with out any improvement in the objective function.

problm

(6) Solve graphically maximize $Z = 40x_1 + 100x_2$

Subject to the constrains $2x_1 + x_2 \leq 500$, $2x_1 + 5x_2 \leq 1000$

and $x_1, x_2 \geq 0$

Soln Replace all inequalities constrains into equations

$\therefore \quad 2x_1 + x_2 = 500$ —(1)

$2x_1 + 5x_2 = 1000$ —(2)

By considering eq(1)

$2x_1 + x_2 = 500$ —(1)

put $x_1 = 0 \Rightarrow (0, 500)$

put $x_2 = 500$

put $x_2 = 0$ $(250, 0)$

$x_1 = 250$

$\therefore$ (0,500), (250,0) are coordinates of eq(1)

Similarly:

By considering eq(2)

$$2x_1 + 5x_2 = 1000$$

put $x_1 = 0$ $\Rightarrow$ (0, 200)

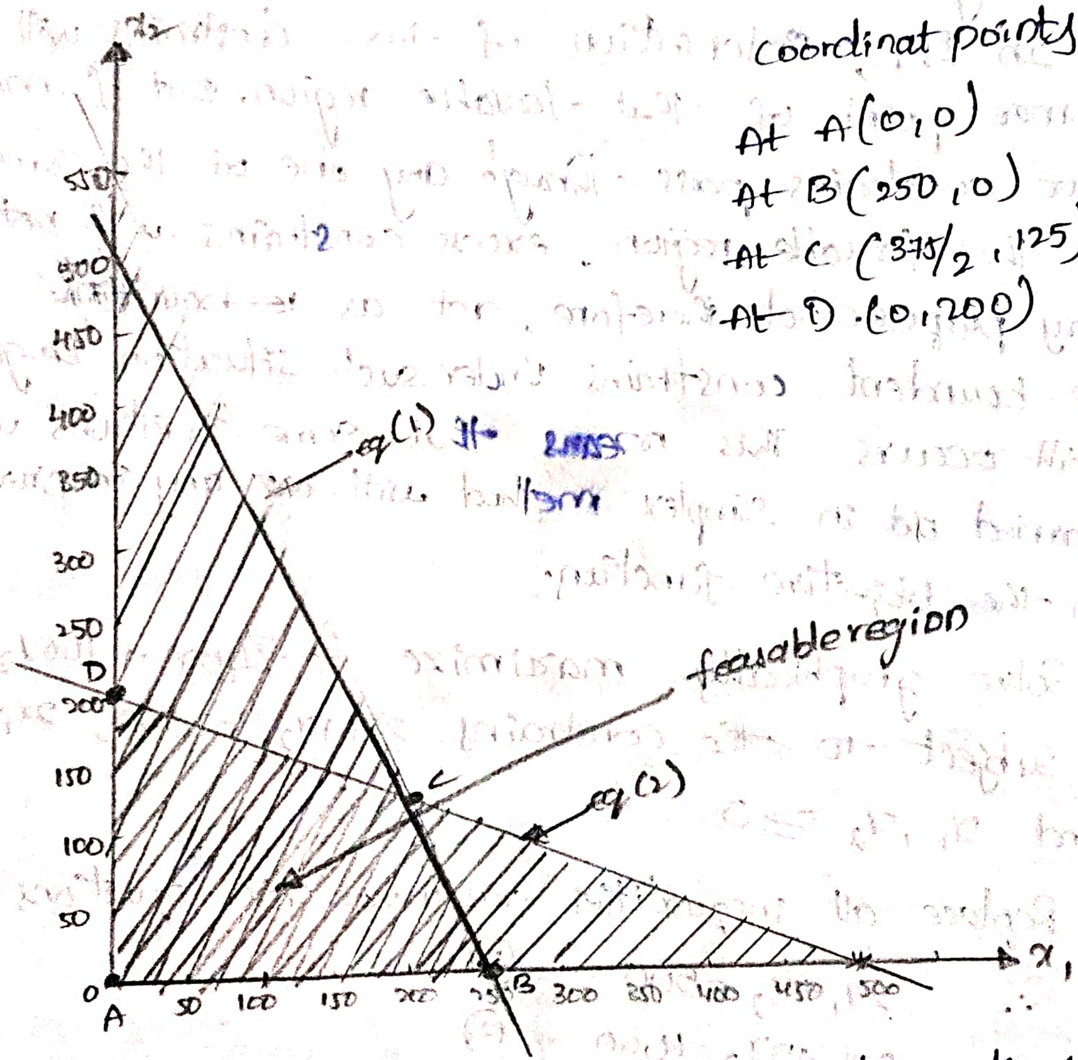
$x_2 = 200$

put $x_2 = 0$

$x_1 = 500$

(500, 0)

$\therefore$ The coordinates from eqn (2) are (0,200) & (500,0)



coordinat points

At A(0,0)

At B(250,0)

At C (375/2, 125)

At D (0,200)

By solving eq(1) and (2) we get coordinate point at C

$$2x_1 + x_2 = 500$$
$$2x_1 + 5x_2 = 1000$$
$$\overline{-4x_2 = -500}$$
$$\boxed{x_2 = 125}$$

$2x_1 + 125 = 500$

$$\boxed{x_1 = \frac{375}{2}}$$

The point C coordinates $(375/2, 125)$

objective function :-

Max $z = 40x_1 + 100x_2$

At point B $= 40 \times 250 + 100 \times 0$
$= 10,000$

At point C $= 40 \times 375/2 + 100 \times 125$
$= 20,000$

At point D $= 40 \times 0 + 100 \times 200$
$= 20,000$

It is a Multiple optimal solution.

The maximize value of z occurs at points D and C
The coordinates are $(375/2, 125), (0, 200)$ the z value is
20,000.

pblm Solve graphically maximize $z = 3x_1 + 4x_2$ subject to the constrains $-3x_1 + 2x_2 \leq 6$, $3x_1 + x_2 \geq 6$ and $x_1 + x_2 \leq 8$ and

$x_1, x_2 \geq 0$

Replace all inequalities Constrains into equations

$-3x_1 + 2x_2 \leq 6$ —— ①

$3x_1 + x_2 \geq 6$ —— ②

$x_1 + x_2 \leq 8$ —— ③

By considering eq ①

$-3x_1 + 2x_2 = 6$

put $x_1 = 0$
$x_2 = 3$ $\Rightarrow (0, 3)$

put $x_2 = 0$
$x_1 = -2$ $\Rightarrow (-2, 0)$

$(0, 3)$ & $(-2, 0)$ are coordinates of eq ①

similarly :

By considering eq ②

$3x_1 + x_2 = 6$

put $x_1=0$ $\Rightarrow (0,6)$
$x_2=6$

put $x_2=0$ $\Rightarrow (2,0)$
$x_1=2$

$\therefore (0,6)$ & $(2,0)$ are the coordinates of eq ②

Similarly :

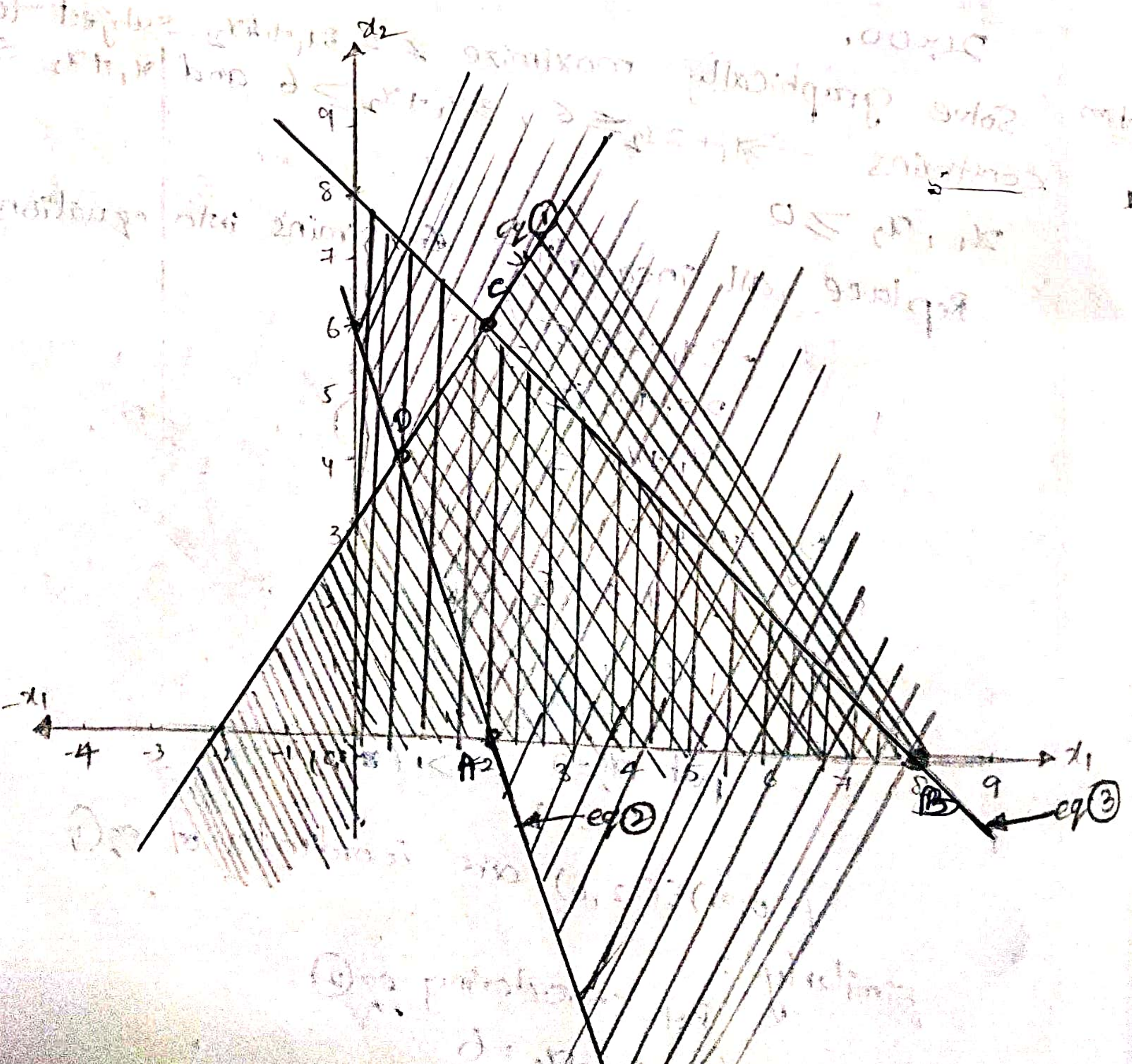
By considering eq ③

$x_1+x_2=8$

put $x_1=0$ $\Rightarrow (0,8)$
$x_2=8$

put $x_2=0$ $\Rightarrow (8,0)$
$x_1=8$

$\Rightarrow$ The coordinates of eq ③ $(0,8)$ & $(8,0)$.



Coordinates at point A (2,0)

B (8,0)

At point C

By solving eq① & ③

eq③×2

$-3x_1 + 2x_2 = 6$

$\underset{(-)}{2x_1} \underset{(-)}{+ 2x_2} = \underset{(-)}{16}$

$-5x_1 = -10$

$\boxed{x_1 = 2}$

eq③ ⇒ $x_1 + x_2 = 8$

$2 + x_2 = 8$

$\boxed{x_2 = 6}$

At point C cooridinates are C(2,6)

At point D

By solving eq① & eq②

$-3x_1 + 2x_2 = 6$

$3x_1 + x_2 = 6$

$3x_2 = 12$

$\boxed{x_2 = 4}$

eq② ⇒ $3x_1 + x_2 = 6$

$3x_1 + 4 = 6$

$3x_1 = 2$

$\boxed{x_1 = 2/3}$

At point D cooridinates are D(2/3, 4)

Objective function :-

Max $z = 3x_1 + 4x_2$

At point A = $3(2) + 4(0)$

$= 6$

At point B = $3(8) + 4(0)$

$= 24$

At point C = $3(2) + 4(6)$

$= 30$

At point D = $3(2/3) + 4(4) = 18$

Result :-

$\therefore$ The maximize value 'z' occurs at point C and the coordinates are (2, 6). The z value is '30'.

$\therefore$ This is a optimal solution.

(3) Solve the following LPP graphically maximize $z = 4x + 7y$ subject to the constrains $x+y \le 60$, $y \le 40$, $x \le 40$ and $x, y \ge 0$.

Replace all inequality constrains into equations

$x+y = 60$ —— (1)

$y = 40$ —— (2)

$x = 40$ —— (3)

By considering eqn (1)

$x+y = 60$

put $x = 0$ (0, 60)

put $y = 0$ (60, 0)

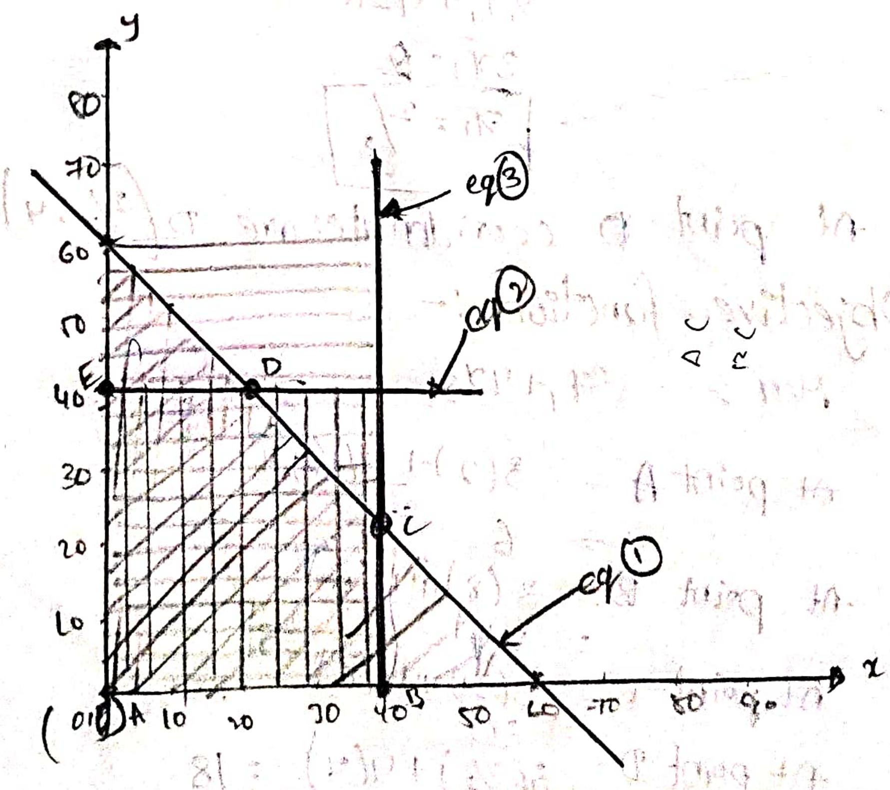
$\therefore$ (0, 60) and (60, 0) are the coordinates

By considering eqn (2)

$y = 40$ (0, 40)

By considering eq (3)

$x = 40$ (40, 0)



Coordinates at point A $(0,0)$
point B $(40,0)$
point C $(\cancel{40,0})$
point D $(\cdot \quad)$
point E $(0,40)$

By considering eq ① & ③
solve eq ① & eq ③

$$x+y=60$$
$$x=40$$
$$\boxed{y=20}$$

eq ① $\Rightarrow$ $x+y=60$
$x+20=60$
$\boxed{x=40}$

$\therefore$ point C $(40,20)$

By solving eq ① & eq ②

$$x+y=60$$
$$y=40$$
$$x=20$$

from eq ① $x+y=60$
$20+y=60$
$\boxed{y=40}$

$\therefore$ point D $=(20,40)$

Objective function :-
Max $Z=4x+7y$

At point A $=0$
At point B $=4(40)+7(0)$
$=160$
At point C $=4(40)+7(20)$
$=160+140$
$=300$
At point D $=4(20)+7(40)$
$=80+280$
$=360$
At point E $=4(0)+7(40)$
$=280$

Result :-

∴ The maximize value 'z' occurs at point D and the coordinates are (20, 40). The z value is '360'.

∴ This is an optimal solution

* Solve graphically maximize $z = 2x_1 + 3x_2$ subject to the constrains $2x_1 + x_2 \le 2$, $3x_1 + 4x_2 \ge 12$ and $x_1, x_2 \ge 0$

Replace the inequalities into equations

$2x_1 + x_2 = 2$ —— (1)

$3x_1 + 4x_2 = 12$ —— (2)

By considering eq (1)

$2x_1 + x_2 = 2$

put $x_1 = 0 \Rightarrow (0, 2)$

put $x_2 = 0 = (1, 0)$

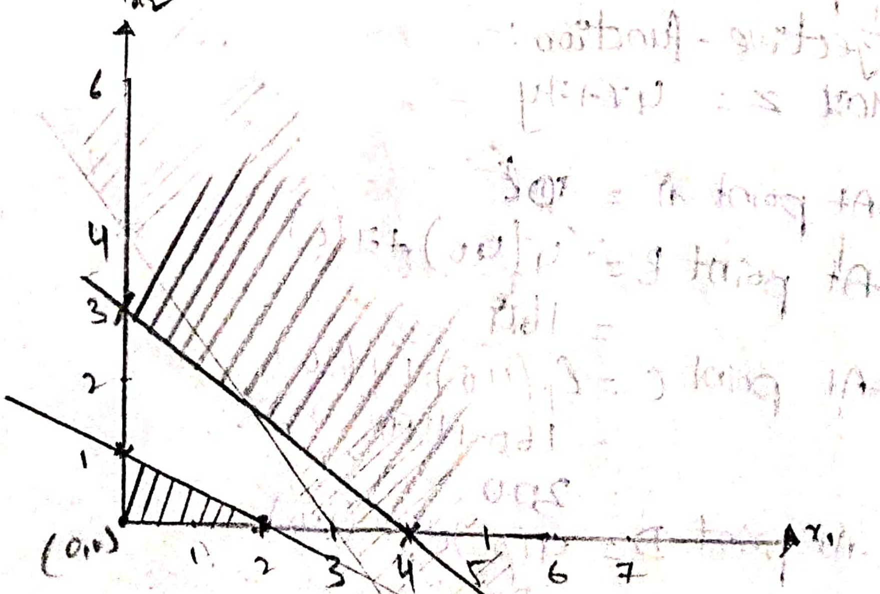
Similarly :

from eq (2)

$3x_1 + 4x_2 = 12$

put $x_1 = 0 \Rightarrow (0, 3)$

put $x_2 = 0 = (4, 0)$



from the graph we find that there is no common region between the two.

Hence there is no feasable solution.

Thus the given LPP has no solution.

* A company produces two products x and y. The cost of producing one unit of x is Rs.40/- and one unit of y is ₹30/-. The production must satisfy the following constrains $x+y \geq 10$, $2x+y \geq 15$ and $x, y \geq 0$
objective minimize $Z = 40x + 30y$

Sol:
Convert inequalities into eqns

$x+y = 10$ — ①

$2x+y = 15$ — ②

considering eq ①

we have $x+y=10$ $\quad x \quad y$

put $x=0$ $\quad (0,10)$

put $y=0$ $\quad (10,0)$

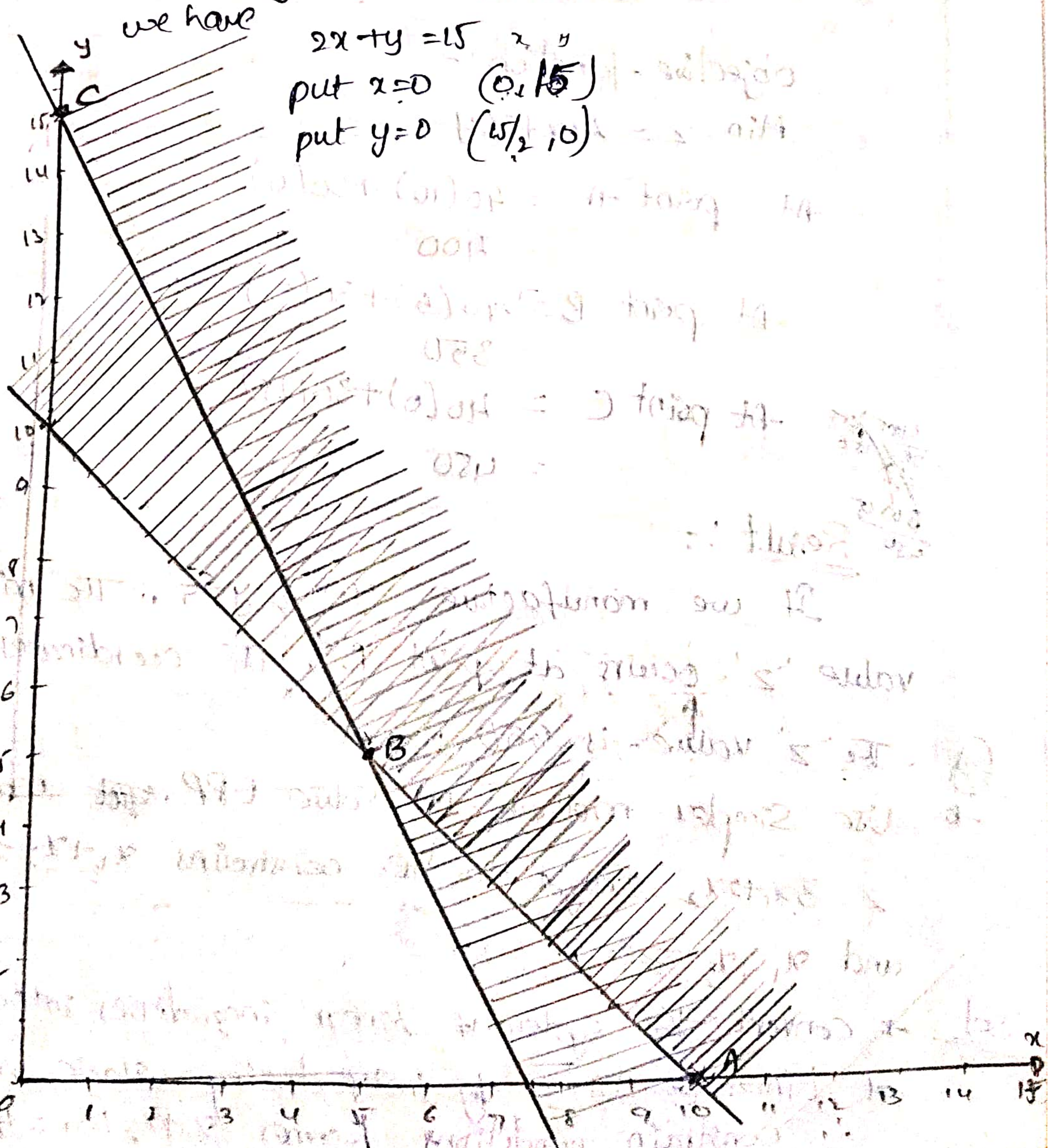
Considering eq ②

we have

$2x+y = 15$ $\quad x \quad y$

put $x=0$ $\quad (0,15)$

put $y=0$ $\quad (15/2, 0)$



coordinates at point at A (10, 0)
~~"~~ " " " " B ( )
" " " " " C (0, 15)

~~considering eq (1)~~

~~x+y = 10~~

~~10~~

Solve eq (1) & eq (2) we get B ( )

$$x+y = 10$$
$$2x+y = 15$$
$$\overline{-x = -5}$$
$$\boxed{x = 5}$$

eq (1) $\Rightarrow$ $x+y = 10$

$$5+y = 10$$
$$\boxed{y = 5}$$

$\therefore$ The coordinates at point B (5, 5)

Objective function :-

Min $Z = 40x + 30y$

At point A $= 40(10) + 30(0)$
$= 400$

At point B $= 40(5) + 30(5)$
$= 350$

At point C $= 40(0) + 30(15)$
$= 450$

400 300 200 150 30×15 450

Result :-

If we manufacture $x=5$ & $y=5$ .. The minimize value 'z' occurs at point 'B'. The coordinates are (5, 5).

The 'z' value is '350'.

(8) Use Simplex method to solve LPP. maximize $Z = 3x_1 + 2x_2$ subject to the constrains $x_1 + x_2 \leq 4$ & $x_1 - x_2 \leq 2$ and $x_1, x_2 \geq 0$

Sol → Convert the system of linear inequalities into a system of Linear equations by introducing slack variables.

$\therefore$ Constrain conditions becomes $x_1 + x_2 + S_1 = 4$ & $x_1 - x_2 + S_2 = 2$

* Sfm $x_1+x_2+S_1=4$ —(1)

$x_1-x_2+S_2=2$ —(2)

and $x_1, x_2, S_1, S_2 \geq 0$

Note:-
(S_1, S_2 are the basic variables can be changable and x_1, x_2 are non basic variables at initial point).

Let objective function can be written as:

$$Max\ z = 3x_1 + 2x_2 + 0S_1 + 0S_2$$

we have to form a table IBFS

Initial Basic feasable solution (IBFS):-

		CJ	3	2	0	0	
Basic variable BV	Coefficient of Basic variable C_B	Solution values SV	x_1 key column	x_2	S_1	S_2	Minimum Ratio(MR) MR= SV / key column element; MR= SV / KCE
S_1	0	4	$\boxed{1}$	1	1	0	4/1 = 4
S_2	0	2	1	-1	0	1	2/1 = 2*
key row	ZJ	0	0	0	0	0	
	ZJ-CJ	.	-3*	-2	0	0	

Least

key element

$4 - \frac{1\times 2}{1}$

old value − [Product of K.E × corre key Row Elem / K Element]

Iteration-1

		CJ	3	2	0	0	
BV	CB	Sv	x_1	x_2	S_1	S_2	MR= SV / KCE
S_1	0	2	0	$\boxed{2}$	1	-1	2/2 = 1
x_1	3	2	1	-1	0	1	—
	ZJ	6	3	-3	0	3	
	ZJ-CJ		0	-5	0	3	

↑ least

Iteration-2

		CJ	3	2	0	0	
Bv	CB	Sv	x_1	x_2	S_1	S_2	MR
x_2	2	1	0	1	1/2	-1/2	
x_1	3	3	1	0	1/2	1/2	
	ZJ	$\boxed{11}$	3	2	5/2	1/2	
	ZJ-CJ		0	0	5/2	1/2	

Max z = $3x_1+2x_2$

$= 3\times3+2\times1 = 11$

By using Graphical method :-

$$x_1 + x_2 \le 4$$
$$x_1 - x_2 \le 2$$
$$Z = 3x_1 + 2x_2$$
and $x_1, x_2 \geq 0$

Replace all inequality constrains into equations.

$x_1 + x_2 = 4$ —— ①
$x_1 - x_2 = 2$ —— ②

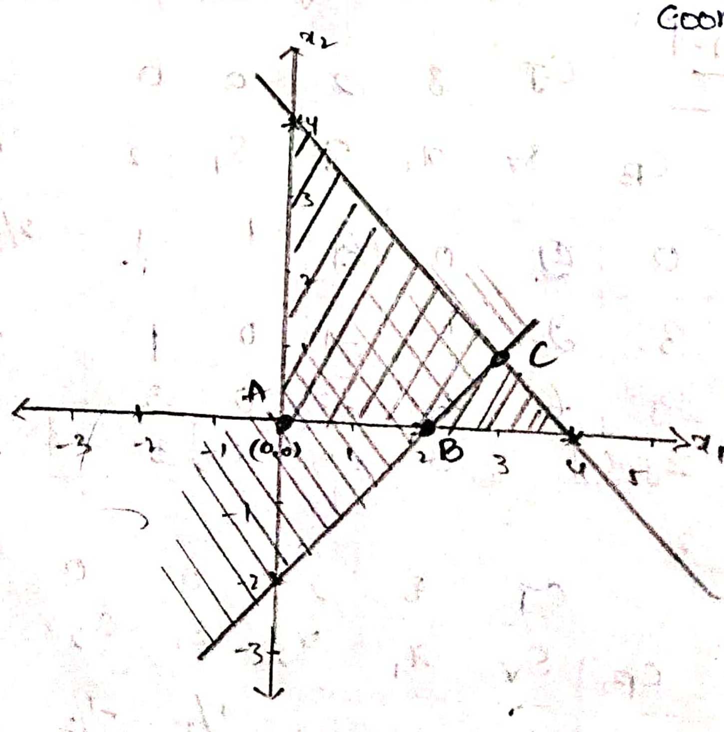
By considering eq ①

$x_1 + x_2 = 4$
put $x_1 = 0$ $(0,4)$
put $x_2 = 0$ $(4,0)$

Ily By considering eq ②

$x_1 - x_2 = 2$

put $x_1 = 0$ $(0,-2)$
put $x_2 = 0$ $(2,0)$



Coordinates at point $A(0,$
at point $B(2,0)$
at point $C(3,1)$

By solving eq ① & eq ②

$$x_1 + x_2 = 4$$
$$x_1 - x_2 = 2$$
$$2x_1 = 6$$
$$\boxed{x_1 = 3}$$

From eq ①

we have $x_1 + x_2 = 4$

$$3 + x_2 = 4$$
$$\boxed{x_2 = 1}$$

∴ The coordinate at point C $(3, 1)$

Objective function :-

$$\text{Max } z = 3x_1 + 2x_2$$

At point A Max $z = 0$

At point B Max $z = 3x_1 + 2x_2$

$$z = 3(2) + 2(0)$$
$$z = 6$$

At point C Max $z = 3(3) + 2(1)$

$$= 11$$

Result :-

∴ The maximum z value occurs at point C and the coordinates are $(3, 1)$. The 'z' value is '11'.

∴ This is an optimal solution.

* Maximize $z = x_1 - x_2 + 3x_3$ subject to the constrains $x_1 + x_2 + x_3 \leq 10$, $2x_1 - 3x_3 \leq 3$, $2x_1 - 2x_2 + 3x_3 \leq 0$ and $x_1, x_2, x_3 \geq 0$

Sol Convert all inequality constrains into equations by introducing slack variables.

$$x_1 + x_2 + x_3 + s_1 \leq 10 \text{ ——— ①}$$
$$2x_1 - 3x_3 + s_2 = 3 \text{ ——— ②}$$
$$2x_1 - 2x_2 + 3x_3 + s_3 = 0 \text{ ——— ③}$$

and objective function is

$$\text{Max } z = x_1 - x_2 + 3x_3 + 0s_1 + 0s_2 + 0s_3$$

Form IBFS

$C_J =$ 1 -1 3 0 0 0

B_v	C_B	S_v	x_1	x_2	x_3	S_1	S_2	S_3	$MR = \frac{SV}{K.C.E}$
S_1	0	10	1	1	1	1	0	0	$10/1 = 10$
S_2	0	3	2	0	-1	0	1	0	—
S_3	0	0	2	-2	3	0	0	1	$0/3 = 0$
	$ZJ =$	0	0	0	0	0	0	0	
	$ZJ - CJ =$		-1	1	-3	0	0	0	

↑ least

Artificial variable Method another name

Iteration-1 :-

$C_J =$ 1 -1 3 0 0 0

B_v	C_B	S_v	x_1	x_2	x_3	S_1	S_2	S_3	MR
key row → S_1	0	10	$1/3$	$5/3$	0	1	0	$-1/3$	6
S_2	0	3	$8/3$	$-2/3$	0	0	1	$1/3$	-
x_3	3	0	$2/3$	$-2/3$	1	0	0	$1/3$	-
	$ZJ =$	0	2	-2	3	0	0	1	
	$ZJ - CJ$		1	-1	0	0	0	1	

↑ key column

Iteration-2 :-

C_J 1 -1 3 0 0 0

B_v	C_B	S_v	x_1	x_2	x_3	S_1	S_2	S_3	MR
x_2	-1	6	$1/5$	1	0	$3/5$	0	$-1/5$	
S_2	0	7	$14/5$	0	0	$2/5$	1	$1/5$	
x_3	3	4	$4/5$	0	1	$2/5$	0	$1/5$	
	ZJ	6	$11/5$	-1	3	$3/5$	0	$4/5$	
	ZJ-CJ		$6/5$	0	0	$3/5$	0	$4/5$	

$x_1 = 0, x_2 = 6, x_3 = 4$ $\quad Z = x_1 - x_2 + 3x_3$

Max $Z = 0 - 6 + 3 \times 4 = 6$ //

Artificial variable techniques — Two Phase Method :- Extention of simplex method.
(or) Big M — Method another name

Maximize $z = 5x_1 + 3x_2$ subject to the constraints $2x_1 + x_2 \leq 1$, $x_1 + 4x_2 \geq 6$ and $x_1, x_2 \geq 0$

($\leq$) slack
($\geq$) surplus

Sol:- Convert inequalities into equations. By using surplus

$2x_1 + x_2 + S_1 = 1 \longrightarrow (1)$

$x_1 + 4x_2 - S_2 + A_1 = 6 \longrightarrow (2)$

and objective function can be return as

$\text{Max } Z = 5x_1 + 3x_2 + 0S_1 + 0S_2 - A_1$

Auxiliary objective function :-

$\text{max } z = 0x_1 + 0x_2 + 0S_1 + 0S_2 - A_1$

phase-1 :-

	C_J		0	0	0	0	-1	
B_V	C_B	S_V	x_1	x_2	S_1	S_2	A_1	MR
key row → S_1	0	1	2	1	1	0	0	$1/1 = 1$
A_1	-1	6	1	4	0	-1	1	$6/4 = 3/2$
	ZJ	-6	-1	-4	0	1	-1	
	ZJ-CJ		-1	-4	0	1	0	
				↑ key column				

Iteration-1 :-

	C_J		0	0	0	0	-1	
B_V	C_B	S_V	x_1	x_2	S_1	S_2	A_1	MR
x_2	0	1	2	1	1	0	0	
A_1	-1	2	-7	0	+4	-1	1	
	ZJ	-2	7	0	4	1	-1	
	ZJ-CJ		7	0	4	1	0	

Since all $Z_J - C_J \geq 0$, and optimum feasable solution to the Auxiliary LPP is obtain. But max $\ngtr 0$ and an Artificial variable A_1 is in the basis at a positive level, The original LPP doesn't posess any feasable solution...so, we can't proceed phase 2.

(9) Minimize $Z = 12x_1 + 20x_2$ subject to the constraints
$6x_1 + 8x_2 \geq 100$, $7x_1 + 12x_2 \geq 120$ and $x_1, x_2 \geq 0$.

Sol
Phase-I Convert all inequality constrains into equations

$$6x_1 + 8x_2 - S_1 + A_1 = 100 \quad \text{--- (1)}$$
$$7x_1 + 12x_2 - S_2 + A_2 = 120 \quad \text{--- (2)}$$

Then objective function can be written as to get max objective function we have to multiply with '-'.

$$\text{Min } z = 12x_1 + 20x_2$$
$$\text{Max } z = -12x_1 - 20x_2 + 0S_1 + 0S_2 - A_1 - A_2$$

Auxiliary objective function :-

$$\text{max } z = 0x_1 + 0x_2 + 0S_1 + 0S_2 - A_1 - A_2$$

Phase-I :-

Bv	CB	Sv	x_1 (CJ 0)	x_2 (CJ 0)	S_1 (CJ 0)	S_2 (CJ 0)	A_1 (CJ -1)	A_2 (CJ -1)	MR
A_1	-1	100	6	8	-1	0	1	0	100/8 = 12.5
A_2	-1	120	7	12	0	-1	0	1	120/12 = 10
ZJ		-220	-13	-20	1	1	-1	-1	
ZJ - CJ			-13	-20	1	1	0	0	

key row

↑ key column

8

Iteration-1

Here A_2 will replace so we will neglect it and as our will

Bv	CB	Sv	x_1 (CJ 0)	x_2 (CJ 0)	S_1 (CJ 0)	S_2 (CJ 0)	A_1 (CJ -1)	A_2 (CJ -1)	MR
A_1	-1	20	4/3	0	-1	2/3	1	-2/3	15
x_2	0	10	7/12	1	0	-1/12	0	1/12	17.1
ZJ		-20	-4/3	0	1	-2/3	-1	2/3	
ZJ-CJ			-4/3	0	1	-2/3	0	5/3	

↑ key column

Iteration-2

	C_J		0	0	0	0	-1	-1	
BV	CB	SV	x_1	x_2	S_1	S_2	A_1	A_2	MR
x_1	0	15	1	0	$-3/4$	$1/2$	$3/4$	$-1/2$	
x_2	0	$5/4$	0	1	$7/16$	$-3/8$	$-7/16$	$9/24$	
	ZJ	0	0	0	0	0	0	0	
	ZJ-CJ		0	0	0	0	1	1	

Since All $Z_J - C_J \geq 0$

$\therefore$ we obtained the phase-1 solution and an optimimum solution to the Auxiliary LPP has been obtained, Also maximum $Z_J = 0$ with no Artificial variable in the Basis.

$\therefore$ we can proceed phase-2.

Phase-2 :-

	CJ		-12	-20	0	0	
BV	CB	SV	x_1	x_2	S_1	S_2	MR
x_1	-12	15	1	0	$-3/4$	$1/2$	
x_2	-20	$5/4$	0	1	$7/16$	$-3/8$	
	ZJ	-205	-12	-20	$1/4$	$3/2$	
	ZJ-CJ		0	0	$1/4$	$3/2$	

Since All $Z_J - C_J \geq 0$

$\therefore x_1 = 15$ & $x_2 = 5/4$

from objective function

$$\text{Max } z = -12x_1 - 20x_2$$
$$= -12 \times 15 - 20 \times \frac{5}{4}$$
$$= -180 - 25$$
$$= -205$$

we want min z. so, we should multiply with '−' for Max z

$\therefore$ Min z = −(Max z) ⟹ −(−205)

Min z = 205 //

*) Solve by 2-phase method maximize $Z = x_1 + 5x_2 + 3x_3$ subject to the constrains $x_1 + 2x_2 + x_3 = 3$, $2x_1 - x_2 = 4$ and $x_1, x_2, x_3 \geq 0$

phase-1

Given

$$\text{Max } Z = x_1 + 5x_2 + 3x_3$$

Constrains ~~equation~~ are

$$x_1 + 2x_2 + x_3 = 3 \longrightarrow (1)$$

$$2x_1 - x_2 = 4 \longrightarrow (2)$$

convert constrains into equations

$$x_1 + 2x_2 + x_3 + A_1 = 3 \longrightarrow (1)$$

$$2x_1 - x_2 + A_2 = 4 \longrightarrow (2)$$

Auxiliary objective function :-

$$\text{Max } z = 0x_1 + 0x_2 + 0x_3 - A_1 - A_2$$

		CJ	0	0	0	-1	-1	
BV	CB	SV	x_1	x_2	x_3	A_1	A_2	MR
A_1	-1	3	1	2	1	1	0	$\frac{3}{1} = 3$
key row → A_2	-1	4	2	-1	0	0	1	$\frac{4}{2} = 2$
	ZJ	-7	-3	-1	-3	-1	-1	
	ZJ-CJ		-3	-1	-1	0	0	

↑ key column

Iteration-1

		CJ	0	0	0	-1	
BV	CB	SV	x_1	x_2	x_3	A_1	MR
key row → A_1	-1	1	0	5/2	1	1	$\frac{1}{5/2} = 2/5$
x_1	0	2	1	-1/2	0	0	$\frac{2}{-1/2} = -4$
ZJ		-1	0	-5/2	-1	-1	
ZJ-CJ			0	-5/2	-1	0	

↑ key column

Iteration-2:-

B_V	C_B	C_J S_V	0 x_1	0 x_2	0 x_3	MR
x_2	$-\cancel{1}$	$2/5$	0	1	$2/5$	
x_1	0	$11/5$	1	0	$1/5$	
	Z_J	0	0	0	0	
	$Z_J - C_J$		0	0	0	

$\therefore$ Since All $Z_J - C_J \geq 0$

$\therefore$ we obtained the phase-1 solution and an optimum Solution to the Auxiliary LPP has been obtained, Also maximum $Z_J = 0$ with no Artificial variable in the Basis.

$\therefore$ we can proceed phase-2,

phase-2

B_V	C_B	C_J S_V	1 x_1	5 x_2	3 x_3	MR
x_2	5	$2/5$	0	1	$2/5$	$11/5 - 3$
x_1	1	$11/5$	1	0	$1/5$	$\frac{11}{5} - 3$
	Z_J	$21/5$	1	5	$11/5$	$\frac{11-15}{5} = \frac{-4}{3}$
	$Z_J - C_J$		0	0	0	

Since all $Z_J - C_J \geq 0$

$\therefore x_1 = 11/5$ & $x_2 = 2/5$ & $x_3 = 0$

from objective function

$$Z = x_1 + 5x_2 + 3x_3$$

$$= \frac{11}{5} + 5\left(\frac{2}{5}\right) + 3(0)$$

$$= \frac{11}{5} + \frac{10}{5}$$

$$= \frac{21}{5}$$

$$\therefore \text{Max } z = \frac{21}{5}$$

Unit - 2 Transportation :-

1) Solve the following transportation problem

	D_1	D_2	D_3	D_4	Availability
S_1	6	8	8	5	30
S_2	5	11	9	7	40
S_3	8	9	7	13	50
Demand	35	25	35	25	120 / 120

Here Demand = Availability

North west corner rule :- NWCR

I

	D_1	D_2	D_3	D_4	Avali
S_1	$\boxed{30}$ 6	8	8	5	30
S_2	5	11	9	7	40
S_3	8	9	7	13	50
D	5 35	25	35	25	120 / 120

II

	D_1	D_2	D_3	D_4	A
S_2	$\boxed{5}$ 5	11	9	7	35 40
S_3	8	9	7	13	50
D	5	25	35	25	90 / 90

III

	D_2	D_3	D_4	A
S_2	$\boxed{25}$ 11	9	7	10 35
S_3	9	7	13	50
D	25	35	25	85 / 85

IV

	D_3	D_4	A
S_2	[10] 9	7	10
S_3	7	13	50
D	35	25	60 / 60

V

	D_3	D_4	A
S_3	[25] 7	13	50
D	25	25	50 / 50

VI

	D_4	A
S_3	[25] 13	25
D	25	25 / 25

The final allocation table is :-

	D_1	D_2	D_3	D_4	Av
S_1	[30] 6	8	8	5	30
S_2	[5] 5	[25] 11	[10] 9	7	40
S_3	8	9	[25] 7	[25] 13	50
D	35	25	35	25	120 / 120

∴ The Initial feasable basic solution (or) Initial transportation cost is

$\Rightarrow 30\times6 + 5\times5 + 25\times11 + 10\times9 + 25\times7 + 25\times13$

$\Rightarrow 180 + 25 + 275 + 90 + 175 + 325$

$\Rightarrow$ Rs 1070

LCM :- Least Cost Method :-

	D_1	D_2	D_3	D_4	Ava
S_1	6	8	8	5	30
S_2	[35] 5	11	9	7	5 40
S_3	8	9	7	13	50
D	35	25	35	25	120 / 120

	~~D1~~ D2	D3	D4	A
S1	8	8	$\boxed{25}$ 5	30 5
S2	11	9	7	5
S3	9	7	13	50
D	25	35	25	85 85

	D2	D3	A
S1	8	8	5
S2	11	9	5
S3	9	$\boxed{35}$ 7	50 15
D	25	35	60 60

	D2	A
~~S1~~	$\boxed{5}$ 8	5
S2	11	5
S3	9	15
D	25	25 25

	D2	A
S2	11	5
~~S3~~	$\boxed{15}$ 9	15
D	20	20 20

	D2	A
~~S2~~	$\boxed{5}$ 11	5
D	5	5 5

The final allocation table :-

	D1	D2	D3	D4	A
S1	6	$\boxed{5}$ 8	$\boxed{25}$ 8	5	30
S2	$\boxed{30}$ 5	$\boxed{5}$ 11	9	7	40
S3	8	$\boxed{15}$ 9	$\boxed{35}$ 7	13	50
D	35	25	35	25	120 120

* It is not in a loop structure
* So, it is a independent problem

$\Rightarrow m+n-1$

$\Rightarrow$ no. of rows + no. of columns -1

$\Rightarrow 3+4-1 \Rightarrow 6$

$\therefore$ The Initial transportation cost is

$\Rightarrow 5\times8 + 25\times5 + 35\times5 + 5\times11 + 15\times9 + 35\times7$

$\Rightarrow$ Rs 775 /-

VAM Method (or) Penality :-

I

	D_1	D_2	D_3	D_4	A	Penality
S_1	6	8	8	5	30	1
S_2	[35] 5	11	9	7	5 40	2
S_3	8	9	7	13	50	1
D	35	25	35	25	120 / 120	
Penality	1	1	1	2		

II

	D_2	D_3	D_4	A	P
S_1	8	8	[25] 5	5 30	3
S_2	11	9	7	5	2
S_3	9	7	13	50	2
D	25	35	25	85 / 85	
P	1	1	2		

III

	D_2	D_3	A	P
S_1	8	8	5	0
S_2	11	9	5	2
S_3	9	[35] 7	15 50	2
D	25	35	60 / 60	
P	1	1		

IV

	D_2	A	P
S_1	[5] 8	5	
S_2	11	5	
S_3	9	15	
D	25 20	25 25	
P			

V

	D_2	A
S_2	11	5
S_3	[15] 9	15
D	5 20	20 20
P	1	

VI

	D_2	A
S_2	[5] 11	5
D	5	5
P		

Final Allocation

	D_1	D_2	D_3	D_4	A
S_1	6	[5] 8	8	[25] 5	30
S_2	[35] 5	[5] 11	9	7	40
S_3	8	[15] 9	[35] 7	13	50
D	35	25	35	25	120 / 120

$\therefore$ The ITC $= 8\times5 + 5\times25 + 5\times35 + 11\times5 + 9\times15 + 7\times35$

$=$ Rs $775/-$

* Find Initial Basic feasable solution :-

	D_1	D_2	D_3	D_4	D_5	A
S_1	50	80	60	60	30	800
S_2	40	70	70	60	50	600
S_3	80	40	60	60	40	1100
D	400	400	500	400	800	2500 \ 2500

Here Demand = Availability.

NWCR

I

	D_1	D_2	D_3	D_4	D_5	A
S_1	$\boxed{400}$ 50	80	60	60	30	800 → 400
S_2	40	70	70	60	50	600
S_3	80	40	60	60	40	1100
D	400	400	500	400	800	2500 \ 2500

II

	D_2	D_3	D_4	D_5	A
S_1	$\boxed{400}$ ~~80~~	~~60~~	~~60~~	~~30~~	~~400~~
S_2	70	70	60	50	600
S_3	40	60	60	40	1100
D	400	500	400	800	2100 \ 2100

III

	D_3	D_4	D_5	A
S_2	$\boxed{500}$ 70	60	50	600 → 100
S_3	60	60	40	1100
D	500	400	800	1700 \ 1700

IV

	D_4	D_5	A
S_2	$\boxed{100}$ ~~60~~	~~50~~	~~100~~
S_3	60	40	1100
D	400 → 300	800	1200 \ 1200

V

	D_4	D_5	A
S_3	$\boxed{300}$ 60	40	1100 → 800
D	300	800	1100 \ 100

VI

	D_5	A
S_3	$\boxed{800}$ ~~400~~	~~800~~
D	800	800 \ 800

~~Initial~~ The final allocation table is :-

	D_1	D_2	D_3	D_4	D_5	A
S_1	$\boxed{400}$ 50	$\boxed{400}$ 80	60	60	30	800
S_2	40	70	$\boxed{500}$ 70	$\boxed{100}$ 60	50	600
S_3	80	40	60	$\boxed{300}$ 60	$\boxed{800}$ 40	1100
D	400	400	500	400	800	2500 / 2500

The Initial transport cost =

$\Rightarrow 50\times400 + 80\times400 + 70\times500 + 60\times100 + 60\times300 + 40\times800$

$\Rightarrow 20000 + 32000 + 35000 + 6000 + 18000 + 32000$

$\Rightarrow 143000$

Least cost Method :-

	D_1	D_2	D_3	D_4	D_5	A
S_1	50	80	60	60	30	800
S_2	40	70	70	60	50	600
S_3	80	40	60	60	$\boxed{800}$ 40	300 1100
D	400	400	500	400	800	2500 / 2500

	D_1	D_2	D_3	D_4	A
S_1	50	80	60	60	800
S_2	$\boxed{400}$ 40	70	70	60	200 600
S_3	80	40	60	60	300
D	400	400	500	400	1700 / 1700

	D_2	D_3	D_4	A
S_1	80	60	60	800
S_2	70	70	60	200
S_3	$\boxed{300}$ 40	60	60	300
D	100 400	500	400	1300 / 1300

	D_2	D_3	D_4	A
S_1	80	$\boxed{500}$ 60	60	300 800
S_2	70	70	60	200
D	100	500	400	1000 / 1000

	D2	D4	A
S1	80	$\boxed{300}$ 60	300
S2	70	60	200
D	100	$\boxed{100}$ 400	500 \ 500

	D2	D4	A
S2	70	$\boxed{100}$ 60	200
D	100	100	200 \ 200

	D2	A
S2	$\boxed{100}$ 70	100
D	100	100 \ 100

Final Allocation table :-

	D1	D2	D3	D4	D5	A
S1	50	80	$\boxed{500}$ 60	$\boxed{300}$ 60	30	800
S2	$\boxed{400}$ 40	$\boxed{100}$ 70	70	$\boxed{100}$ 60	50	600
S3	80	$\boxed{300}$ 40	60	60	$\boxed{800}$ 40	1100
D	400	400	500	400	800	2500 \ 2500

6	$\boxed{5}$ 8	8	$\boxed{25}$ 5	30
$\boxed{35}$ 5	$\boxed{5}$ 11	9	7	40
8	$\boxed{15}$ 9	$\boxed{35}$ 7	13	50
35	25	35	25	120 / 120

conditions for stage-2

1) $m+n-1 =$ No. of allocations

$3+4-1 = 6$ ✓

2) Allocations are in independent positions, $\therefore$ It has non degeneracy.

Stage-2 :-

In stage 2 two methods are there

1. Modi [Modified distribution input method] ⇒ (UV method)
2) Stepping stone Method

Optimality test (UV method)

1) Cost matrix for allocated cell :-

	D_1	D_2	D_3	D_4	
S_1		8		5	$U_1=0$
S_2	5	11			$U_2=3$
S_3		9	7		$U_3=1$
	$V_1=2$	$V_2=8$	$V_3=6$	$V_4=5$	

2) Cell Evaluation for unoccupied cells (U_i+V_j)

	D_1	D_2	D_3	D_4	
S_1	2		6		$U_1=0$
S_2			9	8	$U_2=3$
S_3	3			6	$U_3=1$
	$V_1=2$	$V_2=8$	$V_3=6$	$V_4=5$	

3) All $C_{ij}-(U_i+V_j) \geq 0$

	D_1	D_2	D_3	D_4
S_1	4		2	
S_2			0	-1
S_3	5			7

we have to improve our allocation table

Here the S_2 & D_4 cell is negative so, we want to Reallocate the table.

Reallocation Table

	D_1	D_2	D_3	D_4	A
S_1	6	[10] 8 $+\theta$	8	[20] 5 $-\theta$	30
S_2	[35] 5	$-\theta$ 11	9	$+\theta$ [5] 7	40
S_3	8	[15] 9	[35] 7	13	50
D	35	25	35	25	120 / 120

$\theta=5$

least allocation cost is equal to 'θ'

$\therefore \boxed{\theta=5}$

conditions

1) $m+n-1 = 3+4-1 = 6$ ✓

2) Allocations are in independent positions, Therefore has non degeneracy

Optimality Test

1) Cost matrix for allocated cells

	D1	D2	D3	D4	
S1		8		5	$u_1=0$
S2	5			7	$u_2=2$
S3		9	7		$u_3=1$
	$v_1=3$	$v_2=8$	$v_3=6$	$v_4=5$	

2) cell evaluation for un occupied cells (u_i+v_j)

	D1	D2	D3	D4	
S1	3		6		$u_1=0$
S2		10	8		$u_2=2$
S3	4			6	$u_3=1$
	$v_1=3$	$v_2=8$	$v_3=6$	$v_4=5$	

3) All $c_i-(u_i+v_j) \geq 0$

	D1	D2	D3	D4
S1	3		2	
S2		1	1	
S3	4			7

$\therefore$ All $c_i-(u_i+v_j) \geq 0$ is satisfied.

The final allocation table is

	D1	D2	D3	D4	A
S1	6	[10] 8	8	[20] 5	30
S2	[35] 5	11	9	[5] 7	40
S3	8	[15] 9	[35] 7	13	50
D	35	25	35	25	120 / 120

$TC = 8\times10 \times 5\times20 + 5\times35 + 7\times5 + 9\times15 + 7\times35$

$TC = 770/-$

* Find the optimum solution to the following transportation problem

①

I

Factory	D	E	F	G	Capacity	Penality
A	1	2	1	4	30	0
B	3	3	2	[10] 1	40 (50)	1
C	4	2	5	9	20	2
Demand	20	40	30	10	100 / 100	
P	2	0	1	3		

II

	D	E	F	Capa	P
A	[20] 1	2	1	10 (30)	0
B	3	3	2	40	1
C	4	2	5	20	2
Dem	20	40	30	90 / 90	
P	2	0	1		

III

	E	F	Capa	P
A	2	1	10	1
B	3	2	40	1
C	[20] 2	5	20	3
Demand	20 (40)	30	70 / 70	
P	0	1		

IV

	E	F	Capa	P
A	2	[10] 1	10	1
B	3	2	40	1
Demand	20	20 (30)	50 / 50	
P	1	1		

V

	E	F	capa	P
B	3	[20] 2	20 (40)	1
Dem	20	20	40 / 40	
P				

VI

	E	cap
B	[20] 3	20
Dema	20	20 / 20

Final allocation table :-

Factory	D	E	F	G	Capacity
A	[20] 1	2	[10] 1	4	30
B	3	[20] 3	[20] 2	[10] 1	50
C	4	[20] 2	5	9	20
Demand	20	40	30	10	100 / 100

Initial Transportation cost $= 1\times20 + 1\times10 + 3\times20 + 2\times20 + 1\times10 + 2\times20$

$= \text{Rs } 180/-$

optimality test :-

1) $m+n-1 =$ No. of allocations

$3+4-1 = 6$ ✓

2) Non-degeneracy and it is independent.

1) Cost Matrix for allocated cells :-

	D	E	F	G	
A	1		1		$U_1 = 0$
B		3	2	1	$U_2 = 1$
C		2			$U_3 = 0$
	$V_1 = 1$	$V_2 = 2$	$V_3 = 1$	$V_4 = 0$	

2) Cell evaluation for unoccupied cells :- $(U_i + V_j)$

	D	E	F	G	
A		2		0	$U_1 = 0$
B	2				$U_2 = 1$
C	1		1	0	$U_3 = 0$
	$V_1 = 1$	$V_2 = 2$	$V_3 = 1$	$V_4 = 0$	

3) All $C_i - (U_i + V_j) \geq 0$

	D	E	F	G
A		0		4
B	1			
C	3		4	9

$\therefore$ since all $C_{ij} - (U_i + V_j) \geq 0$

$\therefore$ The Initial transportation cost is optimal and also an alternate optimal solution is

* Solve the following transportation problem

	X	Y	Z	A
A	8	7	8	60
B	8	8	9	70
C	11	8	5	80
D	50	80	80	210

VAM :-

	x	y	z	A	P
A	8	7	3	60	4
B	50 3	8	9	70 20	5
C	11	3	5	80	2
D	50	80	80	~~210~~ 210	
P	5	4	2		

	y	z	A	P
A	7	3	60	4
B	8	9	20	1
C	80 3	5	80	2
D	~~80~~	80	~~160~~ 60	
P	4	2		

1

	z	A
A	60 3	60
B	9	20
D	20 80	~~80~~ 80
P	6	

	z	A
B	20 9	20
D	20	~~20~~ 20

Final allocation table :-

	x	y	z	A
A	8	7	60 3	60
B	50 3	8	20 9	70
C	11	80 3	5	80
D	50	80	80	210 210

∴ The Initial Transportation cost $= 3\times60+3\times50+9\times20+3\times80$

$= 180+150+180+240$

$= 750$

Optimality test :-

$m+n-1 \neq$ no. of allocations

1) $3+3-1=5 \neq$ no. of allocations

2) It is degenarace to convert Non-degenarace we have to use 'ε'-Epslone it indicates almost zero but not equal to zero it should be allocated on least cost of non allocated cells.

	x	y	z	A
A	8	7	60 3	60
B	50 3	8	20 9	70
C	11	80 3	ε 5	80
D	50	80	80	210 210

conditions

1) $m+n-1 \Rightarrow 3+3-1=5$ ✓

2) It is independent and non-degenarace.

1) Cost matrix for allocated cells :-

	x	y	z	
A			3	$u_1=0$
B	3		9	$u_2=6$
C		3	5	$u_3=2$
	$v_1=-3$	$v_2=1$	$v_3=3$	

2) Cell evaluation for un occupied cells :- (u_i+v_j)

	x	y	z	
A	-3	1		$u_1=0$
B		7		$u_2=6$
C	-1			$u_3=2$
	$v_1=-3$	$v_2=1$	$v_3=3$	

3) All $c_i-(u_i+v_j) \geq 0$

	x	y	z
A	11	6	
B		1	
C	12		

$\therefore$ Since all $c_i-(u_i+v_j) \geq 0$

$\therefore$ The Initial transportation cost itself optimal soluti

*) Solve the following transportation problem:-

	M_1	M_2	M_3	M_4	a_i
w_1	8	10	7	6	50
w_2	12	9	4	7	40
w_3	9	11	10	8	30
b_j	25	32	40	23	120 / 120

Sol:-

	M_1	M_2	M_3	M_4	a_i	P
w_1	8	10	7	6	50	1
w_2	12	9	40 4	7	40	3
w_3	9	11	10	8	30	1
b_j	25	32	40	23	120 / 120	
P	1	1	3	1		

	M_1	M_2	M_4	a_i	P
w_1	8	10	[23] 6	[27] 50	2
w_3	9	11	8	30	1
b_j	25	32	23	80 / 80	
P	1	1	2		

	M_1	M_2	a_i	P
w_1	[25] 8	10	2 27	2
w_3	9	11	30	2
b_j	25	32	57 / 57	
P	1	1		

	M_2	a_i
w_1	[2] 10	2
w_3	11	30
b_j	32 / 30	32 / 32

	M_2	a_i
w_3	[30] 11	30
b_j	30	30 / 30

The Initial transportation cost $= 25\times8 + 2\times10 + 23\times6 + 40\times4 + 30\times11$
$=$ Rs 848/-

Condition:-

i) $m+n-1 =$ No. of allocations
$3+4-1 = 6$ ✗

By using 'ε' the allocation table is

	M_1	M_2	M_3	M_4	a_i
w_1	[25] 8	[2] 10	7	[23] 6	50
w_2	12	9	[40] 4	[ε] 7	40
w_3	9	[30] 11	10	8	30
b_j	25	32	40	23	120 / 120

i) $m+n-1 = 6$
ii) It is Non-degeneracy & independent

Optimality test:-

1) cost Matrix for allocated cells:-

	M_1	M_2	M_3	M_4	
w_1	8	10		6	$u_1 = 0$
w_2			4	7	$u_2 = 1$
w_3		11			$u_3 = 1$
	$v_1 = 8$	$v_2 = 10$	$v_3 = 3$	$v_4 = 6$	

ii)

ii) Cell evaluation for unoccupied cells :- $u_i + v_j$

		3		$u_1 = 0$
9	11			$u_2 = 1$
9		4	7	$u_3 = 1$
$v_1 = 8$	$v_2 = 10$	$v_3 = 3$	$v_4 = 6$	

iii) All $c_{ij} - (u_i + v_j)$:-

		3	
3	-2		
0		6	1

There are 3 factories A, B and C. ~~with~~ To supply goods for 4 dealers D_1, D_2, D_3 and D_4. The production capacity of these factories are 1000, 700 and 900 units per month respectively. The requirement from the dealers are 900, 800, 500 and 400 units per month respectively. for unit return (excluding transportation) are 8, 7, 9 at three factories. The following table given unit transportation cost from factories to the dealers. Determine the optimal solution to maximize the total returns.

Sol:

	D_1	D_2	D_3	D_4
A	2	2	2	4
B	3	5	3	2
C	4	3	2	1

Sol:- Profit = Return − Transportation cost

we can form a transportation table with profit as follows

By using VAM:-

The profit matrix is   8, 7, 9 ⇒ given profit

$\boxed{8-2=6}$

	D_1	D_2	D_3	D_4	A	P
A	8−2 6	6	6	4	1000	2
B	7−3 4	700 2	4	5	700	2
C	9−4 5	6	7	8	900	1
R	900	100 800	500	400	2600 / 2600	
P	1	4	2	1		

	D_1	D_2	D_3	D_4	A	P
A	6	6	6	400 4	600 1000	2
C	5	6	7	8	900	1
R	900	100	500	400	1900 / 1900	
	1	0	1	4		

	D_1	D_2	D_3	A	P
A	6	6	6	600	0
C	900 5	6	7	900	1
R	900	100	500	1500 / 1500	
P	1	0	1		

	D_2	D_3	A
A	6	500 6	100 600
R	100	500	600 / 600

	D_2	A
A	100 6	100
R	100	100 / 100

The Initial Transportation Cost =

$2\times700 + 4\times400 + 5\times900 + 6\times500 + 6\times100$

$\Rightarrow 1400 + 1600 + 4500 + 3000 + 600$

$\Rightarrow 11,100.$

Optimality Test :-

1) $m+n-1$ = no. of allocations

$\Rightarrow 3+4-1 = 6$ ×

By using "ε"

	D_1	D_2	D_3	D_4	A
A	6	6 [100]	6 [500]	4 [400]	1000
B	4 [ε]	2 [700]	4	5	700
C	5 [900]	6	7	8	900
R	900	800	500	400	2600 \ 2600

1) Cost Matrix for allocated cells :-

	D_1	D_2	D_3	D_4	
A		6	6	4	$U_1 = 0$
B	4	2			$U_2 =$
C	5				$U_3 =$
	$V_1 =$	$V_2 = 6$	$V_3 = 6$	$V_4 = 4$	

* Solve the following transportation problem to maximize the total profit. (The entry denote profit from sale of a unit product).

	M_1	M_2	M_3	M_4	supply
S_1	15	51	42	33	23
S_2	80	42	26	81	44
S_3	90	40	66	60	33
Demand	23	31	16	30	100 / 100

Sol:- Convert this problem into minimization problem by subtracting each element from highest value. Now table converted as follows:

VAM :-

	M_1	M_2	M_3	M_4	supply	P
S_1	75	39	48	57	23	9
S_2	10	48	64	9	44	1
S_3	23 0	50	24	30	33 10	24
Demand	23	31	16	30	100 / 100	
P	10	9	24	21		

	M_2	M_3	M_4	supply	P
S_1	39	48	57	23	9
S_2	48	64	30 9	44 14	39
S_3	50	24	30	10	6
Dem	31	16	30	77 / 77	
P	9	24	21		

	M_1	M_2	M_4	supply	P
S_1	75	39	57	23	18
S_2	10	48	9	44	1
S_3	0	50	30	9	30
D	23	31	30		
P	10	9	21		

22
31
30
83

	M_2	M_3	supply	P
S_1	39	48	23	9
S_2	48	64	14	16
~~S_3~~	~~50~~	10 ~~24~~	~~10~~	26
Dem	31	16	47 / 47	
P	9	24		

64
48
16

M_1 M_2 supply
S_1
S_2
S_3

	M_2	M_3	Supply	P
S_1	39	48	23	9
~~S_2~~	14 ~~48~~	~~64~~	~~14~~	16
Dem	31 17	6	37 / 37	
P	9	16		

64
48

23
17
6

	M_2	M_3	supply
S_1	17 39	48	23 6
Demand	17	6	23 / 23

	M_3	supply
~~S_1~~	6 ~~48~~	6
Dem	6	6 / 0

The Initial Transportation cost $= 22\times0 + 9\times30 + 24\times10 + 48\times14 + 39\times17 + 48\times6$

$= \cancel{240+240+672+663+288}$

$= 2133$

	M_1	M_2	M_3	M_4	Supply
S_1	75	(17) 39	(6) 48	57	23
S_2	10	(14) 48	64	(30) 9	44
S_3	(22) 0	50	(10) 24	30	33
Dema	23	31	16	30	100 / 100

optimality test :-

<u>Conditions</u> :-

1) $m+n-1 =$ no. of allocations
   $3+4-1 = 6$ ✓
2) It is non-degenarace and independent

1) Cost matrix for allocated cells :-

	M_1	M_2	M_3	M_4	
S_1		39	48		$u_1 = 0$
S_2		48		9	$u_2 = 9$
S_3	0		24		$u_3 = -24$
	$v_1 = 24$	$v_2 = 39$	$v_3 = 48$	$v_4 = 0$	

ii) Cell evaluation for unoccupied cells :- $(u_i + v_j)$

	M_1	M_2	M_3	M_4	
S_1	24			0	$u_1 = 0$
S_2	33		57		$u_2 = 9$
S_3		15		-24	$u_3 = -24$
	$v_1 = 24$	$v_2 = 39$	$v_3 = 48$	$v_4 = 0$	

$$\begin{array}{r} 75 \\ -24 \\ \hline 51 \end{array}$$

iii) All $c_{ij} - (u_i + v_j)$

51			57
-23		7	
	13		54

* Unbalanced transportation cost :-

(2) Solve the following transportation problem

Factory	Ware House				Capacity
	W_1	W_2	W_3	W_4	
F_1	11	20	7	8	50
F_2	21	16	10	12	40
F_3	8	12	18	9	70
Demand	30	25	35	40	160 / 130

Sol :-

* Since Total Demand is equal to 130 it is less than the total supply i.e, 160. Therefore the given problem is an unbalanced Transportation problem.
* we convert this into a balanced Transportation problem by introducing dummy warehouse (W_5) with cost '0', and assigning demand equal to 30 unit.
* There for the modified table as follows.

VAM

Factory	Ware House					Capacity	P
	W_1	W_2	W_3	W_4	W_5		
F_1	11	20	7	8	0	50	7
F_2	21	16	10	12	30 0	10 40	10
F_3	8	12	18	9	0	70	8
Demand	30	25	35	40	30	160 / 160	
P	3	4	3	1	0		

	W_1	W_2	W_3	W_4	Supply	P
F_1	11	20	7	8	50	1
F_2	21	16	10	12	10	2
F_3	8	25 12	18	9	45 70	1
D	30	25	35	40	130 / 130	
P	3	4	3	1		

	W_1	W_3	W_4	Supply	P
F_1	11	35 7	8	50 15	1
F_2	21	10	12	10	2
F_3	8	18	9	45	1
D	30	35	40	105 / 105	
P	3	3	1		

	W_1	W_4	Supply	P
F_1	11	8	15	3
~~F_2~~	~~21~~	10 ~~12~~	10	9
F_3	8	9	45	1
D	30	40 30	70 / 70	
P	3	1		

	W_1	W_4	Supply	P
F_1	11	8	15	3
F_3	20 8	9	45 15	1
D	30	30	60 / 60	
P	3	1		

	W4	supply
F1	~~$\boxed{15}$ 8~~	~~15~~
F3	9	15
D	$\boxed{15}$ 30	~~30~~ 30

	W4	supply
F3	$\boxed{15}$ 9	15
D	15	~~15~~ 15

ITC = 7×35 + 8×15 + 12×10 + 0×30 + 8×30 + 12×25 + 9×15

= 1160/-

optimality test :

1) m+n-1 = 3+5-1 = 7 ✓

2) Non-degeneracy ✓

1) Cost matrix for allocated cells :-

	w_1	w_2	w_3	w_4	w_5	
F_1			7	8		$U_1 = 0$
F_2				12	0	$U_2 = 4$
F_3	8	12		9		$U_3 = 1$
	$V_1 = 7$	$V_2 = 11$	$V_3 = 7$	$V_4 = 8$	$V_5 = -4$	

2) Cell evaluation for unoccupied cells :- $(U_i + V_j)$

	w_1	w_2	w_3	w_4	w_5	
F_1	7	11			-4	$U_1 = 0$
F_2	11	15	11			$U_2 = 4$
F_3			8		-3	$U_3 = 1$
	$V_1 = 7$	$V_2 = 11$	$V_3 = 7$	$V_4 = 8$	$V_5 = -4$	

3) $C_{ij} - (u_i + v_j)$

4	9			4
10	1	-1		
		10		3

Since All $C_{ij} - (u_i + v_j) \ngeq 0$

∴ we want to re allocate the table.

Reallocation table :-

	w_1	w_2	w_3	w_4	w_5	Supply
F_1	11	20	$\boxed{35}$ $-\theta$ 7	$\boxed{25}$ $+\theta$ 8	0	50
F_2	21	16	$\boxed{10}$ $+\theta$ 10	$-\theta$ 12	$\boxed{30}$ 0	40
F_3	$\boxed{30}$ 8	$\boxed{15}$ 12	18	$\boxed{15}$ 9	0	70
D	30	25	35	40	30	160 / 160

optimality test :-

1) m+n-1 = no. of allocations ⇒ 3+5-1 = 7

2) Non degeneracy.

1) cost matrix for allocated cells :-

	w_1	w_2	w_3	w_4	w_5	
F_1			7	8		$U_1 = 0$
F_2			10		0	$U_2 = 3$
F_3	8	12		9		$U_3 = 1$
	$V_1 = 7$	$V_2 = 11$	$V_3 = 7$	$V_4 = 8$	$V_5 = -3$	

2) Cell evaluation for unoccupied cells :-

	W_1	W_2	W_3	W_4	W_5	
F_1	7	11			-3	$u_1=0$
F_2	10	14		11	·	$u_2=3$
F_3			8		-2	$u_3=1$
	$v_1=7$	$v_2=11$	$v_3=7$	$v_4=8$	$v_5=-3$	

3) All $c_i-(u_i+v_j)$

4	9			3
10	2		1	
		10		2

∴ since All $c_i-(u_i+v_j)\geq 0$

~~∴ The~~

The final allocation table is

	W_1	W_2	W_3	W_4	W_5	supply
F_1	11	20	$\boxed{25}$ 7	$\boxed{25}$ 8	0	50
F_2	21	16	$\boxed{10}$ 10	12	$\boxed{30}$ 0	40
F_3	$\boxed{30}$ 8	$\boxed{15}$ 12	18	$\boxed{15}$ 9	0	70
D	30	25	35	40	30	160 / 160

$TC = 7\times25 + 8\times25 + 10\times10 + 0\times30 + 8\times30 + 12\times15 + 9\times15$

$= 1030$

Topic **Assignment :-**

=> 2) Flowcharts

Assignment is also known as Hungarian Algorithm
In order to get the best solution

Problem :-

1) There are 6 operators to be allocated to each machine the time taken by an operator on a specific machine to do a job is given in the following Table. Find out an allocation of operation so has to minimize operation time.

Machines

Operator	M_1	M_2	M_3	M_4	M_5	M_6
A	10	12	8	10	8	12
B	9	10	8	7	8	9
C	8	7	8	8	8	6
D	12	13	14	14	15	14
E	9	9	9	8	8	10
F	7	8	9	9	9	8

Sol:- Step-1

Row minimisation :- least value of row = 8
10-8, 12-8 like that

	M_1	M_2	M_3	M_4	M_5	M_6
A	2	4	0	2	0	4
B	2	3	1	0	1	2
C	2	1	2	2	2	0
D	0	1	2	2	3	2
E	1	1	1	0	0	2
F	0	1	2	2	2	1

All rows having zero's.
But one column is not having for that we want to do C.M

Column minimisation

	M_1	M_2	M_3	M_4	M_5
A	2	3	$\boxed{0}$	2	4
B	2	2	1	$\boxed{0}$	2
C	2	0	2	2	$\boxed{0}$
D	$\boxed{0}$	0	2	2	2
E	1	$\cancel{0}$	1	$\boxed{0}$	2
F	$\cancel{0}$	$\boxed{0}$	2	2	1

Now Assingment

14
9
23

Assignment

	M_1	M_2	M_3	M_4	M_5	M_6
A	2	3	$\boxed{0}$	2	$\cancel{0}$	4
B	2	2	1	$\boxed{0}$	1	2
C	2	$\cancel{0}$	2	2	2	$\boxed{0}$
D	$\boxed{0}$	$\cancel{0}$	2	2	3	2
E	1	$\cancel{0}$	1	$\cancel{0}$	$\boxed{0}$	2
F	$\cancel{0}$	$\boxed{0}$	2	2	2	1

Result :-

operator		Machine		Time
A	$\longrightarrow$	M_3	$\longrightarrow$	8
B	$\longrightarrow$	M_4	$\longrightarrow$	7
C	$\longrightarrow$	M_6	$\longrightarrow$	6
D	$\longrightarrow$	M_1	$\longrightarrow$	12
E	$\longrightarrow$	M_5	$\longrightarrow$	8
F	$\longrightarrow$	M_2	$\longrightarrow$	8

49 hrs //

2) A company has 5 jobs to be done on 5 Machines any job can be done on any machine the cost of doing the jobs in different machines are given below. Assign the job so has to minimise the total cost.

Machines

Jobs	A	B	C	D	E
1	13	8 (8.8)	16 (16.8)	18 (18.8)	19 (19.8)
2	9 (9.9)	15 (15.8)	24	9	12
3	12	9	4	4	4
4	6	12	10	8	13
5	15	17	18	12	20

13-8

Row minimization :-

	A	B	C	D	E
1	5	0	8	10	11
2	0	6	15	0	3
3	8	5	0	0	0
4	0	6	4	2	7
5	3	5	6	0	8

Assignment :-

	A	B	C	D	E
1	5	$\boxed{0}$	8	10	11
2	~~0~~	6	15	~~0~~	3
3	8	5	$\boxed{0}$	0	~~0~~
4	$\boxed{0}$	6	4	2	7
5	3	5	6	$\boxed{0}$	8

Least open element - all open elements
&
Least open element + intersecting elements

Assignment

	A	B	C	D	E
1	8	$\boxed{0}$	8	13	11
2	~~0~~	3	12	~~0~~	$\boxed{0}$
3	11	5	$\boxed{0}$	3	~~0~~
4	$\boxed{0}$	3	1	2	4
5	3	2	3	$\boxed{0}$	5

Job	Machine	Time
1	→ B	8
2	→ E	12
3	→ C	4
4	→ A	6
5	→ D	12
		42

3) Different jobs on 4 different machines the following matrix shows the cost in rupee, for producing the Jobs on 1, 5 how should Jobs assign to various machines so that the total cost is minimize

	J_1	J_2	J_3	J_4
M_1	5	7	11	6
M_2	8	5	9	6
M_3	4	7	10	7
M_4	10	4	8	3

Row minimization

	J_1	J_2	J_3	J_4
M_1	0	2	6	1
M_2	3	0	4	1
M_3	0	3	6	3
M_4	7	1	5	0

Column minimization

	J_1	J_2	J_3	J_4
M_1	0	2	2	1
M_2	3	0	0	1
M_3	0	3	2	3
M_4	7	1	1	0

Assignment

	J_1	J_2	J_3	J_4	
M_1	$\boxed{0}$	2	2	1	✓
M_2	3	$\boxed{0}$	~~0~~	1	
M_3	~~0~~	3	2	3	✓
M_4	7	1	1	$\boxed{0}$	
	✓				

	J_1	J_2	J_3	J_4	
M_1	~~0~~	1	1	~~0~~	✓
M_2	4	$\boxed{0}$	~~0~~	1	
M_3	$\boxed{0}$	2	1	2	✓
M_4	8	1	1	$\boxed{0}$	✓
	✓			✓	

	J_1	J_2	J_3	J_4
M_1	~~0~~	~~0~~	~~0~~	$\boxed{0}$
M_2	5	$\boxed{0}$	~~0~~	2
M_3	$\boxed{0}$	1	~~0~~	2
M_4	8	~~0~~	$\boxed{0}$	~~0~~

Job	Machines	time
M_1 →	J_4 →	6
M_2 →	J_2 →	5
M_3 →	J_1 →	4
M_4 →	J_3 →	8
		23 hrs

12/11/23

4) There are 4 jobs to be assigned to the machine only 1 job could be assigned to 1 machine. The amount of time in hours required for the jobs in the matrix are given in the following matrix. Find an optimum assignment of jobs to minimize the total processing time. Find which machine

no job is assigned what is the total processing time to complete all the jobs.

Jobs	A	B	C	D	E
1	4	5	6	2	7
2	10	12	11	14	16
3	4	3	2	1	5
4	8	7	6	9	5

Sol:- Row minimization:-

	A	B	C	D	E
1	2	1	4	0	-5
2	0	2	1	4	6
3	3	2	1	0	4
4	3	2	1	4	0

Column minimization.

	A	B	C	D	E
1	2	0	3	0	-5
2	0	1	0	4	6
3	3	1	0	0	4
4	3	1	0	4	0

Assignment

	A	B	C	D	E
1	2	$\boxed{0}$	3	~~0~~	-5
2	$\boxed{0}$	1	~~0~~	4	6
3	3	1	$\boxed{0}$	~~0~~	4
4	3	1	~~0~~	4	$\boxed{0}$

Job	Machines	time
1	B	3
2	A	10
3	C	2
4	E	5

Sol:- Here we have to add dummy job

~~Row minimization:-~~

	A	B	C	D	E
1	2	1	4	0	-5
2	10	12	11	14	16
3	0	2	1	4	5
4	8	7	6	9	5
5	0	0	0	0	0

Row minimization:-

	A	B	C	D	E
1	2	1	4	0	-5
2	0	2	1	4	6
3	3	2	1	0	4
4	3	2	1	4	0
5	0	0	0	0	0

Assignment :-

	A	B	C	D	E
1	2	1	4	$\boxed{0}$	-5
2	$\boxed{0}$	2	1	4	6
3	3	2	1	~~0~~	4
4	3	2	1	4	$\boxed{0}$
5	~~0~~	$\boxed{0}$	~~0~~	~~0~~	~~0~~

Assignment :-

	A	B	C	D	E
1	1	~~0~~	3	$\boxed{0}$	-4
2	$\boxed{0}$	2	1	5	6
3	2	1	$\boxed{0}$	~~0~~	3
4	3	2	1	5	$\boxed{0}$
5	~~0~~	~~0~~	~~0~~	0	~~0~~

Job	Machine	time
1 -	B -	2
2 -	A -	10
3 -	C -	2
4 -	E -	5
		19 hrs

UNIT - 3

Sequencing (or) Job Sequencing.

Sequence :-

* The problem is to find sequence of job on 1st machine till the completion of the last job on the last machine this is refered as the sequencing problem.

* In the case 3 jobs and 3 machines the total no. of possible sequences will be $(3!)^3 = 216$ ways. Hence it requires lots of computational time and calculation time. In order to overcome such types job sequence technique had been introduced by S.N. Johnson and R. Bellman.

Definition :-

Processing time :

The time required by each job on each machine is known as processing time.

Ellapsed time:

The time will be the start of first job on first machine and the completion of last Job on last machine is called as Ellapsed time (or) Total Ellapsed time.

Ideal time :-

The time for which a machine remains ideal during total Ellapsed time is called an ideal time of a machine.

Assumptions :-

* No machine can process more than one operation at a time.
* Each operation ones started is to be performed upto its completion.
* Processing time of each job on each machine is known and it doesnot change.
* The time required to transfer of jobs between machines is negligible.
* Ones job has begin on a machine it must be completed before another job can begin on the same machine.

find the sequence of job that minimizes the total Elapsed time (hours) required to complete the following jobs on two machines in the order $M_1 - M_2$. Also find ideal time of the machines.

Job	A	B	C	D	E
M_1	5	1	9	3	10
M_2	2	6	7	8	4

Sol: The smallest processing time is '1' hour for Job 'B' on machine-1. So, we scheduled the job B first as shown the below.

M_1	B					M_2

By Eliminating the job B the reduced set of processing time is as follows.

Job	A	C	D	E
M_1	5	9	3	10
M_2	2	7	8	4

The smallest processing time is 2 hrs for 'Job A' on Machine-2.

M_1	B				A	M_2

By Eliminating the job A the reduced set of processing time is,

Job	C	D	E
M_1	9	3	10
M_2	7	8	4

The smallest processing time is 3 hrs for job D on Machine-1

M_1	B	D			A	M_2

By Eliminating Job D the reduced set of processing time is,

Job	C	E
M_1	9	10
M_2	7	4

The smallest processing time is 4 hrs for 'Job E' on machine-2

M_1 | B | D | | E | A | M_2

Job	C
M_1	9
M_2	7

M_1 | B | D | C | E | A | M_2

Job sequence	Machine-1		Machine-2	
	Time In	Time out	Time In	Time out
B	0	1	1	7 (1+6)
D	1	4 (3+1)	7	15 (7+8)
C	4	13 (4+9)	15	22 (15+7)
E	13	23 (13+10)	23	27 (23+4)
A	23	28 (23+5)	28	30 (28+2)

Result :-

Total Ellapsed time = 30 hrs

Ideal time of M/C 1 = 30 - 28 = 2 hrs

Ideal time of M/C 2 = 1+1+1 = 3 hrs

2) Determine the sequence which minimizes the total time for processing 5 Jobs on 3 machines namely Machine A, Machine B, Machine C. the following table gives the processing time.

Jobs	1	2	3	4	5
M_A	8	10	6	7	11
M_B	5	6	2	3	4
M_C	4	9	8	6	5

Sol:- Conditions

(1) Min processing time of MA $\geq$ Max processing time of M_B

(1) Min MA $\geq$ Max MB

(2) Min M_C $\geq$ Max M_B

(1) $6 \geq 6$ ✓

(2) $4 \geq 6$ ✗

Take 2 dummy Machines by adding MA, MB, Mc

	Jobs	1	2	3	4	5
MA+MB	MD	13	16	8	10	15
MB+Mc	ME	9	15	10	9	9

$\Rightarrow$ MD | 3 | | | | | ME

Jobs	1	2	4	5
MD	13	16	10	15
ME	9	15	9	9

$\Rightarrow$ MD | 3 | | | | 4 | ME

Jobs	1	2	5
MD	13	16	15
ME	9	15	9

$\Rightarrow$ MD | 3 | | | 1 | 4 | ME

Jobs	2	5
MD	16	15
ME	15	9

$\Rightarrow$ MD | 3 | | 5 | 1 | 4 | ME

Jobs	2
MD	16
MC	15

$\Rightarrow$ MD | 3 | 2 | 5 | 1 | 4 | MC

Jobsequence	Machine A		Machine B		Machine C	
	Time In	Timeout	Time In	Time out	Time In	Timeout
3	0	6	6	8(6+2)	8	16(8+8)
2	6	16 (10+6)	16	22(16+6)	22	31(22+9)
5	16	27 (16+11)	27	31 (27+4)	31	36(31+5)
1	27	35 (27+8)	35	40(35+5)	40	44(40+4)
4	35	42 (35+7)	42	45 (42+3)	45	(45+6)

Result :-

* Total Ellapsed time = 51 hrs //
* Ideal time of M_A = 51 - 42 = 9 hrs //
* Ideal time of M_B = (51-45)+6+8+5+4+2 = 31 hrs //
* Ideal time of M_C = 8+6+4+1 = 19 hrs //

3) The processing time of 6 Jobs on 3 Machines i.e, M_1, M_2, M_3 are given below find the total ellapsed time and Ideal time of Jobs.

Jobs	M_1	M_2	M_3
1	3	8	13
2	12	6	14
3	5	4	9
4	2	6	12
5	9	3	8
6	11	1	13

Sol:- Conditions (1) Min MA $\geq$ Max MB $\Rightarrow 2 \geq 8$ ✗

(2) Min MC $\geq$ Max MB $\Rightarrow 8 \geq 8$ ✓

Jobs	Mp	Me
1	11	21
2	18	20
3	9	13
4	8	18
5	12	11
6	12	14

⇒ Mp | 4 | | | | | | Me

Jobs	Mp	Me
1	11	21
2	18	20
~~3~~	~~9~~	~~13~~
5	12	11
6	12	14

⇒ Mp | 4 | 3 | | | | | Me

Jobs	Mp	Me
1	11	21
2	18	20
~~5~~	~~12~~	~~11~~
6	12	14

⇒ Mp | 4 | 3 | | | | 5 | Me

Jobs	Mp	Me
~~1~~	~~4~~	~~21~~
2	18	20
6	12	14

⇒ Mp | 4 | 3 | 1 | | | 5 | Me

Jobs	Mp	Me
2	18	20
~~6~~	~~12~~	~~14~~

⇒ Mp | 4 | 3 | 1 | 6 | | 5 | Me

Jobs	Mp	Me
2	18	20

Mp | 4 | 3 | 1 | 6 | 2 | 5 | Me

Jobsequence	Machine -1		Machine -2		Machine -3	
	Time In	Timeout	Time In	TimeOut	Time In	Timeout
4	0	2	2	8 (2+6)	8	20 (12+8)
3	2	7 (2+5)	8	12 (8+4)	20	29 (20+9)
1	7	10 (7+3)	12	20 (12+8)	29	42 (29+13)
6	10	21 (10+11)	21	22 (21+1)	42	55 (42+13)
2	21	33 (21+12)	33	39 (33+6)	55	69 (55+14)
5	33	42 (33+9)	42	45 (42+3)	69	77 (69+8)

* Total Ellapsed time = 77 hrs //

* Ideal time of M[illegible] = 77 - 42 = 35 hrs //

* Ideal time of M_2 = (77 - 45) + 2 + 1 + [illegible] + 3
  = 32 + 2 + 1 + [illegible] + 3
  = 49 hrs //

* Ideal time of M_3 = ~~8 + 8 + 9 + 20 + 22 + 27~~ (8 + 0)
  8 hrs //

UNIT-04
GAME THEORY.
ASSIGNMENT-04

25/04/25.

Part-A.

1. What is a saddle point?

A. If the maximin[$\underline{v}$] and minimax[$\bar{v}$] values are equal then the game is said to have 'Saddle point'.

2. Define the value of the game.

A. When each player plays his optimal strategy the resulting payoff is called the "Value of the game".

3. What is a fair game?

A. If $\underline{v} = \bar{v} = 0$. The game is said to be fair.

4. What is the optimum strategy for a player?

A. The stratergy which optimizes a players pay-off's is called "Optimum Strategy".

5. Define pay-off.

Sol: A payoff is the outcome of a game that depends on the selected strategies of the players.

6. What is strategy?

A. In game theory, A strategy is a complete plan of action that a player will use in a game, specifying what they will do in any possible situation.

7. What is dominance rule (or) dominance property in game theory?

A. The Dominance property (or) dominance rule, refers to a situation where one strategy consistently yields a better outcome for a player, regardless of the opponent's choice compared to another strategy.

08. What is two persons zero-sum game?

A. A game with two players A and B in which A gain for A is equal to the loss for B. (Total sum is zero) is called 'Two persons zero-sum game'.

09. What is non-zero sum game?

A. A non-zero sum game is a situation where the combined outcome for all players does not equal zero.

10. Explain maximin and minimax principle or strategy

A. In Game Theory, Minimax strategy focuses on minimizing potential maximum losses, while maximin strategy aims to maximize the minimum guaranteed payoff.

Part - B.

1. Solve the following game

$$\begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{array}{c} \begin{array}{cccc} B_1 & B_2 & B_3 & B_4 \end{array} \\ \begin{bmatrix} 8 & 2 & 4 & 0 \\ 3 & 4 & 2 & 4 \\ 4 & 2 & 4 & 0 \\ 0 & 4 & 0 & 8 \end{bmatrix} \end{array}$$

Sol:

	B_1	B_2	B_3	B_4	Row minimum
A_1	3	2	4	0	0
A_2	3	4	2	4	2
A_3	4	2	4	0	0
A_4	0	4	0	8	0
Column maximum	4	4	4	8	

Maximin $(\underline{v}) = 2$

Minimax $(\bar{v}) = 4$

$\therefore \underline{v} \neq \bar{v}$

$\therefore$ It is Mixed Strategy.

⇒ As compared to A1 and A3. A1 is inferior strategy. Therefore, A1 is deleted. The resultant matrix is

$$\begin{array}{c|cccc} & B_1 & B_2 & B_3 & B_4 \\ \hline A_2 & 3 & 4 & 2 & 4 \\ A_3 & 4 & 2 & 4 & 0 \\ A_4 & 0 & 4 & 0 & 8 \end{array}$$

⇒ As compared to B1 and B3. B1 is inferior strategy. Therefore, B1 is deleted. The resultant matrix is

$$\begin{array}{c|ccc} & B_2 & B_3 & B_4 \\ \hline A_2 & 4 & 2 & 4 \\ A_3 & 2 & 4 & 0 \\ A_4 & 4 & 0 & 8 \end{array}$$

⇒ It doesn't satisfies Dominance rule. Therefore, we apply Convex combination.

Therefore, A1 is deleted. The resultant matrix is

$$\begin{array}{c|ccc} & B_2 & B_3 & B_4 \\ \hline A_3 & 2 & 4 & 0 \\ A_4 & 4 & 0 & 8 \end{array}$$

⇒ Now, by applying convex combination rule, Delete B2 column. The resultant matrix is

$$\begin{array}{c|cc} & B_3 & B_4 \\ \hline A_3 & 4 & 0 \\ A_4 & 0 & 8 \end{array}$$

$$\begin{array}{c|cc} & B_3 & B_4 \\ \hline A_3 & a_{33} & a_{34} \\ A_4 & a_{43} & a_{44} \end{array}$$

$$P_3 = \frac{a_{44} - a_{43}}{(a_{44}+a_{33})-(a_{34}+a_{43})} = \frac{8-0}{(8+4)-(0+0)} = \frac{\cancel{8}^{2}}{\cancel{12}_{3}} = \frac{2}{3}$$

$$P_4 = 1 - P_3 \Rightarrow 1 - \frac{2}{3} = \frac{1}{3}$$

$$q_3 = \frac{a_{44}-a_{34}}{(a_{44}+a_{33})-(a_{34}+a_{43})} = \frac{8-0}{12} = \frac{8}{12} = \frac{2}{3}$$

$$q_4 = 1-q_3 \Rightarrow 1-\frac{2}{3} = \frac{1}{3}$$

$$v = \frac{(a_{44} \cdot a_{33})-(a_{34} \cdot a_{43})}{(a_{44}+a_{33})-(a_{34}+a_{43})}$$

$$= \frac{(8\times 4)-(0\times 0)}{12} = \frac{32}{12} = \frac{16}{6} = \frac{8}{3}$$

$$\therefore v = \frac{8}{3}$$

Result:-

The optimal strategy are $S_A = \begin{bmatrix} A_1 & A_2 & A_3 & A_4 \\ 0 & 0 & \frac{2}{3} & \frac{1}{3} \end{bmatrix}$ and

$$S_B = \begin{bmatrix} B_1 & B_2 & B_3 & B_4 \\ 0 & 0 & \frac{2}{3} & \frac{1}{3} \end{bmatrix}$$

Value of the game $(v) = \frac{8}{3}$

2. Solve the following game

$$\begin{array}{c|cccc|c} & B_1 & B_2 & B_3 & B_4 & \text{Row minimum} \\ \hline A_1 & 5 & -10 & 9 & 0 & -10 \\ A_2 & 6 & 7 & 8 & 1 & 1 \\ A_3 & 8 & 7 & 15 & 1 & 1 \\ A_4 & 3 & 4 & -1 & 4 & -1 \\ \hline \text{Column Maximum} & 8 & 7 & 15 & 4 & \end{array}$$

Minimax $\underline{v} = 1$

Maximin $\bar{v} = 4$

$\therefore \underline{v} \neq \bar{v}$

$\therefore$ It is Mixed Strategy.

⇒ As compared to A_1 and A_3. A_1 is inferior strategy. Therefore A_1 is deleted. The resultant matrix is

$$\begin{array}{c|cccc} & B_1 & B_2 & B_3 & B_4 \\ \hline A_2 & 6 & 7 & 8 & 1 \\ A_3 & 8 & 7 & 15 & 1 \\ A_4 & 3 & 4 & -1 & 4 \end{array}$$

⇒ As compared to A_2 and A_3. A_2 is inferior strategy. Therefore A_2 is deleted. The resultant matrix is

$$\begin{array}{c|cccc} & B_1 & B_2 & B_3 & B_4 \\ \hline A_3 & 8 & 7 & 15 & 1 \\ A_4 & 3 & 4 & -1 & 4 \end{array}$$

⇒ As compared to B_2 and B_4. B_2 is deleted. The resultant matrix is

$$\begin{array}{c|ccc} & B_1 & B_3 & B_4 \\ \hline A_3 & 8 & 15 & 1 \\ A_4 & 3 & -1 & 4 \end{array}$$

⇒ Now, by applying convex combination rule, B_1 is removed. And the resultant matrix is

$$\begin{array}{c|cc} & B_3 & B_4 \\ \hline A_3 & 15 & 1 \\ A_4 & -1 & 4 \end{array}$$

$$\begin{array}{c|cc} & B_3 & B_4 \\ \hline A_3 & a_{33} & a_{34} \\ A_4 & a_{43} & a_{44} \end{array}$$

$$p_3 = \frac{(a_{44}-a_{43})}{(a_{44}+a_{33})-(a_{34}+a_{43})} = \frac{4+1}{(4+15)-(1-1)} = \frac{5}{19}$$

$$p_4 = 1-\frac{5}{19} = \frac{14}{19}$$

$$q_3 = \frac{(a_{44}-a_{34})}{(a_{44}+a_{33})-(a_{34}+a_{43})} = \frac{4-1}{19} = \frac{3}{19}$$

$$q_4 = 1-\frac{3}{19} = \frac{16}{19}$$

$$V = \frac{(a_{44} \cdot a_{33}) - (a_{34} \cdot a_{43})}{(a_{44} + a_{33}) - (a_{34} + a_{43})}$$

$$V = \frac{60+1}{19} = \frac{61}{19}$$

Result:-

The optimal strategies of player A and B

$$S_A = \begin{bmatrix} A_1 & A_2 & A_3 & A_4 \\ 0 & 0 & \frac{5}{19} & \frac{14}{19} \end{bmatrix} \text{ and } S_B = \begin{bmatrix} B_1 & B_2 & B_3 & B_4 \\ 0 & 0 & \frac{3}{19} & \frac{16}{19} \end{bmatrix}$$

3a). In the following game is determinable find the limits of the value of λ.

$$\begin{array}{c|ccc} & B_1 & B_2 & B_3 \\ \hline A_1 & \lambda & 6 & 4 \\ A_2 & -1 & \lambda & -7 \\ A_3 & -2 & 4 & \lambda \end{array}$$

Sol: Given, matrix is,

$$\begin{array}{c|ccc|c} & B_1 & B_2 & B_3 & \text{Row minimum} \\ \hline A_1 & \lambda & 6 & 4 & 4 \\ A_2 & -1 & \lambda & -7 & -7 \\ A_3 & -2 & 4 & \lambda & -2 \\ \hline \text{Column Maximum} & -1 & 6 & 4 & \end{array}$$

Maximin $[\underline{V}] = 4$

Minimax $[\overline{V}] = -1$

$\therefore \underline{V} = V$

$\overline{V} = V$

$\therefore \lambda$ lies in between -1 and 4

03. If the saddle point is (2,2) in the following pay-off matrix. Find the range of values of p and q.

$$\begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array}\begin{array}{c} \begin{array}{ccc} B_1 & B_2 & B_3 \end{array} \\ \begin{bmatrix} 2 & 6 & 5 \\ 10 & 7 & q \\ 5 & p & 8 \end{bmatrix} \end{array}$$

Sol: Given matrix is,

$$\begin{array}{c|ccc|c} & B_1 & B_2 & B_3 & \text{Row minimum} \\ \hline A_1 & 2 & 6 & 5 & 2 \\ A_2 & 10 & \boxed{7} & q & 7 \\ A_3 & 5 & p & 8 & 5 \\ \hline \text{Column maximum} & 10 & 7 & 8 & \end{array}$$

Maximin $[\underline{v}] = 7$

Minimax $[\bar{v}] = 7$

$\therefore q \geq 7$

$p \leq 7$

$\therefore$ It is a Pure Strategy.

04. Solve the following game.

Sol:

$$\begin{array}{c|cc|c} & B_1 & B_2 & \text{Row minimum} \\ \hline A_1 & 8 & -3 & -3 \\ A_2 & -3 & 1 & -3 \\ \hline \text{Column maximum} & 8 & 1 & \end{array}$$

$\underline{v} = -3$

$\bar{v} = 1$

$\underline{v} \neq \bar{v}$

$\therefore$ It is Mixed Strategy.

$$P_1 = \frac{(a_{22} - a_{21})}{(a_{22}+a_{11}) - (a_{12}+a_{21})} = \frac{1+3}{(1+8)-(-3-3)} = \frac{4}{9+6} = \frac{4}{15}$$

$$P_2 = 1 - P_1 = 1 - \frac{4}{15} = \frac{11}{15}$$

$$q_1 = \frac{(a_{22} - a_{12})}{(a_{22} + a_{11}) - (a_{12} + a_{21})}$$

$$= \frac{1+3}{15} = \frac{4}{15}$$

$$q_2 = 1 - q_1 = 1 - \frac{4}{15} = \frac{11}{15}$$

Result:-

The optimal strategies are $S_A = \begin{bmatrix} A_1 & A_2 \\ \frac{4}{15} & \frac{11}{15} \end{bmatrix}$ and $S_B = \begin{bmatrix} B_1 & B_2 \\ \frac{4}{15} & \frac{11}{15} \end{bmatrix}$

$$v = \frac{(a_{22} a_{11}) - (a_{12} \cdot a_{21})}{(a_{22} + a_{11}) - (a_{12} + a_{21})}$$

$$= \frac{(1 \times 8) - (-3)(-3)}{15} = \frac{8-9}{15} = \frac{-1}{15}$$

5. Solve the following game

Sol:

$$\begin{array}{c|ccc|c} & B_1 & B_2 & B_3 & \text{Row minimum} \\ \hline A_1 & 1 & 7 & 2 & 1 \\ A_2 & 0 & 2 & 7 & 0 \\ A_3 & 5 & 1 & 6 & 1 \\ \hline \text{Column Maximum} & 5 & 7 & 7 & \end{array}$$

Maximin $\underline{v} = 1$

Minimax $\bar{v} = 5$

$\therefore \underline{v} \neq \bar{v}$

$\therefore$ It is mixed strategy.

$\Rightarrow$ By applying Dominance rule, we remove B_3 column

$$\begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{c} \begin{array}{cc} B_1 & B_2 \end{array} \\ \begin{bmatrix} 1 & 7 \\ 0 & 2 \\ 5 & 1 \end{bmatrix} \end{array}$$

$\Rightarrow$ Again By applying Dominance rule, we remove A_2 row.

$$\begin{array}{c} \\ A_1 \\ A_3 \end{array} \begin{array}{c} \begin{array}{cc} B_1 & B_2 \end{array} \\ \begin{bmatrix} 1 & 7 \\ 5 & 1 \end{bmatrix} \end{array}$$

$$P_1 = \frac{(a_{22}-a_{21})}{(a_{22}+a_{11})-(a_{12}+a_{21})}$$

$$= \frac{1-5}{(1+1)-(7+5)} = \frac{+\cancel{4}^{2}}{+\cancel{10}_{5}}$$

$$= \frac{2}{5}$$

$$P_2 = 1 - P_1$$

$$= 1 - \frac{2}{5}$$

$$= \frac{3}{5}$$

$$q_1 = \frac{(a_{22}-a_{12})}{(a_{22}+a_{11})-(a_{12}+a_{21})}$$

$$= \frac{1-7}{-10} = \frac{+\cancel{6}^{3}}{+\cancel{10}_{5}} = \frac{3}{5}$$

$$q_2 = 1 - q_1$$

$$= 1 - \frac{3}{5}$$

$$= \frac{2}{5}$$

$$V = \frac{(a_{22} \cdot a_{11})-(a_{12} \cdot a_{21})}{(a_{22}+a_{11})-(a_{12}+a_{21})} = \frac{(1\times 1)-(7\times 5)}{-10} = \frac{1-35}{-10} = \frac{-\cancel{34}^{17}}{-\cancel{10}_{5}} = \frac{17}{5}$$

Result:

The optimal strategy are $S_A = \begin{bmatrix} A_1 & A_3 \\ \frac{2}{5} & \frac{3}{5} \end{bmatrix}$ and $S_B = \begin{bmatrix} B_1 & B_2 \\ \frac{3}{5} & \frac{2}{5} \end{bmatrix}$

6. Solve the following game graphically.

$$\begin{array}{c|cc} & B_1 & B_2 \\ \hline A_1 & -6 & 7 \\ A_2 & 4 & -5 \\ A_3 & -1 & -2 \\ A_4 & -2 & 5 \\ A_5 & 7 & -6 \end{array}$$

Sol: let $S_B = \begin{bmatrix} B_1 & B_2 \\ q_1 & q_2 \end{bmatrix}$

$$E_1(q_1) = a_{11}q_1 + a_{12}q_2$$

$$= -6q_1 + 7q_2$$

$$= -6q_1 + 7(1-q_1)$$

$$= -6q_1 + 7 - 7q_1$$

$$E_1(q_1) = -13q_1 + 7 \longrightarrow (1)$$

$$E_2(q_1) = 4q_1 - 5q_2$$

$$= 4q_1 - 5(1-q_1)$$

$$= 4q_1 - 5 + 5q_1$$

$$E_2(q_1) = 9q_1 - 5 \longrightarrow (2)$$

$E_3(q_1) = -1q_1 - 2q_2$

$= -1q_1 - 2(1-q_1)$

$= -1q_1 - 2 + 2q_1$

$E_3(q_1) = q_1 - 2 \rightarrow (3)$

$E_4(q_1) = -2q_1 + 5q_2$

$= -2q_1 + 5(1-q_1)$

$= -2q_1 + 5 - 5q_1$

$E_4(q_1) = -7q_1 + 5 \rightarrow (4)$

$E_5(q_1) = 7q_1 - 6q_2$

$= 7q_1 - 6(1-q_1)$

$= 7q_1 - 6 + 6q_1$

$E_5(q_1) = 13q_1 - 6 \rightarrow (5)$

From equation (1)

Put $q_1 = 0 \Rightarrow E_1(0) = -13(0)+7 = (0,7)$

Put $q_1 = 1 \Rightarrow E_1(1) = -13(1)+7 = (1,-6)$

$\therefore E_1 = (7,-6)$

From equation (2)

Put $q_1 = 0 \Rightarrow E_2(0) = 9(0) - 5 = (0,-5)$

Put $q_1 = 1 \Rightarrow E_2(1) = 9(1) - 5 = (1,4)$

$\therefore E_2 = (-5,4)$

From equation (3)

Put $q_1 = 0 \Rightarrow E_3(0) = 0 - 2 = (0,-2)$

Put $q_1 = 1 \Rightarrow E_3(1) = 1 - 2 = (1,-1)$

$\therefore E_3 = (-2,-1)$

From equation (4)

Put $q_1 = 0 \Rightarrow E_4(0) = -7(0)+5 = (0,5)$

Put $q_1 = 1 \Rightarrow E_4(1) = -7(1)+5 = (1,-2)$

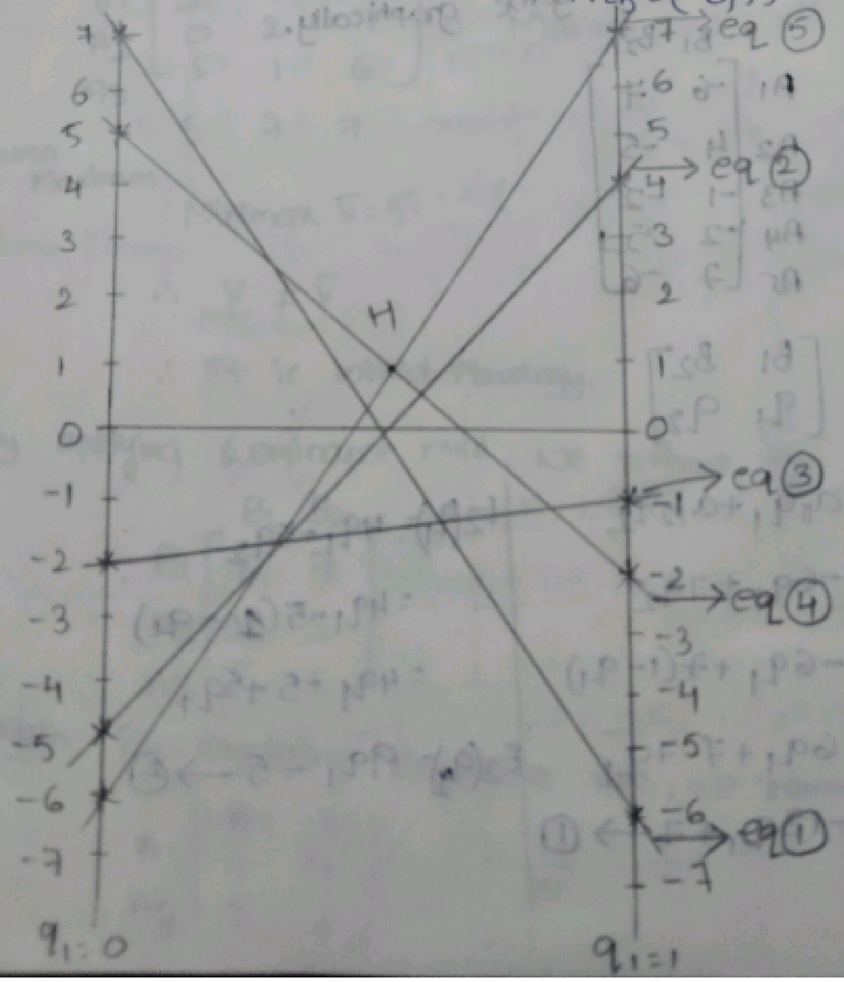
$\therefore E_4 = (5,-2)$

From equation (5)

Put $q_1 = 0 \Rightarrow E_5(0) = 13(0) - 6 = (0,-6)$

Put $q_1 = 1 \Rightarrow E_5(1) = 13(1) - 6 = (1,7)$

$\therefore E_5 = (-6,7)$



$P_4 =$

$P_5 =$

$q_1 =$

q_2

$\therefore v$

Result :-

The optim

Value of

7. Solve th

Sol: Let

$E_j(P_1$

$E_j($

$E_1($

$E_j($

$(0) = -13(0)+7 = (0,7)$

$) = -13(1)+7 = (1,-6)$

$,-6)$

$: 9(0)-5 = (0,-5)$

$9(1)-5 = (1,4)$

$, 4)$

$0-2 = (0,-2)$

$-2 = (1,-1)$

$,-1)$

$-7(0)+5 = (0,5)$

$-7(1)+5 = (1,-2)$

$\therefore E_4 = (5,-2)$

$13(0)-6 = (0,-6)$

$3(1)-6 = (1,7)$

5

$$\begin{array}{c|cc} & B_1 & B_2 \\ \hline A_4 & \multicolumn{2}{c}{\begin{bmatrix} -2 & 5 \\ 7 & -6 \end{bmatrix}} \\ A_5 & & \end{array}$$

$$P_4 = \frac{-6-7}{(-6-2)-(5+7)} = \frac{-13}{-20} = \frac{13}{20}$$

$$P_5 = 1 - P_4 \Rightarrow 1 - \frac{13}{20} = \frac{7}{20}$$

$$q_1 = \frac{-6-5}{-20} = \frac{-11}{-20} = \frac{11}{20}$$

$$q_2 = 1 - q_1 = 1 - \frac{11}{20} = \frac{9}{20}$$

$$\therefore V = \frac{12-35}{-20} = \frac{-23}{-20} = \frac{23}{20}$$

Result :-

The optimal strategy $S_A = \begin{bmatrix} A_1 & A_2 & A_3 & A_4 & A_5 \\ 0 & 0 & 0 & \frac{13}{20} & \frac{7}{20} \end{bmatrix}$ and $S_B = \begin{bmatrix} B_1 & B_2 \\ \frac{11}{20} & \frac{9}{20} \end{bmatrix}$

Value of the game $(V) = \frac{23}{20}$

7. Solve the following game graphically

$$\begin{array}{c|cccc} & B_1 & B_2 & B_3 & B_4 \\ \hline A_1 & 2 & 1 & 0 & -2 \\ A_2 & 1 & 0 & 3 & 2 \end{array}$$

Sol: Let $S_A = \begin{bmatrix} A_1 & A_2 \\ P_1 & P_2 \end{bmatrix}$

$$E_j(P_1) = a_{1j}P_1 + a_{2j}P_2$$
$$= a_{1j}P_1 + a_{2j}(1-P_1)$$
$$= a_{1j}P_1 + a_{2j} - a_{2j}P_1$$
$$\boxed{E_j(P_1) = P_1(a_{1j} - a_{2j}) + a_{2j}}$$

$E_1(P_1) = P_1(2-1)+1$

$E_1(P_1) = P_1 + 1 \rightarrow (1)$

$E_2(P_1) = P_1(1-0)+0$

$E_2(P_1) = P_1 \rightarrow (2)$

$E_3(P_1) = P_1(0-3)+3$

$E_3(P_1) = -3P_1+3 \rightarrow (3)$

$E_4(P_1) = P_1(-2-2)+2$

$E_4(P_1) = -4P_1+2 \rightarrow (4)$

From Equation (1)

Put, $P_1 = 0 \Rightarrow E_1(0) = 0+1 = 1\ (0,1)$

Put, $P_1 = 1 \Rightarrow E_1(1) = 1+1 = 2\ (1,2)$

$\therefore E_1 = (1,2)$

From Equation (2)

Put $P_1 = 0 \Rightarrow E_2(0) = 0 = (0,0)$

Put $P_1 = 1 \Rightarrow E_2(1) = 1 = (1,1)$

$\therefore E_2 = (0,1)$

From Equation (3)

Put, $P_1 = 0 \Rightarrow E_3(0) = -3(0)+3 = 0+3 = 3 = (0,3)$

Put, $P_1 = 1 \Rightarrow E_3(1) = -3(1)+3 = 0 = (1,0)$

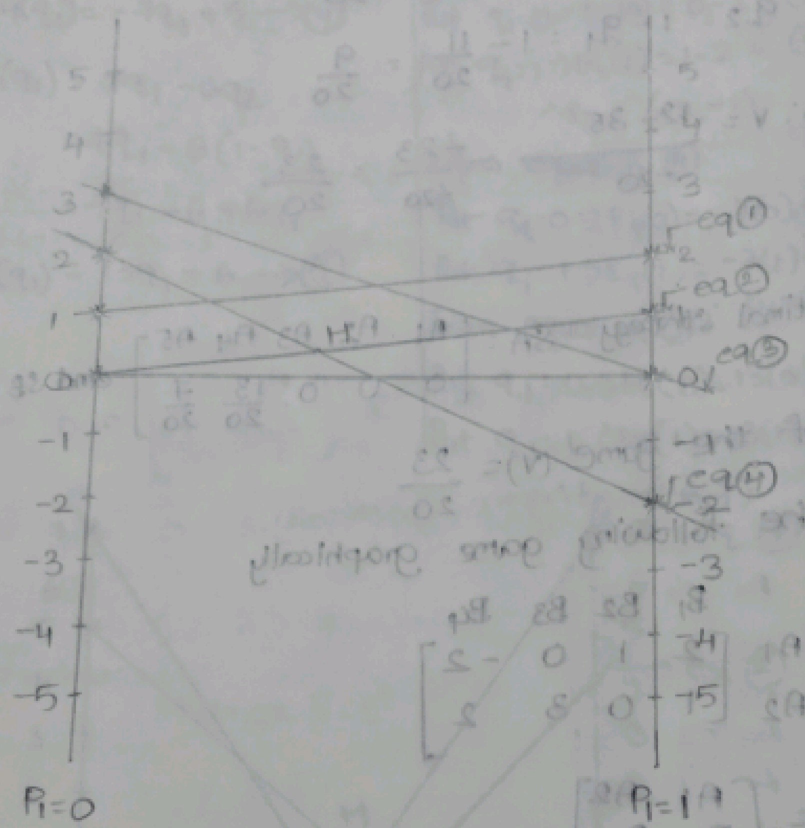
$\therefore E_3 = (3,0)$

From equation (4)

Put, $P_1 = 0 \Rightarrow E_4(0) = -4(0)+2 = 0+2 = 2\ (0,2)$

Put, $P_1 = 1 \Rightarrow E_4(1) = -4(1)+2 = -4+2 = -2\ (1,-2)$

$\therefore E_4 = (2,-2)$



$$\begin{array}{c|cc} & B_2 & B_4 \\ \hline A_1 & 1 & -2 \\ A_2 & 0 & 2 \end{array}$$

$P_1 = \frac{2-0}{(2+1)-(-2+0)} = \frac{2}{5}$ | $q_2 = \frac{2+2}{5} = \frac{4}{5} \Rightarrow q_4 = 1-q_2 \Rightarrow 1-\frac{4}{5} = \frac{1}{5}$

$P_2 = 1-P_1 = 1-\frac{2}{5} = \frac{3}{5}$ | $V = \frac{2-0}{5} = \frac{2}{5}$

Result:-

The optimal stategy $S_A = \begin{bmatrix} A_1 & A_2 \\ \frac{2}{5} & \frac{3}{5} \end{bmatrix}$ and $S_B = \begin{bmatrix} B_1 & B_2 & B_3 & B_4 \\ 0 & \frac{4}{5} & 0 & \frac{1}{5} \end{bmatrix}$

value of the game $(v) = \frac{2}{5}$

UNIT-05

PROJECT MANAGEMENT

ASSIGNMENT-05

26/4/25

Part - A

1. Distinguish CPM and PERT.

A.

CPM	PERT
1. CPM stands for Critical Path Method.	1. PERT strands for Project Evaluation and Review Technique.
2. It is activity oriented technique.	2. It is event oriented technique.
3. CPM manages the predictable activities	3. PERT manages unpredictable activities
4. It focus on cost optimization.	4. It is focused on time control.
5. It is a Deterministic model	5. It is a probability model.

2. What are the three time estimates used in PERT Network? Define them.

A. As in the case of PERT, there are 3 main techniques.

i) Optimistic time (to)

ii) Pessimistic time (tp)

iii) Most likely time (tm)

i) Optimistic time (to): This is the shortest possible time required to complete an activity, assuming everything goes perfectly and ideally. It represents the best-case scenario

ii) Pessimistic Time (tp): This is the longest possible time required to complete an activity, assuming everything goes wrong and there are numerous delays or setbacks. It represents the worst case scenario

iii) Most Likely Time (tm):- This is the most probable time required to complete an activity under normal circumstances, without any major issues or unexpected delay. It represents the most realistic estimates.

3. When do you use dummy activity ?

A. Dummy activities in project network diagrams are used to maintain the logical flow and precedence relationships between tasks when there's no real activity between two events.

4. Define critical path.

A. ⇒ A path along the network in which earliest finish time and the latest finish time are equal, this path is known as 'Critical path'.

⇒ It is represented by the double line or thick line.

⇒ Critical path is the longest path in the network diagram. It indicates the duration of the project.

5. Applications of CPM | PERT

A. CPM Application:-

1. Project planning: Identifies the longest path of dependent tasks (critical path) that determines the shortest project duration.
2. Time Management: Highlights tasks that cannot be delayed without affecting the overall project.
3. Resource Allocation: Focuses attention on critical tasks to ensure timely completion.
4. Monitoring and Control: Helps in tracking progress and adjusting timeline as needed.

PERT Applications:-

1. Project Scheduling: Uses three time estimates (optimistic, most likely, pessimistic) to account for uncertainty
2. Risk Management: Analyzes the probability of project completion within a given time.
3. Decision Making: Helps project managers evaluate alternative schedules.
4. Time Forecasting: Offers a weighted average duration for activities to improve planning accuracy.

6. Define Float.

A. It is time between the latest finish time and earliest finish time on critical path.

7. Why is identifying the Critical path important in Project Management?

A. The critical path is the longest sequence of tasks in a project that ~~neess~~ must be finished on time for the whole project to be completed on schedule. If any task on this path is delayed, the entire project will be delayed.

8. Define Earliest Start Time (EST) and Earliest Finish Time (EFT)

A. Earliest Start Time (EST):- It is the earliest possible time at which an activity can start.

EST = Earliest Finish Time − Duration.

Earliest Finish Time (EFT):- It is the earliest time at which an activity can finish.

9. State any two rules for constructing a Network Diagram.

A. i) Each activity must be represented by a single arrow or node that shows its start and end.

ii) Activities must follow a logical sequence, meaning no activity can start until all its preceding actities are completed.

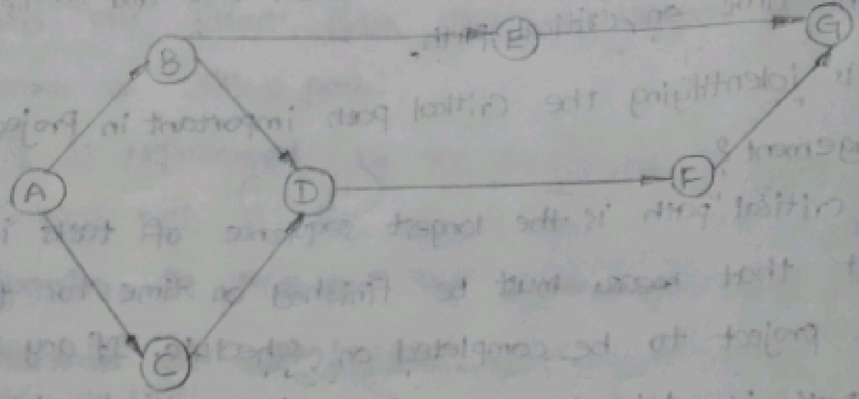
10. Define Network Analysis in Project Management.

A. Network Analysis in project management is the process of planning, scheduling, and analyzing a project using a visual diagram (network) of interconnected activities. It helps identify the sequence of tasks, estimate timeframes and determine the critical path for successful project completion.

Part - B.

1. Draw a network diagram for the following set of activities: A<B, C; B<D,E; C<D; D<F; E<G; F<G.

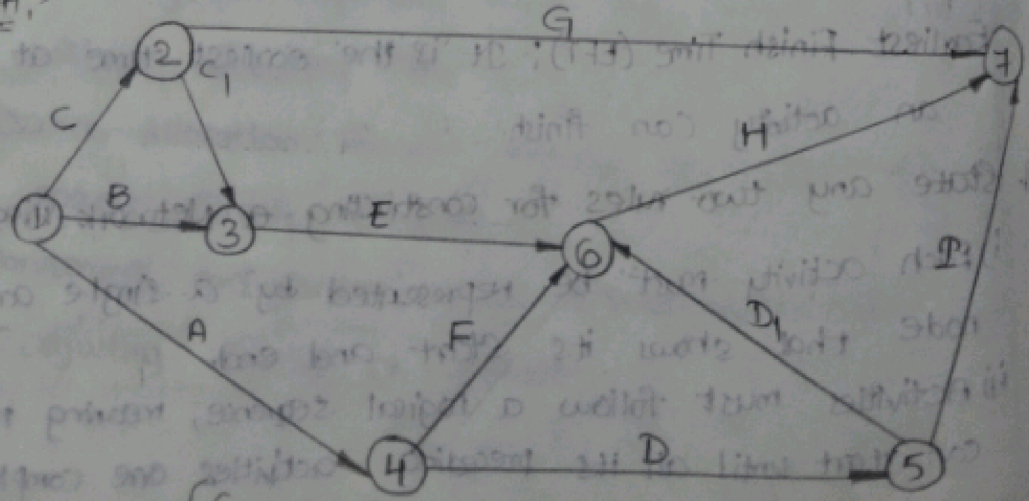
Sol: AON:-



2. Draw a network diagram for the following set of activities

Activity	A	B	C	D	E	F	G	H	I
Immediate Predecessor	–	–	–	A	B,C	A	C	D,E,F	D

Sol: AOA:-

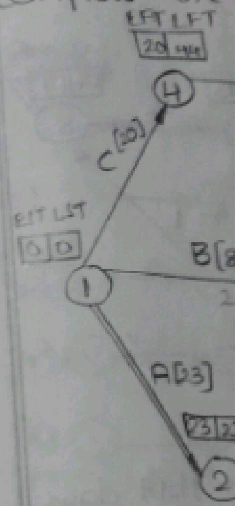


(C_1 and D_1 are dummy)

3. A project consists ...
such that A<D,E ...
The time of ...

Job	A	B
Time (days)	23	8

Find the critic...
complete the ...



Result:-

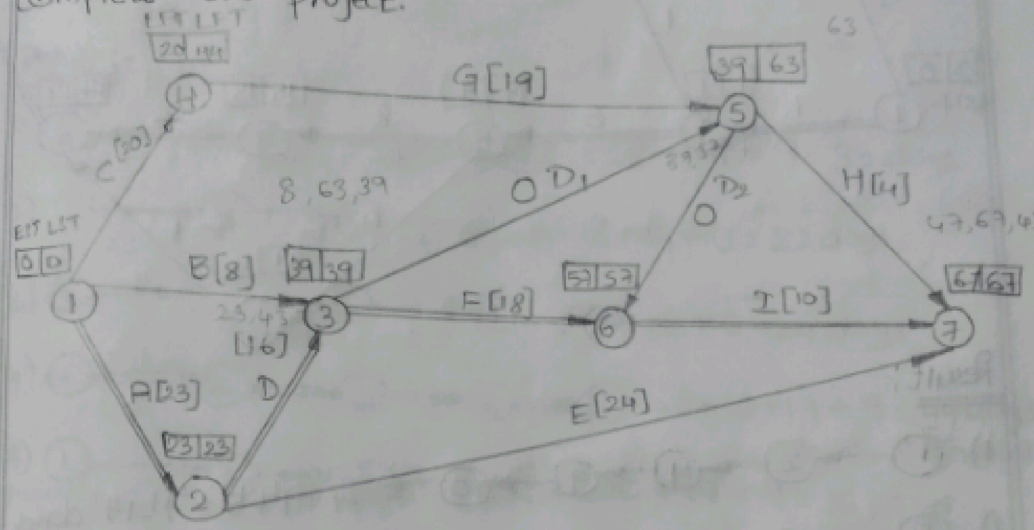
1) 1
2) 1
3) 1
4) 1
5) 1
6) 1
7) 1
8) 1

∴ The ...
is ...

3. A project consists of a series of jobs A, B, C, D, E, F, G, H, I such that A<D,E ; B,D<F ; C<G ; B<H ; F,G<I.
The time of completion of each job is given below.

Job	A	B	C	D	E	F	G	H	I
Time (days)	23	8	20	16	24	18	19	4	10

Find the critical path and the minimum time required to complete the project.



Result :-

1) (1) → (4) → (5) → (7) = 20+19+4 = 43 days

2) (1) → (3) → (5) → (6) → (7) = 8+0+0+10 = 18 days

3) (1) → (3) → (6) → (7) = 8+18+10 = 36 days.

4) (1) → (2) → (3) → (6) → (7) = 23+16+18+10 = 67 days [Critical path]

5) (1) → (2) → (7) = 23+24 = 47 days

6) (1) → (3) → (5) → (7) = 8+0+4 = 12 days

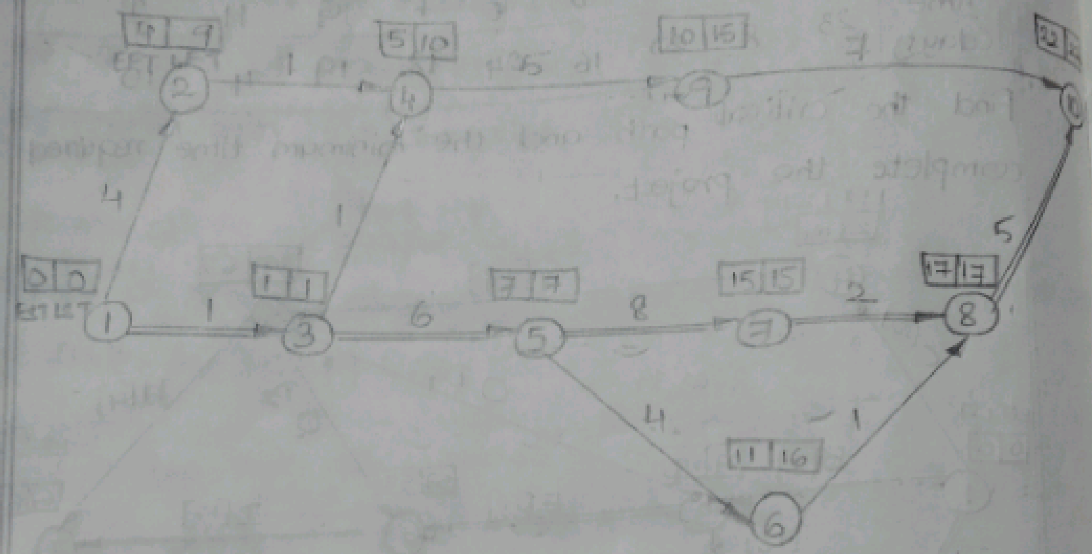
7) (1) → (2) → (3) → (5) → (7) = 23+16+0+4 = 43 days

8) (1) → (2) → (3) → (5) → (6) → (7) = 23+16+0+0+10 = 49 days

∴ The minimum time required to complete the project is 67 days.

4. Find the critical path of the project with the following acti

Job	(1,2)	(1,3)	(2,4)	(3,4)	(3,5)	(4,9)	(5,6)	(5,7)	(6,8)	(7,8)	(8,10)	(9,10)
Time	4	1	1	1	6	5	4	8	1	2	5	7



Result:-

1) ① → ② → ④ → ⑨ → ⑩ = 4 + 1 + 5 + 7 = 17 days

2) ① → ③ → ④ → ⑨ → ⑩ = 1 + 1 + 5 + 7 = 14 days

3) ① → ③ → ⑤ → ⑦ → ⑧ → ⑩ = 1+6+8+2+5 = 22 days [critical path]

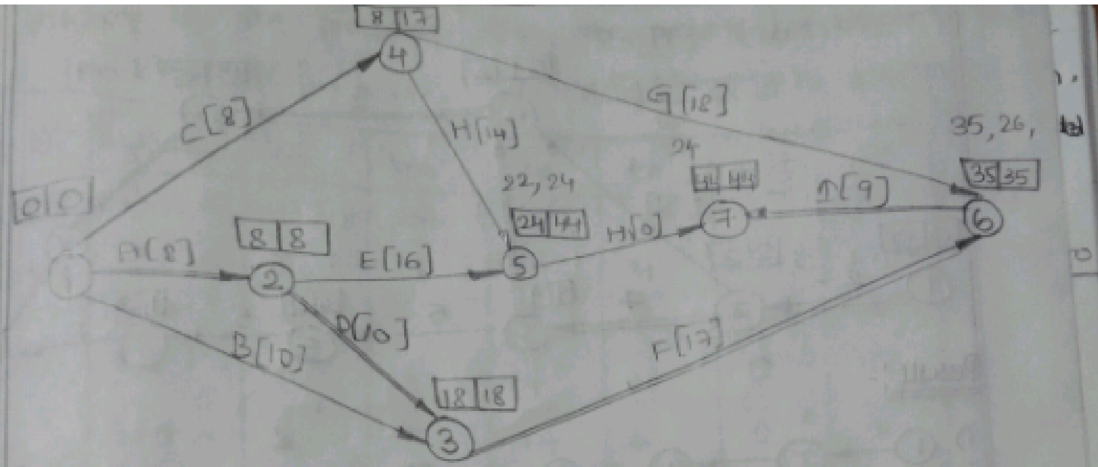
4) ① → ③ → ⑤ → ⑥ → ⑧ → ⑩ = 1+6+4+1+5 = 17 days

∴ The minimum time required to complete the project is 22 days.

5. A project consists of jobs A, B, C, D, E, F, G, H, I such that A<D; A<E; B<F; D<F; C<G; C<H; F<I; G<I.
The time taken for each job is given below:-

Job	A	B	C	D	E	F	G	H	I
Time (days)	8	10	8	10	16	17	18	14	9

Draw the network diagram. Find the critical path and minimum time of completion of the project.



Result:-

1) (1) ⟶ (4) ⟶ (6) ⟶ (7) : 8 + 18 + 9 = 35 days

2) (1) ⟶ (4) ⟶ (5) ⟶ (7) = 8 + 14 + 0 = 22 days

3) (1) ⟶ (2) ⟶ (5) ⟶ (7) = 8 + 16 + 0 = 24 days

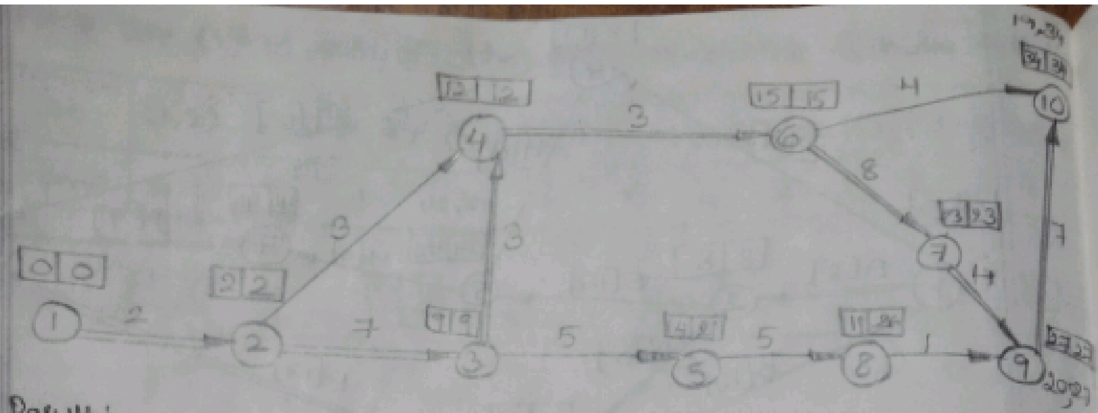
4) (1) ⟹ (2) ⟹ (3) ⟹ (6) ⟹ (7) = 8 + 10 + 17 + 9 = 44 days [critical path]

5) (1) ⟶ (3) ⟶ (6) ⟶ (7) = 10 + 17 + 9 = 36 days.

∴ The minimum time required to complete the project is 44 days.

6 Find the critical path of a project having the tasks as given below.

Job	Time
(1,2)	2
(2,3)	7
(2,4)	3
(3,4)	3
(3,5)	5
(4,6)	3
(5,8)	5
(6,7)	8
(6,10)	4
(7,9)	4
(8,9)	1
(9,10)	7



Result:-

1) 1 → 2 → 4 → 6 → 10 = 2+3+3+4 = 12 days

2) 1 → 2 → 4 → 6 → 7 → 9 → 10 = 2+3+3+8+4+7 = 27 days

3) 1 → 2 → 3 → 4 → 6 → 7 → 9 → 10 = 2+7+3+3+8+4+7 = 34 days [Critical path]

4) 1 → 2 → 3 → 5 → 8 → 9 → 10 = 2+7+5+5+1+7 = 27 days

∴ The minimum time required to complete the project is 34 days.

7. A project consists of the following activities and time estimation

Activity	Estimated duration in weeks		
	Optimistic (t_o)	Most Likely (t_m)	Pessimistic (t_p)
(1, 2)	1	1	7
(1, 3)	1	4	7
(1, 4)	2	2	8
(2, 5)	1	1	1
(3, 5)	2	5	14
(4, 6)	2	5	8
(5, 6)	3	6	15

a) Draw the network

b) Find the expected time and variance for each activity.

c) What is the probability that the project will be completed 4 weeks earlier than the expected time.

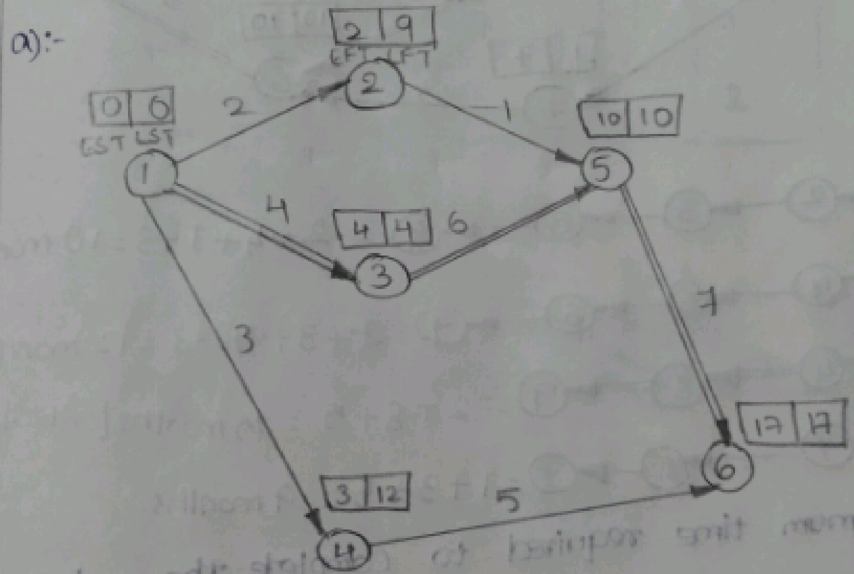
d) What is the probability that the project will be completed in 19 weeks?

$t_e = \frac{t_o + 4t_m + t_p}{6}$ $\sigma = \frac{t_p - t_o}{6}$

Sol: b):-

Activity	t_o	t_m	t_p	t_e	σ	σ^2
1-2	1	1	7	2	1	1
1-3	1	4	7	4	1	1
1-4	2	2	8	3	1	1
2-5	1	1	1	1	0	0
3-5	2	5	14	6	2	4
4-6	2	5	8	5	1	1
5-6	3	6	15	7	2	4

a):-

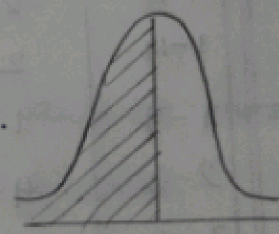


$\sigma^2 = 1+4+4$

$\therefore \sigma^2 = 9$

Result:-

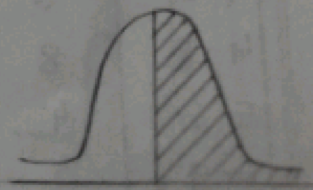
$\therefore$ Total Expected Time (T) = 17 weeks.



c) t = 13 weeks (17 weeks − 4 weeks)

$$z = \frac{t-T}{\sqrt{\sigma^2}} = \frac{13-17}{\sqrt{9}} = \frac{-4}{3} = -1.33$$

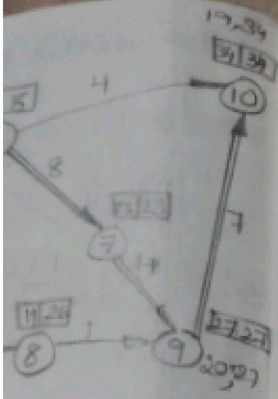
z = −1.33 (0.0918) from Normal Distribution table.



d) t = 19 weeks, T = 17 weeks

$$z = \frac{t-T}{\sqrt{\sigma^2}} = \frac{19-17}{\sqrt{9}} = \frac{2}{3} = 0.66$$

z = 0.66 (0.745[illegible]) from Normal Distribution table.



-4 = 12 days

3+3+8+4+7 = 27 days

) = 2+7+3+3+8+4+7 = 34 days [Critical path]

+7+5+5+1+7 ... 7 days

project is

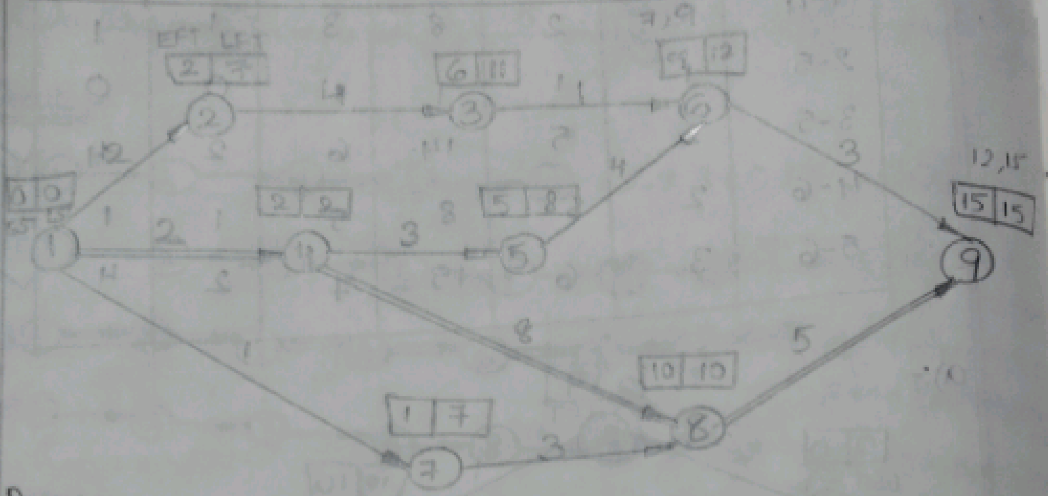
time estimation

ctivity.

be completed

8. For a certain project the data is given below. Draw the network diagram, identify the critical path and compute the project duration (in months) and also find the total float

Activity	1-2	1-4	1-7	2-3	3-6	4-5	4-8	5-6	6-9	7-8	8-9
Time	2	2	1	4	1	3	8	4	3	3	5



Result:-

1) ① → ② → ③ → ⑥ → ⑨ = 2 + 4 + 1 + 3 = 10 months

2) ① → ④ → ⑤ → ⑥ → ⑨ 2 + 3 + 4 + 3 = 12 months

3) ① → ④ → ⑧ → ⑨ = 2 + 8 + 5 = 15 months [critical path]

4) ① → ⑦ → ⑧ → ⑨ = 1 + 3 + 5 = 9 months.

The minimum time required to complete the project is 15 months.

Activity	Expected time (In Months)	EST	LST LFT-D	EFT EST+D	LFT LST+D	Float = LFT-EFT
1-2	2	0	5	2	7	5
1-4	2	0	0	2	2	0
1-7	1	0	6	1	7	6
2-3	4	2	7	6	11	5
3-6	1	6	11	9	12	3
4-5	3	2	5	5	8	3
4-8	8	2	2	10	10	0
5-6	4	5	8	9	12	3
6-9	3	9	12	15	15	0
7-8	3	1	7	10	10	0
8-9	5	10	10	15	15	0

09. A small pro... estimates in ... What is ... at least

Activity
1-2
1-3
1-4
2-5
3-5
4-6
5-6

Result:-

Tota...

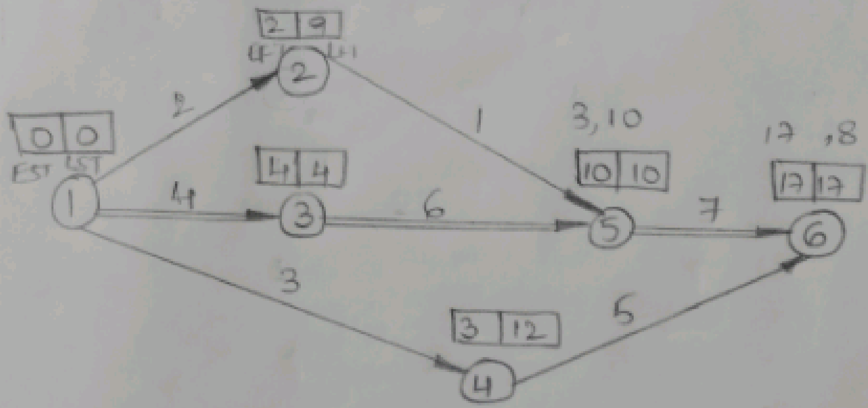
To fi...

4 w...

09. A small project is composed of seven activities whose time estimates in weeks are given below. Find the critical path. What is the probability that the project will be completed at least four weeks earlier than expected? $t_e = \frac{t_o + 4t_m + t_p}{6}$

$\sigma = \frac{t_p - t_o}{6}$

Activity	t_o	t_m	t_p	t_e	σ	σ^2
1-2	1	1	7	2	1	1
1-3	1	4	7	4	1	1
1-4	2	2	8	3	1	1
2-5	1	1	1	1	0	0
3-5	2	5	14	6	2	4
4-6	2	5	8	5	1	1
5-6	3	6	15	7	2	4



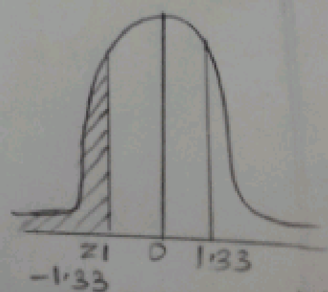
$\therefore \sigma^2 = 1+4+4 = 9$

Result:-

Total Expected Time $(T) = 17$ weeks.

To find the probability that the project would be completed 4 weeks earlier. Given that, $t = 17-4 = 13$ weeks.

$$z = \frac{t-T}{\sqrt{\sigma^2}} = \frac{13-17}{\sqrt{9}} = \frac{-4}{3} = -1.33$$



(0.0918) from Normal distribution table