



**SREENIVASA INSTITUTE OF TECHNOLOGY AND MANAGEMENT STUDIES,
CHITTOOR (AUTONOMOUS)**

Approved by AICTE, New Delhi, Affiliated to JNTUA, Ananthapuramu.

DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING



Name of the Subject: Power Systems –II

Year/ Branch : III Year/EEE

Regulation : R23

Prepared By : Mrs. B. K. Shalini, Assistant Professor-EEE.

DEPARTMENT VISION:

To excel in the domains of electrical and electronics engineering through academic excellence, research innovation, actively addressing challenges, developing sustainable solutions, and making a Positive societal impact, with the aim of further strengthening research and promoting holistic development among students.

DEPARTMENT MISSION:

- Provide quality education and training in electrical and electronics engineering.
- Foster research and innovation in the field.
- Address real-world challenges with sustainable solutions.
- Promote holistic development and ethical values.
- Enhance industry collaborations and practical learning opportunities.

PROGRAM OUTCOMES (POs):

-  PO1- Engineering Knowledge
-  PO2- Problem Analysis
-  PO3- Design/ Development of Solutions
-  PO4- Conduct Investigation of Complex Problems
-  PO5-Modern Tool Usage
-  PO6-The Engineer and Society
-  PO7-Environment and Sustainability
-  PO8-Ethics
-  PO9-Individual and Team Work
-  PO10-Communication
-  PO11-Problem Management and Finance
-  PO12-Lifelong Learning

PROGRAM EDUCATIONAL OBJECTIVES (PEOs)

After few years of graduation, the, graduates of Electrical and Electronics Engineering shall

- PEO1: To cultivate graduates of the EEE program with a strong foundation in electrical and electronics engineering principles and practices, equipping them with the knowledge and skills necessary for successful careers or further studies in the field (Competent Professional)
- PEO2: To develop graduates who can conduct research, innovate, and apply their knowledge effectively to address real-world challenges in electrical and electronics engineering, contributing to technological advancements and sustainable solutions (Innovative Engineers)
- PEO3: To foster graduates who demonstrate professionalism, ethical conduct, leadership qualities, effective communication, and teamwork skills, enabling them to make a Positive societal impact, engage in lifelong learning, and adapt to the evolving needs of the industry. (Professionally Ethical and adaptable Leadership)

PROGRAM SPECIFIC OUTCOMES (PSOs)

- PSO1: Graduates will demonstrate proficiency in applying electrical and electronics engineering principles and practices to design, analyze, and implement solutions in diverse application areas.
- PSO2: Graduates will Possess research aptitude, innovation skills, and the ability to address real-world challenges by developing sustainable solutions in the field of electrical and electronics

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POWER SYSTEMS-II

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PRE-REQUISITES: Power Systems-I

COURSE EDUCATIONAL OBJECTIVES:

1. To study about line parameters and constants
2. To study the performance of transmission lines
3. To know about overhead line insulators, corona, sag and tension in transmission lines
4. To study about symmetrical components and different types of faults in power system.
5. To understand the concept of voltage control, compensation methods

UNIT –1: Transmission line parameters:

(9)

Types of Conductors - Calculation of Resistance for Solid Conductors, Bundle Conductors, Skin effect, Proximity effect, Concept of GMR & GMD- Transposition of Power lines-Calculation of inductance for single phase and three phase, Single and Double circuit lines, Symmetrical and asymmetrical conductor configurations with and without transposition. Calculation of Capacitance for 2 wire and 3 wire systems, effect of ground on Capacitance, Capacitance calculations for symmetrical and asymmetrical single and three phase, single and double circuit lines, Numerical Problems

UNIT –2: Performance of transmission lines:

(9)

Classification of Transmission Lines-Short, medium and long line and their models representation - Nominal-T, Nominal- π and A, B, C, D Constants for symmetrical networks, Numerical Problems and solutions for estimating regulation and efficiency of all types of lines. Ferranti effect and Charging Current

UNIT –3: Overhead line insulators:

(9)

Types of Insulators, String efficiency and Methods for improvement, – Voltage Distribution, Calculation of String efficiency, Capacitance Grading and Static Shielding., Numerical Problems.

Sag and Tension: Sag and Tension Calculations with equal and unequal heights of towers, Effect of wind and ice on weight of conductor, Stringing chart, Sag template and its applications Numerical Problems.

Corona: Corona- factors affecting corona, critical voltages and Power loss due to Corona. Radio Interference.

UNIT –4: Power System Transients (9)

Types of system transients-Travelling or propagation of surges- attenuation, distortion, reflection and refraction coefficients- termination of lines with different types of conditions – open circuited line, short circuited line, T-junction (numerical problems)-Bewley Lattice diagrams (for all cases mentioned with numerical examples).

UNIT –5: Voltage Control and Power Factor Improvement

(9)

Methods of voltage control, shunt and series capacitors / Inductors, tap changing Transformers, synchronous phase modifiers, power factor improvement methods. Compensation in Power Systems: Concepts of Load compensation Load ability characteristics of overhead lines – Uncompensated transmission line – Symmetrical line – Radial line with asynchronous load – Compensation of lines.

Total Hours: 45

COURSE OUTCOMES:

On successful completion of the course, students will be able to		POs
CO1	Analyse the transmission lines and obtain the transmission line parameters and constants	PO1, PO2, PO3, PO4
CO2	Analyse transmission line performance	PO1, PO2, PO3, PO4
CO3	Design transmission lines to meet the day-to-day power requirements.	PO1, PO2, PO3
CO4	Understand and apply Per Unit System for Fault Calculations	PO1, PO2, PO3, PO4
CO5	Apply load compensation techniques to control reactive power.	PO1, PO2, PO3, PO4

TEXT BOOKS:

1. C.L. Wadhwa, —Electrical Power Systems, New Age International Pub. Co, Third Edition, 2001.
2. D.P. Kothari and I.J. Nagrath, —Modern Power System Analysis, Tata Mc Graw Hill Pub. Co., New Delhi, Fourth edition, 2011.
3. B.R.Gupta, —Power System Analysis and Design, S.Chand Publishing, 1998.

REFERENCE BOOKS:

1. Chakrabarti, M.L. Soni, P.V. Gupta, U.S. Bhatnagar, —A Text book on Power System Engineering, Dhanpat Rai Publishing Company (P) Ltd, 2008.
2. John J. Grainger & W.D. Stevenson, —Power System Analysis, Mc Graw Hill International, 1994.
3. Hadi Sadat, —Power System Analysis, Tata Mc Graw Hill Pub. Co. 2002.
4. W.D. Stevenson, —Elements of Power system Analysis, McGraw Hill International Student Edition.

REFERENCE WEBSITE:

1. https://onlinecourses.nptel.ac.in/noc22_ee17/preview

CO-PO MAPPING:

CO-PO	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12
CO1	3	3	3	3	-	-	-	-	-	-	-	-
CO2	3	3	3	3	-	-	-	-	-	-	-	-
CO3	3	3	3	-	-	-	-	-	-	-	-	-
CO4	3	3	2	3		-	-	-	-	-	-	-
CO5	3	3	2	3	-	-	-	-	-	-	-	-
CO*	3	3	2.6	3	-	-	-	-	-	-	-	-

OBJECTIVES OF POWER SYSTEMS-II:

- To understand the fundamental concepts and parameters of electrical transmission lines.
- To analyze the performance of short, medium, and long transmission lines.
- To study the effects of Ferranti effect, charging current, and transmission line capacitance.
- To understand the construction and performance of overhead line insulators and improve string efficiency.
- To analyze sag and tension in overhead transmission lines under different conditions.
- To study corona, critical voltages, corona power loss, and radio interference.
- To understand the causes and characteristics of power system transients and surges.
- To analyze surge propagation using Bewley lattice diagrams.
- To study various methods of voltage control and power factor improvement.
- To understand load compensation and reactive power compensation techniques for improving power system performance.

INTRODUCTION:

Power Systems-II deals with the study and analysis of electrical power transmission systems and their operating characteristics. The course is organized into five units covering the following major areas:

- **Unit-I – Transmission Line Parameters:** Study of transmission line conductors, resistance, inductance, capacitance, GMR, GMD, skin effect, proximity effect, and transposition of conductors.
- **Unit-II – Performance of Transmission Lines:** Analysis of short, medium, and long transmission lines using different mathematical models, along with Ferranti effect and charging current.
- **Unit-III – Overhead Line Insulators:** Study of types of insulators, voltage distribution, string efficiency, capacitance grading, sag and tension, corona, corona power loss, and radio interference.
- **Unit-IV – Power System Transients:** Understanding travelling waves and surge propagation, attenuation, distortion, reflection, refraction, and analysis using Bewley lattice diagrams.
- **Unit-V – Voltage Control and Power Factor Improvement:** Study of voltage control methods, shunt and series capacitors and inductors, tap-changing transformers, synchronous phase modifiers, power factor improvement, and load compensation techniques.

Overall, the course enables students to develop a strong understanding of **transmission line analysis, overhead transmission systems, power system transients, voltage regulation, power factor improvement, and compensation techniques** for efficient and reliable power system operation.



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POWER SYSTEMS-II

UNIT-1: Transmission line Parameters

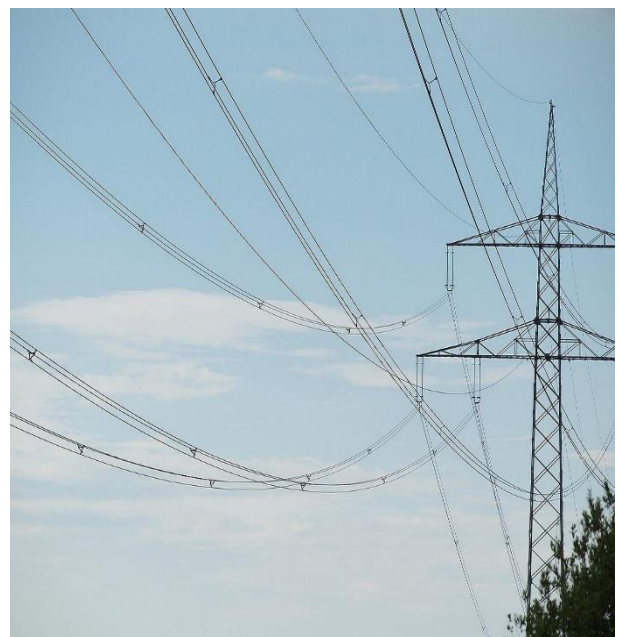
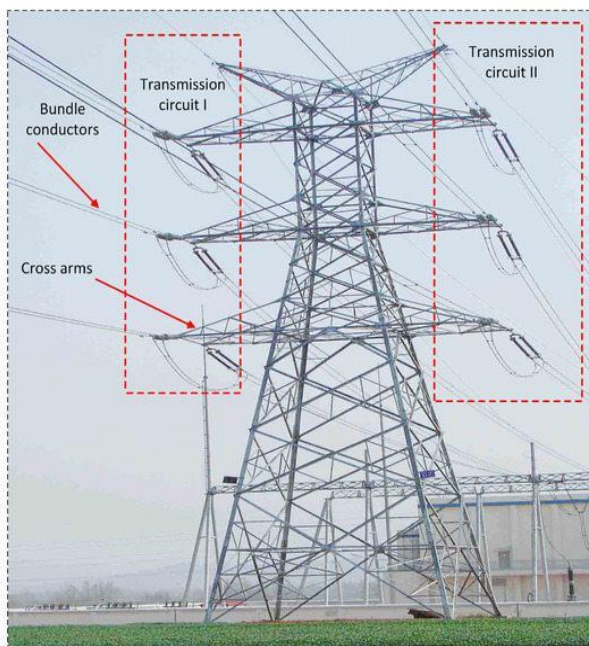
Types of Conductors – Calculation of Resistance for Solid Conductors, Bundle Conductors, Skin Effect, Proximity effect, Concept of GMR & GMD – Transposition of Power Lines – Calculation of inductance for Single phase and three phase, Single and Double circuit lines, Symmetrical and asymmetrical conductor configurations with and without transposition. Calculation of Capacitance for 2 wire and 3 wire systems, effect of ground on Capacitance, Capacitance calculations for symmetrical and asymmetrical single and three phase, single and double circuit lines, Numerical Problems.

INTRODUCTION TO TRANSMISSION LINE PARAMETERS:

- A Transmission Line has four parameters: Resistance, Inductance, Capacitance, and Shunt Conductance.
- The Series Resistance is very important in Transmission efficiency evaluation and in economic studies.
- The Power Transmission capacity of a line is mainly governed by the series inductance.
- Shunt Capacitance causes a charging current to flow in the line and is assumed importance in medium and long lines.
- Shunt Conductance accounts for leakage currents flowing across the insulators and ionized pathways in the air.

TYPES OF CONDUCTORS:

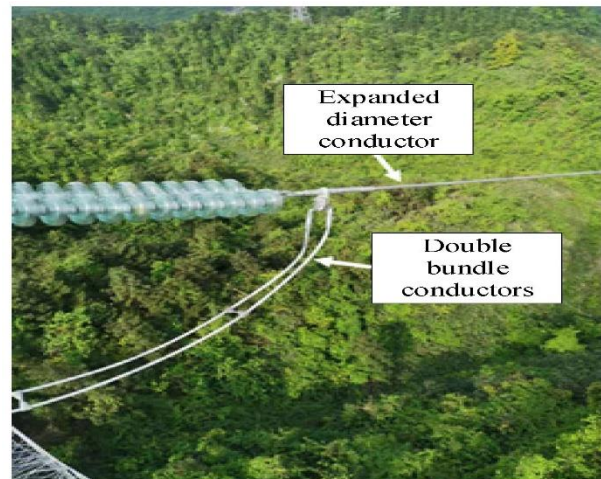
A conductor is a material used in transmission lines to carry electrical energy from generating stations to substations and consumers. The selection of a conductor depends on factors such as electrical conductivity, tensile strength, weight, cost, and resistance to corrosion.





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1. Copper Conductors

Copper was the first material widely used in transmission lines because of its excellent electrical properties.

Construction

- Made of stranded copper wires twisted together.
- Stranding increases flexibility and mechanical strength.

Properties

- Very high electrical conductivity.
- High tensile strength.
- Good resistance to corrosion.
- Heavy in weight.

Advantages

- Low resistance and lower power losses.
- Long service life.
- Good mechanical strength.

Disadvantages

- High cost.
- Heavier than aluminium conductors.
- Requires stronger support structures.

Applications

- Used in underground cables and special applications.
- Rarely used in modern overhead transmission lines due to high cost.



2. AAC (All Aluminium Conductor)

AAC consists entirely of aluminium strands without any reinforcement.

Construction

- Made of several aluminium wires stranded concentrically.

Properties

- Lightweight.
- Good conductivity.



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- Lower tensile strength than copper.

Advantages

- Economical.
- Easy to install because of low weight.
- Good corrosion resistance.

Disadvantages

- Lower mechanical strength.
- Larger sag due to lower tensile strength.
- Not suitable for long spans.

Applications

- Urban distribution systems.
- Short-distance transmission lines.



3. AAAC (All Aluminium Alloy Conductor)

AAAC is made from aluminium-magnesium-silicon alloy.

Construction

- Entire conductor consists of aluminium alloy strands.

Properties

- Better strength than AAC.
- Good conductivity.
- Excellent corrosion resistance.

Advantages

- Higher strength-to-weight ratio.
- Better corrosion resistance.
- Longer service life in coastal regions.



Disadvantages

- Slightly lower conductivity than pure aluminium.
- More expensive than AAC.

Applications

- Coastal areas.
- Industrial and polluted environments.
- Medium transmission distances.

4. ACSR (Aluminium Conductor Steel Reinforced)

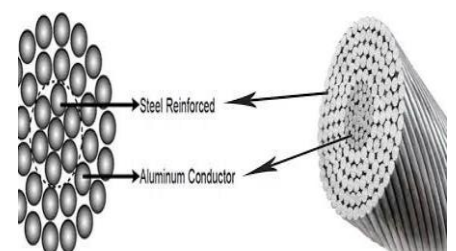
ACSR is the most commonly used conductor in overhead transmission systems.

Construction

- A central steel core provides mechanical strength.
- Aluminium strands are wound around the steel core to carry current.

Properties

- High tensile strength.
- Good electrical conductivity.
- Suitable for long spans.





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Advantages

- Can support long distances between towers.
- Lower sag compared to AAC.
- Economical and reliable.

Disadvantages

- Steel core increases weight.
- Steel may corrode in highly corrosive environments.

Applications

- Long-distance overhead transmission lines.
- High-voltage and extra-high-voltage systems.

Example

A typical ACSR conductor may have:

- Steel core: 7 strands
- Aluminium layer: 26 strands

5. ACAR (Aluminium Conductor Alloy Reinforced)

ACAR combines aluminium strands with an aluminium alloy core.

Construction

- Outer layers consist of aluminium strands.
- Inner core consists of aluminium alloy strands.

Properties

- Better conductivity than ACSR.
- Good mechanical strength.
- Lightweight.

Advantages

- Higher current carrying capacity.
- Better corrosion resistance than ACSR.
- Lower weight.

Disadvantages

- More expensive than ACSR.
- Less commonly used.

Applications

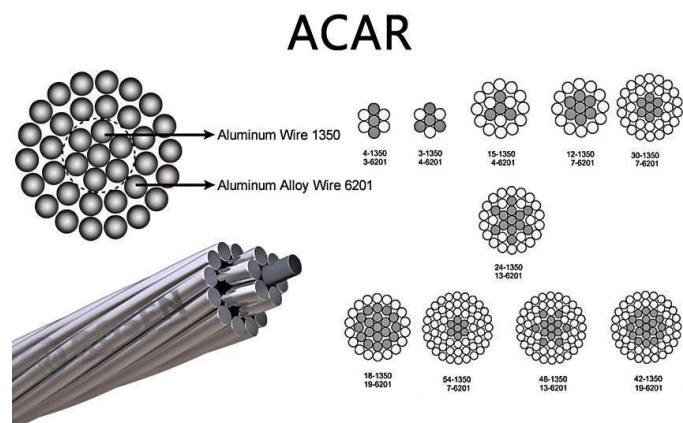
- Medium and long-distance transmission lines.
- Areas requiring high conductivity and moderate strength.

6. Bundle Conductors

Bundle conductors consist of two or more sub-conductors per phase separated by spacers.

Construction

- Each phase contains multiple conductors connected electrically in parallel.
- Spacers maintain fixed spacing between sub-conductors.





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Types

- Twin bundle conductor (2 conductors per phase)
- Triple bundle conductor (3 conductors per phase)
- Quad bundle conductor (4 conductors per phase)

Advantages

- Reduces corona loss.
- Reduces radio interference.
- Increases current carrying capacity.
- Reduces inductive reactance.
- Improves transmission efficiency.



Disadvantages

- Higher installation cost.
- Requires spacers and additional hardware.

Applications

- Extra High Voltage (EHV) lines (220 kV, 400 kV).
- Ultra High Voltage (UHV) transmission systems.

Note: Among all types, ACSR is the most widely used conductor for overhead transmission lines because it provides an excellent combination of high conductivity, very high mechanical strength, medium cost, and suitability for long-distance transmission.

Comparison of Conductors:

Conductor Type	Conductivity	Mechanical Strength	Weight	Cost	Typical Use
Copper	Very High	High	High	Very High	Underground cables
AAC	High	Low	Low	Low	Distribution lines
AAAC	High	Medium	Low	Medium	Coastal areas
ACSR	High	Very High	Medium	Medium	Long-distance transmission
ACAR	Very High	High	Low	High	Special transmission applications
Bundle Conductor	Very High	Very High	High	High	EHV and UHV transmission

CALCULATION OF RESISTANCE FOR SOLID CONDUCTORS:

- The Resistance of a Solid conductor is calculated using the formula;

$$R = \frac{\rho L}{A}$$

Where: R= Resistance of the conductor (Ω)

ρ (rho)= Resistivity of the conductor material (Ω -m)

L= Length of the conductor (m)

A= Cross-sectional area of the conductor (m^2)

- For Solid Circular Conductor:

$$A = \frac{\pi d^2}{4}$$

Where: d=Diameter of the conductor (m)



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- Therefore, the resistance formula becomes:

$$R = \frac{4\rho L}{\pi d^2}$$

Factors Affecting Resistance:

- 1) Length (L): Resistance increases with increase in the Length.
 - 2) Area (A): Resistance decreases with increase in cross-sectional area.
 - 3) Material Resistivity (ρ): Different materials have different resistivities.
 - 4) Temperature: Resistance of conductors increases with temperature
- For Small Change in Temperature, the resistance increases linearly, and the resistance at a temperature T is given by
 - $R_T = R_o(1 + \alpha_o \cdot T)$
where, R_T = resistance at $T^\circ C$
 R_o = resistance at $0^\circ C$
 α_o = temperature co-efficient of resistance at $0^\circ C$.
 - By using the above equation, the resistance R_2 at a temperature $T_2^\circ C$ can be found if the resistance R_1 at a temperature $T_1^\circ C$ is known, i.e.,
$$\frac{R_2}{R_1} = \frac{T_2 + 1/\alpha_o}{T_1 + 1/\alpha_o}$$

for aluminium $1/\alpha_o = 228^\circ C$
 - So above equation can be written as

$$\frac{R_2}{R_1} = \frac{T_2 + T}{T_1 + T}$$

Example:

1. A Solid Cylindrical aluminium conductor 20 km long has an area of 397500 circular mils. Obtain the conductor resistance at a) $20^\circ C$ and b) $40^\circ C$. The Resistivity of aluminium at $20^\circ C$ is $2.8 \times 10^{-8} \Omega m$.

Sol: Given data:

Resistivity of Aluminium $\rho = 2.8 \times 10^{-8} \Omega m$

Length L = 20 km = 20 x 1000m = 20000m

Area = 397500 circular mils [1 circular mils = $\frac{1}{1973.5 \times 10^6} m^2$]

$$= \frac{397500}{1973.5 \times 10^6} m^2 = 201.418 \times 10^{-6} m^2$$

$T_1 = 20^\circ C, T_2 = 40^\circ C$

a) Resistance $R = \frac{\rho L}{A} = \frac{2.8 \times 10^{-8} \times 20000}{201.418 \times 10^{-6}} = 2.78 \Omega$

b) Resistance at $40^\circ C$ (R_2) = $R_1 \left(\frac{T_2 + T}{T_1 + T} \right) = 2.78 \left(\frac{228 + 40}{228 + 20} \right) = 3.0042 \Omega$



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CALCULATION OF RESISTANCE FOR BUNDLE CONDUCTORS:

A Bundle conductor consists of two or more sub-conductors connected in parallel and used as a single phase conductor in high voltage transmission lines. Since the sub-conductors are connected in parallel, the total resistance of the bundle is reduced.

If a bundle contains 'n' identical sub-conductors, each having resistance R,

$$R_b = \frac{R}{n} = \frac{\rho L}{nA}$$

Where: R_b = Resistance of Bundle conductor

R = Resistance of One conductor

n = Number of Sub-conductors in the bundle.

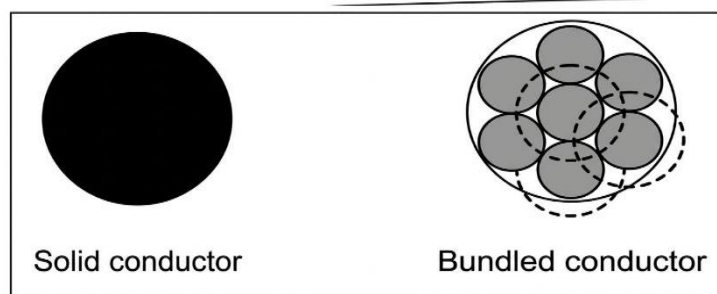
SKIN EFFECT:

The Tendency of alternating Current to concentrate near the surface of a conductor is called **Skin Effect**.

The Direct Current distribution in a conductor is uniform. But the Alternating current distribution in a conductor is not uniform. The Current density near the surface is more than near to the center of the conductor.

It is affected by the frequency of the current. If the frequency of current is more, the current distribution is more non-uniform. This effect is called as **Skin Effect**.

Consider a solid current carrying conductor and bundled conductor occupy the same cross sectional area as shown



Let R be the resistance of a solid conductor.

Let I be the current through the conductor.

Loss of a solid conductor = $I^2 R$

Let 'n' be the number of strands.

Resistance of each strand = nR [Ω]

Current through each strand = $\frac{I}{n}$ Amp

Let us assume half of the conductors ($n/2$) carry currents of $\left(\frac{I}{n} + \Delta I\right)$ Amp and other half of the conductor ($n/2$) carry current of $\left(\frac{I}{n} - \Delta I\right)$ Amp.

Total Loss = $\frac{n}{2} \left[\frac{I}{n} + \Delta I \right]^2 nR + \frac{n}{2} \left[\frac{I}{n} - \Delta I \right]^2 nR$



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$$\begin{aligned} &= \frac{n^2 R}{2} \left[\left(\frac{I}{n} \right)^2 + \frac{2I\Delta I}{n} + \Delta I^2 + \left(\frac{I}{n} \right)^2 - \frac{2I\Delta I}{n} + \Delta I^2 \right] \\ &= \frac{n^2 R}{2} \left[2 \left(\frac{I}{n} \right)^2 + 2 \Delta I^2 \right] = I^2 R + n^2 R \Delta I^2 > I^2 R \end{aligned}$$

Thus the effective resistance of A.C will be more than D.C if current distribution is not uniform.

The Inductance of each strand vary according to its position. The strand near the Centre are surrounded by a greater magnetic flux and hence larger inductance. Flux linking the strands decreases as we move towards the outer strand. Therefore the outer strands conduct more A.C. than the inner strand.

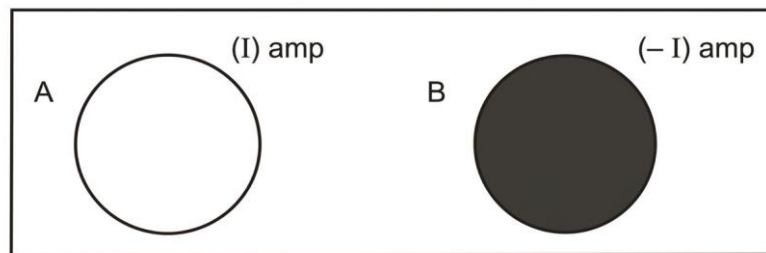
Skin effect is negligible for frequency < 50 Hz and the conductor diameter < 1 Cm.

Skin effect depends on the conductor size, frequency, resistivity and permeability of the conductor material.

PROXIMITY EFFECT:

Proximity effect is the phenomenon in which the alternating magnetic field produced by current flowing in a nearby conductor causes a non-uniform distribution of current in a conductor. This results in circulating currents within the conductor and an increase in its effective resistance.

Consider two conductors carrying alternating currents. The magnetic flux produced by one conductor links with the neighbouring conductor and affects the current distribution inside it. As a result, the current density becomes higher in some regions of the conductor and lower in others instead of remaining uniform. Due to this uneven distribution, the AC resistance of the conductor becomes greater than its DC resistance.



The proximity effect is generally negligible in overhead transmission lines because the conductors are spaced far apart. However, it becomes significant in underground cables where conductors are placed close to each other.

The severity of proximity effect depends upon:

- Distance between conductors
- Size of the conductor
- Frequency of the current
- Resistivity of the conductor material
- Magnetic permeability of the conductor material



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CONCEPT OF GMR & GMD:

1. Geometric Mean Radius (GMR): The Geometric Mean Radius (GMR) is the equivalent radius of a conductor used in the calculation of inductance in transmission lines. It takes into account the effect of the magnetic flux inside the conductor.

For a solid conductor of radius r , the GMR is given by:

$$r' = r e^{-1/4} = 0.7788r$$

Where:

- r = actual radius of the conductor
- r' = equivalent radius or GMR or D_s

The factor **0.7788** accounts for the internal flux linkage within the conductor.

Importance of GMR

- Used in the calculation of **inductance** of transmission lines.
- Important for determining the inductive reactance of the line.
- For bundled conductors, GMR helps in calculating the equivalent conductor radius.

2. Geometric Mean Distance (GMD): The Geometric Mean Distance (GMD) is the equivalent spacing between conductors in a transmission line. It is also called the Mutual Geometric Mean Distance (MGMD).

For three phase transmission line with conductor spacing's D_{ab} , D_{bc} , and D_{ca} , the GMD is given as

$$(D_{ab} * D_{bc} * D_{ca})^{1/3}$$

Where:

- D_{ab} = distance between conductors A and B
- D_{bc} = distance between conductors B and C
- D_{ca} = distance between conductors C and A

For Equilateral spacing $GMD = D$

Where (D) is the spacing between any two conductors.

Importance of GMD

- Used in the calculation of **inductance and capacitance** of transmission lines.
- Represents the average distance between conductors.
- Helps determine transmission line parameters accurately.

$$\begin{aligned} \text{Therefore, Inductance per phase per meter} &= L = 2 * 10^{-7} \ln \left(\frac{GMD}{GMR} \right) \\ &= 2 * 10^{-7} \ln \left(\frac{D}{D_s} \right) \quad \text{H/m} \end{aligned}$$

$$\text{Loop Inductance or Total Inductance} = 2 * L = 4 * 10^{-7} \ln \left(\frac{D}{D_s} \right) \quad \text{H/m}$$

$$\text{Inductance per phase in mH/km} = 2 * 10^{-7} \ln \left(\frac{D}{D_s} \right) * \frac{1000}{10^{-3}} = 0.2 \ln \left(\frac{D}{D_s} \right) \quad \text{mH/km}$$

$$\text{Loop Inductance} = 2 * L = 0.4 \ln \left(\frac{D}{D_s} \right) \quad \text{mH/km}$$

$$\text{Reactance per phase in ohm/m} = 2\pi f L = 2\pi f * 2 * 10^{-7} \ln \left(\frac{D}{D_s} \right) \quad \text{ohm/m}$$



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Problem:

1. Find the Loop inductance and reactance per km of a single phase overhead line consisting of two conductors, each 4.4755cm diameter. The Spacing between conductors is 3 m.

Sol: Diameter = 4.4755 cm;

Spacing or Distance between two conductor is $D = 3$ m

Radius(r) = Diameter/2 = 4.4755/2 = 2.23755 cm = 2.23755×10^{-2} m

Inductance of conductor/ph, $L_a = 2 \times 10^{-7} \ln \left(\frac{D}{D_s} \right) = 2 \times 10^{-7} \ln \left[\frac{D}{r'} \right]$ H/m

$$= 2 \times 10^{-7} \ln \left(\frac{D}{0.7788 r} \right)$$
$$= 2 \times 10^{-7} \ln \left[\frac{3}{0.7788 \times 2.23775 \times 10^{-2}} \right]$$

$$= 2 \times 10^{-7} \times 5.1483 \text{ H/m}$$

$$= 1.0296 \times 10^{-6} \text{ H/m}$$

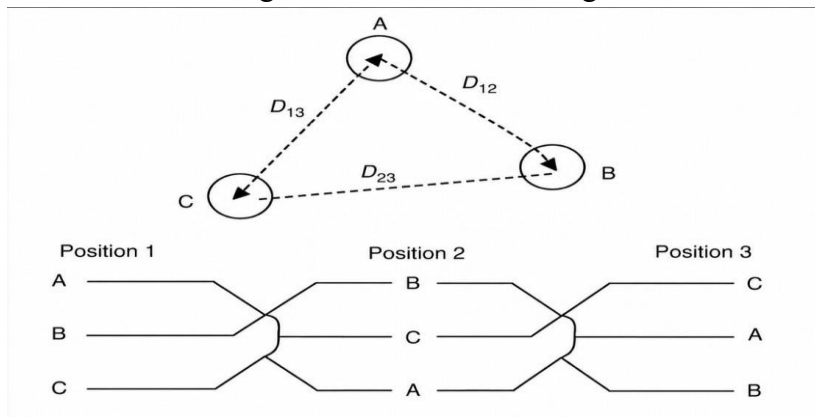
$$= 1.0296 \text{ mH/km}$$

Loop Inductance = $2 \times L_a = 2 \times 1.0296 = 2.0592 \text{ mH/km}$

Reactance per kilometre (X) = $2\pi \times 50 \times 2.0592 \times 10^{-3} \text{ H/km}$
 $= 0.647 \text{ } \Omega/\text{km}$

TRANSPOSITION OF POWER LINES:

To Avoid the unbalancing effect due to unsymmetrical spacing, interchanging the position of phase conductors at regular intervals along with the line so that each conductor occupy other position over an equal distance is called as Transposition of Conductors. Transposition of conductor's arrangement is shown in the figure



Transposition makes it possible for each phase conductor to have the same average inductance. Modern transmission lines are not transposed at regular intervals although an interchange in the positions of the conductors is made at the switching stations to balance the inductance.

If the lines are not transposed, the flux linkages and inductance of each phase will not be the same. Hence voltage drop in the transmission line will not be the same even though



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balanced current flows. And also external flux outside the conductors is not zero, so radio interference occurs.

If the line is transposed each line will take all the three positions for the one third length of the line.

$$\text{Average value of inductance } L = \frac{L_A + L_B + L_C}{3}$$

Average value of flux linkages λ is found by calculating the flux linkages for each position occupied by the conductor and then finding the average flux linkages.

Flux linkages of conductor A is in position 1, conductor B is in position 2 and conductor C is in position 3.

$$\lambda_{A1} = 2 \times 10^{-7} \left[I_A \ln \left(\frac{1}{r^1} \right) + I_B \ln \left(\frac{1}{D_{12}} \right) + I_C \ln \left(\frac{1}{D_{13}} \right) \right] \text{ Wb-turn/m}$$

Flux linkages of conductor A is in position 2, conductor B is in position 3 and conductor C is in position 1.

$$\lambda_{A2} = 2 \times 10^{-7} \left[I_A \ln \left(\frac{1}{r^1} \right) + I_B \ln \left(\frac{1}{D_{23}} \right) + I_C \ln \left(\frac{1}{D_{21}} \right) \right] \text{ Wb-turn/m}$$

Flux linkages of conductor A is in position 3, conductor B is in position 1 and conductor C is in position 2.

$$\lambda_{A3} = 2 \times 10^{-7} \left[I_A \ln \left(\frac{1}{r^1} \right) + I_B \ln \left(\frac{1}{D_{31}} \right) + I_C \ln \left(\frac{1}{D_{32}} \right) \right] \text{ Wb-turn/m}$$

Average flux linkages of conductor A,

$$\begin{aligned} \lambda_A &= \frac{\lambda_{A1} + \lambda_{A2} + \lambda_{A3}}{3} \\ &= \frac{2 \times 10^{-7}}{3} \left[3I_A \ln \left(\frac{1}{r^1} \right) + I_B \ln \left(\frac{1}{D_{12} \cdot D_{23} \cdot D_{31}} \right) + I_C \ln \left(\frac{1}{D_{13} \cdot D_{21} \cdot D_{32}} \right) \right] \\ &= \frac{2 \times 10^{-7}}{3} \left[3I_A \ln \left(\frac{1}{r^1} \right) + \ln \left(\frac{1}{D_{12} \cdot D_{23} \cdot D_{31}} \right) + (I_B + I_C) \right] \end{aligned}$$

For balanced condition,

$$I_A + I_B + I_C = 0$$

$$I_B + I_C = -I_A$$

$$\begin{aligned} \text{Therefore, } \lambda_A &= \frac{2 \times 10^{-7}}{3} \left[3I_A \ln \left(\frac{1}{r^1} \right) + \ln \left(\frac{1}{D_{12} \cdot D_{23} \cdot D_{31}} \right) + (-I_A) \right] \\ &= 2 \times 10^{-7} I_A \ln \left[\frac{(D_{12} \cdot D_{23} \cdot D_{31})^{1/3}}{r^1} \right] \end{aligned}$$

Inductance of phase A, $L_A = \frac{\lambda_A}{I_A}$

$$= 2 \times 10^{-7} \ln \left[\frac{\sqrt[3]{D_{12} \cdot D_{23} \cdot D_{31}}}{r^1} \right] \text{ H/m}$$

$$= 2 \times 10^{-7} \ln \left[\frac{GMD}{D_S} \right] = 2 \times 10^{-7} \ln \left[\frac{D_M}{D_S} \right]$$

$$L_B = L_C = L_A \quad [\text{Inductances are same for Transposed lines}]$$

The quantity $\sqrt[3]{D_{12} D_{23} D_{31}}$ is called the equivalent spacing and is denoted by D_{eq} .

Geometric mean distance of the lines = Mutual GMD = $D_M = D_{eq} = \sqrt[3]{D_{12} \cdot D_{23} \cdot D_{31}}$

For equilateral spacing (Symmetrical lines), $D_M = D$

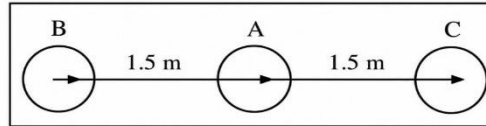


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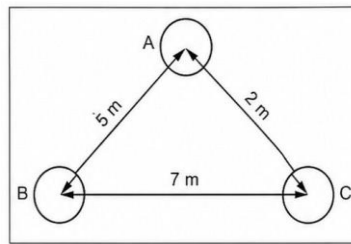
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Practice Problem:

1. Determine the inductance of a 3 phase line, operating at 50Hz and the conductors are transposed and arranged as shown in the figure. The Conductors diameter is 0.7 cm.



2. Calculate the conductance of a 3 phase line operating at 50 Hz and the conductors are transposed and arranged as shown in the figure. The conductor diameter is 2.3222 cm.



CALCULATION OF INDUCTANCE FOR SINGLE PHASE AND THREE PHASE LINES:

1. CALCULATION OF SINGLE PHASE LINES:

Consider the cross sections of two solid round conductor of a single-phase line as shown in the figure.

Let r_1 , r_2 be the radius of phase and neutral conductors.

Let D be the spacing between centre to centre of two conductors.

Let I_1 be the current carried by the phase conductor.

Let I_2 be the current carried by the neutral conductor.

Neutral conductor is the return conductor. Therefore the direction of current flowing in the neutral conductor is opposite to that of phase conductor.

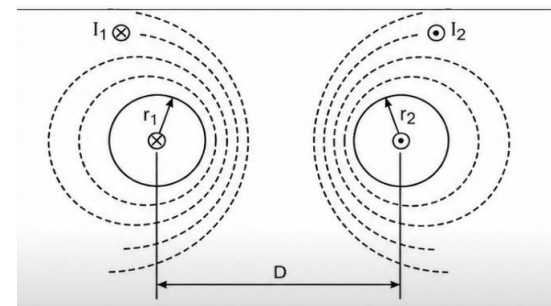
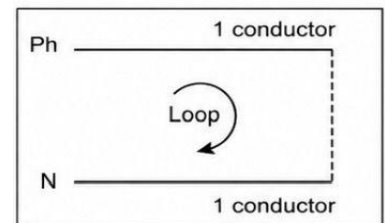
$$I_2 = -I_1$$

Currents I_1 and I_2 always set up magnetic fields. These fields surrounding the two conductors always reinforce each other in the space between them as shown in the figure.

The Two parallel conductors form a rectangular loop of one turn through which flux is produced by currents in the two conductors. Since the flux links the loop, the loop possesses inductance.

Inductance of conductor 1 due to internal flux and due to external flux linkage. The flux beyond D links a net current of zero and does not contribute to the net magnetic flux linkages in the circuit.

$$\begin{aligned} \text{Total flux linkages } \lambda_1 &= \lambda_{int} + \lambda_{ext} \\ &= \frac{\mu_0 I}{8\pi} + \frac{\mu_0 I}{2\pi} \int_{r_1}^D \frac{dx}{x} \\ &= \frac{\mu_0 I}{2\pi} \left[\frac{1}{4} + \int_{r_1}^D \frac{dx}{x} \right] \end{aligned}$$





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Total inductance of conductor 1, $L_1 = \frac{\lambda}{I}$

= {Inductance due to internal flux} + {Inductance due to external flux}

$$= \frac{\mu_0}{2\pi} \left[\frac{1}{4} + \ln \frac{D}{r_1} \right]$$

$$= \frac{4\pi \times 10^{-7}}{2\pi} \left[\frac{1}{4} + \ln \frac{D}{r_1} \right]$$

$$= 2 \times 10^{-7} \left[\ln \left[\frac{1}{r_1 e^{-1/4}} \right] + \ln D \right]$$

Let $r'_1 = r_1 e^{-1/4}$, $L_1 = 2 \times 10^{-7} \left[\ln \frac{1}{r'_1} + \ln D \right]$ H/m

$$= 2 \times 10^{-7} \left(\ln \frac{D}{r'_1} \right) \quad \text{H/m}$$

Similarly, Inductance of Conductor 2 is

$$L_2 = 2 \times 10^{-7} \left(\ln \frac{D}{r'_2} \right) \quad \text{H/m}$$

$$\text{Where, } r'_2 = r_2 e^{-1/4}$$

If the two conductors are identical then,

$$r_1 = r_2 = r \text{ And}$$

$$L_1 = L_2 = L$$

Inductance per phase per meter length of the line $L = 2 \times 10^{-7} \left(\ln \frac{D}{r'_1} \right)$ H/m

$$r'_1 = r e^{-1/4} = 0.7788 r$$

Loop Inductance of conductor = $2L = 4 \times 10^{-7} \ln \left(\frac{1}{r'_1} \right) + 4 \times 10^{-7} \ln D$

From the equation, the first term is only a function of the conductor radius. This term is considered as the inductance due to internal flux and external flux due to conductor 1. The second term is dependent upon conductor spacing (D). This term is known as the inductance spacing factor.

2. CALCULATION OF THREE PHASE LINES:

➤ The three conductors A, B and C of a 3-phase line carrying currents I_A , I_B and I_C respectively.

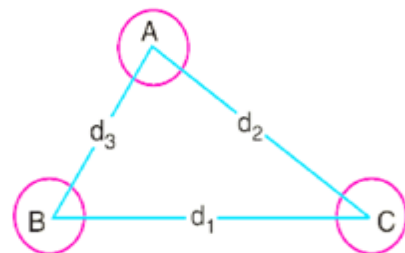
➤ Let d_1 , d_2 and d_3 be the spacings between the conductors as shown.

➤ Let us further assume that the loads are balanced i.e. $I_A + I_B + I_C = 0$.

➤ Consider the flux linkages with conductor A.

➤ There will be flux linkages with conductor A due to its own current and also due to the mutual inductance effects of I_B and I_C .

• Flux linkages with conductor A due to its own current



$$= \frac{\mu_0 I_A}{2\pi} \left(\frac{1}{4} + \int_r^\infty \frac{dx}{x} \right) \quad \dots(i)$$



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- Flux linkages with conductor A due to current I_B $= \frac{\mu_0 I_B}{2\pi} \int_{d_3}^{\infty} \frac{dx}{x}$... (ii)
- Flux linkages with conductor A due to current I_C $= \frac{\mu_0 I_C}{2\pi} \int_{d_2}^{\infty} \frac{dx}{x}$... (iii)

• Total flux linkages with conductor A is $\Psi_A = (i) + (ii) + (iii)$

$$= \frac{\mu_0 I_A}{2\pi} \left(\frac{1}{4} + \int_r^{\infty} \frac{dx}{x} \right) + \frac{\mu_0 I_B}{2\pi} \int_{d_3}^{\infty} \frac{dx}{x} + \frac{\mu_0 I_C}{2\pi} \int_{d_2}^{\infty} \frac{dx}{x}$$

$$= \frac{\mu_0}{2\pi} \left[\left(\frac{1}{4} + \int_r^{\infty} \frac{dx}{x} \right) I_A + I_B \int_{d_3}^{\infty} \frac{dx}{x} + I_C \int_{d_2}^{\infty} \frac{dx}{x} \right]$$

$$= \frac{\mu_0}{2\pi} \left[\left(\frac{1}{4} - \log_e r \right) I_A - I_B \log_e d_3 - I_C \log_e d_2 + \log_e \infty (I_A + I_B + I_C) \right]$$

As $I_A + I_B + I_C = 0$, $\therefore \Psi_A = \frac{\mu_0}{2\pi} \left[\left(\frac{1}{4} - \log_e r \right) I_A - I_B \log_e d_3 - I_C \log_e d_2 \right]$

SYMMETRICAL AND ASYMMETRICAL CONDUCTOR CONFIGURATIONS WITH AND WITHOUT TRANSPOSITION:

i) Symmetrical Spacing:

- If the three conductors A, B and C are placed symmetrically at the corners of an equilateral triangle of side d , then, $d_1 = d_2 = d_3 = d$.
- Under such conditions, the flux linkages with conductor A become :

$$\begin{aligned} \Psi_A &= \frac{\mu_0}{2\pi} \left[\left(\frac{1}{4} - \log_e r \right) I_A - I_B \log_e d - I_C \log_e d \right] \\ &= \frac{\mu_0}{2\pi} \left[\left(\frac{1}{4} - \log_e r \right) I_A - (I_B + I_C) \log_e d \right] \\ &= \frac{\mu_0}{2\pi} \left[\left(\frac{1}{4} - \log_e r \right) I_A + I_A \log_e d \right] \quad (\because I_B + I_C = -I_A) \\ &= \frac{\mu_0 I_A}{2\pi} \left[\frac{1}{4} + \log_e \frac{d}{r} \right] \text{ weber-turns/m} \end{aligned}$$

Inductance of conductor A, $L_A = \frac{\Psi_A}{I_A} \text{ H/m} = \frac{\mu_0}{2\pi} \left[\frac{1}{4} + \log_e \frac{d}{r} \right] \text{ H/m}$

$$= \frac{4\pi \times 10^{-7}}{2\pi} \left[\frac{1}{4} + \log_e \frac{d}{r} \right] \text{ H/m} \quad L_A = 10^{-7} \left[0.5 + 2 \log_e \frac{d}{r} \right] \text{ H/m}$$

- Derived in a similar way, the expressions for inductance are the same for conductors B and C.

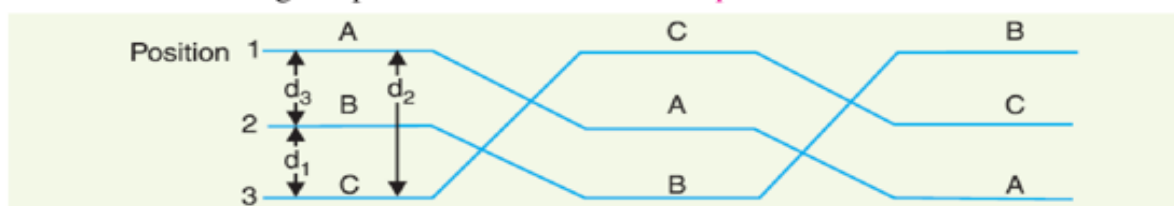


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ii) Unsymmetrical Spacing:

- When 3-phase line conductors are not equidistant from each other, the conductor spacing is said to be unsymmetrical.
- Under such conditions, the flux linkages and inductance of each phase are not the same.
- A different inductance in each phase results in unequal voltage drops in the three phases even if the currents in the conductors are balanced.
- Therefore, the voltage at the receiving end will not be the same for all phases.
- In order that voltage drops are equal in all conductors, we generally interchange the positions of the conductors at regular intervals along the line so that each conductor occupies the original position of every other conductor over an equal distance.
- Such an exchange of positions is known as *transposition*.



- The phase conductors are designated as A, B and C and the positions occupied are numbered 1, 2 and 3.
- The effect of transposition is that each conductor has the same average inductance.
- Above Fig. shows a 3-phase transposed line having unsymmetrical spacing.
- Let us assume that each of the three sections is 1 m in length. Let us further assume balanced conditions *i.e.*,

$$I_A + I_B + I_C = 0.$$

- Let the line currents be :

$$I_A = I(1 + j0)$$

$$I_B = I(-0.5 - j0.866)$$

$$I_C = I(-0.5 + j0.866)$$

- As proved above, the total flux linkages per metre length of conductor A is

$$\psi_A = \frac{\mu_0}{2\pi} \left[\left(\frac{1}{4} - \log_e r \right) I_A - I_B \log_e d_3 - I_C \log_e d_2 \right]$$

- Putting the values of I_A , I_B and I_C , we get,

$$\begin{aligned} \psi_A &= \frac{\mu_0}{2\pi} \left[\left(\frac{1}{4} - \log_e r \right) I - I(-0.5 - j0.866) \log_e d_3 - I(-0.5 + j0.866) \log_e d_2 \right] \\ &= \frac{\mu_0}{2\pi} \left[\frac{1}{4} I - I \log_e r + 0.5 I \log_e d_3 + j0.866 \log_e d_3 + 0.5 I \log_e d_2 - j0.866 I \log_e d_2 \right] \\ &= \frac{\mu_0}{2\pi} \left[\frac{1}{4} I - I \log_e r + 0.5 I (\log_e d_3 + \log_e d_2) + j0.866 I (\log_e d_3 - \log_e d_2) \right] \end{aligned}$$



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$$= \frac{\mu_0}{2\pi} \left[\frac{1}{4} I - I \log_e r + I^* \log_e \sqrt{d_2 d_3} + j 0.866 I \log_e \frac{d_3}{d_2} \right]$$

$$* \quad 0.5 I (\log_e d_3 + \log_e d_2) = 0.5 I \log_e d_2 d_3 = I \log_e (d_2 d_3)^{0.5} = I \log_e \sqrt{d_2 d_3}$$

$$= \frac{\mu_0}{2\pi} \left[\frac{1}{4} I + I \log_e \frac{\sqrt{d_2 d_3}}{r} + j 0.866 I \log_e \frac{d_3}{d_2} \right]$$

$$= \frac{\mu_0 I}{2\pi} \left[\frac{1}{4} + \log_e \frac{\sqrt{d_2 d_3}}{r} + j 0.866 \log_e \frac{d_3}{d_2} \right]$$

∴ Inductance of conductor A is

$$\begin{aligned} L_A &= \frac{\Psi_A}{I_A} = \frac{\Psi_A}{I} = \frac{\mu_0}{2\pi} \left[\frac{1}{4} + \log_e \frac{\sqrt{d_2 d_3}}{r} + j 0.866 \log_e \frac{d_3}{d_2} \right] \\ &= \frac{4\pi \times 10^{-7}}{2\pi} \left[\frac{1}{4} + \log_e \frac{\sqrt{d_2 d_3}}{r} + j 0.866 \log_e \frac{d_3}{d_2} \right] \text{ H/m} \\ &= 10^{-7} \left[\frac{1}{2} + 2 \log_e \frac{\sqrt{d_2 d_3}}{r} + j 1.732 \log_e \frac{d_3}{d_2} \right] \text{ H/m} \end{aligned}$$

- Similarly inductance of conductors B and C will be :

$$L_B = 10^{-7} \left[\frac{1}{2} + 2 \log_e \frac{\sqrt{d_3 d_1}}{r} + j 1.732 \log_e \frac{d_1}{d_3} \right] \text{ H/m}$$

$$L_C = 10^{-7} \left[\frac{1}{2} + 2 \log_e \frac{\sqrt{d_1 d_2}}{r} + j 1.732 \log_e \frac{d_2}{d_1} \right] \text{ H/m}$$

- Inductance of each line conductor $= \frac{1}{3} (L_A + L_B + L_C)$

$$= * \left[\frac{1}{2} + 2 \log_e \frac{\sqrt[3]{d_1 d_2 d_3}}{r} \right] \times 10^{-7} \text{ H/m}$$

$$= \left[0.5 + 2 \log_e \frac{\sqrt[3]{d_1 d_2 d_3}}{r} \right] \times 10^{-7} \text{ H/m}$$

- If we compare the formula of inductance of an unsymmetrically spaced transposed line with that of symmetrically spaced line, we find that inductance of each line conductor in the two cases will be equal if $d = \sqrt[3]{d_1 d_2 d_3}$
- The distance d is known as *equivalent equilateral spacing* for unsymmetrically transposed line.



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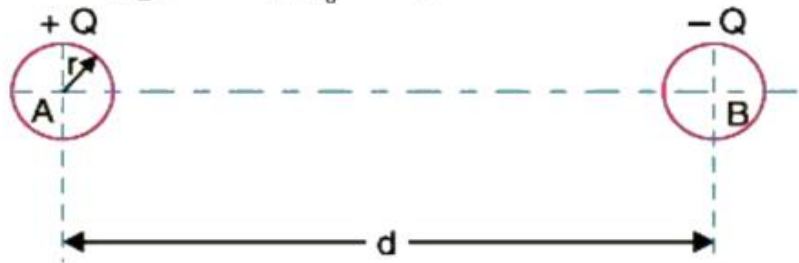
CALCULATION OF CAPACITANCE FOR 2-WIRE AND 3-WIRE SYSTEMS

1. Capacitance of Single Phase Two-Wire Line:

Consider a single phase overhead transmission line consisting of two parallel conductors A and B spaced d metres apart in air. Suppose that radius of each conductor is r metres. Let their respective charge be $+Q$ and $-Q$ coulombs per metre length. The total p.d. between conductor A and neutral “infinite” plane is

$$V_A = \int_r^{\infty} \frac{Q}{2\pi x \epsilon_0} dx + \int_d^{\infty} \frac{-Q}{2\pi x \epsilon_0} dx$$

$$= \frac{Q}{2\pi \epsilon_0} \left[\log_e \frac{\infty}{r} - \log_e \frac{\infty}{d} \right] \text{ volts} = \frac{Q}{2\pi \epsilon_0} \log_e \frac{d}{r} \text{ volts}$$



Similarly, p.d. between conductor B and neutral “infinite” plane is

Activate Win

$$V_B = \int_r^{\infty} \frac{-Q}{2\pi x \epsilon_0} dx + \int_d^{\infty} \frac{Q}{2\pi x \epsilon_0} dx$$

$$= \frac{-Q}{2\pi \epsilon_0} \left[\log_e \frac{\infty}{r} - \log_e \frac{\infty}{d} \right] = \frac{-Q}{2\pi \epsilon_0} \log_e \frac{d}{r} \text{ volts}$$

Both these potentials are w.r.t. the same neutral plane. Since the unlike charges attract each other, the potential difference between the conductors is

$$V_{AB} = 2V_A = \frac{2Q}{2\pi \epsilon_0} \log_e \frac{d}{r} \text{ volts}$$

Capacitance, $C_{AB} = Q/V_{AB} = \frac{Q}{\frac{2Q}{2\pi \epsilon_0} \log_e \frac{d}{r}} \text{ F/m}$

$$C_{AB} = \frac{\pi \epsilon_0}{\log_e \frac{d}{r}} \text{ F/m}$$



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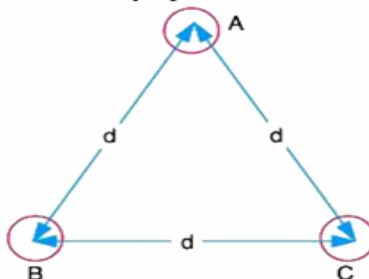
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2. Capacitance of 3-phase overhead lines:

In a 3-phase transmission line, the capacitance of each conductor is considered instead of capacitance from conductor to conductor. Here, again two cases arise viz., symmetrical spacing and unsymmetrical spacing.

(i) Symmetrical Spacing

Fig shows the three conductors A, B and C of the 3-phase overhead transmission line having charges Q_A , Q_B and Q_C per meter length respectively. Let the conductors be equidistant (d meters) from each other. We shall find the capacitance from line conductor to neutral in this symmetrically spaced line. Referring to Fig,



Overall potential difference between conductor A and infinite neutral plane is given by

$$\begin{aligned} V_A &= \int_r^\infty \frac{Q_A}{2\pi\epsilon_0} dx + \int_d^\infty \frac{Q_B}{2\pi\epsilon_0} dx + \int_d^\infty \frac{Q_C}{2\pi\epsilon_0} dx \\ &= \frac{1}{2\pi\epsilon_0} \left[Q_A \log_e \frac{1}{r} + Q_B \log_e \frac{1}{d} + Q_C \log_e \frac{1}{d} \right] \\ &= \frac{1}{2\pi\epsilon_0} \left[Q_A \log_e \frac{1}{r} + (Q_B + Q_C) \log_e \frac{1}{d} \right] \end{aligned}$$

Assuming balanced supply, we have, $Q_A + Q_B + Q_C = 0$

$$\therefore Q_B + Q_C = -Q_A$$

$$\therefore V_A = \frac{1}{2\pi\epsilon_0} \left[Q_A \log_e \frac{1}{r} - Q_A \log_e \frac{1}{d} \right] = \frac{Q_A}{2\pi\epsilon_0} \log_e \frac{d}{r} \text{ volts}$$

$\therefore$ Capacitance of conductor A w.r.t neutral,

$$\begin{aligned} C_A &= \frac{Q_A}{V_A} = \frac{Q_A}{\frac{Q_A}{2\pi\epsilon_0} \log_e \frac{d}{r}} \text{ F/m} = \frac{2\pi\epsilon_0}{\log_e \frac{d}{r}} \text{ F/m} \\ C_A &= \frac{2\pi\epsilon_0}{\log_e \frac{d}{r}} \text{ F/m} \end{aligned}$$

Note that this equation is identical to capacitance to neutral for two-wire line. Derived in a similar manner, the expressions for capacitance are the same for conductors B and C.

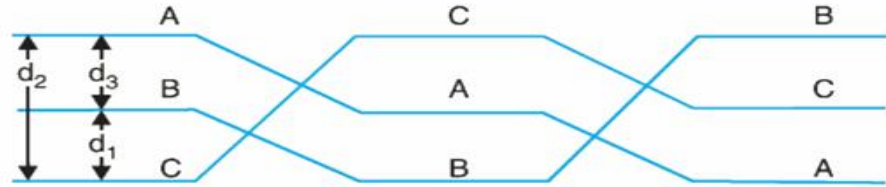


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(ii) Unsymmetrical spacing.

Fig. shows a 3-phase transposed line having unsymmetrical spacing. Let us assume balanced conditions i.e. $Q_A + Q_B + Q_C = 0$.



Considering all the three sections of the transposed line for phase A,

$$\text{Potential of 1st position, } V_1 = \frac{1}{2\pi\epsilon_0} \left(Q_A \log_e \frac{1}{r} + Q_B \log_e \frac{1}{d_3} + Q_C \log_e \frac{1}{d_2} \right)$$

$$\text{Potential of 2nd position, } V_2 = \frac{1}{2\pi\epsilon_0} \left(Q_A \log_e \frac{1}{r} + Q_B \log_e \frac{1}{d_1} + Q_C \log_e \frac{1}{d_3} \right)$$

$$\text{Potential of 3rd position, } V_3 = \frac{1}{2\pi\epsilon_0} \left(Q_A \log_e \frac{1}{r} + Q_B \log_e \frac{1}{d_2} + Q_C \log_e \frac{1}{d_1} \right)$$

Average voltage on conductor A is

$$\begin{aligned} V_A &= \frac{1}{3} (V_1 + V_2 + V_3) \\ &= \frac{1}{3 \times 2\pi\epsilon_0} \left[Q_A \log_e \frac{1}{r^3} + (Q_B + Q_C) \log_e \frac{1}{d_1 d_2 d_3} \right] \end{aligned}$$

As $Q_A + Q_B + Q_C = 0$, therefore, $Q_B + Q_C = -Q_A$

$$\begin{aligned} \therefore V_A &= \frac{1}{6\pi\epsilon_0} \left[Q_A \log_e \frac{1}{r^3} - Q_A \log_e \frac{1}{d_1 d_2 d_3} \right] \\ &= \frac{Q_A}{6\pi\epsilon_0} \log_e \frac{d_1 d_2 d_3}{r^3} \\ &= \frac{1}{3} \times \frac{Q_A}{2\pi\epsilon_0} \log_e \frac{d_1 d_2 d_3}{r^3} \\ &= \frac{Q_A}{2\pi\epsilon_0} \log_e \left(\frac{d_1 d_2 d_3}{r^3} \right)^{1/3} \\ &= \frac{Q_A}{2\pi\epsilon_0} \log_e \frac{(d_1 d_2 d_3)^{1/3}}{r} \end{aligned}$$

$$C_A = \frac{Q_A}{V_A} = \frac{2\pi\epsilon_0}{\log_e \frac{\sqrt[3]{d_1 d_2 d_3}}{r}} \text{ F/m}$$



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Problems :

1. Determine the capacitance and the charging current per km when the transmission line of example 2.2 is operating at 132 kV.

Solution: The radius of conductor = 0.4 cm.

The mutual GMD of conductors, $D_m = 2.015$ metres.

$$\begin{aligned}\therefore \text{Capacitance per phase per metre} &= \frac{2\pi\epsilon_0}{\ln \frac{2.015}{0.4} \times 10^2} \text{ F/metre} \\ &= \frac{10^{-9}}{18 \times \ln \frac{2.015}{0.4}} = 8.928 \text{ pF/metre} \\ &= 8.928 \times 10^{-12} \times 10^3 \text{ F/km} \\ &= 8.928 \times 10^{-9} \text{ F/km}\end{aligned}$$

$$\begin{aligned}\text{The charging current} &= \frac{132 \times 1000}{\sqrt{3}} \times 8.928 \times 10^{-9} \times 314 \\ &= 0.2136 \text{ amp/km. Ans.}\end{aligned}$$

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2. Determine the capacitance and the charging current per km when the transmission line of example 2.5 operates at 220 kV, dia of conductor = 2.5 cm.

Solution:

Mutual GMD = 6.61 metres

$$\begin{aligned}D_{s_1} &= \sqrt{1.25 \times 10^{-2} \times 10.965} = 0.3702 \text{ metre} = D_{s_3} \\ D_{s_2} &= \sqrt{1.25 \times 10^{-2} \times 9} = 0.3354 \text{ metre}\end{aligned}$$

$$\therefore D_s = \sqrt[3]{D_{s_1} D_{s_2} D_{s_3}} = \sqrt[3]{0.045965899} = 0.3582 \text{ metre}$$

$$\therefore \text{Capacitance per km} = \frac{10^{-6}}{18 \ln \frac{6.61}{0.3582}} = 0.019056 \text{ } \mu\text{F/km}$$

$$\begin{aligned}\therefore \text{Charging current per km} &= \frac{220 \times 1000}{\sqrt{3}} \times 314 \times 0.01905 \times 10^{-6} \\ &= 0.76 \text{ amp/km. Ans.}\end{aligned}$$

3. Determine the capacitance and charging current per km of the line of example 2.7 if the line operates at 220 kV, dia = 4.5 cms.

Solution:

$$D_s = \sqrt{2.25 \times 10^{-2} \times 0.4} = 0.094868 \text{ metre}$$

$$\therefore \text{Capacitance per km} = \frac{10^{-6}}{18 \ln \frac{8.19}{0.094868}} = 0.01246 \text{ } \mu\text{F}$$

$$\begin{aligned}\text{The charging current per km} &= \frac{220 \times 1000}{\sqrt{3}} \times 314 \times 0.01246 \times 10^{-6} \\ &= 0.497 \text{ amp. Ans.}\end{aligned}$$

2 Marks Questions:

1. Define the primary parameters of a transmission line and explain their significance.
2. Define GMR and GMD.
3. Write the expression for the inductance of a single-phase two-wire transmission line.
4. Write the expression for the capacitance of a single-phase two-wire transmission line.
5. What is transposition of conductors? Why is it necessary in transmission lines?
6. Define skin effect and mention the factors affecting it.
7. Define proximity effect and state its effect on transmission line performance.
8. Differentiate between GMR and GMD.
9. Explain the effect of conductor spacing on the inductance and capacitance of a transmission line.
10. What are bundled conductors? State their advantages in high-voltage transmission lines.

10 Marks Questions:

1. Discuss the following Conductors
a) Solid Conductor b) Stranded Conductor c) Bundled Conductor
2. Derive the Capacitance equation of 3-phase overhead line with symmetrical and unsymmetrical spacing.
3. Derive an expression for the inductance per phase of a 3-phase unsymmetrical line. Calculate the inductance of each conductor in a 3-phase, 3-wire system, when the conductors are arranged in a horizontal with spacing such that $D_{RY} = 4m$; $D_{YB} = 3m$; $D_{BR} = 2m$. the conductors are transposed and each has a diameter of 2.5 cm.
4. Derive the expression for capacitance of a single phase two-wire line system. Calculate the capacitance of a 100 km long 3-phase, 50Hz overhead transmission line consisting of 3 conductors, each of diameter 2 cm and spaced 2.5 m at the corner of an equilateral triangle.
5. A) Explain the Bundled Conductor with merits and Demerits.
B) Calculate the capacitance of a conductor per phase of a three phase 400 km long line, with the conductors spaced at the corners of an equilateral triangle of side 4 m and the diameter of each conductor being 2.5 cm.
6. Derive the inductance of a three-phase transmission line with unsymmetrical spacing and explain the process of transposition of conductors.
7. Explain the primary parameters of a transmission line—resistance, inductance, capacitance, and conductance—and discuss the factors affecting each parameter.

REFERENCES:

- ❖ C.L. Wadhwa, —Electrical Power Systems, New Age International Pub. Co, Third Edition, 2001.

“The efficiency of a power transmission system begins with the accurate analysis of its transmission line parameters.”



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UNIT-2: PERFORMANCE OF TRANSMISSION LINES

Classification of Transmission Lines – Short, medium and long line and their models representation – Nominal-T, Nominal- π and A, B, C, D Constants for symmetrical networks, Numerical problems and solutions for estimating regulation and efficiency of all types of lines. Ferranti effect and charging current.

CLASSIFICATION OF TRANSMISSION LINES - INTRODUCTION

The important considerations in the design and operation of a transmission line are the determination of voltage drop, line losses and efficiency of transmission. These values are greatly influenced by the line constants R , L and C of the transmission line. For instance the voltage drop in the line depends upon the values of above three line constants. Similarly, the resistance of transmission line conductors is the most important cause of power loss in the line and determines the transmission efficiency. In this chapter, we shall develop formulas by which we can calculate voltage regulation, line losses and efficiency of transmission lines. These formulas are important for two principal reasons. Firstly, they provide an opportunity to understand the effects of the parameters of the line on bus voltages and the flow of power. Secondly, they help in developing an overall understanding of what is occurring on electric power system.

CLASSIFICATION OF OVERHEAD TRANSMISSION LINES

A transmission line has three constants R , L and C distributed uniformly along the whole length of the line. The resistance and inductance form the series impedance. The capacitance existing between conductors for 1-phase line or from a conductor to neutral for a 3-phase line forms a shunt path throughout the length of the line. Therefore, capacitance effects introduce complications in transmission line calculations. Depending upon the manner in which capacitance is taken into account, the overhead transmission lines are classified as :

- (i) **Short transmission lines.** When the length of an overhead transmission line is up to about 50 km and the line voltage is comparatively low (< 20 kV), it is usually considered as a short transmission line. Due to smaller length and lower voltage, the capacitance effects are small and hence can be neglected. Therefore, while studying the performance of a short transmission line, only resistance and inductance of the line are taken into account.
- (ii) **Medium transmission lines.** When the length of an overhead transmission line is about 50-150 km and the line voltage is moderately high (>20 kV < 100 kV), it is considered as a medium transmission line. Due to sufficient length and voltage of the line, the capacitance effects are taken into account. For purposes of calculations, the distributed capacitance of the line is divided and lumped in the form of condensers shunted across the line at one or more points.
- (iii) **Long transmission lines.** When the length of an overhead transmission line is more than 150 km and line voltage is very high (> 100 kV), it is considered as a long transmission line. For the treatment of such a line, the line constants are considered uniformly distributed over the whole length of the line and rigorous methods are employed for solution.



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It may be emphasized here that exact solution of any transmission line must consider the fact that the constants of the line are not lumped but are distributed uniformly throughout the length of the line.

However, reasonable accuracy can be obtained by considering these constants as lumped for short and medium transmission lines.

Important Terms

While studying the performance of a transmission line, it is desirable to determine its voltage regulation and transmission efficiency. We shall explain these two terms in turn.

(i) Voltage regulation. When a transmission line is carrying current, there is a voltage drop in the line due to resistance and inductance of the line. The result is that receiving end voltage (V_r) of the line is generally less than the sending end voltage (V_s). This voltage drop ($V_s - V_r$) in the line is expressed as a percentage of receiving end voltage V and is called voltage regulation. The difference in voltage at the receiving end of a transmission line **between conditions of no load and full load is called **voltage regulation** and is expressed as a percentage of the receiving end voltage.

(ii) Transmission efficiency. The power obtained at the receiving end of a transmission line is generally less than the sending end power due to losses in the line resistance.

The ratio of receiving end power to the sending end power of a transmission line is known as the **transmission efficiency** of the line

PERFORMANCE OF SINGLE PHASE SHORT TRANSMISSION LINES

As stated earlier, the effects of line capacitance are neglected for a short transmission line. Therefore, while studying the performance of such a line, only resistance and inductance of the line are taken into account. The equivalent circuit of a single phase short transmission line is shown in Fig.

Here, the total line resistance and inductance are shown as concentrated or lumped instead of being distributed. The circuit is a simple AC. series circuit.

Let, I = load current

R = loop resistance i.e., resistance of both conductors

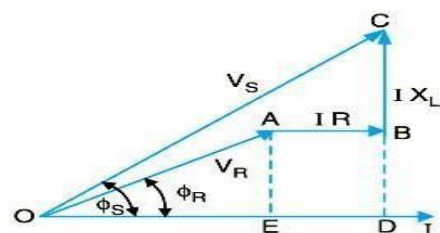
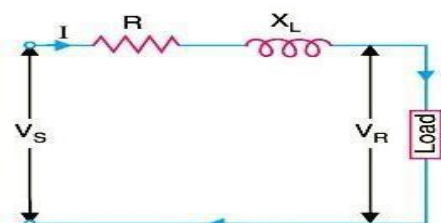
X_L = loop reactance

V_r = receiving end voltage

$\cos \phi_R$ = receiving end power factor (lagging)

V_s = sending end voltage

$\cos \phi_S$ = sending end power factor





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The *phasor diagram of the line for lagging load power factor is shown in Fig. From the right angled triangle ODC, we get,

$$\begin{aligned} \text{or} \quad (OC)^2 &= (OD)^2 + (DC)^2 \\ V_S^2 &= (OE + ED)^2 + (DB + BC)^2 \\ &= (V_R \cos \phi_R + IR)^2 + (V_R \sin \phi_R + IX_L)^2 \\ \therefore V_S &= \sqrt{(V_R \cos \phi_R + IR)^2 + (V_R \sin \phi_R + IX_L)^2} \end{aligned}$$

$$\begin{aligned} \text{(i) \%age Voltage regulation} &= \frac{V_S - V_R}{V_R} \times 100 \\ \text{(ii) Sending end p.f., } \cos \phi_S &= \frac{OD}{OC} = \frac{V_R \cos \phi_R + IR}{V_S} \\ \text{(iii) Power delivered} &= V_R I_R \cos \phi_R \\ \text{Line losses} &= I^2 R \\ \text{Power sent out} &= V_R I_R \cos \phi_R + I^2 R \\ \text{\%age Transmission efficiency} &= \frac{\text{Power delivered}}{\text{Power sent out}} \times 100 \\ &= \frac{V_R I_R \cos \phi_R}{V_R I_R \cos \phi_R + I^2 R} \times 100 \end{aligned}$$

PERFORMANCE OF THREE-PHASE SHORT TRANSMISSION LINES

For reasons associated with economy, transmission of electric power is done by 3-phase system. This system may be regarded as consisting of three single phase units, each wire transmitting one-third of the total power. As a matter of convenience, we generally analyze 3-phase system by considering one phase only. Therefore, expression for regulation, efficiency etc. derived for a single phase line can also be applied to a 3-phase system. Since only one phase is considered, phase values of 3-phase system should be taken. Thus, V_s and V_r are the phase voltages, whereas R and X_L are the resistance and inductive reactance per phase respectively.

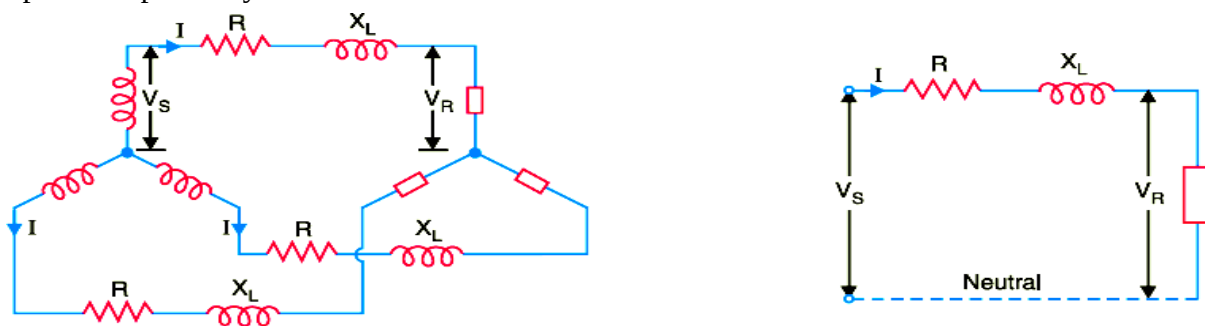


Fig (i): shows a Y-connected generator supplying a balanced Y-connected load through a transmission line. Each conductor has a resistance of $R \Omega$ and inductive reactance of $X \Omega$.

Fig. (ii): shows one phase separately. The calculations can now be made in the same way as for a single phase line.

Effect of Load power factor on Regulation and Efficiency

The regulation and efficiency of a transmission line depend to a considerable extent upon the power factor of the load.

1. Effect on regulation.



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The expression for voltage regulation of a short transmission line is given by :

$$\% \text{age Voltage regulation} = \frac{IR \cos \phi_R + IX_L \sin \phi_R}{V_R} \times 100 \quad (\text{for lagging p.f.})$$

$$\% \text{age Voltage regulation} = \frac{IR \cos \phi_R - IX_L \sin \phi_R}{V_R} \times 100 \quad (\text{for leading p.f.})$$

The following conclusions can be drawn from the above expressions

- i) When the load power factor is lagging or unity or such leading that $IR \cos \phi_R > IX_L \sin \phi_R$ then voltage regulation is positive *i.e.* , receiving end voltage V_R will be less than the sending end voltage V_S .
- ii) For a given V_R and I , the voltage regulation of the line increases with the decrease in power factor for lagging loads.
- iii) When the load power factor is leading to this extent that $IX_L \sin \phi_R > IR \cos \phi_R$, then voltage regulation is negative *i.e.* the receiving end voltage V_R is more than the sending end voltage V_S .
- iv) For a given V_R and I , the voltage regulation of the line decreases with the decrease in power factor for leading loads.

ABCD PARAMETERS:

A major section of power system engineering deals in the transmission of electrical power from one particular place (e.g. Generating station) to another like substations or distribution units with maximum efficiency. So it's of substantial importance for power system engineers to be thorough with its mathematical modeling. Thus the entire transmission system can be simplified to a **two port network** for the sake of easier calculations.

The circuit of a 2 port network is shown in the diagram below. As the name suggests, a 2 port network consists of an input port PQ and an output port RS. Each port has 2 terminals to Connect itself to the external circuit. Thus it is essentially a 2 port or a 4 terminal circuit, having Supply end voltage = V_S and Supply end current = I_S given to the input port P Q.

And there is the Receiving end Voltage = V_R and Receiving end current = I_R Given to the output port R S. As shown in the diagram below.

Now the **ABCD parameters** or the transmission line parameters provide the link between the supply and receiving end voltages and currents, considering the circuit elements to be linear in nature.

Thus the relation between the sending and receiving end specifications are given using

ABCD parameters by the equations below.

$$V_S = AV_R + BI_R \text{ ----- (1)}$$

$$I_S = CV_R + DI_R \text{ ----- (2)}$$

Now in order to determine the ABCD parameters of transmission line let us impose the required circuit conditions in different cases.

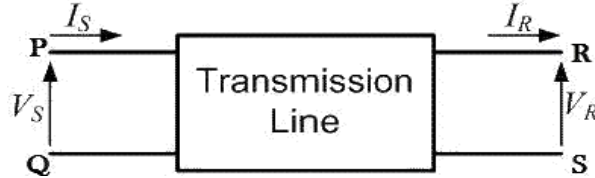


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ABCD parameters, when receiving end is open circuited

The receiving end is open circuited meaning receiving end current $I_R = 0$. Applying this condition to



Equation (1) we get

$$V_S = A V_R + B I_R \Rightarrow V_S = A V_R + 0$$

$$A = \left. \frac{V_S}{V_R} \right|_{I_R = 0}$$

Thus it implies that on applying open circuit condition to ABCD parameters, we get parameter A as the ratio of sending end voltage to the open circuit receiving end voltage. Since dimension wise A is a ratio of voltage to voltage, A is a dimensionless parameter.

Applying the same open circuit condition i.e $I_R = 0$ to equation (2)

$$I_S = C V_R + D I_R \Rightarrow I_S = C V_R + 0$$

$$C = \left. \frac{I_S}{V_R} \right|_{I_R = 0}$$

Thus it implies that on applying open circuit conditions to ABCD parameters, we get parameter C as the ratio of sending end current to the open circuited receiving end voltage. Since dimension wise C is a ratio of current to voltage. Its units is mho. Thus C is open circuit conductance and is given by $C = \frac{I_S}{V_R}$ (U)

ABCD parameters when receiving end is short circuited



Receiving end is short circuited meaning receiving end voltage $V_R = 0$. Applying this condition to equation (1) we get

$$V_S = A I_R + B V_R \Rightarrow V_S = 0 + B I_R$$

$$B = \left. \frac{V_S}{I_R} \right|_{V_R = 0}$$



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Thus it implies that on applying short circuit condition to ABCD parameters, we get parameter B as the ratio of sending end voltage to the short circuit receiving end current. Since dimension wise B is a ratio of voltage to current, its unit is Ω . Thus B is the short circuit resistance and is given by

$$B = \frac{V_S}{I_R} \quad (\Omega)$$

Applying the same short circuit condition i.e. $V_R = 0$ to equation (2) we get

$$I_S = C \cdot 0 + D I_R \Rightarrow I_S = 0 + D I_R$$

$$D = \frac{I_S}{I_R} \Big|_{V_R = 0}$$

Thus it implies that on applying short circuit condition to ABCD parameters, we get parameter D as the ratio of sending end current to the short circuit receiving end current. Since dimension wise D is a ratio of current to current, it's a dimensionless parameter. $\therefore$ The ABCD parameters of transmission line can be tabulated as:-

Parameter Specification Unit

$$A = \frac{V_S}{V_R} \text{ Voltage ratio unitless}$$

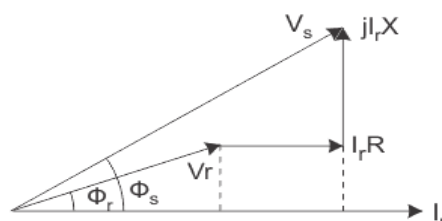
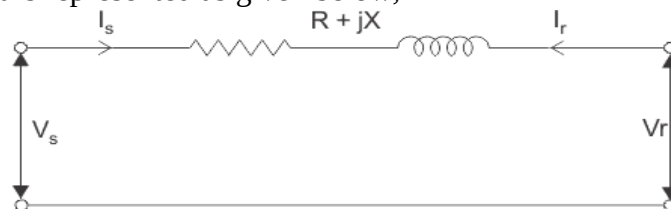
$$B = \frac{V_S}{I_R} \text{ Short Circuit Resistance (ohm)}$$

$$C = \frac{I_S}{V_R} \text{ Open Circuit Conductance (mho)}$$

$$D = \frac{I_S}{I_R} \text{ Current Ratio unitless}$$

SHORT TRANSMISSION LINE

The transmission lines which have length less than 80 km are generally referred as short transmission lines. For short length, the shunt capacitance of this type of line is neglected and other parameters like resistance and inductance of these short lines are lumped, hence the equivalent circuit is represented as given below,





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Let's draw the vector diagram for this equivalent circuit, taking receiving end current I_r as reference. The sending end and receiving end voltages make angle with that reference receiving end current, of ϕ_s and ϕ_r , respectively. As the shunt capacitance of the line is neglected, hence sending end current and receiving end current is same, i.e., $I_s = I_r$.

Now if we observe the vector diagram carefully, we will get, V_s is approximately equal to $V_r + I_r R \cos \phi_r + I_r X \sin \phi_r$ that means,

$V_s \cong V_r + I_r R \cos \phi_r + I_r X \sin \phi_r$ as it is assumed that $\phi_s \cong \phi_r$

As there is no capacitance, during no load condition the current through the line is considered as zero, hence at no load condition, receiving end voltage is the same as sending end voltage. As per definition of voltage regulation,

$$\begin{aligned} \% \text{ regulation} &= \frac{V_s - V_r}{V_r} \times 100 \% \\ &= \frac{I_r R \cos \phi_r + I_r X \sin \phi_r}{V_r} \times 100 \% \\ \text{per unit regulation} &= \frac{I_r R}{V_r} \cos \phi_r + \frac{I_r X}{V_r} \sin \phi_r = v_r \cos \phi_r + v_x \sin \phi_r \end{aligned}$$

$$A = \left. \frac{V_s}{V_r} \right|_{I_r = 0}$$

Here, V_r and V_s are the per unit resistance and reactance of the short transmission line.

Any electrical network generally has two input terminals and two output terminals. If we consider any complex electrical network in a black box, it will have two input terminals and output terminals. This network is called two – port network. Two port model of a network simplifies the network solving technique. Mathematically a two port network can be solved by 2 by 2 matrixes.

A transmission as it is also an electrical network; line can be represented as two port network. Hence two port network of transmission line can be represented as 2 by 2 matrixes. Here the concept of ABCD parameters comes. Voltage and currents of the network can be represented as,

$$V_s = AV_r + BI_r \text{ ----- (1)}$$

$$I_s = CV_r + DI_r \text{ ----- (2)}$$

Where A, B, C and D are different constant of the network. If we put $I_r = 0$ at equation (1), we get

Hence, A is the voltage impressed at the sending end per volt at the receiving end when receiving end is open. It is dimension less.

If we put $V_r = 0$ at equation (1), we get

$$B = \left. \frac{V_s}{I_r} \right|_{V_r = 0}$$



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That indicates it is impedance of the transmission line when the receiving terminals are short circuited. This parameter is referred as transfer impedance.

$$C = \left. \frac{I_s}{V_r} \right|_{I_r = 0}$$

C is the current in amperes into the sending end per volt on open circuited receiving end. It has the dimension of admittance.

$$D = \left. \frac{I_s}{I_r} \right|_{V_r = 0}$$

D is the current in amperes into the sending end per amp on short circuited receiving end. It is dimensionless.

Now from equivalent circuit, it is found that,

$V_s = V_r + I_r Z$ and $I_s = I_r$ comparing these equations with equation 1 and 2 we get, $A = 1$, $B = Z$, $C = 0$ and $D = 1$. As we know that the constant A, B, C and D are related for passive network as

Here, $A = 1$, $B = Z$, $C = 0$ and $D = 1$

$AD - BC = 1$.

$1 \cdot 1 - Z \cdot 0 = 1$

So the values calculated are correct for short transmission line. From above equation (1),

$$V_s = AV_r + BI_r$$

When $I_r = 0$ that means receiving end terminals is open circuited and then from the equation 1, we get receiving end voltage at no load

$$V_{r'} = \frac{V_s}{A}$$

and as per definition of voltage regulation,

$$\% \text{ voltage regulation} = \frac{V_s / A - V_r}{V_r} \times 100 \%$$

Efficiency of Short Transmission Line

The efficiency of short line as simple as efficiency equation of any other electrical equipment, that means

$$\% \text{ efficiency } (\mu) = \frac{\text{Power received at receiving end}}{\text{Power delivered at sending end}} \times 100 \%$$
$$\% \mu = \frac{\text{Power received at receiving end}}{\text{Power received at receiving end} + 3I_r^2 R} \times 100 \%$$



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MEDIUM TRANSMISSION LINES

In short transmission line calculations, the effects of the line capacitance are neglected because such lines have smaller lengths and transmit power at relatively low voltages (< 20 kV). However, as the length and voltage of the line increase, the capacitance gradually becomes of greater importance.

Since medium transmission lines have sufficient length (50-150 km) and usually operate at voltages greater than 20 kV, the effects of capacitance cannot be neglected. Therefore, in order to obtain reasonable accuracy in medium transmission line calculations, the line capacitance must be taken into consideration.

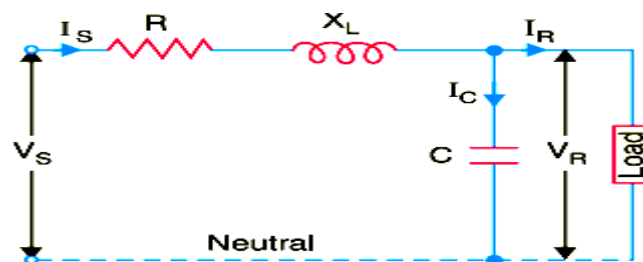
The capacitance is uniformly distributed over the entire length of the line. However, in order to make the calculations simple, the line capacitance is assumed to be lumped or concentrated in the form of capacitors shunted across the line at one or more points. Such a treatment of localising the line capacitance gives reasonably accurate results. The most commonly used methods (known as localised capacitance methods) for the solution of medium transmission lines are:

- (i) End condenser method
- (ii) Nominal T method
- (iii) Nominal π method.

Although the above methods are used for obtaining the performance calculations of medium lines, they can also be used for short lines if their line capacitance is given in a particular problem.

(i) End Condenser Method ;

In this method, the capacitance of the line is lumped or concentrated at the receiving or load end as shown in Fig. This method of localising the line capacitance at the load end overestimates the effects of capacitance. In Fig, one phase of the 3-phase transmission line is shown as it is more convenient to work in phase instead of line-to-line values.



Let, I_R = Load Current per Phase

R = Resistance per phase

X_L = Inductive reactance per phase

C = Capacitance per phase

$\cos \phi_R$ = Receiving end power factor (lagging)

V_s = sending end voltage per phase



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The *phasor diagram for the circuit is shown in Fig Taking the receiving end voltage V_R as the reference phasor,

we have, $\vec{V}_R = V_R + j 0$

Load current, $\vec{I}_R = I_R (\cos \phi_R - j \sin \phi_R)$

Capacitive current, $\vec{I}_C = j \vec{V}_R \omega C = j 2 \pi f C \vec{V}_R$

The sending end current I_S is the phasor sum of load current I_R and capacitive current I_C i.e.

$$\begin{aligned} \vec{I}_S &= \vec{I}_R + \vec{I}_C \\ &= I_R (\cos \phi_R - j \sin \phi_R) + j 2 \pi f C V_R \\ &= I_R \cos \phi_R + j (-I_R \sin \phi_R + 2 \pi f C V_R) \\ &= \vec{I}_S \vec{Z} = \vec{I}_S (R + j X_L) \end{aligned}$$

$$\vec{V}_S = \vec{V}_R + \vec{I}_S \vec{Z} = \vec{V}_R + \vec{I}_S (R + j X_L)$$

Thus, the magnitude of sending end voltage V_S can be calculated.

$$\% \text{ Voltage regulation} = \frac{V_S - V_R}{V_R} \times 100$$

$$\begin{aligned} \% \text{ Voltage transmission efficiency} &= \frac{\text{Power delivered / phase}}{\text{Power delivered / phase} + \text{losses / phase}} \times 100 \\ &= \frac{V_R I_R \cos \phi_R}{V_R I_R \cos \phi_R + I_S^2 R} \times 100 \end{aligned}$$

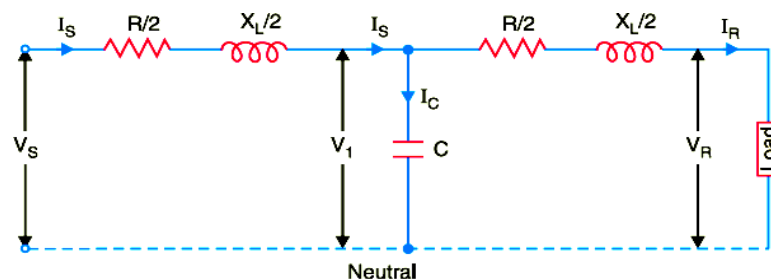
Limitations

Although end condenser method for the solution of medium lines is simple to work out calculations, yet it has the following drawbacks:

- There is a considerable error (about 10%) in calculations because the distributed capacitance has been assumed to be lumped or concentrated.
- This method overestimates the effects of line capacitance

(ii) Nominal T method

In this method, the whole line capacitance is assumed to be concentrated at the middle point of the line and half the line resistance and reactance are lumped on its either side as shown in Fig. Therefore, in this arrangement, full charging current flows over half the line. In Fig. one phase of 3-phase transmission line is shown as it is advantageous to work in phase instead of line-to-line values.





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Let, I_R = Load Current per Phase

R = Resistance per phase

X_L = Inductive reactance per phase

C = Capacitance per phase

$\cos \phi_R$ = Receiving end power factor (lagging)

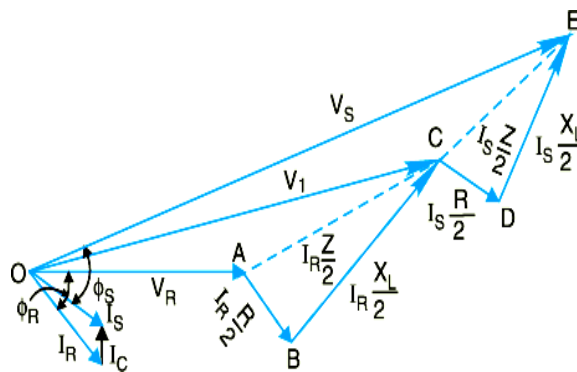
V_S = sending end voltage per phase

V_1 = Voltage across capacitor C

The *phasor diagram for the circuit is shown in Fig. Taking the receiving end voltage V_R as the reference phasor, we have,

Receiving end voltage, $\vec{V}_R = V_R + j 0$

Load current, $\vec{I}_R = I_R (\cos \phi_R - j \sin \phi_R)$



Voltage across C , $\vec{V}_1 = \vec{V}_R + \vec{I}_R \vec{Z} / 2$
 $= V_R + I_R (\cos \phi_R - j \sin \phi_R) \left(\frac{R}{2} + j \frac{X_L}{2} \right)$

Capacitive current, $\vec{I}_C = j \omega C \vec{V}_1 = j 2\pi f C \vec{V}_1$

Sending end current, $\vec{I}_S = \vec{I}_R + \vec{I}_C$

Sending end voltage, $\vec{V}_S = \vec{V}_1 + \vec{I}_S \frac{\vec{Z}}{2} = \vec{V}_1 + \vec{I}_S \left(\frac{R}{2} + j \frac{X_L}{2} \right)$

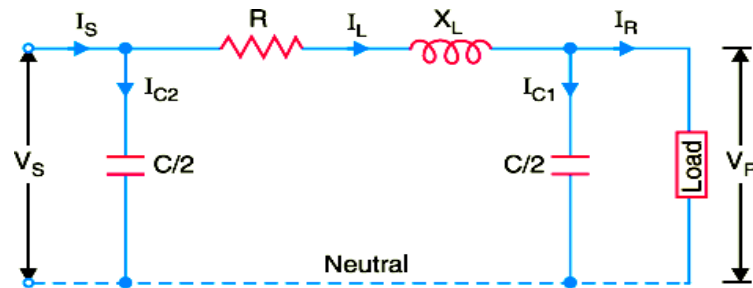
(iii) Nominal π method:

In this method, capacitance of each conductor (i.e., line to neutral) is divided into two halves; one half being lumped at the sending end and the other half at the receiving end as shown in Fig. It is obvious that capacitance at the sending end has no effect on the line drop. However, its charging current must be added to line current in order to obtain the total sending end current.



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Let, $I_R = \text{Load Current per Phase}$

$R = \text{Resistance per phase}$

$X_L = \text{Inductive reactance per phase}$

$C = \text{Capacitance per phase}$

$\cos \phi_R = \text{Receiving end power factor (lagging)}$

$V_S = \text{sending end voltage per phase}$

The *phasor diagram for the circuit is shown in Fig. Taking the receiving end voltage as the reference phasor, we have,

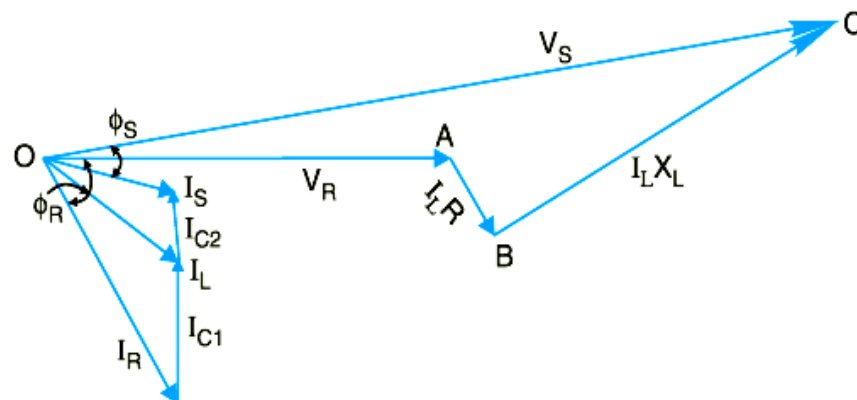
$$\vec{V}_R = V_R + j0$$

Load current,

$$\vec{I}_R = I_R (\cos \phi_R - j \sin \phi_R)$$

Charging current at load end is

$$\vec{I}_{C1} = j \omega (C/2) \vec{V}_R = j \pi f C \vec{V}_R$$



Line current,

$$\vec{I}_L = \vec{I}_R + \vec{I}_{C1}$$

Sending end voltage,

$$\vec{V}_S = \vec{V}_R + \vec{I}_L \vec{Z} = \vec{V}_R + \vec{I}_L (R + jX_L)$$

Charging current at the sending end is

$$\vec{I}_{C2} = j \omega (C/2) \vec{V}_S = j \pi f C \vec{V}_S$$

$\therefore$ Sending end current,

$$\vec{I}_S = \vec{I}_L + \vec{I}_{C2}$$



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MEDIUM TRANSMISSION LINE:

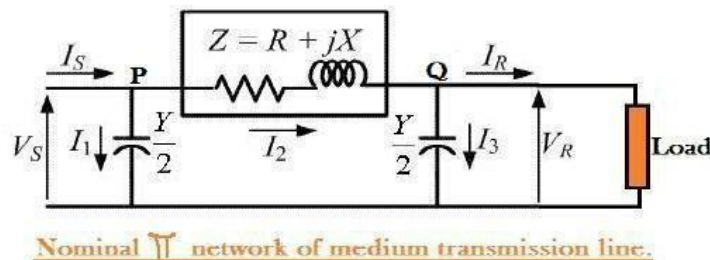
The transmission line having its effective length more than 80 km but less than 250 km is generally referred to as a **medium transmission line**. Due to the line length being considerably high, admittance Y of the network does play a role in calculating the effective circuit parameters, unlike in the case of short transmission lines. For this reason the modelling of a **medium length transmission line** is done using lumped shunt admittance along with the lumped impedance in series to the circuit. These lumped parameters of a medium length transmission line can be represented using two different models, namely.

- (i) Nominal Π representation.
- (ii) Nominal T representation.

Let's now go into the detailed discussion of these above mentioned models.

(i) Nominal Π representation of a medium transmission line

In case of a nominal Π representation, the lumped series impedance is placed at the middle of the circuit whereas the shunt admittances are at the ends. As we can see from the diagram of the Π network below, the total lumped shunt admittance is divided into 2 equal halves, and each half with value $Y/2$ is placed at both the sending and the receiving end while the entire circuit impedance is between the two. The shape of the circuit so formed resembles that of a symbol Π , and for this reason it is known as the nominal Π representation of a medium transmission line. It is mainly used for determining the general circuit parameters and performing load flow analysis.



As we can see here, V_S and V_R is the supply and receiving end voltages respectively, and I_S is the current flowing through the supply end. I_R is the current flowing through the receiving end of the circuit.

I_1 and I_3 are the values of currents flowing through the admittances. And I_2 is the current through the impedance Z .

Now applying KCL, at node P, we get

$$I_S = I_1 + I_2 \quad \text{—————(1)}$$

Similarly applying KCL, to node Q, we get

$$I_2 = I_3 + I_R \quad \text{—————(2)}$$

Now substituting equation (2) to equation (1)

$$I_S = I_1 + I_3 + I_R$$

$$= \frac{Y}{2} V_S + \frac{Y}{2} V_R + I_R \quad \text{—————(3)}$$



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Now by applying KVL to the circuit,

$$\begin{aligned}V_S &= V_R + ZI_2 \\&= V_R + Z\left(V_R \frac{Y}{2} + I_R\right) \\&= \left(Z \frac{Y}{2} + 1\right) V_R + ZI_R \text{ —————(4)}\end{aligned}$$

Now substituting equation (4) to equation (3), we get.

$$\begin{aligned}I_S &= \frac{Y}{2} \left[\left(\frac{Y}{2} Z + 1 \right) V_R + ZI_R \right] + \frac{Y}{2} V_R + I_R \\&= Y \left(\frac{Y}{4} Z + 1 \right) V_R + \left(\frac{Y}{2} Z + 1 \right) I_R \text{ —————(5)}\end{aligned}$$

Comparing equation (4) and (5) with the standard ABCD parameter equations

$$V_S = AV_R + BI_R$$

$$I_S = CV_R + DI_R$$

We derive the parameters of a medium transmission line as:

$$A = \left(\frac{Y}{2} Z + 1 \right)$$

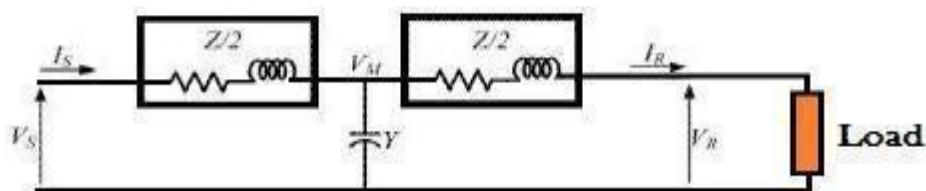
$$B = Z \Omega$$

$$C = Y \left(\frac{Y}{4} Z + 1 \right)$$

$$D = \left(\frac{Y}{2} Z + 1 \right)$$

(ii) Nominal T Representation of a Medium Transmission Line

In the **nominal T** model of a medium transmission line the lumped shunt admittance is placed in the middle, while the net series impedance is divided into two equal halves and placed on either side of the shunt admittance. The circuit so formed resembles the symbol of a capital **T**, and hence is known as the nominal T network of a medium length transmission line and is shown in the diagram below.



Nominal T representation of a medium transmission line.



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Here also V_s and V_r is the supply and receiving end voltages respectively, and I_s is the current flowing through the supply end. I_r is the current flowing through the receiving end of the circuit. Let M be a node at the midpoint of the circuit, and the drop at M , be given by V_m .

Applying KVL to the above network we get

$$\frac{V_s - V_m}{Z/2} = Y V_m + \frac{V_m - V_r}{Z/2}$$
$$\text{Or } V_m = \frac{2(V_s + V_r)}{YZ + 4} \quad (6)$$

And the receiving end current

$$\text{Or } I_r = \frac{2(V_m - V_r)}{Z/2} \quad (7)$$

Now substituting V_m from equation (6) to (7) we get,

$$\text{Or } I_r = \frac{[(2V_s + V_r) / YZ + 4] - V_r}{Z/2}$$

Rearranging the above equation:

$$V_s = \left(\frac{Y}{2}Z + 1\right)V_r + Z\left(\frac{Y}{4}Z + 1\right)I_r \quad (8)$$

Now the sending end current is

$$I_s = YV_m + I_r \quad (9)$$

Substituting the value of V_m to equation (9) we get,

$$\text{Or } I_s = Y V_r + \left(\frac{Y}{2}Z + 1\right)I_r \quad (10)$$

Again comparing equation (8) and (10) with the standard ABCD parameter equations

$$V_s = AV_r + BI_r$$

$$I_s = CV_r + DI_r$$

The parameters of the **T** network of a medium transmission line are

$$A = \left(\frac{Y}{2}Z + 1\right)$$

$$B = Z\left(\frac{Y}{4}Z + 1\right) \Omega$$

$$C = Y \text{ mho}$$

$$D = \left(\frac{Y}{2}Z + 1\right)$$



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LONG TRANSMISSION LINES

It is well known that line constants of the transmission line are uniformly distributed over the entire length of the line. However, reasonable accuracy can be obtained in line calculations for short and medium lines by considering these constants as lumped. If such an assumption of lumped constants is applied to long transmission lines (having length excess of about 150 km), it is found that serious errors are introduced in the performance calculations. Therefore, in order to obtain fair degree of accuracy in the performance calculations of long lines, the line constants are considered as uniformly distributed throughout the length of the line. Rigorous mathematical treatment is required for the solution of such lines. Fig shows the equivalent circuit of a 3-phase long transmission line on a phase-neutral basis. The whole line length is divided into n sections, each section having line constants $1/n$ th of those for the whole line. The following points may be noted:

- (i) The line constants are uniformly distributed over the entire length of line as is actually the case.
- (ii) The resistance and inductive reactance are the series elements.
- (iii) The leakage susceptance (B) and leakage conductance (G) are shunt elements.
- (iv) The leakage susceptance is due to the fact that capacitance exists between line and neutral. The leakage conductance takes into account the energy losses occurring through leakage over the insulators or due to corona effect between conductors. Admittance.

$$= \sqrt{G^2 + B^2}.$$

- (v) The leakage current through shunt admittance is maximum at the sending end of the line and decreases continuously as the receiving end of the circuit is approached at which point its value is zero.

ANALYSIS OF LONG TRANSMISSION LINE (RIGOROUS METHOD)

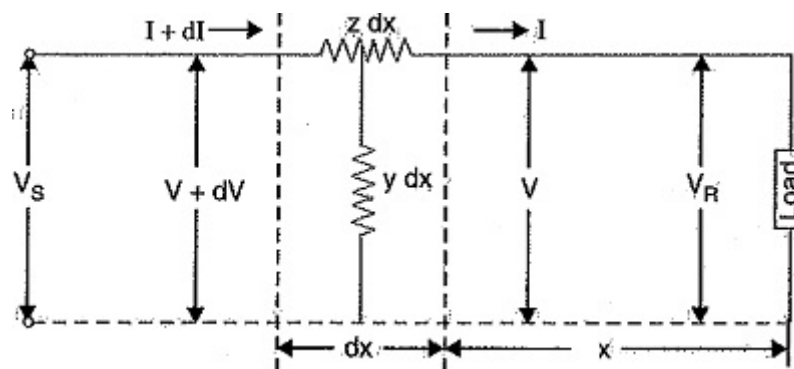


Fig. shows one phase and neutral connection of a 3-phase line with impedance and shunt admittance of the line uniformly distributed. Consider a small element in the line of length dx situated at a distance x from the receiving end.



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Let

z = series impedance of the line per unit length

y = shunt admittance of the line per unit length

V = voltage at the end of element towards receiving end

$V + dV$ = voltage at the end of element towards sending end

$I + dI$ = current entering the element dx

I = current leaving the element dx

Then for the small element dx ,

$z dx$ = series impedance

$y dx$ = shunt admittance

Obviously, $dV = I z dx$

$$\frac{dV}{dx} = I z$$

Now, the current entering the element is $I + dI$ whereas the current leaving the element is I .

The difference in the currents flows through shunt admittance of the element i.e.,

dI = Current through shunt admittance of element = $V y dx$

or
$$\frac{dI}{dx} = V y \quad \dots(ii)$$

Differentiating eq. (i) w.r.t. x , we get,

$$\frac{d^2 V}{dx^2} = z \frac{dI}{dx} = z (V y) \quad \left[\because \frac{dI}{dx} = V y \text{ from exp. (ii)} \right]$$

or
$$\frac{d^2 V}{dx^2} = y z V \quad \dots (iii)$$

The solution of this differential equation is

$$V = k_1 \cosh(x \sqrt{y z}) + k_2 \sinh(x \sqrt{y z}) \quad \dots(iv)$$

or
$$\frac{dI}{dx} = V y \quad \dots(ii)$$

Differentiating eq. (i) w.r.t. x , we get,

$$\frac{d^2 V}{dx^2} = z \frac{dI}{dx} = z (V y) \quad \left[\because \frac{dI}{dx} = V y \text{ from exp. (ii)} \right]$$

or
$$\frac{d^2 V}{dx^2} = y z V \quad \dots (iii)$$

The solution of this differential equation is

$$V = k_1 \cosh(x \sqrt{y z}) + k_2 \sinh(x \sqrt{y z}) \quad \dots(iv)$$



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From equation (ii),

$$\frac{dI}{dx} = yV$$

Substitute the expression for V :

$$\frac{dI}{dx} = y [k_1 \cosh(x\sqrt{yz}) + k_2 \sinh(x\sqrt{yz})]$$

Integrating both sides with respect to x ,

$$I = y \int [k_1 \cosh(x\sqrt{yz}) + k_2 \sinh(x\sqrt{yz})] dx$$

Using

$$\int \cosh(ax) dx = \frac{1}{a} \sinh(ax)$$

and

$$\int \sinh(ax) dx = \frac{1}{a} \cosh(ax),$$

$$a = \sqrt{yz},$$

we get

$$I = \frac{y}{\sqrt{yz}} [k_1 \sinh(x\sqrt{yz}) + k_2 \cosh(x\sqrt{yz})] + C$$

Since the integration constant is already included in the constants k_1 and k_2 , the current equation is written as

$$I = \sqrt{\frac{y}{z}} [k_1 \sinh(x\sqrt{yz}) + k_2 \cosh(x\sqrt{yz})] \quad \dots(v)$$

Final General Solutions of Transmission Line Equations

$$V(x) = k_1 \cosh(\gamma x) + k_2 \sinh(\gamma x)$$

$$I(x) = \frac{1}{Z_c} [k_1 \sinh(\gamma x) + k_2 \cosh(\gamma x)]$$

where

- $\gamma = \sqrt{yz} = \text{Propagation constant}$
- $Z_c = \sqrt{\frac{z}{y}} = \text{Characteristic impedance}$

Equations (iv) and (v) give the expressions for V and I in the form of unknown constants k_1 and k_2 . The values of k_1 and k_2 can be found by applying end conditions as under



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At $x = 0$, $V = V_R$ and $I = I_R$

Putting these values in eq. (iv), we have,

$$V_R = k_1 \cosh 0 + k_2 \sinh 0 = k_1 + 0$$

$\therefore$

$$V_R = k_1$$

Similarly, putting $x = 0$, $V = V_R$ and $I = I_R$ in eq. (v), we have,

$$I_R = \sqrt{\frac{y}{z}} [k_1 \sinh 0 + k_2 \cosh 0] = \sqrt{\frac{y}{z}} [0 + k_2]$$

$\therefore$

$$k_2 = \sqrt{\frac{z}{y}} I_R$$

Substituting the values of k_1 and k_2 in eqs. (iv) and (v), we get,

$$V = V_R \cosh (x\sqrt{yz}) + \sqrt{\frac{z}{y}} I_R \sinh (x\sqrt{yz})$$

and

$$I = \sqrt{\frac{y}{z}} V_R \sinh (x\sqrt{yz}) + I_R \cosh (x\sqrt{yz})$$

The sending end voltage (V_S) and sending end current (I_S) are obtained by putting $x = l$ in the above equations i.e.,

$$V_S = V_R \cosh (l\sqrt{yz}) + \sqrt{\frac{z}{y}} I_R \sinh (l\sqrt{yz})$$

$$I_S = \sqrt{\frac{y}{z}} V_R \sinh (l\sqrt{yz}) + I_R \cosh (l\sqrt{yz})$$

Now,

$$l\sqrt{yz} = \sqrt{l y \cdot l z} = \sqrt{Y Z}$$

and

$$\sqrt{\frac{y}{z}} = \sqrt{\frac{y l}{z l}} = \sqrt{\frac{Y}{Z}}$$

where

Y = total shunt admittance of the line

Z = total series impedance of the line

Therefore, expressions for V_S and I_S become :

$$V_S = V_R \cosh \sqrt{YZ} + I_R \sqrt{\frac{Z}{Y}} \sinh \sqrt{YZ}$$

$$I_S = V_R \sqrt{\frac{Y}{Z}} \sinh \sqrt{YZ} + I_R \cosh \sqrt{YZ}$$

It is helpful to expand hyperbolic sine and cosine in terms of their power series.

$$\cosh \sqrt{YZ} = \left(1 + \frac{YZ}{2} + \frac{Y^2 Z^2}{24} + \dots \right)$$

$$\sinh \sqrt{YZ} = \left(\sqrt{YZ} + \frac{(YZ)^{3/2}}{6} + \dots \right)$$

FERRANTI EFFECT:

On a long transmission line, when the line is open circuited (or) very lightly loaded, the voltage at the receiving end is more than the sending end voltage because of presence of line capacitance in the transmission line. This is known as "**Ferranti Effect**".

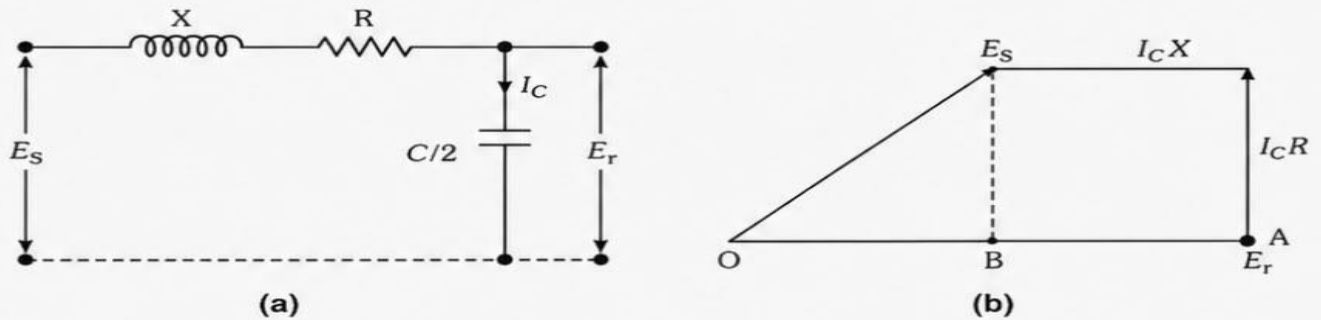


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Derivation of Rise in Voltage at the Receiving end (Ferranti Effect):

The voltage rise at the receiving end in the long transmission lines under open circuit (or) very lightly loaded conditions is due to the *emf* of self inductance of charging current being inphase with the impressed voltage at the sending end of the line.



E_s = Sending end voltage, E_r = Receiving end voltage
 I_C = Charging current
 X = Reactance of the line, R = Resistance of the line
 $I_C X$ = Voltage drop due to line reactance
 $I_C R$ = Voltage drop due to line resistance

Let us consider half of the total capacitance of the line is assumed to be concentrated at the receiving end as shown in Fig. 1.36(a).

From the vector diagram 1.36(b) the sending end voltage (E_s) is less than the receiving end voltage.

$$\therefore \text{Rise in voltage} = OA - OB = I_C X.$$

$$\text{We have } I_C = \frac{E_r Y}{2}$$

$$\text{Hence, the rise in voltage} = I_C X = \frac{E_r Y}{2} \quad \dots\dots\dots (1)$$

$$\text{Where } X = 2\pi fL; Y = WC = 2\pi fc.$$

Substituting X and Y values in equation (1),

$$\text{Rise in voltage} = \frac{E_r 2\pi fL \cdot 2\pi fc}{2} = \frac{(2\pi f)^2 E_r LC}{2}$$

$$\therefore \boxed{\text{Rise in voltage} = (2\pi f)^2 E_r \frac{LC}{2} \text{ volt}}$$

$$\text{We know that } 3 \times 10^5 = \frac{1}{\sqrt{LC}}$$

$$\therefore 9 \times 10^{10} = \frac{1}{LC}$$

$$LC = \frac{1}{9 \times 10^{10}} = 1.11 \times 10^{-11}$$

Where L = Length of the line in km.

C = Capacitance/km/conductor.

L = Inductance/km/conductor.

E_r = Receiving end voltage.

Assuming 50 Hz as frequency

$$\therefore \text{Rise in voltage} = (2\pi \times 50)^2 E_r \times \frac{1.11 \times 10^{-11}}{2}$$

$$= 0.55 E_r \left[\frac{1}{1000} \right]^2 \text{ Volt.}$$





SREENIVASA INSTITUTE OF TECHNOLOGY AND MANAGEMENT STUDIES (AUTONOMOUS)

(Approved by AICTE, New Delhi & Affiliated to JNTUA, Ananthapuramu)
DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING
(NBA Accredited)

CHARGING CURRENT:

Charging Current in Transmission Lines

In transmission lines, having a length over 100 km, the effect of capacitance to produce a current called **Charging Current**.

This current is in quadrature with the voltage and flows through the line even if the receiving end is open circuited. It has the maximum value at the sending end of the circuit and goes on decrease towards the receiving end point.

Due to this charging current there will be power loss in the line even though the line is open circuited.

Power Loss Due to Charging Current

Consider a line of length l kms having a charging current I_C at the sending end. Consider a point P at a distance of x kms from the sending end of the line length ' l ' kms as shown in the Fig.

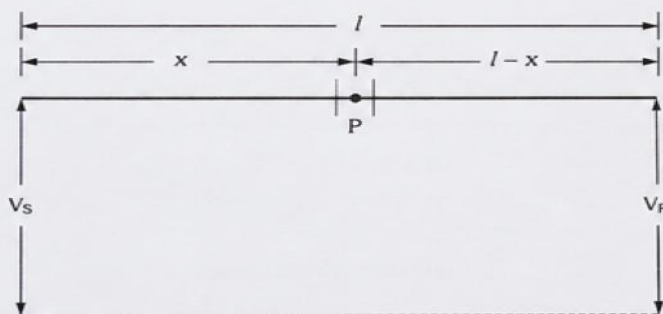
The charging current, at this point:

$$I_P = \frac{I_C}{l} (l - x)$$

If R is the total resistance of the line, the resistance per unit

$$\text{Length} = \frac{R}{l}$$

The resistance of the element



$$dx = \frac{R}{l} dx$$

$$\therefore \text{Loss occurring in the element } dx = I_P^2 \frac{R}{l} dx$$

$\therefore$ Total loss in the entire line

$$\begin{aligned} &= \int_0^l I_P^2 \frac{R}{l} dx = \int_0^l \frac{I_C^2}{l^2} (l - x)^2 \frac{R}{l} dx \\ &= \frac{I_C^2 R}{l^3} \int_0^l (l - x)^2 dx = \left[\frac{I_C^2 R}{l^3} \frac{(l - x)^3}{3} \right]_0^l \\ &= \frac{I_C^2 R}{l^3} \times \frac{l^3}{3} = \frac{I_C^2 R}{3} \end{aligned}$$

$$\therefore \text{The power loss due to the charging current} = \frac{I_C^2 R}{3}$$

2 Marks Questions:

1. Define a short transmission line and state the assumptions made in its analysis.
2. Define a medium transmission line. Name the methods used for its analysis.
3. What is a long transmission line? State the condition under which it is considered a long line.
4. Define voltage regulation of a transmission line and write its expression.
5. Define transmission efficiency and write its expression.
6. What are ABCD (generalized) constants of a transmission line? Write the transmission-line equations.
7. What is the nominal-T method of analysis of a medium transmission line?
8. What is the nominal- π method of analysis of a medium transmission line?
9. Define Ferranti effect. Under what conditions does it occur?
10. What is charging current in a transmission line? Why is it significant?
11. What is meant by surge impedance or characteristic impedance of a transmission line?
12. What is surge impedance loading (SIL)

10 Marks Questions:

1. Explain the performance equation of short transmission lines.
2. A single phase overhead transmission line is delivering 600 KVA load at 2KV. Its resistance and reactance are 0.18Ω and 0.36Ω respectively. Determine the voltage regulation if the load power factor is (i) 0.8 lagging (ii) 0.8 leading.
3. Starting from first principles deduce expressions for ABCD constants of a long transmission line in terms of its parameters. The generalized circuit constants of a transmission line are:
 $A = 0.93 + j0.016$ and $B = 20 + j140$
The load at the receiving end is 60 MVA, 50 Hz, 0.8 power factor lagging. The voltage at the supply end is 220 KV. Calculate the load voltage.
4. Determine the A, B, C, D constants for a three phase 50 Hz transmission line, 250 km long having the following distributed parameters $L = 1.15 \times 10^{-3} \text{ H/km}$, $C = 7.8 \times 10^{-9} \text{ F/m}$, $R = 0.14 \text{ ohms/km}$. $g=0$.
5. Explain the Ferranti effect in transmission lines. Discuss its causes and methods of reduction.
6. Derive the expressions for the charging current of a transmission line and explain its significance.
7. Explain the performance of short, medium, and long transmission lines with suitable equivalent circuits.

REFERENCES:

- ❖ C.L. Wadhwa, —Electrical Power Systems, New Age International Pub. Co, Third Edition, 2001.

“Reliable power transmission depends on the proper design, selection, and performance of overhead line insulators and conductors under varying electrical and mechanical stresses.”

- The overhead line conductors should be supported on the poles or towers in such a way that currents from conductors do not flow to earth through supports i.e., line conductors must be properly insulated from supports.
- The insulators provide necessary insulation between line conductors and supports and thus prevent any leakage current from conductors to earth.

Properties Of Insulators:

- (i) High mechanical strength in order to withstand conductor load, wind load etc.
- (ii) High electrical resistance of insulator material in order to avoid leakage current to earth.
- (iii) High relative permittivity of insulator material in order that dielectric strength is high.
- (iv) The insulator material should be non-porous; free from impurities and cracks otherwise the permittivity will be lowered.
- (v) High ratio of puncture strength to flash over.

Insulator Materials:

The most commonly used materials for insulators are

(i) Porcelain

- It is produced by firing at a high temperature a mixture of kaolin, feldspar and quartz.
- It is mechanically stronger than glass. gives less trouble from leakage and is less effected by changes of temperature.
- It is not so homogeneous as glass. It should be glazed during manufacturing process.
- Also, fault cannot be detected easily as it is not transparent.
- Tension beared by this material is usually weak and does not withstand tensile stresses exceeding 5 kg/mm^2 .
- The dielectric strength of porcelain should be 15 kV to 17 kV for every one-tenth inch thickness.

(ii) Glass

- Glass is cheaper than porcelain in the simpler shapes and if properly toughened and annealed gives high resistivity and dielectric strength of 35 kV for one-tenth inch thickness.

- This property makes it possible to use single piece construction whatever be the operating Voltage.
- Glass is quite homogeneous material and can withstand higher compressive stresses as compared to porcelain.
- Glass being transparent, it is very easy to detect any flaw (defect) like trapping of air etc.
- It has lower coefficient of thermal expansion and as a result, the strains due to temperature changes are minimized.
- The major drawback of glass is that moisture condenses very easily on its surface and hence its use is limited to about 33 kV.

(iii) Steatite:

- It is a naturally occurring magnesium silicate, usually found combined with oxides in varying proportions.
- It has a much higher tensile and bending stress than porcelain and can advantageously be used at tension towers (or) when the transmission line takes a sharp turn.

① Types of Insulators:

The successful operation of an overhead line depends to a considerable extent upon the proper selection of Insulator.

The most commonly used Insulators are

1. Pin type insulators:

→ As the name suggests the pin-type insulator is secured to the cross-arm on the pole. There is a groove on the upper end of insulator for housing the conductor.

→ The Conductor passes through this groove and is bound by the annealed wire of the same material as the conductor.

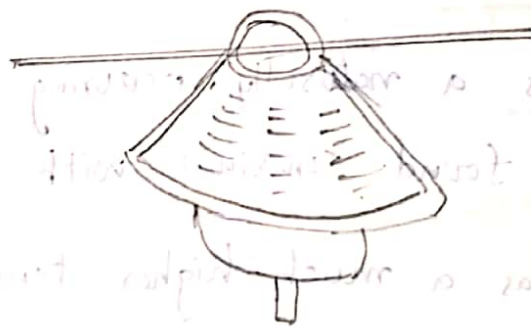
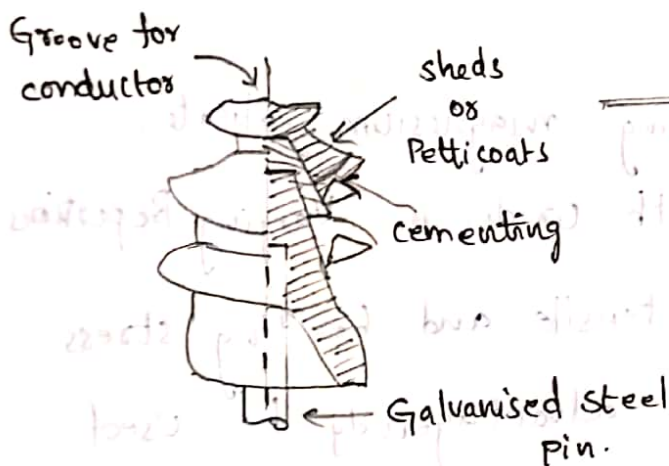


Fig: Pin Type Insulator.

→ Pin type insulators are used for transmission and distribution of electric power at Voltages upto 33 kV.

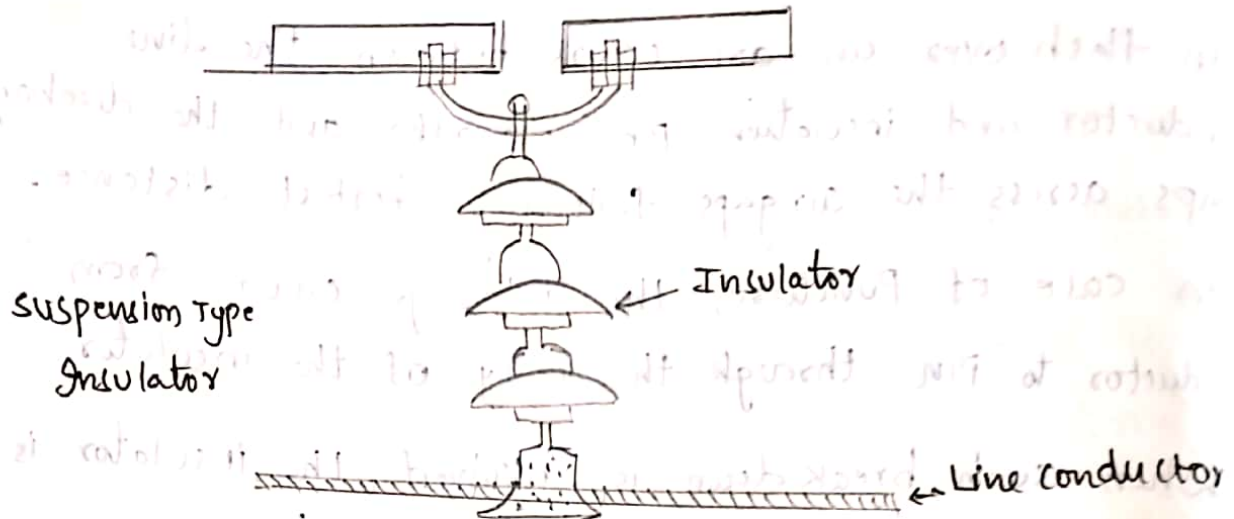
→ Beyond operating Voltage of 33 kV, the pin type insulators become too bulky and hence uneconomical.

causes of Insulation Failure:

- Insulators are required to withstand both mechanical and electrical stresses. Electrical stresses in insulators are due to line voltage and may cause the breakdown of insulator.
 - The electrical breakdown of insulator can occur either by "flashover or puncture".
 - In flash over an arc occurs between the line conductor and insulation pin (i.e. earth) and the discharge jumps across the airgaps following shortest distance.
 - In case of Puncture, the discharge occurs from conductor to pin through the body of the insulator.
 - When such break down is involved, the insulator is permanently destroyed due to excessive heat.
 - In practice, sufficient thickness of porcelain is provided in the insulator to avoid puncture by the line voltage.
 - "The ratio of puncture strength to flash-over voltage is known as 'Safety factor'".
- * Safety factor of Insulator = $\frac{\text{Puncture Strength}}{\text{Flash Over Voltage}}$
- It is desirable that the value of safety factor is "high" so that flash over voltage takes place before the insulator gets punctured.
 - For pin type insulators, the value of safety factor is about 10.

2. Suspension type Insulators:

- The cost of pin type insulator increases rapidly as the Working Voltage is increased. Therefore, this type of insulator is not economical beyond 33 kV.
- For high Voltage ($> 33 \text{ kV}$) it is usual practice to use Suspension type insulators shown below.



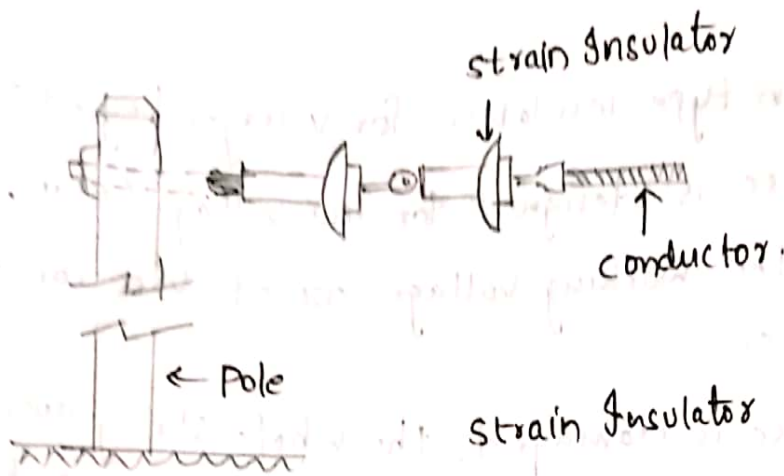
- They consist of a number of porcelain discs connected in series by metal links in the form of a string.
- The conductor is suspended at the bottom of the string while the other end of the string is secured to the cross arm of the tower.
- Each unit or disc is designed for low Voltage, say 11 kV. The number of discs in series would obviously depend upon the Working Voltage. For eg. if working Voltage is 66 kV, then six discs in series will be provided on the string.

Advantages:

- (i) cheaper than pin type insulators for voltages beyond 33kV.
- (ii) Each unit or disc is designed for low voltage (11kV).
Depending upon the working voltage, no. of discs can be connected in series.
- (iii) If any one disc is damaged, the whole string does not become useless because the damaged disc can be replaced by the new one.
- (iv) Provides greater flexibility to the line. String is free to swing in any direction and can take up the position where mechanical stresses are minimum.
- (v) The additional insulation required for raised voltage (any) can be easily obtained by adding desired no. of discs.
- (vi) Suspension type Insulators are generally used with steel towers. This provides partial protection from lightning.

3. Strain Insulators:

- When there is a dead end of the line or there is corner (or) sharp curve, the line is subjected to greater tension.
- In order to relieve the line of excessive tension, strain insulators are used.
- For Low Voltage lines, shackle insulators are used as strain insulators.



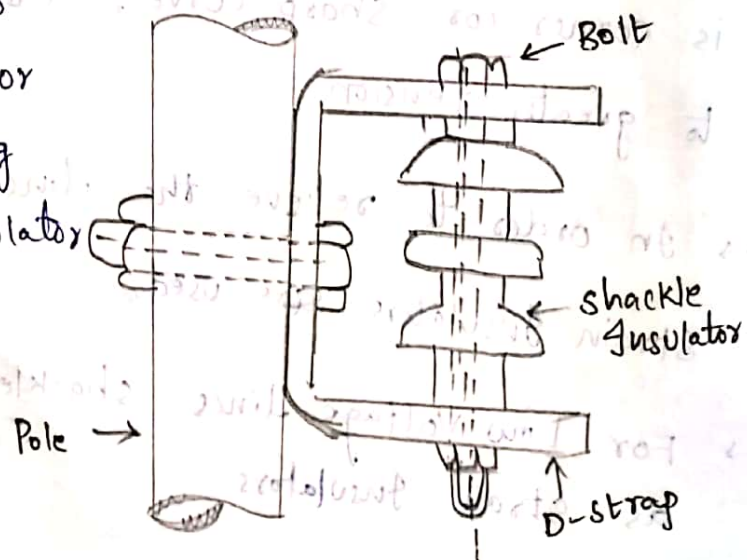
→ However, for high voltage transmission lines, strain insulator consists of an assembly of suspension insulators as shown in fig. The discs of strain insulators are used in vertical plane.

→ When the tension in the lines is exceedingly high, as at long river spans, two or more strings are used in parallel.

4. Shackle Insulators:

→ In early days, the shackle insulators were used as strain insulators. But now-a-days they are frequently used for low voltage distribution lines. Such insulators can be used either in horizontal or in a vertical position.

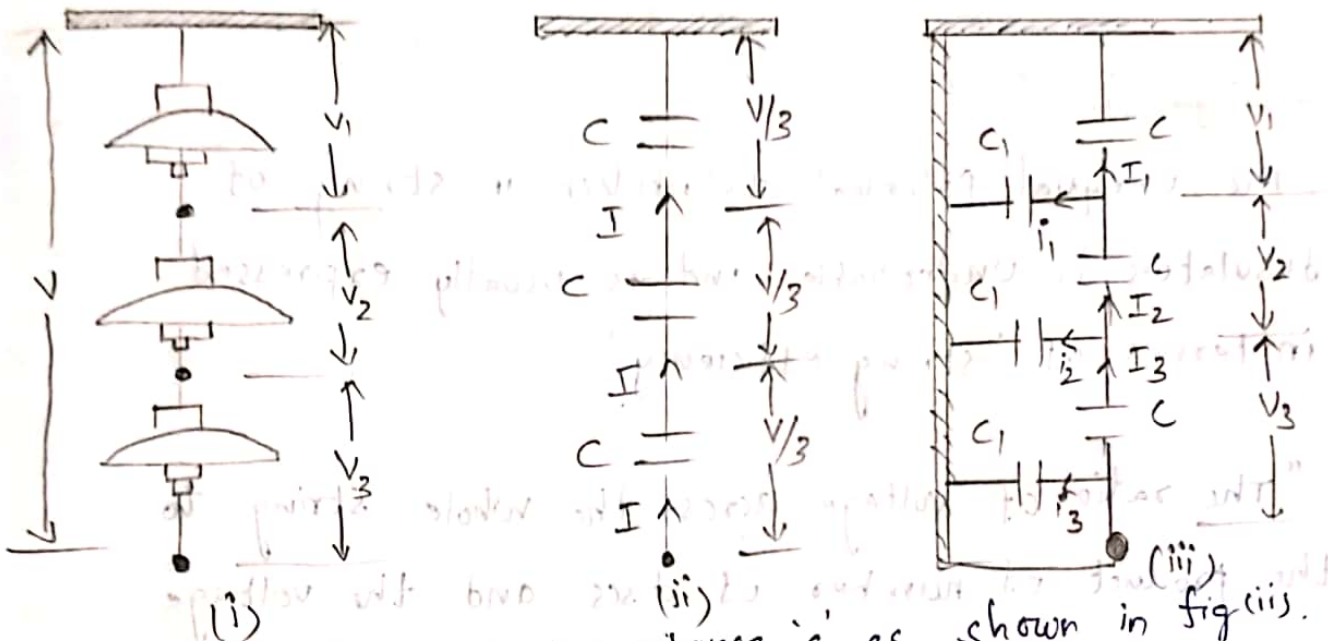
→ They can be fixed to the pole with a bolt or to the cross arm. Fig shows shackle insulator fixed to the pole.



② Potential Distribution Over Suspension Insulator String:

→ A string of suspension insulators consists of a number of porcelain discs connected in series through metallic links.

Fig(i) shows 3-disc string of suspension insulators. The porcelain portion of each disc is in between two metal links.



→ Each disc forms a capacitance 'c' as shown in fig(ii). This is known as mutual capacitance or self capacitance.

→ If there were mutual capacitance alone, then charging current would have been the same through all the discs and consequently, Voltage across each unit would have been the same (i.e $V/3$) as shown in fig(ii).

→ However, in actual practice, Capacitance also exists between metal fitting of each disc and tower or earth. This is known as shunt capacitance (C_1).

→ Due to shunt capacitance, charging current is not the same through all the discs of string as shown in fig (iii). Therefore, Voltage across each disc will be different.

→ Obviously, the disc nearest to the line conductor will have the maximum Voltage. Thus referring to Fig (iii), V_3 will be much more than V_2 (or) V_1 .

String Efficiency:

The unequal potential distribution in string of Insulators is undesirable and is usually expressed in terms of "string efficiency".

"The ratio of voltage across the whole string to the product of number of discs and the voltage across the disc nearest to the conductor is known as string efficiency i.e.,

$$\text{String Efficiency} = \frac{\text{Voltage across the string}}{n \times \text{Voltage across disc nearest to conductor}}$$

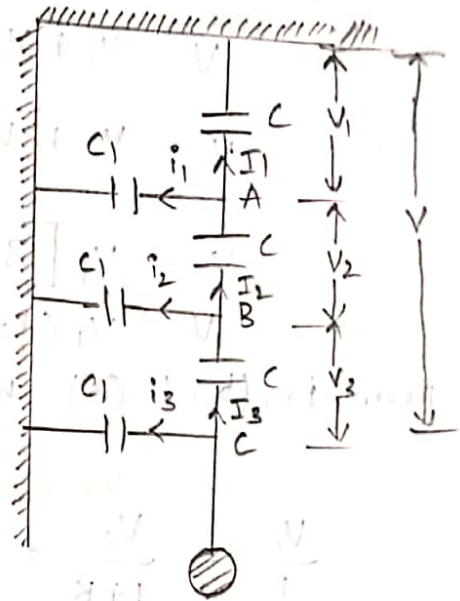
n = number of strings in the string.

→ The greater the string efficiency, the more uniform is the Voltage distribution. Thus 100% string efficiency is an ideal case for which the Voltage across each disc will be exactly the same. Hence, Efforts should be made to improve it as close to 100% as possible.

Mathematical Expression:

Following fig shows the equivalent circuit for a 3-disc string.
 Let us suppose that self capacitance of each disc is 'C'.
 Let us further assume that shunt capacitance C_1 is
 some fraction 'k' of self capacitance i.e. $C_1 = k \cdot C$.

Starting from the cross arm (or)
 tower, the voltage across each
 unit is ' V_1 ', ' V_2 ' and ' V_3 ' respectively
 as shown.



→ Applying KCL at node 'A'

$$I_2 = I_1 + i_1$$

$$V_2 \omega C = V_1 \omega C + V_1 \omega C_1$$

$$V_2 \omega C = V_1 \omega C + V_1 \omega k C$$

$$V_2 = V_1 (1 + k) \quad \text{--- (1)}$$

→ Applying KCL at node 'B'

$$I_3 = I_2 + i_2$$

$$V_3 \omega C = V_2 \omega C + (V_1 + V_2) \omega C_1$$

$$V_3 \omega C = V_2 \omega C + (V_1 + V_2) \omega k C$$

$$V_3 = V_2 + (V_1 + V_2) k$$

$$V_3 = k V_1 + V_2 (1 + k)$$

$$V_3 = k V_1 + V_1 (1 + k) (1 + k)$$

$$(\because V_2 = V_1 (1 + k))$$

$$V_3 = KV_1 + V_1(1+K)^2$$

$$V_3 = V_1 [K + 1^2 + 2K + K^2]$$

$$V_3 = V_1 [1 + 3K + K^2] \rightarrow (2)$$

→ Voltage between conductor and earth

$$V = V_1 + V_2 + V_3$$

$$V = V_1 + V_1(1+K) + V_1(1+3K+K^2)$$

$$V = V_1 [3 + 4K + K^2] \rightarrow (3)$$

$$V = V_1(1+K)(3+K)$$

From (1), (2) & (3) we get

$$\frac{V_1}{1} = \frac{V_2}{1+K} = \frac{V_3}{1+3K+K^2} = \frac{V}{(1+K)(3+K)} \rightarrow (4)$$

$$\% \text{ String Efficiency} = \frac{\text{Voltage across string}}{n \times \text{Voltage across disc nearest to conductor}} \times 100$$

$$\% \text{ String Efficiency} = \frac{V}{3 \times V_3} \times 100$$

Imp. Points:

(i) The max. Voltage appears across the disc nearest to the Conductor. (i.e. line Conductor)

(ii) The Voltage across the string is equal to phase Voltage
i.e. Voltage across string = Voltage between line & Earth = Phase Voltage

(iii) Line Voltage = $\sqrt{3} \times$ Voltage across string.

1. In a 33 kV overhead line, there are three units in the string of insulators. If the capacitance between each insulator pin and earth is 11% of self-capacitance of each insulator, find (i) the distribution of voltage over 3 insulators and (ii) string efficiency.

sol. Fig shows the equivalent circuit of string insulators. Let V_1 , V_2 and V_3 be the voltage across top, middle and bottom unit respectively.

If C is the self capacitance of each unit, then KC will be the shunt capacitance.

$$K = \frac{\text{shunt capacitance}}{\text{Self - capacitance}} = 0.11$$

$$\text{Voltage across string, } V = \frac{33}{\sqrt{3}} = 19.05 \text{ kV}$$

At Junction A

$$I_2 = I_1 + i_1$$

$$V_2 \omega C = V_1 \omega C + V_1 K \omega C$$

$$V_2 = V_1 (1 + K)$$

$$V_2 = V_1 (1 + 0.11)$$

$$V_2 = 1.11 V_1$$

At Junction 'B'

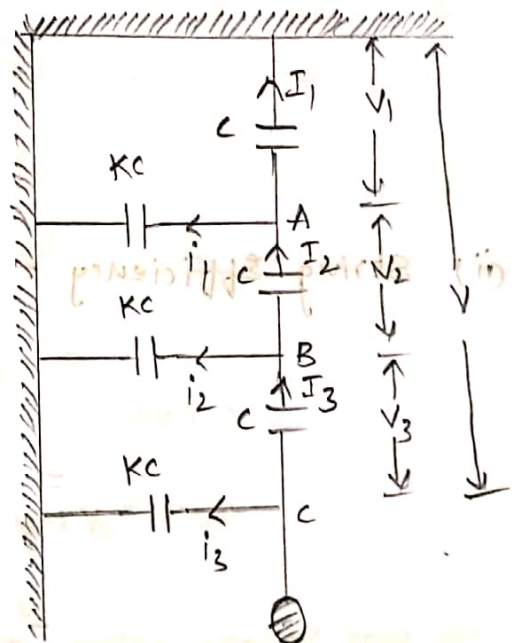
$$I_3 = I_2 + i_2$$

$$V_3 \omega C = V_2 \omega C + (V_1 + V_2) K \omega C$$

$$V_3 = V_2 + (V_1 + V_2) K$$

$$V_3 = 1.11 V_1 + (V_1 + 1.11 V_1) 0.11$$

$$V_3 = 1.342 V_1$$



(i) Voltage across the whole string is

$$V = V_1 + V_2 + V_3$$

$$V = V_1 + 1.11V_1 + 1.342V_1$$

$$V = 3.452V_1$$

$$19.05 = 3.452V_1$$

$\therefore$ Voltage across

$$\text{top unit } V_1 = \frac{19.05}{3.452} = 5.52 \text{ kV}$$

$$\text{Middle unit } V_2 = 1.11V_1 = 1.11 \times 5.52 \\ = 6.13 \text{ kV}$$

$$\text{Bottom unit } V_3 = 1.342V_1$$

$$= 1.342 \times 5.52$$

$$= 7.4 \text{ kV}$$

$$(ii) \text{ String Efficiency} = \frac{\text{Voltage across String}}{\text{No. of insulators} \times V_3} \times 100$$

$$= \frac{19.05}{3 \times 7.4} \times 100$$

$$= 85.8\%$$

2. A 3- ϕ T.L. being supported by three discs insulators. The potentials across top unit (i.e. near to tower) and middle unit are 8 kV and 11 kV respectively. Calculate (i) the ratio of capacitance between pin and earth to the self capacitance of each unit. (ii) the line voltage (iii) string efficiency.

sol. The equivalent circuit of string insulators is shown in fig. It is given that $V_1 = 8 \text{ kV}$ and $V_2 = 11 \text{ kV}$.

(i) Let K be the ratio of capacitance between pin and earth to self capacitance.

If C farad is the self capacitance of each unit, then capacitance between pin and earth $= KC$.

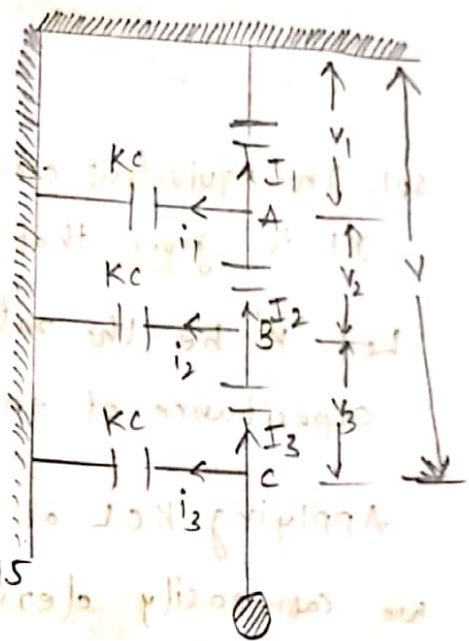
i. Applying KCL at junction 'A'

$$I_2 = I_1 + i_1$$

$$V_2 \omega C = V_1 \omega C + V_1 K \omega C$$

$$V_2 = V_1 (1 + K)$$

$$K = \frac{V_2 - V_1}{V_1} = \frac{11 - 8}{8} = 0.375$$



(ii) Applying KCL at junction 'B'

$$I_3 = I_2 + i_2$$

$$V_3 \omega C = V_2 \omega C + (V_1 + V_2) K \omega C$$

$$V_3 = V_2 + (V_1 + V_2) K$$

$$= 11 + (8 + 11) \times 0.375 = 18.125 \text{ kV}$$

$$V_3 = 18.12 \text{ kV}$$

$$\text{Voltage b/n line and earth} = V_1 + V_2 + V_3 = 8 + 11 + 18.12 = 37.12 \text{ kV}$$

$$\therefore \text{Line Voltage} = \sqrt{3} \times 37.12 = 64.28 \text{ kV}$$

$$\text{(iii) String efficiency} = \frac{\text{Voltage across string} \times 100}{\text{No. of insulators} \times V_3}$$

$$= \frac{37.12}{3 \times 18.12} \times 100$$

$$= 68.28\%$$

3. The three bus-bar conductors in an outdoor substation are supported by units of post type insulators. Each unit consists of a stack of 3-pin type insulators fixed one on the top of the other. The voltage across the lowest insulator is 13.1 kV and that across the next unit is 11 kV. Find the bus bar voltage of the station.

sol. The equivalent circuit of insulators is shown in fig.

It is given that $V_3 = 13.1 \text{ kV}$ and $V_2 = 11 \text{ kV}$.

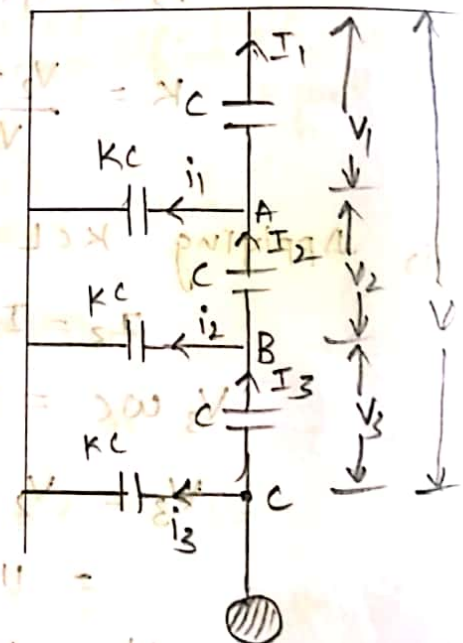
Let 'K' be the ratio of shunt capacitance to self capacitance of each unit.

Applying KCL at junctions 'A' & 'B' we can easily derive the following equations.

$$V_2 = V_1 (1+K)$$

$$V_1 = \frac{V_2}{(1+K)} \rightarrow (i)$$

$$V_3 = V_2 + (V_1 + V_2) K \rightarrow (ii)$$



Substituting eqn (i) in eqn (ii) we get

$$V_3 = V_2 + \left[\frac{V_2}{1+K} + V_2 \right] K$$

$$V_3 (1+K) = V_2 (1+K) + [V_2 + V_2 (1+K)] K$$

$$= V_2 [(1+K) + K + (K+K^2)]$$

$$V_3 (1+K) = V_2 [1 + 3K + K^2]$$

$$13.1 (1+K) = 11 (1 + 3K + K^2)$$

$$11K^2 + 19.9K - 2.1 = 0$$

solving above eqn we get

$$K = 0.1$$

$$V_1 = \frac{V_2}{1+K} = \frac{11}{1+0.1} = 10 \text{ kV}$$

$$\text{Voltage between line and earth} = V_1 + V_2 + V_3$$

$$= 10 + 11 + 13.1$$

$$= 34.1 \text{ kV}$$

$\therefore$ Voltage between bus bars (i.e. line voltage)

$$= 34.1 \times \sqrt{3}$$

$$= 59 \text{ kV}$$

(A)

4. A string of 4 insulators has a self capacitance equal to 10 times the pin to earth capacitance. Find (i) the voltage distribution across various units

* Methods of Improving String Efficiency:

- From earlier discussion, it is clear that Potential distribution across string of suspension insulators is not uniform.
- Maximum Voltage appears across the insulator nearest to the line conductor and decreases progressively as the cross arm is reached.
- If the insulation of the highest stressed insulator break down (or) flash over takes place, the breakdown of other units will take place in succession.
- This necessitates to equalize the Potential across the various units of the string i.e. to improve string efficiency.

The various methods to improve string efficiency are

(i) By using Longer cross-arms.

→ The value of string efficiency depends upon the value of ' K ' i.e. ratio of shunt capacitance to mutual capacitance.

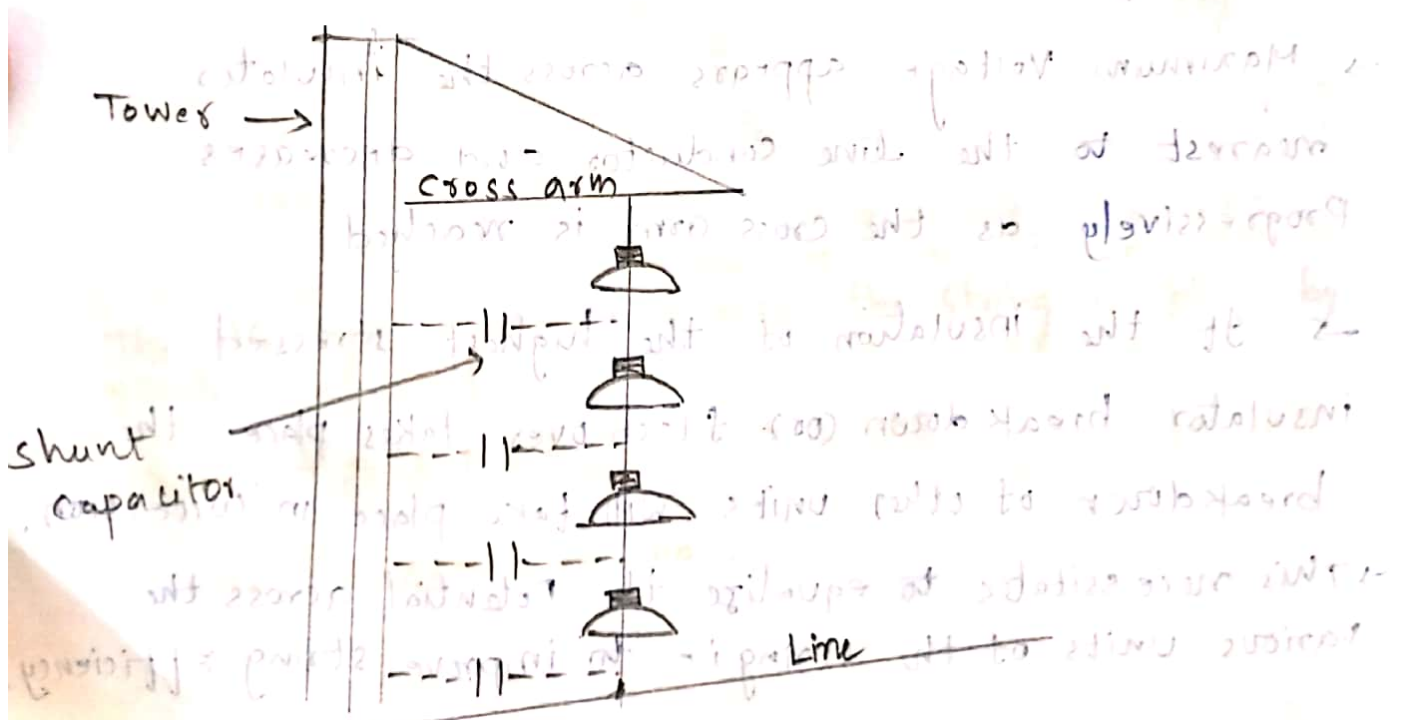
→ The lesser the value of ' K ', the greater the string efficiency and more uniform is the voltage distribution.

The value of ' K ' can be decreased by reducing the shunt capacitance.

→ In order to reduce shunt capacitance, the distance of conductor from tower must be increased i.e. longer cross-arms should be used.

→ However, limitations of cost and strength of tower do not allow the use of very long cross-arms.

In practice $K = 0.1$ is the limit that can be achieved by this method.

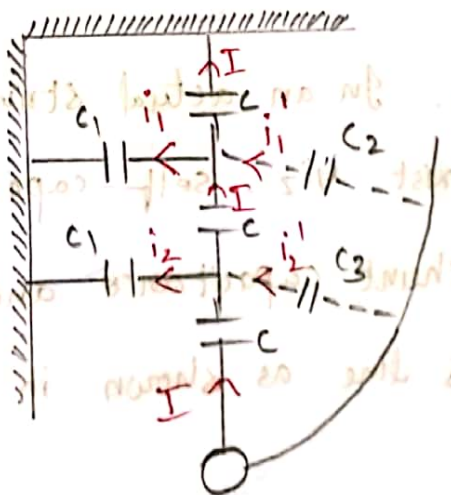


(ii) By grading the insulators.

- In this method, insulators of different dimensions are so chosen that each has a different capacitance.
- The insulators are 'capacitance graded' i.e. they are assembled in the string in such a way that the top unit has the minimum capacitance, increasing progressively, as the bottom unit is reached.
- Since Voltage is inversely proportional to capacitance, this method tends to equalize the potential distribution across the units in the string.
- This method has the disadvantage that a large number of different sized insulators are required.

(iii) By using a guard ring / static Shielding:

- The potential across each unit in a string can be equalised by using a guard ring which is a metal ring electrically connected to the conductor and surrounding the bottom insulator as shown in fig.



→ The guard ring introduces capacitance between metal fittings and the line conductor.

→ The guard ring is contoured in such a way that shunt capacitance currents i_1, i_2 etc are equal to metal fitting line capacitance currents i'_1, i'_2 etc

→ The result is that same charging current ' i ' flows through each unit of string. Consequently, there will be uniform potential distribution across the units.

→ The self capacitance of each unit in a string of three suspension insulators is ' c '. The shunting capacitance of the connecting metal work of each insulator to earth is $0.15c$ while for line it is $0.1c$.

calculate

(i) Voltage across each insulator as a percentage of the line voltage to earth and

(ii) string efficiency.

sol. In an actual string of insulators, three capacitance exist viz., self-capacitance of each insulator, shunt capacitance and capacitance of each unit to line as shown in fig.

At junction A,

$$\text{By KCL} \cdot I_2 + i_1' = I_1 + i_1$$

$$V_2 \omega C + (V_2 + V_3) 0.1 \omega C = V_1 \omega C + 0.15(V_1 \omega C)$$

$$0.1 V_3 = 1.15 V_1 - 1.1 V_2 \quad \text{--- (i)}$$

$$V_3 = 11.5 V_1 - 11 V_2 \quad \text{--- (i)}$$

At junction B

$$I_3 + i_2' = I_2 + i_2$$

$$V_3 \omega C + V_3 \times 0.1 \omega C = V_2 \omega C + (V_1 + V_2) \omega C \times 0.15$$

$$1.1 V_3 = 1.15 V_2 + 0.15 V_1 \quad \text{--- (ii)}$$

Substituting (i) in (ii) we get

$$1.1 (11.5 V_1 - 11 V_2) = 1.15 V_2 + 0.15 V_1$$

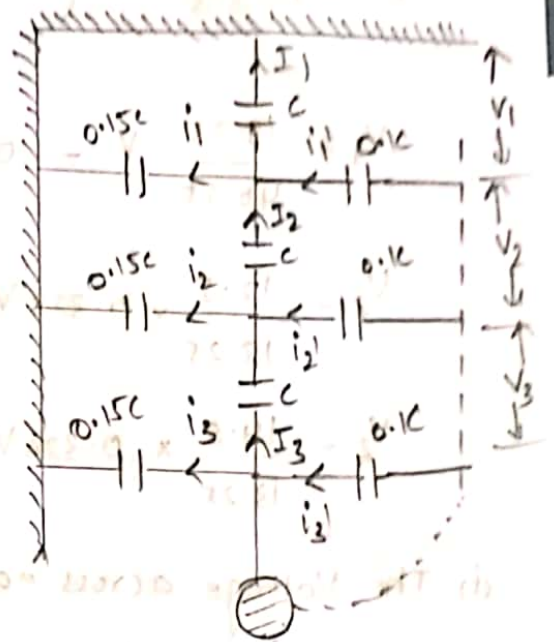
$$13.25 V_2 = 12.5 V_1$$

$$V_2 = \frac{12.5}{13.25} V_1 \quad \text{--- (iii)}$$

Substituting (iii) in (i) we get

$$V_3 = 11.5 V_1 - 11 \left(\frac{12.5}{13.25} \right) V_1$$

$$V_3 = \frac{14.8}{13.25} V_1$$



Now, Voltage between conductor and earth is

$$V = V_1 + V_2 + V_3 = V_1 \left(1 + \frac{12.5}{13.25} + \frac{14.8}{13.25} \right) = \frac{40.55 V_1}{13.25} \text{ Volts}$$

$$V_1 = \frac{13.25}{40.55} V = 0.326 V \text{ Volts}$$

$$V_2 = \frac{12.5}{13.25} \times 0.326 V = 0.307 V \text{ Volts}$$

$$V_3 = \frac{14.8}{13.25} \times 0.326 V = 0.364 V \text{ Volts}$$

(i) The Voltage across each unit expressed as a % of V being

$$\text{Top unit} = \frac{V_1}{V} \times 100 = \frac{0.326 V}{V} \times 100 = 32.6\%$$

$$\text{Second from Top} = \frac{V_2}{V} \times 100 = \frac{0.307 V}{V} \times 100 = 30.7\%$$

$$\text{Third from Top} = \frac{V_3}{V} \times 100 = \frac{0.364 V}{V} \times 100 = 36.4\%$$

$$(ii) \text{ String Efficiency} = \frac{V}{n \times V_3} \times 100$$

$$= \frac{V}{3 \times 0.364 V} \times 100$$

$$= 91.5\%$$

② Grading Of cables:

- The process of achieving uniform electrostatic stress in the dielectric of cables is known as grading of cables.
- Electrostatic stress in a single core cable has a maximum value (g_{max}) at the conductor surface and goes on decreasing as we move towards the sheath.
- The maximum voltage that can be safely applied to a cable depends upon g_{max} i.e., electrostatic stress at the conductor surface.
- For safe working of cables, ^{having homogeneous dielectric} the strength of dielectric must be more than g_{max} .
- If a dielectric of high strength is used for a cable, it is useful only near the conductor where stress is maximum. But as we move away from the conductor, the electrostatic stress decreases, so the dielectric will be unnecessarily over strong.
- The unequal stress distribution in a cable is undesirable for two reasons.
 - * Firstly, insulation of greater thickness is required, which increases the cable size.
 - * Secondly, it may lead to breakdown of insulation.
- In order to overcome this disadvantage, it is necessary to have a uniform stress distribution in cables.
- This can be achieved by distributing the stress in such a way that its value is increased in the outer layers of dielectric. This is known as grading of cables.

The two main methods of grading of cables are

- (i) capacitance grading
- (ii) Intersheath grading.

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Capacitance Grading:

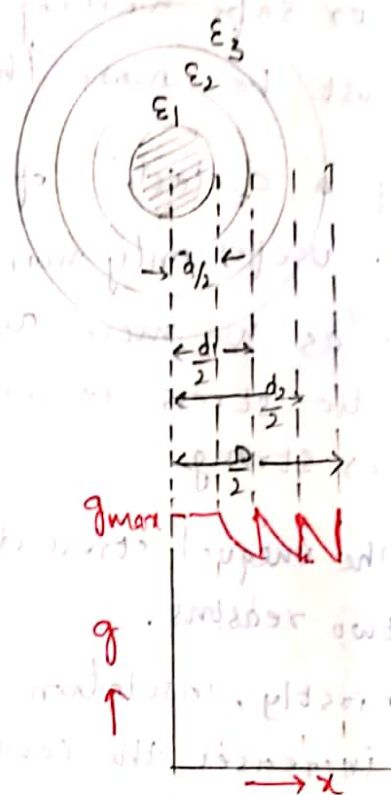
"The process of achieving uniformity in the dielectric stress by using layers of different dielectrics is known as capacitance grading".

→ In capacitance grading, the homogeneous dielectric is replaced by a composite dielectric.

→ The composite dielectric consists of various layers of different dielectrics in such a manner that relative permittivity ϵ_r of any layer is inversely proportional to its distance from the centre.

→ Under such conditions, the value of potential gradient at any point in the dielectric is constant and is independent of its distance from the centre.

→ In other words, the dielectric stress in the cable is same everywhere and grading is ideal one.



the capacitance grading can be explained from figure. There are three dielectrics of outer diameters d_1, d_2 and D and of relative permittivity ϵ_1, ϵ_2 and ϵ_3 respectively.

If the permittivities are such that $\epsilon_1 > \epsilon_2 > \epsilon_3$ and the three dielectrics are worked at the same maximum stress, then

$$\frac{1}{\epsilon_1 d} = \frac{1}{\epsilon_2 d_1} = \frac{1}{\epsilon_3 d_2}$$

$$\text{or } \epsilon_1 d = \epsilon_2 d_1 = \epsilon_3 d_2$$

Potential difference across the inner layer is

$$V_1 = \int_{d/2}^{d_1/2} g \, dx = \int_{d/2}^{d_1/2} \frac{Q}{2\pi \epsilon_0 \epsilon_1 x} \, dx$$

$$= \frac{Q}{2\pi \epsilon_0 \epsilon_1} \log_e \frac{d_1}{d}$$

$$V_1 = \frac{g_{\max}}{2} d \log_e \frac{d_1}{d} \quad \left\{ \because \frac{Q}{2\pi \epsilon_0 \epsilon_1} = \frac{g_{\max}}{2} d \right.$$

Similarly, potential across second layer (V_2) and third

layer (V_3) is given by

$$V_2 = \frac{g_{\max}}{2} d_1 \log_e \frac{d_2}{d_1}$$

$$V_3 = \frac{g_{\max}}{2} d_2 \log_e \frac{D}{d_2}$$

Total P.d between core and earthed sheath is

$$V = V_1 + V_2 + V_3 = \frac{g_{\max}}{2} \left[d \log_e \frac{d_1}{d} + d_1 \log_e \frac{d_2}{d_1} + d_2 \log_e \frac{D}{d_2} \right]$$

If the cable had homogeneous dielectric, then for the same values of d , D and g_{max} , the permissible Potential difference between core and earthed sheath would have been

$$V' = \frac{g_{max}}{2} d \log_e \frac{D}{d}$$

→ A single-core lead sheathed cable is graded by using three dielectrics of relative permittivity 5, 4 and 3 respectively. The conductor diameter is 2 cm and overall diameter is 8 cm. If the three dielectrics are worked at the same maximum stress of 40 kV/cm, find the safe working voltage of the cable. What will be the value of safe working voltage for an ungraded cable, assuming the same conductor and overall diameter and maximum dielectric stress?

Sol: Here $d = 2 \text{ cm}$, $d_1 = ?$, $d_2 = ?$, $D = 8 \text{ cm}$

$\epsilon_1 = 5$, $\epsilon_2 = 4$, $\epsilon_3 = 3$; $g_{max} = 40 \text{ kV/cm}$

Graded cable, As the maximum stress in the three dielectrics is same,

$$\epsilon_1 d = \epsilon_2 d_1 = \epsilon_3 d_2$$

$$5 \times 2 = 4 \times d_1 = 3 \times d_2$$

$$10 = 4d_1 \quad 10 = 3d_2$$

$$d_1 = 2.5 \text{ cm}$$

$$d_2 = 3.34 \text{ cm}$$

Permissible Peak voltage for the cable,

$$\begin{aligned} V &= \frac{g_{\max}}{2} \left[d \log_e \frac{d_1}{d} + d_1 \log_e \frac{d_2}{d_1} + d_2 \log_e \frac{D}{d_2} \right] \\ &= \frac{40}{2} \left[2 \log_e \frac{2.5}{2} + 2.5 \log_e \frac{3.34}{2.5} + 3.34 \log_e \frac{8}{3.34} \right] \\ &= 20 [0.4462 + 0.7242 + 2.92] \\ &= 81.808 \text{ kV} \end{aligned}$$

$\therefore$ Safe working voltage (rms) for cable

$$= \frac{81.808}{\sqrt{2}} = 57.84 \text{ kV}$$

Ungraded cable,

permissible Peak voltage for cable

$$\begin{aligned} &= \frac{g_{\max}}{2} d \log_e \frac{D}{d} \\ &= \frac{40}{2} \times 2 \times \log_e \frac{8}{2} \\ &= 55.44 \text{ kV} \end{aligned}$$

$\therefore$ safe working voltage (rms) for cable

$$\begin{aligned} &= \frac{55.44}{\sqrt{2}} \\ &= 39.2 \text{ kV} \end{aligned}$$

ii) Intersheath Grading:

→ In this method of cable grading, a homogeneous dielectric is used, but it is divided into various layers by placing metallic intersheaths between the core and lead sheath.

→ The intersheaths are held at suitable potentials which are in between the core potential and earth potential.

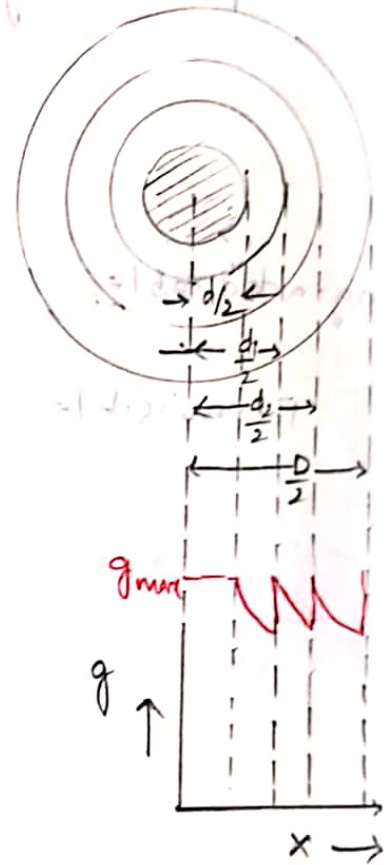
This arrangement improves voltage distribution in the dielectric of cable and consequently more uniform potential gradient is obtained.

→ Consider a cable of core diameter ' d ' and outer lead sheath of diameter ' D '.

Suppose that two intersheaths of diameters d_1 and d_2 are inserted into homogeneous dielectric and maintained at some fixed potentials.

→ Let V_1, V_2, V_3 be the voltage between core and intersheath 1, intersheath 1 and intersheath 2, intersheath 2 and outer lead sheath.

→ As there is a definite potential difference between the inner and outlayers of each intersheath, therefore each sheath can be treated like a homogeneous single core cable.



Maximum stress between core and intersheath₁ is

$$g_{1\max} = \frac{V_1}{\frac{d}{2} \log_e \frac{d_1}{d}}$$

similarly $g_{2\max} = \frac{V_2}{\frac{d_1}{2} \log_e \frac{d_2}{d_1}}$

$$g_{3\max} = \frac{V_3}{\frac{d_2}{2} \log_e \frac{D}{d_2}}$$

Since the dielectric is homogeneous, the maximum stress in each layer is the same. i.e.,

$$g_{1\max} = g_{2\max} = g_{3\max}$$

$$\therefore \frac{V_1}{\frac{d}{2} \log_e \frac{d_1}{d}} = \frac{V_2}{\frac{d_1}{2} \log_e \frac{d_2}{d_1}} = \frac{V_3}{\frac{d_2}{2} \log_e \frac{D}{d_2}}$$

As the cable behaves like three capacitors in series, therefore all the potentials are in phase i.e. voltage between conductor and earthed lead sheath is

$$V = V_1 + V_2 + V_3$$

→ A single core cable of conductor diameter 2 cm and lead sheath of diameter 5.3 cm is to be used on a 66 kV, 3- ϕ system. Two intersheaths of diameter 3.1 cm and 4.2 cm are introduced between the core and lead sheath. If the maximum stress in the layers is the same, find the voltages of the intersheaths!

ol. Here, $d = 2\text{cm}$, $d_1 = 3.1\text{cm}$, $d_2 = 4.2\text{cm}$
 $D = 5.3\text{cm}$: $V = \frac{66 \times \sqrt{2}}{\sqrt{3}} = 53.9\text{ kV}$

$$g_{1\text{max}} = \frac{V_1}{\frac{d}{2} \log_e \frac{d_1}{d}} = \frac{V_1}{1 \times \log_e \frac{3.1}{2}} = 2.28 V_1$$

$$g_{2\text{max}} = \frac{V_2}{\frac{d_1}{2} \log_e \frac{d_2}{d_1}} = \frac{V_2}{1.55 \log_e \frac{4.2}{3.1}} = 2.12 V_2$$

$$g_{3\text{max}} = \frac{V_3}{\frac{d_2}{2} \log_e \frac{D}{d_2}} = \frac{V_3}{2.1 \log_e \frac{5.3}{4.2}} = 2.04 V_3$$

As the max stress in the layers is same

$$g_{1\text{max}} = g_{2\text{max}} = g_{3\text{max}}$$

$$2.28 V_1 = 2.12 V_2 = 2.04 V_3$$

$$\therefore V_2 = \frac{2.28}{2.12} V_1 = 1.075 V_1$$

$$V_3 = \frac{2.28}{2.04} V_1 = 1.117 V_1$$

$$V_1 + V_2 + V_3 = V$$

$$V_1 + 1.075 V_1 + 1.117 V_1 = 53.9$$

$$V_1 = \frac{53.9}{3.192} = 16.88\text{ kV}$$

$$V_2 = 1.075 V_1 = 1.075 \times 16.88 = 18.14\text{ kV}$$

$$\therefore \text{voltage on first intersheath (i.e. near to core)} = V - V_1 \\ = 53.9 - 16.88 \\ = 37.02\text{ kV}$$

$$\text{Voltage on second intersheath} = V - V_1 - V_2 = 53.9 - 16.88 - 18.14 = 18.88\text{ kV}$$

Corona: When an alternating Potential difference is applied across two conductors whose spacing is large as compared to their diameters, there is no apparent change in the condition of atmospheric air surrounding the wires if the applied Voltage is low.

→ However, when the applied Voltage exceeds a certain Value, called Critical disruptive Voltage, the conductors are surrounded by a faint Violet glow called Corona.

→ The phenomenon of Corona is accompanied by a hissing sound, Production of ozone, Power loss and Radio Interference.

→ If the applied Voltage is increased to breakdown value, a flash over will occur between the conductors due to breakdown of air insulation.

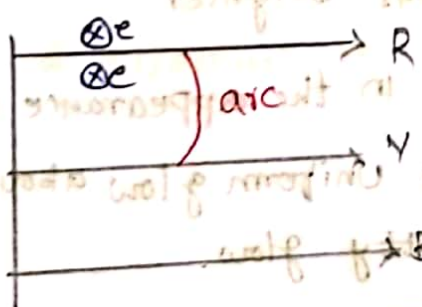
" The phenomenon of Violet glow, hissing noise and production of ozone gas in an overhead transmission line is known as Corona.

→ If the conductors are polished and smooth, the Corona glow will be Uniform throughout the length of the conductors, otherwise the rough points will appear brighter.

→ with dc voltage, there is difference in the appearance of the two wires. The Positive wire has Uniform glow about it, while the negative conductor has spotty glow.

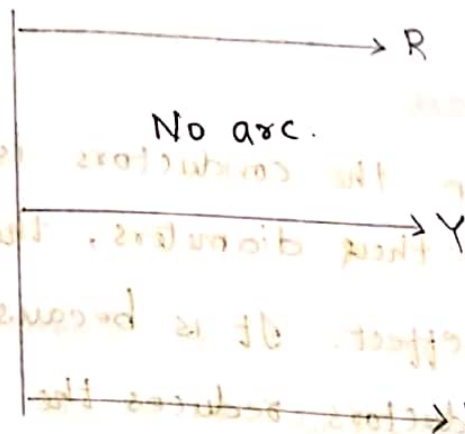
Description of CORONA:

- on economical basis, most of the transmission system is overhead system.
- There are certain free electrons which are available in the space in which some of the electrons are very near to surface of conductor and some of them are away from the surface conductor.
- The electrons near the power conductor will move at a faster rate than that of electrons away from the surface of conductor, because the electric field intensity is maximum at the surface of conductor. The electric field intensity is proportional to the operating voltage of the system.
- Whenever the operating voltage is more than 270KV, the field intensity will also increase so that the mobility of electron will increase.
- The high mobility electron will collide with the other air molecules, which will dislodge the electrons from these molecules.
- The process is continuous and at certain instant the electron avalanche will take place i.e. an arc will strike between the two lines.



→ The arc is an ionized body, which will be treated as line to line fault. The reliability of supply is reduced.

→ In order to eliminate the arc, the distance between the two lines are increased, in a Non-linear manner as the operating voltage increase, so that the field intensity will be restricted to be maximum at the surface of the conductor and almost zero between the conductors.



Factors Affecting Corona:

The phenomenon of corona is affected by the physical state of the atmosphere as well as by the conditions of the line. The following are the factors upon which corona depends:

- (i) Atmosphere: As corona is formed due to ionisation of air surrounding the conductors, therefore it is affected by the physical state of atmosphere. In the stormy weather, the number of ions is more than normal and as such corona occurs at much less voltage as compared to fair weather.

(ii) Conductor size: The corona effect depends upon the shape and conditions of the conductors. The rough and irregular surface will give rise to more corona because unevenness of surface decreases the value of breakdown voltage. Thus a stranded conductor has irregular surface and hence gives rise to more corona than a solid conductor.

(iii) Spacing between the conductors:

If the spacing between the conductors is made very large as compared to their diameters, there may not be any corona effect. It is because large distance between conductors reduces the electro-static stresses at the conductor surface, thus avoiding corona formation.

(iv) Line Voltage: The line voltage greatly affects corona.

If it is low, there is no change in the condition of air surrounding the conductors and hence no corona is formed. However if the line voltage has such a value that electrostatic stresses developed at the conductor surface make the air around the conductor conducting, then corona is formed.

Important Terms:

(i) critical disruptive Voltage:

→ It is the minimum phase-neutral voltage at which corona occurs.

consider two conductors of radii ' r ' cm and spaced ' d ' cm apart. If ' V ' is the phase-neutral potential, then potential gradient at the conductor surface is given by

$$g = \frac{V}{r \cdot \log_e \frac{d}{r}} \text{ Volts/cm}$$

In order that corona is formed, the value of ' g ' must be made equal to the breakdown strength of air.

→ The breakdown strength of air at 76 cm pressure and temperature of 25°C is 30 kV/cm (max) or 21.2 kV/cm (r.m.s) and is denoted by ' g_0 '.

→ If V_c is the phase-neutral potential required under these conditions, then

$$g_0 = \frac{V_c}{r \log_e \frac{d}{r}}$$

Where g_0 = breakdown strength of air at 76 cm of mercury and 25°C

= 30 kV/cm (max) (or) 21.1 kV/cm (r.m.s)

$\therefore$ critical disruptive Voltage, $V_c = g_0 \cdot r \cdot \log_e \frac{d}{r}$

→ The above expression for disruptive Voltage is under standard conditions i.e. at 76 cm of Hg and 25°C . However, if these conditions vary, the air density also changes, thus altering the value of g_0 .

→ The value of g_0 is directly proportional to air density.
 → Thus the breakdown strength of air at a barometric pressure of $\bar{p}$ cm of mercury and temperature of $t^{\circ}\text{C}$ becomes δg_0 where

$$\delta = \text{air density factor} = \frac{3.92 b}{273 + t}$$

Under standard conditions, the value of $\delta = 1$

∴ Critical disruptive Voltage, $V_c = g_0 \cdot \delta \cdot r \cdot \log_e \frac{d}{r}$

→ correction must also be made for surface condition of the conductor. This is accounted for by multiplying the above expression by irregularity factor m_0 .

$$\text{Critical disruptive Voltage, } V_c = m_0 \cdot g_0 \cdot \delta \cdot r \cdot \log_e \frac{d}{r} \text{ kV/phase.}$$

Where $m_0 = 1$ for polished conductors

$= 0.98$ to 0.92 for dirty conductors.

$= 0.87$ to 0.8 for Stranded Conductors

$\frac{b}{a} \cdot \log_e \frac{d}{r} = \log_e \frac{d}{r} \cdot \log_e \frac{d}{r} \therefore$

(ii) Visual Critical Voltage:

It is the minimum phase-neutral voltage, at which corona glow appears all along the line conductors.

It has been seen that in case of parallel conductors, the corona glow does not begin at the disruptive voltage V_c , but at a higher voltage V_v , called visual critical voltage.

The phase-neutral effective value of visual critical voltage is given by the following empirical formula

$$V_v = m_v g_0 \delta \cdot r \left(1 + \frac{0.3}{\sqrt{\delta r}} \right) \log_e \frac{d}{r} \text{ kV/phase}$$

Where m_v is another irregularity factor having a value of 1.0 for polished conductors and 0.72 to 0.92 for rough conductors.

(iii) Power Loss due to Corona:

Formation of corona is always accompanied by energy loss which is dissipated in the form of light, heat, sound and chemical action. When disruptive voltage is exceeded, the power loss due to corona is given by

$$P = 242.2 \left(\frac{f + 25}{\delta} \right) \sqrt{\frac{\delta}{d}} (V - V_c)^2 \times 10^{-5} \text{ kW/km/phase}$$

where f = Supply frequency in Hz

V = phase-neutral voltage (r.m.s)

V_c = disruptive voltage (r.m.s) per phase.

Radio Interference

- Radio Interference is the adverse effect introduced by corona on wireless broadcasting.
- The corona discharges emit radiation which may introduce noise signals in the communication lines, radio and television receivers.
- It is mainly due to the brush discharges on the surface irregularities of the conductor during positive ^{half} cycles. This leads to corona loss at voltages lower than critical voltages. The negative discharges are less troublesome for radio reception.
- Radio interference is considered as a field measured in micro volts per metre at any distance from the transmission line and is significant only at voltages greater than 200 kV.
- The amplitude of RI levels varies inversely as the frequency at which the interference is measured.
- Thus, the services in higher frequency band eg. television, frequency modulated broadcasting, microwave relay, radar etc. are less affected.
- RI is one of the very important factors while designing a transmission line.

Notes

Advantages and Dis-Advantages of corona

Advantages:

- (i) Due to corona formation, the air surrounding the conductor becomes conducting and hence virtual diameter of the conductor is increased. The increased diameter reduces the electrostatic stresses between the conductors.
- (ii) corona reduces the effects of transients produced by surges.

Dis-Advantages:

- (i) corona is accomplished by a loss of energy. This affects the transmission efficiency of the line.
- (ii) ozone is produced by corona and may cause corrosion of the conductor due to chemical action.
- (iii) The current drawn by the line due to corona is non-sinusoidal and hence non-sinusoidal voltage drop occurs in the line. This may cause inductive interference with neighbouring communication lines.

Dividing eqn (iii) with eqn (i)

$$\frac{W}{53} = \frac{(65.2 - 54)^2}{(61.2 - 54)^2}$$

$$W = 53 \times \left(\frac{11.2}{7.2} \right)^2$$

$$W = 128 \text{ kW.}$$

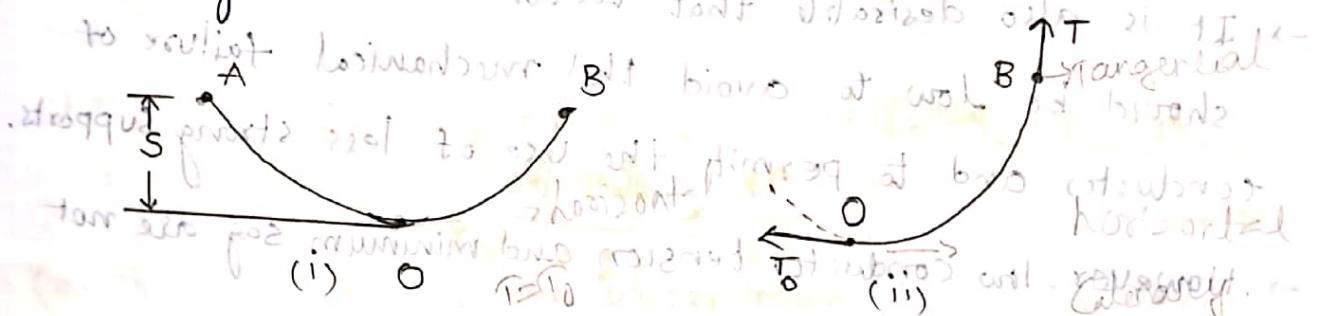
Tension: Force along a length of cable/rope stretching cable tight

Sag in Overhead lines:

The difference in level between points of supports and the lowest point on the conductor is called Sag.

Fig (i) shows a conductor suspended between two equilevel supports A and B. The conductor is not fully stretched but is allowed to have a dip between A and B.

The lowest point on the conductor is 'O' and the sag is 'S'.



The following points may be noted.

- (i) When the conductor is suspended between two supports at the same level, it takes the shape of catenary. However, if the sag is very small compared with the span, then sag span curve is like a parabola.

(ii) The tension at any point on the conductor acts tangentially. Thus tension T_0 at the lowest point 'O' acts horizontally as shown in fig(ii).

(iii) The horizontal component of tension is constant throughout the length of the wire.

(iv) The tension at supports is approximately equal to the horizontal tension acting at any point on the wire.

Thus if 'T' is tension at the support B, then $T = T_0$

Conductor Sag and tension:

→ The conductor sag should be kept to a minimum, in order to reduce the conductor material required and to avoid extra pole height for sufficient clearance above ground level.

→ It is also desirable that tension in the conductor should be low to avoid the mechanical failure of conductor and to permit the use of less strong supports.

→ However, low conductor tension and minimum sag are not possible.

→ It is because low sag means a tight wire and high tension, whereas a low tension means loose wire and increased sag.

Calculation of Sag

(i) When supports are at equal levels

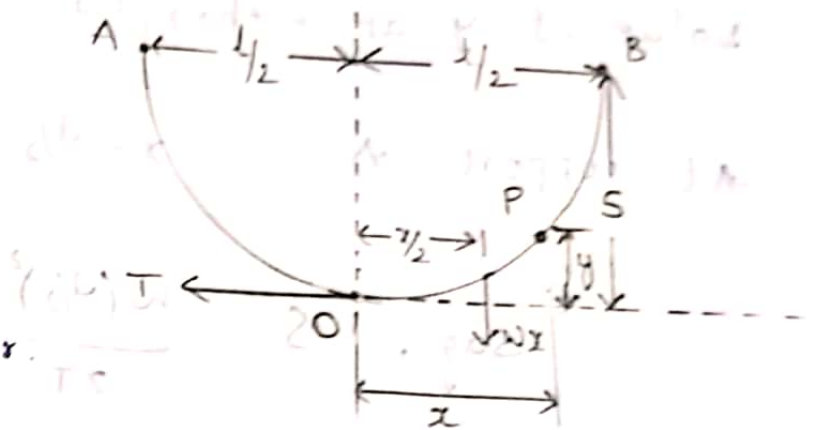
consider a conductor between two equal level supports 'A' and 'B' with 'O' as the lowest point as shown in fig. It can be proved that lowest point will be at the mid-span.

Let

l = length of span

w = Weight per unit length of conductor

T = Tension in the conductor



Consider a point P on the conductor. Taking the lowest point O as the origin, let the co-ordinates of point P be x and y . Assuming that the curvature is so small that curved length is equal to its horizontal projection (i.e., $OP = x$), the two forces acting on the portion OP of the conductor are

(a) The weight wx of conductor acting at a distance $x/2$ from O.

(b) The Tension T acting at O.

Equating the moments of above two forces about point O, we get

$$T \cdot y = w x \cdot \frac{x}{2}$$

$$y = \frac{w x^2}{2T}$$

The maximum dip (sag) is represented by the value of 'y' at either of supports A and B.

At support A, $x = l/2$ and $y = S$

$$\therefore \text{Sag, } S = \frac{w(l/2)^2}{2T}$$

$$S = \frac{w l^2}{8T}$$

(ii) When supports are at unequal levels

→ In hilly areas, we generally come across conductors suspended between supports at unequal levels.

→ Fig shows a conductor suspended between two supports A and B which are at different levels.

The lowest point on the conductor is 'O'.

Let

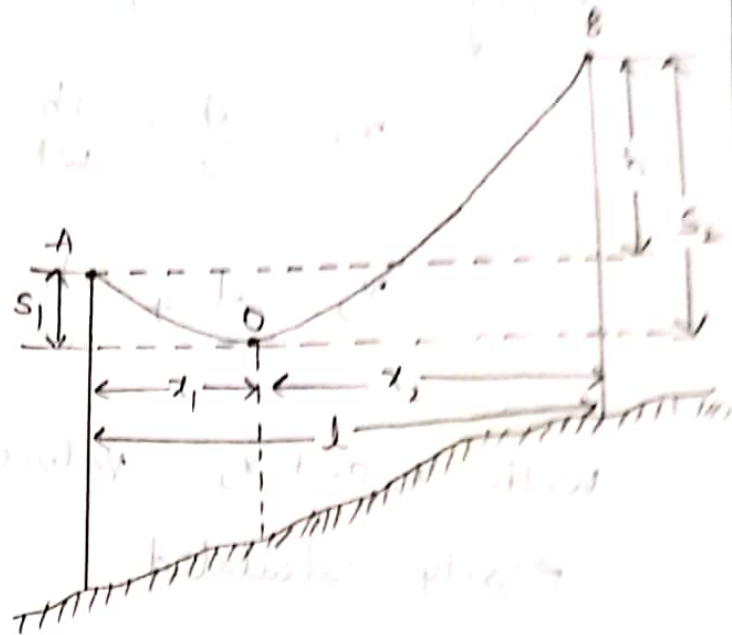
l = Span length

h = Difference in levels between two supports

x_1 = Distance of support at lower level (ie. A) from O

x_2 = Distance of support at higher level (ie. B) from O

T = Tension in the conductor.



If w is the weight per unit length of the conductor, then

$$\text{Sag } s_1 = \frac{w x_1^2}{2T}$$

$$\text{and Sag } s_2 = \frac{w x_2^2}{2T}$$

Also

$$x_1 + x_2 = l \quad \rightarrow (i)$$

$$\text{Now } s_2 - s_1 = \frac{w}{2T} [x_2^2 - x_1^2]$$

$$s_2 - s_1 = \frac{w}{2T} [(x_2 + x_1)(x_2 - x_1)]$$

$$h = \frac{w}{2T} \cdot l (x_2 - x_1)$$

$$x_2 - x_1 = \frac{2Th}{w \cdot l} \quad \rightarrow (ii)$$

$$\begin{cases} a^2 - b^2 = (a+b)(a-b) \\ x_1 + x_2 = l \\ s_2 - s_1 = h \end{cases}$$

Solving (i) & (ii) we get

$$x_1 = \frac{l}{2} - \frac{T h}{w l}$$

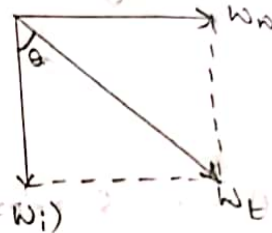
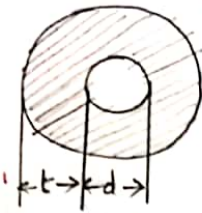
$$x_2 = \frac{l}{2} + \frac{T h}{w l}$$

With x_1 and x_2 , values of S_1 and S_2 can be easily calculated.

Effect of Wind and Ice loading:

- The above formulae for sag are true only in still air and at normal temperature when the conductor is acted by its weight only.
- However, in actual practice, a conductor may have ice coating and simultaneously subjected to wind pressure.
- The weight of ice acts vertically downwards i.e. in the same direction as the weight of the conductor.
- The force due to wind is assumed to act horizontally i.e. at right angle to the projected surface of the conductor.
- Hence the total force on the conductor is the Vector Sum of horizontal and vertical forces as shown in fig.

Ice coating



Total weight of conductor per unit length is

$$W_t = \sqrt{(W + W_i)^2 + (W_w)^2}$$

where * W = weight of conductor per unit length

= conductor material density $\times$ Volume per unit length

* W_i = weight of ice per unit length

= density of ice $\times$ Volume of ice per unit length

$$= \text{density of ice} \times \frac{\pi}{4} [(d+2t)^2 - d^2] \times 1$$

$$= \text{density of ice} \times \pi t (d+t)$$

* W_w = wind force per unit length

= wind pressure per unit area $\times$ projected area per unit length

$$= \text{wind pressure} \times [(d+2t) \times 1]$$

When the conductor has wind and ice loading also,

the following points may be noted.

(i) The conductor sets itself in a plane at an angle ' θ '

to the vertical where

$$\tan \theta = \frac{W_w}{W + W_i}$$

(ii) The sag in the conductor is given by

$$S = \frac{w_t l^2}{8T}$$

Hence S represents the slant sag in a direction making an angle θ to the vertical. If no specific mention is made in the problem, then slant sag is calculated by using the above formula.

(iii) The vertical sag = $S \cos \theta$.

Formulae:

* Working Stress = $\frac{\text{Ultimate Strength}}{\text{Safety factor}}$

* Working Tension = working stress * conductor area.

* $W_t = \sqrt{w^2 + w_w^2}$, $W_t = \sqrt{(w + w_i)^2 + w_w^2}$ (if ice loading is present)

* Sag, $S = \frac{W_t l^2}{8T}$

* $\tan \theta = \frac{W_w}{w}$

* Vertical sag = $S \cos \theta$

* Wt. of conductor / m run, $W = \text{Conductor area} * 100 \text{ cm} * \text{Specific Gravity}$

1. A 132 kV transmission line has the following data:
Wt of conductor = 680 kg/km, Length of span = 260 m
Ultimate strength = 3100 kg, Safety factor = 2
Calculate the height above ground at which the conductor should be supported. Ground clearance required is 10 metres.

sol. Wt. of conductor/metre run, $w = \frac{680}{1000} = 0.68 \text{ kg}$

Working Tension, $T = \frac{\text{Ultimate strength}}{\text{Safety factor}} \times \text{conductor area.}$

$$T = \frac{3100}{2} \times 1 \quad \left\{ \because \text{Assume area} = 1 \text{ cm}^2 \right\}$$

$$T = 1550 \text{ kg}$$

Span length, $l = 260 \text{ m}$

$$\text{Sag, } s = \frac{wl^2}{8T} = \frac{0.68 \times (260)^2}{8 \times 1550} = 3.7 \text{ m}$$

$\therefore$ conductor should be supported at a height of $10 + 3.7 = 13.7 \text{ m}$

2. A transmission line has a span of 150 m, between level supports. The conductor has a cross sectional area of 2 cm^2 . The tension in the conductor is 2000 kg. If the specific gravity of the conductor material is 9.9 gm/cm^3 and wind pressure is 1.5 kg/m length, calculate the sag. What is the vertical sag?

sol. Span length, $l = 150\text{ m}$

Working Tension, $T = 2000\text{ kg}$

Wind force/m length of conductor, $w_w = 1.5\text{ kg}$

Wt. of conductor/m length, $W = \text{Sp. Gravity} \times \text{Volume of 1m conductor}$
 $= 9.9 \times 2 \times 100 \left(\frac{\text{gm}}{\text{cm}^3} \times \text{cm}^2 \times 1\text{m} \right)$
 $= 1980\text{ gm}$
 $= 1.98\text{ kg}$

Total weight of 1m length of conductor is $= \left(\frac{\text{gm}}{\text{cm}^3} \times 100 \right) \times \left(\frac{\text{cm}^2}{10^2 \times 100} \right) = \text{gm} \times 100$

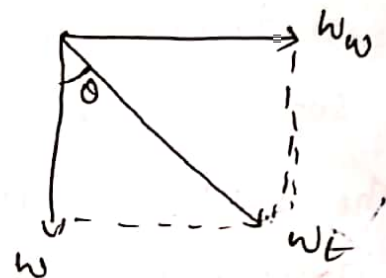
$$W_T = \sqrt{W^2 + W_w^2}$$
$$= \sqrt{(1.98)^2 + (1.5)^2}$$
$$W_T = 2.48\text{ kg}$$

$$\text{Sag, } S = \frac{W_T l^2}{8T} = \frac{2.48 \times (150)^2}{8 \times 2000} = 3.48\text{ m}$$

This is the value of slant sag in a direction making an angle θ with the vertical.

$$\tan \theta = \frac{w_w}{w} = \frac{1.5}{1.98} = 0.76$$

$$\theta = \tan^{-1}(0.76) = 37.23^\circ$$



$$\therefore \text{Vertical sag} = S \cos \theta$$
$$= 3.48 \times \cos 37.23^\circ$$
$$= 2.77\text{ m}$$

3. A transmission line has a span of 275 m between level supports. The conductor has an effective diameter of 1.96 cm and weighs 0.865 kg/m. Its ultimate strength is 8060 kg. If the conductor has ice and coating of radial thickness 1.27 cm and is subjected to a wind pressure of 3.9 gm/cm² of projected area, calculate sag for a safety factor of 2. weight of 1 c.c. of ice is 0.91 gm.

sol. Span length $l = 275$ m, wt. of conductor/m length, $w = 0.865$ kg
conductor diameter, $d = 1.96$ cm. ice coating thickness, $t = 1.27$ cm

Working tension, $T = \frac{8060}{2} = 4030$ kg (area is not given, so considered it as unity)

Volume of ice per metre (i.e. 100 cm) length of conductor

$$= \pi t (d+t) \times 100 \text{ cm}^3$$

$$= \pi \times 1.27 (1.96 + 1.27) \times 100 = 1288 \text{ cm}^3$$

Weight of ice per metre length of conductor is

$$w_i = 0.91 \times 1288 = 1172 \text{ gm} = 1.172 \text{ kg}$$

wind force /m length of conductor is

$$w_w = [\text{Pressure}] \times [(d+2t) \times 100] \quad \left(\frac{\text{gm}}{\text{cm}^2} \times \text{cm} \right)$$

$$= 3.9 \times (1.96 + 2 \times 1.27) \times 100 \text{ gm} = 1755 \text{ gm} = 1.755 \text{ kg}$$

Total weight of conductor per metre length of conductor is

$$w_t = \sqrt{(w + w_i)^2 + (w_w)^2}$$

$$= \sqrt{(0.865 + 1.172)^2 + (1.755)^2} = 2.688 \text{ kg}$$

$$\rightarrow \text{Sag} = \frac{w_t l^2}{8T} = \frac{2.688 \times (275)^2}{8 \times 4030} = 6.3 \text{ m.}$$

4. The towers of height 30m and 90m respectively support a transmission line at water crossing. The horizontal distance b/w the towers is 500m. If the tension in conductor is 1600 kg. Find the minimum clearance of the conductor and water and clearance mid-way between the supports. Weight of the conductor is 1.5 kg/m. Bases of towers can be considered to be at water level.

Sol.

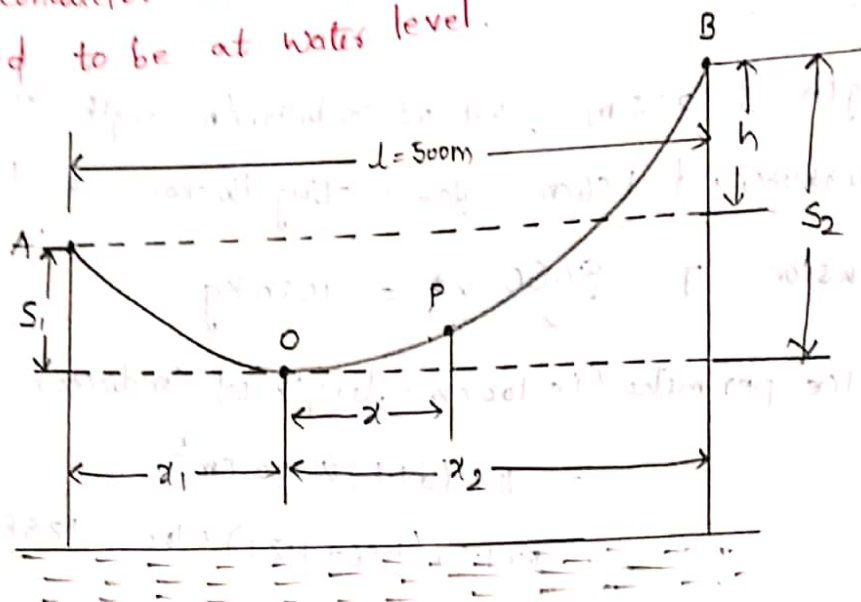


Fig shows the conductor suspended between two supports A and B at different levels with 'O' as the lowest point on the conductor.

Here $L = 500\text{ m}$; $w = 1.5\text{ kg}$; $T = 1600\text{ Kg}$

Difference in level between supports, $h = 90 - 30 = 60\text{ m}$.

Let the lowest point 'O' of the conductor be at a distance x_1 from the support at lower level (i.e. support A) and at a distance x_2 from the support at higher level (i.e. support B)

Obviously $x_1 + x_2 = 500\text{ m} \rightarrow \text{①}$

Now Sag $S_1 = \frac{w x_1^2}{2T}$ and Sag $S_2 = \frac{w x_2^2}{2T}$

$$h = S_2 - S_1 = \frac{w x_2^2}{2T} - \frac{w x_1^2}{2T}$$

$$h = \frac{w}{2T} (x_2^2 - x_1^2)$$

$$h = \frac{w}{2T} (x_2 + x_1)(x_2 - x_1)$$

$$\left\{ \begin{array}{l} a^2 - b^2 = (a+b)(a-b) \\ h = 60 \\ x_1 + x_2 = 500 \\ w = 1.5 \\ T = 1600 \end{array} \right\}$$

$$60 = \frac{1.5 \times 500 (x_2 - x_1)}{2 \times 1600}$$

$$x_2 - x_1 = \frac{60 \times 2 \times 1600}{1.5 \times 500}$$

$$x_2 - x_1 = 256 \text{ m} \longrightarrow \textcircled{2}$$

Solving expressions (i) & (ii) we get $x_1 = 122 \text{ m}$, $x_2 = 378 \text{ m}$

$$\text{Sag } S_1 = \frac{w x_1^2}{2T} = \frac{1.5 \times (122)^2}{2 \times 1600} = 7 \text{ m}$$

Now, clearance of the lowest point 'O' from water level
 $= 30 - 7 = 23 \text{ m}$

Let mid-point P be at a distance 'x' from lowest point 'O'
 clearly $x = 250 - x_1 = 250 - 122 = 128 \text{ m}$.

$$\text{Sag at mid point P, } S_{\text{mid}} = \frac{w x^2}{2T}$$

$$= \frac{1.5 \times (128)^2}{2 \times 1600}$$

$$S_{\text{mid}} = 7.68 \text{ m}$$

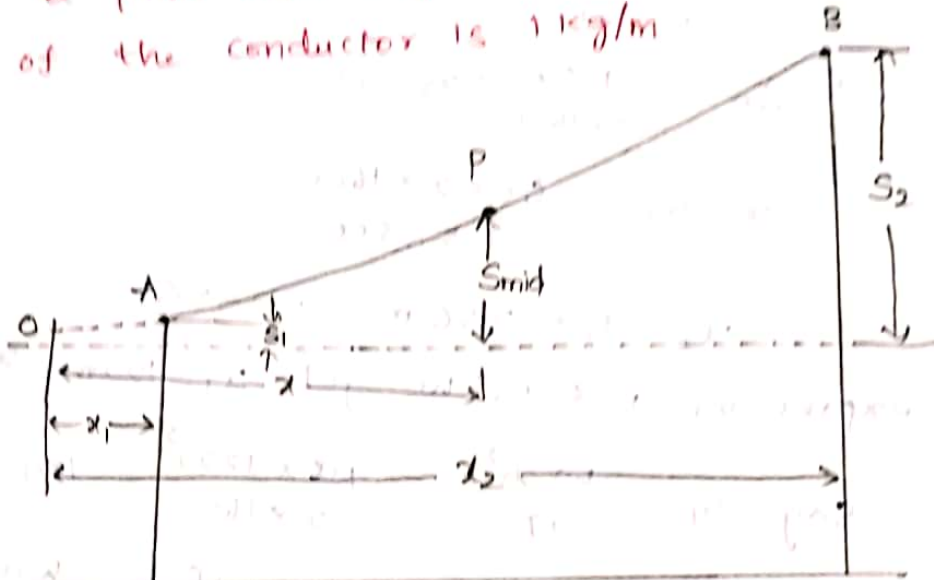
clearance of mid point = P from water level

$$= 23 + 7.68$$

$$= 30.68 \text{ m}$$

Q. An overhead transmission line at a river crossing is supported from two towers at height of 40m and 90m above water level, the horizontal distance between the towers being 400m. If the maximum allowable tension is 2000kg, find the clearance between the conductor and water at a point mid-way between the towers. weight of the conductor is 1kg/m.

Sol.



Here $h = 90 - 40 = 50\text{m}$; $d = 400\text{m}$

$T = 2000\text{kg}$; $w = 1\text{kg/m}$

$$x_1 + x_2 = 400\text{m} \rightarrow \textcircled{1}$$

Now $h = S_2 - S_1 = \frac{wx_2^2}{2T} - \frac{wx_1^2}{2T}$

$$50 = \frac{w}{2T} (x_2 + x_1)(x_2 - x_1)$$

$$(x_2 - x_1) = \frac{50 \times 2 \times 2000}{1 \times 400} = 500\text{m}$$

$$x_2 - x_1 = 500\text{m} \rightarrow \textcircled{2}$$

Solving eqs (i) & (ii), we get $x_2 = 450\text{m}$ and $x_1 = -50\text{m}$

Now x_2 is the distance of higher support 'B' from the lowest point 'O' on the conductor, whereas x_1 is that of lower support 'A'. As the span is 400m, therefore, point 'A' lies on the same side of 'O' as 'B'.

Horizontal distance of midpoint 'P' from lowest point 'O' is

$$x = \text{Distance of A from 'O'} + \frac{400}{2}$$

$$x = 50 + 200$$

$$x = 250\text{m}$$

$\therefore$ Sag at point 'P', $S_{\text{mid}} = \frac{wx^2}{2T}$

$$S_{\text{mid}} = \frac{1 \times (250)^2}{2 \times 2000}$$

$$S_{\text{mid}} = 15.6\text{m}$$

Now, Sag $S_2 = \frac{wx_2^2}{2T} = \frac{1 \times (450)^2}{2 \times 2000} = 50.6\text{m}$

Height of point 'B' above midpoint 'P' = $S_2 - S_{\text{mid}}$

$$= 50.6 - 15.6$$

$$= 35\text{m}$$

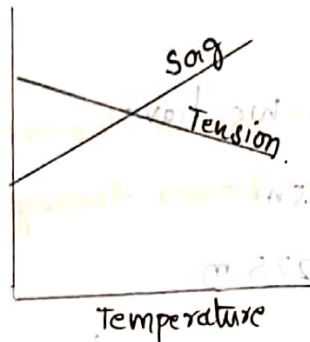
$\therefore$ clearance of mid-point 'P' above water level

$$= 90 - 35$$

(ii) $\therefore$ $\frac{21 \times 0.021 \times 5}{0.008 \times 1} = 55\text{m}$

Stringing charts:

- For use in the field work of stringing the conductors temperature-sag and temperature-tension charts are plotted for the given conductor and loading conditions.
- Such curves are called stringing charts.
- These charts are very helpful while stringing overhead lines.



Sag Template:

Normally there are two types of support being used

- The standard or straight run or intermediate tower
- The angle or anchor or tension tower.

→ While the straight run towers are used for straight run and normal conditions, the angle towers are used at angles, terminals and other points where a considerable amount of unbalanced pull may be thrown on the supports. The angle towers are, therefore

designed to withstand heavy loadings as compared to standard towers.

→ In order to locate the position of the towers, the first step is to know a suitable value for the support height and if there are no special guiding factors which dictate this choice, several alternatives may be tried.

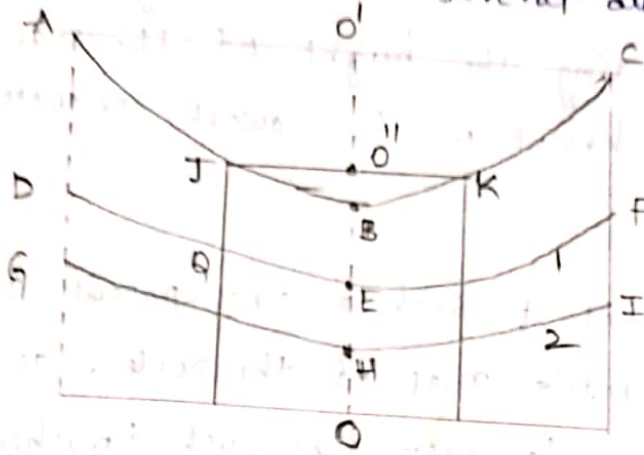


Fig: Sag Template (1) Ground clearance Line
(2) Tower footing Line.

→ The next step is to plot a sag template on a piece of transparent paper which consists of a set of curves as shown in fig. The horizontal and vertical distances represent the span lengths and sags respectively.

* GHI is the tower footing line (2) i.e., this line gives the location of the footing of the tower.

* DEF is the ground clearance line (1) i.e., the minimum clearance of the power conductor from the ground.

This clearance to ground will depend on the operating voltage and is given according to Indian Electricity Rules.

→ Curve ABC is such that with a span length of AC, the maximum sag of the conductor would be OB and with span JK, the max sag is O'B. The curve DEF is obtained from ABC by subtracting Ordinates from ABC equal to the minimum ground clearance required and curve GHI is obtained from ABC by setting off from AB, a distance representing the height of the standard tower, from point of attachment of the lowest conductor to the ground level.

→ It is then clear that if such a transparent template is placed upon a profile map of the route, as indicated such that the ground clearance line just touches the profile as at Q then points G and I, where the 'support foot' line cuts the profile, will indicate the position of the towers represented by GA and IC. The curve ABC will represent the actual shape and position of the lowest conductor, and since JQ represents correct ground clearance, the conductor nowhere approaches the ground by more than the safe amount.

→ In the particular case shown the points of conductor support are upon the same horizontal level, but the same process applies when the route is a steeply sloping one and the shape and position of the conductor will always be represented by the curve ABC as shown in fig.

2 Marks Questions:

1. What is an **insulator**? State its function.
2. List the **types of insulators** used in overhead lines.
3. What is a **pin insulator**?
4. What is a **suspension insulator**?
5. Define **string efficiency**.
6. What is **capacitance grading**?
7. Define **sag** and **tension**.
8. What are the factors affecting **sag**?
9. What is **corona**?
10. What is **corona power loss**?

10 Marks Questions:

1. Discuss the formation of Corona on the transmission lines.
2. A 3-phase, 220 kV, 50 Hz transmission line consists of 1.5 cm radius conductor spaced 2 meters apart in equilateral triangular formation. If the temperature is 40°C and atmospheric pressure is 76 cm, calculate the corona loss per km of the line. Take $m_0 = 0.85$.
3. Explain the methods of improving string efficiency. A single phase overhead line has two conductors of diameter 1cm with a spacing of 1 meter between centers. If critical voltage for air is 21.1 kV/cm, for what value of the line voltage will corona commence?
4. A) Derive the expression for sag and tension of a conductor when the supports are at the same level.
B) Derive the expression for sag and tension when the supports are at unequal levels.
5. Explain the phenomenon of corona in transmission lines. Derive the expressions for critical disruptive voltage and visual critical voltage.
6. A transmission line has a span of 155m between the two supports the line conductor has cross sectional area of 1.55cm^2 and it weighs 100kg/100m. If the breaking stress of the copper conductor is 4110 kg/cm^2 . Find the maximum sag for a safety factor of 4. The maximum wind pressure is 90kg/m^2 of the projected surface?
7. A) What is string efficiency? Why is it necessary to have high string efficiency? How can it be achieved?
B) Determine the sag of an overhead line for the following data: span length 160 meters, conductor diameter 0.95 cm, weight per unit length of the conductor 0.65 kg/meter. Ultimate stress 4250 kg/cm^2 , wind pressure 40 kg/cm^2 of projected area. Factor of safety 5.

REFERENCES:

- ❖ C.L. Wadhwa, —Electrical Power Systems, New Age International Pub. Co, Third Edition, 2001.

“The reliability of overhead power transmission depends on the effective design of insulators, control of sag and tension, and minimization of corona effects.”

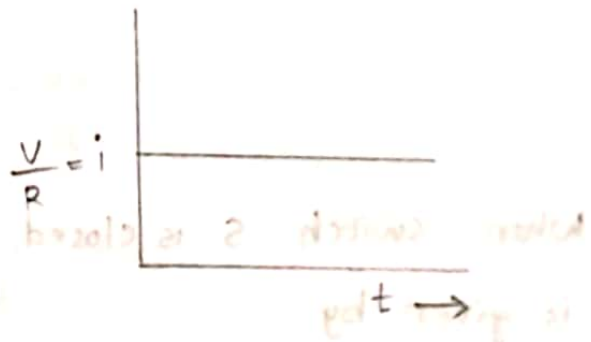
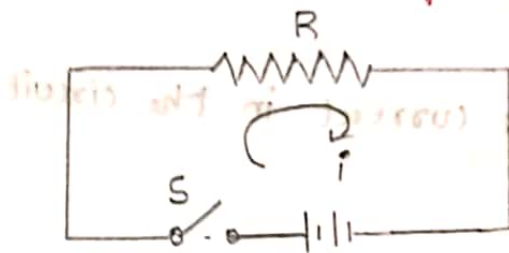
Unit: Power System Transients:

Transients Phenomenon is an aperiodic function of time and does not last longer. The duration for which they last is very insignificant as compared with the operating time of the system. Yet they are very important because depending upon the severity of these transients, the system may result into blackout in a city, shut down of a plant, fires in some buildings etc.

For analysing circuits for transients we will make use of Laplace transform technique which is more powerful and easy to handle the transient problems than the differential equation technique. The transient will depend upon the driving source also i.e. whether it is d.c source or an a.c source.

1. D.C SOURCE

(a) Resistance only.



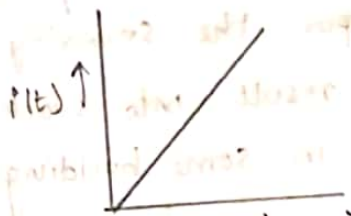
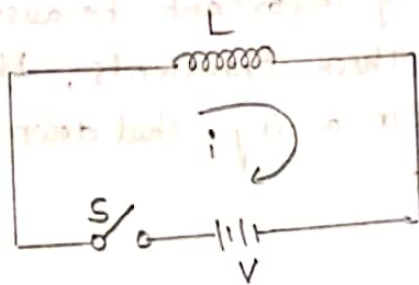
As soon as the switch S is closed, the current in the circuit will be determined according to Ohm's law.

$$I = \frac{V}{R}$$

Now transients will be there in the circuit.

(b) Inductance only:

When a pure inductance is switched onto a d.c. source, the current at $t=0^+$ is zero and this increases linearly with time till for infinite time it becomes infinity.

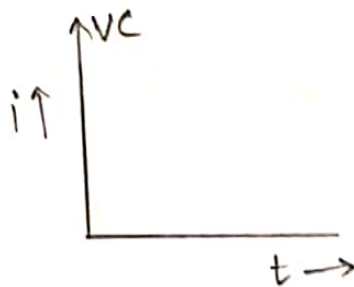
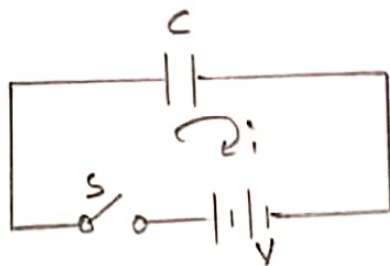


When switch S is closed, the current in the circuit will be given by

$$I(s) = \frac{V(s)}{Z(s)} = \frac{V}{s} \cdot \frac{1}{sL} = \frac{V}{L} \cdot \frac{1}{s^2}$$

$$i(t) = \frac{V}{L} \cdot t$$

(c) Capacitance only:

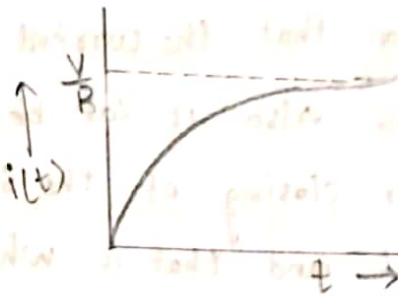
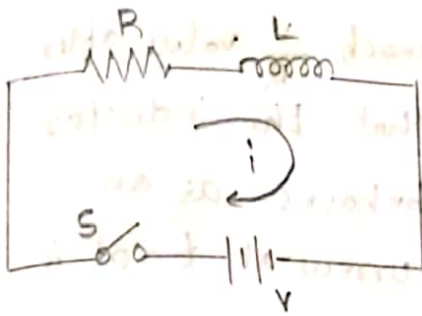


When switch S is closed, the current in the circuit is given by

$$I(s) = \frac{V(s)}{Z(s)} = \frac{V}{s} \cdot \frac{1}{1/sc} = VC$$

which is an impulse of strength (magnitude) VC.

(d) R-L circuit:



When switch S is closed, the current in the circuit is given by

$$I(s) = \frac{V(s)}{Z(s)} = \frac{V}{s} \cdot \frac{1}{R + Ls}$$

Applying partial fraction

$$\frac{V}{s(R + Ls)} = \frac{A}{s} + \frac{B}{R + Ls}$$

$$A(R + Ls) + Bs = V$$

Put $s = 0$, $AR = V \Rightarrow A = \frac{V}{R}$

$$s = -\frac{R}{L}, \quad B\left(-\frac{R}{L}\right) = V$$

$$B = -\frac{VL}{R}$$

$$\therefore I(s) = \frac{V/R}{s} + \frac{-\frac{VL}{R}}{R + Ls}$$

$$I(s) = \frac{V}{R} \left[\frac{1}{s} - \frac{1}{s + R/L} \right]$$

(Divide second term with 'L')

$$i(t) = \frac{V}{R} \left[1 - \exp\left(-\frac{R}{L}t\right) \right]$$

The Variation of current is shown. It can be seen from the expression that the current will reach $\frac{V}{R}$ value after infinite time. Also it can be seen that the inductor just after closing of the switch behaves as an open circuit and that is why the current at $t = 0^+$ is zero.

When $t = \frac{L}{R}$, $i(t) = \frac{V}{R} \left(1 - \frac{1}{e}\right)$

$$= I_m \left(1 - \frac{1}{e}\right)$$

$$= 0.632 I_m$$

At time $t = \frac{L}{R}$, the current in the circuit is

63.2% of the maximum value reached in the circuit.

This time in seconds is called time-constant of the circuit.

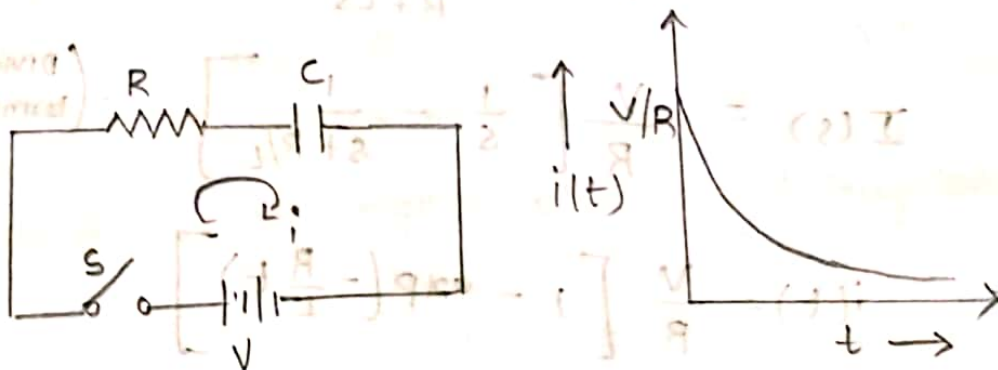
The larger the value of inductance in the circuit

as compared with resistance, the slower will be the

build up of current in the circuit. The energy stored in the inductor under steady state condition will be

$$\frac{1}{2} L I_m^2, \text{ where } I_m = \frac{V}{R}.$$

(e) R-C circuit



After the switch 'S' is closed, current in the circuit is given by

$$I(s) = \frac{V(s)}{Z(s)} = \frac{V}{s} \cdot \frac{1}{R + \frac{1}{sC}}$$

$$= \frac{V}{s} \cdot \frac{sC}{(RCs + 1)}$$

$$= V \cdot \frac{C}{RCs + 1}$$

$$= V \cdot \frac{1}{Rs + \frac{1}{C}}$$

$$= V \cdot \frac{1}{R(s + \frac{1}{RC})}$$

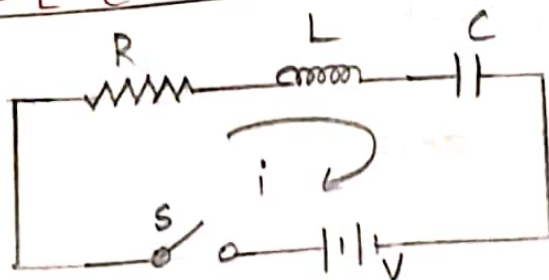
$$I(s) = \frac{V}{R} \cdot \frac{1}{s + \frac{1}{RC}}$$

$$i(t) = \frac{V}{R} \cdot e^{-t/RC}$$

→ It is seen that at $t=0$, the capacitor acts as a short circuit to the d.c. source and the current is $\frac{V}{R}$ limited only by the resistance of the circuit.

→ At $t=\infty$ the current in the circuit is zero and capacitor is charged to a Voltage V . The energy stored by capacitor is $\frac{1}{2} CV^2$.

(f) R-L-C Circuit:



After the Switch S is closed, the current in the circuit is given by

$$I(s) = \frac{V(s)}{Z(s)} = \frac{V}{s} \cdot \frac{1}{R + Ls + 1/Cs}$$

$$= \frac{V}{s} \cdot \frac{Cs}{RCs + Lcs^2 + 1}$$

$$= \frac{V}{s} \cdot \frac{1/L}{s^2 + \frac{R}{L}s + 1/LC}$$

$$I(s) = \frac{V}{L} \cdot \frac{1}{\left\{s + \left(-\frac{R}{2L} - \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}}\right)\right\} \cdot \left\{s + \left(-\frac{R}{2L} + \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}}\right)\right\}}$$

Let $\frac{R}{2L} = a$ and $\sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}} = b$ then

$$I(s) = \frac{V}{L} \cdot \frac{1}{(s+a-b)(s+a+b)}$$

$$= \frac{V}{2bL} \left\{ \frac{1}{(s+a-b)} - \frac{1}{(s+a+b)} \right\}$$

$$i(t) = \frac{V}{2bL} \left\{ e^{-(a-b)t} - e^{-(a+b)t} \right\}$$

There are 3 conditions based on value of b .

(i) If $\frac{R^2}{4L^2} > \frac{1}{LC}$, b is real

(ii) If $\frac{R^2}{4L^2} = \frac{1}{LC}$, b is zero

(iii) If $\frac{R^2}{4L^2} < \frac{1}{Lc}$, b is imaginary.

case (i) When ' b ' is real

$$i(t) = \frac{V}{2\sqrt{\frac{R^2}{4L^2} - \frac{1}{Lc}}} \left[\exp\left\{-\left(\frac{R}{2L} + \sqrt{\frac{R^2}{4L^2} - \frac{1}{Lc}}\right)t\right\} - \exp\left\{-\left(\frac{R}{2L} - \sqrt{\frac{R^2}{4L^2} - \frac{1}{Lc}}\right)t\right\} \right]$$

and Variation of current is as follows



→ waveform when b is real

case (ii) When $b = 0$

$$i(t) = \frac{V}{2bL} \{ e^{-at} - e^{-at} \} \text{ which is indeterminate}$$

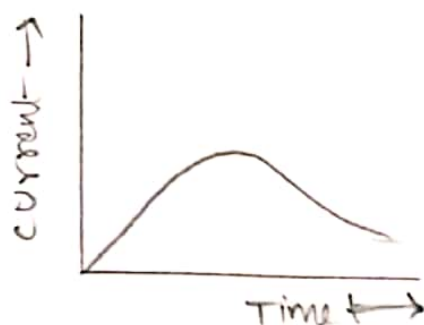
Therefore, differentiating $i(t)$ with respect to b gives

$$i(t) = \frac{V}{2L} \cdot t (e^{-(a-b)t} + e^{-(a+b)t})$$

Now at $b = 0$

$$i(t) = \frac{V}{2L} \cdot t e^{-at} = \frac{V}{L} \cdot t \cdot e^{-(R/2L)t}$$

Variation of current is given as



→ waveform when b is zero.

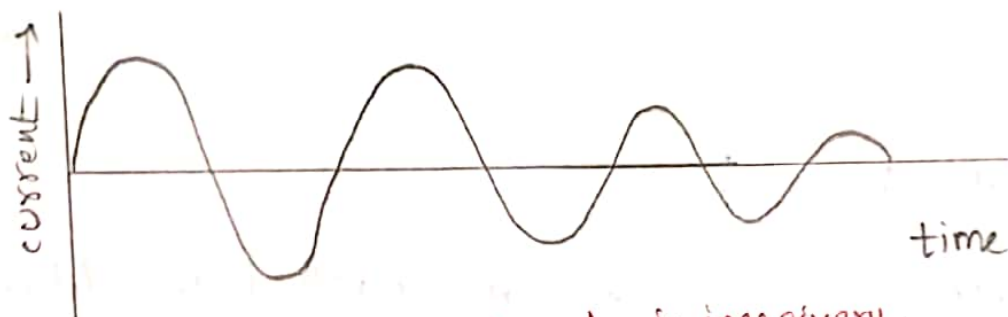
Case (iii): When 'b' is imaginary

$$i(t) = \frac{V}{2bL} \left\{ e^{-at} \cdot e^{jkt} - e^{-at} \cdot e^{-jkt} \right\}$$

$$= \frac{V}{2bL} e^{-at} \cdot 2 \sin kt$$

$$= \frac{V}{2L \sqrt{\frac{R^2}{4L^2} - \frac{1}{Lc}}} e^{-at} \cdot 2 \sin \left(\sqrt{\frac{R^2}{4L^2} - \frac{1}{Lc}} \right) t$$

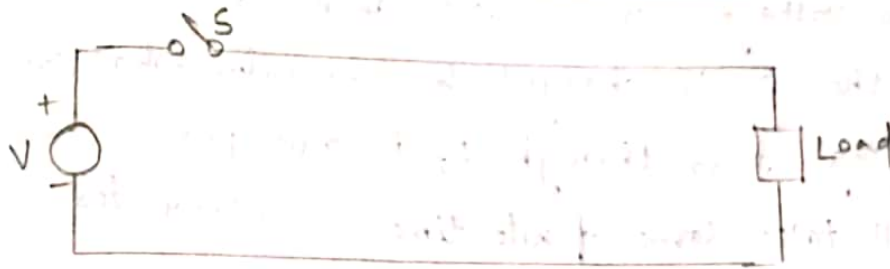
The current waveform is as follows.



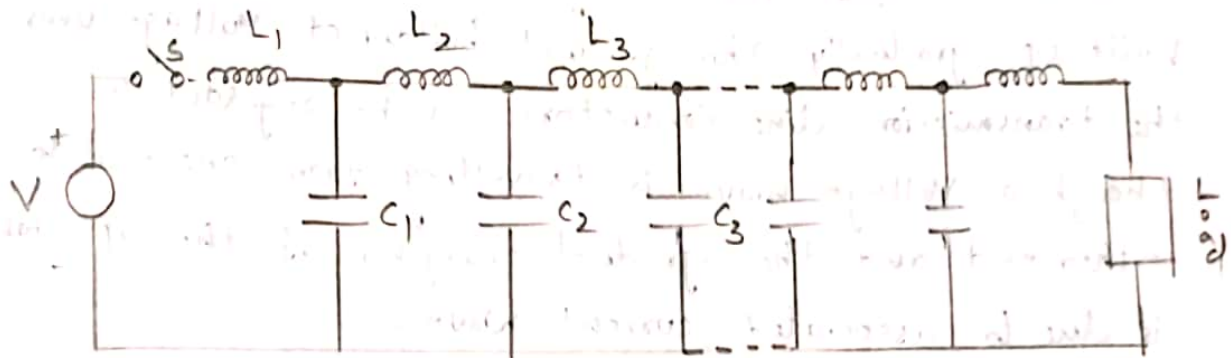
→ waveform when b is imaginary.

When 'b' is positive or zero, the variation of current is non-oscillatory whereas it is oscillatory when 'b' is imaginary. Because of the presence of the capacitance, the current in all the three cases die down to zero value with d.c. source in the circuit.

Travelling Waves on Transmission Lines:



(a) Long transmission line



(b) Equivalent π -section of a long transmission line.

→ To understand the concept of Travelling Wave phenomenon over transmission lines consider fig(a). The line is assumed to be lossless.

→ Let ' L ' and ' C ' be the inductance and capacitance respectively per unit length of the line. The line has been represented in fig(b) by a large number of L and C π -sections.

→ When the switch ' S ' is closed, the voltage does not appear instantaneously at the other end. When the switch ' S ' is closed, the inductance L_1 acts as open circuit and C_1 is short circuit instantaneously.

→ The same instant, the next section cannot be charged because the voltage across capacitance C_1 is zero. So unless the C_1 is charged to some value whatsoever the charging of C_2 through L_2 is not possible which of course will take some finite time. Similarly for 3rd, 4th sections and so on.

→ So we see that the voltage at the successive sections builds up gradually. This gradual buildup of voltage over the transmission line conductors can be regarded as though a voltage wave is travelling from one end to other end and the gradual charging of the capacitance is due to associated current wave.

→ Now it is desired to find out the expressions for the relation between voltage and the current waves travelling over the transmission lines and their velocity of propagation.

→ Suppose that the wave after time t has travelled through a distance x . Since the line is assumed to be lossless, whatever is the value of voltage and current wave at the start is remains same through out the travel.

→ consider a distance dx which is travelled by the wave in time dt .

→ The electrostatic flux is associated with voltage wave and electromagnetic flux is associated with current wave.

The electrostatic flux which is equal to the charge between the conductors of the line upto a distance x is given by

$$q = VCx \rightarrow (1)$$

current $I = \frac{dq}{dt} = VC \cdot \frac{dx}{dt}$

where $\frac{dx}{dt}$ is the velocity of travelling wave over the line conductor. Let this be represented by v .

Then $I = VCv \rightarrow (2)$

→ similarly the electromagnetic flux linkages upto distance ' x ' around the conductor is given by

$$\psi = ILx \rightarrow (3)$$

Voltage $V = \frac{d\psi}{dt} = I \cdot L \cdot \frac{dx}{dt}$

$$V = I \cdot L v \rightarrow (4)$$

→ Divide eqn (4) with eqn (2) we get

$$\frac{(4)}{(2)} = \frac{V}{I} = \frac{I \cdot L v}{VCv} = \frac{I \cdot L}{V \cdot C}$$

$$\therefore \frac{V}{I} = \frac{L}{C}$$

$$\frac{V^2}{I^2} = \frac{L}{C}$$

$$\frac{V}{I} = \sqrt{\frac{L}{C}} = Z_n \quad (Z_n = \text{natural frequency})$$

$$\frac{V}{I} = Z_n \rightarrow \textcircled{5}$$

→ Multiplying eqn $\textcircled{4}$ and eqn $\textcircled{2}$ we get

$$\textcircled{4} \times \textcircled{2} \quad VI = VC \times ILV$$

$$I = LC V^2$$

$$V^2 = \frac{1}{LC}$$

$$V = \frac{1}{\sqrt{LC}} \rightarrow \textcircled{6}$$

→ Now the expression for 'L' and 'C' for O.H.T.L is given by

$$L = 2 \times 10^{-7} \ln \frac{d}{r} \text{ H/m}$$

$$C = \frac{2\pi\epsilon}{\ln d/r} \text{ F/m}$$

→ Substituting L and C value in eqn $\textcircled{6}$ we get

$$V = \frac{1}{\sqrt{LC}}$$

$$V = \frac{1}{\sqrt{2 \times 10^{-7} \ln d/r \times \frac{2\pi\epsilon}{\ln d/r}}}$$

$$V = \frac{1}{\sqrt{4\pi\epsilon \times 10^{-7}}} \quad \left\{ \epsilon = \frac{1}{36\pi} \times 10^9 \right\}$$

$$V = \frac{1}{\sqrt{4\pi \times \frac{1}{36\pi} \times 10^{-9} \times 10^{-7}}}$$

$$V = \frac{1}{\sqrt{\frac{1}{9} \times 10^{-16}}} \quad V = \frac{1}{\left(\frac{10^{-16}}{9}\right)^{1/2}} = \frac{1}{\left(\frac{10^{-8}}{3}\right)}$$

$$\therefore V = 3 \times 10^8 \text{ m/sec}$$

This means that the velocity of propagation of travelling waves over the O.H.TL is equal to the velocity of light.

Open End lines

consider a line with the receiving end open circuited as shown in fig.

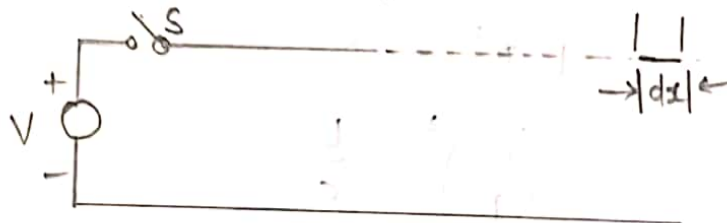


fig: case of an open-ended line.

When the switch 'S' is closed, a voltage and current wave of magnitudes 'V' and 'I' respectively travel towards the open end. These waves are related by the equation

$$\frac{V}{I} = Z$$

where Z = characteristic impedance of line.

→ considers the last element dx of the line, here the impedance is going to change from Z to ∞ impedance as the line is open-ended.

→ The electromagnetic energy stored by the element

$$dx = \frac{1}{2} L dx I^2$$

→ The Electrostatic Energy in the element is

$$dx = \frac{1}{2} C dx V^2$$

→ Since the current at the open end is zero, the electromagnetic energy vanishes and is transformed into electrostatic energy.

→ As a result, let the change in voltage be 'e'

then

$$\frac{1}{2} L dx I^2 = \frac{1}{2} C dx e^2$$

$$L I^2 = C \cdot e^2$$

$$\left(\frac{e}{I}\right)^2 = \frac{L}{C}$$

$$\frac{e}{I} = \sqrt{\frac{L}{C}} = Z$$

$$e = I \cdot Z = V$$

→ This means that potential of the open end is raised by 'V' volts.

→ The total potential of open end when wave reaches this end is given by $V + V = 2V$.

- The wave that starts travelling over the line when the switch S is closed, could be considered as the incident wave.
- After the wave reaches the open end, the rise in potential V could be considered due to a wave which is reflected at the open end, is called as reflected wave.
- Actual voltage at the open end could be considered as "refracted or transmitted wave".

$$\therefore \text{Refracted Wave} = \text{Incident wave} + \text{Reflected Wave}$$

$$= V + V$$

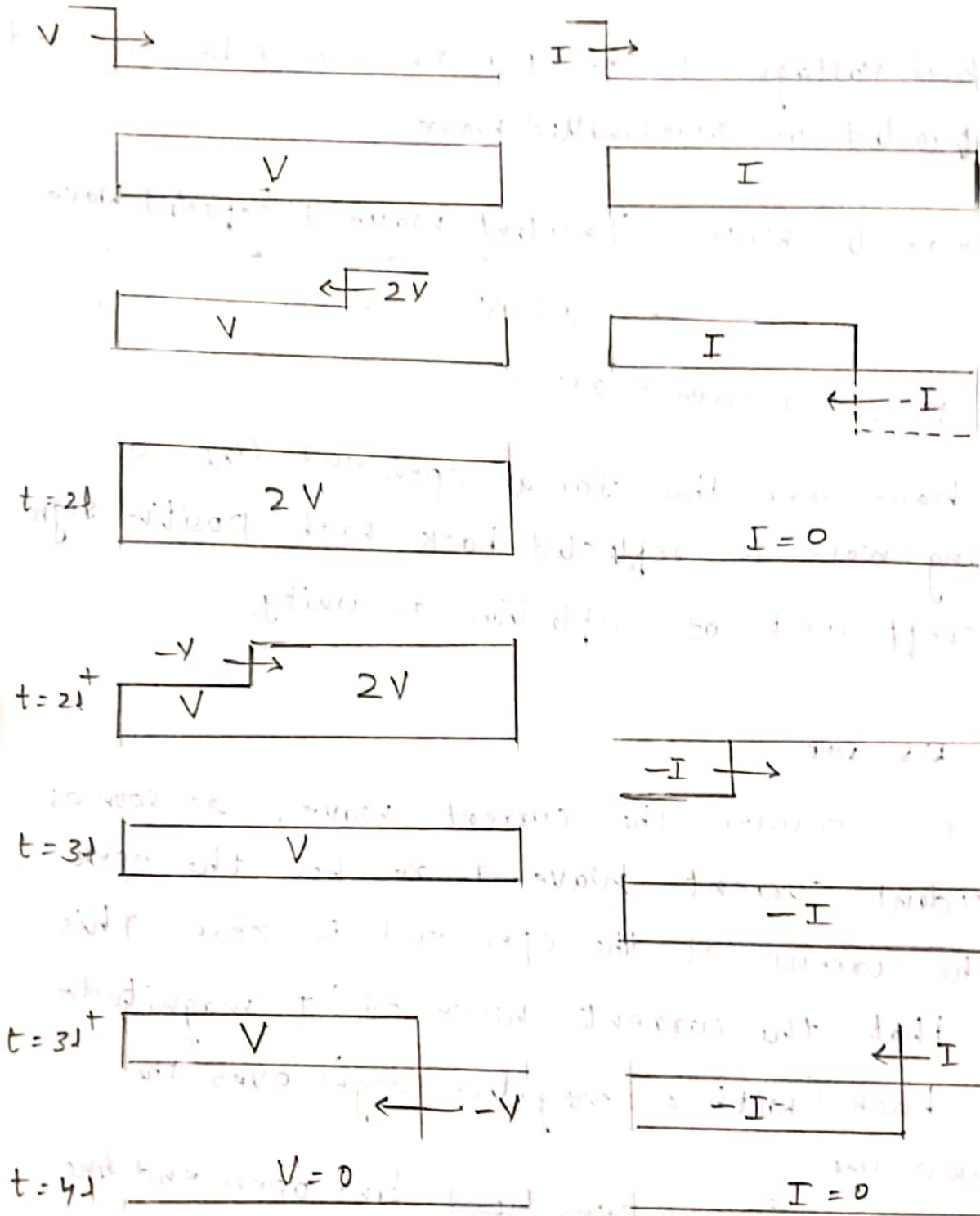
$$\text{Refracted Wave} = 2V.$$

- we have seen that for an open end line a travelling wave is reflected back with positive sign and coefficient of reflection as unity.

Let us see

- Let us consider the current wave, as soon as the incident current wave 'I' reaches the open end, the current at the open end is zero. This means that the current wave of 'I' magnitude travels back (with a negative sign) over the transmission line.
- This means for an overhead line open end line, a current wave is reflected with negative sign and coefficient of reflection is unity.

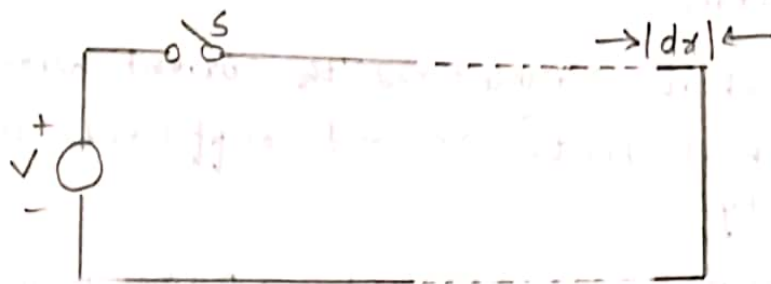
The variation of current and voltage waves over the line is shown in fig below.



Variation of voltage and current in open ended line.

Short-circuited Line

→ consider the line with receiving end short-circuited as shown in fig.



* case of a short circuited line.

→ When the switch 'S' is closed, a voltage wave of magnitude V and current wave of magnitude I start travelling towards the shorted end.

→ consider the last element 'dx', where the electrostatic energy stored by the element is $\frac{1}{2} C dx V^2$ and electromagnetic energy $\frac{1}{2} L dx I^2$. Since the voltage at the shorted end is zero, the electrostatic energy is vanishes and is transformed into electromagnetic energy. As a result, let the change in current be 'i'.

$$\text{then } \frac{1}{2} C dx V^2 = \frac{1}{2} L dx i^2$$

$$C V^2 = L i^2$$

$$\left(\frac{V}{i}\right)^2 = \frac{L}{C}$$

$$\frac{V}{i} = \sqrt{\frac{L}{C}} = Z$$

$$V = i \cdot Z$$

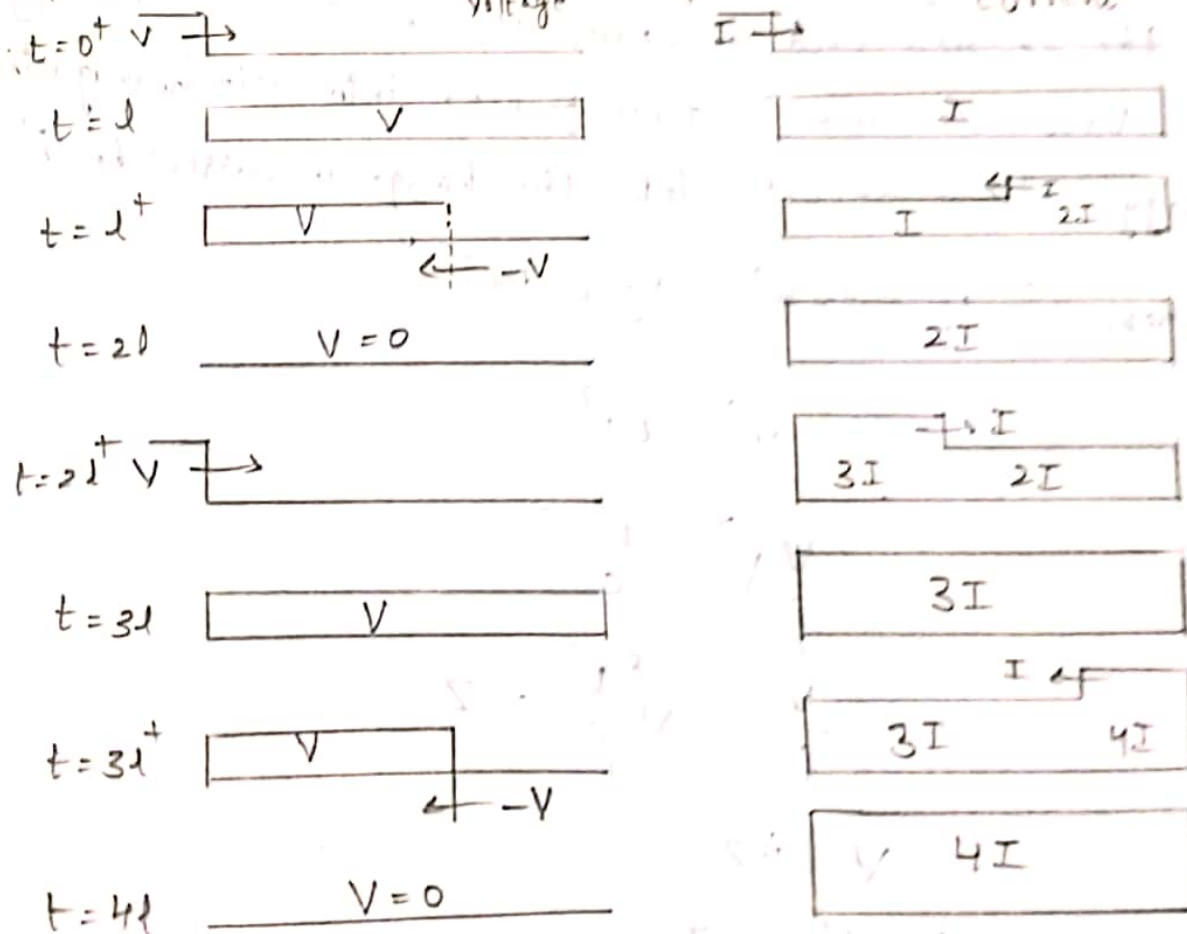
$$i = \frac{V}{Z} = I$$

→ This means that increase in current is 'I' amperes. As a result the total current at the shorted end, when the current wave reaches the end is $(I + I) = 2I$ amperes. This could be considered due to a reflected current wave of magnitude 'I' amperes.

→ Therefore for a short-circuited end the current wave is reflected back with positive sign and coefficient of reflection as unity.

$$\begin{aligned} \therefore \text{Refracted current at shorted end} &= \text{Incident current} + \text{Reflected current} \\ &= I + I \\ &= 2I. \end{aligned}$$

→ Since the Voltage at the shorted end is zero, a voltage wave of $-V$ could be considered to have been reflected back into the line. Voltage wave is reflected back with negative sign and unity coefficient of reflection. The Variation of Voltage and current over the line is shown in fig.



Line Terminated through a Resistance



→ Let 'Z' be surge impedance of the line terminated through a resistance 'R'.

→ Let $V, I =$ ^{Incident} ~~Incident~~ Voltage and Incident Current.

$V', I' =$ Reflected Voltage and Reflected current.

$V'', I'' =$ Refracted Voltage and Refracted current.

→ The relation between reflected Voltage and current wave is given by

$$I' = -\frac{V'}{Z}$$

→ The relation between Incident voltage and Incident current wave is given by

$$I = \frac{V}{Z}$$

→ The expression for refracted current wave when Incident current wave and Incident voltage wave reaches the Resistance 'R' is given by

$$I'' = \frac{V''}{R}$$

$$I'' = I + I'$$

$$I'' = \frac{V}{Z} - \frac{V'}{Z} \quad \left(\because \begin{array}{l} I = \frac{V}{Z} \\ I' = -\frac{V'}{Z} \end{array} \right)$$

$$\frac{V''}{R} = \frac{V}{Z} - \frac{V'}{Z} \rightarrow \textcircled{1}$$

$$\frac{V''}{R} = \frac{V}{Z} - \frac{(V'' - V)}{Z}$$

$$\frac{V''}{R} = \frac{V}{Z} - \frac{V''}{Z} + \frac{V}{Z}$$

$$\frac{V''}{R} + \frac{V''}{Z} = \frac{2V}{Z}$$

$$V'' \left[\frac{1}{R} + \frac{1}{Z} \right] = \frac{2V}{Z}$$

$$V'' \frac{Z+R}{Z \cdot R} = \frac{2V}{Z}$$

$$\boxed{V'' = V \cdot \frac{2R}{R+Z}} \rightarrow \textcircled{2}$$

$$\rightarrow I'' = \frac{V''}{R}$$

$$= \frac{1}{R} \cdot V \cdot \frac{2R}{R+Z}$$

$$I'' = \frac{2V}{R+Z}$$

Multiplying & dividing Z on R.H.S

$$I'' = \frac{V}{Z} \cdot \frac{2 \cdot Z}{R+Z}$$

$$\boxed{I'' = I \cdot \frac{2Z}{R+Z}} \rightarrow \textcircled{3}$$

From Eqn ①
$$\frac{V''}{R} = \frac{V}{Z} - \frac{V'}{Z}$$

$$\frac{V+V'}{R} = \frac{V}{Z} - \frac{V'}{Z}$$

$$\frac{V}{R} + \frac{V'}{R} = \frac{V}{Z} - \frac{V'}{Z}$$

$$\frac{V'}{R} + \frac{V'}{Z} = \frac{V}{Z} - \frac{V}{R}$$

$$V' \left[\frac{R+Z}{Z \cdot R} \right] = V \left[\frac{R-Z}{Z \cdot R} \right]$$

$$\boxed{V' = V \cdot \frac{R-Z}{R+Z}} \rightarrow \textcircled{4}$$

This is equation of deflected wave.

Now we know that $I' = -\frac{V'}{Z}$

$$I' = -\frac{V}{Z} \left(\frac{R-Z}{R+Z} \right)$$

$$\boxed{I' = -I \cdot \frac{R-Z}{R+Z}} \rightarrow \textcircled{5}$$

From equations ②, ③, ④ and ⑤

→ coefficient for the

* Refraction current wave = $\frac{2Z}{R+Z}$

* Refraction Voltage wave = $\frac{2R}{R+Z}$

* Reflection current wave = $-\left[\frac{R-Z}{R+Z} \right]$

* Reflection Voltage wave = $\frac{R-Z}{R+Z}$

For open circuit $R \rightarrow \infty$:

→ The coefficient of refraction current wave

$$= \frac{2Z}{R+Z}$$

$$= \frac{2Z}{\infty+Z} = 0$$

→ The coefficient of refraction Voltage wave

$$= \frac{2R}{R+Z}$$

Ⓜ & ⓓ with 'R' we get

$$= \frac{2R^2}{R(R+Z)} = \frac{2R^2}{R^2(1+\frac{Z}{R})}$$

$$= \frac{2}{1+Z/R}$$

$$= \frac{2}{1+Z/\infty} = \frac{2}{1+0} = 2$$

→ The coefficient for reflection current wave

$$= -\frac{R-Z}{R+Z}$$

$$= -\frac{R[1-\frac{Z}{R}]}{R[1+Z/R]} = -\frac{(1-\frac{Z}{R})}{1+Z/R}$$

$$= -\frac{(1-\frac{Z}{\infty})}{1+Z/\infty}$$

$$= -\frac{(1-0)}{1+0}$$

$$= -1$$

→ The coefficient for reflection Voltage wave

$$= \frac{R-Z}{R+Z}$$

$$= \frac{R \left[1 - \frac{Z}{R} \right]}{R \left[1 + \frac{Z}{R} \right]} = \frac{1 - \frac{Z}{R}}{1 + \frac{Z}{R}}$$
$$= 1.$$

∴ For open circuit line

$$I_{\text{refraction}} = 0$$

$$V_{\text{refraction}} = 2$$

$$I_{\text{reflection}} = -1$$

$$V_{\text{reflection}} = 1.$$

Note:

Similarly the coefficients of reflection and refraction for current and voltage for short-circuited overhead line can be derived by putting $R=0$.

∴ For short circuit line

$$I_{\text{refraction}} = 2$$

$$V_{\text{refraction}} = 0$$

$$I_{\text{reflection}} = 1$$

$$V_{\text{reflection}} = -1$$

Line Connected to a cable:



→ Line connected to a cable.

- A wave travels over the line and enters the cable.
- Since the wave looks into a (cable) different impedance, it suffers reflection and refraction at the junction.

→ Therefore the refracted voltage is given by

$$V' = V \cdot \frac{2R}{R+Z}$$

$$V'' = V \cdot \frac{2Z_2}{Z_2 + Z_1}$$

→ If the impedance of OHTL is 400Ω approximately the impedance of the cable is 40Ω , with these values the voltage entering the cable will be?

$$V'' = V \cdot \frac{2Z_2}{Z_2 + Z_1}$$

$Z_2 \rightarrow$ cable
 $Z_1 \rightarrow$ O.H.T.L

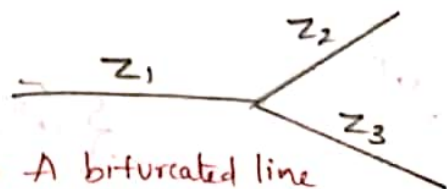
$$= V \cdot \frac{2 \times 40}{40 + 400}$$

$$V'' = V \cdot \frac{80}{440}$$

$$V'' = \frac{2V}{11}$$

Approximately 20% of the incident voltage.

Reflection and Refraction at a T-junction.



- A voltage wave V is travelling over the line with surge impedance Z_1 as shown in fig.
- When it reaches the junction, it looks a change in impedance and therefore suffers reflection and refraction.
- Let V_2'' , I_2'' and V_3'' and I_3'' be the voltages and currents in the lines having surge impedances Z_2 and Z_3 respectively.
- since Z_2 and Z_3 form a parallel path

$$V_2'' = V_3'' = V''$$

$$V + V' = V''$$

Incident $\Rightarrow$ currents $I = \frac{V}{Z_1}$

$$I' = -\frac{V'}{Z_1}$$

$$I_2'' = \frac{V''}{Z_2}$$

$$I_3'' = \frac{V''}{Z_3}$$

$$J + J' = J_2'' + J_3''$$

$$\frac{V}{z_1} - \frac{V'}{z_1} = \frac{V''}{z_2} + \frac{V''}{z_3}$$

$$\{V'' = V + V'\}$$

$$\frac{V}{z_1} - \frac{(V'' - V)}{z_1} = \frac{V''}{z_2} + \frac{V''}{z_3}$$

$$\frac{V}{z_1} - \frac{V''}{z_1} + \frac{V}{z_1} = \frac{V''}{z_2} + \frac{V''}{z_3}$$

$$\frac{2V}{z_1} - \frac{V''}{z_1} = \frac{V''}{z_2} + \frac{V''}{z_3}$$

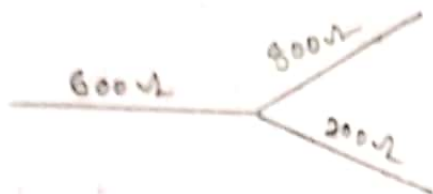
$$\frac{2V}{z_1} = \frac{V''}{z_1} + \frac{V''}{z_2} + \frac{V''}{z_3}$$

$$\frac{2V}{z_1} = V'' \left[\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3} \right]$$

$$V'' = \frac{(2V/z_1)}{\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}}$$

5. A surge of 100 kV travelling in a line of natural impedance 600Ω arrives at a junction with two lines of impedances 800Ω and 200Ω respectively. Find the surge voltages and currents transmitted into each branch line.

sol. Given



$$V'' = \frac{2V/Z_1}{\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}} = \frac{2 \times 100/600}{\frac{1}{600} + \frac{1}{800} + \frac{1}{200}}$$

$$V'' = \cancel{42.01} \text{ kV } 42.04 \text{ kV}$$

$$\rightarrow I_2'' = \frac{V''}{Z_2} = \frac{42.04 \times 10^3}{800} = 52.55 \text{ A}$$

$$\rightarrow I_3'' = \frac{V''}{Z_3} = \frac{42.04 \times 10^3}{200} = 210.2 \text{ A}$$

P. A 3- ϕ T.L has conductors 1.5 cms in diameter spaced 1 m apart in equilateral formation. The resistance leakages are negligible. Calculate

- the natural impedance of the line,
- the line current if a voltage wave of 11 kV travels along the line
- the rate of energy absorption, the rate of reflection and the state and form of reflection if the line is

terminated through a star connected load of 100Ω per phase

(iv) the value of the terminating resistance for no reflection and

(v) the amount of reflected and transmitted power if the line is connected to a cable extension, with inductance and capacitance per phase per cm of $0.5 \times 10^{-8} \text{ H}$ and $1 \times 10^{-6} \text{ uF}$ respectively.

Sol. Inductance per unit length = $2 \times 10^{-7} \ln \frac{d}{r} \text{ H/m}$
 $= 2 \times 10^{-7} \ln \frac{100}{0.75}$
 $= 9.78 \times 10^{-7} \text{ H/m}$

Capacitance per unit length = $\frac{2\pi\epsilon}{\ln d/r} \text{ F/m}$

$= \frac{2\pi \times 8.854 \times 10^{-12}}{\ln \left(\frac{100}{0.75} \right)}$
 $= 1.136 \times 10^{-11} \text{ F/m}$

(i) $Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{9.78 \times 10^{-7}}{1.136 \times 10^{-11}}} = 293.4 \Omega \approx 294 \Omega$

(ii) Voltage per phase = $\frac{11 \times 10^3}{\sqrt{3}} = 6350 \text{ V}$

Line current $I_L = \frac{6350}{294} = 21.60 \text{ A}$

(iii) the Voltage across the resistance terminate is

$$V'' = V \cdot \frac{2R}{R+Z} = 2 \cdot V \cdot \frac{Z_2}{Z_1+Z_2}$$

where Z_1 = Line surge Impedance

Z_2 = Line Terminating Impedance

$$Z_1 = Z_n = 294 \Omega$$

$$Z_2 = 1000 \Omega$$

$$V'' = 2 \times 6350 \times \frac{1000}{294 + 1000}$$

$$V'' = 9814.5 \text{ V} = 9.8 \text{ kV}$$

$$\text{The rate of energy consumption} = \frac{3 \cdot V''^2}{R}$$

$$= \frac{3 \times (9.8 \times 10^3)^2}{1000}$$

$$= 298 \text{ kW}$$

$$\text{Reflection Voltage } V' = V \cdot \frac{R - Z}{R + Z}$$

$$= 6350 \times \frac{1000 - 294}{1000 + 294}$$

$$= 3.465 \text{ kV}$$

$$\text{Rate of Reflected Energy} = \frac{3V'^2}{R (= Z_1)} = \frac{3 \times (3.465 \times 10^3)^2}{294}$$

$$= 122.5 \text{ kW}$$

(iv) Since in order that when incident wave when reaches the terminating resistance does not suffer reflection i.e. the terminating resistance should be equal to natural Impedance of line. $R = Z_n = 294 \Omega$.

(v) surge Impedance of cable $Z_c = \sqrt{\frac{L}{C}}$ ohms.

$$\begin{aligned} L &= 0.5 \times 10^{-9} \text{ H (given)} \\ C &= 1 \times 10^{-6} \text{ F} \\ Z_c &= \sqrt{\frac{0.5 \times 10^{-9}}{1 \times 10^{-6} \times 10^{-6}}} \\ &= 70.7 \Omega \end{aligned}$$

Reflected Voltage $V' = V \cdot \frac{R - Z}{R + Z}$

$$= 6350 \times \frac{70.7 - 294}{70.7 + 294}$$

$$= -3.9 \text{ kV}$$

Refracted Voltage $V'' = V \cdot \frac{2R}{R + Z}$

$$= 6350 \times \frac{2 \times 70.7}{70.7 + 294}$$

$$= 2.46 \text{ kV}$$

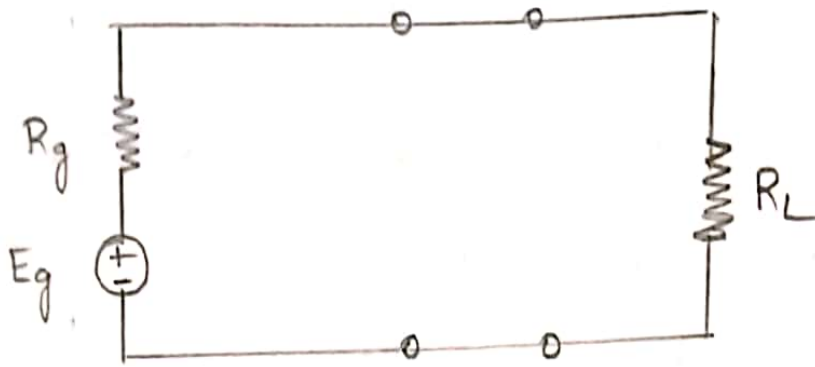
Reflected Power $P = \frac{3V'^2}{R} = \frac{3 \times (-3.9 \times 10^3)^2}{70.7}$

$$= 155 \text{ kW}$$

Refraction power $P = \frac{3V''^2}{R} = \frac{3(2.46 \times 10^3)^2}{70.7}$

$$= 256.7 \text{ kW}$$

Bewley's Lattice Diagram (or) Bewley's Zig-Zag Diagram.



→ The above fig shows a finite transmission line connected to a generator on the sending end and a load on the receiving end.

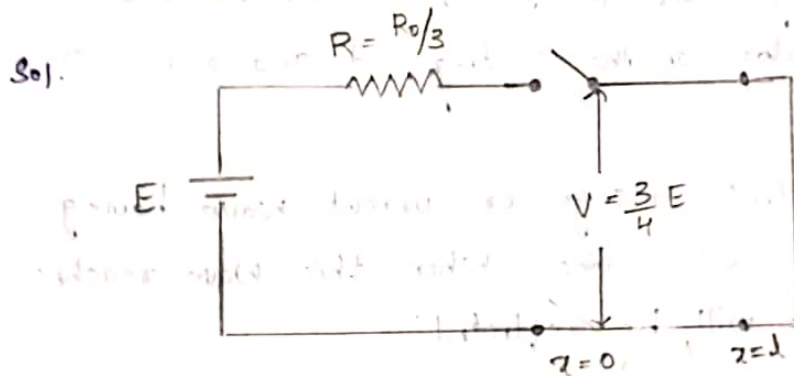
→ Consider an incident voltage or current wave having been launched on such a line. When this wave reaches the load end it will be reflected.

→ The reflected wave on reaching the generator end will again be reflected due to the generator resistance R_g and this process will continue.

→ To keep track of the reflections it is convenient to draw a diagram showing the locations of the leading edge of the wave at each incident of time.

→ Such a diagram is known as Bewley's lattice (or) zig zag diagram.

Q. A battery with an emf 'E' and series Resistance 'R' are connected at $t=0$ to the sending end of the transmission line which is short circuited at the far end shown in fig below. Plot the sending voltage and currents as a function of time for $R = \frac{R_0}{3}$. The time required for a wave to travel full length for the line is 'T'.



Step 1: To find reflection coefficients for the two ends

$$\text{Load End; } K_L = \frac{R_L - R_0}{R_L + R_0} = -1$$

$$\text{Generator End; } K_g = \frac{\frac{R_0}{3} - R_0}{\frac{R_0}{3} + R_0} = -\frac{1}{2}$$

Step 2: To find Sending End Voltage and current

$$E_s = E - \frac{E}{\frac{R_0}{3} + R_0} \times \frac{R_0}{3}$$

$$= E - \frac{E}{4}$$

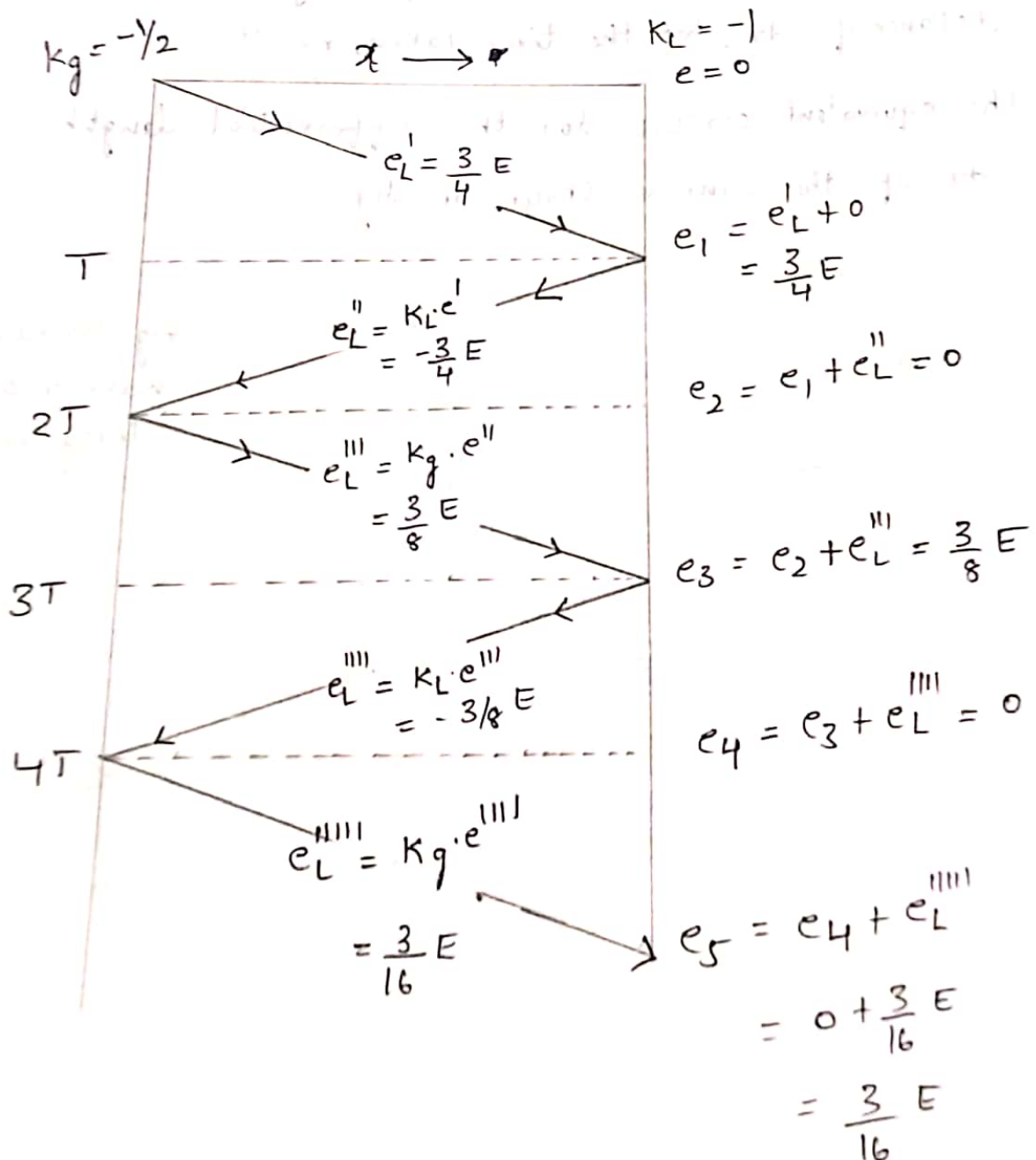
$$= \frac{3}{4} E$$

$$E_s = \frac{3}{4} E$$

$$I_s = \frac{3}{4} \frac{E}{R_0}$$

steps: To draw zig-zag diagram for the voltage and current.

In a zig-zag diagram distance is plotted horizontally and time is plotted downward as shown.



Attenuation of Travelling Waves:

→ Let R, L, C and G be the resistance, inductance, capacitance and conductance respectively per unit length of the line.

→ Let the value of voltage and current waves at $x=0$ be V_0, I_0 . Our objective is to find the values of voltage and current waves when they have travelled through a distance of x units over the overhead line.

→ Let the time taken be t units when voltage and current waves are V and I respectively. To travel a distance of dx , let the time taken be dt .

The equivalent circuit for the differential length dx of the line is shown in fig.

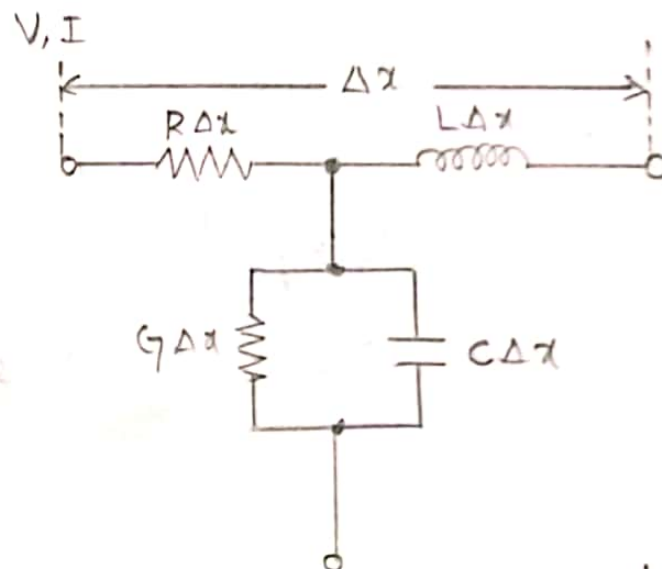
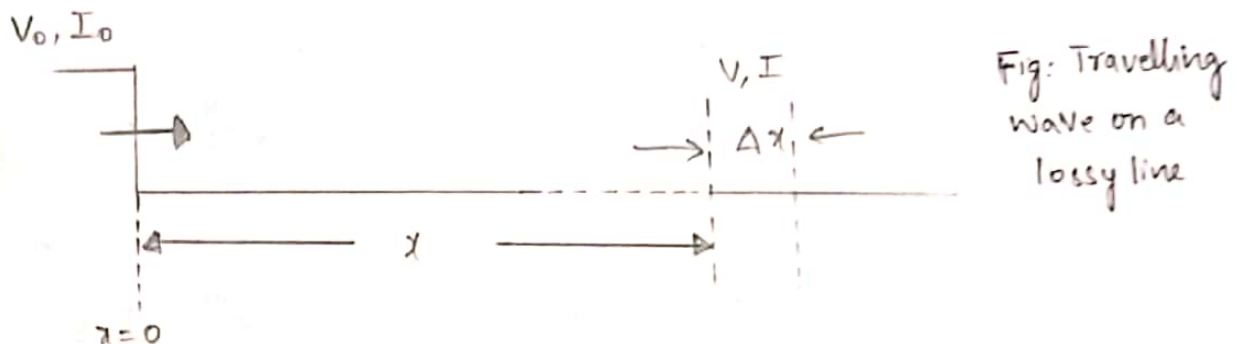


Fig: Differential element of transmission line.

The powerloss in the differential element is

$$dp = I^2 R dx + V^2 G dx \rightarrow (i)$$

Also power at distance x , $V \cdot I = P = I^2 Z_n$

$$\text{Differential power, } dp = -2I \cdot Z_n dI \rightarrow (ii)$$

where Z_n is the natural impedance of the line. Here the negative sign is because there is reduction in power as the wave travels with time.

equating equations (i) & (ii)

$$\begin{aligned} -2I Z_n dI &= I^2 R dx + V^2 G dx \\ &= I^2 R dx + I^2 Z_n^2 G dx \\ dI &= - \frac{I (R + G Z_n^2)}{2 Z_n} dx. \end{aligned}$$

$\left\{ V = I \cdot Z_n \right\}$

$$\frac{dI}{I} = - \frac{(R + G Z_n^2)}{2 Z_n} dx$$

Integrating we get

$$\ln I = - \left(\frac{R + G Z_n^2}{2 Z_n} \right) x + A$$

$$\rightarrow \text{At } x=0, I=I_0 \Rightarrow A = \ln I_0$$

$$\ln \frac{I}{I_0} = - \frac{R + G Z_n^2}{2 Z_n} x = -ax$$

$$\text{where } a = \frac{R + G Z_n^2}{2 Z_n}$$

$$\therefore I = I_0 \cdot e^{-ax}$$

Similarly it can be proved that $V = V_0 \cdot e^{-\alpha x}$.

→ This shows that the current and voltage waves get attenuated exponentially as they travel over the line and the magnitude of attenuation depends upon the parameters of the line. Since the value of resistance depends not only on the size of the conductor but also on the shape and length of the waves.

→ An empirical relation due to Foust and Meuser takes into account the shape and length of the wave for calculating the voltage and current at any point on the line after it has travelled through a distance x units and is given as

$$V = \frac{V_0}{1 + K \times V_0}$$

where x in kms,

V and V_0 in kV. and

K is the attenuation constant of value

$K = 0.00037$ for chopped waves

$= 0.00019$ for short wave

$= 0.0001$ for long waves.

2 Marks Questions:

1. What is meant by a travelling wave in a transmission line?
2. Define power system transients.
3. What is a surge in a transmission line?
4. Define surge impedance of a transmission line.
5. What is meant by surge propagation?
6. What are reflection and refraction of travelling waves?
7. What is meant by attenuation and distortion of travelling waves?
8. What happens to a travelling wave when it reaches an open-circuited end of a transmission line?
9. What happens to a travelling wave when it reaches a short-circuited end of a transmission line?
10. What is a Bewley lattice diagram? State its purpose.
11. Define reflection coefficient and refraction coefficient.
12. What are the causes of power system transients?

10 Marks Questions:

1. Derive the coefficient of reflection & refraction of voltage and current waves when the line is terminated through an inductance.
2. Analyze the reflection and refraction of voltage and current waves for short circuit lines.
3. What is Bewley-Lattice diagram? Explain its utility in the study of travelling waves. Explain the nature of refracted components of voltage and current at the junction of two lines of surge impedances Z_1 and Z_2 .
4. A 5kV surge travels along a line of surge impedance 500 ohms.
 - (i) Find the magnitude of the incident current waves. The line is terminated by resistance of 800 ohms.
 - (ii) Find the rate of dissipation of energy and the values of reflected voltage and current waves.
 - (iii) What if the rate of energy reflection?
 - (iv) What should be the value of the terminating resistance so that energy in the wave is completely dissipated, transmitted and reflected power?
5. A) Discuss the traveling waves on a transmission line.
B) Two stations are connected together by an underground cable having a surge impedance of 50 ohms joined to an overhead line with a surge impedance of 400 ohms. If a surge having a maximum value of 110 kV travels along the cable towards the junction with the overhead line, determine the value of the reflected and the transmitted wave of voltage and current at the junction.
6. Explain the attenuation and distortion of travelling waves in transmission lines and discuss their effects on power system performance.
7. Explain the causes and effects of power system transients and discuss the different types of electrical surges occurring in transmission systems.

REFERENCES:

- ❖ C.L. Wadhwa, —Electrical Power Systems, New Age International Pub. Co, Third Edition, 2001.

“Understanding power system transients and travelling waves is essential for protecting transmission networks against sudden electrical disturbances and ensuring reliable power system operation.”

UNIT-5: Voltage Control and Power Factor Improvement

Methods of Voltage control, shunt and series capacitors/Inductors, tap changing transformers, synchronous phase modifiers, power factor improvement methods. Compensation in power systems: Concepts of Load Compensation Load ability characteristics of overhead lines – Uncompensated transmission line – Symmetrical line – Radial line with asynchronous load – Compensation of lines.

METHODS OF VOLTAGE CONTROL:

In power systems, **voltage control** is primarily achieved through the management of **reactive power flow**. When the reactive power requirement of a load is high, it can lead to voltage drops; conversely, injecting reactive power into the system can raise the voltage.

Based on the sources, the following methods and devices are used for voltage control:

1. Synchronous Phase Modifiers

This is a fundamental method for controlling system voltage by adjusting reactive power flow. A **synchronous phase modifier** is essentially a synchronous machine that acts as a variable compensator:

- **Overexcited:** The machine works as a **capacitor**, generating positive reactive VARs to support the voltage.
- **Underexcited:** The machine works as an **inductor**, absorbing reactive power to lower the voltage.

2. Shunt and Series Compensation

Compensation is used to maximize the utilization of transmission lines and improve reliability.

- **Shunt Capacitors:** These are used to generate reactive power at the point of the load, reducing the net reactive power that must be transmitted over the line ($Q - Q'$), thereby supporting the voltage.
- **Series Compensation:** This involves installing components in series with the transmission line to alter its electrical characteristics and improve stability and voltage regulation.

3. FACTS (Flexible A.C. Transmission Systems)

FACTS devices represent a modern, advanced approach to power system control, including voltage stability. Specific devices mentioned for voltage regulation and protection include:

- **DSTATCOM (Distribution Static Compensator):** Used specifically for voltage regulation and load compensation at the distribution level.
- **DVR (Dynamic Voltage Restorer):** Employed to protect sensitive loads from voltage variations.
- **UPQC (Unified Power Quality Conditioner):** A versatile device used for improving overall power quality, including voltage control.

4. HVDC Transmission Control

In **High Voltage Direct Current (HVDC)** systems, voltage and power flow are controlled through the operation of rectifiers and inverters. Methods include:

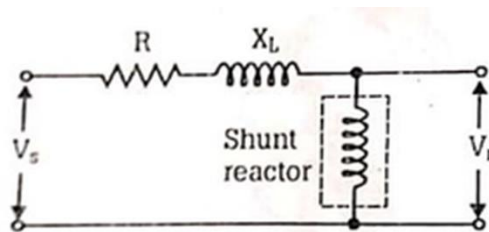
- **Constant Ignition Angle Control:** Managing the timing of thyristor firing.
- **Constant Extinction Angle Control:** Maintaining a specific angle to ensure proper inverter operation.
- **Constant Current Control:** Regulating the current to stabilize the system.

5. Other Methods and Considerations

- **Tap-Changing Transformers:** The sources mention transformers with **fixed voltage ratios** and those that can control the bus voltage near the tap side or impedance side.
- **Load Shedding:** While more of an emergency measure for frequency and voltage stability, **under-frequency relays** can automatically disconnect blocks of load to prevent a complete system shutdown when voltage and frequency drop below safe limits.
- **Excitation Systems:** The excitation of generators is used to manage their reactive power output, which in turn improves synchronous stability and aids in voltage control.

SHUNT AND SERIES CAPACITORS/INDUCTORS:

Shunt Reactors are used in long transmission line at the load end. When the line is lightly loaded as discussed earlier, the line voltage increases due to charging current (i.e., capacitance effect of line known as Ferranti Effect). The Current taken by the reactors is in phase opposition to the current due to the line capacitance. The leading current due to line capacitance is thus cancelled out by the lagging current taken by the reactors.



In the construction, the shunt reactors are identical with a power transformer, except their cores. They have similar windings, tanks, bushings, reactors etc. An air gap is provided within the reactor core to prevent magnetic saturation. Shunt reactors are subjected to overvoltage's like power transformers.

Reactive power absorbed by the Reactor,

$$Q_L = -V \cdot I_L = \frac{-V^2}{X_L} = \frac{-V^2}{\omega L} \text{ VAR/phase}$$

Where, L = Inductance of reactor (Henry)

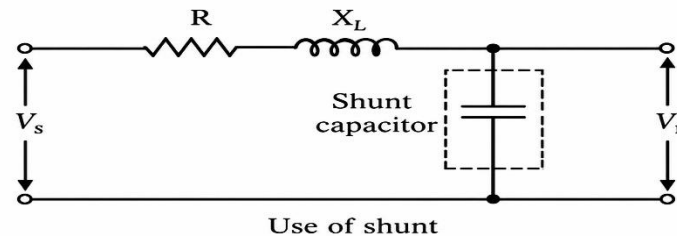
$$X_L = \text{Reactance of reactor} = \omega L$$

V = Phase Voltage

The Negative sign indicates negative power supply i.e. reactive power is absorbed.

Shunt Capacitor:

Although the use of shunt capacitors is mainly for power factor correction at the load terminals of low voltage, small banks of capacitors connected in star/delta are also used on high voltage lines in sub-stations. Such shunt capacitors supply capacitive current and thus produce continuous leading KVAR independent of load variations. In Transmission systems, the capacitor banks are connected to the tertiary delta winding of the sub-station transformers.



Reactive Power supplied by the shunt capacitors,

$$Q_C = V \cdot I_C = \frac{V^2}{X_C} = V^2 \cdot \omega C \text{ VAR/phase}$$

Where, V = Phase Voltage

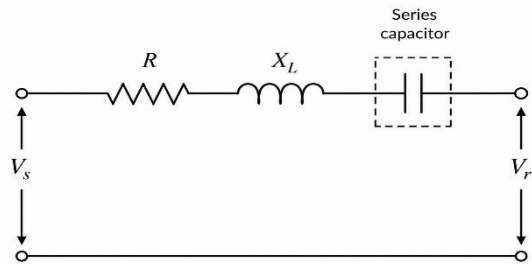
C = Capacitance Per Phase

X_C = Shunt Capacitive Reactance per phase

A serious disadvantage with the shunt capacitors is that at no load or light load, the receiving end voltage may considerably exceed the sending end voltage. To overcome this difficulty, the capacitor bank is provided with fixed and variable elements. With the rise or fall of voltage, the variable elements may be removed from or added to the bank to decrease or increase the capacitance. The voltage control obtained by shunt capacitors is not smooth.

Series Capacitors:

When a capacitor with a capacitive reactance X_C is connected in series with the transmission line. The effective reactance of the line is reduced from X_L to $(X_L - X_C)$. The voltage drop, therefore is reduced by compensating the inductive reactance of the line. For $X_L = X_C$ full compensation is achieved and the circuit becomes resistive. For $X_C = 0$ i.e. when capacitors is removed compensation is zero. In practice, partial compensation is done for stability reasons.



As the capacitors are in series with the line, sometime they have to carry large currents due to overloads or sustained faults. The Overcurrent's will cause excessive voltage drops across the capacitors. To protect the capacitors from such abnormal voltages, protective spark gap is provided across these capacitors.

The series capacitors may be located at the sending end, receiving end or at the center of the line.

Rating of series capacitor,

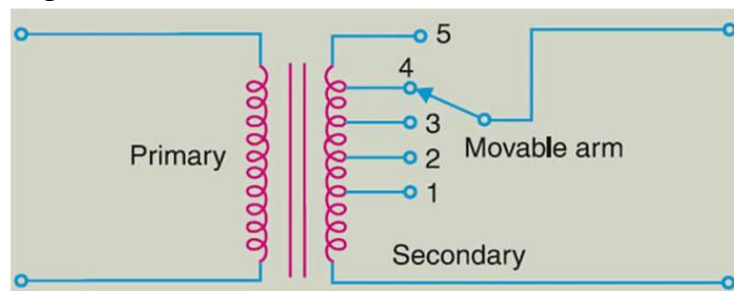
$$Q_C = I^2 X_C \text{ VAR}$$

The Series capacitors control voltage regulation, improve system stability, its use is only limited to H.V. systems.

TAP CHANGING TRANSFORMERS:

When the movable arm makes contact with lower positions such as 1, the secondary voltage is minimum, during the period of light inductive load.

When the movable arm contact with higher position such as 5, the secondary voltage is maximum, during the period of high inductive load.



Advantages of Tap changing Transformer:

- During high system load conditions network voltages are kept at highest practical level to minimize reactive power requirements increase effectiveness of shunt capacitors to compensated reactive power.
- During high load conditions, it is usually required to lower network voltages avoid under excited operation of generators.\

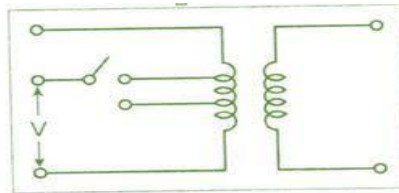
- All power transformers on transmission lines are provide with taps for control of secondary voltage. The tap changing transformers do not control voltage regulating the flow of reactive VARs but by changing transformation ration.

There are two type of tap changing transformers:

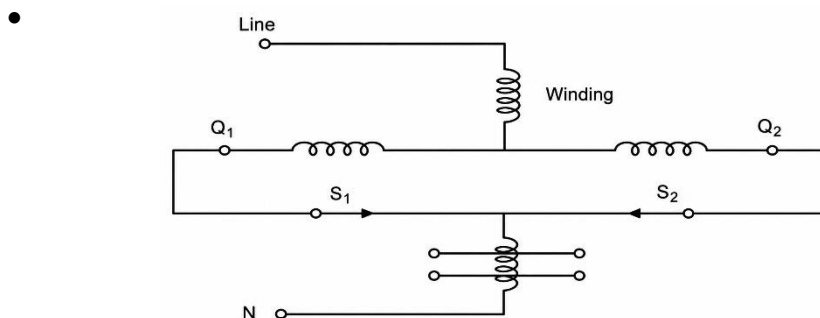
- Off-Load tap changing transformers.
- On-Load(Under-Load) tap changing transformers (OLTC)

i. Off-load tap changing transformers:

The off-load tap changing transformers as shown in figure which requires the disconnection of the transformer when the tap setting is to be changed. Off-load tap changers are used when it is to be operated in frequently due to load growth or some seasonal change.



- On-load tap changing transformer is used when changes in transformers ratio to be needed frequently, and no need to switch off the transformer to change the tap of transformer.
- It is used on power transformers, auto transformers and bulk distribution transformers and at other points of load service.
- The modern practice is to use on-load tap changing transformer which is shown in figure.
- In the position shown, the voltage is maximum and since the currents divide equally and flow in opposition through the coil between Q1 and Q2, the resultant flux is zero and hence minimum impedance.



On-load tap changing transformers (OLTC):

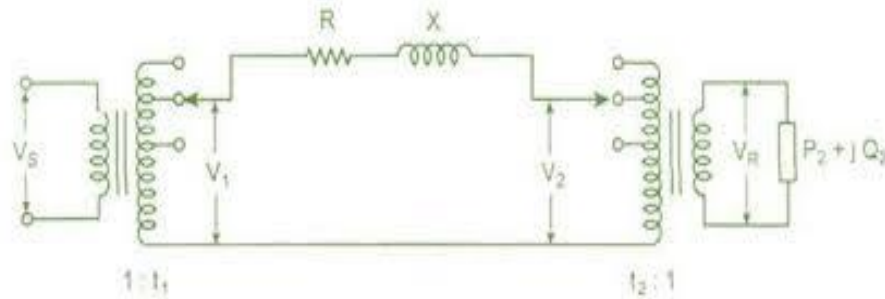
To reduce the voltage, the following operations are required in sequence:

- Open Q1.
- Move Selector switch S1 to the next contact. Close Q1.
- Open Q2.

- Move selector switch S2 to the next contact. Close Q2.
- Thus, operations are required for one change in tap position. The voltage change between taps is often 1.25% of the nominal voltage.

System Level Control using Generator Voltage Magnitude Setting:

- Transformers transfer the reactive power from one side to another side by altering the in phase component of the system voltage. Let us consider the tap changing transformer at both ends of a line is shown in the figure.
- Let t_1 , t_2 be the functions of nominal transformation ratio i.e., tap ratio/ nominal voltage.
- The actual voltage will be t_1V_1 and t_2V_2 . Let V_1 , V_2 be the nominal voltage at the ends of the line. Since, the line has impedance, it is necessary to compensate the voltage drop in the line so that the voltage at the receiving end is maintained at a desired level.



SYNCHRONOUS PHASE MODIFIERS:

The Synchronous phase modifiers is the synchronous motor running without a mechanical load. It is connected with the load at receiving the end of the line. The synchronous phase modifier absorbs or generated the reactive power by varying the excitation of the field winding. It keeps the voltage constant at any condition of the load and also improves the power factor.

It is well known that a synchronous motor can be made to take either a lagging or leading current from the line by alternating its excitation. Idle-running synchronous motor were first employed in connection with power plants to correct for low power factor of the load, and thus reduced the current and the power losses in the feeders and generators. The synchronous machines used for power factor improvement are usually referred to as synchronous condensers as they are always required to take a leading current. However and idle-running synchronous motor can be used for voltage regulating purposes by connecting it in parallel with the load at the receiving end of a line.

Under Excitation: Current lags behind the supply voltage, the motor has a lagging power factor.

Over Excitation: Current leads the supply voltage. The motor power factor is leading.

Advantages:

- It is a convenient device for keeping the receiving end voltage constant under any excitation load conditions.
- It improves the power factor.
- The output can varied smoothly.

Disadvantages:

- It is costly for its installation.
- Maintenance & operations are not easy.
- It is difficult to increase its capacity in order to increase demand.

POWER FACTOR IMPROVEMENT METHODS:

Different Methods Used for Power Factor Correction. The following devices and equipment are used for power factor improvement in an electrical system.

- **Capacitor Banks:** A bank of capacitors can be installed to reduce the reactive power demand of the load, improving the power factor. The capacitors can be fixed or switched, depending on the load requirements.
- **Synchronous Condensers:** A synchronous motor operating at no-load and over-excited, can be used as a synchronous condenser to improve the power factor of the system.
- **Phase Advancers:** Phase advancers are AC exciters connected to the rotor circuit of induction motors to improve the power factor of the motor.
- **Static Var Compensators (SVCs):** SVCs are solid-state devices that use a thyristor-controlled reactor (TCR) and a thyristor-switched capacitor (TSC) to provide continuous reactive power compensation.
- **Active Power Filters:** These filters can correct power factor issues by generating current components that cancel out harmonic distortion in the system.
- **Switched Capacitor Banks:** A switched capacitor bank uses automatic switching devices to vary the reactive power demand based on the load requirements, improving the power factor.
- **Static Synchronous Compensator (STATCOM):** A STATCOM is a voltage source converter that can provide reactive power compensation, harmonic filtering, and voltage regulation.
- **Hybrid Power Filters:** Hybrid power filters combine active and passive filtering techniques to provide a comprehensive solution for power factor improvement and harmonic distortion reduction.

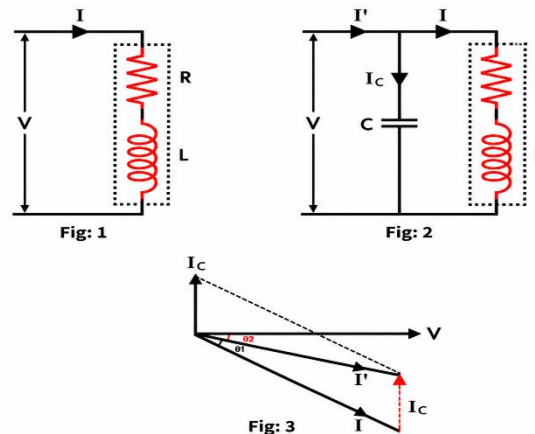
The common methods used for power factor correction as follows:

1. Static Capacitor
2. Synchronous Condenser
3. Phase Advancer
4. Capacitor Banks
5. Static VAR Compensator (SVC)

1. Static Capacitor

We know that most industries and power system loads are inductive, which causes a decrease in the system power factor due to lagging current. To improve the power factor, **static capacitors** are connected in parallel with these devices operated on low power factor.

These static capacitors supply leading current, which balances out the lagging inductive component of the load current. This effectively eliminates or neutralizes the lagging component of the load current and corrects the power factor of the load circuit to enhance the overall efficiency. To enhance system or device efficiency, these capacitors are installed near large inductive loads, like induction motors and transformers, to improve the load circuit power factor.



For example, let's consider a single-phase inductive load shown in

Figure 1, which is drawing lagging current (I), and the load power factor is $\cos\theta$.

Figure 2 shows the load with a capacitor (C) connected in parallel. As a result, a current (I_c) flows through the capacitor and leads 90° from the supply voltage. In other words, the capacitor provides leading current, and in a purely capacitive circuit, the current leads the supply voltage by 90° , which means the voltage lags 90° behind the current. The load current remains (I), and the vector sum of (I) and (I_c) is (I') which lags behind the voltage at θ_2 , as shown in Figure 3.

Figure 3 demonstrates that the angle of $\theta_2 < \theta_1$, implying that $\cos\theta_2$ is less than $\cos\theta_1$ ($\cos\theta_2 > \cos\theta_1$). Therefore, the capacitor improves the load power factor.

It is important to note that after power factor improvement, the circuit current is lower than the low power factor circuit current. Additionally, the active component of current remains the same before and after power factor improvement because the capacitor eliminates only the reactive component of current. Finally, the Active power (in Watts) remains the same before and after power factor correction.

Advantages:

A capacitor bank offers several advantages over other methods of power factor improvement, including:

- Low losses in static capacitors

- No moving parts, therefore requiring low maintenance
- Ability to work in normal conditions (i.e., ordinary atmospheric conditions)
- No requirement for a foundation for installation
- Lightweight, making them easy to install

Disadvantages:

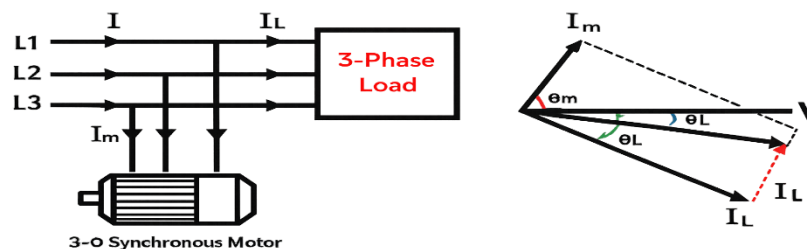
However, there are some drawbacks to using a capacitor bank, which include:

- A shorter lifespan for static capacitor banks (around 8-10 years)
- The need to turn the capacitor bank ON or OFF when there is a change in load, which can cause switching surges in the system.
- Risk of damage if the rated voltage increases beyond its limit
- Expensive repair costs if the capacitors become damaged.

2. Synchronous Condenser

When a synchronous motor operates at no-load and is over-excited, it is called a synchronous condenser. When a synchronous motor is over-excited, it provides leading current and works like a capacitor.

In a synchronous motor, a separate DC source is used to excite the field winding. Therefore, the input supply only provides current to energize the stator, i.e., the current provided is in-phase with the supply voltage. So the power factor remains unity.



The power factor can be adjusted by varying the DC excitation. By increasing the DC excitation, the power factor varies from lagging to unity and leading power factor. When the DC excitation increases, the field windings are over-magnetized. The input supply provides a current component to the stator to compensate for this over-magnetization. This current leads to the supply voltage, causing a leading power factor or generating reactive power.

An inductive load consumes reactive power, causing a lagging power factor, while a capacitive load generates reactive power, causing a leading power factor. A synchronous motor can be used to improve the overall power factor of an electrical system by adjusting the DC excitation. The synchronous motor used specifically for power factor improvement without any mechanical load is called a **synchronous condenser**.

The synchronous condenser is used in parallel with the load to improve the power factor. Improving the power factor reduces the extra current drawn from the source that is wasted in the power lines. Consequently, it helps in the reduction of electricity bills and saves energy.

When a synchronous condenser is connected across the supply voltage (in parallel), it draws leading current and partially eliminates the reactive component. This way, the power factor is improved. Generally, synchronous condensers are used to improve the power factor in large industries.

Advantages:

- Long lifespan (up to 25 years)
- High reliability
- Allows for stepless adjustment of power factor
- Does not generate harmonics or require maintenance for them
- Faults can be easily removed
- Is not affected by harmonics
- Requires low maintenance (only periodic bearing greasing is necessary)

Disadvantages:

- High cost (including high maintenance costs), therefore it is mostly used by large power users
- An auxiliary device is needed for operation as synchronous motors have no self-starting torque
- Produces noise.

3. Phase Advancer

The Phase Advancer is a simple AC exciter that connects to the main shaft of a motor and operates with the motor's rotor circuit to improve power factor. It is commonly used in industries to improve the power factor of induction motors.

Since the stator windings of an induction motor take lagging current 90° out of phase with voltage, the power factor of the motor is low. By supplying exciting ampere-turns from an external AC source, the current does not affect the stator windings, and the power factor of the induction motor improves. This process is done by the **Phase Advancer**.

Advantages:

- Sufficiently reduces the lagging kVAR (reactive component of power or reactive power) drawn by the motor because the exciting ampere turns are supplied at slip frequency (f_s).
- The Phase Advancer can be easily used where the use of synchronous motors is unacceptable.

Disadvantage:

- Using a Phase Advancer is not economical for motors below 200 H.P. (about 150kW).

4. Capacitor Banks

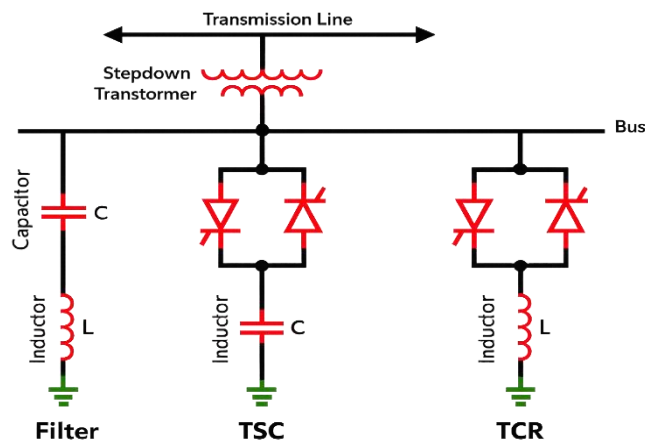
The capacitor bank (published as separate & descriptive article) is connected in parallel to the load, and when the inductive load draws current from the system, the capacitor bank supplies capacitive reactive power to offset the inductive reactive power. The amount of capacitive reactive power

needed to improve the power factor depends on the characteristics of the load, such as the magnitude of the inductance and the phase angle between the voltage and current.

5. Static VAR Compensator (SVC)

We have covered this topic in a separate article describing Static VAR Compensator (SVC) including a circuit diagram, construction, working principles, and applications. You can read the article to learn how SVCs are used for power factor improvement.

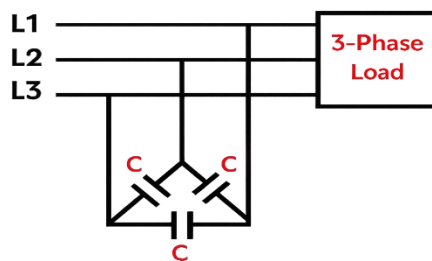
Static VAR Compensator – SVC



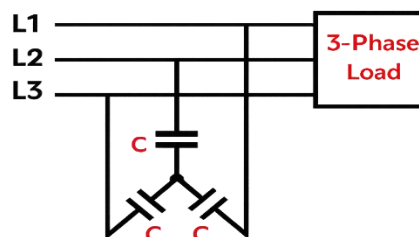
The following figure shows power factor improvement in a three-phase system by connecting a capacitor bank in:

1. Delta connection
2. Star Connection

Methods of Power Factor Improvement



Capacitor Bank Connected in Parallel with 3-Phase Load in **DELTA** Connection



Capacitor Bank Connected in Parallel with 3-Phase Load in **STAR** Connection

COMPENSATION IN POWER SYSTEMS:

Compensation in power systems refers to the intentional addition of reactive power devices (capacitors or inductors) to a network. It is used to stabilize voltage, improve power factor, reduce energy losses, and increase the maximum power-transfer capacity of long transmission lines.

The main techniques for compensation are categorized by how devices are connected to the power grid:

1. Shunt Compensation

Shunt devices are connected in parallel with the transmission line or load.

- **Shunt Capacitors:** Provide reactive power to the system, primarily used to correct lagging power factors and boost local voltage levels.
- **Shunt Reactors:** Absorb excess reactive power. They are essential during light-load or no-load conditions to prevent the Ferranti effect (where receiving-end voltage dangerously spikes).

2. Series Compensation

Series devices are connected inline with the transmission system.

- **Series Capacitors:** Offset the inductive reactance of the transmission line, which effectively reduces the overall electrical length of the line. This significantly increases the active power transfer capability $P_{max} = \frac{V_S V_R}{X_{line}}$

3. Advanced FACTS Devices

Flexible AC Transmission Systems (FACTS) use high-speed power electronics to dynamically adjust compensation based on real-time system demands.

- **STATCOM (Static Synchronous Compensator):** A shunt device that regulates voltage by dynamically injecting or absorbing reactive power.
- **SVC (Static Var Compensator):** Regulates voltage using thyristor-controlled reactors and switched capacitors.

CONCEPT OF LOAD COMPENSATION:

Load compensation is the management of reactive power to improve the quality of supply especially the voltage and p.f. levels. Here the reactive power is adjusted with respect to an individual load and the compensating device is connected to the load itself. There are three main objectives in Load Compensation:

- (i) Better voltage profile (ii) p.f. correction (iii) Load balancing.

The voltage profile must remain within $\pm 5\%$ of the rated value for better and efficient operation of various electrical equipments. We know voltage variation is due to imbalance in the generation and consumption of reactive power in the system. If the generated reactive power is more than what is being consumed, voltage levels go up and vice versa. However, if the two are equal the voltage profile becomes flat.

Unfortunately, the reactive power in a system keeps on varying and hence if the reactive power generation is also simultaneously controlled, a more or less near flat voltage profile could be maintained.

1. One of the obvious methods would be to have the system of large strength *i.e.*, it interconnects large size machines and a large number of lines so that the effective impedance as seen from any point into the network is negligibly small and hence the voltage profile could be improved. However, this would result in high fault levels and would require switchgears of relatively large capacity and hence is uneconomical from view point of improving voltage profile. In fact, the network should be designed based on active power transfer capability and the reactive power should be met locally by installing shunt compensating elements (capacitor and inductors). It is to be noted that these elements do not contribute to the fault level and yet provide and maintain proper balance between the generated and consumed reactive power.
2. It is desirable both economically and technically to operate the system at near unity p.f. Usually p.f. correction means to generate reactive power as close as possible to the load which requires it rather than generate it at a distance and transmit it to the load, as this results not only in large conductor size but also in increased losses. In fact, in order to operate the system at near unity p.f., some of the electric utilities impose certain penalty on account of operating loads at low p.f.
3. A very important concept of load compensation is load balancing. It is desirable to operate the three phase system under balanced condition as unbalanced operation results in flow of negative sequence current in the system and is highly dangerous especially for the rotating machines.

An *ideal load compensator* would perform the following functions: (i) It would provide controllable and variable reactive power almost instantaneously as required by the load; (ii) It should operate independently in all the three phases; and (iii) It would maintain constant voltage at its terminals.

Typical loads which require compensation are as follows: arc furnaces, induction furnaces, steel rolling mills, very large induction motors, arc welders, induction welders etc. These loads can be further classified as non-linear and those which cause disturbance by being switched on and off. The non-linear loads such as arc furnace generate harmonics in the system and hence arc furnace compensators usually have harmonic filters to filter out these harmonics, so that the voltage remains mostly of fundamental frequency. It is to be noted that synchronous motors as loads improve the voltage profile and p.f. of the system as this can be operated as a synchronous capacitor.

It is well known that the approximate p.u. voltage regulation for a series impedance carrying a load current I is given as

$$\Delta V = \frac{IR}{V} \cos \phi \pm \frac{IX}{V} \sin \phi$$

Where ϕ the p.f. angle of the load and V is the load voltage. Here positive sign is for inductive load and -ve sign for the capacitive load. The load can be made capacitive effectively by connecting a capacitance across an otherwise inductive load. With this the

can be so adjusted that $\frac{IR}{V} \cos \phi$ can be made equal to $\frac{IX}{V} \sin \phi$ and thus change in load voltage ΔV can be made equal to zero. Therefore, a purely reactive compensator can eliminate supply voltage variations caused by changes in both the real and the reactive power of the load. Also, it is clear from the above expression that with a reactive compensator it is not possible to achieve simultaneously both the unity p.f. and zero voltage regulation.

An alternative way of studying voltage regulation is through the use of short circuit capacity of the bus, $X : R$ of the system, the active and reactive load powers. The voltage drop across the series impedance is given as

$$\begin{aligned} \Delta V &= (R + jX) \left(\frac{P - jQ}{V} \right) \\ &= \frac{RP + XQ}{V} + j \frac{XP - RQ}{V} = \Delta V_r + j\Delta V_x \end{aligned} \quad (21.2)$$

If a 3-phase short circuit is created at the load bus, the short circuit apparent power

$$S_{sc} = EI_{sc} \frac{E^2}{Z_{sc}}$$

where $Z_{sc} = R + jX$ and I_{sc} is the short circuit current.

$$\begin{aligned} \text{Now } R &= |Z| \cos \phi_{sc} & X &= |Z| \sin \phi_{sc} \\ &= \frac{E^2}{S_{sc}} \cos \phi_{sc} & &= \frac{E^2}{S_{sc}} \sin \phi_{sc} \end{aligned}$$

Substituting for R , and X and letting $E \cong V$, we have

$$\Delta V = \frac{V}{S_{sc}} (P \cos \phi_{sc} + Q \sin \phi_{sc}) + j \frac{V}{S_{sc}} (P \sin \phi_{sc} - Q \cos \phi_{sc}) \quad (21.3)$$

$$\begin{aligned} \text{or } \frac{\Delta V}{V} &= \frac{1}{S_{sc}} (P \cos \phi_{sc} + Q \sin \phi_{sc}) + j \frac{1}{S_{sc}} (P \sin \phi_{sc} - Q \cos \phi_{sc}) \\ &= \frac{\Delta V_r}{V} + j \frac{\Delta V_x}{V} \end{aligned} \quad (21.4)$$

value of capacitance



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Usually ΔV_x is ignored as this does not affect the voltage magnitude much, rather this gives phase shift to the voltage.

Therefore,
$$\frac{\Delta V_r}{V} = \frac{1}{S_{sc}} (P \cos \phi_{sc} + Q \sin \phi_{sc}) \quad (21.5)$$

The above equations are not only valid for any value of P and Q but these are also valid for small changes in P and Q i.e.,

$$\frac{\Delta V_r}{V} = \frac{1}{S_{sc}} (\Delta P \cos \phi_{sc} + \Delta Q \sin \phi_{sc}) \quad (21.6)$$

Normally the real power flow does not affect much the voltage of bus and, therefore,

$$\frac{\Delta V_r}{V} \cong \frac{\Delta Q}{S_{sc}} \sin \phi_{sc} \quad (21.7)$$

and since $\phi_{sc} = \tan^{-1} \frac{X}{R}$ and the ratio $\frac{X}{R}$ is usually more than 4, therefore $\sin \phi_{sc} \cong 1$, and, therefore,

$$\frac{\Delta V_r}{V} \cong \frac{\Delta Q}{S_{sc}} \sin \phi_{sc} \cong \frac{\Delta Q}{S_{sc}} \quad (21.8)$$

This means the per unit change in voltage equals the ratio of change in reactive power to the short circuit capacity of the bus.

LOADABILITY CHARACTERISTICS OF OVERHEAD LINES:

In order to have proper understanding of the power transfer capability of the lines which is affected by voltage levels and line length, a useful concept of line loadability is defined as the degree of line loading permissible expressed in per cent of SIL for a given thermal or voltage drop or S.S. stability limits. This concept was first introduced by St. Clair who developed transmission power transfer capability curves for voltages up to 330 kV and line lengths up to 640 kms. These curves have been useful to power system engineers for quickly estimating the maximum line loading limits.

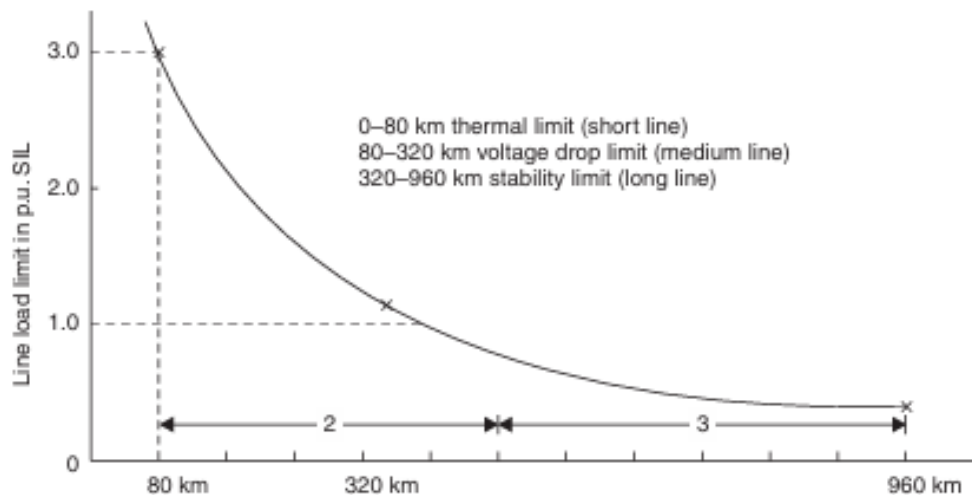


Fig. 21.1 Transmission line loadability curve.



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Figure 21.1 shows the universal loadability curve for uncompensated lines applicable to all voltage levels. The curve shows the limiting values of power that can be transmitted as a function of line length. The curve has been drawn taking into account the maximum allowable voltage drop along the line to be 5% and that the minimum allowable steady state stability margin is 30% for which the load angle is about 44° .

Since the ratio X/R for extra high and ultra high voltages is very high (more than 20), the resistance of these lines is negligible as compared to its reactance and these lines could be considered as lossless lines. Therefore, the parameter β is same for all such lines and, therefore, the loadability expressed in per unit of SIL is universally applicable to lines of all voltage classes. It can be seen from the figure that lines longer than 480 km. have their loadability less than SIL and, therefore, their loadability can be improved by compensating the lines.

From the Fig. 21.1 it is clear that for short length line the steady state limit loading will be greater than the thermal loading and hence the loadability is determined by the thermal loading rather than steady state stability loading.

For medium length uncompensated lines voltage levels are the considerations for line loadings and for long length uncompensated lines steady state stability limit loading is the consideration as the loading is even less than the surge impedance loading.

As mentioned here, it is not desirable to operate transmission lines close to its steady state stability limit and a margin in the power transfer should be allowed to take care of any sudden changes in the system. In determining an appropriate margin, the concepts of dynamic and transient stability are useful. A system is said to be dynamically stable if it recovers its normal operation after it is subjected to a small perturbation. The degree of dynamic stability is expressed in terms of the rate of damping of transient component of voltages, currents and the load angles of the synchronous machines. The rate of damping is of main consideration and, therefore, small perturbation theory and eigen value analysis is normally used to study dynamic stability limits of the system.

The analysis of the system under large perturbation is known as transient state stability analysis. If a system recovers to its normal operating condition after being subjected to a large perturbation such as failure of a major component of the system (failure of a large generator, an important tie-line or a power transformer), the system is said to have achieved transient stability. The transient stability limit of a system is the maximum amount of power that can be transmitted prior to occurrence of the disturbance and still the system recovers to its normal operating condition after the disturbance. Therefore, it can be concluded that whereas the steady-state stability limit of a system is a fixed value for a particular system, the transient stability limit will have different values for different perturbations. For further understanding of the subject the reader is requested to refer to Chapter 17 of this book.

The second basic requirement in a.c. power transmission is to have proper voltage levels. An undervoltage operation results in heavy loading, reduction in steady state stability limit of the system, degradation in the performance of induction motors which forms almost 70% of the total load on the system.



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A power transformer is normally operated at the knee-point of its B-H curve due to economic reasons. If somehow the system voltage increases due to Ferranti effect or due to rejection of load the transformer operates along the saturation region of the B-H curve where the inductance offered by the winding decreases and keeps on changing depending upon the dynamic behaviour of increase of system voltage. Saturation of transformers subjected to over voltages can produce high currents rich in harmonics and in the presence of sufficient capacitance, there is risk of phenomenon of resonance taking place which may result in further increase in system voltages which may prove disastrous for the system insulation. This is known as Ferro-resonance as it is due to the non-linear behaviour of the iron core of the transformer. In such a situation it may be necessary to disconnect shunt capacitors very quickly to reduce the chances of occurrences of Ferro-resonance. Over voltage may also result in flash over and/or puncture of the insulation of the system. A small over voltage if allowed to exist on the system for a long time may not puncture the insulation but it would certainly shorten the useful life of insulation of the system.

Some compensating devices are used to achieve the above mentioned basic requirements in a.c. transmission. Before we do that some characteristics of uncompensated lines will be studied. In order to maintain continuity in the subject, there may be some repetition from Chapter 4 of the book.

UNCOMPENSATED TRANSMISSION LINE:

The basic equation describing a transmission line is given as

$$\frac{\partial^2 V}{\partial x^2} = \gamma^2 V \quad (21.9)$$

$$\frac{\partial^2 I}{\partial x^2} = \gamma^2 I \quad (21.10)$$

and
$$\frac{\partial V}{\partial x} = Iz \quad (21.11)$$

where
$$\gamma^2 = (r + j\omega L)(g + j\omega C)$$

and
$$z = (r + j\omega L), y = (g + j\omega C)$$

where x is the distance from the sending end. If l is the length of the line, the solution of the above equation for a lossless line i.e.,

$$r = 0 \text{ and } g = 0$$

$$V(x) = V_r \cos(\beta(l-x)) + jZ_c I_r \sin \beta(l-x) \quad (21.12)$$

where β is the phase constant i.e.,

$$\gamma = \alpha + j\beta \text{ and for a lossless system}$$

$$\alpha = 0$$

Since
$$\gamma^2 = (r + j\omega L)(g + j\omega C) = -\omega^2 LC$$

$$\gamma = j\beta$$

$$\text{where } \beta = \omega \sqrt{LC} = \frac{2\pi f}{v} = \frac{2\pi}{\lambda}$$

using equations (21.11) and (21.12) we get

$$I(x) = I_r \cos \beta(l-x) + j \frac{V_r}{Z_c} \sin \beta(l-x) \quad (21.13)$$



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β is also known as wave number *i.e.*, the number of complete waves per unit of line length.

At 50 Hz the wave length $\lambda = \frac{3 \times 10^8}{50} = 6000$ kms and β can be expressed as one wave

length per 6000 kms *i.e.*, 360° per 6000 kms or $= \frac{360}{6000} = 0.06^\circ$ per km or 1.0466×10^{-3} rad per km. The quantity βl is the electrical length in radians.

Z_c in equation (21.13) is known as the surge impedance or natural impedance. Its value depends upon the line design. Its value decreases with increase in operating voltage. Also with vertical configuration, the surge impedance is lower as compared to horizontal or delta configuration of the conductors. The usual value of Z_c lies in the range 200 to 400 ohms. Table 21.1 gives various parameters of lines operating at different voltages.

Table 21.1 Line parameters of typical EHV-UHV overhead lines

Characteristics	345 kV		500 kV		765 kV		1100 kV		1500 kV
	Hori.	Delta	Hori.	Delta	Hori.	Delta	Hori.	Delta	Horizontal
r Ω/km	0.04	0.04	0.0016	0.001	0.001	0.001	0.0005	0.0005	0.00045
X_l Ω/km	0.30	0.30	0.30	0.27	0.27	0.27	0.25	0.25	0.25
ωC $\mu\text{S}/\text{km}$	3.8	3.8	3.8	4.3	4.1	4.1	4.66	4.66	4.9
Charging MVA/km	0.45	0.45	0.9	1.08	2.5	2.5	5.8	5.8	10.8
β (rad/km $\times 10^{-3}$)	1.05	1.05	1.05	1.05	1.05	1.05	1.05	1.05	1.05
Z_c in ohm	285	283	287	247	258	257	232	231	225
SIL (MW)	417	420	870	1010	2270	2280	5220	5250	10000
Line current at SIL	700	700	1000	1170	1710	1720	2740	2750	3850

The surge impedance of a line is defined as the apparent impedance of an infinitely long line and is given by the ratio of voltage to the current at any point along it. Now an infinite line can be simulated by terminating any line by its surge impedance. In that case $V_r = I_r Z_c$ and the apparent impedance of the line at any point is

$$\begin{aligned}
 Z(x) &= \frac{V(x)}{I(x)} = \frac{V_r \cos(l-x)\beta + jI_r Z_c \sin \beta(l-x)}{I_r \cos \beta(l-x) + j \frac{V_r}{Z_c} \sin \beta(l-x)} \\
 &= \frac{I_r Z_c [\cos(l-x)\beta + j \sin \beta(l-x)]}{I_r [\cos \beta(l-x) + j \sin \beta(l-x)]} = Z_c
 \end{aligned} \tag{21.14}$$

It can be seen that when the line is terminated through Z_c , the voltage and current magnitude remain same all along the length. Therefore, the line is said to have flat voltage profile. The voltages at the two ends are of equal magnitude but displaced in phase by β_0 , the sending end voltage leading the receiving end voltage. For a 450 kms long line at 50 Hz the angle is $0.06 \times 450 = 27^\circ$.



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From equation (21.14) it is clear that the ratio of $\frac{V}{I}$ everywhere is a real number Z_c and therefore, voltage and current are in phase at all points of the line *i.e.*, power factor is unity at all points. This also means that at every point of the line, the reactive power generated equals the reactive power absorbed. Therefore, whereas the natural active power loading of the line is given by V^2/Z_c the natural reactive power loading is zero.

One of the major advantages of operating the line corresponding to natural loading is that because of flat voltage profile, the insulation all along the line is uniformly stressed.

Suppose the line is open circuited at the receiving end *i.e.*, $I_r = 0$. Therefore,

$$V(x) = V_r \cos \beta(l - x) \quad (21.15)$$

and
$$I(x) = j \frac{V_r}{Z_c} \sin \beta(l - x) \quad (21.16)$$

At $x = 0, V(x) = V_s$ and $I(x) = I_s$
$$\therefore V_s = V_r \cos \beta l \quad (21.17)$$

and
$$I_s = j \frac{V_r}{Z_c} \sin \beta l$$
$$= j \frac{V_s}{Z_c} \tan \beta l \quad (21.18)$$

Suppose the length of the line is 100 km $\cos(0.06 \times 100) = \cos 6^\circ \approx 1.0$ and therefore no Ferranti effect is experienced whereas for say 300 km $\cos(0.06 \times 300) = \cos 18^\circ = 0.95$.

i.e., $\frac{V_r}{V_s} = 1.05$ a rise of 5% of voltage at the receiving end. This could be tolerated. However, if

the length is 800 km $\cos 48^\circ = 0.66$, and $\frac{V_r}{V_s} = 1.494$ a rise of 49.4% in the receiving end voltage as compared to the sending end voltage. Such a high rise in receiving end voltage is unacceptable and some suitable compensating devices must be used to avoid this situation.

In actual practice the receiving end voltage rises more than what is calculated above as the sending end voltage will increase with the increase in charging current due to rise in open circuit voltage at the sending end generator.

From equation 21.18 under no load condition the sending end current (charging current) is $I_s = jI_c \sin \beta l$ and for 300 km long line it will be $I_s = j 1.05 I_c \sin 18^\circ = j 0.324 I_c$ and this will lead the sending end voltage by 90° . Here 1.05 is used, as the receiving end voltage under no load condition will be 1.05 times that under natural loading condition assuming sending end voltage as 1.0 p.u. every time.

When a line is to be energized, the generator has to supply leading vars to the line and the reactive power is given by

$$Q_s = I_m (V_s I_s^*) \quad (21.19)$$

Using value of I_s from equation (21.18), we have $Q_s = -P_c \tan \beta l \quad (21.20)$

For 300 km long line

$$Q_s = -P_c \tan 18^\circ = -0.325 P_c$$



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i.e., the generator should be able to supply so much of leading vars or absorb lagging vars. Under-excited operation of the generator below a certain minimum excitation is not desirable for two reasons.

(i) Lower excitation results in reduced internal e.m.f. of the generator and hence reduced stability limits.

(ii) Lower excitation increases heating of the ends of the stator core.

Therefore, it is the under-excited operation of the alternator rather than voltage rise due to Ferranti effect that stringent limit is imposed on the length of uncompensated lines. Suppose the rating of alternators is P and their maximum reactive power absorption is Q i.e.,

the equivalent power factor of the alternators is $\cos\left(\tan^{-1} \frac{Q}{P}\right)$ leading. Say this p.f. for a typical system is 0.95 lead, therefore $Q = 0.3286 P$ and if the loading of the generators equals the surge impedance loading of the line, then $P = P_c$ and using equation 21.20, we have

$$0.3286 P = -P \tan \beta l$$

or length of the line will be $\frac{18.19}{0.06} = 303 \text{ km}$. i.e., 303 km is the maximum length of transmission

which can be energised by these generators under these conditions or if the p.f. of the alternators remains 0.95 and the length of the line is known to be say 400 kms, then the total generator capacity should be

$$0.3286 P = P_c \tan (0.06 \times 400)$$

$$P = 1.35 P_c$$

That is to charge the line 35% extra generating capacity would be required which is not desirable and hence instead this situation can be met by having the same generator capacity $p = P_c$ but using shunt reactors at the end of the line or at intermediate points of the line during charging.

SYMMETRICAL LINE:

A symmetrical line is one with identical synchronous machines at both the ends and $|V_s| = |V_r|$ and under no load condition these are in phase also indicating that there is no transfer of power (no load condition) otherwise V_s shall lead V_r , phase lead depending upon the loading. With regard to such a line following observations are made:

(i) If the loading is less than the surge impedance loading of the line, the mid-point voltage is higher than the terminal voltage as there is a net of excess reactive power generated by the line.

(ii) However, if the loading is greater than the SIL of the line, due to deficit reactive power on the line the mid-point voltage is lower than the terminal voltages.

(iii) If the loading equals SIL of the line, the line has flat voltage profile. The corrective measures in cases (i) and (ii) above can be taken up by connecting suitable compensating network at suitable locations.



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Assuming the line to be lossless, let us analyse the system when a load of $P_L + jQ_L$ is connected. Using equation (21.12) the voltages at the two ends can be correlated as

$$V_s = V_r \cos \beta l + jZ_c \frac{P_L - jQ_L}{V_r} \sin \beta l \quad (21.21)$$

The above equation holds good irrespective of whether the load is synchronous or asynchronous. Let us assume that the load is synchronous and taking V_r as reference, say V_s leads V_r by an angle δ known as load angle. Therefore,

$$V_s = |V_s| (\cos \delta + j \sin \delta) \quad (21.22)$$

Substituting for V_s in (21.21), we have

$$\begin{aligned} |V_s| [\cos \delta + j \sin \delta] &= V_r \cos \beta l + jZ_c \frac{P_L - jQ_L}{V_r} \sin \beta l \\ &= V_r \cos \beta l + Z_c \frac{Q_L}{V_r} \sin \beta l + jZ_c \frac{P_L}{V_r} \sin \beta l \end{aligned} \quad (21.23)$$

Equating the real and imaginary parts on the two sides, we have

$$V_s \cos \delta = V_r \cos \beta l + Z_c \frac{Q_L}{V_r} \sin \beta l \quad (21.24)$$

and
$$V_s \sin \delta = Z_c \frac{P_L}{V_r} \sin \beta l$$

or
$$P_L = \frac{V_s V_r}{Z_c \sin \beta l} \cdot \sin \delta \quad (21.25)$$

If the length of the line is short $\sin \beta l \cong \beta l = \omega \sqrt{LC} \cdot l$

and $Z_c \sin \beta l = \sqrt{\frac{L}{C}} \omega \sqrt{LC} \cdot l = lL\omega = X = \text{Total reactance of the line. Equation (21.25), therefore, reduces to a very well known equation}$

$$P_L = \frac{V_s V_r}{X} \sin \delta \quad (21.26)$$

Let $V_s = V_r$, then equation (21.25) becomes

$$\begin{aligned} P_L &= \frac{V_r^2}{Z_c \sin \beta l} \cdot \sin \delta \\ &= \frac{P_c}{\sin \beta l} \cdot \sin \delta \end{aligned}$$

For certain geometry and operating voltage of the line P_L is maximum when $\delta = 90^\circ$ and this maximum depends upon the length of the line, the longer the length the smaller is the

maximum value of P_L . The ratio $\frac{P_{L \max}}{P_c}$ for a few lengths is tabulated here.

Table 21.2. $\frac{P_{L \max}}{P_c}$ as a function of length

Length in km	Length in degrees	$P_{L \max}/P_c$
300	18	3.236
600	36	1.701
900	54	1.236
1200	72	1.051
1500	90	1.00



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RADIAL LINE WITH ASYNCHRONOUS LOAD:

Even when the load is asynchronous, there is a maximum power that can be transmitted over a line. This can be calculated as follows for a unity p.f. load. Let us consider that sending end source and line form a voltage source with open circuit voltage V_0 and impedance $(R + jX)$ and at the receiving end a variable resistive load (unity p.f.) R_1 is connected. Our objective is to find out value of R_1 for which the power transfer is maximum.

Now short circuit current $I_{sc} = \frac{V_0}{Z} = \frac{V_0}{R + jX}$ and short circuit p.f. $\cos \phi_{sc} = \frac{R}{Z}$

The load current $I = \frac{V_0}{(R + R_1) + jX}$

The power delivered $= \frac{V_0^2}{(R + R_1)^2 + X^2} \cdot R_1 = P$

The power delivered is maximum when $\frac{dP}{dR_1} = 0$

$$[(R + R_1)^2 + X^2] - R_1 \cdot 2(R + R_1) = 0$$

$$\text{or } R_1^2 + R^2 - 2RR_1 + X^2 - 2RR_1 - 2R_1^2 = 0$$

$$\text{or } R_1 = Z$$

$$\begin{aligned} \text{Therefore, } P_{\max} &= \frac{V_0^2 Z}{(R + Z)^2 + X^2} = \frac{V_0 I_{sc} Z^2}{R^2 + Z^2 + 2RZ + X^2} \\ &= \frac{V_0 I_{sc} Z^2}{2Z^2 + 2RZ} \\ &= \frac{V_0 I_{sc} Z}{2(Z + R)} \\ &= \frac{V_0 I_s}{2(1 + \cos \phi_{sc})} \end{aligned} \quad (21.27)$$

Now V_0 is the open circuit voltage i.e., V_r when $I_r = 0$.

Therefore, from equation (21.12)

$$V_s = V_0 \cos \beta l$$

$$\text{or } V_0 = \frac{V_s}{\cos \beta l} \quad (21.28)$$

Similarly, short circuit current I_{sc} is the value of I_r when $V_r = 0$

Using equation 21.12 again, we have

$$V_s = jI_{sc} Z_c \sin \beta l$$

$$\text{or } I_{sc} = \frac{V_s}{j Z_c \sin \beta l} \quad (21.29)$$

Assuming the line to be lossless $\cos \phi_{sc} = 0$

$$\therefore P_{\max} = \frac{V_0 I_{sc}}{2} = \frac{V_s}{2 \cos \beta l} \cdot \frac{V_s}{Z_c \sin \beta l} = \frac{V_s^2}{Z_c \sin 2\beta l} \quad (21.30)$$

Equation (21.30) represents loci of maximum power for different length of lines at unity p.f.



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Let us consider a general load $P_L + jQ_L$ at the receiving end keeping the sending end voltage fixed. The receiving end current

$$I_r = \frac{P_L - jQ_L}{V_r}$$

Using equation (21.12) and assuming line to be lossless, we have

$$V_s = V_r \cos \beta l + jZ_c \frac{P_L - jQ_L}{V_r} \sin \beta l \quad (21.31)$$

For a fixed V_s and line length, equation (21.31) is quadratic in V_r and thus will have two roots. Figure 21.2 shows relation between $\frac{V_r}{V_s}$ as a function of normalised loading $\frac{P_L}{P_c}$

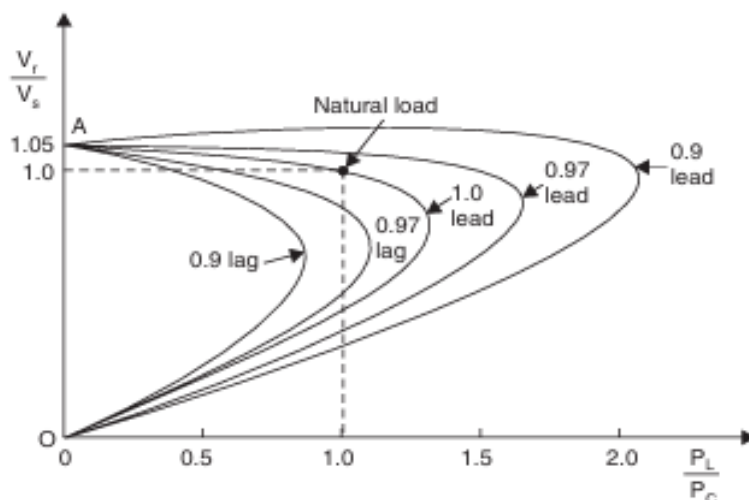


Fig. 21.2 $\frac{V_r}{V_s}$ as a function of normalised loading.

From the figure it is clear that there is a maximum power that can be transmitted for each load p.f. and for any loading, there are two different values of V_r . Normal operation of the power system is along the upper part of the curve where the receiving end voltage is nearly 1.0 p.u. The load is increased by decreasing the effective resistance of the load up to the maximum power, the product of load voltage and current increases and the system is stable. As the maximum power point is reached, a further reduction in effective load resistance, reduces the voltage much more than the increase in current and, therefore, there is an effective reduction in power transmission. The voltage finally collapses to zero and the system at the receiving end is effectively short circuited and, therefore, the power transmitted is zero (point O). From Fig. 21.2, it is again clear that power transmitted is zero both at Point A and O. Point A corresponds to open circuit and O to short circuit and in either case the power transmitted is zero. However, at A the V_r is 1.05 p.u. and at O it is zero.

Whereas short length lines are operated without any problem, long length lines cannot be operated without the use of compensating devices because of (i) Ferranti effect, (ii) under-excited and hence unstable operation of alternators during charging of the line, and (iii) reduction of power transfer capability of the lines which reduces the margin between the stable and unstable operation of the system. Therefore, we not study various compensating devices for compensation of lines.



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COMPENSATION OF LINES:

By compensation of lines is meant the use of electrical circuits to modify the electrical characteristics of the lines such that the compensated lines will achieve the following objectives:

- (i) Ferranti effect is minimised so that a flat voltage profile will exist on the line for all loading condition.
- (ii) Underexcited operation of alternator will be avoided and an economical means of reactive power management will be achieved.
- (iii) The power transfer capability of the system will be enhanced and hence stability margins increase.

In order to assess the effectiveness of a compensated system a performance index in terms of product of length of line and the power to be transmitted is evaluated. This criterion is selected as we know that for longer length lines it is not even possible to load lines to their natural loadings without compensation.

We know that flat voltage profile on the line can be achieved if the loading of the line corresponds to its surge impedance loading. Therefore, to achieve flat voltage profile, the compensating device should be so chosen that the effective or virtual surge impedance Z_c of the line should give virtual natural loading equal to the actual load. Since in actual practice the actual load keeps on changing with time, the compensating devices should also vary without delay so that every time the effective surge impedance matches with the actual loading *i.e.*,

$$P'_c = \frac{V_r^2}{Z'_c} = P_L$$

where P_L is the actual load. The compensating devices are the suitable connection of capacitors and/or inductors on to the lines. Compensation when carried out with the sole objective of modifying the surge impedance of the line, is known as surge impedance compensation or Z_c compensation.

Under-excited operation of the alternator or charging current problem of the line can be avoided by dividing the line into shorter sections and this is known as compensation by sectioning. It is achieved by connecting constant voltage compensators at intervals along the line. Since the power transmitted will be same through all sections, the maximum power will be decided by the smallest section and, therefore, there is increase in power transfer capability of the system and hence stability limit is increased.

The third aspect of the compensation *i.e.*, to increase power transfer capability of the system is achieved by inserting capacitor at suitable location in series with the line so that the net inductive reactance of the line is reduced which is equivalent to reducing the effective length of the line. This method of compensation is known as line-length compensation.

In any long length line, all the three types of compensations can be used if required.

Compensators are classified as passive and active compensators. Shunt reactors and capacitors and series capacitors are the passive compensators and synchronous capacitors and thyristorised controlled capacitors and reactors are the active compensators.

Rapid response excitation system used for synchronous machines also acts as strong compensating system and improves both voltage profile and stability.

Even though compensators are used at discrete locations along the line, it is useful to obtain certain relation assuming the effect of compensators as uniformly distributed. The relations so desired are more or less true for practical system.

We know that,

$$Z_c = \sqrt{\frac{L}{C}} = \sqrt{\frac{j\omega L}{j\omega C}} = \sqrt{x_L x_C} \quad (21.32)$$



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Suppose shunt inductance L_{sh} per unit is used as a compensator, the net shunt susceptance will be

$$\begin{aligned} j\omega C' &= j\omega C + \frac{1}{j\omega L_{sh}} = j\omega C - \frac{j}{\omega L_{sh}} \cdot \frac{\omega C}{\omega C} \\ &= j\omega C \left(1 - \frac{1}{\omega^2 CL_{sh}} \right) \\ &= j\omega C (1 - \gamma_{sh}) \end{aligned} \quad (21.33)$$

where

$$\gamma_{sh} = \frac{1}{\omega^2 CL_{sh}} = \frac{x_c}{x_{Lsh}}$$

and γ_{sh} is known as the degree of shunt compensation.

The modified value of surge impedance will be

$$Z'_c = \sqrt{\frac{j\omega L}{j\omega C(1 - \gamma_{sh})}} = \frac{Z_c}{\sqrt{1 - \gamma_{sh}}} \quad (21.34)$$

However, if shunt capacitance is added, then γ_{sh} will be negative and hence it can be concluded that shunt inductance increases the virtual surge impedance of the line and hence reduces the virtual surge impedance loading of the line and shunt capacitance reduces the virtual surge impedance.

Let us consider the effect of series compensation on the surge impedance loading.

Suppose C_{se} is the series capacitance per unit length for series compensation. Therefore, the series reactance will be

$$\begin{aligned} j\omega L - \frac{j}{\omega C_{se}} &= j\omega L - \frac{j}{\omega C_{se}} \cdot \frac{j\omega L}{j\omega L} \\ &= j\omega L \left(1 - \frac{1}{\omega^2 LC_{se}} \right) \\ &= j\omega L \left(1 - \frac{X_{cse}}{X_L} \right) \\ &= j\omega L (1 - \gamma_{se}) \end{aligned} \quad (21.35)$$

where γ_{se} is known as degree of series compensation. Therefore, virtual surge impedance

$$\begin{aligned} Z'_c &= \sqrt{\frac{j\omega L(1 - \gamma_{se})}{j\omega C}} \\ &= Z_c \sqrt{(1 - \gamma_{se})} \end{aligned} \quad (21.36)$$

Taking into consideration both shunt and series compensation simultaneously, we have

$$Z'_0 = \sqrt{\frac{j\omega L'}{j\omega C}} = Z_c \sqrt{\frac{1 - \gamma_{se}}{1 - \gamma_{sh}}} \quad (21.37)$$

Therefore, the virtual surge impedance loading

$$P'_c = P_c \sqrt{\frac{1 - \gamma_{sh}}{1 - \gamma_{se}}} \quad (21.38)$$



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The wave number β is also modified to

$$\beta' = \beta \sqrt{(1 - \gamma_{se})(1 - \gamma_{sh})} \quad (21.39)$$

The equation derived earlier for uncompensated lines are valid for compensated lines except the equivalent parameters *e.g.*, inductance, capacitance etc., are to be substituted as obtained in this section.

From equations (21.37) and (21.38) it is clear that for a fixed degree of series compensation, capacitive shunt compensation decreases the virtual surge impedance loading of the line. However, inductive shunt compensation increases the virtual surge impedance and decreases the virtual surge impedance loading of the line. If inductive shunt compensation is 100% the virtual surge impedance becomes infinite and the loading zero which implies that a flat voltage profile exists at zero load and Ferranti effect can be eliminated by the use of shunt reactors. However, under heavy loading condition, flat voltage profile can be obtained by using shunt capacitances. Suppose, we want flat voltage profile corresponding to $1.2 P_c$ without series compensation, the shunt capacitance compensation required will be by using equation (21.38).

$$\begin{aligned} 1.2 &= \sqrt{1 - \gamma_{sh}} \\ \text{or } 1.44 &= 1 - \gamma_{sh} \quad \text{or } \gamma_{sh} = -0.44 \text{ p.u.} \end{aligned}$$

Flat voltage profile can also be obtained by series compensation for heavy loading condition. Again using equation (21.38), and assuming shunt compensation to be zero, the series compensation required for a loading $1.2 P_c$ is

$$\begin{aligned} 1.2 &= \sqrt{\frac{1}{1 - \gamma_{se}}} \\ \text{or } 1.44 &= \frac{1}{(1 - \gamma_{se})} \\ \text{or } \gamma_{se} &= 0.306 \text{ p.u.} \end{aligned}$$

However, because of the lumped nature of series capacitors voltage control using series capacitors is normally not recommended. These are normally useful for improving the stability limits of the system.

Next we study the effect of distributed compensation on the line charging reactive power. Rewriting equation (21.20)

$$Q_s = -P_c \tan \beta l$$

where Q_s is the leading reactive power supplied by the generator for a radial line and for a symmetrical line,

$$Q_s = -P_c \tan \beta l / 2$$

However, for compensated lines, these equations become

$$Q'_s = -P_c \tan(\beta l) - \text{radial line} \quad (21.40)$$

$$\text{and } Q_s = -P_c \tan(\beta l / 2)' - \text{Symmetrical line}, \quad (21.41)$$

From equation (21.39) it is clear that by using inductive shunt compensation and or series capacitive compensation, the effective length of the line can be reduced and hence $(\beta l)'$ becomes small so that $\tan(\beta l)' \cong (\beta l)'$ and $\tan(\beta l / 2)' \cong (\beta l / 2)'$. Substituting values of P_c' and $(\beta l)'$ from equations (21.38) and (21.39) into equations (21.40) and (21.41), we have

$$Q'_s = P_c \sqrt{\frac{1 - \gamma_{sh}}{1 - \gamma_{se}}} \cdot \beta l \sqrt{(1 - \gamma_{sh})(1 - \gamma_{se})} \quad (21.42)$$



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$$= P_c \beta l (1 - \gamma_{sh})$$

for a radial line and

$$Q'_s = P_c \frac{\beta l}{2} (1 - \gamma_{sh}) \quad (21.43)$$

for a symmetrical line.

From the above equations, it is clear that series compensation has no effect on the no load reactive power requirements of the generator and, therefore, the series compensated line generates roughly as much line charging reactive power at no load as a completely uncompensated line (for $\gamma_{sh} = 0$) of the same length. However, if the length of the line is large and needs series compensation from stability point of view, the generators at the two ends will have to absorb excessive reactive power and, therefore, it is important that shunt compensation (inductive) must be associated with series compensation.

Let us consider shunt compensation not only uniformly distributed but regulated as well which is more or less similar to compensation by sectioning.

Suppose the line is operating at its natural loading. From equation (21.25)

$$\frac{P}{\sin \delta} = \frac{P_c}{\sin \beta l} \text{ and if } P = P_c, \delta = \beta l$$

i.e., the transmission angle δ equals the electrical length of the line. Considering series compensation to be absent and assuming that the shunt compensation could be regulated continuously such that $P'_c = P$ at all times then $\beta' l = \delta$ at all times. Therefore,

$$\frac{P}{\delta} = \frac{P'_c}{\beta' l}$$

substituting for P'_c and β' from equations (21.38) and (21.39) respectively, we have

$$\frac{P}{\delta} = \frac{P_c \sqrt{1 - \gamma_{sh}}}{\beta \sqrt{1 - \gamma_{sh}} \cdot l} = \frac{P_c}{\beta l} = \text{constant} \quad (21.44)$$

Equation (21.44) shows a linear relation between P and δ and the slope is given by

$$\frac{P_c}{\beta l} = \frac{V_0^2}{Z_c \beta l} = \frac{V_0^2}{\sqrt{L/C} \cdot \omega \sqrt{LC} \cdot l} = \frac{V_0^2}{X_l} \quad (21.45)$$

Power angle relation is given as

$$P = \frac{V_0^2}{X_l} \sin \delta$$

Therefore, it's slope at $\delta = 0$ is $\frac{V_0^2}{X_l}$ and the $P - \delta$ straight line is tangent to $P - \delta$

characteristic of the 100% shunt compensated line as shown in Fig. 21.3 (a) since the line with 100% shunt compensation behaves exactly as a series inductance.

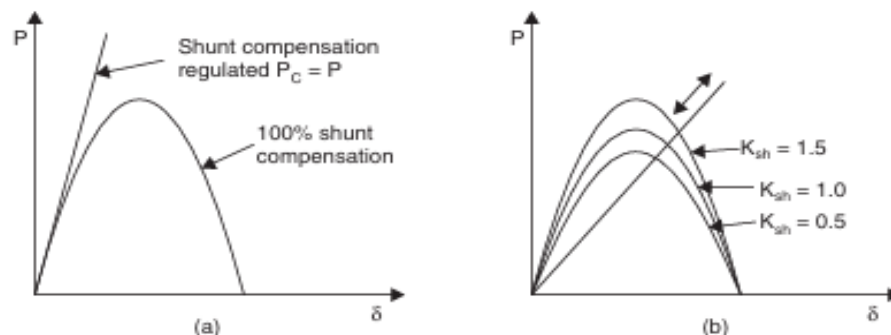


Fig. 21.3 (a) Effect of regulated shunt compensation in power transmission
(b) $P - \delta$ curves for different γ_{sh} .

Figure 21.3 (a) suggests that if shunt compensation could be adjusted continuously, the power transmitted could be infinite.

2 Marks Questions:

1. What is meant by voltage control in a power system?
2. Why is voltage control necessary in a transmission system?
3. What are the different methods of voltage control?
4. What is the function of a shunt capacitor in voltage control?
5. What is the purpose of a shunt reactor in a transmission line?
6. What is a tap-changing transformer?
7. What is meant by power factor improvement?
8. Why is power factor improvement necessary in power systems?
9. What is a synchronous phase modifier?
10. What is meant by load compensation?
11. Differentiate between series compensation and shunt compensation.
12. What are the advantages of reactive power compensation?

10 Marks Questions:

1. Explain the necessity of voltage control in power systems and discuss the various methods of voltage control.
2. Explain the operation of shunt capacitors and shunt reactors for voltage control and reactive power compensation.
3. Explain the construction, working, and applications of tap-changing transformers for voltage control.
4. Explain the working of a synchronous phase modifier and discuss its application in power factor improvement.
5. Explain the different methods of power factor improvement and discuss their advantages.
6. Explain the concept of load compensation and discuss the different methods of load compensation.
7. Explain series and shunt compensation techniques used in transmission lines and compare their applications.
8. Discuss the role of reactive power compensation in improving voltage profile, power factor, and transmission efficiency.
9. Explain the compensation of overhead transmission lines and discuss the effect of compensation on system performance.
10. Explain the compensation techniques for uncompensated, symmetrical, and radial transmission lines with suitable diagrams.

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“Effective voltage control and reactive power compensation are essential for maintaining a stable voltage profile, improving power factor, and ensuring efficient and reliable operation of power systems.”